Submission of Articles

The *Ontario Mathematics Gazette* (*OMG*) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *OMG*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles must be word-processed in MS Word, double-spaced with wide margins, not exceeding 10 numbered pages of text, and prepared according to the *Publication Manual of the American Psychological Association, Sixth Edition*. Figures and diagrams should be drawn by computer, if possible, or drawn in black ink in camera-ready form. Embedded images must also be submitted separately in jpeg or tif format. Proof of the photographer's permission is required, and for **photos of students** under the age of 18, the written permission of a **parent or guardian is required**.

You must submit **one complete copy** of your article, embedded with any tables, figures, and captioned photographs or graphics, to the Editor, Marian Small, along with **separate files for each of the text, graphics, and/or photographs**.

Your name should not appear anywhere in your article, including websites, so that your article can be sent out for blind review. Your name, full mailing address, and e-mail address must be included on a separate sheet. Upon review, you will be notified as to whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected.

The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME.

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The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of OAME.

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Editor

▲ **Marian Small**

Associate Editors

▲ **Anne Yeager**

▲ **Marilyn Hurrell**

***Abacus* Co-Editors**

▲ **Mary Lou Kestell**

▲ **Kathy Kubota-Zarivnij**

Design and Production

▲ **Penny Clemens,** Graphic Designer

Printing & Binding

▲ **Pole Printing,** Box 69, 89 King Street East
Forest, ON N0N 1J0 (519) 786-5112

Advertising Manager

▲ **Robert Sherk**

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Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** at the above address. Courier is recommended to avoid possible delays. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be camera ready and colour advertisements should have no bleeds.

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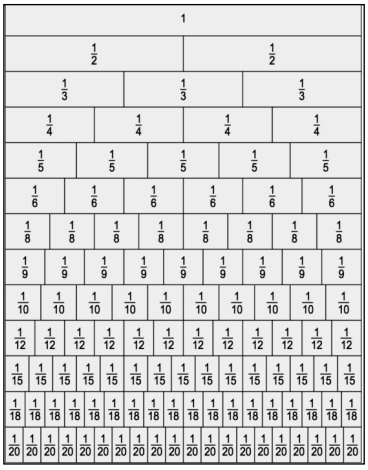
Advertising rates are available by telephoning, e-mailing, or writing to the Advertising Manager.

▲ Editor's Message

MARIAN SMALL



Much of the work I focus on revolves around helping to ensure that struggling students receive the right sorts of mathematical opportunities to grow. Sometimes that involves providing a problem that is accessible from many perspectives, including simple hands-on approaches. For example, it is quite feasible to ask most Grade 7 students to determine groups of fractions that take up the same space (equivalent fractions) if a fraction tower like the one below is provided.



Sometimes that involves providing an open enough problem so that all students can meaningfully participate. For example, it is quite feasible to ask most Grade 5 students, struggling or not, to choose two numbers to multiply, to create three different problems solved using that multiplication, and to explain why those problems are solved by multiplication; but this assumes the students really have a sense of what multiplication is. It is possible that students are missing the fundamental notions that multiplication is about counting the total of equal groups, is about describing the area of rectangles with given dimensions, or that it is a measure of comparison (e.g., I have four times as much as you do.), in which case, even this very open problem is not truly accessible.

In cases like these, where fundamental notions are missing, it makes sense to deal directly with the gaps or misunderstandings and to focus on the critical building blocks that the students missed. Hopefully, if this is successfully accomplished, the students then rejoin their peers in a richer, more contextually based problem-solving program. With this in mind, there is increasing attention to the nature of intervention materials. The

Ministry of Education is currently sharing two packages of materials I had an opportunity to create. They are for Grades 6 and 9, for certain topics, and have the name Gap Closing.

The Grade 6 materials focus on number sense. They might be useful to teachers of struggling students in Grades 7 or 8 as well. They might also be useful to teachers in Grades 4 or 5, not to replace current teaching materials, but to provide alternative approaches or interesting questions they might use with their students. Similarly, the Grade 9 materials, which focus on algebra, number, and measurement, could be useful to teachers of Grades 7 to 12.

All of the Gap Closing materials are available on the Math Gains website **www.edugains.ca**. What you might find interesting about them is that there is the use of open questions as an appropriate intervention strategy, even for struggling students. While there are also more structured learning opportunities, both the structured and the open approaches focus on conceptual gaps, rather than procedural gaps. There are also diagnostic tools to help a teacher choose where a student needs help; not all struggling students require the same type of intervention.

If you haven't had a chance to look at these materials, you might look to see how they could be useful to you. ▲

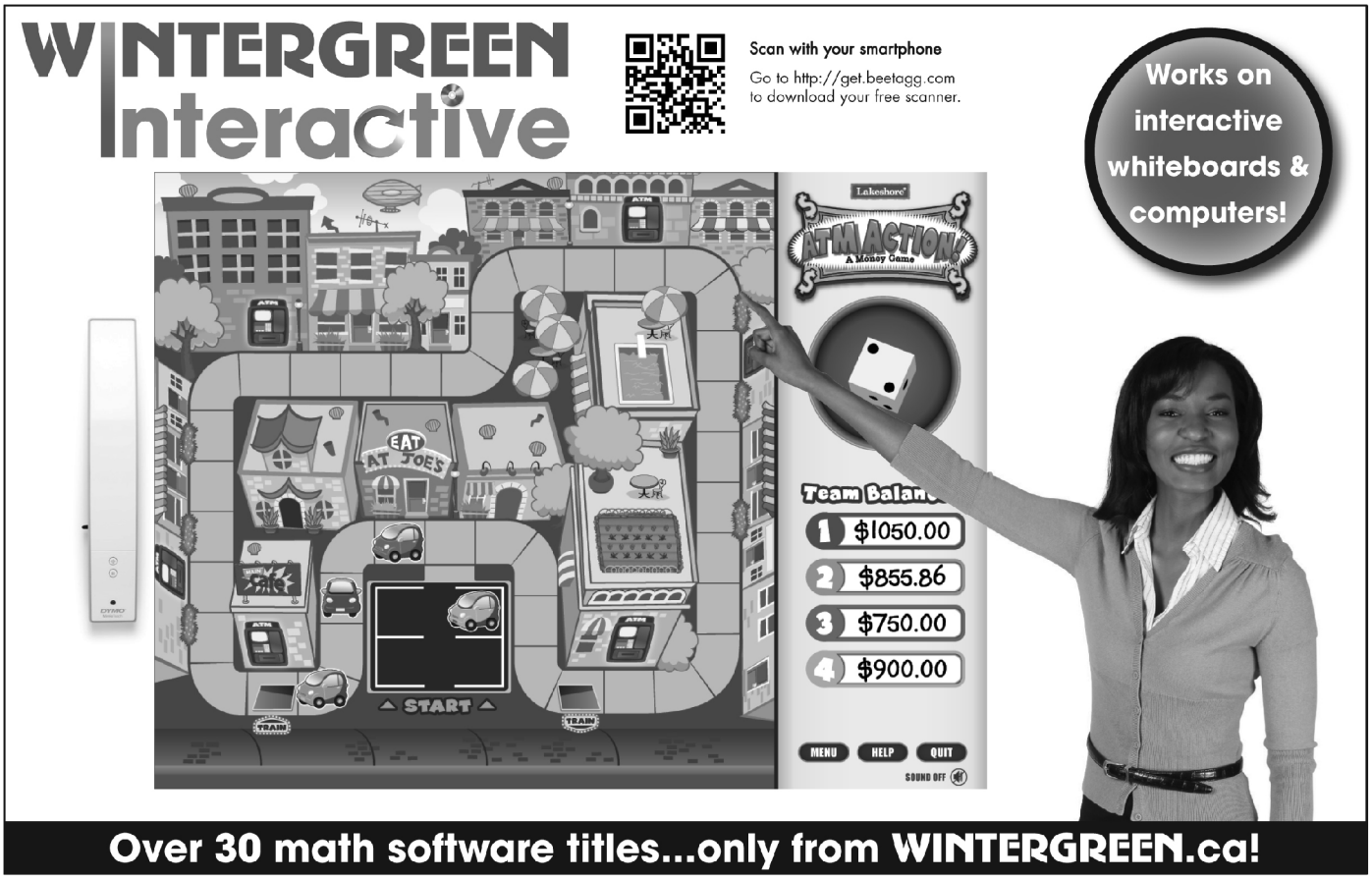
▲ Call for *Gazette* Editor

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▲ President's Message

CONNIE QUADRINI, PRESIDENT



Connie Quadrini is the Grades 7–12 Mathematics Program Consultant for the York Catholic District School Board. Her current work focuses on facilitating networked, job-embedded professional learning involving Grades 4–12 teachers within families of schools, as well as supporting school teams in the collaborative development and implementation of school improvement plans. Connie has served as an OAME director for 6 years and Y4MA executive member for 13 years. She was the co-chair of the OAME 2008 Annual Conference, and was the recipient of the OAME Union Gas Award for Outstanding Leadership in Mathematics Education in 2003. Connie has taken a leadership role in numerous Math GAINS initiatives, including Steering Team Lead for the 2008–2009 Coaching for Math GAINS; co-organizer of Math CAMPPP 2008, 2009, 2010, 2011; and co-facilitator of several professional learning series, including the Mathematics Coaching Cycle, PRIME Leadership Framework Adobe Connect Book Study, and Leadership in K–12 Mathematics System Planning.This year, she presents mathematics leaders with an opportunity to explore assessment within a new OAME Leadership Conference format.

Teachers Preparing for Knowledge Building Classrooms – Aligning, Curriculum, Instruction, and Assessment

In my role as mathematics consultant, I have the privilege of working and learning alongside teachers of mathematics. Through co-planning, co-teaching, and co-debriefing, we work collaboratively to implement teaching approaches and strategies that engage students in rich mathematical experiences, enabling them to develop understanding of concepts as they problem-solve, select tools and strategies, represent their thinking, reason, make connections, and communicate and reflect upon their understanding. During co-planning, we develop a learning goal that reflects the important mathematics or big ideas, and is based on curriculum and mathematical process expectations. Once we articulate the learning goals and anticipated success criteria, we select a problem that will provide an entry point for all students, enabling them to engage in and make sense of the mathematics identified in the learning goal. This problem is crafted to be in the "zone of proximal development" for students, challenging and engaging, yet accessible. In preparation for our co-teaching, we, as learners, do the problem so we can anticipate student solutions. This work naturally prompts us to consider the role of assessment *for* learning, which is, "the process of seeking and interpreting evidence for use by learners and their teachers to decide where the learners are in their learning, where they need to go, and how best to get there" (Assessment Reform Group, 2002, p. 2). As we work to align the learning goal and anticipated success criteria, we prepare "our eyes to see and ears to hear student thinking" (Ontario Ministry of Education, 2005). In our effort to elicit information about student learning, we consider assessment strategies that will help us gather data during the lesson. As we co-teach, we observe students as they engage in the mathematics that emerges while solving the problem. We listen as students pose questions to one another as they clarify and expose thinking. As co-teachers, we then facilitate the analysis of solutions and consolidate the learning through an interactive summary, which we expect will reflect our anticipated success criteria. During the co-debrief session, we triangulate our observations and conversations, and student products (Ontario Ministry of Education, 2010, p. 34) to clearly understand what a student knows and can do. At times, we discover that the student thinking is unclear or lacks sufficient detail for our triangulation. This valuable information serves as a guide for feedback in an effort to move student thinking forward, and prompts us to consider next steps for instruction. As we engage in this process collaboratively, we develop the content knowledge for teaching (CKT-M) needed to assess and respond to students' mathematical thinking.

A Classroom Example

During a co-teaching cycle with a Grade 9 Applied Mathematics teacher, we co-planned a lesson focused on proportional reasoning. In an effort to assess students' representations of proportional thinking and repertoire of problem-solving strategies, we selected and adapted a question from the Grade 9 Applied EQAO Winter Assessment (2006), #2b.

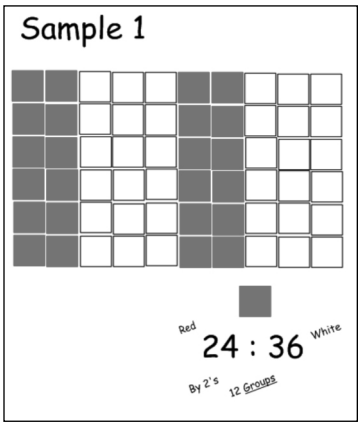
Paul's Quilt

Paul's grandmother is sewing a quilt for him. She asks him to cut red and white pieces. Every 5 pieces in the quilt consist of 2 red pieces and 3 white pieces. If the quilt has a total of 60 pieces, how many pieces are there of each colour? Show your work.

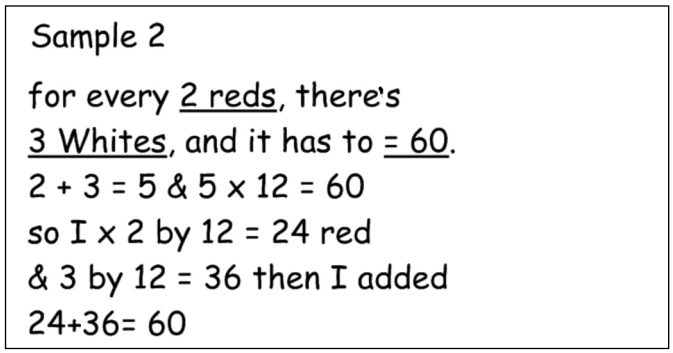
The original problem presented a quilt with a total of 300 pieces. We changed this number to 60, recognizing that this adjustment would enable students to access the problem and solve it, using a variety of tools and strategies.

In preparation for our observations and conversations with students, we considered the way in which we would assess students' thinking.

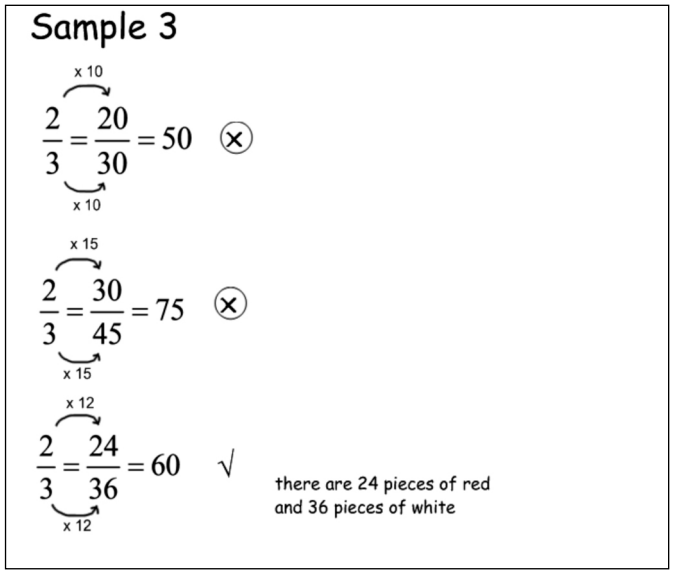
- What mathematical thinking is evident in the solutions?
- What is not clear?
- What questions, if any, should we pose to further elicit student thinking?
- What feedback might we provide?



During the lesson, I circulated as students solved the problem. I observed Maria as she created her model (Sample 1), using red and white tiles. It appeared evident from her model that she understood the proportional relationship (2:3) and the whole (60). In observing Maria, I noticed that she applied her understanding of a proportional relationship by maintaining the proportion to generate the number of red and white pieces. I watched her skip-count to determine the total number of red and white pieces. Maria represented her solution as a ratio, noting "by 2's" and "12 groups." I asked her to share the significance of these notes. Maria indicated that she skip counted by 2's, 12 times in order to determine the number of red pieces. Through observations, conversations with Maria, and by examining her product, it was evident that she had an understanding of the proportional relationship and could solve the problem, using concrete materials. The co-teacher and I would need to support her in further annotating her mathematical thinking on her product.



As the co-teacher and I examined Sample 2, created by Josh, we noticed how he used the given parts to create a whole that would be multiplied by 12 to find the total number of pieces in the quilt. This scale factor was used to determine the number of red and white pieces in the quilt. Clearly, Josh's solution exposed his proportional reasoning.



As Hakim worked, the co-teacher and I watched as he used the parts given in the problem to create a fraction $\frac{2}{3}$ (Sample 3). I wondered whether he would make the common error of comparing this fraction with a fraction involving the whole. To our surprise, we watched as Hakim created equivalent fractions. In my zeal to understand Hakim's thinking, I ask him, "Tell me about your strategy." He proceeded to describe his "test and check" method that had him use easy numbers to multiply by to find the right total. This systematic trial-and-error strategy, unbeknownst to Hakim, enabled him to find the scale factor, and then solve the problem. As teachers, we knew that we would have to address the precision in mathematical form in his solution and agreed that the discussion would focus on the concept of equality.

During our Consolidation, we took the opportunity to have students compare and contrast their solutions. Students quickly identified commonalities across the solutions. They saw the parts, 2 and 3, the wholes, 5 and 60, and the 12, which we named the scale factor. In prompting students to find the 12 in the model from Sample 1, students suggested that we rearrange the 6 x 10 array to form a 12 x 5 array so that the scale factor would be easily seen. We introduced the concept of variable as a placeholder for the unknown number of pieces of, for example, red pieces. We also introduced a proportion equation, $\frac{2}{5} = \frac{x}{60}$, and asked students to identify the similarities and differences between this representation and the representations presented in the student solutions. Students made connections with ease, once again, identifying parts and wholes, and determined the scale factor, which would then be used to find $x = 24$.

Using Assessment for Learning to Inform Instruction

During the co-debrief, the co-teacher and I discussed the mathematical thinking that was evident within the solutions and how our observations and conversations with students opened the window to students' understanding of proportional relationships. Through further deconstruction and analysis of the solutions, we discussed next steps for instruction which, for some, included formalizing the mathematical thinking that was shared during conversations, for others, precision in communicating mathematical thinking, and for the entire class, further supporting students in generalizing a proportional relationship using an equation. The next day's lesson would then provide students with an opportunity to use an elastometer to further develop conceptual understanding of proportions and enable a transition to representing proportional relationships using an equation. For more information on creating and using an elastometer, visit **www.edugains.ca/resources/ProfessionalLearning/SelfDirectedPLM/ClassroomInstructionalStrategies/index.html?movieID=3**.

Creating the Conditions for Success

As I reflect upon this experience, and the many other co-teaching cycles in which I have engaged, I continue to be in awe of the mathematical thinking that students bring to their learning. As teachers, we are called upon to create the opportunities for students to actively engage in learning that evokes and exposes this thinking. Through observations, conversations, and products, we create the ideal conditions for uncovering what students truly understand and can do. As Fullan, Hill, and Crevola (2006) suggest, "Assessment *for* learning is information gathered today about what needs to be done tomorrow," so let us "prepare our eyes to see, ears to hear" and voice to respond.

References

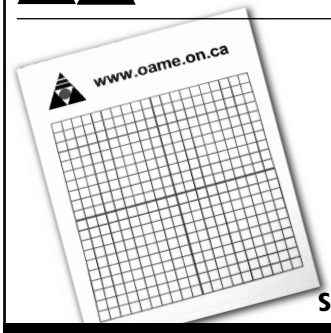
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▲ Technology Corner – iPad in the Classroom

MARY BOURASSA



Mary teaches mathematics at West Carleton Secondary School in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She has presented workshops internationally, authored mathematics resources, is a past VP of OAME and a Past President of COMA. An award-winning teacher, Mary continually strives to learn new and better ways of helping students learn and love mathematics.

I believe that iPads and other tablet computing devices will be in every classroom in the not too distant future. There is great potential in these devices, which is why I have been exploring the uses of my iPad2 in the mathematics classroom for several months now. I am sure much of what I will describe is also available on the BlackBerry® PlayBook™ and other tablet devices. It would be incredible to have iPads for all of my students, but I have yet to manage that, so I am working from the perspective of the teacher with a single iPad.

I teach in a classroom which has a SMART Board™, so the first thing I did was get an app that allows me to remotely connect my iPad to my computer. This is accomplished through Virtual Network Computing (VNC) via a wireless network. A VNC app needs to be on the iPad, VNC software needs to be on the computer being accessed, and the IP address of the computer needs to be known. Once that is configured, you can connect the iPad to the computer, and in my case, this allows me to control the SMART Board remotely. I can be at the back of the room and move the screen shade or write on the Board. Being freed from the front of the room allows such a different dynamic to happen in the classroom. Students could also write their solutions on the iPad if they did not want to go to the SMART Board. And, unlike the SMART AirLiner™ wireless slate, you can see what you are writing on the tablet as you write it.

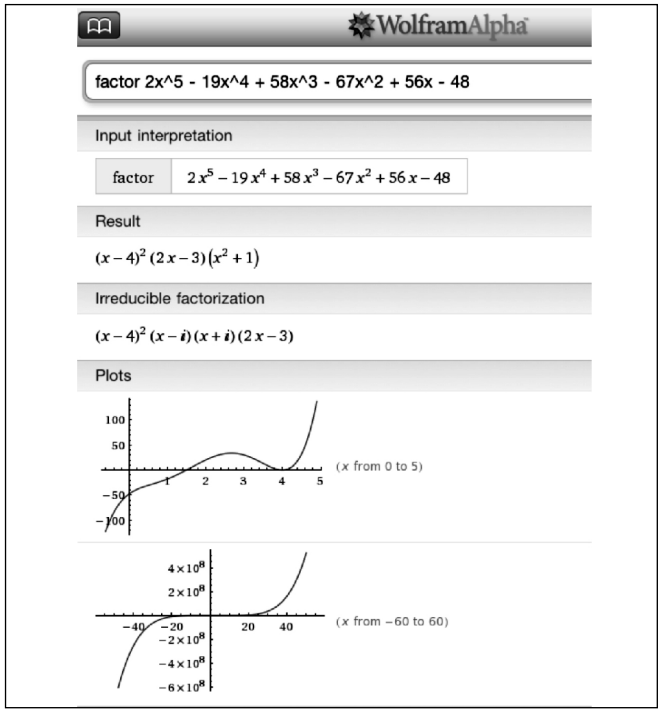
The iPad2 had two cameras on board, so taking pictures of students' work, for example, if they are making posters, is a snap! You can also take videos of presentations or of group work during an exploration or activity or when there are interesting mathematical conversations happening. When students produce brilliant work in a less traditional format, it is great to be able to e-mail it to their parents right on the spot.

There are many, many apps for education and for mathematics. I will share some of my top picks, but more are released every day, so this is by no means a comprehensive list.

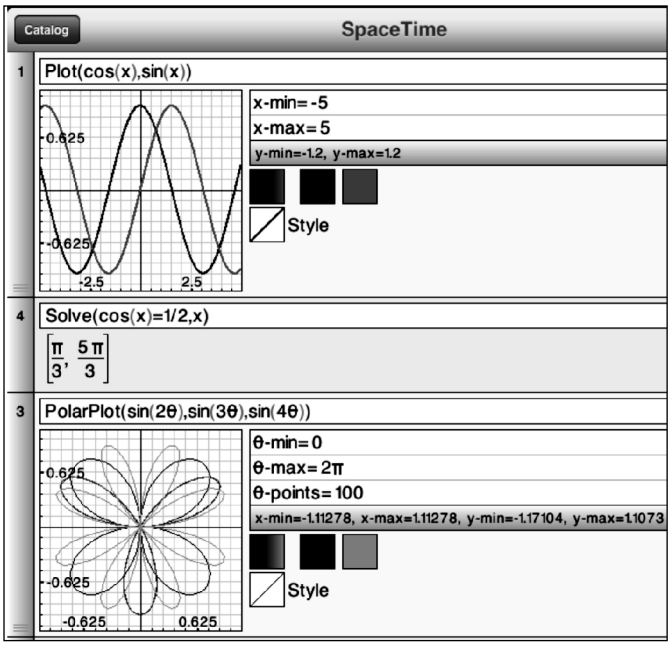
For younger students, there are dot-to-dot apps, counting apps, Tangram apps, math bingo, math with balloons, math with dinosaurs, math with princesses, Ninja math, etc. You get the idea—there is an app for pretty much everything! There are also spelling apps, science apps, history apps, geography apps, etc., all designed for younger children.

Moving on to mathematical apps:

- Wolfram Alpha™ (shown below) is "an engine for computing answers and getting knowledge" **wolframalpha.com**. This is the app for finding out about anything mathematical, from the name of a 19-sided polygon (enneadecagon) to when René Descartes was born (March 31, 1596) to factoring a quintic expression and looking at its graph.



- SpaceTime (shown below) is a powerful mathematics app and graphing calculator, which can do everything from 2-D or 3-D graphing (all in colour) to algebra with CAS built in.



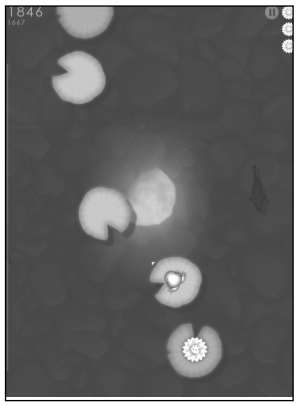
- PocketCAS lite is a free graphing symbolic calculator.
- Quick Graph allows fast 2-D or 3-D graphing.

There are many more excellent mathematics apps, a number of which are free. Hopefully software many of us use, such as The Geometer's Sketchpad and TI-Nspire™, will come out as apps soon.

The iPad2 can be connected to a data projector using a dock connector to VGA adapter so that you can display anything on the iPad (this was not true for the iPad1). Now we can capture students' interest by finding the mathematics in the apps they use. This occurred to me immediately the first time I played Angry Birds. This is a puzzle game in which the player launches birds in an attempt to kill pigs. Where's the math you ask? Each bird's flight path is parabolic! There are so many questions that could follow a screen shot like the one shown in the next column. Students can start by putting the screen shot as the background of a graphing program and determine an equation to represent the parabola shown. Next they can determine equations of parabolas that will free the caged birds, one equation with the vertex below the original and one equation with the vertex above the original. They could calculate the angle at which they need to pull the bird to launch it and determine which path (above or below) has a higher tolerance for error. Other Angry Bird levels have obstacles above the birds, so students would need to factor in where the vertex should be in order to avoid these obstacles. Determining the vertex using various methods is in context and has a purpose. Students could also build their own slingshot/ catapult to test their conjectures, with a picture of a pig as the target, of course! Connections to science can be made by talking about velocity, acceleration, force, and mechanics.



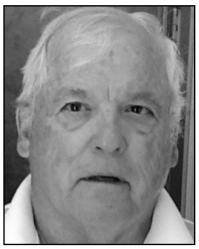
Another app is called Dizzypad HD. A frog rotates on a lilypad and the player must tap to make it jump when the arrow is pointing at the next lilypad or the one following it, which is further away and thus a more difficult move. Students could calculate the time it takes to make one rotation, then calculate the range of angles that will land the frog on the lilypad and how much time they have to tap so that the frog does not end up in the water. A screen shot could be inserted as the background of dynamic geometry software, to explore properties of triangles, and trigonometry could be used to calculate distances or angles after some measurements are taken.



Current textbooks will hopefully become available in a format able to be read on a tablet, be that PDF or as an app. This would allow for interactive content, dynamic links, and real-time updates. I envision a classroom where students no longer have to carry books around, where they can take notes on their tablet, get handouts electronically, and are fully engaged in their learning. There will be those who fear what else students will be able to do with this technology, such as texting, surfing the Internet, and playing games, but students have always found ways to be distracted. It is our job to keep them interested, engaged, and on task and to design lessons, activities, and assessments that reflect how they are learning. The future is here and I am excited to jump in! I welcome comments from those of you already on this journey or those who would like to go down this road, and hope to share what I learn in future columns.▲

▲ OAME/NCTM Report September 2011

TODD ROMIENS



Todd just retired from teaching K–12 methodology in the Faculty of Education at the University of Windsor. He is a past president of OAME and a life member, as well as a previous editor of the Gazette*.*

It is an honour to be asked to be the new NCTM rep to OAME. I'd like to thank Laurie Moher for the splendid job she has done, and I look forward to keeping you all informed of NCTM activities and opportunities.

As you are aware, NCTM provides many publications and professional development opportunities. If you are able, I would highly recommend attending either a regional conference or the annual conference. The time is well spent and you will learn a lot. This fall, regional conferences are being held in the following cities:

Oct 19–20	Atlantic City
Oct 26–27	St. Louis
Nov 2–4	Albuquerque

The 2012 Annual conference will be in Philadelphia, April 25–28. Consult **nctm.org** for more details. If you would like to present at any conference, go to **reconf@nctm.org** to submit a proposal.

Since we are beginning a new school year, it would be a good time to consider joining NCTM (in addition to OAME), if you are not already a member. Full membership for $78 U.S. per year includes your choice of one journal: (printed); *Teaching Children Mathematics* (Grades K–6), *Mathematics Teaching in the Middle School* (Grades 5–9), *or the Mathematics Teacher* (Grades 9–12).The alternative to a printed copy would be to become an e-member for $66 and get your journal online. Journals are published eight times a year, and I am sure you will always find ideas to use in your classroom. Go to **nctm.org** for more information.

As you know from previous issues, NCTM also provides an "Illuminations" site, which has lessons, activities, and web links, as well as a link to learn about the *Principles and Standards* document. As of this printing, there are 106 online activities, 605 lessons for pre-K–12, information about the standards, and hundreds of web links. These are all easily accessed using the navigation bar at the top of the Illuminations home page. I am including four activities, one for each grade band.

Activity for Grades K–2: Bar Graph Investigations

In this activity, students apply what they know about subtracting to compare numbers through constructing bar graphs and using them to answer questions. Students collect data by designing a survey and then completing the bar graph. Tally sheets are used to tabulate the data and graph paper is recommended to draw the bar graph. Examples for the data collection could be favourite fruits, ice cream, school subjects, etc. Students can compare items in the graph by posing questions such as, "How many more students like vanilla ice cream than chocolate?" The "data grapher" outlined in the next activity can also be used to draw the bar graphs.

Activity for Grades 3–5: Data Grapher

This activity can be used to analyze data with bar graphs, line graphs, pie charts, or pictographs. Data can be entered in multiple rows and columns; there is no limit to rows, but only six columns are allowed. Insert your data, choose the type of graph you want, and hit preview. The graph can be saved or printed.

Activity for Grades 9–12: Advanced Data Grapher

This activity extends the use of "Data Grapher" to include scatterplots, histograms, and stem and leaf plots. Students can input their data in multiple rows and columns (maximum of six columns). Labels for the table, *x*-axis, *y*-axis are allowed in order to provide more information about the graph. After the data has been entered, use the "Preview" button to view the graph. The graph can be printed from the preview screen.

Activity for Grades 9–12: Function Matching

This is a great activity to allow students to explore the values of a parameter for any linear, polynomial, exponential, or periodic graph. When a type of graph is selected, one will appear on the screen. A second graph (yours) will also appear. The object is to then select parameters to match your graph to the original. You can check to see if you are correct, zoom in, zoom out, or get hints. Besides being educational, it is fun! ▲

▲ What's the Problem? Doing Fractions Like an Egyptian

Shawn Godin



Shawn is the head of Mathematics, Technology, and Family Studies at Cairine Wilson Secondary School in Orleans, Ontario. The web page for this column is ***whats-the-problem.wikispaces.com****.*

Welcome back to another school year, problem solvers. Last time, I left you with the following problem:

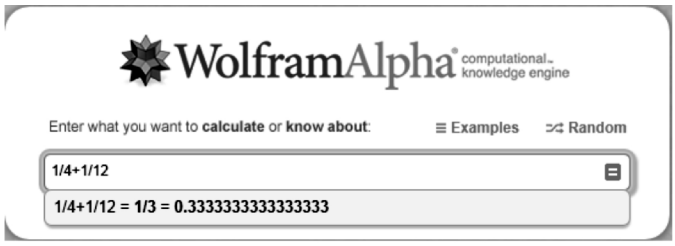
Ancient Egyptians wrote all fractions in terms of distinct unit fractions (that is, in terms of distinct fractions with numerators of 1). For example, instead of writing $\frac{11}{12}$, they would write $\frac{1}{2}+\frac{1}{3}+\frac{1}{12}$. The unit fraction $\frac{1}{2}$ can be written in terms of other unit fractions as $\frac{1}{2}=\frac{1}{3}+\frac{1}{6}$. Find some other unit fractions that can be written as the sum of two unit fractions.

The problem requiring solvers to find an infinite number of unit fractions, each of which could be written as the sum of two unit fractions, appeared as Totten-M1 in the September 2009 issue of *Crux Mathematicorum with Mathematical Mayhem*, and a solution appeared in the April 2010 issue. The problem was dedicated to the memory of my friend, Jim Totten, who was the Editor-in-Chief of *Crux Mathematicorum with Mathematical Mayhem* when I was editing the Mathematical Mayhem section. Jim passed away suddenly shortly after retiring and completing his editorial term and is dearly missed. [You can check out more problems at the *Crux Mathematicorum with Mathematical Mayhem* website **journals.cms.math.ca/CRUX/**.]

Many students, or adults, are not particularly fond of fractions. Yet there are all kinds of interesting mathematical properties associated with fractions. For example, noticing that $\frac{1}{2}=\frac{1}{3}+\frac{1}{6}$ might suggest a possible procedure for writing any unit fraction as the sum of other unit fractions, but more data might be useful. A bit of digging will yield the following examples:

$$\frac{1}{2}=\frac{1}{4}+\frac{1}{4} \qquad \frac{1}{3}=\frac{1}{4}+\frac{1}{12} \qquad \frac{1}{3}=\frac{1}{6}+\frac{1}{6}$$
$$\frac{1}{4}=\frac{1}{5}+\frac{1}{20} \qquad \frac{1}{4}=\frac{1}{6}+\frac{1}{12} \qquad \frac{1}{5}=\frac{1}{6}+\frac{1}{30}$$

You can generate examples a number of ways. As you are searching for solutions of a specific type, going through by hand may not be the best possible strategy. One possibility is to use some sort of computer algebra system. A free example is Wolfram Alpha **www.wolframalpha.com/**. Not only does it act like a mathematized search engine, it also does computations for you. For example, if you enter one of our examples above, it immediately gives the result as a fraction and a decimal. If you press enter, you get more information. It is definitely a site worth showing to your students.



In those examples, $\frac{1}{2}=\frac{1}{4}+\frac{1}{4}$ and $\frac{1}{3}=\frac{1}{6}+\frac{1}{6}$ stand out. From our definition above, they are not Egyptian fractions, since the unit fractions involved are not unique. That's not to say there isn't something interesting going on. In each case, we added two equal fractions together to get a unit fraction whose denominator is half of the original. It shouldn't take much to convince you that we are just playing with equivalent fractions, that is:

$$\frac{1}{3}=\frac{2}{6}=\frac{1}{6}+\frac{1}{6}$$

So we should be able to rattle off many other examples, such as $\frac{1}{11}=\frac{1}{22}+\frac{1}{22}$ and $\frac{1}{123}=\frac{1}{246}+\frac{1}{246}$. We can even take the idea one step further and write $\frac{1}{4}=\frac{1}{12}+\frac{1}{12}+\frac{1}{12}$ or $\frac{1}{5}=\frac{1}{20}+\frac{1}{20}+\frac{1}{20}+\frac{1}{20}$. (Notice that since $\frac{1}{12}+\frac{1}{12}=\frac{2}{12}=\frac{1}{6}$, then $\frac{1}{4}=\frac{1}{12}+\frac{1}{12}+\frac{1}{12}=\frac{1}{6}+\frac{1}{12}$. Does this look familiar?) We can even use the property multiple times to get something like

$$\begin{aligned}\frac{1}{2}&=\frac{1}{4}+\frac{1}{4}\\&=\frac{1}{4}+\frac{1}{8}+\frac{1}{8}\\&=\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{16}\\&=\ldots\end{aligned}$$

I have gone off on a tangent, but that is OK. Mathematics is about exploring and explaining patterns and properties, not just "getting the answer." Discovering the relations above helps solidify conceptual understanding of fractions, as well as giving students the opportunity to communicate mathematically and give you

evidence of the depth of their understanding.

Returning to the problem, we see that the original example has something in common with the new examples:

$$\frac{1}{2}=\frac{1}{3}+\frac{1}{6} \qquad \frac{1}{3}=\frac{1}{4}+\frac{1}{12} \qquad \frac{1}{4}=\frac{1}{5}+\frac{1}{20} \qquad \frac{1}{5}=\frac{1}{6}+\frac{1}{30}$$

Each involves two fractions whose denominators differ by one. Rearranging our original example, we get $\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$, which may shed a little light on our problem. Invoking a little algebra, we see that for any two consecutive whole numbers n and $n+1$, we have

$$\frac{1}{n}-\frac{1}{n+1}=\frac{n+1}{n(n+1)}-\frac{n}{n(n+1)}$$
$$=\frac{1}{n(n+1)}$$

This yields an infinite family of solutions to the original problem, that is $\frac{1}{n}=\frac{1}{n+1}+\frac{1}{n(n+1)}$. You may like to see how you can use this to find results involving three or more unit fractions instead of just two.

For the past several years, I have given my Grade 9 classes a variation of the problem above. I would give them a couple of problems to calculate (to refresh their memory of working with fractions), such as

$$\frac{1}{4}-\frac{1}{5} \qquad \frac{1}{8}-\frac{1}{9} \qquad \frac{1}{21}-\frac{1}{22}$$

I then ask them to look for and describe as many patterns as they can, as well as trying to extend them. Many of the students come up with $\frac{1}{n}-\frac{1}{n+1}=\frac{1}{n(n+1)}$, even though they describe the pattern in words as opposed to algebra. Some students even extend that pattern to the difference between any unit fractions. This family of problems has the strange property that the difference between the fractions is equal to the product. That is,

$$\frac{1}{4}-\frac{1}{5}=\frac{1}{4}\times\frac{1}{5} \qquad \frac{1}{8}-\frac{1}{9}=\frac{1}{8}\times\frac{1}{9} \qquad \frac{1}{21}-\frac{1}{22}=\frac{1}{21}\times\frac{1}{22}$$

It is fascinating to see how many interesting results a problem can yield if you are willing to keep digging after you get "the answer" and you listen to all those little "what if…?" questions.

Now for your homework:

Prove that the quadratic equation $ax^2 + bx + c = 0$ does not have a rational root if a, b, and c are odd integers.

Until next time, happy problem solving! ▲

▲ Hey, It's Elementary – Parents as Teachers: Professional Development as Partnership

LYNDA COLGAN



Lynda Colgan's career has included roles as a classroom teacher, a university professor, and newspaper columnist. Her contributions to mathematics and its teaching have been recognized through awards such as the Marshall McLuhan Foundation Distinguished Teacher Award. Lynda always exhibits a passion for mathematics and views her professional mission as dispelling the myth that math is the bad guy.

As teachers, our lives often feel consumed by school and schooling. So it may come as a surprise to confront the reality that school-age children spend 70 percent of their waking hours (including weekends and holidays) outside of school; therefore, as educators, we must acknowledge that our role is limited to but a small sum of a child's educational opportunity. For children to succeed, we must look beyond our schools to the informal classrooms, opportunities to learn, communities of learners, and enthusiastic learning assistants that abound in our communities and families.

This is not new news: there are decades of substantive research to show that when parents are involved, students have:

- higher grades, test scores, and graduation rates
- better school attendance
- increased motivation, better self-esteem
- lower rates of suspension
- decreased use of drugs and alcohol
- fewer instances of violent behaviour

In fact, family participation in education is twice as predictive of students' academic success as family socio-economic status. Some of the more intensive programs had effects that were ten times greater than other factors, i.e., the more intensely parents are involved, the more beneficial the achievement effects.

The more parents participate in schooling, in a sustained way, at every level—in advocacy, decision-

making, and oversight roles, as fundraisers and boosters, as volunteers and paraprofessionals, and as home teachers—the better for student achievement. And, the earlier in a child's educational process parent involvement begins, the more powerful the effects.

What should make educators sit up and take note, however, is the substantive research that shows that the most effective forms of parent involvement are those which engage parents in working directly with their children on learning activities at home.

Three major factors of parental involvement in the education of their children:

1. Parents' beliefs about what is important, necessary, and permissible for them to do with and on behalf of their children
2. The extent to which parents believe that they can have a positive influence on their children's education
3. Parents' perceptions that their children and school want them to be involved

Although most parents do not know how to help their children with their education, researchers have shown that with guidance and support, they may become increasingly involved in home learning activities and find themselves with opportunities to teach, to be models for, and to guide their children.

If student learning increases when parents are invited into the process by helping at home, then it is clear that by enlisting parents' involvement, educators and administrators are provided with a valuable support system that has the potential to create an expert team that is working for each child's success.

While researchers have shown that the vast majority of parents are willing to assist their students in learning, they have also gathered data that indicates that most of the time, parents are not sure what assistance is most helpful and appropriate.

The implications of this body of research suggests that educators would be well advised to help parents connect to their children's learning, thereby enabling parents to communicate in powerful ways that they value what their children achieve. Whether it's working together with ten frames, displaying student bar graphs at home on the refrigerator, or responding to a particular class assignment about the role of decimals in personal budgets, parents' actions communicate to their children that mathematics education is important and accessible.

Acting upon this extensive body of research, and Joyce Epstein's framework for the six different types of parent involvement, the Community Outreach Centre at the Faculty of Education at Queen's University, has developed and implemented a novel school and family partnership program in Eastern Ontario: *Parent Mathematics Courses*.

Unlike *Family Math* courses that typically provide opportunities for parents and children to engage in mathematics activities together in a low-risk, participatory environment, the *Parent Mathematics Courses* offered by the Community Outreach Centre at Queen's and

Top Five Reasons to Engage Parents

1. Decades of research show that when parents are involved students have:
   - Higher grades, test scores, and graduation rates
   - Better school attendance
   - Increased motivation, better self-esteem
   - Lower rates of suspension
   - Decreased use of drugs and alcohol
   - Fewer instances of violent behavior

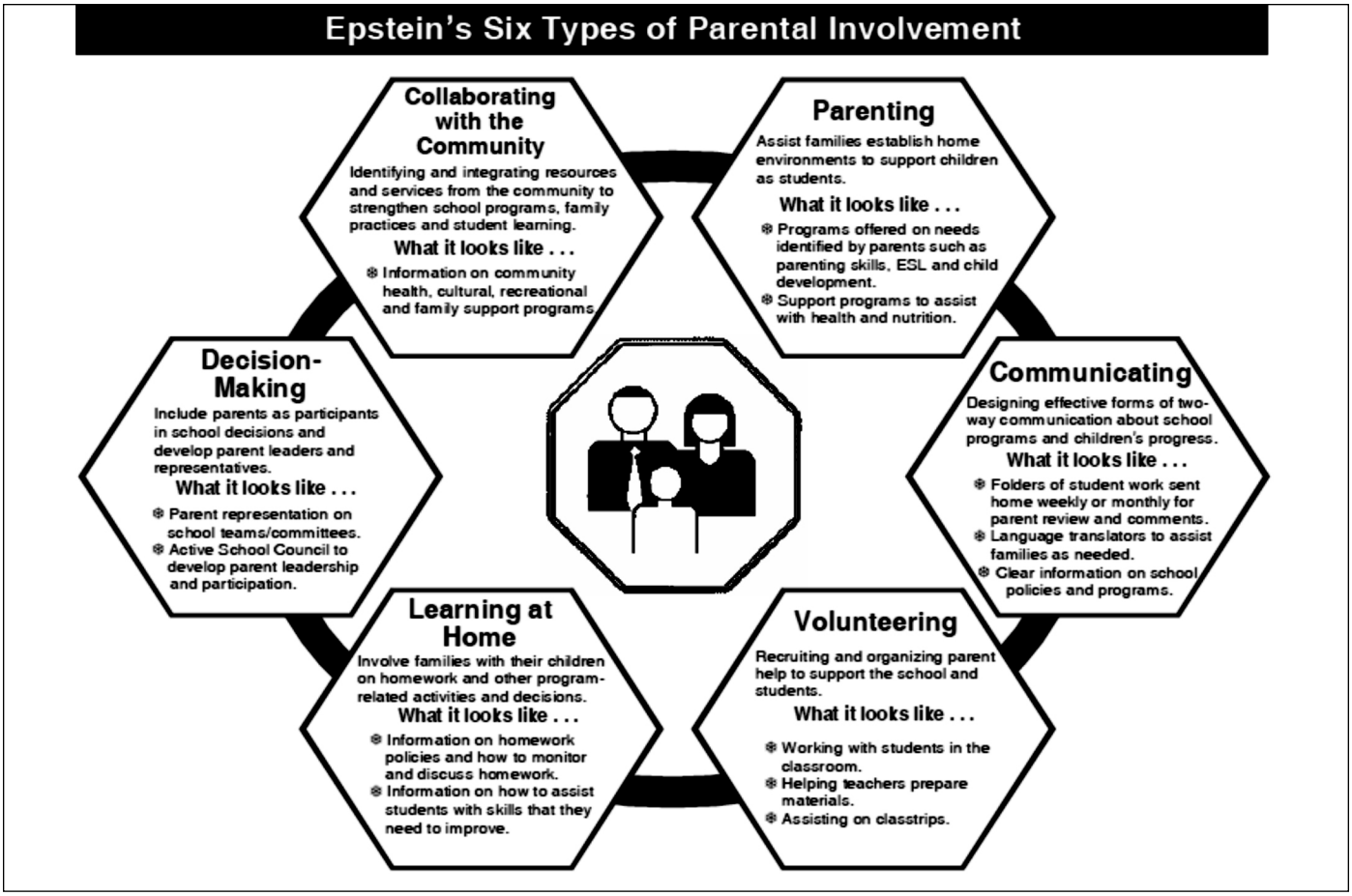
   National Parent Teacher Association
2. Family participation in education is *twice* as predictive of students' academic success as family socioeconomic status. Some of the more intensive programs had effects that were *10 times* greater than other factors.

   Walberg (1984) in his review of 29 studies of school-parent programs.
3. School benefits:
   - Improves teacher morale
   - Higher ratings of teachers by parents
   - More support from families
   - Better reputations in the community

   Henderson & Berla (1994).
4. Parent involvement leads to feelings of ownership, resulting in increased support of schools.

   Davies (1988).
5. Parents express a genuine and deep-seated desire to help their children succeed academically, regardless of differences in socioeconomic status, race, ethnicity, and cultural background.

   Mapp (1999)



through local families of schools (funded by the Ministry of Education's *Parent's Reaching Out Grants*) provide multi-week parent-only courses (differentiated by division, i.e., primary, junior, or intermediate). Parents receive instruction in both mathematics content (e.g., subitizing and fact families, expanded notation and regrouping, theoretical probability, and stem-and-leaf graphs) and pedagogical content knowledge related to mathematics (problem-solving strategies; multiple solutions; and mathematical communication through symbols, numbers, diagrams, models, graphs, and charts).

These courses were developed in response to requests from parents and caregivers who called the Community Outreach Centre (COC) or completed surveys initiated by the COC. Parents were clear—they wanted to understand not only the rationale for the elementary mathematics curriculum, but the content and ancillary manipulatives that comprised elementary mathematics programs. When their child brought home a *ten frame* or a *hundreds chart*, they wanted to know not only the correct procedures for using the resource, but also why it was important to their child's learning. When their child came home and told them that "borrowing" and "carrying" were arithmetic swear words, they wanted to know why this was the case and what procedures should be used to perform multi-digit computations. Parents were clear: they did not want to undermine the teacher, nor negatively impact their child's achievement, by showing them "their way"—typically an algorithm (traditional or invented) that worked, but defied explanation. Neither did they want to face their child's ire or worse: disappointment.

For five weeks, a committed cohort of "parents" (including grandparents, foster parents, older siblings, and caregivers) attended one of the three division-based courses. Each week, there was a lecture about the strand and an overview of the content was provided. In each three-hour class, parents were asked to respond to readings about relational versus instrumental learning or explain what Sputnik had to do with mathematics in school. They were introduced to dot cards or fraction strips; Bloom's Taxonomy or standard problem-solving strategies, rubrics and exemplars or the notion of "big ideas" in mathematics. They had opportunities to learn with rekenreks and sorobons; they were introduced to Egyptian multiplication and Napier's Bones; they measured with string and trundle wheels. In other words,

they relearned elementary mathematics content through the eyes of their child. They had opportunities to learn about and practise questioning techniques and problem-posing strategies. They solved open-ended problems, engaged in lively discussion about the merits of various approaches, and were assigned homework: age-appropriate math problems for the kitchen table. Each week, the task was to solve a problem with children at home and bring the solutions back the following week for discussion and debriefing. In this way, using authentic student samples, parents learned about the value of alternate solutions and the ways in which various solutions would be assessed, evaluated, and graded, and why.

In short, the course was a much abbreviated pre-service mathematics curriculum course. In the same way that a teacher education course provides a novice educator with the research rationale, skills, and opportunity to learn and reflect on practice by enacting theory in the classroom (i.e., modelling and expanding upon practices demonstrated by an expert pedagogue), the COC mathematics courses for parents were aimed at helping every parent to acquire the knowledge, skills, and attitudes to feel empowered and welcome to participate in their child's mathematics education. Using research to inform its practice, the COC sought to design programs that would require parents to serve in an educative role directly with their children by applying curriculum-specific knowledge and pedagogy in the "home" classroom. An explicit goal of the course was to encourage parents to see that they were (at least) equal collaborators in their child's mathematics education and therefore should develop practices that could support the partnership.

As an instructor, I found the experience of teaching a curriculum methods course to parents enormously rewarding. These were students who were intellectually hungry and curious. They were generous in expressing their enthusiasm and surprise. They were unabashed about their confusion and lack of confidence. There was no attrition in these courses… in fact, numbers increased as the courses progressed.

While newsletters, response journals and day planners are effective day-to-day communication tools

Strategy	When to Use it	How To Use It
1. Draw a Diagram	• The problem gives infomation that can be shown in a diagram. • The question asks for a solution that can be found, using the diagram.	1. Find the information needed for the diagram. 2. Draw and label the diagram with information from the problem. 3. Use the diagram to answer the question.
2. Find a Pattern	• The problem gives information that follows a pattern. • The question asks for a solution related to the problem.	1. Find the information that follows a pattern. 2. Find the rule for the pattern. 3. Use the rule to solve the problem.
3. Act It Out	• The problem gives information that can be acted out. • The question asks for a solution that can be found by acting something out.	1. Find information that can be acted out. 2. Act out the information in as many ways as possible. 3. Choose the way that answers the question.
4. Make a Table	• The problem gives more than one set of data. • The question asks for the data to be continued.	1. List the sets of data. 2. Make a table of the data. Circle the information needed to answer the question.
5. Write an Equation	• The problem tells about a situation. • The question asks for an unknown amount.	1. Write what the situation is. 2. Write what the unknown is. 3. Choose a letter for the unknown. 4. Write an equation, and solve for the unknown.
6. Guess and Check	• The problem gives a total amount. • The question asks for the kind and number of items in the total.	1. Find the total amount. 2. Find the different items that make up the total. 3. Make a reasonable guess about the kind and number of items; then check it. Continue guessing and checking until the guess is correct.

for quick messages about events, behaviours, and assignments, the experience of the COC concludes with a clear message: parents want to support mathematics learning, but the changes to both breadth and depth of content, algorithms, pedagogy (including manipulatives and assessment), and classroom environment are staggering developments... even to parents of primary-division children. Therefore, sustained authentic, comprehensive, and respectful professional development is as essential for parents as it is for teachers. If we agree that education begins at home and parents are their children's first and most influential teachers, then supporting and educating parents is a logical strategy. As they say, if you snooze, you lose..., so I encourage mathematics educators to include parents in professional development programs. In terms of transparency and goodwill, there could be no greater generosity: a true enactment of the credo that *it takes a whole village to raise a child*.

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▲ Data Rocks – StatCan's Treasure Trove of Math Resources

Elise Mennie



Elise Mennie manages communications for the "Students and Teachers" section of the Statistics Canada website.

Statistics Canada is a valuable source for teaching statistics and data management in Ontario. In fact, the StatCan website has millions of numbers on everything under the Canadian sun, but how do you find what you need? Just go to the **Students and Teachers** section **www.statcan.gc.ca/learningresources** and click on **Teachers** to find all that's relevant to you, classified by school subject.

You can watch the brief "Just for teachers" video tutorial to orient yourself to our information and services. Then jump right into the Mathematics section and choose your level:

- Kindergarten to Grade 5
- Grades 6 to 8, or
- Grades 9 to 12

In this column, I'll describe the most popular resources and their connection to Ontario math curriculum.

Census at School Online Survey Project



Developed in conjunction with the Fields Math Research Institute, this exciting, active learning project supports the data-management outcomes in Grades 4 to 8 curriculum: "Students will collect and organize categorical, discrete, or continuous primary data and secondary data and display the data in charts, tables, and graphs."

After students answer the **Census at School** online questionnaire, the teacher downloads a spreadsheet of the class responses. These primary data are used to

study the distribution of data, discrete and continuous variation, or mean, median, and mode. Students make different types of graphs, study bias, investigate questions, and draw conclusions.

Increasing numbers of Ontario teachers, particularly in Grades 6 to 8, are taking advantage of the Census at School project. In 2010–2011, Ontario students represented close to 60 percent of about 28 000 Canadian participants.



Census at School results import very nicely into Tinkerplots, Open calc, Excel, and Fathom softwares, so that students can have fun exploring visualizations of their data. Some teachers are using their class results with SMART Boards™ as well.

At the secondary level, Census at School is an excellent resource for applied mathematics for everyday life, such as the Grade 12 College and Apprenticeship Mathematics. As students personally experience the survey design, they better understand how data are collected and are more interested in learning how to analyze data.

In the Grade 12 Mathematics of Data Management course, students use random samples of student responses from Canada and other countries (such as the U.K., Australia, and South Africa) to explore the variability inherent in data. For example, students can investigate the variability in the median and mean of different data samples of equal sizes on a topic like percentage of students who smoke.

Teacher's Guide to Data Discovery

Written by a teacher, this primer on different types of data and graphs is helpful to consult before analyzing Census at School data with your class. The **Teacher's Guide to Data Discovery** gives a good overview to support you in helping students develop basic statistical skills. It specifically covers:

- finding interesting and grade-appropriate Canadian data sets
- choosing appropriate graphs for different kinds of data
- calculating basic statistical measures, with or without statistical software

How to Do a Survey: Statistics, Power from Data!

Make sure to consult our essential reference on how to do a survey: *Statistics: Power from Data!* This online reference covers everything from sampling methods to questionnaire design and from graphing to analytical measures. Each chapter includes exercises and links to related lesson plans.

The new chapter, "Planning a survey," takes you through each step of the survey process, giving the example of how a secondary student council might organize a survey to help with decision making. Popular sections for intermediate- and secondary-level data management are "Data collection," "Types of graphs," and "Measures of central tendency."

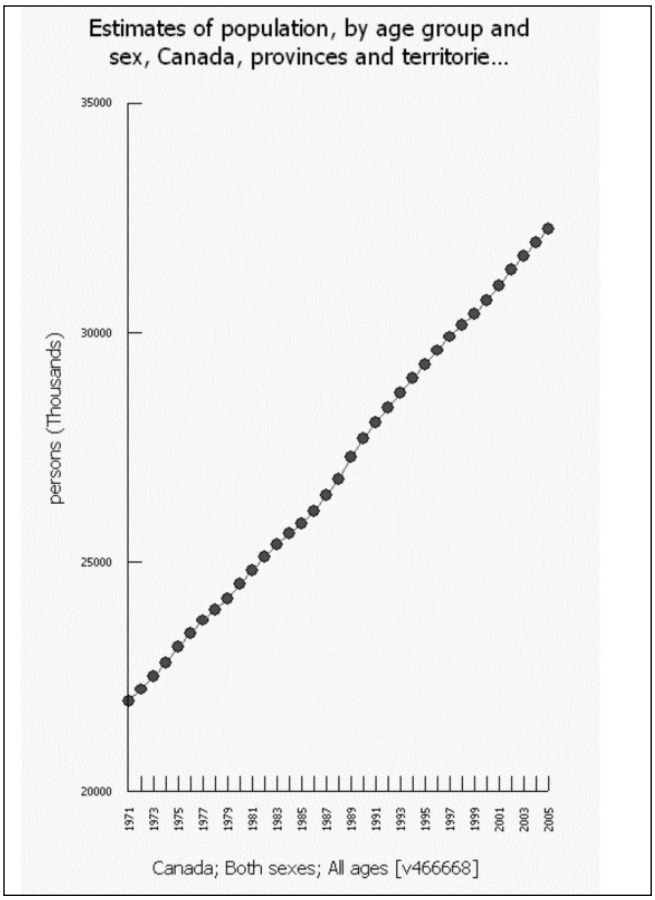
E-STAT and Function Modelling

Whenever you teach functions—in university or college preparation courses, from Grades 9 to 12—the learning expectations include collecting and graphing secondary data to model different types of functions. The **E-STAT** database is an excellent source of real-world data sets for this purpose; it's a huge storehouse of socio-economic time series data from hundreds of StatCan surveys.

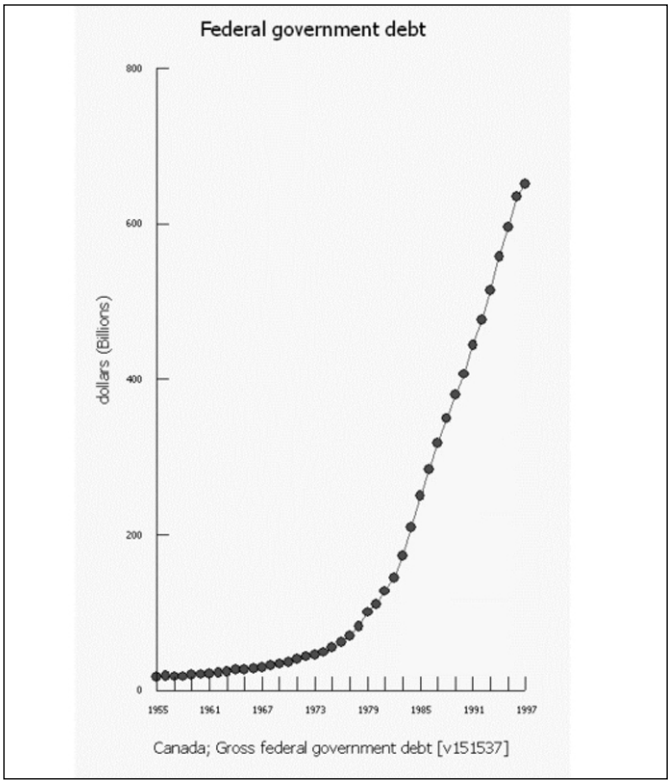
We wanted to save you the trouble of randomly searching for data that can be modelled by a linear, exponential, quadratic, or sinusoidal pattern (or even a logistic, cubic, or combined one). Our **Function Modelling** resource provides over 35 examples of data that can be approximated by these different function models. A lesson plan is included for each type of function. Here are some example models:



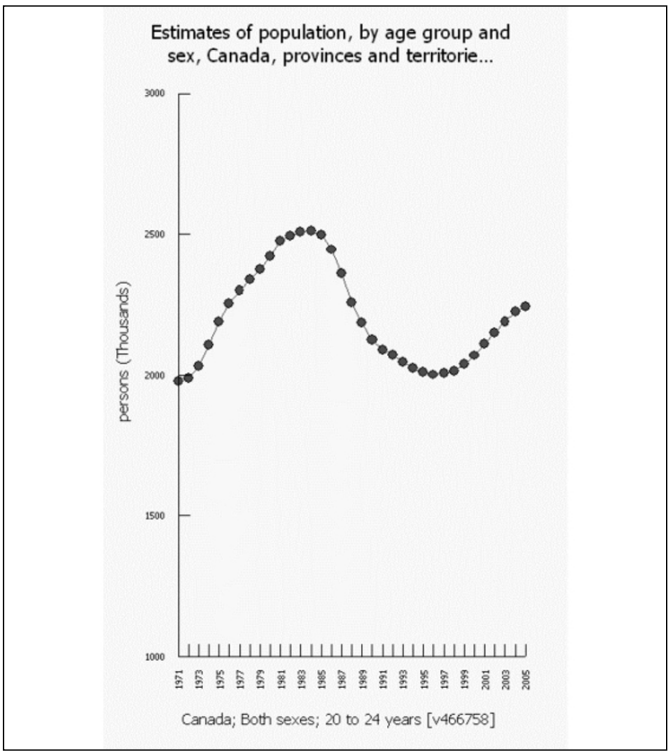
Quadratic: Number of males registered in apprenticeship programs, 1991 to 2001



Linear: Population growth, 1971 to 2005



Exponential: Federal government debt, 1955 to 1977



Sinusoidal: Population aged 20 to 24, 1971 to 2005

Investigating Social Justice Issues

The **Investigating Social Justice Issues** resource suggests a number of data sets and significant questions for students to explore. Found in the Grades 9 to 12 section, it offers topics in areas such as crime, health, the environment, labour, or education. Students can download the data into dynamic analysis software, like Fathom, and look for correlations between variables to test a hypothesis. For example, is there a relationship between literacy and income levels? How do greenhouse gas emissions from different sectors and industries compare and what does this say about our choices as a society?

Teachers are perhaps a student's best resource when it comes to making sense of the numbers about a social justice issue, or exploring what the numbers demonstrate. We encourage you to open a discussion with your students to challenge them to think critically about the circumstance around the numbers and to explore more deeply the environment surrounding the issues.

Intermediate students may be more interested in data that is closely related to them. You can use data of a local nature, such as primary class data on recycling from the Census at School survey.

Data Analysis Fairs



The **Data Analysis Projects and Fairs** section helps you organize and participate in a data fair. Similar to a science fair, the data analysis fair showcases student projects. Students answer a research question, using statistical techniques and graphical presentations, and present their work as a written or electronic report or a poster. The first data analysis fair was organized by Statistics Canada in 2003 to showcase exemplary culminating projects of students taking the Grade 12 Mathematics for Data Management course.

Since then, educators across Ontario have expanded this concept to celebrate student data-management work at other grade levels as well. The Halton District School Board has organized the Halton Peel Regional Data Fair for the past nine years. The data fair involved high school students as well as Grades 7 and 8 students. An enterprising Grades 4–5 teacher at Kleinburg Public School in York Region had her students present projects to the whole school and to parents.

Finding Data for Teaching and Projects

Students can simply click on "**How to...** find data, maps, and more" on the **Students** page, to access sources of data for their projects. On the **Teachers** page, under **Mathematics**, click on the **Data** tab at each grade level to link to sources including: *Summary tables* (on Canada's people, economy, and government), *Community profiles* (population characteristics for every community in Canada), various *Census* databases, as well as the *E-STAT* database. The Grades 9 to 12 **Data** section also offers large microdata sets, including instructions and a lesson plan, which are useful for data-analysis projects.

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▲ In the Middle – To Inspire or Perspire.... That is the Question! Engaging Students, Parents and the Community in Mathematics

CAROLINE ROSENBLOOM



Caroline Rosenbloom is currently on secondment as an Elementary Coordinator in the J/I Midtown Technology Option and Mathematics Instructor in the pre-service program at OISE/UT. She has enjoyed working with intermediate students and teachers as a middle school teacher, Mathematics Instructional Leader for the Toronto District School Board, and Education Officer at EQAO. Caroline is co-author of "Investigating Mathematics Using Polydrons."

Imagine that you are at a party, engaged in conversation. Someone asks what you do for a living. You say, "I'm a math teacher." To which they respond, "Oh, you are one of those people who like math. I wasn't good at math (or I hated math.)." Unfortunately, the attitude underlying this response is easily transferred to their children and then into the classroom.

Imagine turning the situation around, inspiring the adults in hopes of having them transfer mathematical excitement to their children! It's not just the students we want to inspire, but the people outside our classrooms as well. By exposing parents and other community members to mathematics, its curiosities, and its power, perhaps we can cause ripples that will reach all the way back to the classroom, affecting the whole learning environment.

Inspiring the Student

The most effective way to inspire students is to help them gain confidence in math. Students begin to perspire in math class when they don't feel confident they can answer a question. In the intermediate grades in particular, students are very aware of how they appear in front of their peers and often try to hide what they don't know.

> *A learner's experience in the classroom frames the view she/he will have about the subject."*
>
> ~ From the book *Primary Questions & Prompts*

Sometimes questions can be closed, leading to a single strategy that may or may not easily come to a student. Such questions perpetuate the myth that there is one way to solve a problem and that getting the right answer is all that matters. Our classroom questions and prompts should inspire students to reason, think, and express their mathematical ideas. They should model mathematical thinking. We may see this as a futile path, but the persistence and high expectations pay off. As students become more confident, and are able to articulate their thinking, so does their excitement for learning grows!

Anne Watson and John Mason (1998) generalized six types of questions and prompts that could be used while teaching math.

Types of Questions and Prompts	Examples of General Questions and Prompts
Exemplifying/Specializing	• Give one or more examples of… • What makes… an example of? • Why does ½ of ½ give an answer that is less than ½?
Completing/Deleting/Correcting	• What can be added without affecting…? • Tell me what is wrong with the statement, "The area of the polygon is the product of the edges" • What needs to be changed so that…?
Comparing/Sorting/Organizing	• What is similar/different about these solutions? • Sort or organize the following according to…. • Is multiplication a technique for making things bigger?
Changing/Varying/ Reversing/Altering	• Change the definition of a square so that it describes a rectangle. • What if...? • A rectangle has four right-angles. • How many right angles can a • pentagon have?
Generalizing/Conjecturing	• Is it always, sometimes, never true that • between any two rational numbers • there is another rational number?
	• What can change and what has to stay the same so that… is still true?
Explaining/Justifying/ Verifying/Refuting	• Explain why…. • Why do $x + 1 = 3$, $2x + 2 = 6$, $nx + n = 3n$ all give the same answer? • How is… used in…?

Prompting and questioning in this way modifies our instruction. We develop a community of inquirers who talk to one another, investigate, ask questions and struggle with the mathematics in a collaborative environment. Other great questions can be found at **nrich.maths.org** and **http://www.edugains.ca/newsite/math2/dimathcards8x11.html**.

Inspiring the Parents

How many times have you heard parents say, "The math my child does is so different from what I learned" or "I don't know how to help them with math." We often have curriculum nights in September to inform our parents about classroom procedures. We also have Family Math nights to engage whole families in activities. As teachers, we should be mindful of a possible disconnect between parents' math experience and what their child is learning in school. Mathematics itself has not changed; however, the types of questions and ways they are asked may be strikingly different. Today's students may have more, and different, tools available than their parents did. The breadth and depth of understanding that is required today might be different from that required of our students' parents.



Parent-only information sessions in the Fall are a great way to get to know parents, help them understand their own fears in mathematics, and expose them to ways to understand our curriculum. As we do with our students, parents can experience math problems, using the tools and questions we use in class. If a parent is having difficulty helping his or her child, we can teach him or her questioning and investigative strategies that apply, even when one does not know the underlying math. For example, "Why do you think your answer is correct?" and "What happens if we change this number to that number?" or "Just take a guess and then lets see why it's wrong." Websites like *Figure This!, Math Playground, National Library of Virtual Manipulatives* and *Maths Dictionary* are great resources for parents. These sessions are great opportunities for discussion and ways

to provide real hands-on experience for parents. You know you've inspired parents when you hear them say, "I wish I knew this when I went to school!"

Inspiring the Community

Dictionary.com defines busking as *entertaining on the street or in a public place, or to make a showy or noisy appeal*. Math busking, which was seen in Britain, was a way to wow the public with math. Bev Caswell, Mathematics Coordinator for the Robertson Program for Inquiry-Based Teaching of Math and Science at the Dr. Eric Jackman Institute of Child Study, is working with 24 TDSB teachers of Model Schools for Inner Cities in a special AQ course. Teachers entertained the public with life-size math tricks and puzzles at the corner of Spadina & Bloor in what Bev calls Toronto's first *Math Flash Mob*. What a great way to inspire the public!





Schools can also inspire their community by math busking at the local library, at a nearby park, or at the Spring Fun Fair at school. Older students can also math busk in the school playground at recess for the younger students.

Inspiring the community also means having the community involved. Run a math job fair, where parents or members of the community share how they use math in their job, or create a community math blog on your school website.

So get your school team involved to excite your students, parents, and community with math! Our focus on *inspiring* to learn will reduce *perspiring* in mathematics, making learning math more enjoyable for all!

Enjoy the start of the new school year in math!

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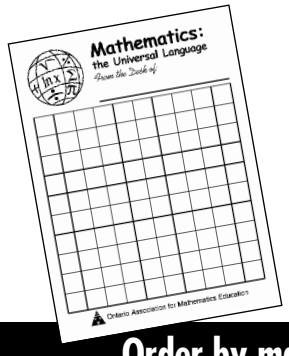
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▲ My Favourite Lesson

CRYSTAL CHILVERS



Crystal Chilvers is currently released from her duties as department head and secondary math teacher at Centre Wellington in the Upper Grand District School Board to work on mathCLIPS.ca. She has contributed to some Nelson mathematics textbooks and teacher resources, led writing teams working on Grades 11 and 12 TIPS resources, facilitated at math CAMPPP, and coached in her board.

My "Favourite Lesson" was borne out of a collaborative effort with a colleague, Shelley Yearley, at Math CAMPPP 2009. We were looking to extend thoughts on a Big Idea discussion, and what started as an investigation turned into an amazing learning experience for teachers and students alike. The lesson has been recreated a number of times, with equally positive results.

I had the pleasure of co-teaching this lesson in many Grade 10 academic math classes during the 2009–2010 school year, co-teaching with four different teachers. Initially, we co-taught the lessons to learn from one another; the department found that co-teaching the lesson was so valuable, that this collegiality has continued to today. They continue to use their prep periods to either co-teach lessons or cover someone else's class so that a teacher can participate in the lesson.

How the lesson works:

MINDS ON!

Students discussed the similarities and differences between various linear growing patterns that they learned last year, using an activity called "Secret Pattern Building Challenge" created by Dr. Ruth Beatty, Lakehead University, Orillia campus. Next, they were asked to compare, using pictorial representations, those familiar growing patterns, to a new non-linear growing pattern: "number of tiles = the area of a square whose side length is the term number." At this point, a full-class discussion ensued, and the term quadratic was introduced. This discussion allowed teachers time to assess where the students were at the beginning of the lesson.

ACTION!

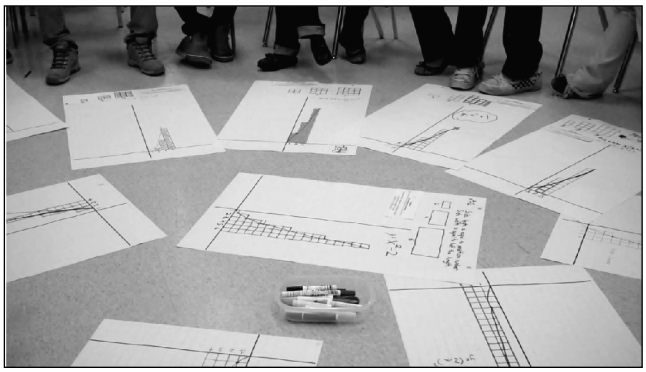
Each pair of students was given a different pattern to build. This series of patterns is provided on BLM 1.1 in Appendix 1. They then discussed similarities and differences between their new pattern rule and the one previously done as a class. This is where the fun began! The teachers circulated to discuss with each pair what assumptions they made, what conclusions they had drawn, and what questions they still had. While circulating, I was able to meet up with the classroom teacher to share observations; that allowed students more time to flesh out ideas, independently.

Next the students turned their pattern rule card upside-down on their desk, and began a "gallery walk." With their partner, students moved from desk to desk, trying to guess the pattern rule that was represented in front of them. They then turned the card over to see if they were right.

When the gallery walk was complete, students recorded four representations of their patterns rules on chart paper—word, pictorial, graphical, and algebraic. Once again, the pairs revisited the assumptions and observations they originally made and compared those to the newly created graphic representations.

CONSOLIDATE/DEBRIEF

We chose to debrief the activity, using a new experience, the community circle share. The desks were pushed aside and chairs were placed in a large circle surrounding the chart papers. Students were then asked to first decide which graphs were most similar to each other. They were to state how they were similar and how they were different. This was treated in a think/pair/share fashion, with students silently looking first, then discussing with an elbow partner, and finally sharing with the entire group. Each class shared unique observations that brought about different conversations. This prompted others to speak, and soon this snowballed into diverse, but thoughtful, moments for all.



My experience with this lesson:

Each time I had the pleasure of co-teaching this lesson, I came out energized. I was energized by the amount of preparation that this lesson provided to start a challenging unit, the power of the pictorial representations and the insights students made from them, and the learning that took place for both teachers and students alike.

Here is some of the learning that I observed:

1) Teachers learned the value of not intervening when students misinterpreted their pattern rule.

Some pairs initially modelled their pattern rule incorrectly. Teachers observed how those misconstructions were handled, without intervening. In some cases, by observing other patterns, students were able to correct their own work. In other cases, students stayed behind at an incorrect representation to discuss the assumptions. The talk in the room was positive and encouraging in all cases. Most importantly, by discussing misinterpretations, students who were initially unsure were then able to go deeper into the pattern discussions. Teachers were impressed with the way the students worked together to ensure that all classmates were successful. In fact, after the lesson, our debriefing turned into a bit of a brag session, with each teacher sharing insights that his or her students had shown.

2) Teachers observed students anticipating symbolic representations.

When students looked at their own pattern rule, they didn't immediately turn that rule into an equation—in fact, most did not create an equation until after they had created the pattern with the tiles or graphed it. So, when they circulated during the gallery walk and began guessing at the pattern rule for each representation, I expected to hear students state a pattern rule in words. However, in most cases, students looked at the representation and stated an algebraic equation. They then flipped over the pattern rule card and translated it into an equation to see if it matched. Students seemed empowered by their own mathematical language capabilities *(e.g. "I think this one is something like* x *plus one squared" card flipped and read aloud, "a square whose side length is one more than the term number. So that's* x *plus one, and it's a square, so* x *plus one squared. I'm right!")*.

3) A Math Talk Learning Community grew organically.

During the gallery walk, students did not need to be told to justify their guesses. They either just started right into their justification because they wanted to prove that they could guess the pattern rule quickest, or because discussion started after another classmate said, "I don't see it!" or "How'd you get that so fast?" In either case, students confirmed or denied what they found. The students who initially didn't understand were the ones who volunteered to guess at the next pattern rule confidently. Teachers allowed for the conversations to expand without worrying about the clock because they saw the value of the math talk that was occurring.

4) Students showed early insight into transformations.

Students observed that the graphs of $y = x^2$, $y = x^2 + 1$, $y = (x + 1)^2$, $y = (x - 1)^2$, and $y = x^2 - 1$ were all pretty much the same, only "moved" 1 unit in some direction. They also noted that the graph of $y = 2x^2$ looked like a "stretched version" of $y = x^2$, and $y = \frac{1}{2}x^2$ looked like a "squished version" of $y = x^2$.

Students were quick to point out that $y = (x + 1)^2$ was the graph $y = x^2$ translated to the left 1 unit. When asked if that seemed counterintuitive (as I have often heard in the past), a student would point out that it made perfect sense if you looked at the pictorial representations of each pattern rule. In term 2, for instance, on $y = x^2$, the area was the same as that for term 1 in $y = (x + 1)^2$. In the past, I've heard students memorizing rules like "If it's inside the brackets go backwards to what you think," but after this lesson, students stopped using rules and used the pictorial model to justify the direction of the transformation.

Clearly, the pictorial model was more impactful than the graphical alone when looking at transformations left and right.

5) Students made powerful connections to prior learning.

One student started out saying that the graphs of $y = x^2$ and $y = x^2 + 1$ had the same slope, but that the slope of $y = 2x^2$ was twice that of $y = x^2$. He brought his understanding of slope to a new concept without the language, but with insight. He was willing to take a risk and basically described the step pattern of a quadratic. He explained that if you were to count on $y = x^2$ how many spaces you go up for each step over, then you would end up following a pattern: 1, 3, 5, etc., and he pointed out that you would see the same pattern for $y = x^2 + 1$. He then moved on to the graph of $y = 2x^2$ and explained that the pattern had doubled: over 1, up 2; over 1, up 6; over 1, up 10. When asked if he could

prove his hypothesis with the other graphs, many other students joined in the discussion, pointing out how this new pattern fit each of the graphs. One student's observations during the circle share grew into a rich whole-class discussion that students eventually referred back to throughout the quadratic unit.

6) Teachers changed teaching methods based on student observations.

Teachers used the circle share materials and experience to drive the lessons on transformations. As the chart papers were posted in the classroom, students were continually reminded of their earlier discoveries. They added notes to them throughout the quadratic unit and created new visuals as needed. By using student-generated materials, teachers really were able to start where their students were (Hume, 2007).

By giving students greater control at the beginning of the quadratics unit, more was achieved in the long run. By not intervening, teachers observed that students spoke more freely with one another about mathematics. Students were more engaged in their learning because of the use of manipulatives and openness of the task. And because of this initial investment, powerful connections were set and maintained throughout the unit. Teachers agreed that the students were more invested in the rest of unit because they were instrumental in the groundwork that was created. I look forward to this lesson each semester with all of the unique moments that occur.

Credit

This lesson was a collaboration of efforts from math teachers at Haliburton Highlands Secondary School and Huntsville High School, in the Trillium Lakelands District School Board, and at Centre Wellington District High School in the Upper Grand District School Board.

Many thanks to Roberta Belfry of the Toronto District School Board for her assistance in writing this article.

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Appendix 1 *BLM 1.1: Secret Pattern Challenge Cards*

SECRET PATTERN-BUILDING CHALLENGE
With your partner, build the first three terms for this rule:
number of tiles = the area of a rectangle whose:
length is the term number
width is one more than the length

SECRET PATTERN-BUILDING CHALLENGE
With your partner, build the first three terms for this rule:
number of tiles = the area of a rectangle whose:
length is the term number
width is two more than the length

SECRET PATTERN-BUILDING CHALLENGE
With your partner, build the first three terms for this rule:
number of tiles = the area of a rectangle whose:
length is the term number
width is twice the length

SECRET PATTERN-BUILDING CHALLENGE
With your partner, build the first three terms for this rule:
number of tiles = the area of a square with:
side length equal to the term number
an additional 1 unit square attached

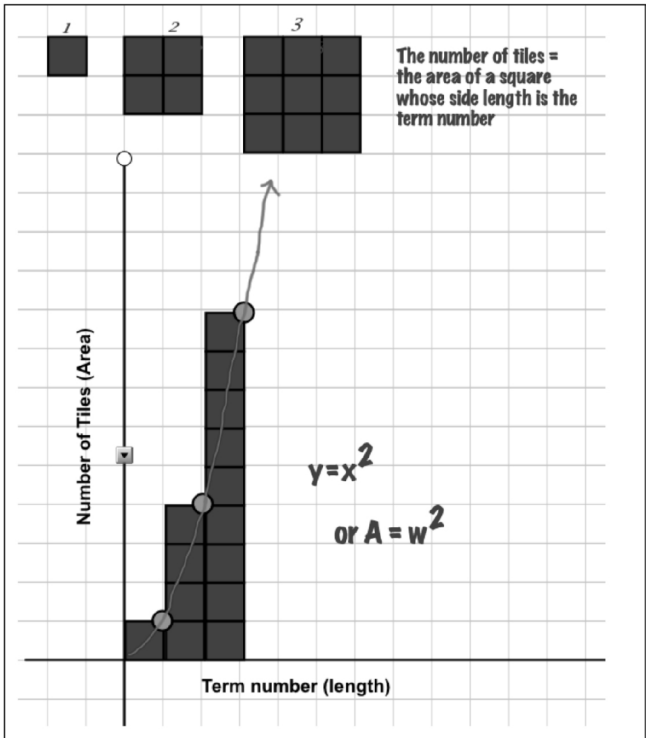
SECRET PATTERN-BUILDING CHALLENGE
With your partner, build the first three terms for this rule:
number of tiles = the area of a square with:
side length equal to the term number
2 additional unit squares attached

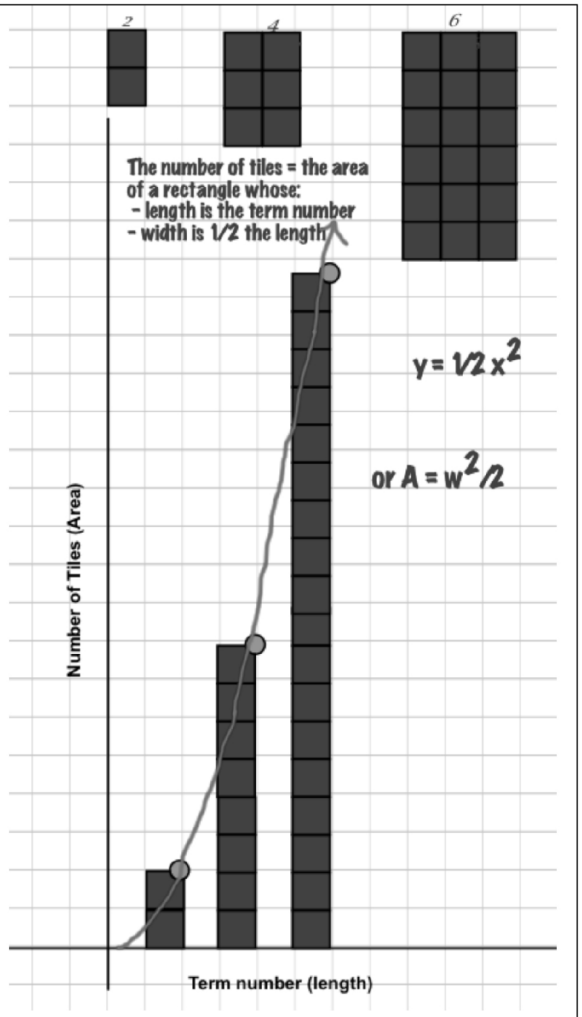
SECRET PATTERN-BUILDING CHALLENGE
With your partner, build the first three terms for this rule:
number of tiles = the area of a square with:
side length equal to the term number
a 1 unit square hole in it

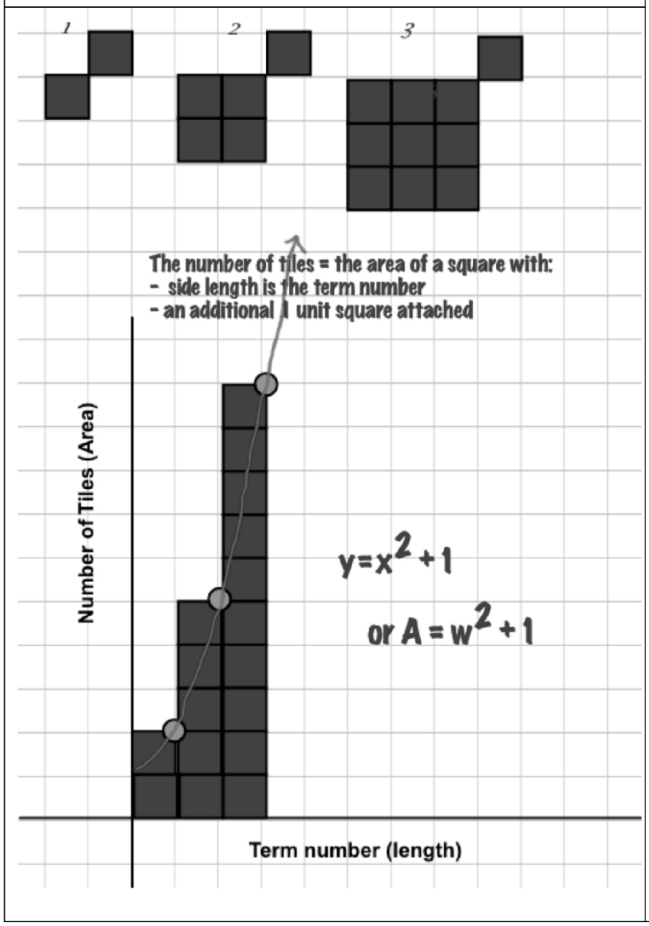
SECRET PATTERN-BUILDING CHALLENGE
With your partner, build the first three terms for this rule:
number of tiles = the area of a rectangle whose:
length is the term number
width is ½ of the length

SECRET PATTERN-BUILDING CHALLENGE
With your partner, build the first three terms for this rule:
number of tiles = the area of a rectangle whose:
length is the term number
width is one less than the length

SECRET PATTERN-BUILDING CHALLENGE
With your partner, build the first three terms for this rule:
number of tiles = the area of a square whose side length
is two more than the term number


1
2
3
The number of tiles = the area of a square whose side length is the term number
Number of Tiles (Area)
$y=x^2$
or $A = w^2$
Term number (length)


2
4
6
The number of tiles = the area of a rectangle whose:
- length is the term number
- width is 1/2 the length
$y = 1/2\,x^2$
or $A = w^2/2$
Number of Tiles (Area)
Term number (length)


1
2
3
The number of tiles = the area of a square with:
- side length is the term number
- an additional 1 unit square attached
Number of Tiles (Area)
$y=x^2+1$
or $A = w^2 + 1$
Term number (length)

Unit #3: Day 1: Building Quadratics		MPM2D1
	MATH LEARNING GOALS • Students will build parabolic models, given geometric pattern rules. • Students will look for ways of classifying parabolic relationships by comparing graphs, pattern rules and possibly equations. • Students will investigate similarities and differences between different quadratic relationships and connect those to alternative representations of those relationships.	MATERIALS • Grid chart paper • Markers • Colour tiles • 1 copy of BLM 1.1 • Sticky notes
MINDS ON…	WHOLE CLASS: BUILDING A SAMPLE PATTERN • Provide each pair of students with some colour tiles and have them build the first three terms in the pattern rule "number of tiles = the area of a square whose side length is the term number." Model this for them. PAIRS: THINK/PAIR/SHARE • Discuss similarities and differences between the graph showing the relationship between the width of a square and the area of the square and the graphs created in the secret pattern challenge for linear growing patterns. WHOLE CLASS: DISCUSSION • Record students' information as they share what they had discussed. Keep this visible for the lesson.	• Students had already done the Secret Pattern-Building Challenge (from Ruth Beatty) for Linear Growing Patterns and other LGP activities before this class. They can be found in the Linear Growing Patterns Section of **MathCLIPS.ca**.
ACTION	PAIRS: ACTIVITY • Provide each pair with one pattern rule from BLM 1.1. Students will represent the first three terms of their pattern rule, using the colour tiles, and label each term with a sticky note. Clarify that most should be building terms 1, 2, 3, but if those terms are not possible, they should build the first three "buildable terms." Circulate to clarify and confirm student thinking. PAIRS: GALLERY WALK • Pairs should turn the pattern rule upside-down at their desks. Have students attempt to guess the pattern rule for the other groups and then flip over the pattern rule to see if they are correct. PAIRS: ACTIVITY • Ask students to predict what the graph of area versus term number would look like (how will it be different from or similar to the one in the Minds On). Provide each pair with a piece of chart paper and markers. Have them record their pattern rule on the graph by realigning the tiles for each term vertically above the term number and outlining the tiles and plotting a point to represent the number of tiles. They should then draw a smooth curve through the data. Students will also need to record the word version on the top of the page, along with a picture of the model with the term number, and a possible equation.	• When handing out cards, note that the later patterns may be more challenging to build. • Ensure that the grid paper is pre-labelled with the axes and scale (1:1), to allow for easier debriefing of the activity. • The colour tiles will align with the grid paper. Students will need to be reminded to place the "dot" on the intersection point, rather than the middle of the space. • Students may want to have their curve pass through the *y*-axis, even though it would difficult to build the 0th term. When they do this, ask them to justify the *y*-intersect that they chose. • These graphs will be useful for the remainder of the unit to highlight connections between different forms of equations, as well as for translations.
CONSOLIDATE DEBRIEF	THINK/PAIR/SHARE: CIRCLE DISCUSSION • Place all the chart paper in the centre of a discussion circle with two markers. • Encourage students to record observations and add notes to chart papers during the discussion, adding new terminology where appropriate. • Ask students to discuss, with an elbow partner, the similarities and differences between what's on their chart paper and the chart paper that we created as a class in the Minds On. Have volunteers share their observations—encourage students to add to or correct inaccuracies on the chart paper. • Examining all the graphs, discuss the following questions: - What similarities and differences do you notice overall? How might those connect to the pattern rule/pictorial representation/equation? - Which graphs would you say are most similar? Why? What makes them different? - Ask students to "order" the chart paper to show how they think they should be displayed on the walls for the rest of the units.	
REFLECTION	FURTHER CLASSROOM CONSOLIDATION INDEPENDENT: EXIT CARD • Select and note two graphs/pattern rules. Record all the similarities and differences you can see for your chosen graphs.	

Appendix 2: *Lesson Plan Template*

▲ Recreating the Greek Temples and Egyptian Pyramids in Google SketchUp

JELENA LONCAR



Jelena began her career at the secondary level, but has since moved into the post-secondary world teaching mathematics.

To support students in developing their understanding of 3-D geometry, it makes sense for them to build things. In this project used in a Grade 9 academic class, students have a chance to recreate history as they develop their understanding of 3-D geometry, using either a technological or physical model, whichever they prefer. The project is adaptable to other grade levels as well.

Features

Google SketchUp is suggested for the technological model. The following features make Google SketchUp an excellent tool for building very detailed 3-D models.

- Protractor: Angles can be specified, measured, and labelled.
- Measuring Tape: Lengths can be specified, measured, and labelled.
- Orbit: Objects can be viewed from all angles.
- Colour and Material: Colour and material can be specified to create a visually appealing model.

The Project

Students approach the project in three parts: 1) beginning with research about the shape, 2) hand-drawing a sketch of their design with all relevant dimensions and scale, and then 3) creating the scale model. It is advisable to ensure that the sketch (component #2) is submitted before step 3 begins and that the students have a clear idea of how they will put together their project. This will minimize "projects going wrong," and will also focus students on the mathematics of the shapes, rather than being distracted by the modelling component.

I generally introduce this project once we have covered enough material in the geometry unit for students to be able to constructively begin working on their project. The deadline is generally just after the unit test so that students have sufficient time to polish the project and study for their test. I have provided two to three days for students to work on this in class. I distribute these into one full day at both the start and toward the end of the project, with perhaps one or two half-days. The amount of preparation time depends on the dynamics of the class. Another option is to provide three dedicated project days at the end of the unit.

Details of the project can be found in Appendix A, and the rubric used for assessment in Appendix B. I have included exemplars of student work at the end of this article.

Setting the Stage

Very little guidance is needed to use Google SketchUp. I have presented students with the basic tools to showcase Google SketchUp capabilities. My example was the design of a chair (you can look up the tutorial for this on the SketchUp website). This example allows you to quickly demonstrate basic 3-D drawing tools and things to look out for. For instance, because we are now working with three axes, orientation becomes important. I have encouraged students to search the tutorial section of SketchUp for videos of more specific features.

Group Work Versus Individual Work

The project can be assigned individually or in pairs. Since I used pairs, the project was primarily done in class, so I was able to see how much work each student put into his or her project. However, this can prove difficult with larger classes.

In the case of group work, I would suggest allowing students to choose their own groups, but holding them accountable to a "contract." The contract is designed by the group, and is signed and dated by each student. The contract clearly outlines the specific responsibilities of each student, as well as specific dates and times of group meetings. You can also have the students keep minutes of their meetings to summarize their accomplishments at each meeting time. If issues arise, have a group meeting with the students, with the teacher acting as the mediator to help refocus students while referring to the agreement in the contract. Alternatively, you can assign the project individually. This option might require a bit more time.

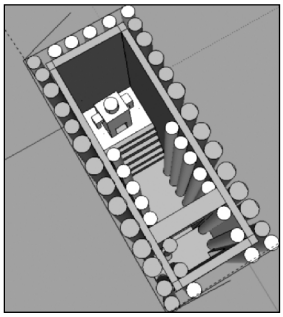
Extensions

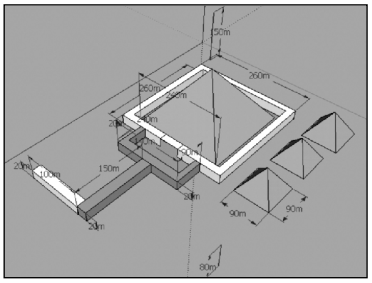
As an extension, I have allowed students to choose their own 3-D infrastructure. However, I have found that you must carefully watch over students' choices, as they sometimes choose awkward structures to be represented with 3-D shapes. To minimize ad-hoc projects, I would suggest having a minimum number of 3-D shapes that must be used in the infrastructure of choice. Also, students will sometimes have really big ideas that are difficult to materialize on a short timeline (e.g., a 3-D model made of cake).

The Greek Temples and Egyptian Pyramids are traditional examples; however, teachers can choose structures that are closer to home, such as the pyramids of Chichen Itza or the Montreal Olympic Village.

Display of Projects, Exhibition Style

On the due date, we have a classroom display of projects. Students bring in their physical models and I project the virtual models as a slide show in PowerPoint. Students then walk around to see each other's work. It is also nice to invite other teachers and principals to visit the class during this time.







Reflections

Students do an amazing job! You get to witness all kinds of talent exhibited by your students, which you may not have otherwise gotten to see in your mathematics classroom. The real-life models can also be very impressive.

Students have enjoyed this project as they were able to work in virtual 3-D. Google SketchUp allows students to view the different shapes they have studied from all different angles, which is helpful to visual learners in particular.

Working in Google SketchUp accommodates all skill (and interest) levels. With a bit of patience, the basics become simple to understand. However, the program allows for much more complicated designs if that is the student's interest.

Keep student work as examples for future courses and to decorate your classroom. ▲

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3-D Creations

For this assignment, you are required to construct a 3-D model to replicate a famous structure of the past.

Your choice of structure may be one of the following:

I The Parthenon

II Great Pyramid of Giza

Your model will be created in one of the following ways:

Option 1: Create the 3-D shapes out of construction paper to construct a model of the pyramids. Label the measurements of each shape (length, height, where appropriate).

OR

Option 2: Recreate the structure, using Google SketchUp. Label the measurements of each shape (length, height, where appropriate).

Download:**sketchup.google.com/download/**

Tutorial: **sketchup.google.com/training/videos/new_to_gsu.html**

For all options, you must:

- research all relevant dimensions for an accurate model and clearly state your source(s)
- create your model to scale
- draw a sketch of your model, including scaled measurements
- calculate the **volume** and the **surface** area of your creation

Final Submission:

The following must be submitted:

- sketch with labelled measurements
- calculations of surface area and volume of each shape used
- your model (if used Google SketchUp, email to teacher by the morning of the due date)

Appendix A

	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
SKETCH	Sketch is mostly not present; many errors.	Sketch is present but incomplete; some to many errors.	Sketch is present; some errors.	Sketch is present, correct, and to scale.
SHAPES	3-D shapes have not been used appropriately/accurately.	3-D shapes have been used appropriately and accurately to a small extent.	3-D shapes have been used appropriately and accurately, but improvements could be made.	3-D shapes have been used appropriately and precisely.
SCALE/ CALCULATIONS OF VOLUME (V) AND SURFACE AREA (SA)	There is no accuracy in measurement based on dimensions given. Scale is inconsistent or has been ignored. V and SA not calculated.	Some accuracy exists; however, it is largely inconsistent. Scale is somewhat consistent. V and SA not accurate or not fully calculated.	Mostly accurate. Scale mostly consistent; however, improvements can be made. V and SA present, but proper form is not used, or errors present.	Accuracy of measurement is exact. Scale is consistent throughout. V and SA present, with proper form. Errors not present.
NEATNESS	No neatness. No effort.	Somewhat neat. Some effort.	Neat. Effort present. Improvements could be made.	Neat recreation; the model is excellent.

Appendix B

▲ Lesson Plays: Learning How to Improvise

NATHALIE SINCLAIR & RINA ZAZKIS



Nathalie is a faculty member in the Faculty of Education and an associate member in the Department of Mathematics at Simon Fraser University. Her primary areas of research have related to the use of digital technologies in mathematics teaching and learning, as well as the role of aesthetics in both mathematics and school mathematics. More recently, she has focused on the use of particular communication choices on student engagement and agency.



Rina is a faculty member in the Faculty of Education and an associate member in the Department of Mathematics at Simon Fraser University. Her research is in the area of mathematics education with a general focus on mathematical content knowledge of pre-service teachers and the ways in which this knowledge is acquired and modified. Teaching, learning and understanding elementary number theory has been a special focus of her studies.

Imagine taping a few minutes of your lesson and then going back and reading the transcript. You might be surprised about what you notice when reading that transcript.

- *What kinds of questions did you ask?*
- *How did you talk about the mathematics involved?*
- *Who was the authority in the classroom?*

In mathematics classrooms where the focus is on student reasoning, on conceptual understanding, and on rich problem-solving tasks, the teacher no longer can plan the exact sequence of questions, responses, and interactions that will ensue. Instead of following a pre-defined script, the teacher improvises. But how can an improviser prepare to teach? In this article, we describe and illustrate a new tool that we have been using (Zazkis, Liljedahl, & Sinclair, 2009) in our work with both pre-service and in-service teachers, a tool that we have found can help teachers prepare to improvise. The Lesson Play is a lesson or part of a lesson presented in dialogue form, featuring imagined interactions between a teacher and her students. In the following section, we outline some of the differences between a Lesson Play and the much more familiar tool of teacher preparation, the Lesson Plan. We then focus on some of the specific ways in which Lesson Plays can draw attention to the important pedagogical and mathematical moves that can support powerful student learning.

Lesson Plan Versus Lesson Play

The "Lesson Plan" has been a staple of pre-service teacher education for many decades. The various templates that have been developed over the years have been criticized in the scholarly literature (see John, 2006; Maroney & Searcey, 1996) for oversimplifying what it means to teach, as well as for failing to consider how teachers actually plan. Those who use them can easily assume that the clear identification and organization of content outcomes will result in the acquisition of this same content by the students.

Why, despite the ongoing criticism and acknowledgement that "real teachers do not plan that way," has the traditional model sustained its popularity? John (2006) suggests several interrelated reasons. He believes that "much of the attraction of this approach to planning lies in its elegant simplicity" (p. 485). Related to this, the model reinforces a sense of control based on prediction and prescription.

In the mathematics education literature, teacher-researchers such as Lampert (2001) have shown how expert planning and preparation for teaching a lesson involves extensive work in connecting particular mathematics to particular students, moving back and forth between mathematics and the structure of tasks appropriate for particular learners. Thus, Lampert begins by designing mathematical tasks, but the implementation of these tasks shifts in accordance with students' responses. Yinger (1980) might describe this type of by-the-seat-of-your-pants teaching as improvisation. In order to improvise well, one must be able to deal with unexpected situations, handle new questions, propose alternate problems, and be able to work with unfamiliar forms of reasoning. What kind of planning can help support this style of teaching?

We think that the "lesson play" might provide a novel juxtaposition to the traditional planning framework as a method of preparing—and even practising—to teach a

lesson. These two methods structure the act of preparation in two fundamentally different ways, each with its own affordances. However, behind the form of the lesson play lies an image of teaching that is closer to rehearsed in-the-moment choices and decisions than it is to predetermined plots and outcomes.

Writing Lesson Plays

We usually begin by offering an initial exchange between a teacher and a student, or a class of students, and invite teachers to imagine what might happen in an ensuing exchange. Teachers often respond quite quickly with statements beginning, "I would say…" or "I would do…." But when writing a Lesson Play, one has to actually choose the words that will fulfill the stated intention, and this can be more challenging—but it is precisely what teachers must do all the time: make on-the-spot decisions about what to say or do, how to respond, whether to tell, explain, wait, elicit, etc.

Here's the prompt (taken from Crespo, Oslund, & Park, 2010):

> *You are in a classroom of about 20 students. You have put the following statement on the board:*
>
> $$8 + 4 = \square + 5$$
>
> *You ask, "What goes in the box?" One student raises her hand and says that the answer is 12.*
>
> *T: Does everyone agree that 12 is what goes in the box?*
>
> *Sts: Yes*

Take a few minutes to extend the interaction. What do you pay attention to? What are you trying to avoid?

Attending to Interaction: Funneling versus Focusing

In her work on mathematics discussions in the classroom, Wood (1998) had identified two forms of classroom interaction: focusing and funneling. Similar to the traditional classroom pattern of Initiation, Response, Feedback (IRF), funneling occurs when the teacher asks a series of questions that guide the students through a procedure or to a desired end. In this situation, the teacher is engaged in cognitive activity and the student is merely answering the question to arrive at an answer, often without seeing the connection among the questions. Consider the lesson play below, created by a prospective teacher in Crespo et al.'s (2010) study, whose interaction pattern funnels students into solving the problem in a particular way:

Lesson Play 1

T: If 12 fits in the box, that means that 12 + 5 is equal to 8 + 4. What is 12 + 5?

Sts: 17

T: And what is 8 + 4?

Sts: 12

T: Now, are 17 and 12 equal to each other?

Sts: No

T: So the number in the box must equal 12 when added to 5. A way we can figure this out is by subtracting 5 from 12. What is 12 – 5?

Sts: 7

In this Lesson Play, the teacher does not find out why the students initially respond "12," which means that even though they were able to provide the correct answer at the end, it is highly likely they have not learned how to solve the problem on their own.

In contrast to funneling, focusing requires the teacher to listen to the students' responses and guide them based on what the students are thinking, rather than how the teacher would solve the problem. Achieving this kind of focusing interaction can be very challenging, and requires the use of moves that go beyond simple initiation and feedback.

Lesson Play 2

T: Why?

Nick: Because 8 + 4 = 12

T: Meghan, can you write the statement Nick just made on the board, under mine?

Meghan: *[writes 8 + 4 = 12 on the board]*

Simin: It looks just like what you wrote, except there's no plus 5.

T: Let's look at Meghan's statement, which says that 8 + 4 is equal to12; both sides of the equation have the same value.

Meghan: So can I put the plus 5 after the 12?

[pause]

Paulino: No, don't do that. Because then it will say that 8 + 4 = 12 + 5.

T: Why can't we say that, Paulino?

Paulino: We said 8 + 4 is the same thing as 12 and…

Veda: 12 + 5 is 17.

Paulino: Yeah, and 12 can't equal 17.

T: You are saying that we can't add 5 here because then we'd have an equation saying that 12 equals 17. Can we look back to the equation above Meghan's? If we want to say that this side *[gesturing to the left side of the equation]* equals this side *[gesturing to the right side of the equation]*, then what should we put in the box?

Amanda: So we need to have 12 on that side too.

Paulino: Yeah, they need to be the same, so we need to have 7 there.

T: Meghan, can you put the 7 in the box and read the sentence out loud?

In this interaction, which we have created and which is admittedly longer, the teacher works with the response given by the students, starting with Nick's assertion and then picking up on Paulino's reluctance to follow Meghan's idea of adding 5. The teacher also gives students time to consider Meghan's idea before jumping in to evaluate it, which gives Paulino a chance to respond. Instead of evaluating the responses herself, she also asks the students to *explain* their reasoning. This Lesson Play is not meant to represent the "best" way of imaging the ensuing interaction, nor is it perfect. However, writing it required trying to imagine how a funneling exchange might be avoided, as well as practising the kinds of "moves" that can help focus the interaction.

Adapting the "talk moves" identified by Chapin, O'Connor, and Anderson (2009), Beth Herbel-Eisenmann (2010) proposes the following set of discursive moves that teachers use in order to promote in students the development of a sophisticated mathematical discourse: inviting, waiting, re-voicing, asking students to re-voice, probing, and creating opportunities for reasoning. Re-voicing was used in Lesson Play 2, when the teacher responded to Paulino's statement—it is used to emphasise a relational way of thinking about equality. In the following lesson play, written by one of our prospective teachers, the teacher has asked why Tracey thinks that 143 is a prime number. Re-voicing is also used to introduce or promote a more sophisticated mathematical discourse:

Lesson Play 3

Tracey: We divided 143 by 2, 3, 4, 5, 6, and 7 and 8 and 9. Each time, we had a number left over.

Teacher: What does that mean when you have a "remainder," a number left over?

Tracey: Well, it means that if I divide 143 by 2 and I get a remainder, then I cannot use 2 times something to get 143. So 2 does not go into 143.

Teacher: So what you are saying is that 2, 3, 4, 5, 6, 7, and 9 are not "factors" of 143?

Tracey: Yes.

Teacher: Who remembers what a factor is from Grade 4?

Kerry:They are, like, when you have 10 and you times 2 and 5 to get 10. The 2 and 5 are factors of 10.

Teacher: Good, factors are numbers that are multiplied together to make another number.

Re-voicing enables the teacher to bridge the students' informal language, "a number left over," with the more mathematical term "remainder." However, re-voicing can have the effect of taking away student agency, that is, diminishing the range of actions students can take and the control they have over the mathematical discussion and exploration. When the teacher re-voices Tracey's explanation of "remainder" by using the word "factors" instead of "go into," she is offering a more mathematically sophisticated discourse, but also threatening Tracey's sense of participation. Re-voicing may then be accompanied by asking students to re-voice. The teacher tries to do this when she asks whether anyone remembers what a factor is. However, by pointing to the fact that factors was a Grade 4 topic, this invitation to re-voice comes across as an invitation to remember something previously known.

In her last turn, the teacher offers a definition of factors based on Kerry's description. In fact, the teacher is trying to model the discourse of generality; instead of offering specific examples, as Kerry does, the teacher would like the students to offer more general statements, perhaps even definitions. But here the teacher does the cognitive work of turning Kerry's statement into a definition (one that is somewhat vague and misleading). Consider this alternate route the teacher could have taken:

Lesson Play 4

Teacher 11: Who remembers what a factor is from Grade 4?

Kerry: They are, like, when you have 10 and you times 2 and 5 to get 10. The 2 and 5 are factors of 10.

Teacher 11: Can someone offer another description of a factor that does not involve a specific example?

This is an example both of inviting and of asking students to re-voice.

These kinds of moves are difficult to plan explicitly, in the context of a Lesson Plan, since they require making explicit word and phrase choices that depend on the context. However, they can be practised in the context of a Lesson Play, where teachers have to pay explicit attention to the way in which they are using language to interact with their students.

Attending to the Mathematics: Operation versus Relation

The previous section drew attention to the different kinds of interactions that can be discerned in Lesson Plays, and the different moves these entail. In this section, I focus more on the mathematical discourse present in Lesson Plays. Using the same prompt as above, compare these two Lesson Plays (both taken from prospective teachers in Crespo et al.'s (2010) study):

Lesson Play 5

T: Well, what does 8 + 4 = ?

Sts: 12

T: What does 12 + 5 = ?

Sts: 17

T: Does 17 = 12 ?

Sts: No

Lesson Play 6

T: Who can explain to me why 12 goes in the box?

Sts: Because 8 + 4 = 12

T: How does 8 + 4 = 12 + 5?

In Lesson Play 5, the focus of the teacher, and probably that of the students, is on the operational nature of the equation; the emphasis is on calculating the sum. The teacher breaks up the two number sentences and invites students to compare the sum. In contrast, the focus of Lesson Play 6 is on the relational nature of the equation, on how the two sides need to be equal.

Research has found that young children tend to associate the equal sign with a request to calculate a result, instead of seeing it as a symbol of mathematical equivalence. Therefore, it is important for teachers to draw attention to the relationship nature of equality. But this can be difficult. As Stephens (2006) shows, even when prospective teachers are aware of the importance of focusing on the relational rather than the operational, and even when they find opportunities to do so, they do not appear to value these opportunities, or think that typical teachers should do so. Further, even when teachers are aware of the importance of the relational approach and value it, they can find it challenging to actually adopt this approach.

Perhaps teachers think they are emphasizing the relational, but their default modes of responding and questioning communicate an operational approach. It is in this sense that Lesson Plays, which allow one to carefully choose and practise certain ways of talking, can help teachers practise a more productive discourse of teaching.

Attending to Authority and Student Agency

Several researchers have pointed to the various ways in which authority is played out in the classroom, particularly in terms of the positioning of various players including the teacher, the students, and the textbook. Teachers can position the mathematics as being the ultimate authority, or the textbook, or even themselves. While sometimes appropriate, these positioning choices can have the effect of severely reducing student agency.

In relation to a prompt where a teacher asks whether 362 is divisible by 4, this Lesson Play, written by one of our prospective teachers, positions the textbook as the authority:

Lesson Play 7

T: What resources can you use to find out the rule?

S3: We can look in the glossary of the textbook...

T: OK. Let's do that.

S4: The text says that the rule for divisibility by 4 is that the last two digits of the number are divisible by 4.

While the rules of divisibility are certainly in the textbook, this Lesson Play might suggest to students that they are rules that are "given" and that are not based on reasoning or sense making. To reposition the students as authority, the teacher might have added at the end of Lesson Play 7 a statement such as, "Let's make sure that rule makes sense."

The following Lesson Play, also written by one of our prospective teachers in response to a prompt related to divisibility, positions the teacher as the authority.

Lesson Play 8

T: So Tim, I believe you have mixed up the divisibility rules for 3 and 4. Can I show you something? OK, so let's try and work this problem out.

Teacher divides 354 by 4 manually and gets 88 remainder 2.

S1: So that means it isn't divisible by 4?

T: That's correct. Because you have mistakenly applied what you learned about the divisibility rule for 3 in this question. Let me show you…

[The play continues with the teacher implementing the divisibility rule for 4.]

Unarguably, teachers are the source of knowledge and the source of authority. However, this knowledge and authority can be implemented in different ways. By looking at Lesson Plays like these, teachers might be able to identify alternate ways of leading activity without asserting the kind of authority that prevents students from engaging in the questioning, verifying, and reasoning that are valued in the classroom.

In addition to these more explicit positioning moves, Herbel-Eisenmann, Wagner, & Cortes (2010) have examined "stance bundles," which are phrases that communicate personal feelings, attitudes, and values, and relate to the ways in which students are assumed to have choice in the discourse and to have obligations. These choices and obligations can be important in studying the extent to which discourse is opened up or closed down in the classroom. In their analysis of secondary school mathematics classroom, Herbel-Eisenmann and her colleagues found pervasive use of stance bundles that could encourage limited student agency.

Most of the stance bundles found in the secondary mathematics classroom involved drawing on personal authority. Using examples from our pre-service elementary teacher Lesson Plays, we can see a similar phenomenon. Consider the following example: The student is asked about the colour of the 80th car of a toy train, where the first car is red, the second is blue, the third is yellow, the fourth is red, the fifth is blue, and the sixth is yellow, and so on—and gives the answer "red" "because the fourth car is red and 80 is a multiple of 4."

Lesson Play 9

Mrs. Brown: Okay, now **I want you** to show me one set of the repeating pattern

[Hannah lines up 3 blocks: Red, Blue, Yellow.]

Okay good. Now I want you to add 4 more to the set.

Mrs. Brown: Why don't you draw out your pattern to 12. Let's see what it looks like.

S: Okay.

[R] [B] [Y]
[R] [B] [Y]
[R] [B] [Y]
[R] [B] [Y]

Mrs. Brown: Okay, now **I want you** to rewrite your pattern, but write it all in a line.

The use of "I want you to" in both examples positions the students as being obliged to the teacher. T7 could have said, "Okay, now rewrite your pattern, but write it all in a line," which would have communicated the same content, but the interpersonal relationship created would have been markedly different. In using the pronouns "I" and "you," the teacher establishes a certain contract that is personal. And the verb "want" emphasizes the desire for compliance on the part of the teacher. Note also that there is no reason given, which makes the sense of obligation even more personal.

A second alternative to personal authority is more subtle because it avoids the strong phrases like *need to*, while still communicating a sense of inevitability about how the future will unfold. Consider this example, also in response to the toy train problem:

Lesson Play 10

Ms. Tooney: I have been walking around the room, and it seems as though a lot of students seem to have different answers for part A. How many of you say that the 80th car is red?

[Out of 22 students in the class, 12 put up their hand.]

Ms. Tooney: Ok, we are going to go through this problem together as a class. We are going to form a train here at the front of the class so we can see exactly what is going on in this problem. Do I have 3 volunteers to start the train?

Ms. Tooney: Luke, you are the first car in our train. What colour are you?

Luke: Red.

Ms. Tooney: Olivia, you are the second car in our train. What colour are you?

Olivia: Blue.

Ms. Tooney: And Michael, you are the third car in our train. What colour are you?

Michael: Yellow.

The use of "we are going to" gives the sense that the speaker knows what will happen—similar to what happened in the "focusing" interactions discussed above. The certainty of expression in these cases can be located in the auxiliary verbs "are" and "am", which express a high certainty. Compare "we are going to" with "we might try to" or "I think we will." In using these bundles, the teacher communicates a sense of his or her being in control, and being sure of what would happen next. As Herbel-Eisenmann et al. note, "This situation was much like the positioning in which the teacher positioned him- or herself in authority over students, though with less concern for politeness: the teacher declared what the students would do instead of asking the students to do something."

The three types of stance bundles described so far position the student as being subjugated to either the teacher or the discipline. But what other types of positions might the student take on in the mathematics classroom? Herbel-Eisenmann et al. found several other types of stance bundles that offered the message of alternatives and have the potential to open up the discourse. These all recognized that there was some choice involved, such as *do I need to* […]?, *I'm not going to, I was going to, or If you want to and are you going to* […]? The former indicate that there is a situation that involves choice, while the latter explicitly refer to students' desires.

Conclusion

We have offered three different lenses through which classroom dialogues can be read: attending to classroom interaction, attending to mathematical reasoning, and attending to authority and student agency. While these lenses are familiar, and have been much discussed by mathematics educators, we think that Lesson Plays offer a particularly useful and detailed tool for examining them. We see Lesson Plays as a potential vehicle for collaborative inquiry, allowing teachers to engage in critical work around teaching, providing them with a focused opportunity to examine the interactions and word choices that are made, considering why and when they might have negative consequences, and imagining alternatives. Lesson Plays might have particular value for teachers as they enhance their skills in consolidating a lesson. We also see them as tools for newer teachers, or teachers trying to change their practices, to experiment with new moves, and practise these moves in the relatively safe and slow-paced context of writing dialogues.

Centrally, the Lesson Play provides an opportunity to imagine the future, being informed by the past, rather than re-examining the past. Its structure is built around the specific conceptions of a particular student, or group of students, learning the details of a mathematical concept, with the preciseness of mathematical language, through the relationship of teaching. It is not a description of how things will occur in the classroom, but an imagined account of how things might occur. We see this kind of interpretive exercise consonant with Maxine Greene's (and others') vision of aesthetic engagement, in which one experiencing the world as if, as a hypothetical world of "what is not yet, or what might, unpredictably, still be experienced" (1995, p. 62). For Greene, such engagement—in which we bring together prior experiences, reconfigure, and reconstruct them, finding in them familiarity and strangeness—opens the doors to transformative teaching and learning.

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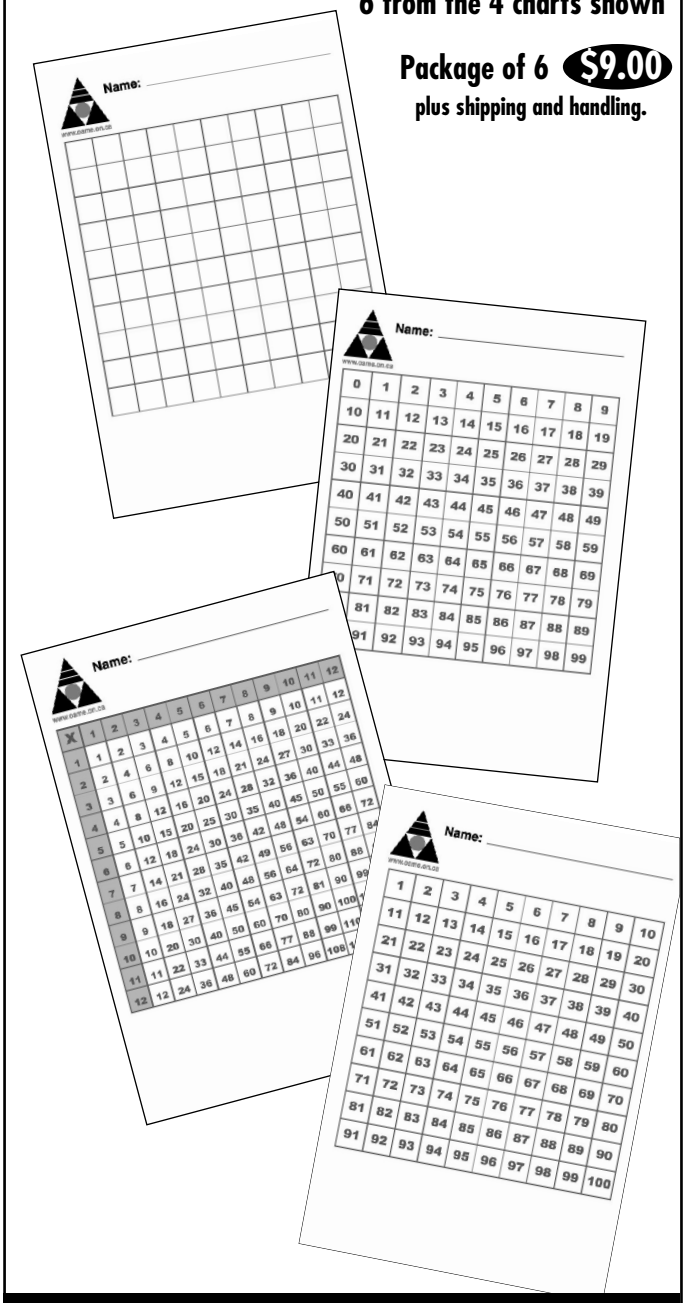
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▲ Homework Help

MANDY MAYBEE



Mandy has been an intermediate math educator for 14 years. Her current work is with the Independent Learning Centre, engaging students in learning mathematics online.

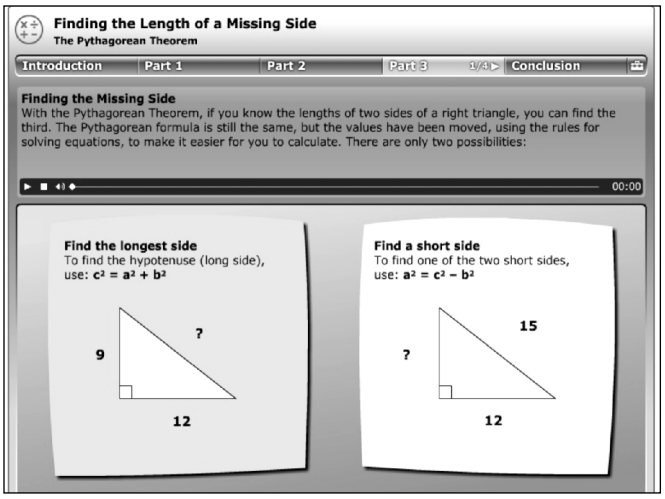
As educators, we have a variety of tools at our disposal to connect with and engage the students in our classrooms. What happens when they get home, crack the books, and just don't get it? They'll check their notes, try to find something on Google, and then give up… discouraged and listening to that voice that lies to them, "You're no good at math." As educators, we work to build the confidence and skills of our students, but how can we continue that support and maintain that momentum outside of the classroom?

I'd like to introduce you to Homework Help (HH). This unique website, **HomeworkHelp.ilc.org**, is administered by the Independent Learning Centre (ILC) and funded by the Ministry of Education. Homework Help provides 24/7 resources, purposed to support the learning of intermediate math students in Ontario. Supporting the efforts of classroom teachers, HH is a multi-faceted differentiated resource that meets students where they need help the most by supporting their learning outside of the classroom.

Resources include:

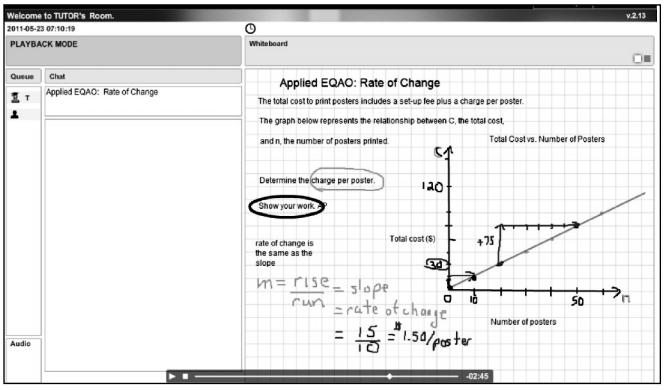
Interactive Tutorials: Dynamic interactive games that develop foundational skills and challenge students to illustrate their understanding of key concepts

Listen and Learn Seminars: Explorations of key topics, intended to address significant gaps in student understanding



Comprehensive Glossary

Best Sessions: A selection of highly effective tutor sessions, supporting the topics specified by the Ontario curriculum. The example shown here illustrates the capability of the chat environment.



Online Calculator

Student Lockers: A repository of student–tutor sessions, saved for quick access to previous learning. Students can also flag the sessions of other students, enabling them to learn from each other.

Personal tutoring by OCTs Sunday through Thursday, 5:30 p.m. to 9:30 p.m. EST.

From one user:

This website is the best thing ever! I missed a few days of school this week, and my teacher does not offer extra help a lot, so I fell behind. I was having a lot of trouble understanding just based on notes. I thought this website wouldn't be very helpful, but it was. Now that I've used it once when I get stuck at home, I can always turn to this. I think that teachers should really push this website on their students because it is a great help, especially for high school students!

HH has been proving its effectiveness through pilot programs in the last four years. In September 2007, the ILC was tasked with designing the after-school math site, including a tutoring chat function. This pilot project has been expanding each year. Positive feedback from students, parents, and teachers, as well as supporting research has prompted the Ministry of Education to expand Homework Help province-wide for the 2011–2012 school year. The ILC provides and maintains the HH website, updating and refreshing resources to meet student needs, and trains certified Ontario teachers as online tutors.

This year, Grades 7, 8, 9, and 10 math students in English school boards across the province will have access to a proven and trusted resource for free math support and tutoring.

French school boards can access similar support at SOS Devoirs, **www.sosdevoirs.org**.

For those students who are stuck at home, frustrated over a math concept, this means having access to resources and even a certified teacher after school hours. For teachers, it means the learning their students engage in during the day will be supported and reinforced, even if the student struggles with his or her homework.

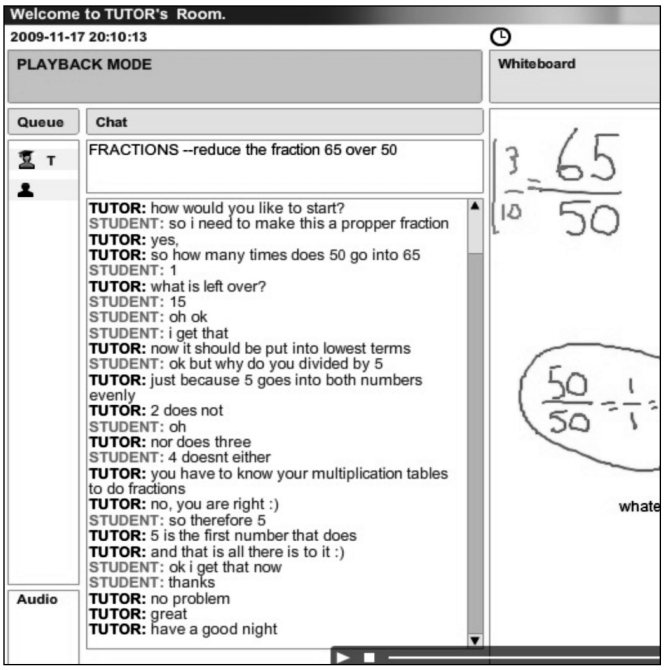
With their learning supported outside of the classroom, teachers are able to more effectively use the time they have inside of the classroom.

How Homework Help supports math learners

Homework Help is a wonderful site for us students. It really takes the pressure off of trying to do a problem that I really don't understand.

We all need assistance to learn—but asking for help isn't always easy, particularly under the judgmental eye of our peers. At **HomeworkHelp.ilc.org**, students can work independently through interactive tutorials, building their skills and confidence, without the social pressures of their peer group. In the Tutor Chat Rooms, students ask questions and develop their comprehension anonymously. Having an experienced, certified teacher as an online tutor provides exceptional support in a safe environment. Since students can only "chat" with teachers, not with each other, the focus is on learning math, without the pressures of a peer group.

In the chat environment, the student usernames are hidden so that users learning in the queue only see that a "student" is interacting with a tutor.



This anonymity provides a safe environment, where students feel comfortable asking questions.

I love this website because usually I feel a bit embarrassed asking my teacher questions so many times, because I feel a bit dumb so this website helps me so much. The people don't know who I am and they can't see me. It's way better.

Resources that target key areas in the curriculum

Homework Help is consistently being updated to reflect the continuing needs of Ontario students. The ILC tracks the content of questions asked in the chat rooms and uses this information to update and bolster the site resources. This guarantees that students will be able to find resources that meet their needs, even when they don't have access to their classroom teacher. For example, the chart below indicates where HH students have struggled most in the past year.

Three key topics students asked about at HH in 2010/2011:

7	8	MFM 1P	MPM 1D	MFM 2P	MPM 2D
Fractions	Fractions	Ratios	Linear equations	Substitution and Rates	Properties of quadratics
Algebraic expressions	Algebraic equations	Algebra	Simplifying polynomials	Algebraic equations	Factoring polynomials
Integers	Integers	Integers	Characteristics of $y = mx + b$	Trig ratios	Systems of Equations

The continuing struggles students face with fractions, algebra, and integers have been identified through HH metrics. This data is used to support students by providing numerous site resources for each topic with which students struggle. These include interactive tutorials and Best Sessions with varying levels of difficulty that employ multiple strategies to find the solution. Five nights per week, students can engage in dynamic interactions with tutors. The Ontario certified teachers ensure students can build their skills in an environment which caters to their needs and learning style.

Grade 9 transition

The HH program purposefully spans the transition from elementary to secondary school. Students who work with HH in Grade 8 benefit from the continuity of HH's support as they adjust to secondary courses. Grade 9 tutors identify the skills students already have from Grade 8 and build on this foundation, supporting students as they progress through the Grade 9 curriculum. The site resources, available 24/7, focus on the foundational skills students develop in Grades 7 and 8 and often need to refresh in Grades 9 and 10.

I was failing Grade 9 math, and since I've started using homework help, I'm getting 80's!!! Thanks so much :)

Learning retention from one semester to another

Semestered secondary school students encounter another challenge that is mediated by the HH program. After completion of MFM1P, a first semester Grade 9 student may not have math instruction again for a full calendar year. HH usage indicates that these students struggle with the retention of the Grade 9 skills and use the Best Sessions and Tutorials on the site to review concepts at the beginning of Grade 10.

How Homework Help supports classroom teachers

HH doesn't provide the answers

Tutors are trained to work with students to develop an understanding of concepts and employ a variety of strategies to engage and confirm student learning. Every tutor is a certified Ontario teacher; they may even teach in the classroom next to yours!

Amazing experience! The teachers are so nice and didn't give me the answer... but helped me understand each step... Whomever I talked to right now encouraged me to go to the next step... and I actually understood... so overall, an awesome experience.. Now I feel prepared for tomorrow's lesson as well.

Maintaining the momentum

In the classroom, teachers work hard to engage and inspire Ontario math students. The HH team is motivated to support the classroom teacher. Each topic in the curriculum is covered by a number of different site resources so students can find a model that is similar to what they learned in class. Tutors are trained to begin coaching students with the information the student already learned. Starting with this foundation means that when a HH student returns to your classroom, he or she will have successfully completed the homework, and his or her understanding with be directly linked to what you are doing in your classroom.

HH tutors refine their teaching practice

HH tutors are equipped to instantly address questions from across the curriculum while employing numerous strategies to engage and instruct our online students. What teachers experience is that being an online tutor with HH refines their teaching practice. HH hires great teachers to work online and the skills they learn and develop go back with them to benefit their classrooms, colleagues, and families of schools. By being exposed to what is going on in classrooms across the province, HH tutors are able to bring these experiences, questions, strategies, and approaches into their own teaching environments and benefit their students.

How can you and your students benefit from HH?

1) Log onto **HomeworkHelp.ilc.org** and register as an educator. – Find out first hand what this resource has to offer you and your students.
2) Register your class in September. – Each student can register using their OEN (Ontario Education Number) and Date of Birth. These two pieces of data are used as "keys" to ensure only Ontario intermediate math students are able to access the resource. This data is kept confidential and not available to tutors or the ILC beyond registration.
3) Encourage intermediate math students to use HH. – The resource has had significant impact on student users for the past four years. Encourage the Grades 7 to 10 students you know to check it out: HYPERLINK "http://HomeworkHelp.ilc.org" HomeworkHelp.ilc.org
4) Work as a HH tutor. – Homework Help is interested in hearing from teachers with a passion for math, who are extremely comfortable with the intermediate curriculum. ▲

ABACUS: A COMPILATION

Volume 46 – This resource for teachers includes the concepts of addition, subtraction, multiplication, and division of whole numbers and decimals, alternative algorithms, mental math strategies, manipulatives, and links to literature. It will be useful for exploring these aspects of teaching and learning personally or with a team of teachers. Includes one two-coloured bead string.



$10 EACH VOLUME, PLUS SHIPPING AND HANDLING

NEW

Volume 47 – This resource for teachers includes the concepts of equality and relational thinking, algebra as generalized arithmetic, algebra as functional relationships and expressing generality using multiple representations. With links to manipulatives and literature, it will be useful for exploring these concepts personally or with a team of teachers.

Order by mail, fax, or online:

OAME
or order online www.oame.on.ca
Do not send cash or cheque. You will be invoiced, with shipping and handling costs added.

▲ Call for Manuscripts

The *Ontario Mathematics Gazette* is inviting manuscripts for all grade levels.

Instructions for submission of manuscripts are found on page 1 of any *OMG*.

Contact the Editor for further details.

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