Submission of Articles

The *Ontario Mathematics Gazette* (*OMG*) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *OMG*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles must be word-processed in MS Word, double-spaced with wide margins, not exceeding 10 numbered pages of text, and prepared according to the *Publication Manual of the American Psychological Association, Sixth Edition*. Figures and diagrams should be drawn by computer, if possible, or drawn in black ink in camera-ready form. Embedded images must also be submitted separately in jpeg or tif format. Proof of the photographer's permission is required, and for **photos of students** under the age of 18, the written permission of a **parent or guardian is required**.

You must submit **one complete copy** of your article, embedded with any tables, figures, and captioned photographs or graphics, to the Editor, Marian Small, along with **separate files for each of the text, graphics, and/or photographs**. Please e-mail all files to Marian Small at **marian.small@gmail.com**.

Your name should not appear anywhere in your article, including websites, so that your article can be sent out for blind review. Your name, full mailing address, and e-mail address must be included on a separate sheet. Upon review, you will be notified as to whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected.

The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME.

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▲ Table of Contents

Up Front

▲ Editor's Message /Marian Small 2
▲ President's Message /Connie Quadrini 4

Regular Features

▲ The *Abacus* . . . . . .8-Page Insert
▲ Technology Corner – Have You Seen It? /Mary Bourassa 6
▲ OAME/NCTM Report June 2011 /Laurie Moher 8
▲ What's the Problem? Count 'em Up /Shawn Godin 9
▲ Data Rocks – A Social Justice Data Fair /Joel Yan/Hina Kausar-Ahmad 11
▲ In the Middle – Sparking Curiosity: Summer Fun Exploring Math /Caroline Rosenbloom 16
▲ What's New at EDU? /Jeff Irvine 19
▲ Hey It's Elementary – Inside, Out.... Unusual Mathematics Classrooms /Lynda Colgan 20
▲ Fields Institute – MathEd Forum Notes /Stewart Craven 24
▲ Directors' Dialogue /Ann Michele Stenning 25

Articles

▲ "Messy Time" Transition to Reform-Based Mathematics Teaching and Learning /Dan Jarvis/Doug Franks 28
▲ Teaching Through Problem Solving in Grade 9 /Robert Todd 34
▲ Historical Approaches to Linear Equations /Vera Sarina 37
▲ Assessment for Learning in a Problem-Solving Context /Elizabeth Hoerath 39

Notices

▲ OAME T-Shirts 5
▲ OAME Elementary Sticky Chart Pads 9
▲ OAME Sticky Graph Pads 25
▲ OAME Memo Pads 36
▲ *Abacus:* A Compilation Volume 46, Volume 47 40
▲ OAME Board Members . . . . . .Inside Back Cover

Advertisements

▲ Wintergreen 3
▲ Homework Help / ILC 7
▲ Nelson Eduction PRIME . . . . . Back Cover

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▲ About the *Ontario Mathematics Gazette*

The *Ontario Mathematics Gazette*—ISSN 0030–3211—is indexed in the Canadian Education Index and is published four times per year. Its Canadian Publication Mail Product Sales Agreement Number is 40051074.

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▲ Editor's Message

MARIAN SMALL



Consultants and administrators around the province are trying to support teachers in adopting the three-part lesson structure in their mathematics instruction. However, it is important for teachers to realize that there are similarities, but some differences, in what people interpret that three-part lesson to be. Some of the differences are in terminology, but others are more substantial.

Some of the differences are in what people name the three parts of the lesson. The different terminology might or might not suggest different ideas about what happens in those parts of the lesson. Some call the three parts of the lesson *Minds-On, Action, Consolidate*. Others use the terms: *Before, During, After*. Still others use the terms *Getting Started, Working On It, Reflect and Connect*. Others use the terms *TIPS lesson* or *MATCH template*.

But what might be different in intent about what happens in the parts of a lesson? Let's begin by thinking about a **problem-solving-based lesson**.

Part I

Although almost all of those who are committed to the three-part lesson see the first part as an opportunity to activate prior knowledge and gain valuable assessment for learning data, some intend Part I to be a stand-alone activity that would accomplish those goals, some intend that it specifically be an opportunity to revisit ideas or strategies from a previous related lesson, while others intend that it is an opportunity to get students ready for a very specific problem or task by modelling a similar, simpler problem or by orienting the students to the specifics of the Part II task.

Part II

There is often agreement that Part II of the lesson should involve students working, usually collaboratively with others, on a rich problem or task. During this time, teachers circulate, observe, and facilitate student discussion, but in a problem-solving- focused lesson, do not stand at the front providing direct instruction. The complexity of the problem will vary, of course, depending on the grade level and other relevant factors. Students normally use this time not only to solve the problem, but also to represent and communicate their thinking in some form that can be shared with others.

Part III

There might be the most divergence in the interpretation of what Part III of the lesson is. Although most of those who advocate a three-part lesson see it as an opportunity to consolidate the learning of the lesson, the approaches to that consolidation vary. Some would suggest that what happens here is that some or all students present their work to share and that's the end of it. While some see this sharing as having students specifically compare the various presented strategies, others do not. Some teachers would include a practice element, in some form, whether still collaboratively or individually, as another part of that consolidation, and some would not.

Some would require that certain points related to the main goal of the lesson are specifically identified within student work, sometimes by the teacher listing them, but often by the teacher asking pointed questions to make those points visible. Some would demand a self-reflection element, and others would not.

Some teachers use a preplanned consolidating question of a metacognitive nature to generate a class discussion on the main goal of the lesson; others would use a pre-planned consolidating question that is more of an application-type question on a more individual basis, perhaps as an exit pass.

A Lesson That Is Not Problem Based

On some occasions, our instructional focus might be skill practice, whether playing games, working on individual exercises, etc. Or we might decide that a more direct-instruction approach is appropriate in a particular situation. Some ask me, "Is there still a three-part lesson?"

It seems to make sense to have a first part of the lesson where we engage students, activate prior knowledge, and gather assessment for learning information. And it continues to make sense to find a way to pull the lesson together to consolidate what students learned, so there might still be a three-part lesson.

In Summary

So what really matters? I think that what really matters are not how many parts our lesson has nor how we define those parts. I think that what really matters is that we each think deeply and purposefully about what student learning we intend to evoke in our lesson, that we design a lesson that maximizes the possibility for this to happen, and that each of us respects the value in others' viewpoints as well. ▲



▲ President's Message

CONNIE QUADRINI, PRESIDENT



Connie Quadrini is currently the Grades 7–12 Mathematics Program Consultant for the York Catholic District School Board. Her current work focuses on facilitating networked, job-embedded professional learning involving Grades 4–12 teachers within families of schools, as well as supporting school teams in the collaborative development and implementation of school improvement plans. Connie has served as an OAME director for 6 years and Y4MA executive member for 13 years. She was the co-chair of the OAME 2008 Annual Conference, and was the recipient of the OAME Union Gas Award for Outstanding Leadership in Mathematics Education in 2003. Connie has taken a leadership role in numerous Math GAINS initiatives, including Steering Team Lead for the 2008–2009 Coaching for Math GAINS; co-organizer of Math CAMPPP 2008, 2009, 2010; and co-facilitator of several professional learning series including the Mathematics Coaching Cycle, PRIME Leadership Framework Adobe Connect Book Study, and Leadership in K–12 Mathematics System Planning.

A Focus on Professional Learning Keeping Inquiry at the Heart

We continue to learn and grow for the sake of the children we teach. ~ Mona Mirshak, 1996

In my early years of teaching, I had the privilege of working alongside this extraordinary educator. An educator of over 30 years, Mona had experienced a variety of roles including teacher, department head, consultant, and secondee to the Ontario Ministry of Education. As a lead learner, Mona kept inquiry at the heart of her work. She engaged so many of her colleagues, including myself, in grounding professional learning in support of student learning. To this day, it is Mona's vision that inspires me to continue to refine my inquiry focus and seek out collaborative professional learning opportunities and networks to support my growth as an educator.

Professional Learning in Mathematics

As a mathematics community, we have been blessed with many opportunities to engage in meaningful professional learning. OAME offers us the Annual Conference, Leadership Conference, and local chapter mini-conferences. Many provincial opportunities have been afforded to us, including various Math GAINS (Growing Accessible Interactive Networked Supports), PLMLC (Professional Learning for Mathematics Leaders and Coaches) series, Math CAMPPP (Collaborative Actions for Mathematical Precision, Personalization and Professional Learning), and CIL-M (Collaborative Inquiry for Learning: Mathematics). Many of you may also have participated in mathematics professional learning within board networked learning series or school-based professional learning communities.

Deborah Ball suggests that content knowledge for teaching (CKT-M) is critical for teachers of mathematics. What does this mean and what are the implications of the development of CKT-M within professional learning?

Over the past years, we have seen professional learning in Ontario evolve. For example, collaborative inquiry and co-teaching have enabled knowledge-building environments that support the development of CKT-M by providing educators with "opportunities to practice the kinds of mathematical thinking, reasoning, and communicating used in teaching" (Deborah Ball, 2008). By engaging in mathematical problems and tasks within professional learning, educators unpack important mathematical ideas and make connections among them. As educators study students' mathematical thinking, they may uncover potential misconceptions, partial understanding, and/or struggles that students bring to the task. Through collaborative analysis of student learning, educators "see" mathematics through the eyes of students, enabling them to consider ways in which to respond to their work.

Whether we are considered a "specialist" in mathematics or not, we are called to deepen our own understanding of the mathematics we teach through the development of CKT-M so that we can best serve our students. It is through such purposeful actions and interactions within inquiry-based professional learning that educators can develop this expertise.

Our Provincial Opportunity

Over the past few years, professional learning in Ontario has presented us with new ways of thinking about mathematics instruction. Whether a focus on teaching through problem solving, using open questions, engaging students in purposeful mathematics talk, or using manipulatives and technology effectively, educators have explored new approaches and strategies

within instruction to support student learning. Yet with this new learning comes new dilemmas, in particular those involving assessment. With the release of the Growing Success document, we are presented with a renewed opportunity to explore assessment and its interconnection with instruction and CKT-M.

The upcoming OAME Leadership Conference entitled "Creating Communities of Assessment Inquiry and Practice: A Vision for Leadership in Mathematics" will take place on October 27, 28, and 29, 2011. With a focus on Assessment Leadership from NCSM's PRIME Leadership Framework, the conference will engage participants in exploring actions and interactions that focus on the following assessment indicators:

- Every teacher uses student assessments that are congruent and aligned by grade level or course content.
- Every teacher uses formative assessment processes to inform teacher practice and student learning.
- Every teacher uses summative assessment data to evaluate mathematics grade level, course, and program effectiveness.

The conference will provide an exciting "year-long" opportunity for collaborative inquiry. Once registered, delegates will have the opportunity to post assessment dilemmas on the Leadership Conference wiki and explore those posted by their colleagues. Prior to the October conference, delegates will engage in a virtual session that will prime them for action in preparation for the conference. The conference will provide an opportunity for learning with K–12 plenary groupings, as well as within grade band and by role breakouts. The seamless interconnections between plenary groupings and breakouts, as well as across breakouts, will provide rich professional learning experiences for delegates. Following the October conference, delegates will engage in another virtual session that will provide an opportunity for sharing new learning and new dilemmas. Delegates will then be invited to reconvene at the OAME 2012 Annual Conference to continue their collaborative inquiry and/or share their year-long learning with others.

An Invitation

As I enter my year of presidency and reflect upon Mona's powerful vision of learning, I invite you to reflect upon your inquiry for the year. What will your focus be? How will you embed CKT-M? Whom will you seek to work with as you journey through this inquiry? What professional learning will you seek out and how will you share your learning and new insights with others? I hope that you will join me at the upcoming OAME Fall Leadership Conference and look forward to our collaborative assessment inquiry.

References

Ball, D. w/Bass, H., Boerst, T., Cole, Y., Jacobs, J., Kim, Y., Lewis, J., Sleep, L., Suzuka, K., Thames, M., & Zopf, D. (2008). Developing teachers' mathematical knowledge for teaching. Mathematics Focus Session, presented at the University of North Carolina, Chapel Hill, NC, October 21, 2008. ▲

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▲ Technology Corner — Have You Seen It?

MARY BOURASSA



Mary teaches mathematics at West Carleton Secondary School in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She has presented workshops internationally, authored mathematics resources, is a past VP of OAME and a Past President of COMA. An award-winning teacher, Mary continually strives to learn new and better ways of helping students learn and love mathematics.

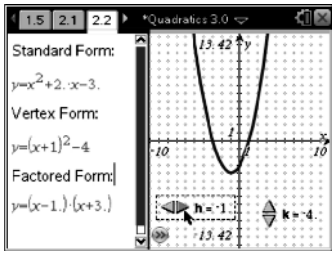


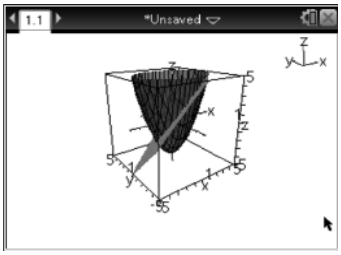
In April, Texas Instruments released version 3.0 of their TI-Nspire™ CX and TI-Nspire™ CX CAS software and handheld. The new handheld, which is sleeker and lighter, features a full-colour backlight display.

The TI-Nspire CX allows you to insert real-world images to model or to help demonstrate part of an activity. It is now possible to insert a picture of a bridge whose arch is parabolic and have students determine its equation. Questions can easily be added to the activity, as the variety of question types which can be inserted has been greatly improved. These can be set as "self-check" so that students can work through an activity at their own pace and receive formative assessment along the way.

Connections between the different representations of functions continue to be made with the ability to split the screen to show a graph with parameters controlled by sliders and dynamic equations at the same time. I was very excited to find a new command on the CAS version: complete the square. Interactive math boxes provide the ideal method of showing values dynamically. In the example shown, the equation is f1(x) = (x – h)2 + k. The input for the first math box is y = expand(f1(x)), which produces the equation in standard form. The input for the second math box is y = completeSquare(f1(x),x), which produces the equation in vertex form. The input for the third math box is y = factor(f1(x)), which produces the equation in factored form. The input for each math box is hidden so all the user sees is how each equation changes as the h and k parameters are changed.

Other mathematical improvements include the addition of 3-D graphing. These graphs can also be rotated, providing students with a 360° view of 3-D functions. This should be an excellent handheld addition to any Calculus and Vectors course.





Data collection has been significantly improved, thanks to the Vernier DataQuest™ application. A multitude of probes can now be used with the TI-Nspire handheld (including the microphone), and the interface is extremely user-friendly. This is a great way to bring real data into the mathematics classroom.

PublishView is a new piece of software that comes with TI-Nspire 3.0, that allows you to insert TI-Nspire applications, but also add text boxes, images, video, and links. The potential of this software for making interactive handouts for students to work through is great, but greater still is the idea that students can use it to create complete mathematical reports.

There are many other improvements. The handling of categorical data and the addition of colour makes a huge difference. Differential equation graphing has been added. You can have multiples of pi on the tic marks for graphs. The list goes on. Check it all out on TI's website or at one of their summer workshops.

Another very exciting technology that every teacher should see is Wolfram Alpha (**www.wolframalpha.com/**). It is available free online, or as an iOS app for $1.99. It provides answers to all kinds of mathematical and non-mathematical questions ranging from factoring, graphing, and solving equations to researching famous mathematicians. If you need to know the population of Canada, it will give it to you with a population graph going back to 1970. If you want to know something, go to Wolfram Alpha.▲

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▲ OAME/NCTM Report June 2011

LAURIE MOHER



Laurie is Numeracy Consultant, JK–8 for the Kawartha Pine Ridge District School Board. She has served on the Board of Directors of OAME for over 12 years.

NCTM is continually developing resources for teachers on a variety of topics that span the learning continuum from Pre-K to Grade 12. In keeping with this support, they have just released three new resources. The first resource is a book in the Essential Understanding series, designed to help teachers develop content knowledge and to present the featured mathematics in multiple ways. The target audience for this book is elementary teachers and it is called *Developing Essential Understanding of Addition and Subtraction for Teaching Mathematics in Pre-K–Grade 2*. The book is organized around two big ideas and shows connections to the smaller, essential understandings for this topic. This book is important because what students learn about addition and subtraction lays the foundation for their later learning of multiplication and division.

The second book is called *Focus in High School Mathematics: Fostering Reasoning and Sense Making for All Students*. This book offers strategies for including reasoning and sense making in the daily mathematics experiences of all students. In accordance with NCTM's Equity Principle, all students, whether they are high achieving or low achieving, have learning difficulties or are gifted, deserve to experience the power of mathematics through reasoning and sense making, and that is the focus of the resource. Visit the NCTM's website to view other books in these two series.

The third book is called *Disrupting Tradition: Research and Practice Pathways in Mathematics Education*. This book examines what happens when partnerships are formed between researchers and mathematics education professionals. It offers examples of the role that research plays in supporting and improving school mathematics. It also raises questions such as, "What lessons have been learned from these researcher/educator partnerships? What types of activities have researchers and educators engaged in together?"

Other Support

Beginning in June 2010, NCTM President Mike Shaughnessy began posing a monthly "Problem to Ponder." These problems focus on reasoning and sense making. The solutions for March's Problem to Ponder will appear around mid-April along with a new "Problem to Ponder." To see past problems and solution ideas, see the President's Corner.

As one strategy to engage today's technology-focused learners, NCTM is recommending the use of mathematics games in the classroom. To assist teachers and parents with this strategy, NCTM has put together a list of the benefits of using games, and some tips for evaluating the effectiveness of the games, in the form of reflective questions. Following the tips, links to several websites (identified as elementary, middle school, or high school) are provided.

Illuminations Spotlight

The following lessons and activities focus on Algebra through the grade bands.

Pre-K–2 Lesson: Graphing Trash Material

In this lesson, students work in groups of two or three to gather and categorize data about the garbage they would normally throw away on any given day. From the group's collection, ten items are selected at random. These items are graphed using a bar graph. Students share their graphs with another group and then the class data is examined. At the end of the investigation, students come up with a plan for recycling or reusing some of their items and share their plan with the class.

Grades 3–5 Lesson: Chairs around the Table

In this variation of a classic problem, students investigate the number of chairs that can be placed around an arrangement of square tables. In this lesson, three related problems are presented and each yields a different linear relationship for the students to explore and discover. Students are able to investigate this problem using pattern blocks and a computer applet.

Grades 6–8 Activity: Equations of Attack

In Equations of Attack, students represent ships by plotting points on a coordinate grid before playing a graphing equations game with a partner. Points along the *y*-axis represent cannons and slopes, and are randomly selected to determine the line and equation of

attacks. In a game of math skill and strategy, students work to sink their partner's ships. After the game has been completed, an algebraic approach to the game is explored.

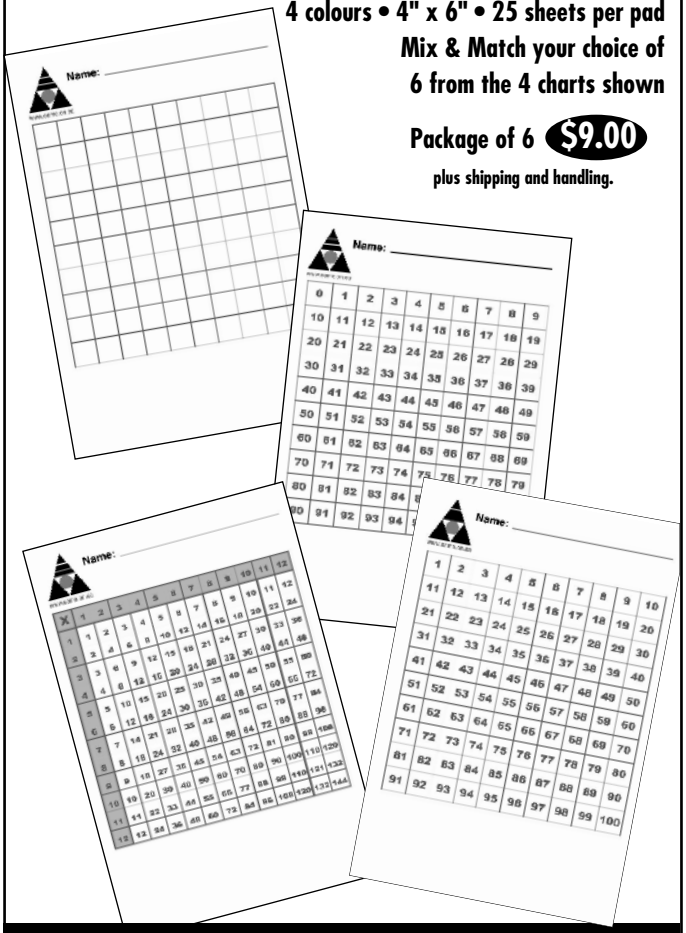
Grades 9–12 Lesson: Barbie Bungee

In this lesson, students use a Barbie® doll and rubber bands to model a bungee jump. Students collect data using a rubber band bungee cord, use the data collected to construct a scatter plot, and generate a line of best fit. They then predict how many bands are needed for Barbie to jump safely from a pre-determined distance. ▲

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▲ What's the Problem? Count 'em Up

SHAWN GODIN



Shawn is the head of Mathematics, Technology, and Family Studies at Cairine Wilson Secondary School in Orleans, Ontario. The web page for this column is ***whats-the-problem.wikispaces.com****.*

Welcome back problem solvers. Last time, I left you with the following problem:

> *Alice, Barney, and Carl are playing a game with 100 counters, although they were not using all of them. They each had a pile of counters in front of them. All at the same time, Alice passed a third of her counters to Barney, Barney passed a quarter of his counters to Carl, and Carl passed a fifth of his counters to Alice. After this, they all had the same number of counters. With how many counters could each have started?*

For the last three years, I have been part of a group of teachers working together to plan lessons, discuss strategies, and develop resources. The group has been supported by the Ottawa Carleton District School Board through grants from the Ministry of Education's GAINS project. (You can read more about GAINS at its website, **www.edugains.ca**.) The group that I am involved with has teachers from Cairine Wilson SS, as well as three of our feeder schools—Emily Carr MS, Henry Larsen ES, and Terry Fox ES.

The problem this month was presented, in a modified form (we used 50 counters instead of 100), to Melanie Baker-LaSage's Grade 8 class at Emily Carr MS. The first part of the discussion centres on what we observed the students doing.

Students were working in groups of four and had access to manipulatives, chart paper, markers, and a couple of copies of the problem. All groups very quickly came to the conclusion that the number of chips must be a multiple of three, since they each end up with the same number and there are three people. From there, a number of different strategies were used to solve the problem.

One common strategy used, which did not work, was working backwards. Students reasoned that if they all started with the same amount, then giving back the portion they had been given would give the starting number of chips. That is, since Alice passed a third of her counters to Barney, then, when working backwards, Barney would give a third of his counters to Alice. None of the students noticed that a third of Barney's final stack may not be the same as a third of Alice's starting stack. What some groups noticed was that they didn't have enough chips to do the problem this way. In order to be able to take a third, fourth, and fifth of the same number, each ending pile had to be a multiple of 60, which meant they would have to start with at least 180 chips. Those students were encouraged to explore whether if they were allowed to have 60 each at the end, they would have a solution to the problem. Most students assumed they would, but when they worked it out, they found out that they didn't and had to turn to another strategy.

The other most common strategy was trial and error, using manipulatives (counting chips). It was interesting to see that since they were told they had 50 chips, they assumed most of the chips were needed. Starting with a total number of chips that is a multiple of three (for example 48), they divided them so that they could give a third of one group, a fourth of a second group, and a fifth of the last (for example 12, 16, and 20). Some groups found a solution and stopped, later to be prompted, "Is that the only solution?" Some groups that were struggling were given a smaller number to start with, which allowed them to find a solution.

In the end, all groups came up with at least one solution. Some groups came up with all the possibilities. In discussion, the groups that got multiple solutions reasoned that if you have a solution, doubling (or tripling) the numbers will work. It seemed they were working on intuition rather than a deep understanding of the problem. It also seems to be rooted in the idea that in mathematics, the answer is the only thing that is important. It is the focus of the group this year to look at learning through problem solving, so that students are given many opportunities to work on problems and reflect on the problem, solution, and "what if..." questions.

Another method of solving the problem, for more advanced students, would be to deal with it algebraically. That is, since Alice, Barney, and Carl must pass a third, a fourth, and a fifth of their stacks, respectively, then they must start with $3a$, $4b$, and $5c$ chips, for some whole numbers a, b, and c. Thus, after they pass the chips, they end up with $2a + c$, $3b + a$, and $4c +$ b chips, respectively. All these quantities must be equal, thus

$2a + c = 3b + a$

so

a = $3b$ – c.

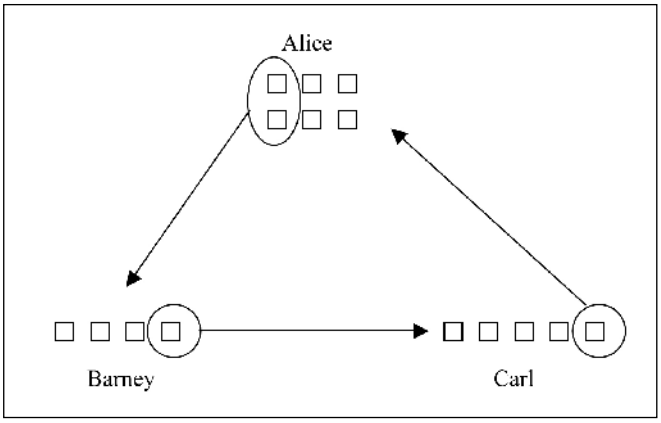
Substituting into the other equation for a, we get

$$3b + a = 4c + b$$
$$3b + (3b - c) = 4c + b$$
$$5b = 5c$$
$$b = c$$

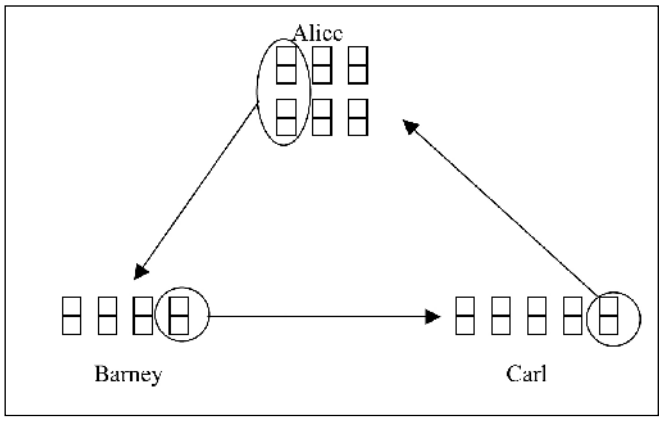
If we introduce a parameter t, we see that Barney and Carl start with $4t$ and $5t$ chips, respectively. Since $a = 2b$, $3a = 6b$, Alice starts with $6t$ chips. They each end with $5t$ chips (Alice has $6t - 2t + 1t$, Barney has $4t - t + 2t$, and Carl has $5t - t + t$ chips).

One nice thing with the algebraic solution is that we see that there is one "class" of solutions. It confirms the students who reasoned that once they had 6, 4, 5 as a solution, 12, 8, 10 and 18, 12, 15 would also be solutions and provides other alternatives as well.

A visual representation of the algebraic solution can make the reason why new solutions can be multiples of existing ones even clearer. That is, if students were working with linking cubes as their counters and they showed the first solution as



then the next as



we see the solutions have the same structure. We could think of the first diagram as being boxes, rather than individual chips, where each box had the same number of chips in it. Then, no matter how many chips are in each of the boxes, it will lead to a solution to the problem.

My thanks to the GAINS group: Angela Burns (ECMS), Claire Slater (ECMS), Dianne Morris (HLES), Jean-Paul Beaulieau (TFES), Kim Engelbrecht (HLES), Lynn Miller (CWSS), as well as the learning coaches from the board office, Anne Holness and Robin McAteer. I especially want to thank Melanie Baker-LeSage and her class at Emily Carr MS for allowing us to test our lessons with them and to watch them at work. Being allowed to watch students interact with a lesson, without having to worry about your role as their teacher, is a wonderful learning experience. You get to see things you would have missed otherwise (good and bad) that allow you to think deeply about your own lessons. If you have a chance to be part of a group like our GAINS group, I would highly recommend it. You can check out some of the fruits of our labour at our wiki: **cw-ec-hl-tf-math-gains.wikispaces.com/**.

Now it is time for your summer homework.

Ancient Egyptians wrote all fractions in terms of distinct unit fractions (that is, in terms of distinct fractions with numerators of 1). For example, instead of writing $\frac{11}{12}$, they would write $\frac{1}{2}+\frac{1}{3}+\frac{1}{12}$. The unit fraction $\frac{1}{2}$ can be written in terms of other unit fractions as $\frac{1}{2}=\frac{1}{3}+\frac{1}{6}$. Find some other unit fractions that can be written as the sum of two unit fractions.

Have a great summer, and happy problem solving. ▲

▲ Data Rocks – A Social Justice Data Fair

Joel Yan & Hina Kausar-Ahmad



Joel Yan is a mathematician who is keen to help educators use Canadian data in mathematics teaching and learning. He is currently working on a contract to organize a data fair at Statistics Canada.



Hina Kausar-Ahmad is an elementary teacher at Kleinburg Public School with the York Region District School Board, and is passionate about social justice and student engagement.

Question: What do you get when you ask Grades 4 and 5 students to research an issue that is important to them, using statistical data, and then to present their findings to other students, teachers, and parents?

Answer: The very successful Social Justice Data Fair, held at Kleinburg Public School in York Region (located just north of Toronto) on February 28, 2011, with exhibits from a single Grade 4/5 class taught by Hina Kausar-Ahmad.



Students sharing their Tap Water Challenge

Along the back wall of the gym, students presented their findings on the amount of sugar and caffeine consumed by staff and students at their school—creatively displayed as a concrete bar graph using sugar cubes and a striking visual showing the amount of sugar in one can of pop.

A map compared the world hunger rate to the world's population: Sub-Saharan Africa has half as many red stickers (representing malnourished people) as white stickers (representing total number of people), while North America has only a fraction of red stickers compared to white ones. Students have studied undernourishment rates as they look at the average weekly spending on food of families in nations around the world. They point out that the average weekly spending varies from $500 in Germany to under $2 in the Republic of Chad.

Another group looked at obesity rates to show the high amount of food waste in Western countries.

On the other side of the gymnasium, two girls have surveyed the entire school and created graphs to show how many students wear clothing made in factories where child labour is used (based on their Internet research). To their surprise, they have found that child labour exists in North America as well.

Another group has studied the problem with overflowing landfills and increasing garbage in Ontario, and find that the top four components of household garbage (mixed paper, food waste, newsprint, and cardboard) can all be recycled or put into the green bin. They are passionate to rectify this situation.

Two groups stand proudly behind their projects and their video game systems displays as they convince teachers and parents of the positive effects of video games.

Another team wondered if female and male literacy rates were equal around the world, and if not, why not? They used data to show that girls perform as well as boys in many educational tests, and have higher graduation rates than boys in Canada.

On another table, the results of a comprehensive gas consumption study are illustrated, with graphs showing total number of Kleinburg students coming to school by car. This team surveyed the whole school, factored in the fuel efficiency of each type of vehicle (from the Internet), and then measured the distance from school to each home (using Google maps) to compute the total gas consumption and cost of driving to school in Excel. These students creatively use an empty red plastic gas can labelled "gasoline statistics" to pull interesting gas consumption facts out during their presentation.

Question: How did the teacher guide these young students to develop such exciting and relevant projects?

Answer: After reading together *If the World Were a Village* (Smith, 2006), the class brainstormed topics such as world hunger, garbage, world population, literacy rates, and energy. A few boys wanted to analyze some aspect of video games. When the teacher insisted on a social justice angle, they agreed to study the impacts of video games on student health and learning.

Children worked in assigned project pairs, choosing the topic that interested them the most. A Grade 5 student served as a big brother/big sister mentor to the Grade 4 student. Several students with special needs were fully integrated into teams, based on their interest in the topic.

For young students to manage such a large, complex project, it was essential to have a very structured process with formative feedback. Hina divided the project into six subtasks and provided ongoing feedback, both oral and written, through daily conferences. She focused on the process of data collection and analysis, rather than the end product, to enhance learning.

The six subtasks listed below were each assigned points, for a total of 100.

Phase 1: Planning (5 points) Students complete a worksheet listing their topic, their survey/research question, an outline of how they are going to collect their data, and how they are going to sort and creatively display their results.

Phase 2: Planning Conference with the Teacher (5 points) A detailed meeting with each group resulted in a positive feedback statement giving the expected benefits of the project and a list of next steps from the teacher. This statement was taken home and signed by both the student and parent.

Phase 3: Research and Data Collection (25 points) Students provided one typed page outlining background information on their topic and why it is a social justice issue. Students also completed all data collection by doing research (mainly Internet) and/or their own statistical surveys.

Phase 4: Creative Display (30 points) Students found creative ways of displaying their data. They were encouraged to use different types of graphs and three-dimensional artifacts.

Phase 5: Reflections (10 points) Students answered the question "so what?" by writing a 250-word reflection on what they learned and how this was relevant to improving the world. They analyzed graphs, and any bias or error in their data collection.

Phase 6: Putting It All Together (25 points) This final step was due four weeks after the students started the project. Here is the text from the assignment sheet:

You have done all of this meaningful work. It is now time to raise awareness by presenting your findings. Presentations should be 5–7 minutes long. Please use the following framework:

a) Survey/Research Question

b) Background Information (Why did you choose this topic?, Why did you share the facts/statistics?, Why is this a social justice issue?)

c) Procedure for Data Collection (How did you collect the information you needed?, How did you choose your sample without being biased?)

d) Results /Graph Analysis (Present your findings by analyzing your graphs. What do they tell you?)

e) So what? Please reflect on how you can be the change that you want to see in the world, based on your findings.

Feedback on the Student Projects

The students were passionate about their project results and very eager to present them. The data fair provided strong evidence that project-based learning addressing real-world issues that students care about can stimulate powerful learning and fully engage students in their learning.

Parents, students, teachers, and a Statistics Canada official were all very impressed by the quality of the student displays and presentations. All could see how the students were confident in their results, partly because they had used mathematics to come up with their important findings.



A laughable moment as students share which staff member consumes the most caffeine!

Here are some comments from adults who attended this fair:

"I thought the data fair was very successful. I was impressed by all of the projects and the level of understanding in the students. This project brought an application of mathematics to life for the students. They learned how to collaborate with their peers, how to deal with people in order to gather accurate data, how to present that data (including the power in graphing it), and how to draw conclusions from the data. This project is a real-life, concrete application of mathematics."
Parent of student participant

"The day of the actual data fair, the level of excitement, anticipation, and energy was contagious!" Classroom volunteer

"I thought the data fair was simply excellent. The topics the children dealt with were such major social issues. It was surprising that even at this young age, the students could realize the importance of such global issues. To be able to tie this in with this math unit really excited the students. Capping this off with a data fair 'presentation day,' where the children showed their understanding and hard work to their school, was a fitting and rewarding end."
Parent of student participant

"The level of the statistical analysis was very high for Grades 4 and 5 students. The quality of both the visual displays and the oral presentation were exceptional. And, most important, several student groups discovered important results that have potential to improve the quality of life in our world. Absolutely inspiring!"
Joel Yan, Mathematician,
StatCan Resource Champion

All the students at the school were encouraged to visit the project displays in the gym over the lunch hour. Several of them expressed the desire to do similar projects in their classes.

High Visibility for the Projects

Since the projects had generated a positive buzz among all those who attended the data fair, the principal suggested they be prominently displayed in the school's foyer. Several educators came to see the projects, including a professor from OISE. After viewing the student planning sheets and taking a video of the projects, which he plans to use in his teaching, the professor provided the comments below.

"The Kleinburg PS Data Fair provided a powerful example of how authentic learning can have a transformative impact on students. The Data Fair was one of the most impressive examples of teaching math that matters that I have seen. It was very interesting to see the wide range of ways math was used to convey the data and the messages students wanted to send... it was evident that the students' understanding of their world, their place in it, and the ways they can contribute to improving the world had greatly been enhanced through their collection, interpretation, and application of the data gathered."

Garfield Gini-Newman,
Education Professor, OISE, University of Toronto

Hina's Reflections: My Rationale for This Social Justice Project

As a student, math was a subject where we worked mostly out of the textbook and had to find the answers to non-contextual mathematical word problems, such as gathering data on ice cream preferences. As a teacher, I realize more than ever how mathematical concepts can be quite abstract for students to grasp. The Data Fair enabled students to develop confidence in their learning. In response to the question, "As a user of data, what did you learn?," here is what some students said:

"By using this data, it made me more comfortable with making line graphs, which we just learned this year."

"Even though our graphs show that the male literacy rate is higher, it doesn't mean that girls are not as smart. Some women in the world, like Afghanistan, aren't allowed to go to school—and that's not fair."



A student shares her data on female and male literacy rates around the world with her grandmother.

I witnessed similar high levels of sensitivity when students were presenting in class as formative practice and conducting peer assessment. I started crying when two students that were presenting said, "As global citizens, how can we continue to take long hot showers when we know that there are kids our age who don't even have clean drinking water?" Her teammate added, "So you can't blame them (children with no access to clean water) for not having an education, because they are busy fetching water all day in villages."

From developing and revising their survey/research questions, to collecting data and displaying it in graphs using technology, students had ownership of their learning as they co-constructed it with peers, teachers, parents, and volunteers.

"I think it was a very good way to extend learning to gain more awareness of global issues. It generated excellent discussions and higher level of thinking." Parent of student participant

I teach from a social justice framework and thread this throughout the curriculum. All of my students are working toward becoming global citizens, and the Data Fair projects enabled them to not only develop a better conceptual understanding of mathematics, but sensitized them to the reality of some pressing issues in the world. Students were not merely looking for answers, but collecting and analyzing data concerning domestic and international issues that allowed them to talk about important issues such as racism and inequality. What was powerful for me was to see the ability of students to decipher data, and then begin to question the validity of their data, and identify biases in their own data collection. Throughout the process, they considered questions relating to bias and skewed data, such as the temperature of the water, to ensure that the data were valid. When the students realized they had included a bias by unconsciously serving the different waters at various temperatures, they subsequently retested after eliminating this bias. The learning that took place from that one class discussion about reliability and validity was amazing. About this, a student expressed this view: *"As a consumer of data, I learned that sometimes the information is skewed and not accurate and that you have to be careful when reading data." When deciding which type of graph would be most fitting for displaying their data, a student said: "If we use a bar graph instead of a pie graph and spread the numbers in our scale, it will seem as if the intermediate (students) drink the most pop."*



Grades 4/5 class from Kleinburg PS with Ms. Kausar (right), Mr. Joel Yan (Statistics Canada representative), Mrs. Latner (Special Education Resource Teacher), Lesley Oliva (classroom volunteer), Mrs. Carusi (parent).

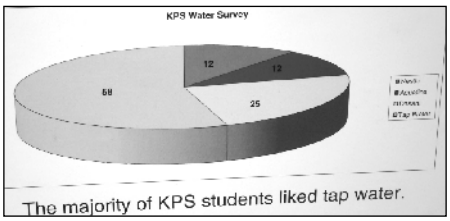
When one of my students, who was analyzing the impact of video games on one's health, came to me and said, "Ms. Kausar, you know video games can actually make you smarter," I was skeptical. However, he impressed me as he used the data his team had gathered to persuade me, "because you have to problem-solve and think of strategies to win and learn and follow all the rules…."

On Thursday mornings, my students have music, and they came back from music, and without any instruction from me, went directly to working on their Data Fair projects. The level of engagement was at an all-time high.

Probably the greatest benefit for students was the opportunity to talk about their mathematical analysis work. They achieved a noticeable feeling of empowerment by having succeeded in addressing a major issue that they care about and often having come up with proposals to improve the situation related to their topic. From this experience, students have learned that their projects have value both in and outside the classroom.

Here are some quotations from students' presentations on the day of the Data Fair:

Tap Water Challenge: *"Plastic bottles are filling up our landfills and killing our environment. The funny part is that most students said they preferred bottled water, but when we conducted the Tap Water Challenge, the results showed that people prefer tap water."*



The majority of KPS students liked tap water.

Child Labour: *"I am wearing the clothes that children are making in factories instead of going to school. They aren't in school, they are working in factories. That makes me sad."*

World Hunger: *"There is hunger in North America, there may be kids in my community/school that don't have enough to eat, and go to school hungry. Today, in order to help, I think we should have more food drives than just the one around Christmastime. There are children in the world that don't have enough to eat, and have to work for their family, they can't go to school."*

Reflection of a student on his project about gas consumed in driving students to school:

"One thing I like about this (project) was how I had a partner that was fair and values my opinions… I liked my topic, which was gas, since I already knew so much about it. I liked going to class; it was really fun. Overall, I think that this project is really cool. I learned so much about gas and I think that I will learn so much more."



Students showing how they selected their sample from the school to collect their primary data.

Data Fair Students Seem to Be More Actively Engaged in Improving the World

My goal for my students is to make them be the change that they want to see in the world. After the Data Fair, one of my students and her mom, who makes custom jewellery, came to me with a proposal to sell bracelets to raise funds to adopt an orphan. The group who researched the growing problem of our overflowing landfills, said, "I think one of the reasons that people do not recycle is not because they don't know, but because the ice rink where I play hockey doesn't have a recycling bin." His teammate, who also plays hockey, said, "I told my coach that we should have a recycling bin in the arena." Seeing my students transform their thinking is moving—this is social justice by the numbers. Shortly after the disaster in Japan, my class started making

1000 paper cranes to send a positive message of peace and support to the students of Japan. The students wanted to do even more for the people of Japan. I just received a petition, signed by all the students, asking if we could conduct a fundraising bake sale in the class (with no nuts guaranteed to be used) to raise money for Japan.

Background on Data Fairs in Ontario

The idea of a Data Fair to celebrate and share the student data analysis projects was piloted by Statistics Canada in conjunction with Ottawa area boards in 2003 (Suurtamm et al., 2004). In 2008, Statistics Canada published a general resource on its Learning Resources website for math teachers, called **Data analysis projects and fairs**. At this site, teachers are given information and resources to help organize a Data Fair. Students and teachers can also use this site to find tips on collecting primary data or finding and using secondary data. It was from this website that Hina got the idea of organizing an in-school Data Fair to showcase the excellent work of her students.

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Fred or Lynda Ferneyhough

▲ In the Middle – Sparking Curiosity: Summer Fun Exploring Math

CAROLINE ROSENBLOOM



Caroline Rosenbloom is currently on secondment as an Elementary Coordinator teaching mathematics in the pre-service program at OISE/UT. She has enjoyed working with intermediate students and teachers as a middle school teacher, Mathematics Instructional Leader for the Toronto District School Board, and Education Officer at EQAO. Caroline is co-author of "Investigating Mathematics Using Polydrons."

> *"I often whine to myself about the state of education, elementarily or secondarily speaking. And when teachers ask me how they can make things like math more fun, my internal whining is all but deafening. Math, I say, mostly to myself, but occasionally to someone who is actually listening, is already fun. You don't have to make it fun. You have to make the fun of it apparent, accessible."* ~ Bernie DeKoven, Game Developer and Fun Coach

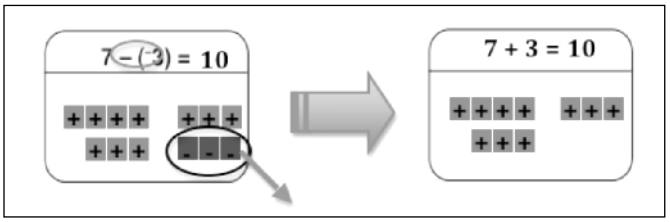
The above quotation motivates some reflection: "Am I making the fun apparent in the mathematics I teach?" We worry about covering the curriculum expectations, accepting that we don't have time to do fun, motivating math. So we need to ask ourselves, "What are they really gaining from my class? Are they learning that math is fun?"

In English classes, we want students not only to know how to read, but to enjoy reading, so we take special care in choosing appealing texts. We can, similarly, focus on more than just content in math class. We can emphasize the enjoyment of mathematics. This article discusses a few enjoyable ways to explore math with your students.

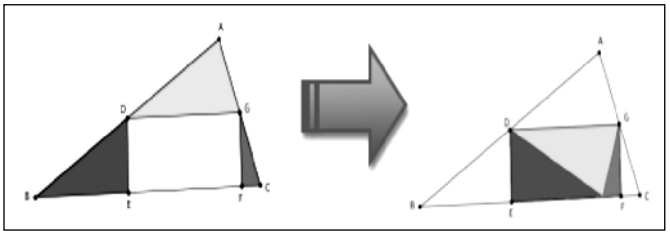
Considering different ways of measuring events in our lives is both interesting and entertaining. For example, bring the song *Seasons of Love* from the Broadway musical *RENT* to math class. This song repeatedly mentions "five hundred twenty-five thousand six hundred

minutes." It is interesting to investigate what this number actually represents. This naturally leads to considering different ways of measuring and comparing the duration of other events in our lives. How many hours will students spend in school this year? How does this compare with the number of hours they will spend surfing the Web this year? How many minutes will they spend in the shower this year? How many full days would that be equivalent to? How many days are there in the average life of a person in Canada? How much of it will be spent in traffic?

Sometimes rules are introduced without understanding what these rules mean. In fact, understanding why the rules are what they are is part of the fun. A student might ask, "I wonder *why* two negatives = a positive?" In the March issue, we were able to represent the mathematical expressions, using tiles and the notion of the zero principle to show that the expression 7 – (-3) = 7 + 3.



We also want to encourage students to wonder about other rules or topics they experience in math. Considering the construction where the corners of the triangle are folded in below, students might ask the following:



- *I wonder if the area of the rectangle is half the area of the whole triangle?*
- *I wonder if ∆ADG is proportional to ∆ABC?*
- *I wonder why DG is half of BC?*
- *I wonder, if I double the length of AB, will that also double the area of the rectangle?*

These are great questions for students to explore, using tools like Geometer's Sketchpad®.

On occasion, students might ask, "Why do I need to know this math?" Sometimes the way to motivate students is to show how math relates to the world. *Math By Design* is a website that includes a video series called *Math in Action*. These videos show how professionals in different fields use math in their work. In one video, a cake artist describes how she uses volume to determine the number of servings for each cake and how she uses proportional reasoning to create a tiered cake. Other videos explore jobs that involve sculptures, architecture, urban planning, and landscape architecture.

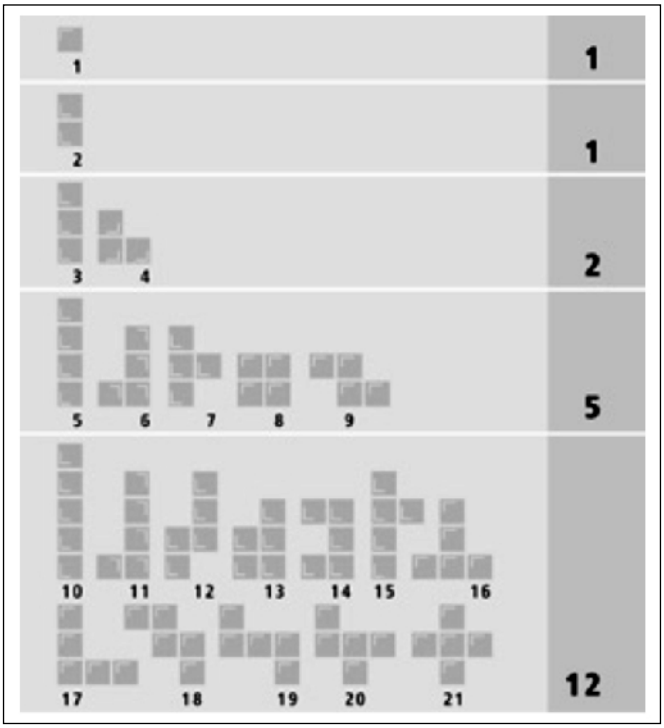
In the same way that English and reading does not end when the school year ends, we want to give students the opportunity to explore mathematics over the summer. Below are suggestions for a few fun and interesting games and activities with a decidedly mathematical flavour…

Play Games



Blokus is a strategic game that borrows concepts from the famous "Four Colour Theorem," and exercises spatial reasoning and logic skills. It uses 21 pieces that are made up of 1 monomino, 1 domino, 2 triominoes, 5 tetrominoes, and 12 pentominoes.

Shown below are all of the pieces used in *Blokus*. Not only is the game intriguing, but students can also investigate whether there are any other edge-connected arrangements of 1, 2, 3, 4, or 5 squares that do not appear in this list.



Explore Fermi Problems

Fermi problems are estimation problems designed to teach dimensional analysis, approximation, and the importance of clearly identifying one's assumptions. Students can ask a variety of fun Fermi questions, including ones to investigate social issues in their community:

- *How many books are there in a bookstore?*
- *How large a bowl would you need to hold a million goldfish?*
- *What is the weight of all the trash produced in your house in a year?*
- *If the land area of the earth were divided up equally for each person on the planet, about how much would you get?*

In the lyrics of the song *In the year 2525* by Zager & Evans, there is a line that states "man has cried a billion tears." We can ask, how many litres of water is a billion tears? It is interesting to consider other songs with mathematical lyrics and investigate what they imply.

Mathematical Doodling with Vi Hart

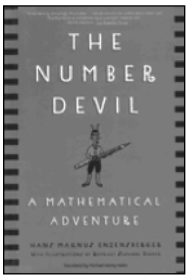
What student doesn't like to doodle? Vi Hart created videos investigating doodling, with really neat connections to induction, recursion, topology, knot theory, graph theory, parity, number theory, and just plain interesting art and drawings. Students will love to try this!

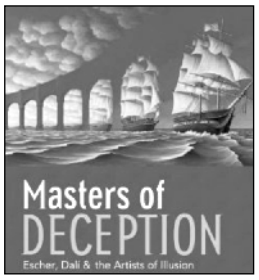
Summer Reading

Students can explore some summer reading by exploring these interesting books about mathematics.

These use mathematics in a humorous and entertaining way.

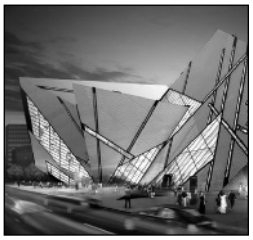






Algebra Unplugged by Kenn Amdahl & Jim Loats; *The Number Devil: A Mathematical Adventure* by Hans Magnus Enzensberger; *Masters of Deception: Escher, Dali & the Artists of Optical Illusion* by Al Seckel.

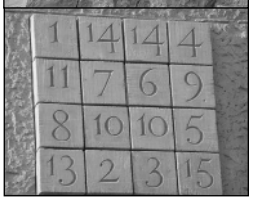
Look for Math in Your Travels



Whether you are travelling along the streets of Toronto and admiring the architecture of the Royal Ontario Museum or travelling outside the country, there are many opportunities to experience math.



For example, we may see magic numbers when visiting the façade of *Sagrada Familia* in Barcelona, Spain.



Or see the *Swatch Internet time*, which divides the day into 1000 beats, at the Centre Pompidou in Paris, France. That is, at 15:03:29 in Paris, the time is 585 beats all over the world!



You can also visit the *Math Midway*, which is a travelling exhibition that is part of the upcoming Museum of Mathematics. This museum dedicated to mathematics plans to open in its new home in Manhattan in 2012.

So let's not worry kids about formulas and definitions this summer, but have students enjoy and appreciate mathematics for the amusement and wonder that is gives all of us.

Have a wonderful summer!
Mathematically Yours,
Caroline

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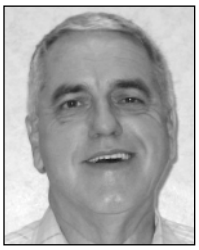
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Swatch Clock @ Centre Pompidou (n.d.). Flickr. [Photo]. Photo posted to **www.flickr.com/photos/clockspot/ 4645032349/** ▲

▲ What's New at EDU?

Jeff Irvine



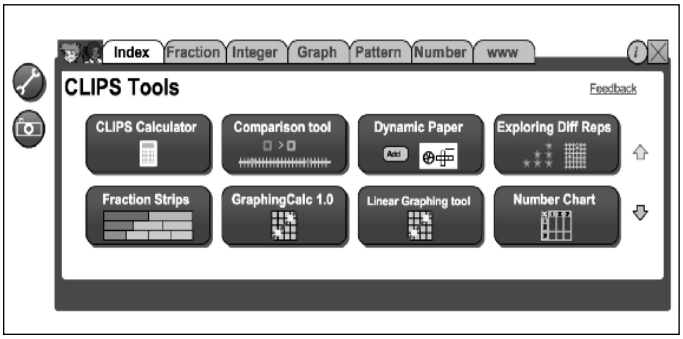
Jeff is an Education Officer at the Ministry of Education.

Math CAMPPP 2011

The 2011 Math CAMPPP (Collaborative Action for Mathematical Precision, Personalization, and Professional Learning) will be held August 15 through August 19 at the Kempenfeldt Centre in Barrie. This year's summer institute will provide opportunities to deepen teacher understanding of both Proportional Reasoning and Algebraic Thinking. The main focus will be on effective instruction and assessment strategies for differentiating responses to students' mathematical thinking. Plenary sessions will be led by Dr. Ruth Beatty (Lakehead University), Dr. Cathy Bruce (Trent University), Amy Lin, and Dr. Marian Small. For more information and registration forms, go to the EduGAINS website.

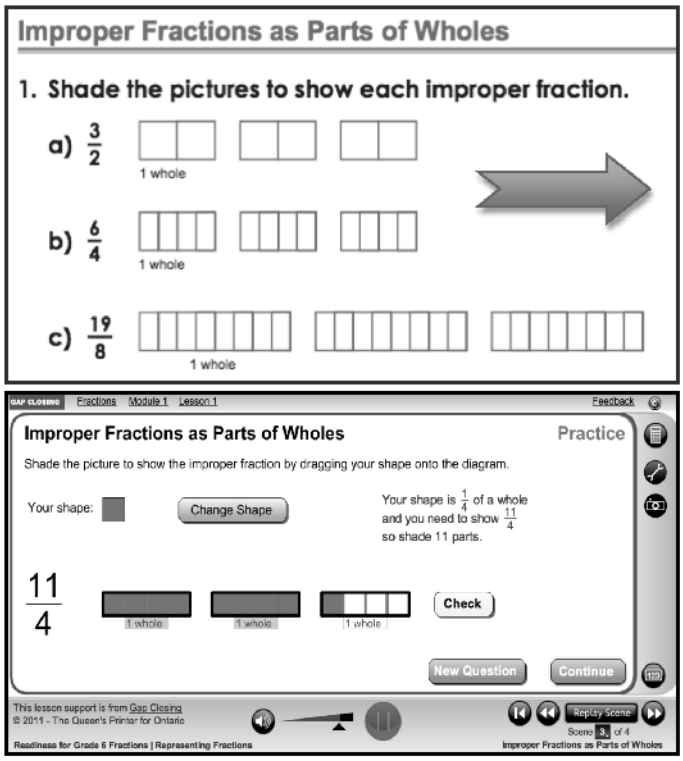
New CLIPS Tools

As of April 2011, a new download is available through the CLIPS website. The *CLIPS Tools* collection is a set of interactive tools including links to high-quality websites with multi-purpose tools. These include the National Library of Virtual Manipulatives, NCTM's Dynamic Paper applet, and the OAME 3D graphing tool, as well as an ever growing collection of tools designed specifically for CLIPS activities. The CLIPS tools can be used to model different types of numbers, arithmetic operations, and mathematical objects like patterns and graphs. The collection is a single SWF file that can be downloaded onto any computer or in an Interactive whiteboard lesson. Click on the wrench icon when you are inside CLIPS, or go directly to the tools by visiting **oame.on.ca/CLIPS/TOOLS**.



New Interactive Supports for Grade 6 Gap Closing

Interactive e-learning activities are being developed as a supplement to the popular Think Sheets from the Grade 6 Gap Closing material. Each e-activity will address the same learning goals as the corresponding Think Sheet and will be accessible 24/7. When using these interactive versions either independently or with a learning partner, students will get immediate feedback, work at their own pace, and have multiple opportunities for success.



GAP CLOSING Think Sheet / GAP CLOSING "Interactive Think Sheet"

Please address all questions and feedback to **jeffrey.irvine@ontario.ca**. ▲

▲ Hey It's Elementary – Inside, Out.... Unusual Mathematics Classrooms

LYNDA COLGAN



Lynda Colgan's career has included roles as a classroom teacher, a university professor, and newspaper columnist. Her contributions to mathematics and its teaching have been recognized through awards such as the Marshall McLuhan Foundation Distinguished Teacher Award. Lynda always exhibits a passion for mathematics and views her professional mission as dispelling the myth that math is the bad guy.

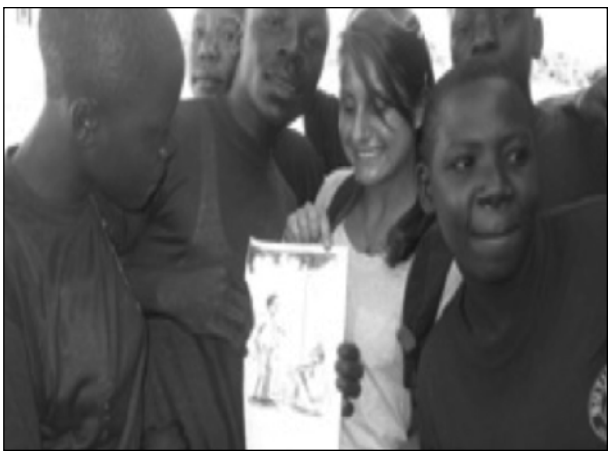
One of the unique features of the B.Ed. program at the Faculty of Education at Queen's University is the *Alternate Practicum*. This mandatory three-week field experience requires all teacher candidates (both Intermediate/Senior and Primary/Junior) to work in an educational setting that is far beyond the walls of the "typical" classroom.

Queen's B.Ed. students experience rich and unique field-based opportunities, from designing curriculum units for museums to teaching in outdoor classrooms in rural villages in Tanzania. Some teacher candidates gain field experience in classrooms at children's hospitals or long-term care facilities, and others learn about schooling at sea, aboard tall ships. Fortunately for me, and for some 3000 children across the Greater Kingston Area, about 50 teacher candidates opted to complete their Alternate Practicum experience at the Community Outreach Centre at the Faculty of Education, for which I am coordinator. The goal of the Community Outreach Centre is to improve the outcomes for students who are "at risk" in literacy, math, science, & technology education. The Community Outreach Centre is determined to make a positive impact by carefully planning and implementing activities and research which allows for the participation of all families, including those from lower socio-economic backgrounds, minority groups, and those with at-risk students. We want to engage parents, teachers, community agencies, and volunteer associations who contribute to the education of children both inside and outside of classroom walls. We want to help all caregivers, who play an important role in one way or other, to learn how to support the development of positive attitudes toward literacy, mathematics, science and technology, and to support formal and informal learning that may have a positive impact on student achievement in these domains.

Given the mandate of the Community Outreach Centre, some of the projects for which teacher candidates elected to participate were in areas other than mathematics. These included participating in a pilot project using Hanen Oral Language strategies in Kindergarten classrooms, partnering with teachers in a response-to-intervention early language program, working one-on-one with adolescent single parents to bring reading and early mathematics into the home environment, developing curriculum units for interactive displays at the Museum of Health Care, the MacLachlan Woodworking Museum, the Pump House Steam Museum, and the Miller Geology Museum, delivering interactive science workshops in local Junior Division classrooms (*Alka-Seltzer Rockets with an Environmental Twist, Windmills, Bouncy Balls,* and *Pen Lid Submarines*), and preparing displays for *Science Rendezvous Kingston!* to celebrate the International Year of Chemistry.







Happily, more than half of the teacher candidates who completed their Alternate Practicum at the Community Outreach Centre worked on mathematics-related projects. Fifteen of these early career teachers worked with Stewart Craven (former mathematics coordinator, Toronto District School Board) to produce four *Math Murder Mystery* summer camp programs for students in Grades 6 to 8. The curricula will be field-tested over the fall and next summer; the Community Outreach Centre will run four camps for local children, offered at no charge to children in needy communities. By targeting children who enjoy mathematics and could benefit from the stimulation of enrichment activities that include graphing calculators, dynamic geometry software, and math field trips, but whose parents cannot afford the registration fees for typical Queen's summer camp programs, we hope to stimulate interest in mathematics and its application beyond the textbook.

Six teacher candidates worked with Gary Rasberry, a local musician and songwriter. The outcome of their three-week collaboration is a collection of 12 math action songs for children. We will launch the CD in the fall, promoting it to local schools, on Queen's Radio CFRC 101.9 FM, and through local print, audio, and visual media. The songs, lyrics, and teacher notes will be available for purchase on the Community Outreach Centre website (currently under reconstruction, but soon to be available at **community.outreach.educ.queensu.ca**) over the summer. You will be able to preview tracks and write reviews.



The *Math Murder Mysteries* and our *Mathemagical* CD will be enormous successes when implemented later this year and next, but the Alternate Practicum project with a mathematics focus that had the greatest immediate impact was *Mathakazam*!

Under the direction of Annie Milne, Artistic Director of the Gentlewings Puppet Theatre in Kingston, Ontario, four teacher candidates rehearsed the play Mathakzam! for one week and then performed it for 27 schools (representing more than 2000 students) over an exceptionally busy, two performances per day, two-week schedule.



To provide a context for the play (which I wrote), meet 10-year-old twins ***Math***ew and ***Math***ilda.

Mathew and Mathilda got their names because their mother loves numbers, shapes, patterns, and formulas so much that she named her precious children after her most favourite thing in the whole world: mathematics.

Mathew and Mathilda's mother is a genuine genius, but a little eccentric in her dress and behaviour. Her entire wardrobe is mathematical: the blouses she wears are embroidered with triangles and tessellations, and her jeans are covered with glittery numbers and formulas. Each one of her ten fingernails is brightly painted with operation signs and symbols: + − < = > ≅ # and %. Her toenails spell out the names of famous mathematicians like "Pythagoras"—five letters on each foot. She wears oversized bracelets and earrings and necklaces and hair barrettes, all made of brilliantly coloured homemade papier mâché beads that are in 3-D shapes like prisms, stellated dodecahedrons and buckyballs, and 2-D cut-outs of triangles, kites, and circle graphs. Even the wire frames for her glasses are mathematical, bent into the shape of trapezoids one day and chevrons the next.

*Mathew and Mathilda's mother is a mathematics professor who sees mathematics everywhere—in the patterns of the flowers in the garden, in the bricks on the driveway, on cereal boxes, in people, and even in baseball diamonds. Wherever she goes, she carries a huge and ancient carpet bag (that once belonged to her great-great-great-great-grandfather: Matricks [pronounced **matrix**] the magician). These days, the very worn, but still beautiful magician bag is overflowing with*

protractors, rulers, calculators, blocks, a computer, a mini-blackboard, pencils and paper of all sorts—squared, blank, origami, and tissue. The children's mother is never, ever without her enormous bag because as she says, "You never know when you could come upon some new mathematics just waiting there to be examined or discovered." "Without my old magician's bag filled with special math tools, what would I do if I accidentally stumbled upon a new number, measurement, pattern, or equation?" she reasoned to the twins.

Mathew and Mathilda and their mother spend a lot of time together because their father died when they were very, very young. The children's mother hopes that she can convince Mathew and Mathilda to follow in her footsteps and become mathematicians one day just like her. The good thing is that she shares her love of mathematics with them every day—on walks in the woods, through the neighbourhood, and to the library and museum, all the time pointing out the math in nature, in buildings and structures, and even toilet paper. The twins find her impromptu math lectures to be very interesting, but a little embarrassing. The bad thing is that the twins' mother stops every few steps, opening her notebook and her carpet bag to record her latest math discovery and perform her experiments, always getting strange looks from passersby who stare at her odd clothing and the collection of math "stuff" pouring out of the tattered magician bag onto the ground around her feet. The sad thing is that because of their mother's unusual clothes and her habit of shouting "Mathakazam" every time she spots some new mathematical example, Mathew and Mathilda often get teased by the other students at school and in the neighbourhood.

Mathew and Mathilda love their mother very much. They don't like the fact that their classmates make fun of her and say that she is, well, weird. And they really, really don't like the fact that the other children tease them about how nerdy they are, pointing at them, and making loud jokes about how the three of them must be the geekiest family in the whole, wide world. Mathew and Mathilda are worried, sad, and hurt, but they really don't know what to do.

Then one day, while their mother was engrossed in an experiment to figure out how many jelly beans really do fit into a jar, the children noticed that a big, black notebook has fallen out of a secret compartment in their great-great-great-great-grandfather's magician bag. After reading just a few pages, Mathew and Mathilda realize that their great-great-great-great-grandfather, Matricks, was no run-of-the-mill "pull the rabbit out of a hat" kind of magician. His tricks put him in a class by himself. Matricks, you see, was a mathymagician—the secret behind all his trickery was math of all kinds—numbers, shapes, and equations. And that gave them an idea.

The twins shouted loudly, "Mathakazam!" as they waved two pussywillow branches (their temporary magic wands).

"Whaaa...?? Who said that?" Their mother was so surprised to hear her special word coming from Mathew and Mathilda that she let go of the jar—thousands of jelly beans burying her legs all the way up to her knees.

"Mother, will you help us to learn all of great-great-great-great-grandfather's mathymagical hocus pocus? Those meanies at school won't be laughing when they see how we can outsmart them," asked Mathew.

Mathilda added, "With these tricks, we'll be the ones with all the secret powers. As great-great-great-great-grandfather Matricks said, 'Give me a number. Give me a thumb. Give me seven geometric shapes. I can outwit anyone. ***Mathakazam! Mathakazee! Bring on the problems. You cannot stump me.****'"*

And with that, Mathew and Mathilda helped their mother dig her way out of the mountain of jelly beans, pack up the ancient mathymagic bag, and head home.

Now, every day, just after dinner, their mother helped them to understand one of Matrick's math tricks. Then at school, in the playground, or at a friend's house (for Mathew and Mathilda were now the most popular kids in the neighbourhood, and their mother, though still considered to be, well, unusual, was considered to be unusual in a really cool way), Mathew and Mathilda would perform their mathymagic tricks, closing their show with their family's trademark ***Mathakazam****! and leaving everyone shaking their heads and wondering aloud, "How'd they do that?" No matter how many times the twins were asked that question, they*

always gave the same answer: "I don't have any special powers. I just know the secret." And now you can know the secrets too.



In the stage adaptation of the play, Mathew and Mathilda are bullied at school because math is their favourite subject and because in the eyes of their peers, "math is not a talent." They set out to prove their peers wrong, calling upon the magic of Matrix, and in the process, meeting applied mathematicians like Snowflake Bentley, Leonardo da Vinci, Alexander Graham Bell, Jonathan Swift, and Harriet Tubman. In the end, the twins help their peers win a contest by choreographing a dance using math to guide the steps and the actions. They become school heroes because their knowledge of math saves the day.



To prepare classes for the field trips to the Upper Canada Academy of the Performing Arts (the home of the Gentlewings Puppet Troupe), teachers were provided with age-appropriate background material that included diagrams, explanatory material, and an annotated bibliography detailing children's books appropriate to the topic.

Although the program evaluation of **Mathakazam**! is still in progress, preliminary results suggest that the puppet play was an enormous success with parents, students, and teachers. The play simultaneously addressed the topics of bullying of high-achieving students in mathematics and the applicability of mathematics beyond the textbook. Long after the performance, teachers report that children in their classroom refer to some of the examples of math featured in the play during lessons about measurement, geometry, and patterning. The teachers reported that they have continued to refer to the play and the ancillary materials to complement and supplement curriculum in lively and unusual ways.

You will be able to access the play through the Community Outreach Centre website before the new school year, September 2011. It will be supported with teaching notes and classroom resources to maximize its effectiveness as a teaching tool.

A new puppet play, *The 3D Little Pigs*, is in production for the Alternate Practicum, 2012, and there are plans for a second mathematics CD; new support materials (including new camp curricula [e.g., math and optics]), resource books (and websites), and other creative materials (e.g., Kaleidoscope instructions for *Geometer's Sketchpad*®) are planned for the 2011–2012 academic year.

The 50 teacher candidates who participated in Alternate Practica at the Community Outreach Centre will enrich and be enriched by the opportunity to teach in Ontario schools, and I sincerely hope that each and every one of them secures a classroom "home." They will become members of your community, thriving in opportunities to work as classroom teachers and leaders of extracurricular activities, and they will transfer the learning from the rich Alternative Practicum opportunities to learn to support the growth of themselves and others as educators and professionals. It is wonderful to know that gifted individuals like these will be leading the way, improving the learning experiences of students of all ages and stages.

In his opening remarks at Granada, Carlo Sequin (at the conference of the *International Society of Art, Mathematics, and Architecture* [ISAMA] conference and *Bridges: Mathematical Connections Between Art, Music, and Science*) commented that a conference is successful if one takes home at least one good idea—something which becomes the germ of something new. By that criterion, the 2011 Alternate Practicum at the Faculty of Education at Queen's University was a most successful field experience because many people enjoyed a feast of ideas to take home… to the classroom or the kitchen table, and that is what community outreach is all about. ▲

▲ Fields Institute – MathEd Forum Notes

STEWART CRAVEN



Stewart is a senior mathematics teacher recently retired from the Toronto District School Board. He continues to teach at York University and actively participates in the Fields Institute Mathematics Education Forum. He was the previous editor of the Gazette*.*

On Saturday, March 26, 2011, we were treated to three wonderful presentations that addressed the history of the "probabilistic revolution" and the ways in which we can integrate probabilistic thinking into the mathematics curriculum.

Sandy Zabell, a Professor of Mathematics and Statistics at Northwestern University, led the way with a talk entitled "The Taming of Chance: Twenty Years Later." Sandy's description says it best.

> *In the ancient world chance was often contrasted with the absence of cause (Polybius) or Fate's unalterable necessity (Tacitus). Despite the widespread use then of randomization (and somewhat surprisingly for a civilization capable of producing Ptolemy's Almagest), there was no calculus of chances. After the 1654 Fermat–Pascal correspondence, mathematical probability finally emerged, but at first only in the restrictive setting of equally likely cases.*
>
> *In the early nineteenth century, Laplace gave chance a purely subjective view of its nature: the universe unfolds in a purely deterministic way; probability only expresses our ignorance of the underlying causes. Over the next century, however, this view became widely challenged. There were many reasons for this: the discovery of statistical regularities in social data, the rise of Mendelian genetics, the development of quantum physics. Ian Hacking's 1990 book* The Taming of Chance *gave a fascinating and provocative analysis of the origins of this conceptual revolution, and sought to explain its distinctive character. In this talk, after first briefly reviewing the prehistory of the probabilistic revolution, I go on to critically examine the theses advanced by Hacking in his book, and discuss their subsequent reception (or lack of it.)*

Georges Monette, Department of Mathematics and Statistics at York University, and Alison Gibbs, Department of Statistics at the University of Toronto, followed Sandy. First, Georges addressed "Causality for Good Citizens."

> *Samuel Wilks (1950): "Statistical thinking will one day be as necessary for efficient citizenship as the ability to read and write," paraphrasing H.G. Wells (1903).*
>
> *Surely that day has come! But what statistical thoughts are needed for efficient citizenship? One could argue that uncertainty about causality is at the root of most current social, political, and scientific controversies. Does human activity cause climate change? How will changes in fiscal policy affect economic recovery? Does using cellphones increase the risk of cancer? In almost every case, controversy subsists because causal information is needed from data that are largely observational as opposed to experimental. Students of statistics learn the dictum "correlation is not causation," a truism that doesn't help them develop the finer judgment necessary to evaluate imperfect evidence from observational data that is often all that is available to inform crucial and urgent issues. What more can statistics say? We would like to explore whether a good citizen's version of causality can be made simple enough to teach in high schools yet not so simple that it merely breeds belief in new fallacies. We distinguish between causal versus predictive inference and between observational versus experimental data. Experimental data is ideally suited for causal inference and observational data for predictive inference. For causal questions from observational data, we try to find an approach that leads to a balance between uncritical gullibility and blind skepticism.*

Alison's speech centred on "Informal Statistical Inference."

> *Continuing with the theme of the importance of statistical thinking for efficient citizenship, we appeal for increased emphasis in our mathematics curricula on randomness, risk, and reasoning rationally in the presence of uncertainty. In particular, we will consider ideas for expanding the*

study of statistics in schools beyond graphical and numerical summaries of data, and we will show some important recent work about how we can support sophisticated inferential reasoning in very young students.

These three presentations engendered a lively conversation during the afternoon session. There appears to be consensus that our students need to learn more about ideas involving probability and indeterminism. As we in Ontario continue to think about "big ideas" in the mathematics curriculum, the issues addressed at this meeting certainly need to be considered.

For information about future meeting dates and agendas, please log into our website at **www.fields.utoronto.ca/programs/mathed/**. ▲

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▲ Directors' Dialogue

ANN MICHELE STENNING



Ann Michele Stenning is a mathematics teacher at South Collegiate Institute with teh Thames Valley District School Board, She enjoys mentoring new teachers and learning new ways of how to address the various learning styles of her students.

OAME Board of Directors Meeting February 4–5, 2011

Friday, February 4

- David Zimmer, OAME's president, welcomed everyone to the meeting. David Petro, this year's OAME annual conference co-chair, announced that Saturday sessions will be available for parents and there are specific sessions on Friday for consultants, math coaches, and superintendents.
- Linda Ferneyhough, OAME's executive director, announced that OAME's Leadership Conference hosted a record number of delegates this year.
- Beverly Farahani, a member of an ad hoc curriculum review committee, which includes members from OMCA, the Fields Institute, and OAME, is soliciting curriculum feedback from educators. To facilitate feedback on curriculum and other topics, Greg Clarke provided board members an introduction to wikis. Greg has set up a wikispace for all OAME members at **oame.wikispaces.com**. He encourages OAME members to visit the space. Greg also asked participants to post using a name that people will recognize.

Saturday, February 5

Members began work within one of their assigned committees. These committees were: Communications, Outreach, Strategic Planning and Renewal, and Curriculum.

A general meeting of the board of directors followed morning work in committees. Minutes from the last general meeting were approved and Lynda Ferneyhough presented the financial report. Connie Quadrini reported on the Fall Leadership 2011 Conference.

Chapter Reports

COMA

Our annual mini-conference, named after our former president, Ronald C. Bender, was held on Saturday, November 20, 2010 at the University of Ottawa. The conference, which was co-sponsored by our chapter and the University of Ottawa, was a great success, despite lower registration numbers than usual. Teachers from both elementary and secondary panels were treated to morning coffee and snacks, two workshop presentations of their choice, an exhibitors' display, and lunch at the university pub. Overall, this was a prime opportunity for cutting-edge professional development, as well as a chance for quality networking with colleagues.

At the mini-conference, we also introduced the latest instalment in our "Pay-It-Forward" campaign. We drew seven names of conference attendees, who won up to $750 each to reimburse costs associated with attending the March 2011 Leadership Conference. An additional three winners were drawn from a pool of names that had been submitted via email just before Christmas.

The next event we will be hosting is an Elementary Forum featuring a presentation on "Integrating the Five Math Strands in Elementary Classrooms" by Jennifer Capitani on Monday, February 28, 2011. This annual forum will focus on teaching elementary mathematics in a more holistic way by integrating strands under big ideas. This will provide another wonderful opportunity for networking and professional dialogue.

We look forward to celebrating the Carleton-Ottawa Math Olympics again this year. They will be taking place on April 16, 2011. A Secondary Forum will also follow in the spring.

GOLDEN

Our committee has been enjoying some down time after hosting last year's annual conference. We will soon be starting to plan this year's mini-conference. It will be around the end of April at Brock University. Delegates will be able to choose two sessions from a group of workshops offered and then have dinner. There will also be a publishers' display area for delegates to explore.

ISOMA

On November 4, 2010, Royal St. George's College hosted the Fall ISOMA Math Conference. The theme of the evening was "Knowledge Integration." Delegates enjoyed an evening of great presentations, dinner, and a keynote address by Professor Ed Jernigan. Thank you to Gerry Doerksen and his colleagues at Royal St. George's College for organizing this year's conference and offering teachers great professional development opportunities.

We are now preparing for the 2011 ISOMA Math Olympics. This year, the Olympics will be held at Greenwood College. We look forward to a day filled with exciting relays, individual challenges, and group activities.

MAC^2

Things have been very busy at MAC^2 over the past few months. MAC^2 recently helped with a junior math road rally that was hosted by the Simcoe County District School Board. Members volunteered their time during the day to help students enjoy a wonderful day of math activities. We are currently in the process of planning our spring mini-conference that will be held in April. Next on the list is organizing our regional OMO competition to decide who shall go to the provincial meet.

NOMA

The most significant event since the last Directors' Meeting is undoubtedly the NOMA Fall Conference, held on October 29 and 30, 2010. Important relationships were forged and renewed in the Northern mathematics community with this conference. Laurentian University was a site we used to build bridges between mathematics educators at all levels, add prestige to the event, and accommodate 176 people. Dr. Marian Small, who gave the keynote address, also facilitated workshops throughout both days.

Friday was a leadership event that drew participants from numerous boards across Ontario. Saturday was the open conference day. All reviews since the event have been very positive.

Although our chapter has a very small active core, these people learned a lot and worked extremely hard organizing our conference. We hope to expand our participation base as we move forward so that one day in the future we might be able to put on events of a greater scale.

At our last chapter meeting before the break, we dealt with the matter of chapter executive positions. Cheryl Geoghegan is both chapter president and chapter rep, our treasurer is Sue Melville, and our OMO coordinator is Sheridawn Maloney. Our membership secretary is Marc Poirier, our secretary is Krista Sarmatiuk, and our publicity team is comprised of Lois Vincent and Joe Snofl. Soon we will start planning Spring Workshops and the OMO Playdowns.

NWOAME

Once again, NWOAME has had a busy year. We are building on the success of the last two years and once again planning "Pi-Day" celebrations for Friday, March 11, 2011. Junior students will be visiting host schools, where they will travel in groups to several stations and engage in games and activities all related to Pi. The Grade 8 students in the host schools will run the activities. This year, we hope to add Algonquin Public School to our list of host schools.

On April 27, we have planned a math evening social with Florence Glanfield as guest speaker. Florence is an associate professor of mathematics education at the University of Alberta, is of Métis ancestry, and has a rich history of First Nations mathematics experiences, both as a learner and as a teacher. NWOAME is hosting this evening in affiliation with the Faculty of Education at Lakehead University.

Additionally we will once again host our regional Math Olympics, scheduled for April 16, 2011. We look forward to bringing regional winners to the provincial OMO competition in Toronto on the weekend of June 4. In and amongst our planned activities, NWOAME executive members continue to interact with local and regional school boards, engaging in constant dialogue and playing an active role in math-related education and professional learning in Northwestern Ontario.

PRMA

Members of the PRMA Chapter have been working this year to write a newsletter for classroom mathematics teachers. The first edition, which was released in December, provided some of our favourite activities to use with students when we find that we have an extra ten minutes here or there. Currently, we are working on a newsletter to highlight our favourite mathematics education sites. You can find our newsletters, along with information related to the "happenings" of PRMA, at our website: **chapters.oame.on.ca/prma/**.

PRMA members are also working to plan our "Playdowns" for the Ontario Mathematics Olympics. PRMA West will be hosting its "playdown" at Durham College on May 5 and 6. For more information, please contact the PRMA West OMO Coordinator, Leah Franklin PRMA East will be hosting its "playdown" event at the PVNC CDSB Catholic Education Centre on April 28. For more information on this event, please contact the PRMA East OMO Coordinator, Sandra Connolly

TEAMS

TEAMS is officially hosting OMO 2011. If any TEAMS members are interested in participating in organizing this event, please contact the co-chairs, Wayne Erdman or Laurissa Werhun

Educators, do you know a Grade 7 or 8 student who would enjoy problem solving in a group setting? We are planning our TEAMS Math Olympics, to be held on March 26, at the Catholic Education Centre, 80 Sheppard Ave. East. Winning students go on to the Ontario Math Olympics. If you are interested in fielding a single student or a team of four students (one male and one female Grade 7, and one male and one female Grade 8), please contact Michele Goveia or Jane Silva.

TEAMS held a very successful mini-conference in October at Dante Alighieri Academy (Beatrice Campus) and Bathurst Heights Learning Centre, on "Linking Assessment and Instruction." Over 200 members attended. Door prizes included sponsoring one member to the OAME conference in May.

The TEAMS Executive would like to encourage elementary teachers and Instructional Leaders from each board to become more involved in our chapter. Please come out to our meetings. We meet monthly at the Catholic Education Centre and look forward to seeing you.

WOMA

WOMA will offer a spring mini-conference in London for the low price of $10. The host school is equipped with SMART Boards™ in every room. The school's gym turns into a dining area where educators view publishers' displays and enjoy supper following two professional development sessions. Attendees register through MCIS and are eligible to win free registration to OAME's annual conference. ▲

▲ Call for Manuscripts

The *Ontario Mathematics Gazette* is inviting manuscripts for all grade levels.

Instructions for submission of manuscripts are found on page 1 of any *OMG*.

Contact the Editor for further details.



▲ "Messy Time" Transition to Reform-Based Mathematics Teaching and Learning

DAN JARVIS & DOUG FRANKS





Dan Jarvis and Doug Franks are Associate Professors of mathematics and graduate education in the Schulich School of Education, Nipissing University, North Bay, Ontario. *In working with mathematics teachers, and with teachers of mathematics, in their various research and service capacities, they are continually impressed by the ability and willingness of practitioners to embrace the messiness and initial frustration that often characterizes rich, reform-based mathematics teaching and learning.*

The Challenge of Effective Mathematics Teaching in 2011

In an era of reform-based mathematics education (National Council of Teachers of Mathematics, 2000; Ontario Ministry of Education, 2005a, 2005b, 2007), in which key elements such as problem-based learning (PBL), co-operative group work, manipulatives, technology, and varied types of assessment are being emphasized, it is not difficult to understand why teaching mathematics remains a complex undertaking. The twenty-first-century mathematics educator must possess at least four overlapping areas of specialized knowledge and related competencies (see Figure 1):

- He or she must have a thorough understanding of the ***mathematics content*** (e.g., Ontario Curriculum expectations as listed/described within the five mathematics strands).
- He or she must also master, and be able to utilize with confidence, an ever-expanding range of ***mathematics resources for teaching***, such as manipulatives (e.g., linking cubes, GeoBoards, Pattern Blocks, Algebra Tiles) and technologies (e.g., graphing/CAS calculators, computer software, Internet web resources, interactive whiteboards).
- He or she must develop what Deborah Ball and colleagues (Ball & Bass, 2000; Thames & Ball, 2010) have described as unique ***"mathematics knowledge for teaching"*** (MKT), or a pedagogical awareness of how mathematical topics are connected, and how students acquire, organize, and communicate their mathematical thinking (including an awareness of common student misconceptions and alternative solution strategies).
- He or she must incorporate *balanced instructional planning*, featuring both the more traditional methods of teacher-directed lessons and skill-based practice (e.g., notes, textbook questions), combined with regular opportunities for rich, interactive problem-based learning (PBL).

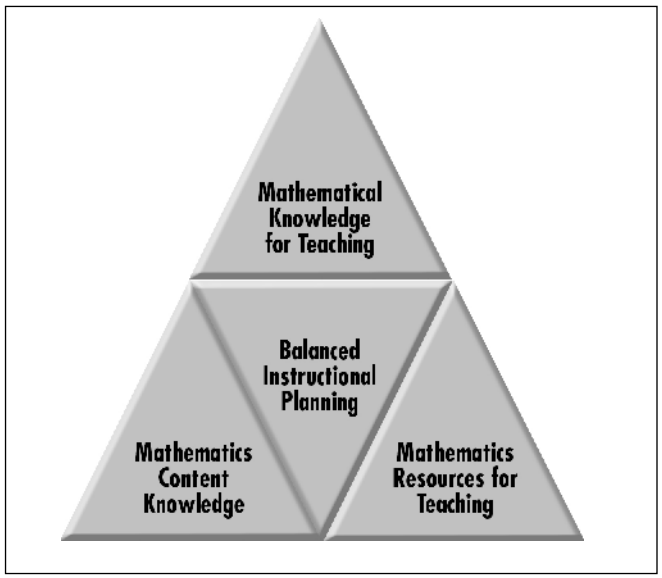


Figure 1. *Four components of effective mathematics teaching and learning*

While all four of the components described in Figure 1 are significant aspects of teacher practice, for the purposes of this article, and based on the themes emerging from the collected research data, we will focus primarily on Mathematics Content Knowledge and Balanced Instructional Planning[1].

Problem-Based Learning (PBL)

[1] *For more information regarding the use of, and research relating to, Mathematics Resources (manipulatives, technology, Internet), the reader may wish to refer to the Canadian Council on Learning's report entitled* Lessons in Learning: Promising Practices in Primary Mathematics Instruction *(Canadian Council on Learning, 2009) and to the insightful, regularly occurring* Gazette *articles/columns (e.g.,* Abacus *activities,* Technology Corner*). Canadian research in the area of Mathematics Content Knowledge (Kajander & Mason, 2007; Kajander et al., 2010) is also recommended for further reading.*

Medical professor Dr. Howard Harrows originally developed problem-based learning at McMaster University in the 1970s. Problem-based learning (PBL), although not synonymous with "problem solving" in the literature, often features overlapping principles or characteristics (e.g., learning is based on challenging, open-ended problems; students often work in collaborative groups; teachers often assume the role of "facilitator" of learning).

Problem solving has certainly received an increasing amount of attention within the reform-oriented mathematics education agenda. For example, The National Council of Teachers of Mathematics (NCTM) strongly argued for a problem-solving focus in its *Principles and Standards for School Mathematics* (2000) document. Further, the *Ontario Revised Curriculum: Mathematics* (Ontario Ministry of Education, 2005a, 2005b, 2007) lists problem solving first among its seven core mathematical processes. It has been proposed that all seven of these processes (i.e., problem solving, reasoning and proving, reflecting, selecting tools and computational strategies, connecting, representing, and communicating) can be meaningfully practised within a classroom where PBL is regularly being implemented (Consortium of Ontario School Boards, 2005; Jarvis, 2008; MacMath, Wallace, & Chi, 2009). For the purposes of this paper, we would like to suggest that PBL often encompasses the three stages of the popular Van de Walle "3-part lesson." These three parts are listed as: the introduction of the activity (verbally and/or via demonstration), the exploration/activity itself (which may occur individually, with a partner, or, more commonly, in small groups; also, which may occur during one class period/session, or which may extend over several sessions), and the whole-class debriefing/discussion that follows (Van de Walle & Folk, 2008).

Collaborative Action Research in Mathematics Education

In 2008–2010, we were involved in a provincial initiative entitled *Teachers Learning Together: The Math Journey*, which was implemented by the Elementary Teachers' Federation of Ontario (ETFO). As such, we were part of one of the research teams who provided support to school-based teacher teams, and who were empowered to create, implement, and report on collaborative action research projects throughout the province. Many of the action research projects focused on problem-based learning in mathematics classrooms. The remainder of this article will highlight two key concepts that have emerged from the project cycle as we have conducted interview conversations with teacher teams, namely, what we'll refer to as the "messy time" of problem-based teaching transition, and the "balanced instructional programming" with which every teacher must arguably struggle in order to find a personal/professional comfort zone which they feel is effective in their daily work of teaching mathematics (Ontario Ministry of Education, 2004, pp. 7–8).

Teacher Beliefs

Each teacher team selected, or had appointed for them, a "Lead Teacher" who would act as facilitator and general organizer for the group in supporting the ongoing research project. During the Lead Teacher meeting in February 2009, one of the participants drew our attention to the notion, based on her own observations, that when teachers begin to move toward a more PBL-based curriculum, they often experience an uncomfortable, second-guessing, "messy time" of sorts. This *messy time* is presented as natural and necessary, as teachers work through the process of being less directive in their methods, and more open to the structured unpredictability of both the mathematics activity and the debriefing that follows. The original exchange follows:

Interviewer: I think you said something earlier along the lines of when you started for the first month or two, it was "messy" or "fuzzy" or did not have a lot of movement? You were thinking that this was going nowhere with your teachers, but in retrospect, this time was necessary?

Teacher: You know what? It reflects two things that I said. One thing is that I notice that in my classroom…, when you start this whole problem-solving thing, it's not very neat and tidy. When we started at the beginning with our team, everybody was a little bit in, and a little bit out. People were happy to talk, but… they're not sure they're engaged… That was such a very obvious shift in their belief about the role of the teacher… we could talk the talk, and we could all have that conversation… it might not be every day that they're doing problem solving in their classrooms, but the shift is massive. So that piece is really exciting.

In subsequent interviews, by way of follow-up, we presented this idea of "messy time" to teachers within the various case study teams, asking for their comments on this idea. One team shared the following thoughts on this particular issue:

Teacher: I think I felt at first like, When am I actually teaching? They're doing a lot of problem solving, and they're showing me a lot of things, but when am I really giving the explicit instruction, and how am I actually teaching? At some points, I really felt like, they're just running the show and why am I here?

Teacher: It takes a lot of time too. You have to give them time to work on the problems, and to talk about it, so there is less of us talking. I don't think we're used to that.

Another significant factor contributing to teacher uneasiness with a move toward PBL instructional strategies is that it often reveals how little teachers sometimes know about the mathematical content, in terms of being able to understand student solutions and/or recognizing student misconceptions. For example, in the following rich exchange, several teachers discuss their discomfort with their perceived absence of understanding of math which they feel is lacking in their own experience and yet necessary for successful PBL implementation:

Teacher: We have had this conversation about the content knowledge being something that we lack. We weren't taught with understanding. We're being told to teach the problem solving and we're really moving in that direction, but unlike a lot of other subject areas, we're missing some content knowledge.

Teacher: So, because we weren't taught for understanding, we're missing some of those pieces, and so far, our board hasn't paid much attention to mathematics at all, so we're not developing our content knowledge in that way… I'm pretty much able to get my head around Grades 3 and 4 concepts, but I'm still, every day, thinking, "Wow. That makes a lot of sense." Just realizing after 15 years of teaching it and having obviously passed Grades 3 and 4 that, " I never thought of that before."

Teacher: Yes. I'm sitting there thinking, "I'm not smart enough to do this myself, so how can I expect these kids to do it?"

Regardless of the frustrations regarding weak content knowledge, awkward classroom implementation, and a perceived lack of control, all of which were shared as part-and-parcel of the reform-based mathematics teaching journey, several of the teachers in the study indicated that, in retrospect, their beliefs had indeed shifted throughout the year. The following excerpt notes a teacher's changing beliefs regarding both mathematics teaching and math education research, in which he had now played a part:

Teacher: People are doing research all the time. I've never thought of ever participating in anything like that before until I had this experience. Obviously now, I think I understand why people are doing it… Because we have been focused and concentrating on our problem-solving situation, my whole opinion has changed about math, about teaching, about what children can do, about what the expectations are, where we're going. I think for that reason in itself, it's given me some indication that research is really important and it's a good thing that people are doing it other than from the personal point of view… We do have something to share with people.

Not only are teacher beliefs crucial in the examination of "messy" time, but also those of students.

Student Attitudes

The "messy time" of adjustment, as described in the last section, also affects and challenges students in different ways, regardless of their prior achievement in the more traditional classroom. For example, weaker-performing students can often rise to the occasion of creative problem solving and ultimately find themselves in roles of leadership and receiving peer admiration and positive feedback for their PBL contributions. Other students, who have perhaps been the highest achieving within a more directed approach, often find themselves deeply confused and upset, as not only the classroom experience has drastically changed, but different aspects of learning are now being valued in private conversations (teacher/student) and in public (carpet time) discussions.

As one teacher explained, "It's usually hardest for the ones who have traditionally done well. They're not accustomed to understanding. They have no need for that, and they want that direction from you. They want to follow your instructions." Another teacher noted, "I have children who have done extremely well in math… the way I was teaching math last year. A student was getting Level 4's in absolutely everything. Her test papers were perfect scores—even the bonus question, and she is struggling with this kind of work." A third teacher shared similar observations, "The smart ones—the ones who are traditional math students, who are good at memorizing and following the procedure, those are the ones who object the most, even in Grade 3."

The following teacher documents a perceived progression in student attitudes toward PBL:

Teacher: [T]he kids were feeling discombobulated at first because I wasn't showing them how to do something. As a result, I had some students a couple months ago saying, "Are we going to use the textbooks soon?" That's what they were comfortable with, which was really difficult for me to hear. But after, I would say a four-month period, boom! We're sliding and we're just speeding along and everybody is happy, and we're content. I would say it was kind of a steep learning curve at first. They didn't like the fact that… I wasn't telling them how to do things, and that made them very uncomfortable.

One teacher explained how PBL with very young children sometimes led to emotional responses, even among the stronger students within the class:

Teacher: I teach kids that are much younger than the rest, and I just found that as soon as I asked them to explain what they did, they started erasing. They immediately thought, "What I've done is wrong; otherwise why would she ask me to explain it?"… Ironically with my stronger students in my class, I had a lot of tears… They're seven years old, so there are a lot of tears in Grade 2. I found it wasn't the weaker kids who you might expect are lost… It was the older kids who are used to getting everything, and they're used to having A's, and suddenly you're not telling them what to do, so they have to just figure it out. When they start finding it difficult, or the solution isn't obvious to them, they aren't used to having to just power through. It took… about the same—probably about the end of November [until] it was starting to get better.

Students' reaction to this very different way of experiencing mathematics learning was not entirely unexpected. As one teacher noted, if a student is "not used to being in a classroom where student thinking is encouraged and respected, then that is something that has to be a part of the training process for any children, in any grade really. If they're not used to having that kind of approach, then there is a lot of retraining and reteaching them that."

Notwithstanding the above frustrations for students, a number of teachers attributed increased overall engagement in mathematics learning to the problem-based approach:

Teacher: I find they're much more engaged, and for us it's been very collaborative—often they're working on problems together, so they're exchanging thinking… They've had to shift how they think about math, but it hasn't been a difficult change for them. I'd say that they're enjoying math and they think of themselves as good problem-solvers.

For some teachers, what most stood out was the delight in seeing individual students who had not met with mathematical success in the past really starting to shine in the new environment. One teacher described a student who was new to the school and told her at the start of the year the he did not like math. But things changed during the year: "It's very interesting because he's doing extremely well with [problem-based mathematics], and he has all the inventive strategies, so it's really nice to see him moved around to different groups because he's helping the thinking of others."

And a teacher with another case study team noted: "[I]t's quite amazing. She's really come a long way. Her confidence—you can see it in the classroom. So, kids will go to her [for support]."

Further, one participant asserted that PBL is advantageous for all students, even those with special needs, as it allows multiple entry points and ways of thinking, representing, and communicating:

Teacher: Even our special education people who are the advocates of traditional [math]… they've been part of the process of learning and going to different locations to see problem solving in action, and they're slowly converting to the fact that quite the opposite is true. People definitely need… to know number facts because you don't see relationships between numbers if you don't know those number facts. [But] if all your problem-solving energy goes into a basic fact, you have no brainpower left to do the real mathematics, the real problem solving at those levels. Everybody has an entry point. Those kids who typically struggle aren't disengaged anymore because they're all involved… I think when students didn't do well, in the back of my mind, I heard that comforting expression, "Oh, well, this doesn't work for some kids." I wasn't doing it right, that's why—but I don't believe that anymore at all. I haven't seen a student who doesn't benefit, although the ones who typically do well, do resist. They don't respect other people's ideas. They think they know more, but when they come around, they do well… It's a real life skill to listen to other people.

A move to PBL not only affected the teachers involved, but also their students who daily observe their teachers' reading, planning, experimenting, and communicating with colleagues.

Teacher: Even my students, and their [colleagues'] students, knowing that we do this together on a regular

basis has a huge impact on kids' understanding—that learning is real and authentic. Not just something that you do at school. It's lifelong and teachers don't have all the answers. They work together and they know exactly what we talk about because we go into each other's classrooms and we share that with the kids. That makes a big difference. There's a real sense of community value to this.

Transitioning to a curriculum delivery model that incorporates PBL experiences not only affects the beliefs and attitudes of teachers and students involved, it also includes a very basic, pragmatic question: How often do I plan for PBL-type activities and how does this relate to overall instructional planning?

Striking a Balance in Instructional Programming

Problem-based learning is not new for many teachers in Ontario. Recently, a series of professional development activities have focused specifically on this teaching strategy. Boards are at different stages of implementation and teacher readiness. As one teacher described, the expectations from the Ministry of Education are now clearly along these lines, and, in her particular case, she felt that the head start that her school had experienced regarding PBL methods, had placed them, as a result, in a relatively good position in terms of the encouraged reform.

Teacher: One of the things we had at the end of the year… all of the schools in the board sent one teacher to one of two sites, to a session where they were introduced to the problem-solving model. And the message at that session—so it was board-wide—was, this document—depending on whether you're K–3 or Grades 4, 5, 6—is now four to six years old. So now it's not a matter of, "Are you comfortable with it?," it's an expectation in Ontario that you *will* teach through problem solving—not occasionally do problem solving — and that the three-part lesson is to be happening in all classrooms… We're in an advantageous situation now because we're not going to have that same awkward, uncomfortable feeling about having to put into practice something that now is going to be a board expectation, so that's a really nice place for us to be.

One of the most difficult parts of implementing a PBL approach to mathematics teaching and learning is to know how to find "balance" in one's planning. Teachers struggle with questions of how often, when, and to what degree PBL should be used in their classroom. Does drill-and-practice have any place within a reform-based curriculum? How should difficult questions from parents/guardians be handled when the topic is the content and flow of the mathematics curriculum, particularly in light of Grades 3 and 6 EQAO assessment? How does one plan for split-grade classes? What about students with special needs and ESL/ELD students—do the proposed advantages of PBL apply to all student populations?

Interviewer: Is every day a problem-solving-oriented lesson, or do you have a couple of times a week where… it's going to be standard skills development?

Teacher: I'm in both places… [M]y last actual test that I did, the whole test was just problem solving... Basically, in our long-range plans, we've always gone with the expectations and checked them off. I'm bouncing around. I'm doing both. Where I find that the children aren't really that knowledgeable, I have gone back to doing the standard algorithms for multiplication. We've done a lot of inventive strategies, but the document states that they do need to be able to use it with manipulatives, or whatever, and do a four-digit by a two digit. We've been struggling with that, but I've done a lot of the hands-on and a lot of the manipulatives. I have found that for those who didn't have the understanding… I've been using words like "groups of" and "sharing." I've been doing that for years, so I don't understand when the kids don't get that. I guess I'm just bouncing back and forth, but I'm doing probably more so now—more problem-solving approach most of the time.

When we use the term "balance," we would like to be clear that we do not necessarily mean *equal division of time* between traditional and reform strategies/activities; nor do we wish to imply that an achieved focal "balance" would be *fixed* in the case of each individual teacher—rather, it would likely be more *variable* in nature, depending on other significant factors such as students' abilities/interests, the teaching context, and available resources. Whereas some teachers are convinced that all learning should be done using a PBL model (no doubt a minority of teachers), others argue that striking a "balance" between direct teaching and PBL activities requires ongoing classroom experimentation and reflection. Ideally, a reform-based teaching approach would be implemented by all teachers of mathematics on a continual basis in some form or another, recognizing that each teacher needs to regularly (each year/course) establish her or his own "reality" of what is working best for students and himself or herself.

Teacher: You can incorporate that [practising basic skills] into your problem-based learning to allow you to do some more guided math, so that they're… practising it in the class.

Teacher: Exactly. I think it's back to… Do you have to do a problem every day to be teaching through problem solving? No. You don't have to be doing a problem every single day, but I think it's hard to feel like you're in between. I think people feel like you're either here or there.

Time required to prepare PBL lessons, particularly when one is first beginning to use this approach, was cited as being one of the main challenges in striking the curricular "balance."

Teacher: I find that it's taking me more time to prepare because I have to rethink the way I did stuff. So, I'm trying to do more stuff where it's more verbal and fewer paper-pencil tests. So, it's because it's new—I know the math program, but to find new ways to do it. We have a lot of good resources, but it's to go through the resources and find what I'm looking for, for what I want to teach, in a different way—I find that quite time-consuming.

Sharing of developed resources within schools, or across schools within District School Boards, was therefore seen as highly beneficial, something that could be facilitated within existing or targeted professional development initiatives.

Concluding Thoughts

In using the term "messy time," we are not specifically referring to the messiness of the classroom during a problem-based learning activity, although this may very well be the case when compared to the tidiness of the direct teaching classroom. Nor are we referring to the "ordered chaos" that may also characterize a classroom in which PBL activities are happening—the active communication of students and teachers during such a session may indeed seem relatively noisy to the observer. Rather, we refer specifically to the "messy" transition—the perceived frustration and internal questioning—that we believe often takes place within the minds of teachers who are brave enough to begin to implement these changes in their classrooms. Having observed instances of this transition in schools, and having talked to teachers who are experiencing both the excitement and discomfort of this transition process, we believe that "messy time" may represent an essential, and ultimately beneficial, aspect of teacher instructional transition. This disconcerting, and perhaps even painful, process serves to: (i) solidify teacher beliefs/attitudes regarding reform-oriented practice; (ii) echo the intellectual discomfort (i.e., "cognitive dissonance") of students experiencing PBL in their classes; and (iii) reify the PBL claims and rhetoric found in the mathematics education literature through actual classroom-based cycles of implementation and reflection.

How can administrators and coordinators help *mediate the messiness*? The teachers that took part in the collaborative action research project that has been described in this paper would no doubt recommend some form of ongoing peer support mechanism, whether funded (release time) or at least encouraged by the administrator and local board. Reassuring teachers that the first few months of PBL implementation may indeed be accompanied by feelings of frustration and doubt, but that these are normal, beneficial, and often accompanied by "breakthrough" moments of student insight and overall increased student engagement, represents a simple yet perhaps highly significant method of bolstering teacher risk taking:

Teacher: I think that's the huge shift in moving from very teacher-centred instruction to… more focused on the needs of individual children. Rather than just having a blanket lesson… [wherein] you're making the assumption that everybody is entering it at the same point, and you lose a lot of children that way, in the problem-solving approach, you can meet the needs of individual children. You may be doing it in your explicit instruction one-on-one. You may pull a little group together when you see they're having a little bit of a problem. I think that the big shift for us as teachers is to move away from that teacher-centric kind of approach.

Helping teachers who have begun the PBL journey to strike the elusive "balance" in their daily instructional planning is perhaps a much more difficult endeavour—one that will no doubt require an equivalent "messy time" transition experience for those in administration and teacher support positions.

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▲ Teaching Through Problem Solving in Grade 9

ROBERT TODD



Rob has been a high school math teacher for the past 10 years and is currently a math coach and department head at St. John's College in the Brant Haldimand Norfolk Catholic District School Board in Brantford, Ontario.

The first thing I want to address in this article is what the phrase *teaching math through problem solving* means. The answer is… I don't really know what other people mean, but in this article, I would like to share some of the things I have learned about what it looks like and feels like to me. I want to talk about the issues that require attention, and concerns that people have that might not really be problems after all. I want to share with you some of the problems that I used most successfully. Finally, I want to convince you to take the plunge and try it.

The first thing I want to share is that I was scared to death of teaching this way when I started. At one point this past semester, I was more than a week behind the other Grade 9 academic classes. I stopped asking where everyone was, as I was getting worried—kind of like jumping off the scales before it gets to the weight you don't want to be. Ignorance is bliss. When I realized how far behind I was getting, I almost gave up.

Jumping ahead, that's probably the day I learned the most about teaching through problem solving. I remember clearly thinking, "enough of the fun and exploring and getting to the core of the math, we have to start zipping through this stuff so that they see it all." At that moment, part of my brain yelled, "NO! Getting to the core of the math is what I want to do."

(Are you worried that I am advocating not covering some of the curriculum? Well, I'm not. Nor was I advocating it on that day. I had just decided I was committed to seeing my experiment through.)

Anyhow, I am happy to say that it has all worked out very nicely. I am extremely proud of what the students learned, and I taught through problem solving whenever I could. Let me explain that statement. In no way am I claiming I did this every day; sometimes I went a week or

so without teaching this way. I did it whenever I could come up with a problem I felt good about.

So, that's the first point. **The time investment is more than worth it.** At some point, the students became hyper-engaged and we started to mow through ideas faster than I thought possible. When the last of the students bought into the problem-solving method, the learning power of the classroom became so high that we did not need the time we did previously to get through curriculum. I think most of you already have had this thought—if the students would just use all the power available to them, we could get through the curriculum easily. And we did.

Secondly, teaching through problem solving is fun. I get to see the students enjoying math. I get to see all sorts of cool ways that they are thinking about questions, they feel like their thinking is valued, and the engagement level is so high—they almost can't help but learn a lot. I really believe that. If they are interested, the students will learn more than we think is possible, and make it look easy doing it.

Here is an example. The day before I went to teach exponents, I did an exit slip with three questions on it. The three questions were:

Simplify: 3^4

Simplify: $2^5 2^{11}$

Simplify: $\frac{5^{23}}{5^7}$

Even though my students were Grade 9 academic and had seen these types of questions in Grade 8, I was surprised how few of them had these ideas mastered. Many knew the answer to the first question, so I knew they knew what an exponent was, but almost none could do the next two questions.

The next day I went to the board and wrote the question:

Simplify: $\frac{3^8\left(3^{14}\right)}{3^{17}}$

I allowed them much of the period to work on it. I gave this much time because I really think of the exponent laws as a "light bulb" concept. Once you get it, you get it, and no "laws" are needed. In fact, I kind of think someone who needs the "laws" probably doesn't "get it." So the objective of this lesson was for the students to really see what exponents were and how they worked. I wanted the students not to need to rely on notes talking about exponent "laws."

So in that period, the students used all kinds of strategies. They often checked with others. And in the end, we shared all we had done. The solutions were rarely straightforward and mostly not anything I would have said or taught. And I walked out with a huge smile on my face and thought, "Well, that's that; they do understand!" The rest of the lessons were tying up loose ends. The results on the test were amazing and I was not in the least bit surprised. (I will not even start into the assessment side of things here. I knew what they knew because of the way I taught it, or rather, the way we learned it.)

Now I will explain what I did. I put a question on the board that I had not taught the students how to do; this is what makes it a problem. I explained the question very carefully so that it was very clear what they have to try to accomplish. But I provided no scaffolding worksheet. You know what I am talking about—the sheet where we give the table, graph, picture, or whatever it is that guides them to the method we would like them to use. Don't misunderstand. I still use these types of sheets sometimes. I am just trying to say that is not what I think of now when I think of teaching through problem solving.

I gave the students a lot of time to attempt the problem, while I wandered around, encouraged their ideas, and fixed little errors, but I never told them what to do. The students sometimes worked alone, sometimes in groups, and sometimes switched back and forth. I made no real rules about how they would work.

Mostly I cheered them on. At some point, I started giving letters to students with interesting solutions. Then once I believed that I had many types of solutions represented, I asked the people with letters to put their solution on the board. (Lately I keep track of who has been at the board so that I use as many different people as I can.) The letter "A" is reserved for the person who just worked it out—no fancy methods, maybe a lot of work, and very easy to understand what he or she did. And the higher letters are for people who used a more abstract approach. Then I went up and described what each person did, and assigned the homework. (Sometimes a note follows if a "procedure" has been developed.)

Assigning the students letters just allowed me to stay organized. There really was no need for this. What it allowed me to do was be able to take up the solutions in an order that went from easiest to understand (and maybe the one that took the longest), to the most abstract (and maybe the one that was the quickest). So

there was nothing "better" about different letters—it's just a way to describe a solution type. Sometimes I abandoned the letters, and just worked off what I saw on the board after the solutions were up, but that was after I had become more comfortable.

Classroom management was an issue sometimes, but not in the way you might expect. Some students could do the problems quickly and easily, so I had to be prepared for them. Often this meant simply asking them to show how to get the answer in another way. Other times, I had another problem ready to go that was more challenging, but on the same topic (often chosen from previous math contests). Another simple solution was just to allow them to take a look at any homework that was to be assigned for the day.

One other classroom-management issue was students who would refuse to try or did not know how to get started. To be honest, I did not have this problem for long. Once the students saw how much I valued any effort at solving problems, they were always willing to try. As a side note, basic methods were often the ones I really needed to have available to help make sense of the more abstract methods that were presented. I made it clear to the students that I valued all the different strategies.

There were many problems that worked out very well. Some examples:

1. *Snoopy is flying his airplane at a height of 1000 m and he spots his arch-enemy, the Red Baron, flying at a height of 805 m. Immediately, Snoopy starts descending at a rate of 45 m/s. At the same moment, the Red Baron starts descending at a rate of 30 m/s.*

   a) *When will Snoopy be at the same height as the Red Baron?*

   b) *If both planes continue to descend at these rates, when will each plane crash into the ground?*

   c) *Snoopy's guns have a range of 80 m. At what times will Snoopy be in range of the Red Baron?*

2. *Find the value of* $3\frac{3}{8} - 1\frac{2}{3}$.

I presented these types of problems in the first or the second lesson of the unit, even though they were the questions that I normally would have addressed near the end of a unit. Asking students to try these on the first day allowed us to really see where we were going with the unit, and gave us something to work toward throughout the unit. I think they were also successful because they are the type of questions that can be handled in many different ways, and were not problems that needed some special "trick" to get the answer.

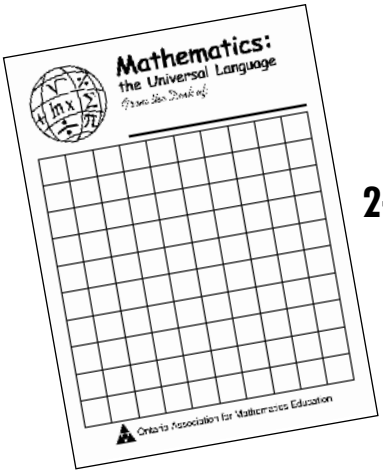
One difficulty that came up is when students were absent. I was quite fortunate to not have to deal with this all that often, but it was a huge problem when it did happen. Missing these lessons could have a huge impact on a student's understanding. In the end, I did the best I could to pass along what the student had missed. Another difficulty that did come up was students being so proud of the method they used on the day of the problem solving lesson, that they became closed-minded to other methods. This problem faded as the semester went on, and students gained some experience about what they should do with the "power" I had given them to choose their method.

And that's it. I don't do it every day, but on many days. I have been known to prod someone along who has done something easy and I want a little more from him or her. I am teaching Grade 9 academic this semester, but I fully intend to give this a run with my Grade 9 applied class next semester.

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▲ Historical Approaches to Linear Equations

VERA SARINA



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Old mathematical texts can provide insight into the concepts we teach in our classrooms. The Scottish mathematician Philip Jourdain wrote a hundred years ago: "But it is as true as it is natural that we should find that the best way to become acquainted with new ideas is to study the way in which knowledge about them grew up" (Jourdain, 2000). History of mathematics can help teachers develop deeper understanding of the origins and development of algebra, provide alternative strategies for students, and make the development of mathematics more real to students.

This article is an attempt to highlight some historical approaches that might be of interest in intermediate or secondary math classrooms.

Egyptian Hau Calculations

Classical algebra is concerned with determining the values of unknowns. Although symbolic algebraic notation (letters for variables, signs for operations, etc.) is a relatively young invention of Western European mathematics of the sixteenth and seventeenth centuries (Cajori, 2007), cultures around the world produced a variety of methods for determining unknown numbers without using a variable. The ancient Egyptians called an unknown quantity or number "hau," meaning heap. Among the methods of "hau" calculations they developed, of particular interest to us might be the method of "false supposition" that formalizes the "guess and check" approach suggested in the Ontario curriculum documents (OME, 2005, pp. 83, 94).

This method can be summarized in two steps:

- Make an arbitrary assumption as to the value of the unknown.
- Determine the true value by comparing the result of substituting the wrong value in the original expression with the actual data.



An example of using the method of "false supposition" is problem 24 from the Rhind papyrus (Rhind or Ahmes papyrus is dated around 2000 BC. It is a set of practical (with a few exceptions) problems with solutions and is one of the main sources of our knowledge of Egyptian mathematics):

"A quantity and the seventh part of it have 19 as a sum. What is the quantity?"

Ahmes, the scribe of the papyrus, suggests the following solution to the problem:

- *Assume the quantity is 7*;
- *Then substitute 7 in the problem:* $7+\frac{1}{7}\times 7=8$;
- *8 is not 19; so 8 must disappear*;
- *Thus divide 8 by 8* $\left(\frac{8}{8}=1\right)$;
- *Then multiply* (the result of division) *by 19*: 1 x 19 = 19;
- So the quantity is 7 divided by 8 times 19 (Repeat the "adjustment" operations with the assumed answer to find the quantity): $\frac{7}{8}\times 19=\frac{133}{8}$.

Whereas our approach to guess and check is likely to have students look at the result of 8 and decide whether their next guess should be higher or lower than their first guess of 7, the Egyptians' method was a formal procedure for using the first guess to move directly to the correct answer. Explaining why their method works might be an interesting enrichment problem for students who are algebraically strong. [It is built on the notion that if we want to solve $a+\frac{a}{7}=19$ and we know that $b+\frac{b}{7}=k$, dividing through the second equation by the number k and applying the distributive principle suggests that $\frac{b}{k}+\frac{\left(\frac{b}{k}\right)}{7}=1$ and $\frac{19b}{k}+\frac{19\left(\frac{b}{k}\right)}{7}=19$ and so a solution to the original problem is $19\frac{b}{k}$ in this case where b = 7 and k = 8.]

Why *x*?

Formulas require symbols. The first recorded systematic attempt to talk math in symbols and signs was made by the Greek mathematician Diophantus

(3rd Century AD) in a series of books called *Arithmetica* (Bashmakova, 1997). For example, in his notation $\mathrm{K}^{\mathrm{Y}}\ \alpha\varsigma\beta\ \pitchfork\ \mathring{\mathrm{M}}\ \gamma$ means $x^3 + 2x - 3$. Though imperfect, Diophantus' algebraic notation had opened doors to symbolic algebra, but it took mathematicians around a thousand years to get through these doors.



The French mathematician François Viète (1540–1603) was one of the first to improve upon Diophantus' notation (Klein, 1992). He suggested using vowels for unknown and consonants for known quantities. But what if an equation contains more variables than the number of vowels in the Latin alphabet? René Descartes, a philosopher and father of analytic geometry, introduced later the modern variant where unknowns are denoted by the letters from the end of the alphabet (x, y, z…) and known quantities by the letters from the beginning of the alphabet (a, b, c…).

"To leave no problems unsolved"

In 1591, François Viète published *Introduction to the Analytic Art*, creating the first system of algebraic notation. His treatise is dedicated to treatment of equations. I start the unit on Equations in my classes with a citation from this book: "Finally, the analytical art… appropriates to itself by right the proud problem of problems, which is: TO LEAVE NO PROBLEMS UNSOLVED." This citation highlights the main purpose of equations to students, which is to be a tool for solving problems.

The following passage from Chapter 2 of Viète's book (Klein, 1992) provided me with another insight on how to teach the procedures of solving equations:

> *The Analytic Art assumes as manifest the better known* ***stipulations governing equations*** *... such as are:*
>
> 1. *The whole is equal to its parts.*
> 2. *Things which are equal to the same thing are equal to each other.*
> 3. *If equals are added to equals, the sums are equal.*
> 4. *If equals are subtracted from equals, the remainders are equal.*
> 5. *If equals are multiplied by equals, the products are equal.*
> 6. *If equals are divided by equals, the results are equal.*

François Viète starts his treatise on working with equations by listing the mathematical facts upon which the whole edifice of equations is built. We often teach algebraic procedures without clarifying their arithmetic basics. We often assume that these rules are discoverable to students through guided or discovery learning in fairly short periods of time. But what if that is not the case? If one of the world's most famous mathematicians thought that he had to reiterate these rules before applying them to algebraic procedures, our students definitely need to be explicitly aware of these underlying assumptions.

Conclusion

NCTM Standards recognize that "teachers need to understand the big ideas of mathematics and be able to represent mathematics as a coherent and connected enterprise" (NCTM 2000, p. 16). Knowing history of mathematics helps immensely in the daunting task of representing mathematics as "a coherent and connected enterprise."

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▲ Assessment for Learning in a Problem-Solving Context

ELIZABETH HOERATH



Elizabeth Hoerath is an Instructional Program Leader with the Halton District School Board. She has enjoyed teaching and learning French and mathematics for the past 12 years.

When math teachers use a problem-solving approach, and students are encouraged to use a range of strategies, processes, and solutions in the development of their understanding of math concepts, what does *assessment for learning* look like? How can *success criteria* be developed that reflect the potential scope of student work? What kinds of feedback are meaningful to improve student results when each student's process and product may be unique?

Math educators who design a lesson around a rich problem or open-ended question want students to construct their own understanding of the mathematical concept. Students' construction of this knowledge may involve some struggling with the concept, trying out old and invented strategies, talking through their thinking with peers, and being exposed to more than one approach to the same problem. So assessment *for* learning, rather than assessment *of* learning, would be the focus of responding to students in this type of situation.

Some descriptions of assessment for learning strategies involve the teacher sharing the learning goal with students at the beginning of a lesson, as well as success criteria that describe the elements of quality work. In the constructivist math classroom, the teacher would have the learning goal in mind while designing the problem, but would likely *not* share the learning goal with students at the beginning of the lesson. The learning goal may (or may not) be revealed later in the consolidation part of the lesson, as the teacher plans questions to highlight the development of the math concept. Success criteria that describe steps or procedures for doing math may be inaccessible to some students or they may limit the thinking of others. They may even rob a student of that satisfying "aha" moment when one finally comes to grips with a concept that was previously just beyond the limit of his or her understanding.

I propose that we consider the ways in which mathematical knowledge is constructed in the classroom, and look for opportunities to help students define success for themselves as they make meaning of mathematical problems and processes. Students' interactions with the mathematics may be viewed through the lens of three zones: experiencing mathematics, deconstructing mathematics, and sharing with others. Within each zone, math teachers can find multiple opportunities for assessment for learning.

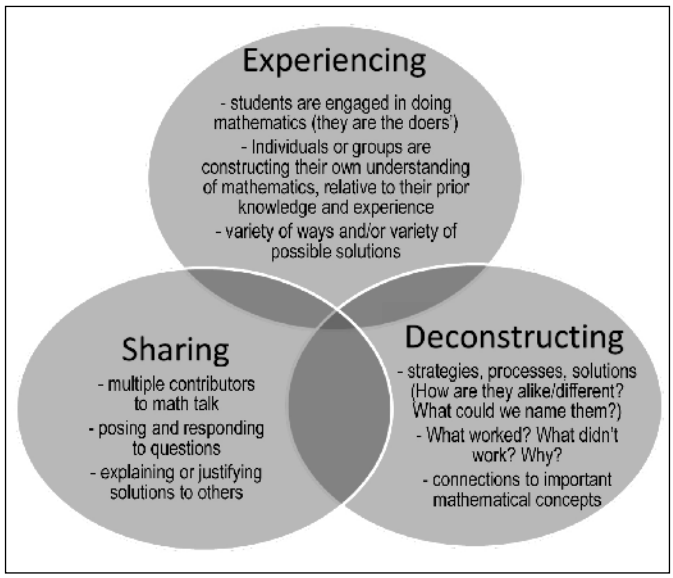


Figure 1. *Zones of Mathematical Work*

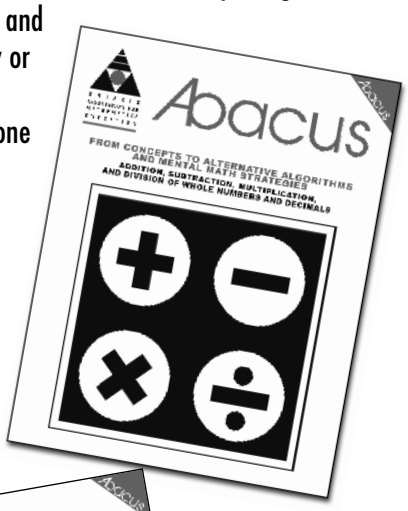
The following chart suggests some specific ways that teachers can use two components of assessment for learning, namely success criteria and effective feedback, in the context of a problem-solving lesson and the three zones of mathematical work. Ideas from this chart can be included in classroom tools such as anchor charts, checklists for use by students and teachers, reflection or goal-setting documents, math journals, math portfolios, or student–teacher conferences. The "TIP!s" are intended to identify how each section can be used strategically to inform ongoing instruction and promote student independence and confidence in their problem-solving skills and understanding of math concepts.

	Success Criteria *What does "problem solving" look like and sound like?*	Effective Feedback Samples *How can teachers, students, and their peers identify what is going well, what to improve, and how to improve it?*
EXPERIENCING	✓ I understand the problem. ✓ I can ask questions to clarify my understanding of the problem. ✓ I can select tools (e.g., manipulatives, grid paper, calculator) to help me solve this problem. ✓ I can tell if my strategy is working. ✓ If my strategy is not working, I can try another way or talk to a classmate about another way. ✓ I can think of another problem that was similar to this one. *TIP! Using checklists for success criteria promotes independence because students can use them to self- or peer-assess their work in a timely fashion. The statements focus on what students **can** do and promote self-confidence.*	• I see you solved the problem using (name of tool/manipulative/strategy). How did that help you find your answer? • Is there another (tool/manipulative/strategy) that you could use to solve this problem? Would this always work? • Would you choose the same strategy if you were given a similar problem tomorrow? Why? *TIP! Using **questions** as feedback prompts promotes student accountability because they are expected to act upon the feedback (i.e., answer the question).*
SHARING	✓ I can explain my solution to others. ✓ I can use math words, symbols, or drawings to show my work. ✓ I can prove my answer. *TIP! Co-creating the criteria and using student-friendly language will deepen student understanding of the criteria.*	• I think your [number line] is a good strategy for showing [5 groups of 7]. How could you [add marks to your number line to show me how you counted the groups]? • I see your group worked out that [the area of the shape is 50 square units]. Why is it important to [include the units in your solution]? *TIP! Ask students to generate their own feedback, using sentence stems. I understood your solution because…. Symbols helped me understand and communicate because….*
DECONSTRUCTING	✓ I can describe how my solution is the same or different from other solutions. ✓ I can identify other strategies that worked for my classmates. ✓ I am can use ideas that worked to solve this problem to solve some other problems. *TIP! Naming strategies and processes helps develop a common mathematical language in your class.*	• Both Shane and Anika [used a calculator to solve this problem], but they have different solutions. Why do you think that is? *TIP! "Feedback" questions can be used during any part of the three-part lesson, as well as a check for understanding (assessment). Track evidence over time by recording questions and responses (e.g., exit pass, journal entry, checklist, portfolio entry with reflection).*

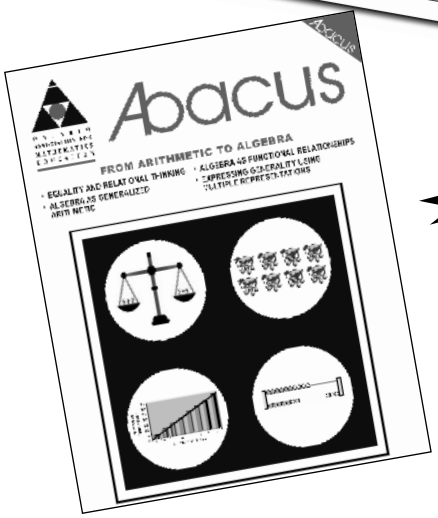
Figure 2. *Assessment for Learning Strategies*

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