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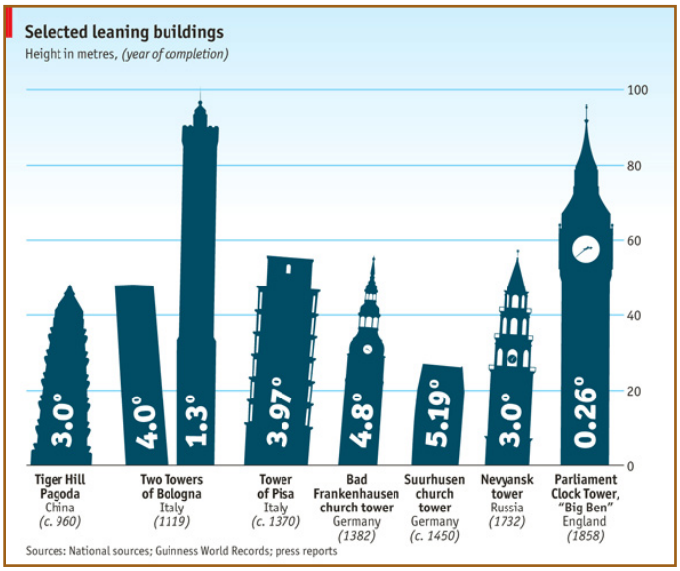
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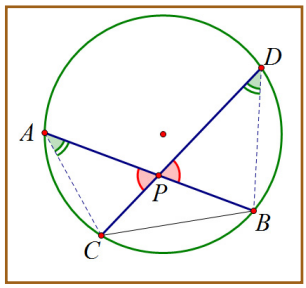
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December 2019

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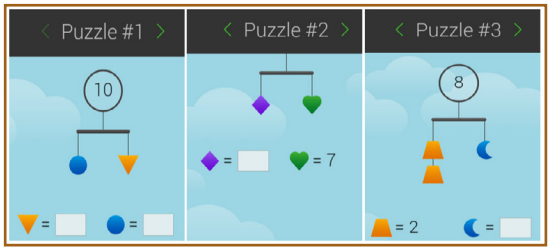
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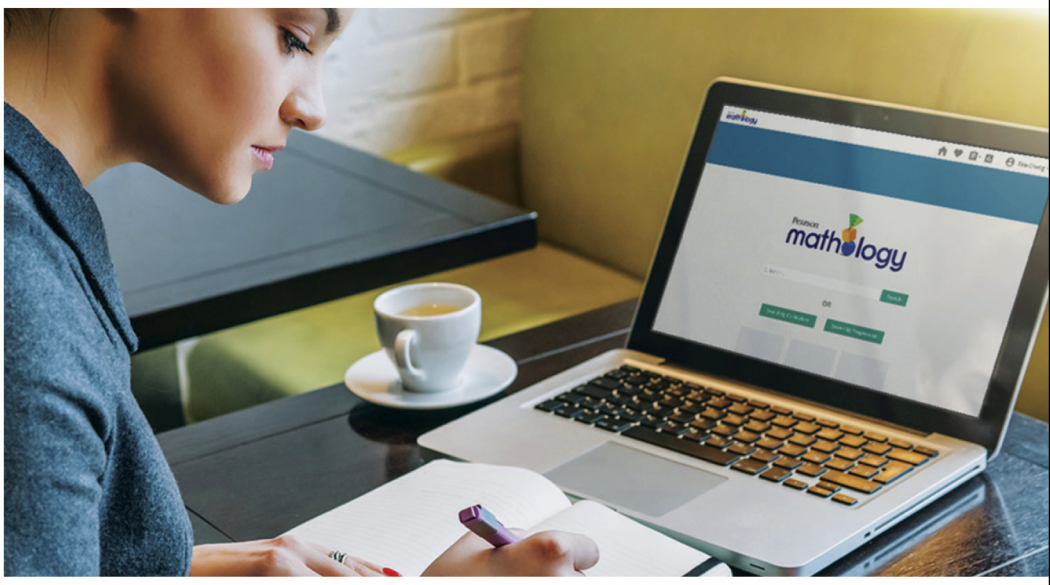
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Math

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- Please take this in the sense of good fun it is intended! Ed.

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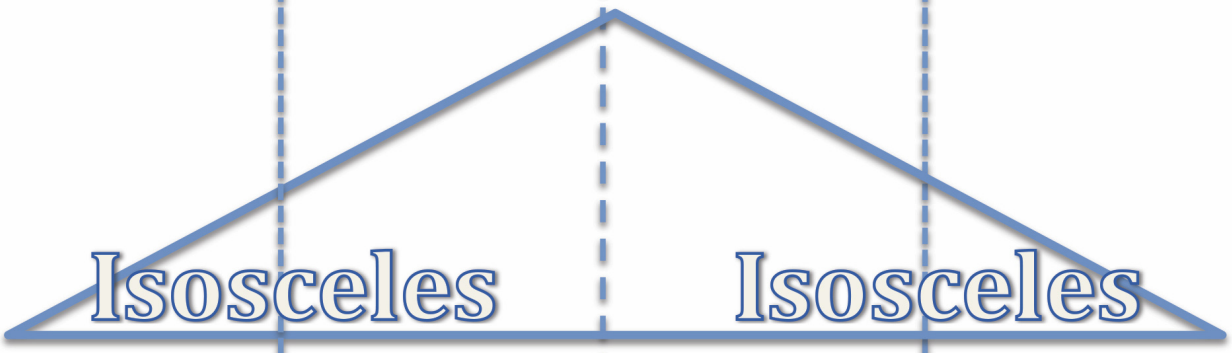
Nifty but definitely very bad ideas!

Donuts with jalapeno icing!

Squishing a rectangle to look like a mare?



Teaching Transform-ations using chalk and talk and talk and chalk, that isn't with foldables



Notice the folded triangle change!

Season reindeer burgers with road salt?

Use "yo yo" or "hey" as Greetings!

Submission of Articles

The *Ontario Mathematics Gazette* (OMG) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *Gazette*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles are to be word-processed, MS Word is preferred, and prepared according to the *Publication Manual of the American Psychological Association*, Sixth Edition (2009). However, please use single-line spacing (not double) and only one space after each period. Articles should not exceed five numbered pages of text, and figures, images, and photographs should be placed in the text close to where they belong, with captions. The photographer's permission is required, and for photos of students under the age of 18, the written permission of a parent or guardian is required.

Please submit your article in one blind file (i.e., identity of author is not evident), and include author names, contact information including email and mailing addresses, photos, biographies, and all content removed for blinding in a second file. Please email these two files to Tim Sibbald at gazette@oame.on.ca.

Upon review, you will be notified whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected. The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME.

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The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of OAME.

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▲ Editor's Report



TIMOTHY SIBBALD, OCT, PhD
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Tim Sibbald holds the Universities Director position on the OAME Board of Directors. He is a former president of OAME. He is an associate professor in the Schulich School of Education, Nipissing University, with a focus on mathematics education.

Take a moment to consider what, if any, distinction you perceive among *formal mathematics, applications, interdisciplinary mathematics,* and *numeracy*. I use the term "numeracy," rather than "mathematical literacy," because I maintain there is an important distinction. For this column, I will remain focused on the four terms mentioned.

What prompted this editorial was an article that, as I was reading, I found was drawing different distinctions than I do. The article (Beckman, 2008) was a conference paper that served as a springboard into thinking about the communication issue that emerges if such terms are used with an assumption that we all have the same understanding. In fairness, however, a few remarks will help tailor considerations of the terms. The notion of formal mathematics is within the context of the classroom as opposed to the overall field of mathematics. Applications is a notion that was emerging as the Ontario curriculum was changing in the the early 2000s, when technology was being endorsed and more focus was given to understanding computed results.

It is not my intention to take anyone to task; rather, I will offer an opinion so that you can contrast it with your own interpretation. Or, should we happen to agree, it may serve to help articulate the opinion so that you can converse with your colleagues to find how much variation exists within the understanding of such fundamental notions.

In math education, formal mathematics is, in my opinion, the conceptual foundation and standards of practice for conveying those concepts that are embedded in the curriculum and teachers' pedagogy. Examples are the perimeter of a circle in terms of the radius, the quadratic formula, and the Pythagorean trigonometric identity. The defining features are inclusion of the mathematical concept within the curriculum expectations (or enrichment teachers provide) and significance of the mathematics from the larger mathematics community. The latter is important, as there are

trends in mathematics, and an example is the "new math" in the 1960s that was axiomatically based. It entered and was later dismissed from school curricula (see Gord Nicholls' column in issue 29(3)).

Applications are problematic, even though we endorse this notion in Ontario. The notion of an application is the utilization of mathematics for a practical purpose. The difficulty is in how the practicality is contextualized. A classroom teacher can provide an example that entails determining the navigation of an airplane to reach a forest fire. Clearly this is not going to be done with an actual airplane or fire. That is reasonable. However, many teachers avoid tactile scenarios for applications for a myriad of reasons, yet, if applications never involve hands-on experiences, then it raises doubts about the authenticity of the connection that is made between the mathematics and an application. This challenge is essentially a modern interpretation of the pure/applied divide that has existed in mathematics for a very long time, albeit the distinction is being made as authentic versus hypothetical applications. Both are doubtlessly necessary, but I sometimes hear teachers express a bias about this detail that may be reflected in their classroom practices.

Interdisciplinary mathematics is, in my opinion, evolving. At one time, it was descriptive of applications and meant that teachers should promote a variety of uses for mathematics. In essence, it was to avoid a myopic promotion of mathematics, such as exclusively using financial applications or, perhaps, using applications that promote the need for further mathematics. The development of an interdisciplinary vision in Ontario has included the notion that all destinations need to be supported and that a wide variety of contexts need to be adopted in classroom teaching. This is challenged by the notion of authenticity that is further complicated by the difficulties with accessing contextual uses of mathematics. In my experience, a quintessential example is what happens when I have inquired about math connections to physical education. High school students recognize the existence of sports statistics, and when pushed to design an experiment, choose tasks such as the probability of getting a basket when shooting a basketball from different locations on a court. Teacher candidates often come up with reading and interpreting food labels or, especially those with a kinesiology background, various technical measures of performance. For this example, however, understanding how professional athletes utilize and are informed by mathematics is elusive to many teachers.

In my research, the notion of interdisciplinary mathematics has shown two prominent features. First is the extension of knowledge across disciplinary boundaries, such as the way the application of mathematics in physical education was revealed. Similarly, in STEM fields, there is considerable push to look at drawing on the larger scope of the combined fields so that what is taught may be more representative of the workplace. This is why coding, as one example, is seeing renewed interest, albeit substantially different from the rudimentary days of programming. The research challenge, for those who might wonder, is that interdisciplinary mathematics needs significant work regarding its theoretical basis.

Jack Lesage

The obituary in the June 2019 *Gazette* did not mention that Jack was a Descartes Medal winner, as well as a recipient of a Life Membership in the OAME in 1986.

The second interdisciplinary feature demonstrated by my research is that some mathematical notions readily migrate across many disciplines. Our elementary teachers know this and can doubtlessly point to the many uses of basic math skills through the various subjects they teach. The belief that a math skill applies across many disciplines is, I claim, a necessary feature of numeracy. It is also necessary that the skill has utility in a variety of contextual applications to establish its place within the scope of numeracy. However, this proposed defining feature of numeracy is exactly that, a proposal. It has been presented internationally and seems to resonate, but there is no singular consensus on what numeracy actually is. There are quite a few definitions, and a few curricula (Wales and Ireland, for example) include it explicitly. Many definitions are circumstantial—if you learn the curriculum, you are numerate; it is the mathematical analogy of literacy; it is math in context that has daily usage—and lack specific details that allow numeracy to be directly clarified within mathematics education.

Whether you hold any level of agreement with the espoused notions of *formal mathematics, applications, interdisciplinary mathematics,* and *numeracy* is a matter for your professional judgment. The intended point of this editorial is that the variety of interpretations that persist is an issue that requires our attention. We can talk about applications and share ideas, but without addressing our interpretations and details such as authenticity, do we develop our professional thinking or simply address day-to-day needs? The same is true of all four notions discussed in this column, and likely to many others in mathematics education. Our area of focus, mathematics education, continues to evolve, and we need to attend to our professional thinking by engaging with the details and

engaging our colleagues to further our collective thinking.

A Thousand Words:

Figure 1 is provided to provoke thinking about the transition from high school to first-year university. It comes from Jungic and Lowvric (2019) in the *First-Year Math and Stats in Canada (FYMSIC) Newsletter* (to subscribe, contact newsletter@firstyearmath.ca). The unlabelled 4.4% category at the top represents instances where categorization of courses was left blank.

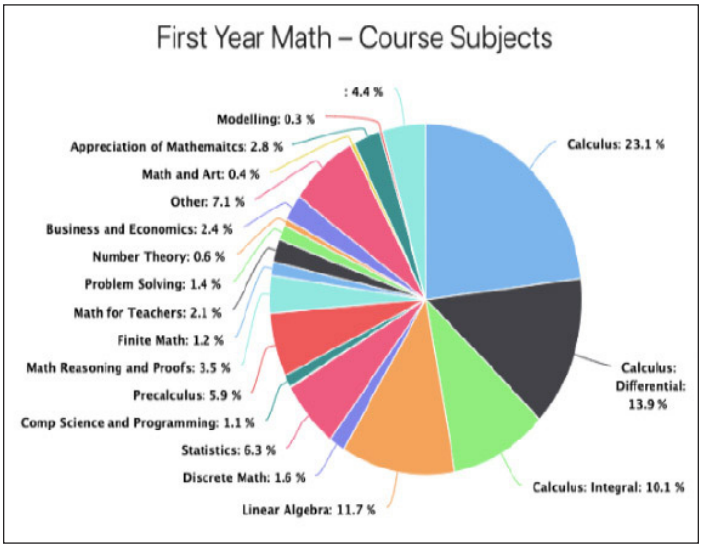


Figure 1: *From the First-Year Math and Stats in Canada Newsletter (Jungic & Lovric, 2019)*

In This Issue

This issue has had unique challenges, and I appreciate the professionalism of the editorial team. Anne Yeager continues to provide sage advice when new issues arise. Jacquie Foster has a keen eye and always finds a way to improve articles in a timely fashion. Gitta, our beloved proofreader, had a long-deserved holiday in Europe and rapidly returned to the fray when she got back. Penny, as always, pulls everything together and assembles what you know as the *Gazette*. Again, I owe thanks to Dan Jarvis, who oversaw the final steps of this issue so that my spouse, Susan, and I could go on an adventure. It is wonderful to see the team at work and to gain insight into the details of mathematics education communication. I would also like to thank the Communications Committee, part of the Board of Directors, for overseeing and providing guidance regarding the *Gazette*.

Paul Alves, OAME/AOEM President, has written an interesting column. He would have you believe he may not have figured some things out, yet he demonstrates awareness and clarity regarding the questions and mysteries he contemplates. *Angelica Mendaglio* speaks of a MathEd Forum focused on two books. *Jacqueline Hill* brings the latest from the NCTM and likes to mix in some shout-outs around the province.

Shawn Godin finds the centre without going around in circles. Assessment Abby returns with thoughts on validity and reliability in assessment. *Carly Ziniuk*'s column is slanted toward a world of leaning buildings and inclined towers (dare I say she is not *tilting at windmills*). *Mary Bourassa* keeps things in balance with equation-solving mobiles and other puzzles. *David Costello* provides the second part of his column with literacy strategies that will improve communication in the mathematics classroom. *Ann Arden* and *Jennifer Holm* will return in the next edition of the *Gazette*.

Reference

Beckman, A. (2008). *Mathematical literacy–Through scientific themes and methods. Proceedings of the 2nd international symposium on mathematics and its connections to the arts and sciences (MACAS2)*. Odense, Denmark: Centre for Science and Mathematics Education, University of Southern Denmark.

Jungic, V., & Lovric, M. (2019). Mathematics courses enrolment numbers. *First-Year Math and Stats in Canada (FYMSiC) Newsletter, 1*(2), 2–7. ▲



President's Message



PAUL ALVES
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Paul Alves, current OAME/AOEM President, is department head of math at Fletcher's Meadow Secondary School in the Peel District School Board.

Greetings from OAME/AOEM wherever you may be reading this December *President's Message*. Although you may be reading this message "Decemberish," I am typing away on a lovely autumn day to meet the strict submission deadlines of our respected and much-appreciated editor. These columns are valuable opportunities for your humble president to reflect on what has happened in the early part of the year and consider the challenges ahead—not only personally, but professionally.

I find myself writing my second December message as I serve my second term as OAME/AOEM President. One would think that with experience comes some sort of perspective and learning gained through that experience. I can distinctly see what is portrayed as the finish line in my teaching career, and it is my hope that by this point in my teaching career, the years spent in the classroom and in various roles supporting classroom teachers, that I would have figured some things out. But I find that I have more questions now than answers, and little time left to resolve those mysteries. This situation brings to mind a quotation from John Dewey that I archived when reading Tara Westover's 2018 book *Educated*. At the start of one chapter, she quotes Dewey: "I believe finally, that education must be conceived as a continuing reconstruction of experience; that the process and the goal of education are one and the same thing." Whether you are a student or a teacher, this quotation resonates and is a reminder that learning is a continual journey, and that when we assess our own learning and that of our students, we are just perceiving where all of us are at that moment in the journey.

Not only is this true of the individual, but one can argue that communities such as OAME/AOEM are also continually reconstructing their experience in light of who they are and what they have learned. At the October Annual General Meeting, the new mission, vision, and strategic priorities were passed by the majority of members present. If you haven't had a chance to read through the new guiding principles, please take a moment and go to the website and select the Guiding Principles from the About OAME tab on the left side of the page. The new Guiding Principles are the culmination of a long process of consultation and work undertaken by the Board of Directors and will guide OAME/AOEM further along its "journey of experience" for the near future until it needs to "reconstruct its experience" again.

Reflecting back on what has been happening around OAME/AOEM in the first half of the school year, we have had some superb learning opportunities locally, provincially, and digitally. Locally, I attended mini-conferences hosted by the Grand Valley Mathematics Association (GVMA chapter) and Toronto Educators Association for MathematicS (TEAMS chapter). These were marvellous events that allowed teachers to participate in high-quality professional learning close to home. Also, the Scarborough Association for Mathematics Education (SAME chapter) has been hosting a book study for OAME/AOEM 2019 that featured speaker and *Gazette* columnist, David Costello's *Using What Works: Strategies for Developing a Literacy-Rich Environment in Math*. I want to thank all of the volunteers and speakers who take time to make these local events valuable learning experiences for all educators. Are you

OAME/AOEM Mission Statement

The mission of OAME/ AOEM is to promote, support, and advocate for excellence in mathematics education throughout the Province of Ontario.

OAME/AOEM:
A Vision for Learning Mathematics

OAME/AOEM envisions an equitable mathematics education community in which everyone experiences high-quality, research-informed, and engaging activities that promote critical thinking, while fostering a positive attitude toward mathematics.

OAME/AOEM Strategic Priorities

OAME/AOEM is committed to the following Strategic Priorities:

- Promoting and advocating for equity in mathematics education
- Supporting all OAME members by providing value for membership, including quality professional learning, resources, and opportunities to network
- Advocating for high-quality, research-informed practices, while building a community to support mathematics education.

interested in checking out what may be happening in your area? Go to the OAME/AOEM website and select Chapter Conferences from the Conferences/Events tab on the left side of the page.

I want to thank the speakers from the OAME/AOEM 2019 Leadership Conference. They were Marian Small, Doug Duff, Hiren Mistry, Nat Banting, Em Del Sordo, Brian Aspinall, and Natasha Henry. My goal with the event was to engage leaders across the province in continuing significant work that has been started in many regions around instruction, assessment, equity, and innovation. Each of these speakers provided a passionate call to continue the valuable work that has been going on in many regions, despite some of the challenges we have been facing this year.

If you have been unable to make it to a local event or the Leadership Conference, there are still opportunities for you to continue your own process of education. There are three webinars coming in the second half of this school year in the OAME Talks Webinar series. Go to https://talks.oame.on.ca/ for a list of the remaining webinars and registration details. The first four webinars, which included our first French-language webinar, and the remaining three, provide a chance to hear some of the speakers from OAME/AOEM 2019. Lastly, the big date on many math educator calendars is OAME/AOEM 2020 in Oshawa. I am looking forward to attending the keynote and featured speaker sessions, as well as the usual diverse selection of program sessions. Registration details will most likely be available early in 2020. Don't hesitate to register, as sessions fill up quickly.

In conclusion, I will reflect on a piece by NCTM President Robert Q. Berry III. In his August 2019 blog post entitled "Starting the School Year by Unpacking Your *Why*," Dr. Berry prompts us to reflect on this question: *Why do you teach?* I recommend reading Dr. Berry's blog post and watching the video from comedian Michael Jr. linked in the post to emphasize the power of knowing your why. To quote Dr. Berry, "Until you are clear on your *why*, it may be difficult to think intentionally about your *what*." Although we are well past the start of the school year, I would encourage you to answer this question as we approach the winter break, and take some time to reflect on what has passed and re-energize ourselves for what is to come. Personally, my why involves the energy that I feel when students are engaged in doing meaningful mathematics and working with committed and passionate educators who are committed to promoting, supporting, and advocating for excellence in mathematics education throughout the province. On behalf of the OAME/AOEM Board of Directors and Executive Committee, I want to wish you a restful and joyful winter break and all the best for $100\sqrt{404-2^2}+\sqrt{404-2^2}$. ▲

▲ Technology Corner: Mathematical Puzzles



MARY BOURASSA
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Mary Bourassa teaches mathematics at West Carleton Secondary School in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She has presented workshops internationally, authored mathematics resources, is a past VP of OAME, and a Past President of COMA. An award-winning teacher, Mary continually strives to learn new and better ways of helping students learn and love mathematics.

Puzzles are a delight to mathematicians, young and old. Mathematical puzzles can help students develop problem-solving skills, logic, and perseverance. Though many puzzles are recreational, others can be tied more directly to the mathematics curriculum, as they help students build up their skills around a particular topic. One of many sites dedicated to puzzles was inspired by the research and development of both *Think Math!* and *Transition to Algebra*. These puzzles were developed at Education Development Center, Inc. and can be found at https://solveme.edc.org/.

SolveMe puzzles feature three types of online puzzles: *Mobiles, Who Am I?*, and *MysteryGrid*, as shown in Figure 1. Within each puzzle type, you have the option to play or build your own. The puzzles get more complicated as you progress through levels Explorer, Puzzler, and Master.

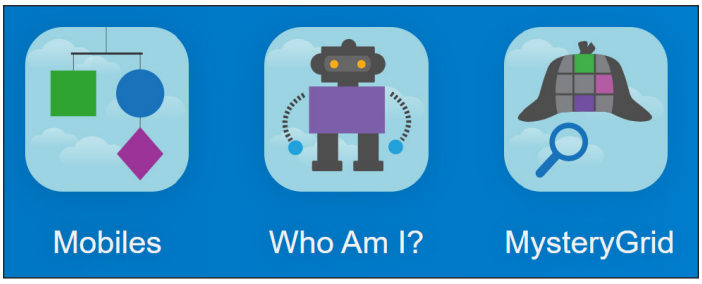


Figure 1

Mobiles present you with a mobile puzzle that you must keep balanced by discovering the weight of each shape. The goal is to help develop and support students' algebraic reasoning. The first few puzzles are quite simple, getting students accustomed to the idea of balance. In Puzzles #1 and #3 shown in Figure 2, the total weight is shown. In Puzzle #2, students are given the weight of one shape and must deduce the other.

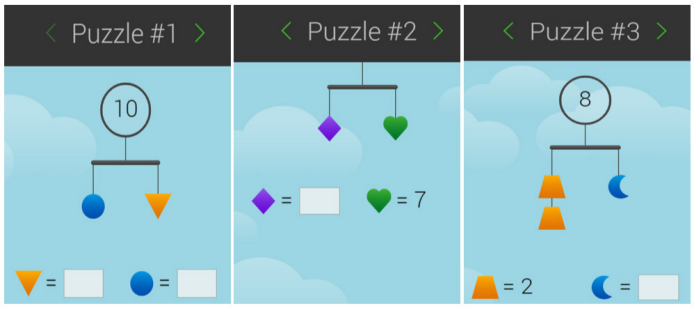


Figure 2

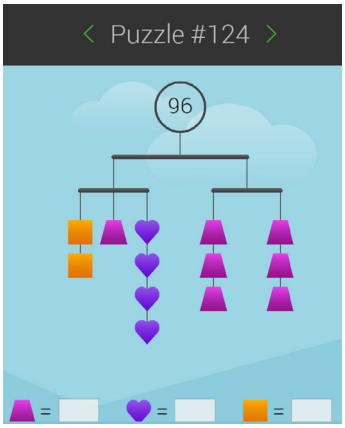


Figure 3

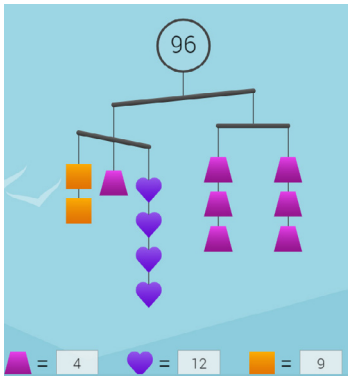


Figure 4

The level of difficulty increases as you work through the puzzles. Puzzle #124 is shown in Figure 3. Students must determine the weight of one square, one trapezoid, and one heart so that the sum of the weights of all the shapes shown in the mobile is 96.

Figure 4 shows what happens when you enter an incorrect answer: the mobile is no longer balanced.

There are writing tools available where by students can break down what they know to help them solve the puzzle, as shown in Figure 5. These are also helpful to the teacher when he or she goes over the solution with students. These tools are available from the toolbar.

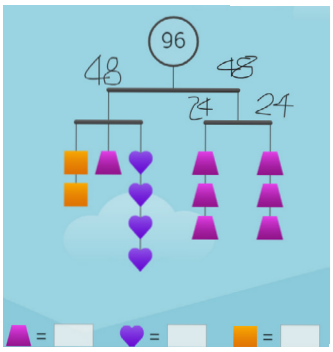


Figure 5

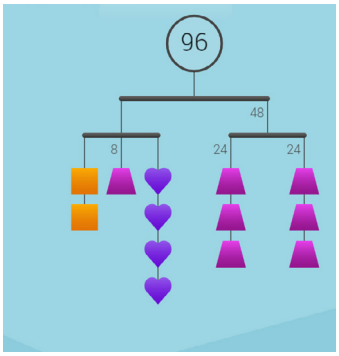


Figure 6

If you have the "Show Numbers in Mobile" setting turned on and enter one or more weights in the answer boxes, then the total weight for any string with only those shapes will be displayed. This is shown in Figure 6.

If you grab a beam and drag it off the mobile, the corresponding equation will be shown. Figure 7 displays two equations that could help students determine the weight of the remaining shapes.

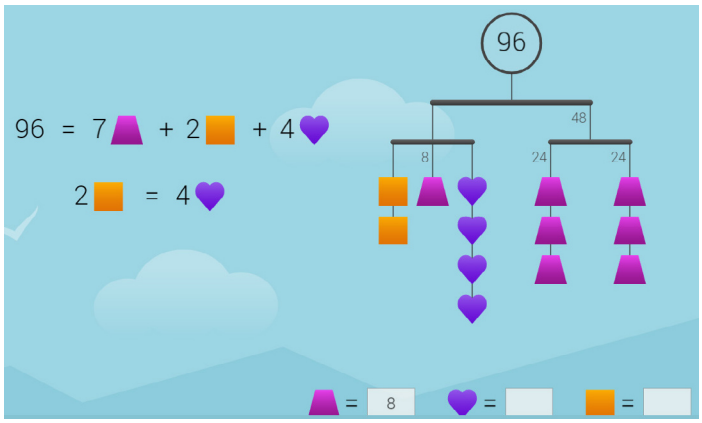


Figure 7

A number of mathematical operations are now available to you. You can subtract one shape from each side of the equation by dragging that shape down. You can divide by a constant by clicking on the coefficient so that its prime factors are shown and then drag off the common factor. This is shown in Figure 8.

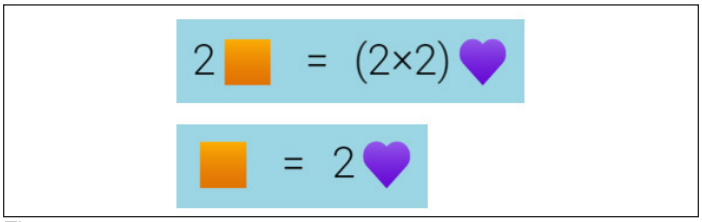


Figure 8

You can also substitute. In our case, we know that the square is worth two hearts, so we can drag that equation to each square on the mobile, and each square will be substituted with two hearts. The result is shown in Figure 9.

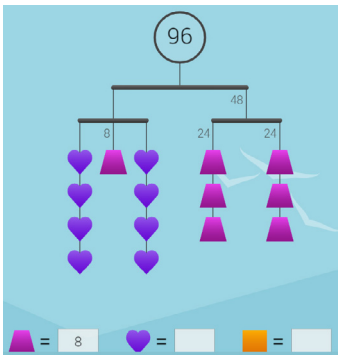


Figure 9

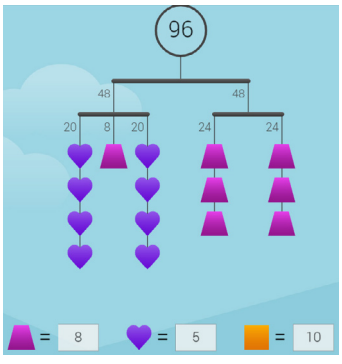


Figure 10

This should all lead to the correct solution, and the mobile staying balanced, as shown in Figure 10.

The next type of puzzles is called “Who Am I?” In these puzzles, a number robot presents you with clues to its number. These range from two-digit to five-digit numbers, including decimals. Each digit has a letter associated with it based on its place value. Figure 11 shows the explanation from the site.

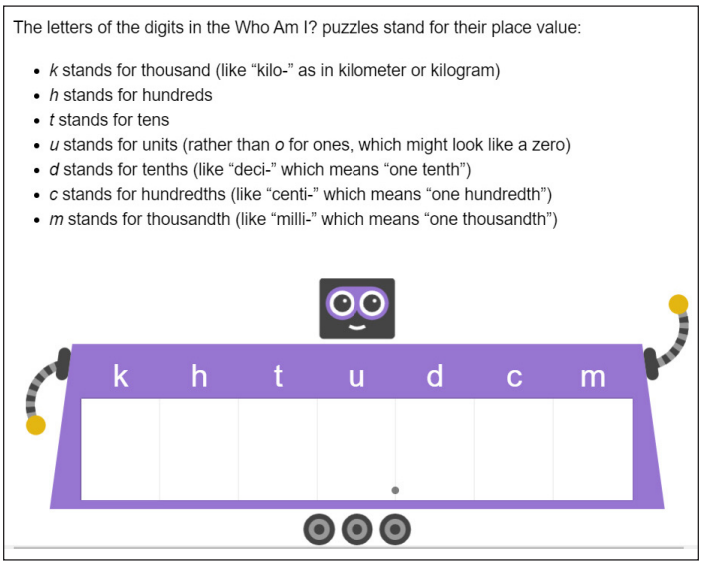


Figure 11

As with the mobiles, these puzzles start out simple and become more complex. Figure 12 shows two puzzles at the beginning of the Explorer series.

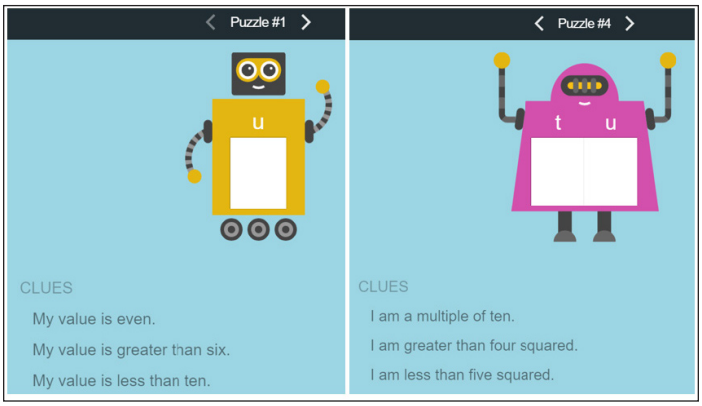


Figure 12

At the other end of the spectrum, Figure 13 shows Puzzle #200. Students will need to have good number sense, understand place value, and have good problem-solving skills to correctly solve this puzzle.

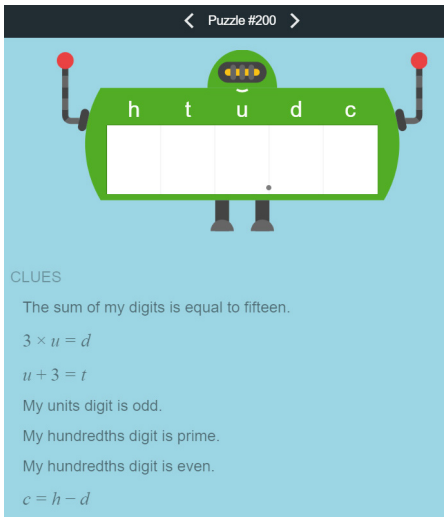


Figure 13

Another option in the “Who Am I” puzzles is Bingo, where students are presented with one clue at a time and must choose the number that matches that clue from an array. This could be 3 x 3, 4 x 4, or 5 x 5, and you can choose which digits to include from three options, as shown in Figure 14.

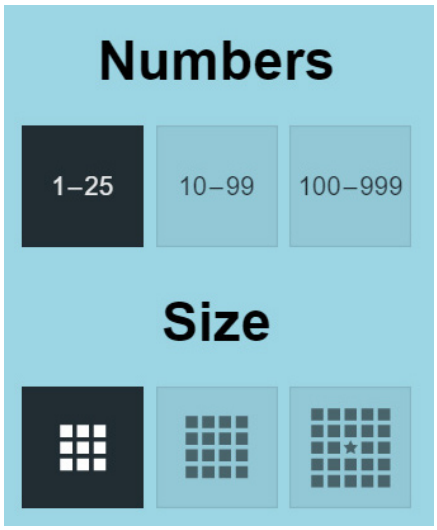


Figure 14

The goal is to get five in a row, horizontally, vertically, or diagonally. The clues appear one at a time, and the player should choose all the numbers that fit the clue. Errors will be marked with a red line through the number, and three errors on the same clue will force the player to move on to the next clue. Figure 15 shows the initial board and the first five clues. At the end of the game, your score is shown (number of clues), along with the best score possible.,

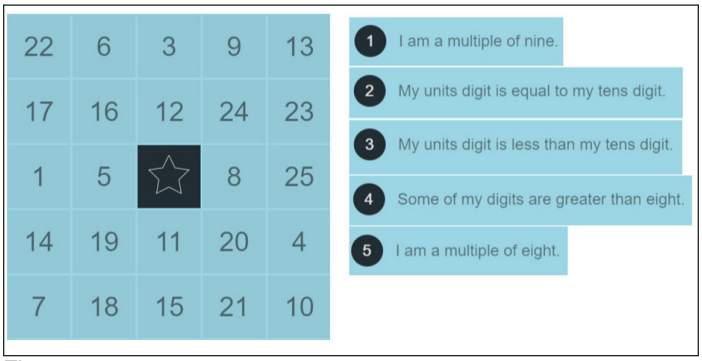


Figure 15

The third type of puzzle is called Mystery Grid. The goal is to fill the grid so that each row and each column contains

exactly one of each type of tile and fits the constraints. Figure 16 shows the first puzzle, which provides a lot of information, along with the solution.

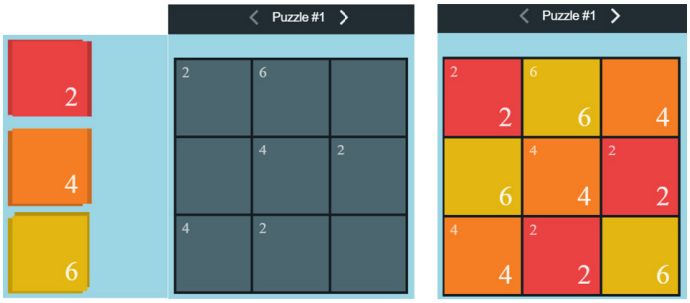


Figure 16

As with each of the other types of puzzles, these become more challenging as you proceed. The inequality symbol is introduced quite early. Figure 17 shows one such puzzle, where the player must place the digits 1, 2, and 4. The top row must therefore be 4, then 2, then 1, to make the inequalities true.

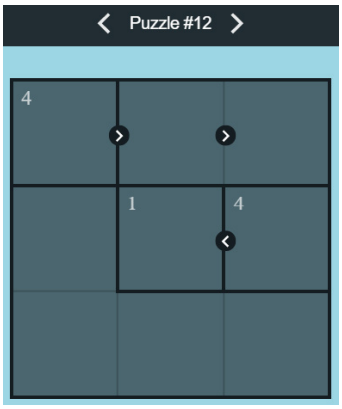


Figure 17

Cages (multiple cells with a dark border around the group) are also introduced, along with targets and operations. The group of numbers within the cage must equal the target number, using the operation shown. For example, Figure 18 shows a cage on the top row, where two numbers must have a sum of 13, with the larger number on the right. The full solution to Puzzle #25 is also shown.

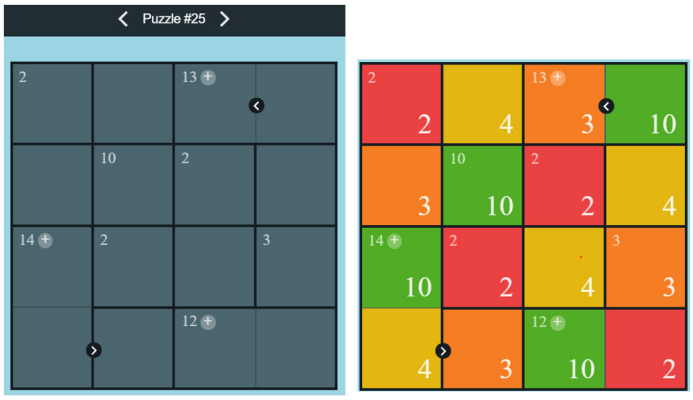


Figure 18

These puzzles include integers, fractions, and variables. The operators in the cages can be addition, subtraction, and multiplication. Though these puzzles begin with simple numbers that need to be placed on the grid, they become very challenging algebra puzzles, allowing students to practise working with like terms and multiplying polynomials. Figure 19 shows one of the more challenging Mystery Grid puzzles.

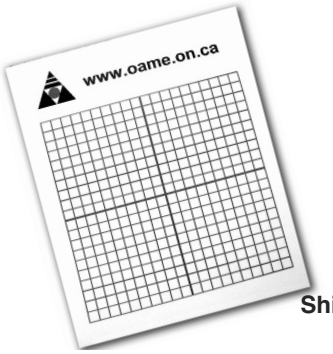


Figure 19

The puzzles at SolveMe.edc.org can be used in the classroom at many levels and in many ways. Teachers and students can also create and share their own puzzles. I encourage you to check them out! ▲

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▲ Linking Literacy and Math: Writing Strategies to Support Mathematical Understanding (Part Two)



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Dr. David Costello is an author who has recently published Using What Works: Strategies for Developing a Literacy-Rich Environment in Math. *David is a vice-principal, who has held many roles. He taught in the primary division before assuming roles of numeracy interventionist, numeracy coach, numeracy leader, and curriculum consultant for Prince Edward Island. David has also instructed university courses focused on curriculum, differentiation, mathematics, and language arts. He facilitates professional learning sessions in the areas of mathematics and school development. His research situates mathematics within a literacy perspective and examines leveraging literacy-based instruction in mathematics.*

This column is an extension of my column published in the previous issue of the *Gazette*. In this column, I continue to emphasize the importance of writing in mathematics as a way of framing, supporting, and consolidating mathematical understanding. This column is about supporting teachers in leveraging successful instructional practices from language arts to mathematics.

When considering writing, it is important to make a distinction between writing to demonstrate knowledge and writing to support learning. One type of writing common in math classes is writing to demonstrate knowledge (Scardamalia & Bereiter, 1987). Students write for a specific audience with a specific purpose. This writing is focused on students showing what they now understand after engaging with a concept or idea. Within this writing, students will provide evidence of their understanding. A second kind of writing, possibly less common in math classes, is writing to support learning. This kind of writing involves writing short, informal pieces that allow students to clarify stumbling blocks. It allows students to frame and support their thinking as they work through a problem (Knipper & Duggan, 2006; Scardamalia & Bereiter, 1987). It is meant to provide students with an exploration into both their thinking and the details of concepts and ideas being explored.

The following are three writing strategies—*graphic organizer, journaling,* and *post it*—that can be applied to mathematics. Each of these strategies can be applied when writing to demonstrate knowledge or when writing to support learning.

Graphic Organizer

Graphic organizers support students throughout the problem-solving process. Organizers assist students in planning their approach and carrying it out to completion by allowing students to arrange their thinking within a supported structure. Graphic organizers can have many forms and functions, which allows many different designs to support students. Examples of supports are working through a problem, sequencing their thoughts, and/or noting similarities and differences.

One strength of a graphic organizer is the visual support it provides for students. Students can refer to checkpoints within the graphic organizer while working through a problem. It can help to categorize key concepts while removing distractions and/or irrelevant information. However, it is important that students not rely too heavily on graphic organizers as a form of support for all problem-solving experiences. While a graphic organizer can assist students, it is not desired that students become dependent on always using one form of support when working through a problem. It is important for students to experience a variety of supports, as one organizer will not necessarily assist in solving all problems. Flexibility is key when problem-solving.

The following are examples of graphic organizers supporting primary-aged students through various problem-solving experiences.

Frayer Model

An example of a Frayer model is shown in Figure 1. In this example, the Frayer Model was used to assist in framing a child's understanding of polygons.

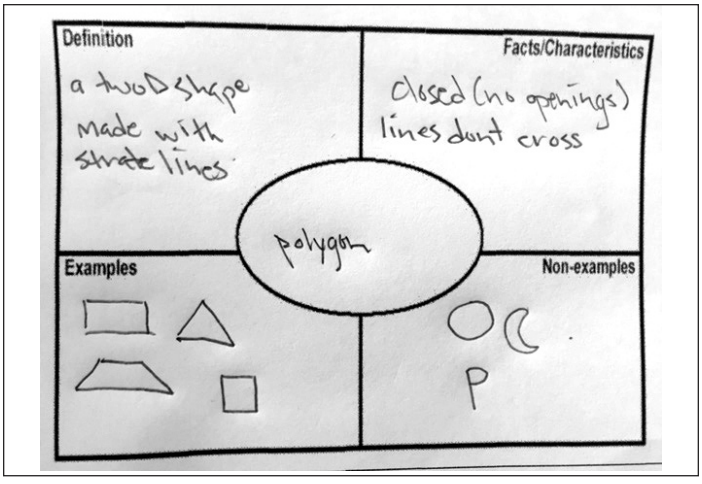


Figure 1: *Example of Frayer model*

Venn Diagram

An example of a Venn diagram is shown in Figure 2. In this example, the Venn Diagram is used to assist in supporting and consolidating a child's understanding of numbers. The circle on the left contains odd numbers, and the circle on the right contains numbers more than 100. The values 233 and 111 are placed so that they are in both circles because they exhibit both properties. The value 100 is outside of both circles, since it has neither property.

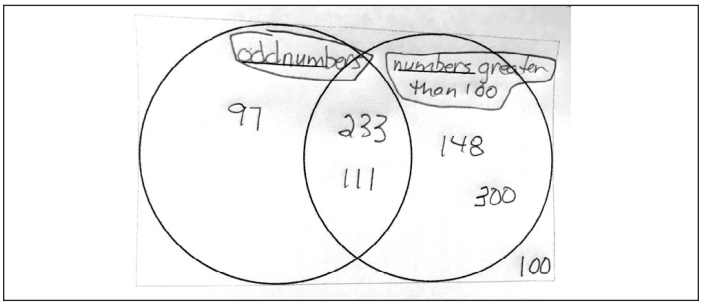


Figure 2: *Example of a Venn diagram*

Flowchart

Figure 3 shows an example of a flowchart. In this example, the flowchart is being used to show a child's sequencing of tasks when problem-solving.

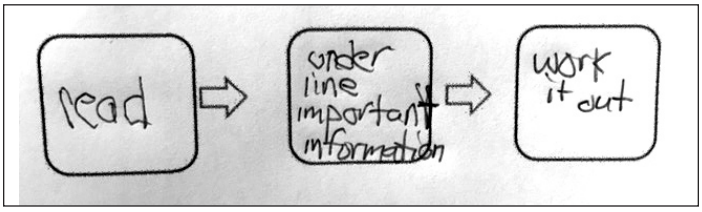


Figure 3: *Example of a flowchart*

Placemat

In Figure 4, the Placemat is used to support and consolidate a child's understanding of addition.

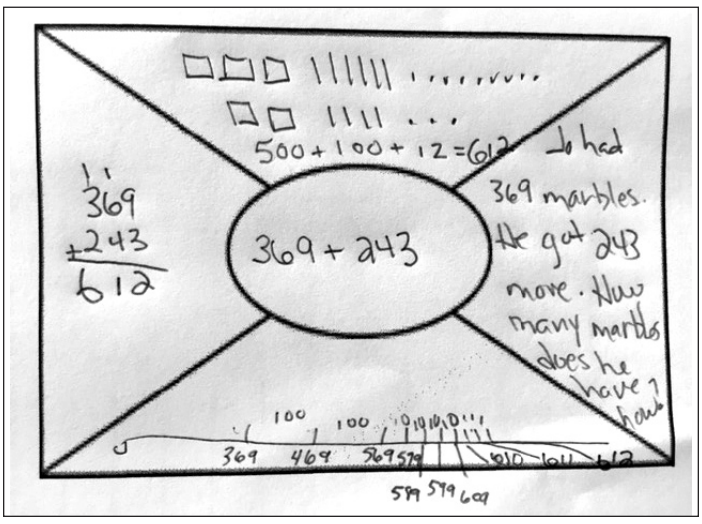


Figure 4: *Example of a Placemat*

In the four examples above, the teacher is able to assess the student's ability to enter, work through, and complete a problem. It is through the graphic organizer that the teacher can determine how the student undertakes a task and completes it. Also, it provides opportunities to identify next steps (in regards to instruction and learning).

Journaling

Journaling serves as a record of student's thinking. It is a place where students can work through problems and record their thinking in the process. All too often, journals are used at the conclusion of the problem-solving process. What I am proposing, in this column, is that journaling be used throughout the entire problem-solving process. As students frame the problem, they can brainstorm a list of concepts and/or strategies applicable to the problem at hand. The journal can record their thinking as they try the brainstormed ideas. When students consolidate their thinking, the journal can record thoughts and points of clarification that may be helpful for them to reflect upon at a later date in time.

Journals are the "canvas" as students work through a problem. It is a place where students can record their thinking, questions, and points of clarification. It is a source of support for the problem at hand. Journals are personal to the student, as they reflect student thinking throughout a variety of problems and/or prompts.

Students can keep a journal in different ways. Traditionally, students have a notebook, where they write journal entries. Students regularly add their thoughts to this journal, which can be used as a tool while problem-solving. As an alternative, students could keep a digital journal, using a digital classroom application. Students could add to their digital journal during allocated times and keep all of the documents in one place. With a digital journal, teachers can check in with students' writing on a regular basis because all of the work is easily accessible.

Figure 5 is an example of journaling from an elementary-aged student.

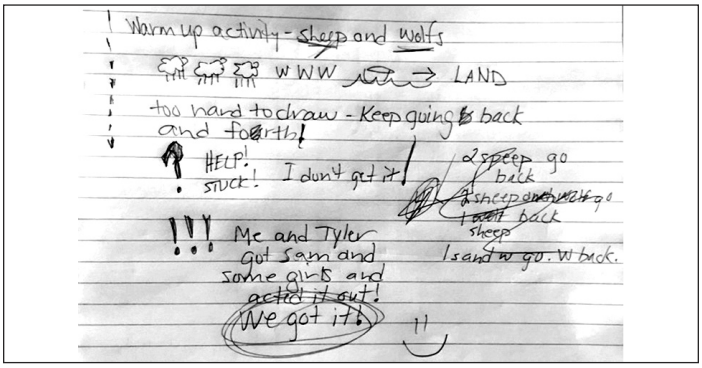


Figure 5: *Example of journaling*

In this task, the student's journaling shows his or her thinking, and having that documented allows the student to return to a certain point in the problem if he or she needs to. Within this particular journal entry, it is apparent how the student organized his or her thinking about the approach to, and work through, the problem. The student now has a record of the work and may be able to refer to the journal entry when he or she encounters similar problems in the future.

Post-It

An important process in mathematics is the ability to examine concepts and/or strategies, and then communicate your understanding. In the post-it strategy, students examine other students' work samples and record questions along with similarities and differences to their own work. In addition to these connections, students also record points offering and seeking clarification on sticky notes. It is a process of articulating what they see and don't see in the work of others.

The purpose of the post-it strategy is to provide students with the opportunity to examine the work of others and then to articulate their thinking in a succinct, targeted manner. By having students record their thoughts on a sticky note, it constrains the available space forcing them to be direct and purposeful. Students will need to first think about their response and then refine it to fit the parameters of the task.

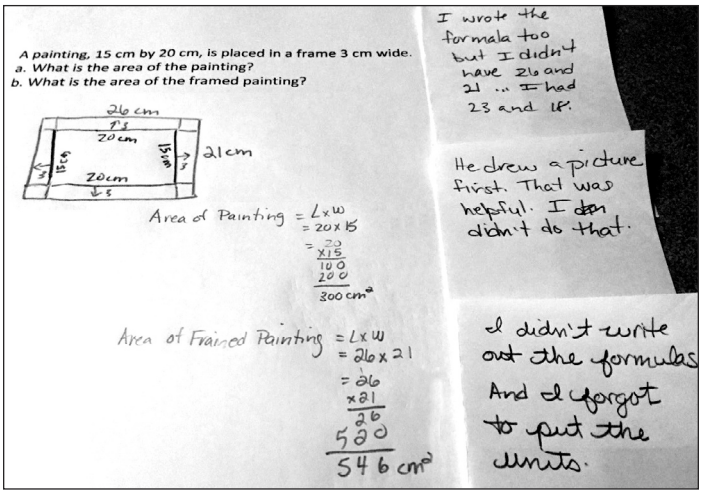


Figure 6: *Example of the post-it strategy*

Figure 6 shows an example of the post-it strategy applied to the work of an elementary-aged student. In this example, students posted notes to the work sample identifying similarities and differences. Such an activity is a critical and reflective endeavour, where students must assess their work relative to that of their peers. From a teacher's perspective, the post-it strategy provides the opportunity to see how students analyze work samples from the perspective of their own work. It is a beneficial self- and peer-assessment practice.

Final Thoughts

The three writing strategies provided in this column—graphic organizers, journaling, and post-it—can support mathematical understanding as students work through a problem. Exploration of the writing strategies encourages students to frame, support, and consolidate their understanding. It also provides a platform to make their thinking visible to both themselves and the teacher. When we move beyond the idea of writing as merely part of a summary or concluding process, we move student learning forward.

What I am advocating is that writing strategies not only be considered for language arts, but also for mathematics. When viewed in this manner, educators and students can realize the importance of writing as a variety of ways to demonstrate knowledge and support learning. I look forward to continuing the discussion of linking literacy and math via Twitter (@dr_costello) or email.

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▲ Fields Institute MathEd Forum Report



ANGELICA MENDAGLIO
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Angelica Mendaglio is an instructional designer at Vretta Inc. in Toronto, Ontario, where she helps to create interactive digital mathematics lessons and activities for middle school students.

The Fields Institute MathEd Forum started off the academic year with a different kind of meeting in the form of a book club. The organizers of the Forum selected two math-centric books written for a general audience: *Weapons of Math Destruction: How Big Data Increases Inequality and Threatens Democracy* by Cathy O'Neil, and *King of Infinite Space: Donald Coxeter, the Man Who Saved Geometry* by Siobhan Roberts. The goal of the meeting was to discuss how math books written for a general audience can inform our teaching of math. These books were chosen as a jumping-off point for the day's activities.

The selected books are quite different from each other—*Weapons of Math Destruction* was written by a former data scientist in the financial sector. It details several examples of algorithms that make use of Big Data, with dangerous outcomes due to a lack of understanding or even malicious intent. *King of Infinite Space*, on the other hand, is a biography of Donald Coxeter, one of the greatest mathematicians of the twentieth century. Roberts uses Coxeter's life and work in mathematics as a lens through which to explore topics in geometry, the history of math, and the importance of a visual and intuitive perspective in mathematics.

The Forum meeting started off with small-group discussions. Attendees split into groups and were each given an excerpt from one of the books, as well as questions to spark discussion around our main topic. Following the small-group discussions, each group shared their thoughts with the rest of the Forum. In the afternoon, a panel discussion was held with panelists from public school, post-secondary, and the private sector, including Siobhan Roberts, the author of *King of Infinite Space*.

One of the questions that was posed to the panel was how Big Data and algorithms could revolutionize mathematics education, and how likely it was to happen. Patrick Robinson, one of the panelists and himself a data scientist, suggested that it could change what kinds of math we are teaching. During the morning discussions, several groups brought up the importance of statistics and critical thinking in the work of a data scientist. The ability to think critically, select appropriate tools, and interpret results is crucial when working with Big Data, which will likely be even more pervasive when our students enter the workforce. Robinson gave the example of taking the class average. This is done frequently to represent the grades of all the students in the class. However, the same class average could be representing a variety of different situations. If the class average is 60%, you might assume that the marks are roughly normally distributed around that mean. However, it would be a very different story if the class average was 60% because every single student received a mark of exactly 60%, or if half of the students received 30% and the other half received 90%. The mean may not tell you the whole picture, and it will not always be the best way to represent a data set. This is where critical thinking becomes important. In the world of Big Data, you need to be able to assess a tool's usefulness as a representation, and interpret the results that it gives you. *Weapons of Math Destruction* was an excellent window into the work of data science, and the mathematics required.

The question of "What mathematics do mathematicians actually do?" was raised at a few points during the day's proceedings. *King of Infinite Space* also offers insight into this question. Much of the book is focused on the tension between geometry and algebra, between visual and symbolic representations in the math community throughout the twentieth century. At this time, geometry was unfashionable as a mathematical field—it was believed to have already been mined for all of its useful knowledge, and was treated as a historical curio, rather than an active field of study. There was also a more active movement, headed by a group of French mathematicians known as Bourbaki, to remove visualizations from mathematics entirely. They held the belief that visual representations can be misleading, and the rigour of algebra was the only way to gain true mathematical knowledge. However, Coxeter continued to use his own heavily visual methods to pursue his studies in geometry, and eventually showed mathematicians that both geometry and visualizations could still contribute to mathematics. In our schools, geometry and the visual often take a back seat to algebra and symbolic representations. This can leave students with the wrong impression of what it means to be "doing mathematics," which can drive them away from the field. Rachel Allen, one of the panelists and an elementary school teacher with the Toronto Catholic District School Board, observed that manipulatives and visualizations are used more in primary grades, but the

attitude is often that students can use them "if they need to." This sends the message to students that visuals are a crutch, and that when using them, they aren't involved in the work of "real mathematicians." *King of Infinite Space* demonstrates clearly that this is not the case.

During the panel, Walter Whitely from York University said that there is a need to be honest with students about what mathematicians do. One Forum attendee described a mathematician's work as looking for beautiful patterns, but what a mathematician actually does is not common knowledge. One potential cure for this could just be these math-centred books created for a general audience. They offer insight into the diverse world of mathematics in an accessible way. Siobahn Roberts described storytelling as a fundamental way of conveying information. Other panelists echoed this sentiment—narratives and math are not often put together, but when they are, the result can reveal the human nature of mathematics. It can prevent the subject from seeming unconquerable, like a set of rules and decrees that cannot be understood. Learning about the history of math can reveal that math is, as Whitely put it, the result of a set of human choices. Students would be surprised to hear that math has evolved over time, that mathematicians have always fought and continue to fight over aspects of their field. Books like *Weapons of Math Destruction* and *King of Infinite Space* could help our students see the human side of mathematics, and maybe even to appreciate the beauty in it that mathematicians see.

The Fields Institute MathEd Forum meets on the last Saturday from September to November, and January to April at the Fields Institute in Toronto. If you are not able to travel to Toronto to join in person, you can watch the presentations remotely through the Fields website. ▲



Greenwich meridian was a decision about where to put zero. Photo taken in Greenwich, UK.

NOMINATIONS FOR OAME/AOEM BOARD OF DIRECTORS 2020-2021

Elections are currently being held for a Vice President (three-year term) and two Directors (up to a three-year term), of the OAME/AOEM.

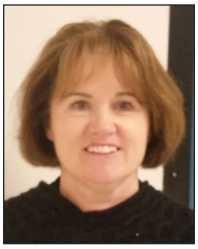
- Due to recent changes in the OAME Constitution, By-Laws & Terms of Reference, the term of office for the OAME President has been changed to two years BEGINNING on September 1, 2020.
- OAME/AOEM members running for Director will declare their intent to work on either Grades JK–6 or Grades 7–12, based on their personal experience, expertise, and interests.

Nominations for these positions are to be submitted electronically by completing an online Nomination Form by 5:00 p.m. on January 31, 2020.

A nominee must be a member of the OAME/AOEM. By March 2020, the Nomination Committee will publish a slate of candidates, including candidate bios, for consideration by the membership. Voting is open to all OAME/AOEM members. The election results will be released at the Annual Conference of OAME/AOEM in May 2020.

Please contact Connie Quadrini (connie.quadrini@oame.on.ca) for a link to the Nomination Form or for more information.
Thank you for submitting your nomination!

▲ OAME/NCTM Report: Helping You Reach Your Mission



JACQUELINE HILL
jacqueline.hill@bell.net

Jacqueline Hill is a Grades 9–12 teacher of mathematics, as well as an online instructor for York University. She is a Past President of OAME and OMCA, as well as the recipient of the award for Exceptional Teaching in Secondary Mathematics. She also wrote the "Director's Dialogue" for the Gazette *for a number of years.*

Happy 2019, Math Gang!

As a math teacher and mathematician, it is important to us here in Ontario to ensure that we reach every child in our classroom (Kindergarten through university). It is our mission to make sure that regardless of where students start in our classroom, we help each and every one of them move forward in their learning. Being an NCTM member helps you reach your mission.

Membership helps you stay on top of the latest math education practices. It gives you access to an online community, set up to help you share your ideas, pose questions, discuss challenges you face in mathematics and the teaching of mathematics, read and share blogs, enjoy webinars and recent recordings. As a member of NCTM you are able to pick the print journals that will help you the most in your classroom. The Essential membership tier includes one subscription to a school journal, with print and digital access, while Premium members enjoy print and digital access to all three school journals as well as NCTM's research journal, the *Journal for Research in Mathematics Education*.

Premium members also receive one FREE e-book per year after renewal. Enjoy a member-only discount at the NCTM Store (20% for Essential members and 30% for Premium members), and stock up on books, e-books, and other professional publications and products.

I truly enjoy the *Mathematics Teacher* oriented to teachers of Grades 8–12 and beyond. It has articles on research, pedagogy, and exciting student-relevant articles.

This column is a shout-out to the following communities in the province of Ontario: Red Lake, Fort Albany, Timmins, and Lindsay. With this shout-out comes a challenge! We are dealing with students who are digitally literate. In stating that, I would just like to add that perhaps we need to help them navigate their journey. I challenge you to give your students the following challenge.

Find two stars (state two things that are positive) and a wish (one thing you wish you would have observed) from the OAME website and the NCTM website. My students found this to be a rewarding experience, and were thankful for being directed to the two websites for mathematics organizations.

NCTM important dates to note:

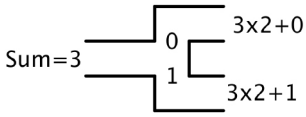
- Centennial Annual Meeting in April 2020.
  March 13, 2020: One-day conference in Los Angeles—Supporting Students' Productive Struggle (Grades K–5)

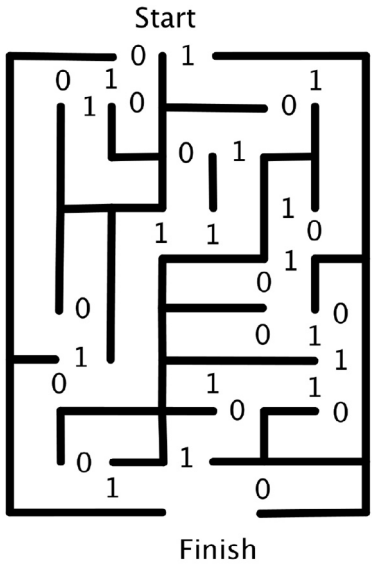
Yours in Mathematical fun,

Jacqueline ▲

Binary Maze

As you do this maze, do the following calculation to determine a hidden value. Start with a sum of zero. Each time you reach a value multiply the sum by two and add on the value for the direction you choose to go. For example, if your sum is 3 when you get to junction, if you choose to go in the zero direction your new sum is 2 x 3 + 0, but if you choose to go in the one direction the sum becomes 2 x 3 + 1. Continue the calculation, with the new sum until you get to the end. This maze has four solutions leading to four different sums.





Secret Values = ___ , ___ , ___ , ___

▲ In the Middle: Lean into Learning about Inclined Towers



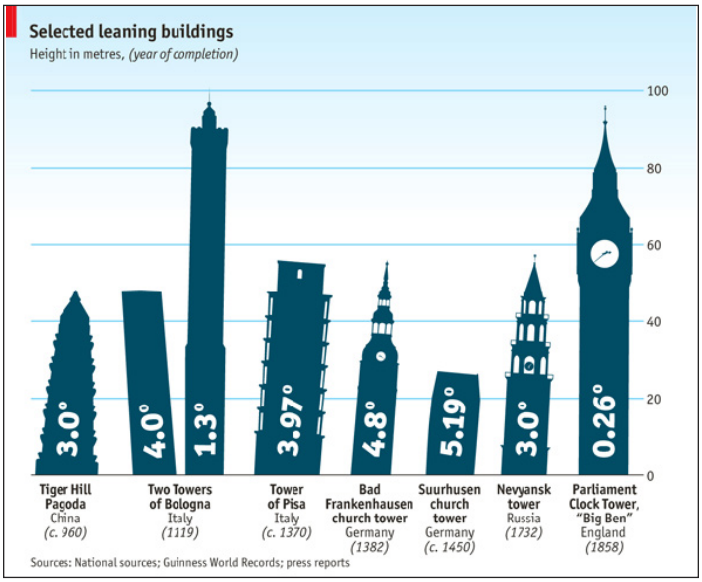
CARLY ZINIUK
carlyziniuk@gmail.com

Carly Ziniuk teaches Grade 9 Mathematics, Grade 12 Data Management, and Advanced Placement Statistics at the Bishop Strachan School in Toronto, Ontario, Canada. She is very active in adopting real-life data to engage her students in solving problems.

On a recent trip to Montreal, my teenage sons and I visited Parc Olympique. The park features the Montreal Tower, which is a popular attraction. At 165 m, it is the tallest inclined tower in the world, leaning on a 45-degree angle.



The advertising for the Tower repeatedly reminds visitors that its lean is more than 40 degrees greater than the famous Leaning Tower in Pisa (see www.parcolympique.qc.ca/en/what-to-do/the-montreal-tower/). I remember seeing an infographic of different leaning towers in *The Economist*, October 24, 2011, responding to a 2011 UK Parliamentary report identifying Big Ben's lean as 0.26 degrees, or 43.5 cm.



(From www.economist.com/graphic-detail/2011/10/24/ tottering-towers)

The infographic does not include the Montreal Tower. Notice the significant difference between the leaning angle of Big Ben and the other towers in the infographic as compared to the Montreal Tower. Looking further at the details presented in the infographic, the buildings presented seem much older and much shorter than the Montreal Tower. These details piqued my curiosity and a multi-strand activity began to percolate. Is the Montreal Tower unique? I was keen to share this trigonometrically intriguing rabbit hole with my colleagues and students. I set about looking for other leaning and inclined buildings worldwide. Table 1 includes data for 14 buildings in addition to the 8 presented in *The Economist*.

The unusual lean of buildings draws students in. Furthermore, I have been exploring opportunities for student choice and differentiation within similar problem-solving contexts. The buildings allow students to examine different structures, while collaborating through sharing their thinking, rather than answers. Since there are 14 new buildings, individual students or pairs of students can explore different towers as starting points for the same questions.

Name of the Building/Tower	Location	Year of Opening	Height from Ground to Pinnacle (in m)	Angle (in degrees)
Buildings in *The Economist* Infographic				
Big Ben	UK	1858	96	0.26
Two Towers of Bologna, Asinelli	Italy	1119	97.2	1.3
Nevyansk tower	Russia	1732	58	3
Tiger Hill Pagoda	China	c. 960	48	3
Tower of Pisa	Italy	c. 1370	57	3.97
Two Towers of Bologna, Garisenda	Italy	1119	48	4
Bad Frankenhausen church tower	Germany	1382	56	4.8
Suurhusen church tower	Germany	c. 1450	25.705	5.19
Buildings Not in *The Economist* Infographic				
Veer Towers, Las Vegas	USA	2010	150	5
Oude Kerk, Delft	Netherlands	1246	79	5.5
Huzhu Pagoda	China	1079	20	7.1
Caerphilly Castle, SE Tower	Wales	1271	15	10
Altair	Sri Lanka	2019	209	13.8
Two Towers, Puerta de Europa	Spain	1996	114	15
Leaning Tower of Gingin	Australia	2008	45	15
AC Hotel Bella Sky Copenhagen	Denmark	2011	76.5	15
Sharps Island Lighthouse	USA	1838	11	15
Montjuïc Communications Tower	Spain	1991	136	17
Capital Gate Tower	UAE	2011	165	18
Montreal Olympic Tower	Canada	1987	165	45
Museum of the Second World War	Poland	2008	40	56
Alamillo Bridge	Spain	1992	140	68

TABLE 1: *Leaning and Inclined Buildings*

One activity is to have the students extend *The Economist's* infographic by adding the additional buildings into the visual, using the following instructions and questions:

1. Choose a building from the chart below, note the details for this building, and find a picture of the building.

Printing the infographic on different sizes of paper (11"x17" or 8"x14" or 8"x11") offer different opportunities to work with scale. Using a projected image of the infographic onto a screen is another option here.

2. What is the vertical scale of the infographic you are working on? How tall will the scaled model of your building need to be?

Another option is to have a small group of students explore the same building. Each student in the group will use proportion and scale to create a model that is the appropriate size for the different printed infographics. Middle school students can utilize similar triangles with a constant angle and proportional heights. This prepares students for the use of similar triangles that are often used as an introduction to trigonometry and primary trigonometric ratios in high school.

3. Create a model of your building that would fit on your infographic. Remember to note its angle and its height. Create a label to place below your building in a style similar to those on the horizontal axis of the infographic.

If possible, it would be interesting to create a gallery by photocopying the students' models without disrupting their further work.

4. Look at the gallery infographic. What decisions are required to add your new buildings to extend the original infographic?

The horizontal axis of the infographic is ordered by year and does not use a precise scale. I have organized the table (with one minor exception) by angle lean, rather than by year of opening. Adding the new buildings to the infographic requires manipulating the placement of some of the given buildings. On paper, this can be done by cutting perpendicular to the horizontal axis between the buildings and adding in new building details between the existing buildings.

5. Using your model, how high is the building (from base to pinnacle)?

This also provides the opportunity to discuss the difference between perpendicular height and (diagonal) length (i.e., "slant height") of a building. The length is emphasized with the Montreal Tower, since it has a funicular going from the ground to the top.

6. What is the horizontal distance of the triangle for your model, that is, the horizontal projection created by its lean?

In a UK Parliamentary report, Big Ben's lean was described as both 0.26 degrees and 43.5 cm (www.parliament.uk/business/news/2011/october1/big-ben-tilt/). The pinnacle's horizontal deviation from perpendicular is called,

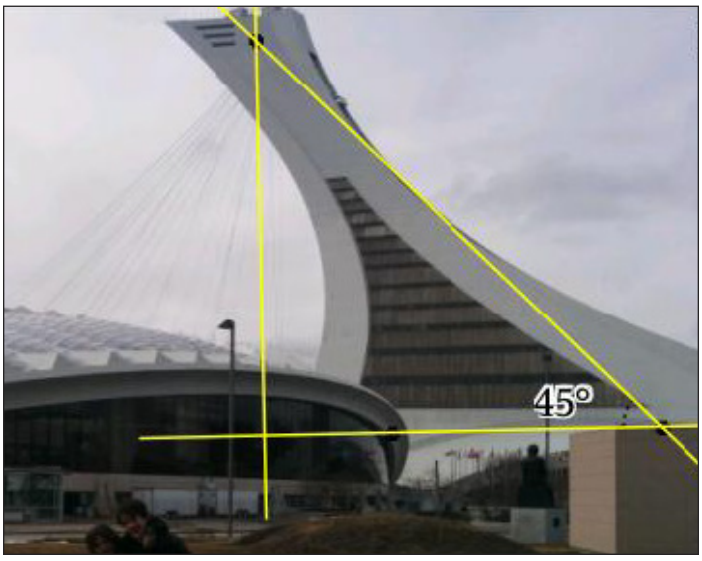
in physics, the horizontal projection. Big Ben's horizontal projection was 43.5 cm.

7. What is the horizontal distance of the triangle for your building? What is the actual horizontal projection created by its lean?

Middle school students can verify the Pythagorean theorem for measurements on their models as well. High school students can use primary trigonometric ratios for both the model and the real building.

8. Use your scale and your calculations above to determine the actual height of your building.

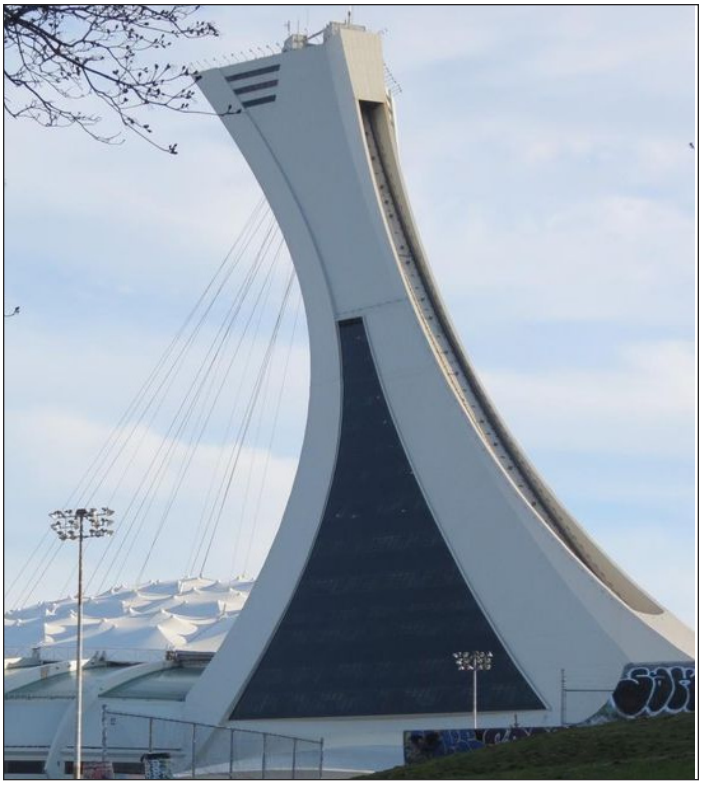
Since the Montreal Tower has a 45-degree lean, its projection and perpendicular height form an isosceles right triangle. I used TI-Nspire™ to create an isosceles right triangle on a side-view photo of the Montreal Tower. Since height is a perpendicular measure, photos of these buildings can be used to practise primary trigonometric ratios and Pythagorean theorem.



9. What is the fewest number of measurements needed, using a ruler to determine the three-side lengths?

The perspective on this photo is not perfect, but since these leaning buildings are frequently visited by tourists, there are many photos available that allow students to determine similar measures of the lean. This is another differentiation opportunity for students who find varied photos of these buildings.

10. What are the side lengths of the triangle formed by the Montreal Tower in this picture?
11. What is the scale of the photograph compared to the real-life tower?



12. What is the angle of the Montreal Tower as shown in the photo from the *Toronto Star,* June 7, 2019 issue? Why isn't it 45 degrees? (www.thestar.com/life/travel/2019/06/07/visit-the-leaning-tower-of-montreal.html)

The Guinness Book of Records indicates the most leaning tower is actually an attraction from Puzzling World in New Zealand. Unfortunately, the perpendicular height of the building is not given, but the Puzzling World website proudly proclaims the building's 53-degree lean. (www.puzzlingworld.co.nz/news/leaning-tower-world-title-stays-in-wanaka/)

13. What information could you use to determine the building's height?



14. If the person "holding" up the tower is at least 175 cm, how tall would the building be if she managed to stand it upward (i.e., perpendicular to the ground)?

The Nationale-Nederlanden building in Prague, designed by Frank Gehry, has a significant tilt as part of the design, but not the entire building. Table 1 includes the Alamillo Bridge, with an asymmetrical cantilever cable bridge with a 68-degree leaning section. Another bridge that could be included with similarly extreme lean and cable-stay variation is the Signature Bridge in New Delhi, which opened in November 2018.

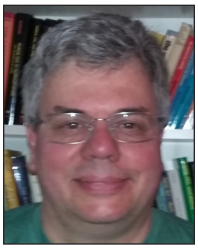
Students will likely differentiate between old buildings that are leaning because of foundation erosion or settling and new buildings that have been designed with leaning as anaesthetic feature. Recently, however, a student told me about Millennium Tower, a skyscraper in San Francisco that is also leaning due to shifting bay mud and anchoring in the bedrock. Millennium Tower, at 197 m tall, is one of the highest buildings in the region. It has sunk over 40 cm and is leaning over 20 cm to the northwest since its opening in 2009 (www.sciencealert.com/a-sinking-58-storey-sky scraper-in-san-francisco-now-has-a-massive-crack).

Creating multiple different-sized infographics and having each student create a proportional building that fits appropriately is only one way of differentiating this activity. The wide variety of building types allows students to work together and independently with similar prompts like those presented here. Having students do additional research to discover buildings they have seen in their travels or have seen on social media can add more options. Working with design technology and other STEM, teachers can provide another opportunity to extend learning and give students another way to see similarity, proportional thinking, and the relationship between angles and sides in right-angle triangles. Extending this activity beyond math class could lead to consideration of how architects incorporate leaning into modern design, and how their work is informed by physics. Just lean into some of these great learning opportunities with your students! ▲



Newton measuring the universe in sculpture by Eduardo Paolozzi outside the British National Library.

▲ What's the Problem? Searching for the Centre

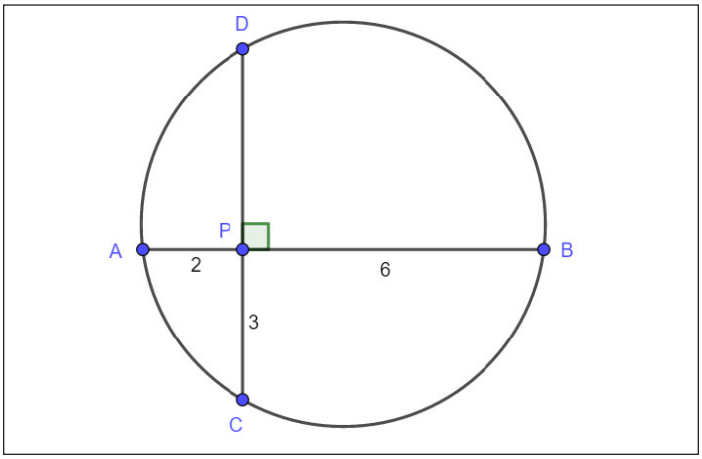


SHAWN GODIN
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Shawn teaches and is a department head at Cairine Wilson Secondary School in Orleans. He strongly believes in the central role of problem solving in the mathematics classroom. He continues to be involved in mathematical activities: presenting workshops, writing articles, working on local projects, and helping create mathematics contests.

Welcome back, problem solvers. Last time, I left you with the following problem:

In the diagram below, AB and CD are perpendicular chords of a circle that intersect at P. Given that PA = 2, PB = 6, and PC = 3, determine the radius of the circle.



I found this problem on the YouTube channel MindYourDecisions. The video is titled *How To Solve for the Radius. Challenging 1970s Math Contest!* It was apparently a junior math contest problem from the 1970s, sent to the channel by someone from Edmonton. MindYourDecisions looks at solutions to math problems from around the world and is worth checking out.

This one calls out for exploration, so let's fire up our favourite dynamic geometry software and give it a go. Keep in mind, all we know for sure is that *A*, *B*, *C*, and *D* are on a circle; *AB* is perpendicular to *CD*, at *P*; and we know the lengths of *AP*, *BP*, and *CP*. We will start with *A*, *B*, *C*, and *P* in the correct configuration and try to find a circle that passes through them. In Figure 1, you see my sketch from the Geometer's Sketchpad®, where I have introduced a free circle. I have put the circle through point *B*, but the centre,

O, is free. From this configuration, we can play until the circle passes through the three points and then measure the radius. When I did it, I got about 4.04 units.

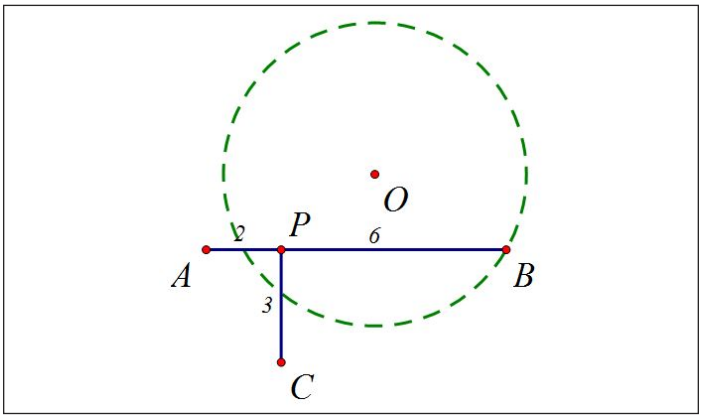


Figure 1: *Experimenting with GSP*

If we want to get an exact answer, we are going to have to use some properties of circles. In Grade 10 Academic math, we may discover that points on the perpendicular bisector of a segment are equidistant from the endpoints of the segment. This can be applied to a circle to see that the perpendicular bisectors of a chord of a circle must pass through the circle's centre. We know that AB and AC are chords of our circle, so we can use our sketch to find the centre of the circle. Figure 2 shows that the perpendicular bisectors of AC and AB meet at O, which is the centre of the circle that passes through A, B, and C. Sketchpad tells us the radius is about 4.03 units, so my original approximation is reasonably good.

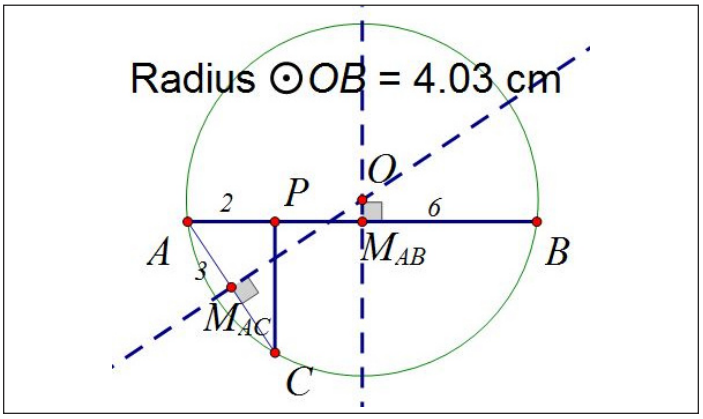


Figure 2: *The exact location of the centre*

It would be nice to be able to determine this radius exactly, using some math that we know. Since we have hinted at Grade 10 Academic (MPM2D), let's attach a coordinate system to our problem and put P at the origin. Thus, we have $A(-2,0)$, $B(6,0)$, $C(0,-3)$, so we can determine the coordinates of the midpoint of AB is $M_{AB}(2,0)$ and the midpoint of AC is $M_{AC}\left(-1,-\frac{3}{2}\right)$. We can determine that the slope of AC is $-\frac{3}{2}$, so the perpendicular bisector of AC has slope $\frac{3}{2}$. We can now work out that the equation of the perpendicular bisector of AC is $y=\frac{2}{3}x-\frac{5}{6}$. Since AB is horizontal, its perpendicular bisector is vertical with equation $x = 2$. Thus, the two perpendicular bisectors intersect at the centre of the circle $O\left(2,\frac{1}{2}\right)$. We can now use the Pythagorean theorem to calculate the radius, OB, to be

$$r^2=\left(0-\frac{1}{2}\right)^2+(6-2)^2$$

$$r^2=\frac{65}{4}$$

$$r=\frac{\sqrt{65}}{2}\approx 4.03$$

Nice! There are some other ways we can solve this, using some mathematics that was in the curriculum many years ago, back when I was in school. I will introduce a couple of ideas from the past and use them in a solution.

Back when I was in school, not only did we have more geometry in the curriculum, we also did compass and ruler constructions. I remember making the connection between the act of doing the construction to the generalizations presented with the particular theorem involved. The first dynamic geometry software that I ever saw was a program called C.a.R., which stood for Compass and Ruler. It allowed you to make free points and do compass and straightedge constructions based on them. Early in my teaching career, I played with this a lot, although I have never used it with students. I did an Internet search and the program is still out there, free to download. I suggest giving it a look.

Our second solution will depend on a theorem that we will prove, but we will need another theorem from my high school days, which we will "discover." Since we have Sketchpad open, let's do a bit more playing. Construct a circle with chord AB. Construct two points on the circumference on the same side of AB, labelled P and Q in Figure 3. Finally, create the triangles ABP and ABQ. Notice that if we measure the parts of the triangles $\angle P = \angle Q$, they stay that way, even if P or Q are moved. This is true in general. If you are interested in proof, you can check out an old geometry textbook, or (Godin, 2019). We would say that inscribed angles subtended by the same arc are equal. Angles $\angle P$ and $\angle Q$ are inscribed in a circle, that is, their vertices and points on each arm lie on a circle. We say they are subtended by the same arc because the arc of the circle, AB, goes across the opening of each angle. You may enjoy doing this with compass, straightedge, and protractor. I encourage you to draw some circles, measure some angles, and convince yourself that the theorem holds.

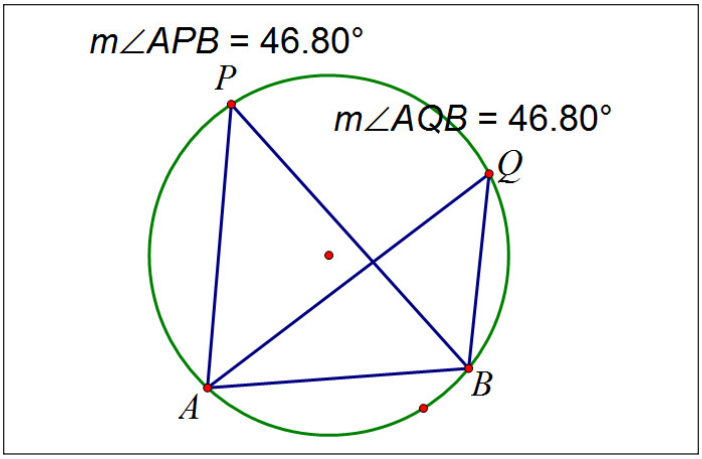


Figure 3: *Equal angles in a circle*

This theorem can be used to prove the intersecting chords theorem, which states (using the original problem Figure): if two chords of a circle, *AB* and *CD*, intersect at *P*, then *AB* x *PB* = *CP* x *PD*. To prove this, join *A* to *C* and *B* to *D* to form triangles *ACP* and *DBP* as in Figure 4. Clearly $\angle ACP = \angle DBP$ as they are opposite angles, and $\angle CAP = \angle BDP$ as they are inscribed angles subtending the same arc. From this, we conclude that our triangles are similar, and hence $\frac{AP}{DP} = \frac{CP}{PB}$, from which the result follows.

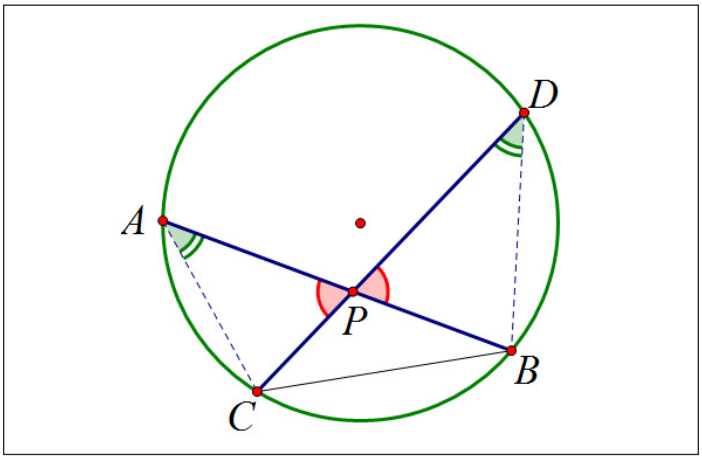


Figure 4: *Intersecting chords theorem*

Using this with the original problem, we get:

$$AP \times PB = CP \times PD$$
$$2 \times 6 = 3 \times PD$$
$$PD = 4$$

Now, if we construct a chord, *EF*, perpendicular to *AB*, so that it intersects at a point 2 units from *B*, as shown in Figure 5, then because of the symmetry of a circle, we have a rectangle *CDFE*. Constructing diagonal *DE*, we create a right-angled triangle, which means that *DE* is a diameter of the circle (see Godin, 2019). We can now get our result by using the Pythagorean theorem to determine the diameter is $\sqrt{65}$ units, so the desired radius is $\frac{\sqrt{65}}{2}$, as we had previously concluded.

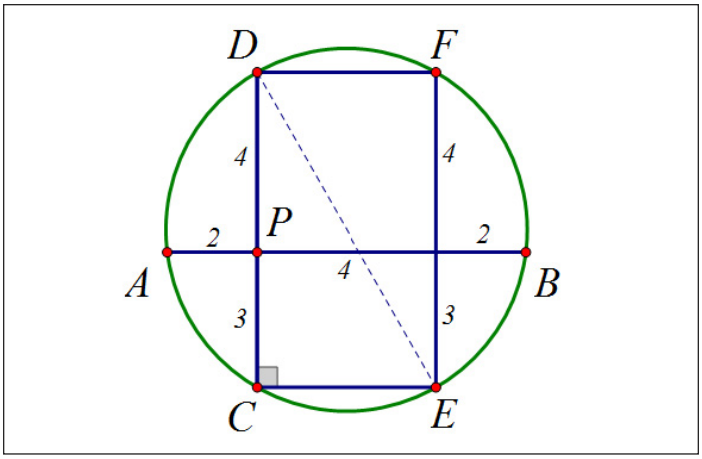


Figure 5: *Finding the diameter*

Over the years, the curriculum has changed many times. Topics have moved from course to course, and grade to grade. Some topics have disappeared entirely. There is a lot of geometry that students used to be exposed to that is no longer in any school mathematics course in Ontario. Yet, with the aid of dynamic geometry software, many of these are ready to be discovered and appreciated by our students. In the original video, one of the solutions explored makes use of a theorem about perpendicular chords and their relation to the radius, which is very appealing. I suggest you try to discover it for yourself and then check out the video.

Now, time for your homework:

The numbers from 1 to 576 are written in a 24 x 24 grid so that the first row contains the numbers 1 to 24, the second row contains the numbers 25 to 48, and so on, as shown in the diagram below. Determine if it is possible to draw an 8 x 8 square around 64 of these numbers, such that the sum of the four corner numbers is 1646.

1	2	⋯	23	24
25	26	⋯	47	48
⋮	⋮	⋮	⋮	⋮
553	554	⋯	575	576

Until next time, happy problem solving!

Reference

Godin, S. (2019). *Problem-solving vignettes: Going in circles. Crux Mathematicorum, 45*(8), 452–456. Retrieved from www.cms.math.ca/crux/v45/n8 ▲

"Mathematics is a place where you can do things which you can't do in the real world."
Marcus du Sautoy

▲ Assessment Abby Collecting Valid Assessment Data



ASSESSMENT ABBY
EMAIL: assessmentabby@oame.on.ca

Ask Assessment Abby A³ is a regular column in the *OAME* Gazette, where teachers can share concerns and best practices about assessment, evaluation, and reporting of mathematics. Please send your questions to Ask Abby at **assessmentabby@oame.on.ca**.

Dear Assessment Abby,

I recently picked up the June 2018 edition of the OAME/AOEM *Gazette* that I saw on someone's desk. I must thank you for the excellent magazine, by the way. So much good stuff in it. Anyway, the reason for my letter to you is that I was reading the Assessment Abby column on student stress around testing. In it, you mention that "repeated use of pencil-and-paper tests may not give valid data about what students know and can do," and I was wondering what you mean by that? I always get a student mark (data) from a test, so can you explain more about this idea, please?

Signed,
Curious to know more

Dear Curious,

First, thank you for your kind words about the *Gazette*. We are very proud of our publications, and we appreciate hearing from our readers that they are getting useful information from it.

I completely understand that this statement you identify may be a bit confusing or that it may have left you (and others) wondering. Let me try to explain by connecting student assessment to a more concrete analogy.

Suppose you are creating a mixture and you need to measure the weights of some of the ingredients. A scale would be a useful tool to assist you in evaluating weight. Now, if your scale has been incorrectly calibrated, it will still give you a number when you go to weigh something. This tool, the improperly calibrated scale, is a perfectly reliable tool because it always gives you the same number when you put the same weight on it. However, we know that this number is not valid—it does not accurately represent the weight of the ingredients—because I have told you that the scale has not been properly calibrated. So, the scale gives us data that is not valid, even though the scale itself is reliable. We can go ahead and use data that is not valid because it came from a reliable tool, but we are unlikely to have our mixture turn out as expected, and that mixture may not be able to be used for its intended purpose, (You might end up with concrete that won't set, for example.)

In this way, pencil-and-paper tests are like a scale. They always give you a number. The question we need to ask ourselves is whether or not that number is valid, or might this be an assessment tool that is reliable, but provides data that may not be valid. Does using only pencil-and-paper tests give you a true sense of what each of your students knows and can do? Does using just this one tool allow all students to demonstrate their knowledge of content, their understanding of concepts, and their ability to use their knowledge and understanding in assorted ways? Does a collection of marks generated only by pencil-and-paper tests truly capture the wide array of ways we know that students demonstrate how they think mathematically or to what extent they are able to communicate their thinking?

Have any of these situations happened to you? You have a student who regularly raises his or her hand to offer a fantastically clear explanation to a solution, yet seems to consistently get marks of 3 out of 6 in the Communication category of achievement on tests. This is an indication that the test may not be a reliable tool for assessing that student in the Communication category. Students may be able to explain something verbally, but struggle when writing their thoughts.

It may be difficult to become less reliant on a tool that has been so reliable for us for so long, but always keep in mind that the purpose of assessment and evaluation is to improve student learning. To correctly identify next steps for improvement, you, as well as your students, need valid data.

We all feel the crunch of time in our classrooms, so ensuring that when we are devoting time toward collecting assessment data, we can depend on the reliability of our assessments is important. How to collect valid data through reliable assessment tools is a question we often get asked about, so in our next column, we will explore some alternate tools that are reliable and may help you collect data with better validity.

I hope this helps clarify what I was saying back in 2018.

Assessment Abby

[Note: For those interested in that column, it can be found by visiting the *Gazette* archives and searching for the June 2018 issue (Volume 56, Number 4). If you are interested in other *Assessment Abby* columns, you can type "assessment abby" into the search bar to get a list of all the article titles.]

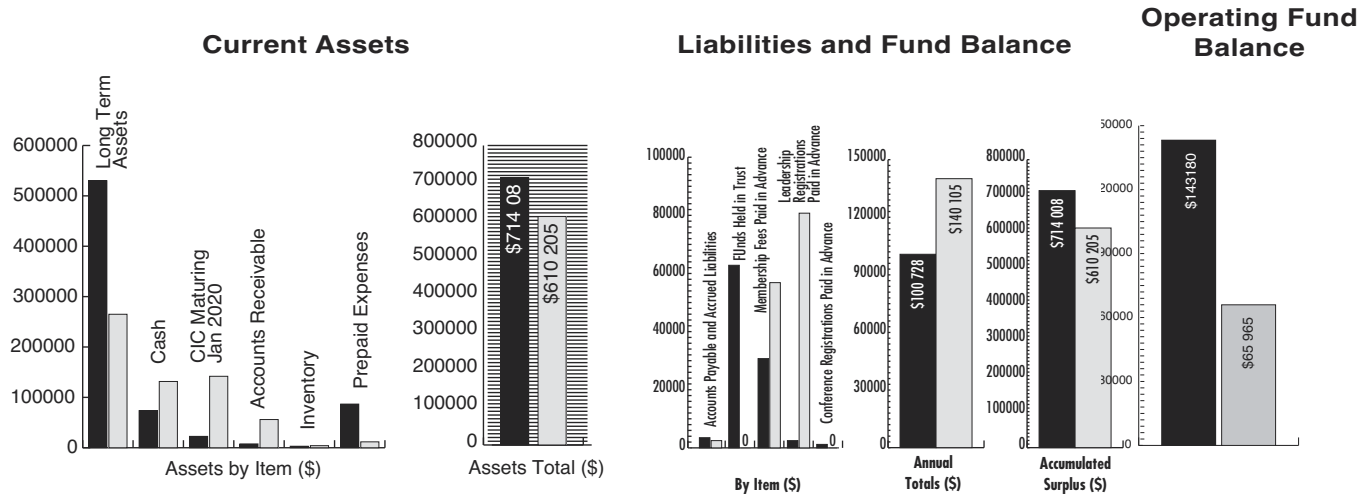
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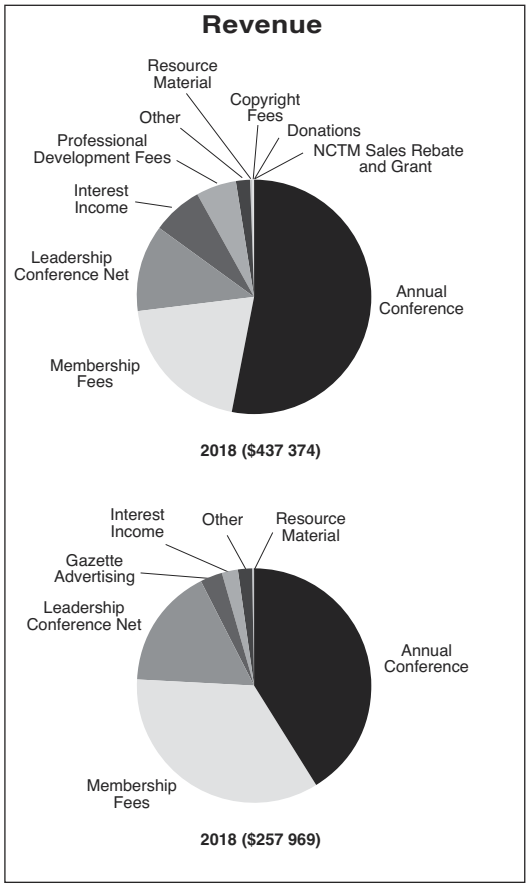
▲ OAME Financial Statement 2018-19

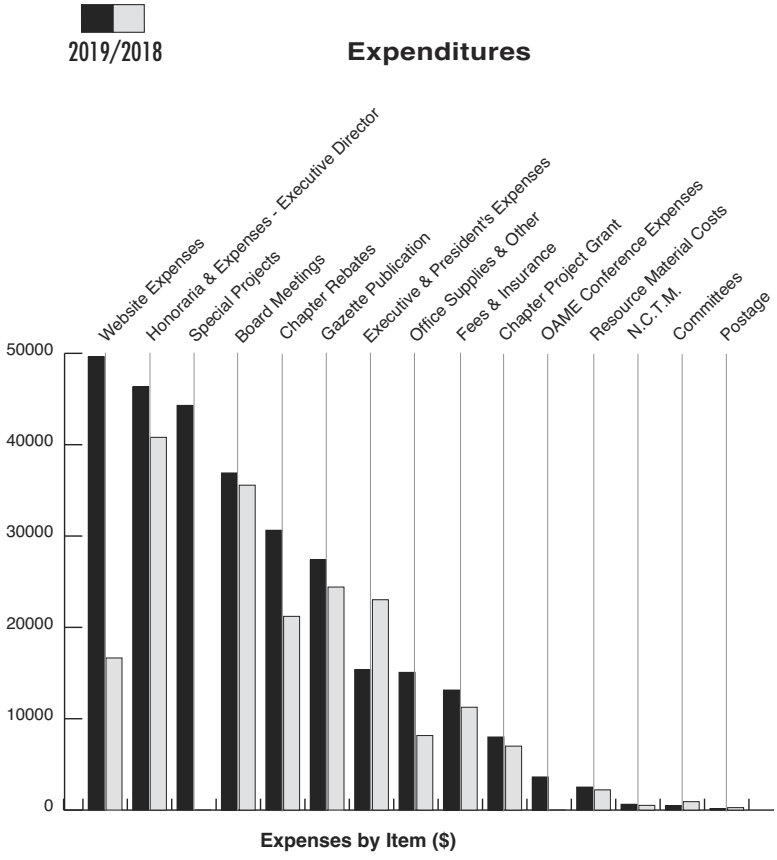
Notice to Readers

On the basis of information provided by management, we have compiled the statement of financial position of Ontario Association for Mathematics Education (Association Ontarienne pour L'Enseignement des Mathématiques) as of August 31, 2019, and the statement of operations, changes in net assets, and cash flow for the year then ended. We have not performed an audit or a review engagement in respect of these financial statements and, accordingly, we express no assurance thereon. Readers are cautioned that these financial statements may not be appropriate for their purposes.

London, Ontario, September 12, 2019, SKRYPNYK GROUP, Professional Corporation, Authorized to practise public accounting by CPA of O.







▲ Mathematical Snapshots: Finding Math in My Home Town



RON LANCASTER
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Ron Lancaster is an Associate Professor at the University of Toronto, where he teaches mathematics courses for pre-service middle and high school teachers. He has over 20 years of experience teaching Grades 7–12 mathematics. Ron's professional activities include consultations and conference presentations in North America, Asia, England, the Middle East, Africa, India, and Europe. Ron is an author for the NCTM (The Mathematical Lens) and member of the Advisory Board for the Museum of Mathematics in New York. He is the recipient of the 2015 Margaret Sinclair Memorial Award Recognizing Innovation and Excellence in Mathematics Education, awarded by the Fields Institute.

In this issue, *Mathematical Snapshots* pays a visit to Chatham, Ontario. I was born in Chatham and lived in the area until I was about 8 years old. I still have many relatives in Chatham and Pain Court. When I visit Chatham, I like to have lunch with my Aunt Marlene at the Chilled Cork in the Retro Suites Hotel. My Aunt Marlene is the only person in the world who calls me by the name Ronnie instead of Ron. This is one of two reasons for why I chose ron2718@nas.net for my personal "e"mail address many years ago (ron2718 = ron"e"). Of course, to appreciate this, you will need to know that the number *e* is approximately equal to 2.718....

In Photograph 1, notice how the digits from 1 to 9 have been arranged in rows. Wonder what pattern has been used. Wonder if the same number of digits has been used. If not, wonder which digit appears most often. Wonder how many digits have been used.

In Photograph 2, notice the price of a bag of popcorn. Wonder how this compares to the price of a bag today. Wonder when the last time a bag of popcorn cost 10 cents. Wonder about the size of a bag used at this popcorn machine and how it compares to a bag used at a movie theatre today.



Photograph 1: *Art in the lobby of the Retro Suites Hotel, Chatham, Ontario*



Photograph 2: *Popcorn machine in the lobby of the Retro Suites Hotel, Chatham, Ontario*



Photograph 3: *Design on floor in the lobby of the Retro Suites Hotel, Chatham, Ontario*

In Photograph 3, notice the geometric shapes that create the design on the floor. Notice the curves. Wonder if they are arcs of circles. Wonder how many colours are used in the design. Wonder about the area of the individual components of the design. Wonder about their areas and which colour was used the most or the least in the design.



Photograph 4: *Stacked cannonballs on corner of Chatham Armory, Chatham, Ontario*

In Photograph 4, notice the stacked balls at the corner of the building. Wonder why these balls are there. Wonder how many balls there are altogether. Wonder how many balls would be needed to add an extra level at the bottom of the stack. Wonder if there is a similar structure at each corner of the building.

Notice that the next few pages provide some ideas for classroom follow-up regarding these snapshots. You may need to adjust the questions, depending on the grade doing the exercises. Furthermore, students could do their own "I wonder" snapshots and create corresponding questions and exercises to share with their classmates.

Question 1

Table 1 shows the first four rows of numbers in the artwork in Photograph 1. There are 13 rows altogether. The first row has 16 numbers, and all other rows have 14 numbers. The numbers in the first row follow the pattern 1, 2, 3, 4, 5, 6, 7, 8, and 9 repeatedly from left to right, and continue into the second row, then the third row, and so on.

Table 1

1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7
8	9	1	2	3	4	5	6	7	8	9	1	2	3		
4	5	6	7	8	9	1	2	3	4	5	6	7	8		
9	1	2	3	4	5	6	7	8	9	1	2	3	4		

(a) Starting with the second row, consider the numbers that appear at the start of each row:

8, 4, 9,...

What is the first number in the:

(i) 5th row?

(ii) 10th row?

(iii) nth row, where $n \geq 2$?

(b) Starting with the second row, consider the sum of the numbers in each row:

73, 68, 75, 64,...

What is the sum of the numbers in the:

(i) 5th row?

(ii) 10th row?

(iii) nth row, where $n \geq 2$?

Question 2

Suppose that in 1950, the cost of a 2-ounce bag of popcorn purchased from a machine such as the one shown in Photograph 2 was 10 cents. Figure 1 shows how much that bag of popcorn would cost in 2019, taking into account inflation.

(a) Using the average annual rate of inflation provided in Figure 1, do a fact check of the cost of a bag in 2019, and the percent change in the cost from 1950 to 2019.

(b) Suppose the cost of the bag of popcorn went up by 3.53% per year from 1950 to 2019. Develop a mathematical model for the cost of the bag of popcorn n years after 1950.

(c) Use your model from part (b) to determine the cost of the bag of popcorn in 2000. Determine the cost, using the online calculator. Do both methods produce the same answer? Should they? Why or why not?

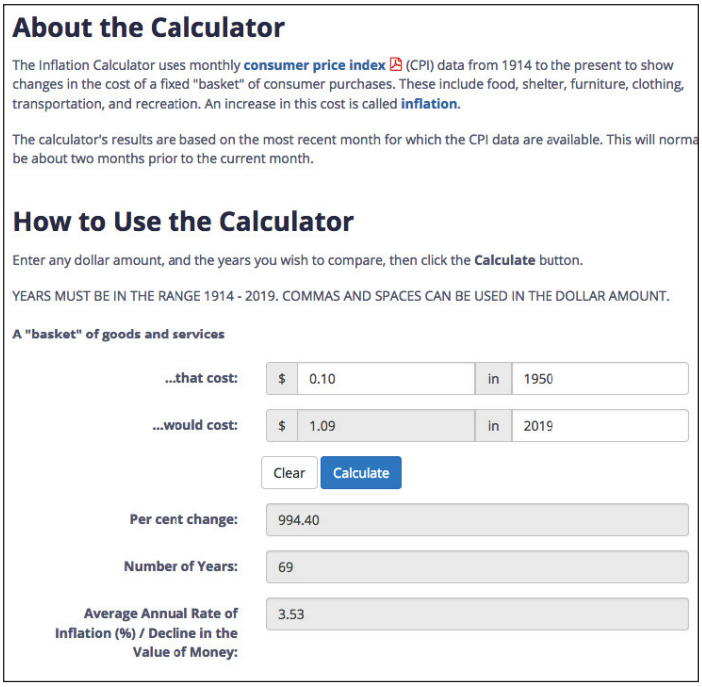


Figure 1: *www.bankofcanada.ca/rates/related/inflation-calculator/*

Question 3

Suppose that Figure 2 is an exact representation of the design on the floor shown in Photograph 3. The specifications for the bottom half of the design are given below. Assume that the design has horizontal, vertical, and rotational symmetry.

Centre of the design is at point *R*.

AB = *BC* = *CE* = *EF* = *FG*

ABIH is a square.

DJ = *AH*/2

N is the point of intersection between the quarter-circle, with centre *A* and radius *AC* and the line that passes through *A* and *R*.

O is the point of intersection between the quarter-circle, with centre *D* and radius *DN* and the line that passes through *D* and *R*.

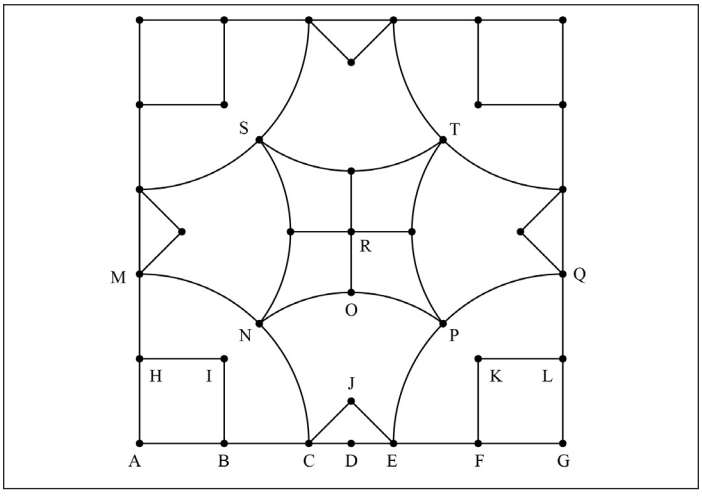
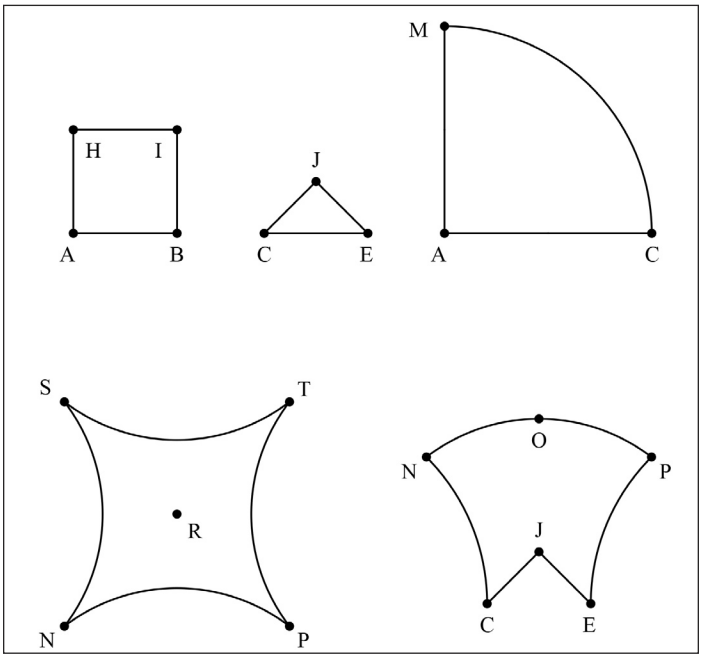


Figure 2

If *AB* = 1 unit, find the area of each of the following regions.



Question 4

(a) Count the number of balls on each level of the structure, from the top down, and record your results in Table 1. Determine a rule (algebraic or verbal) for this sequence.

Table 1

Level	Number of Cannonballs
1	
2	
3	
4	
5	

(b) Suppose additional levels of cannonballs were added to the bottom of the structure. Use your rule from part (a) to determine the number of cannonballs on the:

(i) 6th level

(ii) 10th level

(iii) *n*th level

(c) How many cannonballs are there altogether on the five levels? How many cannonballs would there be altogether if the structure had ten levels? Generalize to *n* levels.

▲

"Logic and mathematics are nothing but specialized linguistic structures." Jean Piaget

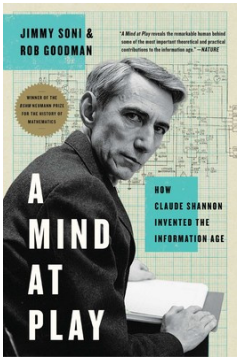
▲ BOOK REVIEW: How Information Transformed the World: A Review of *A Mind at Play*



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Soni, J., & Goodman, R. (2017). *A mind at play: How Claude Shannon invented the information age.* New York, NY: Simon and Schuster.



Every so often, one comes across a compelling book in unusual circumstances. In my case, finding a biography of Claude Shannon while vacationing was serendipitous. I was aware of the father of information theory and some of his ideas, but the book placed the ideas in context—a context that has seen the impact of those ideas over my entire lifetime.

From the beginning, this book sheds light on mathematics. Many students would benefit from classrooms fitting the description: "A math laboratory of that era [(about a hundred years ago)] was 'well stocked with clay, cardboard, wire, wood, metal and other models and materials'—and graph paper" (p. 29). The motivation is to provide an image of how accomplishments before the existence of the computer often used mechanized models that contained parts that acted out the equations being modelled. It describes this as conceptual in the sense that the models were effectively miniature versions of the process, regardless of whether that process was a model of a star or an atom.

As computers were developed, the process of optimizing circuits to have a minimal number of parts was not established. This was the problem Shannon addressed in his master's thesis. He realized that the on/off nature of electronic components (i.e., binary circuitry) allowed the basic structures to be modelled algebraically. When applied to complex structures, simplification of the algebra clarifies the most efficient design. The result was a transformation of design from arbitrary rules with limited results to an algebraic process that produced the best solution.

Bell Labs hired Shannon in part due to his fascination with tinkering. He had a workshop, where throughout his life, he built machines that were informed by mathematics. One example was a machine that could bounce-juggle four balls. In this respect, the book serves to give math educators pause to consider how manipulatives can inform all levels of mathematics education. Consider, for example, that Morse code is described as having been designed using an estimate of letter frequency, where more common letters have shorter sequences of dots and dashes. The frequency of letters, in the days before computers, was estimated by examining the wear on a typesetter's box of iron letters.

What was Claude Shannon's primary achievement? A model of communication that is the foundation of all modern communications. It distinguished encoding and decoding as separate from the transmission of the signal and associated noise. This has informed virtually every communication device our readership has used. "The beauty of this stripped-down model is that it applies universally. It is a story that messages cannot help but play out—human messaged, messages in circuits, messages in neurons, messages in the blood" (p. 139). The encoding and decoding refers to two elements. The first is the removal of redundant information for efficient communication (e.g., I could have said "cmmnctn," and you likely would have figured out the word because of the redundant use of letters in the word), and the second is error correcting. An example of the latter is provided, where encoding A = 00, B = 01, C = 10, D = 11 is compared to encoding A = 00000, B = 00111, C = 11100, D = 11011. In the first version, any error in a single digit would not be evident, but in the second encoding, any single-digit error is detectable. Error-detection algorithms have advanced considerably in the digital age, and are simply assumed these days because they are built into cellphones and Internet communication, but their recognition was a major milestone of the computer age.

For computer science aficionados, there is a mention of Shannon and Alan Turing meeting, but largely being unable to talk about what they were working on during the war. There is a mention of Turing using a "pulse in the hooter" (p. 108) to help with programming. This refers to having a line in a computer program that would make a noise on a speaker so that the operator would know when the computer reached the line. Those among us who are old enough to

Continued on page 33

▲ Connections between Math and Literacy



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Jeff Irvine has been a secondary math teacher in Waterloo and Peel District School Boards, a secondary math department head, and a secondary vice-principal. Jeff has taught at three faculties of education and at Sheridan College. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. He is currently an instructor of mathematics education at Brock University, where he is pursuing a PhD.

Mathematical thinking will one day be as necessary for efficient citizenship as the ability to read and write.
~ H.G. Wells (1866–1946)

Recently I was asked by a local middle school principal to do a series of professional development workshops with his staff. The school board had mandated that all middle school teachers would teach core courses, including mathematics. Since this mandate is diametrically opposed to the Ontario Ministry of Education's Revised Math Strategy, which recommends math specialists for these grades, the school and board will remain nameless. The mandate resulted in teachers with more than 20 years of teaching language arts being required to teach intermediate mathematics as well. There was a high degree of math discomfort, anxiety, and phobia among the staff. In the PD workshops, I emphasized the relationships between teaching mathematics and teaching language arts, using strategies and activities that I had developed over my years in the math classroom. Note that I'm not referring to developing math talk learning communities that place an emphasis on students explaining their reasoning and justifying their thinking. My goal was to achieve baby steps; I wanted to give these teachers some "comforting" activities that related to using language in math class.

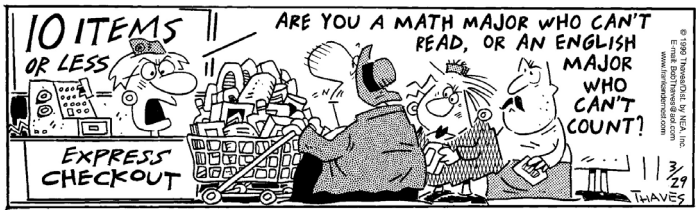


Figure 1: *FRANK & ERNEST © 1999 Thaves. Used By permission of ANDREWS MCMEEL SYNDICATION. All rights reserved.*

Developing these workshops led me to take a look at research linking math and literacy. A couple of pertinent research articles are shown below.

Sammons (2011) identifies similarities between readers and mathematicians.

Characteristics of Readers	Characteristics of Mathematicians
They call upon their prior knowledge to make meaning from text.	They call upon prior knowledge to understand concepts and solve problems.
They are fluent readers.	They are procedurally fluent.
They have a mental image of what they are reading.	They create multiple representations of mathematics concepts and problems.
They monitor understanding as they read.	They monitor understanding as they solve problems.
They can clearly explain their interpretation of the text to others.	They can clearly explain their mathematical thinking to others.

We want to develop our students' understanding of mathematics and move them into the "Characteristics of Mathematicians" column. Integrating literacy strategies into our math classrooms can help us do that.

Bickmore-Brand (1993) has identified seven language strategies that support the learning of mathematics:

- Creating a meaningful and relevant context for the knowledge, skills, and values of mathematics
- Realizing the starting point of interest in mathematics is the knowledge base of the student
- Providing opportunities for the learner to see the skills, processes, and values of mathematics by the teacher's modelling
- Continuing to build on the knowledge base and challenging the students by scaffolding
- Facilitating the metacognition of the student by helping the student identify the learning processes and how he or she learns
- Assisting the learner to accept the responsibility for

the construction of knowledge

- Building a community of learners in a risk-free learning environment

While the impetus for the workshops was middle school, a number of strategies are included to extend the scope and facilitate the ideas across Grades 6 to 12. Here are some classroom-tested activities that I have used. This is not an exhaustive list; there are many more strategies that relate math and literacy, but the strategies described below are all classroom tested, and I know that they work.

Definitions and Twitter

Mathematical definitions are a natural link between math and literacy, but can often be seen as boring by students. For schools that have permission to use Twitter, I have found that this is one way to combat the boredom that definitions sometimes engender. Note that I am not suggesting that teachers use Twitter to communicate with parents or students, but I suggest care to consider equity of access with your students. Here are some ways that Twitter could be incorporated into a discussion about definitions:

- Tweet the mathematical term out to the class and have groups respond with a definition.
- Tweet a definition and ask for the appropriate mathematical term.
- Tweet out only a starting stem and have groups progressively add attributes. For example, the teacher tweets "A triangle with…"; a student tweet adds "all three angles equal…"; and a second student (or group) adds "is called an *equilateral triangle*."

This version can lead to some interesting debates. The following Twitter exchange illustrates the need for mathematical definitions to be precise:

Teacher tweet: "A quadrilateral…"

First student tweet: "with two pairs of equal sides…"

Second student tweet: "and one right angle…"

Third student tweet: "is called a *rectangle*."

One student group argued that the final tweet should be "is called a *square*." The group pointed out that a square meets the conditions set out in the previous tweets, and so should also be considered correct. The debate led to the need to either add the word *exactly* to the first students' tweet; or to agree that both rectangle and square were valid answers (and that *square* is a subset of *rectangle*).

Definition tweeting can also lead to a discussion of necessary versus sufficient conditions. The Twitter exchange below led to a lively debate.

Teacher tweet: " A triangle…"

First student tweet: "with no angles equal…"

Second student tweet: "and no sides equal…"

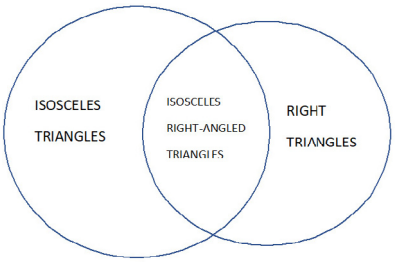
Third student tweet: "is called a *scalene triangle*."

The debate occurred because a student group pointed out that the second student tweet was redundant, since if all the angles of a triangle are of different sizes, then all the sides must be different lengths. This resulted in students proposing two definitions for a scalene triangle, one using the different angle measures and a second definition using the different side lengths. They concluded that if either condition was satisfied, the triangle must be scalene. The diagram added to our word wall for a scalene triangle that contained two triangles, one emphasizing the different side lengths and another emphasizing the different angle measures.

While the Twitter examples here involve geometric definitions, building and refining definitions using Twitter can also be used in algebra (polynomials, like terms, degree); trigonometry (trig ratios, sine law, cosine law); data management (mean, median, mode; probability terms such as *independent* or *dependent*, *and*, *or*; statistical distributions such *as uniform, normal*). Any part of mathematics content that involves definitions can be made more memorable by incorporating technologies that are part of students' daily lives.

Venn Diagrams and Graphs

Adding a visual element can be useful in mathematics. A diagram or a graph can make written terms come alive. For example, a discussion about the relationship of the slopes of perpendicular lines as negative reciprocals gains clarity with a simple graphical example. Venn diagrams add understanding to the relationships. For example, in the triangle family or the quadrilateral family. I found that having student groups construct the graphs or Venn diagrams, and then sharing through a gallery walk, led to the most precise visuals being identified for posting in the classroom. While this probably takes more time than the teacher simply giving the class the diagram, student constructions lead to better understanding.



Dear Delta

This is modelled on the newspaper column *Ask Amy* or *Dear Abby*, where readers pose questions and answers are provided. If it helps modernize the concept for students, they can write their pieces as blog posts or Twitter tweets. Initially, the teacher posts the *Dear Delta* letter, and students write responses, usually in groups. As the students become comfortable, they then write the *Dear Delta* letters, and pass them to a different group, which writes a response. Before using *Dear Delta* letters in your classroom, it is useful for the teacher to generate a sample response, which can be used as an exemplar, and also ensures that the original *Dear Delta* letter is clear.

Dear Delta:

I don't get integers. I really don't get integer subtraction. I'm OK with questions like 3–8 or –3–8. I can use a number line to start at 3 (or –3) and go 8 steps left. But then the teacher asked us to do –3–(–8) and I'm lost. She says that it's the same as –3+8, that you just "change the sign and add," but I don't know why it's the same. Is it just some rule that I have to memorize? I don't like rules.

~ Signed, Baffled in Brampton Student.

Generated responses can demonstrate the students' conceptual understanding, as well as their explanatory skills. Student-generated *Dear Delta* letters are an excellent way to check for understanding in a creative and non-traditional way.

Responding to blog or newspaper mis-steps

Challenging students to find errors in newspapers or blog posts generates a lot of engagement. It can be a really rich area for student-generated responses. Teachers ask students to write a response, explaining the error and giving the corrected version. For example,

March 17, 2017: Headline in an Internet post on college math preparation in the United States: "Data show that more than half of incoming students at some post-secondary institutions require remediation."

First line of the article: "More than 42 percent of students who arrived at the state's community colleges in 2014 required at least one remedial course."

Students love writing letters to the editor, pointing out this type of error. My classes frequently write responses that are actually sent to the source of the error, after minor editing by me.

Word Puzzles

These are traditionally called word problems, but I find the designation "problem" is a problem. Specifically, I have found that the word "problem" is considered to have a negative connotation by students. Two ways of generating word puzzles that are more motivating for students are shown below. Both of these strategies involve blending mathematics and literacy.

One way that integrates word puzzles and coordinate geometry is to have students generate ordered pairs that fit a specific criterion stated in words. The second and third sentences below give an example of the power of a comma, changing the order of operations (by inserting a bracket). Relating coordinate geometry and word puzzles may also address the needs of some students' learning modalities by adding a visual element. Deeper understanding by all students is another benefit of linking the two modes of representation.

Student instructions:

For each sentence below, find some ordered pairs (x, y) that fit the story; graph the ordered pairs; find a graphical relationship: write an equation of a line that passes through the points.

- The second number is three times the first number.
- The second number is three times the first number, plus four.
- The second number is three times the first number plus four.
- The two numbers add up to ten.

Word Puzzles Based on Real Data

A powerful source of student engagement is to base word puzzles on real data and ask students to develop and critique mathematical models based on the data. Initially students can be asked to research a specific situation, using the Internet. For example, students might choose to investigate an endangered species, depletion of a natural resource such as oil, or data related to carbon dioxide pollution. Students are asked to find at least two data points in the form (year, amount). Students then construct a mathematical model that fits their data. After constructing the model, students critique their own models by analyzing how

well the model fits the current data; how useful the model is for predicting future data; flaws or inconsistencies in using the model to predict. Several samples of these activities are shown below. The type of mathematical model depends on the grade level. Grade 9 students typically use linear models; Grade 10 students use linear and quadratic models; as the grade level increases, the students have a wider range of model choices as they encounter other functions such as exponential, logarithmic, sinusoidal, and so on. The examples below show the excerpted data points and the model question.

Example 1:

In 2001, the adult population of the Algonquin wolf was estimated at 488. In 2015, the adult population was estimated to be 154. Construct a linear model of this relationship and use your model to estimate the adult wolf population in 2020. Discuss how well you feel that your model fits reality for the Algonquin wolf population. Provide some reasons as to why you think the wolf population is declining.

Example 2:

A honey bee colony typically has between 20 000 and 80 000 bees. Recently the honey bee population has been hurt by increased pesticide use and by cold winters. In 2014, there were 112 800 honey bee colonies in Ontario. By 2017, that number had shrunk to 105 244 colonies. Construct linear and quadratic models that fit these data. Use your models to estimate the number of honey bee colonies in 2020. Which model do you think is better for forecasting the honey bee population in the future? Justify your answer.

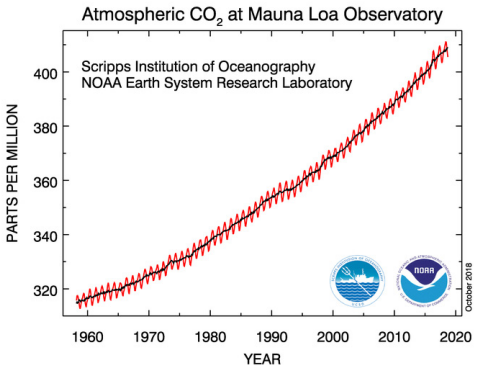
Example 3:

In 1971, there were an estimated 1800 pairs of adult Canada Geese capable of breeding in Ontario. By 2012, there were 80 000 pairs. Select and develop a model that fits these data. Justify your choice of model and estimate the number of adult pairs in 2030. Why is a mathematical model useful in this situation?

The example below can be used as a rich assessment task or a rich learning task for MHF4U. Students are expected to locate the data, using the Internet. [The model is sinusoidal with a positive linear or exponential growth term.]

Example 4:

The Mauna Loa research centre in Hawaii has tracked worldwide carbon dioxide emissions for over 50 years. Construct a model of the data. Justify your model and use it to predict carbon dioxide emissions in 2050. Does your model support the need to take steps to reduce emissions?



What's Behind the Mask?



I was shown this strategy by an ESL teacher in my school, who used it to build reading skills for ESL students, and I adapted it for use in mathematics.

- Students are presented with a paragraph or problem in which some key questions, information, diagram, or situation is covered or obscured by something (I use the mask symbol on the left to cover the information).
- Students then predict the missing information.
- Use think/pair/share or another co-operative group strategy to share possible predictions.
- Reveal the masked data and compare to predictions.
- Discuss feasibility/consistency of student predictions.

Useful for:

- identifying important information
- making and verifying predictions
- identifying extraneous information
- identifying contradictory information
- increasing student interest by making the problem an open problem that requires student-generated data.

Here is a brief illustration of Behind the Mask.

The distance between two local towns is  km. If Johan walks the entire way, it takes him  hours. If he jogs the entire way, he can make the trip in 2 hours. How far apart are the towns?

Signal Words

Signal words are words that have been explicitly clarified in the classroom and imply instructions about what students

are expected to do. Because mathematics is so precise, frequently an instruction in language arts is significantly different from the same word used in mathematics. It is important that we clarify for students exactly what is required because interpreting instructions uses part of the student's working memory that is better used to focus on the problem and its solution (Irvine, 2016).

Some common signal words in mathematics are: *explain, compare, define, list, describe, determine, justify, predict, identify, summarize,* and *show* your work. This list includes many terms used in EQAO assessments. However, the reason to engage students with the definitions is to support student thinking about what is being asked and goes beyond just EQAO assessments. The list contains general words and should be augmented with topic-specific signal words. For example, words like *expand, simplify,* and *factor* are just a few among many others. Ensuring that all students receive clear and unambiguous instructions helps them focus on the "doing" of the mathematics.

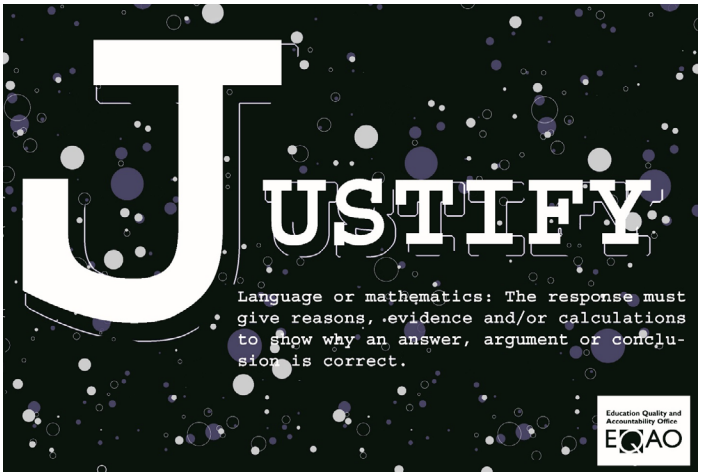


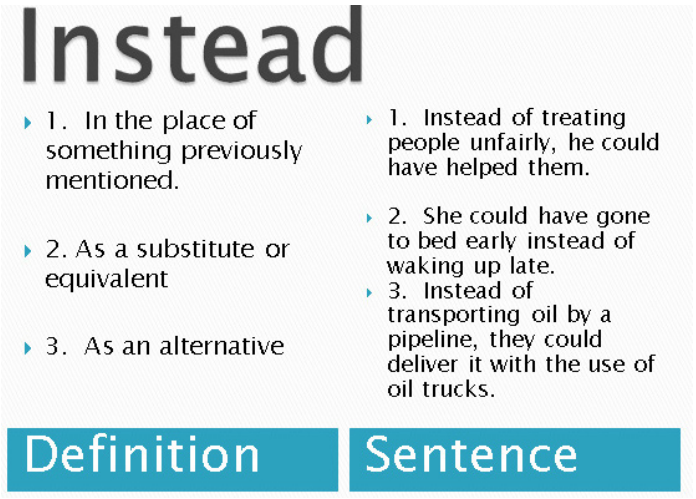
Figure 2: *A sample poster for signal words*

We cannot assume that every student interprets "justify" in the same way. To help achieve consistency and clarity, I asked our art department to construct large posters (see Figure 2) with each of the signal words and their definitions/descriptions. The definitions highlighted both math and literacy meanings of the signal word. The posters were placed in every classroom in our school, as well as every classroom in each of our feeder schools. In this way, every student was able to clarify exactly what they were expected to do.

It would have been an excellent activity to have student groups develop their own definitions for the signal words and then compare to the standard definitions. We didn't do that, but if I had it to do over, having students engage with the words before getting the standard definitions would have provided a better learning opportunity.

Duggleby Connecting Words

This is an activity that I got from my friend, Sandra Duggleby, an instructor at the University of Calgary. Each connecting word card contains the word, its definition, and some examples of its use. Each group decides on a discussion topic (e.g., Pythagorean theorem). The deck of connecting words is shuffled and placed face down. A student takes the top card and begins his or her statement with the word on the card. The next student takes the next card and begins his or her statement with that word. This activity forces students to think critically and formulate coherent statements.



Sandra's research has shown that using this activity resulted in students exhibiting higher-order thinking skills. Sandra developed 76 different cards, broken into five categories, which I usually print on assorted colours. Some sample words are shown below. The deck can be edited by removing certain cards if the teacher finds some of the words too difficult for students, or inappropriate to the topics under discussion. The activity can be used in any subject, but I have used it successfully in math classes in Grades 9 through 12 and all the levels (variation in language may be necessary depending on the needs of the students in your class!).

Connective Category	Word Examples
Additive	also, for example
Adverb	typically, certainly
Adversative	although, instead
Temporal	when, before
Causal	because; if…, then….

Sandra has kindly allowed the OAME to post her resources in our members only area.

I have found that choosing the mathematics topic for discussion is key to success in this activity. Below is a brief

example of Duggleby Connecting Words used for the discussion of the topic Pythagorean theorem. Each sentence is from a different student.

"**If** in a triangle with sides *a,b,c* $a^2 + b^2 = c^2$, **then** the triangle is a right triangle."

"Does that relationship **only** work for triangles with integer sides?"

"**Apparently**, but can the sides also be decimals?"

"I found that the relationship **also** works for some sides that are decimals."

"**When** I tried it with sides 0.3, 0.4, and 0.5, it worked."

"**If** it **only** works for some decimals, **then** we should be able to find a pattern."

…

These are only some of the math–language activities. There are many more. Many of you have had students read stories containing mathematics (such as mysteries solved by the probability concepts of DNA matching), or write stories based on mathematical ideas (permutations and combinations are a favourite for me). One intriguing geometry-related story is Edgar Allen Poe's *The Pit and the Pendulum*. Try having students read this story in "chunks" and construct, then modify, the shape of the room in which the prisoner is held as they read further. For example, Poe's initial description of the wall might imply that the room was cylindrical. "My outstretched hands at length encountered some solid obstruction. It was a wall, seemingly of stone masonry—very smooth, slimy, and cold. I followed it up; stepping with all the careful distrust with which certain antique narratives had inspired me. This process, however, afforded me no means of ascertaining the dimensions of my dungeon; as I might make its circuit, and return to the point whence I set out, without being aware of the fact; so perfectly uniform seemed the wall." Upon reading further, the student discovers that the room has corners and other details that allow a sketch of the room to be constructed, and subsequently revised as the student reads more of the story.

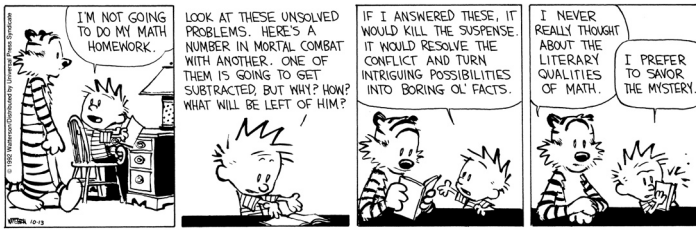


Figure 3: *CALVIN AND HOBBES © 1992 Watterson. Reprinted with permission of ANDREWS MCMEEL SYNDICATION. All rights reserved.*

This article also contains a couple of newspaper comics. I have found that using comics as a teaching aid has resulted in greater student engagement and interest. Comics can simply lighten up the tone of a test or problem. They can also be used as the basis for teacher-generated or student-generated math problems. However, care must be taken to meet copyright requirements.

Integrating literacy and language strategies into our math classrooms is not meant to replace the learning of mathematics. Rather, these strategies and activities can support our students and enhance their learning. For teachers who are not math specialists, I have found that using these strategies provides a comfort level, and this comfort level gets passed on to students.

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Continued from page 27 – Book Review

have used computers 20 to 30 years ago in Ontario classrooms may remember ASCII character seven that caused an audible beep. While programming has evolved considerably since then, it is an example of the sort of historical features that are sprinkled throughout this book. I was reminded of a period years ago when there was debate about whether computers could ever be programmed to "think," and it is interesting to consider Shannon's point of view: "I'm a machine and you're a machine, and we both think, don't we?" (p. 199).

The book has humorous components, while continuing to be biographical. The *ultimate machine*, which Shannon constructed, is described as "a box and switch, which when turned on, produced a whirring of gears and a mechanical hand that emerged from the box, flipped the switch off, and disappeared again" (p. XIV). There is an equation for modelling juggling—a lifelong passion of Shannon's. As a whole, the book conveys a spirit of activity, with games of chess and hex being commonplace in the common areas at Bell Labs. And it was that sense of fun, while being educational, that made this book serendipitous as a holiday read. ▲

▲ The Citee Gamers



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Chantal Buteau is Professor of Mathematics at Brock University, where she has taught, for over ten years, a course in which students learn to design, program computers, and use interactive environments to investigate mathematical concepts, conjectures, theorems, or real-world applications. Her research interests include the use of digital technologies, such as programming, in mathematics learning.



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Meghan Monaghan is an undergraduate student in the Mathematics Integrated with Computers and Applications (MICA) Co-op Honours Program, with a minor in French Studies at Brock University. She is an aspiring future high school Mathematics and French teacher. Meghan believes it is possible for anyone to learn and appreciate math if it's made relevant and taught in a way that suits their learning style.



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Eric Muller is Professor Emeritus at Brock University and a Fellow of the Fields Institute for Research in Mathematical Sciences. It is his experience that students can be motivated to learn mathematics and be creative in its application when they engage in programming. For many students, technology provides a less judgmental environment in which to develop and share their knowledge of mathematics.



ELISA ROCCI
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Elisa Rocci is a recent graduate from the Concurrent Education program at Brock University, holding a Bachelor's degree in Mathematics as well as Education. Her background in programming for mathematics was obtained through a sequence of programming-based mathematics courses at Brock, one of which was for future mathematics teachers only. She is currently a secondary school teacher in Hamilton, Ontario.

> *"I have stressed this distinction because it is an important one. It defines the fundamental difference between probability and statistics: the former concerns predictions based on fixed probabilities; the latter concerns the inference of those probabilities based on observed data."* ~ attributed to Leonard Mlodinow

In the staff room, I enquired if anyone had met the three girls and two boys from our downtown feeder school, who called themselves the *Citee Gamers*. Those who had, agreed that they are not very different from many of the children who use the latest technology gadgets. They are full of dynamic youthful ideals and enthusiasm, and think that they know it all. It's a new generation, not very different from ours at that age, critical of past generations, and not appreciative of what they have accomplished. In a half-hearted way, the Citee Gamers did admit to me that the technology they so masterfully handle had been developed by previous generations. The five students are in my Grade 9 math class and have difficulty fitting into the mathematics curriculum that our generation has developed. The five of them are a bright bunch and often finish the classwork ahead of most other students. They then work hard at keeping busy to avoid having to do more math.

I'd love to get them involved in some mathematics they could enjoy, and from which they could flourish. They happily discuss with me the challenges they face in the latest game, Planet X723. They tell me that most transgalactic pilots need to refuel on Planet R34, where Zyphon, an entrepreneur, has set up a plant to liquefy the deadly planet's atmospheric gas, which makes excellent fuel. But beware, Zyphon is a devious entrepreneur and he is also a gamer. When a pilot pays for a container of fuel, he is not guaranteed to get it, as he must choose from one of three very large intergalactic containers. One is loaded with valuable fuel, while the other two are filled with useless pieces of machinery. Zyphon enjoys his game, which he describes to the pilot as follows: "*Having paid for your fuel, you first choose one of the three containers; then I will reveal the useless pieces of machinery in one of the other two containers; and finally, you will have the opportunity to change your selection to the other unopened one.*"

This intrigues me, as I have seen a similar problem before, and I ask them: *"What is your strategy when purchasing fuel on Planet R34?"*

Citee Gamers: *"We stay with our original choice of*

container, as it makes no difference. We are faced with two containers; it is therefore 50-50 that the fuel is in the one we've selected."

I ask: "Do you know that some mathematicians have questioned your conclusion? It may be worthwhile for you to investigate the situation further. I would be willing to help you with this investigation."

Citee Gamers: *"Okay."*

I propose: *"I suggest you use three cards to model and experiment with the situation."*

The next day, they bring me a hands-on model, which they describe as follows: *"We first divided ourselves into two groups: A and B. Group A picked three cards: the ace of diamonds for the fuel container and two jokers for the containers with useless pieces of machinery. Group A then placed the three cards face down in a row, noting the position of the ace of diamonds. Group B pointed to the card they picked, and group A turned over one of the other two cards, a joker, and invited group B to change their choice. We explored two different cases, one when group B never changed their choice, and another when they always changed their choice."*

I comment: *"I like your simulation. What were your results?"*

Citee Gamers: *"Of 25 repetitions, when group B never changed their choice, they had the ace of diamonds 9 times and a joker 16 times, and of the 25 other repetitions, when they always changed their choice, they had the ace of diamonds 18 times and a joker 7 times."*

I comment: *"Are you now convinced that you should consider changing your choice?"*

Citee Gamers: *"Somewhat, but we would need many more repetitions to be really convinced. However, we've had enough of this boring repetitive stuff."*

I point out: *"You have all this tech. Can that help?"*

Citee Gamers: *"We have no idea how to do that. We could search online."*

I suggest: *"Let me start you off using Excel. We have to break down your model into a set of unambiguous steps and translate them into 'Excel' language that the software will be able to carry out."*

In the Appendix, Citee Gamers and I have included the details of how we developed our Excel simulation program. We chose to repeat the selection 550 times and produced the following graph.



Citee Gamers commented: *"It would be trivial for us to repeat it 5000 times or more. We're now convinced that next time we buy fuel from Zyphon, we'll change our original choice of container."*

I raise a further question: *"I still have a question that would concern me when bargaining with Zyphon. We've only looked at the two extreme cases where the pilot always changes or never changes selection of container. Could she do better by changing sometimes? For example, by tossing a coin and changing when heads and not changing when tails, or rolling a die, changing if she rolled a six, but not otherwise? I'll leave this for you to consider."*

A good week went by before the Citee Gamers came to me with another graph. In their simulation, they had considered what would happen if they changed their selection of container based upon the result of rolling a die, and had modified the original algorithm, which they then programmed. They had run the program 600 times for each probability, and because the number of wins increased from never changing to always changing, they told me that they were now convinced that their best choice was to always change their choice of container.

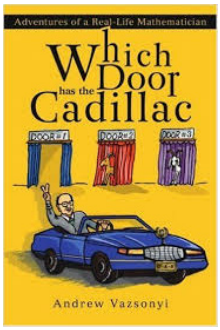


I asked Citee Gamers to describe the changes they made to the original program—interested readers can access the details of this program. It can be found in the members only area of the website under *Files and Links*.

I was surprised when Citee Gamers approached me some weeks later and told me that they had come across the Scratch programming environment, which they found was a lot of fun. They used it to create a guided exploration of the Zyphon problem through five tasks. These have also been posted in the members only area of the website under *Files and Links*.

Since working on this project with Citee Gamers, I have shown them that there are many problems in the present mathematics curriculum that they can attempt by using programming. They are able to design explorations, simulate complex problems, model and simulate real-world situations, visualize mathematical concepts, etc. I'm confident they'll think of other uses for their programming talents.

Reflections



You may have recognized that Zyphon's crooked game is a form of the Monty Hall problem. Marilyn von Savant (1990/91) published it and its solution in her column of *Parade Magazine* (e.g., https://en.wikipedia.org/wiki/Monty_Hall_problem). The outcome of her publication was a lot of mail by individual mathematicians with differing opinions about its solution. This is part of the attraction of this problem; its solution is counter-intuitive. Andrew Vázsonyi, a pioneer in Operations Research, wrote the book *Which Door Has the Cadillac: Adventures of a Real-Life Mathematician* (2002). He was a friend of Paul Erdös, one of the most prolific twentieth-century mathematics problem solvers. They had grown up together in Hungary and were both successful mathematicians. Vázsonyi (1999) recalls mentioning to Erdös the Monty Hall problem with its solution. To his surprise, Erdös said, "No, that is impossible, it should make no difference" (p. 18). Vázsonyi then showed him the results of a simulation of the problem, to which "Erdös objected that he still did not understand the reason why, but was reluctantly convinced that I was right" (p. 18). The simulation provided Erdös with an insight he had not gained from his extensive experiences in mathematics problem solving.

In his *Gazette* article "Auston Matthews and Monte Hall," Nat Banting (2016) analyzed the probabilities in the NHL Entry Draft of a team being first or second in the order of selecting a new prospect. He then discussed the Monte Hall problem and argued that, "Examined together, each offers insight into the other through their structural similarities" (p. 35). We get insights into solving a mathematics problem in different ways. Some are based on our interests and experiences.

It is worth summarizing the Citee Gamers' engagement. They first developed a **model** of the problem, using tactile items (cards) and a set of instructions that they used to **simulate** the problem. They then translated these instructions into a language, a **program**, which a computer could implement. Finally, they communicated through graphs and tables the results of the computer simulation—a **visualization**. Their objective was to find the best, **optimal**, solution in their quest for fuel in their intergalactic travels. In their work, Citee Gamers were involved in twenty-first-century mathematics. This way of doing mathematics has become so prevalent that the European Mathematical Society (made up of about 60 national mathematical societies in Europe) has identified that "…a Third pillar of scientific inquiry of complex systems has emerged in the form of a combination of **modelling**, **simulation**, **optimization** and **visualisation**" (EMS, 2011, p. 2 – bold font added by the authors).

In parallel with this development, the broad education community has developed the concept of Computational Thinking (CT), which can be described as "the thought processes involved in formulating a problem and expressing its solution(s) in such a way that a computer—human or machine—can effectively carry out" (Wing, 2014, p. 5). Citee Gamers engaged in these thought processes as they designed and programmed in Excel a simulation to the probabilistic situation, and used it to explore its solution. In short, Citee Gamers were engaged in CT for a mathematics investigation, and they are not alone. You are invited to read about the activities of one of Citee Gamers' Grade 3 cousin in her story of "Bhanu and the Chocolate Cupcake," which appears in this issue of the *Gazette*. By the way, the Citee Gamers were pleased to tell me that they were now rebranded as the CT Gamers.

Use of programming has started to take place in our Ontario math classrooms, some being described on the Ministry of Education-funded website, *Mathematics Knowledge Network* (www.mkn-rcm.ca/ct/). As a final note, Eric Muller included many of the ideas in a Plenary Presentation at the 40th Annual Meeting of the Canadian Mathematics Education Study Group (2016), which can be retrieved from the Proceedings (www.cmesg.org/wp-content/uploads/2017/07/CMESG-2016.pdf).

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Appendix 1

Excel Program to Simulate Zyphon's Game

1. Citee Gamers: *"First, let's name the three containers 1, 2, 3, and direct the computer to select one of these to contain the fuel and to place its number in the first column A. To do this, we select the cell A3 and type in the code* =RANDBETWEEN(1,3) *so as to randomly select an integer from 1, 2, and 3."*
2. Note: We have typed CONTAINER in cell A1 and WITH FUEL in cell A2. We will continue to type in names in the cells in lines 1 and 2.

   I: *"What should we program next?"*
3. Citee Gamers: *"Get the computer to select the container that the pilot chooses by typing* =RANDBETWEEN(1,3) *in cell* B3."

   Note: Don't worry if the numbers in previous cells change as we enter new instructions.
4. I: *"Great, let me give you the code for the next step, where the computer is given instructions to select the container that Zyphon will choose to open and reveal useless machinery. We wish to have that container in cell C3."*

   =CHOOSE(A3,IF(B3=1,RANDBETWEEN(2,3),IF(B3=2,3,2)),IF(B3=2,IF(3*RAND()<1.5,1,3),IF(B3=1,3,1)),IF(B3=3,RANDBETWEEN(1,2),IF(B3=1,2,1)))

   Let me give you a few pointers with this new language. Brackets play a central role and always come in pairs; for example, the first bracket after CHOOSE is paired with the very last bracket at the end. For our purpose of understanding, let me write the code on separate lines:

   =CHOOSE(A3,
   IF(B3=1,RANDBETWEEN(2,3),IF(B3=2,3,2)),
   IF(B3=2,IF(3*RAND()<1.5,1,3),IF(B3=1,3,1)),
   IF(B3=3,RANDBETWEEN(1,2),IF(B3=1,2,1))
)

   The program will CHOOSE *to implement one of the three IF lines based on the value of* A3. *For example, when* A3 *is 3 (the third container), it will implement the third* IF *sentence, namely* IF(B3=3,RANDBETWEEN(1,2),IF(B3=1,2,1)), *which says that if the pilot has chosen the third container* B3=3, *then Zyphon can choose to open either container 1 or 2,* RANDBETWEEN(1,2); *however, both cases where the pilot chooses either container 1 or 2 are implemented by* IF(B3=1,2,1) *(in English if* B3=1, *choose container 2, otherwise* (B3 *is not equal to 1 and we already know that* B3 *is not 3) choose container 1. I will now leave you to explain the details of the remaining code.*
5. *The pilot is offered to change to the other unopened container. Click on cell D3 and type:* =CHOOSE(A3,IF(B3=1,IF(C3=2,3,2),1),IF(B3=2,IF(C3=1,3,1),2),IF(B3=3,IF(C3=1,2,1),3))

   (This indicates which container the pilot can change to if he so chooses.)
6. a) *If the pilot decides not to change, then: Click on cell* E3 *and type:* =B3

   b) I*f the pilot decides to change, then: Click on cell* F3 *and type:* =D3

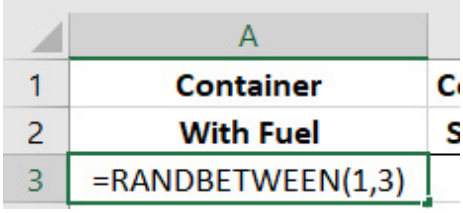
   (Column E *indicates which container is selected if the pilot does not change his mind, and Column* F *indicates which container is selected if the pilot does change his mind.)*
7. We now want the computer to count the number of times the pilot gets the fuel (a win which we denote by "1") and the number of times the pilot gets a container full of useless parts (a loss which we denote by "0").

   a) *If the pilot decides not to change, then: Click on cell* G3 *and type:* =IF(E3=A3,1,0)

   b) *If the pilot changes to the other unopened container, then: Click on cell* H3 *and type:* =IF(F3=A3,1,0)

   (Column G *will indicate whether the pilot wins or loses if he does not change his container selection, and Column* H *will indicate whether the pilot wins or loses if he does change his container selection.)*

	A	B	C	D	E	F	G	H
1	Container	Container	Container	Alternate			Win=1	Win = 1
2	With Fuel	Selected	Opened	Container	Stayed	Changed	Stayed	Change



8. Now repeat this 550 times. You can easily apply the code to other rows by selecting cells A3 to H3, then click and drag the black square in the bottom right corner of the cell down however many rows for which you want to repeat the experiment.

	A	B	C	D	E	F	G	H
1	Container	Container	Container	Alternate			Win=1	Win = 1
2	With Fuel	Selected	Opened	Container	Stayed	Changed	Stayed	Change
3	3	2	1	3	2	3	0	1

The coding above will result in something like this:

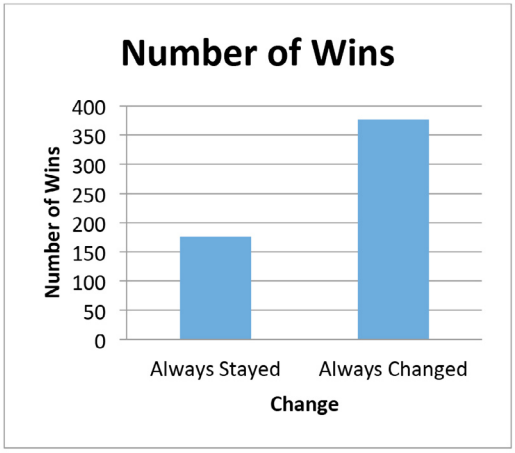
*NOTE: It may not look exactly the same because of the randomization of containers selected.

	A	B	C	D	E	F	G	H
1	Container	Container	Container	Alternate			Win=1	Win=1
2	with fuel	selected	opened	container	stayed	changed	stayed	changed
3	3	2	1	3	2	3	0	1
4	2	3	1	2	3	2	0	1
5	2	3	1	2	3	2	0	1
6	3	1	2	3	1	3	0	1
7	2	2	3	1	2	1	1	0
8	3	2	1	3	2	3	0	1
9	3	3	2	1	3	1	1	0
...	...	...	...	...	...	...	...	...
							=Sum (G3:G552)	=Sum (H3:H552)

9. We now get the computer to sum the number of wins in columns G and H, and plot the results.
10. Create a new table in Excel that looks like this:

Always Stayed	Total # of wins
Always Changed	Total # of wins

11. Highlight this new table, then click “Insert.” Then under “Charts,” select the bar graph to obtain a graph as follows:



▲

▲ Bhanu and the Chocolate Cupcake



CHANTAL BUTEAU
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Chantal Buteau is Professor of Mathematics at Brock University, where she has taught, for over ten years, a course in which students learn to design, program computers, and use interactive environments to investigate mathematical concepts, conjectures, theorems, or real-world applications. Her research interests include the use of digital technologies, such as programming, in mathematics learning.



ERIC MULLER
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Eric Muller is Professor Emeritus at Brock University and a Fellow of the Fields Institute for Research in Mathematical Sciences. It is his experience that students can be motivated to learn mathematics and be creative in its application when they engage in programming. For many students, technology provides a less judgmental environment in which to develop and share their knowledge of mathematics.



ELISA ROCCI
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Elisa Rocci is a recent graduate from the Concurrent Education program at Brock University, holding a Bachelor’s degree in Mathematics as well as Education. Her background in programming for mathematics was obtained through a sequence of programming-based mathematics courses at Brock, one of which was for future mathematics teachers only. She is currently a secondary school teacher in Hamilton, Ontario.

“Many students asked if they were doing coding or math? They really had not previously considered that the two were connected.” (J/I teacher who used Scratch in her teaching of probability, April 2018)

Before getting to the cupcakes!

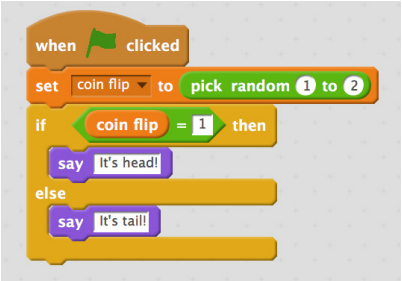
On one hand, we love cupcakes. On the other hand, coding is on our mind basically every day (and so are the cupcakes). Then, there is an intriguing probability problem that we love. What do they have in common? Bhanu's story.

Her story engages you in mathematical activities that we developed with two goals in mind: first, by participating (possibly for the first time) in mathematical work in the Scratch programming environment; and second, exploring ideas of probability through coding and simulation. We prepared the task, integrating a youthful approach. Have you noticed how they easily "click and try" and "retry"? So, as you read through Bhanu's story and work with the Scratch activities, put yourself in the shoes of a young student: click and try! Click and try! Then alter the code and see…. We will be there all along with you, interpreting the mathematical results. The icing on the cupcake comes at the end when we get a taste of an unintuitive probabilistic situation.

The Story

Some years back, I had saved a jar full of one-cent coins. I was surprised that some of my Grade 3 students had never seen one. Some students suggested we should also get rid of the five cent coin. And one of the students, Bhanu, responded: *"I travelled to New Zealand last summer, and they actually have no 5-cent coin there! The smallest coin is worth 10 cents."* We focused back on our mathematical work and used the pile of 1-cent coins for a review of probability. We tossed the coins, recorded, tabulated, graphed, and then discussed the results as more and more data were obtained.

As part of this review, I introduced the class to block-based Scratch programming, freely available from www.scratch.mit.edu/. The developers say (on the website), "With Scratch, you can program your own interactive stories, games, and animations—and share your creations with others in the online community. Scratch helps young people learn to think creatively, reason systematically, and work collaboratively—essential skills for life in the 21st century." My students had fun developing their program to simulate the tossing of coins. I explained that this is called a computer simulation because they are only pictures of the real things. Figure 1 presents a screenshot of what the class completed in Scratch to explore whether a coin would come up heads or tails. Click on www.scratch.mit.edu/projects/252371559/ to run the program (by clicking on the green flag).



When we toss a coin, there are two possible outcomes: heads or tails. Therefore, in this program, we can let the number 1 represent heads and the number 2 represent tails, and allow our program to randomly choose either the number 1 or 2 to simulate flipping a coin.

Figure 1: *Scratch program ready to run to simulate flipping a coin*

A Few Days Later

It was only a few days later that Bhanu came to me very excited about what she had done the previous afternoon. I should explain that, because her mother works in the afternoons, Bhanu has a sitter called Amira. Bhanu really likes Amira and looks forward to those days when she has a trick or surprise for her. Yesterday afternoon she came home to find two identical sealed boxes on the kitchen table. Amira explained that one of the boxes contained her favourite cupcake, a chocolate cupcake with chocolate icing, while the other contained broccoli, not Bhanu's favourite vegetable! Amira told her to pick one of the boxes, and whatever was inside would be her snack. Bhanu was not allowed to open or touch the boxes. Excitedly Bhanu told me that she thought her situation was similar to what we had done in Scratch and had developed a simulation with her dressed in pink (see Figure 2).

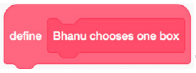


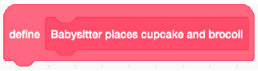
Figure 2: *Bhanu's set-up for her simulation*

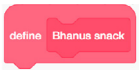
Of course, she not only wanted to show the class her Scratch simulation set-up, but also share with them how she

had amended her program from coins to cupcake and broccoli. This is how Bhanu displayed it for the class:

- She opened her www.scratch.mit.edu/projects /243992883/ program and clicked on 

to open the code.
- She clicked on Bhanu  (bottom right – you might need to scroll down to see it among the other "sprites") to show the code displayed in the middle, and also clicked  (top right) to have more of the code displayed.
- She read the code under 

.
- She completed the code under 

and under 

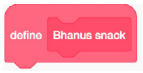
by dragging the missing pieces.

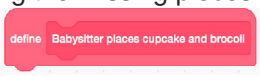
First, she clicked on  (top right) to see the simulation she just programmed, in full screen. Then she proudly ran the program many times by clicking the green flag.

Now you do it. What do you find? How many times does Bhanu get the chocolate cupcake? How many times does she get the broccoli? Bhanu now decided that she would try to have her program simulate not only one trial in one click, but many. To show the class, she opened her other Scratch project, www.scratch.mit.edu/projects/243974530/, and clicked on 

to open the code. Her program, which keeps track of her simulation of five attempts to get a cupcake, was almost done. You can now help Bhanu complete her program:

- Click on Bhanu  to see the code displayed in the middle.
- Read the code under 

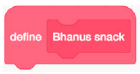
. You will notice that the overall code will simulate five attempts and will keep track of the success (that's the cupcake!) and failure (the broccoli).
- Bhanu's code to select one box on the table is already done for you. Complete the code under 

by dragging the missing pieces. And fix the code under 

.

Now run the program. Does it work? This program runs five times on one click and keeps track of her snacks. Run it many times. What do you find? How many times does Bhanu get the chocolate cupcake? How many times does she get the broccoli?

And now modify the code to run 100 times (check the code under 

). You might want to run a "faster" simulation and change the waiting times in 

to 0 seconds….

What did you find? How many times does Bhanu get the chocolate cupcake? How many times does she get the broccoli? Repeat this simulation by clicking on the green flags ten times. Are the results: i) exactly the same, ii) approximately the same, or iii) very different?

In front of the class, Bhanu showed that she had about a 50-50 chance (i.e., a probability of 0.5) of choosing the box with the chocolate cupcake. She concluded, as we had shown with the coins, that it was a fair game as we had defined it in probability. Nevertheless, she said it really sucked because on the previous night, she had picked the box with the broccoli. Fortunately, Amira was so impressed with what Bhanu had done in Scratch that she gave her a piece of chocolate.

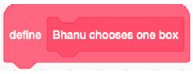
And the Next Day

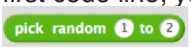
I was surprised when Bhanu came to class the next day even more excited. On the previous evening, Amira had set up a different challenge for Bhanu, as she found three identical sealed boxes on the table. Amira told her that her favourite chocolate cupcake was in one of the boxes, and that the other two boxes contained broccoli. Again, Bhanu had to pick a box and accept whatever was in it for a snack. Bhanu hoped that her chance of getting the chocolate cupcake was still about 50-50, but wanted to explore the problem using yesterday's Scratch simulation.

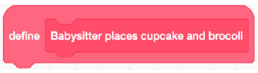
You can help her modify the program to simulate the new situation with three boxes, as follows:

- Click on , (then click on 

to place a third box on the table. Do the same with the second broccoli: click on the icon, and click on 

to give the babysitter a second broccoli.
- Click on Bhanu  to see the code displayed again.
- Modify the code under 

to add the possible choice of a third box by dragging the code pieces to the right. Then, don't forget to look at the very first code line; you must modify something therein: 

.
- Modify (only one thing to change!) the code under 

.

Now run the simulation 100 times.

Are you surprised that the chance to get the chocolate cupcake has changed so much? Run the simulation another

100 times. What do you conclude is Bhanu's approximate chance to pick the box with the chocolate cupcake?

In front of the class, Bhanu showed that she had about a 1 in 3 chance (i.e., a probability of 1/3) of choosing the box with the chocolate cupcake, and she concluded that the game was not a fair game (and definitely not to her "sweet tooth" advantage!).

Wow, and the Following Day

Apparently Bhanu's sitter, Amira, had been so impressed with Bhanu's explorations with Scratch that she challenged her with a situation that she had found counter-intuitive. Bhanu was again faced with three sealed identical boxes—one with Bhanu's favourite chocolate cupcake and the other two with broccoli. However, Amira changed the way that Bhanu was to select a box. This is how it worked: Bhanu selected a box and then Amira, who knew which box contained the chocolate cupcake, opened one of the two boxes that Bhanu had not chosen, and that contained broccoli. Amira, always so kind, allowed Bhanu to change her selection, if she wanted to. The BIG question for Bhanu was whether she would have a better chance of getting the chocolate cupcake if she stayed with the first box she had selected, or if she switched to the other unopened box. What do you think she should do?

Bhanu told Amira that, since there are two unopened boxes in front of her, and one of them contains the cupcake and the other the unwanted broccoli, she was in the same situation as she had been on the first day: she felt she had a 50-50 chance of getting the cupcake, no matter what. But Amira's suspicious look suggested she should rethink her conclusion. To do this, Bhanu decided to once more use Scratch to simulate and explore the situation. As she was starting, Amira whispered, "Maybe it depends on what happens before you get to two boxes…."

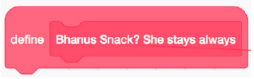
Bhanu found this situation rather challenging. Following Amira's hint, she decided to look at the problem from the start and divide it into two: i) when she would always keep her first choice, and ii) when she would always change her choice to the other unopened box.

We suggest you follow her two sets of instructions.

1. First part – Bhanu always keeps her first choice.

Open Bhanu's "BhanusCupcake-Version3.sb2" program, click on 

to open the code, and amend the program to simulate it as follows:

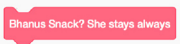
- Click on Bhanu  to see the code displayed.
- Complete the code in 

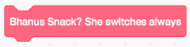
about whether Bhanu gets a cupcake or not, depending on her initial box selection, by dragging the code pieces. Read the comments to help you.
- Check that it works.
- Modify the code to simulate 100 attempts at getting a cupcake. Run it a few times. What do you think? Has Bhanu improved her chances of choosing the chocolate cupcake compared to Day 2? Are you surprised?

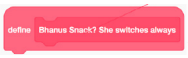
2. Second part – Bhanu always changes to the other unopened box.

Continue amending the code to simulate this situation:

- Bring "Bhanu's Snack? She switches always" in the code under 

by switching the two code blocks, 



.
- Complete the code in 

by dragging the pieces found underneath it. Read the comments to help you. What happens if Bhanu initially picked the box with the cupcake?
- Check that it works.
- Modify the code to simulate 100 attempts at getting a cupcake. Run it a few times. What do you think? Has Bhanu improved her chances of choosing the chocolate cupcake compared to Day 2? Are you even more surprised?

So, if you were Bhanu, what would you do? Would you keep or switch your first choice? Does it matter? It turns out that yes, it matters. And even numerous mathematicians have been surprised by this counter-intuitive result, that changing to the unopened box gives you a two-thirds probability of getting the cupcake… or getting a Cadillac for participants on the *Let's Make a Deal* game show, hosted in the 1960s to 1990s by Canadian Monte Halparin, aka Monty Hall. This situation is now known as the Monty Hall problem, which Marilyn von Savant (1990/91) published along with its solution in *Parade Magazine*. The class looked up the Monty Hall problem online and were surprised that mathematicians could have differing reactions. For example, the "Reflections" section in "The Citee Gamers" in this issue of the *Gazette*, details the reaction of a famous mathematician, Paul Erdös, who only through computer simulation came to accept the result, while stating that he did not grasp it.

One way Marilyn argued for thinking through the situation was (put into Bhanu's context) to consider 1000 boxes, 999 of them containing broccoli and 1 containing a delicious chocolate cupcake with Belgian chocolate icing. And you go with a first pick, say box #234. The host opens 998 boxes, all of which contain broccoli, and leaves box #816 unopened. What will you do? Would you keep box #234, or would you

change to box #816?

Thinking while enjoying the cupcake….

In terms of probability, this adventure allowed us to engage in curriculum expectations where we actively conducted probability experiments and simulated Bhanu's situation, using Scratch programming. These activities exposed us to probability concepts and the language of probability that begin in Grade 2. Perhaps the Monty Hall problem may have taken us beyond the Grades 1–8 curriculum (but what fun!). Scratch programming provided an environment for us to explore probability concepts and for problem solving, where we broke down the problem into smaller pieces, used variables, and were exposed to the Scratch coordinate system—all of which highlighted the mathematical aspect of engaging in programming. In fact, for example, Francis and Davis' (2018) research concluded that "programming robots to move might support the [children's] development and integration of powerful instantiations of number, arithmetic and multiplication" (p. 71), and that their "findings suggest that children's understanding of numbers, and their transitions from additive to multiplicative thinking, can be powerfully supported by engaging in [robotic] practical tasks rather than practice exercises" (p. 71).

Use of programming, also called coding, has started to take place in our Ontario elementary math classrooms. Interested readers can turn to a number of publications, including the following:

Francis, K., & Davis, B. (2018). Coding robots as a source of instantiations for arithmetic. *Digital Experiences in Mathematics Education, 4*(2–3), 71–86.

Gadanidis, G., Brodie, I., Minniti, L., & Silver, B. (2017, April). Computer coding in the mathematics K–8 curriculum? Retrieved from www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/Computer_Coding_K8_en.pdf

Discusses important issues and provides examples of applications with Scratch.

Math+Code 'Zine. (2019, March). Retrieved from www.researchideas.ca/mc/

Has useful teaching stories and resources.

Mathematics Knowledge Network. (n.d.). Retrieved from www.mkn-rcm.ca/ct/

Contains interesting examples of in-class applications, e.g., "Computational Modelling in Elementary Mathematics Education Making Sense of Coding in Elementary Classrooms" by Gadanidis and Caswell, or the Grades 7–8 "Probability & Scratch" task by Dreise and Couturier.

TVO Teach Ontario. (n.d.). Coding & computational thinking in the classroom. Retrieved from www.teachontario.ca/community/explore/coding-in-ontario-classrooms/pages/main

Provides worthwhile materials and discussions.

Please note that to facilitate the in-class use of the Bhanu activities, we have posted a PDF file entitled "Choose Your Cupcake Carefully!" It can be found in the members only area of the website under *Files and Links*. ▲

▲ What is: Strategic Planning and Renewal Committee (SPaRC)

BY THE SPARC 2019/2020 COMMITTEE

SPaRC is one of the many driving forces of OAME/AOEM. We are one of the smallest committees and currently have four members, but the work we do is very important to the ongoing health of OAME/AOEM.

It begins with the organization's backbone—the Constitution. The Constitution is the governing document of OAME/AOEM. SPaRC takes its direction from the Board of Directors and the Executive Committee, and recommends revisions to the Constitution, By-Laws, and Terms of Reference, as needed. "Each of these three sections serves a specific purpose. The Constitution is intended to be a clear set of statements about the mission, goals, and objectives (i.e., the *raison d'être*) of the organization. The By-Laws are the rules and regulations that govern what the organization actually does; think of it as you would a city's/town's/municipality's by-laws—the laws. The Terms of Reference contain how those laws are to be carried out (i.e., the nuts and bolts)" (Otto, 2017, p. 42).

Generally, we work between Board meetings on the directions given to us from the various committees at the most recent Board meeting. We meet face-to-face and collaborate remotely, using a variety of tools.

If you would like more information about SPaRC and its function, please see Otto (2017) in the *Gazette* archives. Bill continues on the SPaRC committee, having become the inaugural Director Emeritus Bill Otto.

Reference

Otto, B. (2017). Nothing like a good constitution. *OAME/AOEM Gazette, 56*(1), 42–43.

OAME/AOEM AWARDS FOR EXCELLENCE IN MATHEMATICS EDUCATION



The purpose of the OAME/AOEM Awards is to formally recognize individuals and educators who demonstrate an outstanding contribution to mathematics education in Ontario. The successful candidate for each award enhances the learning environment for all students, providing opportunities for students to do, see, hear, and touch mathematics in a profound and meaningful way. If you know someone who qualifies for this honour, please nominate her/him/them for one of the following awards.

OAME/AOEM Life Membership Award: This award recognizes an educator who has contributed in a significant way to OAME/AOEM, demonstrates outstanding leadership in mathematics education, and has accumulated 10 or more years of membership in OAME/AOEM.

Award for Exceptional and Creative Teaching in Elementary Mathematics: This award recognizes an exceptional and creative elementary educator who demonstrates excellence in mathematics education and contributes to the overall development of students.

Award for Exceptional and Creative Teaching in Secondary Mathematics: This award recognizes an exceptional and creative secondary school mathematics educator who demonstrates excellence in mathematics education and contributes to the overall development of students.

Elementary School Staff Award for Exceptional and Collaborative Mathematics Teaching: This award recognizes an elementary school (a school accommodating any or all of grades K–8) whose staff fosters collaborative and excellent mathematics teaching, contributes to the overall development of student learning of mathematics, and demonstrates leadership in the mathematics community.

Secondary School Mathematics Staff Award for Exceptional and Collaborative Teaching: This award recognizes a secondary school mathematics staff which fosters collegiality, team work, and excellent classroom teaching, contributes to the overall development of students, and demonstrates leadership in the mathematics education community.

Award for Leadership in Mathematics Education: This award recognizes an educator who has demonstrated leadership by contributing in a significant way to the development of mathematics educators and the enhancement of mathematics education in Ontario through innovation and/or research related activities.

Secondary School Department Award for Exceptional and Collaborative Mathematics Teaching: recognizes a secondary school mathematics staff, who foster collegiality, teamwork, and excellent classroom teaching; contributes to the overall development of students; and demonstrates leadership in the mathematics education community.

Award for Outstanding Contribution to OAME/AOEM and Mathematics in Ontario: This award recognizes an individual who has made a significant contribution to OAME/AOEM and contributed directly or indirectly to the improvement of mathematics education in Ontario but who is not necessarily an educator or a member of OAME/AOEM.

OAME/AOEM Awards Nomination Process

1. Complete the **Award Nomination Cover Page** appropriate to the specific award, found at www.oame.on.ca (look for the **Awards** tab on the left of the page) or contact the Executive Directors at eds@oame.on.ca for an electronic version. You may include a one page letter of recommendation.
2. Include with the nomination cover page up to three **letters of support**, of no more than two pages each. Please ensure all letters of support:
   - Address the criteria for the award as outlined in the nomination cover page.
   - Include the name and position of the supporter to the application.
   - Include the nominee's name and the award in the letter.
3. Include the **Curriculum Vitae**, of up to five pages, for award nominees except for Elementary and Secondary Staff awards.
4. Email or mail all **completed** packages (cover page, supporting letters and CV) by the deadline advertised below to the Executive Directors. Email address and mailing address are included in the nomination cover page.

DEADLINE: Nominations are due by January 31, 2020.

The awards will be presented at the Annual OAME Conference in May 2020.

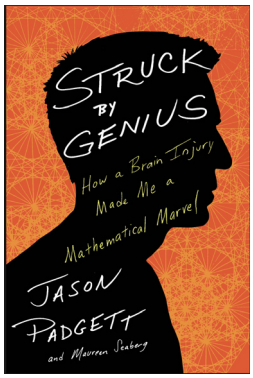
▲ BOOK REVIEW: MATH ABILITY AND THE INJURED BRAIN



TIM SIBBALD
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Padgett, J., & Seaberg, M. (2014). *Struck by genius: How a brain injury made me a mathematical marvel.* Toronto, ON: Harper Collins Publishers Ltd.



The capacity of the human brain is, by all accounts, remarkable. In the case of Jason Padgett, he and co-author, Maureen Seaberg, describe his journey up to and after a brain injury that produced acquired synesthesia and savant syndrome. The challenge for the book is conveying such a fundamental alteration in cognitive processing. To give a sense of this, consider the cognitive implications of the following post-injury description: "The structure of flowing water vibrates in a specific geometric form and frequency to me, and if it were to freeze mid-stream, I'd see a web, but one made up of tiny crystals rather than spider's silk" (p. 3). He later describes seeing numbers as geometrical shapes and strands of numbers as "crystal grids" (p. 46).

A test for synesthesia is described as having a square containing many 5's with 2's among the numbers. People with synesthesia can identify the 2's very quickly because they associate numbers with an additional sense, such as color. That allows them to spot the different values more rapidly. However, if you try this in your classes, be forewarned that synesthesia occurs in only four percent of people.

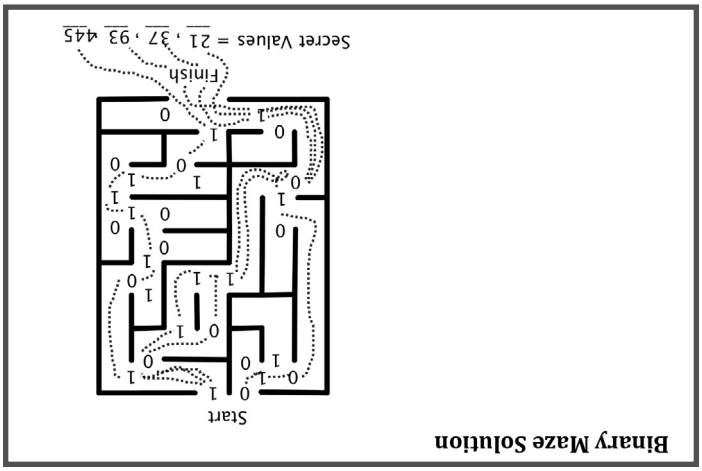
Some aspects of the book are challenging to read because they lack mathematical precision or details to clarify events. For example, pi being described as an "infinite number" (p. 75) lacks precision—it is a finite number with an infinite number of non-repeating digits. An example lacking details was Padgett being flown across the United States to be consulted by people making a planetarium. Subsequent name-dropping suggests the authors are trying to negotiate the validity of his mathematical ability by associating it with famous people. Feedback he recounts seems to rapidly escalate to imply he is performing at a very high level, on par with unique historical figures. Unfortunately, while there is little doubt that the savant aspect of his condition allowed him to accomplish remarkable things, the associations with famous people, and seemingly generous interpretation of his accomplishments, creates a tension for the reader—wondering where to draw the line between achievements and the psychology of confidence after trauma.

The authors speak to teachable moments. For example, Padgett recounts showing a teacher a formula he "created" to approximate pi by splitting a circle into triangles. The formula is presented as

$$f(x)=\frac{x\sqrt{2-2\cos\left(\frac{360}{x}\right)}}{2},$$

and he describes his teacher using a graphing calculator to see that as x gets larger, the value of $f(x)$ converges to pi. It can be derived using the cosine law and is an interesting activity suited to Grade 10 or 11 trigonometry or as an instance of a horizontal asymptote in Advanced Functions (MHF4U). However, the reason it is interesting in terms of the book is that Padgett realized the formula as an adult taking math classes, and it speaks to a moment of importance to him as a student.

The book as a whole is a worthwhile read for those who are not aware of synesthesia and the apparent hidden capacity of the human brain. This book challenges teachers to consider how students with brain injuries experience recovery. It also provides an interesting context in which to consider how confidence evolves after a traumatic event—something every teacher needs to mull over now and again ▲.



Binary Maze Solution

▲ Opportunities to Address Metacognition and Higher-Order Thinking Skills When We Teach Trigonometry



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Metacognition is often defined as "thinking about thinking" (Flavell, 1979). While a catchy phrase, this definition doesn't really tell us much. Metacognition involves concepts like goal setting, planning, monitoring solution progress, and monitoring for clarity and accuracy. A somewhat related concept is Higher-Order Thinking Skills (HOTS). Both metacognition and HOTS are important for deep learning, where students make connections and integrate knowledge into internal cognitive networks (Campbell & Cabrera, 2014). Deep learning is an important dimension for students' understanding and retention of knowledge, and so it is a critical goal of mathematics education. The good news is that deep learning doesn't need to be another "add-on" to an already challenging curriculum. We can use what we already do in math classes to build our students' deep learning, as will be demonstrated in this article.

HOTS are often identified using a taxonomy of learning, such as a Revised Bloom's Taxonomy. This taxonomy is shown in Figure 1 in the Appendix. The top three levels of Revised Bloom are typically designated as HOTS, along with the Implementing level of Apply (applying knowledge to new situations). Revised Bloom doesn't address metacognition, so a different taxonomy, Marzano's Taxonomy (Appendix, Figure 2) is more useful, since it explicitly includes metacognition. HOTS levels for Marzano's Taxonomy are also shown in Figure 2 (Appendix). So how do we address these deep learning concepts in math, without adding a lot of new content and added time? Consider some sample activities from trigonometry. Each basic activity is described briefly, followed by how you can address deep learning ideas, using the basic activity. All these activities are classroom tested and work for promoting both metacognition and HOTS.

Introduction to Trigonometric Ratios

This activity was designed by Texas Instruments for use with the list feature of their graphing calculators, but it transitions well to any spreadsheet program like Excel. The activity introduces terminology (opposite, adjacent, hypotenuse), as well as the three basic trig ratios. It emphasizes that the ratios for any angle are the same for any similar triangles with the same angles. The activity fits well in MPM2D or MFM2P.

Materials: paper (lined paper is best), rulers, protractors, a set of numbered slips of paper (1 cm, 2 cm, up to about 25 cm—do not exceed the width of the paper).

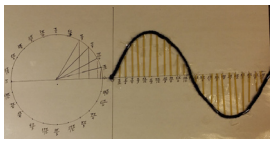
1. Each student selects a slip of paper.
2. Construct a line segment of the given length.
3. At one end of the line segment construct a right angle.
4. At the other end of the line segment construct a 30º angle.
5. Complete the triangle by extending the arm of the angle to meet the perpendicular at the other end of the line segment.
6. Measure and record the lengths of the three sides of the triangle.
7. Build a class chart of the measurements, under the headings "opposite," "adjacent," and "hypotenuse."
8. Enter the information into the first three columns of a spreadsheet. Be sure to set the number of decimal places to 3.
9. In the next three columns of the spreadsheet, compute the ratios opposite/hypotenuse, adjacent/hypotenuse, opposite/adjacent.
10. Surprise! No matter what the lengths of the sides of the triangle, the values for the ratios are the same (or at least they should be, if no measurement errors occurred).
11. Rename the ratios as "sine," "cosine," and "tangent."

To get this activity into HOTS territory, use "What If?" questions: What if we had used a 60º angle? What would be the same? What would be different?

What if we had used a 45º angle? What if we had used the other (complementary) angle in the triangle? (This gives

a great introduction to the concept that the co-ratios of complementary angles are equal.)

Constructing Sine Curves



This activity is based on the article "Spaghetti Sine Curves" from a 2013 *The Mathematics Teacher*. The HOTS are built into the activity (e.g., predict) and by using "What If?" questions that are posed by the students. Typical "What If?" questions include changing the diameter of the wheel, changing the amount that the wheel is buried in the ground, changing Wanda's direction, the impact of pauses if Wanda gets tired, "squashing" the wheel to produce an ellipse.

The Saga of Wanda Worm

Wanda is in the worm playground. One of the rides is a wheel that is partially buried in the ground. Wanda starts at ground level (0), and crawls counter-clockwise around the wheel. When she gets to ground level, she eats her way around the portion of the wheel that is buried in the ground until she gets back to her starting point. This is a good introduction to sinusoidal function graphs. If used in Grade 11, you may wish to change the radian measures to degrees.

1. Predict the shape of the curve representing Wanda's height above the ground versus central angle of the wheel.
2. Sketch your prediction.
3. For each angle from $\frac{\pi}{12}$ to 2π in steps of $\frac{\pi}{12}$, construct a right-angle triangle. Then measure the vertical height above the ground (or below the ground, if necessary), and break a piece of spaghetti to the correct length.
4. Tape each spaghetti piece above the appropriate angle on the graph.
5. Lay a piece of yarn along the tops of the spaghetti pieces to represent Wanda's trip around the wheel.
6. Does the shape of the yarn match your prediction?
7. What is the relationship between the maximum height on your graph and the radius of the wheel? Why does this happen?
8. Construct at least three "What If?" questions related to this activity. Share with your group.

Going in Circles

This is a familiar activity that was popular when graphing calculators were the dominant classroom technology. It is one of the few that I still use, making use of the motion sensor (CBR). The metacognition and HOTS are embedded in the activity (hypothesize, verify, refute, pose, justify). This is an introductory activity for graphs of periodic functions. It would usually be done in Grade 11.

Materials needed: CBR, graphing calculator, hula hoop, measuring tape.

One person from the group will walk around the outside edge of a hula hoop placed on the floor while carrying a CBR pointed at the wall. The CBR will record the walker's distance from the wall versus time.

1. Hypothesize the shape of the distance–time graph that will be produced, and write your hypothesis in your notebook.
2. The walker will walk around the outside of the hula hoop, starting closest to the wall.
3. The front edge of the hula hoop should be 1 m from the wall.
4. Run the Ranger program and set the CBR to run for 15 seconds.
5. Have an observer count the total number of circles walked during the 15 seconds.
6. Sketch a graph of your results in your notebook.
7. Compare your sketch to your original hypothesis. Do you need to make any changes to your hypothesis?
8. Perform a sinusoidal regression on the data, and record the resulting equation in your notebook.
9. Identify amplitude, period phase shift, vertical shift (if any).
10. Write an equation that has a 4-second period, a hula hoop radius of 1 m, and begins 5 m away from the wall. Use this equation as your base equation to analyze each situation below. For each situation, hypothesize the shape of the graph, and then write its equation.
11. Situation 1. The walker starts 10 m away from the wall.
12. Situation 2. The walker walks around the circle twice as fast, compared to your base equation.
13. Situation 3. The radius of the hula hoop is doubled.
14. Situation 4. The walker walks twice as fast and the radius is doubled.
15. Situation 5. The walker starts walking one-quarter of the way around the circle instead of at the point closest to the wall.
16. For the last two situations, respond in sentence form. No equation is necessary. Don't forget to hypothesize first. Situation 6. During the walk, the walker changes the direction of walking around the circle.
17. Situation 7. The walker walks around an elliptical path instead of a circular path.
18. Each member of your group is to pose two other situations for the group to generate hypotheses. Write your situations and hypotheses in your notebook, together with justification for your decisions.

Trigonometry Anticipation Guide

Anticipation guides are an excellent metacognitive tool. They ask students to think about what would happen prior to doing any work with the materials. It is important to revisit the anticipation guide at the end of the unit to confirm or refute the student's original conjectures. Note that an anticipation guide is not a review of previously learned material; it asks students to conjecture about future learning based on their current level of knowledge. To use this activity in Grade 11, change the radian measures to degrees.

Transformational Trigonometry

Use your knowledge of transformations and what you know about to conjecture answers to the questions below:

$y = \sin x$	Amplitude _____	Period _____
	Phase Shift _____	Vertical Shift _____
$y = 3 \sin x$	Amplitude _____	Period _____
	Phase Shift _____	Vertical Shift _____
$y = 3 \sin x + 2$	Amplitude _____	Period _____
	Phase Shift _____	Vertical Shift _____
$y = 3 \sin (x - \frac{\pi}{2}) + 2$	Amplitude _____	Period _____
	Phase Shift _____	Vertical Shift _____
$y = 3 \sin 4x + 2$	Amplitude _____	Period _____
	Phase Shift _____	Vertical Shift _____
$y = 3 \sin 4(x - \frac{\pi}{2}) + 2$	Amplitude _____	Period _____
	Phase Shift _____	Vertical Shift _____
$y = 3 \sin (4x - \pi) + 2$	Amplitude _____	Period _____
	Phase Shift _____	Vertical Shift _____

Summarize your conjectures here:

$y = A\sin k(x - d) + c$

The *A* affects the _____, by _____.

The *k* affects the _____, by _____.

The *c* affects the _____, by _____.

The *d* affects the _____, by _____.

Keep this page and we will refer back to it at the end of this unit of study.

Slinky Standing Waves

This is an excellent interdisciplinary activity, especially with the Physics curriculum. The HOTS and metacognition are built into the activity (conjecture, verify, refute, monitor for clarity, monitor for accuracy). The activity works well in Grade 11 or Grade 12.

Materials required: large Slinky®, measuring tape, smooth floor.

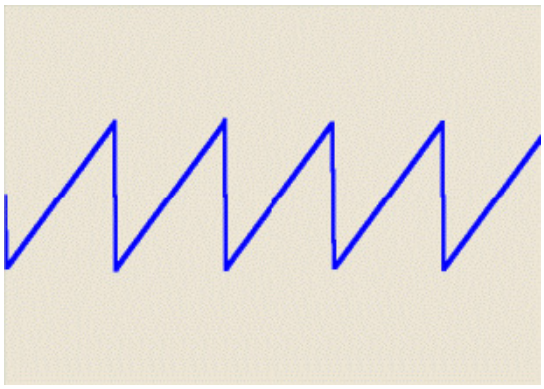
1. On a smooth floor, stretch the Slinky between two people. Be sure to hold the ends of the Slinky on the floor.
2. The third member of the group will use the measuring tape.
3. The fourth member of the group will record information.
4. One person holding the end of the Slinky will give the end sharp sideways snaps. The objective is to generate a large amplitude *standing wave*. This is a wave that does not appear to be moving down the length of the Slinky.
5. Use the measuring tape to measure the amplitude of the wave and the period, in cm.
6. Write an equation for the standing wave.
7. Generate a standing wave with an amplitude one-half as large as the first wave. Without measuring, write the equation for this new standing wave.
8. Conjecture a relationship between the amplitude and the period of a standing wave.
9. Verify your conjecture by generating two other standing waves with different amplitudes, and write their equations.
10. Conjecture how/whether your results would be changed if a smaller-diameter Slinky (more dense) were used.
11. Verify or refute your conjecture, using the smaller-diameter Slinky.
12. Summarize your results. Note that your results are only for *standing waves*. Any relationships that you found may not apply to other sinusoidal functions.

Challenge Questions

These questions challenge students to analyze the problem, plan a strategy, carry out the strategy, and then refine or revise as needed. The planning and revising are metacognitive concepts, while carrying out the strategy requires HOTS. I would not require all students to do these, but the analysis and planning are valuable for all, while perhaps only a few will carry out the plan. These questions would usually be given in MHF4U.

Sawtooth Wave

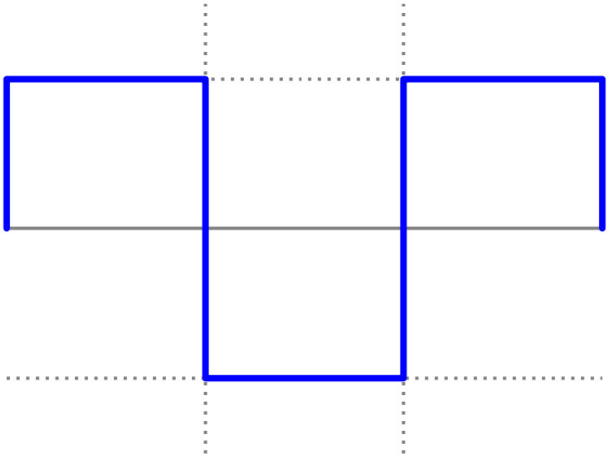
Use combinations of periodic functions to produce a sawtooth wave.



Build a Square Wave

Square waves have many applications in technology, electronics, and communication.

Use combinations of periodic functions to build a square wave.



Metacognition and higher-order thinking skills are critically important for deep learning. It is possible to address these critical skills without making them an added burden by interlacing them with activities that we already use in our teaching practice. The ideas given above for trigonometry can be carried over to virtually any curriculum topic. The power of "What If?" problem posing and conjectures/hypothesizing is almost limitless. I have used anticipation guides in numerous units across a wide variety of grades. Every time I use one of these strategies, I am surprised by the unique situations posed by students who are thinking about their mathematics in interesting ways.

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Appendix

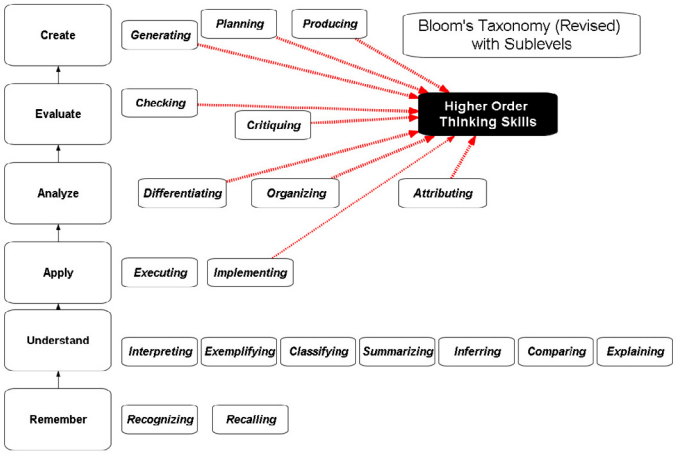


Figure 1: *Revised Bloom showing HOTS*

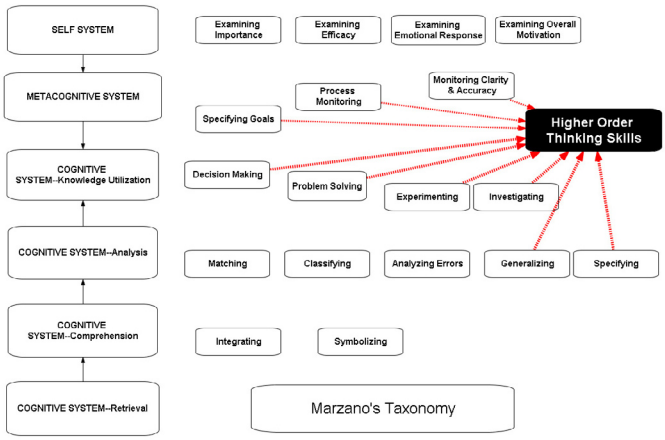


Figure 2: *Marzano's Taxonomy showing HOTS*

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