8-PAGE INSERT ELEMENTARY MATH
ABACUS

Ontario Mathematics Gazette

OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

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December 2011

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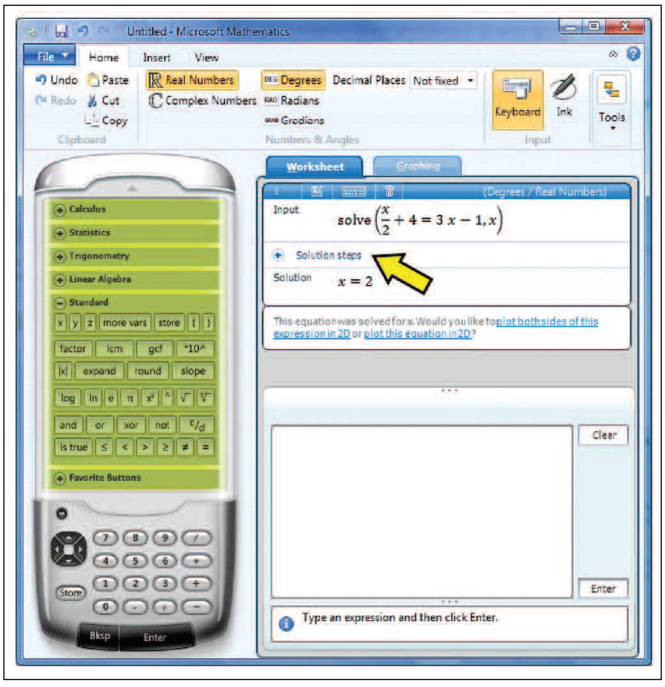


See What's Interactive, Engaging, and Undervalued (Okay, It's Free)?... Math Clips



See OAME Awards 2011 *Kenneth D. Fryer Award for a Secondary Math Department* Recipient





See Ontario Math Olympics 2011

See Technology Corner – Numeric-Algebraic-Graphical Tools

Submission of Articles

The *Ontario Mathematics Gazette* (*OMG*) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *OMG*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles must be word-processed in MS Word, double-spaced with wide margins, not exceeding 10 numbered pages of text, and prepared according to the *Publication Manual of the American Psychological Association, Sixth Edition*. Figures and diagrams should be drawn by computer, if possible, or drawn in black ink in camera-ready form. Embedded images must also be submitted separately in jpeg or tif format. Proof of the photographer's permission is required, and for **photos of students** under the age of 18, the written permission of a **parent or guardian is required**.

You must submit **one complete copy** of your article, embedded with any tables, figures, and captioned photographs or graphics, to the Editor, Marian Small, along with **separate files for each of the text, graphics, and/or photographs**.

Your name should not appear anywhere in your article, including websites, so that your article can be sent out for blind review. Your name, full mailing address, and e-mail address must be included on a separate sheet. Upon review, you will be notified as to whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected.

The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME.

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Submission of Advertisements
Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** at the above address. Courier is recommended to avoid possible delays. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be camera ready and colour advertisements should have no bleeds.

Advertising Rates
Advertising rates are available by telephoning, e-mailing, or writing to the Advertising Manager.

▲ Editor's Message

MARIAN SMALL



As I deliver professional sessions across the country, I am repeatedly asked whether I think students need to know their addition and multiplication facts and why so many of them don't. Because this seems to be such a pervasive question, I thought it might warrant attention in this column.

The short answer, for me, is "Yes." I believe that if students leave Grade 6 still stumbling over many of the addition or multiplication facts, it will be very difficult for those students to engage easily in the types of problem solving we expect of them in Grades 7–12, and it might even jeopardize their potential mathematical success in these grades. I also think that if students do not know their facts, estimation in both mathematics and everyday life becomes virtually impossible. For example, in order for a student to estimate the number of seats in 34 buses that each hold 52 students, he or she could reason that the total must be a bit more than 1500, but it is only possible to know this by knowing that 3 x 5 = 15. Without estimation, there is no way to check that calculations done using technology are reasonable, and without facts, there is limited ability to estimate.

The question that is often asked is why students don't know these facts, since children seemed to be able to memorize them "in the old days." It is also important to consider what the arguments against the position that students need this knowledge might be, and what we can do about the issue, both in the primary and junior years, at the time when students are learning the facts, and in the intermediate years, when teachers are frustrated with that lack of knowledge.

Probably the main reason that so many students do not know their facts is that nobody is demanding it. Students tend to be practical; they respond to what is most pressing. If nobody seems to value knowing the facts, they just don't bother memorizing them. That said, I suspect some students really do know them, but use their calculators anyway to be sure.

The difficulty for many primary and junior teachers is that they are torn between a curriculum and educational culture that values that students understand the meaning

of statements like 3 x 4 = 12, using strategies based on known facts, and a public culture that wants things to be like they always were—children should just memorize their facts! In fact, although the current Ontario curriculum in Grades 1–4 never mentions memorizing facts, the curriculum does mandate the use of mental strategies in Grades 2–4 (not just pictorial or technology tools), and although the curriculum in Grade 5 doesn't mention facts anymore, it should be clear that multiplying two-digit by two-digit numbers using estimation, student-generated or standard algorithms, as is mandated in the curriculum, is not really feasible without knowing facts. I believe that there is an implicit understanding that students who do not have significant memory issues should be using mental math, either through direct recall or fairly quickly using strategies, rather than calculators or pictures of objects, to determine sums and products of single-digit numbers by the end of Grade 5.

What is the argument against this position? Some fear that teachers/parents/administrators will see the issue as an "either/or" situation—that if we want students to construct single-digit products fairly quickly, we have to abandon the work on strategies. I disagree. Although in Grade 2, for addition facts, and Grades 3 and 4, for multiplication facts, we should be encouraging students to use pictures, we should also be encouraging mental visualization and mental numerical strategies as students are ready.

Others would argue that with technology, there is no need for this sort of memorization anymore, but, as was mentioned before, there is no way to check the reasonableness of calculations and no way to estimate without knowing facts. There are, indeed, students whose memory capacity simply does not allow them to recall these facts, but it is not nearly as large a percentage of the student population as teachers report for the percentage of students who seem not to know their facts.

What can a primary or junior teacher do? I do not believe that it's a return to flash cards or "mad minutes," whether on paper or digital, that we should be envisioning. I think we need the continued commitment to making calculations make sense, encouraging students to use both visualization and numerical strategies. But we can combine this with opportunities for students to participate in quick-paced games where they are motivated to know facts fairly quickly, and with problems where students need to use lots of facts and are motivated not to stumble over each one as they seek to solve the problems. The more we use something, the easier it is to remember it; this applies to addition and multiplication facts too. We can do regular short lesson warm-ups where students use facts. As teachers, we can also help by reading calculations more meaningfully. For example, rather than saying "4 times 2," we can say "4 groups of 2," helping students by providing a mental image of what 4 x 2 means and what kind of answer makes sense.

If a student in Grade 5 or 6 is still stumbling over facts, we can use parallel tasks and, while some students are working on problems involving two-digit multiplication, students who need it get additional time to consolidate their fact knowledge. And there may be some students who actually enjoy and are eager to respond to digital flash-card-type activities, after they have had ample time to make meaning of what these calculations are.

Students who reach the intermediate years without knowing their addition and multiplication facts should be expected, if at all possible, to become quicker at these facts, whether using strategies or straight recall (although instantaneous is probably never necessary) through a variety of approaches. Some might respond to intrinsic motivation; by helping them understand that knowing the facts could help them be more successful in math, some students might make the effort to learn them. Some might respond to an external "reward"; perhaps a teacher could find a way to benefit the student in some way if he or she chooses to learn the facts; it might be exemption from some assignment or some other type of external reward. The practice of facts can be accomplished through games, through problems or puzzles that require the use of many facts, or through direct memorization.

In summary, it is time to fix the problem. We certainly want students to have good understanding of what they are doing, but there are certain fundamentals that should become second nature for most students for their ultimate success. ▲

▲ Call for Manuscripts

The *Ontario Mathematics Gazette* is inviting manuscripts for all grade levels.

Instructions for submission of manuscripts are found on page 1 of any *OMG*.

Contact the Editor for further details.

▲ President's Message

CONNIE QUADRINI, PRESIDENT



Connie Quadrini is currently the Grades 7–12 Mathematics Program Consultant for the York Catholic District School Board. Her current work focuses on facilitating networked, job-embedded professional learning involving Grades 4–12 teachers within families of schools, as well as supporting school teams in the collaborative development and implementation of school improvement plans. Connie has served as an OAME director for 6 years and Y4MA executive member for 13 years. She was the co-chair of the OAME 2008 Annual Conference, and was the recipient of the OAME Union Gas Award for Outstanding Leadership in Mathematics Education in 2003. Connie has taken a leadership role in numerous Math GAINS initiatives, including Steering Team Lead for the 2008–2009 Coaching for Math GAINS; co-organizer of Math CAMPPP 2008, 2009, 2010, 2011; and co-facilitator of several professional learning series, including the Mathematics Coaching Cycle, PRIME Leadership Framework Adobe Connect Book Study, and Leadership in K–12 Mathematics System Planning. This year, she presents mathematics leaders with an opportunity to explore assessment within a new OAME Leadership Conference format.

USING RICH ASSESSMENT TASKS

Growing Success: Using Multiple Sources of Evidence to Evaluate Student Achievement

With the release of the *Growing Success* document (Ontario Ministry of Education, 2010), mathematics educators have been provided with the opportunity to re-explore assessment practices and consider what these look like specifically within the context of mathematics.

In the Evaluation section of the policy document, we are presented with what it means to evaluate student achievement. "Evidence of student achievement for evaluation is collected over time from three different sources—observations, conversations, and student products. Using multiple sources of evidence increases the reliability and validity of the evaluation of student learning. 'Student products' may be in the form of tests or exams and/or assignments for evaluation. Assignments for evaluation may include rich performance tasks, demonstrations, projects, and/or essays." (Ontario Ministry of Education, 2010, p. 39)

Rich performance tasks provide teachers with the opportunity to observe process and examine product. But what might a rich performance task look like in mathematics?

A Look at Criteria for Rich Assessment Tasks

NCTM's *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions* (Stenmark, 1991, p. 16), presents us with some suggested criteria for performance tasks. The list, which is adapted from one developed by Steve Leinwand and Grant Wiggins, offers us a number of characteristics for examining and developing such tasks. As we explore the characteristics presented, I invite you to consider rich "*assessment*" tasks as our broader lens for exploring these characteristics, rather that just "*performance*" for evaluation.

ESSENTIAL
- The task fits into the core of the curriculum.
- It represents a "big idea."

vs. TANGENTIAL

Rich *assessment* tasks offer teachers with the opportunity to focus on a "big idea" through the clustering of grade/course expectations. Carefully designed, these tasks enable students to make connections based on important mathematics.

AUTHENTIC
- The task uses processes appropriate to the discipline.
- Students value the outcome of the task.

vs. CONTRIVED

Rich *assessment* tasks engage students in the actions of mathematics, otherwise known as the mathematical processes. Through problem-solving, reasoning and proving, reflecting, representing, connecting, selecting tools and computational strategies, and communicating, students appreciate the utility and power of mathematics as they apply their knowledge and skill to solve meaningful problems. They value the time that they are given to engage in their best mathematical thinking.

RICH
- The task leads to other problems.
- It raises other questions.
- It has many possibilities.

vs. SUPERFICIAL

As students engage in such *assessment* tasks, they uncover the complexity of mathematical thinking that is needed to solve problems. As students pose questions, such as, "What if…?," "How would… affect…?," "Why does this work?," "Do we need to consider…?," they set the stage for their inquiry and envision the possibilities.

ENGAGING
- The task is thought-provoking.
- It fosters persistence.

vs. UNINTERESTING

An engaging *assessment* task is one that will stimulate student interest and engage students in a "good struggle," fostering motivation, perseverance, and resilience.

ACTIVE
- The student is the worker and decision maker.
- Students interact with other students.
- Students are constructing meaning and deepening understanding.

vs. PASSIVE

In contrast to a quiz or test, rich *assessment* tasks offer students the opportunity to interact with other students. As students work collaboratively to explore the task presented, students "bump up" ideas with others. The teacher can focus his or her attention on listening to students as they clarify their mathematical thinking, present conjectures, and share reasoning. In addition, he or she can pose questions to help students make their thinking explicit. Rich *assessment* tasks enable students to be active in the work as they negotiate meaning and deepen their understanding.

FEASIBLE
- The task can be done within school and homework time.
- It is developmentally appropriate for students.
- It is safe.

vs. INFEASIBLE

Rich *assessment* tasks are safe and offer an entry point for all students, while preserving the intent of the assessment of curriculum expectations. As students continue to construct meaning by making connections, they seek out opportunities to revise and/or refine their work. Naturally, this can provide opportunities for work on the task within school and home environments.

EQUITABLE
- The task develops thinking in a variety of styles.
- It contributes to positive attitudes.

vs. INEQUITABLE

In honour of the diverse ways in which students think and make sense of mathematics, it is important to design an *assessment* task that reflects this diversity. Whether students are systematic in their approach to solving problems or "out of the box" thinkers who cycle through different processes before solving a problem, the task must be accessible to a range of diverse learners. The design of the task offers opportunities for students to enter into the problem, using ways of thinking that are familiar or comfortable to them, and encourages students to explore different ways of thinking. Students develop confidence in their ability to approach and solve problems, and as a result, develop positive attitudes toward mathematics.

OPEN
- The task has more than one right answer.
- It has multiple avenues of approach, making it accessible to all students.

vs. CLOSED

Rich assessment tasks that are open give permission to students to think creatively and divergently. Given that there is more than one way of solving the problem and more than one right answer, students recognize that understanding mathematics is not about memorizing rules and procedures, but about developing the flexibility and mathematical habits of mind that will equip them with the tools needed to solve problems within and beyond the classroom. When we create assessment tasks that provide multiple avenues of approach, we honour the mathematical knowledge and understanding that students bring.

Developing an Assessment Task

On page 17, Stenmark (1991) offers educators some suggestions for developing assessment tasks.

Start with an idea.	What existing resources could I use? With whom might I work to brainstorm such ideas?
Test the idea.	Does it meet the criteria for Rich "Assessment" Tasks? Will my students engage in the context?
Begin converting the idea.	Where does it fit in the curriculum? What will my students have to know? Describe the task and furnish information to students on the criteria. Will these be clear to students?
Consider response formats.	Will I require written and/or oral components? What group discussions and activities are needed?
Develop teacher notes.	What do students need to know ahead of time? What materials and equipment are needed? What amount and kind of guidance should I provide?
Draft an assessment approach.	Look for process and product. How will I assess these? What attitudes and attributes do I hope to see? Why are these important? How will I assess these? Define levels of performance, as appropriate. Do these reflect the categories of achievement?
Try out the task.	Which colleagues will I ask to review the task and provide feedback? Administer the task in a few classrooms and seek out feedback from students. How will I use this feedback?
Revise where necessary.	Based on the feedback, what revisions should I make?

Some Final Thoughts

As you reflect upon how, when, and why you might incorporate rich assessment tasks into your program, I invite you to consider the implications for you as an educator and for your students.

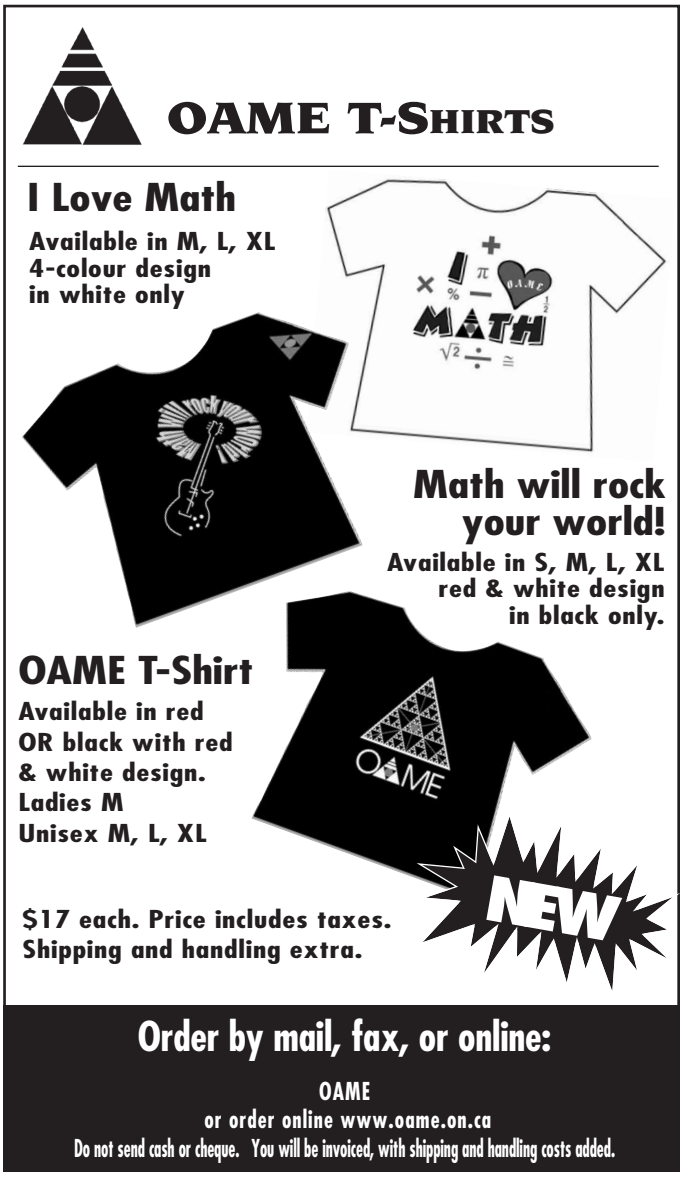
- How might these tasks reveal student thinking in a different way than a test or quiz?
- What kind of support do you anticipate that students will need when they begin working on such tasks?
- What support will you need as you create and assess such assessment tasks?

I wish you well as you consider implementing rich assessment tasks in your grade/course, and look forward to hearing from you about your journey.

References

Ontario Ministry of Education. (2010). *Growing success: Assessment, evaluation, and reporting in Ontario schools – first edition, covering grades 1 to 12*. Toronto: Queen's Printer.

Stenmark, J.K. (Ed). (1991). *Mathematics assessment: Myths, models, good questions, and practical suggestions.* Reston, VA: National Council of Teachers of Mathematics. ▲



▲ Technology Corner – Numeric-Algebraic-Graphical Tools

MARY BOURASSA



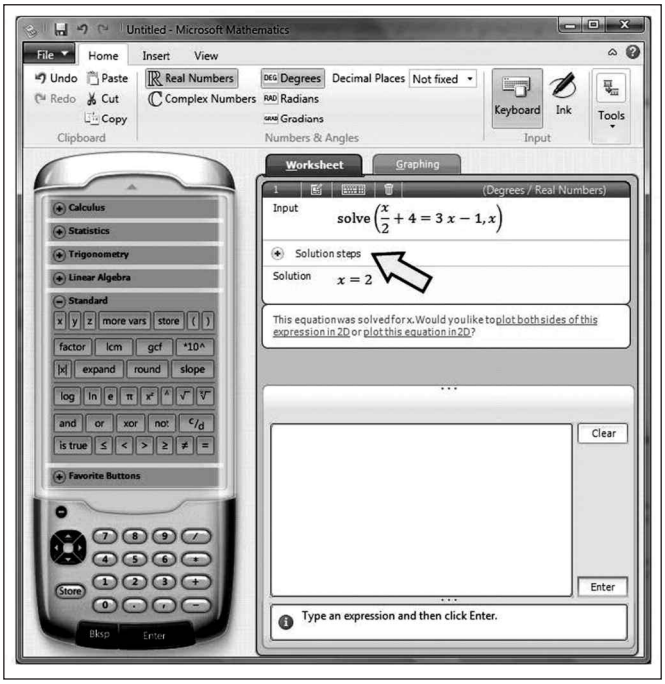
Mary teaches mathematics at West Carleton Secondary School in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She has presented workshops internationally, authored mathematics resources, is a past VP of OAME and a Past President of COMA. An award-winning teacher, Mary continually strives to learn new and better ways of helping students learn and love mathematics.

The Ontario Mathematics Curriculum places importance on connecting numeric, algebraic, and graphical solutions to help strengthen student understanding. In Grade 9, students create tables of values and graph equations so the numeric-graphical connections are generally well established. Throughout high school, they learn about finite differences and their connections to graphs of polynomial functions. Technology can play a big part in furthering these connections and can provide tools to help students develop strong algebraic skills.

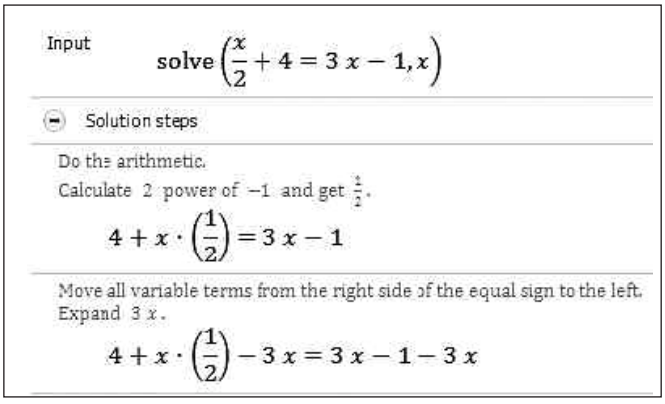
The Ontario Mathematics Curriculum lists using CAS (Computer Algebra System) in many expectations as one of several ways of teaching a variety of concepts. CAS is a phenomenal tool for teachers. I use TI-Nspire™ CAS to create all kinds of questions, and it gives me the flexibility to assign far more open-ended questions where students develop their own problems, which are all different. Having the ability to quickly do things such as expand, factor, and solve allows me to mark their work in a reasonable time frame. The question that arises is: How can students use CAS without it being a "black box" that simply gives them the answer?

Choosing an expectation from the MFM2P curriculum: "solve first-degree equations involving one variable, including equations with fractional coefficients (e.g., using the balance analogy, computer algebra systems, paper and pencil)," there are several approaches in which CAS can help with student understanding.

Using Microsoft™ Mathematics 4.0 software (available free from **www.microsoft.com/download/en/details.aspx?id=15702**), students can see correct step-by-step solutions. I see students using this to help them with a question that they have attempted with paper and pencil, but either their answer is incorrect or they get stuck and don't know how to proceed.



Enter the equation to be solved in MS Math and it will produce the solution. That's the black box and it's really not useful to students. However, if a student gets the wrong answer, he or she can figure out where the error(s) was made by clicking on "Solution steps," as shown. This provides not only the mathematical steps to arrive at the solution, but a written explanation of what is being done.



Collect like terms.

Combine $x \cdot \left(\frac{1}{2}\right)$ and $(-3)\,x$ to get $\left(-\frac{5}{2}\right)x$.

$$4 + \left(-\frac{5}{2}\right)x = 3x - 1 + (-3)\,x$$

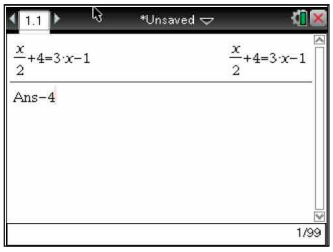
Collect like terms.

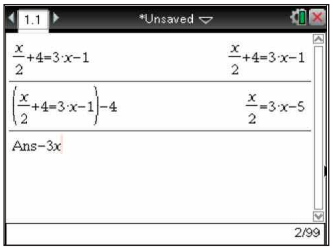
Combine $3x$ and $(-3)\,x$ to get 0.

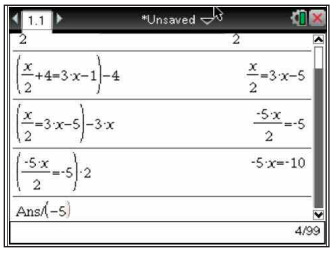
$$4 + \left(-\frac{5}{2}\right)x = -1$$

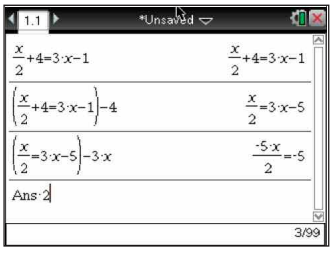
A sample of the solution steps is shown. Although some of the explanations seem more helpful than others, it does walk students through the process of finding the solution. Being able to find and fix errors is a big part of learning and helps develop critical-thinking skills.

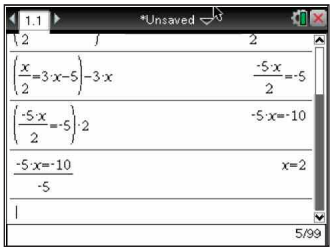
An alternative approach is to use CAS software, such as TI-Nspire™ CAS, to allow students to solve the equation without making any arithmetic errors. CAS is doing the grunt work for them, but not the thinking, as was the case in the example above. The screenshots below show the progression of the student's solution.



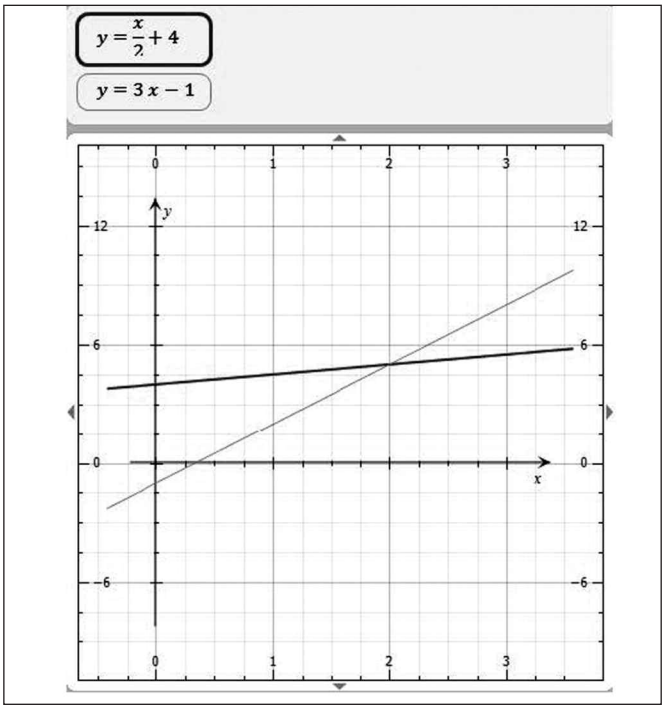








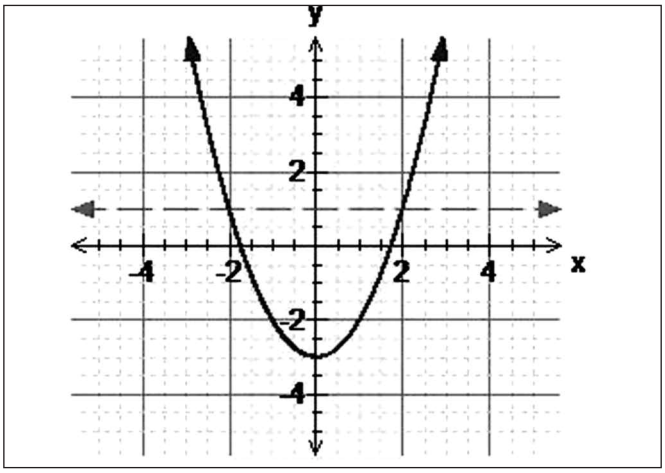
One benefit of this method is that students who perform the wrong operation will notice that their equation did not get simpler to solve. They then need to use critical-thinking skills to reason what they did incorrectly and how to fix it. Using CAS in this way also builds student self-confidence, as the calculator is taking care of the arithmetic, letting them focus on the process of solving the equation and arriving at the correct answer. Another benefit is that students can work at their own pace, understanding every step of the way.

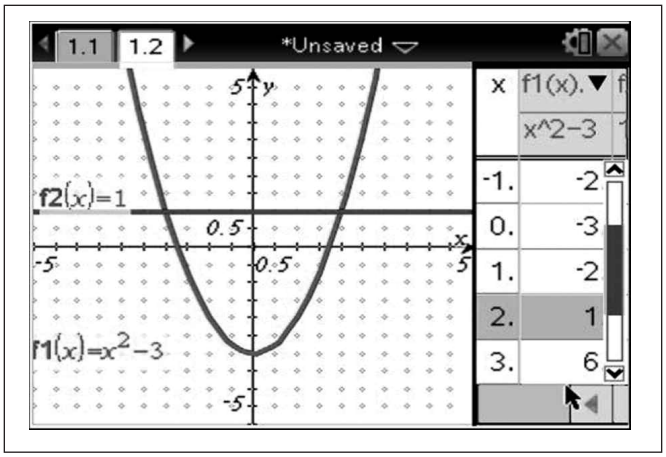


It is important that students understand the meaning of their solution—what does it represent graphically? MS Math will graph the left and right sides of the equation separately, and students can see that they intersect at $x = 2$, which is the solution found algebraically.

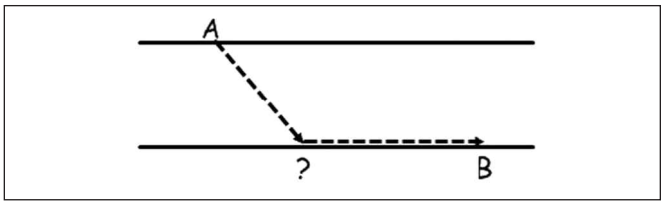
This approach, starting with a simpler example when one side is equal to a constant, is important in making the link to the algebraic solution. Carrying this approach over to a course such as MHF4U gives inequalities much more context, as students truly understand what they are looking for; they are not simply following an algorithm.

For example, if we want to solve a simple inequality such as $x^2 - 3 < 1$, we are looking for the interval of x-values, where the y-value of the parabola is less than 1. These can be shown graphically or can be found in a table of values.





Some of the best problems are those that students can solve numerically or graphically in Grade 9, but can't solve algebraically until they reach calculus. It is so nice to go full circle with the students. These include 3-D geometry optimization problems from Grade 9, which students can solve algebraically once they reach calculus. An example is one where someone needs to get from point A to point B, and students must determine where they should land based on given conditions, e.g., the condition that the first leg is slower or more expensive than the second leg. This problem comes costumed in a variety of contexts, but it is really the same problem.



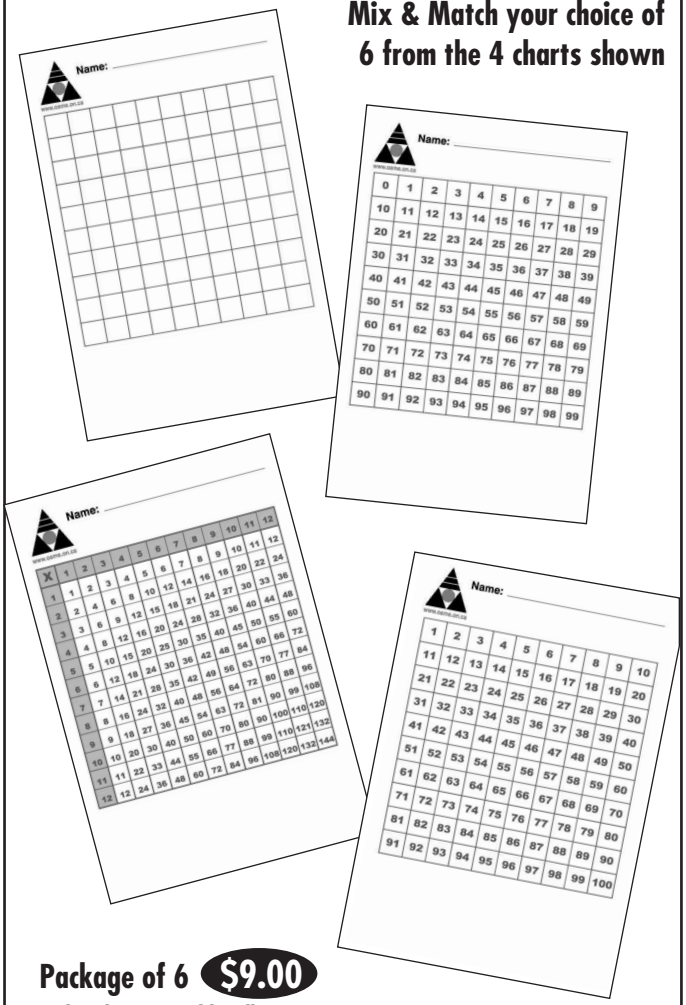
Fred Ferneyhough wrote about solving this problem in a variety of ways in this very column many years ago. He said that Grade 9 students can solve it numerically using a spreadsheet or lists on a graphing calculator and then put the formulas used in the spreadsheet together to come up with an expression for the cost of getting from point A to point B. Graph that equation, using technology, as it is quite a complex equation for Grade 9's, and find the optimal value from the graph, using the technology. Once students reach calculus, they will work with the same equation and solve the problem algebraically. Starting with a refresher of what they may have done to solve this problem in Grade 9 would tie it all together nicely and would help some students better understand how to determine the equation representing cost.

These are only a few examples that demonstrate that technology, paired with good instruction, is a powerful tool. It can help develop skills and understanding, show connections, and improve critical thinking, which will help our students in math class and beyond.

A comprehensive comparison of computer algebra systems can be found at **www.en.wikipedia.org/wiki/Comparison_of_computer_algebra_systems**. ▲

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Only selected candidates will be contacted by email and/or phone.

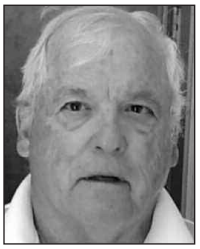
tvo TVO administers the Independent Learning Centre (ILC), the province's designated provider of distance education.

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▲ OAME/NCTM Report December 2011

TODD ROMIENS



Todd just retired from teaching K–12 methodology in the Faculty of Education at the University of Windsor. He is a past president of OAME and a life member, as well as a previous editor of the Gazette.

Now that the school year is in full swing, NCTM has a lot to offer both beginning teachers and seasoned veterans. The website **www.nctm.org** offers teaching tips and other ideas in the areas of Lessons and Teaching, Problems, and Parent and Family Resources.

A focus that has recently been emphasized at all levels of education is "differentiated instruction." I believe that teachers have been differentiating their instruction to accommodate students' learning for quite some time. However, NCTM has produced a new release to help those who may be struggling in their individual classrooms, to help students with varying needs understand the mathematics that teachers are teaching.

The new release is *Using Formative Assessment to Differentiate Mathematics Instruction: Seven Practices to Maximize Learning.* The author is Leslie Laud (2011), a staff development expert and former math teacher. The book focuses on the strategies of using formative assessment to differentiate mathematics instructions for students in Grades 4–10. The cost is $30.95 for non-NCTM members and $24.76 for members.

Through the "Illuminations" part of the NCTM website, there are a number of lessons pertaining to current curriculum that can be explored. The NCTM library has a set of two lessons called "Count on Math," which are accessed through the lessons package for Grades 3–5. Search under "Number and Operations."

The lessons give ideas about teaching the Grade 6 expectation involving a million. The first is titled "Every Breath You Take" and is an open-ended exploration of how many breaths are taken in a specific time. Students estimate, experiment, and graph real data. There is an activity sheet provided to record data. A nice discussion around estimation is encouraged.

The second lesson is titled "Making Your First Million." It is useful to get students to understand the magnitude of the number 1 000 000. Included is a very useful activity to allow students to estimate and investigate ideas related to a million. I have used this activity sheet in many of my classes when teaching about large numbers. A nice resource that gets students involved in this concept is David Schwartz's book *How Much Is a Million?* (1985). In it he pursues many of the same ideas, having students estimate and compute. Many of the questions are very open-ended and allow students to find a variety of approaches to solving the problem.

One way to begin discussion on large numbers is to give students a situation about you. For example, tell them, "As of today, I am 32 467 526 _____ old. Their job is to determine the unit. This is a nice problem, as there are a variety of ways to determine the solution. Once the unit is determined, another nice question is to have them determine your birthdate. If your original number is accurate, they should be able to determine the exact date.

Illuminations is now a channel on YouTube. You can get ideas about how teachers are using the site and other free ideas in their classrooms. Follow **www.youtube.com/nctmilluminations**. There will be lesson tutorials, contests, and teacher success stories.

NCTM continues to provide a wealth of free materials for teachers to tap into. I hope all teachers will take advantage of this resource.

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Laud, L. (2011). *Using formative assessment to differentiate mathematics instruction, grades 4–10.* Reston, VA: National Council of Teachers of Mathematics.

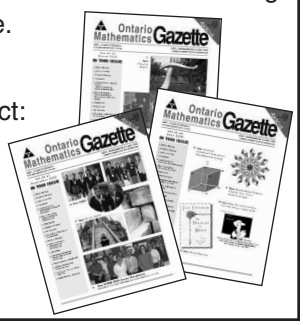
Schwartz, D. (1985). *How much is a million?* New York: Lothrop, Lee & Shepard Books. ▲

▲ Call for Gazette Editor

The *Ontario Mathematics Gazette* is taking applications for a two-year term as editor commencing with the September 2012 issue.

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For more details, please contact:
Fred and Lynda Ferneyhough



▲ What's the Problem? That's Odd

SHAWN GODIN



Shawn is the head of Mathematics, Technology, and Family Studies at Cairine Wilson Secondary School in Orleans, Ontario. He is a past and present member of the OAME Board of Directors and is currently Editor-in-Chief of Crux Mathematicorum with Mathematical Mayhem.

Welcome back, problem solvers. I hope your year is shaping up well. Last time, I left you with the following problem:

Prove that the quadratic equation $ax^2 + bx + c = 0$ does not have a rational root if a, b, and c are odd integers.

This problem appeared as Totten-M3 in the September 2009 issue of *Crux Mathematicorum with Mathematical Mayhem*, and the solution appeared in the April 2010 issue. You can check out more problems at the Crux Mathematicorum with Mathematical Mayhem website, **www.journals.cms.math.ca/CRUX/**.

I love questions like this. Your gut reaction, before you have had time to think or play, is "that can't be right!" Let's play around with this a bit to see what's happening.

Suppose we pull out one of the biggest hammers in our toolbox, the quadratic formula. It tells us that the (possibly complex) solutions to our equation are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Since a and b are odd integers, then x will be rational if (and only if) the discriminant is a perfect square. To see if this was possible, I used Fathom and did some experimenting.

I defined a, b, and c as 2·randominteger(1,100) + 1 to guarantee that they were odd integers, and the discriminant, D, was defined as above. The *square* is a Boolean variable with a value of

$$(D > 0) \text{ and } \left(\sqrt{|D|} = trunc\sqrt{|D|}\right)$$

which will be true if the discriminant is non-negative (which ensures the roots are real) **and** the square root of the absolute value of the discriminant (so that Fathom doesn't get upset if D is negative) is unchanged if we truncate it (that is, drop everything after the decimal), and false otherwise.

My first bit of experimenting yielded the following results for 100 values of a, b, and c.

Collection 1

	a	b	c	D	Square
=	2 randomInte	2 randomInt	2 randomInt	$b^2 - 4a \cdot c$	(trunc ($\sqrt{\|D$
1	197	31	85	-66019	false
2	167	43	179	-117723	false
3	65	137	131	-15291	false
4	3	197	67	38005	false
5	169	121	181	-107715	false
6	177	131	43	-13283	false
7	185	201	137	-60979	false
8	79	125	73	-7443	false
9	125	83	185	-85611	false
10	151	191	13	28629	false
11	179	81	19	-7043	false
12	43	85	113	-12211	false

Frequency of Square

Notice that all values of square were false. I could re-randomize my collection to pick 100 more values, or pick a larger sample, but I would always get the same results. This doesn't prove that our hypothesis is true, but it gives us some hope. (For fun, I changed it so that the numbers could be even or odd, and out of 1000 cases, there were always less than 10 true values. The moral of the story is: factorable quadratics are very rare!)

So let's go back and think about our problem. If we want the roots of $ax^2 + bx + c = 0$ to be rational, then it must be factorable. Thus, we should be able to find integers m and n such that $m + n = b$ and $m \times n = a \times c$, if we are going to factor it. Since a and c are odd, m and n must also be odd (based on the second relationship between m, n, a, and c), but this is impossible because $m + n$ would then be even (based on the first relationship between m, n, and b); this doesn't work, since b is odd, not even. Thus, the equation $ax^2 + bx + c = 0$ does not have a rational root if a, b, and c are odd integers. Similarly, if you think of the algebra tiles representation, we would have

	odd number of x's (dx)	odd number (e)
odd number of x's (fx)	ax^2	some x's
odd number (g)	some more x's	c

Since *a* is odd, *d* and *f* are odd. Since *c* is odd, *e* and *g* are odd, so the number of tiles making up our factors are made up of an odd number of x^2 tiles and an odd number of number tiles. *fe*, the number of *x* tiles in the upper right of the rectangle, and *dg*, the number of *x* tiles in the bottom left of the rectangle, must each be odd. Therefore there must then be an even number of *x* tiles in total, but *b* is supposed to be odd—a contradiction.

My dear old adopted uncle sent me a solution very similar to mine, but with a slight twist.

If we assume that there are rational roots, then we can write the original equation in factored form like this:

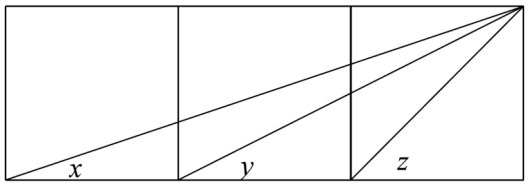
$$(mx + n)(sx + t) = 0$$

where *m*, *n*, *s*, and *t* are integers. This means that $ms = a$, $nt = c$, and $mt + ns = b$.

Now, if $ms = a$, then both *m* and *s* have to be odd. And if $nt = c$, then both *n* and *t* have to be odd. This makes $mt + ns$ even, and we have a contradiction.

Thanks for the solution, uncle, and I would love to receive any correspondence about the problems from other readers. Now for your homework:

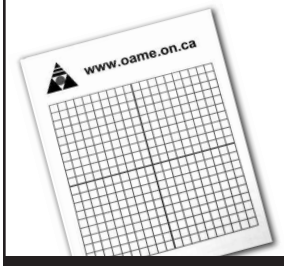
The diagram shows three squares and angles *x*, *y*, and *z*. Find the sum of the angles *x*, *y*, and *z*.



Until next time, happy problem solving. ▲

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▲ Hey, It's Elementary – Music is the Art of Thinking with Sounds (Jules Combarieu)

LYNDA COLGAN



Lynda Colgan's career has included roles as a classroom teacher, a university professor, and newspaper columnist. Her contributions to mathematics and its teaching have been recognized through awards such as the Marshall McLuhan Foundation Distinguished Teacher Award. Lynda always exhibits a passion for mathematics and views her professional mission as dispelling the myth that math is the bad guy.

For many years, it was *au courant* to jump on the "Mozart Effect" bandwagon or join the "Baby Einstein" culture. Each camp claimed that it was possible to enhance children's cognitive growth by listening to specific genres of music (often accompanied by gentle animated images). Popular opinion dominated for several decades: classical music made children smarter. When large-scale research such as Upitis' and Smithrim's (2003) study of 6000 Canadian children showed that at the end of a three-year Grade 6 *Learning through the Arts* program, students scored significantly higher on tests of computation than students in control schools, it seemed possible that popular opinion was correct—there seemed to be many benefits to learning associated with, through, and from music.

Research about the power of music to affect memory is enormously intriguing. Most significant is the fact that memory as a mental system that receives, stores, organizes, and recovers information from sensory input (Coon, 1997) has shown to be affected by many different factors, including music.

Several studies have demonstrated that music enhances the memory of Alzheimer's and dementia patients. In fact, a study at the University of California, Irvine, showed that scores on memory tests greatly improved when Alzheimer's patients listened to music (Tufts University, 2001). Other studies have recognized the therapeutic effect of music on health and education

as an alternate therapy. Oliver Sacks (2006), physician, bestselling author, and professor of neurology and psychiatry at Columbia University Medical Center, argues that music actually alters the physiology of our bodies and makes us healthier, less stressed, and smarter.

Sacks contends that because music activates the whole brain (both right and left hemispheres) simultaneously, learning is maximized and information is retained. In fact, the information being learned through, with, and from music, activates the left brain, while the music activates the right. Activities that engage both sides of the brain at the same time, according to Sacks, cause the brain to be more capable of processing information.

Further research suggests that listening to music facilitates the recall of information because music can act as a great key for recollection (Mammarella, Fairfield, and Cornoldi, 2007). Information learned while listening to a particular song can often be recalled simply by hearing or playing the song mentally.

This is where a recent project by teacher candidates at the Community Outreach Centre, Faculty of Education, Queen's University comes in. By pairing original music with mathematics content, the teacher candidates' goal was to engage students fully in meaningful opportunities to learn. On one hand, this gifted team of early career educators envisioned students tapping their toes and humming catchy tunes. On the other, they anticipated that students would be recalling fundamental mathematics definitions and ideas. Helping students to experience the sometimes complex world of mathematics in a fun, lighthearted, and "fine-tuned" manner was the overall goal of their rich and creative endeavour: *Mathematical Melodies*. To this end, they collaborated to write and record 12 fabulous songs.

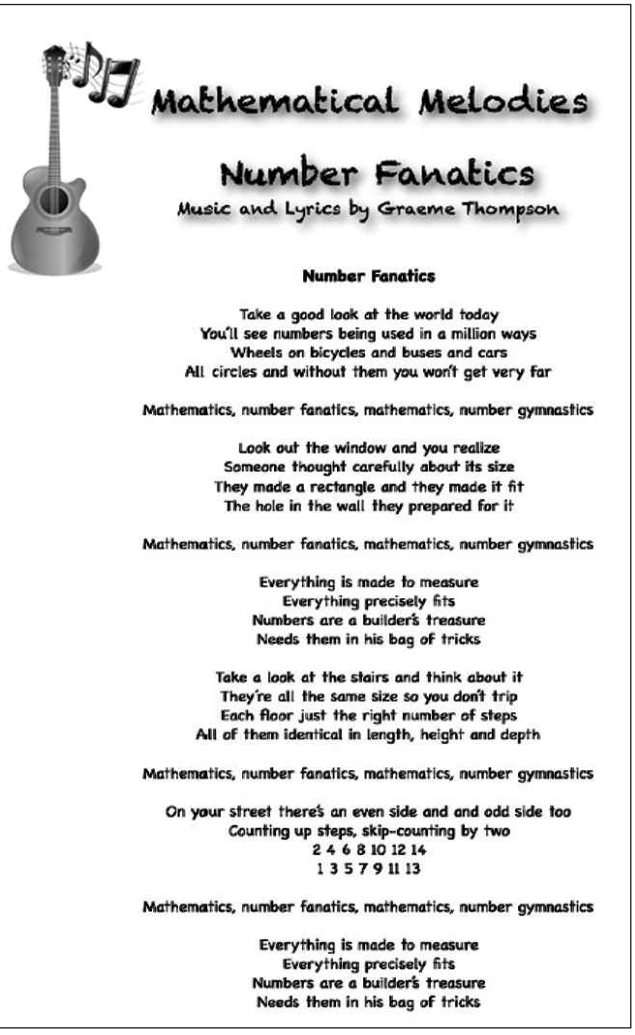
Mathematical Melodies is the product of an amazing collaboration. Six teacher candidates—Paul Hanash, Freeyon Chung, Graeme Thompson, Ruth Jean Charles, Lisa Fenwick, and Allison Hambleton—worked with Gary Rasberry, a local musician and songwriter. The outcome of their three-week collaboration is a collection of math action songs for children and a rich set of teaching resources to integrate the songs into curricula.

The songs address many concepts across the primary and junior divisions, from telling time to distinguishing among mean, median, and mode. They can be downloaded as mp3 files from the Queen's Faculty of Education website, **www.educ.queensu.ca/index.html**, by clicking on the Community tab and navigating to the "Resources for Parents and Teachers" page. On the same page, you will find the link to download the accompanying teacher resources.

The package for each song includes:

1. the lyrics for the song
2. an overview of the "Big Ideas" and Curriculum Connections embedded in the song
3. a "launch" activity, game, and hands-on activity suggestion
4. a cross-curricular teaching idea
5. live links to multimedia resources (websites, public access videos)
6. thematic suggestions and extensions
7. an annotated bibliography of children's literature, both fiction and non-fiction.

To give you a sense of what you can access, I have provided a preview of a great song by Graeme Thompson called *Number Fanatics*.



Mathematical Melodies

Number Fanatics

Music and Lyrics by Graeme Thompson

Number Fanatics

Take a good look at the world today
You'll see numbers being used in a million ways
Wheels on bicycles and buses and cars
All circles and without them you won't get very far

Mathematics, number fanatics, mathematics, number gymnastics

Look out the window and you realize
Someone thought carefully about its size
They made a rectangle and they made it fit
The hole in the wall they prepared for it

Mathematics, number fanatics, mathematics, number gymnastics

Everything is made to measure
Everything precisely fits
Numbers are a builder's treasure
Needs them in his bag of tricks

Take a look at the stairs and think about it
They're all the same size so you don't trip
Each floor just the right number of steps
All of them identical in length, height and depth

Mathematics, number fanatics, mathematics, number gymnastics

On your street there's an even side and and odd side too
Counting up steps, skip-counting by two
2 4 6 8 10 12 14
1 3 5 7 9 11 13

Mathematics, number fanatics, mathematics, number gymnastics

Everything is made to measure
Everything precisely fits
Numbers are a builder's treasure
Needs them in his bag of tricks

Please let us know what you think! A new team of young, gifted musicians will begin work on our second suite of songs in March 2012 and they will use your input and suggestions to inform their work.

References

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▲ Data Rocks – Statcan's Treasure Trove of Math Resources

ANGELA McCANNY



Angela McCanny is a mathematics educator currently working for Statistics Canada. She has been sharing her knowledge with schools since 2006, helping teachers find exciting resources for teaching the math and social sciences curriculum.

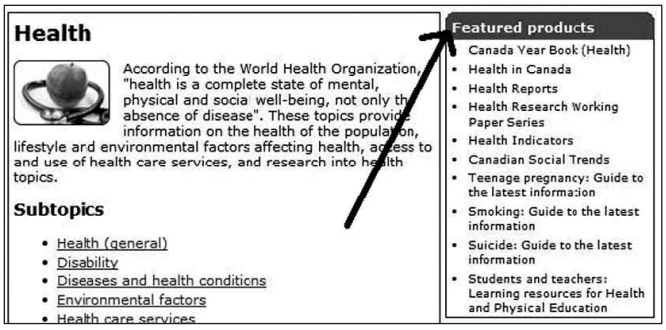
New online videos for MDM4U student data projects: Virtual Statistics Canada support in the classroom!

You know the situation—you've covered all the big topics in the MDM4U curriculum—developing a thesis, forming open and closed data questions, collecting primary and secondary data, and analyzing the data. Now it's time for the end-of-the-term project. Your students choose a topic, start to look for data, and inevitably they get stuck.

Granted, choosing a topic that is of interest and finding the data for it can be the most challenging part of a data project. Now there is help at hand.

Statistics Canada has launched a series of seven new video tutorials, ***So you have to do a data project! A step-by-step student guide to finding data***, at **www.statcan.gc.ca/edu/edu07/edu07_0000a-eng.htm**. Ranging from 5 to 10 minutes in length and available in both English and French, the videos guide students through the process of finding and using data from the **Learning Resources** website, **www.statcan.gc.ca/learningresources**.

Using the Students page of the Statistics Canada website as a starting point, students are shown how to search for data within the website: browsing by subject; searching *The Daily*; and finding data tables in CANSIM, using E-STAT. Students will find links to **Featured products** that often spark interest in further research into issues related to smoking, pregnancy, suicide, and crime, for example.



Throughout the video tutorials, students are encouraged to evaluate periodically, whether they are finding suitable data for their original topic idea or whether they need to branch out to a new or related topic for which there is more data.

Once they have located data for their topic of interest, students learn how to create tables and graphs to paste into their projects, or how to import the data into a spreadsheet program for analysis (Excel or OpenOffice Calc), or dynamic-analysis software such as Fathom.

Using the videos in your lessons

You can assign the viewing of the video tutorials as homework or use them to run your own in-class workshops. A worksheet for students to record their search results is available by emailing the author at **angela.mccanny@statcan.gc.ca**. Here is a suggestion for a three-day plan:

Day one – View tutorials 1, 2, and 3

Tutorial 1: Choosing a topic

Discover resources for data projects from the Statistics Canada student page.

Tutorial 2: Searching for data, using Browse by subject

Start broadly by choosing the subject, then select subtopics to narrow down your search. Find articles, summary tables, and original CANSIM tables related to your subject.

Tutorial 3: Searching for data, using *The Daily*

*Use **Search by topic** within **The Daily** to see brief articles that have been written about your topic by Statistics Canada. You will find data links at the end of each article.*

Ask your students to use the search technique on a topic they have in mind. Be sure to have them record the search terms used and the table numbers they find as they go along.

Day two – View tutorials 4 and 5

Tutorial 4: Searching for CANSIM data in E-STAT

*Search for CANSIM data in E-STAT, using the **Table of contents**, or search by **topic** or **table number**. **CANSIM data** comes from over 350 ongoing social and economic surveys in Canada.*

Tutorial 5: Searching for census data in E-STAT

*E-STAT will let you search for more than 300 years of **Census data**, from the most recent Canadian census results (2006) back to 1665. You can look at variables like the age of marriage, occupations, languages spoken, and the gender balance in the population.*

Again, have the students try out the search techniques on their topic. Encourage them to consider whether they need to revise their topics to match the data available.

Day three – View tutorial 6

Tutorial 6: Creating tables and graphs in E-STAT

*Use E-STAT to create **tables** or **graphs** that you can copy and paste into your project.*

Let students try creating graphs and customizing tables, using the data they have previously found related to their topics. If you are using a spreadsheet program or dynamic-analysis software such as Fathom with the students, view tutorial 7 and practise importing the data.

Tutorial 7: Downloading data from E-STAT into software

Download data from E-STAT into other programs to analyze it, create graphs, and find trends.

Your students will now have a solid set of tools for beginning their data projects.

More project resources – ready-to-use!

Other excellent project resources can be found on the Grades 9–12 Mathematics resource page, **www.statcan.gc.ca/kits-trousses/courses-cours /edu05_0018aeng.htm**, including:

- **Investigating social-justice issues** (topics with associated data tables and critical-thinking questions spur students to ask meaningful questions)
- **Data-analysis projects and fairs** (links to pages for finding and using data)
- **Podcasts and audio learning** (two series for your auditory learners: Dr. Data guides students through finding data in E-STAT and student interviews about data projects)

- **Statistics: Power from data** (a handbook for conducting surveys)

You can also help your students visualize a completed project by viewing the following:

- **Project ideas for senior Math projects** (ideas in the three subject areas – health, crime and economics): **www.statcan.gc.ca/edu/edu06i_0003-eng.htm**
- **Examples of students' projects** (students describe developing a thesis, finding and analyzing data, and making conclusions): **www.statcan.gc.ca/edu/edu02_0005-eng.htm**
- Summaries of exemplary student projects (seven students write about planning their own projects): **www.statcan.gc.ca/edu/edu06i_0007-eng.htm**

Data fairs let them show what they know

Do your students think that their data project is just one more assignment, while you know that it is one of the most formative experiences for life in the real world? Help students discover the interest others have in their area of expertise by hosting a school or board-wide data fair. Similar in model to a science fair, the data-analysis fair showcases student projects in mathematics. It provides a chance for students to present their projects and receive comments from teachers, other students, and the public. Students are excited when they see that their data discoveries are valued outside the classroom.





Renpeng Sun presents his project to fellow students.

> *"The data fair made me realize how much I enjoyed statistics. So I started my honours Co-op degree in statistics at Carleton the following year...."* **Shirin Roshanafshar,** former Grade 12 student, now pursuing a career at Statistics Canada

Statistics Canada has had enthusiastic attendance at six data fairs since 2004, bringing students and federal statisticians together for animated data discussions.

If you would like to organize a data fair for your students, you can learn about getting started, read other schools' experiences and obtain judging guidelines at **www.statcan.gc.ca/edu/edu06i_0005-eng.htm**.

Let the data rock! ▲



▲ In the Middle – Lights, Camera.... Action! Math in Movies and Movies in Math

CAROLINE ROSENBLOOM



Caroline Rosenbloom is currently on secondment as an Elementary Coordinator in the J/I Midtown Technology Option and Mathematics Instructor in the pre-service program at OISE/UT. She has enjoyed working with intermediate students and teachers as a middle school teacher, Mathematics Instructional Leader for the Toronto District School Board, and Education Officer at EQAO. Caroline is co-author of "Investigating Mathematics Using Polydrons."

"Math is part of life!" We hear that claim all the time. We claim that what we teach our students is relevant. Part of what we can do as an educational community is to help students and the community see the connections between math and everyday life. In this article, I point out some fun connections between movies and mathematics. I also encourage students to create their own educational videos and point out tools they can use to do this.

Our students may be surprised to discover how often mathematical concepts appear in movies they have enjoyed. ***Math in Movies*** is a collection of math-related clips created by the Department of Mathematics at Harvard University.

In the movie ***Father of the Bride***, Steve Martin plays George Banks, a father faced with the prospect of the wedding of his eldest child. In one scene, George is at the grocery store, purchasing hot dogs and buns. This clip shows George's frustration with the hot dog and bun packaging. He wonders why hot dogs are sold in packages of 8 and hot dog buns in packages of 12. This is a great clip to show when teaching multiples.

A question to ask our students would be:

How many packages of hot dogs and packages of hot dog buns would George need so that there are no leftover buns?



One scene in ***Shrek the Third*** has Prince Charming questioning Pinocchio. Pinocchio knows he has to answer truthfully, since his nose reveals any deceptive answers. Pinocchio uses logical manipulation to comically answer truthfully, yet disguise his answer and confuse Prince Charming. For example, when Prince Charming asks, "Where is Shrek?," Pinocchio replies, "I don't know where he's not!"



The ***Phantom Tollbooth*** is another great movie that deals with several topics in the intermediate grades, such as:

- addition/subtraction of integers
- Platonic solids such as dodecahedra
- measures of central tendency
- rational and irrational numbers
- Fermi questions – estimation questions

For example, in this movie, the Mathemagician claims to have 4 827 659 hairs on his head. Students can investigate whether it is possible for a person to have this many hairs on his or her head. Other math topics are mentioned and would be interesting to explore in more depth.



Haley Joel Osment plays Trevor McKinney in ***Pay It Forward***. This young boy comes up with a plan to change the world, using the magic of exponential growth. In one clip, Trevor explains how his "pay it forward" plan works. If he does a good deed for three people, and each of them does a good deed for three people, then, quickly, many people will be affected in a positive way. Although he feels that his plan is not working, this demonstration of exponential growth shows how fast and how many people he can impact.

These movie clips can be used to identify math concepts that are then examined further in class. Using movie clips in a **Minds On** task is a great way to set the context as to what will be taught.

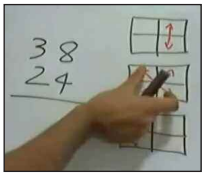
Communicating one's understanding of mathematical concepts is an important aspect of mathematics. It forces an author to clearly focus on the key concepts needed to convey an idea to another person. One might say that you clearly understand an idea only when you can explain it to another person.

An interesting way students can communicate mathematical concepts is by creating screen casts. For example, students can use applications like **QuickTime** to create a screen recording of their desktop. They can use QuickTime together with Geometer's Sketchpad™ to record how they would create a hexagon, using transformational geometry, or demonstrate their understanding of Pythagorean theorem. This combination of tools allows students to narrate their investigations and later proudly share them with their fellow students.



Students can also use QuickTime to create a movie to teach a concept. A movie can be:

- a recording of oneself explaining or teaching a math concept
- a math mystery, where students can create a skit to solve a problem
- photos used to create a slide show to highlight a big idea
- a music video with lyrics to describe a mathematical process



Comic Life and **Bitstrips** are other great tools used to create math stories and problems. These are both Ministry-licensed software.

You'll find that when given the opportunity to demonstrate their thinking in many ways, students become more invested in what they are doing. Additionally, they reinforce their learning by having to summarize their thinking, using various media.

Setting the context in math by bringing in movies and identifying mathematics helps students see connections to their own lives. It encourages students to ask those "I wonder why…?" questions, like George Banks does with the packaging problem. These connections are what help students see the relevance in what they are learning. So ask your students about the next feature film they see. I bet they'll be looking at movies in a whole different way.

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Knill, O. (2006). *Mathematics in movies.* Retrieved from **www.math.harvard.edu/~knill/mathmovies/**

The Math Lab. (2005). Retrieved from **www.themathlab.com/Algebra/exponential%20growth%20and%20decay/payit.htm** ▲

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- Dan Meyer
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- Joan McDuff
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▲ Directors' Dialogue

ANN MICHELE STENNING



Ann Michele Stenning is a mathematics teacher at South Collegiate Institute with the Thames Valley District School Board. She enjoys mentoring new teachers and learning new ways of how to address the various learning styles of her students.

OAME Board of Directors Meeting June 3–4, 2011

Friday, June 3, 2011

Connie Quadrini, OAME's president, began the board meeting by welcoming new board members and chapter representatives. As president, Connie connected OAME's vision statement to her focus on assessment for the year. Lynda Ferneyhough, one of OAME's executive directors, announced that OAME membership had dropped and was currently at 1006 members. Todd Romiens, the NCTM representative, announced that a number of regional NCTM conferences were being planned for the fall. Those interested in attending could register at **www.nctm.org**.

On behalf of SWOAME, David Petro reported on the OAME 2011 conference held this past May in Windsor. Connie thanked David for a great conference. Henry Santos gave an update on the plans for OAME 2012 in Kingston, Ontario at St. Lawrence College. Promotional material such as T-shirts, bracelets, and pens were available to board members. Marc Husband reported that OMO 2011 was in progress during the first weekend in June. Connie Quadrini commented on how well organized the event was. David Zimmer reported on the OAME Leadership Conference, devoted to Big Ideas, that was held in early March. He mentioned that the large turnout was an encouraging indication that people are interested in leadership in mathematics. David noted Lynda's excellent organizational skills in operating the conference, and Connie thanked David for his leadership as well.

Plans for the next Leadership Conference were announced by Connie. A wiki has been set up, where participants can present assessment dilemmas; these can be addressed during the conference and in Adobe Connect sessions throughout the 2011–2012 school year. During the conference, participants will go from plenary sessions to breakout groups, organized by grade or position. The executive approved the purchase of the NCSM Prime resource for each board member; these will be used at next year's Leadership Conference. Connie reminded board members that registration by groups of four or more will receive a discount.

After announcements, board members worked in pairs discussing principles of assessment.

Saturday, June 4, 2011

Business Meeting

- Approval of the agenda was moved by Bill Otto and seconded by David Zimmer. Carried.
- Approval of the minutes from the Board meeting of February 4, 2011 was moved by David Zimmer and seconded by Tom Boland. This was carried.
- David Zimmer noted that membership in each of the subject councils has dropped. NCTM memberships have dropped as well.
- David Zimmer announced that the Minstry's scope and sequence on financial literacy had been developed. This includes the insertion of financial literacy into every subject area. OAME has been approved to offer financial literacy institutes this summer. Connie Quadrini noted that OAME will offer institutes for various panels to focus on financial literacy. Board members asked how OAME executive would select the focus for each institute, and some wondered how financial literacy would fit into specific courses. Connie suggested that members be given an opportunity later in the day to look at some of the specific institutes that had been planned.
- Lynda and Fred Ferneyhough, OAME's executive directors, presented the financial report. Acceptance of the financial report was moved by Bill Otto and seconded by Kyla Kadlec. Carried.
- Fred Ferneyhough then described the current organization of the board and the reasons for the Committee of the Whole. Bill Otto recommended that the Finance Committee form a policy to address the costs of supply teachers to replace Board members attending OAME meetings. Additional ideas for reducing costs were presented
- The board members completed their committee meetings to re-address questions regarding financial literacy institutes. Connie elaborated on one of the proposed institutes. David Zimmer moved that, "The

Board authorizes the Executive Committee to negotiate and engage in a contract with the Ministry of Education for the development of resources to support implementation of the Financial Literacy Curriculum." This was seconded by Sonia Ellison. Carried.

- Bill Otto moved that the board meeting be adjourned.

Written Reports

- David Zimmer reported that there is a federal and international initiative to support financial literacy and that the scope and sequence material will be online soon. There will be a template for creating financial literacy resources, using a three-part format, and both elementary and secondary video clips in French and English are being developed for Fall release in 2011.

Chapter Reports and Updates

CHAMP Chapter Update

CHAMP did Manipulative Madness, where participants attended one session for their own panel, and one for a different panel. CHAMP is hosting a local competition for high school and elementary students in addition to applying to host OAME 2014.

COMA Chapter Report

COMA has continued to support the growth of our local math community. COMA subsidized ten delegates to attend the Leadership Conference in Toronto. By all accounts, winners were thrilled to attend and thoroughly enjoyed the valuable professional development they received in their sessions. Despite being rescheduled due to poor weather, the COMA Elementary Forum on March 7 was a resounding success! Jennifer Capitani, a local elementary teacher, presented an interactive workshop on Integrating the Five Math Strands in Elementary Classrooms to approximately 55 attendees. The great turnout (and great snacks!) provided many people with a chance for valuable networking and professional dialogue. The Carleton-Ottawa Math Olympics was once again a wonderful event, hosting 46 teams to a full day of math fun. With a plethora of brain-busters, a pizza lunch, and lots of awesome prizes for students and teachers alike, everyone walked away a winner. COMA is delighted to sponsor two COMO teams to attend OMO 2011, hosted by TEAMS. Thanks to the committee members and high school student volunteers for making COMO 2011 such a memorable experience. Approximately 15–20 COMA members also made the trek to OAME 2011 in Windsor in May. Many presented sessions, eager for the opportunity to share their expertise and network with math educators from across the province.

MAC^2 Chapter Update

Instead of having a keynote speaker address participants in person, MAC^2 used a 15–minute YouTube video, *TED Talk,* by Stephen Wolfram.

NOMA Chapter Report

NOMA set a goal of increasing participation at all events and from all geographical locations that it represents. So we are thrilled to report that we have met our goal and then some! The new members bring innovative ideas to the table and have re-energized the group. Participation has expanded to include members from outside the K–12 mathematics community and occasional teachers. Thanks to a few tech-savvy members, we are also able to include participants from across the north, including North Bay and Huron Superior.

Thanks to enthusiastic recruiting and exciting new prizes, we more than doubled the number of teams participating in our local Sudbury OMO playdowns, including a team from North Bay. The top two teams went off to the provincial OMO, proudly representing NOMA.

On May 26, NOMA was pleased to host an after-school session facilitated by Dr. Marian Small. She had the K–12 participants actively engaged in mathematical thinking, with a focus on reasoning and proving. Dr. Small reminded us it's not sufficient to answer the question. It is also important for students to articulate the mathematical reasoning behind the answer—and there's no need to use big words! This free session was just one of the "perks" that came with OAME membership. Groups of participants from four locations across Northeastern Ontario were able to join in via Adobe Connect. NOMA hopes these northern colleagues will be able to host future events and allow Sudbury members to participate via Adobe Connect.

Our final spring meeting will be a celebration and an opportunity to generate ideas for our fall conference. Watch for upcoming details.

NWOAME Report

NWOAME is wrapping down a busy year. Our Pi-day celebrations and Regional Math Olympics were both very successful, with four students attending the Provincial OMO competition in Toronto during the weekend of June 4–5. On April 27, we hosted a successful math evening and social with Florence Glanfield as guest speaker. Florence is an associate professor of mathematics education at the University of Alberta, is of Métis ancestry, and has a rich history of First Nations mathematics

experiences, both as a learner and as a teacher. All participants enjoyed the event.

NWOAME is presently looking at ideas for next year. We are investigating ways to encourage new membership and encouraging leadership from within our ranks. I look forward to talking to other Chapter Reps and Directors about any new ideas that their chapters may have tried recently. NWOAME executive members are also looking to develop a more meaningful partnership with local and regional school boards. We continue to engage in constant dialogue, positioning ourselves as an important voice in math-related education and professional learning in Northwestern Ontario.

PRMA Update

PRMA is looking at ways to increase membership. One way to address this issue may be to distribute mini-conferences throughout the PRMA region.

QSLMA Update

QSLMA had a very successful OMO regional competition. The event was so successful that the organizers had to limit the number of teams that could compete. A recent event with Marian Small was the best mini-conference they have had. There is talk of hosting a fall social to help everyone gear up for the year, along with preparations for OAME 2012. There are a lot of math contests occurring at the high school level, so there is hope that these math coaches will join QSLMA. In fact, QSLMA's website will post the results for these contests.

SAME Update

SAME has changed the way they did the OMO contest for the chapter. Some of the contest was written and then dinner followed. The remaining part of the contest was written and the participants, along with their parents, were treated to a musical math theatre presentation while the contest papers were being marked. A briefer awards ceremony followed by acknowledging only the finalists. Certificates and prizes were sent to the schools after the event.

SWOAME Update

Regional playoffs for the Ontario Math Olympics started with 125 students, which was reduced to 30. While SWOAME enjoys the positive memories from their work on OAME 2011, SWOAME is looking at ways to promote their chapter.

TEAMS Update

TEAMS hosted OMO on June 3–4 and are preparing for OAME 2013 at Seneca College in the Finch Avenue East and Don Valley Parkway area. ▲

NOMINATIONS FOR OAME/AOEM BOARD OF DIRECTORS 2012–2013

Each year, elections are held for a President-Elect, a Vice President (3-year term), and 6 to 9 Directors (up to a 3-year term) of OAME/AOEM. OAME/AOEM members running for Director will declare their intent to work on either Grades JK–6 or Grades 7–12 issues based on their personal experience, expertise, and interests.

Nominations for these positions are to be submitted by 4:00 p.m., January 14, 2012, to:
David Zimmer, Past President

Include the nominee's qualifications with the Nominations Form. A nominee must be a member of OAME. By March 2012, the Nomination Committee will publish a slate of candidates on the website, including qualifications, for consideration by the membership. Voting will be on the website, and is open to all OAME/AOEM members. The election results will be released at the Annual General Meeting of the OAME/AOEM in Windsor, May 2012.

NOMINATION FORM

I nominate:_____

for the position(s) of:

☐ President-Elect ☐ Vice-President ☐ Director

If for Director, area of personal experience/expertise, interest:

☐ Grades JK–6 or ☐ Grades 7–12

Nominee's Address: _____

_____

E-mail: _____

Telephone: Home (_____)_____

Work (_____) _____

Nominator's Name : _____

Nominator's Address: _____

_____

E-mail: _____

Telephone: Home (_____)_____

Work (_____) _____

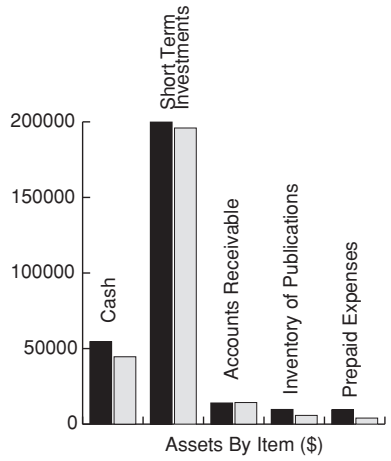
▲ OAME Financial Statement 2010-11

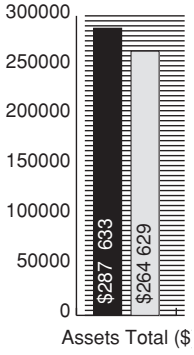
Notice to Readers

On the basis of information provided by management, I have compiled the balance sheet of the Ontario Association for Mathematics Education (Association Ontarienne pour L'Enseignement des Mathematiques) as at August 31, 2011 and the statement of operations and operating fund balance for the year then ended. I have not performed an audit or a review engagement in respect of these financial statements and, accordingly, I express no assurance thereon. Readers are cautioned that these statements may not be appropriate for their purposes.

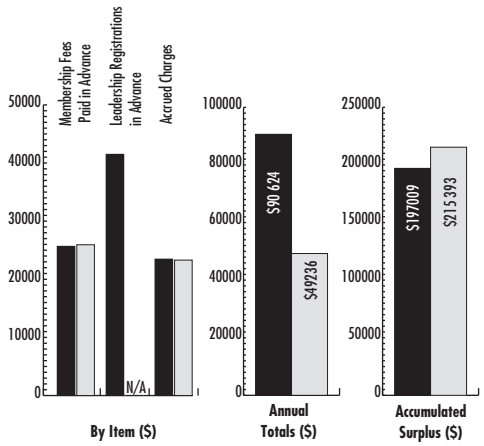
London, Ontario, August 31, 2011, PHILIP J. GRIFFIN Chartered Accountant, Licensed Public Accountant

Current Assets

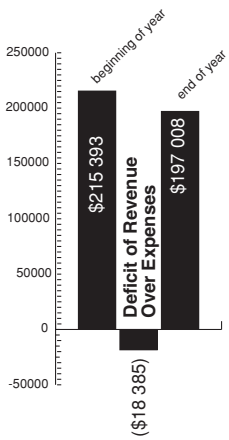




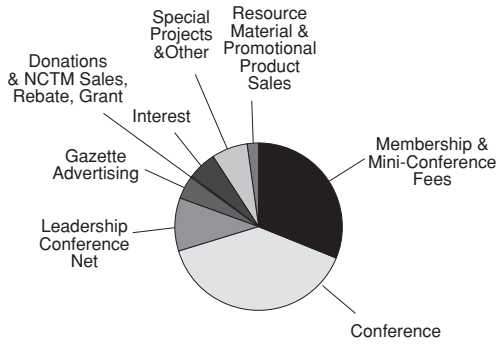
Liabilities and Fund Balance



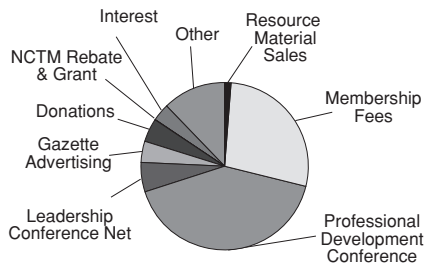
Operating Fund Balance



Revenue



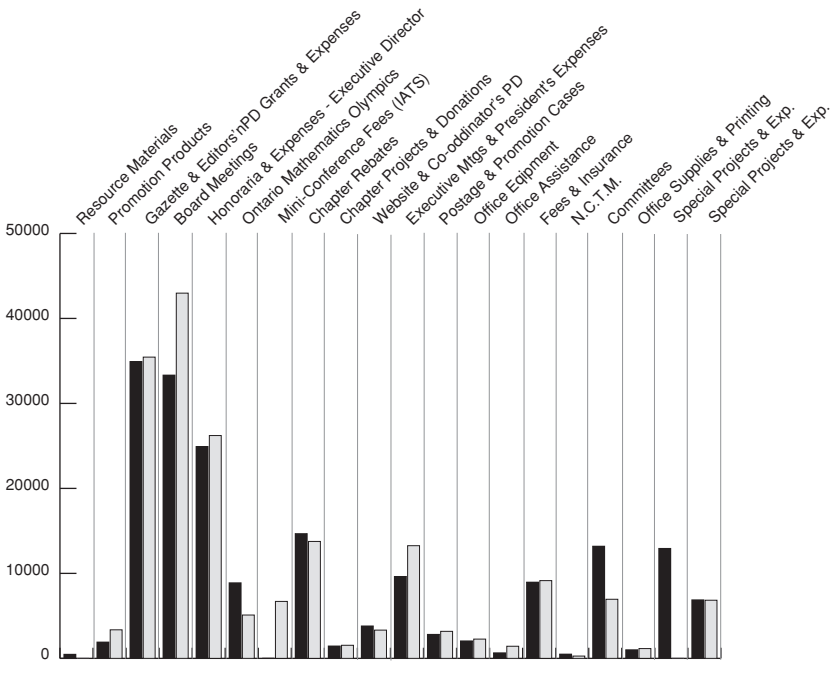
2011 ($164 548)



2010 ($203 241)

2011/2010

Expenditures



▲ Fields Institute – MathEd Forum Notes

STEWART CRAVEN



Stewart is a senior mathematics teacher recently retired from the Toronto District School Board. He continues to teach at York University and actively participates in the Fields Institute Mathematics Education Forum. He was the previous editor of the Gazette*.*

The first session of the forum on Saturday, September 24 addressed the theme *Gifted Mathematics Education*. The presenters included Dr. Viktor Freiman, Professeur agrégé, Université de Moncton; Dr. Zohreh Shahbazi, University of Toronto at Scarborough; and Mr. Jeff Kuindersma and Mr. Andrew Macpherson, both teachers in the Waterloo Catholic District School Board.

The abstract for Dr. Freiman's presentation was as follows:

> *How to challenge mathematically gifted students in our classrooms: from research to practice*
>
> *Research on mathematical talent and giftedness emphasizes that these students, starting from early grades, need to be stimulated and nurtured in order to develop their mathematical ability. At the same time, studies show that educational needs of mathematically gifted students are not always taken into account by school curricula and teaching practices, thus leaving them on their own in their desire to learn more about mathematics. How can we help those students to realize their full intellectual potential? In the first part of our talk, we will review recent state of research and practice on mathematical giftedness from the international and Canadian perspective. In the second part, we will discuss examples of challenging situations that can be developed and implemented by teachers based on research data and best practices.*

Mr. Kuindersma and Mr. Macpherson presented next. The focus of their conversation was based on their experiences and observations about enriched high school mathematics classes that they have taught.

Dr. Shahbazi briefly explained the strategies that she uses in her regular undergraduate math courses to ensure that gifted students are engaged in meaningful activities for them, while she still attends to the needs of the rest of the class. She also outlined, in her capacity as the Coordinator of the Math and Stats Learning Centre, a number of initiatives that provide mathematical challenges to the broad range of undergraduate students at the University of Toronto at Scarborough, such as problem-solving seminars, students' seminars, and problems of the week.

The discussion that followed was characteristically energetic and informative. New questions were posed and ideas were challenged. Perhaps the most interesting moment in the meeting was a conversation between our regular participants and a Grade 7 boy, who is deeply interested in mathematics, and his parents. They were describing their frustration with finding schools in the public system that would satisfy their son's need for more challenging mathematics.

Our next meeting takes place at Brock University on October 29. For details, please consult the website **www.fields.utoronto.ca/programs/mathed/**. ▲

▲ OAME IS TURNING 40!!!

So, what is so special about 40?

All registrants at the annual conference in Kingston will receive a copy of the 40th-anniversary edition of the *Gazette*.

For more information, you can go to **www.oame.on.ca** and follow the links (conferences/events, annual conferences, etc.).

Here (thanks to Ron Lancaster) is a problem about 40.

In number theory, a polite number is a positive integer that can be written as the sum of two or more consecutive positive integers. All other positive integers are impolite.

The first few polite numbers are 3, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 17,....

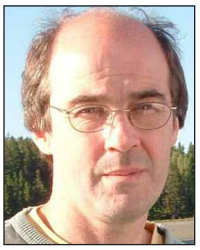
Source: **www.en.wikipedia.org/wiki/Polite_number**

Is 40 a polite number? What property do all impolite numbers have in common?

The answers will be posted online on the OAME website.

▲ Playing with Palindromes, Reversals and Number Puzzles

JOHN GRANT MCLOUGLIN



John is a professor of mathematics education at the University of New Brunswick (UNB). He coordinates mathematical outreach with education students and teachers in libraries and schools around New Brunswick.

Playing with numbers, looking for patterns, and figuring out puzzles offer ways of developing number sense. Looking a little deeper, beyond specific examples, may bring attention to the mathematical underpinnings of number patterns and properties. This article focuses attention on doing and understanding mathematics through the window of number play. The numbers at the heart of this piece include palindromes and reversals.

The ideas presented will be integrated into discussions of what is generally labelled *number theory*, with particular emphases on divisibility, place value, and number sense. Essentially these will be considered in a manner that can sharpen thinking skills through reasoning, with the latter serving as a basis for both mathematical explanation and proof.

Palindromes

Palindromic numbers read the same forwards and backwards in that we see the same ordering of digits, whether starting at the left or right of the number. For example, 23 455 432 and 989 are palindromes. Observe that 989 is one less than 990, a multiple of 11, and so 989 is not divisible by 11. It follows that not all palindromes are divisible by 11. On the other hand, without checking, I am confident that 23 455 432 is divisible by 11. You can check this before proceeding further.

Begin by writing down any number and then appending that same number backwards to it to produce a palindrome with an even number of digits. (Mathematicians use the word *concatenate* to indicate such an adjoining of numbers.) For example, suppose you begin with 765. Then the palindrome formed would be 765 567. It is divisible by 11, for sure! Why must this be the case? The answer lies in the basic place value representation of the number.

$$765\,567 = 7(100\,000) + 6(10\,000) + 5(1000) + 5(100) + 6(10) + 7(1)$$
$$= 7(100\,001) + 6(10\,010) + 5(1100),$$

using the distributive principle.

It is evident that 5(1100) must be divisible by 11, although the other components of the sum are not so clear in this regard. Here is where an interesting number is helpful to know, namely, 1001.

The number 1001 has a remarkable property in that it equals the product of three consecutive prime numbers, 7 x 11 x 13. Of note here is the fact that 1001 is divisible by 11. How will this help us? Observe in the sum above that we have 6(10 010) = 6 x 1001 x 10 and hence, that component is divisible by 11. Finally, the number of 7's is 100 001; however, 100 001 = 100(1001) – 99 and must also be divisible by 11, since both 1001 and 99 are multiples of 11.

In general, any palindrome with an even number of digits must be divisible by 11. As teachers, it is easy to make a number that is divisible by 11 by simply writing any number and then concatenating it with its reversal. That may be handy for use in an example.

The divisibility test to check if a number is divisible by 11 requires that the difference between the alternating sums of digits be divisible by 11. For example, using 765 567, the alternating sums would be 7 + 5 + 6 and 6 + 5 + 7, respectively. These sums are identical, giving a difference of 0 (a multiple of 11) and hence, the number 765 567 is divisible by 11. This will always be the case. This can be explained by observing that every other power of 10 is 1 greater than a multiple of 11 (e.g., $10^2 = 11 \times 9 + 1$, $10^4 = 11 \times 99 + 1$, etc.), and the other powers are 1 less than a multiple of 11 (e.g., $10^1 = 11 \times 1 - 1$, $10^3 = 11 \times 91 - 1$, $10^5 = 11 \times 9091 - 1$).

It should be pointed out that 121 and 484 are examples of palindromes that are divisible by 11, as well as being perfect squares. While most palindromes with an odd number of digits (palindromes that pivot on a single middle digit) are not divisible by 11, there are many exceptions. Another less obvious palindromic multiple of 11 would be 29 392, as the difference between the alternating sums of digits would be (2 + 3 + 2) – (9 + 9), or –11. Note that any positive or negative multiple of 11 satisfies the divisibility requirement for 11.

Reversals

Instead of looking at numbers that are self-reversals, it is also interesting to consider numbers that are reversed as a result of a computation. Consider a problem: "Find a four-digit number with four distinct digits which, when multiplied by 4, results in its reversal." This can be expressed as follows:

ABCD x 4 = DCBA

This problem is commonly shared in my class as a teaching example that invites us to think out loud while problem-solving as a class, thus learning more about mathematical thinking while applying number sense to succeed in solving this problem. Placing this problem in a class context facilitates the growth of both confidence and proficiency in mathematics, while enhancing awareness that this is not an easy question for others present either. Typically some of the earlier observations on the problem centre upon what is not possible. For instance, D cannot be 0 or 1, as D x 4 ends in A, or A cannot be large, since multiplying by 4 must leave us with a product less than 10 000. Take a moment before reading further, if you wish to solve the problem.

Here are some insights that move us toward a solution.

- The value of A is either 1 or 2, as ABCD must represent a number less than 2500, so that 4 x ABCD is a four-digit number. If it is noted that A must be even, as all multiples of 4 end in even digits, then A = 2 falls out.
- When we know that A = 2, it is commonly observed that D must be 3 or 8, as multiplying D by 4 must produce a number ending in 2.
- Then it follows that D = 8, since the resulting product of DCBA > 2000 x 4.
- Noting that 8 x 4 = 32 will result in an extra 3 in the tens place and that 4 x C must be even, then we have that B must be odd. Further, B must not exceed 2, as 2300 x 4 > 9000. Therefore, B = 1.
- Now C x 4 + 3 must end in 1, so C = 2 or 7. C must be 7, since the product is greater than 2100 x 4.
- The desired product is 2178 x 4 = 8712. The required number is 2178.

R-numbers

R-numbers are defined as numbers that are multiples or divisors of their reversals. Numbers such as 2178 or 8712 have this property. Problem solvers may wish to explore some of the ideas briefly mentioned here concerning R-numbers. Concatenating any R-number with its reversal results in a number that is divisible by 11, as previously shown. One might ask whether the R-number itself is divisible by 11. (We exclude here, and in what follows, the trivial case of R-numbers that are themselves palindromic, such as 161, which clearly is a multiple of its reversal, as it is identical to itself, or 1 x 161.)

It can be shown that R-numbers are always divisible by 11. In fact, it is also true that R-numbers are always divisible by 9. It follows then that all R-numbers, regardless of the number of digits, are divisible by 99 (since 11 and 9 are relatively prime). You can check this with 2178 or 8712, for starters. Other four-digit examples will result from solving the challenge below.

Interestingly enough, there are only two four-digit numbers that share the reversal property, that is, a four-digit number that when multiplied by a single digit, results in a reversal of itself. The other involves multiplying by 9 as in EFGH x 9 = HGFE. This is a much easier number to find, given the immediate restriction that E = 1 so as to keep the product below 10 000. Rather than sharing the value of EFGH directly, I will suggest a routine that results in its value. (If you wish to solve the problem, please refrain from reading the next portion of this article at this time.) This "number trick" is particularly appropriate in the junior/intermediate grades to review mathematical skills, and the ability to follow instructions. Don Fraser introduced this example to me, details of which appear in his 1987 publication, *Mathemagic*.

Other sorts of reversals

Other interesting problems involve calculating with numbers that are reversals. For example, begin by writing down three different digits. (While 0 can be used, it is easier to initially illustrate the idea with a class by suggesting the digits be from 1 to 9.) Now order these digits to form the largest three-digit number, and then again to form the smallest three-digit number. For example, with 2, 4, and 5, you would have 542 and 245. Proceed to do the subtraction to determine the positive difference between these numbers.

When you have your difference, take that number and reverse it. Now add the reversal to the difference you obtained. What is your result? Compare with your neighbour. Aha! The result should be the same for all of us.

Now for a little extension, take that result and multiply by 9. Observe what happens. (Students notice that the product is the reversal of their result.)

How is it that everyone gets the same result? Some readers will want to formally play with the algebra to convince themselves. This can readily be done by adapting the following specific example to a general example, using abc as the three-digit number, with $a > b > c$. An abbreviated form of the explanation is first shared here, again based on place value, but using a specific example of 2, 4, and 5 as the digits. A generalization follows.

Observe that 542 – 245 represents the difference between two numbers for which the middle digit is identical and the hundreds and ones digits have swapped places. Hence, the middle digit in the ordering is irrelevant to the result. One way of representing the difference is shown here:

$$542 - 245 = (500 - 200) + (40 - 40) + (2 - 5) = 300 + 0 - 3 = 297$$

Note the result is going to involve 5(100 – 1), since the largest digit appears in the positive position in the hundreds place and is reduced by a number in which it is the ones digit. This amount is decreased by 2(100 – 1), as the smallest digit appears in the hundreds place of the subtrahend (the amount being subtracted) and the ones place of the minuend (the initial number that will be subtracted from). The difference is 5(99) – 2(99) = 3(99), or 297.

That takes care of the difference part.

So what happens when we reverse that result and add the number to its reversal, as in 297 + 792, for our example? In fact, it is always the case that the reversal will produce the complement of 11 groups of 99. That is, in the case of 297 being 3 groups of 99, the reversal of 729 will be 11 – 3, or 8 groups of 99. Hence, the sum always becomes 11 x 99 = 1089 (and yes, 1089 x 9 = 9801).

A generalization can be expressed as follows, where we begin with the three-digit number

$abc = 100a + 10b + c$, where $a > b > c$.

Let $n = abc - cba$.

Since $c < a$, observe that the middle digit of n must be 9.

Since *abc* and *cba* have the same digits, the sums of their digits, which are the remainders when the numbers are divided by 9 (perhaps added to 9 or 18), are equal. That means that the sum of the digits of their difference, n, must be divisible by 9. Since the middle digit of n is 9, the first and last digits of n must add to 9.

Hence, the sum of n and its reversal is

$900 + 2(90) + 9 = 1089$.

Returning to the idea of R-numbers, the significance of 1089 extends beyond merely being an example. It is also connected to some larger examples including 10 989—an example of a five-digit R-number formed by placing a 9 in the middle of 1089. You may wish to determine if all numbers of the form 1099…9989 are R-numbers, where the 10 and 89 are separated by any number of 9's in the middle of the number.

Summary

Playing with mathematics leads people to discover, for themselves, interesting connections and patterns that are waiting to be found. As teachers, we can draw upon our own playful spirit, or the ideas of others, and bring this spirit into the teaching and learning environments. Here I have just scratched the surface in the hope of both awakening mathematical curiosity and bringing out underlying mathematical principles in novel ways. Any individuals interested in sharing ideas on this article are encouraged to write.

Postscript

Upon completing this article, I browsed on the Internet and found that there are many pieces on digital reversals. I was aware previously that there was a five-digit example. Eager problem solvers may wish to determine ABCDE so that when multiplied by 4, it becomes EDCBA, again with distinct letters representing distinct digits. (The similarity of the result to earlier discussion here is a bit uncanny, thus suggesting there is more going on than meets the eye.) Indeed, I had not realized that it was G.H. Hardy in his famous mathematical writing, *A Mathematician's Apology*, who noted that no other four-digit numbers shared the qualities of 2178 (8712) and 1089 (9801), depending on the perspective of factors (multiples) of reversals. This note and many more details (and advanced mathematics) underlying related problems and extensions appear in an article by Lara Pudwell (2007).

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#anyqs: Math in the Time of Twitter (the not-so-short and incomplete story of Dan Meyer)

NICO ROWINKSY



Nico loves teaching, learning, and writing about math. He has been a teacher with TDSB for over ten years.

In May 2010, TED.COM posted a video that would soon become one of the most popular education-related video on the site. The video (557 139 views as of the writing of this article) was titled *Math class needs a makeover* and was a lecture given by a young, energetic San Francisco Bay Area math teacher (who came prepared with sweater vest and all) named Dan Meyer. Mr. Meyer exemplified a young/cool math teacher, and his talk was full of energy, humour, and insight into his primary area of interest, which as an oversimplification, was that math textbooks have several design problems. In the video, he breaks down a typical textbook math question, illustrating the problems on a few levels. The video in and of itself was inspirational, but it also prompted me to follow Dan online at his **blog.mrmeyer. com**, and his consistent tone encouraged me to constantly question my lessons and delivery as a math teacher.

WCYDWT

Hopping on to the dy/dan movement, the first thing I saw from his blog was the acronym WCYDWT. The WordPress tag known as "what can you do with this" was a sort of calling card and acted as a direct response to one of his major critiques of textbooks, something he called "pseudo-context."

As math teachers, we all know this phenomenon. It's when a textbook problem begins with, "Look at the numbers on the shirts of this soccer team," preceded by some bad clip art and some questions about primes or maybe factors. The issue (and a call to action for Dan) is that the soccer team has no relevance to the actual math work. The context of the soccer shirts is so unrealistic that it's almost laughable. You can almost hear your students saying, "When would this ever happen?" or "Who would actually look at soccer shirt numbers and count primes?" The answer, of course, is it would be very unlikely to happen, and probably no one would count primes in this situation—hence, pseudo-context. His blog even included an open invite to submit found problems in textbooks that fit the criteria, so that he could broadcast perpetrators and expose publishers. You can still find it under pseudocontextsaturday.

Dan began to wave his WCYDWT flag right in the face of pseudo-context by both finding and creating clean, simple, and short videos (sometimes stills) that would lead students to questions that could be solved mathematically. His videos led to intuitive, mathematical reactions. Everything created or found was posted on his blog and was offered to teachers with the simple question, "What could you do with this?" The idea of sharing his discoveries and creations with other teachers, exposing us to math with a real context, and also leaving it open for discussion, was a welcome break from the mounds of static resources we all have filed and piled in our classrooms. These new resources were open, alive, and in colour. They included:

- a clip from the Disney movie *Holes*
- a video of Dan with Russian stacking dolls
- an edited clip from *The Daily Show*
- four separate videos mashed together of a toaster on four different settings, called "Toaster Regression"

Where am I going with this? Well, that's just it. It's up to you and the students. The rule for all these videos (even the one of him melting cheese in the microwave, which he openly admits is a work in progress) is that after showing the clip, you ask the students, "Do you have any questions?"

#anyqs

As well as being on ted.com, Dan found himself doing one-off segments for *CNN, Good Morning America*, and some other major network news shows. His promotion of mathematical inquiry in the everyday was very television friendly. His Good Morning America segment had him comparing wait times between two lines at the grocery store. He compared a line with one more person versus a line with 17 more items. Some linear relations ensued.

Dan's popularity grew with each exposure[1], and it's impressive to see how he used his new-found popularity

[1] As far as I know, his popularity has not been graphed, but I'm sure he could come up with a nice Time v. Followers if asked nicely. In fact, at the end of each year, he puts together a collection of infographics that recap the year that was.

(something he fittingly called his "brain trust"), especially as it related to Twitter.

I'll spare you the brief history of Twitter and focus on one of its subtle nuances, the hashtag. The symbol #, called a hashtag in the "twitterverse," is used to categorize tweets. Here are the three things you need to know about hashtags, straight from the Twitter help page itself:

- People use the hashtag symbol # before relevant words in their tweet to categorize those Tweets to show more easily in Twitter Search.
- Clicking on a hashtagged word in any message shows you all other tweets in that category.
- Hashtags can occur anywhere in the tweet.

To clarify the rather esoteric help page, here's an example from Dan's Twitter:

> *I made a quick #anyqs video in honor of Hurricane Irene. What's the first question that comes to mind?* ***www.vimeo.com/28283102*** *#scichat*

And with that simple post, Dan has invited math teachers around the world to watch a video he's made, and in turn, see if there were any perplexing questions that arose from the 47-second clip. He's sharing a resource, and at the same time, testing it out to see if it works. To help find those resources, he's used two hashtags. The first, #anyqs, was used to categorize it as a video/picture meant to induce questions. The second, #scichat, was used because of the scientific nature of the video, as opposed to #mathchat. When I go on Twitter and search #anyqs, I find several videos of this same ilk. Free resources! Mind you, it's a bit of a hit-and-miss experiment, but you quickly learn whom to follow and whom to miss. In my experience, it's more of a hit, hit, hit, then miss.

The responses from other Twitter users sometimes come fast and furious, some with a jokey tone, or a sarcastic angle, and some genuinely inquisitive, but either way, the experiment accurately mirrors the results you would get in the classroom. And that's the point. The #anyqs movement allows teachers to share ideas and prepare us for our collective and inevitable reality: the classroom. I've even posted some #anyqs videos myself along, not that I'm asking you to follow me **@NicoRowinsky**. :)

[2] *The Apple equivalent (some would say superior counterpart) to PowerPoint.*

[3] *Yes, it does mirror our very own Minds On, Action, Consolidation/TIPS, or other Japanese-inspired lessons.*

The graphing stories.com experiment

Dan extended his career as a short-film mathematical storyteller by teaming up with buzz.com and creating something they called "graphing stories." The premise was simple. Film something for 15 seconds, graph your story, send it in, and Dan would create a video that would superimpose your graph onto your video. (The example was 15 seconds of Dan swinging on a playground swing superimposed by a Height off ground versus Time graph that looked suspiciously like a sine wave.) Take something simple, film it, and graph it. And that's Dan's modus operandi. Simplify. Declutter. Get away from the textbook and hit the playground, the basketball court, or even the kitchen. Make it real. And clean.

Dan emphasizes design and is constantly reformatting his Keynote[2] presentations with his audience in mind. Graphing stories was his attempt to share his talents by converting submitted videos to presentations that can be shown to students. There was an entrepreneurial angle to graphingstories.com, but the initial experiment was free to all teachers, and he spent some significant unpaid time creating some 160 videos to share with interested teachers. The digital stories that resulted could easily be shown to students, and since students were likely involved, the whole "selling them on the idea" was all too easy.

Math stories in three acts: The digital version

Dan's blog recently started breaking down digital stories into three acts.[3] The idea has been around for as long as I've taught, but he breaks it down (in a movie context) thusly:

> ACT 1. **Introduce the central conflict of your story/task clearly, visually, viscerally, using as few words as possible.**
>
> ACT 2. **The protagonist/student overcomes obstacles, looks for resources, and develops new tools.**
>
> ACT 3. **Resolve the conflict and set up a sequel/extension.**

His constant questioning of the efficacy of his delivery is welcoming. He often cites past failed attempts and refers readers to other educators in a consistent nod to "people smarter than me."

We all learned this in Kindergarten, I suppose. It's all about sharing. And teachers are beginning to perfect their craft by sharing their resources, their experiences, and even their mistakes online. Dan is just one example who

happens to have taken full advantage of his sharing. In fact, one could say that his craft is no longer that of a Grade 9 math teacher[4], but that of someone who is learning from every one of his shared experiences. He's got sharing down to an art. He's reformatted it into the perfect presentation. He's edited it down to a clean 15-second clip. The sharing itself is his craft. But perhaps his Twitter bio says it the best, and as you'd expect, it's clean and simple.

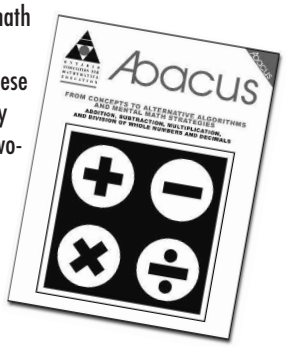
I like to teach.

By the way, Dan Meyer is one of the keynote speakers at this year's OAME Annual Conference – Kingston 2012 - CON.NETWORKS. ▲

[4] This is fact, since he now works as a consultant to Soomo Publishing, along with scheduling regular math conferences and curriculum talks. Oh yea, and he worked for Google.

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▲ WHAT IS "FAIR"?

TIM SIBBALD



Tim is a secondary math teacher in the Thames Valley District School Board. He also teaches pre-service senior math candidates at the University of Western Ontario.

Until recently, I have always felt I had a good understanding of what it means to split something in a fair fashion. As a child, I recall splitting a piece of pie with a sibling and learning the strategy where one person cuts and then the other chooses his or her piece. This simple strategy addresses the possibility of an unequal cut by allowing the chooser to select the larger piece. While this simple strategy works with pie, there are more serious issues that require attention to the concept of fairness.

One of the examples used to discuss fairness is one involving weighted averages a teacher might use to produce grades. This example is used, recognizing that the Ministry's *Growing Success* document (Ontario Ministry of Education, 2010) provides much more thoughtful guidance about how grades should be assigned, because it provides an interesting mathematical example related to the concept of fairness that might well be of interest to students.

Other examples involving inheritance splitting and deciding where to place a new facility are also considered. These illustrate that the determination of a fair solution reflects our choices about how to measure. As a starting point, a student concern about fairness of marks is explored.

Fair Marks?

A marking issue arises from the use of a weighted average across the categories of achievement. It can, and does, happen that a student can receive a mark that exceeds, but diminishes, his or her overall course mark! In simple terms, a student can perform above average on an assignment and it can lower her or his average. As an example, consider a student who has 60% in the Knowledge category of achievement, 90% in the

Application category, 60% in the Communication category, and 60% in the Thinking category. (Note that the KACT order is used throughout this article.) The category weightings are somewhat flexible, but a 35-35-15-15 weighting is used to demonstrate the phenomenon. The overall course mark is:

$$0.35 \times 60 + 0.35 \times 90 + 0.15 \times 60 + 0.15 \times 60$$

or 71%. Suppose the student achieves 75% on his or her next assignment in the Application category. Since 75% exceeds the overall course mark of 71%, students often expect their overall mark to increase. However, the 75% will reduce the Application category mark from 90% and, therefore, reduce the overall course mark. Suppose there is only one assignment in the Application category (the 90% mark) and that the new assignment marked at 75% is of equal weight. The new Application category mark, using an average, would be 82.5%, and the new overall course mark is

$$0.35 \times 60 + 0.35 \times 82.5 + 0.15 \times 60 + 0.15 \times 60.$$

An overall course mark of 68%, or approximately a 3% drop in the overall course mark (which students pay attention to), results, in spite of a mark that was higher than the original overall course mark. The assumptions mean this is most likely to arise in a noticeable fashion early in a course when there is less statistical stability because of there being fewer contributions to the marks. It should be noted that an assignment below the overall course mark can also cause the overall course mark to increase under the right circumstances.

The discrepancy in marks is due to several factors. First, there has to be a discrepancy among the marks for the different categories of achievement. This is not an unreasonable possibility, since the categories were enacted to address the distinctions among the different types of achievements. Second, there has to be a discrepancy between achievement on a new assignment and prior assignments within the category. Again, this is credible and many teachers have observed fluctuations in student responses to different assignments. Third, the new mark has to fall in the gap between the overall course mark and the overall category mark. This is less likely as a course progresses and the number of assessments increases. Lastly, when the situation arises, the impact will be very small if the value of the assignment, in terms of the course, is low. In the illustration, the 3% drop arises from an assignment that is worth half of the 35% Application category (because there are only two assignments in the application category).

The assessment issue typically gives very small discrepancies, and few students notice them. However, it does shine light on the issue of perceived fairness that can be used as a teachable moment about the difference between an average and a weighted average. It also provides a twist for discussing the mean in Grade 9 academic classes, where discussion of how overall marks are calculated is a fitting topic for students.

Fair Inheritance?

Note that I have chosen to express the following in terms of inheritance, but it could equally well be expressed as resolving a bankruptcy or establishing a divorce settlement. The option to include a social dimension is an extra benefit of exploring this area of mathematics.

Suppose that an estate of \$500K (\$500 000) is to be split between two adults who are supposed to receive \$200K and \$400K each. A common interpretation of fairness, in the face of the shortfall, suggests

$$\text{First person: } \$\left(\frac{200}{600}\right) \times 500K = \$167K$$

$$\text{Second person: } \$\left(\frac{400}{600}\right) \times 500K = \$333K$$

However, an alternate interpretation that Szpiro (2010) attributes to the Talmud, a Jewish text of laws and traditions, has a different approach.

This example is explained in words and also given in tabular form below where amounts owed are in brackets. The first adult can, at most, claim \$200K, so the remaining \$300K is allocated to the second adult; that leaves \$200K to be divided. Although the first adult is theoretically entitled to \$200K, the second adult is still entitled to \$100K (\$400K minus the \$300K already allocated) of the \$200K remaining. Therefore, the first adult now has an incontestable claim of \$100K. This

	First Person	Second Person	Total
Initially	(200)	(400)	500
Incontestable	nil	300	(300)
Result	(200)	(100)	200
Incontestable	100	nil	(100)
Result	(100)	(100)	100
Equal Amounts Owed	50	50	(100)
Final Result	(50)	(50)	0
Total Received	(50) – (200) = 150	(50) – (400) = 350	

leaves $100K to be divided, with both parties having theoretical entitlements of an additional $100K. Given the equivalence of the entitlements, the remaining money can now be split evenly and each party receives $50K of it. This gives the two adults totals of $150K and $350K, respectively.

The approach has utility in game theory because it has the property that each party receives the same level of inequity (the final result line in the table has the two people owed the same amount). When the money is split as $167K and $333K, the first person is short $33K, while the second is short $67K. The second approach splits the money as $150K and $350K, which leaves both parties short $50K. This is indicative of an underlying assumption about whether the inequity of the division applies to the promised $600K when there is only $500K. Should the non-existent $100K be divided up according to the rules for dividing the estate or, since it is outside the domain, should it be treated differently?

Fair Locations?

Suppose a new facility is being planned that will service three towns. One interpretation of fairness would set the facility equal distances from each of the three towns. A second version of fairness would weight the distances according to the population of the towns. The first scheme inconveniences everyone by the same amount, while the second inconveniences the total collection of all facility users by the overall minimum amount. The difficulty in interpreting fairness is equivalent to the problem of how to measure the values of inconvenience.

The same problem is well suited to a rich geometry task where Grade 10 academic students could be challenged to find the centre of a triangle that is equidistant from the three vertices (an example is shown in Figure 1). This is the circumcentre, since the centre of a circle that passes through each vertex is exactly one radius away from each vertex. However, introducing the idea of weighting the vertex distances according to population can make a good enrichment challenge. It could also facilitate debate about fairness.

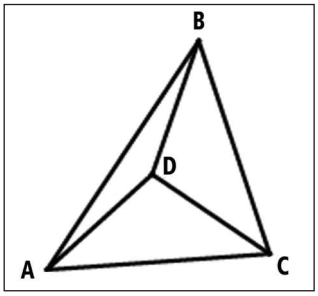


Figure 1 – *Circumcentre*

Suppose three towns—A, B, and C—have populations of 1, 1.04, and 1.1 units, respectively, and want a location that is proportional to population. This is a good task for experimentation using the The Geometer's Sketchpad™ because it can validate the hypothesis that higher populations draw the solution point closer. An experimental result for the same triangle as Figure 1 is shown in Figure 2. The level of impact of population differences on the geometric location is drastic, suggesting the level of impact on the people in the smaller towns will not be as "fair" as they might like.

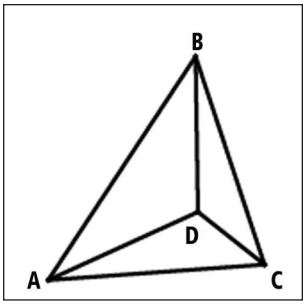


Figure 2 – *Weighted Centre*

As an enrichment activity, this facilitates some choice for students because they can look at the best way to measure the centre and then investigate the consequences through exploration. It also allows a tie between weighted averages and geometric interpretations.

Closing Thoughts

The three examples in this article highlight that choosing how one measures can affect the end result. The triangle example is a good example that can be used to think about social concerns such as car emissions versus equal access. The inheritance example highlights different cultural approaches. The marking example demonstrates how one can explain to students the unusual phenomenon that arises now and again.

For teachers who find the marking example uncomfortable, let me close by assuring that it is intended, in no way, to diminish our assessment practices. In practice, the effect arises rarely and early in courses when there are only one or two marks in each category. Subsequently, statistical stability is increased, making the impact of the phenomenon negligible.

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▲ Ontario Math Olympics 2011

WAYNE ERDMAN



Wayne is a retired math head after 33 years in the classroom, and is now an independent math consultant. His involvement includes chairing OMO 2011, co-chairing OAME 2013, authoring textbooks with McGraw-Hill Ryerson, advising teacher candidates, and he is a sessional lecturer at UPEI.

On June 3 and 4 of this year, 120 Grades 7 and 8 students travelled to Victoria University in the University of Toronto for the 2011 Ontario Math Olympics: Math Through the Ages and Beyond, hosted by the TEAMS chapter of OAME. The Ontario Math Olympics is a two-day event in which teams from various OAME chapters come to a central site to compete in a series of problem-solving activities. "Mathletes" are selected by taking part in local playdowns sponsored by their local chapter. The Ontario Math Olympics is the culminating event, with many of the top Grades 7 and 8 math students from across the province taking part in this fun and challenging competition.

The students began arriving on Friday morning. After registering and getting the keys to their rooms, the teams randomly selected a mathematics term and were given 30 minutes to design a chant incorporating their math term and their chapter name. They performed their chants in front of a set of judges, teacher candidates from the Ontario Institute for Studies in Education.



The teams were judged on their creativity, movement, involvement of their members, and of course, the use of their math term. All teams put on excellent performances, as everyone could see when videos of their performances were shown at the opening ceremonies. Congratulations to MAC^2 team 2 (Chahat Sharma, TJ Yang, Brooke Hilton, Twesh Upadhyaya) on winning the award for best chant.

After completing their chants, the mathletes had free time and took part in a number of icebreakers, designed to build team spirit and collaborative problem-solving skills. Students participated in activities such as a giant competition of "Rock, Paper, Scissors," "Two Truths and a Lie," "Two-Faced Monster," "Ten Count," "Detective," "Birds of a Feather," and "Noah's Ark." There were lots of laughs shared as the students quacked, honked, made faces, and cheered the winners of the "Rock, Paper, Scissors" competition.

Then, the mathletes trekked into Burwash Hall, which reminded everyone of Hogwart Hall in the Harry Potter movies. At the opening ceremonies, the mathletes were welcomed by the president of Victoria University, Dr. Paul Gooch, the president of the OAME, Connie Quadrini, and the president of TEAMS, Monica Rohel. A fabulous roast beef dinner followed, with homemade pie for dessert. Needless to say, the excellent food served was a highlight of the weekend.

Following dinner, everyone walked over to the Royal Ontario Museum, where the mathletes engaged in a scavenger hunt.



Clues were given using math-related information connected to the displays throughout the museum. Unfortunately, the students only had an hour and a half to complete the scavenger hunt, with many wishing they could have stayed longer. Congratulations to Golden team 2 (Senbo Fan, Miko Jovkovic, Hillary Burgi, Ffion Hughes) on being named the best scavenger hunters.

After a good night's sleep in the residence and a healthy breakfast, the mathletes got down to work by writing the individual test. The questions were designed with the theme of Math Through the Ages and Beyond in mind. Congratulations to David Francis (ISOMA team 1), CoCo Zhang (ISOMA team 2), Austin Wu (Y^{4}MA team 2), and Sophia Song (Y^{4}MA team 2) on taking away the awards for top students in the individual test. A rotation of two paired activities and a group activity followed. In one paired activity, students had to use patterning and algebra to build tables from geometric shapes.



The second paired activity required geometry skills and spatial sense to describe how to cut squares and

rearrange them into other shapes. The group activity was a relay, requiring students to solve math problems taken from various world cultures over the centuries, receiving a new problem once one was completed. Congratulations to NWOAME (Anastasia MacDonald, Milan DiGiusepe, Jane Luft, Mitchell Letwin) on winning the paired activities and to TEAMS team 1 (Iris Redinger, Ahzam Ali, Deborah Ooi, Jacob Jackson) for their superior skills in the group event.

The closing ceremonies were an exciting time for the mathletes. Leading up to the ceremonies, the students were busy reminiscing, perhaps saying farewell to some new friends, and trying to anticipate who won the various events, and of course, the medals for the overall results. Thank you's were expressed to the chef and his staff, the volunteers, the coaches, chaperones and parents, the sponsors, and finally, to the organizing committee. The results were then announced and the mathletes came up to receive their medals. Congratulations to our winning teams:



Gold medalists Y⁴MA team 2 – Sophia Song, Austin Wu, Vanessa Du, Reyno Tilikaynen



Silver medalists SAME team 2 – Lily Yang, Ricky Chow, Jing Li, Derek Tao Descency



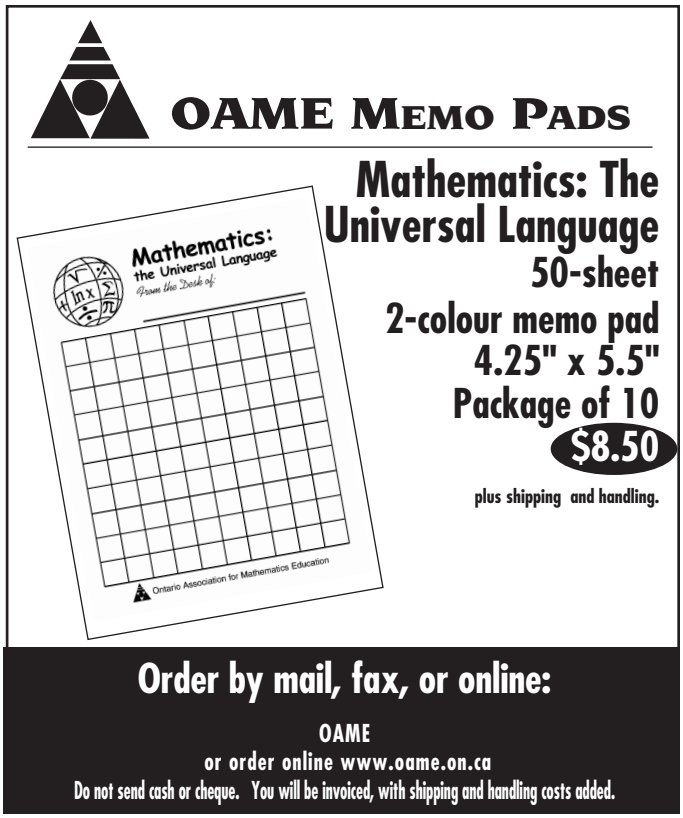
Bronze medalists ISOMA team 1 – Safiya Rajwani, Spencer Zhao, Ioanna Burtea, David Francis

Individual results are not given out after the OMO, other than announcing the winners of the events and naming the top three finishing teams. The rationale is that the students compete because of their love of mathematics, but also to not embarrass students who might not fare well on some of the activities. As we see each year, the students demonstrated superb abilities in mathematics and expressed an appreciation that this type of event exists, allowing them to compete against other top math students from across the province, in a friendly, collaborative setting. Educators are encouraged to get involved at both the local chapter level or by helping with the provincial competition when it comes to your community.

TEAMS thanks all of our sponsors for making the 2011 OMO possible: OAME, Research in Motion, TD Canada Trust, Victoria University in the University of Toronto, Texas Instruments, Maple Leafs Sports & Entertainment, Pizza Pizza, McGraw-Hill Ryerson, Spectrum Educational, Watters Marketing, World's Finest Chocolates, Nelson Education, Mastermind Toys, Scholar's Choice, Geri & Gene Puritch, Michele Goveia, Mary Card, and Wayne Erdman. Finally, TEAMS wishes to thank all the mathletes, the coaches, and chaperones for their support and enthusiasm, and hope they have many positive memories of OMO 2011. We wish everyone the best of luck at OAME 2012 in Windsor.

All the results, photos, and copies of the activities are available on the TEAMS website **chapters.oame.on.ca /teams/ontario-math-olympics/**.

All photos by Mary Card ▲



▲ What's Interactive, Engaging, and Undervalued (Okay, It's Free)?... Math Clips

HEATHER BOYCHUK & LAURIE-ANN LIELKALNS



Heather is a secondary teacher for the Rainbow District School Board. She currently teaches intermediate/senior mathematics, is a NOMA executive member, and is Program Leader of Mathematics at Lasalle Secondary School.



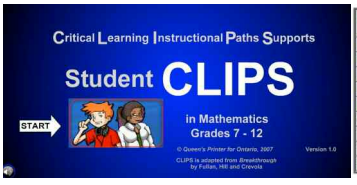
Laurie-Ann is a Coordinator of Special Education with the Rainbow District School Board. She has past experience in teaching intermediate/senior mathematics.

The following piece was adapted from "Teachers Want to Know: A Tip Sheet to Support Instruction and Assessment Practices That Improve Mathematics Achievement (September 2010).

This article is the first of three that will highlight CLIPS, a deep, rich, and interactive learning tool available at no cost. We hope that you will play a role in mobilizing CLIPS provincially.

What are CLIPS?

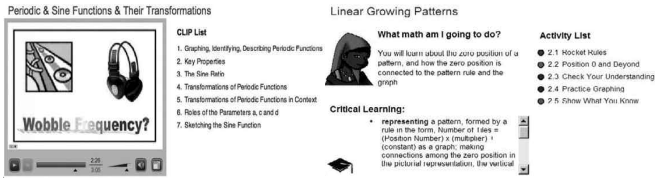
"Critical Learning Instructional Paths Supports" (CLIPS) are multimedia learning objects focused on key topics identified through research as needed by a significant percentage of students.



Index	Key Ideas	7	8	9	10	11	12
Fractions	>>	>>	>>				
Functions							
Integers		>>	>>				
Plane Geometry							
Linear Relations				>>			
Quadratic Relations							
Trigonometry						>>	>>
Transformations						>>	
Patterning and Algebra		>>	>>	>>			

Website: www.mathclips.ca

This web-based learning tool was developed by mathematics teachers and researchers. It offers interactive lessons based on the Ontario curriculum, Grades 4–12. CLIPS tasks and instructions are sequenced to build from one key idea to the next so that they make cumulative sense to students. Focused interactions allow students to engage in learning targeted concepts and skills, and use self-checking practice and immediate feedback to gain confidence that learning has happened.

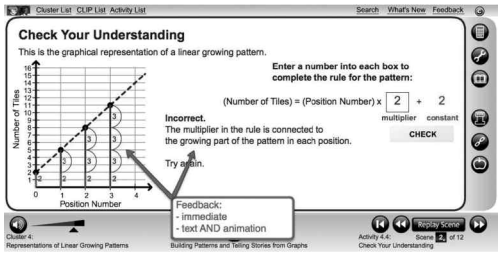


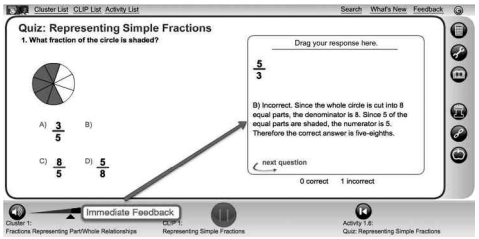
"I liked the sections that allowed you to drag around points, variables and shift the graphs around. It was a lot more interactive and faster than waiting around for the TI-83 to draw the graphs. Overall, it was pretty fun to do." (Grade 11 student)

"I liked it when I gave an answer and right away it told me if I was right or wrong and why." (Grade 8 student)

Why should you use CLIPS in your classroom?

Students interact with powerful animations that bring math to life and naturally engage learners. By filling in numeric values, dragging objects, and playing games, students gain a deep understanding of key math concepts. In today's fast-paced digital world, CLIPS offer students the opportunity to engage in dynamic activities. This empowers students to shape their own learning through continuous and specific feedback.





"I liked how it was talking at you at the same time, but it had everything down to the side, so if you didn't quite understand it, you could reread it yourself and understand it from that." (Grade 7/8 student)

CLIPS facilitates differentiating with respect to readiness, interests, and learning preferences. The audio narrations and dynamic animations appeal to multiple learning styles and help students focus their attention.

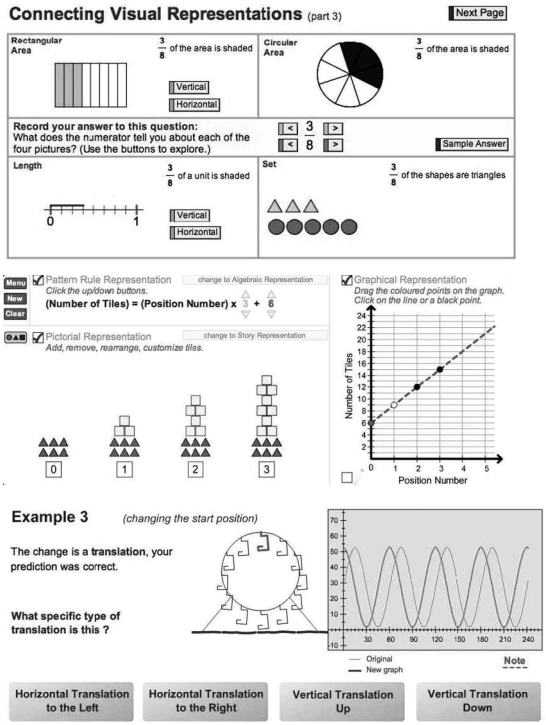
Students work at their own pace, with the immediacy of being able to review any part of an activity at the time it is not understood. Enriching and extending activities to differentiate instruction for disengaged and bored students are available through Resource and Teacher Notes areas.

> "All my students with learning disabilities were doing what everyone else was doing – all the same lessons. And they're doing fine. That's huge—that these kids can engage in the same activities and communicate their thinking to the class. They all did the exact same thing and I did not accommodate any student at any time for this. And everyone did well." (Grade 8 teacher)

How do you use CLIPS in your classroom?

Dr. Cathy Bruce, Dr. John Ross, et al. (2010) suggest that, "In order to maximize the use of CLIPS with students (for the greatest achievement gains), teachers should strongly consider using the CLIPS in conjunction with (and integrated with) classroom instruction focused on the same concepts. Teachers can use the CLIPS tasks or demonstrations as springboards to engage students in mathematics discourse during activation and consolidation phases of the three-part lesson."

The purposeful selection of CLIPS activities (see examples below), integrated with regular classroom teaching, involves using a variety of student groupings, including whole-group activation or consolidation on an interactive whiteboard (IWB), pairs collaborating on an activity, and individuals self-assessing with a game, quiz, or activity. Activities can be downloaded as Adobe Flash (swf) files that can be imported for use in a lesson that uses an IWB. Alternatively, once activities are downloaded on your computer they can be used with a stand-alone projection unit and you no longer require Internet access. While students work on CLIPS, teachers observe students, have focused conversations, and diagnose areas that need further work. The colour-coded Activity List includes introductory (Minds On) activities (green), interactive action tasks (red), consolidation activities (blue), and a Show What You Know list of low-prep ways for students or a group of students to show what they learned in a variety of ways (brown). This aligns with the TIPS 2.0 three-part lesson template.



Getting Started

To become familiar with the materials and what is known about their effectiveness, consult the following sites:

- CLIPS: **www.mathclips.ca**
- Take a *Guided Tour of CLIPS Wrapper* at **www.oame.on.ca/clips/lib/common/supports/wrapper_tour/index.ht**) to get an overview of all the components of the CLIPS screen and how to navigate the various levels to get to the activities you need, or download a one-page guide to the CLIPS screens and icons (**www.oame.on.ca/clips/lib/common/supports/NavigatingCLIPS.pdf**).
- 12-minute virtual tour of clips: **edugains.ca/resources/LearningMaterials /CLIPS/index.htm**
- Research reports and articles: **www.tmerc.ca**; **www.legacy.oise.utoronto.ca/research/field-centres/ross/vita.htm**

Stay Tuned…

Look for two more upcoming articles that will focus on elementary/secondary perspectives and experience with CLIPS.

References

Ross, J.A., Bruce, C.D., Scoffin, S., & Sibbald, T. (2010). *Connecting practice and research: CLIPS project final report to the Ontario Ministry of Education.* Retrieved from **www.tmerc.ca/pub/FractionsCLIPSFinalReport.pdf**

OAME/AOEM AWARDS FOR EXCELLENCE IN MATHEMATICS EDUCATION



The purpose of the Ontario Association for Mathematics Education's Awards is to formally recognize individuals and educators who demonstrate an outstanding contribution to mathematics education in Ontario. The successful candidate of each award enhances the learning environment, providing opportunity for students to do, see, hear, and touch mathematics in a profound and meaningful way. If you know someone who qualifies for this honour, please nominate her/him/them for one of the following awards.

OAME AWARDS

OAME/AOEM Life Membership Award recognizes an educator who has contributed in a significant way to OAME/AOEM; demonstrates outstanding leadership in mathematics education; and has accumulated ten or more years of membership in OAME/AOEM.

Award for Outstanding contribution to OAME/AOEM and Mathematics in Ontario recognizes an individual who has made a significant contribution to OAME/AOEM, but who is not necessarily an educator or a member of OAME/AOEM.

Award for Exceptional and Creative Teaching in Elementary Mathematics recognizes an exceptional and creative elementary teacher who demonstrates excellence in mathematics education and contributes to the overall development of students.

Award for Exceptional and Creative Teaching in Secondary Mathematics recognizes an exceptional and creative secondary teacher who demonstrates excellence in mathematics education and contributes to the overall development of students.

Award for Leadership in Mathematics Education recognizes an educator who has demonstrated leadership by contributing in a significant way to the development of mathematics teachers and enhancing mathematics education in Ontario.

Kenneth D. Fryer Award* recognizes a secondary mathematics department which fosters collegiality, teamwork, and excellent classroom teaching; contributes to the overall development of students; and demonstrates leadership in the mathematics education community.

*This award was established in memory of the late Kenneth D. Fryer and is co-sponsored by the Grand Valley Mathematics Association.

Elementary School Staff Award For Exceptional and Collaborative Mathematics Teaching recognizes an elementary school (a school accommodating any or all of Grades JK–8) whose staff fosters collaborative and excellent mathematics teaching; contributes to the overall development of student learning of mathematics; and demonstrates leadership in the mathematics community.

OAME/AOEM Awards Nomination Process

1. Complete the Award Nomination Cover Page appropriate to the specific award, found at **www.oame.on.ca** or contact the Executive Directors for an electronic version (**EDs@oame.on.ca**). You may include a one-page letter of recommendation.
2. Provide up to three letters of support, of no more than two pages each. These supporting documents may be gathered from a variety of sources to reveal a full picture of the nominee. The support documents will provide precise information about the nominee's contributions as they relate primarily to the specific award.
3. Include the nominee's Curriculum Vitae of up to five pages. (C.V. not required for Kenneth D. Fryer award and Elementary School Staff award)
4. Send the completed cover page and supporting documents by January 14, 2012 to: OAME Executive Directors

Incomplete or inaccurate information may cause a nomination to be withheld from consideration.

DEADLINE: Nominations are due by January 15, 2012 to the Executive Directors (contact info above).

The awards will be presented at the Annual OAME Conference in May 2012.

▲ OAME Awards 2011

CATHY HALL

Cathy Hall is President-Elect of OAME and past Chair of the Outreach Committee.

OAME Life Membership Award



Greg Clarke has been an active OAME member since 1991, both in his local chapter MAC^2 and as a member of the OAME Board of Directors. He has also gained respect over the years as an extraordinarily gifted teacher, chairperson, consultant, and co-ordinator in his local school board, Simcoe Muskoka Catholic Distict School Board, and at the provincial level, as part of the CLIPS writing team for the Ministry of Education.

Greg "has a heart" for all the OAME chapters and has worked diligently to help chapter reps learn the latest technology in mathematics education, to share ideas for mini-conferences, and to effectively communicate with their chapter members and with each other. Greg's expertise and inviting manner have encouraged the chapters to flourish!

Greg has also changed the public face of OAME in the last ten years. In his role as OAME Webmaster since 2004, he has revolutionized the way OAME communicates and does business. Among other accomplishments, Greg developed an online voting system and posted archived copies of the *Ontario Mathematics Gazette*, as well as grade-specific resources and secure assessment files. Possibly Greg's greatest website contribution has been his development and refinement of MCIS, the Mathematics Conference Information System, used for online registration at the annual OAME conference and many chapter mini-conferences. This innovation has revolutionized the registration process and provided invaluable data for conference committees, including online evaluations after the conference.

Throughout his career, Greg has demonstrated his outstanding leadership in Ontario mathematics education to teachers and students alike. At the provincial level, he has presented workshops on current math technology at many OAME annual conferences and served as Registrar for both OAME 2002 and 2007 in Barrie. In 2010, he served as Co-Chair of the Ontario Mathematics Olympics, where he unveiled his newly designed online OMO registration process.

As a Simcoe Muskoka CDSB Math and Computer Coordinator, he eagerly shared his knowledge and insights with his board department heads and teachers. As a member of the Ministry of Education CLIPS (Critical Learning Instructional Paths Supports) writing team, Greg has been able to share his expertise and passion for math with all teachers in Ontario.

Clearly, Greg is on the cutting edge of technology and is influencing how mathematics is taught and learned in Ontario. He has made a significant impact on numeracy education provincially. Yet, with all his accomplishments, he has remained a humble, caring, and dedicated educator. Congratulations, Greg, on a job well done!

Award for Leadership in Mathematics Education

This year, we are privileged to have two recipients of this award. These individuals have demonstrated leadership by contributing to the development of math teachers and enhancing math education in Ontario through innovation and research-related activities.



As a true lifelong learner and math leader, **Laurie Moher** is constantly striving to improve her own mathematics understanding and to provide opportunities for growth for math educators around the province. Her active involvement on the OAME Board of Directors and as an NCTM representative has kept her on the forefront of mathematics thinking and she willingly and enthusiastically brings that knowledge back to educators and students in the Kawartha Pine Ridge District School Board and beyond. Laurie's curiosity and commitment to enhancing mathematics instruction and understanding has led her to embark on research projects, provincial initiatives, and the production of professional development materials designed to move mathematics instruction in Ontario from content knowledge to process-based, math talk communities. As an engaging and respected workshop facilitator and instructional coach, Laurie shares her love of math with teachers and their students, inspiring them to experience math in meaningful ways. Over the years, while wearing her various hats— as classroom teacher, consultant,

instructional coach, OAME Director, PD facilitator, researcher–Laurie has shown herself to be an outstanding educator and an exemplary leader within the Ontario math community. Our hats are off to you, Laurie!



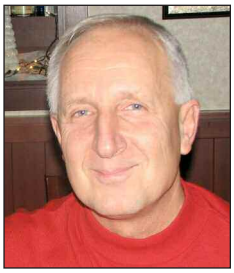
In the words of one of his fans, "Not only does **David Petro** stand out as a leader, but also as a team player. He is one of us, in the trenches of education, and truly believes that we are stronger when we work together." David's collaborative work ethic has helped to foster the success of teachers and students alike. At Cardinal Carter Secondary School in Leamington, he works to ensure a high level of consistency and accountability. David's commitment to math education has inspired other teachers to elevate their own level of teaching/learning and has fuelled their desire to bring mathematics to life for their students. "The time and energy he invests in enhancing math education is immeasurable. Math education is not just a job for David Petro; it is a calling." David's dedication to his profession is also evident in the many initiatives he has participated in over the course of his career (e.g., course profile writer; workshop presenter; textbook author; website designer; technology guru; involvement with Ministry of Education and EQAO; and mentoring/training on topics such as differentiated instruction, assessment and evaluation). Thank you, David, for your leadership in fostering best practices related to teacher and student success in math education in Ontario!

Award for Exceptional Creative Teaching in Secondary Mathematics

There are also two recipients for this award this year. Both individuals are exceptional and creative teachers who demonstrate excellence in math by contributing to the overall development of students.



In the words of his nominator, "**Claudio Attanasio** is passionate about the art and science of teaching and learning mathematics. He is constantly looking for innovative and creative ways to help his students understand mathematics." Claudio has been instrumental in setting the tone of a very innovative and student-centred approach to secondary education at Cardinal Ambrozic Catholic Secondary School in Brampton. He has a keen interest in project-based learning and performance-based assessment. He has written tasks for MHF4U for the Ministry, as well as numerous CLAs (contextualized learning activities). He has also embraced the potential of interactive whiteboards to benefit his students and willingly shares his expertise on this exciting technology with colleagues at his school and across the board by presenting professional learning sessions in the board's "Math Menu." Claudio's rapport with his students and fellow teachers is one of his greatest strengths. Visitors to his classroom see students who are engaged and working collaboratively, doing meaningful mathematics in an inviting environment, where student work is proudly displayed on the walls. Manipulatives and technology (e.g., IWB, document camera) are readily available and used regularly. Claudio is a prime example of an effective and well-respected math educator who regularly opens his classroom to other teachers. He enthusiastically embraces new challenges that may prove to be of benefit to his students and is first to volunteer whenever a new board initiative is seeking a pilot classroom to try some new strategy or technology. In conclusion, Claudio is a valued member of his school community as a leader, educator, mentor, and role model. Congratulations, Claudio!



Steve Chevalier, from Assumption College Catholic High School in Windsor, not only teaches his students subject content, but more importantly, he impacts other areas of their life e.g., developing work habits and time-management skills, and fostering qualities related to hard work and perseverance. His commitment to lifelong learning is a stellar example for his students and colleagues alike. According to his school principal, "One of the qualities that makes Steve so special and well respected is not just being an expert in his subject, but also his willingness to fight for what he believes is best for students. He is constantly advocating for our students and programs and is dedicated to the success of all students." In his classroom, he leads students through problems and concepts by encouraging questioning and discourse. Steve emphasizes the practicality of math concepts in real-life situations, encouraging his students to develop their critical thinking skills and curiosity about how math relates to their everyday world. Steve's impact on his students is showcased in the final words of a letter of support for his nomination, signed by 55 of his

students: “Mr. Chevalier is insightful, dedicated, and an exceptional teacher whose ideas and instruction clearly promote the development of his students as inquisitive problem solvers and motivated learners.” Well done, Steve!

Kenneth D. Fryer Award for a Secondary Math Department

This award is given to a mathematics department that fosters collegiality, teamwork, and excellent classroom teaching. The department must also contribute to the overall development of students and demonstrate leadership in the mathematics education community. **Rideau District High School** has just such a math department! Located in Elgin, Ontario within the Upper Canada School Board, the school serves students from Grades 9–12. Under the enthusiastic direction of department head, Kelly Blair, this math team has worked tirelessly to incorporate innovative technological practice in their classrooms in an effort to meet the varied needs and learning styles of their students. The teachers in this department have moved beyond co-operation and collegiality, into the zone of deep collaboration, resulting in engaging and challenging lessons and assessment pieces for students. The department is results oriented and committed to active professional inquiry and reflection on best practice and pedagogy. Differentiated instruction is at the forefront of their commitment to meeting the needs of all learners. The department has also supported the school’s literacy initiative by embedding literacy strategies in their daily math lessons. These teachers are extremely giving of their time, collaborating with teachers from other secondary schools, visiting elementary schools to present demonstration lessons, working with their elementary colleagues to facilitate a successful transition plan in math, working on board committees, and conducting PD workshops during the school year and summer for local educators. Way to go, team!



Elementary School Staff Award

William Burgess Public School is a JK to Grade 5 school in the Toronto District School Board. Its staff is a diverse group of educators with varying backgrounds and experiences. However, the common thread is that each teacher brings to the table these two things: a desire to collaborate to help children problem solve in all areas of their lives and a desire to be a great mathematics teacher! The school’s Professional Learning Teams gather together on a regular basis for pathway meetings, to plan quality three-part lessons, to do co-teaching lesson studies, and to moderate student work samples. In the words of one of the teachers, “What I appreciate most is that you can truly share, inquire, and be vulnerable with the inherent messiness of learning, without feeling as though you should know it completely.” This speaks volumes in describing the genuine sense of collaboration and mutual support evident within this staff. Teachers at William Burgess are also eager to share their collaborative mathematics inquiry experiences by opening up their classrooms as demonstration classrooms for teachers within their Family of Schools and across their board. As well, theirs is one of the lead schools in the Ministry of Education Primary Collaborative Inquiry Project. Their school principal pays this staff the ultimate compliment when she says, “It is so hard to describe in writing the amazing group of educators I have the pleasure to work with and learn with each and every day. I am amazed at the mathematics learning I see when I walk through our classrooms each day. If you could come for a visit, you would be impressed with how much our kids love math! That is a true reflection of the great educators who teach them!” Our congratulations go out to the teachers of William Burgess Public School! ▲

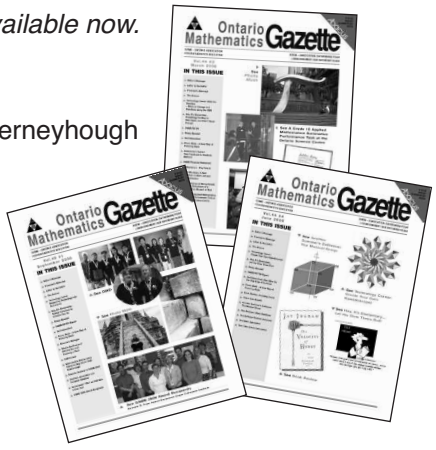


▲ Call for Gazette Editor

The *Ontario Mathematics Gazette* is taking applications for a two-year term as editor commencing with the September 2012 issue.

Job shadowing available now.

For more details, please contact:
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2010-2011 OAME Board

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