8-PAGE INSERT ELEMENTARY MATH ABACUS

Ontario Mathematics Gazette

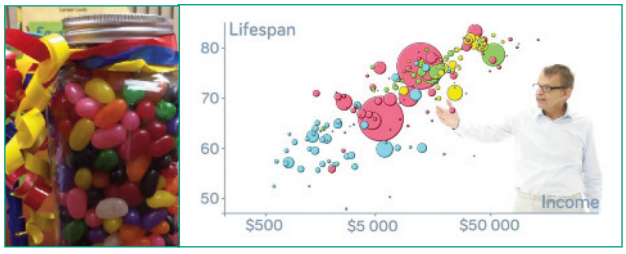
OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

Vol. 55 #3
March 2017

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Magna Tiles® is a popular building toy among Year 1 and 2 kindergarten students.

See Provincial Digital Learning Resources – What's New? Colour Tiles



See The Importance of Increasing Spatial Reasoning Instruction in Kindergarten Classrooms … but Within Reason!

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GLOBAL MATH PROJECT

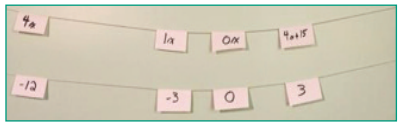
Uplifting Mathematics for All



See Global Math Project – Uplifting Math for All

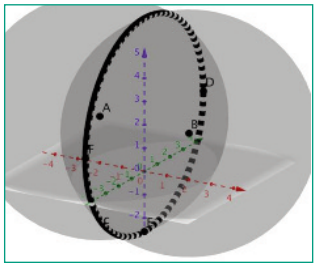


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See Mb4T (Mathematics by and for Teachers): Clothesline Math Adapting a Number-Line Model to Solve Equations

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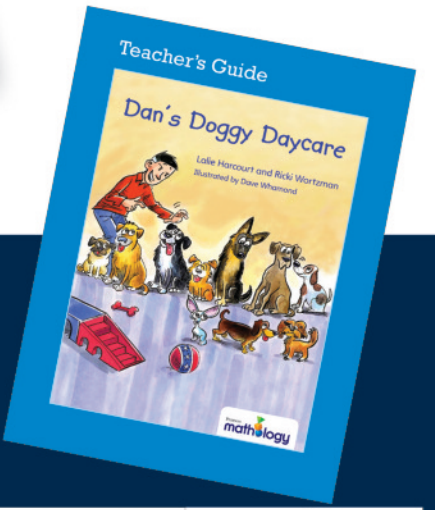
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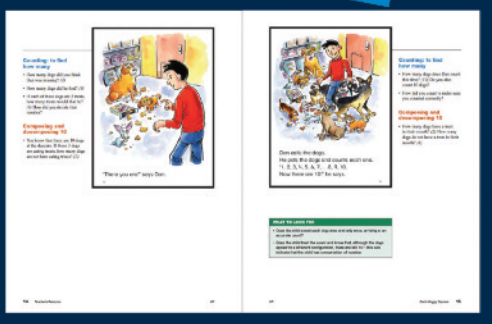


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Submission of Articles

The *Ontario Mathematics Gazette* (*OMG*) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *OMG*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles must be word-processed in MS Word, double-spaced with wide margins, not exceeding 10 numbered pages of text, and prepared according to the *Publication Manual of the American Psychological Association, Sixth Edition*. Figures and diagrams should be drawn by computer, if possible, or drawn in black ink in camera-ready form. Embedded images must also be submitted separately in jpeg or tif format. Proof of the photographer's permission is required, and for **photos of students** under the age of 18, the written permission of a **parent or guardian is required**.

You must submit **one complete copy** of your article, embedded with any tables, figures, and captioned photographs or graphics, to the Editor, Amy Lin, along with **separate files for each of the text, graphics, and/or photographs**. Please email all files to Amy Lin at **gazette@oame.on.ca**.

Your name should not appear anywhere in your article, including websites, so that your article can be sent out for blind review. Your name, full mailing address, and email address must be included on a separate sheet. Upon review, you will be notified as to whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected.

The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME.

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ABOUT THE *ONTARIO MATHEMATICS GAZETTE*

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EDITOR'S MESSAGE



AMY LIN
EMAIL: amy.lin@oame.on.ca

Amy Lin's background in the sciences and the arts helps explain why her passion in mathematics education is to develop both the logical and creative aspects of the subject. She is currently working on her PhD at Brock University in math education and is a Vice-Principal at Applewood Heights Secondary School. She is often asked to speak on mathematics teaching and learning around the province and beyond, and she is an author of a number of mathematics resources for students and teachers at both the elementary and secondary levels. She has initiated and successfully led many professional development projects in school boards in Ontario.

I was saddened to hear the news that Hans Rosling died at the age of 68. For me, he represented a scientist who was constantly curious, demonstrating kindness and empathy, with the ability to bring statistics to life.

Rosling was a physician and an epidemiologist, who was able to influence people with facts. He was a trusted counsellor and speaker to United Nations leaders, billionaire executives such as Facebook's Mark Zuckerberg, and politicians including Al Gore and Fidel Castro. Rosling also has several video lectures and Ted talks on global health and economics, which moved him up to his viral celebrity status. Melinda Gates of the Bill & Melinda Gates Foundation said, "To have Hans Rosling as a teacher is one of the biggest honours in the world."

He has always been committed to the poorest people in the world, since his early days spent in the Democratic Republic of the Congo. Hans Rosling graduated from Uppsala University in statistics and medicine in 1974, studied public health at St. John's Medical College in Bangalore, India, and then worked as a doctor in northern Mozambique from 1979 to1981. His work on a paralytic disease in the Congo led him to earn his Ph.D. from Uppsala University in 1986. In 1997, he became a professor of global health at Sweden's Karolinska Institute, and in 2005, he co-founded the Gapminder Foundation with his son, Ola, and daughter-in-law, Anna Rosling-Ronnlund, working toward fulfilling his dream of a fact-based global outlook, determined to fight misconceptions about global development.

Rosling became famous with his Trendalyzer software with bouncing bubbles and animated statistics and his Ted talks and TV documentaries such as *The Joy of Stats* and *Don't Panic – The Truth about Population.* This elevation in status allowed Rosling to make things happen and influence people. During the Ebola epidemic, he was able to mobilize funds and create an Ebola course for international aid workers.

Personally, I saw Hans Rosling as the ultimate teacher—after all, what statistician would rather present information and ideas with multicoloured Lego® blocks than a PowerPoint or Excel spreadsheet? Rosling called himself an "edutainer" and was known for his animated data visualizations, his humour, and the use of objects such as children's toys, cardboard boxes, and teacups to bring to life data on wealth, inequality, and populations. The use of visuals as a powerful way of portraying data and information has put him in the same category as Edward Tufte and David McCandless, the pioneers of infographics, striving to make statistics accessible and easy to understand. Similar to Tufte and McCandless, Rosling uses infographics to highlight well-known misconceptions. For example, if asked, men will tell you that they are better drivers than most women. Rosling uses statistics to show that men have almost double the number of fatal accidents.

In a time where news and information presented can be difficult for us to decide what is factual and authentic, Rosling's fact-based world view on Gapminder is both timely and necessary. Not surprisingly, Time magazine had named him as one of the world's 100 most influential people in 2012, stating that his "stunning renderings of the numbers… have moved millions of people worldwide to see themselves and our planet in new ways."

Hans Rosling: "I don't consider myself an optimist or a pessimist, but rather a possibilist."

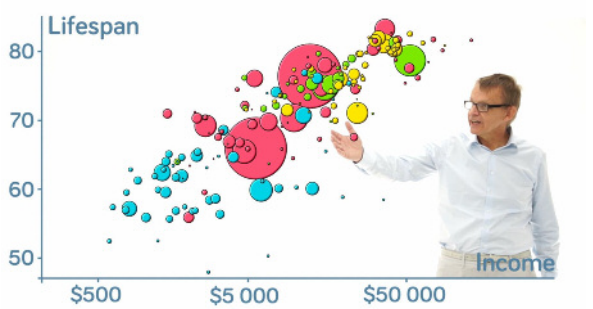


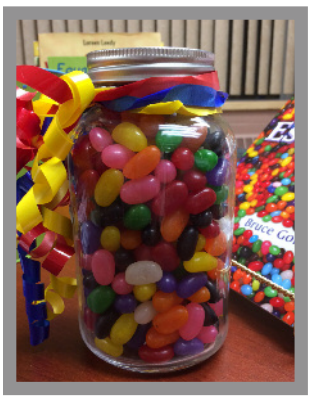
Image based on a free chart from **www.gapminder.org**

We are quickly approaching the OAME 2017 Annual Conference—And Now for Something Completely Different—to held in Kingston, Ontario on May 11–13.

I encourage everyone to make their way to beautiful Kingston and take in some wonderful and engaging presentations. Ontario math educators cover a wide range of age and grade levels, mathematics content and pedagogy, assessment opportunities, classroom and outside-classroom experiences, and stories that will inspire each and every one of us.

In this issue, author Edward Schroeter has written an article about teaching spatial reasoning in primary *classrooms entitled The importance of increasing spatial reasoning instruction in Kindergarten classrooms… but within reason!* Kelly McKie and Chris Suurtamm have written an article in this *Gazette* about their collaborative-inquiry experience with Grade 9 Applied teachers. The article *Checking the Accuracy of GPS Data* by Tim Sibbald provides some food for thought on a mathematical application. A feature article can be found in this month's *Gazette* by Sunil Singh, which highlights the exciting *Global Math Project*.

Our regular columns feature exciting new technology lessons and updates, and our guest columnist for MB4T this month is Jules Bonin-Ducharme. As usual, this issue also features the rich contribution of ideas for elementary math teaching as found in the *Abacus* insert, co-edited by Mary Lou Kestell and Kathy Kubota-Zarivnij.



This month, the jellybean problem has solutions provided from children of varying ages throughout this issue.

Here is the problem: How many jellybeans are in the jar? How do you know?

(Actual: 405 jellybeans) ▲

Editor's Note:

Manipulatives and Analogies for Teaching Integers I An Example of a Grades 7 and 8 Unit.

First author: **Immaculate Kizito Namukasa**

Second author: **Vera Sarina**, Pleasant View Public School

Third author. **Chloe Weir**, Western University

President's Message



JUDY MENDAGLIO
EMAIL: judy.mendaglio@oame.on.ca

As I am writing this, the holidays are over and the United States has a new President. Though it has been unnaturally warm, it has also been very grey. I am anxious to get to the other side of winter's darkness. However, I know that by the time you read this, the days will be longer, the sun stronger, and spring will be upon us. Soon it will be time for us to meet again, at OAME 2017, hosted by the QSLMA chapter of OAME. I am hoping that they will have allowed the Ministry of Silly Walks to recruit conference organizers to demonstrate Silly Walks through the corridors of St. Lawrence College.



If you were wondering what I have been reading recently, I have returned to reading fiction. I am just finishing Margaret Atwood's Hag-Seed. This novel is about a man, Felix, who has had a successful career as director of a theatre festival. There is a line in this book that has stuck with me since I read it. For me, it resonates as being fundamentally applicable to teaching. Felix thinks to himself, "The play's the thing. His job is to help the actors help him execute it." In a classroom, the learning is the thing. How can we help our students help us teach them? If you have suggestions, we'd love to hear from you.

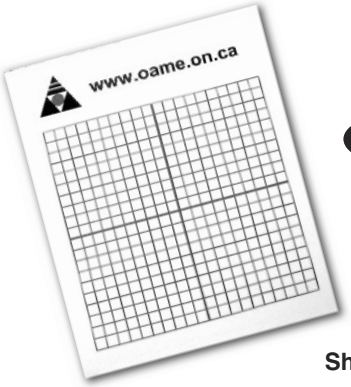
Before the holidays, I borrowed an interesting book with a very intriguing title, *Why Don't Students Like School?: A Cognitive Scientist Answers Questions About How the Mind Works and What It Means for the Classroom*, by Daniel T. Willingham. Daniel Willingham is a cognitive psychologist at the University of Virginia. This book was written in 2010, but the content seems very current. Although the context of the book is not specifically about teaching and learning mathematics, there was much that would be relevant to teachers of mathematics. Those of you who know about the work of Carol Dweck, Jo Boaler, and others, will be happy to see Willingham offer the following advice to teachers: "You want to encourage your students to think of their intelligence as under their control, and especially that they can develop their intelligence through hard work. Therefore, you should praise processes rather than ability." We still see too many students who believe that, if they are not "math people," there is nothing to be gained by working hard. Yet, I know many math people who became math people through hard work and perseverance. We need to continue to share these important messages with our students, as well as with their parents, guardians, and caregivers.

By the way, while carrying around this book, which I borrowed from the Ontario College of Teachers' (OCT) Margaret Wilson Library, it became apparent that many teachers are unaware of the OCT library and its holdings. Did you know, for example, that you can order a book to be sent directly to your home or school, free of charge? Did you know that they have e-books and DVDs? Were you aware of the large number of research publications that you can access through your OCT membership? We know that OAME members value their own learning, as well as the learning of their students. If you read anything that you would like to share with your OAME colleagues, consider submitting an article for publication to the Gazette or Abacus. We continue to be very proud of our award-winning publications.

See you in Kingston! ▲

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▲ In the Middle: Roll or No Roll

CARLY ZINIUK



Carly Ziniuk teaches Grade 9 Mathematics, Grade 12 Data Management, and AP Statistics at the Bishop Strachan School in Toronto, Ontario, Canada. She is very involved in using real-life data to engage her students in solving problems.

There was a series of really good probability activities that OAME produced in 2001, with the Linking Assessment and Instruction in Mathematics: Middle Years work. I have used these with my students for some years and have enjoyed watching my students on investigative tasks. With the recent looks at Conversations and Observations in Math classrooms, these activities have been very helpful to watch students create some interesting unique opportunities for learning.

This activity has students create dice that are either fair or not fair, after exploring the Sicherman dice. With some research, you can learn about Sicherman dice, which have 1,3,4,5,6,8 and 3,3,4,2,2,1 on the faces.* There are a lot great other trick dice that you can find at magic shops and children love to explore these. Most recently, my colleagues and friends have been exploring Maker spaces and Design-Tech, and have been playing with building some dice as well using 3-D printing.

By considering what makes both standard and Sicherman dice interesting mathematically, you might want to consider making your own with your classes. What follows is the assignment and evaluation rubric I have used with my students, adapted from the experiment that originated in the 2001 OAME work!

Create a unique pair of dice, named after yourself, different from both the standard and the Sicherman.

With your simple dice game, you hope to become a high roller by selling your idea to a games' manufacturer for a new twist on games like Monopoly and Clue. As a probability expert, you need to *demonstrate whether rolling your new dice versus the standard dice produces a fair game, and prepare a report for marketing*. Your report should include both experimental evidence gathered during a game completed with a partner and theoretical probability. Since the dice used will make it a unique game, you will need to use your thinking about whether your dice will be fair, or give the edge to one player (by how much) and what that will mean to the game.

* You many want to explore more about Sicherman Dice at **mathworld.wolfram.com/SichermanDice.html**.

Experiment Instructions

- Player A uses the two standard dice, while Player B will use the your name here dice.
- Both players roll their dice and calculate the sum of the roll.
- The player with the highest sum wins. If the scores tie, place a "T" in the results column.
- Complete 30 rounds of dice rolling, and record the results below.
- After 30 rounds, tally the points (number of wins) for Player A and Player B.

Round	1	2	3	4	5	6	7	8	9	10
A's total										
B's total										
Winner (A, B, or T)										
Round	11	12	13	14	15	16	17	18	19	20
A's total										
B's total										
Winner (A, B, or T)										
Round	21	22	23	24	25	26	27	28	29	30
A's total										
B's total										
Winner (A, B, or T)										

Written Report

K, T, A, C

The final report, explaining whether a game of your new dice versus the standard dice is fair, is to be completed individually. The report must provide a detailed written explanation, incorporating both experimental and theoretical data. Use graphs, charts, and/or tree diagrams to help you to analyze the data and to support your written arguments. A rubric follows for your reference.

TEACHER ASSESSMENT – ACHIEVEMENT RUBRIC

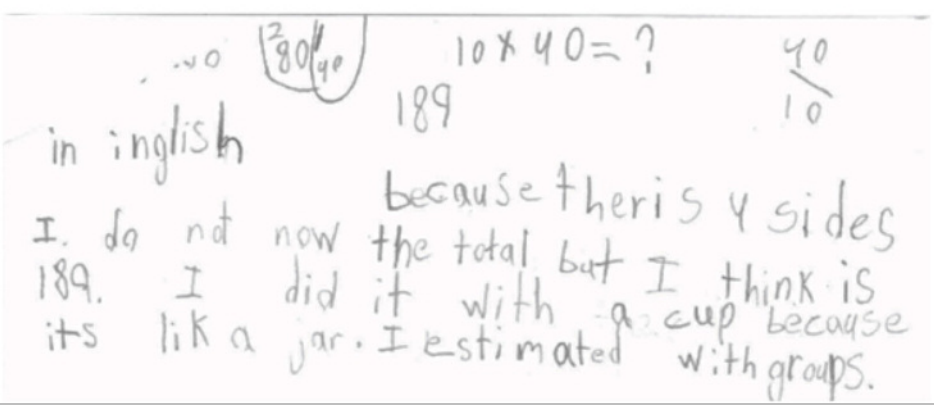
	Level 1 –Novice	Level 2 –Apprentice	Level 3 – Practitioner	Level 4 –Expert
Thinking, Inquiry, and Problem Solving	• Final solution in incorrect. • Conclusions regarding fairness of game are made on "superficial" criteria.	• Solution is correct. • Strategy for determining solution is unclear.	• Solution is correct. • Strategy is clear	• Solution is correct and complete. • Strategy is clear. • Relevant information is applied to give a clear, complete, correct answer.
Knowledge, Understanding, and Skills	Very limited understanding of theoretical and experimental probabilities is demonstrated	• Some knowledge of experimental probability is demonstrated. • Limited knowledge of theoretical probability is demonstrated.	• Some knowledge of theoretical and experimental probability is demonstrated.	• A thorough understanding of theoretical and experimental probabilities is demonstrated.
Application	• Limited attempts to identify the possible outcomes for the two sets of dice are made. • Errors in set-up of outcomes, leading to completely incorrect sample space.	• Possible outcomes for the two sets of dice are identified, although incorrect. • Minor errors or no organization of the outcomes for comparison.	• All possible outcomes for the two sets of dice are found correctly. • Minimal minor errors and organizes the results, although ineffectively, to compare them.	• All the possible outcomes for the two sets of dice are found correctly. Sample spaces for the two sets of dice are compared correctly.
Communication	• Limited and/or incorrect use of diagrams or charts. • Limited and/or incorrect use of the language of probability. • Written descriptions are unclear throughout.	• Unclear or incomplete diagrams or charts. • Limited use of the language of probability. • Written descriptions are unclear in some areas.	• Complete tree diagrams or charts. • Effective use of the language of probability. • Written descriptions are unclear in only a few small areas.	• Precise and clear tree diagrams or charts. • Arguments effectively integrate the language of probability. • Written descriptions are clear throughout.

Teacher's Comments: _____

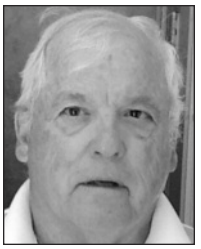
_____

▲

Age 8



▲ OAME/NCTM Report



TODD ROMIENS
EMAIL: todd.romiens@oame.on.ca

Todd recently retired from teaching K–12 methodology in the Faculty of Education at the University of Windsor. He is a past president of OAME and a life member, as well as a previous editor of the Gazette*.*

Another issue that is drawing the attention of NCTM is that of homework. There are questions about how much students should do, at what grade they should start, whether students should do any at all, and what the role of parents (including guardians) is?

Matt Larson, outgoing president of NCTM, has written an article entitled *The Need to Make Homework Comprehensible*, which addresses these issues facing teachers of all grades in the United States and Canada.

Here is what he said:

> Whether you are an elementary, middle-level, or high school teacher, you are likely to have had parents say to you that they can't help their children with their math homework. At the secondary level, the difficulty is often the content itself; at the elementary level, however, it is often a function of parents' unfamiliarity with the instructional strategies that we use today to build conceptual understanding.
>
> In recent years, as new standards and instructional strategies have been implemented at the school and classroom levels to build students' conceptual understanding, in addition to helping them meet traditional procedural fluency goals, parental concerns about math homework, particularly at the elementary level, have increased. Social media provide a new vehicle by which parents rapidly share examples and express their concerns about math homework and instructional strategies that they find confusing or unnecessarily complicated.
>
> The issue is not new. Mathematics homework has long been a conundrum for students, teachers, and parents alike. Today some teachers, schools, and school districts are going so far as to limit, or even ban, homework in certain grades. To some degree, this is part of a historical cycle in the United States: homework falls in and out of favour every 15 to 20 years. Clearly, however, for many parents today, it is a hot topic that we need to address more effectively.
>
> Despite these calls to reduce or ban homework, most math teachers continue to assign it because they recognize that success—whether in mathematics, music or athletics—depends on at least two common components: practice and perseverance. Too many adults, and students for that matter, tend to view success in mathematics as dependent on a talent with which someone either is or is not born. Several highly regarded researchers, including Carol Dweck and Jo Boaler, have demonstrated that simply is not the case. Each and every student can learn more mathematics in the same way that an athlete improves his or her running time or a musician masters the technique required to play a piece of music—by receiving appropriate and effective instruction in a supportive environment by responding to feedback, and by expanding effort—trying again and again, while using different strategies; in short, by engaging in meaningful practice.
>
> However, we need to make the practice that goes home as straightforward and comprehensible as possible for parents. We need to recognize that while we may discuss multiple-solution methods in class, the goal is not for students to practise and master multiple methods; rather, it is for students to practise and develop proficiency with their preferred, well-understood, and efficient method.
>
> Making homework comprehensive means, in part, that we need to be responsible for the types of tasks that we send home. Many of the instructional and homework tasks that frustrate parents reflect instructional strategies used to develop student understanding of underlying mathematical concepts that are different from what they know or remember.
>
> Finally, we need to show parents the solution strategies that we are using in the mathematics classroom. Evidence suggests that once parents are shown, engaged in thinking about, and come to understand the strategies in use, their support increases. NCTM has a number of resources that can support parents in understanding school mathematics today, including the recent publication, *It's Elementary: A Parent's Guide to Elementary Mathematics*, which offers clear explanations of many of the instructional strategies that parents may not have experienced when they were in school themselves.
>
> As mathematics teachers, it is our responsibility to share with parents the instructional strategies we use in the classroom and make the practice that we send home comprehensible so that parents can engage with their children in constructive ways and support their mathematical learning."

Don't forget the NCTM annual conference in San Antonio, Texas from April 5–8, 2017. ▲

OMO 2017 – Constructing Mathematical Understanding



OAME
2017 Ontario Mathematics Olympics
Friday, May 26th, 2017 – Saturday, May 27th, 2017
Humber College Lakeshore Campus,
3199 Lakeshore Boulevard W., Toronto, Ontario

Details will be sent to each chapter, for more information contact:

Earl.totten@tcdsb.org
Bart.Vanslack@tcdsb.org

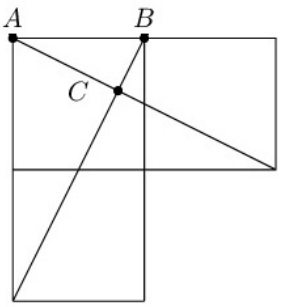
▲ What's the Problem? Squaring a Triangle

SHAWN GODIN



Shawn is the head of Mathematics, Business, Law, and Computer Science at Cairine Wilson Secondary School in Orleans, Ontario. He is a past member of the OAME Board of Directors, and past Editor-in-Chief of Crux Mathematicorum. *He currently writes problems for the Fryer, Galois, Hypatia, and Canadian Open Mathematics Challenge contests.*

Welcome back, problem solvers. Last time, I left you with the following problem:



Three unit squares and two line segments connecting two pairs of vertices are shown.

What is the area of Δ*ABC*?

This problem appeared as question 15 on the 2012 AMC10A contest. The AMC10A (and AMC10B) is a 25-question multiple-choice mathematics competition run by the MAA American Mathematics Competitions (AMC). The MAA AMC holds a number of mathematics competitions in which Canadian students can participate. More information about their contests is available on their website, **www.maa.org/math-competitions**.

As with any geometry problem, we can attack this with the dynamic geometry software package of your choice. If you do so, you will get the answer "0.2 square units" quite quickly. Having found the "answer," hopefully you will say to yourself something like, "Hmm… that is interesting! I wonder *why* we get something that seems to be one-fifth."

So how else could we solve the problem? We are looking at a triangle where one of the vertices, *C*, is the point of intersection of two lines. So if we throw our problem onto a coordinate system, with the bottom left corner of the bottom square at the origin, we get the configuration shown in Figure 1. We can see that our two lines have equations $y = 2x$ and $y = -\frac{1}{2}x + 2$; hence, when we solve the system of equations, we get the point of intersection $C(0.8, 1.6)$. Looking at our triangle, if we use *AB* as our base, then the height will just be 2 – 1.6 = 0.4, and so our area calculation again gives us 0.2 square units, but we still have no insight as to why.

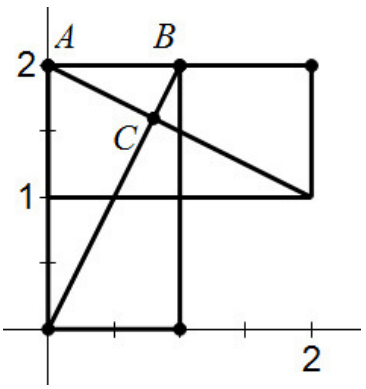


Figure 1

Perhaps we should investigate the actual *geometry* of the situation. If we label a few more points as in Figure 2, we can see that Δ*AXY* and Δ*BPQ* are congruent right triangles. Since we started with three squares, we can see that ∠*ABC* and ∠*BPQ* are complementary; however, by the congruence, ∠*PBQ* = ∠*XAY*. Observing that ∠*XAY* and ∠*BAC* are just the same angle, we can see that our triangle, Δ*ABC* is similar to both Δ*AXY* and Δ*BPQ*. Thus, the ratio of the lengths of the sides in Δ*ABC* is going to be $1 : 2 : \sqrt{5}$. Since we know that the hypotenuse, *AB*, has length 1 unit, the sides of Δ*ABC* will be $\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}$, and 1, so the area will be $A = \frac{1}{2}\left(\frac{1}{\sqrt{5}}\right)\left(\frac{2}{\sqrt{5}}\right) = \frac{1}{5}$.

Alternately, we could have used the fact that if the ratio of the lengths of sides of similar figures is 1 : *k*, then the ratio of their areas is $1 : k^2$. Then, since the areas of the big triangles, Δ*AXY* and Δ*BPQ*, are both 1 square unit, and the ratio of the sides of the big triangle to the small is $1 : \frac{1}{\sqrt{5}}$, the areas will be in the ratio is $1 : \frac{1}{5}$.

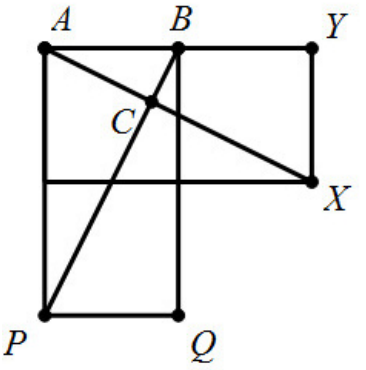


Figure 2

That seems more satisfying! At least we found a connection to the geometry of the problem… but there is still something bugging me. Since the area was one-fifth of a square unit, I should be able to fit five copies of the triangle in the square (with some possible… alterations). Let's see if we can make this work.

Even if students haven't formally studied similarity, they will probably be able to appreciate that our triangle is a smaller version of the bigger triangles, like in the last solution. As such, our triangle is a right triangle, whose legs are in the ratio of and whose hypotenuse is 1 unit long. Thus, we should be able to fit four copies of our triangle inside a unit square as in Figure 3.

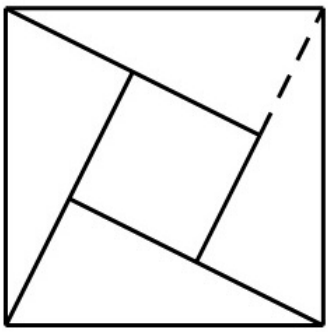


Figure 3

From this diagram, we can tell that the area of the triangle is less than a quarter of a square unit because we still have the square in the middle. If we could somehow show that the square had the area of one of the triangles, we would be done. Since the ratio of the legs of our triangle is 1 : 2, the short side is half as long as the long side. So in Figure 3, the broken line is half as long as the long side of the triangle, which means it is the same length as the side of the square. So if we put the square and triangle together as in Figure 4, and we calculate their areas we get,

$$A_{square} = b^2 \qquad a_{triangle} = \frac{1}{2}(b)(2b) = b^2,$$

so they are the same, which means that the triangle has one-fifth the area of the large square.

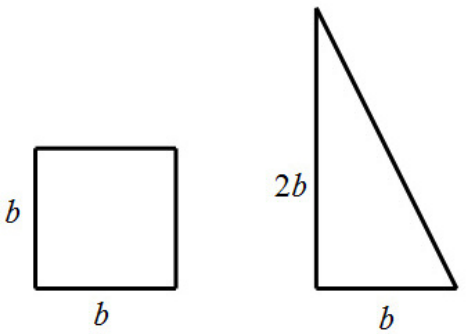


Figure 4

We could have also done something a bit more satisfying as in Figure 5. If we join one of the vertices of the square to the midpoint of one of the non-attached sides, the resulting shaded triangle is similar to our triangle in question. If we rotate the shaded triangle about the indicated point, it fits squarely on top, creating our desired triangle, showing that the square does have the same area as our triangle. You should, just to be thorough, take a bit of time to convince yourself that the figure on the right is actually a triangle. I will leave that to you as an exercise.

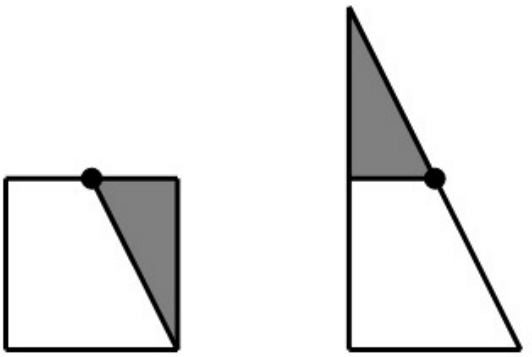


Figure 5

As always, "the answer" shouldn't necessarily mark the end of the problem-solving process. Different solutions will sometimes give you different insights into the problem, or different insights into the connections between mathematical concepts. Once we understand a problem, in some cases, we can further abstract it by generalizing. For example, in this case, instead of having a configuration with 2 squares up and 2 squares to the right, what if we had 3 or 4 or *n*? What if we break the symmetry and go up and across? Do we get any interesting results with these generalizations, or do they give us some insight into why this particular problem had an interesting answer. Mathematics isn't just an endless set of dreadful computations, but a beautiful collection of concepts that are interconnected in fascinating ways. We should try to spend as much time as we possibly can, playing around with those patterns that are just waiting there to be played with. Occasionally, we should probably let our students in on the fun as well.

Now that I have stepped down from my soap box, I suppose I should give you some homework:

Determine all functions that turn addition into multiplication. That is, determine all functions f such that

$$f(x + y) = f(x)f(y).$$

Until next time, happy problem solving! ▲

Provincial Digital Learning Resources – What's New? Colour Tiles



MARKUS WOLSKI

AGNES GRAFTON

ROSS ISENEGGER

GREG CLARKE







*Ross (Near North District School Board), Agnes (Brant Haldimand Norfolk Catholic District School Board), Markus (Bluewater District School Board), and Greg (Simcoe Muskoka Catholic District School Board) are all currently on assignment with the Ontario Ministry of Education as Provincial Math Leads, working on the CLIPS and **mathies.ca** projects.*

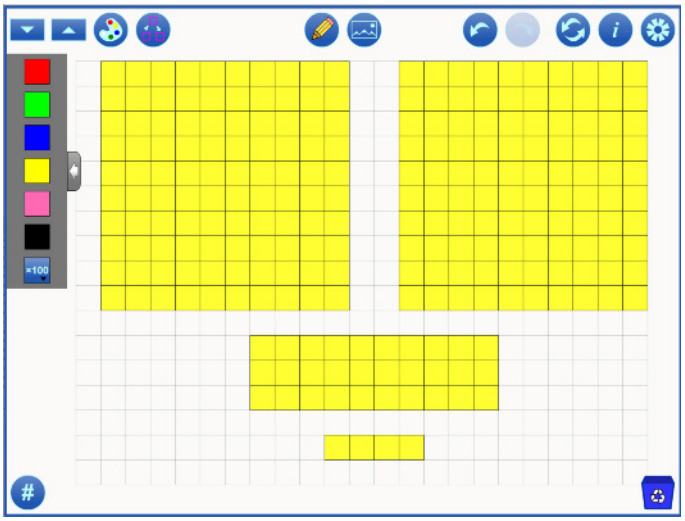
Since the previous issue, the **Set Tool** and **Colour Tiles** apps have been released for mobile on mathies.ca, Colour Tiles has received a significant update, and the **Catch Ball Ops** game has been released for desktop and mobile. This article will focus on sharing the Colour Tiles updates.

Colour Tiles



Colour Tiles now opens with a choice of three sets of tiles. Original 6 will set the tool up very much as version 1.0 would have, with square tiles having a thin black outline available in six colours. As of version 1.1, many more colours are available, as well as tiles that are circles or rounded squares. There are many more settings available in the updated version, including the ability to remove the outline.

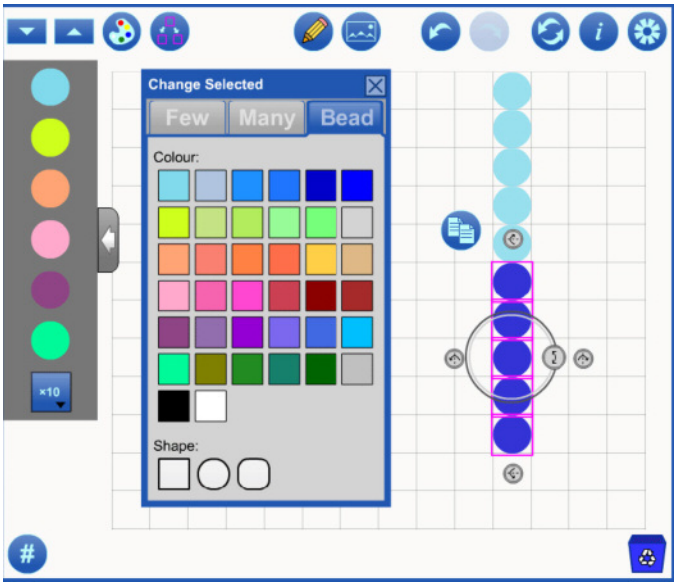
Once a choice is made, the tool opens up and tiles can be dragged from the panel at the left. Tiles can be dragged out in groups of 1, 2, 5, 10, and 100.



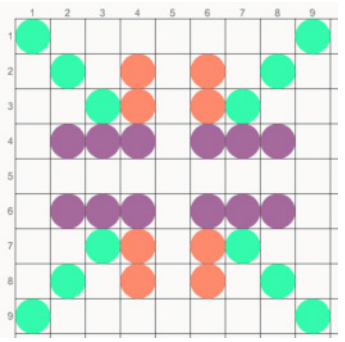
As with base-10 materials, the workspace above can represent 234 or 23.4 or 2.34 or 0.234, depending on the agreed value of a tile.

Operating on Selected Tiles

You can select tiles by drawing a marquee rectangle around them or by simply clicking on specific tiles. The selected set, highlighted by magenta rectangles, can be copied, moved, reflected, or rotated. Click on the painter's palette to open the Change Selected Dialog, then click on a colour or shape to change the selected tiles.

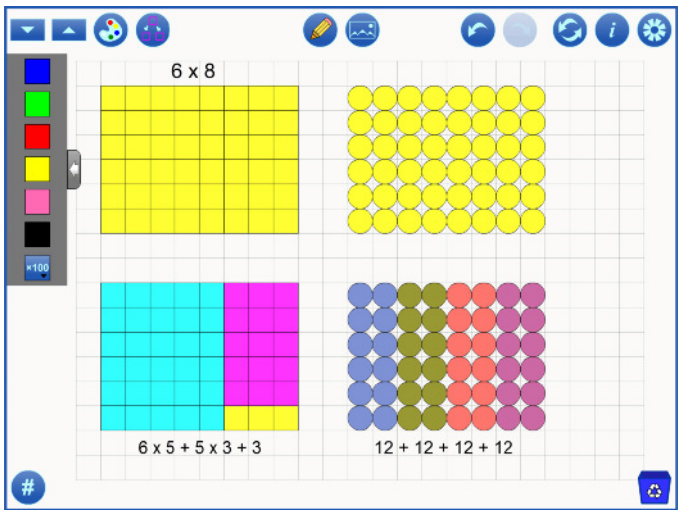


The reflection and rotation buttons allow symmetry to be explored. The design below was created using four reflections. Use your spatial reasoning skills to visualize how this might have been done. How could a rotation have been used instead? When would it be helpful to have a numbered grid displayed?



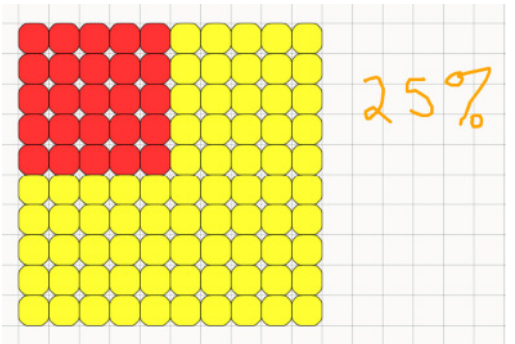
Modelling Multiplication and Percent

In the following example, 6 times 8 is modelled in two ways. The model on the right is more unambiguously an array model.



Colour, together with the annotation tools (available from the pencil icon), is used to share thinking.

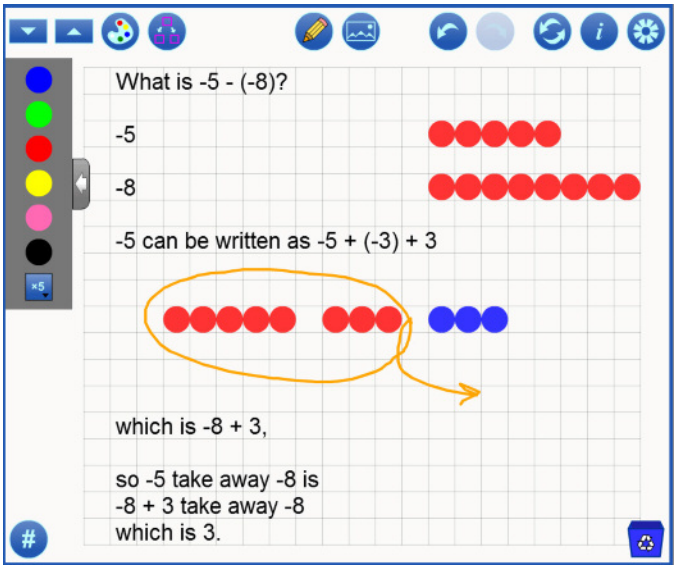
Colouring tiles from a set could be used to represent percents or fractions.



The colouring of 25% above helps visualize its equivalence to one-fourth.

Representing Integer Operations

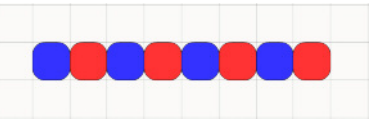
Colour Tiles can be used as virtual Integer Tiles.



Choose your favourite shape and colours for positive and negative to represent integers.

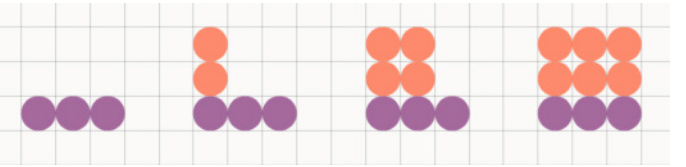
Beading and Other Patterns

The Colour Tiles tool can be used to create a variety of patterns. The following is a simple AB repeating pattern:



Students could be challenged to create this pattern with the least number of steps possible: creating the core and using the Copy button to extend it.

Linear growing patterns can also be created. The ability to copy selections allows the constant and growing parts to be built quickly.



The Beading set of tiles grew out of the “Connecting Indigenous and Western Mathematical Ways of Knowing” research project that Dr. Ruth Beatty, Lakehead University, and Danielle Blair, Provincial Math Lead, have been doing in partnership with select First Nations, Métis, Inuit communities, and Ontario school boards. Students can use the tool to design their loom bead work and engage in mathematical discourse around pattern, number, symmetry, and transformations.



Eganville District Public School student using v1.0 of Colour Tiles as part of loom bead study

Here, the pattern core has been identified using annotation rectangles. Students can now quickly try out different colours for their work, using the Extend Selection button.

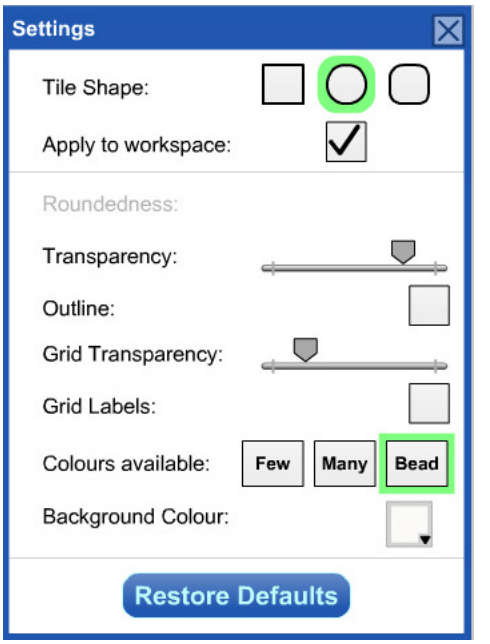
They select one tile on their workspace, press the button, and all tiles of that colour are added to the selection, so that the colour can be changed using the Change Selected Dialog. If no tiles are selected, pressing the button selects all tiles.

Students might identify the pattern core by moving a copy to the next position to see if it overlaps exactly. They can also copy the core to an empty location and repeatedly copy it to see if the same design results. Students can use the grid labels to make near and far predictions of the content of columns and/or cells. They can use algebraic reasoning to determine the total number of beads of each colour required for their design. They can use proportional reasoning to determine how many columns are required for a wrist with circumference 16 cm if there are five size 10 beads per centimetre. They can also adjust their conclusion for someone with a different wrist circumference.

To find out more about using Colour Tiles with loom beading, or about establishing a partnership with a local First Nation, Métis, or Inuit community to explore mathematics in a culturally responsive way, email **contact@mathies.ca**.

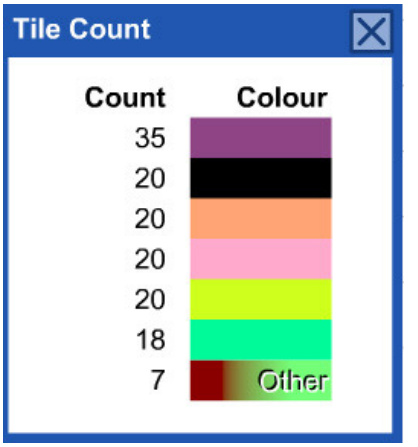
Settings and Count Dialogs

There are more settings available in the updated version which are available using the button.



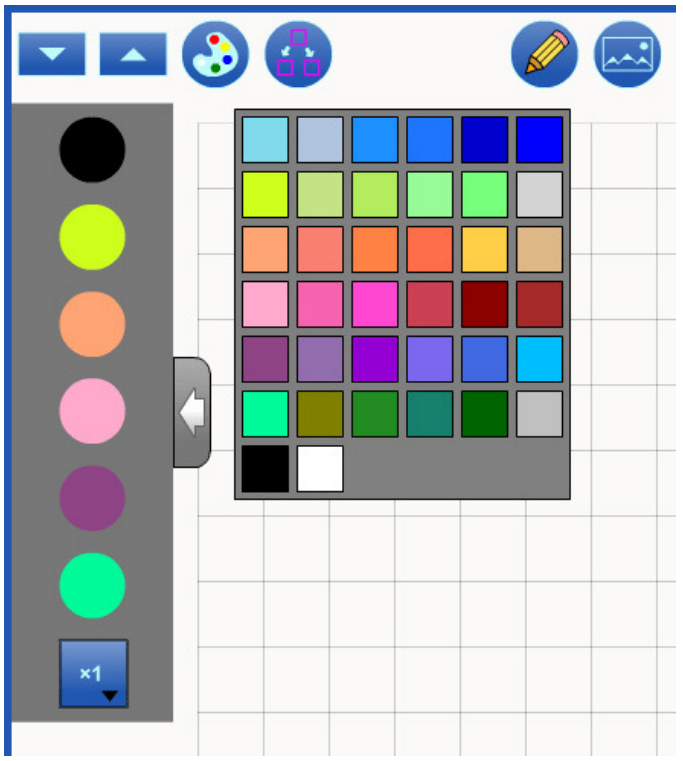
You can choose from among the three shapes. If *Apply to Workspace* is checked, then all shapes will change. If it is not checked, only the tiles in the selection panel will change. The extent of the roundedness of rounded squares can be changed. Tiles can be made more transparent or completely opaque. An outline around each tile can be shown or hidden. Number labels can be added to the grid. The transparency of the grid is configurable. You can also change the colour palette and background colour from within Settings.

Since there are so many more colours available, the Count Dialog (accessed from the # button) does not try to name them, but rather, shows you a swatch for each colour used.



Other Changes

You can click on a tile in the selection panel at the left to change its colour, so that all subsequent tiles dragged from the panel are the new colour.



If you have overlapped tiles, marquee select only picks up the top layer. Selection is now recorded in your undo/redo history. Using undo and redo is noticeably faster, especially on mobile.

Feedback and Future Requests

Please feel free to send us your feedback about any mathies tool, using the Feedback Form button inside the Information Dialog, accessed from the *i* button.



You can also send your comments to **WhatsNew@oame.on.ca**. The Support Wiki has a video of all the new features of the Colour Tiles tool as well as examples and feature explanations. You can share your experiences on Twitter using the hashtag #ONmathies. There is an increasing set of interesting posts of student and teacher work there. To be among the first to find out about the latest digital tool developments, sign up for our email list at **http://mathclips.ca/WhatsNewEmailList.html**.

Here's hoping that the increased flexibility of the Colour Tiles Learning Tool helps you visualize a wide array of interesting mathematics! ▲

▲ Technology Corner: Blogs



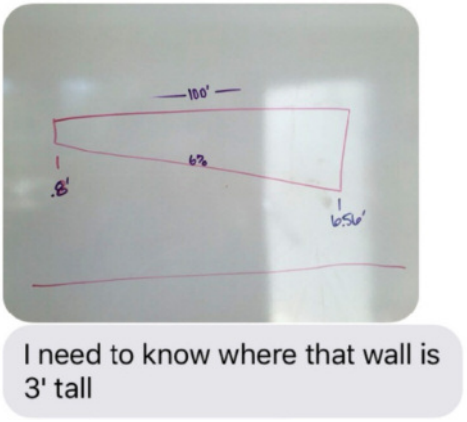
MARY BOURASSA
EMAIL: mary.bourassa@oame.on.ca

Mary teaches mathematics at West Carleton Secondary School in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She has presented workshops internationally, authored mathematics resources, is a past VP of OAME and a Past President of COMA. An award-winning teacher, Mary continually strives to learn new and better ways of helping students learn and love mathematics.

Earlier this week, I saw this tweet from Brian Miller:



The link provided brings you to Brian's blog post, where he details the process of planning to build a retaining wall.



It includes the rationale behind the activity along with student work and Brian's reflections.

I have found that some of the most useful resources for the classroom have been shared on teachers' blogs, just like the example above. There is a multitude of different math

teacher blogs with an equally diverse list of topics. You can learn more about subjects relating to educational research, to education books, tips and tricks for the classroom, how to plan, how to question along with classroom-ready activities. The latter is where I spend most of my blog-reading time, as I can either use these activities in my own classroom or share them with others. These bloggers not only provide activities that you can use, but provide perspective that is invaluable as you prepare to try something new. What did they do to prepare for the activity? How did the activity unfold? What questions arose? What worked really well? What would they change? This is like someone has done a trial run for you and you can now use all that information to adapt the activity and make it run as smoothly as possible.

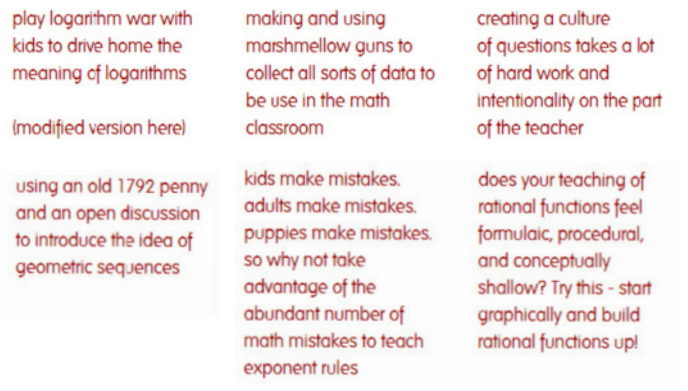
The sheer number of math teacher blogs that exist can be overwhelming, so you need to find ones that meet your needs. If you are looking for something specific, you can do a search. For example, doing a Google search for "MPM2D blog" will return links to posts from Jon Orr (**http://mrorr-isageek.com**), Brandon Grasley (**https://bgrasley.wordpress.com**), and Kyle Pearce (**https://tapintoteenminds.com**), among others. However, sometimes it is hard to sort through the results of such a search. You can narrow things down by using the MTBoS (Math-Twitter-Blog-o-Sphere) search, created by John Stevens (**http://bit.ly/MTBOSS**). This will only search through blogs written by teachers who identify themselves as being part of the MTBoS.

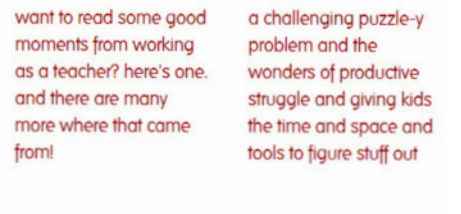


Further to this, you can explore some of these blogs through the MTBoS website, **http://mathtwitterblogosphere.weebly.com/a-few-good-blogposts.html**.



Here is a sample of what you can read about:



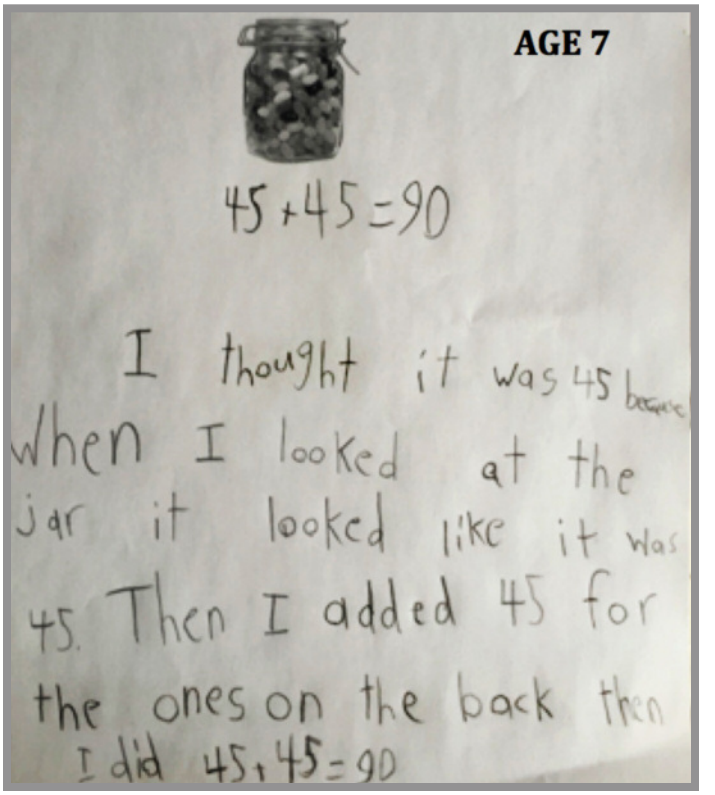


If you find a blog that you really enjoy reading, you may be able to subscribe to it by email, or you can add it to an RSS (Rich Site Summary) reader such as Feedly. RSS readers allow you to add sites to your feed so you can read blogs whenever you have time. You do not have to visit each blog individually, saving you time, and you don't have to remember which blogs you want to read.

If you want to start populating Feedly, the MTBoS website can help you find blogs that may be of interest. Blogs are listed by academic level and interest: **http://mathtwitterblogosphere.weebly.com/list-of-tweeps-and-blogs-by-category.html**.

Another way to find interesting blog posts is to join Twitter, as many math teachers will tweet out links when they blog. Following the MTBoS Blogbot (@MTBoS_Blogbot) is a simple way to use Twitter to learn about new bloggers.

Hopefully, reading blogs will inspire you to start your own. It is a great way to share the cool things going on in your classroom, as well as the days that don't go as planned. Writing a blog allows you to reflect on your practice and helps others at the same time. It's a win-win idea! ▲

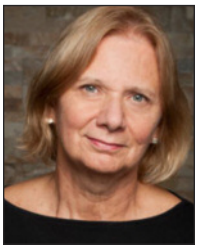


OAME Grade 9 Applied Collaborative Inquiry



KELLY MCKIE
2013-2016, MA (ED) MATHEMATICS EDUCATION, UNIVERSITY OF OTTAWA

After several years working in the Oil and Gas industry Kelly opted to change directions and returned to school to pursue a career in mathematics education. While studying at the University of Calgary she partnered with the Galileo Educational Network Association (GENA where she was introduced to the lesson study model. After moving to Ottawa Kelly pursued a Masters degree in mathematics education at the University of Ottawa under the direction of Dr. Chris Suurtamm and became part of the OAME Grade 9 Applied Collaborative Inquiry project as a research assistant. Kelly continues to work with the OAME, helping to create the math4thenines portfolio of learning resources.



CHRIS SUURTAMM
Email:Christine.Suurtamm@uottawa.ca

Dr. Chris Suurtamm is a Professor of Mathematics Education at the University of Ottawa. Her research focuses on the complexity of mathematics teaching, and the use of formative assessment to improve student learning. She has been involved in many large-scale research projects and is dedicated to connecting research and practice. She has been the Canadian Representative on NCTM, Co-Chair of Assessment Topic Study Groups at International Congresses for Mathematics Education, and a member of the Ontario Ministry of Education Curriculum Council.

Meeting the needs of all students in mathematics in Ontario is an important objective for the mathematics community. Through both school data and provincial assessments, it appears that there are a number of students in the Grade 9 Applied Mathematics course who are not meeting expectations. To help address this issue, the Ontario Ministry of Education provided funding during the 2014/2015 and 2015/2016 school years to the Ontario Association for Mathematics Education (OAME) to develop and pilot a professional learning initiative that would support ten school teams across Ontario. The main objectives of this project were for the school teams to focus on enhancing their understanding of the Grade 9 Applied Mathematics curriculum, and to focus on implementing the curriculum in ways that would best meet the needs of their students. The Ontario Ministry of Education identified the following goals of this collaborative team inquiry:

- Increase student achievement in Grade 9 Applied Mathematics
- Increase teacher knowledge of the curriculum, both in terms of mathematics content knowledge and mathematics knowledge for teaching
- Identify effective teaching and learning strategies
- Share implementation strategies with stakeholders
- Foster teacher leadership in mathematics

During the two years of the OAME Grade 9 Applied Project, school Professional Learning Communities (PLCs) were provided the opportunity to meet once a month for a full day to work on self-defined problems of practice. The purpose of this project was to document strategies for implementing and teaching that would deepen teacher understanding of the intended curriculum and ultimately would enhance teacher practice. School PLCs varied in composition, but generally consisted of two or three Grade 9 Applied-level mathematics teachers, the Student Success teacher, the Special Education Resource teacher, the Math Department Head/Chair, an administrator, and a Board Math Lead. A research assistant was assigned to each school team and supported them throughout the two years of the project.

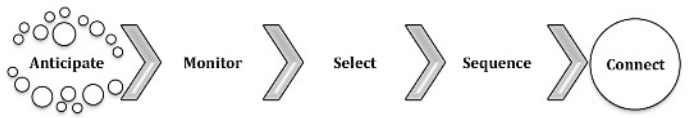
As the project progressed, the research team, led by Dr. Chris Suurtamm, noticed similarities in PLCs' areas of focus and the ways they worked as teams. Areas of focus that emerged included:

- making thinking visible
- incorporating new assessment strategies
- rearranging the curriculum
- use of rich tasks
- use of technology

It is the use of rich tasks that is the focus of this article. Over the last several decades there has been a shift in the role of the mathematics teacher (LaChance & Confrey, 2003; Suurtamm, 2013). More emphasis is being placed on providing students with opportunities to productively struggle with rich mathematical problems that lead to deeper understanding of specific concepts (NCTM, 2014; Warshauer, 2014). Creating these opportunities, and understanding how to guide students through them, can be challenging for teachers (Darling-Hammond & Ball, 1998). Observations from this project prompted the research team to find ways to support teachers as they navigate these specific challenges.

The Five Practices. Between Year 1 and Year 2 of the project, participants were invited to attend a three-day summer institute in London, Ontario. Teams shared their learning and attended several sessions on various topics that the research team felt would address some of the teams' identified needs. One session in particular focused on the use of The 5 Practices for Orchestrating Productive Mathematical Discussions (Smith & Stein, 2011). Essentially, the five practices are:

- ***anticipating*** likely student responses to challenging demanding mathematical tasks
- ***monitoring*** students' actual responses to the tasks (while students work on the tasks in pairs or small groups)
- ***selecting*** particular students to present their mathematical work during the whole-class discussion
- ***sequencing*** the student responses that will be displayed in a specific order
- ***connecting*** different students' responses and connecting the responses to key mathematical ideas (Smith & Stein, 2011, p. 8)



As Year 2 of the project progressed, the impact of the *5 Practices* session and the use of the *5 Practices* articles and/or book became apparent. The different PLCs began using the *5 Practices* in various ways during their monthly meetings and in their own classrooms.

One school PLC used the *5 Practices* as a framework for a lesson study. Teachers began the process by focusing on the *anticipating* phase of the framework. They would bring potential rich tasks to their meetings and would set to completing the tasks themselves in order to *anticipate* some of the different solutions their students might produce.

Together the participants would engage in the mathematics and discuss different solutions they felt students would produce. The next phase of their work involved observing students engaging with the task in the classroom. In sessions following the class, the PLC would debrief as a team. This debriefing allowed the participants to reflect on their anticipation of student responses and discuss unanticipated responses and observations that the teacher may not have been able to observe while teaching. This process would be repeated again at their next PLC meeting with a different task, teacher, and classroom, and carrying forward lessons the PLC learned from engaging with the previous task.

An administrator from another school describes how the *5 Practices* helped her school's PLC work toward their original problem of practice:

> We started with a goal of wanting to increase discussion in the Grade 9 Applied class and also with doing more rich problems. We ended up working on mindset, but marrying all three of those ideas really was the *5 Practices*. It was the catalyst for us to bring all these ideas together. The goal of the *5 Practices* is to get discussions, to get students talking to other students and it provided us with a framework for that" (Vice-Principal).

A participant from a third PLC describes how the book helped them to enhance their use of rich tasks in the classroom:

> We saw [*The 5 Practices*] as a way to meld our… just work on the craft of teaching. We had engagement in our class, we have all these great activities…, but things were kind of a bit rough around the edges. This *5 Practices* book brought another expert into our lesson study group that we didn't have before, and that's an important part of what happened" (Mathematics Department Head).

The response to the methods described by Smith and Stein (2011) by the teachers involved in the OAME Grade 9 Applied Collaborative Inquiry leads us to believe that others outside the project might be interested in exploring the ideas in the book. Moving forward, we hope to continue to explore ways that teachers in Ontario can engage with the methods described in the *5 Practices*. We are currently developing several Adobe Connect sessions based on the *5 Practices* that will be available for teachers in the upcoming months.

References

Darling-Hammond, L., & Ball, D.L. (1998). *Teaching for high standards: What policymakers need to know and be able to do* (CPRE Joint Report Series, JRE-04). Philadelphia, PA: Consortium for Policy Research in Education and the National Commission on Teaching & America's Future.

Lachance, A., & Confrey, J. (2003). Interconnecting content and community: A qualitative study of secondary teachers. *Journal of Mathematics Teacher Education, 6*(2), 107–137.

National Council of Teachers of Mathematics. (2014). *Principles to actions.* Reston, VA: Author.

Smith, M.S., & Stein, M.K. (2011). *5 Practices for Orchestrating Productive Mathematics Discussions.* Reston, VA: National Council of Teachers of Mathematics.

Suurtamm, C. (2013). Mathematics teacher education. In L.H. Meyer (Ed.,) *Oxford bibliographies of education.* New York, NY: Oxford University Press.

Warshauer, H.K. (2014). Productive struggle in teaching and learning middle school mathematics. *Journal of Mathematics Education, 17*(4), 3–28. ▲

▲ Fields Institute MathEd Forum Notes

STEWART CRAVEN



Stewart Craven presently teaches courses for the Continuing Education Department and the Transition Years Program at York University. He is an active participant in the Fields Mathematics Education Forum.

The following talks given at the Fields Education Forum meeting on January 28 provided a look at topics that are presently being researched by mathematics educators in Ontario and across Canada:

Students' and Teachers' Mental Solving of Algebraic Equations: From Differences to Challenges: Jérôme Proulx (L'Université du Québec à Montréal)

This paper presents students' and teachers' strategies for mentally solving algebraic equations. The enactivist notions of problem-posing offer conceptual grounds to engage in analysis of students' and teachers' strategies, and in their comparisons, leading to the exploration of differences in the nature and origin between the solving processes of students and those of teachers.

ICT Use to Improve the Self-Regulation Skills of Students Taking a College Foundational Mathematics Course: Carol Carruthers (Seneca College)

This investigation looked to the potential gained by a learning intervention employing the affordances of information and communication technologies to improve the self-regulation skills of 17 college students taking a foundational mathematics course.

Implementing Interactive Texts in Undergraduate Mathematics: Margo Kondratieva (Memorial University)

In this talk, I will discuss our project on constructing interactive texts on "functions' transformations" and summarize students' feedback concerned with clarity and usefulness of the text, and its ability to engage and develop interest in the subject. This is an ongoing project, and I hope to receive suggestions and comments from the audience.

Achieving Excellence in Primary–Junior Mathematics Pre-Service Teacher Education: Steven Khan (Brock University, Faculty of Education)

In 2015, pre-service teacher education programs in Ontario moved to implement two-year Consecutive Teacher Education programs. This presentation will offer a critical appraisal through reflection on my experiences as a (non-tenure track) limited-term faculty member collaboratively designing, teaching, assessing, and evaluating the two math methods courses that now comprise our Consecutive B.Ed. preparation for Primary–Junior candidates.

Attempting to Improve Health Numeracy Through an Online Instrument: Andie Burazin (University of Toronto, Mississauga), Taras Gula (George Brown College), Miroslav Lovric (McMaster University), and Asia Matthews (Quest)

There is plenty of evidence suggesting that the lack of adequate health numeracy is a significant problem. Canadian Council of Learning reported that "60% of adults in Canada lack the capacity to obtain, understand, and act upon health information and services and to make appropriate health decisions on their own." This collaborative action research project is guided by two questions: (1) What are the core elements of health numeracy for health sciences students and health care professionals? (2) How might their health numeracy be most effectively improved using an on-line, interactive instrument?

Climate Change in Mathematics Classrooms: Yasmine Abtahi (L'Université du Québec à Montréal) and Richard Barwell (University of Ottawa)

Climate change is one of the most urgent global issues. Mathematics is used to describe and predict climate change as well as to communicate it to scientists, politicians, the general public, and children. Hence, the teaching and learning of mathematics is important to not only get a sense of climate-change messages, but also to be critical about them. We are a research team of teacher educators at the University of Ottawa and Bergen University College, Norway, who implemented a survey to get a sense of elementary and secondary school teachers' activities and aims in relation to climate change. Here, we present how mathematics teachers in Canada and Norway use climate change as a topic in the mathematics classrooms, from varying degrees of critical view, for different aims and from different perspectives.

As you can see, mathematics education research covers a wide range of issues, and attending Mathematics Education Forum meetings will help you stay up-to-date with current trends. You are more than welcome to join us!

The Steering Committee for the MathEd Forum seeks input for new themes and would appreciate your suggestions. (Drop me an email at **numeratecitizen@mac.com** and I will pass your ideas along.) Agendae and information can be found at **www.fields.utoronto.ca/programs/mathed/forum/**. ▲

Checking the Accuracy of GPS Data

TIM SIBBALD



Tim is an Assistant Professor of mathematics education in the Schulich School of Education, Nipissing University. He has previously taught high school and been a mathematician in industry. He currently serves as the President of the OAME.

It is incumbent upon on all math teachers to encourage numeracy. A significant part of being numerate is checking whether data that is provided is reasonable. Increasingly data sources are used in daily life, and students need to be taught routines of checking the mathematical details. In some cases, checking the reasonableness of data relies on understanding the geometry of how the data was collected.

One tool that has evolved significantly in the last 20 years is "GPS." The Global Positioning System (GPS) was developed by the U.S. military to provide a standardized method for navigation. It has been very effective for navigation, and non-military commercial applications have arisen with navigation map systems for cars, and many cellphones having GPS options. It is not very surprising that many other nations have created their own GPS systems, and that speaks to GPS systems being considered a matter of national security. Students rarely think of navigation as having that level of importance, and asking students why could be a communication task. It may also be a chance to co-operate with history teachers, who likely can come up with examples of historical navigation challenges (such as Columbus thinking he had reached India, or the Vikings who could determine longitude much more accurately than latitude). Given these historical issues, it is reasonable to check if GPS coordinates agree with the location of the school according to a classroom atlas, globe, or a topographical map.

It is important for students to understand that GPS is a system that can be turned off, as it was during the Gulf war in 1990. This is not simply the GPS device being turned off, but the source of the signals can be changed to be unintelligible by any regular devices. It may also have the accuracy reduced using a process known as *selective availability* (see Unavco, 2016). While it is less likely these days, the consequences of possible disruptions make it necessary to learn other navigation techniques, and that can facilitate a variety of math activities (for example, using compass work and bearings in the schoolyard to see how an angle and distance defines a route from one point to another).

The purpose of this article is not to discuss navigation in a general sense, since it is an immense topic. The purpose is to look at activities related to GPS navigation. Through the use of activities, students can have a better appreciation of how the system works and the geometry involved in it. In terms of the curriculum, this facilitates connections to data management and geometry across grade levels. The geometry that is involved uses circles, but can also be used to bring in three-dimensional geometry of spheres. It supports the inclusion of geography, maps, and the interpretation of the mathematics that arises in terms of a geographical context (for example, using proportional reasoning to improve the accuracy of estimates of the coordinates for the school when using an atlas or globe). There are STEM opportunities as well, since the engineering and technology side of GPS can be engaged along with the associated mathematics. In secondary grades, it can be used for experimental investigations using GeoGebra.

Consider a trip from Moose Factory to North Bay in February of 2015. Data was gathered using a sport GPS designed for hikers, who are not necessarily on roads, and has software for downloading tracks. The route is shown in Figure 1, but poses a challenge for GPS systems that utilize electronic maps because the northern-most part of the route was on an ice road that Google maps and GPS map systems do not recognize. This choice of example was opportune, but also chosen as a reminder that road maps often do not suffice in remote areas that are within driving distance of southern Ontario. In many instances, in northern Ontario, ice roads or logging roads are used and they are not on standard road maps.

In Figure 1, there are two flags on the short vertical section of road that is at the western-most (left) portion of the route. The bottom flag, at the bottom of the vertical section, is in Smooth Rock Falls, which is the southern-most town on the winter road. The ice road actually begins at the northern flag, at the top of the short vertical section, which is at the Otter rapids hydroelectric dam. There is a scale in the lower right of the map that allows one to determine distances (the cities Ottawa, Montreal, and Quebec are also marked). This is a teachable point because many students will assume that highway speeds can be applied. In fact, the trip south was held up by two hours because of a mishap where a transport truck had its trailer slide into the soft snow at the side of a downhill section of the road. It is not unusual for the 150 km of ice road to take five hours (i.e., 30 km/h

average speed) because of the conditions of the road.

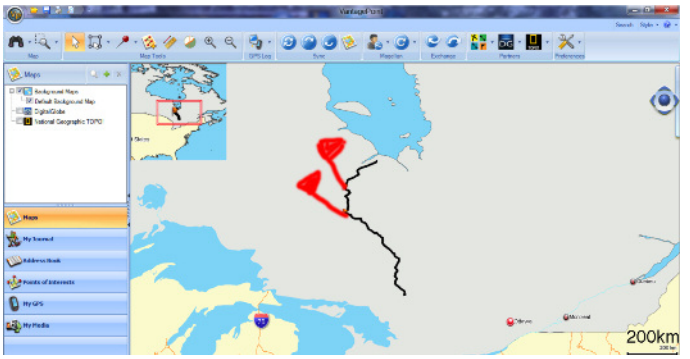


Figure 1: *Trip from Moose Factory to North Bay*

In terms of the map, there are a variety of questions that can be developed from this map or a similar map that suits your neighbourhood. What is the direction when the trip began in Moose Factory, which is in the Moose River close to James Bay? (Ans.: South) What direction, approximately, occurs during any two different stretches of the trip that a student may choose? Use the scale to determine the distance *as the crow flies* (i.e., straight line) from Moose Factory to North Bay? (Ans.: 545 km) What percentage of the actual distance travelled is the straight-line distance? (Ans.: Roughly one-quarter of the trip is close to the direct line joining Moose Factory to North Bay.)

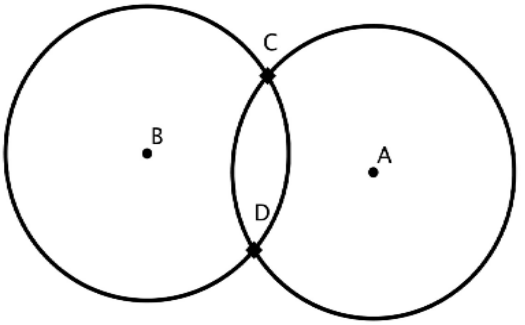


Figure 2: *Two GPS satellites (A, B) gice two possible locations (C, D)*

The route information appears two-dimensional, just like a road map, and provides for an interpretation of the information a person may want when travelling. It looks like a graph on flat paper that is similar to a regular road map. This provides a teachable starting point with respect to how GPS works. This is where the math teacher can start to build an explanation of how satellites are used to determine a person's position on the Earth. GPS is based on satellites providing range information. In effect, for teaching, it suffices to say that the signal from the satellite indicates the distance from the satellite. The distance from a single satellite defines a sphere in three dimensions. That is, if you take the satellite signal anywhere on a particular sphere centred on the satellite, it will tell you the distance is constant. When you introduce the Earth where the sphere of constant distance hits the Earth defines an approximate circle on the Earth's surface. If you imagine getting the range from one satellite while travelling on the circle, the satellite would say you are always the same distance from the satellite. When two satellites, A and B in Figure 2, are available, two circles showing constant range from each satellite leads to two possible ground locations, C or D, for the GPS device. A third satellite can resolve which point you are at.

To provide a tangible demonstration in elementary grades, consider having one student be a satellite who holds a skipping rope to show the GPS device's range (another student). The student representing the device will soon realize that he or she can hold the skipping rope taut anywhere on a circle around the satellite student. The demonstration can then introduce a second satellite as another student with another skipping rope. Each satellite student holds a skipping rope that shows the range of the person using the GPS device. Then ask the student who is representing the device to see how many places he or she can hold both skipping ropes taut.

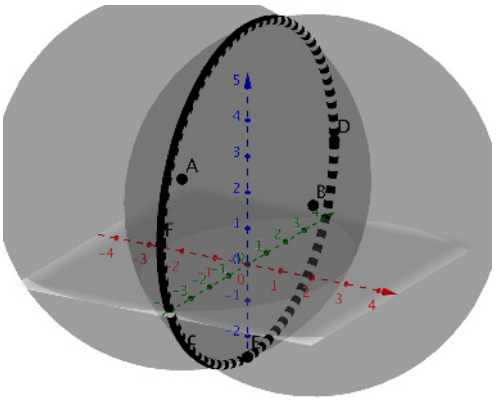


Figure 3: *Intersection of two spheres representing satelittle signals*

In three dimensions (appropriate to senior grades), two spheres, A and B in Figure 3, intersect in a circle, which can be shown using GeoGebra (a file is posted under "Resources" on the website **http://faculty.nipissingu.ca/timothys/**). If you imagine the travel route to be on a flat Earth, then a third satellite allows the location of the GPS to be determined exactly. However, as the Earth is not flat, and the route had elevation changes, and we live in three dimensions, a third satellite actually resolves to two points with different elevations, and it takes a fourth satellite to be certain of the GPS position.

A key idea here is that the third dimension is resolved by using four satellites. This means that GPS devices can determine altitude as well as latitude and longitude. The

altitude was available for the route and is shown in Figure 4. However, it is immediately clear that it is noisy and cannot be accurate—do you need convincing? Consider the lowest point, near the left side of the graph, is –50 m, which is 50 metres *below sea level*. The data listed on the right of the figure demonstrate other issues with the data, such as the unrealistic 414 km/h speed, and the time does not reflect stops that took place along the way.

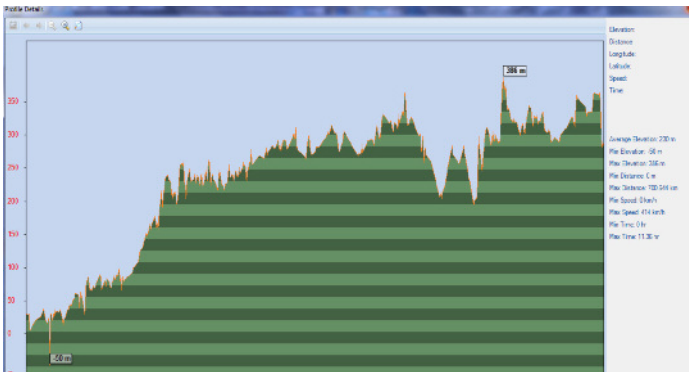


Figure 4: *Altitude above sea level for the trip*

To combat the noise, students can be asked to make suggestions about what one might do to reduce the impact of it. Students might suggest using the general shape of the graph and can be encouraged to explain, using math language, how they could determine the general shape. Ideas might be to only accept points that have a degree of consistency with other local points. The lowest point, being an error, suggests the consistency must be determined over a duration of time (that appears as a larger width on the diagram), and one might only accept intervals that do not have major jumps. An alternative that is a little more elaborate is the idea of only using the average of short intervals of points. This is a realistic strategy that is used for cancelling noise and has applications in stock market prediction. It might be proposed that using the median over short intervals might be even better. Students could investigate both approaches and are likely to find that using the mean is faster to compute than the median.

The idea of errors in the data is important. It is vitally crucial to the accuracy of the GPS system where information is coded so that the decoding only gives access to a certain degree of accuracy. If your device isn't very good, you might only get 300 m accuracy, but a better device with a stronger decoder might get you accuracy of 10 m or better. This means that you can potentially move some distance, without the GPS device recognizing you have moved. The phenomena can be modelled by reconsidering the accuracy of intersecting circles on a plane. To introduce the error, assume that the radii are only known within a range of values; that is, all points between two concentric circles are deemed to be the same distance from the centre of the circle. When there are two satellites, this leads to a region of intersection. For example, suppose two circles have radii that are between 10 cm and 11 cm, then how big of a circle is needed to completely enclose the region where the two circles overlap? This idea is shown in Figure 5, where two concentric circles are used to reflect the inaccuracy of the signals. One only knows they are between the two concentric circles. A second satellite indicates you are between two other concentric circles. The intersections of the four circles indicate that you are in one of two regions bounded by four arcs. If you know which of the two regions you are in, then you can find the four vertices that arise from the intersections of the two pairs of concentric circles.

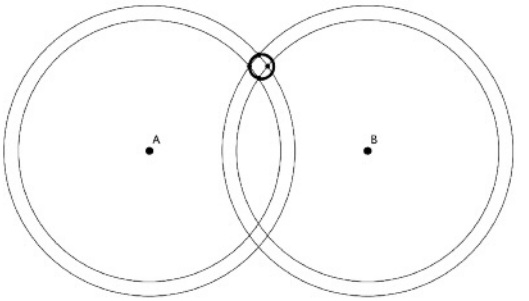


Figure 5: An approach to looking at the accuarcy issue

In Figure 5, the accuracy of the position is modelled using a circle that contains the region. It has been found using three of the vertices, and one vertex remains visible inside the region. This is a good construction task and can be used to investigate the impact of accuracy numerically. This can also be supported in a tactile manner by having a class of students map out an area in the schoolyard where the GPS coordinates are reportedly the same. This may not resemble the simplified geometry of Figure 5 because there can be four or more satellites involved. Certainly the shape should have a boundary that is approximately a polygon.

An algebraic examination of this problem is feasible, but requires the centre of one pair of concentric circles to not be the origin. It is, therefore, going beyond the Grade 10 academic curriculum and is not explained here because it is not clear that an analytic analysis of the errors is going to be particularly meaningful. The numerical approach shows that there is a relationship, and it is not clear that the application warrants the level of algebraic effort required if one wants to push it further. That is to say, there are better problems for formal study of errors. This problem, however, is quite strong for its practical implications and meaning for humans who simply want to know where they are.

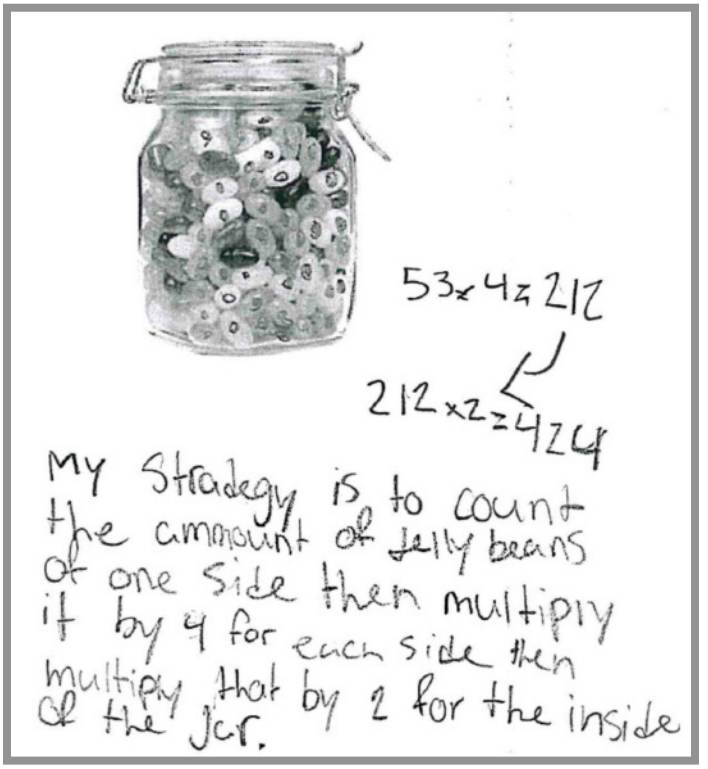
What is intriguing about using GPS in math classes is the range of opportunities and the capacity to tier the instruction

to suit many purposes. It can be used as a contextual reason justifying the initial investigation of circles in elementary grades or introducing three-dimensional geometry in senior grades. The data considerations are couched in daily life and again provide a realistic context. There is room for growing the ideas in this article by collecting GPS data from other trips and making connections to the environment in other ways. For example, a previous Gazette article (Sibbald, 2014) about the height of a hill could be augmented by using GPS data to profile the shape of a hill. The GPS profile of the hill could be another means of measuring the height of a hill, or it could be used in calculus to provide a practical context for talking about the slope of a function.

References

Sibbald, T.M. (2014). Consistency checking the math of a hill. *OAME Gazette, 53*(2), 27–29.

Unavco. (2016). *Tutorial: Selective Availability and the GPS Receiver.* Retrieved from **www.unavco.org/education/resources/tutorials-and-handouts/tutorials/ selective-availability-gps.html** ▲



▲ Hey, It's Elementary: Attention All OAME 2017 Delegates! You are Invited



LYNDA COLGAN

Lynda Colgan's career has included roles as a classroom teacher, a university professor, and newspaper columnist. Her contributions to mathematics and its teaching have been recognized through awards such as the Marshall McLuhan Foundation Distinguished Teacher Award. Lynda always exhibits a passion for mathematics and views her professional mission as dispelling the myth that math is the bad guy.

…to be my guest at the "Mathematics Midway" at *Science Rendezvous Kingston* on Saturday, May 13, 2017 at the Rogers K-ROCK Centre and The Tragically Hip Way!



As part of my responsibilities related to STEM community outreach at Queen's, for the past seven years, I have coordinated *Science Rendezvous Kingston*, which now, along with *Science Rendezvous* events across the country, is the flagship activity for the Natural Sciences and Engineering Research Council of Canada's (NSERC's) national Science Odyssey celebrations.

Founded by Dr. Dwayne Miller from the University of Toronto, *Science Rendezvous* (*SR*) is an annual public education "happening" that highlights and promotes science,

technology, engineering, and mathematics (STEM) research, researchers, and careers. Through interactive displays and first-hand experiences, *SR* seeks to broaden and deepen students' engagement with STEM; promote an appreciation for an interest in the pursuit of science in school and in daily life; and help learners to imagine what it might be like to do science in the world, either as a professional or a hobbyist.

Since its inception in 2011, *Science Rendezvous Kingston* (*SRK*) has grown to become the largest (counted) *Science Rendezvous* event in Canada, with 4220 visitors and 400 volunteers logged in 2016. In SRK's six years, there have been many dynamic and illuminative examples of science (biology, physics, chemistry), engineering (mining, mechanical, computer), technology (computer and broad-based), and mathematics (primarily applications of geometry and computations). Over the years, we have featured "The Giant Colon," "The Incredible World of Bugs," "The Canadian Raptor Conservancy," "A Quarantine Tent," and "A Coding Dojo." Families have had opportunities to attend a "Chemistry Magic Show," try their hand at laparoscopic surgery, extract their DNA, control nature in an augmented reality sandbox, operate Rube Goldberg Machines, meet Nobel Laureate Dr. Art McDonald, and be inspired by the experiences of a local Grade 12 student, Will Sanderson, who has been an environmental researcher in the Arctic and Antarctic.







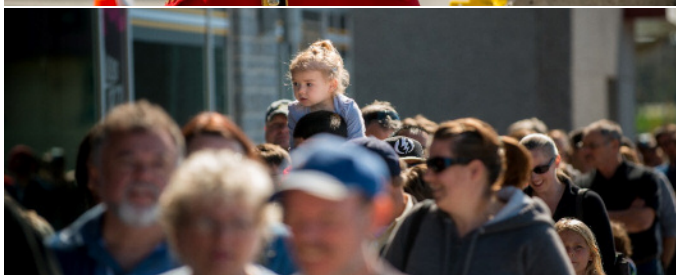






YGK's "pop-up" Science Discovery Centre at The Roger's K-ROCK Centre and out on The Tragically Hip Way (where we feature Solar Telescopes, races with the Queen's Baja Team's Dune Buggies, demonstrations by the Kingston Police Canine Unit) has become ***the*** place to be, annually, on the Saturday before Mother's Day. This year, our event coincides with the 2017 OAME Annual Conference, and we could not be more excited to invite you to join us for a mathxtravaganza.





Funded by NSERC PromoScience and NSERC Science Odyssey grants, in 2016, we launched “The Mathematics Midway” at SRK. Inspired by the wonderful book *A Day’s Adventure in Math Wonderland*, by Jin Akiyama and Mari-Jo Ruiz, and The Museum of Mathematics in New York, our “Mathematics Midway” featured square-wheeled tricycles, and large-scale tangram, pentomino, and Tower of Hanoi puzzles. Three teams of six first-year engineering students crafted three tricycles and our very own tri-colour inverted catenary roadway. Over the course of SRK and as part of promotional events at Kingston’s Market Square, in local schools, and in Ottawa at Constitution Square, the trikes were ridden by more than 1000 people. The pentomino, tangram, and Tower of Hanoi puzzles continue to be part of the Discovery Centre at Kingston’s own Pump House Steam Museum, where they continue to challenge and engage visitors of all ages.





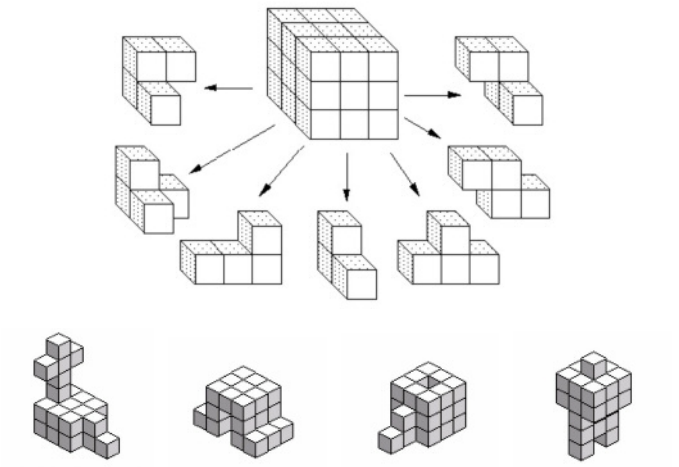




The great news is that “The Mathematics Midway” is expanding for 2017, just in time to be enjoyed by delegates attending the 2017 OAME Annual Conference, *And Now for Something Completely Different*, in Kingston, Ontario from May 11–13.

Did you know that just after your last session ends, you can take a short drive and meet world-renowned mathematician and sculptor George Hart, **and** be a part of history!

Prior to *SRK*, George will be coming to Kingston to work with local students in Grades 7 to 12, to help them to learn about the mathematics of Soma Cubes, the famous solid dissection puzzle invented by Piet Hein in 1936. As mathematics educators, you will know that just as with tangrams, for each Soma Cube, there are seven puzzle pieces. However, for the Soma Cube puzzle, the pieces are made out of unit cubes and are three-dimensional. Six of the pieces are made of four-unit cubes, and one contains just three. The challenge is to assemble all seven pieces into a 3×3×3 cube (there are 240 solutions) or any one of a large number of other 3D shapes (including a Sphinx, bridge, turtle, and robot).



As part of the day-long Soma Cube experience, George and the students will also construct Super Soma cube puzzle pieces so that at SRK, members of the public can experiment with trying to make the cube or other shapes on a large scale! In the case of the Super Soma puzzles, each "unit" is a cubic cardboard box (2' x 2'). Come and try your hand at finding a solution or just come to watch the fun.





In addition, George and a team of students will lead sessions with members of the public to assemble and then reassemble the reconstructable sculpture called "Amazing Acrobats." The sculpture is composed of 60 identical cartoon-like plastic "acrobats." Precisely designed lengths and angles ensure that the parts fit together exactly, while weaving in and out through each other in three dimensions.

As George says, "Overall, this is a unique interdisciplinary experience in which the participants find the precision of geometric thinking, the beauty of wooden sculpture, the engineering of architectural structure, and the fun of an intricate assembly puzzle."



Also in the afternoon, George will lead a group of invited distinguished guests to assemble an original sculpture to commemorate the 175th anniversary of Queen's University. The 1.5 m diameter sculpture (which will be designed to reflect the body of work by Nobel Laureate Dr. Art McDonald, who has done pioneering work about neutrinos) will come to life as the piece is assembled collaboratively for all to see. While the actual sculpture pieces and the ultimate design will not be unveiled until *SRK*, I have included another example below to pique your interest.

The piece below is called "Crystal Flowers in Halls of

Mirrors: Mathematics Meets Art and Architecture" and hangs at class at Aalto University in Helsinki, Finland. The piece is comprised of 4 mm plywood and zip ties. The plywood is stacked, screwed together, and run through a band saw to create the armatures; then holes are drilled for connection points. Once cut out, George and his co-sculptors built the first piece in 1.5 hours.



A second new mathematics feature at SRK's "Mathematics Midway" will be the large-scale kaleidoscopes and the dynamic, beautiful "art-meets-science" geometric patterns that they generate. Three large-scale kaleidoscopes will illuminate the connection between mathematics (e.g., angles, reflections) and geometric visual art (e.g., rotational symmetry). Because the mirror system is the crucial element of a kaleidoscope and the variation in the mirror shape, the number of mirrors used, and the angle at which the mirrors are joined, will produce a diverse range of kaleidoscopic images. The three interactive kaleidoscopes will have different configurations of these design factors, to invite experimentation and exploration by members of the public. The kaleidoscopes (three-mirror, two-mirror, twin-mirror, and four-mirror) will afford participants of all ages, backgrounds, and *mathitudes* to create and photograph original artwork by spinning the kaleidoscopes' canisters and object chambers. It is intended that the intricate mandala images generated will concretize the creativity connection between STEM and the Arts & Design.

There is no doubt in my mind that your trip to downtown Kingston will be worthwhile and fun, but I am closing with an offer that I hope you cannot refuse! You will go home with teaching resources! Teams of B.Ed. candidates are busily developing curriculum materials to elucidate the mathematical and optics principles that underlie the operation of the kaleidoscope, tying them to the Ontario curriculum. They will also work with a video-production team from GAPC (Ottawa) to create a professional 3-minute documentary-style video that reports on the history, science, and art and design of kaleidoscopes. You will receive both the resources and the video on a web key to take home… free!!!





I believe activities like puzzles, sculpture, and the art of symmetry are important "hooks" for many students, and I am supported by others. Recent research has suggested that the Arts are not only important subjects in their own right, but an important vehicle that can serve as an entry point to STEM for underrepresented students. By engaging students' strengths through diverse arts-based learning activities, researchers have seen increased motivation among diverse and disenchanted learners, and more opportunities for access to STEM success. It is noted that STEAM (integration of Arts & Design into STEM subjects) projects also provide rich and diverse opportunities for communication and expression. In these ways, the addition of the Soma Cubes, geometric sculptures, and kaleidoscopes at the "Mathematics Midway" at *Science Rendezvous Kingston* may promote positive mathematics dispositions; contribute to increased mathematical content knowledge; heighten awareness about real-world connections; and, in turn, improve engagement and achievement scores, and reduce drop-out rates.

Hope to see you there!

Reference

Ge, X., Ifenthaler, D., & Spector, J.M. (2015). Moving forward with STEAM education research. In X. Ge, D. Ifenthaler, & J.M. Spector (Eds.), *Emerging technologies for STEAM education: Full STEAM ahead* (pp. 383–395). New York, NY: Springer. ▲

Ask Assessment Abby: Learning Goals and Success Criteria

ASSESSMENT ABBY
EMAIL: assessmentabby@oame.on.ca

Ask Assessment Abby A³ is a regular column in the OAME *Gazette*, where teachers can share concerns and best practices about assessment, evaluation, and reporting of mathematics. Please send your questions to Ask Abby at **assessmentabby@oame.on.ca**.

Dear Assessment Abby,

I know that it is important to develop learning goals and to identify success criteria, but I am still unclear on how to do this in my math class. Can you please give me some suggestions?

Confused about the Goals

Yes, it certainly is important for teachers and students to have a common understanding of math learning goals and to establish clear criteria for what constitutes success.

Growing Success (Ontario Ministry of Education [OME], 2010) states that learning goals are based on curriculum expectations and are shared in student-friendly language at or near the beginning of a learning cycle. When stating goals, consider using phrasing like "We are learning to… [action words]" and including only one criterion (e.g., "add" or "subtract") so that achievement is easier to identify. So goals might include, for example:

- "We are learning to add polynomials with up to two variables."
- "We are learning to subtract polynomials with up to two variables."
- "We are learning to investigate properties of slopes of lines and line segments."
- "We are learning to identify properties of slopes of lines and line segments."

It is also important to discuss and clarify goals with students (e.g., defining terms, considering examples, making connections to what students already know and can do). Once goals are identified, they may be recorded (e.g., in student notes, displayed in the classroom).

Growing Success also states that the Achievement Chart for Mathematics should guide the development of success criteria. Again, use student-friendly language. Consider phrasing like "I can… [action words]," for example:

- "I can identify different types of quadrilaterals based on their attributes."
- "I can classify different types of quadrilaterals, using correct mathematical terminology."

Students should be involved in developing success criteria, including the processes of "identifying, clarifying, and applying those criteria in their learning" (OME, 2010, p. 33). The class may look at samples of student work, for example, and brainstorm success criteria for adding polynomials, perhaps by focusing on Knowledge and Understanding and/or Communication categories of the Achievement Chart. Also keep in mind that as students progress toward achieving learning goals, success criteria may change; the criteria should be open to "review and revision" (OME, 2010, p. 33).

For more about learning goals and success criteria, check out resources at the EduGAINS website (**www.edugains.ca/newsite/aer/aervideo/learninggoals.html**).

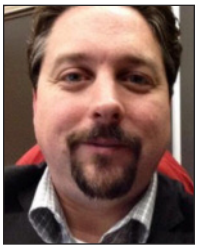
Keep math rich with students in mind,

Assessment Abby

If you have any questions or comments for Assessment Abby, email **assessmentabby@oame.on.ca**. ▲

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▲ MB4T (Mathematics by and for Teachers): Clothesline Math Adapting a Number-Line Model to Solve Equations



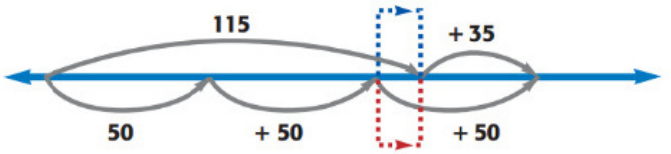
JULES BONIN-DUCHARME

Jules Bonin-Ducharme is a Grades 7–12 Provincial Math Facilitator for the CFORP (Le centre d'innovation pédagogique), supporting the Ontario Ministry of Education in its Renewed Mathematics Strategy.

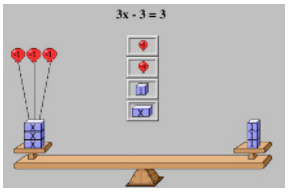
I first met Andrew Stadel (@mr_stadel) at his NCTM talk in Boston. He presented a double-clothesline model to solve equations. I was amazed at the possibilities this model presented. When I spoke to my elementary colleagues, I was surprised to see that this was not something new. They have been using an open double number line to develop algebraic reasoning, but I have never seen it adapted to solving complex equations.

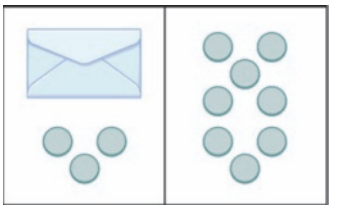
115 + 35 = 50 + 50 + 50

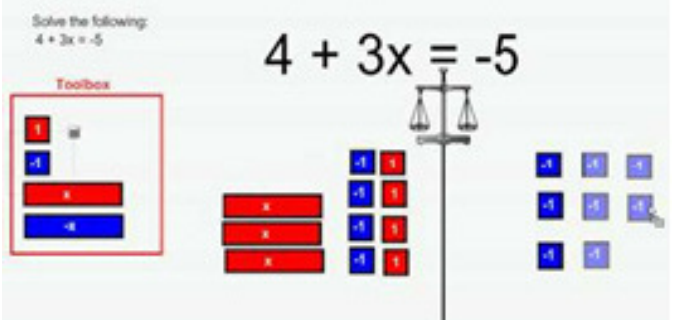
15 more than 50 + 50



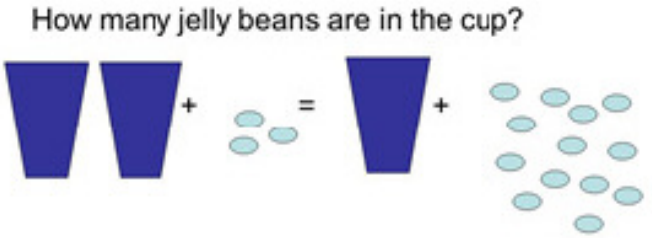
15 less than 50





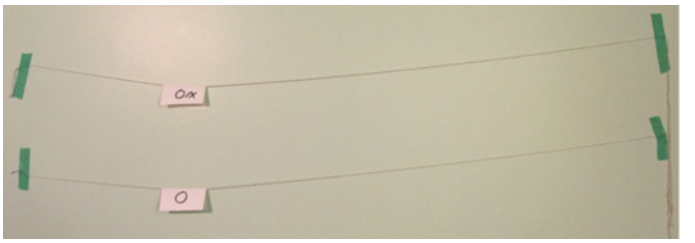


There are many models that explain how the algorithm for solving equations works: balance, cups and counters, algebra tiles, envelopes, and money. The all have a limit, especially working with negative numbers and fractions, and they are great at demonstrating the steps needed when algebraically solving equations using the common algorithm.

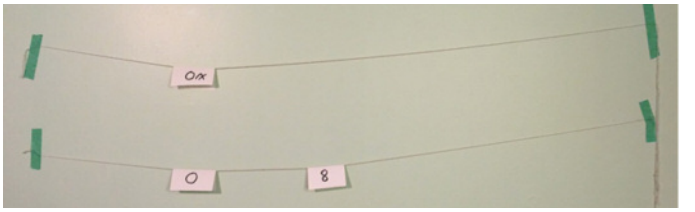


The clothesline model offers lots of reasoning that other models don't do well. It develops number sense, spatial reasoning, proportional reasoning, and algebraic reasoning to help the student visualize and understand the meaning of what solving an equation looks like and means, not just the steps to the algorithm.

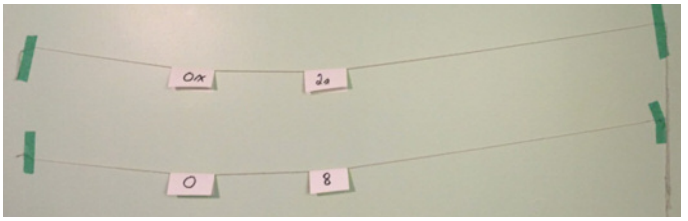
Let's start with the fundamentals of the clothesline. The first thing to know about a double clothesline is that a card is never set in stone. It is movable on the clothesline, making the number line dynamic. Also, any vertically stacked cards are a situation of equality. They must be moved together. On a number line, the 0 is THE reference point. When working with an open number line, it's important to find where 0 is. I will use the bottom line for integers and my top line for working out variables. So, if I were to place the 0x on the top line, it would have to be in line with the bottom line 0.



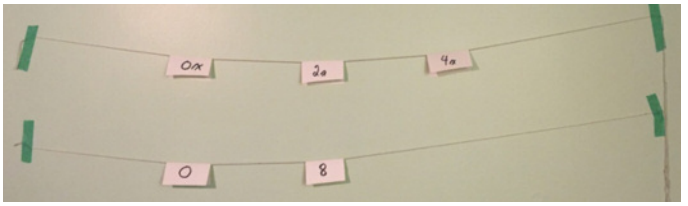
Let's explore some relationships with this double clothesline. Suppose I want to place the number 8. I must place it to the right of the 0, but I can choose anywhere to the right I want. I make my own spacing.

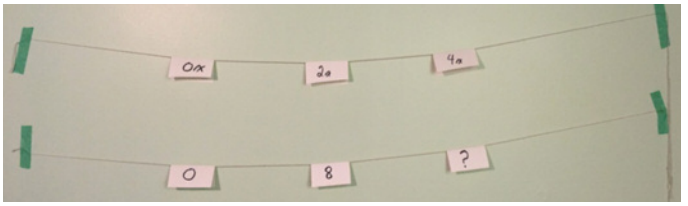


I'm going to create an equality situation by adding $2x$ to the top line. I now have $2x = 8$.

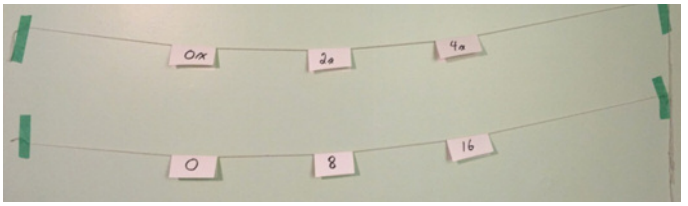


Let's play around with that relationship. Where would I put 4*x*? To place the 4*x*, I must understand that the spacing between 0*x* and 2*x* must remain the same as between 2*x* and 4*x*. I now know where to put my 4*x*.

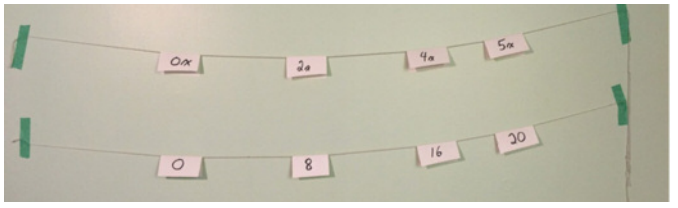


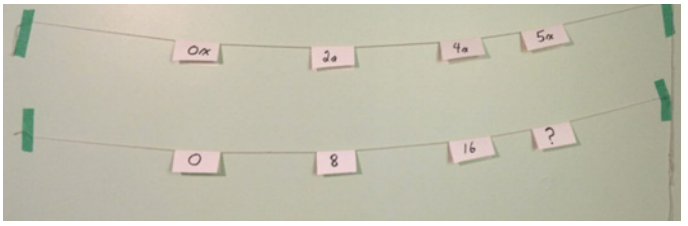


But what number does that correspond to on the bottom line? The spacing must be the same between 0 and 8 as 8 and the number we are trying to determine. That number must be another 8 units away: 16!



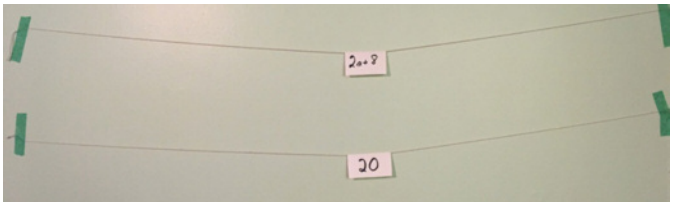
Now let's place 5*x*. It must be at half the distance of the space between 2*x* and 4*x* to the right. Can we find the number at 5*x*? The reasoning is the same: half the distance between 8 and 16 and to the right.



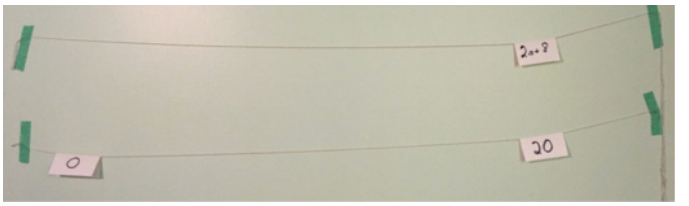


We now have four relationships of equality that can't be moved. If we were to move one situation, we would have to proportionally move all the other equalities.

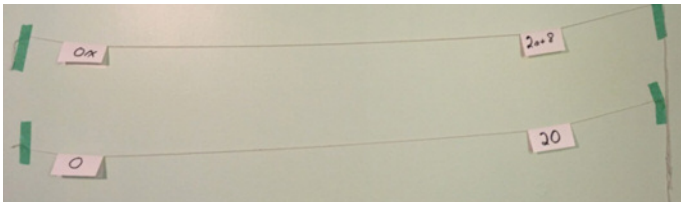
We are now ready to solve a two-step equation. Let's solve the equation $2x + 8 = 20$. We have our first situation of equality.



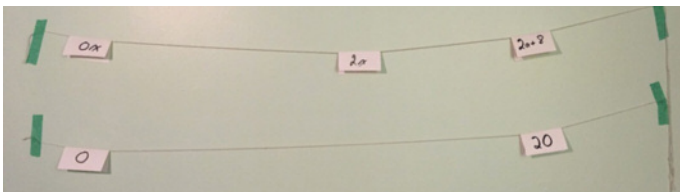
We know where to put our 0 on the bottom line. Since 20 in positive, the 0 must be to the left of it.

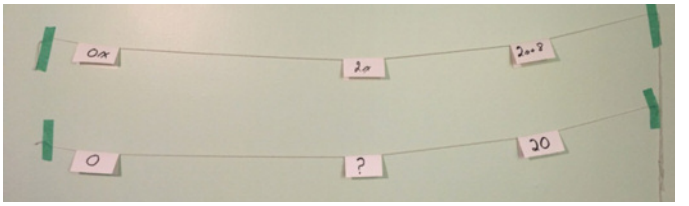


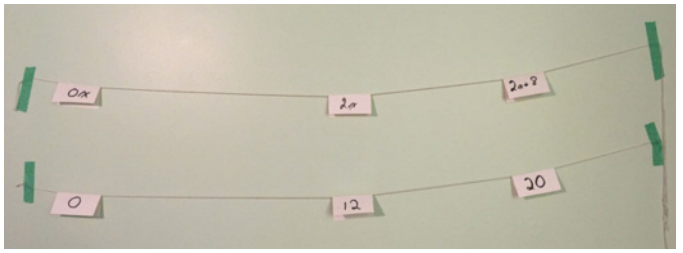
We can now place our 0*x*.



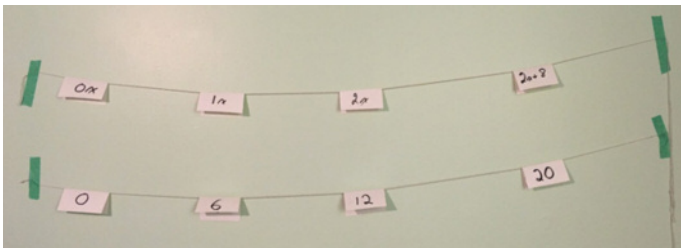
I now wonder where I could place my 2*x*. I would question the students as to whether I should place it to the right or the left of $2x + 8$. $2x$ must be to the left, since if we were to add 8 to $2x$, it would be to the right on the number line. We now know that $2x$ must be 8 units to the left of $2x + 8$. With some spatial and proportional reasoning, we can figure out exactly where that is, since we have already established a 0- to 20-space. However, what value does $2x$ have on the bottom line? We can figure that out by calculating 8 units to the left of 20.





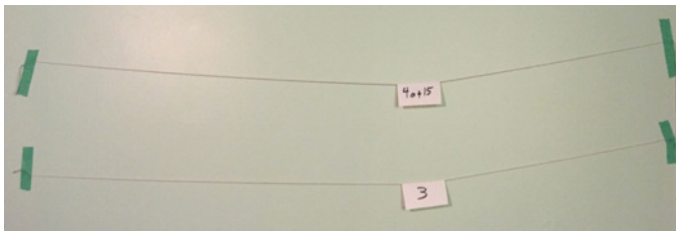


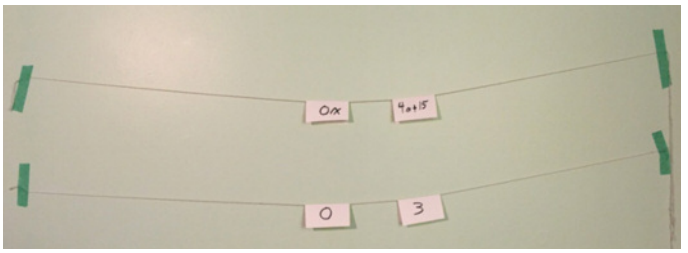
Now we can think about placing the 1x. Where would it fit in? Proportionally, it's halfway between 0x and 2x, and since we can figure out halfway between 0 and 12, we have our solution of 6.

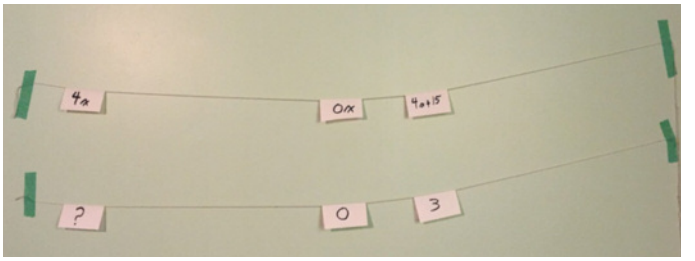


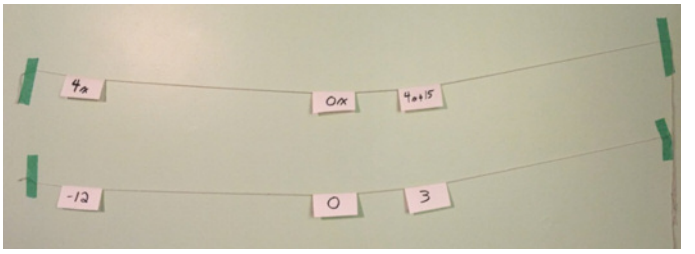
We can retell the story by listing all our equality situations.

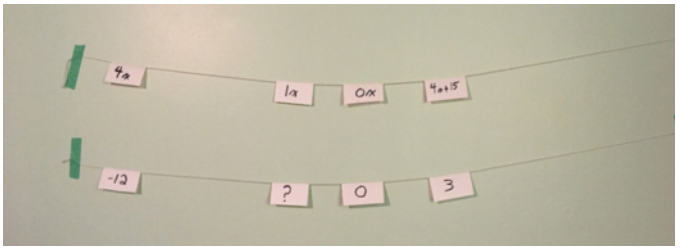
Let's see what happens when we decide to play with negative numbers now. Solve the equation $4x + 15 = 3$. We have to establish our 0, which must be left of the 3. Where do we place our $4x$? It must be 15 units to the left of $4x + 15$, so we are now on the negative side of the number line. Students can visualize the 15-unit space needed and figure out that we must be at -12 on the bottom line. We now have to establish where to place our $1x$. We know our spacing between $4x$ and $0x$. We can partition that space into 4 and place $1x$. We can therefore partition the -12- to 0-space into 4 and come up with -3.

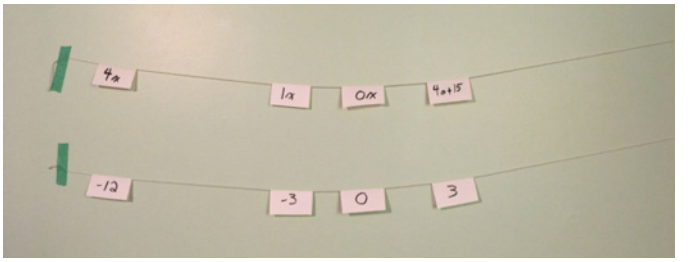












Another important factor with this model is that we have several situations to verify that -3 is the solution. We can verify it at every step of the story.

By visualizing, manipulating, and making sense of the solution, the student can generalize the solving process for himself or herself. It's also possible to work with equations that have variables on both sides, or even equations with fractions. My suggestion is to play around with the model. Test its limits. Visualize, Verbalize, and Verify with your students. How far can you take this model? Continue the discovery of this model at the following sites:

- **https://clotheslinemath.com/**
- **www.estimation180.com/clothesline.html**
- **https://mathwithmeaning.wordpress.com/tag/clothesline/**
- **http://tmerc.ca/m4yc/peterborough.html** ▲

The Importance of Increasing Spatial Reasoning Instruction in Kindergarten Classrooms ... but Within Reason!

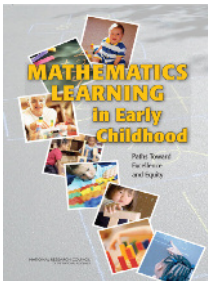


EDWARD SCHROETER
MILLBROOK-SOUTH CAVAN P. S.

Edward Schroeter, B. J., B. Ed., OCT, is a Reading Specialist and Kindergarten Specialist, with qualifications and experience in Special Education, French as a Second Language, and Teacher Librarianship. He was trained and worked as an newspaper reporter and documentary video producer before he began his teaching career. He has taught Grade 4, Core French, French Immersion Senior Kindergarten and Grade 3, Kindergarten Year 1 and 2, and been a Special Education Resource Teacher and Teacher Librarian for the Kawartha Pine Ridge District School Board or its predecessor board since 1992. His current plan is to learn more about teaching and learning early math.

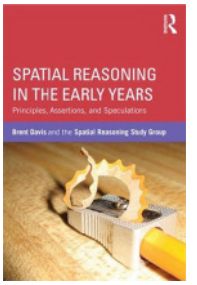
Is there a problem with spatial and geometric skills teaching in Kindergarten?

Researchers Cathy Bruce, Joan Moss, and John Ross in Canada (2012), and Douglas Clements and Julie Sarama in the United States (2011), report that geometry seems to receive the least amount of instructional time in North American elementary school classrooms compared to other strands of mathematics (Bruce, Moss, & Ross, 2012; Clements & Sarama, 2011).



This is not an ideal situation if the conclusions of the 2009 report by the U.S. National Research Council's (NRC) Committee on Early Childhood Math is any indication. *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity* provides a convincing argument for spending more instructional time in Kindergarten on geometric and spatial skills. The report recommends that mathematics experience and instruction for preschool and Kindergarten children aged 3 to 6 years of age concentrate on three strands of mathematics: Number Sense and Operations, Geometry and Spatial Skills, and Measurement. The primary instructional focus of these three strands should be Number Sense and Operations, while the secondary should be Geometry and Spatial Skills. Although measurement ideas in these grades are important, the NRC report advises that it is only necessary to spend a small amount of time on them as broad concepts. Meanwhile, patterning, probability, and data management can be woven into work in the first three strands, with only a small investment of class time (NRC, 2009; Fuson, Clements, & Beckmann, 2010).

What is spatial reasoning?



In 2006, the U.S. National Research Council defined spatial thinking, or spatial reasoning, as involving the location and movement of objects and ourselves, either mentally or physically, in space. It is not a single ability, skill, or process. It incorporates numerous concepts, processes, and tools to represent and communicate ideas about space and spatial relationships (NRC, 2006). In their 2015 publication *Spatial Reasoning in the Early Years*, researchers Yukari Okamoto, Donna Kotsopoulos, Lynn McGarvey, and David Hallowell identify four key components of spatial skills:

- **visualization and representation, i.e., maps, models, and coordinates** – being able to "see" the relationship among stationary objects in reality and/or in graphic representations; e.g., Kindergarten students should be able to build models of the classroom or playground with toys such as doll furniture and blocks; make simple maps of local areas, using felt shapes; draw maps of routes; represent the position of objects relative to landmarks; and locate objects after they are moved.
- **navigation, i.e., perspective taking and self-locomotion** – being able to see the changing relationships among moving objects; e.g., Kindergarten students should be able to use relational words of proximity, such as "beside," "next to," and "between"; begin to use the relational language of "right" and "left"; begin to create symmetrical figures; begin to plan and discuss different routes on maps and state which landmarks they would pass by on their route; and explain which would be "the best" route to take and why.
- **mental rotation and transformation of static and dynamic shapes and objects** – e.g., Kindergarten students should be able to create pattern-block designs with multiples of 60 degrees and 120 degree angles.
- **recognizing, composing, and decomposing**

INTERESTED IN EXPERIENCING MORE ABOUT HOW MATH SHAPES OUR WORLD?

Participate in 2017 International Mathematical Modeling Challenge (IM^{2}C)

The IM^{2}C is an international contest that challenges **high school** students to apply mathematical tools and concepts on **real-world problems** in biology, engineering, finance, medicine, etc.

Teachers can register a team of **up to 4** students from the same school. Each team works over **5 consecutive days** to solve a practical problem and submit solution online.

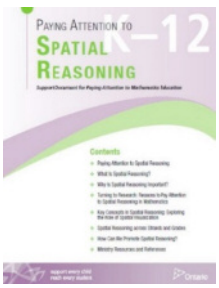
The next Canadian contest is open to **all public, catholic, and private high schools in Ontario during March 13 – April 26, 2017**.

im2c-canada.math.yorku.ca

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stationary geometric shapes and objects – e.g., Kindergarten students should be able to recognize and sort a wide variety of shapes by number of sides, length of sides, or corners, regardless of size or orientation; create compositions with pattern blocks, using a variety of shape sets; systematically complete puzzles, recognize and name common three-dimensional shapes; and build simple three-dimensional structures from pictorial models. (Okamoto et al., 2015)

The importance of spatial skills and geometry concepts



The 2014 Ontario Ministry of Education monograph, *Paying Attention to Spatial Reasoning*, makes a powerful argument that spatial skills are extremely important for successful mathematics performance across curriculum strands and for conceptual understanding of numeric quantities, arithmetic, word problems, measurement, geometry, algebra (patterning), and calculus. To support its argument, the document cites a 2013 longitudinal study of 3-year-olds, by Farmer and his co-authors, from which they conclude that spatial reasoning skills at age 3 are more important then early mathematics or vocabulary skills at predicting math performance at age 5 (Farmer et al., 2013, cited in Flynn & Hawes, 2014). Kelly S. Mix and Yi-Ling Cheng take the conclusion one step further and assert that "the relationship between spatial ability and mathematics is so well established that it no longer makes sense to ask whether they are related" (Mix & Cheng, 2012, p. 206).

Brian N. Verdine and three of his colleagues, Roberta Michnick Golinkoff, Kathryn Hirsh-Pasek, and Nora S. Newcombe, summarized a large body of research about the importance of spatial reasoning instruction for pre-Kindergarten and Kindergarten students in their 2014 article, "Finding the Missing Piece: Blocks, Puzzles, and Shapes Fuel School Readiness." The four authors conclude from their own research and their examination of 80 articles that more sophisticated spatial reasoning appears to improve the quality of children's early mathematics performance (Verdine et al., 2014). They write that spatial ability seems to be connected to understanding numeric quantities (i.e., subitizing) and to early numeracy performance (Verdane et al., 2014). Verdane and his co-authors theorize that children who develop better spatial representations of number earlier

than other children may be able to build on this knowledge to learn other numerical concepts, such as number-line estimation, magnitude comparison, numeral identification, counting, and place value (Verdane et al., 2014).

Spatial thinking also seems to support reasoning in areas which aren't obviously spatial. In their 2010 article, "Early Education for Spatial Intelligence: Why, What, and How," published in *Mind, Brain, and Education*, Nora S. Newcombe and her colleague, Andrea Frick, point out that, for example, Venn diagrams can be used to solve logical problems. Graphs, tables, and other data displays are tools for thinking that allow children to find patterns and make comparisons. Maps have the capacity to visually show the distribution of variables such as population density or natural and economic resources (Newcombe & Frick, 2010).

Why spend increased time on spatial skills in pre-Kindergarten and Kindergarten?

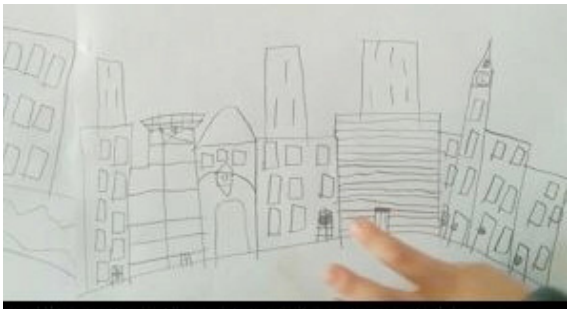
According to W.S. Barnett in his 2011 article for Science, "Effectiveness of Early Educational Intervention," numerous studies find that preschool intervention in spatial- skills instruction is surprisingly effective and is an optimum time to expand them (Barnett, 2011). Verdane and his three co-authors, mentioned earlier in this article, expand on this idea. They quote convincing evidence that spatial skills can be improved with practice, that these improvements endure over time, and that improvements on one type of spatial-reasoning task frequently transfer to other types of spatial tasks, even at relatively young ages. They also write that research suggests that there are straightforward, appealing, and effective play-based, inquiry learning activities, such as building toys, puzzles, and shape sorters, which clearly improve spatial skills in young children (Verdane et al., 2014). Another compelling reason to give priority to spatial-reasoning instruction in Kindergarten classrooms is that the school setting seems to be an ideal place to practise these skills. In their 1998 study, published in the August issue of *Child Development*, Janellen Huttenlocher, Susan Levine, and Jack Vevea show that children's spatial abilities grow during the school year, but stall during the summer months (Huttenlocher, Levine, & Vevea, 1998; cited in Newcombe & Frick, 2010). Finally, improving spatial skills has also been shown to overcome gender and socio-economic barriers to success in mathematics at school and to gainful employment in the labour force—especially certain well-paying careers such as those in science, technology, engineering, architecture and art, and mathematics—(STEAM) careers (Flynn & Hawes, 2014).

Ways to improve spatial reasoning



Magna Tiles® is a popular building toy among Year 1 and 2 kindergarten students.

The research literature indicates that the spatial-reasoning abilities of children tend to improve when they interact with adults during play with spatial toys such as blocks, puzzles, Duplo®, Magna Tiles®, pattern blocks, and attribute blocks. The key factor influencing spatial-skills development in this case are the spatial words, gestures, questions, and challenges that the toys and the play evoke from adults (Verdine et al., 2014). Temple University researcher Nora Newcombe adds that educators can support young children's spatial-reasoning skills by reading them "spatially challenging picture books." These are books that examine scenes from various angles or perspectives, include maps and spatial language, or whose illustrations require close attention to decipher their meaning (Newcombe & Frick, 2010). Some researchers have investigated the use of video programs, video games, and tablet and phone apps for the spatial training of young children. In their 2014 article, Verdine, Golinkoff, Hirsh-Pasek, and Newcombe urge caution about the use of video games, programs, and apps for children aged 2 to 6 because of the importance of adult interaction, which is what precipitates the development of spatial language and gestures (Verdine et al., 2014).



"This building is shaped like a cylinder. You can't see the bottom floor that is sticking out because I am on the sidewalk. To see the bottom floor that is sticking out, you would have to be above the building in an airplane or helicopter looking down, or be a bird." – *SK Student*

What spatial skills deserve the attention of educators?

In the classroom, there are two places where preschool, Kindergarten, and early-grade educator interventions make quantifiable improvements. The first domain is maps and



"My house is close to the school." – *SK student*



This is a Star Wars ship. It's made out of triangles. – *SK student*

models. According to Yukari Okamoto, Donna Kotsopoulos, Lynn McGarvey, and David Hallowell, in their 2015 work published in *Spatial Reasoning in the Early Years: Principles, Assertions, and Speculations*, research shows that intentional educator interactions have a moderately high ability to improve this spatial skill. The second area that warrants attention from educators is the mental transformation of objects, such as rotating, folding, cross-sectioning, and deforming them. Additional instruction in these spatial skills have been demonstrated to make moderate gains in them (Okamoto et al., 2015).



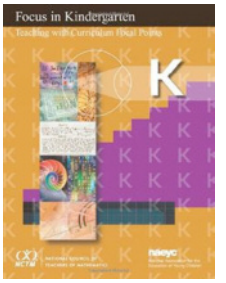
"Mr. Schroeter! I zipped up my own backpack. I rotated my lunch bag like you said and put my message bag at the back." – *SK student*

There are two other areas of spatial-skill instruction, which Okamoto and his colleagues are confident can be improved with training, but which need further study to prove that they can be improved by teachers. Researchers have not yet been able to verify the effect of teacher intervention on these skills. One of them is navigation and perspective taking. This spatial skill includes perspective taking, self-locomotion, spatial language, and gesturing, as well as direction, distance, and angle in 2D surfaces and 3D environments. The second is training children in 2D shapes and 3D objects (Okamoto et al., 2015).

What does a research-based Kindergarten mathematics curriculum look like?



The Ontario Ministry of Education (OME) recognizes the importance of providing spatial and geometric education in the early years. In July 2016, it published its new Kindergarten document, *The Kindergarten Program 2016*. Aligned with current research on spatial reasoning in the early years, it states on page 75 that "spatial thinking skills and geometric reasoning play a critical role in the development of problem-solving skills, mathematical learning and reading comprehension" (OME, 2016, p. 75). During the course of its 32 pages, the program sets out Ontario's two-year math curriculum for Year 1 and Year 2 Kindergarten students. It lists 28 specific curriculum expectations, five of which are spatial and/or geometric in nature. It also lists eight age-appropriate conceptual understandings for spatial and geometric thinking (OME, 2016). Like many provincial curricula, its shortcoming is that it fails to explain in what measure and to what extent these mathematics outcomes should be taught. As every educator knows, not every curriculum expectation has been created equal. However, educators can turn for advice to the U.S. National Council of Teachers of Mathematics (NCTM) and the U.S. National Association for the Education of Young Children (NAEYC). In 2010, these two organizations co-published a series of slim, but well researched, informative, and easy-to-read handbooks for early years' educators, which distill and transform the 398 pages of *Mathematics Learning in Early Childhood* into age-appropriate pre-Kindergarten and Kindergarten mathematics programs.



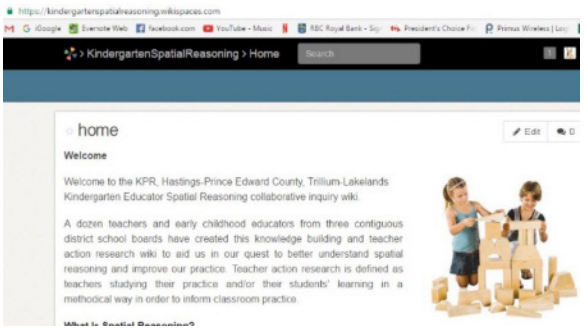
Focus in Kindergarten: Teaching with Curriculum Focal Points and *Focus in Pre-K: Teaching with Curriculum Focal Points* were co-written by three experts on early mathematics learning—Karen Fuson, Douglas H. Clements, and Sybilla Beckmann. In their two 79-page, four-chapter books, the co-authors set out, explain, and illustrate evidence-based, age-appropriate mathematics programs. They elaborate improvements to spatial- and geometric-thinking instruction in the broader context of an entire Kindergarten math program so as not to inadvertently overemphasize one strand of instruction at the expense of another important one. For example, the authors of *Focus in Kindergarten* spend 36 pages on Number and Operations (chapter two), 12 pages on Geometry and Spatial Reasoning (chapter three), and six pages on

Measurement (chapter three) (Fuson et. al, 2010). Using a crude application of content analysis, a research method for systematically analyzing written, spoken, or visual communication with a quantitative description, the number of pages devoted to each math strand in the Fuson-Clements-Beckmann curriculum breaks down as follows:

- 66% Number Sense and Operations
- 22% Geometry and Spatial Reasoning
- 12% Measurement
- Patterning, Data Management, and Probability integrated into the first three topics (Colorado State University, 1993–2017)

The point of the above exercise is not to provide an exact formula for teaching mathematics in Kindergarten, but to remind educators that not all curriculum expectations are equal, and that emphasizing the need for improving one area of instruction, e.g., Spatial and Geometric Reasoning, does not mean abandoning another. In other words, we should not emphasize spatial and geometric thinking instruction at the expense of Number Sense, despite the importance of spatial reasoning.

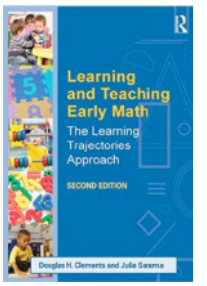
Evidence-based information from our classrooms to yours



A group of a dozen Kindergarten teacher and early childhood educator colleagues and I, from three contiguous district school boards, have joined forces and taken up the challenge to help our young students improve their spatial and geometric thinking—but within a balanced numeracy program. We have created an open, public knowledge-building, and classroom-action research wiki at **https://kindergartenspatial reasoning.wikispaces.com/** as a first step. We hope to receive a Teacher Learning and Leadership Program (TLLP) project grant from the OME that will enable us to conduct and disseminate evidence-based classroom research in 2017–2018. In the meantime, we are identifying, reviewing and posting useful professional books and articles on our wiki, as well as effective spatial picture books, toys, and activities that promote spatial thinking and reasoning and that can be used as provocations to inquiry learning in the discipline of mathematics.

Some recommended professional resources are:

- *Learning and Teaching Early Math* by Douglas H. Clements and Julie Sarama – If you need to know about the developmental progression of spatial



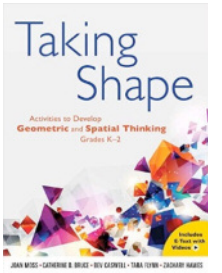
reasoning in young children from ages 1 to 8 or are looking for instructional activities to help them make gains in specific skills, you can turn to the 2014 second edition of this book. This 380-page, black-and-white book in small font provides a detailed and comprehensive look at all aspects of early mathematics learning, and includes five chapters on Spatial and Geometric Reasoning. At the end of each chapter is a developmental continuum that describes the typical skills found among children aged 1 to 8 years. It also provides up to 12 instructional tasks known to develop these skills. The tasks are fully explained, and black-and-white photographs or diagrams are often included. The book is designed specifically for teachers and teachers-in-training (Clements & Sarama, 2014).

- *First Steps in Math: Space – Developing Geometric Understandings* by the Department of Education of



Western Australia – This is another comprehensive source of information about spatial reasoning developmental milestones and instructional strategies for elementary school students in Grades K–8. Published in 2013 under the direction of team leader Sue Willis of Murdoch University, the two-colour, 266-page book is organized into chapters that outline detailed learning outcomes and sample learning activities. It is illustrated with ample diagrams, charts, tables, subheadings, and lists, and is available as a free download through the Creative Commons (Department of Education of Western Australia, 2013).

- *Taking Shape: Activities to Develop Geometric and Spatial Thinking* by Joan Moss, Catherine D. Bruce,



Bev Caswell, Tara Flynn and Zachary Hawes – The focus of this research-based resource is spatial and geometric instruction in K–2. It provides a series of five to seven lesson plans that support five areas of spatial instruction, background information, instructional techniques, and online digital resources and videos. The 2016 publication is organized much like a traditional

teacher's manual, but with a modern typographic style. The 276-page, wide-format book is produced with large print, vivid colour photographs, insets, text boxes, and diagrams (Moss et al., 2016).

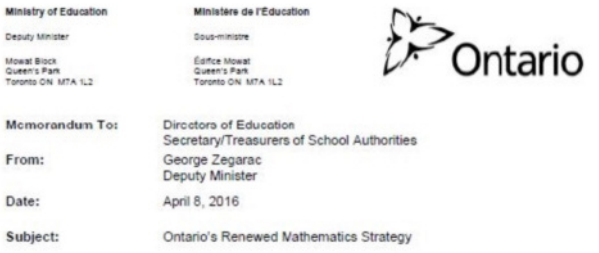
Some recommended picture books are:

- *Elephants Aloft* by Kathi Appelt. Illustrated by Keith Baker. (Ages 4–7) – Rama and Raji, two Indian elephants, travel in a hot-air balloon between mountains, behind waterfalls, and across the seas. The author tells the journey of the elephants with positional language and minimal text, which will no doubt prompt a variety of discussions about spatial themes.
- *Big Bug* by Henry Cole. (Ages 2–6). This picture book illustrates that size is relative and is a matter of perspective. The opening two pages feature a ladybug so large, it doesn't quite fit on them, but the next page zooms out to show the ladybug is now small in relation to the leaf on which it sits. The book continues in this vein until the reader arrives at an aerial view of a farm, at which time, the comparisons reverse. The minimal text invites children to discuss what they are seeing in each scene.
- *Lucy in the City: A Story About Developing Spatial Skills* by Julie Dillemuth (Ages 4–8) – A lost raccoon named Lucy is trying to find her way home with the help of a kindly owl. Readers follow Lucy's journey looking down on the city from the owl's perspective while the owl gives directions. On the journey, the story introduces three fundamental mapping and navigation strategies: interpreting a bird's-eye-view map, paying attention to one's surroundings, and retracing one's steps.
- *The Shape of Things* by Dayle Ann Dodds. Illustrated by Julie Lacome. (Ages 3–5) – This introduction to shapes shows children visually and in verse how circles, squares, and triangles are parts of objects that they see every day. The creative team illustrates shapes and objects such as houses and kites with paper-collage art.
- *Henry's Map* by David Elliot. (Ages 4–8) – Henry, an extremely organized pig, makes a map of the farm to ensure everything is in the right place. The story introduces readers to the concept of a map as a tool to show where objects are located, and also makes the idea of drawing maps.
- *Shrinking Mouse* by Pat Hutchins. (Ages 2–6) – Fox, Rabbit, Squirrel, and Mouse notice that their friend Owl gets smaller and smaller as he flies away from them, and they logically and comically conclude that he is shrinking. One by one, the animals set off in pursuit, and the same thing happens each time, but they grow bigger when they return.
- *Rosie's Walk* by Pat Hutchins. (Ages 2–5) – This 50-year-old classic story follows a hen named Rosie and a fox stalking her around a barnyard. The text uses directional words to describe Rosie's journey as she walks around the pond, over the haystack, and past the mill, completely oblivious to the fox who's pursuing her, and the humorous setbacks he faces on each page.
- *Actual Size* by Steve Jenkins. (Ages 4–8) – In this book, Steven Jenkins uses his trademark cut-paper animals and a single line of text to explore the concept of scale. Everything appears at actual size, so while petite critters fit comfortably on the page, larger ones appear as evocatively cropped bits and pieces: a gorilla's massive hand; a Siberian tiger's snarling face; and the unnerving, basketball-sized eye of a giant squid.
- *Mouse Shapes* by Ellen Stoll Walsh. (Ages 4–6) – Three little mice running from a cat hide among a heap of shapes. Temporarily distracted, they make an assortment of pictures from the shapes. For example, they put a triangle on top of a square to create a perfect house. Eventually, the mice hatch a clever plan to use the shapes to scare the cat away.
- *When a Line Bends… a Shape Begins* by Rhonda Gowler Greene. Illustrated by James Kaczman. (Ages 4–7) – Ten shapes are presented in picture and verse on a double-page spread packed with visual examples. There are even some extras for observant children to discover.

Increased professional support for Kindergarten educators

It's wonderful that educators are surrounded by this wealth of information. Despite its availability, preschool and Kindergarten educators are going to need specific professional development and classroom support to deliver improved spatial- skills instruction. Verdine, Golinkoff, Hirsh-Pasek, and Newcombe, often quoted in this article, have highlighted the importance of educator professional development in this area. They wrote: "Improving access to a spatial education is as much about how we teach these skills as it is about how much we teach them" (Verdine et al., 2014, p. 12). This same quadrumvirate of researchers point out that educators who lack exceptional spatial skills are likely to have greater anxiety about teaching the topic, and research has shown that spatial anxiety among educators leads to lower spatial-skill scores in students (Verdine et al., 2014). Nobody wants the mathematics education of our

young students to be a casualty of teacher shortcomings.



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Ontario

Memorandum To: Directors of Education
Secretary/Treasurers of School Authorities

From: George Zegarac
Deputy Minister

Date: April 8, 2016

Subject: Ontario's Renewed Mathematics Strategy

Help may be on its way—at least for some Ontario educators. On April 8, 2016, the Deputy Minister of Education for Ontario, George Zegarac, announced Ontario's Early Years to Grade 12 Renewed Mathematics Strategy in a memorandum to directors of education. The announcement promises $60 million in dedicated funding to focus on improving mathematics achievement. The memorandum even promises help for Kindergarten educators. It states: "Release time would also provide opportunities for teachers and Early Childhood Educators (ECEs) to work as a team to plan, implement, monitor, and assess mathematics learning in our Kindergarten classrooms" (Zegarac, 2017, p. 7).

As the old adage goes, the proof of the pudding is in the eating. Kindergarten educators in Ontario are curious to know whether this new strategy will be flexible enough to accommodate the needs of early years' educators and students. In the meantime, educators and parents from across the country can join the quest to better understand the importance of, and how best to teach, spatial and geometric reasoning to Kindergarten students by adding their own ideas, questions, thoughts, resources, and research to our wiki at **https://kindergartenspatialreasoning.wikispaces.com/**.

References

Anderson, S.E., & Jaafar, S.B. (2003). *Policy trends in Ontario education: 1990–2003*. The Ontario Institute for Studies in Education of the University of Toronto. Retrieved from **http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.560.2360& rep=rep1&type=pdf**

Ansari, D. (2013). Disorders of the 'mathematical brain': Developmental dyscalculia and mathematics anxiety. Keynote address. Ontario Association for Mathematics Education. Seneca College, Toronto, ON.

Audet, W., Barnes, M., Clegg, M., Heggie, L., Jamieson, D.G., Klinger, D.A., Levine, M.E., Martinussen, R., & Wade-Woolley, L. (2009). *Evaluation report: The impact of the Literacy and Numeracy Secretariat: Changes in Ontario's education system.* Canadian Language and Literacy and Resource Network. Retrieved from **www.edu.gov.on.ca/eng/document/reports/OME_Report09_EN.pdf**

Barnett, W.S. (2011). Effectiveness of early educational intervention. *Science, 333*, 975–978. Retrieved from **http://2015.laschool4education.com/wp-content/uploads/2014/10/effectiveness-of-early-educational-intervention.pdf**

Boyer, T.W., Levine, S.C., & Huttenlocher, J. (2008, September). Development of proportional reasoning: Where young children go wrong. *Developmental Psychology, 44*(5), 1478–1490; doi:10.1037/a0013110

Bruce, C., Moss, J., & Ross, J. (2012). *Survey of JK to grade 2 teachers in Ontario, Canada: Report to the Literacy and Numeracy Secretariat of the Ministry of Education.* Toronto, ON: Queen's Printer for Ontario.

Clements, D.H., & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach* (2nd ed.). New York, NY: Routledge.

Coe, R. (2002, September). *It's the effect size, stupid: What effect size is and why it is important.* Paper presented at the British Educational Research Association annual conference, Exeter, UK. **www.cem.org/attachments/ebe/ESguide.pdf**

Colorado State University. (2017). *Writing@CSU guide: Content analysis.* Fort Collins, CO: Author. **https://writing.colostate.edu/guides/guide.cfm?guideid=61**

Department of Education of Western Australia. (2013). First steps in mathematics: Space: Developing geometric understandings. Perth, Western Australia: Author. **http://det.wa.edu.au/stepsresources/detcms/navigation/first-steps-mathematics/**

Farmer, G., Verdine, B.N., Lucca, K., Davies, T., Dempsey, R., Hirsh-Pasek, K., & Golinkoff, R.M. (2013, April). *Putting the pieces together: Spatial skills at age 3 predict to spatial and math performance at age 5.* Presented at the Society for Research in Child Development, Seattle, WA.

Flynn, T., & Hawes, Z. (2014). *Paying attention to spatial reasoning K–12: Support document for paying attention to mathematics education.* Retrieved from **www.edu.gov.on.ca/eng/literacynumeracy/LNSPayingAttention.pdf**

Fuson, K., Clements, D.H., & Beckmann Kazez, S. (2010). *Focus in kindergarten: Teaching with curriculum focal points*. Reston, VA: National Council of Teachers of Mathematics (NCTM) and the National Association for the Education of Young Children (NAEYC).

Fuson, K., Clements, D.H., & Beckmann Kazez, S., (2010). *Focus in pre-K: Teaching with curriculum focal points*. Reston, VA: National Council of Teachers of Mathematics (NCTM) and the National Association for the Education of Young Children (NAEYC).

Huttenlocher, J., Levine, S., & Vevea, J. (1998). Environmental input and cognitive growth: A study using time-period comparisons. *Child Development, 69*(4), 1012–1029. Retrieved from **www.researchgate.net/publication/13516642_Environmental_Input_and_Cognitive_Growth_A_Study_Using_Time-Period_Comparisons**

Mix, K.S., & Cheng, Y-L. (2012). The relation between space and math: Developmental and educational implications. In J.B. Benson (Ed.), *Advances in Child Development and Behavior, 42*, 197–243.

Mix, K.S., Prather, R.W., Smith, L.B., & Stockton, J.D. (2013, December). Young children's interpretation of multidigit number names: From emerging competence to mastery. *Child Development, 85*(3), 1306–1319. doi:10.1111/cdev.12197

Moss, J., Bruce, C.D., Caswell, B., Flynn, T., & Hawes, Z. (2016). *Taking shape: Activities to develop geometric and spatial thinking, grades K–2.* Toronto, ON: Pearson.

National Research Council. (2006). *Learning to think spatially: GIS as a support system in the K–12 curriculum.* Washington, DC: The National Academies Press. doi: 10.17226/12519

National Research Council. (2009). *Mathematics learning in early childhood: Paths toward excellence and equity.* C.T. Cross, T.A. Woods, & H. Schweingruber (Eds.). Washington, DC: National Academies Press. doi: 10.17226/11019

Newcombe, N.S., & Frick, A. (2010, August). Early education for spatial intelligence: Why, what, and how. *Mind, Brain, and Education 4*(3), 102–111. doi: 10.1111/j.1751-228X.2010.01089.x

Okamoto, Y., Kotsopoulos, D., McGarvey, L., & Hallowell, D. (2015). The development of spatial reasoning in young children. In B. Davis and The Spatial Reasoning Study Group, *Spatial reasoning in the early years: Principles, assertions, and speculations*. Toronto: Routledge Press, pp. 15–28. Retrieved from **www.spatialresearch.org/sites/default/files/Research/Okamoto-et-al2015.pdf**

Ontario Ministry of Education. (2005). The Ontario curriculum, grades 1–8: mathematics. Retrieved from HYPERLINK **www.edu.gov.on.ca/eng/curriculum/elementary/math18curr.pdf**

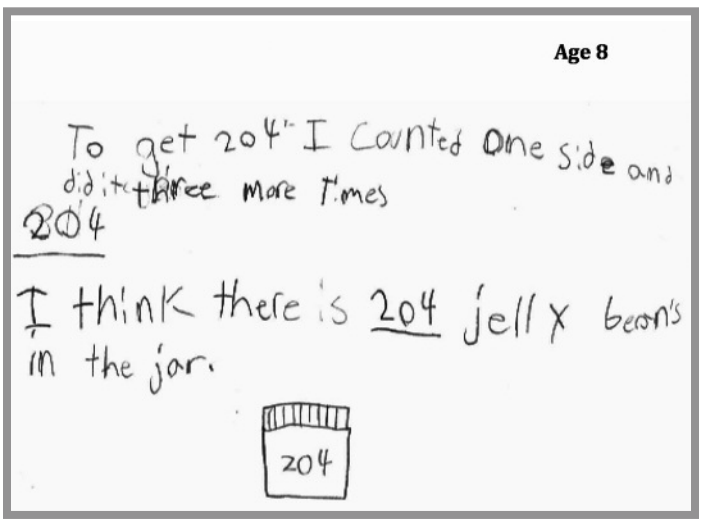
Ontario Ministry of Education. (2016). *The kindergarten program 2016.* Retrieved from **https://files.ontario.ca/books/kindergarten-program-en.pdf**

Roseberry, S., Hirsh-Pasek, K., Golink, R.M. (2013, September). Skype me! Socially contingent interactions help toddlers learn language. *Child Development, 85*(3), 956–970. doi:10.1111/cdev.12166

Verdine, B.N., Golinkoff, R.M., Hirsh-Pasek, K., & Newcombe, N.S. (2014, March). Finding the missing piece: Blocks, puzzles, and shapes fuel school readiness. *Trends in Neuroscience and Education 3*(1), 7–13. Retrieved from **www.elsevier.com/locate/tine**

Verdine, B.N., Golinkoff, R.M., Hirsh-Pasek, K., Newcombe, N.S., Filipowicz, A.T., & Chang, A. (2013, September). Deconstructing building blocks: Preschoolers' spatial assembly performance relates to early mathematics skills. *Child Development, 85*(3), 1062–1076. doi:10.1111/cdev.12165

Zegarac, G., Deputy Minister, Ontario Ministry of Education. (2016, April). Memorandum to Directors of Education: Ontario's Renewed Mathematics Strategy. Toronto, ON: Author. Retrieved from **www.edu.gov.on.ca/eng/policyfunding/memos/april2016/dm_math_strategy.pdf** ▲



▲ Global Math Project – Uplifting Math for All

SUNIL SINGH



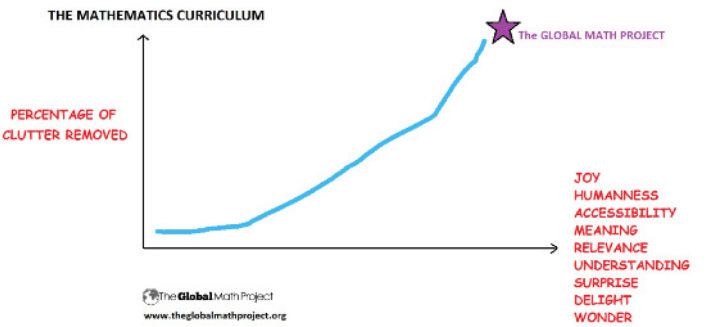
A self-proclaimed Mathematical Jester, Sunil is transforming the way mathematics is revealed with his video blog work at Scolab and his traveling math show for K-12 schools across North America. He was a math and physics teacher for 20 years, including teaching at an International IB school in Switzerland. It was there that he took a math team to the British School of Paris for a European Middle School Math Competition in 2006. In the last decade he has given over 50 workshops on creative mathematics all over Ontario–from schools to faculties of education to even the Royal Conservatory of Music in Toronto! He is a regular contributor to The New York Times *Numberplay Blog and has written a book called* Pi of Life: The Hidden Happiness of Mathematics *(July 2017).*

GLOBAL MATH PROJECT

Uplifting Mathematics for All

It has been six years since a calendar year has been a prime number. While almost all of these occurrences of mathematics' "irregular heartbeat" come and go without much notice or fanfare, it is quite different this time.

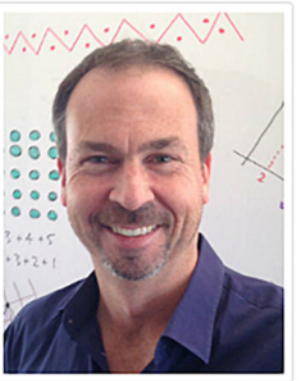
2017 will mark the inaugural year for the project that Dr. James Tanton, one of the founders of the project, calls "audacious and ambitious." When the goal of your endeavour is to galvanize all parts of the world around one unifying—and dazzling—piece mathematics, and have one million people sign up to do this, then perhaps it is fitting that this project have the loftiest aspirations.



Webinar

"Global Math Project - Episode 1: Join the Ride!"

by Samuel St-Amour view profile page



James Tanton

Believing that mathematics really is accessible to all, James Tanton (PhD, Mathematics, Princeton 1994) is committed to sharing the delight and the beauty of the subject. James currently serves as the Mathematician-at-Large for the Mathematical Association of America. He is in demand all over the world for his teacher and student workshops.



Sunil Singh

Over his 20 years of classroom experience and a decade of leadership, Sunil has acquired a deep amount of knowledge, particularly around the four distinct curricula: Common Core, Canadian, Cambridge IGCSE and the International Baccalaureate. He contributes on a regular basis to The New York Times Numberplay Blog. He is a Global Math Project Ambassador and has a book called "Pi of Life: The Hidden Happiness of Mathematics" coming out in Fall 2017.

Event agenda

OVERVIEW

Join us in this first webinar for an introduction to the **unique and amazing Global Math Project**! Your hosts James and Sunil will discuss about the following topics:

- Introduction to the "GMP"
- Where's the idea coming from?
- What's the goal?
- How will it evolve through the year?
- What should people expect?

* First Name * Last Name

* Email Address

A collaboration of GLOBAL MATH PROJECT buzzMath

By clicking this button you submit your information to the webinar organizer, who will use it to communicate with you regarding this event and their other services

*If you have already registered and can't locate your "registration confirmation email" click here

Register now

With just months away from the official opening of the registration page for The Global Math Project, the current participation of Ambassadors from all around the world is truly *uplifting*! There are currently over 120 Ambassadors from over 40 countries, each one promising to share the excitement of the project in his or her part of the world. That's it. No other requirement, except to foster contagious enthusiasm for mathematics and spread the simple joy and happiness of doing it.

And guess what? Ontario has the highest regional representation of Ambassadors with nine... and counting! Through a combination of miraculous timing and shared mathematical philosophies, I was part of a small group of five that were the first Ambassadors. Seeing the project grow from its initial stages to a truly global movement has been a wonderfully affirming experience in the power of mathematics.

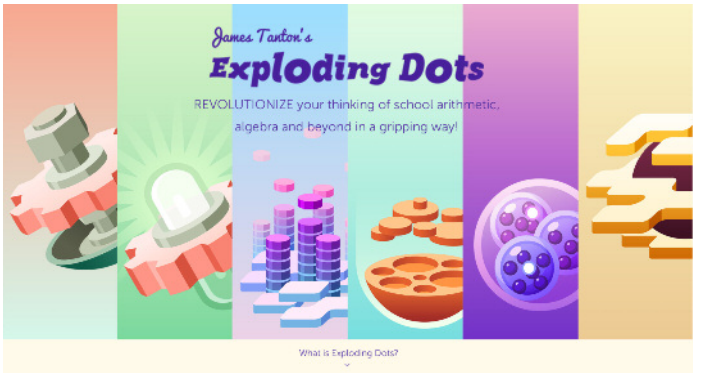
The rollout activity for The Global Math Project will be Exploding Dots, now an official trademark with the U.S. government. It is a mind-blowing journey, from playful arithmetic all the way to polynomial division and infinite series—just through the power of dots and concrete visualization! James Tanton, who received his PhD in mathematics from Princeton and is a Visiting Scholar at the Mathematics Association of

America, has been giving Exploding Dots workshops for years, and now is travelling all over the world, unveiling the deep conceptualization of fundamental mathematics through the joy and delight with dots—literally going "kaboom" and "kapow" in his presentations!



Some of Ontario's Global Math Project Ambassadors

While The Global Math Project is an American initiative, spearheaded by James Tanton, a Canadian company in Montreal, Scolab, is designing the interactive experience and building the Exploding Dots "machine" for the entire world to see and use! The company currently creates the only bilingual digital math platform in Canada, called Netmath.



Landing Page for 2017's Exploding Dots—Coming Soon!

A series of monthly webinars, hosted by James Tanton and myself, will occur right up until the official launch of The Global Math Project in the week of October 10—the date, 10/10/2017, was specifically chosen to eliminate any confusion about different calendar conventions of day/month around the world. Michael Jacobs, a GMP Ambassador from Durham Catholic District School Board, was quick to point out that "10/10" sounds remarkably close to "Tanton." Some things were just meant to be!

The goals of these webinars are to introduce the global audience to the specifics of The Global Math Project and as well, to explore the different approaches to mathematics in various countries/cultures around the world. Every month, there will be a guest Ambassador from a different country, who will speak to the theme of that particular webinar. The hope is that the momentum of these webinars and the project itself will create a measured paradigm shift as to how mathematics is taught. Not only will The Global Math Project be an annual event, but its larger goal is to perhaps create a new international math curriculum that houses the current emphasis on deep number sense, while ensuring that the learning occurs in environments that honour equity, joy, and simply being human.

There is nothing more powerful than an idea whose time has come. ~Victor Hugo

The time for the world to celebrate mathematics with meaning and connection has come. For more information and to learn about how to become a Global Math Project Ambassador, please visit **www.theglobalmathproject.org**.

▲ Vote During the Month of March

ALL MEMBERS TAKE NOTE!! OAME ELECTIONS ARE ONLINE

Annual elections for positions on the Executive and Board of Directors will be online during the month of March. To vote, you must activate your online account on the website at

www.oame.on.ca if you have not already done so. Click on "Members Only" and follow the directions to activate your account. You will find your membership number and expiry date on the envelope of your *Gazette*.

During March, you will find "OAME Elections" in the links on the left side, after you have logged in using your username and password, which you created when you activated your account. Just follow the instructions. Short biographies for all candidates are included. If you need assistance, contact the Executive Directors by e-mail: **EDs@oame.on.ca**

▲ 2016-2017 OAME Board

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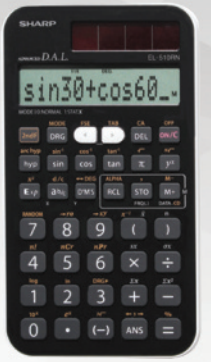
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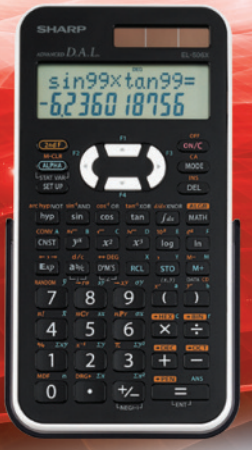
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