Submission of Articles

The *Ontario Mathematics Gazette* (*OMG*) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *OMG*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles must be word-processed in MS Word, double-spaced with wide margins, not exceeding 10 numbered pages of text, and prepared according to the *Publication Manual of the American Psychological Association, Sixth Edition*. Figures and diagrams should be drawn by computer, if possible, or drawn in black ink in camera-ready form. Embedded images must also be submitted separately in jpeg or tif format. Proof of the photographer's permission is required, and for **photos of students** under the age of 18, the written permission of a **parent or guardian is required**.

You must submit **one complete copy** of your article, embedded with any tables, figures, and captioned photographs or graphics, to the Editor, Marian Small, along with **separate files for each of the text, graphics, and/or photographs**. Please e-mail all files to Marian Small at **marian.small@gmail.com**.

Your name should not appear anywhere in your article, including websites, so that your article can be sent out for blind review. Your name, full mailing address, and e-mail address must be included on a separate sheet. Upon review, you will be notified as to whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected.

The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME.

PERMISSION TO REPRINT: Permission to reprint any part of this publication for instructional use or for inclusion in an affiliate or other publication must be obtained through the Editor, Marian Small. Full credit must be given to the author and to the *Ontario Mathematics Gazette*.

The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of OAME.

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Submission of Advertisements

Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** at the above address. Courier is recommended to avoid possible delays. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be camera ready and colour advertisements should have no bleeds.

Advertising Rates

Advertising rates are available by telephoning, e-mailing, or writing to the Advertising Manager.

▲ Editor's Message

MARIAN SMALL



A number of schools and school districts around the province are focusing attention on the mathematical communication of their students. Sometimes motivated by poor student communication scores on EQAO, teachers are concerned about the struggle their students have communicating their mathematical thinking. But the question is—what is good communication? Whereas many teachers, based on the Ontario achievement chart rubrics, focus on student use of mathematical terminology, I have to wonder if it's terminology or primarily the clarity of expression and the depth of justification when justification is involved, that we should be assessing.

Suppose, for example, a student is told that a square picture has a side length of 8 cm and is asked what length of ribbon would be necessary to go completely around it. A student explains that the sides are all the same length and there are 4 sides, so there are 4 sets of 8 cm, or 32 cm, needed for the border. In my opinion, it is not really necessary for the student to use the word perimeter to describe the distance around to achieve a Level 4 score.

I think clarity as well as depth of justification should matter most when justification is involved. Imagine, for example, that a teacher asks students to explain why (-5) + (-6) is negative. Once students have learned "rules," it is reasonable that their explanation might simply be a citation of the rule, in this case, that the sum of two negatives is negative. But initially, until students more deeply understand, and indefinitely if we want a true justification, we should probably be expecting more complete communication. Compare these two responses, for example:

Student A: When you add two negative integers, the sum is negative.

Student B: The negatives are on the left. If you go left 5 from 0 and left another 6, you are still left of 0.

Response A uses more mathematical language, but response B actually explains.

So how do we help students become better communicators? In my mind, we show them the

difference between two responses like A and B above. We can show the value of using mathematical vocabulary and of summarizing ideas, as Response A does. But we should also help students see how Response B explains why, whereas Response A does not.

Consider the questions you ask that require communication and think about what kind of responses you think would exhibit good communication as opposed to merely recitation of rules. Think, too, about whether or how much specific math vocabulary you think is really required in the response. Below are some samples to get your thinking started.

- Explain why 50 x 84 has to be the same as 100 x 42 without actually getting the answers.
- Explain why there are 10 000 cm^2 in 1 m^2.
- Explain why a line with a positive slope that goes through the point (3,8) cannot go through the point (2,10). ▲

▲ Vote Mar 3–May 13

ALL MEMBERS TAKE NOTE!! OAME ELECTIONS ARE ONLINE

Annual elections for positions on the Executive and Board of Directors will be online from March 3–May 13. To vote, you must activate your online account on the website at **www.oame.on.ca** if you have not already done so. Click on "Members Only" and follow the directions to activate your account. You will find your membership number and expiry date on the envelope of your *Gazette*.

From March 3 to May 13, you will find "OAME Elections" in the links on the left side, after you have logged in using your username and password, which you created when you activated your account. Just follow the instructions. Short biographies for all candidates are included. If you need assistance, contact the Executive Directors

OAME members may also vote at the OAME booth at the Annual General Conference in Windsor, May 12 and 13.

▲ President's Message

DAVID ZIMMER, PRESIDENT



Shades of Grey

When I first started teaching high school math, the world of mathematics education seemed very simple. My first course outline was a ditto made from a photocopy of the index of the textbook I was assigned. There were check marks beside the lessons I was to teach and questions I was to assign. We assumed that each textbook lesson would require one 40-minute class to complete, including taking up the previous day's homework. Lesson planning was a breeze. I didn't even know that there was a provincial mathematics curriculum document. I thought the course was completely defined by the textbook.

"Marking" and calculating grades were similarly easy to understand. Final marks were to be determined using a weighted total based on term tests, quizzes, examinations, homework checks, and "effort and attitude." Student solutions were easy to score—each correct step garnered a mark. We gave part marks for work done correctly up to the first error and then tried to follow the reasoning to see if any new errors had been introduced after that. That made problems with longer, more involved solutions more challenging to score, but fortunately, there weren't many of those.

Students could guess the approximate number of steps required to solve a problem by looking at the problem's mark value. We taught our students to gauge their time on final exams by calculating the time value of each mark on the exam and applying that to the value of each question.

It all seemed black and white and I was pleased with myself as a teacher. What I was supposed to teach was clear and unambiguous. I was pretty sure that every math teacher in the province had a common understanding of what we had to get our students to learn. Even the universities seemed happy with the situation. Math was supposed to be hard enough to separate those who could succeed in the science and engineering courses from those who could not, and we seemed to be quite good at doing that.

As I gained more experience and had a chance to interact with other math educators, I began to realize

how hollow, insensitive, and counter-productive that was.

"Differentiating instruction," "Assessment Of, As and For Learning," "Big Ideas," "Effective and Evocative Questioning," and "Teaching through the Mathematical Processes" have all become central issues in our planning of learning experiences in mathematics. Using these perspectives, we endeavour to provide the greatest possible opportunity for the largest number of students to be successful. We are beginning to see empirical evidence that the strategies associated with these initiatives work and that a much greater percentage of our students than we had previously thought can be successful in meaningful ways in our mathematics program.

It has taken a long time for these initiatives to take hold to the extent that they have. It seems that it takes the educational infrastructure much longer to implement empirically justified teaching and learning strategies than it takes the health infrastructure to implement new and proven treatments in medicine. Perhaps this is because such innovations cause situations and issues that teachers used to believe were black and white and easy to manage, to take on a more complex range of shades of grey and are more professionally challenging.

I recall a quotation from an interview with David Suzuki. During that interview, he was asked how growing older had affected his abilities as a scientist. He replied that when he was young, he was sure of everything. His reasoning was crisp and uncluttered. He bemoaned the fact that as he grew older, learned more, and gained greater experience, it wasn't so easy to make crisp decisions. Things were no longer as well defined as they had been in his youth, and there were many more possibilities of which he had become aware. He was embracing this complexity, however, because it had greater potential for innovation and success.

It is very gratifying to see how the learning and teaching of mathematics in Ontario and across Canada has evolved during my teaching years and to recognize the role of OAME in promoting, supporting, and leading that evolution here. There is a recognition implicit in curriculum and policy documents, in professional learning opportunities and resources, and even in published textbooks and digital learning materials, that our responsibility is to ensure the success of as many of our math students as possible. We have begun to realize that our purpose isn't so much to teach math as to teach children.

Math educators have begun to embrace the complexity of teaching human students rather than the content of a discipline. Using shades of grey paints a much more interesting picture and captures a lot more detail than just using black and white. I wonder what will happen in the next evolutionary step. ▲

▲ Letter to the Editor

Dear Editor,

I have recently had the task of perusing the past issues of the *Gazette* to write a brief report on the development of this journal over the time it was written for OAME.

The journal was originally distributed in 1962 by the Ontario Mathematics Commission (OMC) and the Ontario Association of Teachers of Mathematics and Physics (OATMP). Later OATMP became the Ontario Association of Teachers of Mathematics (OATM). The *Gazette* then became the journal for OAME.

I have noticed a marked improvement in the quality of the journal, reflecting both the development of technology, and the addition of new departments and other ideas as an evolution and reflection of new editors. It has become an excellent vehicle for communicating the business of the organization to its members.

The one area in which I have noticed a decline in the number of articles is in mathematics itself. There is no longer a regular feature on problems and solutions; however, this is covered by the excellent journal *Crux Mathematicorum*.

The occurrence of articles on interesting Mathematics is also rare now. This is presumably due to there being not so many articles submitted to the editor. I therefore join the editor in a plea for articles on interesting ideas in mathematics for the *Gazette*.

I intend to take my own advice and write one. I might also suggest that you consider reprinting some of the earlier published articles. Many of them are excellent and well bear repeating.

Tom Griffiths

Response from Editor:

My goal as editor of the *Ontario Mathematics Gazette* is to serve the needs of our readers. In an attempt to support all teachers from K–12 as well to focus on the teaching of mathematics, there may be fewer articles focused strictly on content than was once the case. In fact, external advice has been for us to include a greater proportion of articles focused on pedagogy and more articles based on educational research. I should note that there is a column called What's the Problem that does focus on math problems. If Mr. Griffith's opinion is held by many readers, it is important for us to know. If others are more comfortable with the current mix of articles, it would be useful to know that too. We intend to upload a small survey to gather your opinions on this issue. ▲

▲ Technology Corner — Exponential and Logarithmic Models

MARY BOURASSA



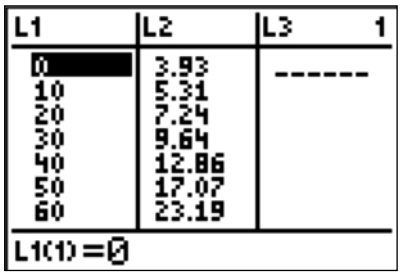
Mary teaches mathematics at West Carleton Secondary School in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She has presented workshops internationally, authored mathematics resources, is a past VP of OAME and a Past President of COMA. An award-winning teacher, Mary continually strives to learn new and better ways of helping students learn and love mathematics.

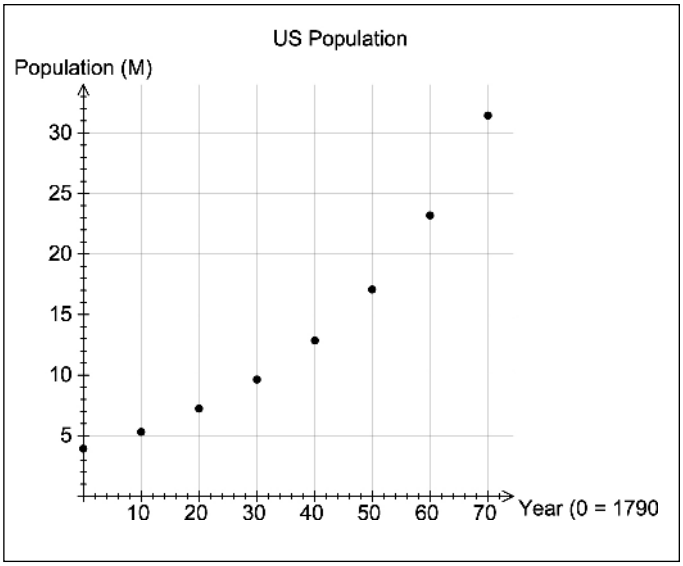
Mathematical modelling is a great way of relating functions studied in class to real-world data. Graphing technology is very helpful with data modelling, as it allows us see how well the model fits the data. Although a graphing calculator can determine the equation of a line or curve of best fit, working through the process of developing the right model is a wonderful exercise for students. Finite differences can be used to determine polynomial models. Different strategies must be employed to model exponential and logarithmic data.

Here is an example of data that grows exponentially. In the years before the Civil War, the population of the U.S. grew rapidly, as shown in the following table. Determine a model for this growth.

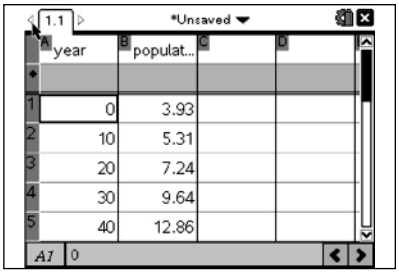
Year	1790	1800	1810	1820	1830	1840	1850	1860
Population in millions	3.93	5.31	7.24	9.64	12.86	17.07	23.19	31.44

Using a graphing calculator to plot the data, let's make 1790 year 0 and enter this new year data and the population data. On a TI-84, enter the data in lists 1 and 2. On a TI-Nspire, enter the data in a Lists & Spreadsheet page. Set up a scatter plot and look at the shape of the plot.





The shape suggests either a polynomial graph of even degree or an exponential graph. Populations generally grow exponentially, so we should check for a common ratio to determine if this case is exponential.



$\frac{5.31}{3.93}=1.35$	$\frac{7.24}{5.31}=1.36$	$\frac{9.64}{7.24}=1.33$	$\frac{12.86}{9.64}=1.33$	$\frac{17.07}{12.86}=1.33$	$\frac{23.19}{17.07}=1.36$	$\frac{31.44}{23.19}=1.36$

Since the ratios are almost constant, we can use an exponential equation to model this data.

We will use the model $y = c \bullet a^x$, where c is the initial value and a is the growth factor.

The common ratio is somewhere between 1.327 and 1.363. Taking the average of the values suggests a common ratio of 1.35. To determine a, we must take into account the fact that the data points are 10 years apart. This means that to get from the first data point to the second data point, you would have to multiply by a 10 times:

$$3.93 \bullet a \bullet a \bullet a \bullet a \bullet a \bullet a \bullet a \bullet a \bullet a \bullet a = 5.31$$

$$3.93 \bullet a^{10} = 5.31$$

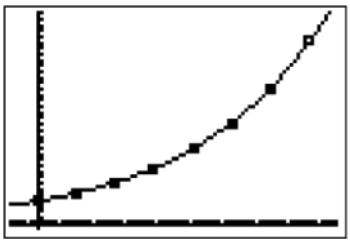
Therefore we need the 10th root of $\frac{5.31}{3.93}$, or 1.35.

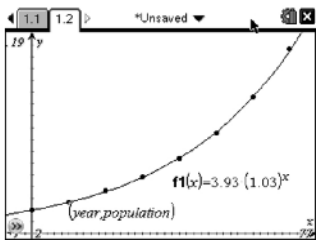
$$\sqrt[10]{1.35} \doteq 1.030465312$$

Store this value in your calculator.

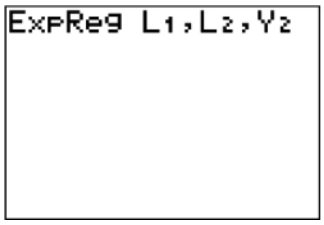
The model is now $y = c \bullet 1.03^x$. Our initial value is 3.93, so we can use that for c. If we did not have the y-intercept, then we could use one data point for x and y

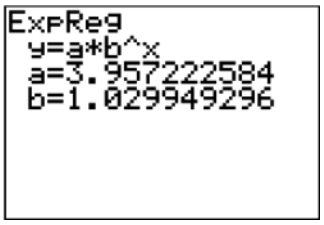
and calculate the value of c. For example, using (60, 23.19), $c = 3.94$, our model is now: $y = 3.93(1.03)^x$. Using a graphing calculator, we can see how well our model fits the data:

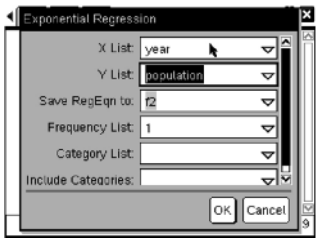


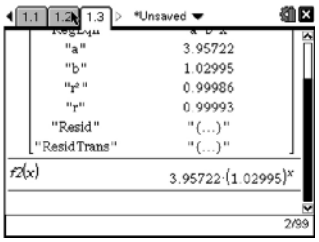


The graphing calculator can also provide an equation for us by doing an exponential regression:







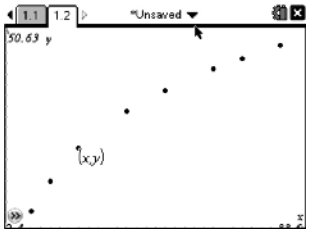


Let's now look at logarithmic data. Determine a logarithmic model, if appropriate, for the following data:

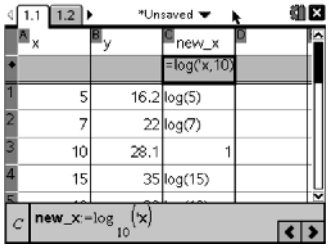
x	5	7	10	15	19	24	27	31
y	16.2	22	28.1	35	39	43	45	47.5

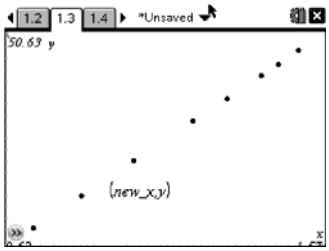
Enter the data and plot them.

To determine if a logarithmic model is appropriate, you can linearize the data. If y represents a logarithmic function, taking the logarithm of the data in x will create a linear relationship. If the graph of the new plot appears to be linear, then a logarithmic model is a good choice for this data.

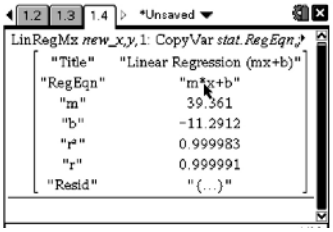


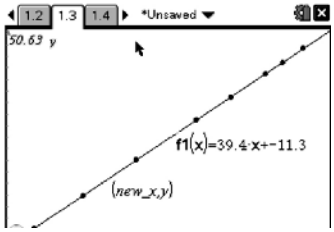
You need to plot log(x) and y. In your lists, make the 3rd column equal to log (1st column); then set up your scatter plot.



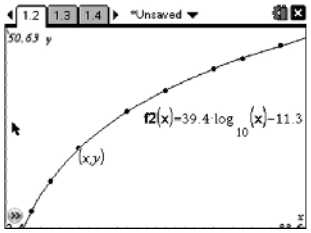


Does the graph look linear? If so, perform a linear regression on it and look at the correlation coefficient to see how well it fits the data. The correlation coefficient is high in this case.





The linear equation is $y = 39.4x - 11.3$. Since we replaced x with log x, we need to replace x with log x in our linear equation. The result is the logarithmic equation that fits the original data.



Logarithmic equation: $y = 39.4\log x - 11.3$. ▲

▲ Vote Mar 3–May 13
ALL MEMBERS TAKE NOTE!!
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▲ OAME/NCTM Report March 2011

LAURIE MOHER



Laurie is Numeracy Consultant, JK–8 for the Kawartha Pine Ridge District School Board. She has served on the Board of Directors of OAME for over 12 years.

As reported in the fall issue of *The Gazette*, The National Council of Teachers of Mathematics Professional Development Focus of the Year for 2010–2011 is "Geometry: Constructing and Transforming Perspectives." Teachers, principals, and teacher leaders who wish to expand their understanding of geometric thinking and how it can be developed across the grade levels can use the resources on the NCTM website to support their professional development in this area. The Geometry Standard in NCTM's *Principles and Standards for Schools Mathematics* serves as the framework for this professional development. To read more about why NCTM has chosen Geometry as this year's professional development focus, select "Professional Development" from the top of the NCTM home page. Then click on the link entitled "Focus of the Year."

In addition to providing their rationale for the focus on geometry, entitled, "Why Geometry?," NCTM has compiled a number of resources connected to its focus under this section. These resources have been categorized according to three levels: Elementary School, Middle School, and High School. Once you have selected a level to investigate, you will find a number of resources that include the following:

- Journal articles (*e.g., Teaching Children Mathematics for Elementary*)
- ON-Math articles
- *Journal for Research in Mathematics Education* (JRME) articles
- Online resources – Illuminations lessons and Illuminations activities
- E-Examples
- Publications

Some of the articles included have the symbol "pd" beside them. Those articles are called "Reflection Guides" and were formally called "Enhanced Articles." The Reflection Guides are a protocol to use with a group when looking at the article, and include thought-provoking questions for discussion. Past "Focus of the Year" resources can also be found in the Focus of the Year section of the website.

Illuminations Spotlight

The Illuminations website has released a new applet entitled "Function Matching." This applet challenges students to determine the function expression that matches a generated function graph and check to see if they are correct. Students can choose from several function types, or they can let the computer choose for them. This applet can be found at **illuminations.nctm.org/ActivityDetail.aspx?ID=215**.

See the Illuminations site for other revised tools and hundreds of lessons and activities for students to explore. Lessons are detailed and include the following:

- Questions for students
- Assessment options
- Extensions
- Teacher reflection questions
- NCTM standards and expectations
- References

Featured Lessons

Pre-K–2: Puzzling Relationships

In this lesson, students begin to explore spatial relationships, using Tangram Puzzles. In the first part of the lesson, students use concrete Tangram Puzzle pieces to develop spatial relationships, vocabulary, and problem-solving skills prior to working with interactive Tangram Puzzle pieces. In the second part of the lesson, students work in pairs to complete electronic Tangram Puzzles. The lesson includes a Tangram Puzzle Activity sheet and a link to the online Interactive Tangram Puzzles. **illuminations.nctm.org/LessonDetail.aspx?id=L757**

Grades 3–5: Geometry in the World of Art

In this series of six lessons, students work with fractional parts of the whole (one-half, one-fourth), and translations, reflections, rotations, and symmetry, to make a quilt square. Students work in teams and use their quilt square to make a four-part mini-paper quilt. There are many opportunities for communication as students describe how they created their design and

then used it to create their mini-quilt. Further communication occurs as students analyze the mathematical similarities and differences among their quilt designs. There are many wonderful children's literature titles that can be used in conjunction with these lessons, and social studies connections are applicable for some grades. Activity masters are included with the appropriate lessons. **illuminations.nctm.org/LessonDetail.aspx?ID=U104**

Grades 6–8: Corner-to-Corner

In this lesson, students explore the relationship between the lengths of the sides and diagonals of a square. Students will use their discoveries to predict the diagonal length of any square. Other resources for this lesson include the Shape Cutter applet. It also includes a teacher resource—a link to Teacher's Lab. This lesson is designed to take one period. **illuminations.nctm.org/LessonDetail.aspx?ID=L683**

Grades 9–12: Embroidered Math

In this lesson, students are introduced to basic graph theory and Euler Circuits. They stitch paths and circuits to create their own graphs. Students are challenged to solve problems as they design a route that creates the same pattern on the front and the back of the canvas. This lesson includes an activity sheet and several overheads to accompany the lesson. **illuminations.nctm.org/LessonDetail.aspx?id=L814** ▲



▲ What's the Problem? Getting Things Squared Away

SHAWN GODIN



Shawn is the head of Mathematics, Technology, and Family Studies at Carrie Wilson Secondary School in Orleans, Ontario. The web page for this column is ***whats-the-problem.wikispaces.com****.*

Welcome back, problem solvers. Last time, I left you with the following problem:

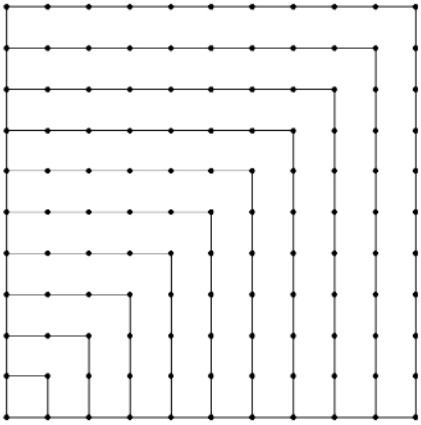
A square is constructed on an 11 by 11 pin geoboard. What are all the possible areas for the square?

The answer is: 1, 2, 4, 5, 8, 9, 10, 13, 16, 17, 18, 20, 25, 26, 29, 32, 34, 36, 37, 40, 41, 45, 49, 50, 52, 53, 58, 64, 65, 68, 81, 82, 100.

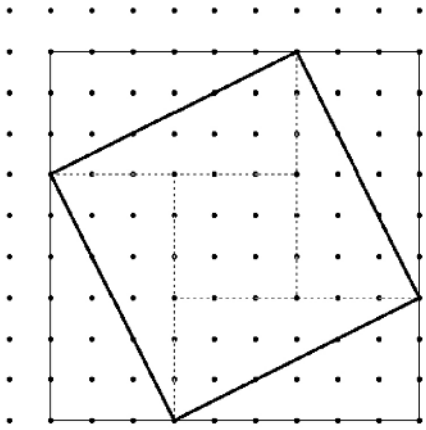
Now what fun was that? Mathematics isn't about the answer; it is about all the things you do that lead to the answer and all the interesting patterns and questions that come up along the way. If nothing interesting happens during the process, then you are not working on a very interesting problem.

I used this problem at the beginning of the semester with my Grade 9 classes. I was hoping to have them recall some of the things they had learned last year. Also, it gave me an opportunity to assess their problem-solving and communication skills. In addition, I was foreshadowing some of the things we would study more deeply later in the semester. We spent a whole class looking for solutions and discussing what they had found. For the next several days, I kept reassigning it to them for homework to see if they found anything new. The students came up with some of the patterns discussed below.

Let's start where most students will start, that is, with squares of area 1, 4, 9, 16, 25, 36, 49, 64, 81, and 100, pictured on the next page.



When the hands go up, you can give them the positive feedback, "Excellent, you've made a great start!" The students need to think outside the box a little to get the rest of the solutions. Sooner or later, a student will come up with something like the solution below (thick).

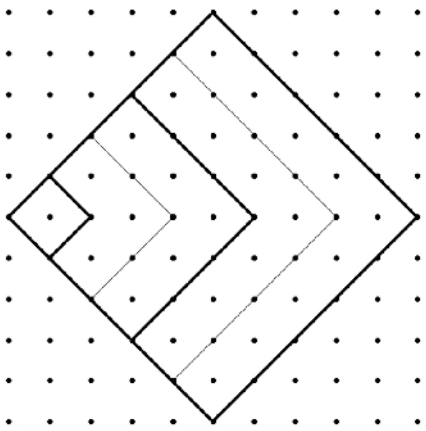


There is some interesting mathematics here. First of all, how do we know if this figure is actually a square? If we construct four right-angled triangles on the outside, as shown with the thin lines in the diagram, we see that they are all congruent, so the quadrilateral is a rhombus. If we look at a point where two triangles meet, at a vertex of our supposed square, we get two non-corresponding angles of congruent right triangles, so they must be complementary. Since the three angles form a straight angle, the angles in our quadrilateral must be a right angle; hence, the quadrilateral is a square.

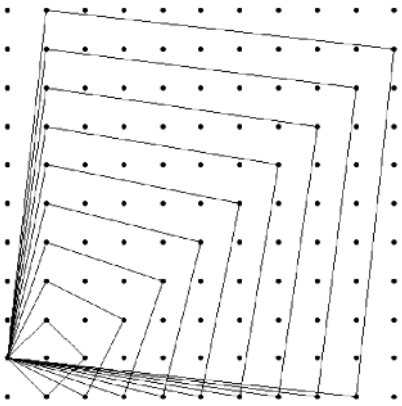
Now that we know the angles are right, what else can we notice? If we look at the slopes of the sides of the square, we notice that two have positive slopes and two have negative slopes. It shouldn't be hard to convince ourselves that if the square is tilted on the board, this will always be the case. Calculating the slopes, we find that they are $\frac{3}{6}=\frac{1}{2}$ and $-\frac{6}{3}=-\frac{2}{1}=-2$; that is, they are negative reciprocals. We can convince ourselves that this is always the case by using the same argument that we did above to show the figure was a square. Another interesting way to come up with this relationship, using a geoboard, is to construct any non-horizontal or non-vertical line segment on the board and calculate the slope, then rotate the board 90° and calculate the slope of the "new" segment. You will be able to see how the rise and run interchange, with one of them "changing directions."

Next, how do we actually calculate the area of this figure? Using the diagram at left, we can subtract from the areas of the large square (thin) the areas of the four triangles that we used earlier. We could also cut the square up into four triangles and a square, as shown by the dotted lines. Or we could calculate it directly by using the Pythagorean Theorem to determine the length of the sides. In this case, the side lengths are $\sqrt{45}$, so the area of this square is 45 square units.

What else can we see? When we start looking at the solutions, we can arrange them into "families." For example, in the diagram below, we have the solutions 2, 8, 18, 32, and 50. They are grouped together because they are the only solutions with sides of slope 1 and -1. Hopefully you see something else going on here. If you look at our solutions a bit more closely, we get: 2×1^2, 2×2^2, 2×3^2, 2×4^2, and 2×5^2. They are all multiples of some of our "original" solutions. It would be an interesting exercise to have students discover and explain why these patterns hold. Can you see how the family 5, 20, and 45 could be embedded in the first tilted example, in the previous column? See if you can create the list of all solutions by grouping into families with sides of equal slope.



What if we grouped solutions into different families? The family below models the solutions whose sides have slopes n and $-\frac{1}{n}$ for positive integer n. The diagram exhibits a number of interesting patterns. Notice how consecutive vertices of the squares are related. The areas are 2, 5, 10, 17, 26, 37, 50, 65, and 82. If we look at the differences between these values, we get 3, 5, 7, 9, 11, 13, 15, and 17, which implies that the second differences are constant, so we have a quadratic relation (using the position in the list as the independent variable). It shouldn't take too much searching to see that for members of this family, $A = n^2 + 1$, where A is the area and n is the position on the list.



Similar families could be constructed. Notice that if we have a square with sides with slopes $\frac{a}{b}$ and $-\frac{b}{a}$ such that, by the Pythagorean Theorem, the lengths of the sides are $\sqrt{a^2 + b^2}$, then the area is $a^2 + b^2$. Thus, if we had an infinite geoboard, the possible areas of squares would be all the numbers that can be written as the sum of two squares of integers (The perfect squares come when one of the integers is zero). It would be interesting to see if you can observe any patterns in these numbers. Determining the integers that can be written as the sum of two squares is a classic problem in number theory. Check out my wiki for some more information on this problem. A similar theorem states that all positive integers can be written as the sum of the squares of four integers (some of which may be zero).

Now for your homework. Alice, Barney, and Carl were playing a game with 100 counters, although they were not using all of them. They each had a pile of counters in front of them. All at the same time, Alice passed a third of her counters to Barney, Barney passed a quarter of his counters to Carl, and Carl passed a fifth of his counters to Alice. After this, they all had the same number of counters. How many counters could they each have started with? Until next time, happy problem solving! ▲

▲ Data Rocks – Working with Many Random Samples

Yves Saint-Pierre



Yves Saint-Pierre has worked at Statistics Canada in many capacities since 1990: questionnaire design, data editing and imputation, analysis, and publication. He has been sharing his knowledge with schools since 2002.

The Census at School project gives you access to a very special tool: a random sampler that allows you to take many samples of different sizes from the population of Canadian students who participated in each year's survey. Here are some ideas of things you can do with different random samples:

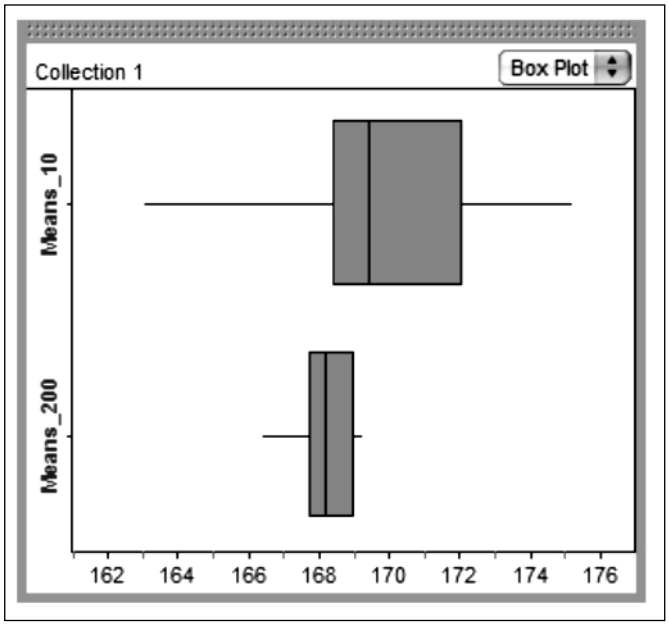
1) Verify the statistical law on the variability of the mean of samples of different size.
2) Compare the mean of various samples to the population mean.
3) Compare your class results to samples of Canadian students.

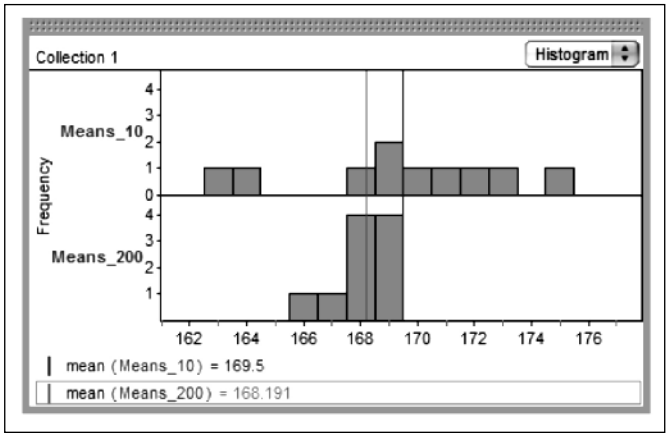
Ask your students to retrieve their own random samples of size 10 and 200. Compile the sample means of the class in one table, like this (here as if for a small class of 10 students):

Mean Height_n=200	Mean Height_n=10
167.715	164
167.17	172.9
166.35	169.6
168.845	169.1
169.145	171.4
168.035	168.4
168.225	169.3
169.05	172.1
169.27	175.2
168.11	163

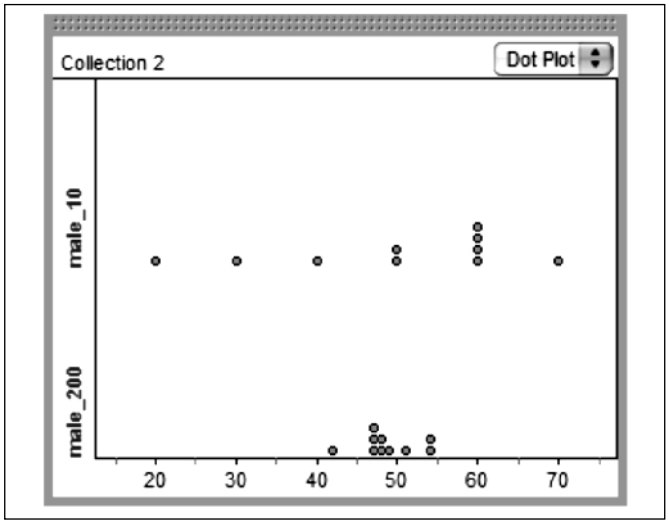
There is a statistical law at work here. The variability of the mean diminishes as $\frac{1}{\sqrt{n}}$, where n is the sample size. We are able to observe this law and check to see how well the law pans out.

The following graphs created in Fathom show means of height from Census at School samples for ten samples of size 10 and ten samples of size 200. The box plot and the histogram both show a much wider distribution for means of the smaller size sample. The spread in this distribution can be quantified by the inter-quartile range or the standard deviation.





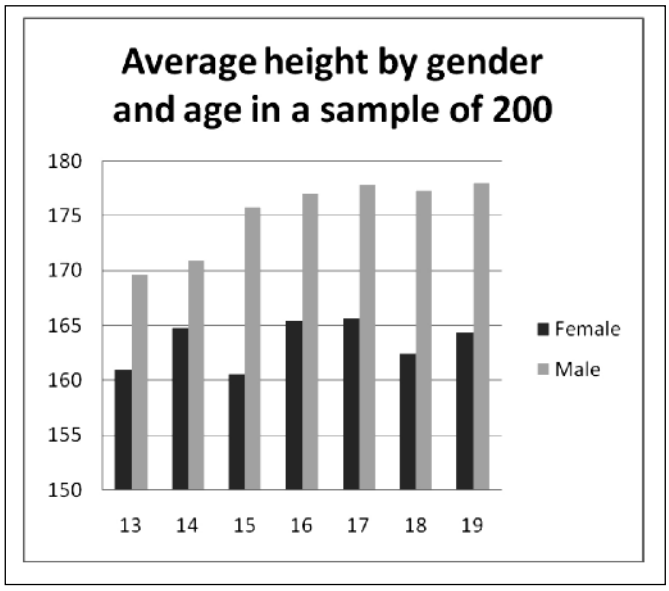
Even categorical data can be compared in a similar way. The next graph shows the percentage of males in each sample. The percentages in the smaller samples vary a lot more than those in the larger samples.



You could also ask students to compare their own sample means to the population mean, available from Census at School, and try to find out how many standard deviations their mean is from the population mean. Students could examine what characteristics of their sample might explain some of the differences in the means.

For example, looking at my samples of size 10, I can see that one of them has a mean height of 163, and at the other extreme, one has a mean height of 175. Since these are samples of students in Grades 9 to 12, I wonder if one of them has more boys than girls. I also wonder if one of these has a greater percentage of older students. Let us find out. The average age is 15.6 years for students in the sample with the highest mean height and is 15.3 for the sample with the lowest mean height. The sample with the lowest mean height has 20% boys, whereas the sample with the highest height has 60% boys.

One of the samples of size 200 shows that boys are taller than girls at all ages, and that the differences in heights are greater for older students. Do all of the larger samples reflect that? Likely yes, because larger samples show more consistency, but you could check that with your students. (See graph below.)



Students could consider what other variables besides gender and age in this sample could be associated with height.

If your class participated in the Census at School survey, your students could also compare their results to a random sample of Canadian students.

In any case, now you have a few ideas of things to do with random samples. If you can think of more, don't hesitate to write to me at **mdmd4u@statcan.gc.ca**.

And now here is how to find these resources:

1) Random Samples: **www.censusatschool.ca** > Data and Results > International Results and Random Data selector > random data selector > ... then follow the instructions to retrieve random samples of Canadian students from the latest year available. Have your students choose samples of various sizes.
2) Population results: **www.censusatschool.ca** > Canadian Summary Results > ... there are tables there for every question in the Census at School. ▲

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▲ In the Middle – From Models to Numbers: Making Connections in Mathematics

CAROLINE ROSENBLOOM



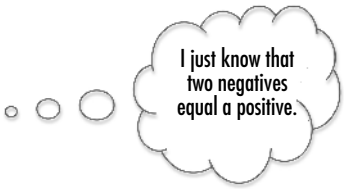
Caroline Rosenbloom is currently on secondment as an Elementary Coordinator teaching mathematics in the pre-service program at OISE/UT. She has enjoyed working with intermediate students and teachers as a middle school teacher, Mathematics Instructional Leader for the Toronto District School Board, and Education Officer at EQAO. Caroline is co-author of Investigating Mathematics Using Polydrons.

Students solve mathematical problems in a variety of ways. Some students need to touch and manipulate cubes or tiles to help them visualize the problem. Some may use software to investigate and explore different models. Some may use symbols to represent their solution algebraically.

But how do we get those students who are at the concrete stage to represent their models symbolically? Do students who represent their solutions symbolically really understand what their formal expressions mean, or have they just learned to manipulate symbols?

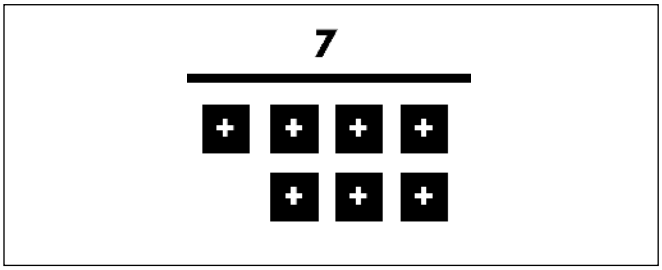
Intermediate mathematics should emphasize making connections between the concrete and algebraic models. If we rush to teach rules or algebraic manipulation without first imparting an understanding of what these rules or expressions mean, then what are students really learning? The purpose of making connections is to understand the development of the rules, the formulas, and the algebraic expressions.

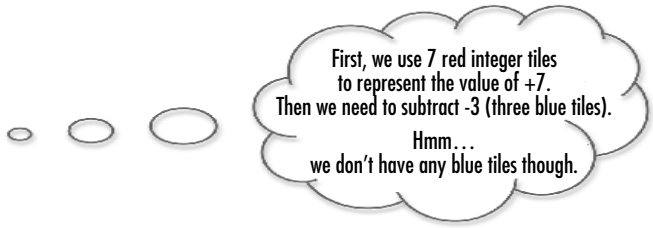
In Grade 7, students are expected to learn how to add and subtract integers. When they are presented with a question like, "What is the value of 7 – (-3)?," we may get a solution that looks like this…

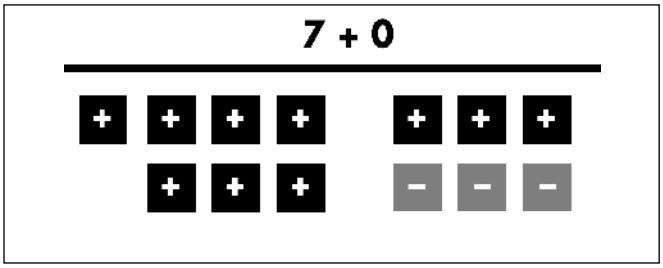


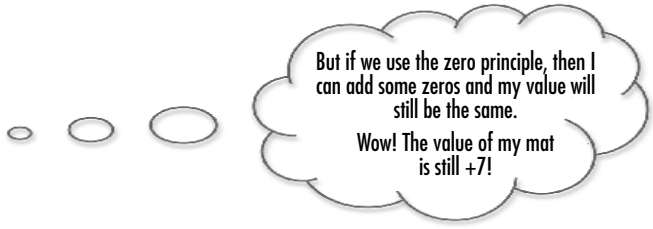
$$7 - (-3)$$
$$= 7 + 3$$
$$= 10$$

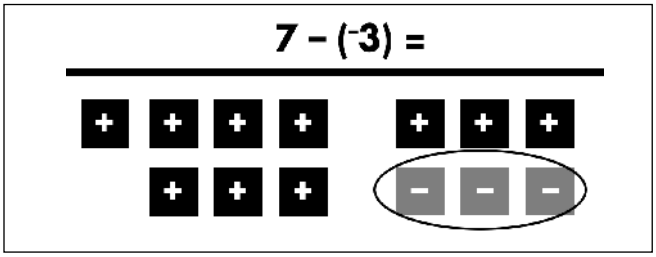
Although the solution is correct, what does this student really know? If, instead of paper-and-pencil work, we use tiles and an integer mat, we can focus on and assess students' understanding of the zero principle.

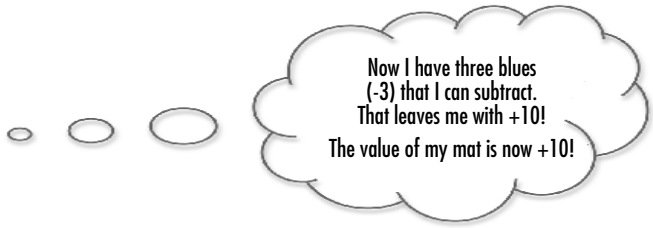










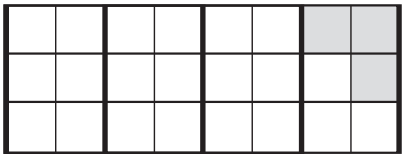


By going through many examples and seeing the pattern of subtracting a negative, students begin to understand what the rule "two negatives = a positive" means.

We can also use visual or concrete models as tools to aid in the understanding of problems in areas such as fractions. In the following problem, the grid below mirrors the calculations below. The grid is used to not only generate the solution $\frac{1}{8}$, but to add meaning to each step of the symbolic derivation below it.

Simaya's dad cut her birthday cake into 12 equal parts and Simaya ate one of the pieces. Later on, there were only a few pieces left, so her dad cut each of the original pieces in half. Simaya then ate 1 of those smaller pieces. What fraction of the cake did she eat in total? Explain your reasoning.

Visual – Dividing the whole cake into 24 pieces.



Calculate half of the original piece.

$$\frac{1}{12} + \left[\left(\frac{1}{12}\right) \div 2\right]$$
$$= \frac{1}{12} + \frac{1}{24}$$
$$= \frac{3}{24}$$
$$= \frac{1}{8}$$

The ½ is one of the big pieces and the 1/24 is one of the little ones.

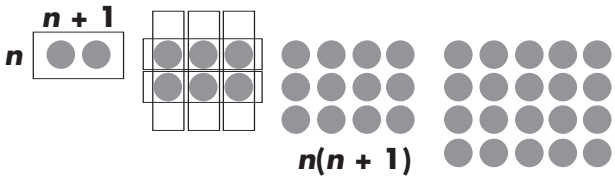
In this patterning and algebra question, students are asked to complete the chart and determine the 20th term in this growing pattern.



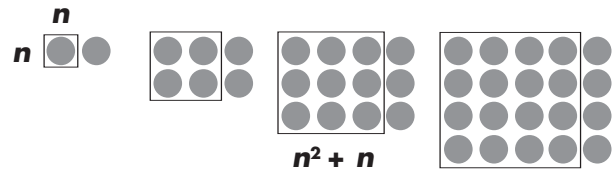
Frame	1	2	3	4	5	6	7	...	20
# of dots	2	6	12	20				...	

1. Complete the chart.
2. Descirbe your pattern in words.
3. What rule can you make to get from one frame to the next?

Some students may ignore the pattern of dots and focus on developing a rule which generates the sequence of numbers (2, 6, 12, 20,…) at the bottom of the chart. While their rule may work, the connection between the numeric sequence and the dots sequence is lost. They would have difficulty justifying their rule. By focusing on both the pattern of numbers and the pattern of dots at the same time, a student connects a dot rectangle with a number in the sequence and can potentially derive and justify a general expression for the nth term in both sequences. Below are two different models that help students visualize and justify two "different" expressions for the nth term in the sequence.



Area model.
Row (*n*) times column (*n* + 1).



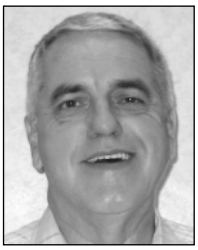
Making connections naturally leads to better mathematical intuition and an understanding of the derivation of rules, formula, and algebraic manipulations. A focus on the final answer to a problem, without a motivating model to connect the solutions steps, is like seeing only the last ten minutes of a movie. The end doesn't make nearly as much sense compared to watching the movie from start to finish. Besides, watching the whole movie is more fun; something interesting usually happens along the way.

References

Los Angeles County Office of Education. *MathStar: Integers*. Retrieved from **mathstar.lacoe.edu/newmedia/integers/welcome.html** ▲

▲ What's New at EDU?

Jeff Irvine



Jeff is an Education Officer at the Ministry of Education.

New Gap Closing Resources and Supports from the Ministry of Education

Gap Closing: Grade 6 – Number Sense

- A Student Book and Facilitator's Guide for students needing more support in the Number Sense strand of Grade 6 Mathematics.
- Released in Fall 2010.
- Research from the beta test in Spring 2010 showed:
  - gaps closed for students in both conceptual understanding and content knowledge
  - gaps closed between males and females in mathematics achievement
  - improvements in student self-assessment skills

  A summary of the beta test research can be found on the EduGAINS website.
- A number of boards have begun implementing these resources. The Ministry of Education is providing supports for this implementation, including:
  - identification of researchers to attach to board implementations
  - provincial mathematics coaches
  - Adobe Connect professional learning series

Gap Closing: Grade 9 Applied – Number Sense and Algebra

- For students in Grade 9 Applied Mathematics who require support in the critical Number Sense area.
- Student Book and Facilitator's Guide.
- Modelled on the successful Grade 6 materials.
- Released in Winter 2011.
- The Ministry of Education is providing supports for boards that have chosen to implement these resources on a system-wide basis.

All the Gap Closing resources are or will be available on the EduGAINS website. Colour print copies for Grade 6 are also available free of charge through Publications Ontario at **www.ServiceOntario.ca/ publications**.

Professional Learning for Mathematics Leaders and Coaches Series (PLMLC)

- Professional learning activities for teachers, coaches, instructional leaders, school administrators, and superintendents.
- Topics identified by an advisory panel that includes OAME, OMCA, and Ministry of Education.
- Face-to-face or Adobe Connect formats.
- Registration information can be found on the EduGAINS website.

EduGAINS website (www.edugains.ca)

The MathGAINS page has been updated to provide more intuitive access to content, as well as access by role and responsibility. The site provides:

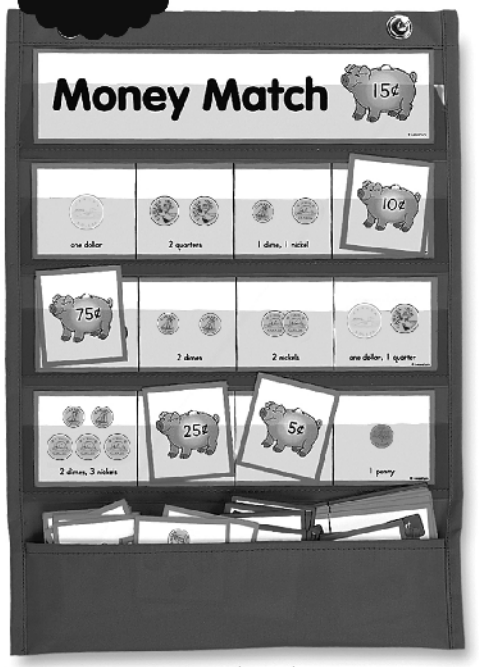
- new and existing resources in Microsoft Word or pdf
- downloadable video resources
- information on how boards or schools are implementing resources
- registration information for PLMLC sessions

Take advantage of the free RSS feed to keep you posted on additions and updates to this website.

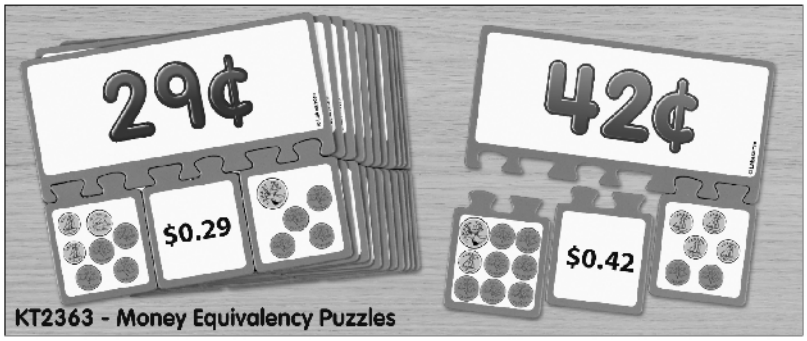
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▲ Hey It's Elementary – The Elephant in the Room

LYNDA COLGAN

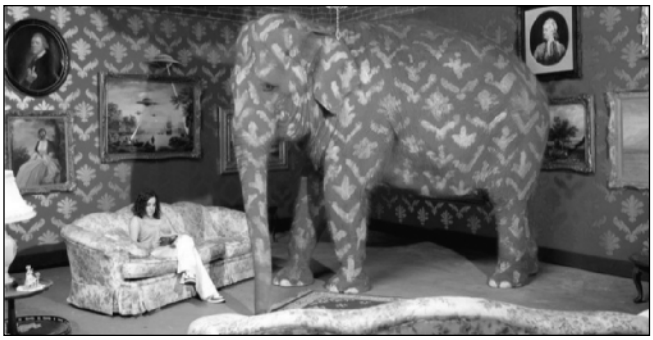


Lynda Colgan's career has included roles as a classroom teacher, a university professor, and newspaper columnist. Her contributions to mathematics and its teaching have been recognized through awards such as the Marshall McLuhan Foundation Distinguished Teacher award. Lynda always exhibits a passion for mathematics and views her professional mission as dispelling the myth that math is the bad guy.

Over the past few months, you will, no doubt, have *seen* ads in the media about the *elephant in the room*. In this recent ad campaign, the elephant is alcohol—more specifically, the issue of impaired driving. The ads are a powerful illustration of one (sadly) all-too-common example of silence and denial in everyday life: everyone at a house party, from the hosts to all of the guests, strive to ignore the elephant (figuratively, as well as literally, since an enormous inflatable pachyderm takes over the room) when one guest, who has consumed too much alcohol, pulls out her car keys and announces that she is going home.

The phrase "elephant in the room" is an English idiom for an obvious truth that is being ignored or goes unaddressed. The idiomatic expression also applies to an obvious problem or risk that no one wants to discuss.

The phrase is based on the idea that an elephant in a room would be impossible to overlook; thus, people in the room who pretend the elephant is not there have made a choice to concern themselves with irrelevant issues rather than deal with the large one.



When the elephant is in the room, one needs to work hard **not** to see it. This was the message of British artist Banksy's September 2006 exhibit, *Barely Legal*. For this living performance art show, Banksy painted a 38-year-old female Indian elephant, named Tai, in the same bold pattern as the wallpaper in a couple's living room. By doing so, Banksy concretized the meaning of the phrase *the elephant in the room* by both making the elephant more obvious and by giving those who chose to ignore it (like the woman in the tableau) an opportunity to pretend that it had blended into the wallpaper background.

Bansky gives us a glimpse into two common responses: avoiding the elephant by dancing around it or denying it by saying, "What elephant? I don't know what you're talking about."

A second message is that there is always evidence related to the elephant in the room in one's immediate environment. You only have to look up to see the elephant or find evidence of its existence, but even then, you may have grown so accustomed to seeing it, that you need a fresh set of eyes to see the situation at hand clearly.

Banksy wanted to communicate his belief that the only way to destroy the elephant is to face it squarely to finally "see" and accept what is really going on. For this show, Banksy wanted visitors of his warehouse installation to face the elephant of poverty and social injustice.

What do impaired driving and performance art have to do with elementary mathematics education? Elephants, of course.

In March 2010, there was a large conference in New York entitled *A Celebration of Teaching and Learning*. At the conference, in a keynote address, PBS journalists David Brancaccio and Jim Simons[1] presented a talk called *The U.S. Math-Teaching Crisis: 'The Elephant in the Room.'* The talk was an attempt to answer the question, "Why we [the U.S.] do so poorly as a nation in math?"

The speakers began their talk by saying that they actually think the answer to that question is straightforward: "We don't have enough teachers of math and science who actually ***know*** math and science," they said bluntly. "It's the elephant in the room." They went on to ask: "Why are we passing poor math students through unchallenging courses that result in teaching degrees?" They also questioned why teacher education programs focus more time on "methods" teaching instead of

1 Jim Simons is the founder of a teacher-recruitment program called Math for America and the mathematician who made a fortune as the CEO of Renaissance Technologies, a private investment firm.

content based on the belief that "all the skills and teaching methods in the world are useless if you do not have a firm grasp of what you are teaching."

The pair also wondered, why it is, for example, that many elementary-level teachers can casually and openly admit to not liking or not being good at math. How would we react, they asked, if teachers said that about reading?

They also raised the issue of teacher accountability: why is no one insisting that teachers employ the most advanced instructional techniques that have been so clearly identified and supported by research over the last 30 years?

Brancaccio and Simons may have disturbed the cognitive tranquility of many conference delegates by challenging them to confront a small number of "professional" elephants and consider their own compliance in several conspiracies of silence. However, one keynote address to a limited (and elite audience) about these important issues is not enough to remove the elephants from the room.

Once awareness of the elephants' existence has been accomplished, the next logical step is to raise the issue in a public forum by inviting those most affected to do the following:

1) **Broach the issues**. No matter how uncomfortable it may be, no one can move ahead without confronting the elephant and its reach.
2) **Tell it like it is**. There is no value in using euphemisms or vague language to describe the elephant, no matter how malodorous.
3) **Describe their emotions** surrounding the elephant.

This may sound impossible. Well, it's not. These steps were taken by a significant number of leading language educators across Ontario as part of a recent visionary initiative in literacy education coordinated by the Ontario Association of Deans of Education (OADE).

The OADE invited Dr. Lesly Wade-Woolley[2] to chair an innovative and important meeting of reading and literacy instructors in early December 2010 at the Ontario Institute for Studies in Education at the University of Toronto. The goal of the two-day working meeting was to reach consensus and recommend a common curriculum and set of learning outcomes to be used by all elementary language instructors at all Ontario Faculties of Education for their BEd language curriculum courses.

The OADE recognized that a broad consensus now exists among researchers and educators regarding the knowledge and skills that children need in order to read, the experience that influences the development of such knowledge and skills, and the basic components of reading instruction. Thus, they charged Dr. Wade-Woolley to ensure that all prospective teachers in Ontario are provided with comparable opportunities to learn about the practical applications and implications of reading research so that they can make a difference where it matters most—in the classroom.

The goal of the meeting was to design a reading program for Primary/Junior teacher candidates that communicates the important consensus that exists within the scientific community about the teaching of reading. The meeting was about how to design a teacher education program that integrates good research (e.g., that effective reading instruction compensates for risk factors that might otherwise prevent children from becoming successful readers) and the practical considerations of the components of an effective reading program.

This project was an important first step in capturing the best knowledge currently available to design curriculum for Ontario's teacher candidates so that all early career teachers, regardless of their choice of teacher education program, will have experienced instruction in how to implement common early reading instruction. In turn, this means that all district school boards and principals can expect a certain common denominator in terms of expertise from all novice teachers, which will, in turn, assist curriculum leaders who are designing local professional development programs, and which should assure parents about the quality of their children's language program.

Why is this initiative so revolutionary? Just as all elementary classroom teachers are bound by the provincial Literacy curriculum document, the OADE was trying to establish a provincial Faculty of Education literacy curriculum document that would guide and inform all elementary language curriculum "methods" courses: a first in Ontario (and perhaps, in Canada).

In Ontario, the elementary school curriculum is determined by and measured using policy documents governed by the Ministry of Education (MOE). The primary mandate of the MOE is to oversee and coordinate all aspects of schooling in publicly funded

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[2] Lesly Wade-Woolley, PhD is a professor of Language and Literacy at the Faculty of Education, Queen's University, and was a member of the panel who developed The Early Reading Strategy: The Report of the Expert Panel on Early Reading in Ontario.

schools in Ontario. There is a common curriculum for all subject areas in all grades in all schools and in all individual classrooms. While delivery of instruction is left to the discretion of the individual classroom teacher, there are many checks and balances and systems of accountability, including provincial report cards.

Faculties of Education fall under the jurisdiction of the Ministry of Colleges and Universities, not the Ministry of Education. Faculties of Education, as schools within universities, have their own elephants: tenured versus adjunct faculty members, academic freedom, individual programs of research, and offshore competition. Currently, each BEd instructor is free to design his or her own course. Some courses have required textbooks and assigned readings; some do not. Some Faculties of Education have examinations (in both content and pedagogical content knowledge); some do not. Some Faculties of Education require that teacher candidates complete courses in all elementary subject areas; some do not. Some teach elementary mathematics education in large lecture settings (sans benefit of hands-on experience with manipulatives and technology); some do not. Some elementary mathematics curriculum courses include content; some do not. Some universities have 36-hour curriculum courses; some do not. There are more differences than similarities when it comes to teacher education programs across Ontario, and there are fewer checks and balances. While all Faculties of Education must be accredited by the College of Teachers and meet the academic requirements of their individual universities with respect to teaching, research, and service, there are no common standards and few forums for faculty members to meet or share resources and ideas.

The OADE initiative, under Dr. Wade Woolley's leadership, is an acknowledgement of the fact that Ontario is the only province to have an eight- month BEd program and that every effort must be made to make the most of the extremely limited number of instructional hours available to "prepare" future teachers by equipping them with the knowledge, skills, and attitudes that they will need to be successful instructors of reading. A first step is to articulate essential learning outcomes.

In 1905, poet and philosopher George Santayana said, "Those who cannot remember the past are condemned to repeat it." This is a most appropriate quotation, given that it is often said that research in mathematics education trails that in language by some 50 years, and that mathematics educators have often walked in the missteps of language researchers.

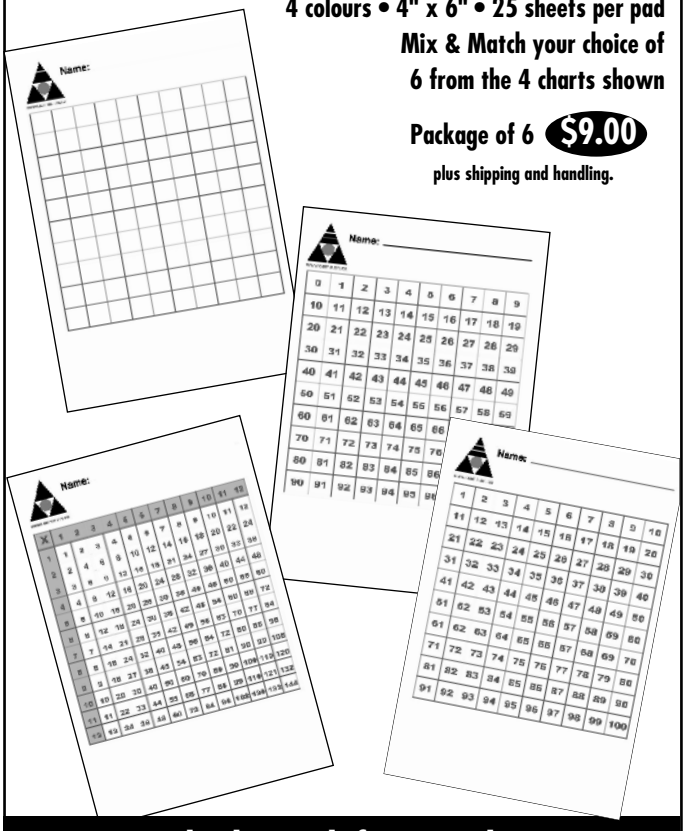
I urge OADE to strike a committee to begin a BEd articulation process in mathematics curriculum that parallels the one begun in language. Given the increased rigour of the elementary mathematics curriculum, the demands of a conceptually-based instructional program, and the logistics and expert knowledge required to enact dynamic, formative, and summative assessment, there is a need for the explication of a common denominator for all elementary mathematics education programs at all Faculties of Education in Ontario.

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▲ Fields Institute – MathEd Forum Notes

STEWART CRAVEN



Stewart is a senior mathematics teacher recently retired from the Toronto District School Board. He continues to teach at York University and actively participates in the Fields Institute Mathematics Education Forum. He was the previous editor of the Gazette*.*

You may be surprised by the number of organizations in Ontario devoted to improving the teaching and learning of mathematics. The OAME (Ontario Association for Mathematics Education), the OMCA (Ontario Mathematics Coordinators Association), and various branches of the Ontario Ministry of Education are all examples of such organizations.

Less well known is the Fields Institute MathEd Forum. One Saturday each month, between 30 and 50 keenly interested individuals representing university faculties of education, mathematics, and statistics; community college mathematics departments; elementary and high school teachers; graduate students in education and mathematics; and a potpourri of other individuals from business, engineering, psychology, and other disciplines, as well as parents, administrators, and publishers, gather to participate in our open forum. The topics of discussion are diverse, covering everything from the use of technology in the classroom to the role of gesturing in teaching mathematics.

This new column will report on the discussions and activities that take place at meetings of the MathEd Forum. We welcome everyone. Meetings take place from 10:00 a.m. until 2:00 p.m. at 222 College Street in Toronto. Upcoming meetings are scheduled for January 29, February 26, March 26, and April 30. Breakfast snacks and a light lunch are provided at each meeting. This column will provide a short synopsis of the topics discussed at the first three meetings of the 2010–2011 academic year.

Our first meeting (in September) started with the question: "What (event, emotion, experience) inspired you to dedicate yourself to mathematics education?" Participants convened in small groups to share their stories and identify recurring themes. When the whole group came back together, it was clear that regardless of the inspiration for doing mathematics, all participants were passionate about mathematics. This led us to ask: "How can we help many more students become emotionally disposed toward engaging in mathematics?" Following that discussion, we were treated to a wonderful presentation and conversation about "How Our Hands Help Us Think about Math," conducted by Susan Goldin-Meadow from the University of Chicago.

In October, our meeting took place at the University of Windsor (once a year, we schedule a meeting outside of Toronto). The initial theme for this meeting was "Equity in Mathematics Education." The presenters and panelists included: Pat Rogers (University of Windsor), Florence Glanfield (University of Alberta), Immaculate Namukasa (University of Western Ontario), and Richard Barwell (University of Ottawa). After a short break, Brenda Gunderson (University of Michigan) addressed the topic, "Using Instructional Technology to Enhance Teaching." Dr. Gunderson focused on the use of interactive lecture notes, personal response systems (or "clickers"), pre-lab lessons, and "lecture capture" in a university setting. Many of the secondary school teachers in attendance showed a keen interest in these ideas.

Our meeting in November focused on "Humanistic Mathematics." Gizem Karaali (Pomona College, Claremont, CA), one of the editors of the reinvented *Journal of Humanistic Mathematics*, which will debut in January 2011, led off by posing and discussing the following three questions:

1) Why are we (as the Journal's editors) doing what we are doing?

2) What does the current world of humanistic mathematics look like?

3) What do we mean by humanistic mathematics, anyway?

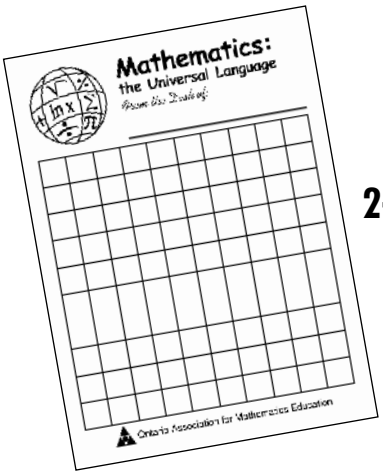
George Gadanidis (University of Western Ontario) followed with a presentation entitled "Why Can't I Be a Mathematician?" in which he used children's drawings of mathematicians at work, media images, and music (i.e., live performances of songs about mathematics) to generate a response.

Miroslav Lovric (McMaster University) and Iain Brodie (Toronto District School Board) used drawings by university and elementary school students to show what students think mathematicians do at work. Shirley Dalrymple (York Region District School Board) used video interviews to find out what high school students think humanistic mathematics is.

Burke Brown (University of Toronto) then spoke to the issue of "Branding Humanistic Mathematics." He made a passionate case for the essential and indispensable role of logos/brands/talismans to successfully establish the humanistic mathematics movement within the mathematics and mathematics education disciplines.

From this short overview, you can see the diversity of topics that we explore at meetings of the MathEd Forum. It is our hope that some of you will take part in a few (or all) of our meetings. For information about future meeting dates and agendas, please log into our website at **www.fields.utoronto.ca/programs/mathed/**. ▲

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▲ Using Reform Mathematics Methods to Teach Area Concepts in a Grade 7 Classroom

SARAH GARRETT



Sarah Garrett is a teacher in the Upper Grand District School Board in the Intermediate division at King George Public School in Guelph, Ontario. She has taught Design and Technology, Mathematics and is currently teaching Instrumental Music. She is a Mathematical Science graduate from McMaster University and recently received her Master of Education degree from Lakehead University.

The Call for Reform

The National Council of Teachers of Mathematics [NCTM] has been calling for changes to instructional methods for more than two decades. The NCTM publication *Professional Standards for Teaching Mathematics* (1991), a companion document to the curriculum document *Curriculum and Evaluation Standards for School Mathematics* (1989), reflected the NCTM's position that teaching needs to change in order to improve student learning. The Ontario Ministry of Education revised its Grades 1–8 Mathematics Curriculum in 2005, providing content and teaching direction to match the philosophy of the NCTM direction. These curriculum changes, commonly known as *reform* teaching methods, are promoted by many educators (e.g., Battista, 1999; Baturo & Nason, 1996; Cady, 2006; Fosnot & Dolk, 2001).

Battista (1999) describes reform in the following way:

> *In the classroom environment envisioned by NCTM, teachers provide students with numerous opportunities to solve complex and interesting problems; to read, write, and discuss mathematics; and to formulate and test the validity of personally constructed mathematical ideas so that they can draw their own conclusions. Students use demonstrations, drawings, and real-world objects—as well as formal mathematical and logical arguments—to convince themselves and their peers of the validity of their solutions. (p. 427)*

Battista contends that this classroom environment supports the learning of all students and concentrates on teaching for understanding, which results in a stronger conceptual grasp of the topics studied. This proposed classroom environment and process has implications for teaching all mathematical concepts, including the topic of area.

In January, 2010, I conducted a study in a Grade 7 classroom at an elementary school in southern Ontario. The study was designed to determine the efficacy of using the reform teaching method to teach the concept of area. The purpose of this article is to provide an example of strategies used within the classroom that support the reform method of teaching.

Reform in Practice

The study spanned a two-and-a-half-week period, and included eight lessons, for a total of 10 hours. For research purposes, a pre-test was administered prior to beginning the unit, and a post-test was administered at the end, and the results were analyzed to provide data for the project. The test included items that measured the students' preferences for arithmetic or geometric problems, asked for the calculation of the area of traditional plane figures as well as amorphous shapes, and investigated the relationship between area and perimeter. An outline of the lessons is provided in the table below.

Lesson Sequence	Topic Covered
1	The concept of covering
2	Exploring figures using the geoboard
3	Investigating dynamic properties of area (the foot problem)
4	Looking at objects in different ways
5	Estimating areas (upper and lower bounds, average)
6	Determining the area of a trapezoid
7	Trapezoid #2
8	Synthesis

I began each lesson by posing a problem (Appendix 1) without providing instruction on the particular methods students should use to solve the problem. Over the course of the instructional period, students were given the opportunity to work in homogeneous pairs. By pairing students who were at the same mathematical level, "full participation of both students" (Lawson, 2007, p. 3) was encouraged. Following the partner work, students came together to participate in what Fosnot and Dolk (2001) refer to as a "math congress" (p. 3). During the math congress, student mathematicians "communicate their

ideas, solutions, problems, proofs, and conjectures to one another" (p. 29). The learners, not the teacher, are the ones who are responsible for defending their thinking (Fosnot & Dolk, 2001; Fosnot & Perry, 2005). During the congress, I facilitated discussions, addressed particular solutions that supported the goal of the lesson, and supported individual learners.

Classroom discourse was a consistent element in this reform-teaching environment. Student-led discussions provided the students with an opportunity to evaluate different approaches, recognize problems with their own approach, and gain a deeper understanding of the topics. I elicited responses using the "five productive talk moves" discussed by Chapin, O'Connor, and Anderson (2003). These included: re-voicing, having students restate someone else's reasoning, asking students to apply their own reasoning to someone else's reasoning, prompting students for further participation, and using wait time (pp. 12–15). Student pairs worked through their solution on chart paper, then presented their solutions to the class in the math congress, as described above. Instead of directing questions to me, other students were encouraged to ask the student who was presenting his or her idea for clarification. When necessary, I stepped in to clarify a point and keep the discussion on track.

Consistent with the concept of reform mathematics, numerous opportunities for the use of manipulatives were provided to the students. Hands-on exploration with the geoboard in Lesson 2 allowed students to physically change the area of a figure by manipulating the elastic that was stretched over the pegs of the geoboard. Tactile opportunities allow students to "easily explore and adjust shapes" (Bray, Dixon & Martinez, 2006, p. 132). In Lesson 3, students created their own polygons, using the perimeter obtained from the outline of their foot. They then used this manipulative to explore area and perimeter situations.

Lesson 4 addressed the relationships among the areas of squares, rectangles, triangles, and parallelograms. Although, according to Ontario Curriculum expectations, students in Grade 7 should have been exposed to these relationships in previous grades, much of the knowledge in the case of my class was simply formula recall. The first five lessons laid the groundwork for the students to deepen their understanding of the formulas they knew through mathematical discourse. The lesson sequence culminated in problem-oriented lessons that focused on the determination of the area of a trapezoid.

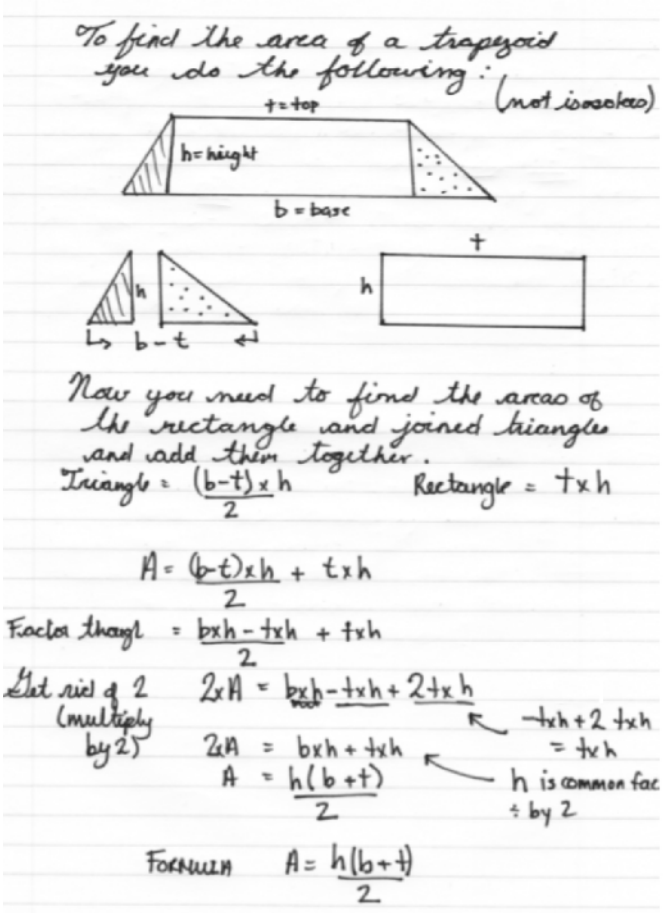
Results

The impact of the reform teaching method was observed and evaluated in a variety of ways. I examined pre- and post-test results, gathered student work after each class, noted classroom observations in a journal, and took videotape records of students at work during each instructional class. Progress was also monitored through observation and evaluation of classroom work. Students' attitudes about geometry, and in particular measurement of area, were determined by asking them directly about their preferences with respect to the types of problems they liked to solve, and by observing their performance on oral and written work. Student response to the reform teaching method was assessed through their participation in, and reaction to, the problem-solving activities presented in each lesson.

Throughout the unit, students' attitudes toward mathematics and the concept of area improved. On the pre-test, only 40 percent of students indicated a preference for solving area problems. Results on the post-test indicated that 60 percent of students preferred to solve area problems. In addition, students wrote comments in their journals, describing their positive attitudes toward the teaching method, hands-on learning, and partner work.

I found generally that there was appreciation of the reform method of teaching, that this teaching method led to the students' adopting a wider range of strategies to solve problems, that students were able to communicate and justify their ideas to others, and that improvement with respect to the understanding of area concepts was observed. Improvement when comparing pre-test and post-test results was experienced by all students. In some cases, students improved as much as 65 percent from the first test to the next.

One of the specific expectations listed in the Grade 7 Mathematics Curriculum is that: "students will determine… the relationship for calculating the area of a trapezoid and generalize to develop the formula" (MOE, 2005, p. 101). On the pre-test, no students were able to correctly calculate the area of a trapezoid, which is not unexpected. Post-test results showed that 63 percent of the students in the class could correctly calculate the area of a trapezoid, so the teaching was both successful for the majority of students and enjoyed. The following is an example of student work demonstrating how this expectation was achieved.



Conclusion

The reform method of instruction is a wonderful way to inspire students to study geometry. Learning through reform teaching allows students to develop a conceptual understanding of the concept. In addition, they enjoy developing formulas, justifying their conclusions, presenting their results, and using manipulatives to solve problems.

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Appendix 1 – Problems Posed to Begin Lessons

Lesson 1

1. Without calculating the area of the three given rectangles, determine which has the biggest area and which has the smallest area. *(Provide students with one of the following sets of rectangles: 4 x 4, 3 x 5, 2 x 9 OR 3 x 6, 2 x 10, 5 x 5 OR 8 x 3, 5 x 5, 6 x 4. Rectangles should be drawn to scale, but no side measurements should be provided.)*
2. These two rectangles have the same area. How could you prove this without calculating the area? *(Provide students with a pair of rectangles in the following dimensions: 2 x 6, 3 x 4 OR 4 x 6, 3 x 8 OR 4 x 4, 2 x 8.)*
3. Given these rectangles, determine their area in square centimetres. Refrain from using any known formulas. *(Provide each group with rectangles that are 8 cm x 4 cm and 3.5 cm x 6 cm.)*

Lesson 2

Using the geoboard, find as many polygons as possible that have an area of 12 square units. Record each new polygon on geoboard dot paper. Be ready to justify why each of your polygons has an area of 12 square units.

Lesson 3

Draw three different shapes on square centimetre paper, following these three rules:

a. Each shape must have a perimeter of 30 cm.

b. Stay on the lines when you draw (no diagonals).

c. You must be able to cut your shape out and have it all in one piece.

d. What do you notice about the area of the shapes?

Lesson 4

Provide students with a 10 cm x 10 cm square of paper.

1. Cut your paper in half. Determine the area of each rectangle, and answer the following questions on chart paper:
   a. What relationship, if any, do the rectangles have with each other?
   b. What relationship, if any, do the rectangles have with the original square?
   c. Do you notice any other relationships between the rectangles and the original square?
2. Cut one rectangle along the diagonal. Determine the area of each of the resulting triangles, and answer the following questions on chart paper:
   a. What relationship, if any, do the triangles have with each other?
   b. What type of triangles are they? How do you know?
   c. What relationship do the triangles have with the rectangle?
3. Take the two triangles and rearrange them into a parallelogram. Determine the area of the parallelogram, and answer the following question on chart paper:

   What do you notice about the area of the parallelogram and the area of the rectangle? The area of the triangles and the area of the parallelogram?

Lesson 5

1. You're making fried eggs for breakfast. Your rectangular frying pan has an area of 500 cm^2 and measures 20 cm wide by 25 cm long. Each egg, when cracked, forms a shape like the one below. Justify your estimate of the largest number of eggs that can be cooked in this frying pan without there being any overlap.

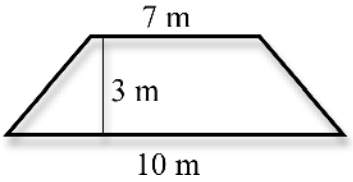


2. There is a circle with a diameter of 7 m painted on a gym floor. You have mats that are 2 m x 2 m.
   a. Estimate the number of mats that can be placed so that they fit entirely within the circle. M. Georges will not let you cut the mats!
   b. Estimate the number of mats that would be necessary to cover the circle completely. Again, you cannot cut the mats.
   c. What is the approximate area of the circle?

Lesson 6

Provide students with a cut-out of a trapezoid. Use a variety of trapezoids.

1. How would you describe your trapezoid? Think of size, shape, angles, as well as relationships to other figures we have looked at during this unit. Record the properties on your chart paper.
2. You are helping your dad paint a section of a wall in your attic. It has the following shape: base 10 m, top 7 m, height 3 m. If paint comes in cans that will cover 38 m^2, how many cans of paint will you need to paint the wall? What is the area of a wall you could cover with the leftover paint?

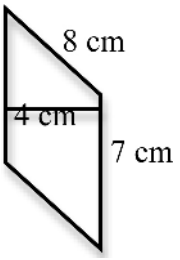


Lesson 7

You have a trapezoid with unknown dimensions. Develop a formula to calculate the area of this trapezoid.

Lesson 8

1. A square has an area of 36 cm^2. What is the area of a square whose sides are twice as long?
2. What is the area of the parallelogram pictured below? (Explain your thinking!)



3. A trapezoid ABCD, with AD parallel to BC, has a height of 4 m. The length of DC is equal to 5 m, the length of BC is equal to 6 m, and the area of the trapezoid is equal to 40 m^2. Calculate the length of AD. Show your work. ▲

▲ Creating and Maintaining a Differentiated Secondary Mathematics Classroom: Examining Educational Theory in Practice

KARRIE A. JONES, JENNIFER L. JONES, AND PAUL J. VERMETTE

Karrie teaches Grades 10 and 11 at Tapestry High School in Buffalo, NY; Jennifer L. teaches Grade 7 at Emmet Belknap Middle School in Lockport, NY; Paul is a professor of Education at Niagara University in Niagara University, NY.

Differentiated Instruction: A Working Definition

Like many terms accepted into everyday lexicon, the concept of differentiation has become increasingly complex in the realm of secondary education. In this article, the term "differentiation" involves the intentional planning and delivering of student-centred lessons which (by design) engage all students in developing their own, negotiated understanding of essential concepts (Benjamin, 2002; Heacox, 2002). Differentiation is not the "individualization instruction" of the 1970s, where each student gets his or her own unique lesson plan, but rather is a way of thinking about student learning as active, personalized, and dynamic.

At the core of the approach to successful differentiation discussed in this article are the theories of Howard Gardner, Grant Wiggins & Jay McTighe, Daniel Goleman, and William Glasser. Though different in their realms of expertise, each provides key insights into successful implementation of differentiated instruction. Understanding the need for student-centred instruction is one thing, but successful implementation is another. For many educators, the conundrum of application is the more pressing, yet largely unanswered, question. To combat this issue, the remainder of this piece seeks to provide practical and theoretically sound ideas for implementation in secondary mathematics.

Differentiation in Action: A Look into Mr. Ryan's High School Mathematics Classroom

At the start of Mr. Ryan's Grade 11 Functions and Applications class, he draws his students' attention to the learning goal for the lesson: "I can use the quadratic formula to get the roots of a quadratic equation." He begins their lesson with the presentation of a context. In supposing that they were given business plans for a concert, showing how operating cost and ticket sale income relate to ticket price x, they are given two functions:

Cost: $C(x) = 30\,000 - 74x$

Income: $I(x) = 2000x - 25x^2$

Using a think/pair/share model, students individually record their own thinking and then talk together about how $I(x) = C(x)$ means that the concert promoters "break even," $I(x) > C(x)$ means that the concert promoters make a profit, and $I(x) < C(x)$ means the concert promoters lose money. First they hypothesize in their small groups how to solve these equations and inequalities, using technology and reasoning about the function rules. Students work attentively on their task, forming their own opinions and questions.

Then students discuss two key questions:

"What ticket prices will lead to maximum profit?"

"What ticket prices will produce no profit at all?"

In their base groups, students find the maximum income by factoring the income function $2000x - 25x^2$. They then consider the "break even" function, a function created by setting the income function equal to the cost function. (Since $2000x - 25x^2 = 30\,000 - 74x$, the function is $f(x) = 25x^2 - 2074x + 30\,000$.) Students are instructed to set this function equal to zero and then factor, expecting to complete a procedure similar to ones used before. Soon, however, student teams realize that they cannot solve this equation by factoring, and begin to question each other and Mr. Ryan about alternatives to factoring.

During the debrief, Mr. Ryan encourages student teams to explain their concerns and observations, specifically asking them to describe how this function was different from the income function, and how this function is easier or harder to work with than others they have seen. Students are once again directed to the learning goal on the board that states: "I can use the quadratic formula to find the roots of a quadratic equation." Mr. Ryan explains that the purpose and function of the quadratic equation is to determine the roots of "unfactorable" equations. He asks students to think about what is implied by labelling an equation as "unfactorable" and why it matters in the context of the

problem. As he writes the quadratic formula on the board, he tells students that they will have been successful if, by the end of class, they can use this formula to determine the roots of a quadratic equation like the "break even" function.

After setting up the rationale and purpose for the quadratic formula, students are led through a brief whole-group discussion about how the quadratic formula can be used to determine the roots of this equation. Just before allowing student teams to consider applying their new understanding to a sample quadratic, Mr. Ryan sings them a song he created (to remember the quadratic formula), sung to the song of "Pop Goes the Weasel." This breaks the tension that is mounting over this very intimidating formula and reassures students that remembering this will not be nearly as difficult as they think. The class sings the verse together in an effort to help students remember the algorithm.

In returning to the original context, the teacher debriefs student calculations by calling on a student to give him values of *a*, *b*, and *c*, for the break-even function, and asking questions such as, "How would your equation be different if…" or "How would your thinking need to be modified if…." Students are then given 5 minutes to complete two practice problems with their base groups, and then a quick two-question individual reflection on their day's work. The reflection questions were: "What obstacles did you have to overcome in order to successfully complete this investigation?" and "What do you think the hardest part of using the quadratic equation will be for other students?" During this time, the teacher works the room to help students with their calculations.

Analyzing Mr. Ryan's Decision Making in Light of Cognitive Differentiation

This piece of high school mathematics teaching brings to light several foundational principles of learning. Discussion of these learning principles must precede any analysis of effective differentiation, as they could be viewed as forming the conditions in which differentiation can occur.

1) Learning in this classroom is active and authentic. Learning is based on transferable, real-life conceptual understanding—not the memorization of rote factors or algorithms. In Mr. Ryan's class, students are not just presented with the quadratic formula and lectured about how to use it; rather, they develop their own rationale for its usage before it is interpreted and applied. In the discussion of differentiation that follows, a classroom in which students are required to (1) think deeply about transferable concepts, to (2) produce products visually and aurally, and (3) provide meaningful evidence of their understanding is expected.
2) Student objectives and learning goals are the minimum competency that must be demonstrated by every student, and the goals should be focused on essential understandings. In Mr. Ryan's class, for example, every student needed to provide evidence that he or she could use the quadratic formula to find the roots of a quadratic equation by the end of the lesson. In an ideally differentiated classroom, all students should be held to high standards.

Differentiation to Maximize Student Academic Gain

Multiple Intelligences Theory: Howard Gardner

Though it has long been recognized that intelligence is not a fixed entity, it is only within the last 25 years that the definition of "school" intelligence has expanded from simply verbal/linguistic and logical/mathematical intelligence—those intelligences easily measured by classic IQ tests—to eight different intelligences. By taking a broader perspective of intelligence, the theory of Multiple Intelligences (Gardner, 1983, 2006) legitimizes non-traditional outlets as effective tools for promoting and recognizing intellectual ability.

From a differentiation standpoint, implementation of Multiple Intelligences theory allows for instruction that deliberately accommodates each of the eight intelligences. In the previous vignette, Mr. Ryan intentionally attends to five of the eight intelligences during the lesson. These intelligences include the use of language to express ideas (verbal/linguistic intelligence); using mathematical reasoning to arrive at solutions (logical/mathematical intelligence); using music and rhythm to learn a new equation (musical/rhythmic intelligence); students interacting and discussing their conclusions with others (interpersonal intelligence); and reflecting on their new understandings (intrapersonal intelligence). Though this range of instructional strategies may be unorthodox, differentiating classroom instruction through Gardner's Multiple Intelligences provides eight potential pathways for student success, therefore providing a litany of options to ensure a positive learning outcome.

It should be noted that Gardner did not intend to suggest that all eight intelligences should be used in every lesson. To clarify, intentional planning with Gardner's intelligence framework in mind means that teachers differentiate student instruction by teaching to the needs, strengths, and talents of their students on a regular basis.

Six Facets of Understanding: Grant Wiggins & Jay McTighe

Another theory utilized in Mr. Ryan's classroom that is fundamental to a discussion of differentiation is Wiggins & McTighe's model of the Six Facets of Understanding. Wiggins & McTighe (2005) state that there are six, equally legitimate ways to demonstrate conceptual knowledge: explanation, interpretation, application, perspective taking, empathy, and self-knowledge. Wiggins & McTighe define each of their six facets as "a common and distinct meaning or connotation of the word 'understand,' with direct implications for how such understanding is revealed" (Wiggins & McTighe, 2005). Incorporation of this theory into practice can be accomplished in numerous ways, including designing assessment options based on the Six Facets, or asking students questions which force them to demonstrate a flexible understanding of their knowledge.

In Mr. Ryan's classroom, differentiation via the Six Facets of Understanding is explicitly used during his small-group and whole-group debriefing. Though this vignette only describes one lesson, the following (Six Facets-based) questions could easily be incorporated into this lesson to ensure maximum student success:

Explain	- Why is that so? - How can we prove it? - How does this work? - What is implied?
Interpret	- What does it mean? - Why does it matter? - What makes sense?
Apply	- How does this make sense in this context? - How should you modify your thinking if…?
Perspective	- Is there adequate evidence? - Is it reasonable? - Is it plausible? - How is this different from…?
Empathy	- What do you think will be the hardest part of this lesson for other students?
Self-Knowledge	- What obstacles did you have to overcome in order to successfully complete this problem/investigation?

Finally, it should be noted that, unlike Bloom's Taxonomy and other hierarchical thinking models, both Gardner's and Wiggins & McTighe's models of differentiation provide multiple equally legitimate and valuable ways of demonstrating understanding. As with Gardner's framework, Wiggins & McTighe state that all Six Facets of Understanding are different ways of demonstrating conceptual knowledge and should be used to provide choice within assignments and assessments (Wiggins & McTighe, 2005). In the classroom, these two models provide countless instructional options for educators. In doing so, they also open doors for increased student engagement and ownership in learning.

Differentiation to Maximize Student Affective Gain

Apart from academic gain, growing bodies of research suggest that Social and Emotional Learning (SEL) is a fundamental component of effective schooling and students' overall well- being. As with the educational theories explored earlier, keys for making differentiation work in the inclusive secondary classroom lay in the implementation of several theories of social and emotional learning that will be considered below.

Emotional Intelligence: Daniel Goleman

Daniel Goleman's widely popularized *Emotional Intelligence: Why It Can Matter More Than IQ* (1998) provides a starting place for consideration of how non-academic differentiation can be used to increase student gains. This text asserts that the competencies of self-awareness, self-regulation, motivation, empathy, and social skills are of greater importance and influence than IQ, as they are fundamental for success in business and beyond. Goleman states that education should provide students with opportunities to practise "the skills we all need to handle ourselves, our relationships, and our work, effectively and ethically" (Collaborative for Academic, Social and Emotional Learning, 2002). They state that Social and Emotional Learning (SEL) does not mean teaching more content, but teaching the same content with a focus on developing the whole child.

In Mr. Ryan's class, concern with affective gains leads to implicit discussion and reflection of group dynamics and personal growth. In his classroom, students are given regular, predictable opportunities to collaborate with others, reflect on team processes, and self-assess. This student team approach allows students to experience working with a variety of individuals with varying levels of expertise.

In addition to working on their social competencies, students in Mr. Ryan's class are afforded the opportunity

to develop their intrapersonal skills though their completion of the seven-question exit pass at the end of the lesson. These exit passes are extremely useful from a differentiation perspective, as they provide students with a flexible opportunity to demonstrate their mathematical skills, while simultaneously providing Mr. Ryan with cognitive and affective student feedback. When used regularly, these passes serve as outlets for student reflection, as well as formative assessment to guide future instruction.

Choice Theory: William Glasser

Unlike previous understanding of human behaviour, Choice Theory posits that humans play an active role in their behaviours and always act in a way that best satisfies their need for fun, freedom, sense of belonging, and/or power. In the classroom, Choice Theory implies that students learn best when their need for fun, freedom, sense of belonging, and/or power are fulfilled and when students are empowered with the freedom of choice.

In the previous vignette, deliberate attention is provided to each of Glasser's four motivators. Students experience fun by working in teams and singing the quadratic equation song (a novel way to learn mathematics); sense of belonging through Mr. Ryan's intentional focus on building and sustaining working relationships; and both power and freedom through the numerous opportunities for choice and competence.

One specific example of this type of differentiation that can be easily implemented in the inclusive secondary classroom is choice within assignment and assessment. As demonstrated in the previous vignette, when student teams choose "any five practice problems with their base groups," giving students options from among tasks which accomplish the same learning goal, is easy to do and very motivating. Not only does regular choice within assignments empower students by fulfilling their need for power and freedom, but it also ensures students are able to select tasks they can complete successfully.

Conclusion

Cited by Gould (2010) as "one of the most important skills for teachers to develop," the ability to differentiate instruction requires thoughtful, intentional lesson planning and delivery. By examining the works of Gardner, Wiggins & McTighe, Goleman, and Glasser through Mr. Ryan's vignette, insight into how teachers can better meet the needs of diverse student populations has been highlighted.

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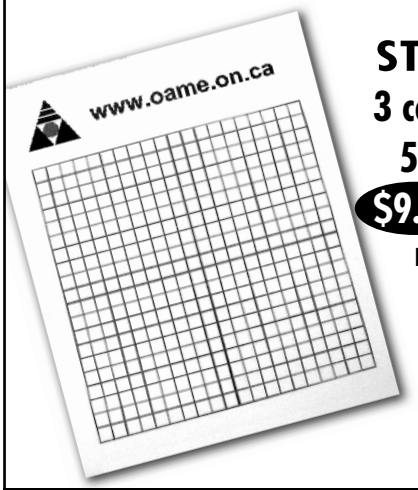
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▲ Weird Maths

Tom Griffiths



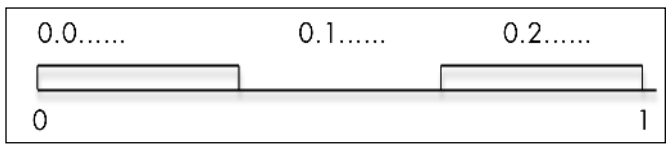
Tom is a retired Ontario teacher. He taught for 49 years at the elementary, secondary, and university levels. He served on the executive and as president of the OAME and is a life member. He has been highly involved in the Math Challenge program at the University of Western Ontario for 15 years since his retirement and was specially recognized for that work at UWO.

A strange result I discovered recently is attributable to Georg Cantor, a Russian-born German mathematician of the nineteenth century, whose ideas were rejected as totally counter-intuitive, and even heretical, well into the twentieth century. He came up with the argument that an infinite set of numbers can take up no space on a number line.

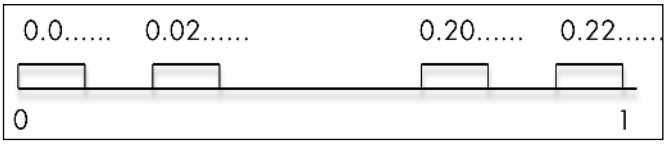
Consider the numbers from 0 to 1, expressed in base 3 decimal form. In base 3, the digits used are 0, 1, and 2. In base 10, we wait until 10 to regroup and so use 10 digits; in base 3, we wait until 3 to regroup. Therefore, our standard numbers 1, 2, 3, 4, 5, 6, 9, 8, 9, 10 are written 1, 2, 10_3, 11_3, 12_3, 20_3, 21_3, 22_3, 100_3, 101_3. In the same way that fractional numbers from 0 to 1 can be written in decimal form in base 10 (e.g., 0.3259), so fractional numbers can be produced in base 3. For example, a base 3 number would be 0.1020012_3.

Imagine that, for some reason, the digit 1 is excluded from numbers between 0 and 1. Numbers such as 0.02202022002_3... are allowed, but numbers like 0.022121_3... are not.

Consider only the first decimal place. Decimals of the form 0.0... would describe the first third of the number line from 0 to 1, and the decimal of the form 0.2... would describe the last third. Hence, no number would be in the middle third, and all numbers would be in the first or last third of this part of the number line.



Now consider the second decimal place. Omitting the possibility of a 1 would exclude the middle thirds of the two sections remaining, based on the first decimal place.



Continue excluding all the middle sections. The Cantor set is the set of points not excluded, but there is nothing left, so the Cantor set is 0% of the line.

To confirm that there is nothing left from the number line from 0 to 1, remove the thirds from 1, using the sum of an infinite geometric series $\frac{1}{(1-r)}$, if the first term is 1 and $r=\frac{1}{3}$.

From 1 remove the middle third, then remove the two middle thirds of the first and last third, etc.

$$1-\frac{1}{3}-\frac{2}{9}-\frac{4}{27}-\frac{8}{81}-.....$$
$$=1-\frac{1}{3}\left(1+\frac{2}{3}+\frac{4}{9}+\frac{8}{27}+.....\right)$$
$$=1-\left(\frac{\frac{1}{3}}{1-\frac{2}{3}}\right)$$
$$=1-1$$
$$=0 \text{ as required.}$$

Therefore the infinite number of numbers described takes up no space on the number line.

Cantor's brilliant step now is to change all the 2's in the base 3 case to 1's, and consider these numbers in base 2 (in binary form), where the only digits are 0 and 1. This gives every possible number, base 2, in the region from 0 to 1, since it produces every possible combination of 0's and 1's in base 2 decimal form between 0 to 1.

Isn't it weird that we showed that an infinite set can take up no space? The exclusion process is very much like a one-dimensional form of the Sierpinski Triangle (see **math.rice.edu/~lanius/fractals/**); note that the Sierpinski Triangle is the two-dimensional equivalent, where one-quarter, rather than one-third, of the figure is omitted at each step. What starts full and dark ends up mostly white and empty.



I think that students should be exposed to such strange notions, which are within their understanding. Their minds will be opened to unusual ideas. Even if we cannot explain the whys of some mathematics, it is still valid and fascinating to explore them. I have shown this to my students in the Math Challenge program from Grades 7 to 12, and they have understood the arguments. They are aware of numbers in different bases, although the idea of fractions in different bases was new to them. The concept of the sum of an infinite series is not difficult to introduce to Grades 7 to 10 students, especially if the lead term is 1, and it certainly will make sense to students in some Grade 11 courses.

There are many more fascinating and reasonably understandable ideas from mathematics of the past 100 or so years, some of them even from this century. I think we owe it to our students to show them that mathematics is a living discipline, and that new ideas within their ability to understand are being produced now. We hope that some of them will go on to produce new mathematics themselves.

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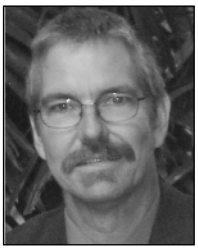
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▲ Math in the Media

TIM SIBBALD



Tim is a secondary math teacher in the Thames Valley District School Board. He also teaches pre-service senior math candidates at the University of Western Ontario.

While reading the October 23, 2010 edition of the *Globe and Mail*, an article about the Afghanistan mission caught my eye. Specifically, it was a picture of a helicopter landing and a graph of defence spending that I realized contained several opportunities for secondary math. The intention of this article is to illustrate some opportunities that emerged, rather than to focus on any single math topic. While the contents of this article may be viewed as what happens when a mathematician reads the newspaper, it could also be considered a call to find and use more images in our math classes.

A photograph of a helicopter (shown on the next page) shows a significant dust cloud, with troops and a vehicle with a trailer awaiting the helicopter's arrival. The picture highlights several mathematical shapes, but requires a grid for students to determine graphical features. The creation of a grid and reading points off the grid could be used for review of graphing in Grade 9. The choice of axes can be a discussion point. For this article, a grid has been added to the photo, using the front rotor of the helicopter as the origin and a *y*-axis passing through both rotors. The *x*-axis was made perpendicular to this, and a square grid size was chosen with half the distance between the rotors as a unit of measure. Positive *y* is interpreted as being behind the helicopter and positive *x* toward the bottom of the photo. This set-up allowed an analysis of the edge of the dust cloud as a quadratic that is opening up. Note that students can infer that the helicopter is moving because the dust cloud is not circular (a circular pattern example, suitable for modelling in MPM2D, can be seen at **www.flickr.com/photos/defenceimages/5036597936/**).

The quadratic shown at the front of the dust cloud was generated using quadratic regression based on the points marked with a small "x." The equation that resulted was:

$y = .14x^2 + .09x - 1.7 \quad r = .99$

This could be developed without using regression by examining the *y*-intercept and roots. In addition to the quadratic, a series of almost parallel equally-spaced "lines" are observable in the dust and so parallel, equally-spaced line segments have been drawn to model the structure. The *y*-intercepts form an arithmetic progression. Students in Grade 9 or 10 could derive the equations for the lines on the graph; they are:

$$y = \frac{2x}{3} + 1.7n \qquad n = 1,2,3,4,5$$

A variety of questions facilitate a deeper conversation about the interpretation:

- What is taking place in the photo? (e.g., The helicopter is coming in to pick up troops.)
- Will the vehicle with the trailer fit inside the helicopter? [Note that the helicopter has a hydraulic ramp to facilitate loading vehicles (see **en.wikipedia.org/wiki/Boeing_CH-47_Chinook for a picture**).]
- Was it reasonable to decide that the lines were parallel? Suggest a reason why parallel lines might be observed. [A teacher can use this for developing the idea of rotation of the blades causing a repeated pattern. Taking a slice, such as the *y*-axis, allows one to look at the pattern as a periodic function, a topic for some Grade 11 courses.]
- The arithmetic progression has been stopped after the fifth line. Is that reasonable and should greater values have been included?
- Would, or how would, the mathematical results be different if positive *y* were taken to be in front of the helicopter and positive *x* were taken to be toward the top of the photo?

Some concepts can be developed that introduce the third dimension. Perhaps a good starting point is to ask students to roughly estimate the direction of the top of the photo. This can be ascertained as roughly south because of the orientation of the shadows and knowing that Afghanistan is in the northern hemisphere. An estimate of time of day can be achieved, but requires skills beyond the secondary curriculum.

Shadow measurement, with accuracy, requires that one is specific about the direction of the sun. To determine this direction, draw a line from the hub of the front rotor of the helicopter to the location of the hub in



Photo by MCPL Craig Wiggins

the shadow on the ground. All measurements of shadows should be parallel to this on the photo. The vehicle can be used to measure the elevation of the sun; surprisingly, the shadow and height of the vehicle appear to be equal, suggesting that the sun has an elevation of 45°.

An elevation of 45° means the height of the helicopter off the ground is the same as the horizontal length of the shadow on the ground and, therefore, can be estimated. Firstly, measure the distance from the centre of the rotor hub to the shadow of the rotor hub in millimetres, and note that this is geometrically the same as the height of the helicopter because a 45° elevation gives a right isosceles triangle. However, a scale factor is needed to convert the measurement into real-world units, topics covered in MFM2P and MAP4C. This requires recognizing that this is a Chinook helicopter, where the rotor hubs are separated by 11.9 m (according to www.airforce-technology.com). So a measurement of the distance between the rotors on the photograph provides the conversion factor to metres. Using this method, an estimated height of the helicopter of approximately 28 m can be found. However, this is the height of the rotor hub above the ground. Since the helicopter has a height of 5.8 m, the wheels are approximately 22 m above the ground. The same calculation can be used to estimate that some of the soldiers are approximately 1.7 m tall ("Is that reasonable?").

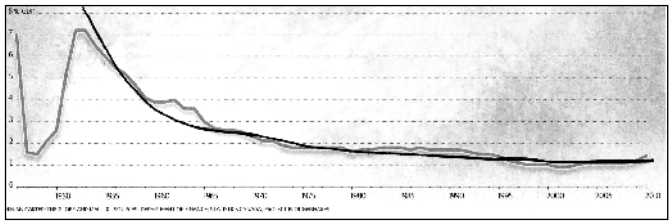
A second aspect of the *Globe and Mail* article was a graph of defence spending since the Second World War as a percentage of GDP. A selection of points spread from 1953 to 2000 were used to determine a curve of best fit. The curve that is illustrated is a reciprocal power law:

$$y = 17.07(\text{year}-1950)^{-.684} \quad r = .99$$

This could be determined using a graphing calculator or, without regression, by taking a pair of points and solving for the coefficient and exponent. It may be preferable to do this as a practical example of solving for the value of an exponent.

Some questions for students:

- How do you choose an appropriate model for the graph? (Note that a quadratic regression implies that spending will increase significantly in the future. Exponential decay may be a valid alternative to the model used here, but requires more thought about the value of y in the horizontal asymptote—as is done in MHF4U.) Students may need help to understand that a fixed percent of GDP does not mean that defence spending stays the same.
- Having chosen a model, which regression model should be used on the graphing calculator?
- The graph has periods of time where the spending exceeds the modelled value. Do these correspond to events that involved the Canadian military? Do they correspond to declines in GDP, such as recessions?
- The newspaper title for the graph says spending has "turned higher in recent years." Is this evident from the graph? (Note that GDP has been increasing.)
- Social-justice questions can be addressed by raising questions about the allocation of monies in relation to GDP and, perhaps, investigating standard of living since 1950. These discussions might be appropriate in MDM4U and MBF3C courses.



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Both the photograph and the spending graph have the potential to evoke a mathematical response, with attention to different details appropriate in different courses. The approach builds on the idea of photo math, with which *Gazette* readers are familiar, and relies on the analysis skills that we develop in the secondary math curriculum. With the increasing availability of projectors and SMART Boards™, teachers have an opportunity to utilize images and can embed higher-level mathematical concepts within the analysis. ▲

▲ Call for Manuscripts

The *Ontario Mathematics Gazette* is inviting manuscripts for all grade levels.

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Contact the Editor for further details.

▲ Mathematics Improvement Process: A Principal's Perspective

DOUG DUFF



Doug is a principal with the Thames Valley District School Board in London, Ontario. He is currently the principal of Parkview School. Formerly, he was a consultant and learning supervisor. He has taught mathematics education to pre-service students at the University of Western Ontario. Most recently, he worked with Nelson Education as a Senior Consultant, author, and professional learning presenter.

> *"A broad consensus now exists among researchers and educators on the knowledge and skills children need in mathematics, the experiences that advance the development of mathematical skills and understanding, and the basic components of an effective mathematics program. But for many teachers, both new and experienced, there continues to be a gap between theory and practice. How can current research about teaching and learning mathematics be brought to life in the classroom? What skills and knowledge can best help teachers meet their commitment to help every child become mathematically proficient?"*
>
> (Ontario Ministry of Education, 2006, p. v)

Despite the best intentions and hard work by mathematics associations and consultants, wide-spread attention to what we know constitutes good mathematics instruction is not as evident as one would hope. As an elementary school principal, several key moments highlighted this "gap" in practice. The first was two years ago as I started the process of school-wide math improvement in my new assignment. In one of my first meetings with the staff, we undertook a data-analysis activity to determine the direction for our school-improvement plans. It was felt by the staff that we should continue our literacy goal, as we had not yet achieved our targets. We had many literacy data markers to analyze and there was certainly some merit in continuing the goal. What stood out to me, however, was the need to improve mathematics scores; yet the only data source I could find was the current EQAO data.

The staff was by no means resistant, but they had no experience dealing with a math goal. They had always worked on literacy; the Board and Provincial mandate was focused solely on literacy, and all of the previous in-service had been on literacy. At the same time, I had spent a lot of time in classrooms observing excellent teachers conducting math lessons that were not reflective of current practice. I had also consulted my colleagues and asked them about their school goals and how they relate to school improvement. Each one I consulted had a progressive plan for their ongoing improvement and initiatives to improve literacy. When asked about mathematics, most did not have math goals; therefore, the discussion ended abruptly, and others, when I persisted, would list the top three answers such as more "hands-on," problem solving, or manipulatives. The discussion was completely opposite to the well-developed scope of their literacy-improvement efforts.

The same scenario occurred when I was hiring for an open teaching position. When I asked the candidates to describe their literacy program, to a person, the answers were involved, thorough, and included all the parts of a good balanced literacy program—including what an exemplary lesson would include. Similarly, when I asked the candidates to describe their math program, the responses were often tangential and most had no idea where to begin. Frequently the same phrases—"hands-on," problem solving, or manipulatives—were offered as words that needed to be mentioned, but with no specific connection to student achievement. "What is your path through your program?"

This basic depth, breadth, and scope, planning question had no prepared approach. "Describe your mathematics program" remains a difficult question for teachers to articulate because the work on numeracy has been so compartmentalized.

Finding a Place to Start

In order to make change pervasive in a school, it seems that changing the culture is a critical factor. Staff and students have to realize that mathematics instruction is important and has to be a part of what makes the school successful. All schools have some area that they are known for, whether it is literacy, sports, music, or safe schools. My job was to connect mathematics to this existing culture and market the new focus of Literacy, Numeracy, and Technology. This triumvirate became the

driving force at staff meetings, parent nights, newsletters, and school council meetings. Because the Board and Ministry were continuing to move forward the literacy agenda, mathematics became the school's domain, and technology would be a means to support teachers and engage students in math curriculum instruction. In short, I needed some math champions.

Creating the Role of the Math Coach/Champion – Numeracy Prep Time

In schools in Ontario, there are numerous strategies to cover preparation time. In most contracts, preparation time for staff has to be covered by another teacher in the school or a teacher who is designated as a "prep teacher." Often these teachers delivering preparation time would provide "coverage" in areas of the curriculum that can be easily separated and reported on, such as drama, media, or technology. Instead of this compartmentalization, in our timetable, I created a "Numeracy Prep" teacher. These teachers were instrumental in building the new culture and they would become the champions of math curriculum and instruction. The vision for these teachers was for each to be a math coach for students in numeration. Their mandate was to build primary students' number sense through a focus on the mathematical processes. There was no fragmentation of the math curriculum because the numeracy teachers would reinforce the number concepts connected to whatever strand the classroom teacher was instructing—for example, connecting place value to linear measurement.

The first step was to support these teachers to be able to provide high-quality support for students. There was willingness by these teachers to extend their knowledge in mathematics education. They attended summer institutes, Board/Ministry sessions on effectiveness guides, and PRIME (Small, 2005) training. As well, they became involved in Board writing and coaching teams wherever possible. I was able to gain release time for these teachers to spend time early in the school year with our Board coaches and consultants in order to outline their plans for the year, to review resources, and analyze their early data collection. With this training, the numeracy prep teachers were able to plan a course of study that supported students in strengthening primary number concepts as they were applied to what was happening in the regular classroom, for example, skip counting by five and telling time, linear patterning and early proportional reasoning, money, and perimeter and operation meanings.

The regular classroom teachers became completely supportive of the initiative when they saw the benefits of the programming. Many primary math teachers believe that there is too much to cover in the number strand and little time to provide students with the constant reinforcement with new number concepts they need. As well, they have little time to provide the differentiation that would support the needs of all of their students. All of these challenges became part of the numeracy prep teachers' instruction. With this ongoing support, the classroom teacher could continue to focus on the big ideas in the mathematics curriculum without spending all their time on review and practice of number and operational skills. Regular class teachers could concentrate on providing three-part problem-solving-based lessons that exposed the big ideas in the curriculum.

Mathematics Professional Learning Communities – PLCs

The classroom teachers were becoming interested in the mathematics movement in the school and decided to meet as a primary team to discuss math growth in the school, review professional articles, and learn about new approaches. As the team and expertise evolved, we began to delve deeper. Concepts such as regrouping, skip counting, composing/decomposing of numbers, or the difference between the quotative and partitive interpretations of division are both sophisticated and challenging mathematically (Ma, 1999), and teachers benefited from collegial discussions about them.

Being careful not to overlook the importance of all the strands, we started our primary team PLC sessions with patterning and, in particular, the connection between assessment and instruction. Often in mathematics, we just teach a concept or topic without comprehending what constitutes understanding. In mathematics, we have them "do" or "finish" the question. If it is correct, the student understands; if it is not, we reteach. When planning for a math lesson, the team was challenged to focus on the intention of the lesson(s) and the evidence instead of merely "covering" a topic.

When we are concerned with what constitutes evidence of understanding, instruction and assessment are connected (Figure 1). As we built our background knowledge through reading, in-service, talking, and experience, we were able to identify and categorize the indicators of understanding and use them to better teach, assess, and differentiate instruction.

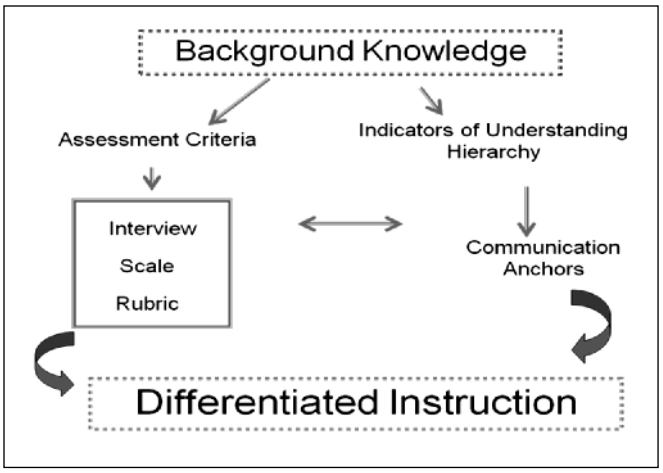


Figure 1: Indicators of Understanding/Assessment Criteria

We became better able to recognize understanding when we saw it or realized what questions still needed to be asked. We focused on building communication anchors and exemplars and through examining learning trajectories, building open and parallel questions to differentiate instruction. For example, one way to describe development in primary students' development of sorting concept is as follows:

- Sorting using a single attribute
- Inferring a sorting rule based on a single attribute
- Sorting and inferring a sorting rule, using more than one attribute when the attributes are related
- Sorting and inferring a sorting rule, using independent attributes (Small, PRIME 2005)

Working with this continuum, we provided learning situations that moved students along the sorting continuum, constantly moving toward a more sophisticated indicator.

Instrumental in these sessions were the use of resource materials such as the *Guides to Effective Instruction in Mathematics* (Ontario Ministry of Education, 2006), *PRIME Phases and Indicator Maps* (Small, 2005), and *Making Math Meaningful* (Small, 2009).

The capacity building resulting from these PLCs was instrumental in broadening the teachers' and students' comprehension of big ideas in elementary mathematics. The original math improvement team has now multiplied from two prep teachers and the principal to the whole primary team. As Ministry release time was introduced for various improvement cycles, we were ready to formalize the learning process with our primary team and prepare for phase two with the junior and intermediate teachers. Teacher collaboration was becoming more evident as a result of the growing commitment toward a goal and the connection with the Ministry's improvement models.

Role of Technology

Another important factor in improving math education and in enhancing collaboration was the introduction of technology to all classes in the school, beginning with the Primary division. Paired with the acquisition of more manipulatives, interactive whiteboards were purchased to give teachers access to instant manipulation of data, questions, graphics and models for students to consider. The greatest immediate effect was the enthusiasm to learn and discover new concepts that the technology delivered. With the help of the math champions and our technology leaders, interactive whiteboard sessions were being offered at lunch hour and after school on a drop-in and invitational basis.

It is crucial that the technology is introduced strategically in coordination with a mathematics goal. Instead of the orientation workshops being "how to use" or "which button to press," the technology was introduced as part of the three-part lesson structure, complete with virtual manipulatives, lesson construction, and problem modelling. The in-service allowed us to build upon the PLCs to promote conceptual reasoning and translate theory into practice. As well, we could model digital data walls and post them on our school conference so that teachers could participate immediately. Lessons, activities, models, and applets were posted to our intranet. The enthusiasm was infectious and it created a positive momentum for informal sharing among the staff.

As a second phase and to sustain the momentum, overhead document cameras, affectionately known as ELMOs (Electricity Light Machine Organization), were purchased to help with immediate sharing of solutions and models, both from students and teachers. Teachers and students could display printed material or handwritten items in colour for the whole class to see on the screen. As well, a textbook, graphing calculator, or resource guide could be placed under the camera. Students shared their mathematical ideas on the document camera to justify and defend their thinking. Some of the standard lines in the classroom that have come out of this implementation are: "ELMO will like that one," "ELMO needs to see that one," "Can you justify that with ELMO?" Students are constantly asking if their model or solution can be shared on ELMO. Teacher questioning shifted to focus on mathematical processes,

including connecting, reflecting, reasoning, and communicating, when explaining strategies or answers. The student models and solutions could be saved and downloaded to create an instant "bansho" (a method of organizing and displaying students' work that is connected to each other conceptually and highlighting the common ideas). Student engagement was also fostered when using technology. The document camera often eased the reluctance of students to share responses.

Lastly, to reinforce the importance of doing mathematics at home, largely the domain of literacy, school-wide licences were purchased for each student for an online math program. This online system enables students to reinforce hundreds of Ontario curriculum math topics. There is also a real-time competitive section where students can compete against students from all over the world. Classes and students are ranked in the world and Canada. Most importantly for us, mathematics became a focus for the whole school beyond the classroom. The energy and competition are engaging and add to the positive culture of mathematics, but also connects parents to the school's initiative.

Conclusion, Reflection, and Next Steps

What is reassuring and motivating is the notion that when you focus and set targets toward a goal, change will occur. Measurable increases in all data sets were realized through the PLC process. Students' flexibility with numbers has improved dramatically on all markers. After the first year of implementation, teachers were quick to credit the numeracy prep teachers for the massive gains in EQAO scores in Primary mathematics. The school went from 57% Levels 3 and 4 to a staggering 91% over the course of the year. Further analysis did not suggest that the cohort significantly affected the high score; however, the pressure was firmly in place for the following year. Again in the second year of implementation, all data markers validated that growth had continued across the division, as progressive teaching methods and use of technology were refined. In the second year, EQAO percentage was sustained at 87%, and improvements were also noted in literacy.

In summary, we were attempting to mimic the literacy movement and create common messaging and pedagogical strategies that became part of the regular routine at the school, instead of an event that ended following the PLC. The new curricular culture had its roots and needed to be constantly nurtured. Hiring practices also had to be considered as a key factor in continuing the improvement. Any time teaching positions became open, numeracy background became part of the advertised skill set and played an important role in the interview.

Change, no matter how positive, doesn't come without its problems. The corollary for us was also true; what you don't focus on suffers. In the second year of implementation, our Grade 6 mathematics scores dropped significantly. Instead of making excuses or questioning our practice, we knew it was time to begin the next phase of the mathematics plan formally with our Junior division. This decision came directly from the Junior teachers, as they had seen the work by the Primary team and understood that they were receiving higher-achieving students. Both the pressure and support are in place, and the Junior phase will pick up from the Primary teachers, without the growing pains and stumbling blocks. The resources, technology, and systems are in place, and we are cautiously optimistic that the gains will be translated across the school.

We are now in the third year of implementation and all of this work does not happen at once. There were no course outlines to follow and our team had to develop new approaches to ensure conceptual understanding. The process continues to evolve with more grades and more training. Instilling a culture of mathematics in a school cannot be willed into existence or demanded by the leader. Instead, it is a strategic mission that builds upon small successes and a few champions.

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▲ UWO Math Challenge Program

TOM GRIFFITHS



Tom is a retired Ontario teacher. He taught for 49 years at the elementary, secondary, and university levels. He served on the executive and as president of the OAME and is a life member. He has been highly involved in the Math Challenge program at the University of Western Ontario for 15 years since his retirement and was specially recognized for that work at UWO.

In the June 2002 *Gazette*, a report on the Math Challenge@Western program was published. This piece is an update on the program, which had 414 registered students as of May 2010. There are 12 staff members, 10 of whom are teachers, who hold monthly two-hour classes on Saturdays for Grades 7 and 8 in the morning and Grades 5 and 6 in the afternoon. Every Monday evening, there are classes for Grades 9 and 10 for two hours, and Tuesday evening classes are held for Grades 11 and 12 for three hours. In addition, there are 1½-hour classes for those students preparing for Olympiads on Monday and Tuesday afternoons; there are usually six students in each of these groups.

All the classes are held at the University of Western Ontario Mathematics Department, which provides a lecture theatre and photocopying facilities. Many of the students come to more than one of the class levels, some up to four. There are even some Grades 2 to 4 students attending. Students are free to choose which class levels to attend. The full schedule can be found at **www.math.uwo.ca/math-challenge** together with a registration form and a description of the program.

Students pay $30 a year for registration, allowing access to as many of the class levels as they please. This pays for equipment, snacks for the elementary classes, books for the library, and conference expenses for the staff, as well as an appreciation dinner for the staff and spouses. Books on problem solving and T-shirts with the program's logo are also available for purchase.

The teachers are all retirees, with one exception. There are three teachers from the elementary level: David McMillan, Margaret Kemp, and Erica Morrill. The secondary teachers are: Peter O'Hara, Paul Fennell, Dennis Kemp , Charles Scollard, and Tom Griffiths. The university-level staff members are Irvine Robinson and Eddy Smet. Marlene Griffiths looks after the registration, attendance, statistics, and the library. Susan O'Hara assists with registration, snacks, and other duties on Saturdays. There are also four or five high school students assisting on Saturday with attendance, snacks, etc., for which they earn community service hours. Every class has at least two teachers involved so that if one is unavailable, the other one is there.

The percentage of female students registered last year is as indicated in the chart below.

Grades	5/6	7/8	9/10	11/12
Female Percent	48%	43%	38%	31%

Average attendance at classes was 67 for the Grades 5/6 class, 66 for the Grades 7/8 class, 65 for the Grades 9/10 class, and 53 for the Grades 11/12 class.

Some students have participated on the Canadian International Olympiad team—Chen Sun three times and Ralph Furmaniak once. They have won an honourable mention, a bronze medal and two silver medals collectively. Many of the students continue to study mathematics or science at university. One comment received from a past student who was studying for his PhD was that until he came to the program, he did not realize that there were math problems that could take more than two minutes to solve.

Much of the teaching is done with the assistance of old OPEN (Sun Life Financial Canadian Open Mathematics Challenge) and CEMC (Centre for Education in Mathematics and Computing at the University of Waterloo) contests. Usually one contest is distributed in a class and taken up the next class. Other teaching, including some of the history associated with the mathematics, is also provided. There is always a storytime about mathematicians and mathematics. In the elementary-level classes, the second hour includes much more hands-on work.

One other activity added since the last report is the McTic Contest. This is the Math Challenge Team Invitational Contest. The contest started five years ago and was held at Huron College in London. Two years ago, the contest was moved to Waterloo, a more central location. The team consists of nine students, and the

contest is modelled after the American Regions Mathematics League (ARML). Peter O'Hara coaches the teams during the third hour of the Tuesday session, and the coaching includes mental arithmetic, which is considered most important.

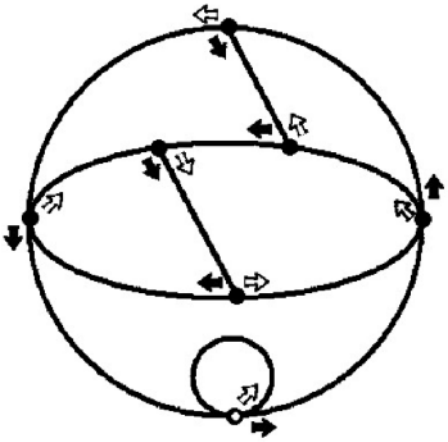
The pattern that has emerged seems to be serving the program extremely well, but it is likely to continue to evolve. There has been some discussion about including classes for parents, teachers, and other interested adults, and classes specifically for the teaching of calculus, but including a broader curriculum than in high school. The goal is still to offer the students classes which stimulate their interest in all kinds of mathematics, including the history of concepts and the story of mathematicians. The students seem to enjoy the experience, and certainly succeed in contests. ▲

▲ Divisibility by 7 Is a Walk on a Graph

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This article is reprinted, with permission from the author, from Tanya Khovanova's Math Blog, where it originally appeared in August 2009.

I have attached a picture of a graph.



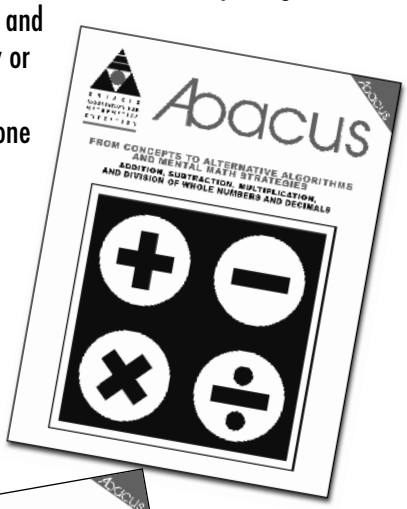
Write down a number *n*. Start at the small white node at the bottom of the graph. For each digit *d* in *n*, follow *d* black arrows in succession, and as you move from one digit to the next, follow one white arrow.

For example, if *n* = 325, follow **three** black arrows, then one white arrow, then **two** black arrows, then one white arrow, and finally **five** black arrows.

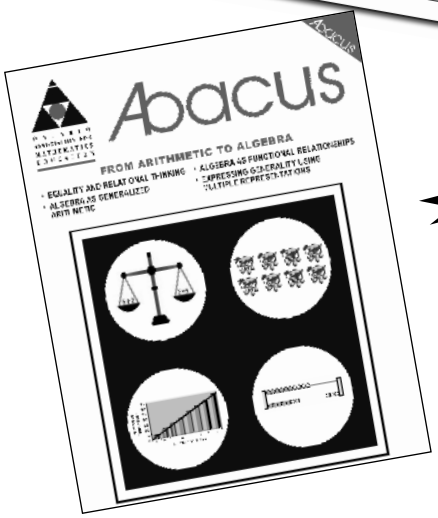
If you end up back at the white node, *n* is divisible by 7. ▲

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▲ Mathematical Images

FRANK MARINGOLA



Frank is an occasional teacher with the Simcoe Muskoka Catholic District School Board.

This feature shares some images around us that are connected to mathematical patterns and concepts. The focus in this issue is on buildings.



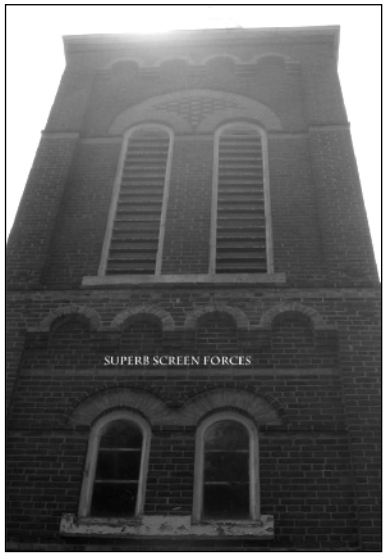
St. Paul's Anglican Church – Coulson's Hill, Bradford West Gwillimbury, Ontario

The symmetry of church architecture is appealing to the eye. It tends to draw one in and centre the mind. The purpose of church buildings is to facilitate reflection. I think the symmetry of geometric shapes is tantamount to balancing out imbalances in the person's life. Church and mass symbolize the need for spiritual health. Mathematics plays a part in this balance and well-being.



Doors of Church – Churchill, Ontario

These doors have symmetry based on the number eight, with eight planks on either side of the door. The planks are beautifully numbered in an arrangement to welcome worshippers into a sanctuary. The number eight is perfect as a reference point in carpentry. Even more perfect is the use of the number in church construction. Eight is a number aligned to the number of planets in the solar system. It is used by multiple religions as a luck symbol. Worshippers want to surround themselves with physical beauty, which can lead to emotional and mental beauty.



Superb Screen Forces – Church in Churchill, Ontario

Visual communication in film and video consists of screen forces. These forces direct the eyes in a direction. This particular photo displays the use of the numbers two (windows) and four (arches) in masonry to direct the worshippers' eyes upward.



Church in Churchill, Ontario

The church spire represents many mathematical concepts used in carpentry, particularly symmetry.

Besides being a testament to a skilled trade, this spire appeals to me through the way it visually focuses the lens of the eye. ▲

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