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ABACUS

Ontario Mathematics Gazette

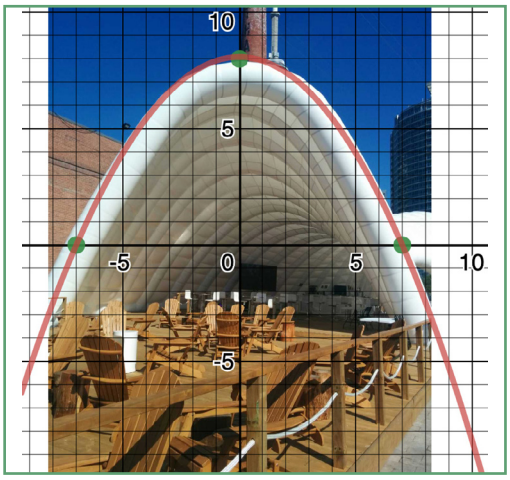
OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

Vol. 58 #3
March 2020

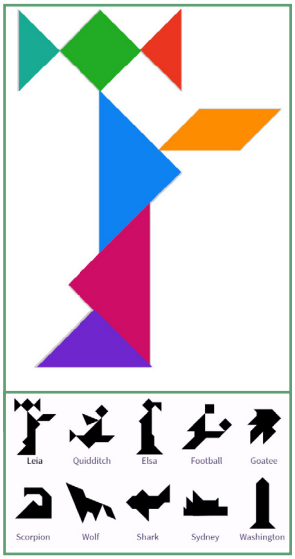
IN THIS ISSUE

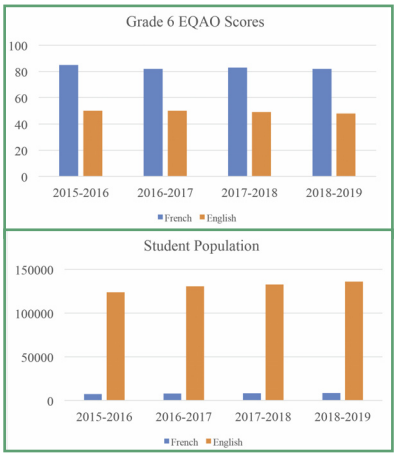
- Editor's Report
- President's Message
- Technology Corner: Mathigon
- Linking Literacy and Math: Applying the Writing Process to Mathematics
- Fields Institute MathEd Forum Report
- OAME/NCTM Report: Surviving and Thriving in Education
- In the Middle: Slow Looking in the Math Classroom
- What's the Problem? Scrutinizing Square Sums
- Mb4T (Mathematics by and for Teachers): Using Data to Mislead
- Book Review: How well do you and your students understand fractions? A Review of *Unpacking Fractions*
- Book Review: Teaching Math Isn't Like It Used to Be! A Review of *Reimagining the Mathematics Classroom*
- Talking about OAMEtalks
- Interview with Dr. Monica Nevins
- Exploring Strategies to Support Primary Students' Communication of Mathematical Thinking – From Theory to Practice
- Some Alternatives to Using Tests and Quizzes for Assessment
- The Treasure Chests: Exploring Relationships among Fractions, Decimals, and Percents



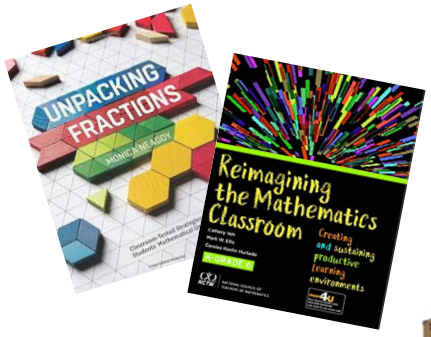
See In the Middle: Slow Looking in the Math Classroom

See Technology Corner: Mathigon





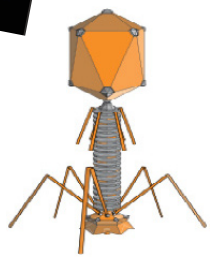
See Mb4T (Mathematics by and for Teachers): Using Data to Mislead



See Book Reviews

See The Treasure Chests: Exploring Relationships among Fractions, Decimals, and Percents





See Editor's Message

Submission of Articles

The *Ontario Mathematics Gazette* (OMG) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *Gazette*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles are to be word–processed, MS Word is preferred, and prepared according to the *Publication Manual of the American Psychological Association*, Seventh Edition. However, please use single-line spacing (not double) and only one space after each period. Articles should not exceed five numbered pages of text, and figures, images, and photographs should be placed in the text close to where they belong, with captions. The photographer's permission is required, and for photos of students under the age of 18, the written permission of a parent or guardian is required.

Please submit your article in one blind file (i.e., identity of author is not evident), and include author names, contact information including email and mailing addresses, photos—head and shoulders, biographies—less than 100 words, and all content removed for blinding in a second file. Please email these two files to Tim Sibbald at gazette@oame.on.ca.

Upon review, you will be notified whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected. The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME.

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The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of OAME.

▲ Table of Contents

Up Front

▲ Editor's Report
by Timothy Sibbald 2
▲ Letters to the Editor 6
▲ President's Message
by Paul Alves 7

Regular Features

▲ The Abacus . . . . . 8-Page Insert
▲ Technology Corner: Mathigon
by Mary Bourassa 9
▲ Linking Literacy and Math: Applying the Writing Process to Mathematics
by David Costello 12
▲ Fields Institute MathEd Forum Report
by Angelica Mendaglio 15
▲ OAME/NCTM Report: Surviving and Thriving in Education
by Jacqueline Hill 16
▲ In the Middle: Slow Looking in the Math Classroom
by Carly Ziniuk 17
▲ What's the Problem? Scrutinizing Square Sums
by Shawn Godin 20
▲ MB4T (Mathematics by and for Teachers): Using Data to Mislead
by Jennifer Holm 41

Special Features

▲ Book Review: How well do you and your students understand fractions? A Review of *Unpacking Fractions*
Reviewed by Lisa Corbett 39
▲ Book Review: Teaching Math Isn't Like It Used to Be! A Review of *Reimagining the Mathematics Classroom*
Reviewed by Lisa Corbett 40
▲ Talking about OAMEtalks
by David petro 45
▲ Interview with Dr. Monica Nevins
by Ann Arden 46

Articles

▲ Exploring Strategies to Support Primary Students' Communication of Mathematical Thinking – From Theory to Practice
by Lisa Cranston 23
▲ Some Alternatives to Using Tests and Quizzes for Assessment
by Jeff Irvine 27
▲ The Treasure Chests: Exploring Relationships among Fractions, Decimals, and Percents
by Justin Hui, Ken Pettigrew 33

Notices

▲ OAME Graph Sticky Charts 12
▲ OAME Board Members . . . . . Inside Back Cover

▲ About the Ontario *Mathematics Gazette*

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Submission of Advertisements
Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** by email or at the above address. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be print ready, and colour advertisements should have no bleeds.

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▲ Editor's Report



TIMOTHY SIBBALD, OCT, PhD
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Tim Sibbald is an associate professor of education with the Schulich School of Education at Nipissing University. He focuses on mathematics education and teaches in the pre-service and graduate programs. He is the editor of the Ontario Association for Mathematics Education (OAME) Gazette*. Two books that he edited and coedited, respectively, are* Teaching Interdisciplinary Mathematics *(Champaign, IL: Common Ground Research Networks) and* The Academic Gateway: Understanding the Journey to Tenure *(Ottawa, ON: University of Ottawa Press).*

A challenge of writing the editorial column is that there is so much going on in math education that it can be difficult to speak to so many disparate items in one column. You will find here a variety of items on different topics and, while I encourage readers to dip into all of it, I also appreciate that headings will facilitate skipping forward to the next topic—but don't judge a topic by its heading!

En français ?

In the days leading up to Christmas, the OAME/AOEM Mission, Vision, and Priorities were translated into French. It is anticipated that, as you read this, they will be on the OAME/AOEM website. I will leave further comment to our OAME/AOEM President. However, I would like to call on our bilingual readers to consider volunteering to provide review and/or translation support for the *Gazette* when French fluency is required. This has not been a significant need in the past, but I sincerely appreciate **Anne Roberge** for some recent translation and **Kyla Kadlec** for verification of a translation. I would also like to acknowledge **Sophie Godon** (Focus First Proofreading) for proofreading services in the last days before Christmas.

Pre-Service Math Proficiency Testing

Over the course of this school year, the province has been enacting a Math Proficiency Test for all pre-service students. As I write this editorial, it has not taken place, but I have no doubt it will. I will not offer a direct opinion, as I see no benefit in doing so, but I will commend the OTF/FEO for providing dialogue through a reasoned position paper in November 2019 about the testing. The position title speaks volumes: *"A recipe for failure: The math proficiency test for beginning teachers."*

I am somewhat alarmed that, at the time of writing this editorial, I have very few details about the testing. There are many questions that remain unanswered. For example, it was specified that the test would cover content up to the Grade 11 level (subsequently changed to the Grade 9 level), but what this means was not clarified. In particular, is this to imply that regardless of what teachers will ultimately teach—meaning anything from Primary–Junior through to Intermediate–Senior, with English and French teachables—they have to be able to explain concepts in math to the Grade 9 level? All versions of the test, unique for each writer, come from one common pool of questions and applies to teacher candidates regardless of division or teachables.

I suspect the content level reflects different pathways that teacher candidates may have taken. Grade 11 at the Essentials level permits financial literacy components such as simple and compound interest. This interpretation is something I could not confirm for pre-service students who are concerned because they have fuzzy recollections of transforming trigonometric function and similar unanswered questions about the testing.

The experience is reminiscent of the first year of EQAO testing, a time when, if my memory serves correctly, I was required to sign a form that said I would not look at the EQAO test, among other things.

Higher Teacher Education

Last October, a new version of the *American Psychological Association Style Guide* was released: "APA-7." Perhaps this did not catch your attention? Perhaps you wonder what computer software it is? Simply put, it is a guide for academic writing—the one used by the *Gazette*. While it fixes several issues that existed in APA-6, I am not writing a column about the details. Suffice it to say the *Gazette* will migrate to APA-7 over the next few issues and I will be shocked if I receive any letters to the editor about it.

The news of APA-7 came to my attention late last summer and caused me to wonder about my own learning of this style that, as editor, is largely routinized. That said, I have to acknowledge Gitta Berg, who proofreads the *Gazette* with attention to style, among other details (Thank you, Gitta!). Using a style guide was one of many things that I learned, or became more versatile with, during my graduate studies in education. It was not a course, but necessary for success because it provided a framework for academic writing that was adopted across all courses. It exemplifies the tip of an iceberg regarding what people learn by pursuing graduate studies, something I dwelled on for months after completing my own master's degree many years ago.

Courses in graduate studies in education generally serve two purposes. The first is to provide a well-rounded knowledge of education so that one can understand one's place in the larger picture. There are, for example, different views of education that have implications for instructional techniques, as well as the psychology of the teacher–student relationship and the sociology of classroom group interactions (whether they include the teacher or not), not to mention an array of policy and leadership approaches. A second purpose is to provide a forum for in-depth investigation of particular topics that are relevant to education. This might be, for instance, a course that digs into the details of issues in math education and looks at what is known. Often courses of these sorts help graduate students understand the scope of what is known, the limitations in what is known, and what could usefully be clarified further about the issue.

There are also substantial developments through personal growth that you won't find outlined in course descriptions or other documentation. An example is substantial learning to write within style guidelines. It does not sound that arduous, and perhaps stretched over a full graduate program, it is not, but there are many details like this that come along with graduate studies. Personally, I found that multi-tasking and time management were two behind-the-scenes details that received considerable cultivation.

Where graduate studies hold significance is in the sum of the parts. No particular detail that I have mentioned sounds transformative in its own right, but collectively, the result of bringing all the parts together and developing coherence of those working parts is significant. To use notions from Self-Determination Theory, graduate studies deliver significant support for intrinsic motivation and autonomy. It is supportive of individual learning that fosters a research-informed way to find one's own way, while being a largely autonomous agent.

Many graduate students make significant decisions along the way: where to pursue graduate studies; and the details of how that will intersect (or interact) with daily work, family, and life in general. There are often considerations of needs and balances; for example, consider a circumstance where you have earned a position as principal, and a graduate degree is required for permanence in spite of the workload of the new role. Perhaps the most daunting decision, for some students, is which route to pursue at the masters level. There are typically three different routes. The most straightforward is strictly coursework, and this is considered a terminal degree, meaning it is not intended for individuals who will pursue

further graduate studies. The middle-of-the-road approach is a Major Research Paper (MRP), and the most research intensive is the thesis route.

An MRP typically carries a weight equivalent to two courses and is oriented to skills-interpreting research. It is often oriented to thinking about implications for a local context and investigating local details.

A thesis, in a masters program, is seen as a prerequisite to doctoral studies, though it is not a commitment to pursue that opportunity. A thesis, in relatable terms, is a short book on a specific topic. It has a structure that begins with explaining the relevance of the topic. This is typically followed by a summary of what is known and leads to defining a particular focal area that requires the research that is reported in the rest of the thesis. In my experience, many people perceive theses as mystical, or mysterious, because of how rarely such things are spoken about. It is significant and it typically carries a weight of four courses. However, looking at theses, talking about them, and realizing there is a high degree of structure speaks to the thesis option being tangible and achievable. Additionally, students have a supervisor, and the element of choice permits a topic of personal interest.

Pragmatically, programs are available throughout Ontario at the various faculties of education. In addition, some tailor their programs to particular needs. The most common is using online methods to address the need for flexible scheduling that many teachers need. My own institution, Nipissing, offers this sort of flexibility along with a doctoral program that only requires residency for six weeks of two consecutive summers. However, while I use this example because it is familiar, I know that there are many variations available, and one is wise to do some exploration before pursuing a particular program of study.

Recently, a regular *Gazette* author, Dr. Jeff Irvine, successfully defended his PhD thesis, and I asked him to contribute a few words about his motivation. His response was:

Good teachers do good things for their students. I am heartened to learn that these "good things" have support in educational research. Very early in my career, I was incredibly fortunate to have an associate teacher named Jean Dubeau, who structured all his classes in student groups. This was amazing to me; he did not follow the "teach as we were taught" paradigm, with no regard to student motivation or student needs. Seeing students working together in groups, posing questions and discussing solutions, was revolutionary. This ignited my interest in instructional strategies that benefited my students and engaged them in their own learning. However, I still didn't know any of the research behind strategies like jigsaw, choice, four corners, or problem posing. While I was interested in learning more about the research, life got in the way. I had a full-time job, my son was a young child, and I completed a couple of master's degrees part-time, but a PhD was too great a commitment.

When I was a department head, I encouraged my teachers to try new strategies, and each department meeting featured a "bring and brag," where a teacher demonstrated a lesson or activity that had worked particularly well. Later, as a vice-principal, I ran a whole-school book study of Robert Marzano's Art and Science of Teaching*. Then I went to the Ministry of Education, where literally everything that we proposed was based on research. As a consequence, I spent a lot of my time evaluating educational research. I was fortunate to read an early version of Marzano's taxonomy of educational objectives. This taxonomy integrates student motivation, metacognition, and cognition; I found the idea really exciting and I wanted to pursue it further. I retired from the Ministry of Education in order to pursue a doctorate (PhD). I learned so much. If I were still a practising teacher, I would have a vast array of instructional strategies that I could use for the benefit of my students.*

When I was in the classroom, I had a large banner across the back of the room that said, "Mathematics is not a spectator sport." Good teachers know that their students must be motivated and active in their learning. An advanced degree provides knowledge to enable teachers to meet these student needs.

I would like to thank Jeff for entertaining my request for his thoughts, and congratulate him on his efforts and success.

It is unusual that the *Gazette* acknowledges outcomes of graduate studies. This is quite simply because we do not hear from as many graduates as we should. If you know someone who has conducted research, whether at the masters or doctoral level, encourage them to consider sharing their work with the *Gazette* readership. It is, in my opinion, unfortunate that we do not hear about the research that graduate students in Ontario are doing. Perhaps some would be arcane or removed from the pragmatism of day-to-day lesson plans, but I would rather we let peer review and the readership decide. What can you do? If you know someone who is pursuing graduate studies, encourage that person to consider the *Gazette* as a forum to connect with math teachers.

Mathematical Biology

Viruses show some remarkable geometric structures. A very common bacteriophage, a virus that infects bacteria,

for instance, has an icosahedron on a cylinder with linear "legs" (see Figure 1). So, it was interesting to read some mathematical considerations in recent research that is progressing toward determining the structure of the HIV-1 virus. Specifically, while not a common topic in math classes, the fact that biologists connect knowing the structure as useful for developing therapies and medications is an important message for our students. The article speaks about evidence that the structure is not simply a hexagonal arrangement ("hexameric") of proteins and is more likely a "6-lune hosohedron"—or beach ball shaped (See Figure 2).

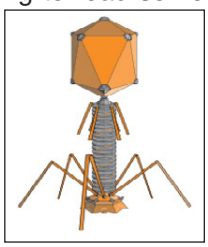


Figure 1: *A bacteriophage (picture from Wikipedia)*

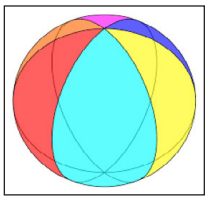


Figure 2: *Hosohedron (picture from Wikipedia)*

Reference: Sun, W., Reyes-Serratos, E., Barilla, D., Santos, J.R.L., Bujold, M., Graves, S., et al. (2019). Mathematical determination of the HIV-1 matrix shell structure and its impact on the biology of HIV-1. *PLOS ONE, 14*(11): e0224965. https://doi.org/10.1371/journal.pone.0224965

Programme for International Student Assessment (PISA) 2018

(Hold on to your hat, there is a lot to digest here!)

Standardized testing is important for research and technical purposes of looking at systems of education. There may be aspects, such as newspaper reporting admonishing specific schools, that are bothersome, but the potential for long-term benefits, in my opinion, outweigh these details. Within standardized testing, I look for evidence of clear analytical thinking (as I have observed with EQAO testing), and I found this tested in the preface of the latest PISA reporting. It was somewhat discomforting to read that education funding increases across the sampled countries of 15 percent in the previous decade have not led to improvements. I find this shortsighted and used a Bank of Canada inflation calculator to find that the inflation rate over that decade in Canada (which is arguably representative of OECD) was 17 percent. However, education is not strictly a dollars-for-sense (pun intended) endeavour, and it is more important to consider what the data on student achievement says. Please appreciate, however, that what you read here is an attempt to extract key details from nearly a thousand pages of details!

According to the PISA reporting, in each country, a representative sample of at least 150 schools was used to address within-country diversity. In Canada, the sample was 22 653 students. This sample was used to generate a group of 42 15-year-old students, who sat through the assessment. In PISA's estimation, the results represent approximately 85 percent of Canada's 15-year-old population. Students around the world do not move through grade levels in the same way, and PISA clarifies the Canadian sample as approximately 0.3 percent Grade 7, 1 percent Grade 8, 10 percent Grade 9, 88 percent Grade 10, 1 percent Grade 11, and 0.1 percent Grade 12.

Results for Canada as a whole are reported in various ways. Compared to equivalent OECD countries, Canada was above average (in a statistically significant way) in Mathematics, Science, and Reading. Canadian performance in mathematics was not significantly different from PISA 2015 for any level of achievement, but the dispersion or spread of scores widened, and there is long-term evidence of a slow decline in math scores. Countries with comparable performance to Canada in mathematics were Poland, Switzerland, Denmark, Slovenia, Belgium, and Finland. Notably, the United States, who provide considerable influence on Ontario teachers, were below average in Mathematics, but they only sampled 4838 students, and caution needs to be exercised when interpreting U.S. PISA results.

It is interesting that the gender gap in math (average scores for boys 514 and girls 510) was measured and reported to be only 17 percent of the corresponding gender gap in reading. The gender gap in math is negligible for low performers and largest for high performers. Additionally, there are gender gaps in relation to competition: boys are much more in favour of competition than girls (by 0.7 standard deviation). Additionally, fear of failure is higher among girls than boys (by 0.5 standard deviation). Roughly 68 percent of Canadian students were found to exhibit a growth mindset, and those students performed better than the students who did not. We have very low segregation of students, considerable social inclusion, and no significant differences in teacher quality in different socio-economic settings.

Returning to mathematics, and within Canada, Ontario's overall math performance was second only to Quebec. Additionally, Ontario and Alberta were very similar.

There is much more in the PISA 2018 reporting, and I have omitted what was reported about achievement by students who are immigrants or first-generation descendants of immigrants (where I would interpret the report as casting Canada in a highly favourable light), bullying, teacher behaviours, and truancy rates. There are also details that were not surveyed in Canada that may be of interest, such as the number of students who reported having a sense of meaning in their lives.

I will offer a personal opinion about the PISA results. Having had children who went to school in Canada, and now having grandchildren who attend school in Canada, I have no concerns about what I read in the PISA report. Of course, there are details that point to areas for attention, but nothing that seemed dire or alarmingly concerning.

Erratum

Page 47 of the December *Gazette* has a box titled "Transformational Trigonometry." A "*sin x*" was inadvertently left out. The instruction should read, "Use your knowledge of transformations and what you know about $y = \sin x$ to conjecture answers to the questions below."

In this issue…

OAME/AOEM President **Paul Alves** speaks about the importance of community in challenging times. **Angelica Mendaglio** reports on a MathEd Forum focused on the Math Proficiency Test for pre-service teachers. **Jacqueline Hill** encourages working together from an NCTM angle, and David Petro speaks about OAMEtalks.

David Costello encourages using a language arts-styled approach to draw out details of the process students use in their thinking. **Shawn Godin** takes us through a problem with a grid on a grid requiring a particular property. **Carly Ziniuk** encourages slow engaged observation. Among our regular columns, **Jennifer Holm** addresses misleading representation of data. Missing from this issue are Mathematical Snapshots and the Interview column. Both should return in June.

It is, once again, my pleasure to acknowledge the ongoing efforts of **Anne Yeager** and **Jacquie Foster**, our associate editors. **Gitta Berg** provides proofreading and is instrumental in making sure there is consistency throughout the *Gazette*. **Penny Clemens**, as she has for many years, brings everything together in the layout you are reading.

Notice

Please note that certain information and text used by the writer of the three *Hey, It's Elementary* columns that appeared in the June 2018 (Vol. 56, Issue 4), March 2019 (Vol. 57, Issue 3), and June 2019 (Vol. 57, Issue 4) editions was used without citation or credit to the original author. This content, which appeared on pages 37–39 of the June 2018 issue, on pages 21–22 of the March 2019 issue, and on pages 15–18 of the June 2019 issue, has now been removed from these issues. If you wish to cite these columns of the *Gazette*, please consult the archival files for the most up-to-date version. If the removal impacts your work, please consult the editor.

▲ Letters to the Editor

Seeing the sidebar about Jack Lesage reminded me that he was the presenter at the first OAME conferences that I attended in 1974. His quotation, "If you only have 15 minutes per day for professional reading, read the comics," spurred me on to rethink teaching strategies for general-level students. I taught a Grade 11 General Business class, using *The Toronto Star* as the textbook. Every day, students would pose and solve problems based on newspaper articles, ads, or editorials. Assessment typically consisted of each group getting a section of the newspaper, formulating several problems (with solutions), and then trading the questions with another group; solving their problems; and assessing the other group's problems, using the solutions that the group had already prepared. The only problem that I encountered was that some of my students became so enthusiastic about math class that they went to the school library and cut out articles from the newspaper to bring to class as the basis for posing problems. I made a deal with the librarian that at the beginning of class, one of the pristine class newspapers (we had a class set each day) would be taken to the library to replace the defaced copy. We hit at least 95 percent of the expectations without ever touching a textbook. The following year, I taught a Grade 12 General Business class, using only comics from the newspaper, again with an emphasis on problem posing. We had a great time!

~ Jeff Irvine

You may have noticed that the *Abacus* is very different this year. In the past, our issues focused on developing conceptual understanding, using models and different calculation methods within problem-solving contexts. Every issue included a full Bansho plan. This year, we decided to focus on building number sense. You will now find an Abacus that includes practice tasks comprised of quick images, puzzles, and games. These tasks provide practice for decomposing and composing numbers to see part–whole number relationships and equivalent number expressions within the context of each number operation (addition, subtraction, multiplication, division). Number representations include set (bar model), linear (number line), area (rectangle), graphical, numeric, and/or algebraic models. The tasks are appropriate for junior and intermediate students. So, if you haven't looked at the middle pages (*) recently, cast your eyes there, and you may be surprised at what you find (* – Note that electronically, the *Abacus* is a separate download, but worth grabbing when you grab your *Gazette*!).

~ Mary Lou Kestell ▲

▲ President's Message



PAUL ALVES
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Paul Alves, current OAME/AOEM President, is department head of math at Fletcher's Meadow Secondary School in the Peel District School Board.

"We must accept finite disappointment, but we must never lose infinite hope." ~ Martin Luther King

A recent vignette from my classroom: At lunch, students invade my classroom to eat lunch, discuss whether the Lakers or Leafs have a real shot at success in the post-season, and to ostensibly play games. My classroom had been advertised as the chess club at lunch, but has slowly transformed itself into a games and socialization club. Games of chess do break out, but you will just as likely find ten students playing UNO, a Connect 4 tournament, or an intense Stratego battle ensuing. There are moments when students come into the room to chat and sometimes it is about math.

On a recent Friday, a student was asking for some help with exponential relations, and we had a good discussion about the way to model relationships where the period is something other than a single period (e.g., one year, one hour). After exploring the effect of changing the period on the growth or decay of an initial amount, the student looked at me and jokingly said, "I understand that. You explained it well. You should do something like teach math." After a week of dealing with what seemed like endless crises, including spending one day completing a four-hour picket, it was a much-needed laugh and reminder of the reason why I love to teach and, in particular, love to teach mathematics. Regardless of the periodic (maybe even constant) maelstrom that surrounds us in our profession, I never lose hope in the work that we do as educators.

Looking back on what seems like the proverbial blink of an eye, but is in fact over 20 years of teaching, it is hard to remember a more turbulent time. Not only has it been a tumultuous school year in terms of the relationship between teachers and the province, but specifically in math education. Changes to what we teach, how we teach, and possibly who will teach math in the province are in the offing. It is in these trying times that we turn to the communities that sustain and help us through all of our days. I am fortunate to work with a stellar group of teachers at Fletcher's Meadow Secondary School in Brampton. From the administrators to the teachers and support staff, we focus on the work that we do so well—being the stewards of an education system many jurisdictions look to as an example of how to teach and prepare students for the challenges that will come in the remainder of the century. In particular, I count myself blessed to work with a department of committed math teachers. They continue to challenge the conventions of what "teaching mathematics" looks like and, in the process, make me look good as the department head of a progressive group of teachers. Most importantly, they make me laugh, especially when I need a boost. I am certain that as you read these words, you will reflect on the community (they may be just a couple of teachers or range to many) that sustains you in the work that you do day to day and that gives you infinite hope.

My hope is that many members look to OAME/AOEM as a community that helps sustain them professionally, regardless of the challenges faced by the provincial

Mission de l'AOEM/OAME

- La mission de l'AOEM/OAME est de promouvoir, soutenir et défendre l'excellence dans l'enseignement des mathématiques dans l'ensemble de la province de l'Ontario.

AOEM/OAME : La vision pour l'apprentissage des mathématiques

- L'AOEM/OAME envisage une communauté d'enseignement des mathématiques équitable dans laquelle tout le monde profite d'activités de haute qualité, fondées sur la recherche et engageantes, qui promeuvent la pensée critique, tout en favorisant une attitude positive envers les mathématiques.

Priorités stratégiques de l'AOEM/OAME L'AOEM/OAME est dévouée aux priorités stratégiques suivantes :

- Promouvoir et défendre l'équité dans l'enseignement des mathématiques
- Soutenir tous les membres de l'AOME/OAME en offrant une valeur ajoutée pour l'adhésion, y compris un apprentissage professionnel de qualité, des ressources et des occasions de réseautage
- Défendre les pratiques de haute qualité fondées sur la recherche, tout en créant une communauté pour soutenir l'enseignement des mathématiques

community. OAME/AOEM remains dedicated to its mission of promoting, supporting, and advocating for excellence in mathematics education throughout the province. Those words are pulled directly from the OAME/AOEM mission statement, which was updated after the annual general meeting in October of 2019. I am pleased to report that in our continuing effort to serve our membership, that the guiding principles for OAME/AOEM have been translated into French and can be found on the website. It is hoped that this is just the start and, in the future, OAME/AOEM will make more content available in both English and French.

One of the dates that provides us with an opportunity to come together and exchange ideas and best practices is the OAME/AOEM Annual Conference. This year's conference, OAME 2020 In Focus, is taking place May 7–8 at Ontario Tech University in Oshawa. At the time of this message, registration will have been open for some time, and if this year is like those in the past, sessions are quickly filling up and the anticipation is growing for another exceptional professional development event. I hope to see you there. Please don't be shy about coming up and starting a discussion about what we can do to improve your experience as a member of OAME/AOEM.

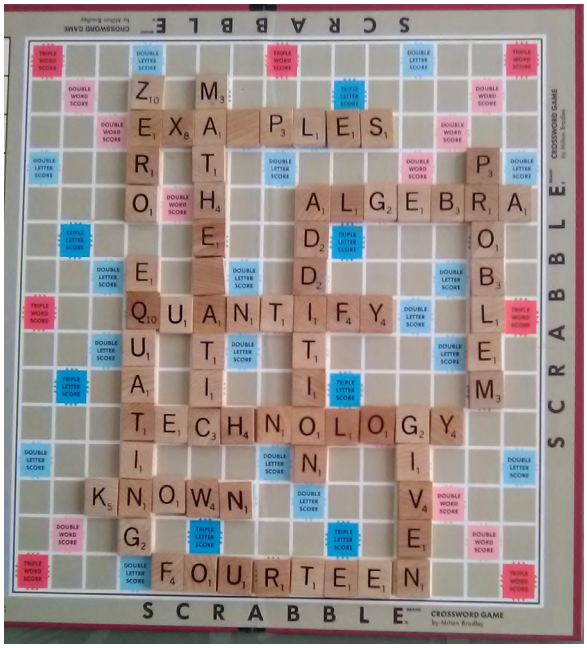
In addition to the conference, OAME/AOEM will have wrapped up the second season of OAMEtalks. The second season provided an excellent lineup of speakers from last year's conference, and an opportunity to hear them speak about what they presented at OAME 2019. The podcast and webinar series is another way that OAME/AOEM is expanding opportunities for learning to the membership. A big thank you to all those involved in the planning and production of the series.

I would also like to give a shout-out to our *Gazette* Editor, Tim Sibbald, and his team, who continue to put together the *Gazette*. Although the publication has been around for some time, Tim and his team have ensured that the quality of the work continues from issue to issue. The *Gazette* (and *Abacus*) is another way that we strive to serve the math community in the province.

A quick reminder before concluding this message: Voting for OAME/AOEM Board of Director and Executive positions is open. Voting is open to all OAME/AOEM members. Please visit the website to view the candidate biographies and to cast your vote!

I want to finish with a mention of a podcast I was listening to that hit on mathematics and the impact it can have on lives. I am a faithful listener of *Heavyweight*. The podcast is the brainchild of Jonathan Goldstein. I have been a fan of his work from the time of the *Wiretap* show on CBC Radio. The subject of *Heavyweight* is rarely mathematics. The stories are a mix of humour and poignancy as someone tries to go back and resolve a moment in their past. The episode that relates to math is "#31 Marie-Claude." Marie-Claude is 50 years old, and her problem, as Jonathan points out, is a "math problem." Marie-Claude then starts relating her history of disappointment and failure with school mathematics. She now wants to confront that problem and, in particular, pass a math exam that would allow her to become a registered realtor. What struck me was the familiar feeling of hearing many stories from adult friends and family about their negative experiences with school mathematics. But what also gave me hope (spoiler alert for those who will want to listen to the episode) is the sense of triumph and validation when those past demons are conquered. I constantly tell my students that they may not understand YET, but they will if they persevere and persist in their desire to learn. It is their persistence and perseverance that gives me hope. Infinite hope. ▲

Daily Numeracy Activity (DNA)



Does adding up the score, with or without special squares, fit with data management skills?

If special squares are ignored, does tallying the overall score support the MAT1L expectation "solve problems requiring estimating and calculating the cost of projects that require the purchase of multiples of the same item?"

Spot the erroneous word!

▲ Technology Corner: Mathigon



MARY BOURASSA
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Mary Bourassa teaches mathematics at West Carleton Secondary School in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She has presented workshops internationally, authored mathematics resources, is a past VP of OAME, and a Past President of COMA. An award-winning teacher, Mary continually strives to learn new and better ways of helping students learn and love mathematics.

Mathigon, the creation of Philipp Legner, is a cutting-edge online tool for learning mathematics. It can be found at https://mathigon.org/ and is currently free for all users. There is also an app that can be downloaded that has some components—only the courses and timeline activity are described below.

There are courses and activities available on the main page of Mathigon. The courses will eventually offer a full Grades 6–12 curriculum (UK and US standards) that promotes active learning, personalizes the order and depth of steps in the learning process, and employs storytelling to weave a narrative around each mathematical topic. New content continues to be developed, but there are several topics that are already complete, which I invite you to explore.

There is a wide variety of activities in Mathigon. One is a timeline with notable figures in mathematics history. Part of the timeline in shown in Figure 1.

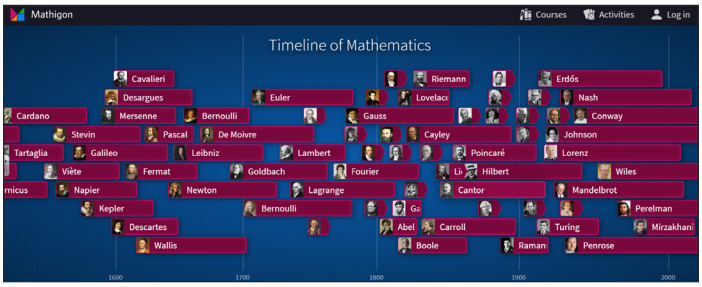


Figure 1: *Notable figures in mathematics history*

When clicked, each person's contribution to mathematics, along with a brief biography, will pop up, allowing students to learn more about both recent and ancient discoveries in mathematics. Katherine Johnson's pop-up is shown in Figure 2.



Katherine Johnson (born 1918) is an African-American mathematician. While working at NASA, Johnson calculated the orbits taken by American astronauts – including Alan Shepard, the first American in space, the Apollo Moon landing program, and even the Space Shuttle.

Her extraordinary ability to calculate orbital trajectories, launch windows and emergency return paths was widely known. Even after the arrival of computers, astronaut John Glenn asked her to personally re-check the electronic results.

In 2015, Johnson received the Presidential Medal of Freedom.

Figure 2: *Katherine Johnson's pop-up among Mathigon's notable figures in mathematics history*

Another activity is Mathigon's Polypad, which is a treasure trove of mathematical goodness. Figure 3 lists the manipulatives within this activity—from polygons and pentominoes to algebra tiles. This collection of interactive digital manipulatives is remarkable in how cleanly each tool functions and how easy each is to use. It produces content that is beautiful to look at and extremely useful for learning. Students and teachers alike can create artwork, visual patterns (as shown in Figure 4), work with fractions, and so much more. Every teacher, whether elementary or secondary, should take a dive into everything that Polypad has to offer. Figure 5 shows a variety of activities that Polypad has to offer.

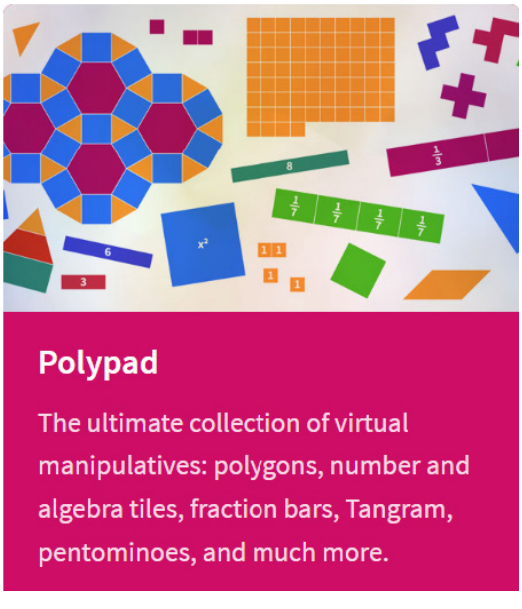


Figure 3: Manipulatives found in Mathigon's Polypad activity

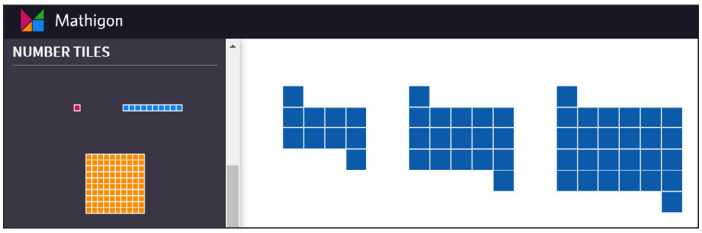


Figure 4: *Sample of visual patterns that students and teachers can create*

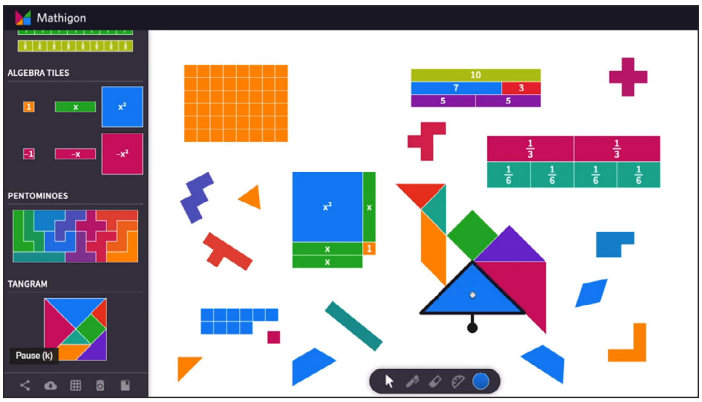


Figure 5: *A variety of activities that Polypad has to offer*

Mathigon also has stunning origami, as shown in Figure 6. It features platonic solids, Archimedean solids, stars and compound shapes, and a category called "beautiful origami." Many of the shapes come with downloadable folding instructions and nets. This provides students with an excellent introduction to origami.

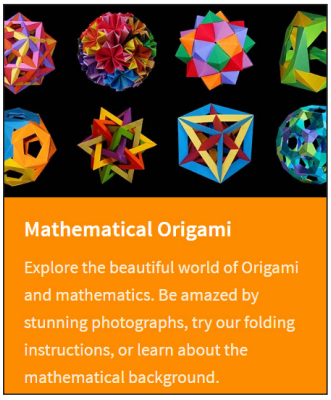


Figure 6: *Examples of Mathigon's origami*

Another activity is called Applications of Mathematics, shown in Figure 7. This activity certainly provides an answer to the perennial student question, "When am I ever going to use this?" The collection currently has 68 applications of mathematics, along with links to allow the user to learn more. Applications range from Glacier Melting to Neurology to Making Chocolate, and so much more. Students could use this as a launch pad to learn more about how mathematics is used in a particular field. As an example, a portion of the Finance and Banking information is shown in Figure 8.

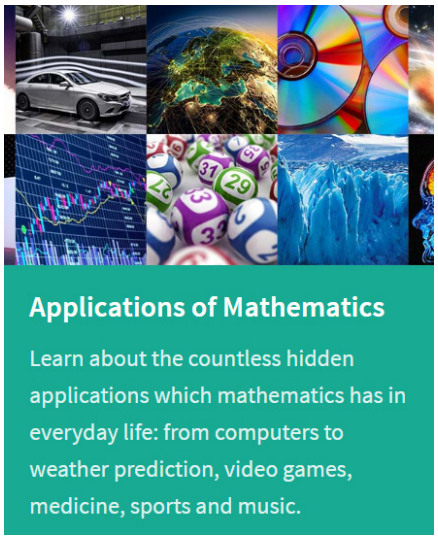


Figure 7: *Mathigon's Applications of Mathematics activity*

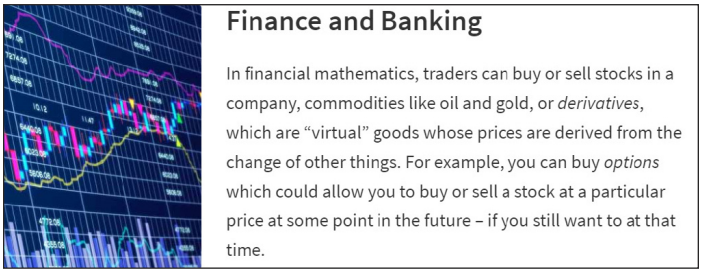


Figure 8: *Example of a portion of Mathigon's Finance and Banking information*

Mathigon also features Problems and Puzzles, as shown in Figure 9. These cover a variety of topics that would make a great addition to your classroom. Students could learn about new topics and challenge themselves with many of these puzzles. A sample is shown in Figure 10.

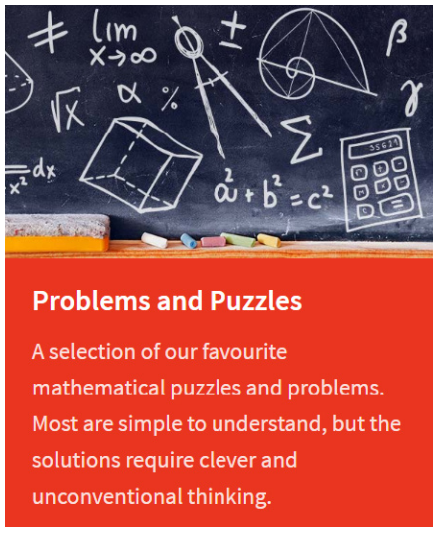


Figure 9: *Mathigon's Problems and Puzzles activity*

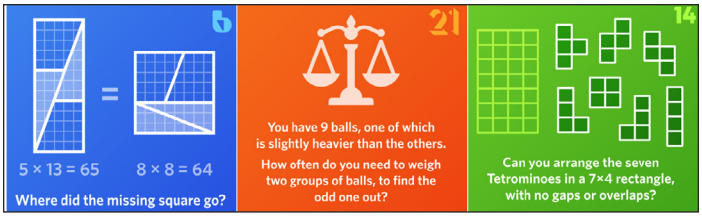


Figure 10: *Samples of Mathigon's puzzles*

Although there are tangrams within the Polypad activity, there is also a separate Tangram Builder, shown in Figure 11. There are basic shapes, animals, geometry, and a section of new tangrams. These are shown in Figures 12 and 13, the latter of which shows a completed "Leia."



Figure 11: *Polypad's Tangram Builder*

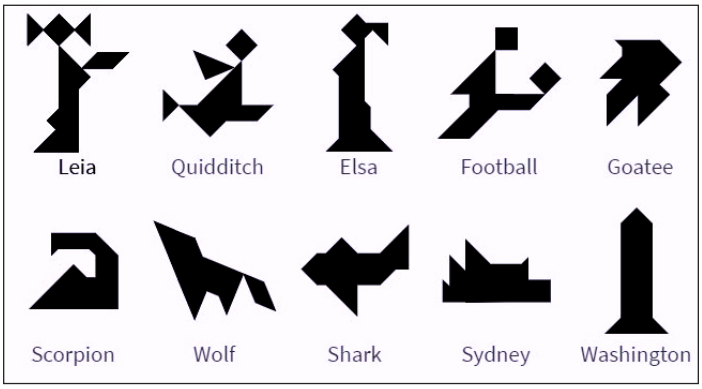


Figure 12: *Basics of a Tangram Builder*

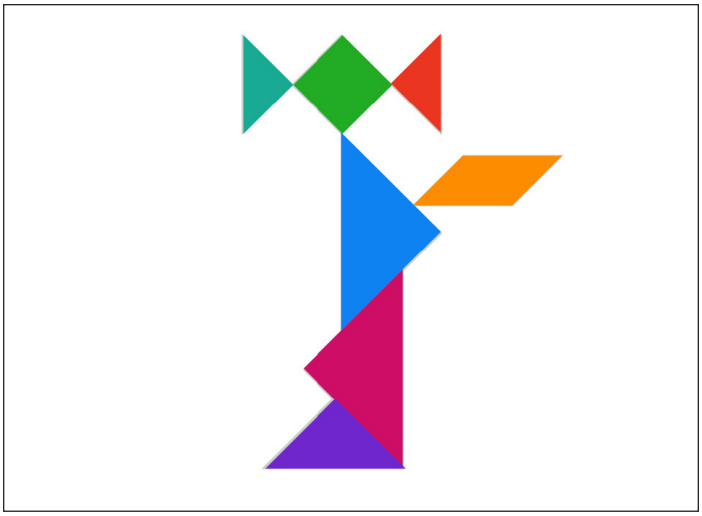


Figure 13: *A completed "Leia"*

The final activity I will feature is called Factris, and be warned, it is addictively fun. Figure 14 shows a game that is similar to Tetris—blocks fall, and when a line is completed, it disappears. The goal is to avoid reaching the top for as long as possible. The difference here is that each block is rectangular and can be rearranged based on its factors within the 16 x 16 grid. For example, a block with area 12 square units can be arranged as 1 x 12, 2 x 6, 3 x 4, 4 x 3, 6 x 2, or 12 x 1. Playing this game helps you learn the factors of many numbers from 2 to 24, as well as what prime numbers are (the dreaded 11 block!). A couple of screens of the Factris game are shown in Figure 15.



Figure 14: *Factris activity*

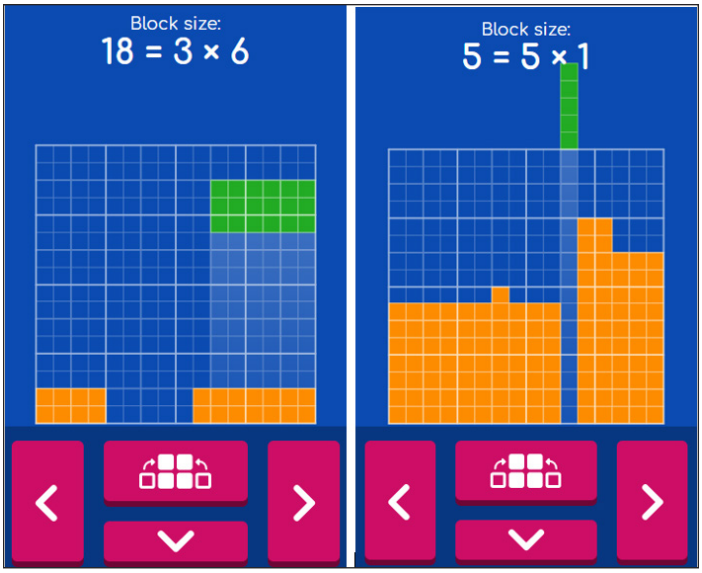


Figure 15: *Screen samples from Factris activity*

In addition to all of the activities listed above, you can explore Exploding Dots, Parallel by Simon Singh, Eureka Magazine, and more, all from the Mathigon website.

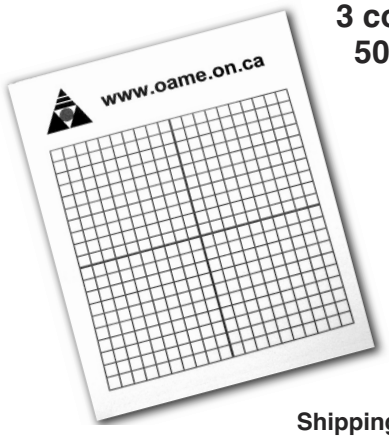
Mathigon's goal is to allow students to engage in, explore, and discover mathematics in an innovative way. There is a wide array of excellent content on this site, presented with stunning graphics. It is well worth your time to discover more of what Mathigon has to offer. Keep up with all the latest on Twitter at @MathigonOrg. ▲

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▲ Linking Literacy and Math: Applying the Writing Process to Mathematics



DAVID COSTELLO
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Dr. David Costello is an author who has recently published Using What Works: Strategies for Developing a Literacy-Rich Environment in Math. *David is a vice-principal, who has held many roles. He taught in the primary division before assuming roles of numeracy interventionist, numeracy coach, numeracy leader, and curriculum consultant for Prince Edward Island. David has also instructed university courses focused on curriculum, differentiation, mathematics, and language arts. He facilitates professional learning sessions in the areas of mathematics and school development. His research situates mathematics within a literacy perspective and examines leveraging literacy-based instruction in mathematics.*

In previous columns, I have taken opportunities to dig into how comprehension strategies and writing strategies can be applied to mathematics as a way for students to strengthen mathematical understanding. While these strategies can provide students with tools to work through a problem, the overall approach to tackling a problem cannot be ignored.

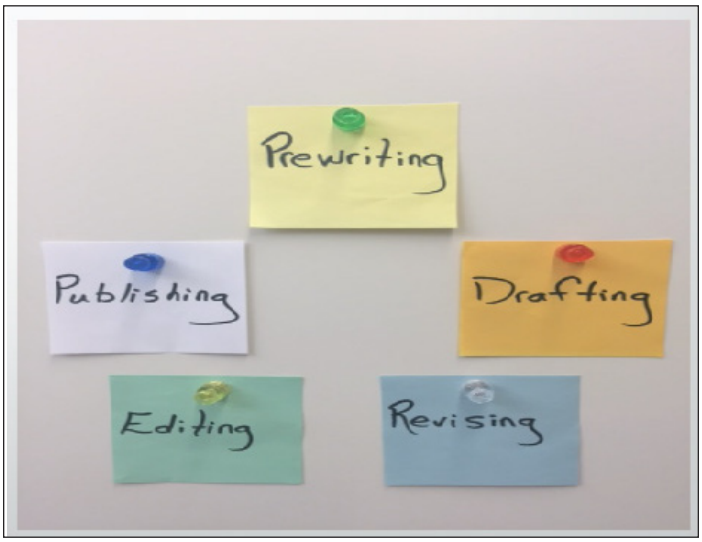
Imagine you are assessing a student's work without acknowledging how he or she worked through the problem, as if, all that matters is the final response. Now imagine that the response was correct, but that the student applied a process that would not be applicable to solving other problems. Imagine this process to be one where it was lucky to have produced a correct answer. The student got the right answer for the wrong reason. Is this something we are fine with?

Product versus process is not a new dilemma. Fortunately, we are moving toward a more comprehensive view of mathematics, which shifts the focus from entirely product based, to considering the process. We are not only interested in how our students achieved the correct answer, but also *how* they arrived at the correct answer. In addition, we are interested in how students arrive at the incorrect answer. We pay attention to how students work through a problem and what thought process is supporting their work. Focusing on process is comparable to the writing process in language arts.

Embedded in the process focus is the notion of communication. It is vital that teachers have a clear picture of student mathematical thinking—how students arrived at their response. Without this picture, it is difficult for the teacher to know how students applied their knowledge to work through the problem—whether they arrived at the correct response or not. If student understanding is the goal of instruction, students need to understand the problems they are engaged with, and judge the reasonableness of their responses. As well, teachers need to clearly see how students engage with the work.

Accuracy has a place in mathematics. The final response, or product, is still important. However, when considered in isolation from the process thinking, it can be misleading because we do not have an understanding of how the student navigated the problem to arrive at the response. If we are interested in mathematical understanding and using student learning as the compass of instruction, we need to actually explore student thinking (the process) and the response (product) together.

What I am proposing in this particular column is that the five-step writing process, used in all grade levels of language arts, can be applied to mathematics as a way for students to frame, support, and consolidate mathematical understanding.



The writing process consists of five phases: pre-writing (rehearsal), drafting, revising, editing, and publishing (Calkins, 1994). Focusing on the writing process shifts the emphasis from a finished product to what writers think and do as they write (Calkins, 1994). We can then support students as writers and work with them as they move through the phases. The table below provides a brief overview of the five writing phases when used in language arts.

Writing Phase	Description of Phase	Example of Phase
Pre-Writing (Rehearsal)	• Planning phase of the writing process. • Consider all ideas on a topic. • Select an organizational structure that will effectively demonstrate their ideas.	• Choosing a topic • Gathering and organizing ideas • Identifying purpose and audience
Drafting	• Actual composing of the text. • Use pre-writing activities to generate a piece of writing in rough form.	• Writing a rough draft • Emphasizing content rather than mechanics
Revising	• Examine the content of their writing for clarity and craft. • Consider changes that would improve the piece. • May keep, delete, move, or change elements.	• Rereading to refine and improve clarification • Adding, removing, changing elements of the text
Editing	• Attend to correctness in conventions. • Undertaken when all revisions to the content are complete. • Proofread to find errors in grammar, usage, mechanics, and spelling; make appropriate corrections.	• Proofreading to locate errors • Correcting mechanical errors
Publishing	• Entails the final preparation of a piece of writing for the intended audience. • Preparing the final draft.	• Sharing finished writing with audience

While the table above presents the five phases of the writing process in isolation, it is important to note that the process of moving between phases is fluid. As writers write, they move through most, if not all, of these five phases, sometimes going back and forth between the phases. I say most because sometimes, depending on the purpose, the writer may choose not to edit or publish the text.

The five phases of the writing process can be applied to student mathematical understanding. When students are engaged with a task or problem, they must frame their thinking, work through to a solution, review their work, and share their thinking, whether it is with the teacher or peers.

Knowing the importance of communication in mathematics, and how writing can support student mathematical understanding (as documented in my previous columns), the table below has the five-phase writing process adapted to reflect a mathematics lens.

Writing Phase	Description of Phase	Example of Phase (in Mathematics)
Pre-Writing (Rehearsal)	• Planning phase of the writing process. • Consider all ideas on a topic. • Select an organizational structure that will effectively demonstrate their ideas.	• Choosing a strategy • Gathering and organizing facts, information, etc. from the problem • Identifying what is required to solve the problem
Drafting	• Actual composing of the text. • Use pre-writing activities to generate a piece of writing in rough form.	• Writing a rough draft of the problem solving • Emphasizing concept rather than details
Revising	• Examine the content of their writing for clarity and craft. • Consider changes that would improve the piece. • May keep, delete, move, or change elements.	• Rereading to refine and improve clarification • Adding, removing, changing elements of the problem solving
Editing	• Attend to correctness in conventions. • Undertaken when all revisions to the content are complete. • Proofread to find errors in grammar, usage, mechanics, and spelling, and then make appropriate corrections.	• Double-checking to locate procedural errors • Correcting procedural errors
Publishing	• Entails the final preparation of a piece of writing for the intended audience. • Preparing the final draft.	• Sharing finished product with audience

This table bears a close resemblance to the table focused on language arts writing. I wanted to have two separate tables to emphasize the similarities and distinctions between writing in language arts and writing in mathematics. The writing process is one that can be transferred to mathematics so that student thinking is prioritized as opposed to the solution. I am not saying the solution is not important, but it will not tell you as much as the thinking surrounding it.

Final Thoughts

Too often, what is valued in mathematics is the product, the final response. In literacy, it would be viewed as the published copy. Omitting the rough drafts, the planning, revisions, and edits diminishes the evidence of student thinking that is necessary for the student to arrive at a solution. When I was going to school, I used scrap paper that was to be thrown out once a good copy could be handed in to the teacher. Within such an approach, there is a loss of revisions and edits, where student thinking is clearly visible and feedback for specific areas of improvement or learning can be provided.

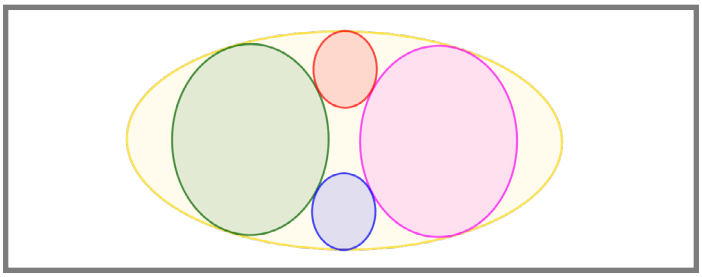
When considering a writing-process approach within mathematics, students can provide documentation of their thinking throughout the problem, and can use this documentation as a form of metacognition when reflecting upon their work. It tracks their thinking throughout the problem. From an instructional point of view, the writing process allows teachers an opportunity to assess student thinking throughout the problem instead of only looking at the good copy. How, as teachers, would we know when students changed their interpretation of the problem, when students changed their strategy, or when students identified a miscalculation in their work if all we saw was the final product?

A writing-process approach to mathematics is rooted in a growth-mindset approach to mathematics. Student thinking is valued instead of just the correct response. Students see the value in their work that leads to them arriving at the correct response. A writing-process approach values the process and the product—a significant shift in moving from a right or wrong approach to learning.

I look forward to continuing the discussion of linking literacy and math via Twitter (@dr_costello) or email.

Reference

Calkins, L.M. (1994). *The art of teaching writing*. Portsmouth, NH: Heinemann. ▲



▲ Fields Institute MathEd Forum Report



ANGELICA MENDAGLIO
angelicamendaglio@gmail.com

Angelica Mendaglio is an instructional designer at Vretta Inc. in Toronto, Ontario, where she helps to create interactive digital mathematics lessons and activities for middle school students.

Note: This is a report on the November meeting of the Fields Institute MathEd Forum, the topic of which was the Mathematics Proficiency Test. As the test is in an ongoing state of design, there may be more recent information than what was shared at the Forum meeting. Please visit www.mathproficiencytest.ca for more information. This column is not a report on the test, but on the events at the Fields Institute MathEd Forum.

The Mathematics Proficiency Test (MPT) that new Ontario teachers will be required to pass starting in 2020 was the centre of discussion at the November meeting of the MathEd Forum. The meeting featured talks by Heather Howell from Educational Testing Service in the United States, Ann Kajander from Lakehead University, and Dragana Martinovic from University of Windsor.

The Forum opened with an introduction on what is known so far about the MPT. The test will have two sections: one of math content and one on pedagogy. The math content questions will cover topics in the curriculum from Grades 3 to 9. The pedagogy questions will include questions on assessment and evaluation practices outlined in the curriculum documents, *Growing Success: Assessment, Evaluation, and Reporting in Ontario Schools, Kindergarten to Grade 12,* and *Learning for All: A Guide to Effective Assessment and Instruction for All Students, Kindergarten to Grade 12*. The MPT will include 75 questions, including 50 questions on math content, 21 questions on pedagogy, and 4 field-test questions (which will not count toward the test score). The first 5 content questions will cover Number Sense and must be completed without a calculator. In order to pass the test, candidates must score at least 70 percent in each section of the test. The first attempt at the test will be free, but a fee will be charged for subsequent attempts. A pilot will be run in early 2020, where teacher candidates can try out the test for free. The MPT is being designed by EQAO, and Faculties of Education will be administering it.

Many concerns with the MPT were voiced by the speakers and Forum attendees. Since it is a requirement for all teacher candidates to pass this test in order to be certified (not only those who will be teaching math), some attendees raised a concern that the test might prevent strong teacher candidates in other fields from becoming certified. Also, as the math curriculum is currently slated for revision later this year, the content of the test may need to change once the revised curriculum is complete.

While speakers at the Forum commented that the Faculties of Education in Ontario cannot give details on what they are doing to help prepare teacher candidates for the test, Ann Kajander's talk gave some information on how Faculties of Education could view the test as an opportunity. At Lakehead University, all Primary/Junior teacher candidates are already required to pass a math competency exam in order to receive their B.Ed. Kajander told Forum attendees how this has resulted in teacher candidates who would typically avoid math courses enrolling in courses to help ensure they pass the test. To make the most of this opportunity, the Faculty of Education offers courses in conceptual knowledge for teaching. Kajander indicated that courses of this nature lead to better outcomes not only for conceptual knowledge for teaching and pedagogical capacity for methods courses, but also for tests on curriculum content like the MPT. She elaborated that this type of course has a deeper impact than a simpler math content course, which would improve performance on a test like the MPT, but would not yield the same benefits for conceptual knowledge for teaching or pedagogical thinking in methods courses. In this way, Kajander encouraged Faculties of Education to view the MPT as an opportunity to reach their teacher candidates who are most in need of support in mathematics.

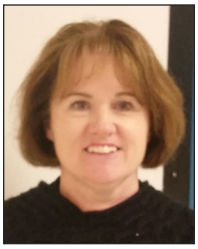
More information on the MPT can be found at www.mathproficiencytest.ca.

The Fields Institute MathEd Forum will convene on the last Saturday in January, February, March, and April at the Fields Institute in Toronto. If you are not able to travel to Toronto to join in person, you can also watch the presentations remotely through the Fields website. ▲



British coins that have how many sides?

▲ OAME/NCTM Report: Surviving and Thriving in Education



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Happy Spring!

A lot happens in the Spring. The flowers begin to peek out from under the snow, Pi day happens (March 14), and it is time to think about new beginnings. In this time of ever-increasing screen time (on your phones, tablets, laptops, and desktops), we may need to consider "how to survive and thrive in your first five and last five years of teaching." With each new season, we need to consider how we connect with colleagues. Once you become an NCTM member, you have access to their online forums, where you can post anything from "How to Teach the Pythagorean Theorem" to "Should students wear hats in class?" The online forum connects you to others who share the same interests as you… while still allowing you to stay tucked away behind your screen.

If we want to stay current with technology, while not allowing the students to "rule the roost," we need to team up, partner up, saddle up, and use all the resources at our disposal. Two ideas that we might try are the following:

1) The online network to pose questions and share ideas and resources (NCTM supported)
2) Buddying up and mentoring (perhaps not a formal mentoring designated by your board office, but a sharing of a course you teach)

The benefits of working with someone face-to-face are outstanding. There is no need to worry about if you have accidently left the "caps lock" on in your last email. You just walk over to their desk and talk to them. If you have ever felt that perhaps you are not "hip" with the latest technology,… put yourself in the shoes of a teacher in his or her first five years. The newer teacher may have the technology down pat, but still struggle with classroom management.

Those "over the water cooler" conversations can give new teachers ideas to try in their classroom, and a sounding board if implementing a new idea doesn't quite work out the first time. And teaching a course at the same time as a technology-savvy teacher will allow you to incorporate online platforms that you may have been reluctant to try in the past.

Teaching is evolving. We need to grow with it, just as that pretty little blade of grass does when it pops its head out of the snow in early spring.

Upcoming Events:

- NCTM turns 100 in 2020, so watch for exciting initiatives and activities.
- April 1, 2020 – Centennial Annual Meeting and Exposition to be held in Chicago

For all you music enthusiasts… NCTM is holding a song contest (see their main page at nctm.org for details).

Yours in mathematical fun,
Jacqueline
▲

Mathematical Word Search: Metric and Imperial units and prefixes

M	O	D	E	T	U	N	I	M	E	A
S	U	E	N	U	M	I	L	E	P	R
F	N	C	E	O	N	A	N	T	O	H
O	C	I	P	K	C	D	E	M	U	E
O	E	T	E	E	I	E	I	E	N	C
T	V	N	D	O	F	L	S	T	D	T
G	E	E	R	T	L	R	O	E	E	O
R	H	C	N	I	Y	M	T	R	H	I
A	I	I	N	G	N	O	L	L	A	G
M	P	H	O	U	R	Q	U	A	R	T

Words and prefixes to find:

CENTI, DECA, DECI, FEET, FOOT, GALLON, GRAM, HECTO, HOUR, INCH, KILO, LITER, METER, MICRO, MILE, MILLI, MINUTE, NANO, OUNCE, PICO, PINT, POUND, QUART, SECOND, TERA

Write the letters that are left over in the spaces below. But write them from left to right and top to bottom.

Metric and imperial units and prefixes have
"_ _ _ _ _ _ _ / _ _ _ _ _ _ _ _ _ _ _ _"

▲ In the Middle: Slow Looking in the Math Classroom



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In the December 2018 issue, I discussed a mathematical exploration of the documentary film *Free Solo* and its subject, climber Alex Honnold. This exploration used Visible Thinking routines from Harvard Graduate School of Education's Project Zero. Recently, I read a new work from Project Zero: Shari Tishman's *Slow Looking: The Art and Practice of Learning Through Observation* (2017). *Slow Looking* is best described by one of its opening lines, "It simply means taking the time to carefully observe more than meets the eye at first glance."

Two Visible Thinking routines, See/Think/Wonder and Think/Puzzle/Explore, indicate a sequence of steps to increase the complexity of student thinking and the mathematical concepts brought to it (www.visiblethinkingpz.org/VisibleThinking_html_files/VisibleThinking1.html). These routines exemplify the concept of Slow Looking. To begin to incorporate Tishman's work, I considered an image that I have used in direct-instruction lessons. By placing axes in different locations on the same image, I can show how the equation of a parabola changes and demonstrate transformations. Figure 1 shows the different axes, quadratic equations, and roots on an inflated arch I photographed at Toronto's Harbourfront.

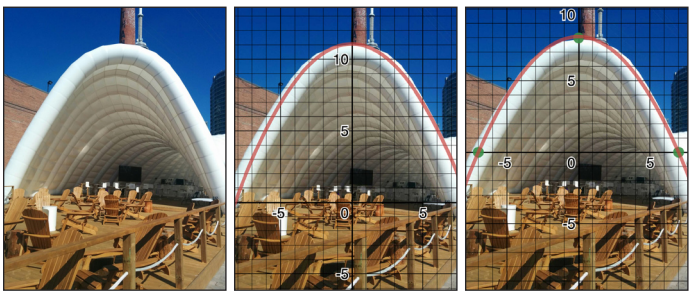


Figure 1: *Harbourfront Centre parabolic arches*

One of Tishman's goals of Slow Looking is perspective-taking. When I led the students through the transformations, I was able to highlight specific changes in the equations. I was interested to see what students might discover once they had time to see, think, and wonder on their own. In small groups, with the images—but without my direction—students identified many other items I had not considered previously. They began to look at the nested parabolic shapes, curious about how those equations would relate. Afterwards, I showed them the original equations I had created and gave them an additional 15 minutes of their own thinking time. I had a motivated class, keen to explore quadratic transformations.

The students considered where else the origin could be placed, and how the equation of the parabola would change based on that choice. Another group wondered whether the scale of the graph "paper" made a difference and how it would change the equations. These were questions beyond what I had originally expected, and I began to consider what we could learn with more time and varied perspectives. Many of their questions eventually connected to later topics, such as finding other kinds of transformations and identifying the roots in different algebraic forms.

Tishman also presents Slow Looking as countering fast decision making. For me, this echoed Thinking, Fast and Slow by Daniel Kahneman (2011). Kahneman, a Nobel Prize winner in Economics, explains how fast thinking leads to overconfidence and mistakes, even when people truly understand what they should do. This includes things like the minor algebraic errors students make when they rush, but also major problems like systematic biases. My feedback to students often contains reminders to think more carefully, to take time to check their steps, and, in general, to go more slowly. Many times, my students understand the basic concepts well, but make errors in a rush to finish their work or get right to the answer. Kahneman showed that slowing down thinking helps to solve more difficult problems, reason more logically, and make better decisions. (Mathematics aside: Michael Lewis' engaging story, *The Undoing Project,* explores Kahneman's academic and personal relationship with Amos Tversky, his lifelong collaborator. *The Undoing Project* has a lot of marvelous examples of probability experiments that can be applied in Data Management.)

I have found that intentionally slowing down learning during math lessons results in improved student learning habits, as they find value in going slower. Increasing the waiting time after questioning during a Socratic lesson noticeably improved my students' engagement and the patience my older students had for others' way of thinking and processes of problem solving. Kahneman indicates that slow thinking both requires and develops disciplined

thinkers. Affected as they are by the fast influences of technology and social media, our students need more opportunities to experience the advantages of taking their time. Implementing structured thinking routines with greater time in math classrooms can help to cultivate that disciplined approach to learning.

Although *Slow Looking* does not present many math-specific examples, it reminded me of Ron Lancaster's Mathematical Snapshots and earlier Photo Math columns in the *Gazette*. Those columns often show that photographs can be motivating prompts for mathematical thinking across the curriculum.



Figure 2: *Spinning maple leaf*

When I first saw a leaf spin in the wind and make these concentric circles, it emphasized for me the definition of a circle, with the leaf's stem as its centre (Figure 2). I watched it spin for a while and waited to see whether the whole circle would be formed with enough wind. As I watched, the leaf only completed part of the circumference. I wondered about the arc length, the fraction of the circle the wind created, and how the radii of the different interior circles were based on the shape of the leaf. I saw this happen again with an oak leaf (Figure 3).



Figure 3: *Spinning oak leaf*

Watching these leaves spin was a lesson in patience and quiet thinking. I was deeply engaged in mathematical Slow Looking. The spinning leaves had me taking the time to marvel at nature and think creatively about different mathematical concepts. As I watched longer, the spinning leaves had me considering both radians and degrees. A Google Image search on "leaf circle snow wind" will yield many wind-spun leaf circles. You can even find a few videos where you can watch the wind form a whole circle.

Using the Visible Thinking routine, called Creative Questions (www.visiblethinkingpz.org/VisibleThinking_html_files/03_ThinkingRoutines/03g_CreativityRoutines/CreativeQuestions/CreativeQuestions_Routine.html), students look over one or more images, first generating and then transforming their questions. With the leaf photos, their fast questions in the first generation were relatively simple: "What is the circumference?" "What fraction of the whole circle is created?" Slower thinking gave them time to transform their questions. The Creative Questions routine provides prompts like, "What would change if…?" or "How would it look differently if…?," and then directs the students to choose a question to extend in this way. Altering their circumference question, my students transformed it to consider whether the distance between circles would always be the same with a given leaf. They asked about the changing relationships of area between the concentric circles and their circumferences.

Bringing in some different leaves with intact stems gave another opportunity to transform and create more questions that relate to these images. My students then asked questions about the area of the circle compared to the area of the leaf, which I had not considered. The routine concludes with reflection, "What new ideas do you have that you didn't have before?" Slow Looking had them extending mathematical ideas (areas within areas) and also engaging in metacognitive strategies.

Think/Pair/Share is another Slow Looking routine that increases silent-thinking time. I used it with linear modelling to compare the age of the "best actress" to the age of the "best actor" over time, and similarly the age of the "best supporting actress" to the age of the "best supporting actor." Figure 4 shows two scatterplots with linear regressions over the history of the Academy Awards. Using the Think/Pair/Share routine, students independently looked at one graph and drew some initial conclusions silently. Then, they explained their graph to their partner; were able to compare and contrast with their partner's related, but different, graph; draw some overall conclusions about age when winning an acting Academy Award; and how that has changed over time. I have also experimented with students writing two conclusions from looking initially at the graphs.

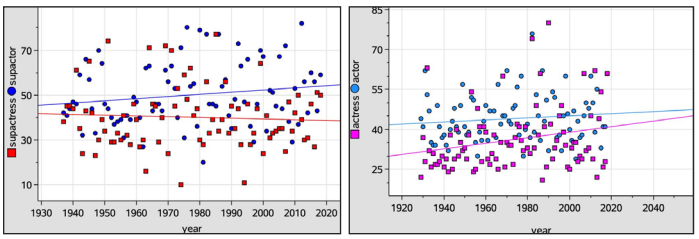


Figure 4: *Scatterplots for: Ages of best supporting actress and supporting actor Academy Award winners by year; and ages of best actor and actress Academy Award winners by year*

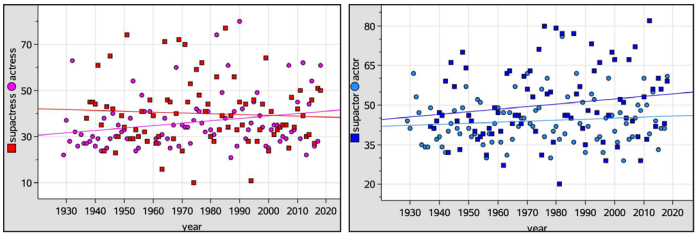


Figure 5: *Scatterplots for: Ages of best supporting actress and best actress Academy Awards winners by year; and ages for best actor and best supporting actor Academy Award winners by year*

I used the same data, but paired differently (Figure 5), and had the partners repeat the Think/Pair/Share routine. Looking at the revisions to their points of view, I was again reminded of Tishman's perspective goal. Kate Tanha has also used data within the Slow Looking framework to draw conclusions involving political and social issues. The motivating images for her looking were Word Clouds, which can be found at https://walktolearn.outofedenwalk.com/2019/07/12/slow-looking-through-a-big-data-lens/.

Tishman's work has motivated me, like Tahna, to incorporate visuals and time for my students in math. The new perspectives and students' engagement have been reminding me to give them more time to think on their own instead of compensating for a faster pace, using more scaffolding—it was interesting to then see what questions and concepts they might put forward. The Visible Learning strategies provided me specific ways to slow down my students' thinking and engage them in some Slow Looking to take in all that meets their eyes. I hope you are able to do the same with your students this year. You might be surprised how much there is to see with just a few more minutes of time.

References

Kahneman, D. (2011). *Thinking, fast and slow.* New York, NY: Farrar, Strauss and Giroux.

Lewis, M. (2016). *The undoing project: A friendship that changed our minds.* New York, NY: W.W. Norton.

List of Academy Award Best Actor Winners by Age. (2019, November 17). In Wikipedia. Retrieved from https://en.wikipedia.org/wiki/List_of_Academy_Award_Best_Actor_winners_by_age

List of Academy Award Best Actress Winners by Age. (2019, December 3). In Wikipedia. Retrieved from https://en.wikipedia.org/wiki/List_of_Academy_Award_Best_Actress_winners_by_age

List of Academy Award Best Supporting Actor Winners by Age. (2019, June 29). In Wikipedia. Retrieved from https://en.wikipedia.org/wiki/List_of_Best_Supporting_Actor_winners_by_age

List of Academy Award Best Supporting Actress Winners by Age. (2019, May 26). In Wikipedia. Retrieved from https://en.wikipedia.org/wiki/List_of_Best_Supporting_Actress_winners_by_age

Project Zero. (2019, December 21). *PZ's thinking routines tool.* Harvard Graduate School of Education. Retrieved from www.pz.harvard.edu/

Tanha, K. (2019, August 31). *Slow looking through a big data lens.* Out of Eden Learn Blog. Retrieved from https://walktolearn.outofedenwalk.com/2019/07/12/slow-looking-through-a-big-data-lens/

Tishman, S. (2017). *Slow looking: The art and practice of learning through observation.* New York, NY: Routledge. ▲

Isosceles versus Equilateral

Isosceles comes from Greek "isos" (meaning "equal") and "skelos" (meaning "legs"). So, an isosceles triangle has equal legs. Contrast this with Latin "aequi" (meaning "equal") and "lateral" (meaning "side"). So, an equilateral triangle is equal-sided.

Did you know? "Billion"

A "billion" is 10002x1000, and a "trillion" is 10003x1000, where "bi" means an exponent of two, and "tri" an exponent of three. However, originally, the British definition of a "billion" was (1000000)2, and only later was it aligned with the modern definition.

Data/Datum

Data is the plural of Datum. These come from Latin, specifically from the root "to give," and is interpreted as "something given." Hence, researchers have to respect that they are being given something when they collect data.

Outlier

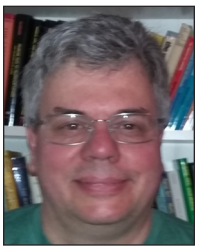
From rock that quarried (hence, "out"), but then left lying (hence, "lie") around unused.

Mathematical Word Search Solution

M	O	D	E	T	U	N	I	M	E	A
S	U	E	N	U	M	I	L	E	P	R
F	N	C	E	O	N	A	N	T	O	H
O	C	I	P	K	C	D	E	M	U	E
O	E	T	E	E	I	E	I	E	N	C
T	V	N	D	O	F	L	S	T	D	T
G	E	E	R	T	L	R	O	E	E	O
R	H	C	N	I	Y	M	T	R	H	I
A	I	I	N	G	N	O	L	L	A	G
M	P	H	O	U	R	Q	U	A	R	T

Metric and imperial units and prefixes have " MEASURED / EVERYTHING "

▲ What's the Problem? Scrutinizing Square Sums



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Shawn teaches at Cairine Wilson Secondary School in Orleans. He strongly believes in the central role of problem solving in the mathematics classroom. He continues to be involved in mathematical activities: presenting workshops, writing articles, working on local projects, and helping create mathematics contests.

Welcome back, problem solvers. Last time, I left you with the following problem:

The numbers from 1 to 576 are written in a grid so that the first row contains the numbers 1 to 24, the second row contains the numbers 25 to 48, and so on, as shown in the diagram below. Determine if it is possible to draw an square around 64 of these numbers such that the sum of the four corner numbers is 1646.

1	2	⋯	23	24
25	26	⋯	47	48
⋮	⋮	⋮	⋮	⋮
553	554	⋯	575	576

This problem is a slight variation of problem A4 from the 2018 Canadian Intermediate Mathematics Contest from the Centre for Education in Mathematics and Computing (CEMC) at the University of Waterloo. You can check out copies of their many mathematics and computer science competitions as well as other resources for teachers and students at the website www.cemc.uwaterloo.ca.

This is a wonderful problem for students to explore and model algebraically. It is also a good example for you, as a teacher, to look at and decide, "How can I adapt this problem for my students?"

A good place to start is to look at an easier problem. If we make some discoveries in the easier version, they might translate to the original problem. How can we simplify this problem? Instead of looking at an 8x8 square on a 24x24 grid, we may look to a 2x2 square on a 10x10 grid. We may have some 100 grids, like the one in Figure 1, lying around that we can work with. If you don't have any, you can easily create your own or find a free version online to download. The familiar grid will make it easier to see the patterns among the numbers, and the smaller square means that the corners are just all the cells in the square.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Figure 1: *One hundred table*

Now let's deal with this systematically. Starting with a 1 in the upper left corner, then moving to 2, and so on, we come up with the following few sums:

$$1 + 2 + 11 + 12 = 26$$
$$2 + 3 + 12 + 13 = 30$$
$$3 + 4 + 13 + 14 = 34$$

We notice that each sum is 4 more than the previous sum. Does this make sense? It is one thing to find a pattern, but it is another to entirely deduce why the pattern exists. In this case, if we move our starting point over by one, each of the numbers in the square also moves over so that instead of writing for the second sum

$$2 + 3 + 12 + 13 = 30,$$

we could have written

$$(1+1)+(2+1)+(11+1)+(12+1)=(1+2+11+12)+(1+1+1+1)$$

and, hopefully, we can convince ourselves that that must be true in general. Having done that, it doesn't hurt to try a few cases to make sure we are correct and haven't made a slight error in logic. From the start of the next row, we get:

$$11 + 12 + 21 + 22 = 66$$
$$12 + 13 + 22 + 23 = 70$$
$$13 + 14 + 23 + 24 = 74$$

And, further down the list, we get:

$$64 + 65 + 74 + 75 = 278$$
$$65 + 66 + 75 + 76 = 282$$
$$66 + 67 + 76 + 77 = 286,$$

which all follow the established pattern. You may have noticed that when we moved from 1 + 2 + 11 + 12 to the next row with 11 + 12 + 21 + 22, each number increased by 10, and the sum increased by 40. If we wanted to, we could even generalize this and imagine we have ten cards, each with a grid of numbers on them. The first card contains 1 to 100, the second 101 to 200, and so on. We could even imagine containing our ten cards in a box, and having ten boxes. The first box would contain 1 to 1000, the second

1001 to 2000, and so on. We could then put ten boxes in a crate, etc. We would see that if we moved our square of four numbers one unit to the right, the sum would go up by 4. If we moved our square down one unit, the sum would go up by 40. If we moved to the next card, our sum would go up by 400, to the next box by 4000, and so on.

We are having all kinds of fun and discovering patterns, but we have gone a bit off track with our original problem. There is nothing wrong with that, and that is exactly what mathematicians do with problems: they play with them, go off on tangents, and change the parameters to see if there is something bigger lurking somewhere in the darkness.

If we return to thinking of the 2x2 square, when the top left corner is 1, the sum of the four corners is 26. Then, as the box is moved, count by 4's and 40's (and possibly 400's and 4000's) until we got to the number we were looking for, or until we got to the edge and realized the bottom right corner was not attainable. That would be a reasonable solution, but we might be more interested in finding a slicker solution.

From our first investigation, we saw that when 1 was in the upper left corner, the four numbers added to 26. Creating a table of values in Figure 2, with n representing the number in the upper left corner and s representing the sum of the four numbers, we quickly recognize this as a linear relation and come up with the equation $s = 4n + 22$ to represent it. A little checking with the other sums we have created shows that this *seems* to work for **all** possible configurations of the square in our set-up. This leaves us with the more mathematically satisfying solution of setting s equal to our desired sum and solving for n. If we obtain a whole number, that tells us the top left corner of the square that gives us our solution (possibly… think about that for a while…), while if we obtain a fraction, we know the sum is not attainable (unless we extend our square to allow for fractions, which in itself could be fun).

n	1	2	3	4	5
s	26	30	34	38	42

Figure 2: *Table for the* 10x10 *array*

At this point, it is extremely important to heed Freddie Mercury's words in the Queen song of the same name and "Don't stop me now" (the next part, "'cause I'm having a good time" is also relevant). It is one thing to get "the answer," but it is much more important to understand *why* the answer is what it is.

If we go back to our 10x10 example, we can imagine a square where the upper left corner is some number n. Then, as we have previously discussed, the number to its right would be $n + 1$, and the number directly below it would be $n + 10$. The last number we can think of in two ways: it is under n +1 and it is to the right of $n + 10$. Notice that both interpretations yield the same result:

$$(n + 1) + 10 = n + 11 = (n + 10) + 1$$

Thus, we obtain:

$$s = n + (n + 1) + (n + 10) + (n + 11) = 4n + 22$$

as we expected, but now we know where it comes from, so we can return to our original problem. Since it is a 24x24 grid, going down one increases the number by 24. If we have an 8x8 square, and we let the upper left corner be n, as before, we can see, as shown in Figure 3, that our sum of the four corner squares must be:

$$n + (n + 7) + (n + 7 \text{ x } 24) + (n + 7 \text{ x } 24 + 7) = 4n + 350$$

So, for our original problem, we set $4n + 350 = 1646$ and get $n = 324$. This means that it is indeed possible, and the four numbers in the corners are 324, 331, 492, and 499.

n	$n+1$	$n+2$	$n+3$	$n+4$	$n+5$	$n+6$	$n+7$
$n+1$ x 24	$n+25$	…					n + 31
$n+2$ x 24	⋮						
⋮			⋮	⋮	⋮	⋮	
$n+7$ x 24	…						$n+7$ x 24 + 7

Figure 3: *The four corners in our original problem*

But wait a minute… is our solution really possible? Here is where we must be careful. It turns out that having a whole-number answer is not enough to guarantee a possible solution. What could go wrong? Suppose I wanted the four corners to add to 430, then solving $4n + 350 = 430$ gives $n = 20$, which is impossible! Why is it impossible? Take a minute to think about it before seeing how this can be resolved below.

The new problem requires that the four corners are 20, 27, 188, and 195. Hopefully you can see the problem now. Since the table is in rows of 24, 20 will be near the end of the first row and 27 will be on the second row. If we had an infinite array of infinite sequences as in Figure 4, we could solve the new problem, but that is not the configuration we have, so we must make sure the solution we arrived at is possible (i.e., is in the domain of solutions). Rewriting and we see that the top two corners fall into the 12th and 19th columns, which means they are on the same line and hence, our solution is valid.

1	2	…	23	24	25	26	…
25	26	…	47	48	49	50	…
49	50	…	71	72	73	74	…
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Figure 4: *An infinite array of infinite sequences*

So, as we can see, there is a lot for the problem solver in this question: precise thinking, algebraic modelling, looking at related problems, reflecting on the reasonableness of our result and, as usual, the possibility of extending this problem. (What would the sum of the four corners of an *axa* square be if the numbers were on a *bxb* grid? What if you were adding the eight corners of an 8x8x8 cube of numbers in a 24x24x24 cubical arrangement of numbers? What about rectangular arrangements?)

There are also a lot of possibilities for the teacher as well. If you are dealing with younger students, putting the numbers on the 10x10 grid to start with helps them to picture the structure of what is going on. You could also reduce the size of the square. To increase the difficulty, you could change the size of the original or inside squares. You could look at the sum of the numbers on the perimeter or all the numbers in the square. You could look at the product of the corners, rather than the sum, or you could ask to find all squares (or rectangles) of any size within the grid whose four corners add up to a given sum.

This reminds me of a problem that a group of us used a few years back in both the Academic and Applied Grade 9 classes. The problem involved being given the sum of three numbers in a row on a 10x10 grid and predicting the middle number. The problem leads to some algebraic modelling and having students come up with their own patterns on the grid and modelling from them. We originally had the students work with a horizontal line, then a vertical line, and then we proceeded to look at when the squares formed a "T." It was interesting because for the lines, the students thought to calculate the average of the three numbers, which gave them the answer. After that point, they were satisfied with the result and unwilling to look at the problem from another perspective. When presented with the "T," they tried averaging them again, but it didn't work, so they were stuck. It took a lot of prodding to get some of the groups to look at the relationship between the squares.

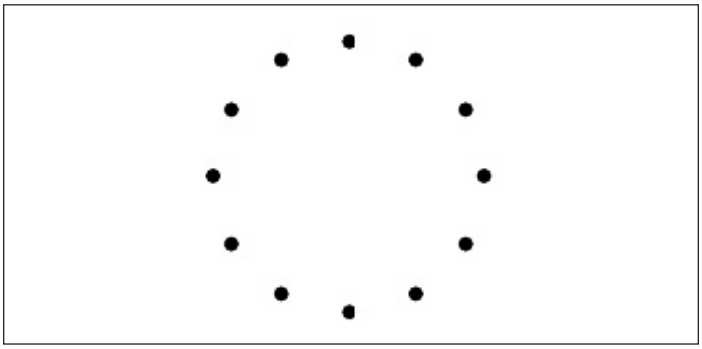
When the group of us got back together, we discussed how we could modify the problem for future use so that the students wouldn't so readily "get" the answer for the first part. Some of us who taught Grade 9 the following year resurrected the problem starting with the "T," which was too much for a first step. An easier starting point may be the "L" shape made of three squares. Maybe the third time will be a charm.

When you find an interesting problem, always be open to how the problem could be modified for use in your classroom. There is no shortage of textbooks, resource books, and contest problems that you can use.

I have been fortunate to work with groups of colleagues from Cairine Wilson Secondary School, curriculum coaches, and teachers from other high schools and elementary schools over the years. Some of the best professional development I have ever had has occurred within these groups. When you are given the opportunity to meet on several occasions on a project that you have helped to develop, it is a wonderfully personalized way to grow as an educator. You get to have discussions with other colleagues about very specific topics, and I always pick up something that I work into my own practice. In many cases, we have used a lesson study model and have tried out our lessons in each other's classrooms. When you are taken out of the teaching role and can observe more closely what specific students do with your problems, you obtain insights that you wouldn't have realized on your own. A shout-out to my colleagues Steve Cowie, Julie Gupta, Kate Harman, and Lynn Miller at Cairine Wilson Secondary School, as well as curriculum coaches Ann Arden (OAME columnist), Seth Rose-Radunsky, and all the other teachers whom I have had the pleasure to work with over the years. If you have an opportunity to work with a group of colleagues, I urge you to take it.

Now, for your homework:

Twelve dots are equally spaced around a circle, like they are on the face of a clock. How many stars can you create by connecting some of the dots?



Until next time, happy problem solving. ▲



Integer elevator!

▲ Exploring Strategies to Support Primary Students' Communication of Mathematical Thinking – From Theory to Practice



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"Jaden created a symmetrical design with the pattern blocks, but when I tried to talk to him about it, he just shrugged his shoulders," sighed one teacher. "When I asked Jasdeep how she solved today's mental math question, she just repeated Mohammad's answer. "I'm sure she had a different strategy, but she just echoes what other students say," responded another. It was the first meeting of our collaborative inquiry group with two consultants from central office and three teachers from three different schools who were all teaching combined Senior Kindergarten/Grade 1 classes. Our area of focus was quickly becoming clear. We were sure our students were doing some quality math thinking, but they weren't able to articulate it to their teachers or to their classmates.

Our group met four times over one school year, from December to June, for a full day each time. At the first meeting, the group collaboratively developed our inquiry question based on our observations of our students: *What are the instructional strategies that support students in thinking about numbers and expressing their own thoughts?* We then dug into professional literature to search for possible answers.

Digging into the Literature to Select Strategies

We began our inquiry with the belief that our students were capable of deep mathematical thinking, but were not able to communicate the range and depth of their thinking to peers, parents (which also includes guardians, caregivers, and other family members), or educators. *The Kindergarten Program* document (Ontario Ministry of Education [OME], 2016, p. 76) reminds educators that "young children come to school already knowing a great deal about mathematics. Children bring with them an intuitive knowledge of mathematics, which they have developed through curiosity about their physical world and through real-life experiences." This aligned with what we were seeing in our classrooms. Teachers observed students demonstrating their mathematical knowledge throughout the day when making comparisons (e.g., "Mine is the biggest.") and asking questions (e.g., "Who has the biggest?" and "How big is it?"). We wondered how we could support students in communicating their intuitive mathematical thinking.

Based on our literature review, we narrowed our focus to five strategies. Each teacher selected the strategies they wished to use based on their knowledge of their students' strengths and their professional judgment. The five strategies we selected, and some of the literature we reviewed, were as follows:

Small-Group Instruction

Small-group instruction is a strategy that elementary teachers are comfortable with using for guided reading, and they understood the benefits of small-group instruction in that context. Marilyn Burns (2000, p. 30) suggests small-group instruction is also appropriate for mathematics instruction. "A small group structure has the potential to maximize the active participation of each student and to reduce an individual's isolation. It provides a setting that values social interaction, a necessary element of children's learning. When organized in small groups, more students have the opportunity to offer their ideas for reaction and to receive immediate feedback." In addition, the small-group setting allows students to draw on the resources and strategies of their classmates, to compare decisions about what information is needed, and to compare their data and solutions (Burns, 2000, p. 159). In addition to the benefits for students, teachers found that small-group instruction in reading gave them better insight into each child's current strategies and understandings, and allowed them to be more precise in their assessment, instruction, and determination of next steps. We anticipated these same benefits would arise from small-group instruction in mathematics.

Use of Technology

While our students were adept at using iPads, we wanted to think about how we could use them to increase students' communication about their mathematical thinking. Applications (apps) that are specifically designed for mathematics often focus on a drill-and-practice approach that simply replaces a paper worksheet with a digital worksheet with animation and/or colour. Some math apps are games with little opportunity for students to develop

problem-solving skills or reflect on their learning (Attard, 2013). We chose to focus on apps that students could use to communicate their thinking, and these may or may not be mathematics apps. Using ChatterPix Kids, students could take photos of the manipulatives that they had used, give the manipulatives a digital mouth, and then record themselves explaining how they used the manipulative to solve the problem. The app would then show the manipulative "speaking" and explaining the math. Use of apps like Explain Everything, Book Creator for iPad, and PicCollage – Your Story Maker were used to record student thinking, and then share with others both in the school and at home. (See Resource Links at the end of the article for more information on these apps.)

Effective Teacher Prompts and Questioning

Klibanoff and colleagues (OME, 2011) found that teacher-facilitated "math talk" in the early years significantly increased children's growth in understanding of mathematical concepts. They noted that, while young children may have a beginning understanding of mathematical concepts, they often lack the language to communicate their ideas. By modelling and fostering math talk throughout the day and across subject areas, educators can provide the math language that allows students to articulate their ideas. In order to facilitate mathematical thinking, rather than direct it, teachers must recognize when student thinking is developing or stalled. If it is developing, the educator observes leaving the students to work through their thinking (Sarama & Clements, 2009, p. 325 in OME, 2011). If it is stalled, probing questions can be asked to provoke thinking about alternate ways to perceive the problem.

Some strategies teachers may employ to provoke thinking include the following:

- Noticing and naming specific math behaviours as they occur
- Asking the children questions that elicit explanations of their mathematical processes in various contexts
- Asking the children to demonstrate, describe, and/or explain their solution to a problem
- Helping the children to make connections between what they already know in mathematics and what they are learning at the moment
- Reviewing children's learning, using pedagogical documentation, discussing it with the children, and negotiating with the children further opportunities that interest them (OME, 2016, pp. 83–84)

Number Talks

Number Talks (Parrish, 2010), a resource for teachers from Kindergarten to Grade 5, supports teachers in engaging students in classroom conversations around purposefully crafted computation problems that children solve using mental math. The problems are designed to elicit specific strategies that focus on "number relationships" and "number relationships and strategies." Number talks can be done in small-group or large-group settings. Students solve the problems on their own, and then are expected to share and defend their strategies and solutions. This allows students to collaboratively build understanding of mathematics, while building connections to key concepts. Number talks are brief and can usually be conducted in five to fifteen minutes. The teacher notes clearly explain the strategy that is expected to emerge, as well as other, less efficient, strategies that students may use. Teachers are instructed to honour these less efficient strategies, while noting connections between the thinking behind these strategies and the more efficient strategies.

Manipulatives

Students need the opportunity to learn by doing, talking, and reflecting on their actions through the use of manipulatives in mathematics. Concrete manipulatives provide students with tactile experiences to help them model, describe, and explore mathematics. *Early Math Strategy: The Report of the Expert Panel on Early Math in Ontario* (2003) emphasizes the need for teachers to provide support for their students through the use of manipulatives. When planning activities with manipulatives, the authors of the report advise that teachers should:

- use a manipulative in such a way that students use it as a thinking tool, enabling them to think about and reflect on new ideas
- recognize that individual students may use the manipulative in different ways
- avoid activities that simply ask children to copy the actions of the teacher
- allow students to use manipulatives to justify their answer as well as solve the problem
- provide opportunities for students to explore the same concept with a variety of manipulatives

Other research that informed our practice explored effective professional learning communities. Katz (2013) reminds us that, to be effective, learning communities must move beyond "story sharing" and into the area of cognitive dissonance and discomfort. While it was important for us to have time to share strategies, we knew we had to move deeper and examine student work to maximize the learning for teachers and students.

Getting Started

In their daily practice, teachers observed and documented the learning of all students, but for this inquiry, project teachers selected four marker students to monitor and observe throughout the inquiry process. Marker students are also called "students of mystery," as they are able to achieve, and yet they are not, and there is no observable reason for their lack of progress (Newman, 2012). Keeping the inquiry question in mind, teachers selected marker students who they expected would most likely be impacted by the instructional strategy or strategies chosen. Information was collected through observation, conversations, and student work samples from the four marker students in each of the three classes.

Collaboratively, we created an interview questionnaire with a combination of open-ended and Likert Scale questions and interviewed our marker students in February and June on their beliefs and attitudes about mathematics. Each of us recorded our observations in the manner which we preferred, such as anecdotal notes, photographs, and digital recording, using the iPad.

When we met and analyzed the data from our initial interviews with the students, we noticed themes emerging. Students mostly referred to Operations and Number Sense, and we wondered if their narrow understanding of "what is math" was impacting their communication. We wondered if the lack of communication about their thinking was because they didn't realize when they were thinking about math. It's hard to communicate math if you can't identify it. We decided we needed to be more explicit about noticing and naming the different kinds of math that we use in our classrooms and at home every day.

During subsequent meetings, each of the three teachers shared strategies that they were using to assist students in communicating about mathematics, samples of student work, observations of our students, responses and questions from parents, and our own reflections on student learning. We read and discussed professional literature that supported us and sometimes challenged us in our thinking about students and their ability to communicate their mathematical thinking. The conversations during these meetings were rich and engaging, and were documented in the minutes for later reflection.

What We Noticed

Student Learning

From the group's observations, conversations, and student work samples, we noted an improvement in the students' ability and confidence in communicating their mathematical thinking. During the June interviews, marker students seemed more confident when speaking with the teachers, their answers were more detailed, and they had much more to say than in the earlier interview. Students now named specific strategies and specific manipulatives that they were using in their work, which they had not done previously.

We found that a range of instructional strategies have a positive impact on the ability for students to communicate their thinking. *Number Talks* (Parrish, 2010) had a profound impact on students' confidence and ability to explain their thinking about numbers. *Number Talks* also supported teachers in naming math strategies for students and parents, and in developing a common math vocabulary. This helped build a community where students and teachers felt safe to share their thinking about math and where it was okay to be wrong.

Teachers also noticed that students' Number Sense vocabulary transferred to other areas of math. For example, number-talks strategies were used by the students when communicating about patterning, which supports connection making in mathematics. Early in the year, the students often answered, "I don't know," when asked to explain their thinking or actions in math. As our work progressed, students were able to explain what they were doing, they could identify strategies as well as the kinds of math they were engaged in, and the use of "I don't know" decreased significantly.

Technology was an effective tool for supporting students in communicating their thinking about numbers, as the technology made their mathematical thinking visible. It was easier for some students to communicate their ideas, using apps like Show and Tell, Pic Collage, and ChatterPix, as they had a concrete representation of their thinking and/or their work to refer to when they were talking. Technology was also used to share the students' mathematical learning and thinking with parents and other colleagues, and these online connections helped to support and sustain us between meetings.

Teachers found that small-group instruction was helpful for supporting students in communicating their thinking about numbers. In the small-group setting, it was easier to ensure all voices were heard and teachers were able to support and scaffold student learning. Small-group instruction also allowed teachers to more accurately assess student understanding, resulting in more precise and personalized feedback that supported planning effective next steps for each student.

Effective questioning was embedded in all that we were doing—number talks, use of technology, and small-group

instruction. By focusing on questioning, teachers were able to support students in communicating their thinking, uncover students' understanding and misconceptions, and make connections to specific mathematical concepts and strategies.

What did we learn? What changed in our practice?

Participating in this inquiry made each of us feel that we had better knowledge and understanding of the mathematics curriculum. When looking at the student work, teachers noticed that sometimes they were teaching concepts that were beyond the Kindergarten/Grade 1 curriculum expectations. As a result, they slowed down and spent more time focusing on key early primary concepts such as developing a solid understanding of the number 10 through the use of ten frames, counting games, and subitizing activities.

In our early surveys, students reported that they seldom talked about math at home or at school with their friends. Their answers to the interview questions indicated that they thought of numbers and operations when they thought of math, and we wondered if their parents had the same narrow definition of "what is math?" The teachers realized that they needed to be explicit with parents and students about the wide range of math—geometry, patterning, spatial reasoning, measurement, and making connections between what was happening at school and at home. As the year progressed, teachers made sure to include different types of math when communicating with parents and students, and also made an effort to include a wider range of math activities to send home for interested families.

Teachers also found that math was more enjoyable for themselves and for the students, as they included hands-on learning and child-directed, play-based learning. After this experience, all three teachers have said that if they were to return to a primary grade other than Kindergarten, they would definitely incorporate more child-directed play-based learning in their program upon seeing the positive impact it had on their Kindergarten/Grade 1 students this year.

At our final meeting, everyone expressed how helpful it had been to be part of this inquiry project, and to have a supportive group with whom they could share their observations, their wonderings, and even their frustrations. The inquiry question was open-ended enough that each participant felt that he or she was able to approach it in a way that met his or her professional learning needs, and the needs and strengths of their students.

Resource Links

Book Creator for iPad – Create, read, and publish simple e-books. https://itunes.apple.com/ca/app/book-creator-for-ipad/id442378070?mt=8

ChatterPix Kids – Use the camera tool on the camera with ChatterPix to make anything talk, including your math manipulatives. www.duckduckmoose.com/educational-iphone-itouch-apps-for-kids/chatterpixkids/

Explain Everything – A tool for iPad that enables educators and students to create and share their own tutorials, stories, and presentations. https://explaineverything.com/education/

PicCollage – Your Story Maker – Create photo collages, using a wide range of templates. https://itunes.apple.com/ca/app/pic-collage-photo-editor/id448639966?mt=8

Baseline Questions for Marker Students

Student: _____

Date: _____

1. How do mathematicians show their thinking?	
2. Tell me about you and math.	
3. What is math?	
4. Do you like to share about math in big groups?	
5. Do you like to share about math in small groups?	
6. Do you like using technology to share your ideas in math?	
7. Do you like to share your ideas, using paper and pencil?	
8. Do you like to use manipulatives to share your math ideas?	
9. Do you talk about math at home?	
10. Do you talk about math with your friends?	

References

Attard, C. (2013). Teaching with technology – iPads and primary mathematics. *Australian Primary Mathematics Classroom, 18*(4), 38–40.

Burns, (2000). *About teaching mathematics.* (2nd ed.). Sausalito, CA: Math Solutions Publications.

Gadanidis, G., & Hughes, J.M. (2011). Performing big math ideas across the grades. *Teaching Young Children Mathematics, 17*(8), 486–496.

Heng, M., & Sudarshan, A. (2013). "Bigger number means you plus!" – Teachers learning to use clinical interviews to understand students' mathematical thinking. *Educational Studies in Mathematics, 83*(3), 471–485.

Katz, S., Crévola, C., & Muhammad, A. (2013). *Leaders in educational thought.* Retrieved from www.thelearning

exchange.ca/projects/leaders-in-educational-thought-steven-katz-carmel-crevola-anthony-muhammad/?pcat=999&sess=1
Newman, E. (2012). *Students of mystery: The student work study teacher initiative* [webcast]. Retrieved from www.thelearning exchange.ca/projects/students-of-mystery-the-student-work-study-teacher-initiative/
Ontario Ministry of Education. (2003). *Early math strategy: The report of the expert panel on early math in Ontario.* Toronto, ON: Queen's Printer for Ontario.
Ontario Ministry of Education. (2005). *The Ontario curriculum, grades 1–8: Mathematics (Revised)*. Toronto, ON: Queen's Printer for Ontario.
Ontario Ministry of Education. (2011, September). *Maximizing student mathematical learning in the early years.* Capacity Building Series. (Special Edition #22). Toronto, ON: Student Achievement Office.
Ontario Ministry of Education. (2016). *The kindergarten program.* Toronto, ON: Queen's Printer for Ontario.
Parrish, S. (2010). *Number talks.* Sausalito, CA: Math Solutions Publications. ▲



British public access to measurement standards. Greenwich Observatory, Greenwich, UK.



Geometric ceiling in King's Cross station, London, UK.

▲ Some Alternatives to Using Tests and Quizzes for Assessment



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Jeff Irvine, PhD, has been a secondary mathematics teacher, department head, and vice-principal. He has taught at three faculties of education and at a community college. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. With over 45 years in education, Jeff is particularly interested in the role of student motivation in mathematics achievement.

To maximize learning opportunities for every student, we recognize that there is no "one size fits all" strategy. Teaching and assessment strategies that work well for some students may not work as well (or at all) for other students. This article looks at several classroom-tested assessment strategies that generally work well for students who possess some subset of the following attributes: low math self-efficacy, negative attitude toward mathematics, weak study skills, lack of persistence, often disengaged in class, frequent low performance, weak work habits, little or no homework completion, negative attitudes toward school, and negative experiences with math textbooks. These attributes may be a consequence of the student's dominant learning modality being addressed infrequently; family or personal factors such as taking care of younger siblings; part-time (sometimes full-time) jobs; peer pressure; lack of prior learning; involvement in out-of-school activities. These factors can manifest themselves in class through poor attendance or frequent lates to class, reluctance to get involved in classroom discussions, and especially performance on traditional written assessments that have time constraints. To suitably address these different learning needs, assessment strategies need to be varied, and that can be challenging for many teachers..

Attitude and engagement have a major impact on mathematics achievement (e.g., Conner & Pope, 2013). Fredricks, Blumenfeld, and Paris (2004) found that a significant percentage of mathematics students lack motivation, have low mathematics self-efficacy, and generally exhibit low levels of engagement, as well as

negative attitudes toward mathematics. This situation is exacerbated by reliance on a transmission style of pedagogy, which encourages students to become passive content consumers, rather than active participants during their learning (Cotic & Zuljan, 2009). To address these issues requires not only changes in instruction, but also changes in assessment strategies. Many of the assessment strategies described below were assessed using co-constructed rubrics. I have found co-construction of rubrics to be a powerful way to encourage student involvement. Students can be taught how to construct a rubric. Here are the steps that I used to do this. Initially, activities were assessed using teacher-constructed rubrics. The positive and negative aspects of these rubrics were discussed in class. Attention was given to the structure of the rubric, the criteria (left column) and the quantifiers (top row). First, we discussed what criteria were appropriate for the activity. Then we talked about how to differentiate among the quantifiers (differences in the levels for each criterion, beginning with Level 3 (what constitutes acceptable work). Then, in groups, students were asked to construct a rubric for a different activity. These initial group rubrics were shared, usually using a gallery-walk strategy. The goal for this part of the activity was to identify which criteria should be included in the final rubric, and how the quantifier levels were to be differentiated. A final version of the class rubric was distilled from the group rubrics and gallery walk. As the teacher, I ensured that critical criteria were included.

Here are some assessment strategies that I have found to be effective.

Student Journals

I have found journals to be an excellent assessment strategy for students who are uncomfortable asking questions in class. Here is how I used journals in my math classes. Every week, the last half hour of Friday's class was devoted to journal writing. The format was always the same, consisting of "This week in math" plus a second prompt that often focused on more metacognitive or personal communication (see the table below for some sample sentence stems). "This week in math" was not expected to be a chronological record of what we did that week, but rather was a chance for students to identify how well they felt they had mastered the week's math concepts. So a student might say, "This week I was OK with everything, except questions like _____" or "I'm fine with this week's ideas, since I did all my homework and didn't have a lot of problems." Supporting metacognitive thinking is important for students to develop self-regulation (Ontario Ministry of Education, 2010).

My commitment to my students was first, that no one except me would see their journal entries, and second, that I would respond to every student in writing. Since I used journals in all my classes, it required that I wrote responses to up to 90 students every weekend. The responses might be brief comments, or I might provide students with additional ways to understand a concept, give them additional practice questions, or recommend that the student come in for extra help at lunch or after school. This was very specific feedback that addressed student needs individually. If you use journals in this way, you have to be prepared that once you gain your students' trust, they may disclose very personal things to you in their journals. While this was not my intent, it did allow students to vent or sometimes reveal other dimensions of themselves. Overall, I found that journals helped address students with low self-efficacy in math, and negative attitudes, and since the journals were written in class, issues with homework completion.

Sample Student Journal Prompts

This week in math +

- My life in math so far: _____.
- When I do something right, I like to be recognized by _____.
- My study plan for next week's test is _____.
- My favourite subject is _____ because _____.
- When/where/how do I do math homework: _____.
- My favourite form of evaluation is _____ because _____.
- I prefer to work [alone, partners, group] because ____.
- When I had to explain _____ to my partner, I _____.
- Convince me that you understand _____.
- Make up a question that has this solution: _____.
- What if? [How would your answer to _____ change if _____ were faster/bigger/twice as big/half as big…?]
- With reasons, predict what your mid-term math mark will be: _____.
- List at least three advantages of group work: _____.

- List at least three disadvantages of group work: _____.
- List at least five occupations that use a lot of math: _____.
- Critically evaluate Tuesday's lesson on _____.
- What was your best day in math class this week? Why? _____.
- Critically evaluate your math notebook: _____.

Portfolios

Student portfolios support students who do not or cannot spend much time on math outside of school, and who may not do well on timed written assessments. I used two types of portfolios: work portfolios and showcase portfolios. Work portfolios were a repository for the student's daily work in math class. At the end of the class, students placed all their work in their own portfolio, along with any handouts or other materials that were used in the class. The work portfolio had a table of contents, in which the student added a record of the day's math work and concepts. The work portfolios were formatively assessed to identify learning difficulties and misconceptions. Students found the work portfolios motivating, since they received private formative feedback. The agreement with my students was that they were expected to put in optimum effort during the class, and then any additional work outside of class time was the student's individual choice. A somewhat unexpected result was that some students chose to do a significant amount of additional work outside of class, probably due to it being their own personal choice and not mandated by me.

The second type of portfolio was the showcase portfolio. To build a showcase portfolio, the student selected four or five pieces of work from his or her work portfolio based on predetermined criteria jointly constructed by the teacher and the students. The objective of the showcase portfolio was to demonstrate growth over time. Students were expected to include a reflective paragraph on their progress, supported by the pieces of work they included. Reasons for including each piece were expected. Some prompts for choosing pieces of work are shown in the table below. The showcase portfolio supported student self-assessment, as well as higher-order thinking and metacognitive skills. The showcase portfolio could be used as a formative assessment, or could be evaluated as a summative assessment for a unit of work.

Sample Showcase Portfolio Prompts

- I chose this piece for my portfolio because _____.
- This shows my strengths in _____ are _____.
- This is my best work because _____.
- This shows that I am getting better at _____.
- I like this because it shows that I can _____.
- From this assignment, I learned _____.
- This assignment was interesting to do because _____.
- When I did this assignment, I worked hard on _____.

Assignments

Take-home assignments with flexible due dates are a way for students who don't do well on timed paper-and-pencil assessments to demonstrate their learning. The assignments need to have some level of personalization to minimize students duplicating the work of others. Here are some assignments that I found worked well. The first assignment was successful for Grades 7 through 9, allowing students freedom to choose pictures that were of personal interest to them. Students found this activity to be engaging and fun, while still requiring them to demonstrate their knowledge. Submission due dates were sufficiently flexible to accommodate students who were involved with out-of-school activities or had personal issues.

The Justified World of Geometry

Student instructions:

- Take pictures of as many different three-dimensional or two-dimensional shapes as you can find.
- Name each shape. Use as precise a name as you can (e.g., **isosceles triangle**, rather than **triangle**).
- Identify the attributes that justify the name you used for each shape (e.g., it is an **isosceles triangle** because it is a **triangle** with two equal sides). Note that you are not expected to measure anything. You may choose to say, "It *appears* to be an isosceles triangle because it is a triangle, and two sides *appear* to be of equal length."



For the clock tower, you might identify a **box** (properly, a **rectangular prism**) as the base, a **square-based pyramid** for the roof, a **rectangle** as the face of the base, a **rectangle** or **parallelogram** for the window, an **isosceles triangle** as the face of the roof, and a **circle** as the clock face. What other shapes can you identify in this picture?

- Try to get pictures of at least 20 different shapes. Be sure to **name** each shape and provide **justification** for each name.
- Submit your files electronically by the due date. Technology is available.

The Perpetual Painters of the Golden Gate Bridge

This assignment worked for most grades, with more depth of thought expected for senior students. Senior students might relate bridge shapes to functions that they had studied, include rate problems, or other connections to curriculum.



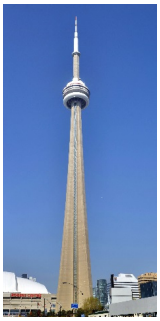
Student instructions:

There is a crew of painters in San Francisco, California, whose entire job is to paint the Golden Gate Bridge. They begin at one end, and paint until they reach the other end. Then they immediately start again at the opposite end of the bridge.

Your task:

Make up at least two problems based on this information. Each of your problems should address at least two different mathematical process expectations. You may want to consult the Internet for additional information about the bridge.

Why Can an Ant Jump off the CN Tower and Survive? A Web-Based Investigation of the Square-Cube Law



This assignment for senior students integrates science and math, which may interest some students. It also allows for some student creativity and higher-order thinking skills. For reference, Bergmann's rule states that larger-sized species are found in colder environments, and smaller-sized species are found in warmer regions. Allometry is the study of relationships among body size and shape, anatomy, physiology, and behaviour. This information can be found on the websites listed in the student instructions. However, not all questions can be answered using only the websites listed in the activity.

Student instructions:

1. Investigate the web page www.world-builders.org/lessons/less/les9/area.html and write a brief report of your findings.
2. What is the square-cube law, and to what two mathematical quantities does it relate?
3. Give some examples from biology of the square-cube law for different species.
4. Investigate the website www.talkorigins.org/faqs/sauropods/sauropods-misc.html and write a brief report of your findings.
5. What is Bergmann's rule and how is it related to the square-cube law?
6. What is allometry and how is it related to the square-cube law?
7. Why can't King Kong exist?
8. Why can an ant jump off the CN Tower and survive?
9. Construct a three- or four-panel cartoon about something you'd like to see if the square-cube law didn't exist. You may use your Bitstrips account, if you wish.

Role Play

Role play generates interest for students who are more involved in drama, and the bodily-kinesthetic modality is a change of pace for all students. Two role-play scenarios that were successful for my students are shown below.

The House Hunt Project (Grade 12 College Preparation)

Scenario:

Two students role-play a couple who wish to purchase a house or condominium. They will be interviewed by a third member of the group, who will role-play a bank loans officer. The teacher will observe and assess with a rubric, looking for evidence of mathematical concepts, accuracy of calculations, adequacy of preparation, and positive group dynamics.

Student instructions:

Background Preparation:

Loan Officer

1. Obtain a blank mortgage application form from a local bank.
2. Determine current mortgage rates.
3. The website www.cmhc-schl.gc.ca/ has a lot of useful information.
4. Learn how to compute mortgage payments, using appropriate technology.
5. Learn how to compute Gross Debt Service Ratio (GDS) and Total Debt Service Ratio (TDS), and investigate cut-off rates used by local banks.
6. Make up a list of questions to ask the prospective house hunters during the interview.

Couple Applying for a Mortgage

. Invent an appropriate background for yourselves (ages, occupations, incomes, amount available for down payment, sources of financing for your home).
. Identify a house or condominium that you would like to buy, using locally available information (newspapers, the Multiple Listing Service®, Internet).
. Generate a budget and a net worth statement to present at the interview. Dollar amounts should be realistic.

The Interview

The interview is run by the loans officer, with time available for the couple to ask questions and provide clarification, if necessary. The mortgage application, the budget, and the net worth statement should be completed prior to the interview and brought with the couple.

Investment Adviser

This role-play scenario fits Grade 10 Applied or Academic and Grade 11 College courses. The activity connects systems of equations to the real world and is motivating for many students, regardless of their dominant learning modality. It also provides a non-threatening method of assessing student knowledge.

Prior Activities:

Before the role play, students completed problems involving asset allocation, weighted average rate of return, and risk scales. Here are some sample problems:

The following clients have come to you for advice on investing. In most cases, clients wish to invest money in two different investments. The lower-risk investment usually carries a lower rate of interest, but greater safety. The higher-risk investment caries a higher rate of return.

Tony Scott wants to invest some money in Treasury Bills—very low risk, 3% annual interest; and some in a balanced mutual fund—medium risk, 10% annual rate of return. He has $18 000 available to invest and wants an annual income of $960. How much should he invest in each choice?

To compute weighted average rate of return, add the annual dollar returns for each investment and divide by the total amount of money invested.

Rita Ritson has $10 000 to invest. You have recommended a mix of low-risk Treasury Bills (annual rate 3%) and balanced mutual funds (medium risk, annual rate of return 8%). Determine how much money Rita should place in each investment if she wants a weighted average annual rate of return of 5.55%.

Investment risk is often rated on a scale of 1 (Low) to 10 (High). Portfolio risk is the weighted average of the risks of the individual investments in the portfolio.

Dani received a windfall of $20 000. Allocate his investment between Guaranteed Investment Certificates (risk level 2) and equity mutual funds (risk level 8) to achieve a portfolio risk level of 6.75.

Role Play

Once students have become proficient at these types of problems, students in groups of three construct an interview scenario. In the scenario, one student will be the investor, one will be the adviser, and the third student will be a peer assessor. Students work collaboratively to prepare for the interview. The student investor formulates several problems involving the three concepts outlined in the above problems; the student adviser investigates current rates of return and risk levels for actual investments, and prepares questions for the investor; together with the other group members, the student assessor co-constructs a rubric to assess each participant in the interview. Everything is transparent to all three students. The students then role-play the interview and prepare a summary to be submitted to the teacher.

Rich Assessment Tasks (RATs)

Rich assessment tasks are a way to assess student learning without traditional paper-and-pencil assessments (or in addition to traditional assessments). They are often done in student groups, especially when they include data collection. Tasks may be closed-ended (sample 1 below) or may be open-ended (sample 2 below). The richness often allows students to demonstrate their creativity and/or deep understanding of concepts. Two rich assessment tasks that I have found to be engaging to students and informative to the teacher are shown below. These can be used for formative assessment, using co-constructed rubrics, or for summative evaluations, using teacher-constructed rubrics.

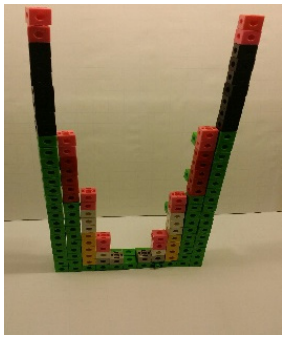
Connecting Cube Quadratics (Grade 10 or Grade 11)

Student instructions:



1. Build a representation of $y = x^2$ for $-5 \le x \le 5$, using three colours of connecting cubes. The representation must show first and second differences.
2. How would you modify your representation for each of the following:
   $y = 2x^2$
   $y = 0.5\ x^2$
   $y = x^2 + 3$
   $y = (x - 2)^2$
   $y = (x + 3)^2 + 2$
3. Justify your answers for #2. Describe how to build a representation for the general quadratic $y = a(x - h)^2 + k$.
4. Describe how you could use connecting cubes to model other polynomial functions such as a cubic, or how you could model a sinusoidal function.

Comments for teachers:



Superficially this activity seems straightforward. The subtle difficulty is the need to represent the second differences. This leads to a very rich discussion among the groups. Students can quickly build a model that shows first differences, using different-coloured cubes. However, more thought is required to illustrate the constant second differences contained within the first differences. The picture shows how this can be accomplished by using a third colour to identify the second differences.

Ecological Footprint (Grades 7 to 9)

This activity should be modified to use a map of your local community. Assess with a rubric, and add a gallery walk to allow students to consider different representations. The instructions were intentionally written on two sides of a single piece of paper. Students were instructed not to turn the page over until instructed to do so. The open-ended nature of the question ("For the city of Brampton, shown in the map below, illustrate this relationship.") allowed for creative solutions and forced students to demonstrate a deep understanding of ratio and proportion.

Front of page:

The land area needed to supply a typical Western city's material, energy and food requirements—its "ecological footprint" is about _____ times larger than the city itself.

Student instructions:

First: Do not turn this page over!

Second: Fill in the blank. (You may wish to discuss with your group.)

Back of page: The land area needed to supply a typical Western city's material, energy, and food requirements—its "ecological footprint" is about 120 to 200 times larger than the city itself. *~ Globe and Mail*

For the city of Brampton, shown in the map below, illustrate this relationship.



Other Questions (will require additional research):

- What factors contribute to this footprint?
- How many "ecological Brampton footprints" could fit into Ontario? Into Canada?
- What is your personal ecological footprint?
- What is the ecological footprint of the earth's population if everyone were to live at a "Western" standard?
- What determines whether the upper or lower range in

the above quotation applies?

- How can the footprint be reduced in size? What are the sociological implications of this?
- Identify what information you need to answer the above questions.
- Indicate where you would find the information you need.

By employing strategies like the ones outlined above, students with some or all of the attributes identified at the beginning of this article can demonstrate their learning in a less stressful environment than traditional assessments. There is abundant research (e.g., Connor & Pope, 2013) that addressing student motivation and affective dimensions has a positive impact on student achievement. This is especially true in mathematics (Fredricks et al., 2004). In addition, using assessment strategies that better address students who possess attributes like the ones identified at the beginning of this article is necessary if teachers are to equitably assess student achievement for all students. Einstein has been quoted as saying, "Insanity is doing the same thing and expecting a different result." However, in teaching, insanity is doing the same thing and expecting the same result. Our students are individuals, with different learning needs and different assessment needs. Using a variety of assessment strategies is imperative for us to allow every student to demonstrate his or her learning in an optimal way.

References

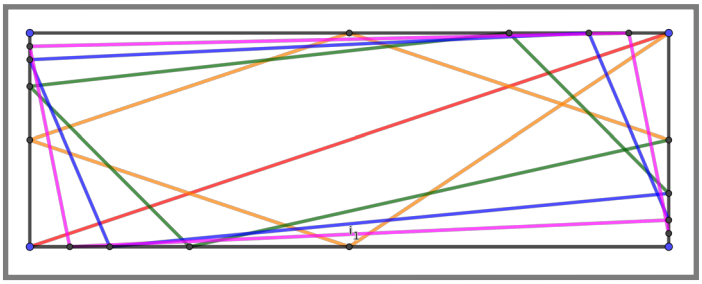
Conner, J.O., & Pope, D. (2013). Not just robo-students: Why full engagement matters and how schools can promote it. *Journal of Youth and Adolescence, 42*(9), 1426–1442. doi:10.1007/s10964-013-9948

Cotic, M., & Zuljkan, V. (2009). Problem-based instruction in mathematics and its impact on the cognitive results of the students and on affective-motivational aspects. *Educational Studies, 35*(3), 297–310. doi:10.1080/03055690802648085

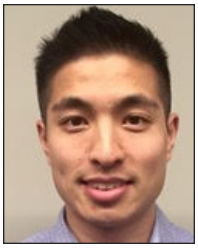
Fredricks, J., Blumenfeld, P., & Paris, A. (2004). School engagement: Potential of the concept, state of the evidence. *Review of Educational Research, 74*(1), 59–109. doi:10.3102/00346543074001059

Ontario Ministry of Education. (2010). *Growing success: Assessment, evaluation, and reporting in Ontario schools, grades 1 to 12.* Toronto, ON: Queen's Printer for Ontario.

Quote Investigator. (n.d.). *Albert Einstein.* Retrieved from www.quoteinvestigator.com/2017/03/23/same/ ▲



▲ The Treasure Chests: Exploring Relationships among Fractions, Decimals, and Percents



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> *Ideally, as students build on their sense of number throughout their elementary schooling, they are given opportunities to make many connections among whole numbers, fractions, decimals, and percents, which support them in deepening their understanding of proportionality and ratio* (Ontario Ministry of Education, 2014).

SETTING SAIL ON THE HIGH SEAS

In this article, we will share our experience with developing a task to support students with applying proportional reasoning and demonstrating effective problem-solving skills. We will explain the steps we took from brainstorming initial ideas, to linking mathematical concepts and curriculum expectations, to engaging students in the learning, to monitoring and consolidating the learning, as well as reflecting on our next steps.

FINDING THE TREASURE MAP

Justin was teaching a Grades 5/6 class of 26 students, which included three English Language Learners, and three students whose special needs were addressed through Individual Education Plans (IEPs). Ken was the principal of the school, who was seeking to develop his own learning in mathematics, in order to better support and lead the mathematical learning and instruction across the school. In the spirit of collaborative learning, Justin invited Ken to co-plan and co-teach a unit of study related to fractions, decimals, and percents. The goal of this collaboration was to refine the practice of developing tasks that purposefully required students to apply skills in the areas of problem solving, articulating reasoned responses, engaging in productive conversations, representing ideas in personally meaningful ways, and reflecting on the connections made between ideas and concepts.

Thinking about fractions requires a critical understanding about numbers and their relationships, and is linked positively with overall progress in mathematical skills development (Torbeyns, Scneider, Xin, & Siegler, 2015). Students use fractions to represent relationships between numbers, which demonstrates an understanding of proportional reasoning. Teachers need to provide students with frequent opportunities to experiment with fractions so that they gain an appreciation that fractions have many meanings. For example, fractions can represent part of whole numbers (e.g., 1/2, represented on a number line, would fall between 0 and 1). Fractions can also represent directions for understanding quantity and magnitude (e.g., half of the basket of apples). Knowing the different meanings of fractions is both important and quite practical. Consider an example where you are cutting tiles for a new bathroom floor. The difference between cutting off a 1/4 inch or a 3/4 inch could result in an embarrassing gap. Or, your choice between eating 1/2 of a pie or 1/6 of a pie could result in quite a stomachache. Ultimately, fractions make sense when talked about in relation to the context in which they are being used. So, in addition to the opportunities to work with fractions, students also need to work with decimals and percents as a means for representing proportional thinking in personally relevant ways (e.g., visual representations or models).

To get started, we anchored our planning in the learning expectations reflected in the Number Sense and Numeration strand of the provincial curriculum. Then, building from these expectations, we identified a learning goal and the underlying big ideas.

Learning Goals:

We are learning to use proportional reasoning to draw conclusions about the relationships among decimals, fractions, and percents.

Big Idea(s):

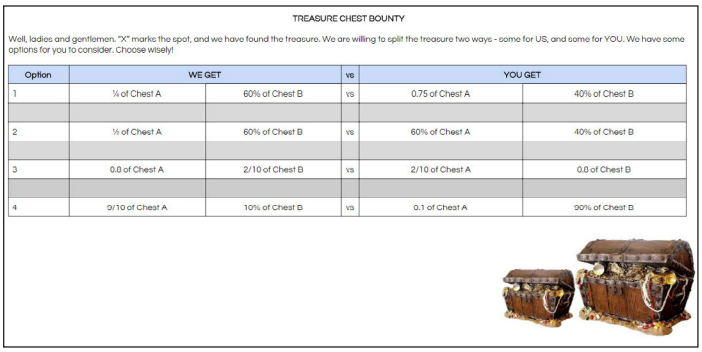
- Proportional reasoning can be used to solve real-life problems.
- Proportional relationships allow students to build connections within and across strands of Mathematics.
- Fractions, decimals, and percents are closely related, and can be used interchangeably when representing parts to a whole (e.g., 1/2 = 0.5 = 50%).
- Decimals, fractions, and percents represent the same concepts, and percents represent hundredths.

We then engaged our students in the process of co-developing a list of criteria to assist them with monitoring their own progress toward success with fractional thinking. We asked students to consider what thinking behaviours they would need to demonstrate and defend in order to be successful. The success criteria were organized around the four domains of learning reflected in the provincial Achievement Chart for assessment.

SUCCESS CRITERIA	
Knowledge & Understanding	☐ I show that I understand that fractions, decimals, and percents are all ways of representing values. ☐ I show that I understand that fractions, decimals, and percents are related, and can be converted to each other. ☐ I show that I understand that fractions can sometimes be simplified.
Thinking	☐ I use a plan of action that focuses on different aspects of the problem. ☐ I use critical- and creative-thinking skills to reflect on the reasonableness of my solution(s), and if necessary, I adjust my approach or strategy. ☐ I justify and prove my answer through ways that make my thinking clear.
Communication	☐ I can choose to use models to represent my thinking about fractions, decimals, and percents. ☐ I represent my thinking in a way that makes sense to me, and which clearly shows how I approached the problem.
Application	☐ I use what I know about numbers and proportional relationships to solve new problems. ☐ I am able to synthesize the learning, and explain how I can apply my learning to new situations.

FINDING THE BOUNTY

Over the course of the year, we had become quite comfortable with brainstorming word problems for our students. When it came to fractions, decimals, and percents, we wanted our students to experience a relatable situation that made these concepts useful for them. We wanted our problems to provoke and challenge mathematical thinking, while inspiring students to apply problem-solving skills in a fun and engaging situation. Pretty soon, the idea of a pirate bounty came to mind, where students would have some choices to make in order to gain some treasure. We threw in a twist, where students were at risk of being bamboozled by their teachers; what student doesn't like the idea of that?



TREASURE CHEST BOUNTY

Well, ladies and gentlemen. "X" marks the spot, and we have found the treasure. We are willing to split the treasure two ways - some for US, and some for YOU. We have some options for you to consider. Choose wisely!

Option	WE GET		vs	YOU GET	
1	¼ of Chest A	60% of Chest B	vs	0.75 of Chest A	40% of Chest B
2	½ of Chest A	60% of Chest B	vs	60% of Chest A	40% of Chest B
3	0.0 of Chest A	2/10 of Chest B	vs	2/10 of Chest A	0.0 of Chest B
4	9/10 of Chest A	10% of Chest B	vs	0.1 of Chest A	90% of Chest B

In our best pirate impersonations, we regaled our crew (well, our students) with a story of our daring adventure on the high seas, where we successfully located and retrieved two treasure chests of different sizes. We then proceeded to present the class with some options for how to share the treasure (including one, Option 2, that is mathematically impossible, but is offered as a way to check how students approach the problem). Of course, being pirates with a penchant for quandaries, we first refused to tell the crew what was actually inside the treasure chests (after all, we would lose our membership in the League of Pirates if we did that). Instead, we proposed a sharing model in the form of mixed representations (e.g., You can have 1/10 of Chest A and 60% of Chest B, while we get 0.9 of Chest A and 2/5 of Chest B).

Rather than front-loading the task with explicit mini-lessons, we wanted our students to work through and develop a conceptual understanding of the interconnected relationships among fractions, decimals, and percents. In this problem, students were primarily focused on making sure that they received their fair share of the treasure. Inadvertently (but quite purposefully on our end), they actually applied their learning about these relationships in the real-life context of avoiding being "ripped-off" by their pirate captains. Through inquiry and engagement, students developed mathematical learning that could be applied beyond the math classroom.

Description of the Task:

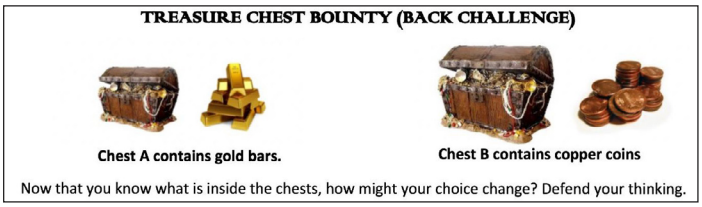
In designing the problem, we anticipated that our students would use conversion strategies in order to judge the reasonableness of the options for sharing the treasure. So, we made sure to present the options in mixed form, including fractions, decimals, and percents.

We also wanted our students to apply proportional reasoning when evaluating and defending their choices. By having the treasure chests being different sizes, students needed to consider the relationship between a portion of treasure they would receive from the small chest versus a portion of treasure that they would receive from the larger chest (e.g., whether 25% of a large chest would yield more treasure than 40% of a smaller chest).



To add another element, we were purposefully vague in identifying the contents of the treasure chests. We wanted our students to think beyond simple numeric values (e.g., 75% yields more than 50%), and to consider the role of the contents in determining value (e.g., 75% of a chest of copper is worth less than 50% of a chest gold). Students were asked to make decisions that would result as much in their favour as possible, regardless of the contents of the chest. This element provided students with an opportunity to think reflectively and to hedge their bets.

As a back-page challenge, we told the students what each chest contained, and asked them to both reflect on their original choice, and to consider how they might have approached the problem differently, had they known the contents ahead of time.



TREASURE CHEST BOUNTY (BACK CHALLENGE)

Chest A contains gold bars.

Chest B contains copper coins

Now that you know what is inside the chests, how might your choice change? Defend your thinking.

We also had a variety of manipulative resources available to students, including:

- treasure chests
- centimetre cubes
- snap cubes (larger)
- Cuisenaire rods
- fraction circles

TELLING THE TALE OF THE ADVENTURE

We loosely followed a three-part math lesson framework to engage students in the task.

1. Minds On

We began by engaging the class in a purposeful conversation in response to these questions:

- What are fractions, decimals, and percents?
- How might they be related to one another?

We were sure to give consideration to what we anticipated we would see or hear from our students during these initial conversations. For example, we anticipated that we would see:

- students reflecting on when they have seen fractions, decimals, and percents outside of the school context (e.g., recipes, temperature, price tags, sharing items, video games)
- students identifying landmark relationships (e.g., 50% = 1/2 = 0.5)

Next, we invited two students up to the carpet and proceeded to offer one student 1/2 of the contents of a treasure chest, and the other student was offered 40%. We then asked each student:

- How do you feel about our offers?
- Why do you feel that way?

Again, we gave consideration to what we anticipated we would hear from students:

- Referring to the situation as fair or unfair
- Connecting 1/2 to 50% and 40% to being less than half

2. Hands On

Next, we presented the students with the images of two treasure chests, one approximately twice as large as the other. We offered to share our bounty with the students—but we wouldn't make it easy for them. We asked the students to choose from a range of choices on how best to share the treasure. We anticipated that students would:

- convert the expressed values into common forms (e.g., fractions, decimals, or percents)
- use place value to understand percents and decimals (e.g., ones, tenths, and hundredths)
- reduce fractions to their lowest common denominator
- use proportional reasoning to estimate the values expressed by the fractions, decimals, or percents in relation to the treasure chests (e.g., 50% of the larger chest yields more treasure than 50% of the smaller chest)
- use models and/or manipulatives to represent their thinking

As students engaged in the task with their math partners, we began to circulate the room to monitor the learning by listening in and observing how students were tackling the problem. When necessary, and to help advance the conversation, we would offer prompts or questions that served to redirect, clarify, or extend thinking. Prompts took the form of the following:

Redirection:

- I noticed that you are focusing on the number values. What other information might be important for making your decision?

Clarification:

- I heard you talk about adding the numerators and the denominators to get your answer; can you represent that using a model to help us see what that might look like?

Extension (of their conceptual understanding):

- How might things change if we found a third treasure chest?
- Which of the options are possible? Can you explain why?
- Which of the options are not possible? Can you explain why?
- What other options might be possible? Impossible?
- Is it possible for Chest B to be worth more than Chest A? How? Why or why not?

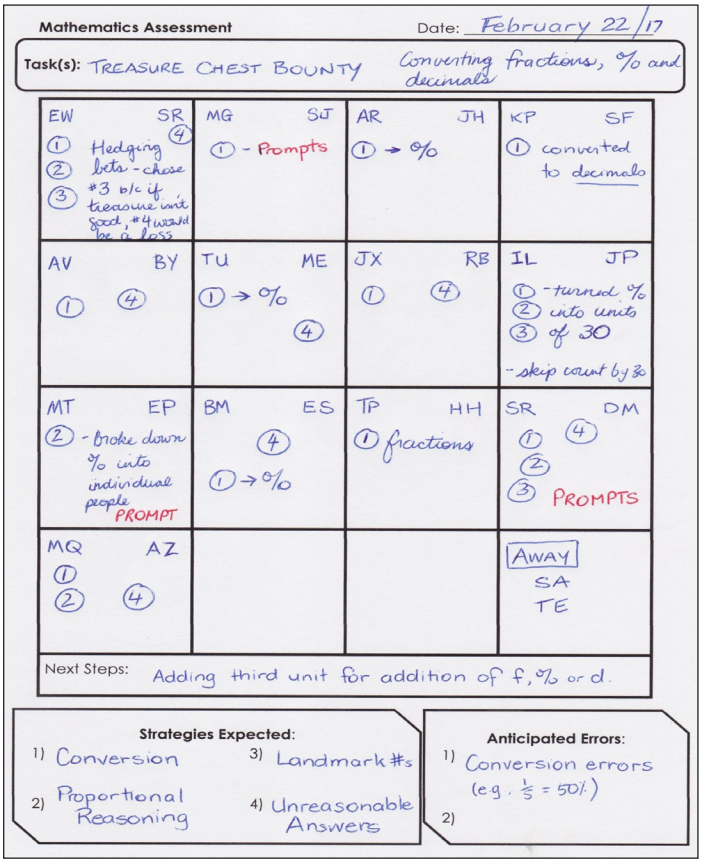
Additional conversation prompts that we used to encourage communication and build mathematical thinking included:

- Talk about your thinking.
- Explain your thinking to your partner.
- What do you already know?
- I wonder if that is true in each of the situations.
- I wonder what that idea might look like.

- Do you agree? Do you disagree? Why?
- Can you repeat what he or she just said in your own words? Would you like to add on?

ASSESSING THE BOUNTY

While we were checking in on each group, we carried with us our assessment grid. In each of the larger squares, we listed the partnerships. To the side, we created a numbered list of the strategies we anticipated observing our students using. By numbering the list, we were able to quickly jot down the corresponding number when we saw or heard the strategy being used. This allowed us to focus our attention on intervention or extension, and to make note of strategies that we did not anticipate. Our notes formed the basis for our choices during the consolidation phase, where we were purposeful in deciding what examples of student work to share, and in what order we should share it in order to build and galvanize understanding.



Consolidation

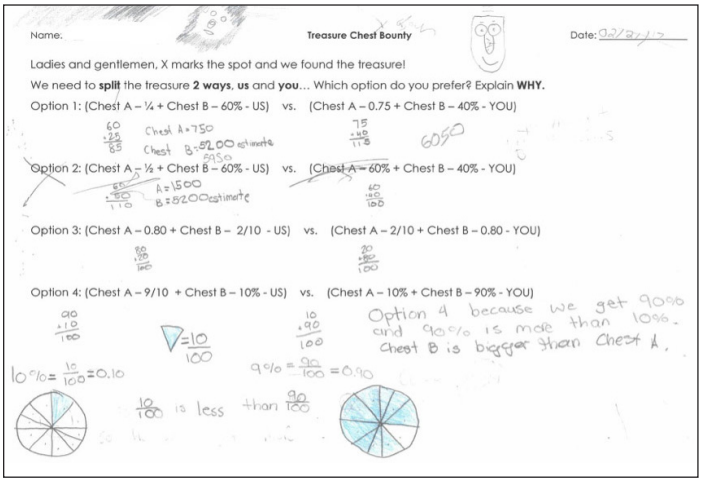
In preparation for having students consolidate their thinking in a math congress, we made purposeful choices regarding which examples of student work to share. As we had monitored the learning in the classroom, we had noticed certain approaches that we felt needed to be highlighted in order to clarify and build on the collective thinking of the class. We had decided to showcase the following key ideas:

Common Errors

We wanted to begin by looking at common errors that students made, so that we could work through strategies to clarify the learning. In some cases, when students were attempting to determine the total amount of treasure they might get from the two chests, they demonstrated a disconnection in their proportional and spatial reasoning skills. We noticed a misconception where students identified both chests combined as the full quantity, made up of the four fractional units presented to them (see Option 3 on the front page of the problem). So, when students opted to take 1/2 of the treasure in Chest A with 60% of the treasure in Chest B, they concluded that they had 110% of treasure. These students appeared to leap to seeing the fractional units as components of an algorithm. Thus, we knew that it would be important to highlight the need for using models and representations to develop a stronger conceptual understanding.

Magnitude over Value

Some students based their choices purely on numeric magnitude (i.e., equating the overall value of the treasure with the magnitude of the portion). For example, one group of students determined that because Option 4 yielded 90% of the larger chest, Chest B, it must be the best choice for yielding the greatest amount of treasure. Students may be inclined to select the larger percentage, without taking into account the absolute value of the total amount. We knew that it would be important to share examples where students recognized that fractions are relational; that is, they represent a part that is related to a particular whole (e.g., 75% of a chest filled with copper yields a lesser value than 25% of a chest filled with gold).



Reasoning

Some students clearly explained how they took multiple factors into consideration before making a choice. For example, some students considered the value of the possible contents of the chest, and weighed the options accordingly. They made choices based on a variety of possible outcomes (e.g., 90% of the smaller chest could be worth more than 90% of the larger chest, if the contents were more valuable). We knew that we needed to share examples where students offered a reasoned choice. One example that was particularly rewarding was from a pair of students who selected Option 1 because they reasoned, "This option is the most fair. We don't want to be greedy. No matter what is in the chests, we will get some of it. We don't need all of it." This brought in a social element and reminded us that there is more to mathematics than just the numbers.

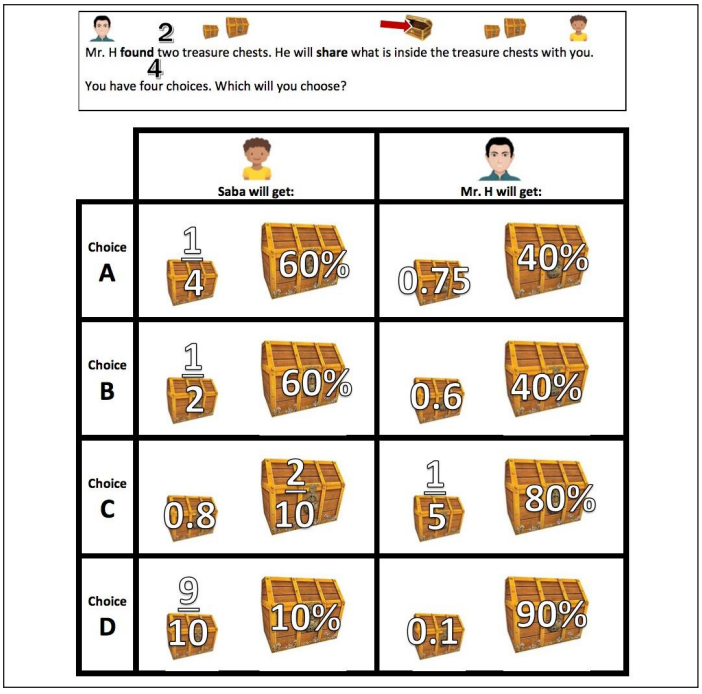
Hedging Their Bets

Some students clearly took into consideration the possible value differentials of the contents of the chests. Their reasoning was based on a desire to avoid investing too much in either chest, just in case one or the other turned out to be filled with something worthless. Some students decided on Option 1, which gave them a relatively good share of each chest. Although this option did not yield them the greatest amount of treasure, it protected them from losing too much if one turned out to be worthless.

SAILING OFF INTO THE SUNSET

Reflection

Afterwards, we reflected on the experience with our students. We were encouraged by the engagement of the students, which was demonstrated by the on-task conversations and behaviours that we observed throughout the 90-minute period. Students were routinely drawing upon the conversation prompts that had been introduced earlier in the year, to engage in mathematical thinking conversations. We were also impressed with the demonstrations of social awareness that were revealed through students talking about the concepts of sharing, being fair, and not wanting to be greedy. We acknowledged that for future use of this problem, we would reword it in order to make it easier to understand, and to use simplified language and images to make it more accessible to English Language Learners. For example, we might present students with a template that looks like the following:



Students could then make use of the manipulative resources available to represent their thinking. Once they have checked off their choice, they could engage in a conference with the teacher, or with peers, to show and explain their thinking.

Final Words

Our students truly enjoyed this task, and we had a lot of fun learning alongside them. For a full 90-minute mathematics period, our students remained on task and engaged in rich mathematics problem solving. We were encouraged by the level of discourse in groups, where students shared ideas, asked for clarification, challenged each other respectfully, and attempted to summarize each other's ideas. This task served as an entry point for further learning about fractions, decimals, and percents. But most of all, it took the mathematics and made it something that students could care about and work with as a foundation on which to build their learning. Ultimately, it is our goal to prepare our students with the right tools, the right skills, and the right map as they set off for mathematical adventure on the high seas!

References

Ontario Ministry of Education. (2014). *Paying attention to fractions: Support document for paying attention to mathematics education.* Toronto, ON: Queen's Printer for Ontario.

Torbeyns, J., Scneider, M., Xin, Z., & Siegler, R.S. (2015, June). Bridging the gap: Fraction understanding is central to mathematics achievement in students from three different continents. *Learning and Instruction, 37*, 5–13. Retrieved from www.dx.doi.org/10.1016/j.learninstruc.2014.03.002 ▲

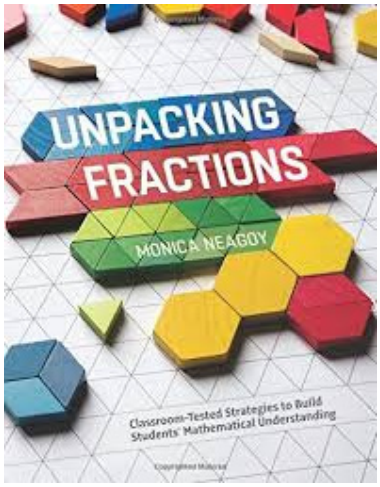
▲ Book Review: How well do you and your students understand fractions? A Review of *Unpacking Fractions*



REVIEWED BY LISA CORBETT
mrscorbettsclass@gmail.com

Lisa Corbett has been teaching since 1999. Her first real job was as a math teacher, which was terrifying, but turned out to be the best career move she could have made. A graduate of Rowan University in New Jersey, Lisa has recently begun working on a Master's degree at Nipissing University. She works for the Near North District School Board as a primary teacher. She lives in North Bay with her husband and two children. She uses Twitter a lot. @LisaCorbett0261

Neagoy, M. (2017). *Unpacking fractions: Classroom-tested strategies to build students' mathematical understanding.* Alexandria, VA: ASCD.



Unpacking Fractions is both an informative text for teachers looking for new ways to teach fractions, and a mini-course in understanding fractions. Dr. Monica Neagoy provides instruction so readers understand fractions, as well as teaching them how to make sure students understand fractions, rather than following memorized rules and procedures by rote. In the foreword, Gaile Burril, Past President of the NCTM, writes, "Perhaps the most critical message in this book is the importance of taking time to build conceptual understanding rather than rushing to teach algorithms to mastery." I feel this message was clearly conveyed.

The book includes eight chapters that focus on topics such as "Chapter 2: Use Visual and Tactile Models," "Chapter 4: Teach the Concept of Equivalence (Not Just the Rule)," and "Chapter 7: Connect Fractions and Decimals." Each chapter furthers the readers understanding of fractions, as well as providing support for improving the instruction of fractions in elementary and secondary schools.

Each chapter of *Unpacking Fractions* is divided into the same sections. First, there is an introduction of the topic, followed by a case study. Teachers and classes Dr. Neagoy has worked with are highlighted, and the reader can see the concept in action. These chapters are well written, and helped to draw me into the experience of what teaching a particular concept would be like in class. This was important to me, as I have mostly been a primary teacher, and many of the ideas in this book are for students beyond Grade 4.

Next, the author includes a "Recognizing Misconceptions" section. She identifies the common reasons students might have difficulty or become confused by the specific topic or idea presented in the book. This will be especially useful to teachers who are newer to teaching math, who want to improve their teaching of fractions, or who follow the "Five Practices for Orchestrating Mathematical Discourse" (Smith & Stein, 2011). Dr. Neagoy identifies common underlying difficulties that become stumbling blocks for students, common errors she has seen from the many students she has worked with, and then shares strategies the teacher might use to help with these misconceptions.

Next, the author teaches us the math behind a particular concept in the "Unpacking the Mathematical Thinking" section. I found this section to be very helpful because when I was taught, memorizing rules and tricks were the norm. Participants in a summer book club, who focused on this book, agreed, recognizing that we (the teachers) don't always have a deep understanding of the math we are teaching. This section of each chapter is packed with examples, explanations, and reasoning; There are many models and explanations about connecting to the math, as well as detailed pictures illustrating the models. I came away from these sections feeling like I had taken a math class this summer!

Finally, each chapter ends with a "Targeting Misconceptions with Challenging Problems" section. For me, this was a very powerful section. There are many example problems that a teacher could use in class to help students develop a deep understanding of fraction concepts. Coming up with meaningful math questions to ask our students can sometimes be a challenge, but this book provides many for you to use. For example, in the chapter titled "Teach the Concept of Equivalence (Not Just the Rule)" (Chapter 4), an example problem is:

> Miriam cut a small pie into eight equal slices and ate two. Isabella cut an identical pie into fourths and ate one piece.

Continued on page 45

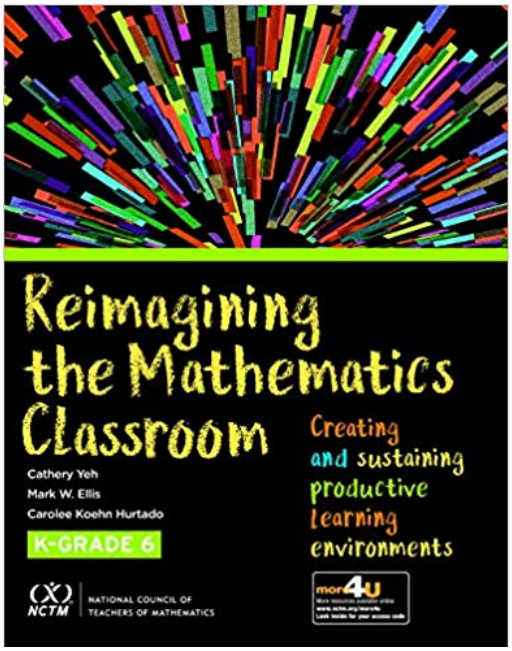
▲ Book Review: Teaching Math Isn't Like It Used to Be! A Review of *Reimagining the Mathematics Classroom*



REVIEWED BY LISA CORBETT
mrscorbettsclass@gmail.com

Lisa Corbett has been teaching since 1999. Her first real job was as a math teacher, which was terrifying, but turned out to be the best career move she could have made. A graduate of Rowan University in New Jersey, Lisa has recently begun working on a Master's degree at Nipissing University. She works for the Near North District School Board as a primary teacher. She lives in North Bay with her husband and two children. She uses Twitter a lot. @LisaCorbett0261

Yeh, C., Ellis, M.W., & Hurtado, C.K. (2017). *Reimagining the mathematics classroom: Creating and sustaining productive learning environments, K–grade 6.* Reston, VA: National Council of Teachers of Mathematics.



Many books have been written recently to assist teachers as we change the way mathematics is taught. Many of these are written by experienced teachers, who want to share the details of a successful year or a practice-changing project that worked well in their own classes. Experiences of our fellow educators are valuable; however, it is important for us to rely on research to support our teaching methods. *Reimagining the Mathematics Classroom: Creating and Sustaining Productive Learning Environments* was written by Dr. Cathery Yeh, an assistant professor in the education department at Chapman University in California; Mark W. Ellis, a professor at California State University; and Carolee Koehn Hutado, an assistant professor in the education department at the University of California. The book outlines several research-based practices an educator can implement in any mathematics classroom in Grades K–6.

Published by the National Council of Teachers of Mathematics, this resource helps us consider elements necessary to create a productive mathematics environment in our classroom. For example, Chapter 1 discusses the physical environment, Chapter 2 encourages rich math talk, and Chapter 5 focuses on involving the family and community in creating a productive learning environment that extends beyond the daily math period. The book highlights three of the eight NCTM "Principles to Action": Principle 1 – Teach Toward the Understanding of Powerful Mathematics; Principle 2 – View Students as Sense Makers with Valuable, Important Ideas; and Principle 3 – Nurture a Mathematics Community of Learners.

The authors include a robust review of current research and literature about what is most important for a teacher to include in creating a powerful learning environment in mathematics classes. Many examples are provided of teachers who have put this research into practice and have found their students to be more engaged and able to exhibit higher mathematical thinking skills than in traditional math classes. An eight-page bibliography provides a varied list of possible sources for further exploration. Many of the ideas in the bibliography are brought together in this book with practical, easy-to-follow examples of teachers creating reimagined math classrooms.

One of the bonuses of this book is access to additional resources through the National Council of Teachers of Mathematics "More 4 U" website. There, readers can use the code provided to watch video examples of lessons and activities highlighted in the book, as well as access student pintables for activities and teacher resources. I found a list of "purposeful questions" to prompt student thinking to be especially useful (Chapter 2).

Each chapter includes a "Peek Inside a Classroom" section. After examining an idea, such as assessment in the classroom, the authors tell us what this has looked like in a real classroom. They have a list of 23 teachers who opened their classrooms and tried out the ideas presented in the research. For people who are at the beginning of their journey of adopting new ways of teaching math, this is invaluable. People who have been moving away from textbook-driven instruction will also find many insights and considerations that will help move their practice forward. ▲

▲ MB4T (Mathematics by and for Teachers): Using Data to Mislead



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Jennifer Holm is an Assistant Professor at Wilfrid Laurier University and works with primary/junior and junior/intermediate pre-service teachers, as well as in the field supporting current mathematics teachers. She is interested in developing mathematics knowledge for teaching and analyzing beliefs about mathematics with both pre-service and in-service teachers. Her current focus is on the beliefs and opinions that pre-service teachers hold about mathematics and teaching along with the connection they have to past experiences. She uses this research to support future teachers in developing beliefs and knowledge that will encourage and support effective teaching practices.

Graphs and data management are expectations in every grade level in the Ontario curriculum. In each grade, students learn how to read and interpret data from different graphs and in different presentations. Given the prevalence of graphs and data in the media, it is an important life skill for all students to be able to look critically at data. However, many adults I have talked with say that statistics give a true picture of the data, and that the information it yields is black and white. Yet, depending on how the statistical data is reported, this could be a false statement, and it points to an important gap in understanding. The focus of this column falls within the Grades 7 and 8 curricula, where teaching can have a powerful impact on how students understand graphs and data. Specifically, this column looks at how graphs and data can be used to manipulate a viewer.

One of the Grade 7 math expectations is: "identify, through investigation, graphs that present data in misleading ways (e.g., line graphs that exaggerate change by starting the vertical axis at a point greater than zero)" (Ontario Ministry of Education [OME], 2005, p. 107). The goal is that students will become savvy consumers of data by understanding how data can be displayed in a way that leads to an exaggerated or false interpretation. Additionally, Grade 8 includes the curriculum expectation: "make inferences and convincing arguments that are based on the analysis of charts, tables, and graphs" (OME, 2005, p. 119). Together these expectations require being able to analyze when data is being used in a misleading way. This column will look at a variety of graphs, as well as different ways to manipulate them, to cause different conclusions to be drawn based on the image.

There are plenty of examples in the media when data is being used in a misleading way to further someone's agenda. Therefore, it is in the best interests of our students to help them learn how to tell when this is happening and to be able to critically examine what is being said. One note for the column reader is that the graphs and data presented herein are not intended to be used in the classroom; they have been selected to be of interest to teachers, while illustrating the point of how data can mislead. Using statistics and information that is relevant and meaningful to the students is more appropriate and would have a stronger impact than the teacher-oriented examples given here. Although I could present some of the statistics from the media, I am going to use a data set I have used with Masters students to illustrate the point and keep any political agenda from interfering with this column.

Consider the descriptive statistics shown in Figure 1 for a set of data that represents percentage achievement on a mathematics test for all the Grade 6 students in the imaginary school. The descriptive statistics highlight the point of how data can be reported to exaggerate or mislead the findings. Reports could state that the school is doing extremely well, since the most common score, or mode, on the test was a 96 percent. Although the mode is 96 percent, the interpretation is misleading, since (using the actual data) only 15 of the 149 students had this score (about 10 percent).

Number of students: 149
Range: 14–99
Average: 65.61
Mode: 96
Median: 73

Figure 1: *Descriptive statistics*

Another interpretation could erroneously note that the Grade 6 students in this school are failing, since the average is below the provincial level of achievement. However, if you look at the median, it shows that half of the students are meeting the provincial level of achievement (73 percent is the listed median, and this is in the Level 3 range). Depending on the statistic selected, it could seem like the school is performing well or not. Measures of central tendency are included in the Grade 8 curriculum, so this would be an interesting exercise to help students understand what the different measures "say" about the data set. Truly, all of the data in the descriptive statistics would be helpful in order to properly understand what is happening at the school, yet sometimes the media only reports one of the statistics to serve a particular purpose.

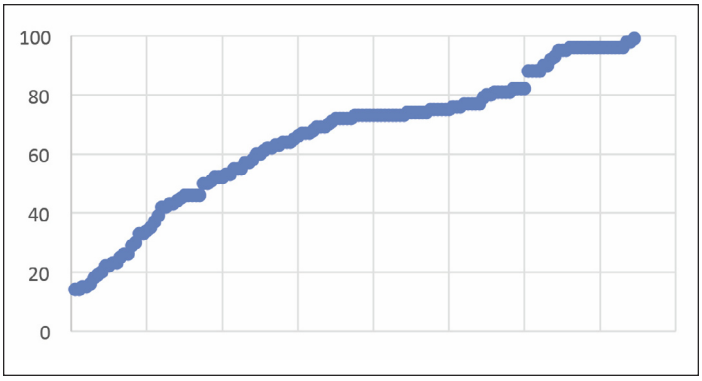


Figure 2: *Student achievement when ordered*

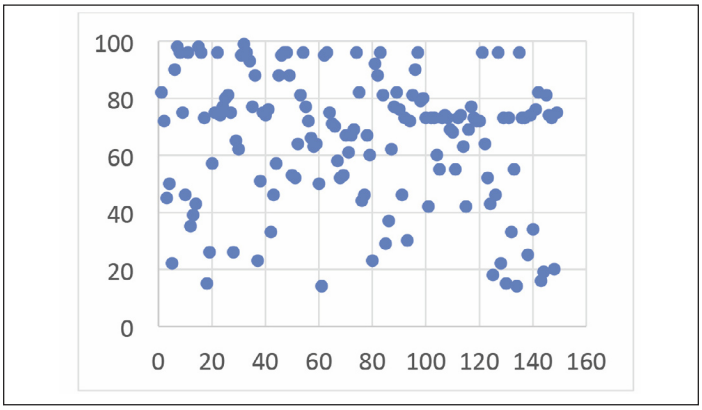


Figure 3: *Student achievement without the ordering*

If you look at the scatter plot in Figure 2, it is challenging to determine what exactly is happening in the school for Grade 6 mathematics scores. Even though the percentage score is listed on the left, the volume of dots makes it difficult to understand what you are observing in terms of data. Without the labels and the notes, this graph makes it seem as if there is an upward trend in student mathematics scores. Figure 3 shows the same data portrayed as a scatter plot where the data points are not in order, and now you can see the different places where there are clusters of scores, but also the ranges where the test scores are high. As you examine the scatter plot, the school seems to be doing well in Grade 6 mathematics, as the majority of the dots are near the top of the graph.

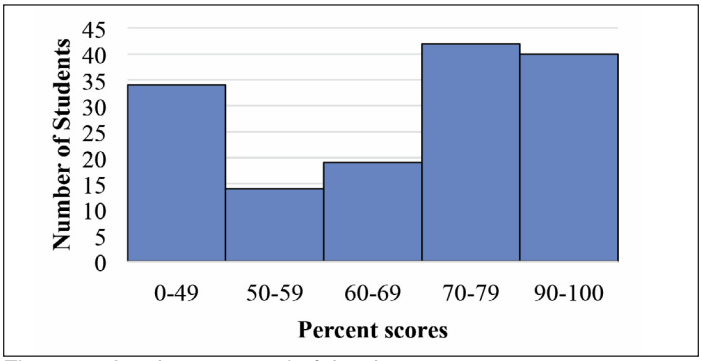


Figure 4: *Another portrayal of the data*

Figures 4, 5, and 6 are all graphing the same data, but show another problematic way that graphs can be manipulated in order to serve a purpose. Figure 4 shows a histogram of the ranges for the data set by the percentage scores associated with the levels in *Growing Success: Assessment, Evaluation, and Reporting in Ontario Schools, Grades 1 to 12* (OME, 2010) (below Level 1 for 0–49, Level 1 for 50–59, Level 2 for 60–69, Level 3 for 70–79, and Level 4 for 90–100). Visual inspection of the graph makes it clear that the school has a substantial number of scores in the Level 3 or 4 range, but there is also a high number of students who fall below the Level 1 range.

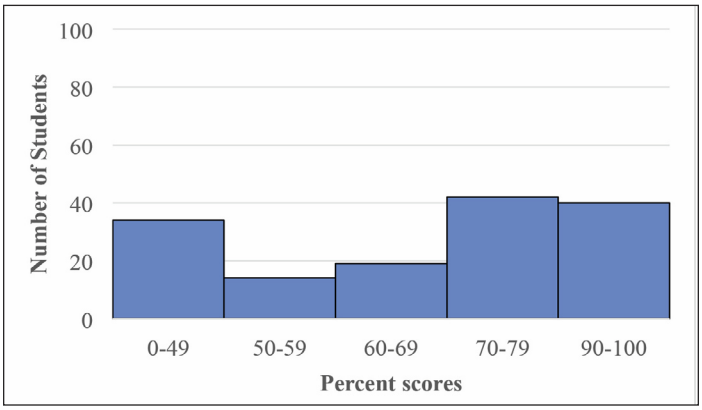


Figure 5: *The data portrayed another way*

Figure 5 shows the same data graphed; however, the range on the vertical axis has been changed to run from 0 to 100. By changing this, the differences between the bars seem to be much less. As you continue to increase this range, you can actually make it seem as if the bars are almost level. In this case, it does not help the school in stating that students are doing well, but it does make it seem like the scores are not quite so disparate.

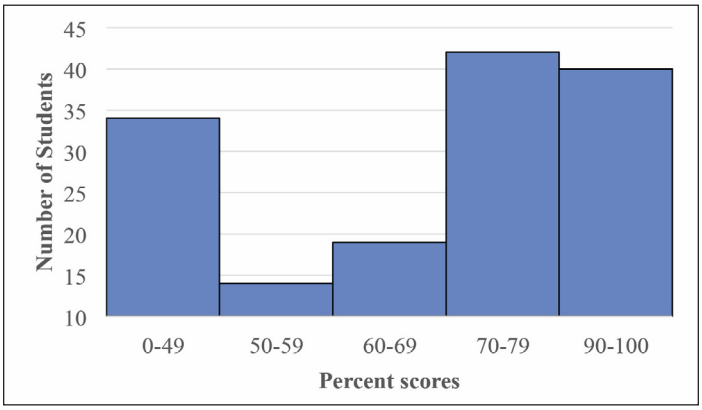


Figure 6: *The data with a different range*

Figure 6 shows another trick of the eye in graphing that makes it interesting. Not starting the graph at 0 on the vertical axis gives the impression that there are very few students in the Level 1 or 2 ranges. This gives the

impression that there is a large number performing below Level 1 or above Level 3, but that there are not many students lying in the middle of the crowd (i.e., Level 1 or 2). A graph like this could be used to say that the school has been able to move those middle-level students to performing at Ministry expectation, which is not entirely the truth of the matter.

For the next portion of the column, I am going to use the 2018–2019 EQAO data reports, which are freely available, in order to make a few other points about graphs. The data was retrieved from a school board report (EQAO, 2019c, p. 3) to show the data for the French-speaking school boards and from the highlights for the English-speaking boards (EQAO, 2019a, p. 2). This data was chosen for purely illustrative purposes in order to convey a few additional ways to use data to mislead readers.

Imagine what happens if you do not consider sample size when looking at trends. Consider the graph in Figure 7. It would seem reasonable to generate a headline that French-speaking students in Ontario are better at mathematics than their English counterparts, according to the Grade 6 EQAO results. Each pair of bars in the graph presents the same data: the percentage of students who are achieving Ministry expectations in Grade 6 mathematics. Looking at the double bar graphs for the last four cycles of the EQAO, it is clear that there is a difference between those students in the French boards when compared to those in the English boards across the province.

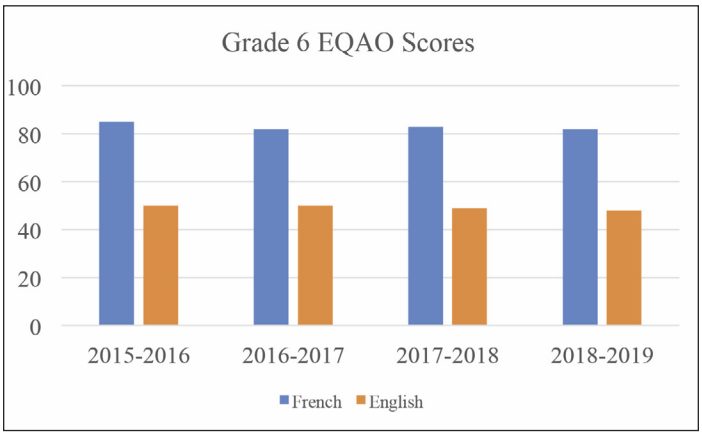


Figure 7: *A misleading comparison*

The graph presented in Figure 7 might suggest that mathematics teachers in the French boards are better than those in the English boards. This would be an inappropriate conclusion to draw from just this graph because there is not nearly enough information to make this comparison (e.g., What other factors influence mathematics performance?). It is important to help students understand that there is more to making direct comparison between two variables than just a single assumption. Students need to understand that the amount of data used is not clear from the graph, and that detail implies one should not draw such a bold conclusion without clarifying the details.

More advanced statistical procedures would need to be applied in order to truly understand if there is a statistical difference between the scores because of the extreme difference in sample sizes between French and English school boards, as shown in Figure 8. Although the advanced statistics portion is not part of the Grade 7 or 8 curricula, a conversation about the effects of sample size should be. In Grade 8, there is a specific expectation that asks students to consider different sample sizes, so this could be a natural extension to that conversation. Having students poll a single group of students versus the whole class (or school) can illustrate the point of considering sample size.

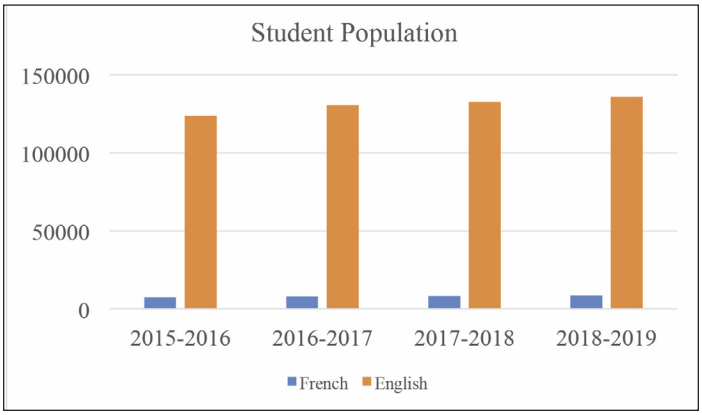


Figure 8. *Data on student population for French-speaking boards from EQAO (2019c, p. 3) and English-speaking boards from EQAO (2019b, p. 34).*

Figure 9 is a line graph that makes it seem as if the Grade 6 EQAO results have taken a dramatic nose dive. When you consider the data points, the actual difference between 2015–2016 and 2018–2019 is two percentage points. Figure 10 makes it seem as if mathematics scores have stagnated, simply by changing the values on the vertical axis. When you consider that 2 percent of the population in Grade 6 in 2018–2019 is about 2722 students, this may seem like an enormous drop off to some, and a relatively stagnant change to others depending on their view of the implications of almost 3000 students falling below the provincial level of achievement. Considering relative sizes of percentage points can also be an important discussion in Grades 7 and 8, since there is a huge different between thinking about 2 percent of the school versus 2 percent of the entire EQAO writing population. Having to consider the "whole" is another important factor in looking at data when given percentages.

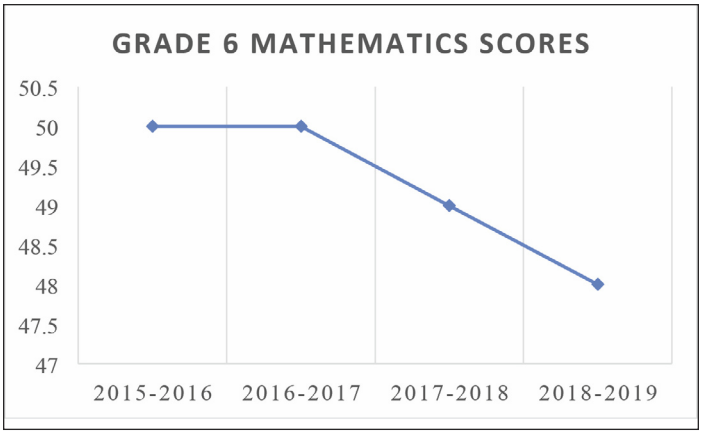


Figure 9: *Line graph of results*

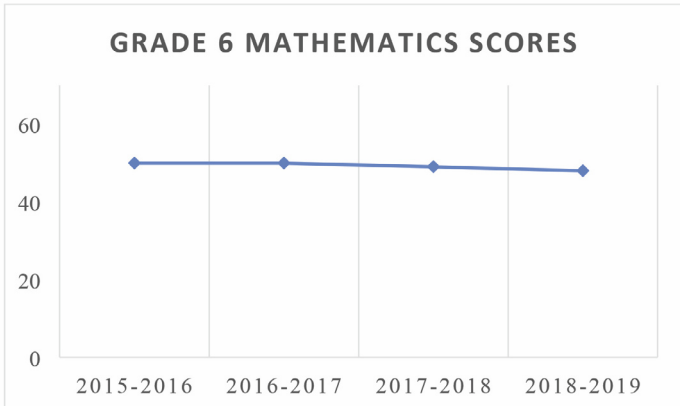


Figure 10: *Alternate version of the line graph*

The media and politicians have become masters of using pieces of data in order to "tell their story" and to serve the agenda of the present. Although the data is correct, the way it is presented does not tell the entire story. Simple tricks such as only reporting the "useful" descriptive statistics or altering the scale on the graphs can lead the reader to see something that actually is not entirely truthful. Burying sample sizes or simply not reporting them can also lead to misleading conclusions being drawn. As well, drawing conclusions about the relationship between two variables without all the information can support a point, while misleading the reader. Supporting students to be critical of the statistics they are being shown, and what those specific statistics imply, will support students in being critical users of data and the media.

References

EQAO. (2019a). *Highlights of the provincial results: Mathematics, English-language students, 2018–2019*. Author: Queen's Printer for Ontario. Retrieved from www.eqao.com/en/assessments/results/communication-docs/provincial-report-highlights-math-2019.pdf

EQAO. (2019b). *Ontario student achievement 2018–2019: EQAO's provincial elementary school report.* Author: Queen's Printer for Ontario. Retrieved from www.eqao.com/en/assessments/results/assessment-docs-elementary/provincial-report-elementary-2019.pdf

EQAO. (2019c). *Report du conseil scolaire : Conseil scolaire Viamonde.* Author: Queen's Printer for Ontario. Retrieved from www.eqaoweb.eqao.com/eqaoweborgprofile/Download.aspx?rptType=PBS&_Mident=66303&YEAR=2019&assessmentType=3&orgType=B&nF=ToAjBPXVmRMmVirMHVk4KUhoARbwnXhuiWII5PDaOM8=&displayLanguage=F

Ontario Ministry of Education. (2005). *The Ontario curriculum, grades 1–8: Mathematics* (Revised). Toronto, ON: Queen's Printer for Ontario.

Ontario Ministry of Education. (2010). *Growing success: Assessment, evaluation, and reporting in Ontario schools, grades 1 to 12.* Toronto, ON: Queen's Printer for Ontario. Retrieved from www.edu.gov.on.ca/eng/policyfunding/growSuccess.pdf ▲

***Harold & Kumar: Escape from Guantanamo Bay* (Warner Bros., 2008)**

1:35 of the movie

I fear that I will always be
A lonely number like root three.
A three is all that's good and right,
Why must my three keep out of sight,
Beneath a vicious square root sign?
I wish instead I was a nine.
For nine could thwart this evil trick,
With just some quick arithmetic.
I know I'll never see the sun
As one point seven, three, two, one
….
Such is my reality,
A sad irrationality,
When hark, what is this I see,
Another square root of a three?
Has quietly comes waltzing by,
Together, now, we multiply,
Form a number we prefer,
Rejoicing as an integer.
We break free from our mortal bonds,
And with a wave of magic wands,
Our square root signs become unglued,
And love for me, has been renewed.

(Note: "R" rated movies, as I understand it, require principal approval. So consider this as mental floss for your own thinking about the state of mathematics in pop culture.)

▲ Talking about OAMEtalks



DAVID PETRO
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David Petro, the current Past President of OAME. He is Vice-Principal of Corpus Christi Middle School in the Windsor-Essex Catholic District School Board. He is a large proponent of exploiting technology for the educational benefit of students.

When Greg Clarke and I first discussed the idea of creating an OAME-based podcast, it was actually more than a decade ago. It was at the end of OAME 2007 in Barrie, where Greg was one of the organizers. I had presented a session on listening to podcasts about math, and we were saying that it shouldn't be too difficult to create one of our own. Over the coming years, we would often say, "Hey, when are we going to create that podcast?" Well, in late 2018, when I became President, it seemed like it was time—and so began OAMEtalks.

So, what is OAMEtalks? When it was conceived, it was meant to be a place where people could get some insightful information that came out of the annual OAME conference. Although we originally thought there would be just a podcast, we expanded the idea to include live webinars at the same time. The premise is that we take some of the featured and most popular sessions from the most recent OAME conference and have those speakers repeat them for our members in an online webinar format.

Greg and I realized that it's not easy for many teachers to physically get to the OAME conference, so we thought this was a way for many of our members to experience the ideas that were heard at the conference, without leaving the comfort of their homes. We set off in our first season to feature Jon Orr and Kyle Pearce, Marian Small, Fawn Nguyen, and Mark Chubb. Each of these speakers repeated their featured session. We have just wrapped up our second season with Jules Bonin-Ducharme, Matt Baker, Jennifer Wilson and Liz Ramirez, Kaneka Turner, Alex Overwijk, and Jimmy Pai, as well as Liisa Suurtamm and Christine LeBeau. Some of these speakers were featured speakers, and we had already posted a recording of their actual OAME 2019 session on the OAME members-only section of the website. For their webinars, speakers repeated the breakout sessions that they had held. All of these webinars were recorded and shared on the OAME website. What this means is that we now have a growing collection of recordings of sessions (and webinars) of the workshops provided at the OAME annual conference.

But I haven't mentioned the podcast yet. The webinars usually run on the second Wednesday of each month and are available for OAME members only. To preview the webinars, we do an interview with the speakers to have them talk about some of the ideas that will be seen in the webinar. This becomes the podcast. We release these on the first day of the month for each podcast, and the previews are available to the entire world. As a bonus, we have been releasing episodes through short interviews with some of the featured speakers at the upcoming OAME conference.

Check out the webinars (live or recorded) in the members-only section of the OAME website, as well as the podcasts, at https://talks.oame.on.ca/. ▲

Continued from page 39

- Model with a drawing the different ways that Miriam and Isablella partitioned their pies, and shade the eaten portions. What can you conclude?
- How might Ann slice an identical pie a third way, yet still eat the same amount as Miriam and Isabella? (p. 144)

The many example problems in the book are a resource for teachers who are trying to provide meaningful problems for their students, as well as serving as models for teachers, showing how to write meaningful problems.

This book is densely packed with information, making it an excellent text for a book club or collaborative inquiry. Despite the large amount of information in the book, I found it easy to read and understand. The personal anecdotes and case studies woven throughout helped me clearly see how teaching the concepts and ideas would look in a classroom. Despite the fact that I have been a primary teacher for much of my career, and serious fraction work doesn't start until later in the Ontario curriculum, I found this book contained a lot of information and teaching strategies that will be useful for me in my classes.

Unpacking Fractions was the subject of a summer book club in the NOMA chapter of OAME. The publisher provided a study guide that was very useful for the book club group. Dr. Neagoy graciously participated in several podcasts as part of the book club. The podcasts were broadcast on VoicEd Radio (voiced.ca), but have since been taken out of circulation.

Reference

Smith, M.S., & Stein, M.K. (2011). *Five practices for orchestrating mathematical discourse.* Reston, VA: NCTM & Corwin Press. ▲

Interview with Dr. Monica Nevins



ANN ARDEN
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Ann Arden is a math teacher in the Ottawa-Carleton District School Board and is currently an instructional coach. She has also taught in the Faculty of Education at the University of Ottawa, and her son's teacher this year was a former student! Ann is very interested in assessment to improve learning, especially through the use of conversational and observational evidence. Ann is on Twitter as @annarden.



Monica Nevins is a Professor (Department of Mathematics and Statistics) and Vice-Dean (Governance and International Relations, Faculty of Science) at the University of Ottawa. She was named a Fellow of the Canadian Mathematical Society in 2019.

For this interview, Ann was joined by two Grade 6 students, Caitlin and Charley, who had never met a mathematician before.

Can you tell me about your experiences with math as a K–12 student? Do you have anything that sticks out for you?

Honestly, everything that I can remember from my K to 12 schooling is probably about math. I remember a time in Grade 6 where there were two bonus questions—one was easy and one was hard. I messed up the easy one, but I got the hard one, and it was this funny mix of chagrin and feeling great. There's lots of that in math for me: when you get something wrong, it hurts, but when you figure out why, it can be as amazing as if you got it right the first time.

Math class was all about the way I like to think—I was always looking for patterns, and everything was nice and interesting in math. I remember asking my teacher in elementary school, "Why do we have math for an hour and then English for an hour—why can't we just have a day of math?"—and they didn't have a satisfactory answer. I have really good days now where I just do math and that's fun.

Do you have any particular memory in your high school experiences that would have encouraged you to apply to be a math major?

In high school, I did very well in all my subjects, but I really like being 100 percent totally right. Math was the only subject where this could happen, and I thought, "That's the best part!" But I wondered, "What can I do with a math degree?" My teachers didn't know either, but the ones I admired had gotten there by following their own interests, so… why not? Maybe I'd be a teacher, too.

When I started university, I thought math, computer science, and physics were all in the right range. In second year, I took a course in group theory and I was so excited: "This is what I've been waiting for!" and let physics go. In third year, the chair of the math department actually called me at home to encourage me to upgrade my course choices for the next year, saying I would do well, and this felt like an amazing vote of confidence that I didn't even realize I needed to hear. That's when I let computer science go. It was the right decision: it set me up to take all the coolest math courses in fourth year. I thought, "I need a little bit more of this before I stop and get myself a serious job," and I applied to graduate school in case I made it into MIT. [Note: Dr Nevins went straight from undergraduate math to complete a PhD at MIT.] I found the hardest courses have the best payoff: when you figure something out, it is so great. That was true for high school too.

What are your research interests?

I do research in two quite distinct areas, and the common theme to my research is about putting algebraic structures to work.

The main one is about p-adic groups: sets of matrices whose entries are not real numbers or complex numbers, but the infinitely more fascinating (from algebraic, topological, and number theoretic perspectives!) p-adic numbers. The big problem that I (and many others!) work on is trying to prove a vast set of conjectures about p-adic groups that would answer deep questions in number theory; we chip away, and it's coming into focus.

My second area of research is in mathematical cryptography, which was in some sense my first love; puzzles fascinate me. Modern mathematical cryptography is the critical, irreplaceable magic to our secure communications on the Internet, for example, but we need to keep ahead of advances in algorithms that can attack this core. Both the core

and the attacks are based on hard mathematical problems arising from various algebraic structures.

Do you think it was your teachers who made you want to be a mathematician, or did you just always know that you wanted to do it eventually? [Charley's Question]

I didn't always know that I wanted to do it, but over time, it became the only thing I really, really wanted. There were some teachers who had a really big impact (like that professor who called me at home); others just conveyed how happy math made them in a way I could really identify with. I think I ended up modelling my career after the teachers I appreciated the most, the ones that were really nice and generous with their time.

When you were growing up in elementary and high school, were you one of the people who always "got it" or one of the people who had to "work through it" each time? [Charley's Question]

"Getting it" is something about when the teacher has a concept in mind and is trying to convey it to you, and once it's clear, it just "clicks" into place. I remember in Grade 10, my teacher said something was true "in general," and I argued intensely back and forth with him ("Is it true all the time or not?"), and then there was a big click when I learned that the mathematical meaning of "in general" is "in all cases." I think I had felt furious because it sounded wishy-washy!

Sometimes I "got it" quickly and was eager to do a problem to test it out, but there were certainly times when it took more work. Integrals in university had me in tears one weekend, but I wanted to understand so badly, I thought, "I better try harder because I really want to get there." That "click" felt AMAZING, because I'd worked so hard for it.

In school, I hated the phrase, "You're really good at math," because I worried it was like something you could lose. What might happen if the next time I got a math problem, I don't know how to solve it? Is it over? Eventually I stopped worrying I would "lose it" and about what other people think—I'm doing the math (and persisting and thus succeeding) because it's fun. To get back to your earlier question, my teachers had a huge influence on how I felt about things. If they gave me a good problem and set me up properly, it became about the math and not about what everyone else was thinking.

I'm still learning more math all the time. My real life is all about learning math. I go to plenty of places where I don't understand something and then I am making a choice: "Okay, do I care enough about this that I'm going to try and figure this out?" If yes, I listen really hard, take some notes, and then get my questions ready. Or maybe I need to go look something up. I know from experience, now, that when I care enough about something, I know I will be able to figure it out. Just give me enough time, and if I really want it, I can have it!

I can see that you really love math! Were there any times that you were really frustrated and maybe didn't want to do this anymore? [Caitlin's Question]

Yes. You get these times of highs and lows, and maybe want to do something easier. But "easier" just isn't as rewarding—those "highs" are what it's all about.

It was a big shock for me to start at MIT. I was a pretty good student, and then I showed up at MIT and it was like, "Whoa, these people are geniuses!" I felt so intimidated. I think that was probably one of my lower points. I thought to myself, "What am I going to do? Okay, I guess I'll do my homework and then I'll go to the seminar because they are all going to the seminar. And it looks like they understand everything, but I don't understand anything!" Actually, the others didn't understand anything either. They were just trying—we were all just trying.

I was afraid that everyone knew more than I did. Sometimes people can sound very convincing, or you can have kids in the class who are super fast and really loud. That wasn't my personality: I always waited until I was sure before I said anything, if at all. It took me a while to realize that that's all it is, personality—it's not good or bad; it's not about how much math they know compared to me.

The most eye-opening thing for me was working with a colleague who was very confident. I was in awe of this person. Then, after spending meeting after meeting with him, I realized he was wrong half the time! He was just willing to say things as if they were true, thinking out loud. It's just like when I do my calculations quietly by myself and I make all these mistakes and have to correct them until I get it right, and the fact that I made the mistakes was a huge part of learning, they're the things that advanced the project, but I'd hate to do it in front of people. My colleague was just more willing to do that out loud and be happy about what we figured out in the end. Realizing this, I felt like I had finally seen behind the curtain, and could stop worrying about whether I was as smart as the confident person next to me.

We made a great team: we're doing the same stuff, but he is "out there" with his math, and someone like me, I'm kind of "in my head" and the math is "in here."

As a teacher, I feel the perceptual part of math in particular is so important. People feel that they're either "in the club" or they're "not in the club." They feel like they can do the math, or they feel that they can't do the math. I find a lot of students doubt themselves. How did you overcome that and what's your advice for teachers to encourage their students?

It's the low-level doubt that keeps me fired up. If you always

felt, "I can do this!," why would you ever bother? It's only because you wonder if you can do it that you actually dive into it. People will say, "You're good at math, you're gifted," implying math is a "thing" you have that can be taken away, that other people don't have. That's a horrible thought—and that's not what this math stuff is like. It is not about a "thing" that I own that could wash off. If a student in Grade 10 feels like they can't do math anymore, I wonder, "What do they think they have been doing all this time?" We are just progressing, and you are building your knowledge. If there's something in Grade 10 that does not make sense, you just have to back up and figure out which piece got lost along the way. It is not like our brains are capable of linear functions, but then reject quadratics.

I see students in first-year university who are incredibly bright. If someone is stuck, it is about, "Let's back up and see where the problem is, and let's build out a plan of what you need to work on to get up to speed and figure this thing out." Students need to be willing to "fess up" that they didn't understand something, and that's very hard (especially if they believe that "you're either good at math or not" spiel).

I think it can become dangerous for students if they have been told they are very good at math because they do not want to ask questions. If they get stuck, they just wait until the usual thing happens, which is that it "clicks." But sometimes they need to be more active, to ask questions and get some help to get it to "click." Students start to realize that sometimes the explanation was not in the language needed to make it "click" for them. They realize they need to keep developing more resources for how to learn math.

As a math professor, what would you like high school teachers to be thinking about in Grades 10 to 12 about communicating to students, encouraging them to go into math or math for business or math for engineering?

This is a very big question. Our big encouraging message, at any stage, is that students should find out what they like and let it lead them. For the ones who love math, I want to communicate that they have a chance to really invest in themselves. By challenging themselves to keep the math in their studies, they're rising to the top, standing out in the playing field, and opening doors they don't even know about yet. AI, quantum computers, big data… we're entering a very math-rich era and they can own it.

I think it helps to be sensitive about the perception that math requires brilliance, which is so reinforced in society and can really turn people off who don't see themselves that way. We can help by instead reinforcing how everything in math builds on everything learned previously; as a math student, you walk into the class, and everything else you learned in math classes beforehand is with you right now. We have vocabulary that is unleashing a whole torrent of information. When you hear the word "probability," just think about how many things you have attached to that word: like coin flipping, fractions, pie charts, statistics, balls pulled out of bags, poker odds, and more! That is what you are bringing with you to class. When a student comes into my first-year class and I say "logarithm," I want that to unleash dozens of thoughts—the graph, the function, base, exponential, equations—I want everything! Those interconnections and that reinforcement take a lot of time, but without them, they forget everything after the exam.

And honestly, the real math (and the real joy) is all the understanding and learning new ways of thinking —it is the process that we went through, not just the result. We have so much emphasis on producing the right answer, but we forget that this is just a proxy for "I have learned this material." I want us to give students a chance to explore and to have those "ups and downs," and to realize that the process is the fun part, the real part of math. I wish we had more time to do that. How do we invent time for high school students (any students!) to be in that headspace and devoted to really thinking things through?

(Then there's a whole tangent to be had about, "How can people these days, with devices, focus for the length of time it takes to figure something worthwhile out?")

Have you seen Dan Meyer's 2010 TED talk "Math class needs a makeover?" I think he calls it an "impatience with irresolution." We as a society want everything to be like a sitcom: 30 minutes with a laugh track and a beginning, middle, and an end.

Right. Another analogy I have that's related to that is that some students do math with the tips of their fingers instead of with their minds. They do this symbol manipulation: "There's the example and I just substitute the symbols."

Tips of the fingers. Wow, I just saw that visual!

Absolutely. It's when there is an example in the book and it's practically the same—maybe the numbers are different. You copy what is there, just moving numbers and symbols around, and get to the end and say, "Well there, I did some math!" And I say, "No math whatsoever happened." Math is when it went in your head, and the stuff that came out of your fingertips onto the page is what came from your mind, processed and thought about. I definitely see students doing symbol manipulation (that is meaningless to them), copying examples, and thinking that this mimicry is what math is, no deeper than a sitcom.

In high school, maybe we don't get to play with the real math as much as we need to. And playing is good!

Thank you very much! ▲

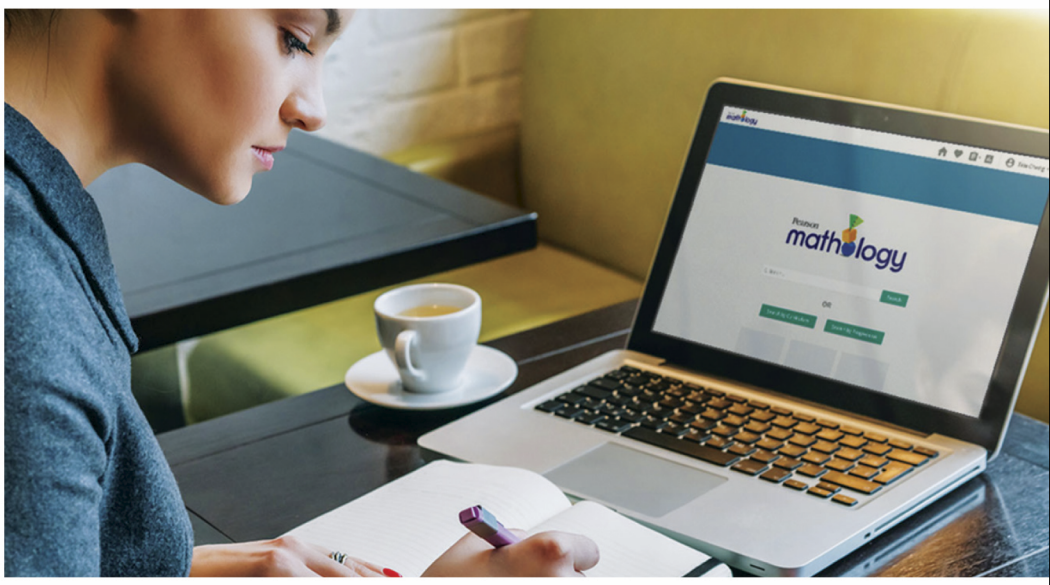
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