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Ontario Mathematics Gazette

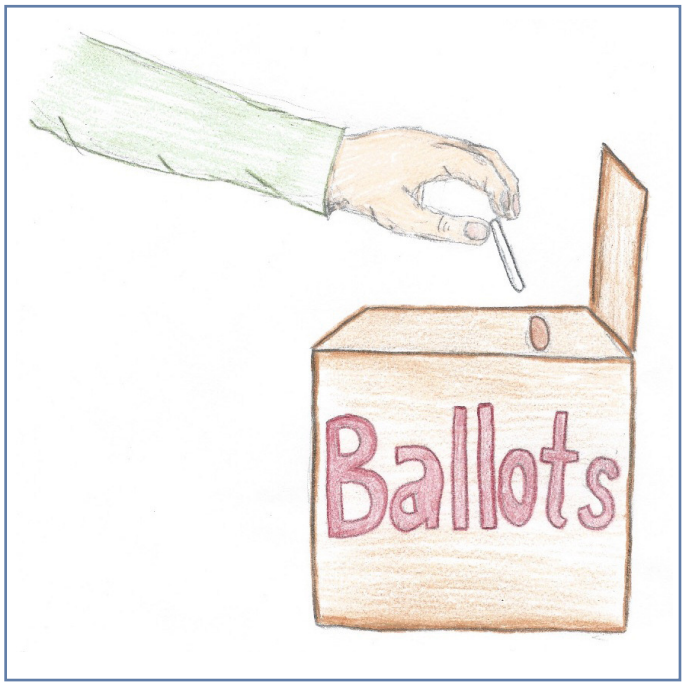
OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

Vol. 56 #3
March 2018

IN THIS ISSUE

- Letter to the Editor
- President's Message
- Executive Directors' Report
- Fields Institute MathEd Forum Report
- Provincial Digital Learning Resources – What's New? Enhanced Annotation Tools
- What's the Problem? Totalling Trains
- Technology Corner: GeoGebra Materials
- OAME/NCTM Report
- Mb4T (Mathematics by and for Teachers): Relating Area and Perimeter
- In the Middle: Do They Make Enough?
- Hey, It's Elementary: "You can't help getting older, but you don't have to get old." (George Santayana)
- Don't Ask Why: The Division between Presentations and Representations
- Math Jams: Students Analyzing, Comparing, and Building on One Another's Work
- The Fields Medal from an Ontario Perspective
- Book Review: And Now for Something Completely (Cursively) Different....
- Book Review: Looking at Math in a Different Way. A Book for Math Geeks and Math-Geek Wannabes



See Executive Directors' Report



See The Fields Medal from an Ontario Perspective

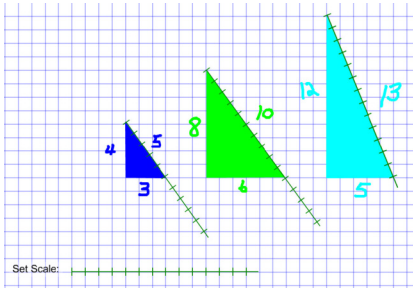


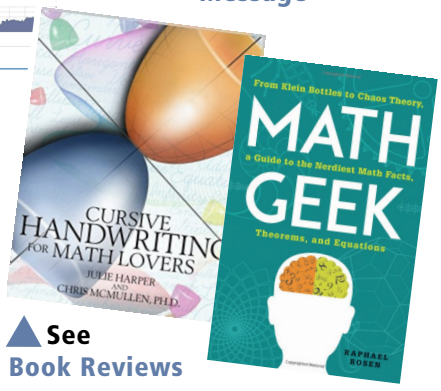
See President's Message

See In the Middle: Do They Make Enough?

See Provincial Digital Learning Resources – What's New? Enhanced Annotation Tools





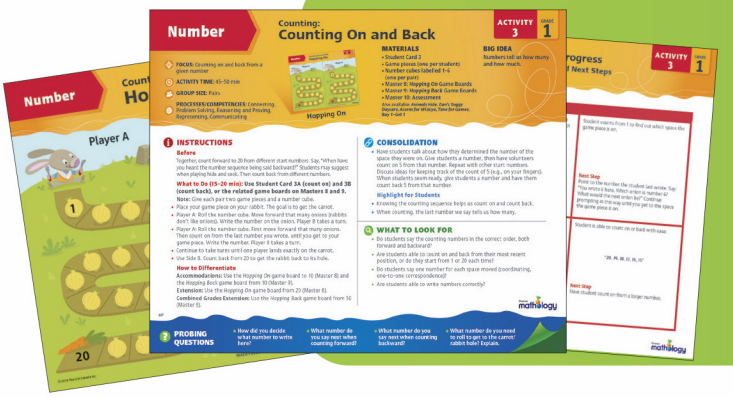


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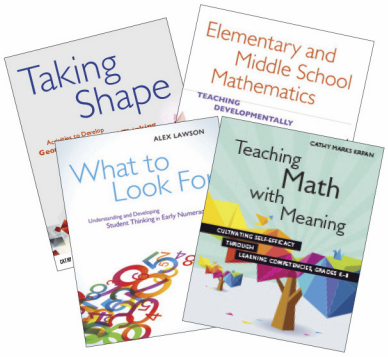
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Submission of Articles

The *Ontario Mathematics Gazette* (OMG) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *Gazette*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles are to be word-processed, MS Word is preferred, and prepared according to the *Publication Manual of the American Psychological Association*, Sixth Edition (2009). However, please use single-line spacing (not double) and only one space after each period. Articles should not exceed five numbered pages of text, and figures, images, and photographs should be placed in the text close to where they belong, with captions. The photographer's permission is required, and for photos of students under the age of 18, the written permission of a parent or guardian is required.

Please submit your article in one blind file (i.e., identity of author is not evident), and include author names, contact information including email and mailing addresses, photos, biographies, and all content removed for blinding in a second file. Please email these two files to Tim Sibbald at **gazette@oame.on.ca**.

Upon review, you will be notified whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected. The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME.

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▲ Table of Contents

Up Front

▲ Editor's Report /Timothy Sibbald 2

▲ President's Message /Jill Lazarus 4

▲ Executive Directors' Report /Lynda and Fred Ferneyhough 6

Regular Features

▲ The Abacus . . . . . 8-Page Insert

▲ Fields Institute MathEd Forum Report /Angelica Mendaglio 7

▲ Provincial Digital Learning Resources – What's New? Enhanced Annotation Tools /Agnes Grafton /Ross Isenegger /Markus Wolski /Greg Clarke 15

▲ What's the Problem? Totalling Trains /Shawn Godin 19

▲ Technology Corner: GeoGebra Materials /Mary Bourassa 21

▲ OAME/NCTM Report /Jacqueline Hill 24

▲ Mb4T (Mathematics by and for Teachers): Relating Area and Perimeter /Jennifer Holm 31

▲ In the Middle: Do They Make Enough? /Carly Ziniuk 33

▲ Hey, It's Elementary: "You can't help getting older, but you don't have to get old." (George Santayana) /Lynda Colgan 36

Articles

▲ Don't Ask Why: The Division between Presentations and Representations /Nat Banting /Ilona Vashchyshyn 10

▲ Math Jams: Students Analyzing, Comparing, and Building on One Another's Work /Naomi Norquay /Tina Rapke 25

Special Features

▲ BOOK REVIEW: *And Now for Something Completely (Cursively) Different….* /Reviewed by Natalie Wilson 8

▲ The Fields Medal from an Ontario Perspective /Judy Mendaglio 9

▲ BOOK REVIEW: *Looking at Math in a Different Way. A Book for Math Geeks and Math-Geek Wannabes* /Reviewed by Christine Sasso 39

▲ Recreation Page 40

Notices

▲ OAME Graph Sticky Charts 14

▲ OAME Board Members . . . . . Inside Back Cover

Advertisements

▲ Pearson . . . . . Inside Front Cover

▲ CCS 24

▲ Infinite Possibilities - OAME 2018 . . . . . Back Cover

▲ About the *Ontario Mathematics Gazette*

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Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be print ready, and colour advertisements should have no bleeds.

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▲ Editor's Report



TIMOTHY SIBBALD, OCT, PhD
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Tim Sibbald is the current Gazette *editor, co-chair of finance for the 2018 annual conference, and a Past President of OAME. He is an associate professor in the Schulich School of Education, Nipissing University, with a focus on mathematics education.*

There is a common concern about the basic skills of math students during the years after the curriculum says they have covered some content. For example, often newspaper articles criticize students for not knowing their multiplication tables. It is not just newspapers that complain; it is not uncommon to hear first-year professors complain about select skills. Recently a colleague expressed frustration about having four students in a first-year class of a hundred, who did not know how to find a common denominator. As a teacher, I am of the opinion that the issues behind these items are different—the first (multiplication skills) speaks about a numeracy skill that is universally used, whereas the second (a common denominator) may reflect a choice, realistic or not, to apply effort to pursue university entrance, even though some skills are weak.

The multiplication claim is presented in the media as if there is no requirement for students to "know" the multiplication table. By "know," they refer to rapid recall and competence with the information. This is known as factual knowledge without the requirement of any contemplation of reason or concept. There are several issues buried in this single issue. First is the claim that the curriculum does not require knowledge of the multiplication table. I have heard this claim reiterated by teachers, pre-service candidates, and others in education who appear to accept what the media has said. But is it true? In the Grade 3 curriculum, Number Sense and Numeration is a sub-strand that incorporates *Operational Sense*. The specific expectation says: students will "multiply to 7 x 7 and divide to 49 ÷ 7, using a variety of mental strategies (e.g., doubles, doubles plus another set, skip counting)." In Grade 4, the corresponding expectation is "multiply to 9 x 9 and divide to 81 ÷ 9, using a variety of mental strategies (e.g., doubles, doubles plus another set, skip counting)." This clarifies that requirements do exist in the curriculum.

Returning to what it means to "know" the multiplication table, the two expectations mentioned facilitate the use of multiple strategies. The expectations are building student

knowledge in a conceptual manner. There is consensus that knowing why something is true is important during the process of learning. However, facilitating multiple strategies for concept development is not necessarily the same as causing rapid recall. This draws out the notion that toward the end of concept development, when one is getting ready to move on, there is a need to consolidate one's learning. This is, in my opinion, where a challenge resides.

Solution to chess problem on page 15 of the December (56(2)) issue: Kasparov vs. Alexander Karman, 2000,

Solution

1. Qf6 Qf6
2. Pf6 Black can no longer defend against Rh8.

The consolidation of conceptual learning is often taken as a classroom appreciation of the broad variety of ways a concept can be understood. The focus is often about consolidating in the sense of summarizing and showing the scope of the concept or approaches to the concept. It is consistent with what is said about allowing multiple-solution methods and allowing students to develop the ability to use different procedural approaches.

A different interpretation of consolidation is to take the aforementioned approach and continue to refine the student's capacity to work with the knowledge. The ambiguity that there can be two interpretations of the intent of "consolidation" creates an issue, in my opinion, for some teachers. Regardless of their solution method related to the concept, students should gain procedural fluency and consistency—that is, an intention within the curriculum. This interpretation refers to the need to be able to determine 9 x 6 without drawing arrays, or using manipulatives, or using a calculator. It is not a concern whether a student performs this as an act of memory recall (i.e., rote), for example "54," or as a rapid process (10 – 1) x 6 = 60 – 6 = 54, or some other technique (9 x 3 x 2 = 27 x 2 = 40 + 14 = 54) , as long as students can complete the 9 x 6 calculation sufficiently fast that it does not hinder its usage as a contributing component for a larger task. It is the increased pace that often diminishes the details of the actual method used because generating an answer is becoming routine. The "routinization" of a concept (i.e., formation of a rapid and routine process) seems to be problematic for some teachers within a curriculum that promotes conceptual understanding.

The readership of the *Gazette* likely appreciates the need for both interpretations of consolidation and likely supports the mental facility of working with numbers in a fluid manner that does not hinder higher-order thinking. I have been taken to task for suggesting that the curriculum, as policy, may not be as clear as it could be about the migration from concept to procedural fluency. And, maybe just as the multiplication table is in the curriculum, I need to go back and take a careful look. Certainly there is no harm in revisiting the curriculum documents; it is, after all, the reason we have ongoing professional learning and the policies readily available to us.

Though it is tangential, the pre-service teachers I work with come from across the province (a key feature of the Schulich School of Education). When asking them about their experiences, I find it is something of a barometer reflecting what occurs across the province. In rough terms, it seems half of students learn multiplication to 10 x 10, and half learn it to 12 x 12. I have had one rare exception, where a teacher candidate had learned up to 16 x 16. There are important considerations behind this detail. Ten by ten meets the curriculum expectation—the minimum that we have to teach. Twelve by twelve is to provide facility for mental math when using imperial measurements such as feet and inches. Since imperial units appear in the curriculum in Grade 9 locally developed and Grade 12 College, there is justification for 12 x 12. Construction and carpentry commonly use imperial measurements to a sixteenth of an inch, and that was the justification for pursuing multiplication to 16 x 16. Among teachers with a strong orientation toward math, there is a general facility for special cases of multiplication as well, such as knowing the squares of numbers beyond 12^2, or special cases such as powers of two.

These sorts of interpretation challenges permeate other parts of the curriculum. I simply use the multiplication table as an example that newspapers find resonates with their broad readership. Consider that the familiarity with algebra supports mental math because it allows application of algebraic truths to specific cases. A classic example found in many books on mental calculation is consideration of questions such as 18 x 22. Many readers will have immediately associated this with a difference of squares, but is that type of example pointed out when factoring is taught? The details are 18 x 22 = (20 – 2) x (20 + 2) = $20^2 – 2^2$ = 400 – 4 = 396. This can go much further by considering that any algebraic binomial multiplication can be applied to specific values, and that choosing from among different possible binomials can make the calculation readily available by knowing squares of values.

This *Gazette* Issue

Within these pages, we have the Executive Directors acknowledging the work of the co-chairs of the annual conference. With the OAME Annual Conference, "Infinite Possibilities," coming in May, it is time to register if you have not done so already. They also draw attention to the voting taking place for the board of directors—take the time to vote! Thank you to Will MacKenzie for the cover artwork promoting attention to voting. Jill Lazarus, the president of OAME, speaks about the Leadership Conference that was held in Ottawa for the first time. She speaks about the

presentation of Dr. Peter Liljedahl, who will also be a keynote at the annual conference.

In the NCTM report, Jacqueline Hill looks at feedback she received from students about homework, and uses that as an example of how resources such as the *Gazette* can be actively used in the classroom. Angelica Mendaglio reports on activities at the Fields Institute MathEd Forum that focuses on the use of social media. Speaking of "Fields," Judy Mendaglio draws our attention of an Ontario connection to a recent winner of the coveted Fields medal.

Our regular columnists continue to inspire with a broad range of topics. Carly Ziniuk uses wealth as a springboard for currency conversion and social comparisons. The Provincial Digital Team explains the use of variable rulers that can assist in understanding various concepts. Shawn Godin discusses a problem involving trains. Mary Bourassa looks at pedagogical uses of GeoGebra, where she brings out different types of questions within the GeoGebra environment. Jennifer Holm considers the connection, or lack thereof, between perimeter and area in the Math by and for Teachers article. Finally, Lynda Colgan brings us a long-term "29-year" perspective on math education.

Natalie Wilson reviews a unique math book that focuses on cursive writing. Christine Sasso also provides a book review of *Math Geek*.

Nat Banting and Ilona Vashchyshyn consider the conceptual basis for division of fractions by fractions. They provide an explanation that is useful for all teachers, whether they actively teach the topic or not.

Naomi Norquay and Tina Rapke explain the design of "math jams." These are a social construction for classroom groups that require students to understand approaches taken by other students. It is interesting how metacognitive thinking is encouraged by students having to build a solution, using another groups' initial work.

Don VanderKlok brings an interesting approach to balancing chemical equations. The approach provides a definitive answer and is a step up from guess-and-check approaches.

Lastly, I would be remiss if I did not thank Anne Yeager, Gitta Berg, and Penny Clemens—the *Gazette* arises because of strong teamwork, and a large part of that strength is due to their efforts. Thank you! I would also like to thank Jacqueline Foster who has engaged in some editorial work for this issue. ▲

Late breaking news!

On Dec. 26, 2017, the largest known prime number became $2^{77,232,917}-1$. That's big!

See **www.mersenne.org** for details.

▲ President's Message



JILL LAZARUS
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Jill Lazarus, the current President of OAME, is a high school teacher in the Renfrew County District School Board. She is also completing a PhD in Education at the University of Ottawa. Jill has a range of interests in mathematics education, including curriculum design and implementation, student communication in mathematics, educational technologies, classroom assessment, and professional learning.

Here we are, at the start of a New Year and mid-way through another school year. Looking back at the first semester, the 2017 OAME Leadership Conference, *Connecting Leading Ideas*, was a highlight for me. The opportunity to connect with leaders in mathematics education, including teachers, coaches, consultants, administrators, and researchers, was both exciting and inspiring. I would encourage you to check out some of the ideas that were captured on Twitter with #OAMElead.

First, there was a thought-provoking *Thinking Classroom* keynote by Dr. Peter Liljedahl (see **www.peterliljedahl.com/presentations**), followed by breakout sessions facilitated by Peter Liljedahl and Judy Larsen, and by educators from across the province: Alex Overwijk, Pierre Tranchemontagne, Jaime Depippo, Jimmy Pai, and Karine Rozon. It was fascinating to see the *Thinking Classroom* experience at this conference. The picture in Figure 1, for instance, shows the breakout session for *leaders* (e.g., administrators, coaches, consultants), with more than 100 people engaged in a problem.



Figure 1: *Breakout session for leaders*

As you can see, like in a *Thinking Classroom*, everyone was engaged in a rich problem, using vertical non-permanent surfaces (VNPS), working in visibly random groups (VRG), and there was no "teacher" at the front of this "classroom." Actually, the facilitator, Peter Liljedahl, is in this picture. Can you find him?

In the same picture, you might also be able to see that the groups are working on dry-erase boards, donated by Wipebook (**wipebook.com**)! VNPS include more than dry-erase boards, however. As you can see in the following pictures, windows worked quite nicely

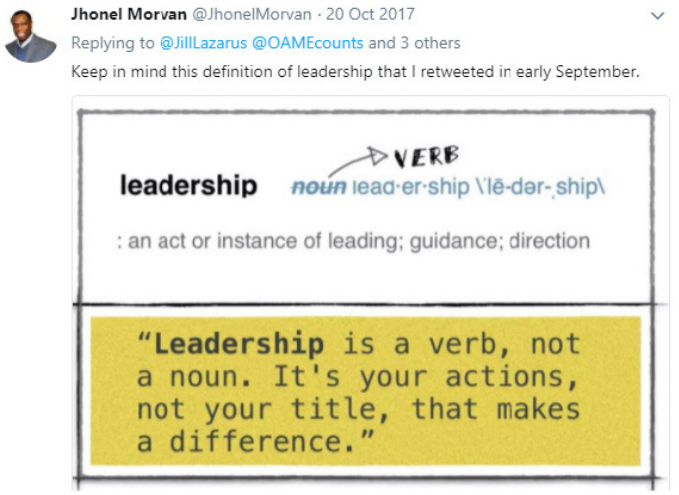
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The learning continued in the afternoon, with a panel discussion facilitated by Jimmy Pai, that connected different perspectives on leadership, from Dr. Chris Suurtamm, Jhonel Morvan, Julie Séguin-Mondoux, and Mary Bourassa. Before the conference, we posed a question on Twitter: "If you had to choose one word to describe 'Leadership' in Mathematics Education, what would it be? Why?" Jhonel Morvan reminded us that leadership is about "your actions, not your title" (Tweet below):



This sentiment was reflected throughout the discussion. For me, some of the takeaway verbs included *connecting, supporting, influencing, sharing, reflecting, modelling, learning, coaching, believing, building community, inspiring,* and *valuing*. For a beautiful Sketchnote that captures some of the ideas shared by these panelists, I would encourage you to check out Laura Wheeler's Sketchnote of this discussion. Laura, and others, also created Sketchnotes to capture some of the other "leading ideas" at this conference (this can also be found at #OAMElead).

Connecting back to the classroom, Dr. Chris Suurtamm, Jon Orr, Kyle Pearce, and Mary Bourassa facilitated elementary and secondary breakout sessions. These OAME members, who model the above leadership qualities through their contributions to the mathematics education community, shared inspiring ideas for supporting practice.

If you attended this conference, you are invited to offer your feedback by completing an evaluation at **oame.on.ca/MCIS/index.php**.

Moving forward, the 45th Annual OAME Conference is coming up May 3–4, 2018. For more information, check out the website at **oame2018.weebly.com** and keep connected with #oame2018 on Twitter.

In order to continue to support the growth of this annual event, an OAME Ad Hoc Committee is currently exploring some of the challenges involved in organizing the annual conference. In November, an email was sent to OAME members to invite their input into some of the questions that this committee has been exploring. Thank you to everyone who responded. The feedback is appreciated by the committee. ▲

▲ Executive Directors' Report

LYNDA AND FRED FERNEYHOUGH
EMAIL: eds@oame.on.ca





Lynda and Fred Ferneyhough have been the Executive Directors of OAME since 2010. They taught in the Peel District School Board for over 30 years and had a three-year stint in the United Arab Emirates. Both of them served OAME as Chapter Representatives for CHAMP, Directors, and Vice-Presidents. During their career, they were Department Heads, and authors for McGraw-Hill Ryerson. They are both certified as instructors for Texas Instruments (TI), and Fred continues to coach for TI in the United States.

In any volunteer organization, there are generally a large number of people whose efforts stand out and make the organization run smoothly. In the previous two articles, we mentioned the contributions of our board members and the presidents of the organization. This time, we're highlighting the work of the annual conference co-chairs.

Without a doubt, the organization as a whole is partially dependent on the funds that are received from the annual spring conference. In the OAME guidelines, the income from that conference is split, with half of the profits staying with the host chapter and half going into the organization.

Over the past few years, attendance at the conference has been high and challenged the capacity of the venues that we have used, resulting in a solid financial base for OAME. In the June issue, we will mention the committee members, but for now, we will focus on the efforts of the co-chairs.

We, the Executive Directors, consider the OAME Conference Chairs with the same respect that we accord the OAME Presidents. We're not exactly sure why they do what they do, but are extremely grateful that their efforts, their time, and their service are there for us. This is no small undertaking. Countless hours go into the planning stages of the annual conferences, usually starting about three years in advance of the event. The conference chairs need to learn, usually on the fly, how to manage 15 committees, learn all of their functions, and ensure that the committee members continue applying their efforts to their prescribed roles in a timely manner. In addition, they take responsibility for finding a venue, striking a budget, and selecting keynote and featured speakers in conjunction with the appropriate committees.

Consider this for a moment. Over my career with the Peel District School Board, I worked under 12 different people as principals, and close to 30 people as vice-principals. In any school, members of the administrative team set the tone for the school. Some of them are incredibly good at their roles. All of them have to manage many departments, and a greater number of teachers. The same is true for the conference chairs. In an ideal situation, the 15 committees work together for the benefit of the entire conference. That can be an incredible feat with so many people clamouring for attention, budget, and/or a recognition that their needs are somehow more important than the needs of others—similar to what we sometime encounter in a classroom. Yet, in the time that we've been attending or assisting with conferences, we've never seen any of our co-chair teams handle those challenges poorly. In most cases, the co-chairs are accorded the same respect that any natural leader commands.

So, here's an acknowledgement to the conference chairs. Nothing we say in this column can adequately express our gratitude for the countless hours that you put into organizing a conference, with the enormity of leading a large group of mathematics teachers and providing excellent professional development for the mathematical community in Ontario.

We would also like to note that this issue of the *Gazette* is being published at the same time that polls are open for our annual elections. There are four openings for candidates this year: President-Elect, Vice-President, Board member (Grades 7 and up) and Board member (K–Grade 6). The online polls are open until the end of March. Please take the time to vote. ▲

Vote During the Month of March

ALL MEMBERS TAKE NOTE!! OAME ELECTIONS ARE ONLINE

Annual elections for positions on the Executive and Board of Directors will be online during the month of March. To vote, you must activate your online account on the website at **www.oame.on.ca**, if you have not already done so. Click on "Members Only" and follow the directions to activate your account.

During March, you will find "OAME Elections" in the links on the left side, after you have logged in using your username and password, which you created when you activated your account. Just follow the instructions. Short biographies for all candidates are included. If you need assistance, contact the Executive Directors by phone at 519-899-4203, or by email: **EDs@oame.on.ca**

▲ Fields Institute MathEd Forum Report



ANGELICA MENDAGLIO
EMAIL: angelicamendaglio@gmail.com

Angelica Mendaglio is an instructional designer at Vretta Inc. in Toronto, Ontario, where she helps to create interactive digital mathematics lessons and activities for middle school students.

The Fields Institute MathEd Forum convened in November for a series of presentations and discussions surrounding the central theme of "Social Media and Mathematics Education." Speakers pondered how teachers can use social media to enhance their teaching in a myriad of ways, from making student thinking visible, to engaging parents and guardians in the classroom experience, to finding inspiration in another teacher's experiences. The variety of ideas shared by the presenters, as well as a rich discussion among the attendees, made for an excellent meeting to close out the year.

The first speaker, Shelly Vohra from the Peel District School Board, presented her research on the impact of student engagement when a teacher uses Twitter as an instructional tool in a Grade 8 math class. Shelly's qualitative study looked at how the teacher leveraged student use of social media to engage and communicate with others to improve student outcomes in the math classroom. She spoke about how the teacher used Twitter, how different artifacts (including tweets) supported student learning, and the perceptions of both the teacher and the student of the impact of Twitter on the learning experience. The study showed that students were encouraged to extend their learning outside of the classroom and turn to other students (and not only their classmates) for support. Shelly also highlighted that the digital platform was seen as a safe place for students who were reluctant to share their thinking in a classroom setting, and that some students were more willing to participate in an online forum than a discussion in the classroom.

Ruthie Sloan, also from the Peel District School Board, discussed her experiences with using a digital portfolio platform, called Seesaw, which has helped her students to document their learning and include parents and guardians in the classroom experience. Using the platform, she has been able to give her students options for how to communicate their thinking in ways that go far beyond what they could do with paper, selecting a medium they felt most comfortable using. The platform also allowed her to communicate with parents/guardians, administrators, and other teachers.

Judy Larsen from the University of the Fraser Valley joined the meeting remotely to share her experiences with the Math Twitter Blogosphere (#MTBoS). What started as a desire to observe the classrooms of other teachers as part of her Master's thesis led to her discovering a wealth of resources, as well as an online community, where teachers share their experiences teaching math by engaging in rich discussion.

One particular tweet that Judy shared in her presentation (see Figure 1) led to an interesting discussion at the forum, which seemed to emulate conversations that would be happening in the MTBoS. Judy encouraged the Forum participants to find the tweet and continue the conversation, which would open it up so that even more people could benefit from the different perspectives available at the Forum. A panel discussion followed, where the Forum participants were able to continue the in-depth discussion with the speakers and other panelists.



Figure 1: This tweet in Judy's presentation sparked discussion at the Forum

The MathEd Forum meets monthly at the Fields Institute in Toronto. You can find more information on upcoming meetings and register to join either in person or through the live stream at **www.fields.utoronto.ca/activities/workshops**.

Source

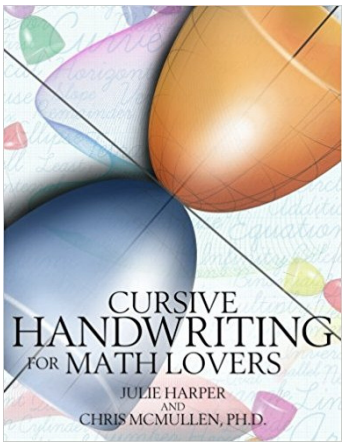
Larsen, J. (2017, November 25). *Belonging to the MTBoS: Tapping into a community of mathematics educators* [Film clip]. Retrieved from **www.fields.utoronto.ca/video-archive//event/1574/2017** ▲

▲ BOOK REVIEW: AND NOW FOR SOMETHING COMPLETELY (CURSIVELY) DIFFERENT....

REVIEWED BY: NATALIE WILSON

Natalie Wilson coordinates volunteer-led literacy and numeracy programs for school-age children on behalf of Frontier College in North Bay. She spends much of her time looking at developing new programs or approaches to engage children in hands-on learning. She also provides a variety of parent-engagement activities with community partners across Northeastern Ontario.

Harper, J., & McMullen, C. (2015). *Cursive handwriting for math lovers.* North Charleston, SC: CreateSpace Independent Publishing Platform. ISBN 9781512181326.



This is not the sort of book one comes across every day. In fact, it would likely be a bit of a surprise to find even one other book with the same content as this one on a bookstore or virtual store shelf. That originality in itself proffers a fair reason why there may be some value in exploring this book.

Cursive Handwriting for Math Lovers is exactly what it says it is: a workbook brimming with cursive writing activities, all firmly steeped in math language and terminology. In the introduction, the authors suggest this book is a way for math lovers to practise cursive writing, but it could be even more popular if used the other way round: as a math book for cursive writing lovers. As many children in the primary and junior grades are particularly motivated to learn the art of writing in cursive, this is an ingenious way for educators and parents to introduce them to math concepts and terminology outside the ordinary array of "math is fun" workbooks, puzzles, and games.

The workbook offers a fabulous range of mathematical terms for writers to explore—in handwriting. The terms are not solely those associated with run-of-the-mill, everyday language (terms such as sum, quotient, product, numerator or denominator); instead, the book offers a mix of math phraseology that is sure to wrinkle a few foreheads, bring about a pause, and prompt the odd writer to ask, "Hey, what does this word mean?"

Indeed, that is the greatest benefit of this book: it frees mathematical language from a textbook and sets it into a format that is seen as fun and less intimidating for a variety of learners. Not only does such a move into a more accessible format familiarize learners with some intriguing new vocabulary, it also provides the same opportunity for parents, whether math oriented or not, to learn (or remember) those terms.

The layout and design of the workbook lends itself as easy and accessible to the cursive writer. Only five lines of replicable text are on each page, and every line (aside from one page that introduces the upper- and lower-case cursive alphabet) features a different mathematical term or equation. An entire page is devoted to the written numeric value of Pi, another to Euler's number e, and yet another to Fibonacci's sequence. Pages are also interspersed with the odd illustration of a mathematical term, often in relation to one of the writing activities on the opposite page.

However, the very things that makes the workbook so appealing can also be regarded as a drawback. The authors do not present suggestions for parents or educators on how to use the book effectively, nor do they suggest any follow-up activities that could be helpful in further extending learner curiosity as they work through this book. They make no reference to the rationale behind the math terminology chosen, nor do they provide any context or connection (other than the writing exercises opposite) to the illustrations scattered throughout the book. While the volume works well as a workbook, its original intent, it offers little opportunity for those not familiar with math to link the writing activities with other math learning, either in the classroom or at home. So, though the book could be easily integrated into a math classroom or math program, where a strong foundation in mathematics already exists, it is unlikely that it would provide as many opportunities for learning in a less-developed math environment. The book's success therefore is dependent on who chooses to use it.

Still, there is opportunity here, particularly for teachers who are interested in exploring new ways to familiarize math language and math concepts with their students. Permission is provided for both parents and teachers to copy the worksheets for personal or class use, so purchasing the book is cost effective. And these worksheets could be just the thing to initiate a discussion on identifying math concepts that learners know, for example, numerator and denominator, versus some that are still new, for example, absolute value, or complex conjugate. Plus, there's cursive writing practice to boot. For math lovers, that is definitely something completely different. ▲

▲ The Fields Medal from an Ontario Perspective

JUDY MENDAGLIO



Judy Mendaglio is Past President of OAME. She has been a member of the Fields Institute's MathEd Forum for many years. Judy hears from many mathematicians who agree with Maryam Mirzakhani and Manjul Bhargava—that mathematics is beautiful and its beauty can and should be experienced by all students.

Have you ever heard of the Nobel Prize? Of course you have. I'll bet your students have too. But have you ever heard of the Fields Medal? Surprisingly, far fewer people are aware of the Fields Medal than are aware of the Nobel prizes, even though the Fields Medal is often called the Nobel Prize for Mathematics. What is particularly astonishing about how little regard people give to the Fields Medal is that this award originated in Ontario, and the Fields Institute for the Mathematical Sciences is on College Street in downtown Toronto. Surely this is a story that every Ontario student should know about?

Some background: The idea for a global award for excellence in mathematics was first proposed at the 1924 International Congress of Mathematicians, which that year, happened to take place in Toronto. Participating in this Congress were 444 mathematicians from 27 countries. The originally proposed name for the medal was "International Medal for Outstanding Discoveries in Mathematics." It was John Charles Fields (1863–1932), a Canadian mathematician from Hamilton, who was serving as secretary of the Congress, who provided the funding for the award, which has since become known as the Fields Medal.

The Fields Medal is awarded every four years at the International Congress of Mathematicians. Originally, only one mathematician could receive the medal. However, in 1966, in recognition of the global expansion of both the amount and quality of research being conducted in various branches of mathematics, the rules were changed to allow up to four medals to be awarded at each Congress. To become a Fields Medallist, you would have to be a mathematician who has made major contributions to mathematics and be under the age of 40 on January 1 of the year of the award. The age restriction is unique to the Fields Medal, and the purpose for the age restriction was to encourage young mathematicians who had a promising future in their field.



The Fields Medal itself is made of 14-karat gold. On the front side, the medal shows the head of Archimedes along with a quotation that has been attributed to him:

TRANSIRE SUUM PECTUS MUNDOQUE POTIRI – Rise above oneself and grasp the world.



On the back is the inscription (which translates as "The mathematicians, having congregated from the whole world, pay tribute for outstanding writings"):

CONGREGATI EX TOTO ORBE MATHEMATICI OB SCRIPTA INSIGNIA TRIBUERE

In the background is a sphere inscribed in a cylinder, which Archimedes used to show that the ratio of the volume of the inscribed sphere to the volume of the cylinder is 2 : 3.

The Fields Medal is recognized as a sign of excellence worldwide. In fact, one of the criteria used as an indicator of excellence by the Academic Ranking of World Universities (ARWU) (known as the Shanghai Rankings) is the number of Fields Medalists the university employs.



In last month's *Gazette*, you may have read about Maryam Mirzakhani, the only woman to have won a Fields Medal to date. Next year, the Fields Institute will be honouring Maryam at their Fields Medal Symposium in Toronto. At the 2016 Fields Medal Symposium, another medalist, Manjul Bhargava (see photo), was honoured. Manjul, having been born in Hamilton Ontario, has been heralded as Canada's first Fields Medal winner. Dr. Bhargava is currently a professor at Princeton University.

Manjul manages to find time to do things other than mathematical research in the field of number theory. For one thing, he is a talented tabla player. To hear him describe the connections between the tabla and mathematics, see **www.youtube.com/watch?v=oCpdAPt7UOs**. Dr. Bhargava also served as associate producer for the Hollywood film starring Dev Patel and Jeremy Irons, *The Man Who Knew Infinity*, about mathematician Srinivasa Ramanujan.

Manjul presents to students as well as to the general public. In his talks, he has two messages that should be relevant to mathematics educators in Ontario. In his own words:

On school mathematics:

> *My suggestion for teaching maths in schools is to really make mathematics fun because mathematics actually is fun. A lot of us really don't realize that when we go to grade school because it is taught in a robotic way. You're given a problem and you're asked to memorize steps to solve it, and then you just blindly apply it and try to be careful so you don't make an error, but you don't really know why you're doing those steps. It shouldn't be like that. It should really about being creative; coming up with those steps on your own, and everyone will come up with a different way to do it.*

On productive struggle, where Manjul is describing when he was growing up and doing math puzzles, how he would go to his mother for help:

> *Can you tell me how to do it, and her answer always was you should figure it out itself (sic: yourself) and then come tell me. And so, I used to just enjoy just playing around with mathematics and if I ever got stuck, my mother was there, as a resource, and that was something very useful.*

You can watch Manjul online, describing some of the ways that he sees beauty in mathematics. His Fields Medal Symposium Public Lecture is available at **www.youtube.com/watch?v=LP253wHIoO8**. A video of a talk he gave to students in Bengaluru, India, titled "Poetry, daisies, and cobras: Math class with Manjul Bhargava," can also be found on YouTube.

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WANTED

The *Gazette* editorial team has an opening for an associate editor. This requires a good eye for details, such as wordings that can be improved, grammar improvements, and proper use of punctuation. Knowledge of APA6 is useful, but not essential.

Interested? Email the editor and, using formal English, please articulate your interest.

▲ Don't Ask Why: The Division between Presentations and Representations



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Nat Banting is a mathematics teacher in Saskatoon, Saskatchewan, and a recent M.Ed. graduate from the University of Alberta. He serves as the co-editor of the Saskatchewan Mathematics Teachers' Society periodical, The Variable, *and tweets his thoughts concerning the teaching and learning of mathematics as @NatBanting.*



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Ilona Vashchyshyn holds a B.Sc. in Mathematics and a B.Ed. (Secondary) from the University of Saskatchewan, and currently teaches mathematics in Saskatoon, Saskatchewan. Ilona is an executive member of the Saskatchewan Mathematics Teachers' Society and a co-editor of the Society's periodical, The Variable. *She tweets her thoughts about mathematics teaching and learning as @vaslona.*

$$\frac{3}{4} \div \frac{1}{2}$$

It may be a tad unsettling to have an article begin so abruptly, with an exercise immediately thrown at the reader. After all, it is customary for articles in math education journals to first introduce a theme, then ground it in a survey of the relevant research literature, and follow this with a description of a problem or a classroom experience. From there, the problem or experience is typically dissected in light of the theory, and implications for practice (or further research) are suggested. In full awareness of this common structure, here we intentionally reverse this process to ground the article in experience—an experience triggered by the exercise presented at the onset.

We are assuming that any warm-blooded teacher of mathematics encountered the problem in an intentional manner. That is, we are making the assumption that you, the reader, interacted with the exercise, despite receiving no prompt that asked you to do so. In addition, we are assuming

that you may have interacted with the exercise in a particular way. In doing so, we are including ourselves in this shared experience, which we do not intend to speak down to or criticize. Rather, this article was borne out of a shared experience we had while confronted with the same possibilities, which will be presented in the following pages.

An Experience

Perhaps your interaction with the exercise went something like this:

$$\frac{3}{4} \div \frac{1}{2}$$
$$= \frac{3}{4} \times \frac{2}{1}$$
$$= \frac{6}{4}$$
$$= \frac{3}{2}$$

Following our presumptive trail, we assume that many readers suffer from a split (pedagogical) personality when confronted with such an exercise. This is the condition common among teachers, in which a teacher loses the ability to repress the urge to be a teacher, to observe every opportunity through the context of a classroom. This condition might lead to the recitation of a familiar dyadic poem:

"Don't ask why, just invert and multiply!"

It is a moniker that is often recited in conversations involving fraction division. It is catchy enough to have withstood the test of time essentially unaltered, and was a part of the grade-school experience for both of us, the authors. However, the poem, and the algorithm it so succinctly encapsulates, carries with it a sort of weight. The phrase has become a flagship for what Skemp (1976) calls *instrumental understanding*, or "rules without reasons." He suggests that this form of understanding is the "possession of such a rule, and ability to use it" (p. 2). Despite the explicit use of carefree language that advocates for relief from the burdensome question of why, the poem also represents an internal battle for teachers. On the one hand is the efficiency purported by the simple algorithmic melody: that is, it produces results. On the other is a sense of guilt that perhaps use of the algorithm, despite its efficiency, does not necessarily reflect that students (and perhaps, teachers) understand the process of dividing fractions beyond an instrumental sensibility. So, we (as teachers) scramble to find an alternate model that frees us from, yet simultaneously upholds the simplicity of, the catchy tune.

Consider the following pictorial model of fraction division that involves the manipulation of the familiar part-whole area model and the common interpretation of division as making equal groups. As a reminder, the problem to be solved is the following:

$$\frac{3}{4} \div \frac{1}{2}$$

We begin with a representation of three-quarters (Figure 1). In this model, the whole (the largest rectangle) is divided into four sections, using vertical lines. One can see this dissection by reading across the top of the model. As we move along the topmost edge of the whole, we can see that three of the four sections have been shaded, representing the fraction three-quarters.

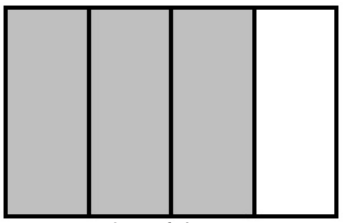


Figure 1: *Representation of three-quarters*

To represent the division by one-half, we must first re-divide the whole into halves. Because the dividend (three-quarters) is pictured along the top of the model, we use the left-hand side of the model to represent the divisor—halves (Figure 2). The whole is then cut using a horizontal line in such a way that reading along the left-hand side of the model (and ignoring the vertical cuts) produces a visual of two sections, one of which is shaded (using diagonal lines in order to distinguish from the previous shading).

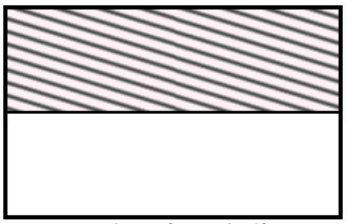


Figure 2: *Representation of one-half*

If we superimpose the two representations (Figure 3), the model now pictures eight sections of identical size, and provides a visual of the numbers of said sections constituting both the dividend and divisor. Three-quarters is represented by the six shaded sections, and one-half is represented by the four sections thatched with diagonal lines.

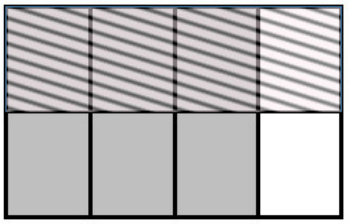


Figure 3: *Representation of one-half superimposed on the representation of three-quarters*

The solution to the task of dividing three-quarters by one-half is achieved by determining how many groups of one-half exist in three-quarters. Outlining groups of one-half (four sections) from the set of three-quarters (six sections) reveals the solution to be one full group of four, with two-fourths toward another (Figure 4). The solution to the exercise, then, is one and two-fourths, or one and one-half.

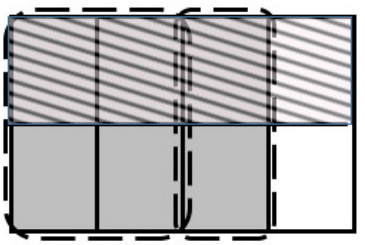


Figure 4: *One and one-half total groups*

This model has some beautiful features. First, it is robust. That is to say, the creation of an array to represent the division of fractions will always work. Although the denominators may grow to unwieldy sizes, they never outgrow the feasibility of the model. In that sense, there is a connection to the universal appeal to the rhyme: It produces results. Second, the model is elegant in its pictorial depiction of the operation of division. By this, we mean that with this model, the operation of division is actually manifested, rather pleasingly, as a dissection and then a regrouping of sections. As such, it manages to simultaneously distance itself from the abstract nature of symbols, while still connecting the division symbol (÷) to the act of division through the common metaphor of making groups. For these two reasons, the pictorial model holds promise for enacting a deeper understanding of the process of division of fractions.

However, although there is a pervasive belief that the presentation of an additional model (such as the part-whole area model) constitutes due diligence on the part of the teacher, the idea that understanding (of a concept such as fraction division) resides within this pictorial model should give us pause. To illustrate our point, and in continuing the theme of reflecting from experience established from the onset of this article, we now propose an alternative, intriguing model of fraction division in the hopes of providing a new perspective on both the operation of fraction division specifically, and on the distinction between presentation and representation in general.

Consider this, if you will, a thought experiment.

An Alternate Experience

In a galaxy far, far away… a teacher is confronted with the following exercise:

$$\frac{3}{4} \div \frac{1}{2}$$

With a familiar dyadic poem ringing in her ears,

"Once the tops are equal size, just take the bottoms and divide!"

the teacher interacts with the exercise, almost involuntarily, despite receiving no prompt asking her to do so. Her thought process in determining the answer goes something like this:

$$\frac{3}{4} \div \frac{1}{2}$$
$$\frac{3}{4} \div \frac{3}{6}$$
$$= \frac{6}{4}$$
$$= \frac{3}{2}$$

Despite the poem's utility as a reminder of the standard (at least, in this galaxy) division algorithm, which involves creating common numerators for the divisor and dividend, it nevertheless represents an internal battle for the teacher. On the one hand is the efficiency purported by the algorithmic melody: that is, the algorithm produces results. On the other is a sense of guilt that perhaps use of the algorithm, despite its efficiency, does not necessarily reflect that students understand the process of fraction division. And so, the teacher arms himself or herself with a pictorial model, which involves dilation of the familiar part-whole area model and the common (at least, in this galaxy) interpretation of division as determining a scale factor.

We begin with a representation of three-quarters. In this model, the whole (the largest rectangle) is divided into four sections, using vertical lines (Figure 5).

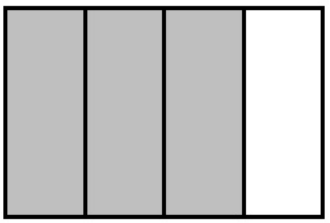


Figure 5: *Representation of three-quarters*

To represent division by one-half, the whole must be "stretched" until the shaded area represents one-half—that is, in such a way that the size of the original sections remains unchanged. In the case of three-fourths divided by one-half, the whole, which originally consisted of four sections, must be stretched until it can be divided into six equal sections, whose size is equal to that of the original sections (Figure 6). (We might also imagine this process as adjoining sections until the shaded area represents one-half.)

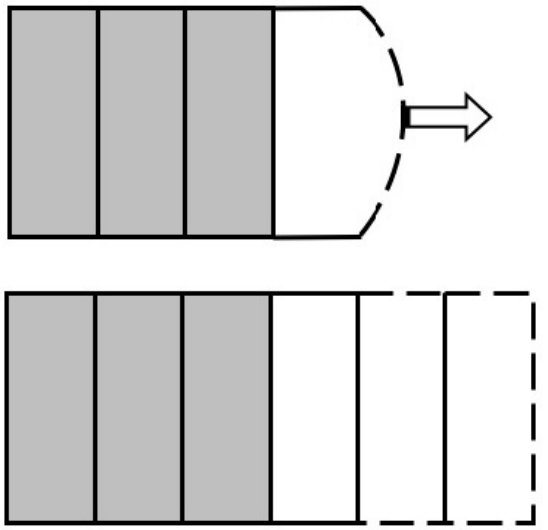


Figure 6: *Representation of a whole "stretched" until the shaded area, originally representing three-quarters, represents one-half*

The model now pictures six sections of identical size, and provides a visual representation of both the dividend and divisor. Three-quarters is represented by the three shaded sections in the original, undilated diagram (Figure 5), and one-half is represented by the three shaded sections following dilation of the whole (Figure 6). The solution to the task of dividing three-quarters by one-half is achieved by determining how many sections constitute the undilated and dilated wholes. The ratio of the total number of sections in the dilated whole (in this case, six) to the total number of sections in the undilated whole (in this case, four) represents the division of, in this case, three-quarters by one-half. The solution to the exercise is, then, six-fourths, or one and one-half, which represents the scale factor required to extend four sections of equal size into six sections of the same size.

This model has some beautiful features. First, it is robust. That is to say, dilation of an area model to represent the division of fractions will always work. (We invite the reader to convince herself or himself that this is indeed true). Second, the model is elegant in its pictorial depiction of the operation of division. By this, we mean that with this model, the operation of division is manifested, rather pleasingly, as stretching or dilation, illustrating its relationship with the inverse process of multiplication. Hence, it manages to simultaneously distance itself from the abstract nature of symbols, while still connecting the division symbol (÷) to the act of division through the common (at least, in this galaxy) metaphor of stretching the whole. For these reasons, the pictorial model holds promise for enacting a deeper understanding of the process of division of fractions.

Discussion

We begin a discussion of the parallel experiences presented above with a clarification: Our presentation of alternative models is neither intended to villainize nor canonize the customary symbolic (invert-and-multiply) or pictorial (dissecting and grouping) models for dividing fractions, nor does it suggest a fix or replacement for the way that fraction division is addressed in schools in the form of the common-numerator algorithm or dilation model. Rather, it is an investigation into how we (collectively, as teachers) *feel* we have addressed it—namely, through the provision of multiple models.

When we first stumbled upon the concept of division as dilation, it took a significant shift in understanding to appreciate the nature of division purported by the model. Accordingly, based on our interactions with this "alternate experience," we assume two things about your experience with the above thought experiment. First, we assume that the mechanism of the model is clear. That is, we assume that you could recreate the procedure to model another, similar problem requiring the division of fractions. Second, we assume that the mathematical justification for such a model is (at best) only partially apparent as a result of the limited interaction you have had with the model. The consequence, then, is an experience that pulls in two directions. On the one hand, you have the ability to reproduce a method that will provide correct results; on the other, you likely have little ground, at this point, on which to justify *why* it seems to do so and to interact with the model in ways that go beyond mechanistic use.

This tension creates the milieu from which we wish to make the following crucial point regarding mathematical representations: A mathematical representation gains meaning as a product of doing, and not as an artifact in and of itself. That is, we wish to draw the distinction between a *representation* and a *presentation* as it applies to the teaching and learning of mathematics. Greeno and Hall (1997) call attention to the tendency for educators to misconstrue notations, pictures, and their manipulations as representations in and of themselves, apart from an agent that is actively representing a given concept or idea. For a model to function as a representation, they argue, "someone has to *interpret* it and thereby give it meaning" (p. 366). This interpretation of meaning is the process by which presentations—or "*potential representations*" (Greeno & Hall, 1997, p. 366)—become representations. The distinction is based on the conviction that every act representation is enacted by a representer, and apart from this interaction, a physical manifestation (or presentation) contains no inherent meaning. In other words, it is through learning action between

a presentation and knower that a representation is brought forth. It follows that an essential component of good (that is, meaningful) representations is that they grow out of children's thinking (Fennell & Rowan, 2001).

While we are in agreement that the clever poem depicting the symbolic algorithm for the division of fractions is not tailored to sponsor knowing interaction on the part of the learner, attending to the distinction between presentations and representations means that simply providing a plethora of alternative models does little to address the concern. Although it appears as though this provision will result in a larger proportion of students meaningfully engaging with the mathematics, "the efficacy of multiple representations in the classroom needs more than the multiple representations themselves" (Amit & Fried, 2005, p. 63)—that is, it relies on the manner in which learners understand them as relevant to the solution of the problem.

This is not to suggest that teachers should never present models in the mathematics classroom, nor that no voice should be given to students to do the same under the fear that there is no hope of others finding meaning within presented models. Rather, teachers need to be aware that while their goal in providing multiple models may be to teach a given concept (e.g., fraction division), the models they present might illuminate little more than how to execute (instrumentally speaking) the procedure involved. As such, in presenting models, teachers should encourage discussions that go beyond the completion of the model and toward discussions underlying the concept the model is designed to represent.

Finally, we feel it pertinent to draw explicit attention to the connection between the readers of this article and learners in mathematics classrooms. Based on your experience of interacting with the dilation model presented above, you may feel that it will make little sense to students, and may therefore decide that it is not suitable for sponsoring understanding of fraction division in the classroom. However, our point here is that the feeling of frustration you may have experienced is possible with any model presented to students—no matter how intuitive it may seem to their mathematics teacher. In presenting an opportunity to interact with an alternative model of the concept of fraction division, we hope that we have sponsored an experience that *represents* (for the reader) the experience of many students with the various models that they encounter in the classroom. Specifically, the matter-of-fact introduction of division as dilation was meant to simulate the presentation of multiple models under the illusion that presentation immediately translates into understanding, without subsequent opportunities to interact the models in connection to the concept at hand. As such, it is our hope that readers of this article—like us—are struck by a nagging curiosity, a need to interpret meaning into the dilation model until it moves beyond a failsafe algorithm and emerges as a representation of division. Of course, in presenting this opportunity, we cannot guarantee that this outcome has actually occurred. Perhaps another experience may help:

$$\frac{1}{2} \div \frac{2}{3}$$

Stuck? Just remember:

"Once the tops are equal size, just take the bottoms and divide!"

References

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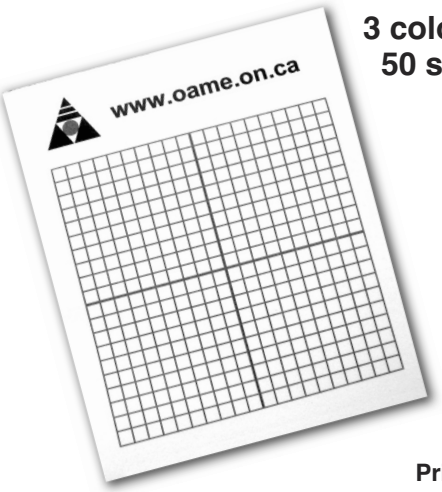
Fennell, F., & Rowan, T. (2001). Representation: An important process for teaching and learning mathematics. *Teaching Children Mathematics, 7*(5), 288–292.

Greeno, J.G., & Hall, R.P. (1997). Practicing representation: Learning with and about representational forms. *Phi Delta Kappan, 78*(5), 361–367.

Skemp, R.R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching, 77*, 20–26. ▲

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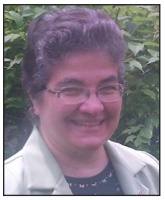
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Provincial Digital Learning Resources – What's New? Enhanced Annotation Tools





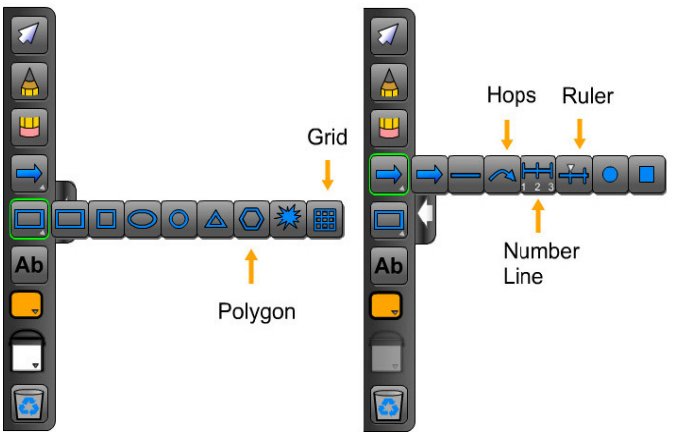


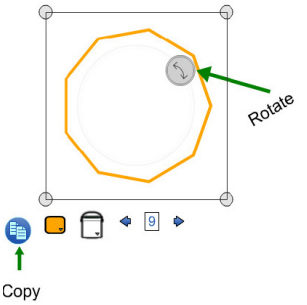


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*Agnes (Brant Haldimand Norfolk Catholic District School Board), Ross (Near North District School Board), Markus (Bluewater District School Board), and Greg (Simcoe Muskoka Catholic District School Board) are all currently on assignment with the Ontario Ministry of Education as Provincial Math Leads, working on the digital resources found at **mathies.ca**.*

Each of the more recent mathies Learning Tools includes a panel of annotation tools that students can use to highlight parts of their solution and communicate their thinking. A major endeavour of the mathies development team has been to enhance this set of annotation tools. Annotation now includes hops and regular polygons in addition to the lines, rectangles, ovals, and other drawing objects that already existed. The new number line, ruler, and grid objects make the annotation tools even more useful for communicating mathematical thinking.





Another exciting addition to annotation is the ability to copy and rotate annotation objects, making the tools much more versatile.

Beginning with Notepad, Rekenrek, and Set, these annotation improvements are being integrated into each of the mathies apps as they are updated.

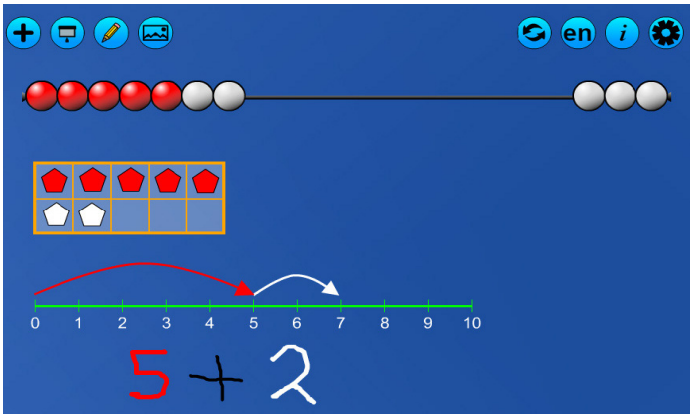
Let's look at some examples of these new features in action!

Multiple Representations

Representing is one of the seven mathematical processes which are foundational to the Ontario Mathematics Curriculum. The front matter of the *The Ontario Curriculum, Grades 1–8: Mathematics* (2005, p. 16) states:

> Learning the various forms of representation helps students to understand mathematical concepts and relationships; communicate their thinking, arguments, and understandings; recognize connections among related mathematical concepts; and use mathematics to model and interpret realistic problem situations. Students should be able to go from one representation to another, recognize the connections between representations, and use the different representations appropriately and as needed to solve problems.

As you can see in this example from the Rekenrek tool, the annotation improvements allow students to work with multiple representations, all within the same workspace.



Here, the Rekenrek tool is used to represent the number 7. The student can readily see that 7 is composed of 5 and 2, since Rekenreks are designed with two colours of beads to support students in understanding anchors of 5 and 10.

The annotation tools can then be used to create related representations. The new grid object and regular polygons

are used to create a ten-frame representation. Hops are used to show the same information on a number line. Finally, the symbolic representation is written using the pencil tool.

Students can now look for connections between the various representations. They could use the text tool and/or the pencil tool to document their thinking. They could even import an image, perhaps a photograph of some work, with a physical manipulative.

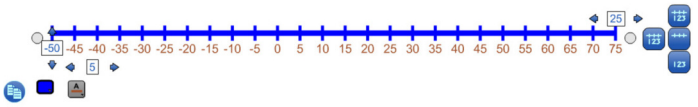
Since this work is being done in a digital environment, it is also possible for students to adjust the various representations in order to explore questions such as, "How would each representation change if the represented value were one smaller? Two bigger?"

Number Lines

Number lines are powerful models that allow students to represent their thinking across strands and grade levels. The mathies annotation number line is designed to equally partition the length into the indicated number of divisions. Here are two number lines of different lengths, both showing the default of ten divisions. Notice that the interval length and labelling automatically adjust in order to maintain the same number of divisions.

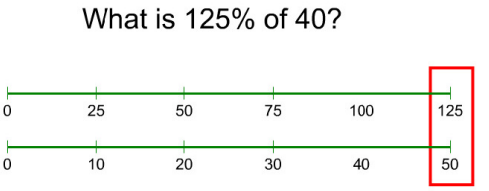


It is possible to customize the number line to suit the problem you are modelling. The steppers can be used to adjust the initial value (-50), the interval size (5), and the number of divisions (25). The two buttons to the right of the copy button can be used to change the line colour/thickness and the label colour, respectively. Use the blue *format number line* button on the right to show either a labelled number line (as shown), just the number line, or simply the labels.



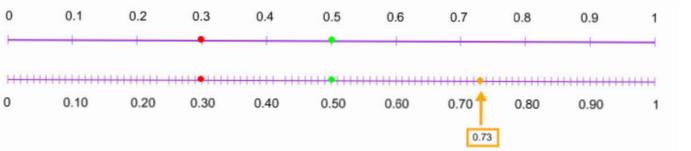
Percent Example:

In this example, the scale on one number line is set to show intervals of 25. This line is then copied, maintaining it's length and spacing. Notice that the copy is automatically placed directly below the first number line to allow for easy comparisons. The scale is then adjusted to model this percentage problem.



Decimal Example:

Currently, the annotation labels only show whole numbers or integers. However, an unlabelled number line, together with some text labels, could be used to create decimal number lines.

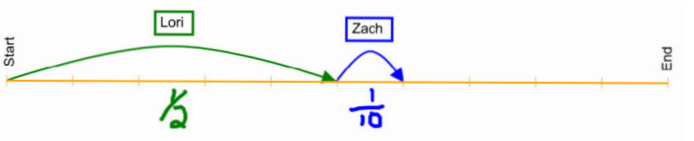


Fraction Example:

An unlabelled number line could also be used to represent fractions and solve problems such as this Relay Race problem, found in the Fractions Learning Pathways Cell OP-D (see **www.fractionslearningpathways.ca**).

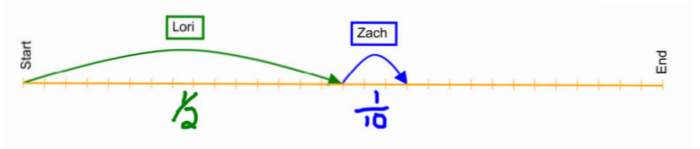
> Lori is putting together a relay team together (Sic.) to run a race.
>
> She will complete ½ of the run. Her friend, Zach will complete ⅒ of the run. If she asks some of her friends to run part of the run, what fraction of the run will each friend need to run?
>
> Use a number line to represent the race and the different distances that were run.
>
> Justify your thinking.

This image shows both Lori's and Zach's run.

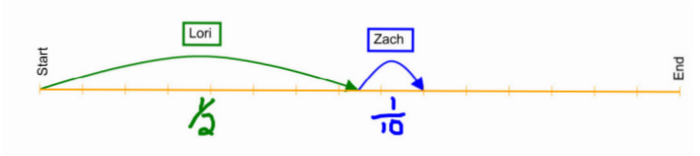


The number line is currently partitioned into ten equal pieces. This might lead students to complete the race, using one friend, who runs the remaining $\frac{4}{10}$ of the distance, or perhaps two friends, who run $\frac{1}{10}$ *and* $\frac{3}{10}$, of the race, respectively. However, the dynamic nature of the number line would allow students to repartition the number line in a variety of ways.

What combinations might this same line yield when partitioned into thirtieths? What might students notice about equivalent fractions?

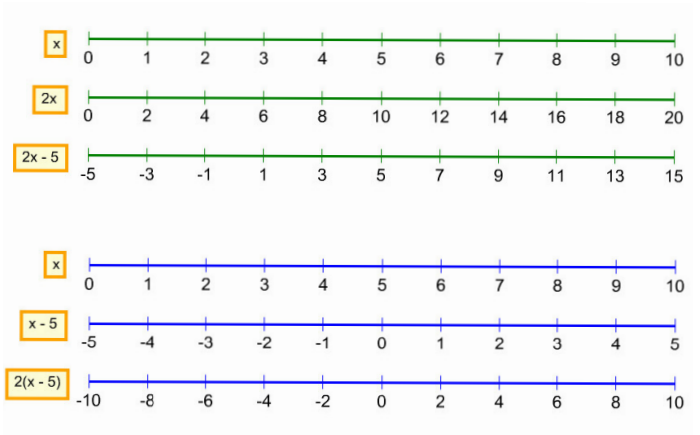


What conversations might emerge if a student chose to repartition the same number line into fifteenths?



Linear Expressions Example:

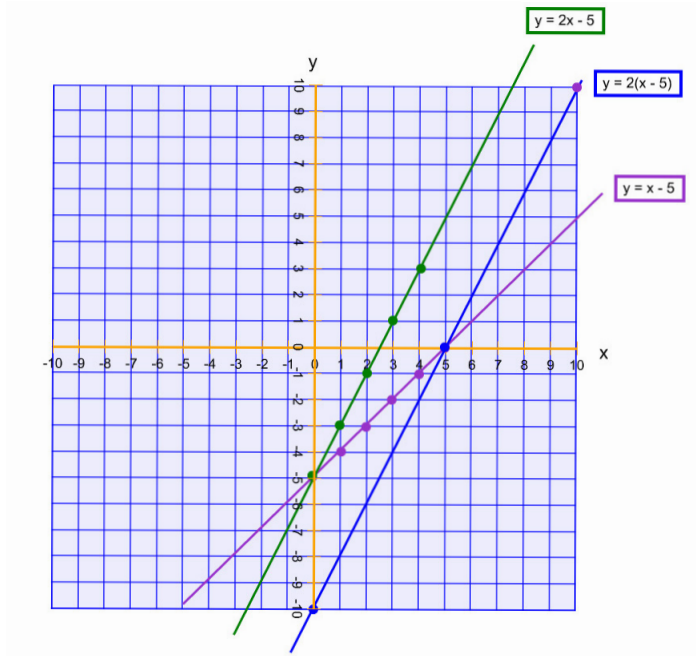
Adjusting the initial value and/or the interval size supports this exploration of linear expressions. Depending on the grade level, you could simplify the task to only show one operation. You could consider this an order-of-operations exploration, or think about it as an efficient way to build a table of values for a linear growing pattern. It could also be used as the foundation for exploring composition of functions.



Comparing the bottom green and bottom blue lines allows students to identify the relationship between $2x - 5$ and $2(x - 5)$.

Graphing Example:

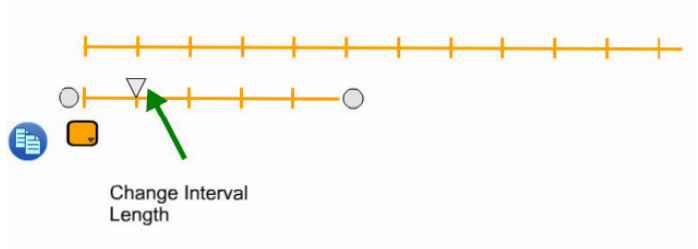
The Notepad tool features a number of different backgrounds (grid, isometric, and lined) in addition to the full set of annotation tools. Here the number line is used to quickly create a pair of axes. The vertical number line was created by setting the starting value to 10, the interval to –1, and the number of divisions to 20. This is a bit tricky, but was done so that the labels would appear on the left side of the axis. The rest of the graph was created using the point and line tools. Once you create one dot, it is easy to copy and reposition it.



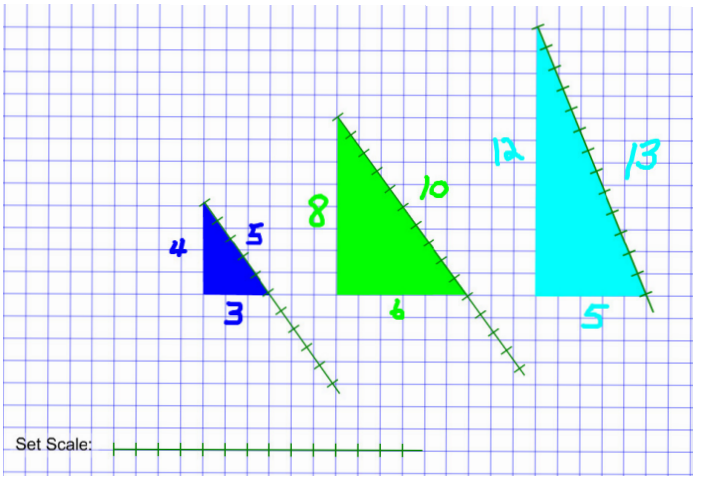
How does this graphical representation support students in thinking about the relationship between $2x - 5$ and $2(x - 5)$ or between $x - 5$ and $2(x - 5)$?

Pythagorean Triples

Annotation rulers behave differently from number lines. Rulers were designed to allow the user to measure objects with non-standard units. As such, the length of the interval remains fixed, regardless of how long the ruler is. The interval length can be adjusted by dragging the triangle located above the first interval tick.



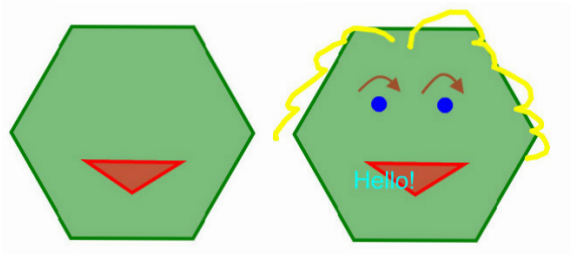
Rulers are useful when exploring the perimeter of shapes, especially those that have slanted sides. This example, created in Notepad, shows how the ruler could be used to determine the length of the hypotenuse of a right triangle. First, adjust the ruler so that it exactly matches the background grid. This establishes one interval on the ruler to be equal to the length of one grid square. Next, make a copy of this ruler (this ensures the scale is identical) and drag it to measure the length of each hypotenuse.



Understanding Annotation Drawing Layers

As you use the annotation tools to create more complex drawings, it is helpful to know a bit about layering.

Essentially, any object from the Shapes menu is created on the shapes layer, with shapes on top of each other in the order they are drawn. Here, the hexagon is drawn first, followed by the triangle on top. If the shapes had been drawn in the opposite order, the hexagon would have covered the triangle.



Text, freehand drawings, and objects from the Lines and Points menu all occupy a second layer, which sits on top of the shapes layer. So, even if the eyes, hair, and text had been created first, they would still be on top of the hexagon and triangle.

As you make adjustments to these sorts of complex drawings, it is helpful to know that clicking on an object brings it to the top, while double clicking on an object sends it to the bottom. Sometimes you may need to temporarily drag an object out of the way in order to make the desired adjustments.

Save and Load

As this article is being written, the team is working on completing an update to Notepad (v 2.0.0), which includes the ability to save and load files. This exciting new feature will allow students to save their work and return to it later. They can then load the file and pick up where they left off. It will also facilitate sharing completed work with peers and teachers. This feature is slowly being added to mathies learning tools as updates occur. Visit **mathclips.wikispaces.com/File+Operations** for more details.

French Versions and Supports

Visit **www.mathies.ca/francais.php** to see a complete list of tools available in French.



The French version of Notepad is called Bloc-notes. They are not really two different tools, but rather, tools that have a language-switching button. Work is also being done to translate various supports into French. The Bloc-notes wiki page can be accessed using the link on the francais.php page or by clicking the Aide de Wiki button found in the information dialog within the Bloc-notes tool.

A French translation of the Set and Colour Tiles tools will be part of the next update.

Flash and Browsers

If you open mathies tools, using an Internet browser, like Microsoft Edge, Google Chrome, Firefox, or Safari, you may notice that permission is required to run them. This is due to the fact that it is using Adobe's Flash player. Sometimes that permission is granted by clicking a link entitled "Get Flash Player," by clicking an Allow button, or by changing the browser settings. If this is problematic in your context, consider using the EXE and DMG installers (if available from the Download button on mathies) or the mobile versions available from the App Store and on Google Play.

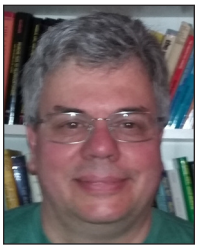
Feedback and Future Requests

Please feel free to send us your feedback about any mathies tool, using the Feedback Form button inside the Information Dialog, accessed from the *i* button. Visit the support wiki page for more examples and detailed descriptions of the functionality of the tool.

You can also send your comments to **WhatsNew@oame.on.ca**. You can share your experiences on Twitter, using the hashtag #ONmathies, and follow or message us at @ONmathies. There is an increasing set of interesting posts of student and teacher work.

To be among the first to find out about the latest digital tool developments, sign up for our email list at **www.mathclips.ca/WhatsNewEmailList.html**. ▲

▲ What's the Problem? Totalling Trains



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Shawn Godin is head of Mathematics, Business, Law, and Computer Science at Cairine Wilson Secondary School in Orleans. He strongly believes in the central role of problem solving in the mathematics classroom. He continues to be involved in mathematical activities: presenting workshops, writing articles, working on local projects, and helping create mathematics contests.

Welcome back, problem solvers. In the last issue, I left you with the following problem:

> *You have a number of 1 by 1 squares and 1 by 2 rectangles. You want to create a 1 by 10 rectangle. In how many ways can you do this?*

This problem is from the "Math 9–12 project: Innovating the high school math curriculum." The project is a research-based study tasked with developing interesting and sophisticated mathematical activities for high school students. The project is led by Peter Taylor of Queens University. You can find further information on the study and check out the current projects at **www.math9-12.ca**.

Let's consider the current problem. In the project, reference is made to making a train out of cars that have lengths 1 and 2. We will use the same language from this point forward. This is an ideal problem to use some manipulatives to help get a sense of what is going on. Square tiles would work fine; I will use linking cubes.

Rather than jump in with a train of length 10, let's try some smaller trains.

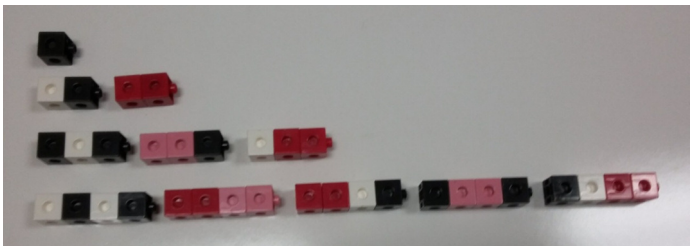


Figure 1: *Trains of lengths 1, 2, 3, and 4*

In Figure 1, we can see the start of the data gathering. If we continue, we can find the values in Table 1.

Length	Number	Examples
1	1	1
2	2	11, 2
3	3	111, 21, 12
4	5	1111, 22, 211, 121, 112
5	8	11111, 221, 212, 122, 2111, 1211, 1121, 1112
6	13	111111, 222, 2211, 2121, 2112, 1221, 1212, 1122, 21111, 12111, 11211, 11121, 11112
7	21	1111111, 2221, 2212, 2122, 1222, 22111, 21211, 21121, 21112, 12211, 12121, 12112, 11221, 11212, 11122, 211111, 121111, 112111, 111211, 111121, 111112

Table 1: *Trains of various lengths*

Upon closer inspection, it seems a pattern is emerging. If we let represent the number of trains of length , then we get

$$t_3 = t_2 + t_1$$
$$t_4 = t_3 + t_2$$
$$t_5 = t_4 + t_3$$
$$t_6 = t_5 + t_4$$
$$t_7 = t_6 + t_5,$$

so it would seem that we have the recursive formula

$$t_1 = 1,\ t_2 = 2,\ t_n = t_{n-1} + t_{n-2},\ n > 2.$$

If this is indeed the case, we could easily complete our problem by calculating the next few terms to determine that there should be 89 trains of length 10. This is indeed correct, but just because "there seems to be a pattern" doesn't actually mean that one exists. We should see if we can justify what is happening. If we look at a couple of examples, it may become clear. In Figure 2, there are two trains of length 10. The top one is 11112112, while the bottom one is 2212111.

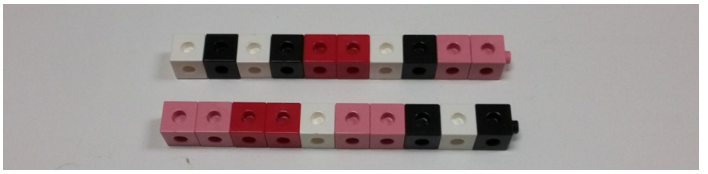


Figure 2: *Two trains of length 10*

One of the trains starts with a car of length 1, and the other starts with a car of length 2. What's the big deal? It turns out these are the only two ways that a train can start. If we remove the starting car, we obtain the configuration in Figure 3.

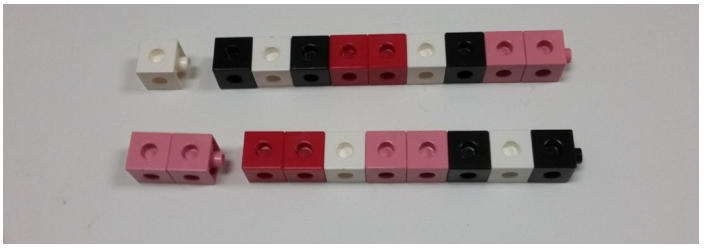


Figure 3: *Removing the first car*

We see that when we remove the first car, we are left with either a train of length 9 when the first car has length 1, or a train of length 8 when the first car has length 2. Thus, the number of trains of length 10 that start with a car of length 1 is the same as the number of trains of length 9. Similarly, the number of trains of length 10 that start with a car of length 2 is the same as the number of trains of length 8. Hence, $t_{10} = t_9 + t_8$, and in general $t_n = t_{n-1} + t_{n-2}$.

Now you may ask yourself, what was so special about the first position? If we look at our two trains again, the one on the top has a car of length 2 occupying positions 5 and 6, while the other one doesn't. That means that the top car can be split into two trains of length 4 and a car of length 2, while the bottom can be split into two trains of length 5, as shown in Figure 4.

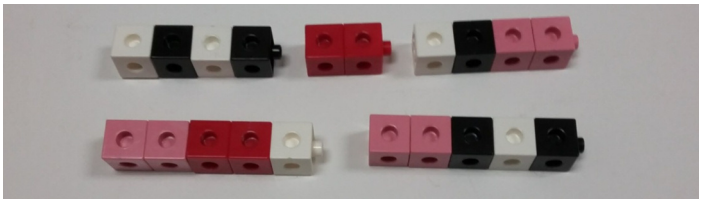


Figure 4: *More splitting of trains*

This means that the number of trains of length 10 should be given by

$$t_{10} = t_4 \times t_4 + t_5 \times t_5$$

A quick calculation shows this is true. You can go through the same process to obtain a number of other ways to calculate t_{10}.

The sequence of the number of trains of various sizes is a subsequence of the well-known Fibonacci sequence 1, 1, 2, 3, 5, 8,…, which is mentioned specifically in the Discrete Functions strand of the MCR3U course. This problem leads to many areas of investigation well worth exploring. The aforementioned project leads the exploration to some Fibonacci identities as well as a tie in with Pascal's triangle.

We will look at one more connection that we can make. If we plot our results from Table 1 with Desmos and add in the trains up to length 10, the graph looks like an exponential function. Getting Desmos to determine the curve of best fit of the form yields , which is quite good, as shown in Figure 5.

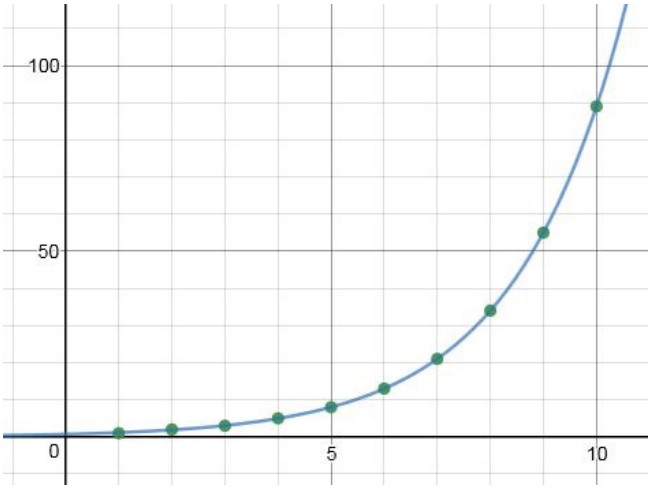


Figure 5: *Graph of number of trains*

If we assume that the terms have the form and insert this expression into the recursion relation, we get

$$a \times b^n = a \times b^{n-1} + a \times b^{n-2},$$

which simplifies to $b^2 = b + 1$ and has solutions $\frac{1+\sqrt{5}}{2}$ and $\frac{1-\sqrt{5}}{2}$. If you assume that the term must then be a linear combination of powers of these two numbers, we can substitute some terms, perform a little computation, and obtain

$$t_n = \frac{5+\sqrt{5}}{10}\left(\frac{1+\sqrt{5}}{2}\right)^n + \frac{5-\sqrt{5}}{10}\left(\frac{1-\sqrt{5}}{2}\right)^n.$$

This is similar to Binet's formula for the Fibonacci numbers. It is beautiful in the sense that the formula is full of radicals, but the result is always an integer. A little checking shows that the first term is close to what Desmos gave us. The base in the second term is smaller than 1 and provides the "correction" needed to bring the result to an integer.

The Fibonacci sequence could provide you with years of exploration, and you would still only scratch the surface. There are many wonderful properties of the Fibonacci sequence that are difficult to prove, but they can often be established with an argument similar to the one applied to the trains. One example, for your amusement, is to show that the sum of the squares of consecutive Fibonacci numbers is itself a Fibonacci number. Now I will provide you with some homework to add a little variety to your life:

On a certain island live only knights, who always tell the truth, and knaves, who always lie. Knights and knaves are indistinguishable by sight. One day, you find 10 island natives standing in a circle. Each one states: "Both people next to me are knaves!" Of the 10 in the circle, what is the minimum possible **number** of knights?

Until next time, happy problem solving! ▲

Technology Corner: GeoGebra Materials



MARY BOURASSA
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Mary Bourassa teaches mathematics at West Carleton Secondary School in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She has presented workshops internationally, authored mathematics resources, is a Past VP of OAME, and a Past President of COMA. An award-winning teacher, Mary continually strives to learn new and better ways of helping students learn and love mathematics.

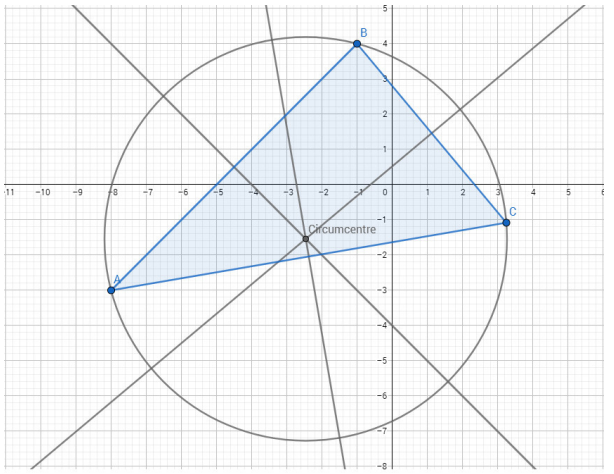


Figure 1: *Construction of the circumcentre*

Many of you may have created simple materials in GeoGebra, like the circumcentre shown in Figure 1. GeoGebra is a very powerful, free tool, and useful for exploring so much more than just geometry. It calls itself "The graphing calculator for functions, geometry, algebra, calculus, statistics, and 3D math!" With over a million free and interactive materials available at **www.geogebra.org**, you can find pretty much anything you might want, without having to create it yourself.

Although there are numerous great creators of GeoGebra work, I am highlighting materials by Steve Phelps (@giohio), whose work can be found at **www.geogebra.org/stevephelps**. He has created a series of challenges that he tweeted out recently, where you are given a target equation and must move the given graph by dragging the two points so that they will be located on the graph of the target function. An example with quadratic equations in vertex form is given in Figure 2. When you click the "Check your graph?" box, your graph will change colour if correct, incorporating the points in your graph.

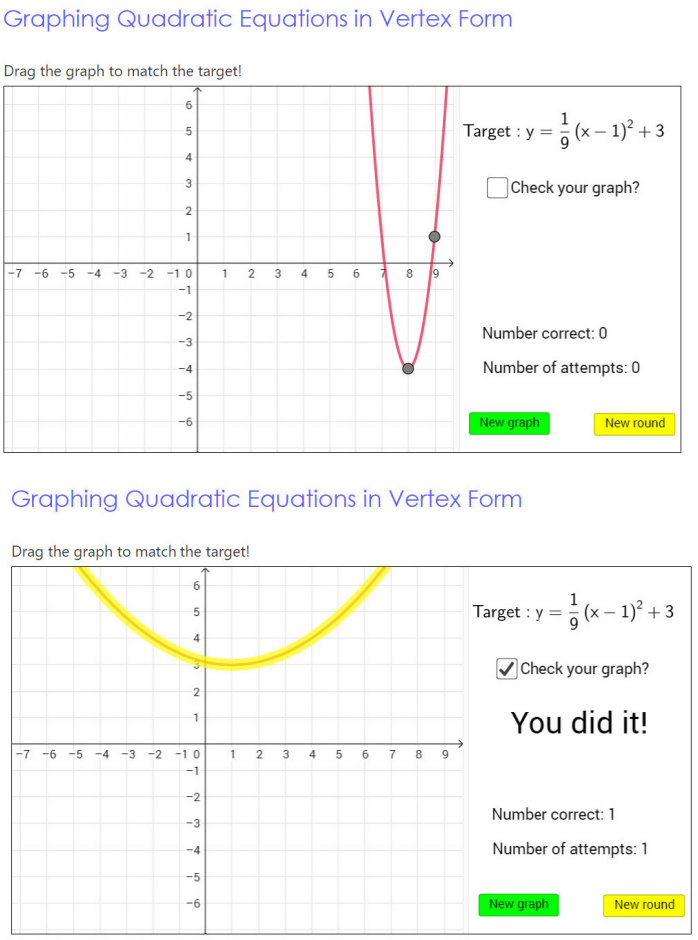
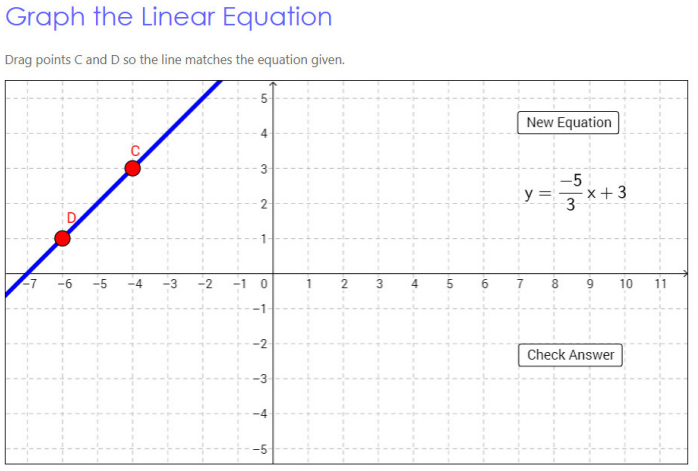


Figure 2: *Graphical matching, with the given at the top and the solution at the bottom*

Similar activities for linear, cubic, and sinusoidal graphs are shown in Figure 3.



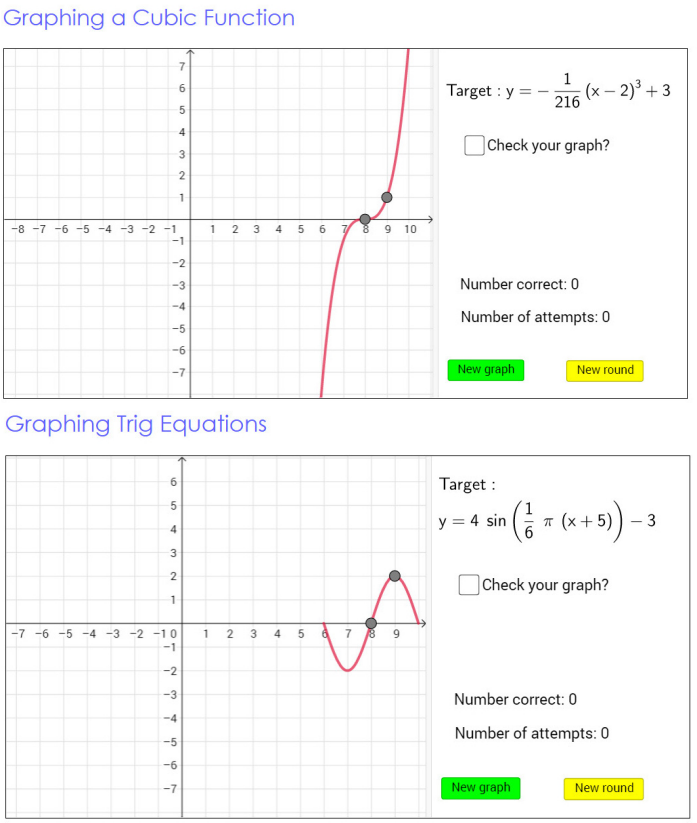
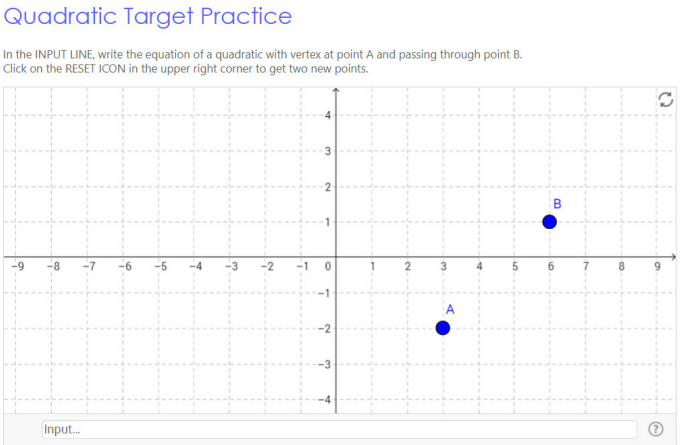


Figure 3: *Examples of linear, cubic, and sinusoidal graphs*

Other materials take the concept up a layer of abstraction by only providing two points, not the entire shape of the graph. The instructions specify that the vertex of the parabola is at point A and that it passes through point B, so there can only be one correct graph. Steve has created linear target practice, quadratic target practice, circle target practice, trigonometric target practice, and even triangle target area. The quadratic target practice looks like Figure 4.



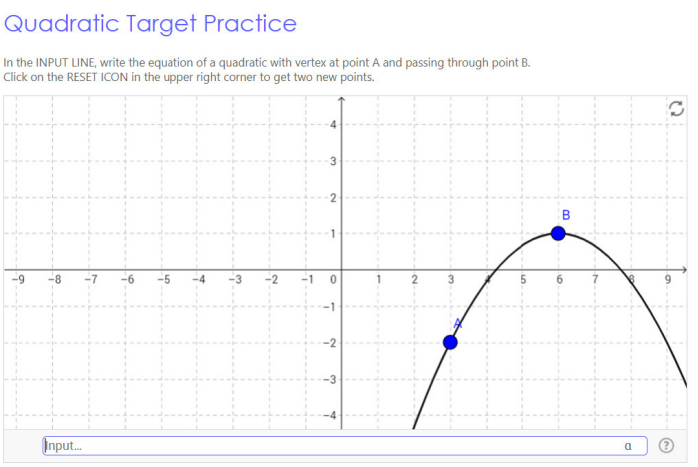


Figure 4: *The quadratic target practice example*

When you perform a search on someone's GeoGebra page, you also obtain related results from other users further down the page. This is how I found the following quadratic targets activity by "wyattmg1" in Figure 5. In this activity, you must determine multiple quadratic equations such that each of the blue points has one parabola that goes through it, while the red points have two parabolas that go through them.



Figure 5: *Fitting multiple quadratics*

Steve has also created a series of "What's My Rule?" materials, where you must determine the relationship between values A and B by dragging A on a number line and observing what happens to B. An example is shown in Figure 6.

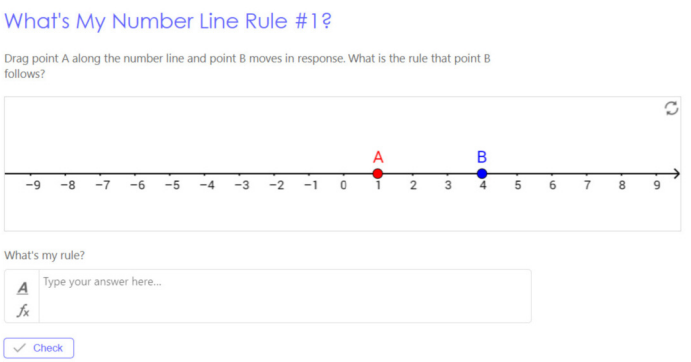


Figure 6: *Investigating a hidden rule*

I found the equation of the graph of the parabola shown in Figure 7 very interesting. Students could notice patterns and how they tie into the "A" value for the equation.

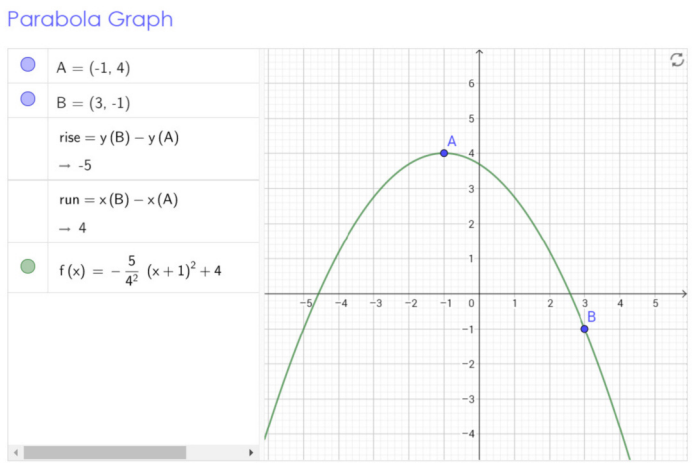
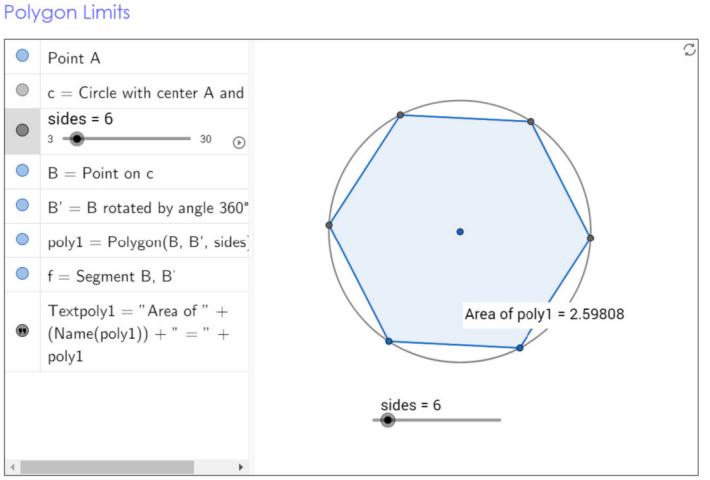


Figure 7: *Make an equation to fit the points*

Those of you teaching calculus can find an abundance of materials, such as the two examples shown in Figures 8 and 9, relating to limits and optimization.



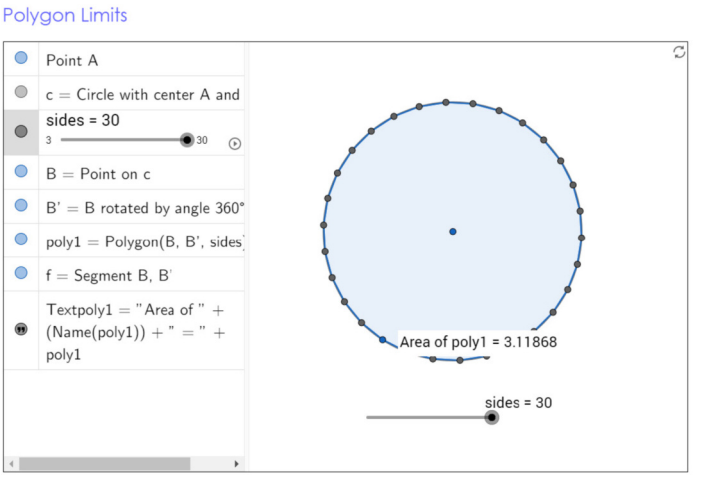


Figure 8: *Example of a limiting process*

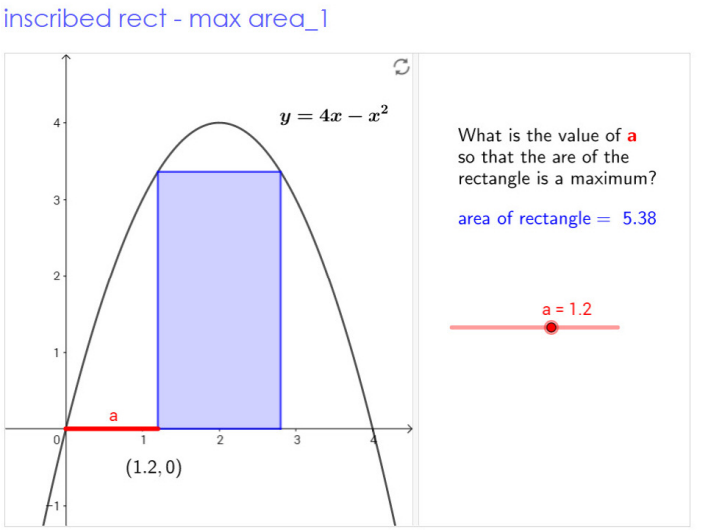


Figure 9: *Example of optimization*

Steve also shows off GeoGebra's ability to take a freehand drawn graph and manipulate it. Check out the cool transformations shown in Figure 10.

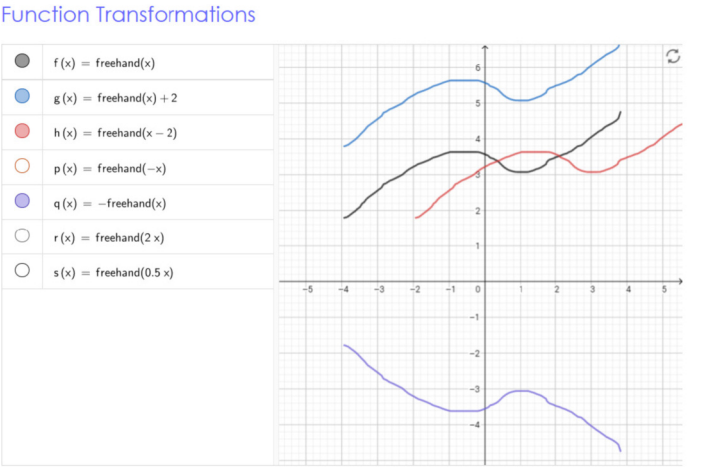


Figure 10: *Example of freehand transformations*

There is so much to explore! If you are on Twitter, be sure to follow Steve Phelps (@giohio) along with Audrey McLaren (@a_mcsquared), John Golden (@mathhombre), and Tim Brezezinski (@dynamic_math), among others. To end, Figure 11 shows an amazing creation from Steve:

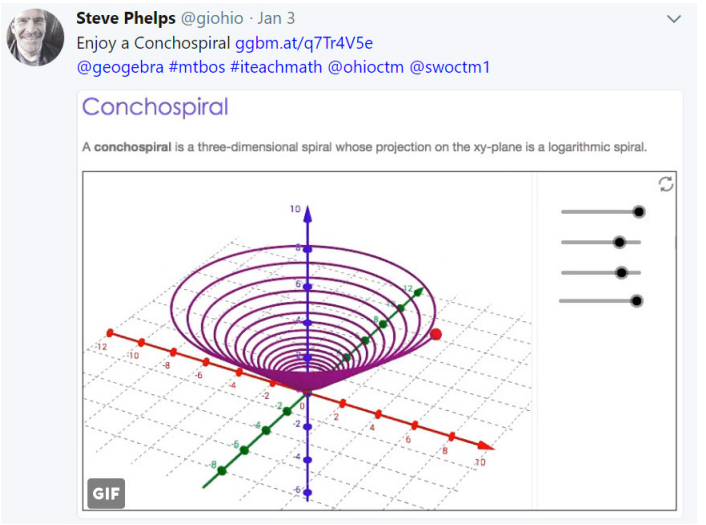


Figure 11: *An interesting creation* ▲

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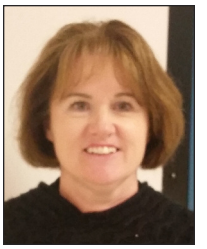
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▲ OAME/NCTM Report



JACQUELINE HILL
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Jacqueline Hill is a Grades 9–12 teacher of mathematics, as well as an online instructor for York University. She is a Past President of OAME and OMCA, as well as the recipient of the award for Exceptional Teaching in Secondary Mathematics. She also wrote the "Director's Dialogue" for the OAME Gazette *for a number of years.*

In 2010, the Ontario Ministry of Education published the *Growing Success* document. It brought clarifications and conversations to schools. Our schools have come to terms with the "homework" issue. Homework that is given to consolidate students' knowledge skills, or in preparation for the next day, cannot count as assignments for evaluation (p. 39). So anytime I read comments with regard to this topic. I stop and take notice. With this in mind, I would like to reflect on Todd Romiens' March 2017 NCTM report. His work always gets to the heart of the issue.

As a classroom teacher, I look forward to my copy of the *Gazette*. I often look for puzzles, computer applications, or articles that I can share with my students. Todd wrote about the NCTM's stand and view on homework. I took the article to my students and asked for their feedback. The assignment was written in their "Orange Book"—the place they ink their thinking, otherwise known as their journal. The assignment was to summarize the article and then write a letter to the author with their thoughts on the topic. Now, I must tell you that my students are very familiar with the *Growing Success* document (in particular, page 39 about homework, and page 43, which discusses the 16 things a teacher should do before assigning a mark of "0" to a student). Here are some of my comments from my Grade 9 Academic students:

> "I don't like it just as much as the next kid, but it has definitely played a role in my understanding of concepts" (D.C.).

> "Unfortunately, I disagree with this method of learning (homework), but I know that it is a better solution for some, but in troubled families or those who lack motivation or time, I don't see this as a very efficient way to practise a subject that can just as easily be done in class, where students can get professional help and have access to better resources" (T.R. Grade 9 Academic).

"Homework should never be stopped, but they could, however, limit the amount of work they bring home" (E.H-D.).

"Overall, I think that reading this article has changed my perspective on my view of homework. Although I already do my math homework, and the *Growing Success* document states, 'assignments for evaluation must not include ongoing homework that students do to consolidate their knowledge,' I will continue to do my homework and can now see the purpose of doing so" (J.J.).

My Grade 11 Workplace class could not see the need for any homework. The students felt that as long as they were able to summarize their notes each day, they would not have to spend any time at home working on concepts. They also felt that since the pace of their course enabled them to do a single concept in three or four days, and since the tests were cumulative, they did not have to study or do homework in order to succeed in the course.

From a grassroots perspective, I suggest that the Gazette can be a tool in your classroom for conversations—just as it was with Todd's column, in my classroom. In this case, the column was inspired by an NCTM position. The NCTM considers many such topics that are relevant and meaningful at all levels of education.

NCTM important dates to note:

NCTM Annual Conference, April 25–28, 2018

Walter E. Washington Convention Center, Washington, DC.

NCTM top three things of interest on the website right now:

1) At a classroom level – Under the Classroom Resources tab, there is a section entitled "Activities with Rigor and Coherence – ARCs." There are K–12 resources that get at the heart of understanding.
2) At a provincial level – Every Student Succeeds Act – ESSA Toolkit is located on the website and is very helpful to teachers in their preparation of lessons and activities.
3) At a university level – January 1, 1970 was issue 1 of the *Journal for Research in Mathematics Education*. Each year, many topics with regard to education are explored and prepared in a meaningful way—everything from how what is taught affects students to what should be selected to teach children.

Reference

Ontario Ministry of Education. (2010). *Growing success: Assessment, evaluation, and reporting in Ontario schools*. Toronto, ON: Queens Printer for Ontario. ▲

▲ Math Jams: Students Analyzing, Comparing, and Building on One Another's Work



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Naomi Norquay is an Associate Professor in the Faculty of Education, York University. She teaches pre-service foundation courses and graduate courses in research methodologies and curriculum theory. Her main research documents a disappeared Black pioneer settlement in Grey County, Ontario. She has been experimenting with Comics Jams *in her pre-service teaching and has welcomed the opportunity to collaborate with Tina Rapke on this project.*



TINA RAPKE
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Tina Rapke is an assistant professor at York University in the Faculty of Education. She views her research and teaching as related, seamless, and complementary. She and Naomi have been having a lot of fun Jamming*!*

Math Jams is a fun, engaging, active way for students to employ deep approaches to learning mathematics at any level, from elementary school to university. The creation of *Math Jams* was inspired by *Comics Jams* and emerging mathematics education research that focuses on effective use of student thinking in the classrooms (e.g., NCTM, 2014; Smith, Hughes, Engle, & Stein, 2009). Here we detail exactly what *Math Jams* are, talk about implementation, and consider why you would want to use *Math Jams* in your own classroom. We also give you some sample *Math Jams* from a Grade 4 classroom and a university education classroom. We hope that after reading this article, you will be inspired to use *Math Jams* in your own classroom and encourage your friends and colleagues to try out *Math Jams*.

What are *Math Jams*?

Math Jams are a spin-off of *Comics Jams*. Let us first explain *Comics Jams*. *Comics Jams* are attributed to Rupert Bottenberg, a Montreal-based illustrator, who established "COMIX JAMS" as a collaborative medium for illustrators and the general public to create comics together. We were

drawn to the collaborative nature of this medium, mindful of the educational uses of comics in the classroom (Bitz, 2009). In a *Comics Jam*, each member of a group—usually three or four people—starts one story by illustrating a single frame (both picture and words). Each subsequent member continues the storyline by first analyzing what has already been contributed, and then building on the initial story by completing the next comic frame. This continues until all group members have contributed one frame to each of the stories in the group. Each comic is then returned to the group member who started that story, and the comics are then discussed by the group. Group members gain insights into how the others took up their ideas. We have been using *Comics Jams* to engage students in weekly readings and lectures. Students craft collective comics on a given topic related to course themes by responding to a prompt such as, "What are you left wondering about?" or, "Create a comic that addresses [a particular theme or idea]."

We have adapted *Comics Jams* to fit within mathematics classrooms. For a *Math Jam*, students are also placed in groups of three or four members, but they work together as a group on a single given mathematics task on paper or whiteboards situated around the classroom. We monitor the mathematical ideas that students are negotiating and recording. Once we see that each group has something that another group can analyze and/or compare to their own strategies and build upon, we tell the students, "It's time to *Jam*!," meaning that each group needs to move to the workspace of another group. We tell students to first try as a group to make sense of what the other group left them and then decide on at least one idea or at least 20 percent of the work to keep and build upon to produce a revised response that clearly demonstrates the mathematical thinking that took place during the solving process. Note that we have only had students move once and only build on one other group's ideas. At the end, students return to their original workspace to reflect on how their classmates have built on their mathematical ideas.

Why should you try out M*ath* J*ams* in your own classroom?

You should try out *Math Jams* in your practice because they: 1) are engaging and active ways for students to employ deep approaches to learning mathematics, and 2) can be used to respond to calls in the research about emphasizing students' mathematical thinking in classrooms.

The benefits of *Math Jams* can be connected to several areas of research, including student engagement, deep approaches to learning, and active learning. Some researchers (e.g., Fredricks, Blumenfeld, & Paris, 2004) discuss student engagement in terms of behavioural engagement and emotional engagement, using descriptors like *enjoyment* and *interest* (e.g., Trowler, 2010). When we have used *Math Jams*, we have observed students laughing and smiling. In one university classroom, dancing actually broke out during a *Math Jam*. This is significant, as students' enjoyment of mathematics is directly related to their achievement in mathematics (García, Rodriguez, Betts, Areces, & Gonzalez-Castro, 2016). *Math Jams* can also be spoken of in terms of Lambert and Sugita's (2016) conceptualization of engagement "not simply as on-task behaviour, but as engagement in the practices that define mathematical activity" (p. 348). They go on to speak about students engaging with the mathematical strategies of other students, which is also indicative of deep approaches to learning. Seeing things from different perspectives is one way to describe deep approaches to learning (Marton, Beaty, & Dall'Alba, 1993). *Math Jams* are designed so that students have to make sense of others' solution strategies (see things from a different perspective), and build on them. *Math Jams* also indicate active learning, as they encourage students to take on an active role in the learning process. In this way, *Math Jams* might be considered antithetical to the traditional lecture, wherein students sit passively, take notes, and formulate pertinent questions to ask (Röj-Lindberg, 2001).

The mathematics education community clearly values students' mathematical thinking in the classroom. This is evident in Ontario mathematics curricula, where thinking is one of the categories in the achievement chart, and where there are questions addressing thinking in Ontario's provincial testing (EQAO). However, the *actual* implementation of using students' mathematical thinking in classrooms has proven to be challenging and complex (Leatham, Peterson, Stockero, & Van Zoest, 2015). It is suggested that "[e]ffective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments" (NCTM, 2014), but there are few practical, explicit, and easily implemented strategies.

Smith and colleagues (2009) provide one of the few exceptions of explicit suggestions for orchestrating productive mathematical discussions. They offer five practices to help teachers use students' mathematical thinking. In the practice of sequencing (one of the five practices), they clearly articulate, "…the goal is to have student presentations build on each other to develop powerful mathematical ideas" (Smith et al., 2009, p. 554). This process is meant to support students in building shared understanding. This is starkly different from students working individually, with potentially only a few students in the

classroom understanding the topic/strategy. However, the focus is on students' presentations and thus on oral communication.

Math Jams have a similar goal of students building on each other's ideas to develop and spread understanding among students; however, *Math Jams* are not focused on oral presentation, but rather on written forms. Specifically, *Math Jam*s make it essential that students interact with and analyze each other's *written* ideas, developing their understandings by ultimately returning to reflect on how their classmates built on their ideas. In fact, NCTM's suggestions about effective teaching, the goal of Smith and colleagues, and our own work with *Comics Jams*, motivated us to create *Math Jams* as an explicit strategy that has students analyze and build on one another's mathematical ideas in writing.

Implementing M*ath* J*ams*

When implementing *Math Jams*, it is helpful to think about the following: the types of problems to use, directions to give when launching, timing, and how to support students to engage with their classmates' work. We suggest using problems with several entry points, as these will allow for several explanations/ideas to be built upon, clarified, and perhaps related to each other. Furthermore, if students do not initially understand the problem, there are many questions the teacher can ask before and after the *Jam*. (We will highlight such questions in the examples we provide.) Having problems with multiple entry points and an attentive teacher who is closely monitoring ideas as they emerge can increase the opportunities for students to communicate and engage with one another's ideas. In terms of timing, we were concerned that if the jam occurred too soon after launching the task, then students would have difficulties finding an idea to build upon, and if too late, then students would have too little to do.

We recommend providing explicit instructions when launching *Math Jams*, as these can help address concerns about students' desire for ownership over their individual ideas (e.g., Bruce, 2007). Indeed, in our implementation of *Math Jams* at the elementary level, students mentioned "stealing ideas," even after the teacher was very purposeful in describing that *Math Jams* required students to share and build on each other's ideas, highlighting the benefits of doing so. For example, our collaborating elementary teacher explained to students that the "basic idea of a *Math Jam* is about sharing ideas. But not only sharing ideas by talking about them, we're actually going to be sharing ideas by trading them." He reiterated that: students were to work together and not alone; when they engaged another group's work, they would decide what part of that group's work they would carry forward to work up with their own ideas; and finally, at least one idea from the other group's work had to be built upon. The teacher then asked the students why they might be doing a *Math Jam* and asked what the students thought would be the benefits of doing a *Math Jam*. One student said, "We could get, like, different ideas… we'll learn from them," and another student said that he could steal ideas. The teacher was very particular in clarifying that "we're not stealing because we know we're passing them around and we're accepting them and giving them freely." We have implemented *Math Jams* in classrooms where teachers have an awareness of collective understanding (as described in Martin, Towers, & Pirie, 2006), which may have played a role in their success. We have experienced *Math Jams* in terms of students sharing responsibility with their group mates, with ideas owned by the group collectively, freely shared and accepted.

When supporting students to engage with one another's work, the concepts of sharing and building upon are important when considering how to alleviate students' anxiety about classmates seeing their work. In the literature on peer feedback, there is discussion about alleviating the anxiety students may have with classmates seeing their work by providing positive feedback first (Topping, 1998). In *Math Jams*, it is anticipated that anxiety may be alleviated by building upon and sharing ideas. When students receive/accept classmates' ideas, the students have already worked on the mathematical problem and are essentially making sense of the ideas in light of their previous engagement with the problem. This means that at first glance, it may appear that students are simply correcting their classmates' work or judging that their ideas are not valuable. However, students are in fact mapping their previous thinking onto the ideas they receive. It may be that a student/group experienced a similar misconception to one present in the ideas they received, and may have chosen to build on the misconception by providing a rationale to explain why it is a misconception. In the language of effective peer feedback (Lu & Law, 2012), students are identifying growing misconceptions and addressing them with concrete ideas about how to move the shared ideas forward.

Example of a M*ath* J*am* in a Grade 4 Classroom

Our first example of a *Math Jam* comes from an inner city Grade 4 class that consisted of students with diverse experiences and ways of knowing. The school acknowledges that many of their students struggle with poverty and language barriers. The task given addressed the Grade 4 Ontario mathematics curriculum expectation on proportional relationship, wherein students must

"demonstrate an understanding of simple multiplicative relationships" (Ontario Ministry of Education [OME], 2005, p. 68). Students were told that 36 cookies cost $12, and that Hanna pays $2, Joanna pays $3, Leanna pays $4, and Nanna pays $3. They were then asked, "How should the cookies be shared fairly based on how much each paid?"

Students worked in groups of two or three on the task. When students had difficulties in how to attempt the problem, teachers asked them about fairness. They asked questions like: "Would it be fair if Joanna and Nanna had a different number of cookies?" "Would it be fair if Hanna had 2 cookies and Leanna had 2 cookies?" "If Hanna had 3 cookies, how many cookies would Leanna have for it to be fair?" These questions demonstrate the multiple entry points the task has. Figure 1 is an example of students' work. The original group wrote in blue and the next group in green.

In the upper left-hand corner (written in blue) is the idea that students decided to keep to build upon from the work that they received. Clearly the students (who received this work) analyzed this work and compared it with their own. This is evidenced as the students analyzed their classmates' work and wrote, "They were doubling the numbers." They then compared it to their own strategies, as is evidenced by them saying, "I used the doubling," meaning that they did not judge the ideas to be invaluable, but actually found them similar to their own initial engagement with the problem. Students are communicating that their classmates' strategy cannot be used to obtain a solution because 4+6+8+6=24, and in order to get 36 (the total number of cookies), they would have to add 12 more (i.e., 24+12=36). The students then offer a correct answer and check their solution (as they also did for their classmates' guess) by adding up the numbers and confirming that they come to 36 cookies. Essentially, students are demonstrating the Grade 4 mathematical process of reflection by "comparing and adjusting strategies used, by explaining why they think their results are reasonable" (OME, 2005, p. 65).

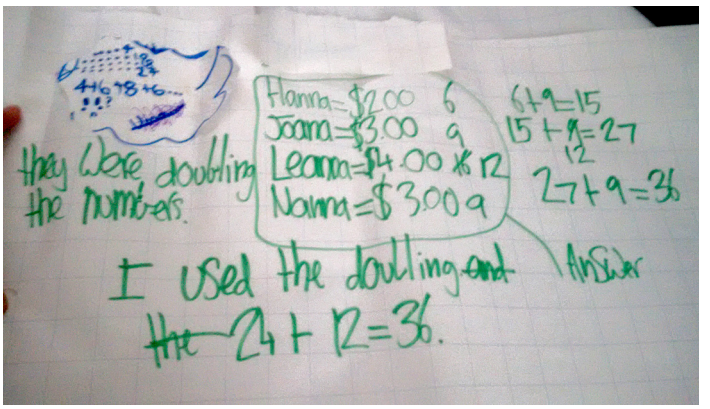


Figure 1: *Example of a Grade 4* Math Jam

Example of a *Math Jam* in a University Classroom

Our second example comes from a mathematics education course taken by future teachers. This problem was initially developed to address the Grade 6 curriculum expectation: "solve problems requiring conversion from larger to smaller metric units" (OME, 2005, p. 90). Students were told that a group of eight students are on a field trip when their bus breaks down 40 km away from their school. A teacher takes four of them back to school in her car, travelling at an average speed of 120 km/h. The other four students start walking toward the school at a steady rate of 6 km/h. The teacher drops the four at school and then immediately turns around and comes back for the others. Students are asked, "When does the teacher reach the second group of students?" They are given five choices (20 mins, 30 mins, 30.5 mins, 38 mins, 39 mins) and asked to indicate and explain their choice. Student worked in groups of three or four.

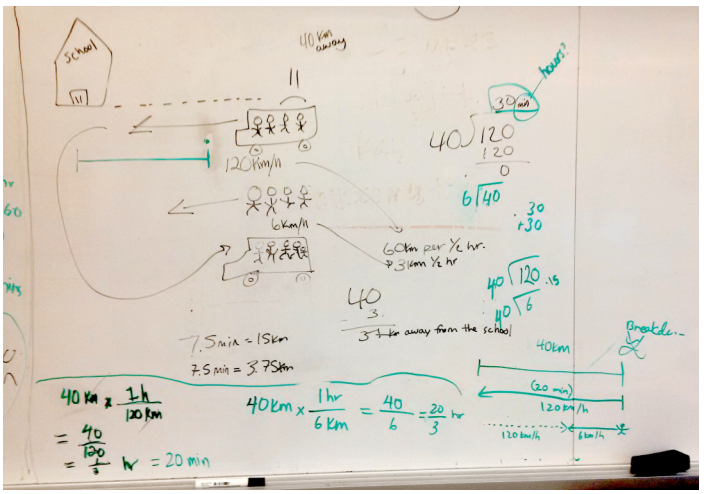


Figure 2: *Example 1 of a* Math Jam *with future teachers*

When students were experiencing difficulties, the instructor asked questions such as: "How long does it take the teacher to travel 60 km?" "How long does it take the teacher to travel 30 km?" "How long does it take the teacher to travel 40 km?" These questions demonstrate the multiple entry points students can take when approaching the problem. The black writing in Figure 2 is from the original group, and the second group wrote in green. Notice in the upper right-hand corner of Figure 2, students have used division and the numbers 120 and 40 to obtain an answer of "30 mins." The second group of students indicate that they analyzed their classmates' work by circling "mins" and writing "hours?" In the bottom left-hand corner, they address/build on ideas about division and the use of the numbers 120 and 40 by offering a different strategy and obtaining a solution of "1/3 hr = 20 mins."

In Figure 3, students have indicated that they have analyzed their classmates' work (the black writing) by extrapolating/explaining some of the ideas. Specifically, in the upper left-hand corner of Figure 3, "38 km" is written in black. Underneath the diagram, they have explained where the 38 km comes from. The students first compute how far the students have walked in 20 minutes (the time for the teacher to drive back to the school), and then explain that the teacher would have to travel back at most 38 km.

Discussion

We note similarities between the Grade 4 class and the teacher education class. Notice that in the *Math Jams* we have provided, students in both classes have improved each other's work, making the thinking more clear and concise (e.g., Figure 3), or they have addressed misconceptions (e.g., Figures 1 and 2). With regard to timing, when students seem to be working faster, or when the *Jam* occurs later in the lesson, students will polish each other's work, adding or deleting text to make the thinking more clear and concise. When the *Jam* occurs soon after the launch of the task, or if students are working more slowly, students tend to identify misconceptions and offer a rationale and a better idea. It seems that, regardless of the timing of the *Jam* or of the level of learning, students still analyze and build upon one another's work.

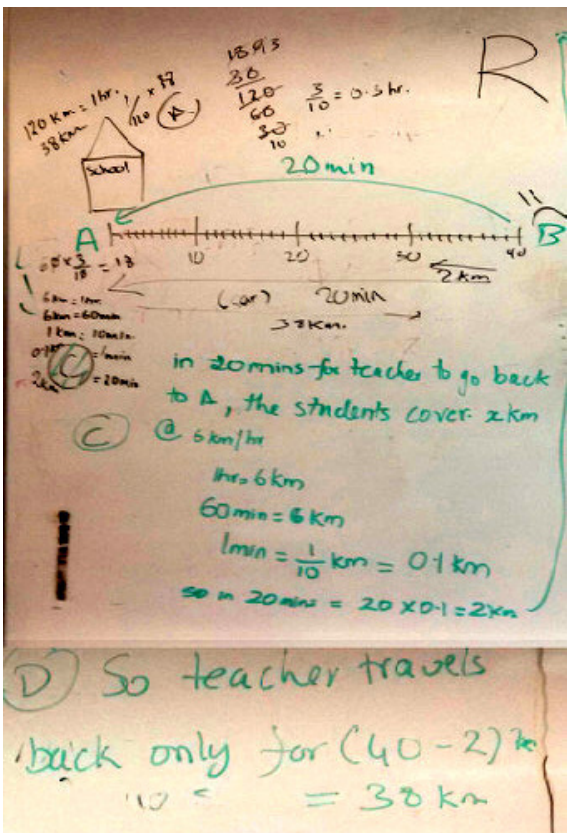


Figure 3: *Example 2 of a* Math Jam *with future teachers*

In Figure 3, students have built on their classmates' ideas by providing annotations to the original work. They have clarified their classmates' ideas, as they have provided details as to why the original picture contains 38 km. Figures 1 and 2 document *Math Jams* where students address misconceptions, showing that they have analyzed their classmates' work by indicating that their classmates' solutions/strategies are incorrect. "Building upon" has occurred in the sense of offering a rationale for why the initial group's work is not a solution (e.g., the total is not 36 cookies), or offering a strategy that clarifies that 3 is related to 1/3 of an hour, which means it takes the car 1/3 of an hour, or 20 minutes (not 30 minutes), to travel 40 km at a speed of 120 km/h. In other words, students have made sense of someone else's solution/perspective in terms of their own thinking by offering a new strategy that involves all the same numbers. Furthermore, *Math Jams* support students to make sense of why their work was incorrect and address their misconceptions by returning to their original workspace to reflect upon how their classmates have analyzed, compared, and built upon their work. This is in line with research findings that students engaging with classmates' mathematical ideas is indicative of their achievement (Ing et al., 2015).

Conclusion

We hope that *Math Jams* can offer you, the reader, a valuable strategy to increase communication and opportunities for your students to interact with one another's ideas. The evidence we have provided here indicates that students in *Math Jams* are engaging with each other's ideas by analyzing, comparing, and building on those ideas. In *Math Jams*, mathematics learning is deep, engaged, and active, with students sharing the responsibility of building mathematical solutions and understanding collectively.

It is notable that as students are taking responsibility and acting collectively as a group, the process is not teacher-centred, and thus can be noisy. A noisy classroom is often understood by teacher candidates to be evidence that the teacher lacks classroom-management skills. Learning classroom-management skills that address the noise associated with groups is often a priority for beginning teachers (Cakmak, 2008; Pereira & Gates, 2013). In comparison to other group problem-solving environments that we have experienced, during *Math Jams*, the room is usually very loud, and it is difficult for the instructor to have students pay attention when the instructor wants to point something out, wrap up the *Jam*, or have students return to their original workspace. In fact, we experienced this when we were doing *Jams* at a conference with university academics! The instructor needs to be comfortable with this

type of environment. Having teacher candidates experience this in a teacher education setting may help them understand that a "noisy classroom" does not equal "lack of classroom-management skills." Obviously, we do not view students being loud and focused on a task as a bad thing, and we hope this might even motivate you further to try out *Math Jams*.

We are also planning to bring some of the ideas of *Math Jams* back to our use of *Comics Jams* and are seeing ways that *Math Jams* and *Comics Jams* can enhance and support each other. Specifically, we will be encouraging students to polish previous comic frames to help clarify the storyline. We also think it would be interesting to focus on and bring forth the benefits of *Math Jams* by having students engage in a *Comics Jam* after they have engaged in a *Math Jam*. In these ways, students can experience jams in multiple disciplines, as jamming holds the potential to weave through disciplines. Even though we only report on preliminary work and have collected a limited amount of data, we hope that we have convinced you of the potential of *Jams*, and that you will also use *Jams* in your own ways and in your own context to maximize the benefits. We look forward to future large-scale studies and hearing and learning from your implementations of *Comics* and *Math Jams*.

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MB4T (Mathematics by and for Teachers): Relating Area and Perimeter



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Jennifer Holm is an Assistant Professor at Wilfrid Laurier University and works with primary/junior and junior/intermediate pre-service teachers, as well as in the field supporting current mathematics teachers. She is interested in developing mathematics knowledge for teaching with both pre-service and in-service teachers. She has begun to focus on the beliefs and opinions that pre-service teachers hold about mathematics and teaching along with their past experiences that have led to these opinions. She uses this research to support future teachers in developing beliefs and knowledge that will encourage and support effective teaching practices.

On the first day of a pre-service teaching-methods course for future elementary teachers, I give them a survey to start conversations about types of mathematics knowledge and beliefs in order to bring attention to some of the issues we will tackle during the semester (see *Perceptions of Mathematics* survey in Kajander, 2007, for more information). There are ten content questions on the survey that have served as interesting spaces for information about what future teachers already know about mathematics. One question (which is conceptually oriented) that has always had a variety of unanticipated responses asks, "Is it true that, as the perimeter of a rectangle increases, so does the area? Explain." Last year, the question was given to 53 pre-service teachers in a single semester, and of those teachers 85 percent were incorrect. The previous question on the survey is about calculating (i.e., procedural knowledge) the area and perimeter of a rectangle, and of the 53 pre-service teachers, 46 were correct, with 2 additional pre-service teachers having an addition error in spite of correct procedures, so the misconception is not an inability to understand how to find the area and perimeter.

The results are surprising when you consider where you find these ideas in the Ontario curriculum (Ontario Ministry of Education [OME], 2005). Area is first mentioned in Grade 1, when students are first exploring area with non-standard units and comparing objects based on the area. Perimeter is introduced in Grade 2, and both area and perimeter are found in Grade 3 expectations. In Grade 4, the specific expectation is that students are "determining area and perimeter relationships for rectangles" (OME, 2005, p. 64). In Grade 5, they are determining and "applying" these relationships (OME, 2005, p. 76). These are all expectations in the primary/junior curriculum, yet misconceptions persist into a pre-service program.

Taking the survey question one step further, I examined the responses to see if I could ascertain where the difficulty lies. Although some of the incorrect responses simply said "yes" or "I don't know," the respondents who attempted to explain their answer show where the misconception lies. The responses point to a causal effect of how expanding one, automatically expands the other. Two examples of such responses are as follows: "Yes, if the permiter (sic) is bigger, you can put more 'stuff' inside. More little squares" (with a drawing of a 3x4 rectangle and 4x5 rectangle filled with squares); and, "Yes, because as the outside edges expand, the middle (area) will expand as well. The bigger the perimeter, the bigger the area." The idea here is that there is a proportional response—when you increase the perimeter, the area also expands. In speaking with my future teachers, I often ask the question, "If I told you the perimeter of a rectangle, could you tell me the area of the rectangle?," and most will immediately respond, "Of course." It is not until after many experiences that focus on the relationship between area and perimeter that these initial conceptual misunderstandings begin to change.

The first task is one that I have seen in countless elementary textbooks, as well as used in my own classrooms with students. I give each group 24 square tiles, ask the groups to create as many quadrilaterals as possible with the tiles, and to calculate the area and perimeter of each of the shapes. Conversations usually begin to turn to the idea that every shape does (and should) have the same area, since the number of tiles does not change. What does change is the perimeter of the shapes. We then talk about what the groups notice about the perimeters. They quickly ascertain that the largest perimeter is achieved by placing the squares in a single line, creating a perimeter of 50 tiles (1 tile by 24 tiles rectangle). The smallest perimeter comes

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with the more compact shape of the 4 tiles by 6 tiles rectangles, with a perimeter of 20 tiles.

The next task changes the problem by giving the groups 24 toothpicks and asking them to create as many rectangles as possible with a different-sized base. Groups are then asked to record the side lengths, perimeter, and area of each of the shapes. Table 1 demonstrates the results obtained, although many pre-service teachers end up with random organizations of the rectangles through creating the shapes by trial and error. Pre-service teachers are then asked to graph the base and area in order to facilitate some conversations about what relationship is observed (see Figure 1). Conversations quickly turn to the fact that this time, the perimeter is constant, but that the area changes. By observing the graph, it becomes apparent that the quadrilateral with the largest area for the same perimeter is the square. This activity is also well suited to having a conversation about quadratics or algebra.

Base length (toothpick)	Height length (toothpick)	Perimeter (toothpick)	Area (toothpick^2)
1	11	24	11
2	10	24	20
3	9	24	27
4	8	24	32
5	7	24	35
6	6	24	36
7	5	24	35
8	4	24	32
9	3	24	27
10	2	24	20
11	1	24	11

Table 1: *Results of the toothpick perimeter problem*

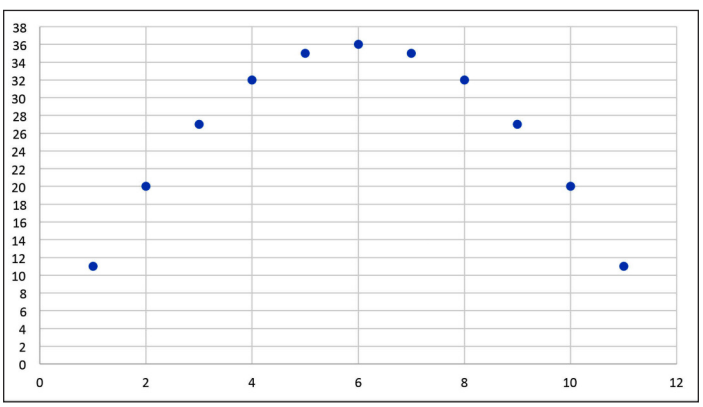


Figure 1: *Graph of area for a fixed perimeter based on the toothpick problem. Base length in toothpicks on the* x*-axis, and area in toothpicks^2 on the* y*-axis.*

The final task that helps to increase the understanding of the relationship between area and perimeter is the following problem (Rectanus, 1997):

> Bill, Kathy, and John are working together to make the garden plot below (see Figure 2) larger.



Figure 2: *Garden for Rectanus (1997) problem*

> Bill says, "We have to buy more fencing because if we increase the area of the garden, our current fence won't go all the way around."
>
> Kathy doesn't think so. "That's not true," she says. "We don't need any more fencing. We can move the fence we have into a different position and still make the area of the garden larger."
>
> John disagrees with both Bill and Kathy. "I know a way that we can make the garden larger," he says, "and use less fencing than we currently have."
>
> Who do you think is right—Bill, Kathy, John, all of them, or a couple of them? Explain your reasoning. Include sketches to explain your answer.

This develops interesting conversation because there are multiple answers that can be defended depending on how deeply the problem is investigated. I usually give toothpicks to the groups to act as the fencing pieces and then let them explore. The conclusion that usually provides the most satisfaction is that John and Kathy are the most accurate, but claims need to be supported with examples.



Figure 3: *Sample solution for justifying Kathy's response in the garden problem*

Some answers I have seen include creating a rectangle like Figure 3 to support Kathy being correct. The rectangle created has the same perimeter of 14 fence pieces, but the area is now 10 squares and larger than the initial 6 squares in the problem. Clearly by rearranging the fence pieces, you can create a bigger area. The conversation often shifts to the idea of not using a four-sided shape, and what would happen if you could curve the sides of the fence instead of leaving them straight. Once the teachers deduce that they are getting closer to a square shape, they realize that John can be right too. By creating the 3 by 3 square in Figure 4, they realize they have used fewer fence pieces, but still have a bigger area than the initial garden.

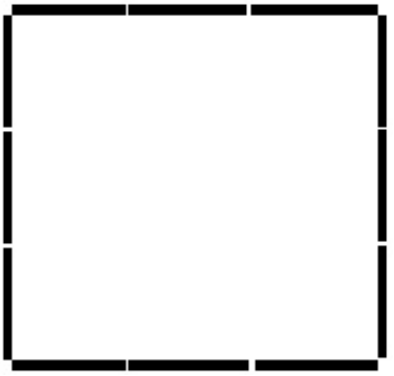


Figure 4: *Sample solution for justifying John's stance in the garden problem*

After talking about the questions, I ask the initial question about whether it is true that, if you change the perimeter, the area also changes again, but have the teachers talk about the question in groups. The conversations end up being much deeper. When I admit that they have been asked the question before, on the survey, with little success, they are really surprised by their own misconceptions. It becomes clear that there is no direct correlation between perimeter and area, and that it is impossible to determine the area or perimeter of the rectangle when only provided with one of these

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▲ In the Middle: Do They Make Enough?



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Carly Ziniuk teaches Grade 9 Mathematics, Grade 12 Data Management, and Advanced Placement Statistics at the Bishop Strachan School in Toronto, Ontario, Canada. She is very active in adopting real-life data to engage her students in solving problems.

I have been inspired by the work of some of my students working on the Trick or Eat Campaign, a student initiative with Meal Exchange at Halloween, to collect food for our local food bank. The students had gone door to door, collecting non-perishable foods. While we were loading up over almost 2000 kg of food for delivery, one of the students wondered how much food her recent concert ticket would have purchased. It was a concert ticket that she treasured! It prompted me to revise an activity I had used in the past that causes students to think through a news article in different ways, using math. With many conversions, the students were interpreting place value, correct order of operations, rates and ratios, and lots of appropriate rounding—clearly a Number Sense-centred activity. The activity connects the incomes of the wealthiest celebrities to philanthropy, with data originating from *Forbes* and *Billboard* magazines (2017 data) and the Daily Bread Food Bank Annual Report (2016/2017, see **www.dailybread.ca/about/annual-reports-and-financial-statements/**).

Each year, *Forbes* magazine compiles *The Celebrity 100* (June 12, 2017), a list of celebrities with the highest income in the given year. Sean Combs (a.k.a. Puff Daddy, P. Diddy, Puffy, Diddy, and now more recently, Brother Love) topped the list with $130 million U.S. dollars (USD) in 2016. The Top Ten also included #2 Beyoncé Knowles with $105 million USD, with approximately $54.7 million USD from her most recent Formation World concert tour, with the remainder from endorsements and streaming revenues. Similarly, #4 Drake, who is Canadian, earned $94 million USD, as the most-streamed artist last year earning $18.1 million from streaming services alone.

To get a sense for understanding these tremendous values, it can be handy for middle school students to put them in terms they can recognize. What follows is a series of questions that endeavour to help students comprehend the meaning of large values. Students can work through these or similar questions for their own favourite celebrity. (You can find the *Forbes* list with suitable data at **www.forbes.com/celebrities/list/**.)

1. The endorsement-streaming figure alone means that Beyoncé earned _____ a day, on average, whether she got on stage or not that day.
2. Considering just her concert earnings (not from endorsements, and ignoring any recording income for the year), Knowles made _____ a minute, where she played 49 concerts, each approximately 2 hours long, during her concert tour last year.
3. if she wanted to save for a Tesla Model X car at a base price of $80 000 USD, it would take about _____ hours, on average, for her to earn enough for the car, considering her total income.

Most of these celebrities are American, and the *Forbes* list is published in U.S. dollars (USD). It is common practice for many Canadians to consider expressing those figures in Canadian dollars (CAD) as a frame of reference. This provides a worthwhile opportunity to discuss the difference between the two currencies and explore the fact that the conversion rate changes daily! Since the Forbes article reports data for the year, it makes more sense to use the annual exchange rate. CanadaForEx (**www.canadianforex.ca**) presents the 2016 annual exchange rate as 1 CAD = 0.7855 USD, which does not allow for simple substitution (i.e., 5 CAD = 5 x 0.7855 USD), and proportional reasoning is needed. This is an opportunity for discussion, and some students may want to use reciprocals to find the conversion rate in the other direction. Another good discussion has occurred with my classes when students have considered why the exchange rate has so much precision in the decimals, much beyond the "cents." Looking at the difference in the context of these celebrity incomes certainly reveals how much difference a tenth of a cent makes in the conversion.

4. The 2016 annual exchange rate is presented as 0.7855 USD for 1 CAD. How many Canadian dollars would Beyoncé get if she converted her total amount into loonies?
5. In fact, even Beyoncé's *total average hourly income* is _____ more than Ontario's $11.40/hr. minimum wage. Her hourly income is minimum wage for _____ people.
6. Assuming Beyoncé sleeps 8 hours a night, on average, she makes _____ every night in Canadian dollars while hoping her newborn twins stay asleep.
7. Humorously, if a Canadian wanted to hand Beyoncé her average total income in loonies and coins, they would have to dole out _____ each second (although we'd have to figure out what to do without the pennies!).
8. In 2016, the Daily Bread Food Bank in Toronto had over 900 000 visitors, many of them children, who were experiencing food insecurity. Daily Bread accepts donations in both non-perishable items and in cash.

   If Beyoncé donated only 1/100 000th of her total annual income, she could buy _____ Daily Bread Food Bank weekly baskets.

Table 1 shows what my students determined for the cost of a weekly basket during this activity, given the items from the Daily Bread website and the local grocery store flyers, for an approximate $50 value.

> J.K. Rowling came in at #3 on the list with the release of the newest book/play, *Harry Potter and the Cursed Child*, and the latest movie, *Fantastic Beasts and Where to Find Them*, totalling $95 million USD. It is estimated that about half that income came from the 4.5 million of the latest print and on-stage adventures of *The Boy Who Lived.*

Annual exchange rates provide a very good opportunity to do multiple conversions between British pounds, US dollars, and Canadian dollars. Considering the multiple nationalities in the *Forbes* list, this can happen most years, as you can see later with Ronaldo as well.

9. Although *Forbes* presented this amount in U.S. dollars, J.K. Rowling lives in the United Kingdom and would want to receive her income in British pounds. The annual exchange rate for British pounds shows £0.7628 for $1 USD. How much would J.K. Rowling make in pounds?
10. What would the exchange rate be from Canadian dollars to pounds?
11. The earnings from the book/play amounted to _____ more than Canadian Drake's streaming income in Canadian dollars.
12. When Drake went to see *Fantastic Beasts*, it cost him $12.75 at a Canadian Cineplex, but he made _____ while he was there because he earned 2 hours of his average total income.

With a brand new contract extension with football (a.k.a. soccer) team Real Madrid, and being named FIFA's best player for the fourth time, Cristiano Ronaldo, was #5 on the list with $93 million USD overall. This includes a record-breaking salary commitment and a new lifetime endorsement deal with *Nike*. Ronaldo only played 20 of Real Madrid's 29 games last year.

Ronaldo gives us an opportunity for conversions into Euros.

13. The conversion from Canadian dollars to Euros is 0.6734. Ronaldo's total income was reported in American dollars, but how much did he get paid in Euros?
14. Ronaldo's total income is _____% of *Forbes*' highest earner's total.
15. While an excited fan paid over €700 (Euros) for a 90-minute FIFA Real Madrid game, Ronaldo, on average, earned _____ during those minutes.
16. If during practice, Ronaldo were to take a stretch break and listen to the newest 4-minute Drake tune, he would make _____ Euros while humming along.

Taking some time to consider why the currency fluctuates daily is a first chance to have middle school students begin to consider a regular microeconomic reality for many people. Supply and demand for a different currency, around the world, causes the currencies to change. Many students will be curious to figure out how currency change is connected to current events in a country. Discussing with students the statement, "If a country is considered politically unstable, then its currency becomes less attractive, and the demand for that currency falls. This changes day to day," is a good starting point to connect a changing rate with the other things students might hear in the news.

You can find the changing daily rates at **www.xe.com**, including graphs like Figures 1, 2, and 3, which connect very well to current events. Relating math to current events, using these rates, is an opportunity for economic connections. See whether you can figure out what happened politically or economically on dates corresponding to some of these spikes or drops?

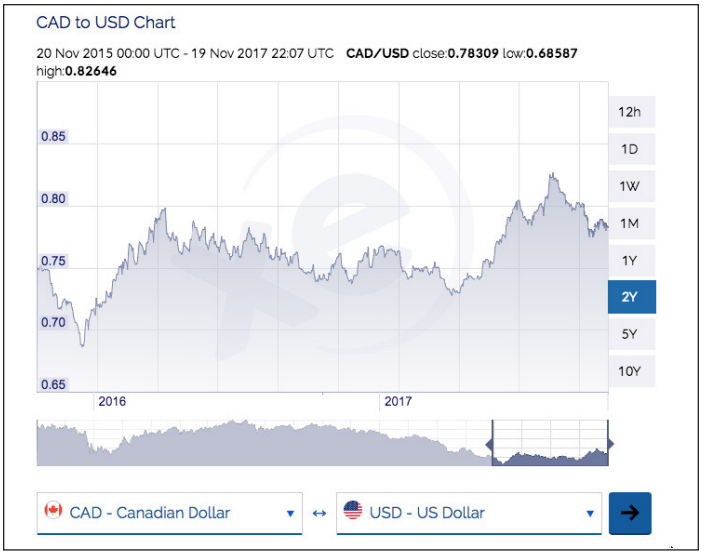


Figure 1: *The value of $1 CAD in USD through time*



Figure 2: *The value of $1 CAD in Euros through time*



Figure 3: *The value of $1 CAD in British pounds through time*

Fortune magazine also notes the most individually wealthy individuals on the planet. Bill Gates (founder of Microsoft Corporation) topped the list with a net worth of $79.2 billion USD, and his friend, Warren Buffett (a stock market investor), not far behind at #3 with $72.7 billion USD in 2016. Gates and Buffett worked with 14 other billionaires to create a Billionaires Giving Pledge (**www.givingpledge.org**), committing to give away half of their wealth to philanthropic causes during their lifetimes.

17. One of the key initiatives of the Gates Foundation's philanthropy is to eradicate polio, (**www.gatesfoundation.org**). The oral polio vaccine (OPV) has an incredibly low cost, between 10 and 15 cents per child, but at least three different doses are required. The OPV carries live virus and is not as safe as the injectable polio vaccine (IPV). However, IPV is much more costly at $5 per child and needs to be delivered to an older child, which leaves infants vulnerable.
    a. If the World Health Organization (WHO) reports 19.5 million children still need polio vaccination, what are the different costs for the different methods?
    b. What percent of Bill Gates' Giving Pledge would be remaining if every child who needed it received the polio vaccination in its IPV form?
18. *The Toronto Daily Bread Food Bank* yearly budget of $19 912 169 CAD represents _____ percent of Buffet's net worth.

ANSWERS

1. $137 808.22
2. $9 302.72
3. 6.7 or 7 hours (if rounded)
4. $133 672 819.86
5. $15 248.05, 1338
6. $122 076
7. $4.24 CAD (Rounding to $4.25 would give her an extra $36/hr.)
8. 26 baskets (so about half a year's worth of food for one person)
9. £0.5991794 for $1 CAD
10. £72 466 000
11. $37 428 389.56 CAD
12. $27 321.69 CAD
13. 79 727 816.68 EUR
14. 71.5%
15. 13 652.03 EUR
16. 606.76 EUR

17a. $5 850 000 USD for 10-cent OPV,
$8 775 000 USD for 15-cent OPV,
$97 500 000 USD for IPV

17b. 99.75%

18. 0.02% (or approximately 1/50th of a percent) ▲

▲ Hey, It's Elementary: "You can't help getting older, but you don't have to get old." (George Santayana)



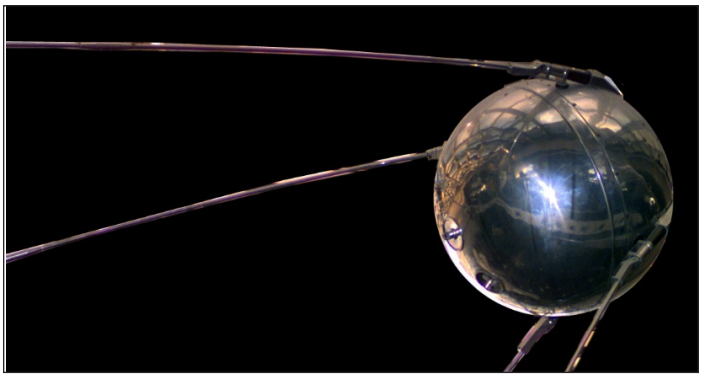
LYNDA COLGAN
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Dr. Lynda Colgan is a Professor at the Faculty of Education, Queen's University. In addition to her teaching responsibilities in the BEd and Graduate programs, she is involved in research and knowledge-mobilization projects with the Council of Ontario Directors of Education, TVO/TFO, the Ontario Ministry of Education, and the Mathematics Knowledge Network.

If you read this, you will be able to approximate my age fairly accurately. Please promise not to let this information go viral by sharing on Facebook, Twitter, or Instagram. To my friends and family, I am 29. Their hearts would break to learn that this is not the truth.

It would appear that the mathematics curriculum in Ontario is going to undergo another major "revision and renewal" process… just one of many during the past 60 years. I thought that I would add some personal and historical context to put things into perspective.

I was a preschooler when Sputnik was launched on October 4, 1957. The image below shows a replica of Sputnik 1, which is on display at the U.S. National Air Force and Space Museum. For those too young to remember, on that day, the Soviet Union successfully launched Sputnik 1, the world's first artificial satellite, about the size of a beach ball (58 cm in diameter) and weighing 83.6 kg. The satellite took about 98 minutes to orbit the Earth on its elliptical path… but it forever changed my life and those of all the children who lived a gazillion miles away and attended Toronto's Bruce Public School in what is now trendily called *Leslieville*.



The Soviet launch of Sputnik sent the United States into panic mode. Clearly, schools needed to emphasize mathematics and science so that the nation would not fall behind the Soviet Union and the "brain-iac" scientists who had obviously won the space race. In 1958, President Eisenhower signed the National Defense Education Act, which poured money into the American education system at all levels... resulting in the "New Math" that included arithmetic using number bases other than 10.

Canada was not immune to the reactions south of the border. Oh, how I remember questions like 10 + 11 in Base 2 (see Figure 1), or being given addition and multiplication tables for base six (see Figure 2).

$$\begin{array}{r} 10 \\ +11 \\ \hline = \ 101 \end{array}$$

- In the ones column, 1 + 0 is 1.
- In the second column, 1+1 is 2, or 10 in base 2

Figure 1: *Addition in base 2*

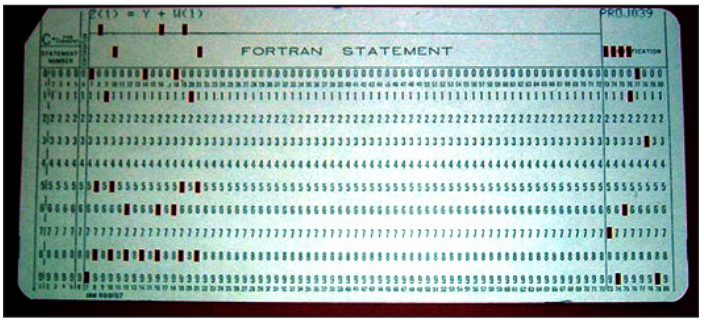
+	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	10
2	2	3	4	5	10	11
3	3	4	5	10	11	12
4	4	5	10	11	12	13
5	5	10	11	12	13	14

×	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	10	12	14
3	0	3	10	13	20	23
4	0	4	12	20	24	32
5	0	5	14	23	32	41

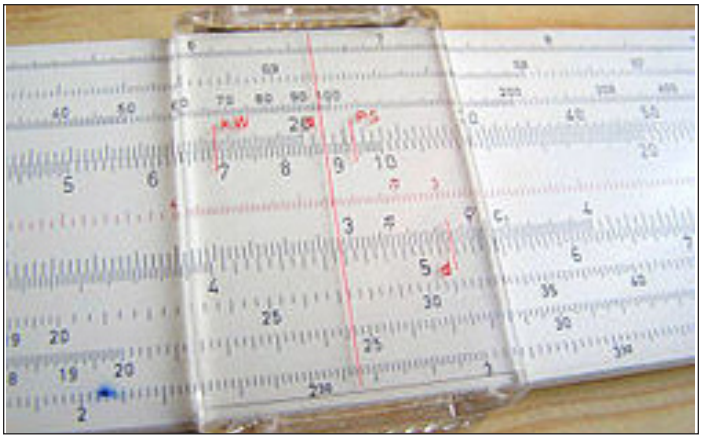
Figure 2: *Addition (left) and multiplication (right) in base 6*

I recall my parents' reaction so very clearly. They were *very* engaged parents and we lived two doors away from the school, so there was no barrier to showing up at Mrs. Harvey's or Mr. Watson's classroom door to ask what the heck the math homework was about. They were unhappy because they couldn't understand my homework, but accepted Mr. Ramsay's explanation that computers operated using the binary system, so the "future" for children in my class was to become part of the computer generation that would put a man on the moon.

A few short years later, I remember being "selected" for a special Computer Programming Class at Riverdale Collegiate Institute. This privilege entailed taking the "Red Rocket" to the Toronto District School Board Head Office at University and College. A small group of us gathered every Saturday morning to learn how to program the computer to count to 100, print out the next 20 leap years, or perform a simple "Pig Latin" translation. We entered our code, using a soft-lead pencil, onto bubble card decks (using Fortran, the "language" of the day) and left the stack with an attendant at the main desk. On Wednesday, the big mailbag returned to the school with our huge computer printouts and punched cards. Eugene, Rosalind, Roland, Nick, and I would sigh as we discovered a small "bubbling" error had caused our program to fail. So on Saturday, we trudged back to the Board Office for our private lesson and the dual tasks of fixing the previous week's program and bubbling anew.



By the time I was in Grade 13, the math program had been repackaged into three courses: *Algebra*, *Calculus*, and *Functions and Relations*. Reflecting the major ideas of the "new math" reform, my teachers, Mr. Saarimaki (a Past President of OAME) and Ms. Robinson, made use of discovery techniques and problem centre activities, television, multiple textbooks, student self-directed activities, and manipulative devices (like the slide rule shown below).



My "off to university" gift was a beautiful slide rule in a leather case, embossed with my name in gold letters. You can imagine my parents' dismay when I saved the income from my part-time job at Laura Secord to buy the first TI electronic 4-function calculator: a plug-in device with two decimal point accuracy. It was forbidden at university, but I used it surreptitiously at home to "double-check" my calculations… likely the first and only time that I could be described as a rebel.

When I became a teacher in the late 1970's, the curriculum had undergone another reform. Statistics and probability were elevated to core topics: flow charts were

used; functions and relations were introduced earlier; and estimation, including geometric transformations, became essential topics. Furthermore, this curriculum stressed the development of computation skills along with a strong recommendation that the problem solving was to be the predominant approach. .

By the time I was the Curriculum Coordinator for the Scarborough Board of Education, I saw the destreaming of all Grade 9 courses to a common curriculum for all students, regardless of ability level. Subsequently (with the change of Provincial Government) in that role, I oversaw the articulation of expected learning outcomes in all grades, the abolition of Grade 13, the reconstitution of the secondary program into two streams (Academic and Applied), the creation of standardized report cards linked to defined learning objectives, and the use of standardized provincial assessments.

"So," you ask?

In the scheme of things, over the last half century, not much has changed. We still "Code" with "Scratch" instead of Fortran or Logo. We still debate the appropriate use of calculators. We still tinker with content and juggle its position and priority. And, we, as mathematics educators, remain solidly at the mercy of political parties and other external forces, from the jury of public opinion to the persuasive sway of loud interest groups, regardless of how well informed their opinions are.

Today, there are calls to throw out the curriculum and start from scratch. There are demands to eliminate assessment, appeals to abandon grades, bids to abolish standards-based provincial testing, and more. As in every other era, there is panic and misinformation.

And, as in every other era, history can teach us some important lessons. I thank my graduate student, Steve Mueller, for this important illustration of the challenge of predicting the future and leading change.

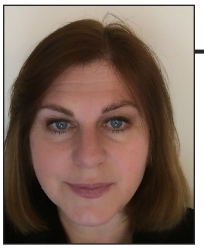
> During the time of WWI, the place of mathematics in the school classroom was beginning to be questioned. The goal of the high school was no longer that of college preparation, but more for varying social goals. This was highlighted in John Dewey's famous 1916 publication of *Democracy and Education* and his view that education should help each individual to develop personal initiative and adaptability. In response to mounting criticism of mathematics education, the National Council of Teachers of Mathematics (NCTM) was founded in 1920 to provide leadership for and by North American mathematics educators. The changing perceptions of the purpose of school at this time was evident in enrolment levels of students taking academic mathematics. Student enrolment in high school mathematics had risen until 1920 and peaked in 1922 at 74.9 percent, before declining to a little more than 50 over the next two decades. As the 1920's turned to the 1930's, education was becoming vocationalized in response to the changing views of the purpose of education and the changing needs of the student body. Traditional mathematical subjects such as algebra and geometry were no longer being seen as having value to a student's future job, and the focus of mathematics courses began a shift toward day-to-day skills such as writing invoices and calculating payrolls. With the Great Depression of the 1930's and the resultant decline of available employment, many students who normally would have left for work were now staying or rejoining the high school system. This influx of supposedly less capable students saw further change to the secondary curriculum with schools intensifying efforts to implement more functionally orientated curriculum policies. These "attacks" on traditional mathematics would continue until a brief reprieve caused by the outbreak of World War II.

No one has a crystal ball. The needs of society will change as local, national, and international events change the course of our lives, collectively and individually. It is folly to try to pronounce predictions about what will or will not be needed in the future—near or far. It is also reassuring to know that thankfully, most things will never change about mathematics because of its universality as a language. There is great comfort in knowing that although the language of mathematics has many dialects—numbers, symbols, graphs, diagrams, and tables, to name but a few—they enrich communication, rather than creating a barrier to it. The letters, words, sentences, paragraphs, and narratives of mathematics will endure, as they have for centuries, in their many forms, in spite of external factors. The pendulum proves this.

In spite of the many "reforms" that defined my early mathematics education as a student and my privileged career as a mathematics educator, I developed and sustained solid mathematical skills, attitudes, and dispositions that allowed me to see the beauty and utility of this amazing subject. May today's students and educators be so fortunate. ▲

Perpendicular: From the French perpendre meaning per "balanced" and pendre to hang (as in hanging a plumb line). Presumably, the implication is a plumb line is perpendre or perpendicular to the surface of the Earth. (Source: **www.etymonline.com**)

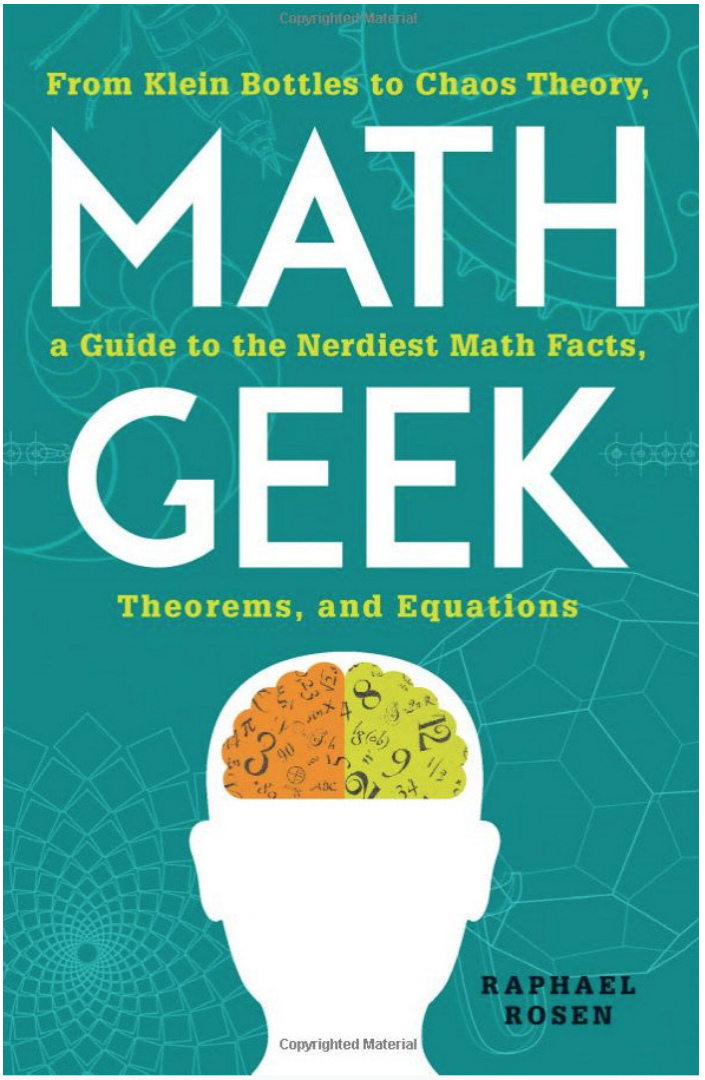
▲ BOOK REVIEW: LOOKING AT MATH IN A DIFFERENT WAY. A BOOK FOR MATH GEEKS AND MATH-GEEK WANNABES



REVIEWED BY CHRISTINE SASSO

Christine Sasso is a retired teacher after 20 years of teaching elementary students for the Windsor-Essex Catholic District School Board. She is currently working as an IT Software Coordinator for a construction company. Christine has served as SWOAME's chapter rep to the OAME board of directors and continues to serve as treasurer for the chapter. Her interest in mathematics education now involves being an online tutor for Ontario's Homework Help.

Rosen. R. (2015). *Math geek: From Klein bottles to chaos theory, a guide to the nerdiest math facts, theorems, and equations*. Avon, MA: Adams Media.



Math Geek is a quick read, divided into four easily skimmed parts. This is definitely a pick-up, put-down, and pick-up-again kind of book, as each chapter is one or two pages in length. Chapters are clearly titled, making it easy to search for a specific topic.

As an educator and more of a math-geek wannabe, rather than a true math geek, I found this book lighthearted and easy to understand, without getting lost in a lot of calculations or long explanations on theory. This makes the book accessible to anyone interested in math, from the novice, to educators of all levels, to full-out math aficionados.

From an educator's perspective, there is a wealth of ideas to introduce a math concept and suggestions for further exploration. The first part on shapes has a variety of interesting tidbits that can be used to develop centres on topics that can be done in most primary grades, from the very first chapter on Romanesco Broccoli to the shape of manhole covers. For the senior of grades, exploring the math behind the Rubik's Cube was of particular interest. The book doesn't provide the how-to of a math lesson, but certainly provides the inspiration.

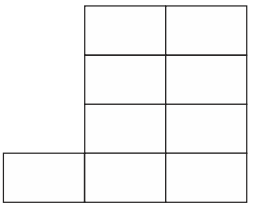
Part two and three on behaviour and patterns, respectively, burst with ideas for demonstrating and testing math, from cutting a cake fairly, to the math behind juggling. I have personally been testing out chapter 37, "The most efficient checkout line." There are also some games that are simple in nature and easily played in the classroom, such as the game "Go." Furthermore, the book provides an introduction to the math behind the game.

The final part focuses on special numbers and is probably the most math specific of the four parts. It still contains some ideas for exploring the math behind the numbers, but is more of a refresh of math concepts such as prime numbers and the golden ratio.

The book is clearly written, and the diagrams that are provided aid in understanding it, but I would have liked a few more diagrams. The chapter on the Koch Snowflake is one example. If someone were unfamiliar with this idea, a picture of one would help. As someone who is interested in math in general, I found the book interesting and thought-provoking, and there were more than a few times that I popped online to get more details. That is the real power of this book. It sparks ideas to explore true "real-life math," it touches on the history behind the concepts, and it leaves you with a trail of breadcrumbs to explore further. ▲

Digital Challenge

Arrange the digits 1 to 9 to make this addition of three numbers work?



Four in a Row

X to move and win.

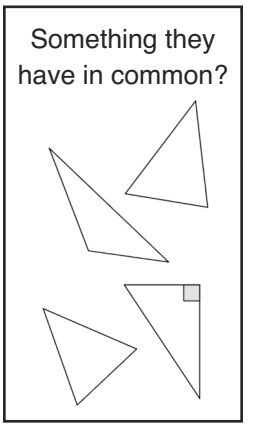
A	O	X	O	
B			X	X
C			O	
D	X			O
	1	2	3	4

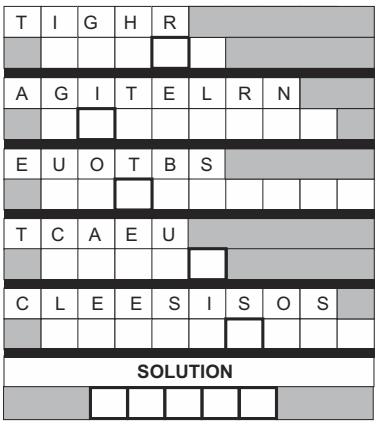
A person stood on two bathroom scales at the same time. One said 26 pounds; the other said 45 pounds. How heavy are they?

A trickster thought he would fool everyone. He stood on two bathroom scales and one scale said 85 pounds and the other said 86 pounds. Everyone knew the trickster weighed less than 100 pounds. Can you figure out the trick? How heavy is the trickster? How heavy is the scale?

Jumble 1

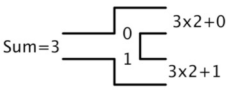
Unscramble each word. Take the marked letters and solve the picture riddle.



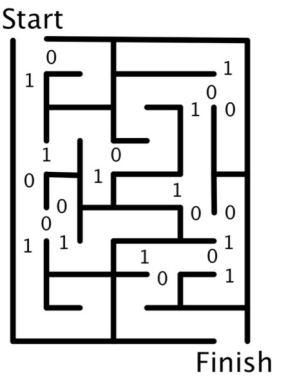


Binary Maze

As you solve this maze, do the following calculation to determine the hidden a-Mazing value. Start with a sum of zero. Each time to reach a junction, multiply the sum by two and add on the value from the junction for the direction you choose to go. For example, if your sum is 3 when you get to a new junction, if you choose to go in the zero direction, your new sum is 2 x 3 + 0, but if you choose to go in the one direction, the sum becomes 2 x 3 + 1.



Continue the calculation, with the new sum until you get to the end. Secret Value = ____



SOLUTIONS

Digital Challenge:

One solution is (How many other solutions are there?)

1	3	5
	4	7
	6	8
	2	0

Four in a Row :

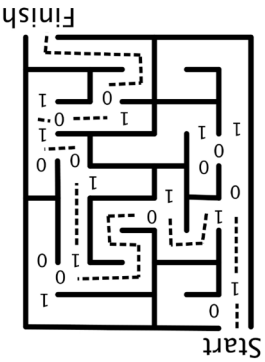
X-B2, O-B1, X-C2, and wins by playing either A4 or D2 on the next move.

Jumble 1:

1. RIGHT 2. TRIANGLE 3. OBTUSE
4. ACUTE 5. ISOSCELES

Something they have in common? "Three"

Binary Maze:



Secret Value = 340