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Ontario Mathematics Gazette

OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

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September 2019

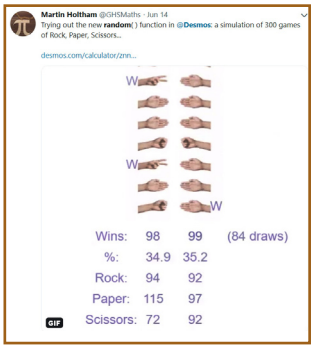
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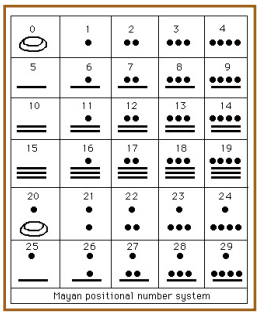
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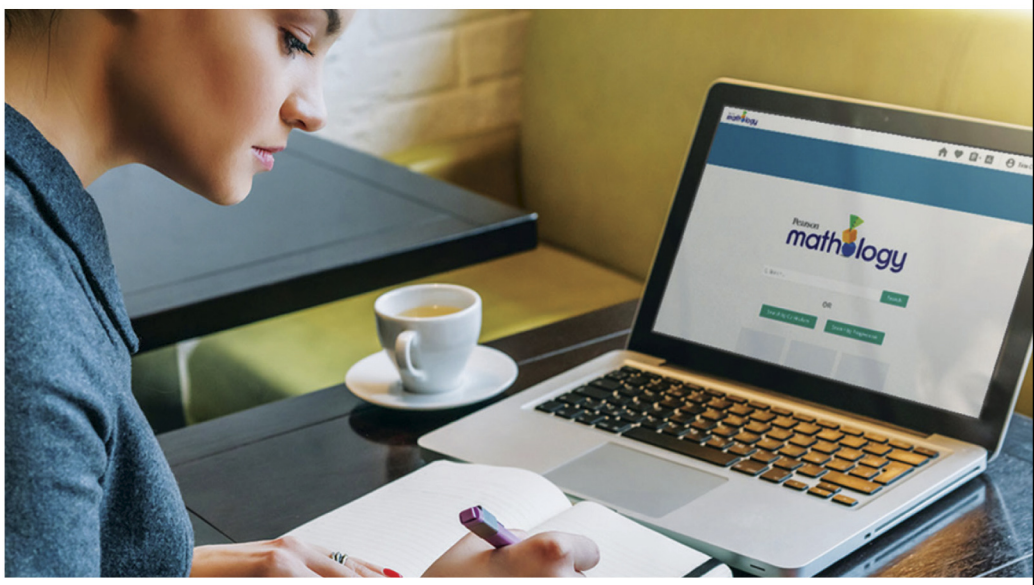
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Please keep in mind the following criteria when making submissions to the *Gazette*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles are to be word-processed, MS Word is preferred, and prepared according to the *Publication Manual of the American Psychological Association*, Sixth Edition (2009). However, please use single-line spacing (not double) and only one space after each period. Articles should not exceed five numbered pages of text, and figures, images, and photographs should be placed in the text close to where they belong, with captions. The photographer's permission is required, and for photos of students under the age of 18, the written permission of a parent or guardian is required.

Please submit your article in one blind file (i.e., identity of author is not evident), and include author names, contact information including email and mailing addresses, photos, biographies, and all content removed for blinding in a second file. Please email these two files to Tim Sibbald at gazette@oame.on.ca.

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▲ Editor's Report



TIMOTHY SIBBALD, OCT, PhD
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Tim Sibbald holds the Universities Director position on the OAME Board of Directors. He is a former president of OAME. He is an associate professor in the Schulich School of Education, Nipissing University, with a focus on mathematics education.

The connection between numeracy and mathematics is exemplary of many issues math education has faced during the last few decades. Yet it also shows the fundamental problematic nature of moving mathematics education forward. It is a complex picture that has many threads that are only loosely connected; still it is compelling because there is a perceived simplicity that defies clarification.

Perhaps as you have been reading, you have accepted the term "numeracy" and feel you have a sense of what it is. However, there is no standard definition of numeracy. Some consider it to be embedded learning within the curriculum (whichever curriculum one happens to work with), but this sidesteps the definition because it essentially says that if one learns from teachers who are using the curriculum, then you will have numeracy skills without actually defining those skills. Others argue that numeracy is a minimal necessary capacity for functional mathematics. The "functional" qualifier refers to there being a realistic context, and there are many different opinions about what constitutes a valid context.

There have been efforts to address realistic contexts within schools. Perhaps the clearest example is the implementation of STEM programs. The acronym STEM is unfortunate, as it is tied to American funding, and a more appropriate vantage point, in my opinion, is to consider interdisciplinary mathematics. That opinion aside, the relevance of such efforts is that it provides a context through the mixing of disciplines for the mathematics being used. Computer programming, for example, uses math skills, but highlights logical and organizational structure within the context of achieving a tangible action by a machine.

The challenge of interdisciplinary approaches (including STEM), as argued by Sibbald (2018), is that it may constitute expensive numeracy, where alternative methods could be equally effective. That opinion should not be taken for all grade levels, and the use of interdisciplinary approaches in some divisions (junior, for example) may be more important

than other divisions. It should be noted that STEM cannot be used directly as a numeracy support because, unlike numeracy, STEM does not have consensus about its practices. British and American views of STEM differ, with the former emphasizing higher grade levels and colleges to be adapting to new materials and systems such as carbon fibre and wind turbines. The American view tends to be oriented to K–12 education with integrated content.

The definitional issue for contextual mathematics, including numeracy, goes much deeper. Educational programs have an idealized goal. This is formalized by a "theoretical framework" that is necessary for effective program evaluation. It is only through program evaluation that one can be sure of whether the idealized goal is genuinely being achieved. This, for example, is necessary to make sure teachers are not simply excited by new equipment and therefore teach differently in their excitement. It is the means of addressing whether genuine change is taking place. For interdisciplinary approaches, including STEM, the only substantial formalized framework was originally developed with the notion of departments in universities co-operating and, while it was republished in 1970, the original publication was in 1949.

This is not to say that having numeracy supported by interdisciplinary approaches should be abandoned. It is to say that the implementation was driven by funding, and much more attention is needed to the foundation on which these programs are built. It is a reflection of many instances where funding is provided, and only after programs are established is the means of evaluating the programs considered.

Numeracy seems to be at the core of arguments about "back to basics" (regardless of the name used for the stance). It promotes mathematics emphasizing capacity for skills, with immediacy in circumstances requiring the skills. This approach seems to be motivated by public perceptions, where the public observe numeracy, particularly when it is executed poorly. A classic example is a cashier who struggles to give the correct change. Often this is oriented to young cashiers who are new to the role and very concerned about making mistakes. They may have learned how to subtract at school and achieved curricular expectations well, but cashiers often use a counting-up strategy as opposed to subtraction. The counting-up strategy begins with the amount owed, and adds small change first, followed by larger amounts until the amount paid is reached. For example, if the amount is $2.35 and the person pays $5.00, then start at $2.35 (with no money in your hand), add a nickel ($2.40), a dime ($2.50), two quarters ($3.00), and a toonie ($5.00)—the cash in hand, .05+.10+.25+.25+2.00 =$2.65, is the required change without explicit use of subtraction. Often new cashiers struggle with changing algorithms, while working a task that is prone to customers taking advantage (there are various fraudulent schemes for confusing cashiers to receive more change than one is supposed to).

The difficulty with "back to basic" strategies is that they assume everyone should be treated according to the needs of the few. It is also built on the faulty assumption that poor numeracy skills observed periodically reflect all students from all schools. The argument is a flagrant abuse of data interpretation and ignores that the contextual circumstances are quite different between paid employment and the classroom.

Yet there are issues within numeracy that mathematics education struggles with. The routinization or automaticity of skills seems to be weaker today as a result of two factors. The first is the effort to be more inclusive of varied strategies that students can come up with and how to instructionally balance that with a need to promote routinization or automaticity. Varied strategies are important for student motivation, creativity, self-efficacy, etc. The second issue is the place of technology. There is broad agreement that calculators should be used for checking after mental math has been used, or in circumstances where the focus benefits

Thank You! To Our Reviewers

The *Gazette* is a double-blind peer-reviewed journal, oriented to practitioners. That is to say, it meets the highest standards, and that is because of the quality of our reviewers. Thank you!

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from expediting the calculation component (an example is how mortgages depend on various factors). The difficulty in this instance is that the proverbial bookends are straightforward, but teachers have to exercise professional judgment in the rather large murky zone in between.

To make matters worse, when technology was endorsed by the math curriculum, it was at a time when computer literacy was on the minds of many. Math was seen as a content area that could help students learn how to use technology. With the passage of time, the curriculum has not changed, but the level of computer literacy our students bring to class certainly has. The decreasing need for math classrooms to support computer literacy does not mean there shouldn't be technology. It does imply a shift from broad technology support as a matter of literacy, to the exercising of professional judgment regarding the pedagogical value of specific technology usages in the classroom.

Did numeracy get lost as this editorial evolved? That is another issue within mathematics education. Contrasting the terms "literacy" and "numeracy" is a fascinating exercise. Literacy claims that in the early years, numeracy must be contained within literacy because being able to read and write numbers is prerequisite to basic math skills. At the other end of the educational spectrum, basic skills education for adults is termed "literacy," and that includes numeracy as a matter of funding. In other words, numeracy is impinged upon by literacy at the two ends of the educational system. In between we have seen a proliferation of terms under the literacy umbrella because of disproportionate funding—Financial literacy is actually numeracy—that arose because "literacy" placed itself as the primary term for the most basic level of competency.

Literacy has also received a broad definition to cross the divides of different mediums. Sometimes the term "multi-literacies" is used to convey an inclusive vision of communication that includes speech, graphic art, gesture, symbols, etc. Numeracy has not generally pursued this line of reasoning, though there is no mistaking a cash register is different from a standard calculator, or that numeracy unmistakably includes reading schedules and sequencing events. Teachers certainly try to engage different contexts and, to a lesser extent, different mediums of numerical information.

The challenge we face in math education is quite simply that the complexity shown here is not unique to numeracy. It seems to be omnipresent through math education, and there are many somewhat disparate views, regardless of the issue.

News

If UNESCO endorses the International Day of Mathematics, then March 14, 2020 will be the first International Day of Mathematics. To prompt your thinking about what your chapter, school, or class might do, a theme of "Mathematics Is Everywhere," a logo (see Figure 1), and a website (www.idm314.org) have been developed. The website includes an interpretation of the theme.



Figure 1: *The International Day of Mathematics logo*

The second news item is that a group has been developing under the name "First Year Math and Statistics in Canada" (FYMSiC). They now have a newsletter and website (www.firstyearmath.ca). The first issue of the newsletter acknowledged Peter Taylor for achieving 50 years of service! Congratulations, Peter!

Congratulations to Canada's second-ever team to compete in the European Girls' Mathematical Olympiad. The team achieved two bronze medals and two honourable mentions at the competition in Kiev, Ukraine. Congratulations to Siyu (Elaine) Liu (Oakville), Anna Krokhine (Toronto), Katie Forbes (Edmonton), and Zixian (Ruby) Wei (Charlottetown), along with the leadership of Dorette Pronk (Dalhousie University) and Sarah Sun (GoldSpot Discoveries Inc.).



Figure 2: *The European Girls' Mathematical Olympiad team*

Other

In each issue, I try to share at least one photo. This time (Figure 3) is of the First Nations Longhouse at the University of British Columbia. The building is demonstrative of geometry in architecture, with roof supports having a sequence of different slopes, and the shape of the roof having a hyperbolic character. It is yet another example of how Indigenous design can be acknowledged in the mathematics classroom. However, having said that, there is a need to be respectful of what one does not know about the cultural significance of a building such as this. Personally I would take the approach of asking my students if any of them know. In that vein, I would welcome a letter to the editor that could provide some context for the architecture.



Figure 3: *The longhouse at UBC*



Figure 4: *Share your thoughts if you use these?*

A question that relates to a device I saw at the annual conference is shown in Figure 4. I would like to invite feedback on experiences with the device in classrooms. If you are using the devices, what has your experience been?

In This Issue

Paul Alves, as the new president (again), brings news of the first OAME Position Paper, the Leadership Conference, and, in a separate feature, his visit to the NCTM Annual Conference.

Angelica Mendaglio reports on the MathEd Forum that focused on gaming. *Jacqueline Hill* introduces us to "book tasting" after having used the approach at an OAME Board of Directors meeting. *Parker Glynn-Adey* and *Ann Arden* give an account of their experiences at the Canadian Math Education Study Group conference, which will be in Ottawa next year.

Carly Ziniuk delves into the divide between math and science and finds opportunities to improve the connection to benefit both disciplines. *Ron Lancaster* ponders shuffleboard and demonstrates how you can use your own collection of photos in your classes. *David Costello* considers writing strategies and how to leverage language arts skills in the math classroom. *Mary Bourassa* shows how data management can be supported using new tools from Desmos. J*ennifer Holm* investigates different ways of looking at a pattern. *Shawn Godin* arranges a mean, median, and mode, while giving homework that may strike a chord. *Lynda Colgan* reflects on the ethnomathematics leadership of *Susan Stuart* (who passed away last Spring). *Ann Arden* interviews *Dylan Wiliam*, who is well known for his work on mathematics assessment.

Lastly, in their final column, the provincial digital team, *Agnes Grafton*, *Ross Isenegger*, and *Markus Wolski*, share their work with virtual algebra tiles. It is sad news that this project has come to an end. I would like to wish Agnes and Ross the best for their retirements and hope we periodically hear from Markus. Their work has been substantial over the years, and their contributions to the *Gazette* have been an important collaboration for getting the word out.

Thank you! Penny Clemens, Gitta Berg, Jacquie Foster, and Anne Yeager. Without your ongoing support, the *Gazette* would not be the high-quality document that it is.

References

Sibbald, T.M. (Ed.). (2018). *Teaching interdisciplinary mathematics*. Champaign, IL: Common Ground Research Networks. doi:10.18848/978-1-61229-958-7/CGP ▲

President's Message



PAUL ALVES
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Paul Alves, current OAME/AOEM President, is department head of math at Fletcher's Meadow Secondary School in the Peel District School Board.

Welcome to another school year and to the first issue of volume 58 of the *Gazette*! Those are the exact same words (except it was volume 53) that led off the *President's Message* during my first term as OAME/AOEM President. I now have the honour of being OAME/AOEM President for a second term, and despite the regular rhythms of a school year, much has changed in the mathematics community across the province in that short passage of time.

Personally, five more years have been added to the total number of years I have been teaching, bringing the grand total to 23. I am the department head of mathematics at Fletcher's Meadow Secondary School in Brampton and am fortunate to work with a dynamic group of educators who are always willing to push their practice forward. Like so many departments and schools across the province, my department has changed over the past year. Some of those changes were voluntary, but others were not. These changes point to what could be a challenging year for many across the province as changes to program and personnel in school boards take affect and we adjust to a new reality. Regardless of the challenges that we may face personally and collectively this year, I am comforted to know the OAME/AOEM will provide stability during these times as it has in the past. During my previous term, my vision was to expand the overall scope of OAME/AOEM and to ensure that we served this expanding base of members. Pondering this second term, my vision is to sustain the growth of the organization over the past few years and honour the work of my predecessors, as well as advocate for the renewed mission and vision of the organization.

Anticipating the challenging climate of this year provided the theme for the OAME/AOEM Leadership Conference: "Paying Attention to Leadership in Mathematics." The conference will be taking place on November 1 at the Pearson Convention Centre in Brampton. The goal of the conference is to look at how we lead and sustain the excellent work that has been done in the past that has made Ontario's education system a model for many across the globe. The day will be kicked off by Marian Small with a talk titled "Making a Difference." Delegates will then be able to select from sessions that address the different aspects of leadership and how we can maintain the positive strides and move forward during this school year. Natasha Henry and Hiren Mistry will address equity, access, and empowerment. This is an area of growth and learning for me during the past few years, and I wanted to ensure that the focus on equity was not lost during a time when we are most likely doing the same if not more with less. Doug Duff and Nat Banting will speak to instructional leadership and how to ensure that we are embedding research-informed practices in our schools. Em Del Sordo will address how to push teacher practice in all areas by looking at the instructional core. Brian Aspinall will provide delegates with an opportunity to consider how we can move beyond the curriculum and look at learners in the twenty-first century. The day will be capped off with "cracker barrel" roundtables. This will provide delegates with an opportunity to network and share their learning from the day. My hope is to continue the learning as the year progresses by providing online learning opportunities for those who attended the leadership conference.



In addition to the Leadership Conference, there will be quite a few learning opportunities available through other OAME/AOEM activities. A good place to start is with your local chapter. Many chapter volunteers are busy planning

events for chapter members to provide learning close to home. Go to the chapter directory on the OAME/AOEM website and check out your local chapter's website for more information. If you are not sure which chapter you are a member of, just log into the OAME/AOEM website and go to the Member Links on the left side. You will see your account information and be able to check for your chapter affiliation.



A date that you can mark on your calendar right now is the OAME 2020 Annual Conference, taking place May 7–8, 2020 at UOIT/Durham College in Oshawa. Congratulations to the organizers of the 2019 conference in Ottawa. It was a highlight for me last year, with so many inspiring sessions and enjoyable events (I took in my first baseball game in the nation's capital), as well as the opportunity to co-present with a new teacher in my department. This year's conference will build on the momentum of last year's conference and preceding years. Just check out the speaker list that has been published up to this point: Dr. Valerie Camille-Jones, George Couros, Fawn Nguyen, Dan Meyer, Ian Vanderburgh, Kamau Bobb, Mary Bourassa, Richard Van Camp, and Marian Small. Add to this lineup the great program of sessions, exhibitors, and other events, and OAME 2020 looks to be another "not to be missed" conference! If you are interested in submitting a proposal, submissions will be opening up this Fall. Speaking at the conference is an opportune way to share the amazing work that happens in schools and boards across the province, as well as a professional learning opportunity.

I want to acknowledge the work of now Past President, David Petro. I read through his *President's Message* from this time last year, and he mentioned that he wanted to make sure that OAME/AOEM members see value in being a member of this organization. I would say that he did substantial work in ensuring that his vision was fulfilled. During his term, he was the primary force behind the OAME Talks podcast. Try listening if you haven't checked them out already. A link to all past episodes can be found on the right-hand side of the OAME/AOEM homepage. In addition, David hosted webinars with some of the podcast guests, which provided a closer exploration of some of the topics discussed in the podcast. These were all free to OAME members. If you missed one or all of them, you can find them in the Member Links section of your account. Behind the scenes, David was instrumental in guiding the organization through a year of change and growth. There were new committees, including the Ad Hoc Advocacy Committee, of which I was a member. This committee is charged with looking at ways to advocate and support the mission and vision of OAME/AOEM. One of the first products is the *Access, Equity, and Inclusion OAME/AOEM Position Statement* , which is published in this issue and can be found online. I want to thank David, on behalf of all members, for his leadership during the past year.

In addition to thanking David, I also want to congratulate him as well as Mark Bouwmeester as the recipients of the OMCA Awards of Excellence. The awards are given to educators to recognize excellence and innovation. Speaking of awards, OAME/AOEM handed out its annual awards at the annual conference. The following were recognized for their outstanding work: Cathy Hall (OAME Lifetime Membership Award); Kerri Evershed (Award for Exceptional and Creative Teaching in Elementary Mathematics); Matt Anderton (Award for Exceptional and Creative Teaching in Secondary Mathematics); and Liisa Suurtamm (Award for Leadership in Mathematics Education). All are deserving winners of their respective awards. If you know an outstanding educator and would like to nominate him or her for one of the awards, visit the Awards page on the OAME/AOEM website to view the descriptions of each award and to access the nomination page.

I also wish to congratulate our new President-elect, Judy Mendaglio. One of the challenges of being President is trying to learn what you are doing and trying to do it in one year. This prompted the organization to change the term of the presidency to two years. I am very happy to know that Judy will be the first President to serve a two-year term. So my presidency marks an end, to a certain degree, as I will be the last President to serve a one-year term. This is a testament to one of the things I value most about being a member of OAME/AOEM—its continual evolution to meet the needs of its members and the challenges that confront it. And so I am not pessimistic about the unknowns that the coming year may present, knowing that collectively, this organization, comprised of dedicated educators, will continue to flourish and stand for what is best in mathematics education in the province. ▲

Technology Corner: Statistics with Desmos



MARY BOURASSA
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Mary Bourassa teaches mathematics at West Carleton Secondary School in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She has presented workshops internationally, authored mathematics resources, is a Past VP of OAME, and a Past President of COMA. An award-winning teacher, Mary continually strives to learn new and better ways of helping students learn and love mathematics.

One of the five strands in the Ontario Grades 1–8 curriculum is Data Management and Probability. Students learn how to collect, organize, display, and analyze data. At the other end of the curriculum, in Grade 12, students can take an entire course in Mathematics of Data Management, University Preparation (MDM4U), where they complete a project demonstrating the skills learned in the course. There are various dynamic statistical tools available to help, but with the increase in the number of Chromebooks, many options are no longer viable. Using Google Sheets is one option; another is Desmos. Desmos recently added a number of statistical tools to its free graphing calculator software, available at www.desmos.com. These can be used at both the elementary and secondary levels.

To demonstrate some of the functionality, I generated 100 random whole numbers from 10 to 50. I gave the list a name by typing "A =" in front of the list, as shown in Figure 1. You can see that I also displayed the median of the list and the mean of the list.

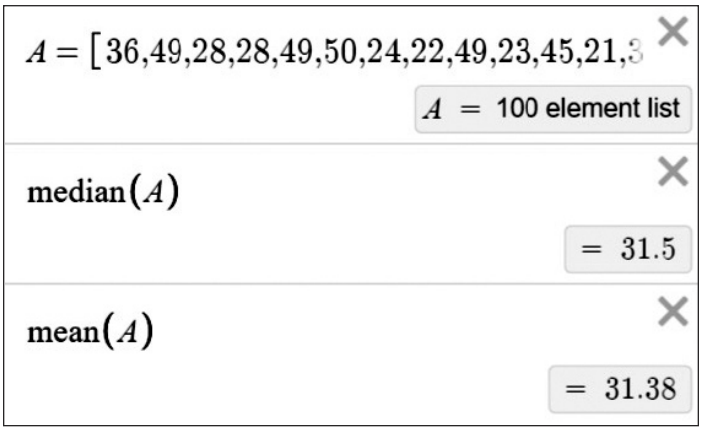


Figure 1

Next, I set up a dot plot for my list, as shown in Figure 2. To view the plot shown in Figure 3, I clicked on the little magnifying glass (bottom left of Figure 2) containing the plus symbol in order to "zoom fit" or adjust the window to fit the data.

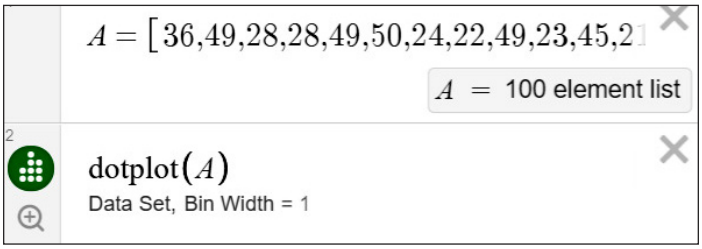


Figure 2

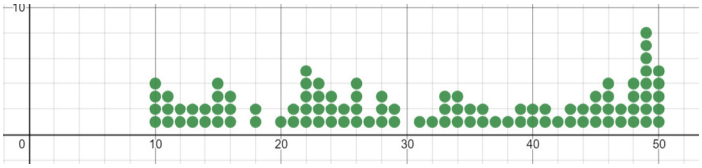


Figure 3

The syntax/options for each tool is shown below it; here you see "Data Set, Bin Width." After you type or select the dot-plot command, these options allow you to specify the data set (here, "A") and the bin width. The default for bin width is 1, but you can change it by clicking on that 1 and entering a new number or directly, within the command, when you set up your dot plot, as shown in Figure 4.



Figure 4

You can also choose to create a histogram of your data, as shown in Figures 5 and 6.

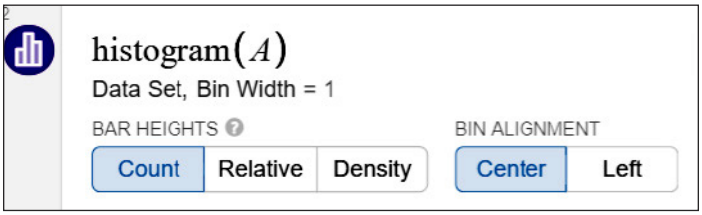


Figure 5

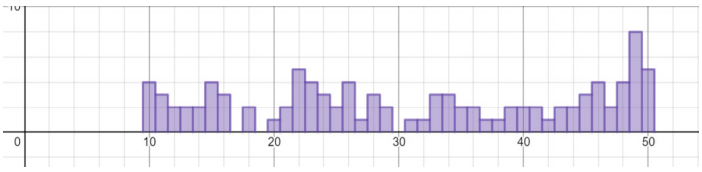


Figure 6

If you change the bin width to 10, you get a different perspective of the data, as shown in Figures 7 and 8.



Figure 7

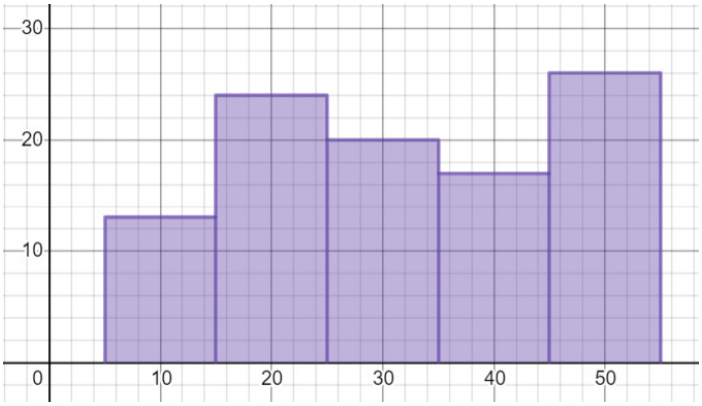


Figure 8

These tools are intuitive and allow students to quickly change the way the data is displayed and focus on exploration.

A box plot can easily be created, as shown in Figures 9 and 10. It can be moved up using the offset, and the height of the plot can also be changed.



Figure 9

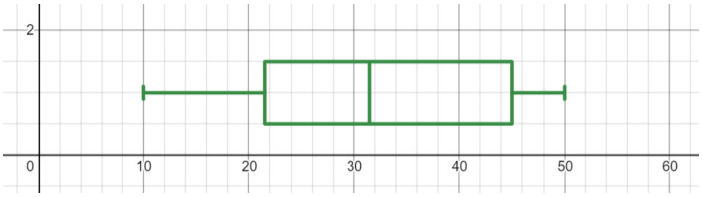


Figure 10

Desmos can create a normal distribution, as shown in Figures 11 and 12. The one shown has been set up to highlight the region between –1.5 and 1.5, with the associated probability of a value being in that range appearing in Figure 11.

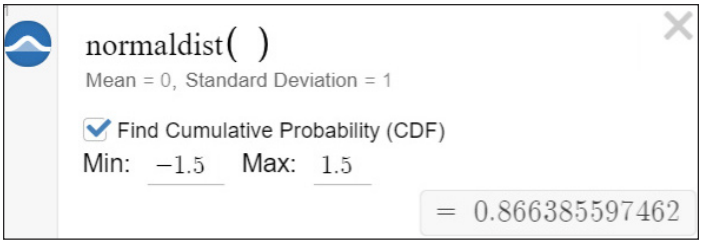


Figure 11

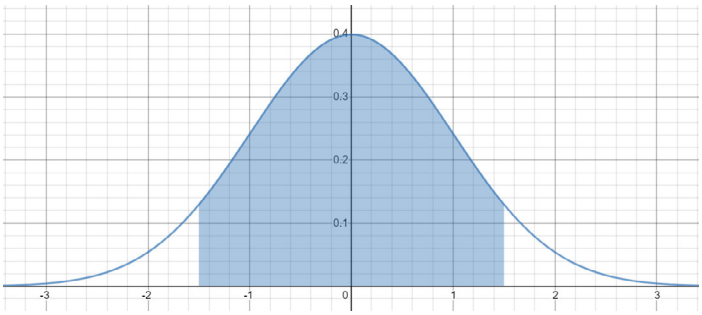


Figure 12

You may notice that there are options for the normal distribution. You can select the mean and standard deviation for your curve, as shown in Figures 13 and 14.

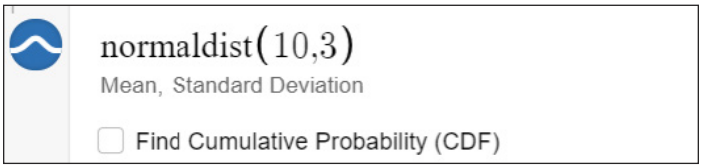


Figure 13

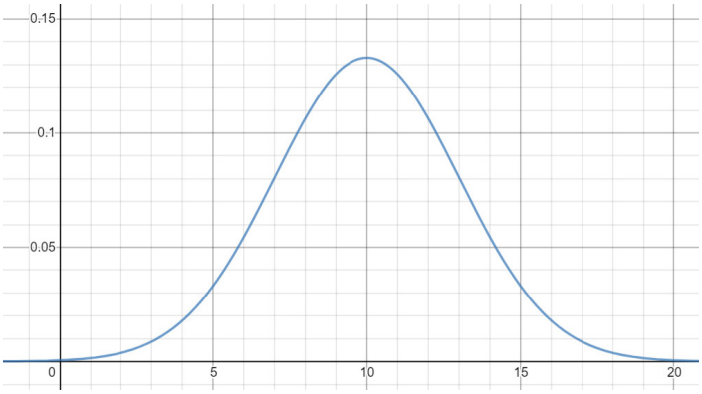


Figure 14

Desmos can also provide binomial distributions, for example, with 40 trials and a 0.3 probability of success, as shown in Figures 15 and 16. Students can learn a lot by investigating what happens when they change each parameter. They could even change the success probability to a variable and add a slider to animate the result.

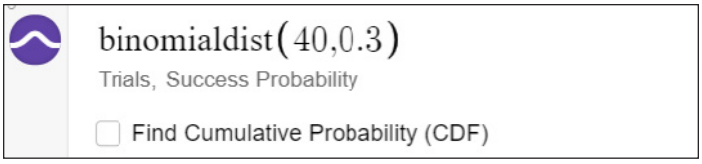


Figure 15

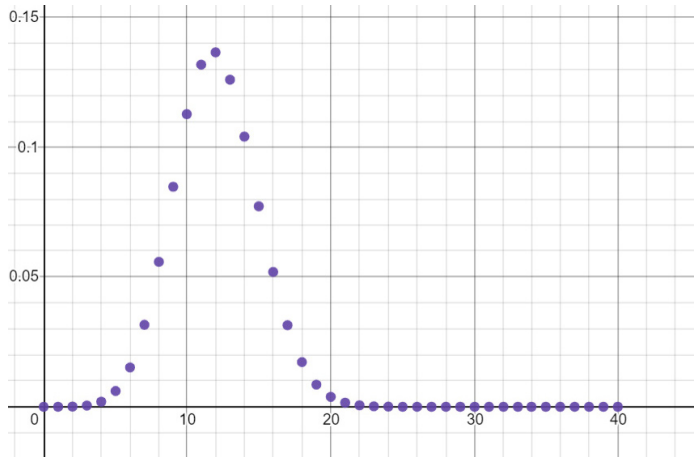


Figure 16

You can find the cumulative probability for a given range of numbers, as demonstrated in Figures 17 and 18. Notice that changes are also reflected in the plot.

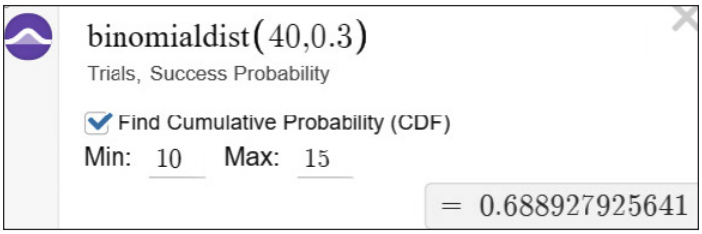


Figure 17

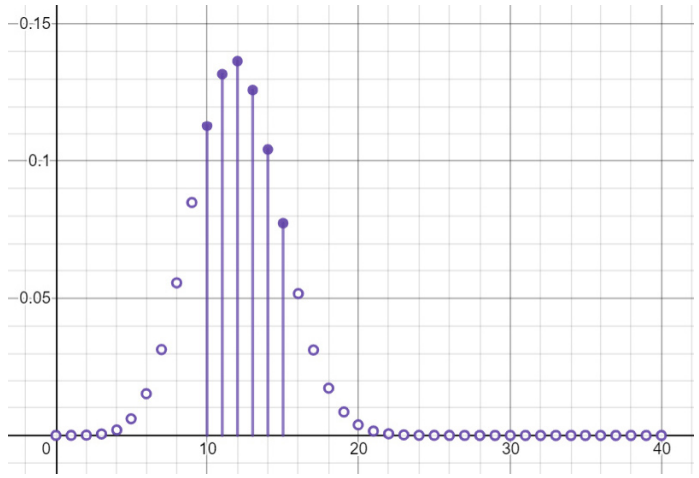


Figure 18

There are many options to explore with all these ways of displaying data, and there are also many other statistical tools and distributions available. Figure 19 shows the "Stats" tab, and Figure 20, the "Dist" tab, both available from the "Functions" menu on the keypad.

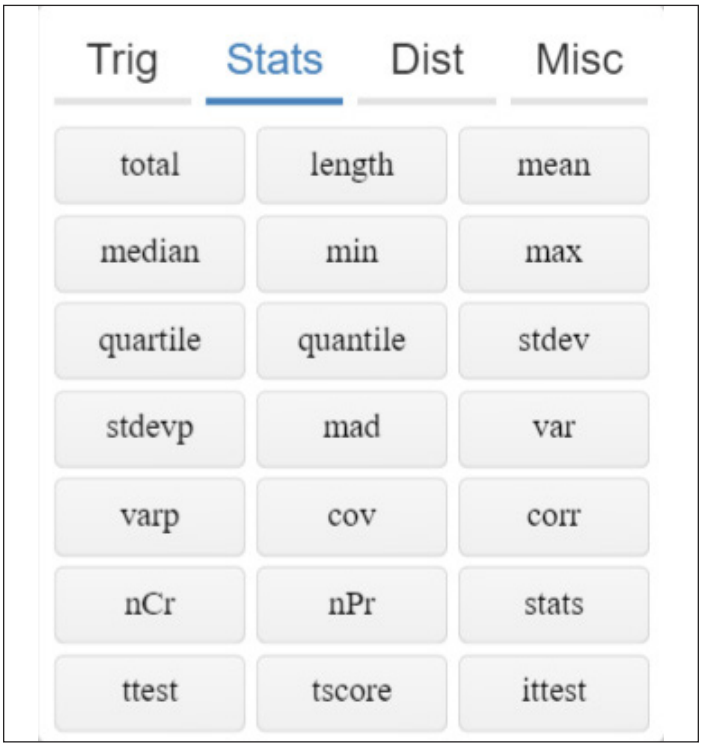


Figure 19

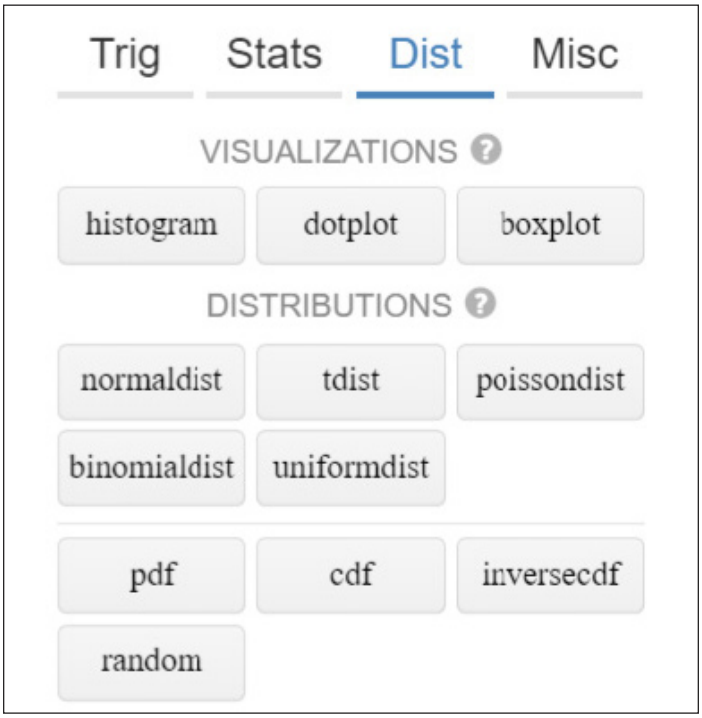


Figure 20

One of the latest additions was the random command. Figure 21 shows an interesting way in which teacher has used the random feature.

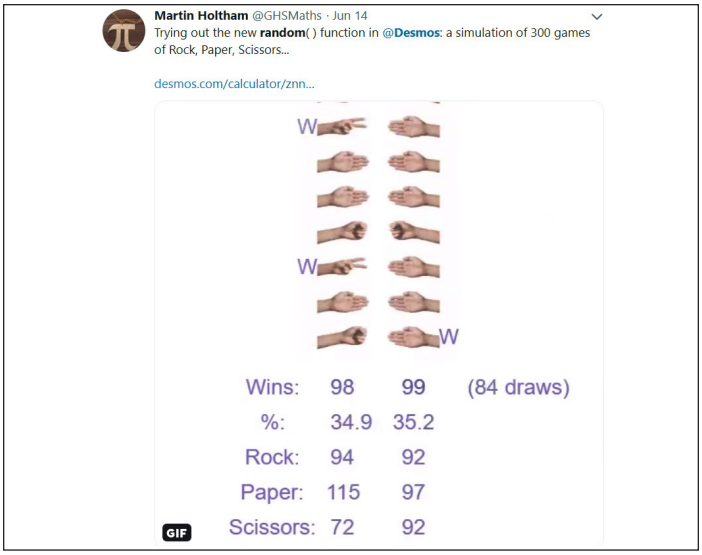


Figure 21

You can also use the random feature to choose values from within a list or distribution. Figures 22 and 23 show one such example, where a sample of 5000 random values were generated based on the specified normal distribution and then plotted as a histogram.



Figure 22

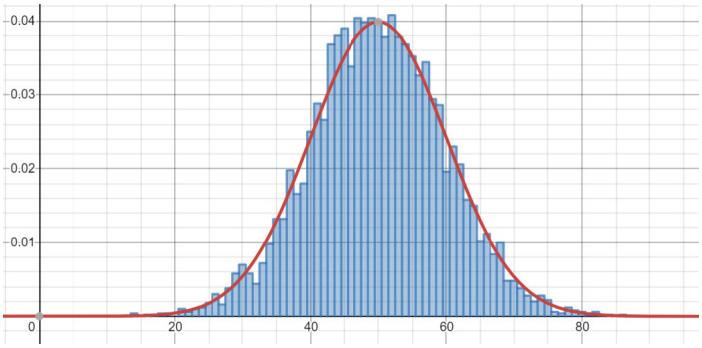


Figure 23

Studying data is both fascinating and important. We want our students to be literate in the information world we live in, and Desmos has made displaying and analyzing data simple. As always, Desmos continues to add new features, so send your requests their way! ▲

▲ Report: Canadian Mathematics Education Study Group 2019 Annual Meeting

PARKER GLYNN-ADEY
ANN ARDEN
EGAN J. CHERNOFF



Parker Glynn-Adey is an assistant professor of mathematics at the University of Toronto, Mississauga Campus. He is interested in using student feedback to make evidence-based teaching choices. Parker is on Twitter as @pgadey.



Ann Arden is a math teacher in the Ottawa-Carleton District School Board and is currently an instructional coach. She has also taught as a part-time professor in the Faculty of Education at the University of Ottawa. Ann is very interested in assessment to improve learning, especially through the use of evidence from conversations and observations. Ann is on Twitter as @annarden.



Egan J. Chernoff (@MatthewMaddux) is a Professor of Mathematics Education in the College of Education at the University of Saskatchewan. Currently, he is the English-language mathematics editor of the Canadian Journal of Science, Mathematics and Technology Education, an associate editor of the Statistics Education Research Journal*, the Book Reviews editor of* The Mathematics Enthusiast*, and sits on the Board of Directors of* For the Learning of Mathematics*.*

This year, the Canadian Mathematics Education Study Group (CMESG) met at St. Francis Xavier University in Antigonish, Nova Scotia from May 31 to June 4.

CMESG is a unique gathering for people working in math education. It attracts math teachers, math education specialists, and mathematicians from across Canada. The meeting is very welcoming, with many opportunities for socializing and networking. The format is refreshingly simple and stress free. There is no frantic rushing around, no deciding between a dozen parallel sessions. Often, there were only two or three choices of parallel session, each of which would appeal to a different kind of math educator. Much of the schedule is devoted to small-group discussion and working groups.

The working groups are the core of CMESG. Every morning, the working groups get together for four hours to focus on a particular issue in math education. Each group gets to hone in on a topic, and develop it over an extended period of time. This year, the working groups were:

- Problem-Based Learning in Post-Secondary Mathematics
- Teaching Primary School Mathematics. What mathematics? What avenues for teacher training?
- Humanizing Data
- Research and Practice: Learning through Collaboration
- Interdisciplinarity with Mathematics: Middle School and Beyond
- Capturing Chaos? Lenses into the K–12 Mathematics Classroom

The working group Parker attended, "Problem-Based Learning (PBL) in Post-Secondary Mathematics," was led by Jeff Hooper (Acadia) and Laura Broley (Concordia). The PBL approach to teaching focuses on students' experience of solving large open-ended tasks. This is not a new idea and, in fact, this year marks the 50th anniversary (!) of the first ever PBL program, which was launched at McMaster University's School of Medicine. The CMESG working group designed a curriculum for "The Problem-Based Learning University," which is a theoretical institution with 8000–10 000 undergraduates, 500 graduate students, with "standard" service courses, and no math program. We took a PBL approach to figure out ways of implementing problem-based learning. Working through a meta-application of PBL over several days was extremely instructive. We got to live the practice.

Parker's take-away from the workshop was the following: "If I were to run a problem-based learning session in one of my courses, then I'd like to go through three levels of testing before trying the session with my students."

- Try the problem on a non-mathematical friend. Is this interesting?
- Experiment with some math friends. What content might it have?
- Take the problem to a math club as a lesson plan. Where do people go with it?

Once I knew that the problem was intrinsically interesting, has some mathematical content, and wouldn't go too wonky, then I would write it up as a lesson plan to be used in a real class.

Ann attended the "Capturing Chaos? Lenses into the K–12 Mathematics Classroom" working group, led by Darien Allan (Collingwood School/Simon Fraser University) and Jean-François Maheux (Université du Québec à Montréal). We began by working on a task in small groups: "You are presented with a collection of coins spread out on a table, and told the number of heads that are visible face up (in this case, yellow). Is it possible (without looking) for you to divide the collection into two groups, with each group having the same number of heads face up?" The images below show Ann's initial attempt to make sense of the problem. Figure 1 shows the case of one head, Figure 2 shows the case of two heads, and Figure 3 shows the case of three heads. It should be noted that any number of coins can (and should) be used.

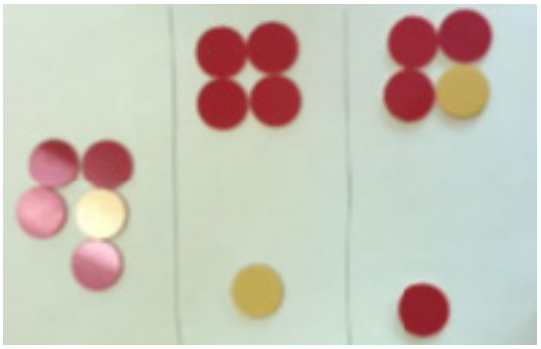


Figure 1: *Case of one head*

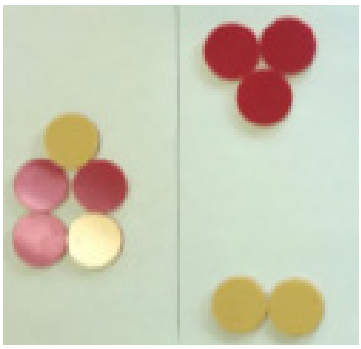


Figure 2: *Case of two heads*

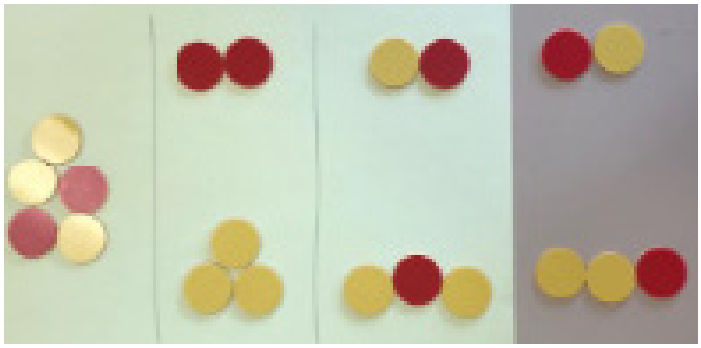


Figure 3: *Case of three heads*

At first, the problem seemed unsolvable, so we thought we had to break the rules that had been set out. When we finally arrived at a solution (no rules needed to be broken), it seemed to have arisen from intuition. A group member

asked questions that probed our small group's solution and built our confidence as we set out to systematically prove our solution. An interesting reflection that our small group had was that we could not trace where the ideas and intuition had come from, which made us think about methodological considerations in mathematics education research. The whole group came to agree that the problem (or any problem) exists in the system of interactions with the material, group members, text of the problem, facilitators, and other factors beyond the four walls of the room we were working in; and the chaos of the mathematics classroom exists in this system of interactions.

Some points and questions that Ann was left thinking about included the following:

- What seems chaotic to one teacher is not necessarily to another teacher.
- We wondered, with the same initial conditions, would we have ended up in the same place?
- What is the role of the teacher/facilitator? What "branches" do we break as we "walk through the forest" of a lesson? Facilitators only have partial understandings and have to make choices about what and who to attend to.
- What am I collecting as a teacher/researcher if I engage in the activity versus if I do not engage in the activity?
- People learn in different ways all the time—what are the boxes that prevent learning from happening/flowing?

Egan attended "Teaching Primary School Mathematics. What mathematics? What avenues for teacher training?," led by Jennifer Holm and Vincent Martin. It was well attended by a diverse group who were all interested in the teaching, learning, and thinking of future elementary school teachers. It was clear, right from the very beginning, that this was a well-organized working group, and a great deal of thought was put into the activities.

There was a definite buzz in the room during the first activities, which required participants to role-play. One participant would play the student and the other would play the teacher. Not so fast, though, the task was for the participant, the one playing the student, to become familiar with a common mathematical error, and then it would be the teacher's job to help that student overcome the issue. The range of errors presented by the organizers was impressive and had quite the range.

Activities on the other days did not disappoint. Our efforts to decipher an approach to counting, which we were not intimately familiar with (ultimately rooted in bases other than 10) was met with different emotions from different people in the room. The different emotions were well handled by the leaders of the working group with the support of the rest of the room, which had been established during the earlier activities. Ultimately, this topic moved beyond discussions concerning the mathematics and branched into other topics related to future elementary school math teachers, such as anxiety, emotion, and other affective aspects.

One other activity worth mentioning, one that garnered a lot of discussion both during the working group and during other parts of the conference, involved reading excerpts from future elementary school math teachers' autobiographies. Given three very different excerpts, Jennifer and Vincent asked us a series of questions, both in our small groups and in a larger group discussion. The varying perspectives that were presented by the participants were, at times, striking to hear. It made one realize how quickly we judge teachers whom we have not even met, and that made for interesting discussions about future teachers, as well as the larger zeitgeist about online anonymity.

In addition to the working groups, there were various other activities that included two plenary speakers. The first plenary talk, "Discovering Mathematics Together with the Students," was by Jean-Marie De Koninck (Université de Laval). Jean-Marie De Koninck is an amazing polymath. He is a prolific researcher in analytic number theory, a world-class swim coach, a math popularizer, and more! He told us about the growth and development of "Science and Math in Action." We got to hear how a chance encounter with a TV crew at a math conference has grown into several full-time productions of theatrical math shows for elementary schools and high schools. Jean-Marie and his team developed ShowMath 1 and 2 (for high schools) and Le Petit ShowMath (for elementary schools). We also learned about the development of an online, multiplayer quiz game (mathamaze.ca), that challenges students to solve interesting math questions and puzzles developed for students by Jean-Marie's team. A screenshot from the game is included below in Figure 4. Jean-Marie's website is www.jeanmariedekoninck.mat.ulaval.ca/en/home/.

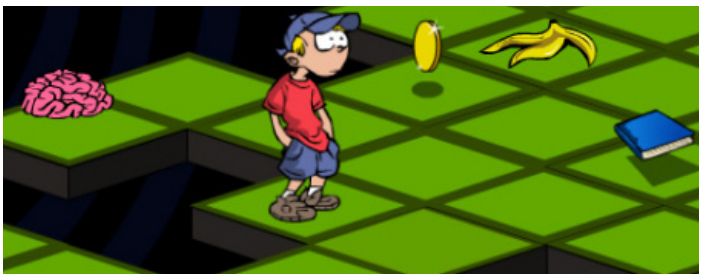


Figure 4: *Opening screen in Mathamaze.ca game* (Image from: https://mathamaze.ca/index.php/about)

The second plenary talk, "Mathematics as Dispossession: Reclaiming Mental Sovereignty by Living Mathematx," was by Rochelle Gutiérrez (University of Illinois). Rochelle talked about her work, which focuses on issues of identity and power in mathematics education, with particular attention to how race, class, and language affect teaching and learning. Her work challenges deficit views of students who are Latinx (a gender-neutral term, instead of Latino and Latina), Black, or Indigenous, and suggests that mathematics teachers need to be prepared with much more than just content knowledge, pedagogical knowledge, or knowledge of diverse students, if they are going to be successful. They need political knowledge. At CMESG, Rochelle shared a holistic ecological approach to mathematics that she calls "mathematx" (pronounced: mathematesh). This approach values an embodied ethics-driven mathematics, and acknowledges the reciprocity inherent in all teaching and learning. This includes seeking, acknowledging, and creating patterns for problem solving and joy (including intuition, aesthetics, and meaning making), and reflecting an ecology of knowings (no universalism). She argued that current forms of mathematics do not honour the ways many nations view the work or create new knowings, and that mathematicians and math educators need to do more than apply mathematics in ethical ways. Rochelle's website is https://education.illinois.edu/faculty/rochelle-gutierrez. If you are interested in learning more about Rochelle's work on Mathematx, here is a link to a video talk, where she discusses some of this work at the Latinx in the Mathematical Sciences Conference 2018: www.youtube.com/watch?v=D266LYIigS0.

Next year's meeting of the Canadian Mathematics Education Study Group will be held at the University of Ottawa in late May or early June. The website for CMESG for information on upcoming meetings and to see the proceedings of previous meetings is www.cmesg.org. ▲



Summer fun: Make the OAME logo!

▲ OAME/NCTM Report: The 2017 NCTM Annual Meeting and Exposition



PAUL ALVES
Paul.Alves@oame.on.ca

Paul Alves, current OAME/AOEM President, is department head of math at Fletcher's Meadow Secondary School in the Peel District School Board.

The NCTM Annual Meeting and Exposition took place on April 3–6 in San Diego. The conference site encompassed the San Diego Convention Centre and the Hilton San Diego Bayfront. If you have never attended an NCTM conference (this was my NCTM rookie year too!), I would highly recommend making it a priority, if possible. How does the NCTM conference differ from other math conferences such as our own OAME/AOEM Annual Conference, and what were the highlights of NCTM 2019?

The first thing that struck me was the scale! The conference spans four days, and those days are packed with plenty to check out for the intrepid delegate. The variety of sessions offered during each time slot was impressive. There were three types of sessions offered at this year's conference: workshops, sessions, and bursts. Workshops were intended to be interactive in nature and lasted 75 minutes. Sessions were lecture-style presentations and lasted 60 minutes. Bursts are a unique option that last 30 minutes and provide the delegate with quick learning on one specific topic. There were also exhibitor workshops offered during each session slot.

The opening plenary was delivered by Dr. Gloria Ladson-Billings, and the title of the talk was "Are We Still Solving for X? The Pedagogical Practices Limiting Students Success in Mathematics." The opening session set the tone for what turned out to be a great few days of learning. Following the opening session, delegates were free to attend the sessions of their choosing. The sessions were streamed into different strands that included: Assessment (eliciting and using student thinking); Building on Students' Strengths (practices that challenge, engage, and empower); Professionalism and Advocacy; Beyond the Classroom Walls (empowerment, access, and equity); Creating Inclusive Classrooms (meeting the needs of each and every student); Building Mathematical Knowledge for Teaching; Enhancing Mathematical Thinking Through Reading, Writing, Speaking, and Listening; For the Love and Joy of Mathematics; and a New Teacher strand.

Word to the wise, take some time and peruse the program. There were printed programs available, but the conference app provides the delegate with an opportunity to filter based on strands and grade bands. The app was also updated in real time, which helps just in case there is a change to the program. Try to plan out your day and have some backups in each time slot. Unlike the OAME Annual Conference, there is no pre-registration for sessions. You will need to arrive a few minutes early to ensure you get a seat to a session you have your eye on.

One unique feature of the conference is the inclusion of a session specifically focused on equity—the Idris Carl Equity Address. This year's address was given by Jose Vilson and was entitled "Equity and Agency from Inside the Classroom." This was my personal highlight for the conference. Jose delivered a powerful talk, touching on issues ranging from the manner in which math is taught ("Math is not and never was neutral."), and its impact for students of colour ("We must all point toward equity and justice."). His address was inspiring. For those interested in learning more, check out Jose Vilson's book, *This Is Not a Test: A New Narrative on Race, Class, and Education* (2014, Haymarket Books), and his work with the organization EduColor, which he founded.

The closing plenary speaker was Talithia Williams, and her talk was titled "A Seat at the Table: Equity and Social Justice in Mathematics Education." The closing session didn't disappoint, and was a fitting conclusion to a great conference experience. Dr. Williams weaved personal stories of her own journey, and the importance of mentors who fostered her abilities and sometimes pushed her to enter arenas that seemed closed to her. Through her personal narrative, the theme of allowing our students to see themselves in the math we teach emerged. I highly recommend watching her Ted Talk and also checking out her book, *Power in Numbers: The Rebel Women of Mathematics* (2018, Race Point Publishing), which touches on many of the same themes she addressed in the closing session. All of the talks mentioned, along with ShadowCon (a unique blend of a live experience and an online sharing), Ignite Talks (short 7-minute talks from inspiring speakers), and the president's address can be found at the NCTM 2019 page on the NCTM website.

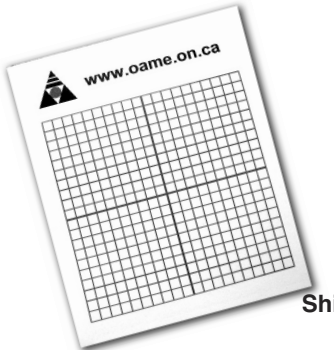
This year, the exhibit hall was located in a large room in the convention centre. Give yourself a good hour to take in all that is available in the exhibits area. You may even want to split this up over two visits, as there is a lot to see and do. Other than visiting the vendor booths and taking advantage of some of the vendor coupons available in the printed program, there were a lot of learning opportunities available in the exhibits hall. There were reflection coves, where delegates could have one-on-one informal discussions with presenters after their workshops. This was a nice feature that avoided the rush to ask questions of presenters before the next session took place in the same space. There were also math circles available for those who wanted to take a deeper look at some of the math presented in some sessions, and a networking lounge.

It is also important to allow some of the learning from sessions to be absorbed, and to just take a break from all that is happening at the conference. Lunch is not included in conference registration, so do some investigating of nearby places to grab a quick bite before getting back into the regular flow of the conference. NCTM has also addressed the need to look after your wellness with wellness workshops and morning yoga. I didn't partake in the morning yoga, but it was still fascinating to see how the many needs of delegates were addressed.

If you are interested in attending the NCTM conference, 2020 is a special year for the NCTM, as it will be the centennial year for the annual meeting and exposition. The centennial will take place April 1–4 in Chicago. This will mark the last year that the conference takes place in the Spring, as it will be moving to the Fall for the following school year. Again, if you can manage it, try to add this event to your professional development calendar for the year. You will come back energized and ready to attend the OAME Annual Conference on May 7–8, 2020 in Oshawa. ▲

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▲ Access, Equity, and Inclusion OAME/AOEM Position Statement (June 2019)

The goal of each OAME/AOEM Position Statement is to outline OAME/AOEM's beliefs on issues surrounding math education by providing transparency and guidance to those involved with mathematics education in Ontario. Any clarification regarding a position or how to proceed in accordance with it can be brought to the attention of the OAME/AOEM Executive Committee.

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

Focal Question

How do we as a community of educators promote access, equity, and inclusion in mathematics education in Ontario?

Summary

In order to promote and sustain an inclusive environment for students and teachers:

- All students should have equitable access to a high-quality, research-informed mathematics education that meets their diverse needs and is provided in a safe and inclusive environment. This includes equitable access to resources and should not be hindered by geographical location, socio-economic status, race, religion, gender, ability, or any other distinguishing feature.
- All teachers should feel empowered to embed practices that are culturally responsive and relevant to all of their students' needs and identities.
- All teachers see themselves reflected in organizations such as OAME/AOEM.

Equity and Access within OAME/AOEM

All educators should feel valued, reflected, and represented within the OAME/AOEM's Board of Directors, Chapter officers, and the associated learning opportunities. The voices of a diverse membership create a strong mathematics learning community.

Elections within OAME/AOEM and its chapters are open to all members (refer to By-Laws 11 and 12 in the *OAME/AOEM Constitution* for more details of elected positions). The OAME/AOEM Board of Directors and Chapter Officers will engage the membership by informing it of the opportunities to run for elected positions, and reporting results of the election in accordance with the *OAME/AOEM Constitution*.

It is important and desirable to have a Board of Directors that is representative of the Ontario population that it serves. OAME/AOEM envisions that all members have opportunities to access professional learning, either in person or via innovative opportunities such as the eConference, webinars, and archived content. OAME/AOEM endeavours to ensure that speakers and facilitators of professional learning reflect the diversity of our membership, students, and the Ontario population. The resources and learning opportunities OAME/AOEM provides will reflect and support the diversity of our membership, students, and the Ontario population.

Equity and Access in Teaching and Learning

All students can succeed in learning mathematics. Teachers play a vital role in ensuring students receive equitable environments to learn mathematics. We recognize that "building relationships is foundational to ensuring mathematics instruction is culturally responsive" (Beatty, 2016). By knowing their students' profiles (lived experiences, cultural perspectives, and capabilities), teachers are better able to develop a diverse and dynamic learning environment for their students. That environment should be inclusive, and provide learning opportunities for all students, regardless of racial, ethnic, religious, linguistic, gender, and socio-economic backgrounds (and any other distinctions that might be exclusionary). The pathways available to students provide options for them to select courses suited to their current readiness and future pathways decisions. All pathways for students should be treated equitably. Regardless of what pathway a student has selected, he or she should have access to high-quality instruction and resources.

We advocate that all learners see themselves in mathematics programming that reflects their voices, choices, capabilities, identities, and experiences. It is the responsibility of all educators to continually be responsive and reflective with respect to their instructional practices in order to further build equity within the mathematics education community. All students should have access to high-quality instruction, regardless of their geographical location, socio-economic status, race, religion, gender,

ability, or any other distinguishing feature. The history and development of mathematics should be honoured in an inclusive way that may include different ways of knowing and doing mathematics.

We encourage teachers to continually examine their practice and resources for biases. These opportunities, for investigating how we teach and how we view our students in the mathematics classroom, are moments for dialogue and professional growth. The OAME/AOEM supports the empowerment of teachers to address equity, access, and inclusion within their roles.

Equity and Access to Resources

Achieving equity and accessibility with regards to resources requires both deconstruction and construction. The goal must be to deconstruct the barriers that impede student success, while simultaneously constructing communities and environments where all parties are included, and all resources are accessible. Teachers, schools, and school boards should have up-to-date policies and resources to ensure that they reflect and honour the community and environment of the student. All students should be able to access resources, regardless of their differences. All teachers should have access to high-calibre professional learning, including the implementation of sound pedagogical resources, that is dedicated to creating a high-quality learning environment for all students. In addition, teachers should have access to professional learning opportunities that allow them to learn how to incorporate these resources in an appropriate manner that is culturally responsive and benefits student learning.

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▲ Interview with Dylan Wiliam



ANN ARDEN
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Ann Arden is a math teacher in the Ottawa-Carleton District School Board and is currently an instructional coach. She has also taught in the Faculty of Education at the University of Ottawa, and her son's teacher this year was a former student! Ann is very interested in assessment to improve learning, especially through the use of conversational and observational evidence. Ann is on Twitter as @annarden.



Ann Arden interviewed Dylan Wiliam, Professor Emeritus, University College, London, England, on March 26, 2019 via Skype. Ann was interested in interviewing Dr. Wiliam because of his foundational work in the area of classroom assessment (reference list included below), particularly related to mathematics education. Dr. Wiliam's work is cited in Ontario's *Growing Success* (2010) document, and he has been published widely in international peer-reviewed journals, NCTM publications, and written several books for teachers. His website is www.dylanwiliam.org.

Thank you for your time today. What were your experiences as a school mathematics student in your Kindergarten to Grade 12 years? Do you have a particular experience that stands out?

I remember very little about my elementary school years. I wasn't particularly successful at school, but when I got to secondary school at age 11 (the age in the UK system). I was pretty good at math. We had 150 kids in the grade, and I was generally in the top three or four in annual examinations. Then my parents moved from Cardiff (Wales) to Manchester (England), and the new school did "modern math." Where I had come from [a school that] did "traditional math." I went from being someone who had been successful at math to someone who was really hopeless. I had no idea what was going on. The school I moved to grouped kids by ability for math, and I was at the bottom of the third group out of five. It was quite an odd experience for me—from having been very successful at mathematics to being very

unsuccessful at mathematics. It took me a couple of years to figure out what was going on. Then eventually I caught up. That is my earliest memory of school mathematics—finding out that none of what I was doing made sense.

I'm really interested in the phrase you used: "I caught up." Do you have an understanding of what that meant for you?

I think I got some private tuition. I think it was mainly familiarity. I understood what the teachers were looking for. At the time, the kids in the second school took their school leaving examinations a year early and I didn't. I was in one of the groups that was not an "express class." So there were loads of kids who were in front of me, or so I thought. When I got to the UK equivalent of the eleventh grade, I discovered I was actually the best kid in the school in math. So, from having been someone who was toward the bottom, I suddenly discovered I found this study easier than everyone else in my class, which was a bit of surprise.

In Ontario, "streaming" of students starts in Grade 9 in subjects like math, science, and English, but we are finding that streaming students into lower tracks often means outcomes are not as good as if we had tracked that same kid into the academic stream.

There's an unpalatable fact that nobody wants to face up to—some kids learn more slowly than others. And this goes back to John (Bissell) Carroll in the 1960s. He was arguing 50–60 years ago that there was no limit about what children could achieve, but some kids would take longer to get there. And what he did, that no one had done before, was to quantify that. He pointed out that a kid at the 95th percentile of achievement is generally learning at five times the speed of someone at the 5th percentile of achievement. So basically, the student is learning in one year what other kids would take five years to learn. And that fits the observed data quite well. But nobody wants to face up to that. So if we want genuine equality of opportunity, then we have to find ways of getting more instructional time to the kids who need it. And that's why we get this huge range of achievement.

We think that teaching all kids the same is equitable, and it's not, because some kids find learning stuff easy, and some kids find learning stuff hard. Let's just look at the elementary grades. The fact is there are some kids in a fourth-grade class who are learning at four times the speed of other kids given the same instruction. The question is: As a teacher, what are you going to do about that? What I see all around the world is a disinclination to face up to that challenge. My solution is very clear. Our job as teachers is to destroy the bell curve.

Kids differ in their mathematical talent. Nobody likes admitting that, but it's true. Some kids find learning math easier than others. If you treat all kids the same, then what you get is mathematical achievement being directly related to mathematical differences in talent. What I'm saying is you can't deduce an "ought" from an "is." So that something is the case doesn't mean that something ought to be the case. In fact, I want to destroy the bell curve by making sure that kids who find learning mathematics more difficult get more instructional support: more time and also the best teachers. Because what we know is that the most effective teachers are about four times as effective as the least effective teachers. So if we systematically assigned the most effective teachers to teach the students who struggled the most, we could actually close the achievement gap. But treating all kids the same makes the achievement gap bigger, and what we do in many schools is we assign the best teachers to teach the highest-achieving kids, which makes the gap even bigger. I don't know what the solutions are. I see what is happening in the rich countries in the world—a refusal to acknowledge the magnitude of the problem.

What if we misidentify kids and put them in a lower track or stream? But these kids have really high potential. Something I've heard from the mathematicians I've talked to is that sometimes they have had to take more time than others, but there is something that, as a society, we don't understand. We look at people who are mathematicians and think they understand things very quickly, but many of the mathematicians who are very successful have actually taken a lot of time to learn things. That isn't part of the narrative that we as a society put forward.

No argument here. One view of mathematics is that it requires memorization and needs to be done quickly. And then there's the view of math from mathematicians, which is that it is a subject that requires elegance and beauty and you never need to do anything quickly. I have no idea what we should be doing. All I'm saying is the empirical evidence shows some kids find learning math more difficult than others. Our education systems don't face up to that. I don't know what it is we should be doing. These issues are more complex than the positions most people take, which are that mixed- ability groups are good and same-ability groups are bad. I don't think the world is that simple. You cannot divorce these things from the quality of professional development for teachers to improve their practice. These things have to sit in a whole range of contextual issues and then also a set of cultural factors about what people value in a system.

Your research is foundational in the field of assessment, whether that is in mathematics education or beyond. Your work with Paul Black

(Black & Wiliam 1998a, 1998b), your work in the research companion for NCTM (Wiliam, 2007), your work in formative assessment (e.g., Wiliam, 2011), and your work with Siobhan Leahy (2005, 2015), to name just a few pieces. You define assessment a bit differently than others. You say, "Assessment is nothing more than a procedure for making inferences." You say, "Assessment is that bridge between teaching and learning." Can you talk about shifting assessment away from the assessment itself to the inferences made?

First of all, I did not learn to speak English until I was ten. I was brought up monoglot Welsh. What is interesting is that Welsh uses the same word for teaching and for learning, and it causes no confusion at all. Many languages don't distinguish between teaching and learning. Maybe I thought more about the difference between teaching and learning because I had to work hard at that distinction. For me, teaching is what teachers do, and learning is this mysterious process that happens inside students' heads. So I've been very critical of schools that have learning and teaching policies because you can't have a learning policy. Learning is this strange process that happens inside students' heads. You can have a teaching policy, and it seems to me that this goes back to the work of David Ausubel in the 1960s. Good teaching starts where the students are. Put together with the fact that students do not learn what we teach, that's why assessment is the central process in teaching. It's only by assessing that you're going to find out if what you did as a teacher has resulted in the learning you intended, which is where that idea of assessment as a bridge between teaching and learning comes from. You can't find out what students have learned without some kind of assessment.

The next part of it has to do with my enculturation as a psychometrician. The work of people like Samuel Messick in the 1980s, and earlier on [Lee] Cronbach in the 1970s, focused on this idea that validity cannot be a property of assessments—it doesn't make sense. Assessment can be valid for one purpose and not for others. For example, a mathematics test with a high reading demand might support inferences about mathematics achievement for good readers, but not for poor readers.

Can you talk about learning intentions and success criteria? These terms are prevalent right now, and I know that I am struggling with how to incorporate success criteria, even when I am quite clear about my intentions for students' learning.

That's one of the things I find disappointing about the Ontario take on our work. One of the things we've been very clear on is that learning intentions (not learning targets, not learning objectives) are the starting point. The absolute minimal starting point is that teaching should be an intentional activity. Teachers should have some idea of what it is students are able to do after some time with the teacher. To me, the broadest, most superordinate set is the learning intention.

You may be able to express this as an objective, or a target, or a goal, but you may also not be able to do that. For me, the learning intentions include all the things teachers might intend, including a francophone idea from people like Linda Allal in Switzerland, the regulation of learning processes. In some cases, you regulate the learning very tightly because you're trying to get every single kid to the same point. In other mathematical situations, you aren't regulating learning very tightly because it's okay if different students explore the same ideas in different ways. In some situations, the teacher wouldn't have the same goal for the whole class, and it would be okay for different students to explore different aspects. Sometimes you want every single kid to be able to balance chemical equations or factor quadratic equations, or sometimes you're happy for different kids to learn different things. And what's really important is that the teacher is really clear on what type of situation this is. You get confusion if the teacher doesn't have a clear idea if this is a convergent or a divergent kind of activity. That's why we ended up with the phrase "learning intentions."

Here's the other problem. Learning is a change in long-term memory, so you cannot establish learning within the instructional episode. Because if nothing has been changed in long-term memory—if kids cannot do it two weeks later—they haven't learned anything. So success criteria are different from learning intentions. Learning intentions are the long-term change in memory you hope will result from students engaging in these instructional activities. Success criteria are the criteria by which you are going to judge the episode today.

In the research I have read, comments are more effective for improving learning than grading and scores (Butler, 1988; Kluger & deNisi, 1996). You say feedback should be forward looking in terms of what is coming, but often "feedback" is the rearview mirror. How do we help teachers get better at giving feedback that moves the learner forward?

In terms of my most recent work, the thing I have found most helpful to say to teachers is this: the purpose of feedback is not to improve the work, it's to improve the student. It's to improve that student's attempt at a task he or she has not yet attempted. In mathematics, it's particularly difficult because all we have to go on is the task the student has just done. What could I say to this student now that

would improve his or her work in the future? When you use that perspective, I think it's very clear that correcting the kid's mistakes is not helpful, but strategies like "five of these are wrong—you find them and fix them" might be. What seems to have resonated with teachers is that feedback should be more work for the recipient than the donor. More importantly, the purpose of feedback is to improve the student, not the work. Too often we focus on the purpose of feedback as improving the work, but the student doesn't benefit at all if the teacher does the intellectual heavy lifting.

How can we, in something like mathematics, use feedback to improve the student and not the work when what's in front of us is the work? One useful strategy is [getting] pairs of math teachers together, bring six student notebooks with you, and think about, discuss, what would be helpful to say to this student right now? Take that process that normally teachers give a minute's thought to and unpack that. Slow that down. Take 10 minutes with each student notebook. What would be most useful to say to each student right now? And what usually happens is teachers say, "I don't know," and I say, "Yes, that's because the task you set them wasn't interesting enough to give you insight into their learning…."

The really important point is you can't give good feedback unless you ask the right questions or set the right tasks in the first place. So for math teachers in particular, one of the reasons they find feedback so hard is because the tasks they set students do not reveal anything useful about students' mathematical thinking. So usually you have to go back to the drawing board and say, "I can't think of anything useful to say to this student right now because I didn't learn anything about his or her mathematical thinking, seeing the task I just gave the student. I need to set better tasks to get better insight into the student's learning."

In 2015, you wrote, "Choosing the tasks to find out what students have learned is far from straightforward" (Wiliam, 2015, p. 251). My question is, how do teachers make sense of, or do better at, choosing tasks? At a classroom level, how do teachers choose these tasks when it is far from straightforward? What advice do you have for us classroom teachers?

First of all, if it's a very straightforward mechanical exercise like, say multi-digit subtraction, you might want to make sure that you are setting tasks that, if students are just subtracting the smaller digit from the larger digit, are going to give you different answers from students who understand how to regroup and subtract properly. The important thing is that your knowledge of the misconceptions students have about a particular topic or piece of work is your best friend in designing these tasks because you can actually then design tasks that elicit evidence of these misconceptions and you can say, "Ah, I see that student made that mistake so I now know what to say to him or her." The whole process is designed back from the feedback. You start with the misconceptions, elicit the evidence of that, and then you know what to say to the students because you designed the task with those misconceptions in mind.

Something I saw in a video on your website… I hadn't considered the difference between multiple-choice questions where there is one correct answer, which are really hard to design, and multiple-choice questions with multiple correct answers as classroom formative assessment. You can actually differentiate and engage students if there is more than one right answer, and you did say they were basically true/false questions—is "A" right or wrong, etc. And you made me think. Can you talk a bit about this strategy?

So, for example, if you've got a semi-circle with a radius of 10 cm, then you ask kids what the area is of the semi-circle, then one of the options could be 50p, which is (pr2)/2, but one option might be (20/2)2p/2. So the point is they're equivalent, but it's much harder to recognize the second answer is correct as well as the first, so that will keep your more high-achieving students on their toes; and the really important point is that other students will be perfectly happy they got one right answer, but you can stretch the high achievers. I call this "inclusive differentiation"—the idea that we differentiate in an inclusive way. We don't make kids feel bad that they didn't get all the right answers, but we also seek to challenge our highest-achieving students.

How do we help teachers improve formative assessment? As you argue, "Students do not learn what we teach" (Wiliam, 2015).

There's a classic paper by Stanley Erlwanger (Erlwanger, 1973). It just documented a student who the teacher assumed was doing well in this mathematics program. In fact, this kid had no idea what was going on. The starting point is you have to understand what's going wrong before you can actually do anything about it, which is why I keep coming back to the idea of evidence elicitation. You can't give feedback until you know what's going wrong, and you don't know what evidence to elicit until you are clear about the learning outcomes. Until the teacher is clear about what learning you intend to occasion, then none of these processes can begin.

Why is math so difficult to learn compared to other things?

It's a discipline that is unyielding. There are not many

other subjects where you can be just flat-out wrong. You can take a history topic and come up with an unusual approach to a particular topic. In English literature, you might argue for a character having these views as opposed to those views. In mathematics, it's just more possible to be flat-out wrong. I'm not saying there's no place for elegance and beauty, but the fact is it's a discipline and you have to learn to work within a set of rules that are pretty unyielding. It's a matter of practice.

Why is mathematics singled out over chemistry, physics, biology, or philosophy?

I think there's a gradation from mathematics to science to history to English. So, for example, in English, almost anything goes. In history, you're trying to explain previous events. In science, you're trying to predict future events. In mathematics, what's probably prized more than anything is internal consistency. You have to be consistent with the axioms. Science is pretty unyielding. There's just less room for inconsistency in mathematics. And then there's this belief that it's your value as a person. Being good at mathematics means you are a smart person. Therefore, all that baggage gets invoked and therefore it brings that price tag. Then what's worse is this societal belief that it's okay not to be good at mathematics. I hear parents saying, "It's okay, I wasn't any good at math either." They would never say that about reading. Basically math is like reading. The more you practise, the better you get, and somehow, we've allowed people to get away with the idea that this might not be your subject in a way that doesn't apply to other subjects. Those are just some of the reasons. There is no simple formula. There's just a whole number of factors that come together.

Thank you so much for your time and this conversation.

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Numerosity of Animals

"Some non-human animals have demonstrated symbolic number representation and ordinality of those signs. Pigeons have demonstrated learned symbolic number representation through peck number on a sign representing that numerosity [2,3]. The combination of ordinality and symbolic number representation has been demonstrated in an African grey parrot [4–6], chimpanzees [7–15] and rhesus monkeys [16]. Alex, a single African grey parrot, was able to learn the names of numerals and subsequently sum the quantities [5,17,18]. In addition, chimpanzees have been able to learn Arabic numerals and order them quantitatively [7–15], and rhesus monkeys have learned to choose the larger of two Arabic numerals to receive a corresponding number of food pellets [16] as well as match numbers of items to the correct Arabic numeral [19]."

From: Howard, S.R., Avarguès-Weber, A., Garcia, J.E., Greentree, A.D., & Dyer, A.G., (2019). Symbolic representation of numerosity by honeybees (Apis mellifera): matching characters to small quantities. *Proceedings of the Royal Society B: Biological Sciences, 286*(1904). doi:10.1098/rspb.2019.0238

▲ Hey, It's Elementary: Remembering Susan Stuart: An Unsung Pioneer in Mathematics Inclusion and Diversity



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As classes end at the Faculty of Education each year, I embark on my annual "cleanup," going through the mountains of paper and assorted detritus that accumulates on my desk and in my office from class to class and meeting to meeting between September and May. Most of the "stuff" is predictable: orphan assignments, extra copies of activity pages, broken calculators, oddball accumulations from tangram and other assorted puzzles, children's books, and, of course, a miscellany of reference books and resources.

Among the 2019 paraphernalia, I unearthed a wonderful surprise: the workshop papers from a 1999 conference held at Queen's for BEd teacher candidates to supplement their math program as part of *The Joy of X* initiative at Queen's Faculty of Education, spearheaded by Dr. William Higginson. What a program it was! Everything from a session on dragon curves by Natalie Sinclair (who now holds the Canada Research Chair in Tangible Mathematics Learning at Simon Fraser University in Vancouver) to an Ethnomathematics workshop by Susan Stuart, (who was at the time) a professor at Nipissing University, with origami, tessellations, and anthropometry in between.

The Ethnomathematics paper was most poignant, given that Sue died on January 3rd this year—a great loss, not only to her family, friends, and community, but to the OAME mathematics education family and beyond.

As I reread Sue's presentation paper, it became clear that it was as timely today as it was 20 years ago. In the conference proceedings article, Susan referred to a seminal paper by Katz (1994), who argued that mathematics ideas emerged from many different cultures because people needed to solve problems. The goal of Susan's workshop was to present content and strategies that supported teachers' efforts to enable students to experience curriculum through an ethnomathematics lens. Susan believed deeply that it was important for students to develop a deeper understanding of how mathematics was invented/utilized across cultures. As a mathematics education leader who had important teacher professional development roles in South Africa, Anguilla, Montserrat, and Bhutan, Susan was committed to applying ethnomathematics principles in the classroom to illustrate that mathematical ideas have never been restricted to any one culture: the basic tenets of inclusion and diversity.

In her background notes, she referred to the original work of Brazilian educator and mathematician Ubiratàn D'Ambrosio, who coined the term "ethnomathematics" in 1977 during a presentation for the American Association for the Advancement of Science, and then went on to write extensively about the topic, beginning in 1985 (D'Ambrosio, 1985). Sue also adapted the teaching suggestions included in Katz's (1998) mathematics history textbook, to make ethnomathematics examples accessible and interesting to children across the elementary grades.

To this end, Susan advocated for teachers to "include mathematics history in lesson planning." She encouraged teachers to give a brief history of the contribution of a particular culture or gender to mathematics in relation to the topic. It was evident from her writing that Susan believed concrete examples and illustrations could help to dispel elitism, i.e., the notion that mathematics ability is limited to only some people based on ethnicity or gender.

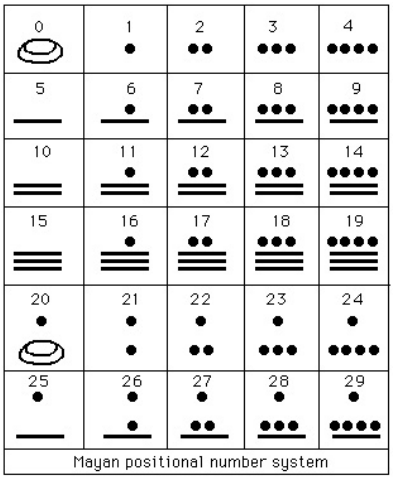
> A careful reading of the history of mathematics generates numerous pedagogical ideas, which can and should be used in today's classrooms. In particular, since many important mathematical ideas grew out of the needs of various cultures around the world, it is vitally important that students in Western nations be exposed to the fact that mathematics is a universal phenomenon. What we call mathematics, was and is present in many civilizations, although not always in explicit form. In particular, various mathematical ideas, which arise in today's courses, were considered by other peoples in the context of their own experiences and values. We can and should use these ideas out of ethnomathematics whenever

possible to illuminate the concepts discussed as well as to demonstrate the universality of mathematical ideas (Katz, 1994, p. 26).

While there is not enough space to give due credit to the diverse and fascinating examples that Susan included in her workshop, I will discuss a few that remain memorable for me after two decades because they positively impacted my own practice and recurred in projects over the years.

Naming Numbers: The Way That Number Words Are Formed

As a former primary educator, Susan recounted a situation in which the parent of a child in her class had been desperately worried about her daughter because she was recording the number thirteen as "31." Susan asked the group of teacher candidates why the child might be doing so, and one replied, "because the *thir* sound comes before the *teen* sound. Thir sounds like *three*, and *teen* sounds like ten, so she is just recording the number phonetically."

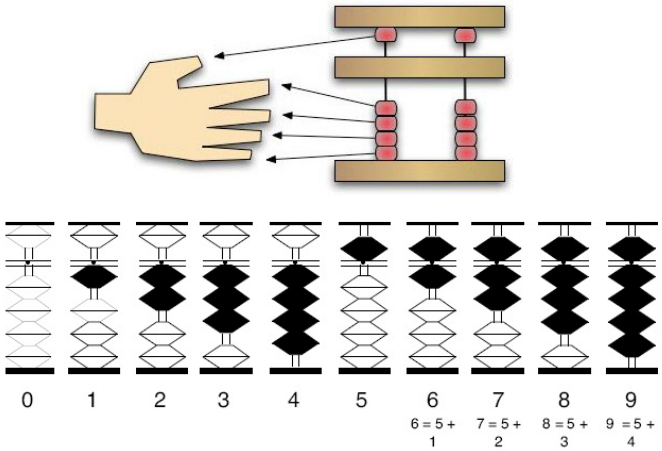


Mayan positional number system

This led to a rich discussion of how numbers are named in various cultures. Susan described how children in both American and Chinese cultures learn unique words for the numbers one through ten. However, while English-speaking children must then memorize another set of unique words—11, 12, 13, and so on—Chinese children use the words they already know to construct new numbers. Instead of 11, they say a word equivalent to "10-one." Twelve translates to "10-two." Using other examples from Marcia Asher's 1994 book, *Ethnomathematics*, Susan went on to fascinate the group of prospective teachers by describing how in many West African cultures (as well as in Mayan, Aztec, and Celtic cultures), people counted their fingers and toes, producing base-20 systems. The Mayan positional number system is shown above.

Finger Counting and the Soroban

Susan's summary paper describes the ways in which multiple systems for counting have been used across time and cultures. Her notes included how to count to 99, using the Korean system of *Chisenbop*, and then transitioned to teaching place value on the Japanese soroban.



Computational Systems for Multiplication

Thinking back to that wonderful Saturday and the responses from the teacher candidates who thrived in an environment rich in interesting lessons sure to spark curiosity in a wide range of learners, I remember that the teacher candidates were almost giddy about learning Egyptian multiplication (used some 2000 years before the Christian era) and the connected "peasant" multiplication systems (used in Russia, Germany, France, and England)—and that they were even more bedazzled when Susan explained that computers were (and are) using related methods.

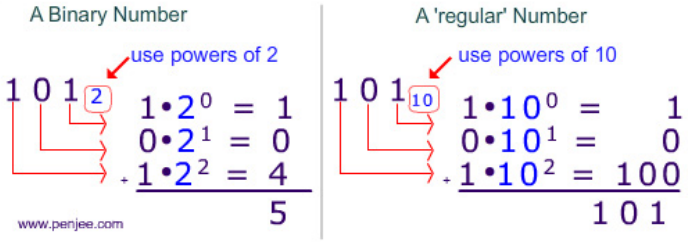
After the initial surprise: Could this method really generate the same product as a calculator? There were requests for more questions to practise… *how did this work?* Susan shared an explanation from *Ask Dr. Math* at *The Math Forum*, which was, in 1999, *the* preeminent, responsive, most reliable, and trusted online mathematics resource freely available to teachers.

Dr. Math explained… for example, to multiply 85 x 18 or 18 x 85, position the multiplicand and multiplier at the top of two columns (the order does not matter). Successively divide the left-hand column by 2 (until a quotient of 1 is obtained), and multiply the right-hand column by 2. When an odd number is divided by 2, discard the remainder. Cross out the rows that have an even number in the left-hand column. Add up the remaining numbers in the right-hand column to obtain the desired product.

85 × 18 = 1530		
85	18	18
~~42~~	~~36~~	
21	72	+ 72
~~10~~	~~144~~	
5	288	+ 288
~~2~~	~~576~~	
1	1152	+ 1152
		1530

18 × 85 = 1530		
~~18~~	~~85~~	
9	170	170
~~4~~	~~340~~	
~~2~~	~~680~~	
1	1360	+ 1360
		1530

This system of multiplication relies on doubling and halving: hallmarks of the binary system (numbers made up of only zeros and ones).



According to Dr. Math, to obtain the binary representation of a number, the number is to be repeatedly divided by two, the remainders recorded and then written in the reverse order, as shown below:

		85	× 18	= 1530
0	1	85	18	18
1	0	42	36	
2	1	21	72	+ 72
3	0	10	144	
4	1	5	288	+ 288
5	0	2	576	
6	1	1	1152	+ 1152
				1530

Dr. Math described the two additional columns: the first indicates the step (and a power of 2); the second contains the binary digits of 85, written from the bottom upwards (the binary digit is the remainder of division by 2: it is 1 for odd numbers and 0 for even numbers). Thus, 85 = 10101012 = 64 + 16 + 4 + 1.

Needless to say, the conference proceedings from that 1999 event have found a new place of honour on my bookshelf. It has been many years since I have seen a workshop about ethnomathematics in Ontario, yet I was struck, all these years later, by what Susan Stuart brought to the experience two decades ago: meaningful mathematics learning, as well as creative authentic opportunities to learn that embraced diversity and inclusion with integrity and rigour.

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▲ Linking Literacy and Math: Writing Strategies to Support Mathematical Understanding (Part One)



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Dr. David Costello is an author who has recently published Using What Works: Strategies for Developing a Literacy-Rich Environment in Math. *David is a vice-principal, who has held many roles. He taught in the primary division before assuming roles of numeracy interventionist, numeracy coach, numeracy leader, and curriculum consultant for Prince Edward Island. David has also instructed university courses focused on curriculum, differentiation, mathematics, and language arts. He facilitates professional learning sessions in the areas of mathematics and school development. His research situates mathematics within a literacy perspective and examines leveraging literacy-based instruction in mathematics.*

In my OAME *Gazette* columns published over the last year, I outlined the strengths of identifying comprehension strategies as thinking strategies versus limiting them to reading strategies. Moving away from reading, my goal with this year's column is to highlight the importance of writing in mathematics as a way of framing, supporting, and consolidating mathematical understanding.

Traditionally, when thinking about writing, we envision language arts classrooms—students writing about a favourite moment, writing a persuasive piece to their teacher or principal, or writing about reading. Many teachers are comfortable with these writing experiences, and we should be, as we have received numerous professional learning opportunities and have read numerous professional learning resources about such. However, I want to stretch your comfort level to approach writing as a valuable tool in mathematics. Once again, we are leveraging what we already do in language arts to mathematics.

When thinking about writing, I often go to the work of Lucy Calkins. Calkins (1994) stressed that it is essential that children are deeply involved in writing, sharing their texts with others, and viewing themselves as authors. These three aspects of writing are not small by any means. Instead, they are broad in that we want students immersed in their writing. We want students to not only record details, but to make significance of them (Calkins, 1994). By making significance

of these details, we move writing beyond a transcription of details to a communication of an author's beliefs, experiences, questions, etc. Thus, writing is not only recording, but is a process of developing an idea (Calkins, 1994).

Let's think about using writing to communicate our thinking. We usually approach this as writing about our thinking when we have reached a decision. I would like to propose that we expand this to include using writing to frame, support, and consolidate our thinking. Writing is a valuable way of providing clarity, both to begin an endeavour and to work through it. The notion that the more a student writes, the more success she or he will have (Calkins, 1994) is important. It is not a quick fix, nor is it something that we can hope will happen on its own. We need to provide students with quality opportunities to write in the classroom, support them as writers, and offer situations where they can share their writing.

When considering writing, it is important to make a distinction between writing to demonstrate knowledge and writing to support learning. One type of writing common in math classes is writing to demonstrate knowledge (Scardamalia & Bereiter, 1987). Students write for a specific audience with a specific purpose. This writing is focused on students showing what they now understand after engaging with a concept or idea. Within this writing, students will provide evidence of their understanding.

A second kind of writing, possibly less common in math classes, is writing to support learning. This kind of writing involves writing short, informal pieces that allow students to clarify stumbling blocks. It allows students to frame and support their thinking as they work through a problem (Knipper & Duggan, 2006; Scardamalia & Bereiter, 1987). This type of writing is meant to provide students with an exploration into both their thinking and the details of concepts and ideas being explored. Writing-to-learn pieces may be journals, learning logs, free writing, and notes. Writing to learn makes students' thinking visible (Knipper & Duggan, 2006). It shows how students are working through a concept and idea that is of interest. Students may share their writing with classmates and the teacher for formative assessment.

It is important to consider the implication of writing about mathematical instructions and student mathematical understanding. When thinking about writing to learn and writing to demonstrate knowledge, both types of writing can be leveraged into mathematics. Writing to learn can support students as they work through a concept and/or strategy. This would be another support for students, and would aid their metacognition (Keene & Zimmermann, 2007) as they work through a task. Writing to demonstrate knowledge is something that we tend to go to first in mathematics. We tend to stress to students to double-check their work for calculation errors and whether their response is reasonable. In addition, we emphasize the importance of showing their work in a method that will be easy for the teacher to follow. What I am stressing here is that while both types of writing are important, we should have more tasks emphasizing writing to learn in math.

The following are three writing strategies (bookmark, eraser free, and got it/not yet) that can be applied to mathematics.

Bookmark

Similar to a bookmark used when reading, a bookmark as a writing strategy serves as a place to re-enter the problem. When students leave notes for themselves so that they can re-enter the problem at another point in time, they do not have to start from scratch. Instead of stressing (implicitly or explicitly) the need to solve a problem in one sitting, bookmarks provide students the option to leave a problem when they encounter a stumbling block, when they have exhausted all strategies available to them, or when time has run out in the class. Bookmarks require students to write a brief note about what they have tried, what questions they have that need to be answered before re-entering the problem, and/or a possible strategy to try upon re-entry.

A great benefit of the bookmark strategy is that students strengthen their resolve to complete a problem, and they understand that a problem does not need to be solved within one sitting. Using the bookmark strategy, students use metacognition in their problem solving when they think and record what they have attempted thus far and what they may need to do upon re-entry.

In Figure 1, the child uses the bookmark strategy as a point of support when re-entering the problem. The child leaves the work already attempted, and provides understanding and suggestions for the next time he approaches the problem. From the teacher's perspective, it provides an opportunity to see how the child moves on from a problem, while providing support for re-entering it later. The suggestion(s) left by the child could be used as a form of learning goal that the teacher monitors over a period of time.

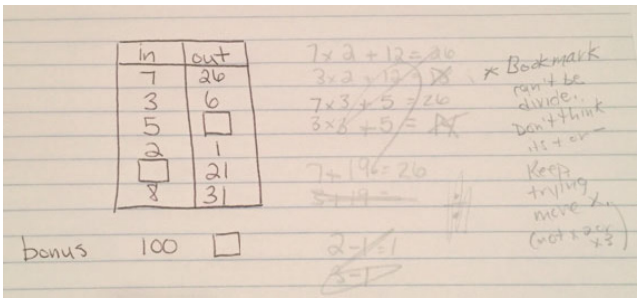


Figure 1: *Example of the bookmark strategy*

Eraser-Free

This strategy can provide the students an opportunity to draft, revise, and edit their work, without losing the thinking behind the changes. Instead of students erasing their work, students will cross it out and move forward. What this provides is documentary evidence of thinking throughout the problem, instead of focusing only on their final response. So, students can use a pencil, with the understanding that they cannot erase their work; students can also use a pen or marker to work so that erasing is not an option.

When student work is erased, so is the record of their thinking to that point. It is vital that their changes in thoughts and points of clarification are maintained so that they can review their thinking throughout and at the conclusion of the problem-solving process. Also, it provides the teacher with an incredible source of information about student thinking. By removing the option to erase, it is the student thinking that is emphasized instead of the final response.

Figures 2 and 3 are examples of using the eraser-free strategy. The former is from a primary-aged child, and the latter is from a junior-division child.

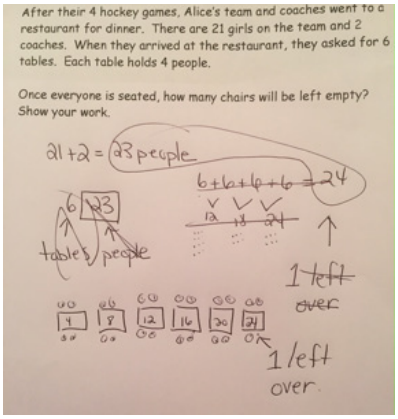


Figure 2: *Primary-aged example of eraser-free work*

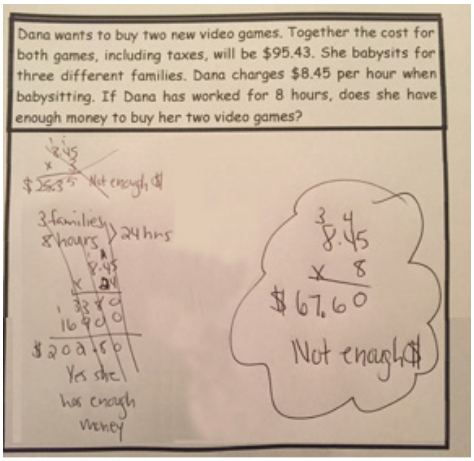


Figure 3: *Eraser-free work by a junior-division student*

In Figure 2, the primary-aged example, the child reviewed her work and identified that she incorrectly framed the problem as groups of six. So, she changed her plan of action to account for the proper approach to the problem (groups of four). She drew a line through her initial work to indicate this change in approach. From the teacher's perspective, he would not have viewed this change of thinking if the child had erased her work and then written the new process. The thinking behind the answer would have been omitted if an eraser had been used.

Figure 3 shows an elementary-aged example, where the child approached the problem twice, before she arrived at a solution. The teacher can determine how the child kept going back to the problem when she hit a stumbling block. In addition to this, the use of a pen enabled the teacher to see all the attempts, and the thinking supporting these attempts.

Got It/Not Yet

Students record their thinking within two columns. The first column, "Got It," is a place for students to record their understanding of the concept and/or strategy. The second column, "Not Yet," is a place for students to record any questions and/or stumbling blocks related to the concept and/or strategy. The Got It/Not Yet strategy is an opportunity for students to clarify their thinking when framing the problem, and a place to continuously refine their thinking as they work through the problem.

The Got It/Not Yet strategy serves as a self-assessment for students. Students identify what they understand and what they need to work toward. In this way, this strategy provides students the opportunity to identify learning goals from the "Not Yet" column.

Figure 4 shows an elementary-aged child's work sample using the Got It/Not Yet strategy. In this example, the child was able to articulate her partial understanding of area, and state what she knew about the problem. In addition, the child could articulate the stumbling blocks she still faced before being able to successfully complete the problem. This provides a great opportunity for the teacher to assess the child's understanding from the perspective of metacognition. The teacher could use this two-column approach as a way to identify next steps for instruction and student learning.

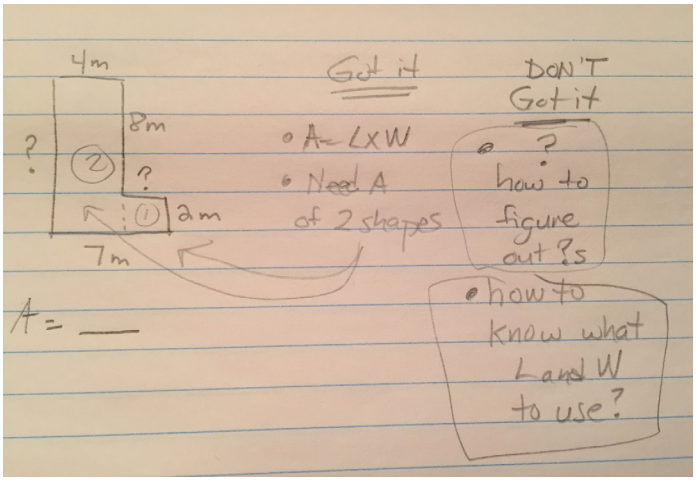


Figure 4: *Got It/Not Yet example*

Final Thoughts

The three writing strategies provided in this column—bookmark, eraser free, and got it/not yet—can support mathematical understanding as students work through a problem. Exploration of the writing strategies encourages students to frame, support, and consolidate their understanding, as well as providing a platform to make their thinking visible to both themselves and the teacher. It is through these strategies that students can use writing to make meaning in the three stages of problem solving: before, during, and after. When we move beyond the idea of writing as merely part of a conclusion process, we move student learning forward.

What I am advocating is that writing strategies not only be considered for language arts, but also for mathematics. When viewed in this manner, educators and students can see the importance of writing as ways to demonstrate knowledge and support learning. I look forward to continuing the discussion of linking literacy and math via Twitter (@dr_costello) or email.

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▲ Mathematical Snapshots: Shuffleboard Mathematics

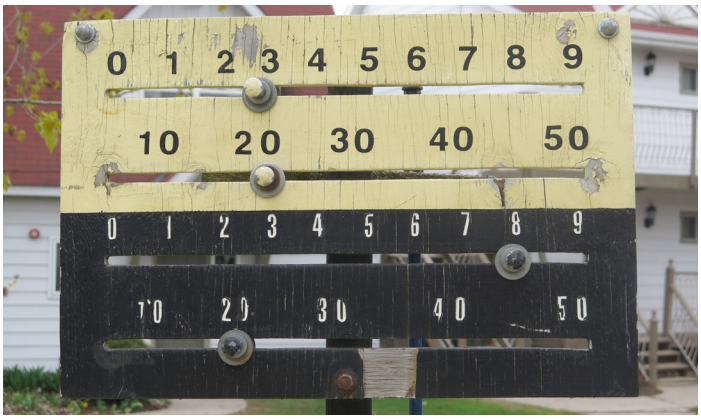


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This column focuses on shuffleboard. The column takes time to notice, wonder, and question with inspiration from Orillia, and a resort near Elgin. The game has a lot of opportunity for mathematics and could be constructed in your own classroom.

In May, I gave a keynote address at the Ontario College Mathematics Association 39th Annual Conference in Orillia, Ontario. After the talk, I spent several hours wandering around downtown Orillia. I saw several shuffleboard courts and thought about how the game can be used in a mathematics classroom for Grades 6–12. Several months later, I found a court drawn on the ground near Chaffey's Lock in Elgin, Ontario, and wondered what instructions students would need to make a regulation-sized court.



Photograph 1: *Shuffleboard scoreboard, Orillia, Ontario*



Photograph 2: *Shuffleboard scoreboard, Orillia, Ontario*



Photograph 3: *Shuffleboard court, near the Opinicon Resort, Elgin, Ontario*



Photograph 4: *Shuffleboard court, near the Opinicon Resort, Elgin, Ontario*

Notice the numbers that appear horizontally in Photograph 1. Notice one set of numbers (repeated in the top and halves) is from 0 to 9, and the other set is from 0 to 50 in steps of 10. Wonder why the one set of numbers stops at 50. Notice the four markers that can be moved from left to right. Notice the positions of the four markers at 3, 20, 8, and 20 from the top to the bottom of the board. Wonder what the score was. Wonder how players score points in shuffleboard.

Notice the numbers that appear vertically in Photograph 2. Notice the numbers are from 0 to 70 again in steps of 10. Notice the two markers. Notice that the marker on the left is below zero. Wonder if this player has a negative score. Wonder how a score could be negative. Wonder if there are any sports where a score can be negative.

In Photograph 3, notice the image of a shuffleboard court. Wonder why there are triangles and trapezoids. Notice the numbers that appear in the shapes. Wonder what the numbers mean. Wonder if there is a standard size for shuffleboard courts.

In Photograph 3, notice the word "Off." Wonder what this means. Wonder if this is used to refer to a player having an off day. Wonder if the areas of the regions marked 10, 8, 8, 7, 7, 10, and "Off" are all the same size. If not, wonder which one has the largest area and why its area is the biggest.

In Photograph 4, wonder about the measure of the three angles in the isosceles triangle. Wonder if the isosceles triangles in all shuffleboard courts are congruent. Wonder if they are similar.

Wonder how to play shuffleboard. Wonder about the average age of a player. Wonder where in the world the game is popular.

Question 1

The basic rules for how players score points in shuffleboard are given below.

Players take turns with the various coloured disks and push them into the marked-out court. Points are awarded for landing in the 10, 8, or 7 sections. Deduct 10 points for landing in the 10 off section. An important objective is to knock the opponents' disks out of the scoring areas—particularly to knock them into the 10 off area. The game may be played up to 50, 75, or 100 points.

The basic rules of shuffleboard and scoring can be found at www.domsports.com/rules&info.htm.

(a) What does a player have to do to obtain a 10, 8, or 7?

(b) What does the word "off" mean, and how does it affect the score?

(c) In Photograph 1, the score appears to be 23 to 28. Suggest ways in which a player could obtain 23 or 28 points.

(d) Is it possible for a player to have a score of 24? What scores are possible?

Question 2

Is the written description of a regulation shuffleboard court given below consistent with the diagram in Figure 1? Why or why not? How does this description and diagram compare with the one shown in Photograph 3 and the part of court shown in Photograph 4?

Floor shuffleboard may be played on decks, pavements, floors, or any suitable smooth and level surface. The regulation outdoor court is 52 feet long and 6 feet wide. A description (details from the website) is quite detailed and has:

- two large triangles that point toward each other in the court, their points being 18 feet apart
- the base of each triangle 9 feet from the point, this base extending across the court (6 feet)
- two lines parallel to the base line that divide the triangle—one line being 3 feet from the base, and the other, 6 feet
- the smaller triangle near the apex, which counts 10 points
- the space behind the 10-point triangle that is divided in half longitudinally, with each half counting 8 points
- another space nearest the base of the large triangle, this space being divided half longitudinally, with each half counting 7 points
- behind the base line of the triangle, another space, 18 inches wide, marked “10 off”; and within this space a separation triangle, 3 inches at its base, running to a point—with ¼-inch lines
- the triangle with a clearance of a ½ inch at the point and base of the triangle from the 10 off space lines (see Figure 1); the outside lines of the court a minimum width of ¾ inch and a maximum of 1 inch
- for inside play or for children, a court 15 feet long and 3 feet wide that is suitable, with one end of the court designated as a head, the opposite the foot

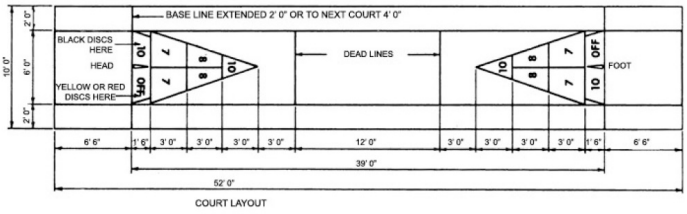


Figure 1: *Basic Rules of Shuffleboard and Scoring from www.domsports.com/rules&info.htm*

Question 3

Figure 2 shows one of the triangles from a regulation shuffleboard court. Strictly speaking, points are only given if a disk is wholly inside a region; that is, no points are received if any part of the disk is touching a line. In the following, this rule is relaxed to make it mathematically more accessible, and the simplification is that points will be awarded based on the location of the centre of the circle, whether the disk crosses a boundary or not.

(a) Find the areas of triangle FDE (worth 10 points) and trapezoids GPOF (worth 8 points), CPOD (worth 8 points), AQPG (worth 7 points), and BQPC (worth 7 points).

(b) If a disk lands randomly with its centre somewhere inside rectangle ABJK, what is the probability that the player obtains:

(i) 10 points?

(ii) 8 points?

(iii) 7 points?

(iv) no points?

(c) Why would it be more challenging to determine the probabilities in part (b) if the rules had not been simplified (and any disk touching a line received no points)?

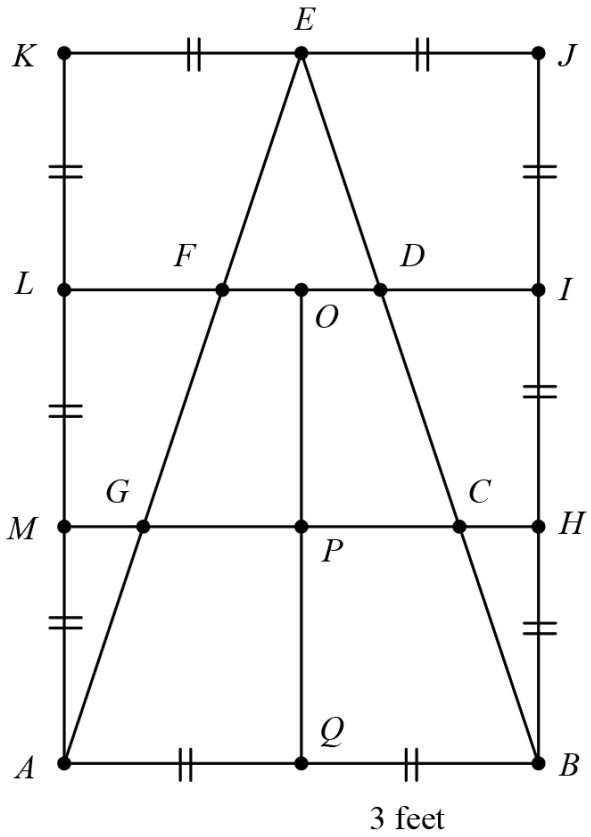


Figure 2: *Geometry of an end of the court*

Question 4

(a) Determine angle DEF in Figure 2.

(b) Figure 3 shows Photograph 4 with triangle DEF constructed within it to coincide with the triangle that contains a 10. Measure or estimate angle DEF. Is this triangle part of a regulation-sized court? How do you know?

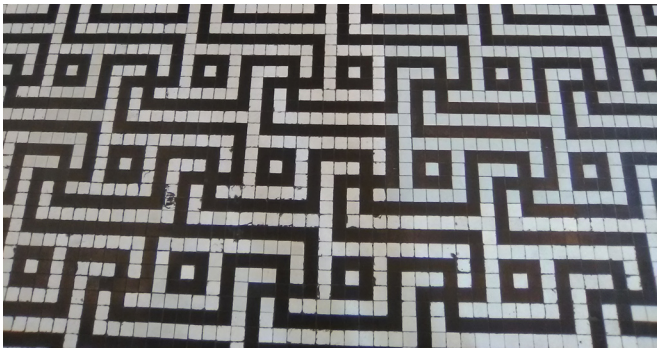
(c) Obtain some coloured chalk and make a regulation-sized shuffleboard court at your school. Play the game with your classmates and try to stay off the "Off."



Figure 3: *Photograph 3 with a triangle added to the interior of the lines*

Final Thought

It would be interesting to have your students build a shuffleboard on their playground. They would have questions to resolve. Wonder what else they might notice constructing the game?▲



Curious tiling in the Tate Museum, London, England.

▲ Fields Institute MathEd Forum Report



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The final meeting of the Fields Institute MathEd Forum for the 2018–2019 year centred on the topic of "Cutting-Edge Trends in Technology." The Forum brought speakers from Humber College, York University, and Microsoft to speak about the use of technology and games in math classrooms, particularly focusing on their potential for learning, as well as aspects teachers should be cautious about.

Some of the presenters spoke about the math present in games that are popular with students, from dot products being used to calculate damage dealt, to monsters in the video game Diablo, to the statistics and graph theory present in the board game Settlers of Catan. Dr. Alexander Gurevich from Humber College also highlighted the similarities between these kinds of games and mathematics—both present a world governed by rules where you must achieve a specific goal or outcome, and having a deeper understanding of the rules and the dynamics between these rules gives you an advantage. By emphasizing this puzzle-like quality of mathematics, or by highlighting the mathematics in games students are already playing, students might experience some of the same engagement when studying math that they feel when playing games.

Other speakers shared some technologies created specifically for education that bring elements of games into the learning experience. This included GeoGebra, free geometry software available online, which facilitates student exploration of geometric properties and constructions. GeoGebra has a library of activities to support teacher usage. Many of the activities allow students to experience geometry in a dynamic and interactive way that is not possible with pencil and paper.

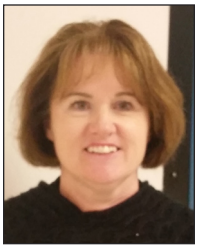
Presenters also shared a few examples of polling software, including Poll Everywhere, Kahoot, and Mentimeter. These applications allow teachers to pose a question to their students that the students are able to answer through their smartphones. They can be used for assessment for learning, assessment of learning, and

assessment as learning. If used during a class that is learning some new concepts, these technologies allow teachers to quickly get a pulse of their students' understanding during class, and informs teachers how to best proceed with the lesson. If used toward the end of a unit, these applications can help students self-assess what they already know, and identify what they still need to work on. They can also be used as a gamified form of summative assessment, bringing an air of competition to the assessment. However, the point was raised at the Forum that Kahoot in particular has a timer, and students must answer each question within a limited time frame. This can be a concern for math educators in particular, as timed assessments have been shown to increase math anxiety in students (Boaler & Dweck, 2016). One attendee said that within his own classroom, students are given the assessment questions ahead of time, and they then use Kahoot to submit their answers. This allows the teacher to encourage slow and deep thinking with reduced anxiety, while taking advantage of the technology's ability to provide the teacher with information on the students' performance.

The 2019–2020 program at the Fields Institute MathEd Forum will start on Saturday, September 28, when the Forum will focus on two books selected by the organizers. The first is *Weapons of Math Destruction: How Big Data Increases Inequality and Threatens Democracy*, by Cathy O'Neil (2018). This book examines several examples of big data algorithms with (frequently unintentional) harmful effects that exacerbate social inequality. O'Neil writes that such algorithms often go unchallenged because they are perceived as being objective. However, because they are programmed by people, they are susceptible to inheriting the biases of their creators and perpetuating pre-existing inequalities in our society. The second book selected by the Forum organizers is *King of Infinite Space: Donald Coxeter, the Man Who Saved Geometry*, by Siobhan Roberts (2006). This is a lively biography of Toronto mathematician Donald Coxeter, praised as the greatest geometer of the twentieth century. Coxeter is credited with bringing the study of geometry back to relevance at a time when mathematical research was concerned only with more abstract and algebraic topics, which threatened to push geometry into obscurity as a field that was only worthy of study in the ancient world. The Forum meeting will look at what we as math educators can learn from these two math books written for a general audience.

(continued on page 32)

▲ OAME/NCTM Report: Happy Fresh New School Year!



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Jacqueline Hill is a Grades 9–12 teacher of mathematics, as well as an online instructor for York University. She is a Past President of OAME and OMCA, as well as the recipient of the award for Exceptional Teaching in Secondary Mathematics. She also wrote the "Director's Dialogue" for the Gazette *for a number of years.*

With a new year come new ideas and responsibilities. Maybe you are trying to freshen up the professional development you are sharing at the local chapter level. The NCTM has had some great ideas about this in the last 12 months. One such idea that came out of a thread of an NCTM email was the idea of "Book Tasting." When I mentioned this idea in my math department, a fresh new teacher said, "Oh! Like speed dating!" Well, needless to say, I have been an OAME member almost as long as I have been married (and in my 30 years of marriage, I had never heard of the term *speed dating*).

The big idea is that people bring their favourite math resource and then share it with the group. The host of the event can bring extra books in case the participants do not bring along their favourite books.

The President of NCTM, Robert Berry, shared his most recent reading list:

1. *Access and Equity: Promoting High-Quality Mathematics in Grades 9–12* (2018), edited by Dorothy Y. White, Anthony Fernandes, and Marta Civil.
2. *The 5 Practices in Practice: Successfully Orchestrating Mathematical Discussion in Your Middle School Classroom* (2019), by Margaret (Peg) Smith and Miriam Gamoran Sherin.
3. *Critical Race Theory in Mathematics Education* (2019), edited by Julius Davis and contribution by Christopher Jett.
4. *We Want to Do More Than Survive: Abolitionist Teaching and the Pursuit of Educational Freedom* (2019), by Bettina L. Love.
5. *Educated: A Memoir* (2018), by Tara Westover. Robert Berry had this to share about this book:

Each year, my school does a "common read" for students, faculty, staff, and administrators. Educated *was recommended for the common read because it touches on many themes, including education, rural communities, religion, income, mental health, family violence, and more. I have seen interviews of the author Tara Westover and want to stretch my thinking about the intersections between her resilience and her identities.*

You might want to start with one or more from his reading list, or go to the NCTM bookstore (online) and peruse a few new titles. My outline for the 1 hour and 15-minute time slot I had at the last OAME Board of Directors meeting to present this idea went as follows. We started with a pile of NCTM books and spread them out on the tables under the following themes: professional development, classroom ready, and equity.

2-minute intro – Think for a minute about a wine-tasting, chocolate-tasting, or beer-tasting party you have attended. Share your thoughts at your table. This evening, we are going to embark on a book-tasting party.

5 minutes – To find people to fill the slots of their speed-dating clock. Once they have found their dates for 12, 3, 6, and 9 o'clock, they are encouraged to return to their tables for themed treats (bottle-cap candy, wine gums, and chocolate).

10 minutes – For the people to read a new book that they have. They can answer questions such as:

- First impressions?
- Did the table of contents pique your interest about the book?
- What are the selling points of this book?
- Would you read this book?

4-minute increments – Find your 3 o'clock (subsequently 6, 9, and 12 o'clock) date. The format is first-person shares, second probes, second-person shares, and first probes, which provides the opportunity to share the book you have begun to read. At each time interval, the dates could record the book, if they felt it would be interesting to take a second reading.

Finally, draw the group together and ask, "Why?" Why do we have a book tasting as an activity? Ask for group ideas. Answers will include things like: "The Internet is not the only resource out there," "Books are a hard copy to be shared and loved," and "Each participant brings a wealth of knowledge."

Resource

Miller, J. *Book tasting: Introduce your students to new books.* Available from www.teacherspayteachers.com/Product/Book-Tasting-Introduce-Your-Students-to-New-Books-2741616

NCTM important dates to note, and a small historical reference:

November 15–16, 2019 in Rochester, New York. Jo Boaler and Steven Strogatz. Contact Lynda Brennan at lyndabrennan123@gmail.com if you need additional information on this event.

The National Council of Teachers of Mathematics is the public voice of mathematics education, supporting teachers to ensure equitable mathematics learning of the highest quality for all students through vision, leadership, professional development, and research. Founded in 1920, NCTM is the world's largest mathematics education organization, with 90 000 members and more than 230 affiliates throughout the United States and Canada (including the OAME).

Yours in mathematical fun,

Jacqueline ▲

(continued from page 31)

The Fields Institute MathEd Forum convenes on the last Saturday of each month from September to November, and from January to April at the Fields Institute in Toronto. Admission is free to all who wish to attend. If you are not able to travel to Toronto to join in person, you can also watch the presentations remotely through the Fields Institute for Research in Mathematical Sciences website.

References

Boaler, J., & Dweck, C.S. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*. San Francisco, CA: Jossey-Bass.

O'Neil, C. (2018). *Weapons of math destruction: How big data increases inequality and threatens democracy.* London, England: Penguin Books.

Roberts, S. (2006). *King of infinite space: Donald Coxeter, the man who saved geometry.* Toronto, ON: Anansi. ▲

Provincial Digital Learning Resources – What's New? Algebra Tiles v1.0.0



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Agnes (Brant Haldimand Norfolk Catholic District School Board), Ross, (Near North District School Board) and Markus were Project Leads, working on the development of the digital resources found at mathies.ca.



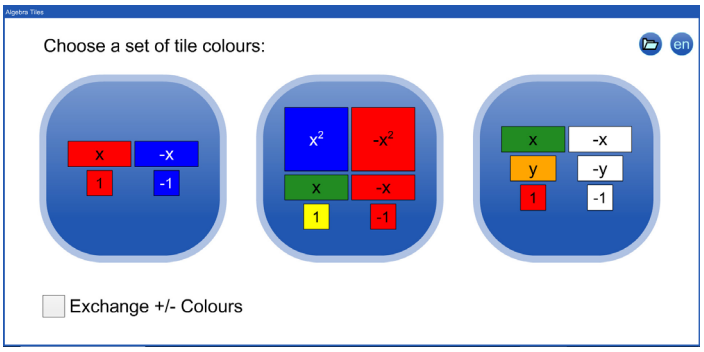
In this issue, we discuss the following new additions to mathies.ca:

- Algebra Tiles tool (Tuiles algébriques)
- Fractions Representation Match game (Jeu d'association – Fractions)
- Whole-Number Representation Match game (Jeu d'association – Nombres naturels)
- Attraper une balle (Opérations) – the French localization of the Catch a Bouncing Ball – Operations game for Whole Numbers
- The *Soutiens à domicile* page with French Home Supports, including WINS and the *Suivi à la maison* printable resources

Algebra Tiles v1.0.0

Algebra Tiles and Integer Tiles are popular manipulatives for use in classrooms, which can be used to:

- represent and perform operations on integers
- represent and perform operations on polynomial expressions
- simplify algebraic expressions and evaluate algebraic expressions for different values of the variable(s)
- factor polynomials
- solve equations
- explore patterning and functions



We are delighted to announce the *mathies Algebra Tiles* tool, available for both official languages. On the opening dialog, you choose from three sets of tile colours (and another set of three with the positive and negative colours interchanged). These choices are meant to mimic physical sets available for purchase.

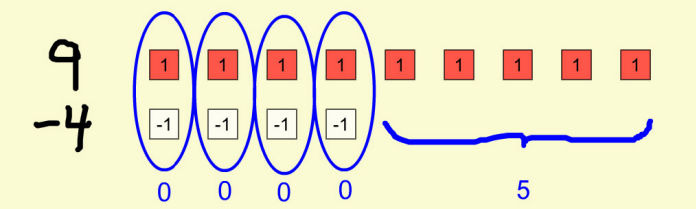
mathies Algebra Tiles offers several advantages:

- Never run out of tiles; just keep dragging more from the selection panel at the right.
- Tiles and segments snap together.
- Adjustable variable values.
- Change the colour of the representations used.
- Use segments to represent lengths, or create frames for multiplication.
- Annotate and save work.
- Use undo/redo to describe the development of the thinking.

Most physical sets of tiles come with instructions for how to model various operations. There are many instructional and learning resources on the Web as well; several are listed at the bottom of the Algebra Tiles support page found at https://support.mathies.ca/en/mainSpace/AlgebraTiles.php. The following examples illustrate how the *mathies Algebra Tiles* tool can be used like the physical manipulatives, as well as how they can extend the power of the representation.

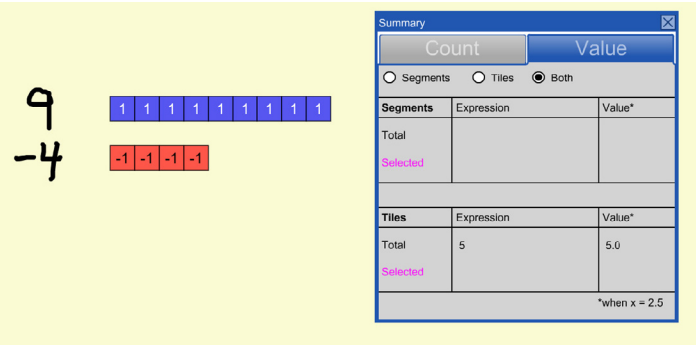
Integers

By using only the 1 and -1 tiles, integers can be represented and operated upon. Typically, students are taught that when 1 and -1 are combined, the result is zero. This is referred to as the *zero principle*. For example, to add 9 and -4, nine 1 tiles are dragged out to represent 9, and four -1 tiles are dragged out to represent -4. Four pairs of 1 and -1 are identified and combined to make 0, leaving the result, 5.



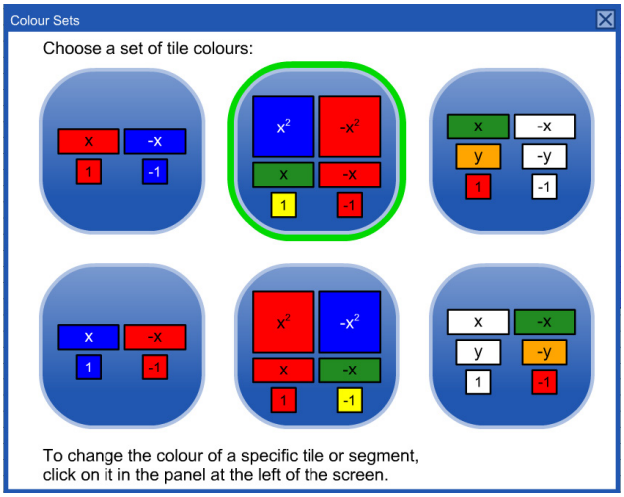
If the four -1 tiles are selected and moved on top of four 1 tiles, the mathies tool will “poof” these opposites; that is, the eight tiles will show a poofing animation, and a sound will be played, literally leaving only the five 1 tiles. If this animation is not to your taste, it can be turned off in the settings.

The Summary Dialog, accessed from the # button, will confirm your count and value.

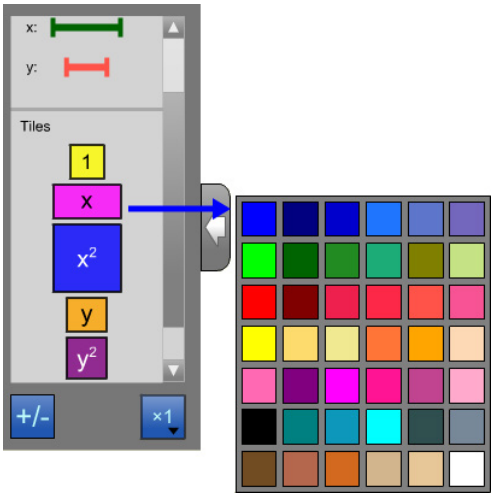


Notice that the Value tab has been displayed and that the colours of the tiles have been changed.

From the workspace, the set of tile colours can be changed by using the colour palette button, without losing any of the representations already created. This would facilitate comparisons of representations between students who initially chose different tile sets.

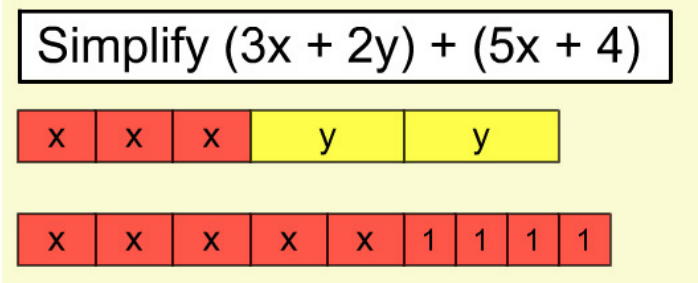


As indicated at the bottom of the dialog, each tile and segment’s colour can be changed by clicking on it in the cloner panel at the left of the screen and choosing one of the 42 available colours.

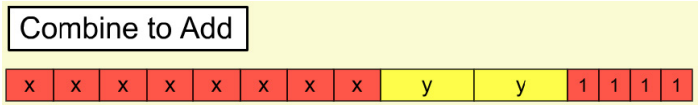


Polynomial Expressions

Once *x* and *y* tiles are introduced, algebraic expressions can be represented and operated upon. In the example below, the two expressions are represented, one on top of the other.

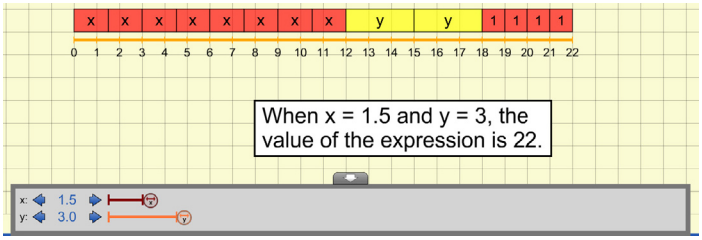


To add the expressions, like terms are combined.



The expression is then named as $8x + 2y + 4$.

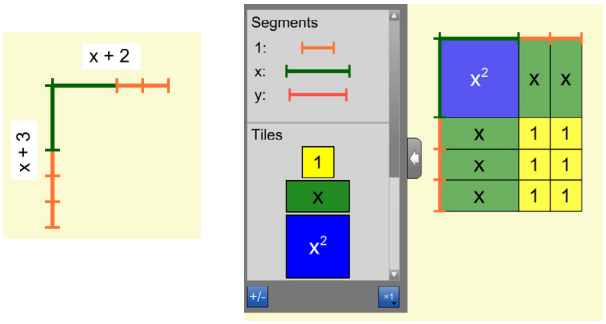
Since the values of the variables can be changed and a unit grid displayed, an algebraic expression can be evaluated for various values of the variables. Here, a number line is used to count the 22 unit squares that the combined expression takes up when $x = 1.5$ and $y = 3$.



The Summary dialog is also useful for checking the tile count and the corresponding value of the expression.

Multiplying Binomials and Factoring Trinomials

An area model can be used to multiply $(x + 2)$ by $(x + 3)$. To create the one dimension of the rectangle, an x and two 1 segments can be arranged horizontally to form the top of the frame. To create the other dimension, an x and three 1 segments can be arranged vertically to form the left edge of the frame. Use the change orientation button to switch selected segments and tiles from horizontal to vertical.



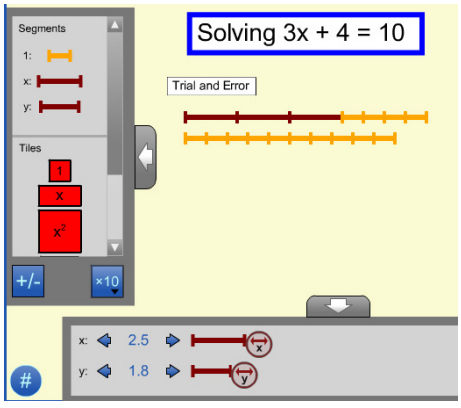
This frame is then filled with tiles to determine the product.

When objects are placed on the stage, they will snap to nearby objects, and a linking animation and stapling sound will play. If the tiles and segments above have been snapped together properly, changing the value of x will still result in a rectangular arrangement of tiles. This helps students to understand that regardless of the value of x, the algebraic expression for the product is $x^2 + 5x + 6$.

Factoring is the inverse operation. To factor $x^2 + 5x + 6$, drag the tiles needed to represent the algebraic expression into the workspace, and then arrange the tiles into a single rectangle. The dimensions of the rectangle determine the factors.

Solving Equations

One way to solve equations using Algebra Tiles is to arrange the left and right side of the equation on top of each other and adjust the variable value(s) until the two sides are the same length.

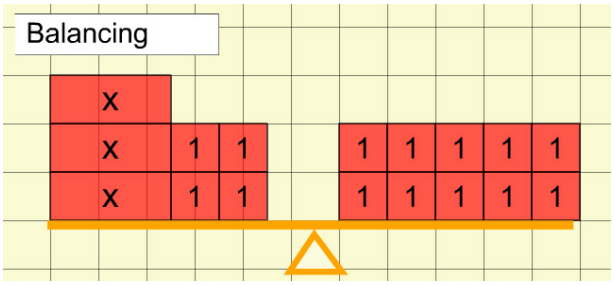


By sliding the value of x, it can be determined that the solution is $x = 2$. There is a setting to change the number of decimal places for the variable values to 0, 1, 2, or 3 according to the precision desired.

Trial and Error can also be used to solve a system of equations like:

- $2x + y = 15$
- $x + 3y = 20$

The Algebra Tiles tool is also useful for solving equations by balancing.



Here, -4 might be added (and poofed) to both sides to simplify the equation to $3x = 6$. The six 1 tiles could then be placed in three equal groups of two to reveal the solution.

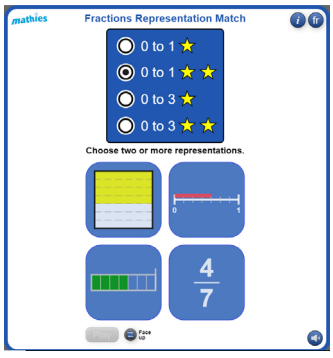
See the support page for more examples of how *mathies Algebra Tiles* can be used.

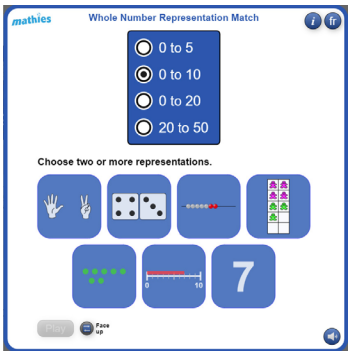
Representation Match Games

Two of the Representation Match Games at mathies.ca/games.php have been replaced with HTML5 versions: Fractions and Whole Numbers. These new versions can be played on a desktop or mobile browser and are responsive to the screen size that you are using.

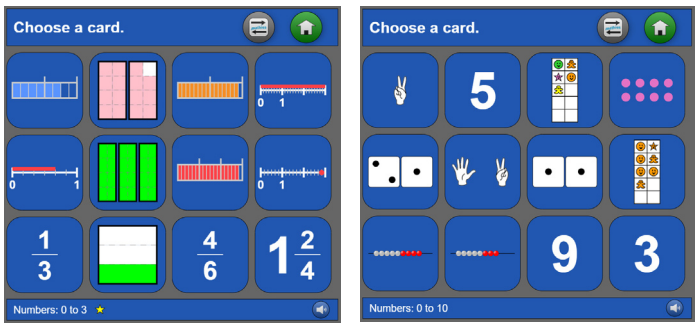
The two games are similar. A range of numbers to use is chosen, followed by at least two different representations. The Fractions game uses fraction arrays, number lines, fraction strips, and numeric representations. The Whole Number game uses slightly different representations for each level, including fingers, dice, dots, numerals, Rekenreks, Ten Frames, number lines, and tallies.

To add a memory component, play the games face down.





Players match two cards that represent the same number.



French Resources

This year there has been an emphasis on increasing the number of mathies resources that are available in French. As new tools have been developed, support pages have been written in both English and French. Many of the support pages found in the Features menu have also been updated. The collection of French support pages can be found at https://support.mathies.ca/fr/mainSpace/Outils_mathies.php.

mathies Français

We have also added a number of learning tools and games to the French resources found at www.mathies.ca/francais.php. These include:

- Tuiles algébriques
- Tableau des nombres
- Attraper une balle (Opérations)
- Jeu d'association (Fractions)
- Jeu d'association (Nombres naturels)
- Jeux de cartes (Fractions et nombres naturels)

mathies Soutiens à domicile

Finally, a new mathies page features a collection of print resources and websites that are available in French. This amalgamation of resources can be accessed by clicking the fr button found in the Home Support row of the home page and include the following two resources.

WINS

The Working in Number Sense (WINS) activities are designed for parents, guardians, and families to engage with their child to build knowledge and skills in Number Sense and deepen understanding of fundamental mathematical concepts. Each activity uses mathies learning tools or games to engage students in a fun, interactive, and meaningful way, while also supporting the development of fundamental skills. WINS activities focus on four key whole-number topics:

- dénombrement (counting)
- le regroupement et la décomposition des nombres naturels (composing and decomposing)
- représenter, comparer, et ordonner des nombres naturels (representing, comparing, and ordering)
- les opérations (operations)

These topics are developed for various grade levels, starting first with skills related to Numbers to 5, and extending to skills needed to work with Numbers to 100.

The Home Connections materials are also now available in French. Home connections are printable Grades 1 to 6 mathematics activities that are designed to connect the learning in the classroom to experience at home. Each activity includes suggested discussion questions to prompt the conversation between adult and child. Activities are available for each of the five strands.

A Fond Farewell

This article will be the last *Gazette* article written by the three of us as a team. New adventures await Ross and Agnes as they enter into retirement, and Markus will be returning to teaching as an instructor in the Computer Science program at Georgian College. It has been our pleasure over the past number of years to have had the opportunity to develop digital math resources such as mathies and CLIPS in collaboration with a number of educators across Ontario. It is rewarding to see the enthusiasm and widespread use of these effective learning tools. Google analytics shows that the number of users this past year is up 90 percent, and that these resources are being well used not only in Ontario, but also across the globe, including in Australia, Europe, and South America. This past year, @ONmathies hit 1000 Twitter followers. It has been so gratifying to see the mathies resources in the hands of students! Thanks to all of you for taking the time to share your learning journey with your students and with the broader math community. Our hope is that you will continue to share creative ways in which students and teachers are using these tools to enhance the learning of mathematics. We are certain that the *Gazette* editor would appreciate any submissions highlighting how you use mathies resources. ▲

Mb4T (Mathematics by and for Teachers): A Focus on Patterns inside Patterns



JENNIFER HOLM
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Jennifer Holm is an Assistant Professor at Wilfrid Laurier University and works with primary/junior and junior/intermediate pre-service teachers, as well as in the field supporting current mathematics teachers. She is interested in developing mathematics knowledge for teaching and analyzing beliefs about mathematics with both pre-service and in-service teachers. Her current focus is on the beliefs and opinions that pre-service teachers hold about mathematics and teaching along with the connection they have to past experiences. She uses this research to support future teachers in developing beliefs and knowledge that will encourage and support effective teaching practices.

When working with different patterns, especially number patterns, sometimes there are interesting hidden patterns that are not obvious at first glance. Take, for instance, the pattern in Figure 1. This is a pattern I have seen many times in classrooms, and the prompt is usually, "How many tiles are in the next row?" Students can build the pattern and see that row 5 would have nine tiles. A discussion could happen around the fact that each row is a consecutive odd number. The discussion could also focus on what an odd number is to address that area of the curriculum. Using this example, students could pair up the tiles and realize that odd numbers have one tile that does not match with another tile in the set (e.g., the row with five tiles contains two pairs and one lone tile). We could also push the Patterning and Algebra strand further by exploring the explicit rule of the pattern by asking, "How many tiles would be in the tenth row?" This would push students beyond building each row to look at the relationship between the row number and the number of tiles. If it does not, you may want to ask how many tiles in the twentieth row or a row with a higher number that makes it frustrating to continue building with the tiles. In this case, when looking at the row numbers in Figure 1, the number of tiles is twice the row number, and then subtract 1 (e.g., Row 3 has $3\times2-1=5$ tiles). Using this procedure, it is easy to see that in the tenth row, there would be 19 tiles ($10\times2-1=19$). A whole discussion about patterning and algebra can occur just by looking at the rows and the number of tiles, but there is a deeper mathematical conversation that can also happen just by manipulating the picture of the pattern.

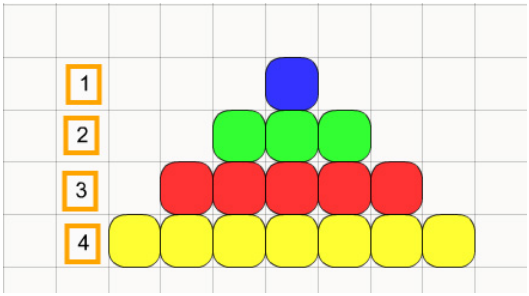


Figure 1: *Initial pyramid pattern*

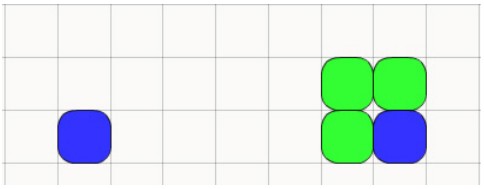


Figure 2: *Rearranging the pyramid to show what happens when you combine the rows into a rectangular shape*

The rows were each given a different colour in order to help build the next part of the discussion around the pattern. In looking at the rows, you could ask, "What happens when you combine each row with the previous rows in a rectangular shape?" Figure 2 depicts the first two examples. You are now exploring what happens when you examine the sums of consecutive odd numbers. The first "row" makes a single square, but when you combine it with the second "row" (it is an L-shaped row), you now have a square that is 2×2. Students can explore what happens when you continue to add on the next row and make observations.

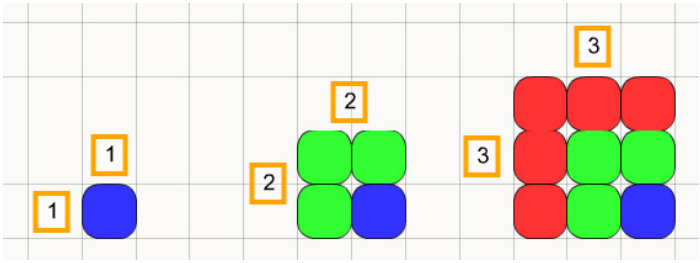


Figure 3: *The first three rows combined into a rectangular shape*

Figure 3 shows the first three stages of the pattern with the sides labelled. A discussion can now happen around the question, "What do you think will happen if you continue adding the rows?" Creating the subsequent parts of the pattern shows that the different odd numbers fit together as a perfect square. A student might notice that the "odd" square in the row of tiles can be interpreted as the corner of the next set of tiles around the square (see Figure 4). In looking at Table 1, you can see that adding consecutive odd

numbers creates a perfect square.

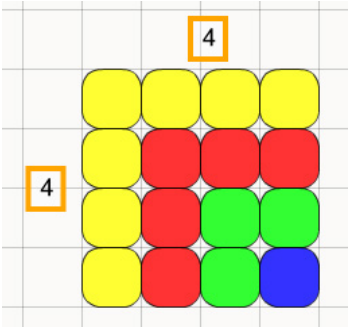


Figure 4: *The first four rows added together. The* X *indicates the "odd" tile that creates the corner.*

Rows	Length of Rows	Size of Square
1	1	1^2
2	1+3	2^2
3	1+3+5	3^2
4	1+3+5+7	4^2
5	1+3+5+7+9	5^2

Table 1: *Table showing the number of rows combined and resulting pattern for the odd-numbers pyramid*

I have found that it amazes students and adults when they realize that adding consecutive odd numbers together creates a perfect square. Looking at the pattern created, as shown in Table 1, you can ask, "How large would the square be if you add the first ten rows together?" Examining the pattern shows that there would be a 10×10 square (or 10^2), with the first ten rows (or odd numbers) added together. The pattern is that the sum of consecutive odd numbers creates a square that is the size of the number of odd numbers added together (e.g., adding the first 15 odd numbers creates a 15×15 square (or 15^2).

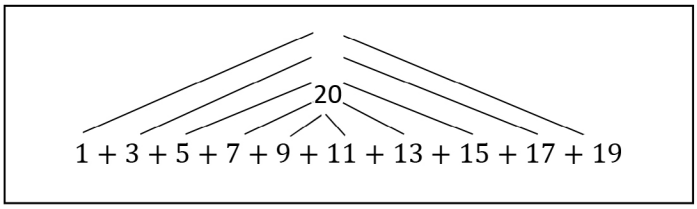


Figure 5: *Image showing the pairs of numbers that add together to equal 20, creating five pairs*

This can now be related to looking at number patterns in order to explore additional patterns within the problem. Take, for instance, what happens when you add the first ten rows (or first ten consecutive odd numbers): 1+3+5+7+9+11+13+15+17+19=100. Just looking at the number sentence can provide some additional conversations about the numbers in the pattern. Students can be asked to verify that these numbers do in fact equal 100 when added together. There are some different ways to approach this problem that lead to some of the observations made by Gauss. Figure 5 shows this way to approach adding the numbers together. In this example, students may notice that when you add the first and last numbers together, you get 20; the second and ninth combined gives you 20; and so on. There are five pairs of "20" in the figure, and this is also consistent with the sum being 100.

Students can be asked how they can predict the answer or use this information to figure out a different sum of numbers. For example, ask students to explore the first 15 numbers (1+3+5+7+9+11+13+15+17+19+21+23+25+27+29) to determine the sum. Now, you can see the numbers add together to make 30, but there is a 15 in the middle that does not have a match. An observation could be that where there is an odd number of digits, you have a single number that is half of the numbers you combine. Students can use this idea and test their predictions to see what they can learn about adding the numbers together in easy ways to find the sum. Students can also explore if there is a way to determine what number will be created without adding each of the numbers or writing out the list. For example, students may notice that the sum they add together is twice the number of odd numbers combined (e.g., with 10, you add 20's together, and with 15, you add 30's). You can also see that the number of times you combine this "magic number" is half the number added (e.g., with 20 numbers, you add 40 for ten times to get 400; with 15, you add 30, 7½ times for 225). As a note, this is only one method, but it does provide some new patterns that can be explored within the original patterning question.

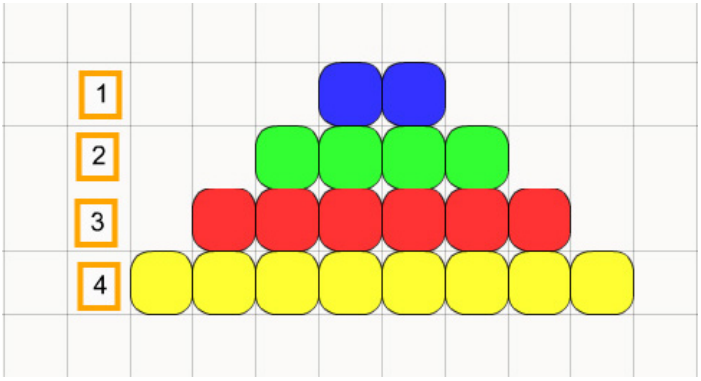


Figure 6: *A pyramid problem using rows of even numbers*

Another thought-provoking question to ask would be to create the same initial pattern, but this time, using consecutive even numbers. Figure 6 shows this new problem to consider. Students can first explore how many tiles would be in the next row. We could again push further to exploring the explicit rule of the pattern by asking, "How many tiles would be in the 10th row?" This would hopefully

push students not to build each row, but instead, look at the relationship between the row number and the number of tiles. In this case, when looking at the row numbers, you see the number of tiles is twice the row number (e.g., Row 3 is 3×2=6). Using this, it is a multiplication by 2 that, in the 10th row, there would be 20 tiles (10×2=20). We can now compare how this differs from looking at the odd-number pattern and why there is a relationship between the two formulas for determining the number of tiles. Each even-number row is one tile more than the odd row on the pyramid, so it makes sense to remove the subtraction part of the formula when looking at the even pyramid.

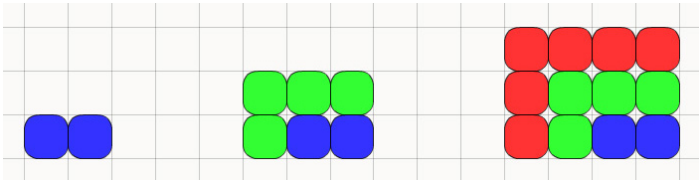


Figure 7: *The first three pictures of the rows combined for the even-numbered pyramid*

I like to next ask, "When we add the rows the same way as the odd number pattern, do we still get squares?" Most students, I have found, instantly say "of course" because they think of even numbers as creating squares. Mathematical conversations come in as they test their initial predictions. Figure 7 shows the first three pictures, showing that now rectangles are created. Table 2 shows the pattern in order to push the idea of how large the rectangle would be if you added the first ten even numbers together. Looking at the table, you can see that the rectangle height is always the number of rows that have been combined, and the base is one more than the rows. For the first ten rows combined, the rectangle would be 10 high and 11 for the base, or 110 tiles total.

Rows	Length of Rows	Size of Rectangle	Total Squares
1	2	1×2	2
2	2+4	2×3	6
3	2+4+6	3×4	12
4	2+4+6+8	4×5	20
5	2+4+6+8+10	5×6	30

Table 2: *Table showing the number of rows combined and resulting rectangular pattern for the even-numbers pyramid*

The deeper mathematics conversations come when you compare the numbers and pictures with the numbers and pictures in the initial problem in Figure 1 and see what happens. Figure 8 shows the first four rows added in both patterns. It becomes really clear that the addition of an even number could only ever create a rectangle and not a square. Again, you can go back to the discussion that the "odd" tile always creates a corner (Figure 4), so not having an odd tile would not allow the even numbers to make a perfect square. When you look at the difference in size between the two, you can also see that the number of tiles in the even pyramid is always more than the odd one with the same number of rows. The difference between the two rectangular shapes is always equal to the number of rows combined. For example, looking at Figure 8, you see that there are four rows combined in each, and there are four more tiles in the even image on the right than the one on the left.

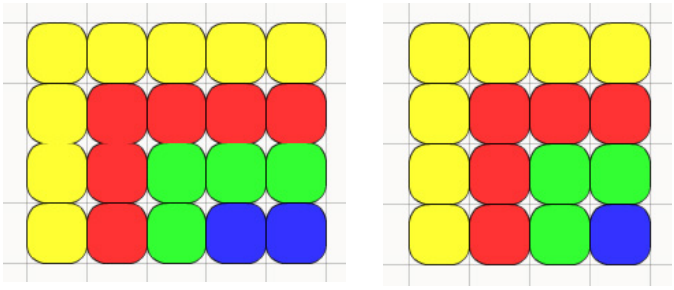


Figure 8: *The image on the left are the first four rows of the odd pyramid, and the figure on the right are the first four rows of the even pyramid*

We can now return to looking at adding the even numbers, like we did in the first example, to see what predictions can be made and if the same ideas hold true when looking at the even numbers being added instead of the odd numbers. To make the comparison the same, we first look at the first ten even numbers being added together: 2+4+6+8+10+12+14+16+18+20. This time (see Figure 9), if you add the first and last, it makes 22, which is not a useful number to use, but the same theory does hold from the odd numbers: you create five sets of 22, which gives us the total of 110. However, if we leave off the last number, the remaining numbers still make three sets of 20, plus the 20 at the end. The number that is now in the "middle" is the 10 (which is the number of tiles that are different between the two pyramids). If we look at the first 15 numbers, we can see if we can find a similar pattern: 2+4+6+8+10+12+14+16+18 +20+22+24+26+28+30. The last number is the "magic" number 30 from the odd numbers, and we can make sets of 30 when we combine the remaining numbers, making seven sets of 30, plus the last 30. What's interesting to see is that the number 15 (which is the difference between the two pyramids) is the number that was added to the one without a match in the odd pyramid, to create another 30. Now students can explore other patterns and numbers that they can discover when looking at the two pyramids or just adding consecutive even numbers. If you return to the observations from the odd numbers, you can see that the last number of

the evens is now the "magic" number and double the number of rows (e.g., with 20 rows, the last number is 40). Students can now look at other observations to see the relationships between the "magic" number and this new pyramid to see if their ideas still make sense and why they change. There are a lot of mathematical discussions that can be generated by comparing the numbers and testing predictions.

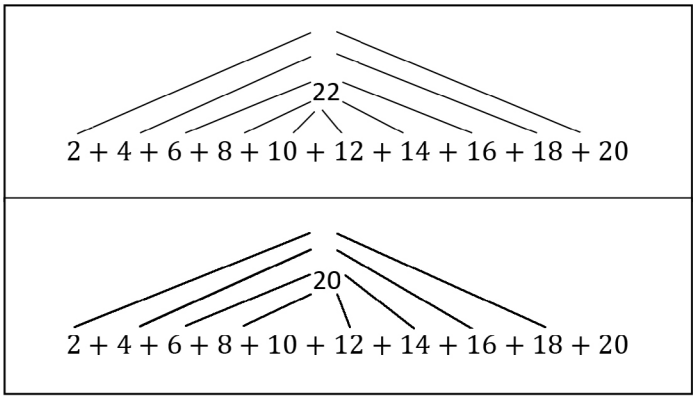


Figure 9: *Image on the top shows the pairs of numbers that add together to equal 22, creating five pairs. The image on the bottom shows the pairs that can be added to make sets of 20.*

When looking at patterning examples, sometimes there are lot of different patterns and conversations that can be created through the use of one single pattern, and then asking different questions. The different questions and predictions can also be used to create other examples to test students' conjectures and understanding of numbers (as the case was here by providing the similar example using only even numbers). The depth of the conversations are sometimes only limited by the time spent on the problems and taking the time to really explore the fascinating number connections that are built within our number system.

All figures were created using Mathies Learning Tools, Colour Tiles: www.mathclips.ca/swfPlayer.html?swfURL=tools/ColourTiles1.swf&title=Colour%20Tiles. ▲



Octagonal cone meets eight parabolic cylinders in Westminster Abbey, London, UK.

▲ In the Middle: Problems in Science



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Carly Ziniuk teaches Grade 9 Mathematics, Grade 12 Data Management, and Advanced Placement Statistics at the Bishop Strachan School in Toronto, Ontario, Canada. She is very active in adopting real-life data to engage her students in solving problems.

A few years ago, a Grade 8 science teacher and I had a conversation about our students, one that I have found myself reflecting on since. He asked me, kindly, why the students we shared were having so much trouble working with the formulas in their physics unit, despite their good understanding of the scientific concepts. We talked longer about specific formulas they were using at the time, relating density to mass and volume, and relating force to mass and acceleration. The required algebraic skills have few operations, straightforward sequencing, and less complexity than what students had done in our math classes. On first glance, it was not clear why they were struggling to apply their algebraic prowess to science.

The Problems

We wondered whether the students' developmental stage made it difficult to transition between the science concepts and using mathematics to rearrange a formula (i.e., changing between different contexts), whether we had timed the sequencing of units poorly, or whether it was just this group of students. After conversations with other Grades 8, 9, and 10 science teachers, we found that the "problem in science" was not just our Grade 8 students, and not just that year.

My colleague and I used a collaborative inquiry model to investigate the situation together. We deliberately noted the specific formulas used, the kinds of questions students were asking, and the algebraic skills that were involved. The sample questions and examples of student work allowed us to identify six prevalent errors and common misconceptions:

1. Most science problems expected students to read more carefully than many math problems of similar complexity. Furthermore, within a given topic, multiple formulas might be used in science, and it was more likely that they would require varied algebraic techniques.

Example 1: *The Canadian Space Agency (CSA) requires an astronaut's mass to be between 50 kg and 95 kg. The gravitational field strength on the moon is one-sixth of that on Earth, or 1.622N/kg. How much would a Canadian astronaut weigh on the moon?*

Example 2: *On a space shuttle, an astronaut is expected to track changes in his or her mass each day due to deconditioning in orbit. If Canadian astronaut Jennifer Sidey-Gibbons' weight on the moon was 92 N, what would you expect her mass on Earth to be?*

2. The science teachers were using the GRASP mnemonic for larger problems.

***G**iven*	What does the problem **give** you?
***R**equired*	What are you **required** to find out?
***A**nalysis*	What techniques or formulas can you use to **analyze** the problem?
***S**olution*	**Solve** the problem.
***P**araphrase*	Write your answer in a **paraphrased** sentence with context and units.

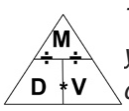
Adapted from UOIT Student Learning Centre's Physics Tip Sheet, https://shared.uoit.ca/shared/department/student-life/student-learning-centre/tip-sheets/physics/grasp-method-uoit-slc.pdf.

Note: UOIT Student Learning Centre has many other useful Math Tip Sheets as well.

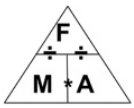
Some students did not use GRASP, and struggled to determine what formula to use, which values went where, and how to sequence the algebra. In math, they did not have as much difficulty approaching algebra sequencing of similar complexity. Even with the applications in algebra done in math class, students involved less context to understand what to do and were more focused on finding the numerical solution.

3. Some students used a graphic organizer called the "magic triangle," since the answer comes out "magically." The triangle did not expect any use of algebra and was not easily generalizable to other formulas. The students memorized each science concept's triangles, and did not connect the various positions on the triangle to the formula's components.

   Example 3: *This triangle relates density, mass, and volume from the formula* $D = \frac{m}{V}$*, where density is the output.*

   The triangle expects you to cover up the variable you are trying to determine and do the calculation that remains. So, for example D = m/V, m = DV*, and* V = m/D *by covering up* D, m, *and* V*, respectively.*

   This triangle, with the same operations, relates force, mass, and acceleration from the formula f = ma*, where force is the output.*

4. Some students tried to memorize steps for solving for each variable, without making the connection to the algebra. In Grade 8, students learned Pascal's Law of Pressure, connecting pressure, force, and area, using questions like this:

   Example 4: *A pressure of 2000 Pa is transmitted throughout a liquid column due to a 200N force applied by a piston. What is the piston's area?*

   For questions like this, we saw students write down the formula, recognize that the formula represented a fraction, and then apply division between the two numbers in the question until they got the correct answer.

5. Units and unit conversion added another layer of confusion. The students not only had to use the formulas correctly, they needed to remember and utilize the conversions between metric and time conversions correctly.

   Example 5: *Jyda lives 3 km away and it took her 40 mins to walk home. What is her velocity in km/h and in m/s?*

   Although we had similar questions in the Grade 9 math course, we rarely asked students to express the same speed in two different units.

6. Students often had to sequence formulas together in science, and keep track of all of the known information to determine more than one variable.

   Example 6: *A car battery labelled 12 V actually supplies a voltage of 12.6 volts. If the battery puts out 950 W of power to run the ignition, radio, and other electronic elements, what is the current provided and what is the battery's resistance?*

   We realized that math questions like this were more commonly broken into two separate parts, finding the current in part a), for example, and the battery's resistance in part b).

Our Solutions

Working together, we understood that a collaborative process between math and science teachers would help both math and science learners. We sought common approaches and actively looked for solutions to the problems. These solutions were our responses to the collection of problems given in the previous list, but each

solution does not correspond to the number of the issue.

1. We wanted to ensure consistency between the courses and give everyone—students and teachers—the chance to identify common formula set-up. We collected all of the formulas from the Grades 8–10 physics components of science courses and endeavoured to include them in our math course algebra unit. In the same way that the math teachers had used for two- and three-dimensional measurement formulas, we developed a single science formula sheet (the bold variable is a common way to identify the formula):

Variables	Formula	Units
d: density m: mass v: volume	$d = \frac{m}{v}$	*Mass measured in kilograms (kg)Volume measured in cubic metres (m^3)* *Density measured in kg/m^3*
F: Force m: mass a: acceleration	$F = ma$	*Mass measured in kilograms (kg)* *Acceleration measured in m/s^2* *Force measured in Newtons (N)*
W: Weight m: mass g: gravity	$W = mg$	*Mass measured in kilograms (kg)* *Gravity measured in N/Kg* *Weight measured in Newtons (N)*
P: Pressure F: Force A: Area	**Pascal's Law of Pressure** $P = \frac{F}{A}$	*Force measured in Newtons (N)Area measured in cm^2* *Pressure measured in Pascals (Pa)*
v: velocity (speed) d: distance t: time	$v = \frac{d}{t}$	*Distance measured in metres (m), or kilometres (km)* *Time measured in seconds (s), or hours (h)* *Velocity measured in km/s or km/h*
P: Power V: Voltage I: Current	$P = VI$	*Voltage measured in Volts (V)* *Current measured in Amperes (A)* *Power measured in Watts (W)*
V: Voltage I: Current R: Resistance	$V = IR$	*Current measured in Amperes (A)* *Resistance measured in Ohms (Ω)* *Voltage measured in Volts (V)*

2. We recognized that GRASP provided a useful problem-solving structure in our math courses, and adopted it for the many science problems throughout our algebra unit. Within that unit, we interchanged problems like *Example 1* with geometry application problems like *Example 7* below, all using GRASP. We encouraged the students to use the variable definitions on the formula sheet and write the formulas and algebraic steps clearly in the analysis section.

   Example 1: *The Canadian Space Agency (CSA) requires an astronaut's mass to be between 50 kg and 95 kg. The gravitational field strength on the moon is one-sixth of that on Earth, or 1.622N/kg. How much would any Canadian astronaut weigh on the moon?*

Given	*mass, lowest m = 50 kg and highest m = 95 kg* *gravitational field strength on the moon, g = 1.622 N/kg*
Required	*Weight on the moon, W =?*
Analysis	*W = mg* *Lowest W = (50)(1.622)* *Highest W = (95)(1.622)*
Solution	W = 81.1 and 154.09
Paraphrase	A Canadian astronaut would weigh between 81.1N and 154.09N on the moon.

We spent time in our classes with a specific discussion about the importance of subscripts, something we had not done beforehand explicitly in Grade 9. This had an unexpected benefit later in the year, when we taught the slope between two points as $\frac{y_2 - y_1}{x_2 - x_1}$: the students were more consistent in their use of subscripts they had struggled with previously.

Example 7: *What is the difference in the volume between an American golf ball with a diameter of 42.7 mm compared to a British ball with a 41.1 mm diameter?*

Given	d_{Am} = 42.7 mm and d_{Br} = 41.1 mm Golf balls are spheres.
Required	$V_{Am} - V_{Br}$ (the difference between the two spheres' volumes)
Analysis	Volume of a sphere $V = \frac{4\pi r^3}{3}$ $V_{Am} = \frac{4\pi\left(\frac{42.7}{2}\right)^3}{3} = \sim 40764.5$ $V_{Br} = \frac{4\pi\left(\frac{41.1}{2}\right)^3}{3} = \sim 36351.7$
Solution	*Difference between the two 4412.9 mm^3* *By dividing each dimension by the conversion factor of 10, this could be converted to 4.412 cm^3.*
Paraphrase	*American golf balls are 4.412 cm^3 larger than British golf balls.*

3. We explained to the science teachers how, in our algebra unit, we used strategies like the *Balance Method* and *Opposite Operations*, and reviewed mnemonics like "*undo BEDMAS*" (sometimes called SAMDEB).

   We discussed why the "magic" triangle approach only worked for one-step equations and did not connect to

any algebraic thinking. Everyone noted that the triangles most often were just given to the students, rather than leading students to properly understanding the connection to an algebraic formula.

We took the time to show the science teachers similar direct variation problems, how we had been explaining to the students why the two equations were equivalent, and how it was necessary to identify the slopes for both forms of the equation, $y = \frac{1}{3}x$ and $y = \frac{x}{3}$. We made notes about the students' discomfort with fractions in both contexts and made a point to practise fluency in both kinds of problems.

4. We noted the specific problems that the students were finding difficult. Most often, these were the ones where the unknown was not on its own, which was atypical in middle school math. We demonstrated solving *Example 2*, as follows.

   Example 2: *On a space shuttle, an astronaut is expected to track changes in his or her mass each day due to deconditioning in orbit. If Canadian astronaut Jennifer Sidey-Gibbons' weight on the moon was 92 N, what would you expect her mass on Earth to be?*

***G**iven*	*Gravitational field strength on the moon, g = 1.622N/kg* *Weight on the moon, W = 92N*
***R**equired*	*Mass on Earth, m =?*
***A**nalysis*	*W = mg* $92 = (m)(1.622)$ $\frac{92}{1.622} = m$ $m \sim 56.720$
***S**olution*	*56.720*
***P**araphrase*	*Jennifer Sidey-Gibbons has a mass of 56.720 kg, which means she is within the CSA requirements of 50 kg to 95 kg.*

In science class, students were using the triangle without understanding its connection to the formula or to algebra. We spent time in math class exploring why $m = \frac{W}{g}$ is the equivalent algebraically to $W = mg$ and encouraged our science colleagues to use this approach, rather than the "magic triangle."

We also learned the students were struggling when the unknown quantity was in the denominator of a fraction. We showed science teachers how to model problems like this for their students, using algebra, rather than a triangle and magic. Some teachers chose strategies they favoured, but most went with illustrating the extra step, as below.

Example 4: *A pressure of 2000 Pa is transmitted throughout a liquid column due to a 200N force applied by a piston. What is the piston's area?*

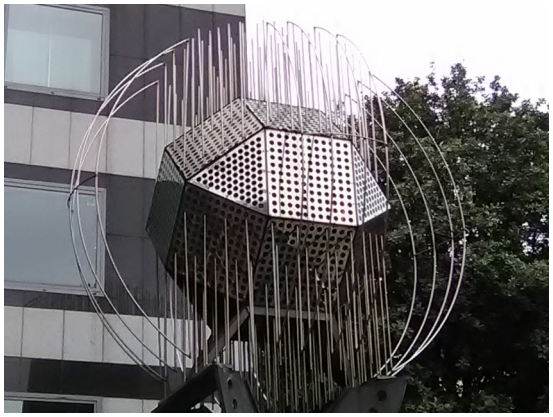
***G**iven*	*Pressure, P = 2000 Pa* *F = 200 N*
***R**equired*	*Area, A =?*
***A**nalysis*	$P = \frac{F}{A}$ $2000 = \frac{200}{A}$ $2000A = 200$ *extra step* $A = \frac{200}{2000}$
***S**olution*	*A = 0.1*
***P**araphrase*	*The liquid column has an area of 0.1 cm^2 applied on the piston.*

5. We agreed that students were struggling with converting units in both math and science. We got metric conversion posters to use as anchor charts for both classrooms, and reinforced this learning by modelling similar problems involving varied units.

For the math teachers, this was a good reminder of the importance of units, and checking the units helped our teaching practice. We reminded our students that their paraphrase statement had to include units when they were solving problems in *both* math and science.

6. To address challenges with problems requiring multiple formulas, we examined whether we could introduce the concept of substitution earlier in the course. Example 6 shows how we were able to do that.

Example 6: *A car battery labelled 12 V actually supplies voltage of 12.6 volts. If the battery puts out 950 W of power to run the ignition, radio, and other electronic elements, what is the current provided and what is the battery's resistance?*

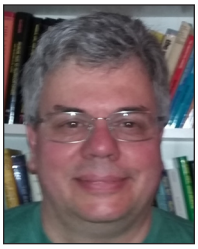


Archimedian solid in sculpture outside the Shaw Theatre, London, UK.

Given	*Voltage, V = 12.6 V* *Power, P = 950W*
Required	*Current, I =?* *Resistance, R =?*
Analysis	*P = VI* $\frac{P}{V} = I$ $\frac{950}{12.6} = I$ *substitution* $V = IR$ $\frac{V}{I} = R$ $\frac{12.6}{\left(\frac{950}{12.6}\right)} = R$ *substitution* *An alternate formula solution was also worked through using exponents.* $\frac{V}{\left(\frac{P}{V}\right)} = R$ $R = \frac{V^2}{P}$
Solution	*I~75.4 and R~0.167*
Paraphrase	*The battery provides 75.4 A in current and 0.167 Ω in Resistance.*

Realizing that our students were having trouble with algebra in physics turned out to be the start of an important and valuable process for all of the teachers involved. Rooted in the understanding that all students had to learn algebra and physics concepts in Grades 8, 9, and 10, our goal was to have the two disciplines more connected explicitly, and agreed as teachers to work on this collaboratively. We investigated the formulas used, the kind of questions students were asking, and the algebraic skills necessary. We connected those directly to our algebra introduction in our Grade 9 math classes. Together we identified teaching incongruencies and the challenges the students were experiencing. We used new strategies like GRASP in our own application problems throughout the math course, and encouraged our science colleagues to replace their "magic triangles" with explicit algebra. This gave us new opportunities to connect with our science colleagues, identify with our students' experiences in science, and learn more about science ourselves. Our students recognized and valued the connections we built together. We look forward to developing this partnership further and building even more connections in the future. ▲

▲ What's the Problem? Messing with Mean, Median, and Mode



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Shawn teaches and is a department head at Cairine Wilson Secondary School in Orleans. He strongly believes in the central role of problem solving in the mathematics classroom. He continues to be involved in mathematical activities: presenting workshops, writing articles, working on local projects, and helping create mathematics contests.

Welcome back, problem solvers. Last time, I left you with the following problem:

> *Melanie computes the mean , the median* M*, and the modes of the 365 values that are the dates in the months of 2019. Thus, her data consists of 12 1's, 12 2's, …, 12 28's, 11 29's, 11 30's, and 7 31's. Let* d *be the median of the modes. Arrange the numbers* μ*,* M*, and* d *from smallest to largest.*

This problem was inspired by problem #12 in the AMC10A contest and problem #7 in the AMC12A contest. The Mathematical Association of America (MAA) sponsors the American Mathematics Competitions (AMC). Each year, they run a contest for students up to Grade 8 (AMC8), two contests for students up to Grade 10 (AMC10A and AMC10B), and two competitions for students up to Grade 12 (AMC12A and AMC12B). All the competitions are used to determine candidates for the United States of America Mathematical Olympiad (USAMO) and the United States of America Junior Mathematical Olympiad (USAJMO). You can find out more about the AMC contests at www.maa.org/math-competitions/about-amc. Past contests and solutions are available on The Art of Problem Solving website at www.artofproblemsolving.com/wiki/index.php/AMC_Problems_and_Solutions.

The focal problem is nice because there are many ways to solve it. You can do all the computations to get your answer directly. Alternately, you can use some logic and properties of the mean, median, and the mode to get at your result more quickly. Getting the result quicker is advantageous if you are writing a contest. Looking at a problem from several points of view helps to strengthen your understanding of the concepts involved.

We will start by calculating things directly. To calculate the mean, we need to calculate

$$\mu = \frac{1+2+3+\ldots+31+1+2+\ldots}{365}.$$

We can save ourselves some time by using the information given in the problem to get

$$\mu = \frac{12\times1+12\times2+\ldots12\times28+11\times29+11\times30+7\times31}{365},$$

which gives a mean of $\mu = \frac{5738}{365} \approx 15.7$.

Since we have 365 numbers, and 365 ÷ 2 = 182.5, it means that if we split our numbers into two equally sized groups, there would be 182.5 numbers in each group. We cannot have part of a number, so we would have to have 182 numbers in each group, with one number left over in the middle, the median. Thus, our median is the 183rd number in the ordered list containing all of the days. Since there are 12 1's, 12 2's, …, calculating 183 ÷ 12 = 15.25 means the 183rd number is past the 15's and into the 16's, so $M = 16$.

Finally, the modes are the numbers that appear most often, which would be 1, 2, 3, …, 28, each of which appears 12 times. To find the median of the modes, 28 ÷ 2 = 14 tells us that we can break the list into two groups of size 14. Hence the 14th and 15th numbers will be our middle two numbers, so $d = \frac{14+15}{2} = 14.5$.

Therefore, arranged from smallest to largest, we have *d,* μ, and *M*. If we want to get at our result a bit more quickly, we need to dig deeper into some properties of the mean, median, and mode.

First, note that we are dealing with consecutive numbers. It is interesting to note that if we have a list of consecutive numbers, the mean and median are the same. Imagine we have an odd number of consecutive numbers, then the median will be the middle number. The mean is the number that, if all the numbers were equal to it, we would have the same sum. In Figure 1, we have a list of an odd number of consecutive numbers (1, 2, 3 on the left and 1, 2, 3, 4, 5 right of the middle), with the median coloured red. For each case, we have the list of numbers, and the list after it has been "levelled out" to the mean (left of middle and on the right).

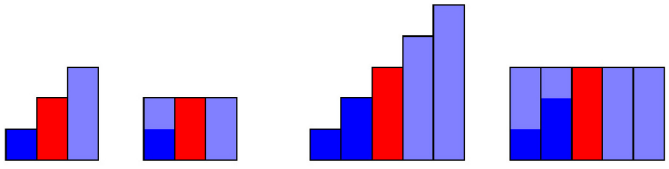


Figure 1: *Mean and median of an odd number of consecutive numbers*

In Figure 2, we have an even number of consecutive numbers (1, 2, 3, 4 and 1, 2, 3, 4, 5, 6), so the median is the average of the two middle numbers. The median is indicated with a red dashed line. As in Figure 1, we have the numbers listed, as well as the list after it has been "levelled out" to the mean. We see in both cases that the median and mean are equal and, hopefully, can see from the diagram that this will be the case for any set of consecutive numbers. The idea generalizes to any list of numbers in arithmetic progression, where consecutive terms differ by the same amount. You may want to try looking at the diagrams that you would get for the list 2, 5, 8, 11, 14 as an example.

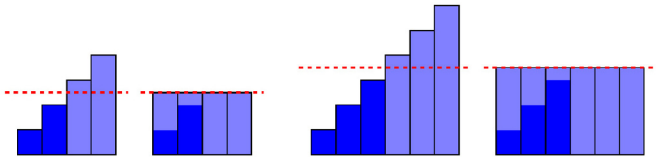


Figure 2: *Mean and median of an even number of consecutive numbers*

In the case where the mean and median are whole numbers, manipulatives, like linking cubes or square tiles, would be useful for exploring this idea. Even when the mean and median are not whole numbers, you could still arrange them to see that they are the same. For example, the case of 1, 2, 3, 4, 5, 6 could be redone as in Figure 3. The alternating pattern of 3, 4, 3, 4, 3, 4 would indicate that we would need to cut the high blocks in half in order to even things out (or think of it as the mean is 3 full blocks and 3 of 6, or ½ of the fourth block). You may want to experiment with other computations of the mean, using manipulatives.

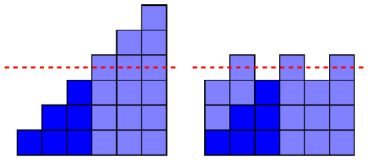


Figure 3: *Mean and median with blocks*

The next important idea is how the mean and median of a set of numbers are affected when we add a new number. The easier one to deal with is the median. The median is the middle number. If we add a number to our list, the middle moves. For example, if we have 1, 2, 3, the middle number is the 2nd number, 2. If we add another number, the middle number will be the average of the 2nd and the 3rd numbers. So, if we add another 2, the list is 1, 2, 2, 3, and the median would remain 2.

If we add a number larger than the median, the median changes, but note that the lists 1, 2, 3, 4 and 1, 2, 3, 400 each have the same median: 2.5. Similarly, the lists 0, 1, 2, 3 and -100, 1, 2, 3 each have median 1.5. It gets more

interesting if the number you add is between 1 and 3, so you may want to play with that. For our problem, we will only consider adding numbers at the end of the list, so the position of the median will only move by 0.5 each time (where the "term 2.5" corresponds to the average of terms 2 and 3).

For the mean, things are different. The size of the new number has a direct affect on the mean. If we want to add a new number to bring the mean to a certain level, the new number must be at that level, *plus* enough to bring all the other numbers up to that level. We will illustrate with a couple of examples.

First, suppose we had three numbers with a mean of 2. This implies we can consider the three numbers as "levelled off" at 2. If I want to add another number to bring the mean up to 3, the new number must be 3 plus the amount to bring the other three numbers up to 3. They all must increase by 1, so the new number must be 3 + 3 x 1 = 6, as shown in Figure 4.

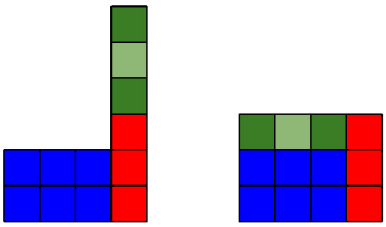


Figure 4: *Moving the mean up by 1*

Now suppose we start with the same situation, but we want to increase the mean from 2 to 4. The new number must increase all the old numbers by 2, so the new number must be 4 + 3 x 2 = 10, as shown in Figure 5.

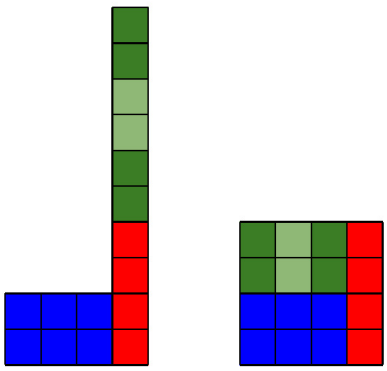


Figure 5: *Moving the mean up by 2.*

Let's go back to the original problem and imagine that we start with 12 1's, 12 2's, …, 12 27's, and 12 28's. Clearly, we must have $\mu = M = d$ (why?). As soon as we add any of the other numbers, μ and M will increase, but d will stay the same. As a result, without any calculations, we know that d must be the smallest number.

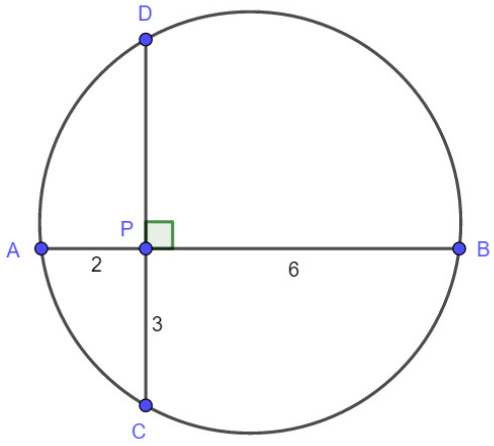
Suppose we have 12 1's, 12 2's, …, 12 27's, 12 28's, 12 29's, and 12 30's, again $\mu = M$. Since there is an even number of numbers, we know the median is the average of the two middle numbers, in this case, 15 and 16. If we add a 31 to the list, the median jumps by 0.5 as it moves to the middle number, the first 16. It will remain there for quite a while and won't move beyond it for the number of numbers we need to add.

For the mean to increase by 0.5, the new numbers would need to add 0.5 to each of the "levelled" numbers, 360 in total, so we would need 180 to be added to the first 360 numbers to bring them up from 15.5 to 16. Changing one of the 29's and one of the 30's both to 31's doesn't affect the median, but gives 3 "extras." That leaves us with only 5 more 31's to add. Since 3 + 5 x 31 < 180, we don't have enough to bring the first 360 numbers up to 16; therefore, we can conclude that $d < \mu < M$.

We could have similarly argued the other way. That is, if we suppose we have 12 1's, 12 2's, …, 12 27's, 12 28's, 12 29's, 12 30's and 12 31's, then the mean and median would both be 16. Removing some of the numbers from the top end will change the median only when 12 numbers are removed, but the mean will drop each time. Hence, if we remove 5 31's, 1 30, and 1 29, the median is still 16, but the mean is lower than 16. It is great when we can use the concepts to solve the problem without having to do all the calculations.

Now, some homework:

In the diagram below, AB *and* CD *are perpendicular chords of a circle. The two chords intersect at* P*. Given that* PA *= 2,* PB *= 6, and* PC *= 3, determine the radius of the circle.*



Until next time, happy problem solving! ▲

OAME Awards 2019

Lifetime Membership Award



Cathy Hall has been an integral member of OAME/AOEM for more than ten years. She served two terms as OAME Director from 2006 to 2011, then bypassed her sixth year as Director to become President-Elect 2011–12. During her year as President-Elect, she chaired the Leadership Conference, "On the Leading Edge: Making Math Happen!," and organized a COMA Coach to ensure delegates from Ottawa had easy travel to the Windsor conference due to the distance involved.

Cathy successfully co-chaired the OAME Annual Math Conference, "Finding the Right MMIX," in 2009, and co-chaired a successful OMO in Ottawa in 2007. She has also served as the COMA secretary for a few years before taking over as chapter treasurer. Although retired, she spent many years as chapter treasurer, attending every meeting, and ensuring good financial planning.

Cathy is an all-around educator. She taught in the Ottawa District School Board as a classroom teacher from Grades K–8, and delivered mathematical professional development from K–10. She opened a new school and helped establish teaching resources to be used, therefore fostering the vision of students engaging in meaningful mathematical experiences through the use of concrete materials and manipulatives, visuals, technology, and other resources. She was a team leader and led student initiatives, which fostered the vision of teachers using a variety of strategies that accommodate different learning styles and use of resources to develop the investigative process in mathematics.

Cathy served as teacher representative on parent/school councils and ran "Math Nights" for parents. Now in retirement, she works as a math consultant in Indigenous communities, where she addresses the vision of learning mathematics, supported through the active and ongoing involvement of students, parents, educators, and the community at large.

Award for Exceptional and Creative Teaching in Secondary Mathematics



Matt Anderton, though committed to innovating in his own teaching, and constantly questioning the impact of current pedagogical practices on today's students in an ever-changing world, is never content to limit his learning to his own classroom. He is driven to share what he learns with other educators, to reach a greater number of students and to challenge our notions of what education should and can look like. Matt openly shares his ideas and learning through research with colleagues at his school, at other schools in his school board, with various online educational communities, and beyond—including the local co-terminus school board. He also willingly listens and considers others' ideas. He builds his teaching practice around collaboration and encourages others to do the same. As department head, he always tries to involve as many people as he can in every initiative, every step toward collective growth, and every important decision. It's clear in all he does that he wants everyone to be better.

Matt exemplifies the life-long learner. He is always reading the latest research regarding mathematics teaching and education in general, and always willing to try—even completely redefine—his teaching in response to what he learns. For example, after reading Peter Liljedahl's research, he has inspired and enabled the staff at St. Benedict Catholic Secondary School in Cambridge, Ontario to embrace the Thinking Classroom model. He has influenced countless other teachers in math, English, and other subjects, to embrace productive struggle, to use visibly random seating, and implement the use of Vertical Non-Permanent Surfaces in the classroom.

Teachers from our feeder schools, our board's numeracy consultant, and staff from all over Ontario have reached out to connect with Matt because of what he is doing. Even when he finds something that works well in the classroom, he doesn't accept that his job is finished. He continues his research, always determined that he and his team—and all teachers—can do better for students in Ontario.

Award for Leadership in Mathematics Education



In **Liisa Suurtamm**'s role as Math Consultant in the District School Board of Niagara, she is responsible for all professional learning by their math teachers. This includes setting the direction and focus for the math learning during the board-wide Professional Development Day, facilitating learning teams at board and school levels, planning and facilitating the bi-monthly Program Leaders' meetings, and providing one-on-one support in classrooms. Outside of the District School Board of Niagara, Liisa is an instructor at Brock University in the junior, intermediate, and senior divisions for concurrent and consecutive pre-service teacher candidates.

Liisa works endlessly for OAME's Golden Section—organizing mini-conferences and having co-chaired the 2010 annual conference, "On the Brink." Furthermore, Liisa has been invited to speak at the Fields Math Education Forum, both provincial and chapter conferences within OAME, and GAINS Summer Math Camp—to name a few. District School Board of Niagara recognized Liisa's dedication to math education with her receiving the Director's Award of Excellence, and the Ontario Mathematics Coordinator's Association (OMCA) Excellence in Math Education Award. Whether working with her teacher candidates, school board teachers, or presenting at a conference, Liisa always grounds the work she does in research. Over the past few years, Liisa has facilitated learning around *From Patterns to Algebra*, *The Five Practices, Spiralling, Teaching Through Problem Solving,* and *Creating Thinking Classrooms*. Also, Liisa has led book studies around *The Five Practices, Principles to Action, Make It Stick,* and *Mathematical Mindsets*.

Through her work with the District School Board of Niagara Math Program Leaders, Liisa promotes their leadership by empowering them to make decisions regarding their own math teams, having them regularly look at their role, and introducing them to new ideas and current research. Liisa further develops teachers' leadership by supporting them in running Teaching Learning and Leadership Program projects.

In 2013, Liisa secured funding to run a five-part series of professional learning across her school board, called "Building Capacity," where she presented research-based practices to over 100 math teachers and administrators.

Award for Exceptional and Creative Teaching in Elementary Mathematics



When I consider the characteristics of what I think makes an exceptional educator—a large repertoire of high-yield instructional strategies and assessment strategies that are incorporated into his or her classroom and beyond; an involvement and participation in his or her profession beyond the classroom; and a continual desire to learn and push his or her understanding of good pedagogy and a deep content knowledge—**Kerri Evershed** embodies each and every one of these.

During her time in the classroom, she was always using instructional strategies that engaged all learning styles and pushed her students' thinking. She is a key member of the local OAME chapter executive, but is also involved in programming for students beyond the school day, that provides opportunities for them to explore other learning opportunities. As an instructional coach, she is committed to co-learning with her peers and sharing that desire to explore the most innovative ways to teach mathematics.

Students in Kerri's class see themselves as risk takers and problem solvers. She provides an environment where all students' thinking is encouraged. Beyond the classroom, Kerri has been the administrator for summer learning camps, where students explore learning opportunities involving technology. She is in charge of all aspects of the program, including hiring, program design, and delivery.

Kerri is also the OMO coordinator for the CHAMP chapter. It would not be an understatement to say that without her, many students would not have had an opportunity to participate in this enrichment activity. She is the critical person that keeps this event running year after year.

She has even challenged herself by teaching a night school course at the secondary level. Not many teachers would be willing to take that risk by teaching a subject beyond their comfort level, but Kerri sees the challenge as an opportunity to bring her creative teaching style to another group of students. ▲

▲ 2019-2020 OAME Board

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