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Ontario Mathematics Gazette

OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

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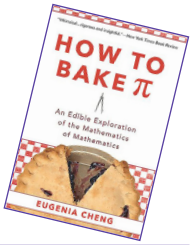
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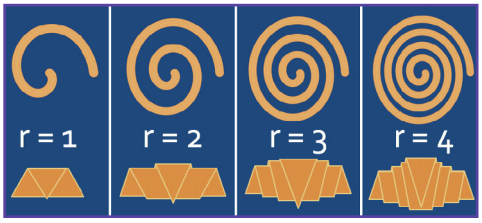
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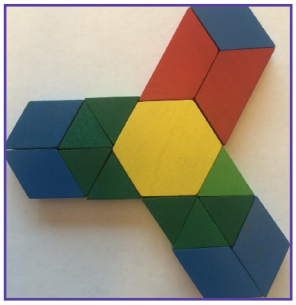


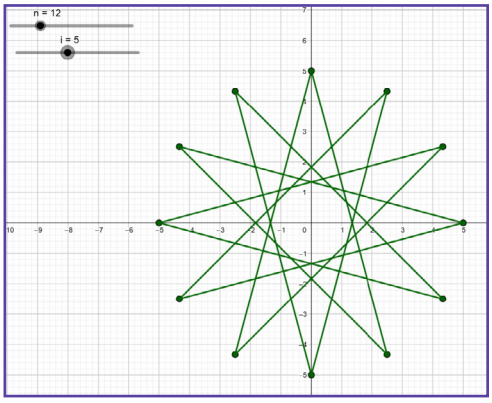


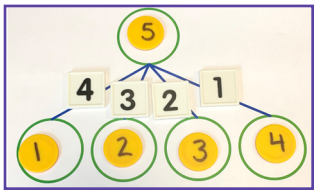












See Supporting Early Geometry and Spatial Reasoning in Kindergarten: A Collaborative Inquiry

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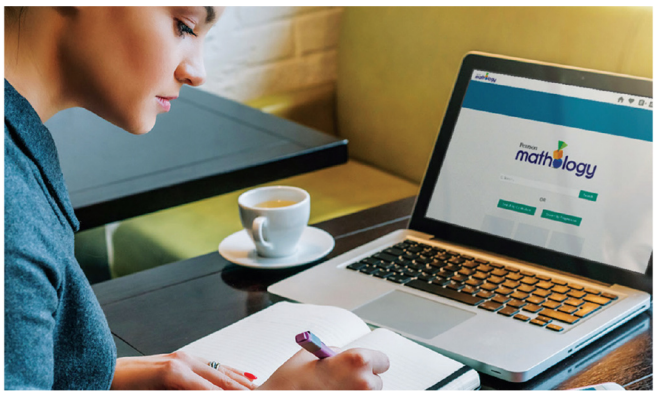
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The *Ontario Mathematics Gazette* (OMG) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *Gazette*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles are to be word–processed, MS Word is preferred, and prepared according to the *Publication Manual of the American Psychological Association*, Seventh Edition. However, please use single-line spacing (not double). Articles should not exceed five numbered pages of text, and figures, images, and photographs should be placed in the text close to where they belong, with captions. The photographer's permission is required, and for photos of students under the age of 18, the written permission of a parent or guardian is required.

Please submit your article in one blind file (i.e., identity of author is not evident), and include author names, contact information including email and mailing addresses, photos—head and shoulders, biographies—less than 100 words, and all content removed for blinding in a second file. Please email these two files to Tim Sibbald at gazette@oame.on.ca.

Upon review, you will be notified whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected. The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME.

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Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** by email or at the above address. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be print ready, and colour advertisements should have no bleeds.

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Editor's Report



TIMOTHY SIBBALD, OCT, PhD
Gazette@oame.on.ca

Tim Sibbald holds the Universities Director position on the OAME Board of Directors. He is a former President of OAME. He is an associate professor in the Schulich School of Education, Nipissing University, with a focus on mathematics education.

Let me begin by assuring readers that while I am aware of the news of late, I have asked columnists, with the exception of our OAME President, to minimize mentions of the topic. For too many people, we find ourselves immersed in the issue daily, and I am hopeful that this edition of the *Gazette* might offer some respite or escape.

I do want to share a small change that has occurred over the last year or two, where two OAME chapters have made significant changes. The first was NOMA, where the chapter logo changed to the new logo shown in Figure 1. The second is a new logo and name change, where $O_3$4ME (sometimes written O34ME) was formerly known as COMA. For those who are not chemically inclined, O_3 is ozone, and the new name is cleverly the O-zone-4-Math-Education, or Ottawa zone for math education. Their new logo is shown in Figure 2.

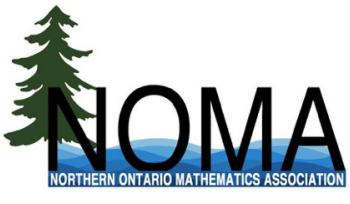


Figure 1: *The new NOMA logo*



Figure 2: *The new O34ME logo*

As we head toward the summer, I would like to share a mental exercise I have revisited many times over the years. The activity is well suited to calm moments of relaxation or down time, when one has fewer stresses than the busier times in the year. It is a form of personal reflection that I like to call deep reflection. The idea is that given all the experience one has in education, you try to learn from the totality of your own education, by which I mean K–12 education, undergraduate studies, pre-service education, and any other formal education. (Am I the only one with a certificate in pollution control?) I also include experience beyond formal

schooling that served to build one's education; this might include interactions with parents, playing games, paper routes, hobbies, cooking, to name a few.

The goal of deep reflection is reconciliation between your past experiences, the evolution of your self as an individual, and the way you teach and envision your teaching currently. That is, you try to comprehend how the experiences fit with who you are, as an educator, today. It is complicated because the experiences are only portrayed the way you recall them.

Begin by exploring various individual memories from your upbringing and education. Ask yourself why you remember the episodes that you recall. What is the nugget of experience that caused that moment to endure in your memory? To provoke such memories, you might consider your earliest recollections, or memories attached to different places or schools where you lived, or which teachers come to mind. The objective initially is to make explicit those episodic memories from your own experience.

When you have explored some memories, begin asking how they reflect who you became as a teacher. What do the episodes have in common with the person you became? You may find some coherence among your reflections that provide personal themes. Some reflections will be representative of human developmental "strands" that come into the braid that is your personal development as a teacher. There are, of course, also sporadic episodes that may be difficult to connect to the rest of your self-image, and those may simply show developmental stages of trying something out, only to decide that maybe there are better approaches. Let me provide some examples to clarify these abstract notions.

A personal theme that I can identify going back a long way is curiosity. It has always been a partner in both my formal and informal education, and many circumstances and occasions were driven by simple curiosity. It continues today, and I have every reason to think that like most ferrets, it is simply a characteristic of who I am, and will be with me throughout my life. However, this dodged the question of how it manifests in my teaching, if it does. This Spring, I was teaching about interdisciplinary mathematics to students with a wide range of teachable subjects. Curiosity facilitated many dialogs with those students about the subjects they will teach in the future, and that is where the curiosity element fit—inquiring about those subjects and learning from my own students.

Curiosity is a grandiose scheme, so let me provide another that I wrestle with. My recollection of teachers is mixed. That has led me to consider the characteristics of the ones I remember and the ones I don't. However, it is difficult to say much about the teachers one does not remember. In my case, there are varying degrees, and I have considered teachers where I remember little, but there are some details. What is clear, as a theme, is that the positive experiences were often moments of interaction that were outside the formal curriculum. Why is that? I continue to mull over whether that is due to the scarcity of such interactions.

As an example of a developmental strand, I look on my development in using language. It evolved through school to a competent layperson level. Through university, the academic components improved through proofs, lab reports, and essays. The learning continued through industry report writing, followed by graduate school, and now as editor (the editorial team is slowly *learning me up!*). It is simply a progression that would amaze my elementary school teachers, given I routinely avoided language arts in favour of tinkering with mathematics in grade school.

Sporadic stages, in my experience, are often somewhat uncomfortable because one knows better now, but at the time, it was ripe for learning. An example is tumbling over the handlebars of a bicycle when I had not properly tensioned the chain after reinstalling a wheel. I still have the scars from that lesson, but only one set of scars—lesson learned! Often in recounting personal moments of this sort, there are awkward times that are difficult to place in context, but perhaps that is part of the commonality they share for me.

Episodic memories, when they are explored, lead to reflections. As one begins to think about a collection of these sorts of reflections, the idea is that commonalities and contrasts highlight personal themes. The notion of deep reflection takes this a step further and asks the educator to consider how those personal themes can be clarified or recognized in your own activities as a teacher. It is an attempt to come full circle; is there a connection between your personal themes and your favoured pedagogical methods? Does the way you address technology with your

Help wanted!

There is an effort underway to improve the indexing of the *Gazette* archive. Members are needed, who are willing to carefully scan through at least three issues and add markers when they see items fitting a list of keywords that is provided.

The goal is to do this in an orderly way so that many hands make light work!

If you are willing, email gazette@oame.on.ca.

students have a connection with your experiences, perhaps as a K–12 student yourself, or as an after-school gamer?

Coming full circle can be an informative realization in one's own teaching. The exploration of my own curiosity inspired a magical lesson, where a friend brought two ferrets to my science class. Students were tasked with considering whether curiosity was genetic, learned, or something else. The recollection of past teachers has provided guidance as a pro-and-con list, where I avoid characteristics I did not like, and endorse those that I can rationalize as beneficial. Other deep reflections have influenced pedagogical choices to an extent that is akin to putting salt on food—there is an impact, but unless you know what you are looking for, it may be difficult to spot.

The drawing together of reflections and experiences to understand who one is today has its own challenges. It is not a simple process to untangle years of experiences when, at first glance, you see only puzzle pieces, and the jigsaw has yet to be assembled. However, over the summer, as you recuperate from a stressful year, I hope you find time to ruminate about your own becoming—that lengthy lifelong learning process, by which you have become the educator you are. Perhaps you will discover that you can learn a thing or two about yourself by simply listening to what your mind has stashed away all these years.

A Member of Our Team

Since I took on the editor role, I have endeavoured to recognize the team that contributes tirelessly to bringing this publication up to the standard it is. You are, of course, aware of the columnists, but my effort was to recognize the editorial team of Anne Yeager (*Gazette 57*(1), p. 3), Jacqueline Foster (*Gazette 56*(4), p. 4), Penny Clemens (*Gazette 57*(3), p. 3), and finally:

Gitta Berg

Gitta Berg has been the *Gazette* proofreader since 2008 (or longer), and more recently, has also served as copy editor. Her first language is German, and she has a French Specialist degree from the University of Toronto. She has travelled within Canada as well as to the United States and Europe, including a stay in Strasburg, France to take summer French courses. Starting in 1982, her family—dogs included—frequently travelled to their Port Charlotte, Florida "home away from home." Gitta started her working life at Procter & Gamble while completing her fourth year of university, and then moved on to The Metropolitan Toronto School Board in 1990 (later, Toronto District School Board). She worked in the area of curriculum-resource development and sales, and after 29 years with the school board, retired at the end of August 2019. She has a daughter who lives in Scotland and runs a business boarding and grooming dogs. Her son passed away suddenly on Christmas Eve, 2015 at the age of 30, two years after her father passed, and she lost her mother on September 1, 2018. Her only living relative in Canada is her sister. Right after Gitta's retirement, she took a UK/Germany vacation, and plans to visit Germany again as soon as air travel resumes. A lover of animals, she hopes to adopt a dog and two cats in the near future. She continues her work with the *Gazette*, along with any other freelance work that comes her way.



Figure 3: *Gitta Berg with Aboriginal psychic medium Shawn Leonard*

In This Issue

Paul Alves brings his final President message addressing the present circumstances. I personally want to express thanks for his grace under pressure, as his message was not an easy thing to craft. Thank you, Paul.

Angelica Mendaglio brings news of the Margaret Sinclair Award from the Fields Institute MathEd Forum. *Jacqueline Hill* applies herself to forward thinking and the role of NCTM. *Carly Ziniuk* challenges us with puff pastry mathematics. *Shawn Godin* looks at methods for counting stars. *Ron Lancaster* tells a horror story that luckily brought out interesting mathematics from Pembroke and Renfrew. *Jennifer Holm* considers the impact of online resources for teaching fraction concepts. *David Costello* considers the writing process as something to strengthen math teaching. Last, but certainly not least, *Jeff Irvine* introduces a new column, *Blast from the Past*, which takes readers back to 1962 and the first issue of the *Gazette*. Jeff's column may inspire readers to explore the riches that can be found in the *Gazette* archive, which is in the members-only area of the website.

We have a good collection of articles about surveys, Kindergarten, elementary EQAO, teaching generalization, and a technology article. Good reading, and hopefully it will inspire you to write an artilce as well.

Last Word

Also in this issue are the hidden talents of the editorial team. Thank you Anne Yeager, Jacqueline Foster, Gitta Berg, and Penny Clemens. A pleasure to work with such an effective team! ▲

▲ Letter to the Editor

I am reading the recent *Ontario Mathematics Gazette* that came in the mail. Interesting as always. There were multiple conversations about problems and problem-solving strategies, and that is worth exploring in every mathematics class. It's a big change from the kind of mathematics education I grew up with—dictated algorithms and practice using those algorithms. But I hope that we are not thinking that problems and problem-solving strategies are antithetical to algorithms and practice. They belong together.

Problems and problem-solving strategies should lead to consolidation, and I missed that in the *Gazette* articles. You have been working on a math topic and "played around" with it, and the class has come up with strategies to solve the problems that were associated with this topic. Now it is time to consolidate—summarize the main sorts of problems you've been working on and then identify the one or two strategies that were most effective at solving those problems. Now practise those strategies so you get good at them, and hopefully the identified strategies are kept in mind in future years of math education, so students keep getting the same message and the same practice year after year.

In our classrooms, we don't always have students who think of strategies or are confident about solving problems. In groups, some students sit back and don't want to say anything. Maybe with the way the classroom discussion is going, or the set-up of the problem that is presented to the classroom groups, they don't "get" the initial problem, let alone try to solve it. For those students, an algorithm is a lifesaver. It helps cut through what was intended to provoke thinking and instead resulted in confusion. The algorithm provides a solid strategy and reference point.

If we start with carefully selected problems and guided "play" with those, hopefully we end with a consolidated summary and an efficient strategy or two (an algorithm or two) that are worth reviewing.

Don Vander Klok
Mathematics Professor, Lambton College ▲

Do you know…

That Robert Langlands, who was born in Canada, is founder of a program of mathematics that is very rich with conjectures and connecting representations. He won the Abel prize in 2018 and he occupies Albert Einstein's old office.

▲ President's Message



PAUL ALVES
Paul.Alves@oame.on.ca

Paul Alves, current OAME/AOEM President, is department head of math at Fletcher's Meadow Secondary School in the Peel District School Board.

"Today was more normal than yesterday. Not because it is, but just because it was as strange as yesterday."

That line came from a poem by Kieran Lennon that was shared on the Heavyweight podcast (Heavyweight Check In 2, March 27, 2020). It is difficult to recall living through a more jarring time than what has been our collective reality for the last few months. I cannot think of a moment where the time that elapses from the time of writing to the moment of reading could be as different as the time that I wrote this—my final *President's Message*. It is a cold April day, and I have just completed my first week of teaching my classes from a distance. Much like every other teacher, administrator, and support staff in the province, I have made this adjustment as best as I could, despite the many challenges that presented themselves. There were many challenges (I truly hope it is "were" and not "are" as you read this). As a community, we had to address the wide range of comfort levels of teachers with teaching online. As a community, we had to address *how* to teach a variety of subjects from a distance. As a community, we had to address the technological needs of students and teachers. Overcoming these obstacles presented logistical challenges, but individually or at the school level or board level, we did our best to meet those challenges. What came to the forefront in the face of these challenges was being present for our students—providing a sense of routine when everything is anything but routine.

There is always the danger, as we live through a collective trauma, that we let the events engulf us and the overriding emotion becomes despair. However, reflecting on this current pandemic and moments of crisis in the past, I am struck by the resilience of the communities that bond us together. I know I have experienced that in my own math department as we met multiple times, and continue to meet more regularly than we ever did to support each other and to connect. I am certain I am not alone and that teachers across this province can be thankful that they have a community, whether it is formal (such as a school or department) or informal (such as a social media group) or

most importantly, families, to lean on during this remarkable moment. It has declared once again the humanity that lies at the heart of all organizations—we are connected, even if that is online, and collectively we can overcome any challenge that presents itself.

This resilience in the face of the current challenges was also evident in the work of the OAME/AOEM Board of Directors over the past few months. Very early on, the Executive Committee recognized the need to act by providing certainty to members and the wider educational community. The decision was made to move all Board of Directors business to virtual meeting platforms. It seems to be too little to just type my gratitude to each and every member who volunteered their time (on top of every other commitment) to meet and work in individual committees and at board meetings. Thank you on behalf of all OAME/AOEM members. Also, through consultation with the OAME/AOEM 2020 conference chairs, the difficult decision was made to cancel the conference. I cannot minimize how difficult a decision this was for everyone involved, but we recognize that this was the right decision. The conference takes three years to plan and that work is done by a host of teachers, who are volunteering their time to ensure that all involved experience substantial professional learning and networking. I want to publicly proclaim my thanks to everyone involved in OAME/AOEM 2020 In Focus. **Every volunteer, every committee member, every committee chair, everyone—THANK YOU!**

In response to the cancellation, OAME/AOEM wanted to still provide professional learning to everyone by capitalizing on the conference proposals that were submitted. The presenters were asked if they were interested in presenting in a virtual OAME/AOEM conference. A small group met quickly, devised and implemented a plan to provide live interactive sessions and access to pre-recorded sessions that covered a wide spectrum of topics and grades. Thank you to those involved in the organizing, to the presenters who were kind enough to give of their time and learning, and to all those who participated in the conference.

We usually honour the winners of the OAME/AOEM awards at the conference. We hope to still be able to do that at one of our future events. I would like to congratulate the Durham Catholic School Coaches—Trish Maecker, Leanne Oliver, Lisa Salamone, Laura Auld, Lynn Hollingworth, Colleen Morgulis, and Joanna Ooms—as this year's recipients of the **OAME/AOEM Award for Secondary School Mathematics Staff Award for Exceptional and Collaborative Teaching**. It was impressive to read about the remarkable work of this group of teachers in improving the learning for students in Durham area high schools. The winner of the **OAME/AOEM Lifetime Membership Award** is Jacqueline Hill. I fondly recall my first OAME/AOEM conference, 2003 Primed for Math, and attending one of Jacqueline's workshops. She made an impact on me and my teaching, and I am certainly not alone. Jacqueline has served OAME/AOEM in a variety of roles, including President in 2007–2008 and currently as NCTM representative. A very deserving winner. If you are reading this and thinking that there may be a math educator or group of educators whom you would like to nominate for an OAME/AOEM award, watch for the call for nominations in November. You can find the full list of awards on the OAME/AOEM website at www.oame.on.ca/main/index.php?lang=en&code=awards.

The OAME/AOEM elections closed at the end of March, and I would like to congratulate and welcome the members elected to the Board of Directors. Sandra Jean Price was re-elected as Vice-President, Bart Vanslack was elected as Director (JK to Grade 6), and Hema Khodai was elected as Director (Grades 7–12). Welcome! I look forward to working with you as I enter my term as Past-President.

This is my final *President's Message*. In fact, this is a bit historic, as it is the final *President's Message* from a President who is serving a one-year term. In the Fall, Judy Mendaglio will be the inaugural two-year term President. Judy will guide this organization ably as we emerge from these uncertain times, and I can't think of anyone else who will ensure that OAME/AOEM will mark a true course forward. Judy is busy planning the OAME/AOEM Leadership Conference for 2020. Although details are fluid at this time, I know it will be a wonderful event, allowing leaders to reconnect and rejuvenate.

I want to take this moment to thank all the members of the executive committee for providing a steady hand guiding this organization during this year of years. You kept me sane when I felt anything but. The executive directors, Fred and Lynda Ferneyhough, for your dedication and service to OAME/AOEM for a decade now (it has been ten years already... happy tin/aluminum anniversary), thank you on behalf of all OAME/AOEM members. To all the members of the OAME/AOEM Board of Directors who serve on committees, you form the foundation of what we do as an organization. Thank you for your flexibility and continued commitment to the work we have done over the past year and, in particular, the past few months. Thank you to our web team. Many educators first interact with OAME/AOEM online, and you have ensured that what they get is varied and always up-to-date. Lastly, thank you to all involved in the publication of the *Gazette* and *Abacus*. The amount of time and work that is required to produce this publication is

truly appreciated by our members. I hope the readers have enjoyed the changes in the *Abacus* this year, with its focus on building number sense. An insightful resource for all members.

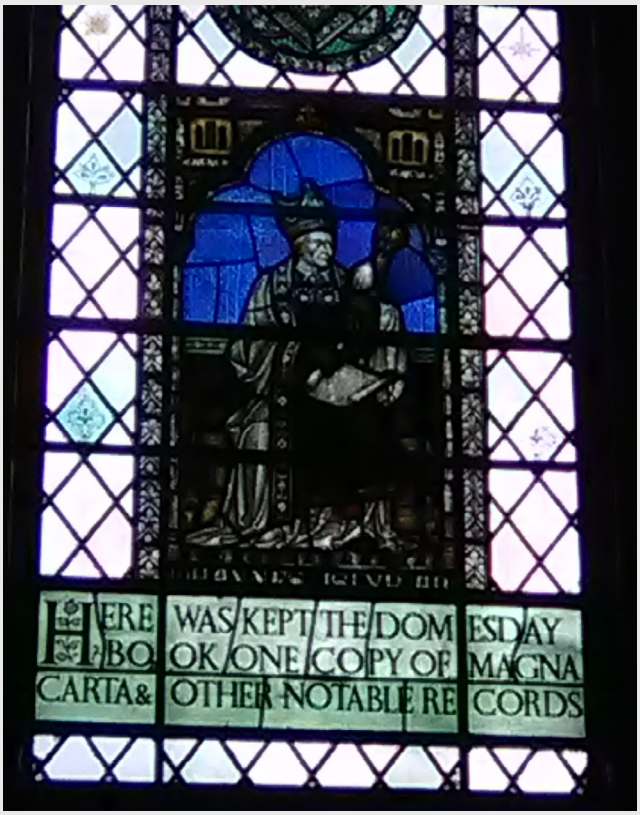
I neglected to thank someone in that list. I neglected to thank you—the OAME/AOEM members, who are the backbone of OUR organization. Emphasis on the OUR. This organization is OUR organization. This is our community. As we look to the future and the possibility of more uncertain times, I am buoyed knowing that we will be together in facing that uncertainty. I wish all of you a restful respite during the summer months, and thank you for the privilege of serving as your President this past year. To borrow a line from my favourite Mountain Goats song, *This Year*:

There will be feasting and dancing in Jerusalem next year
I am going to make it through this year…. ▲

"It wasn't luck, it was math." (time 20:03)

"It's not a risk, it's math" (time 35:10)

Lost in Space, Season 2, Episode 8



The first census, known as the "Domesday book" (pronounced doomsday book), acknowledged in stained glass. Westminster Abbey, London, UK.

▲ Fields Institute MathEd Forum Report



ANGELICA MENDAGLIO
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Angelica Mendaglio is an instructional designer at Vretta Inc. in Toronto, Ontario, where she helps to create interactive digital mathematics lessons and activities for middle school students.

The February meeting of the Fields Institute MathEd Forum celebrated the work of Nat Banting, the winner of the 2019 Margaret Sinclair Award, which recognizes innovation and excellence in mathematics education. The award is presented each year to an educator in Canada, who has demonstrated innovation and excellence in mathematics education at any level, and the recipient of the award delivers a lecture at the Fields Institute in Toronto during the following academic year.

Nat Banting is a mathematics teacher and lecturer focused on mathematics education at the University of Saskatchewan in Saskatoon. Nat is well-known for his online presence, through Twitter and his Musing Mathematically blog, which he hosts on his personal website (www.natbanting.com).

Nat's blog was a focal point of his first talk at the MathEd Forum, titled "Teaching Mathematics on the Hinges of the Immediate Present." In this talk, he reflected on one of his first posts from 2011, titled "Playing with Mean, Median, & Mode." Through this exercise, he parsed key ways in which his approach to teaching and learning mathematics has changed since he started teaching. He highlighted different ways that his early teaching style and current teaching style engage with the present in the classroom.

Nat first identified three notions in his early teaching style: that problems are given, with a focus on fit, and a belief in instructive power. These notions together formed a perspective on the present moment in the classroom—that it is engineered, prophesied, and scripted. In this perspective, problems are given by the instructor to the student, with a particular outcome in mind. The instructor anticipates certain responses from the students, and then listens for those responses in the classroom. Nat pulled quotations from the blog post, which highlighted that at the time, he was encouraging student voice in the classroom, but was only looking for how it fit with what he had previously conceived would arise during the activity. Nat emphasized

that his earlier style of teaching was centred around the "preordained"—everything from the mathematics that would emerge from the problem to the student's reactions was almost on rails, and his role as a teacher was to keep students moving along those rails.

In contrast, Nat tries to be more *present* in his current practice. He now sees problems as being posed (rather than given) to the students, as students will take the same problem in different directions, creating different problems than the one initially posed. In Nat's philosophy, teachers give prompts, but students make the problems for themselves when they engage with those prompts. He makes a distinction between this and the perspective that students may interpret the same problem in different ways—in his view, the students actually make different problems, as problems only exist when they are engaged with by students. He is now listening with his students, rather than listening for specific ideas that he has previously anticipated would arise. This has shifted Nat's practice from working with a *preordained present* to what he calls an *immediate present*.

Nat closed with what he called "an organizing set of aphorisms" that he uses to orient himself within this present mindset. The first, which he often tells his students, is: "The solution to a mathematics problem is an argument." He asks his students to make and defend an argument when engaging with a problem. In his view, a student's argument doesn't indicate what mathematics the student has, but rather, the argument itself is the mathematics. Nat says that this changes the flow of his math classroom, so instead of being composed of discrete and disconnected pockets of problems and solutions, the student's experience becomes more continuous, with more connections being made to previous concepts. The second aphorism, which Nat tells teacher candidates, is: "Teaching isn't about telling; teaching is about triggering." He says this lessens the pressure on the teacher to orchestrate the perfect classroom experience, and shifts the emphasis toward reacting to students in the present moment. Nat's third aphorism is one that he asks himself: "What problem are they solving?" He uses this to remind himself to observe his students as openly as possible, so that he can engage with them on the problems that they have created, rather than try to get them on the rails of what he anticipated they would do.

Throughout Nat's talk at the MathEd Forum, he engaged attendees with an evolving set of problems involving mean, median, and mode, which were very similar to the problems he posed to his students in 2011 and sparked that early blog post. As the talk progressed, the problems gained constraints, and that changed the way the audience engaged with the problem. This approach of manipulating constraints was reminiscent of Nat's featured talk at the OAME 2019 conference, "Creating Mathematical Possibility by Constraining Mathematical Possibility." Nat's talk at the MathEd Forum ended up also providing a first-hand experience of the central claim of that talk, which was that adding constraints can introduce new mathematical possibilities, and are a simple way to add differentiation to a problem.

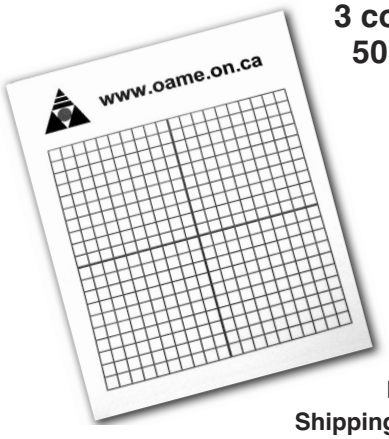
At the February MathEd Forum, the winner of the Margaret Sinclair Award for 2020 was announced to be Chris Suurtamm, professor of mathematics education from the University of Ottawa. Chris has impacted the mathematics education community in Ontario and beyond through numerous projects, initiatives, publications, years as a classroom teacher, and work in helping prospective teachers prepare for their own careers in the classroom.

Nominations for the Margaret Sinclair Award are accepted until December 31 each year. If you would like to nominate someone for this award, visit www.fields.utoronto.ca/honours-and-fellowships/margaret-sinclair-memorial-award for more information.

You can find recordings of previous Forum meetings, including Nat Banting's Margaret Sinclair Award talk, in the *FieldsLive* website. ▲

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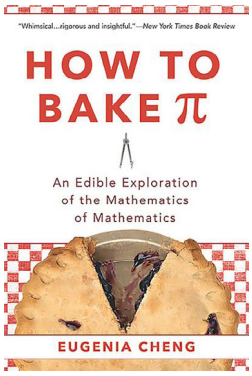
▲ In the Middle: Mathematical Baking



CARLY ZINIUK
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Carly Ziniuk teaches Grade 9 Mathematics, Grade 12 Data Management, and Advanced Placement Statistics at the Bishop Strachan School in Toronto, Ontario, Canada. She is very active in adopting real-life data to engage her students in solving problems.

The *New York Times* 2015 book review (www.nytimes.com/2015/06/14/books/review/how-to-bake-pi-by-eugenia-cheng.html) of Eugenia Cheng's *How to Bake Pi* begins with: "To make mathematics palatable for the lay reader, the author must sweeten the pill." As soon as I read that line, I knew the book was for me. I have been baking, eating, and mathematically investigating recipes with my classes, especially in middle school, since 2001. These were the lessons my students recalled fondly when they came back to visit as adults, raving about how they still used my cookie recipe or couldn't eat a croissant without thinking of all of those layers of butter. When I planned those classes, I knew that I was sweetening the pill of fractions and ratios for my Grade 8 students with different cookie flavours, or making exponential modelling palatable for my Grade 9 students with layers and layers of croissant dough. Given the rapid popularity of baking in the past few weeks, and the opportunity to integrate mathematics with at-home activities, I have been considering some of the ways I have used baking and math.



A *is divisible by* B *means*
A *is a multiple of* B*, which means,*
A = k * B
for some natural (whole) number k.

Figure 1: *Divisibility rules from* How to Bake Pi

How to Bake Pi is largely focused on abstract (pure) mathematics, and often uses the analogy of numerical principles as basic ingredients. Chapter 4 starts with a Puff Pastry, where Dr. Cheng uses the recipe to describe transitivity and links it to basic logic proofs. Figure 1 illustrates a series of her divisibility rules in sequence.

She discusses how divisibility concepts can be used in various ways to produce different outcomes. Cheng connects the two seemingly disconnected ideas of logic proofs and puff pastry as follows:

> There are many different ways of combining these simple ingredients, and most of them will not result in puff pastry. Making puff pastry is a long and precise process, involving repeated steps of chilling, rolling, and folding to create the delicious, delicate, and buttery layers that make puff pastry different from other kinds of pastry. Puff Pastry has a reputation of being difficult to make because of this process (p. 57).

Dr. Cheng explains her mathematical Puff Pastry at www.youtube.com/watch?v=7mxU5NZkKpE.

The process and ingredients for making puff pastry is very similar to the process for croissant dough, which I have often used to explain exponential modelling. In an activity adapted from a paper-folding activity in the 2001 MPM1D Public Course Profiles, students examine the relationship between the number of folds and the number of layers with the recipe for croissants. We consider the first step: one layer of dough with one layer of butter on it, sandwiched between another layer of dough on top. The dough is then rolled out and folded with more butter inside, and so on, with the baker repeatedly rolling it out and folding it. The many layers of butter melt and separate the layers within the dough during baking to make a croissant that is extremely light and tender. As I have demonstrated previously in the column, creating a table (with the variables—number of folds, number of layers of dough, and number of layers of butter)—can help middle school students see the exponential pattern before considering formal mathematical representations of a graph and exponential equation.

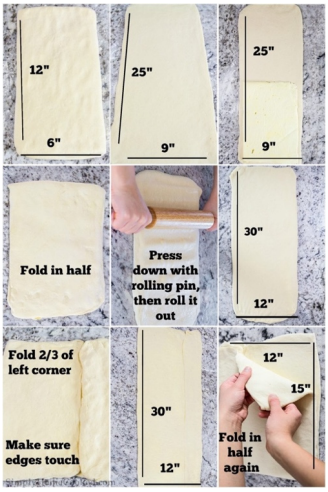


Figure 2: *Croissant dough from Simply Home Cooking blog*

The Simply Home Cooking blog (www.simplyhomecooked.com/almond-croissant/) explores other mathematical opportunities with the image in Figure 2. Sample questions include: How does the area change in the three rectangles of dough in the image? How much butter is there per square inch in the last rectangle?

After rolling and folding the croissant dough, and noticing an exponential number of layers, you then cut it into triangles and roll those as well. In May 2017, *Chalkdust* ("a magazine for the mathematically curious") included an article by Rafael Prieto Curiel entitled "The Croissant Equation" (www.chalkdustmagazine.com/blog/the-croissant-equation/). The article has this tag line: "Each time you eat a croissant, you might be biting more than 500 layers of dough!"

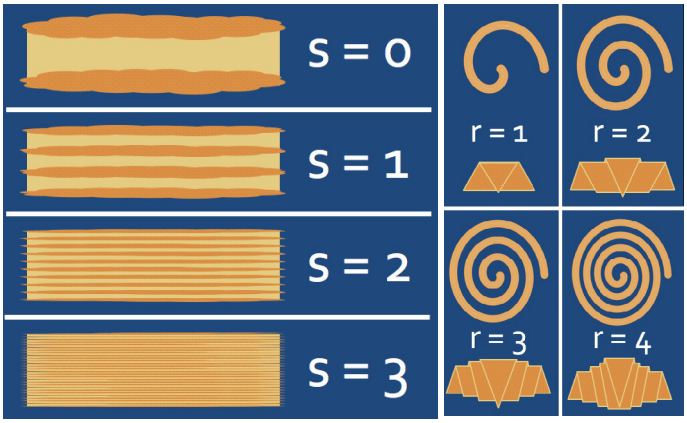


Figure 3: *Chalkdust's layering illustration of variable* s *and variable* r

Curiel produces the following equation: Layers of dough = $2r(3^s) + 1$. Here, *s* represents the number of times the dough is folded (triple folded, i.e., three layers), and *r* represents the number of times the baker rolls each triangular wedge. (Note that when pastry is against pastry without butter in between, then it is assumed the pastry becomes a single layer, and this is why the formula is not $2r(3^s + 1)$.) It is interesting to have students read various recipes online or watch a video to consider what the *r* and *s* values are for a given recipe.



Figure 4: *Anna Olson's Croissant dough from the Food Network*

For example: By looking at the picture Anna Olson presents on the Food Network, or by reading her recipe at www.foodnetwork.ca/recipe/classic-croissants-and-dough/12543/, what would her *r* and *s* values be? What are the *r* and *s* values used by the Buttermilk Pantry, *How to Make Croissants* at www.youtube.com/watch?v=bOQfTGfNLjM?

My favourite challenge for middle school students is to scale recipes or examine the amount an ingredient contributes to the total. My favourite is a basic chocolate chip cookie recipe that can be altered. This version indicates butterscotch chips with nuts, or chocolate cookies with milk and white chocolate, or peppermint dark chocolate cookies. Students can revise this for their own favourite twist.

MS. ZINIUK'S FAMOUS COOKIES – THREE VARIETIES

¾ cup butter or margarine
½ cup (packed) brown sugar
½ cup (granulated) sugar
1 egg or egg replacement
1½ cups flour
1 teaspoon baking soda

And one of these three varieties:

1 teaspoon vanilla
1 cup butterscotch/Skor chips
½ cup nuts (macadamias are great!)

OR

¼ cup unsweetened cocoa
1 tablespoon milk (any type – chocolate, soy, skim)
½ cup white chocolate chips
½ cup milk chocolate chips

OR

1 teaspoon peppermint extract
¼ cup unsweetened cocoa
1 tablespoon milk (any type)
1 cup dark chocolate chips (or mint chips!)

Instructions

- Preheat the oven to 350° F. Lightly grease/spray a cookie sheet.
- Cream together butter or margarine and sugars with an electric mixer at high speed.
- Beat in the egg. Stir in vanilla (or peppermint extract or milk).
- Sift together the dry ingredients (including the cocoa) and add this to the butter mixture along with the chips (and nuts). Stir until well combined.
- Drop by spoonfuls onto the greased cookie sheet. Bake 5–8 minutes at 350° F. Remove immediately from sheet after baking and cool on a rack.

Preparation time: 20 minutes to prepare, lay out, and cook, etc.

Makes 2 dozen regular-sized or 1.5 dozen jumbo-sized cookies.

Conversion Table for Cooking

5 mL = 1 teaspoon	15 mL = 1 tablespoon
240 mL = 1 cup	1 L = 4.2 cups

The recipe's variety of different options gives students some choice (i.e., multiple-solution methods). I have then given them tasks for their choice, such as: Rank the ingredients from most to least and find the total amount of

the ingredients. This is challenging because students must maintain consistency with units and make approximations. Another extension: Determine how many batches to make to serve the entire school, and find how many dozens of eggs or bags of flour we would need to use. We have also considered the largest ingredient by volume and its percentage. Students often ask how much batter makes one cookie, and how much that varies from regular-sized to jumbo. This one activity can review much of the proportional reasoning for elementary students.

Considering how the recipe changes when students adjust it to their own taste presents a classic baking challenge: How much can you alter a recipe and still have it "work"? When does a recipe become so different that it is no longer what it started out as, and, like a copyright problem, when is it no longer "*Ms. Ziniuk's* Famous Cookies"? *How to Bake Pi* discusses a similar quandary: "When does a brownie no longer become a brownie?" How many substitutions and alterations to the recipe would it take? When it is gluten-free? Sugar-free? Vegan? Low-fat? When it has carob instead of chocolate? When it is cut into triangles instead of squares? Dr. Cheng makes a similar argument to consider a *cookie* that is not *cooked*. For those following the raw food movement, she includes a recipe for Raw Chocolate Cookies that are not cooked and do not have sugar in them. She also explores the extent of the exaggeration, "I would eat *anything* dipped in chocolate." Her description of raw brussel sprouts dipped in chocolate had my students howling in disgust (apparently, Cheng asserts they're really tasty!?). Does everything dipped in chocolate improve by the same amount?

Using this to motivate a discussion of mathematical sameness, she compares similar triangles to congruent triangles as chocolate-dipped fresh fruits and vegetables. She uses another baked good, a donut masquerading as a torus, to discuss topological sameness. She also considers the sameness and differences of processes that are not reversible: Melting chocolate can be returned to its previous form by cooling it, but melted ice cream is not restored by just freezing it. Cheng takes this further with an interesting examination of the processes of squaring and un-squaring numbers, which might not return you back to your original form. Squaring 3 leads to 9, but "un-squaring" 9 doesn't necessarily only lead back to 3 (you could have a *negative* experience).

Students have made similar findings when exploring why cutting a recipe in half might not work perfectly. For example: How should you cut an egg in half? Should you beat the egg before you divide it? Could you give it only the yolk or the white, even if the volume were the same? Students who have experienced baking failures and triumphs before will be keen to add in their own mathematical understandings. For many students, the kitchen can be an amazing mathematical classroom. While we are all spending more time at home, watching baking shows, and consuming more homemade treats, *How to Bake Pi* and other baking–math connections can be more than just sweetening a pill; they can be motivation to gobble up lots and lots of delicious mathematical discoveries. ▲

Origami Protractor

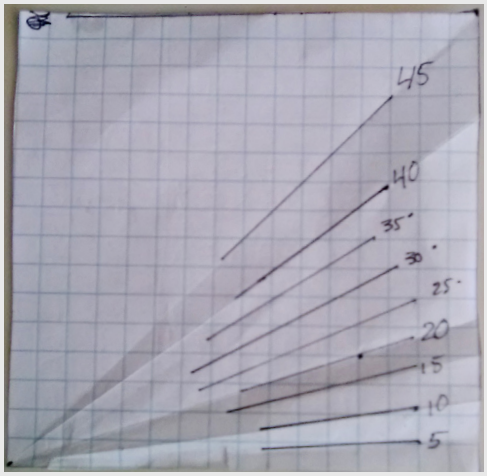
BY TIM SIBBALD

Constructing tools is an important skill that also fits the application category of achievement. Too often students are provided tools without fully appreciating what the tool is doing for them.

An example of a task that can be used to challenge students to make a math tool is the construction of a protractor. However, this has to be done without a protractor!

Take a square of graph paper and place the origin in the lower left corner. Fold it to achieve the diagonal and label 0º, 45º, and 90º. A difficulty arises because further folding (i.e., division by two) does not achieve useful integer angles.

A trick is needed and I will offer two: $\tan(10º) \approx 3/17$ and $\tan(35) \approx 7/10$. Using run = 17 and rise = 3 gives a point that, with the origin, allows a fold for 10º. Similarly, run = 10 and rise = 7 gives a point allowing a fold making 35º. Subsequent folds can generate all multiples of 5º. Here is one I made using the less accurate $\tan(20º) \approx 4/11$:



▲ Mathematical Snapshots: Ottawa Valley Adventures



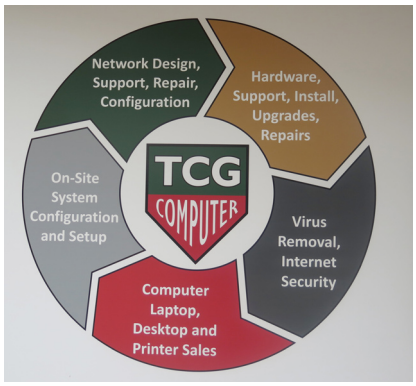
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*Ron Lancaster is an Associate Professor at the University of Toronto, where he teaches mathematics courses for pre-service middle and high school teachers. He has over 20 years of experience teaching Grades 7–12 mathematics. Ron's professional activities include consultations and conference presentations in North America, Asia, England, the Middle East, Africa, India, and Europe. Ron is an author for the NCTM (*The Mathematical Lens*) and member of the Advisory Board for the Museum of Mathematics in New York. He is the recipient of the 2015 Margaret Sinclair Memorial Award Recognizing Innovation and Excellence in Mathematics Education, awarded by the Fields Institute.*

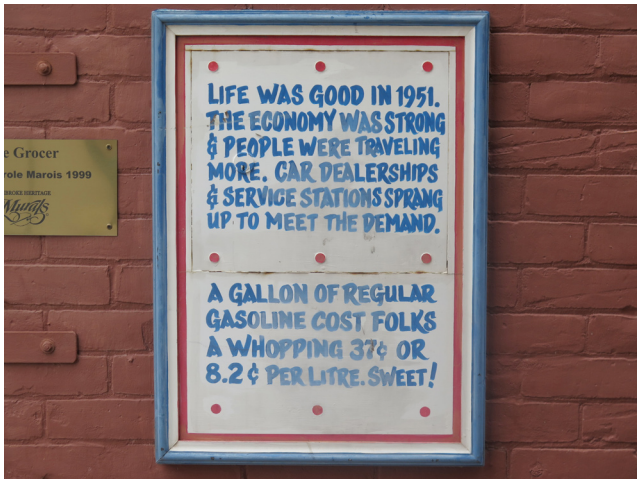
On Wednesday, May 15, 2019, I flew from Toronto to Ottawa to attend the 2019 Annual OAME Conference. Because I had to be in Manhattan on Friday, May 17, I booked a room for one night at the Hilton Garden Inn near the airport in Ottawa. On Wednesday night, I had dinner at Khao Thai Restaurant with a group of teachers, who were also in town to attend the conference. I arrived at my hotel at 11 p.m. and discovered that my reservation was for Thursday night and not Wednesday! Suddenly, what had been a wonderful evening with friends turned into a nightmare. The hotel was full, and due to severe flooding in the Ottawa area and several large conferences, there was not a single hotel room anywhere in Ottawa or any nearby city. I eventually found out that the Holiday Inn in Pembroke, about 150 km from Ottawa, had one room left. I made a reservation for 3 a.m. and drove there very slowly to avoid meeting a moose in the middle of the night. (Notice the moose, wonder if I can stop in time, what is the relationship between stopping distance and speed?)

When I arrived, I found that they did not have a room, and the reservation system assumed I was coming at 3 p.m.! Fortunately, the Best Western next door had a room, and I finally got to sleep at about 3:30 a.m. My workshop on Thursday was later in the day, and the only good news in this story was that I was able to take a leisurely drive back to Ottawa. I spent time in Pembroke (first city in Canada to have electric streetlights) and in Renfrew (officially known as Canada's Hockeytown, with three local boys inducted into the Hockey Hall of Fame). I was tired, but the stimulation of noticing, wondering, and questioning kept me awake and engaged. I took several hundred math-related photographs—six of them appear in this column for you to share with your students and colleagues.

Before looking at the questions that follow, discuss what you notice in the photographs, and suggest reasons as to why I stopped and took these photos. When you look at the photos, what are you wondering about? What questions do you have?



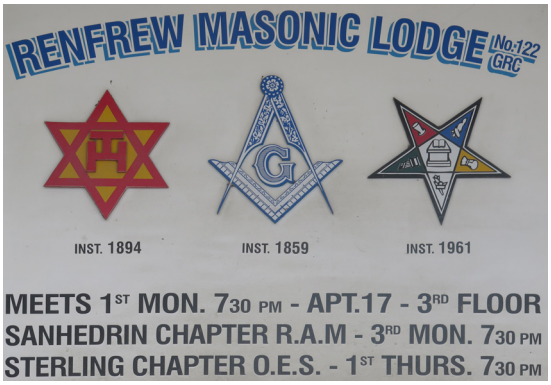
Photograph 1: *TCG Computer Solutions, Pembroke, Ontario*



Photograph 2: *Sign that accompanies a mural by Karole Marois, Pembroke, Ontario*



Photograph 3: *Highway sign along Trans-Canada Highway, near Cobden, Ontario*



Photograph 4: *Sign for Masonic Lodge, Renfrew, Ontario*



Photograph 5: *Letters and numbers formed with small squares in front of O'Brien Apartments, Renfrew, Ontario*



Photograph 6: Windows on the TSA United Church building, Renfrew, Ontario

Question 1

Figure 1 shows a geometrical diagram of the logo for the TGC Computer store that appears in Photograph 1. The diagram matches the logo quite well, except for the removal of the white space between the five identical regions within the annulus. The following assumptions were made in making the diagram (all these are very close to the actual logo).

OF = 1 unit, OD = 2 units (where O is the centre of both circles); angle DEF = 120°; DE = DF.

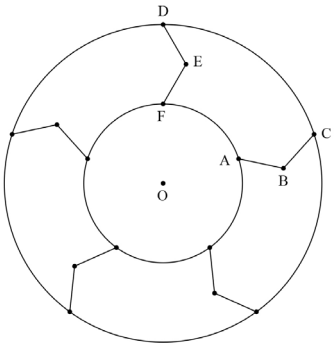


Figure 1

(a) Put together a written description of Figure 1 and give it to someone else. Ask them to draw the figure based on your written description. Compare their diagram with Figure 1. Revise your description if there are any differences.

(b) Does Figure 1 have line symmetry, rotational symmetry, both, or neither?

(c) Compare the area of the circle with centre O and radius OD, with the area of the circle with centre O and radius OF.

(d) Determine the area of the region ABCDEF. Is the area of this region larger, smaller, or the same as the circle with centre O and radius OF? How do you know?

(e) Determine the length of DE.

(f) Count the number of letters and words that appear in each of the five regions in the space between the small and the large circles. Enter all counts into Table 1.

(g) Calculate the average number of characters and words in each section. If each section had the same number of characters and words, would it be more visually appealing? Why or why not? Design your own logo with five sections that has the same number of characters and letters per section. Did you find this easy to do?

Table 1

section	number of letters	number of words
1 Hardware, Support,...		
2 Network Design,...		
3 On-site System,...		
4 Computer, Laptop,...		
5 Virus Removal,...		

(h) Consider the following spelling game. The number of letters in section 1 is 37. There are 11 letters in the

word “thirty-seven.” There are 6 letters in the word “eleven.” There are 3 letters in the word “six.” Then there are 5 letters in the word “three,” followed by 4 letters in the word “five.” There are 4 letters in the word “four.” The game ends here because we will continue to have four letters every time.

Play the spelling game with the text in the other four sections. What do you notice? What are you wondering about? What questions do you have?

Question 2

There are over 30 murals in downtown Pembroke that tell the story of the city. The sign in Photograph 2 is part of a large mural called “The Grocer” by Karole Marois.

(a) What does it mean to say that in 1951, “life was good” and that “the economy was strong”? Ask members of your family or friends to suggest a time when life was not good and the economy was bad. How can we measure these things?

(b) According to the sign, how many gallons are there in a litre? Compare this conversion with another source. Are the two conversions the same? If not, why not?

(c) According to gasbuddy.com, On April 9, 2020, the lowest price for gasoline was 72.9 cents per litre. This is about nine times the cost in 1951. Select several products that you buy on a regular basis. How much do they cost today? How much did they cost in 1951? What assumptions are you making?

(d) Suppose the cost of gasoline in Pembroke went up by m cents per litre per year from 1951 when it was 8.2 cents per litre, to 2020 when it was 72.9 cents per litre. Determine a mathematical model for the price of gasoline as a function of time.

(e) Suppose the cost of gasoline in Pembroke went up by r% per year from 1951 when it was 8.2 cents per litre, to 2020 when it was 72.9 cents per litre. Determine a mathematical model for the price of gasoline as a function of time.

(f) Compare the two mathematical models from parts (d) and (e). Which one is more accurate? How do you know?

Question 3

(a) What information does the sign in Photograph 3 provide to drivers on the highway? Why does a driver need to know this information? (If necessary, see *Gazette 53*(2), 27–29 for an explanation of the sign.)

(b) There is a sign in Hamilton, Ontario that has the text “6%” on it. Compare the angles of descent for the hills in Pembroke and Hamilton.

(c) In the hallway of your school, use a long piece of rope or string to make a physical model of the hill in Pembroke. If you use something that is 10 m long, how high will the string be at one end if the other end is on the ground (Figure 2)?

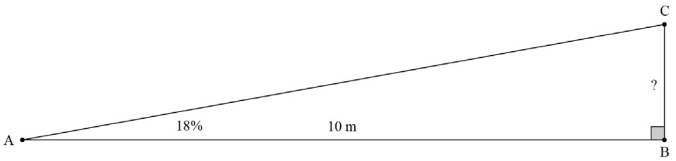


Figure 2

Question 4

Photograph 4 shows the symbols and meeting times for three chapters of the Masonic Lodge.

(a) Is it possible for the chapter that meets on the first Monday of the month to meet on February 8 in any given year? Why or why not?

(b) The data in Table 2 shows the distribution for the day of the month in 2020 for the meetings of the chapter that meets on the first Monday of the month. Add to the table the day of the month for all the meetings in the next nine years. What patterns do you notice in the data? What are you wondering about? What questions do you have?

Table 2

n (the nth day of the month is the first Monday)	N, number of months with Monday on that day
1 (June)	1
2 (March, November)	2
3 (February, August)	2
4 (May)	1
5 (October)	1
6 (January, April, July)	3
7 (September, December)	2

Source: www.timeanddate.com/calendar/

(c) Figure 3 shows geometrical diagrams of two of the shapes that appear in Photograph 4 (the one above the text INST. 1894 and the other above INST. 1961). The areas and perimeters of both shapes are provided in Figure 3 without any identification. Which area goes with which shape? How do you know? Do the same for the perimeters.

area of figure = 20.78 cm^2
area of figure = 16.47 cm^2
perimeter of figure = 24.00 cm
perimeter of figure = 27.83 cm

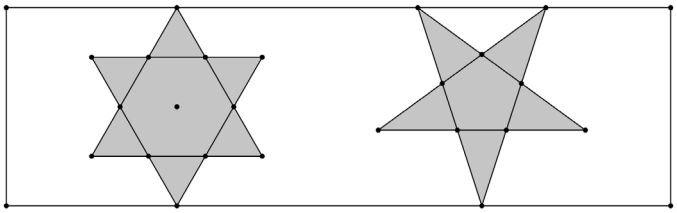


Figure 3

Question 5

Photograph 5 shows the letters O, B, R, I, E, N, and S and the numbers 1, 9, 0, and 9, formed from small squares.

(a) Count the number of squares that were used for each of the letters and numbers. Record your counts in Table 3. Which letter uses the most squares? Which number uses the least? What about the numbers?

Table 3	
letter or number to form the letter or number	number of small squares used
O	
B	
R	
I	
E	
N	
S	
1	
9	
0	
9	

(b) Using graph paper, make all the letters of the alphabet and numbers from 0 to 9 that do not appear in Photograph 5. Make sure the height of all the numbers and letters is the same.

(c) Count the number of squares used for all the letters of the alphabet and numbers that you made in part (b). Combine this data with the data in Table 3. What is the average number of squares used for each letter and number?

(d) The first three stages of a design that appears in Photograph 5 are shown in Figure 4. Extend this design to three more stages. Count the number of white and grey squares at each stage. Describe the pattern in the number of white and black squares, as well as the total number of squares in the growing stages of this design. How many white and black squares are in the *n*th stage? What is the total number of squares in the *n*th stage?

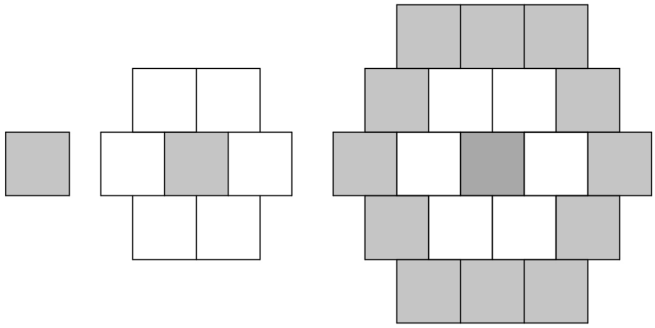


Figure 4

Question 6

Figure 5 shows a geometrical diagram of the circular windows on the TSA United Church building shown in Photograph 6.

Suppose the radius of the small circle is 1 unit and the radius of the large circle is 2 units. Find a region from 1 to 18, or a combination of these regions, that has an area equal to an area listed in Table 4.

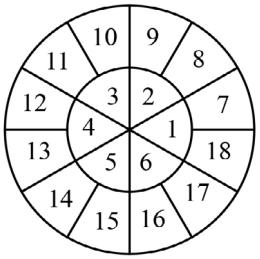
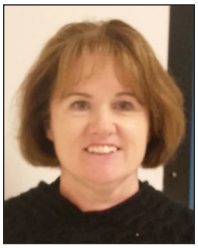


Figure 5

Table 4	
area (square units)	region or combination of regions
π	
2π	
3π	
4π	
$\frac{\pi}{2}$	
$\frac{\pi}{3}$	
$\frac{\pi}{4}$	
$\frac{\pi}{5}$	
$\frac{\pi}{6}$	
$\frac{3\pi}{2}$	

Hope those questions get your creativity going. Wonder what math opportunities my next misadventure will lead to? Wonder what photos you might have waiting for your mathematical questions? ▲

▲ OAME/NCTM Report: Forward Thinking



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Jacqueline Hill is a Grades 9–12 teacher of mathematics, as well as an online instructor for York University. She is a Past-President of OAME and OMCA, as well as the recipient of the award for Exceptional Teaching in Secondary Mathematics. She also wrote the "Director's Dialogue" for the Gazette *for a number of years.*

What do we want teachers, teaching, students, and the world to look like in 25 years?

How can NCTM help construct this vision?

Firstly, we have to work on "us." We have to ensure we are getting enough sleep, relaxation, and time to recharge our batteries. If we are to stand the test of time, we need to take time for ourselves. The website www.nctm.org has many articles to which you, as a member, would have access. The *Journal for Research in Mathematics Education* is an informative way to spend an afternoon. Spend time on you! Carve out time each week just for you! If we want to be happy and healthy in 25 years, we have to have a balanced lifestyle now.

Teaching and students (K–12) in the province of Ontario have moved online at the present time. NCTM has produced a series entitled *100 Days of Professional Learning*, which looks at the direction of education in the coming years, as well as education right now… today! By accessing the www.nctm.org website, you can register for the professional development opportunities that suit you best. Webinars are recorded and made available at www.nctm.org/100.

Consider: What would we like the world to look like in 25 years? Just take a step back in history. One hundred years ago, all mathematical manipulations were done by hand… though the machines, like the enigma machine, were slowly leading toward the early workings of computers. When was the introduction of the slide rule? Looking at the history of mathematics, we know that in 1622, Oughtred invented the first slide rule. These were still used in the 1950's and 1960's. I know teachers out there who still know how to use them. Now consider that if you go way back into history, and consider the parts and pieces that still have fundamental use in today's mathematics, you will find the abacus and Rekenrek. So if we consider 25 years from now, computers will still be a part of the educational systems.

NCTM is dedicated to helping mathematics educators delve deeper into their craft. Whether you are a classroom teacher, retired teacher, university professor, or person in a position of responsibility at a school board or with the Ontario Ministry of Education, NCTM is ready to help you make the most of your summer.

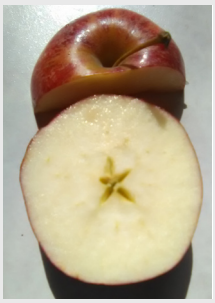
Keep well,
Keep safe,
Keep strong, and
Keep calm.
Respectfully, Jacqueline. ▲

Jumble

Unscramble each word. Take the marked letters and solve the picture riddle.

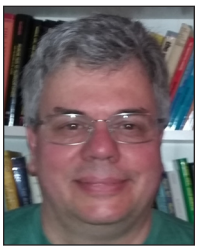
T	O	T	R	A	E				
A	M	R	I	P	Y	S	D		
B	L	A	T	E					
E	P	C	R	N	T	E			
A	T	N	I	R	O	L	A		

What do you find when you cut an apple sideways?



Solution:									
					–				

▲ What's the Problem? Counting Stars

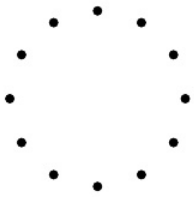


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Shawn teaches at Cairine Wilson Secondary School in Orleans. He strongly believes in the central role of problem solving in the mathematics classroom. He continues to be involved in mathematical activities: presenting workshops, writing articles, working on local projects, and helping create mathematics contests.

Welcome back, problem solvers. Last time, I left you with the following problem:

Twelve dots are equally spaced around a circle, as they are on the face of a clock. How many stars can you create by connecting some of the dots?



This problem was inspired by the YouTube video *Doodling in Math Class: Stars* from Vi Hart's channel. You can find her in the video at https://youtu.be/CfJzrmS9UfY. Vi's channel contains many amusing, unique mathematical videos, which I strongly encourage you to explore.

To facilitate investigation, I have created two resources to accompany this column. In my *What's the Problem?* Google drive folder, www.drive.google.com/drive/folders/0ByDIaUaj8StpanhnUWo2bEV6ZE0?usp=sharing, you will find the document "WTP June 2020 Counting Stars worksheet.pdf." This 18-page document contains pages of points around a circle for you, or your students, to play with. There are pages for 3 points around a circle up to 20 points, that could be used for this or other activities. You will also find documents connected to several other columns in this Google drive folder.

I have also created a GeoGebra sketch, available at www.geogebra.org/m/u4ejc8yg. This sketch allows you to control the number of starting points, n, from 3 to 40, as well as the step size, i, from 0 to n. Figure 1 depicts the sketch using 12 points, $n = 12$, and a step size of 5, $i = 5$. This is also the star we see, later in the article, second from the right in Figure 2.

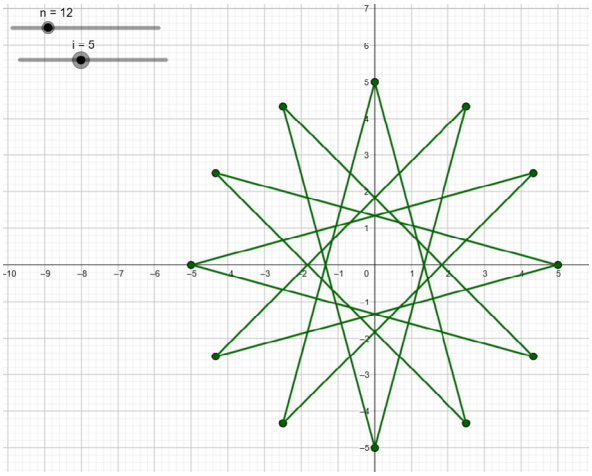


Figure 1: *GeoGebra sketch for investigating our problem*

This problem begs to be played with, but first we need to define what we mean by a "star." I will be working on the assumptions that the "connection rule" is regular and the same for all points, that is, our rule will be of the form "connect all points to the point 3 spaces away clockwise."

Using this definition and some tinkering, it doesn't take long until we get the six solutions shown below in Figure 2.

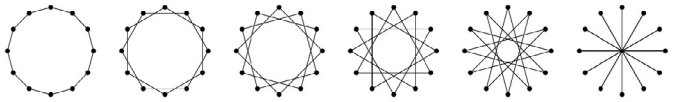


Figure 2: *The five "stars," using all 12 points*

I guess this problem was easier than I thought, until next time….

Hopefully a couple of things have caught your eye at this point. Most strikingly, the diagram at the far right seems to be very different from the others. It is made up of 6 segments, each meeting at a common point at the "centre" of the original 12 points. This is unlike the other stars, where we can follow a path around the star.

If we look a little closer, we might notice something else. Most of the stars we have created, 4 of the 6, are made up of disjoint figures. Going from left to right, we get: a complete path (dodecagon), two hexagons, three squares, four triangles, a complete path, and six line segments. In Figure 3, the complete paths are highlighted in red, and one of the disjoint figures is highlighted in blue for the other figures.

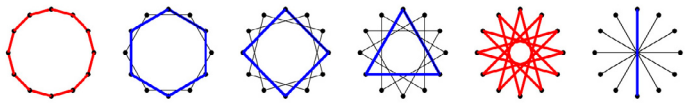


Figure 3: *Stars with closed-figure components*

Thinking about this a bit more, we can see something in the numbers: two six-sided figures ($2 \times 6 = 12$), three four-

sided figures (3 × 4 = 12), four three-sided figures (4 × 3 = 12), and six two-sided "figures" (6 × 2 = 12). Hexagons are formed when skip-counting by 2 around the 12 points, but since 2 is a factor of 12, we return to our starting point without visiting all the points. It is interesting to note that in the hexagons, we go from point to point by moving 2 spaces clockwise or counter-clockwise. We get the same result if we move 10 spaces, with the observation that moving 10 spaces clockwise is the same as moving 2 spaces counter-clockwise. For those of you who know a little modular arithmetic, this is a way to visualize the fact that $10 \equiv -2 \pmod{12}$ and $2 \equiv -10 \pmod{12}$. If you aren't familiar with modular arithmetic, check out Kleiner (1981) in the *Gazette* archives and Ridout (2019), or Godin (2019) in *Crux Mathematicorum.*

It seems that we get disjoint figures when we step by an amount that is a factor of 12. If we keep in mind that moving by 10 and moving by 2 give the same results, we might want to restate our observation, as we get disjoint figures when we step by an amount that has a common factor with 12 that is larger than 1. The only numbers between 1 and 12 that don't share a common factor with 12, that are larger than 1, are 1, 5, 7, and 11. We see the figure created by doing steps of 5 or 7 in Figures 2 and 3. If we step by 1 or 11, we just get a regular dodecagon.

It is one thing to have a theory and another thing to test it. Let's see what happens if we use 10 points instead of 12. Since the factors of 10 that are greater than 1 are 2 and 5, we get the disjoint polygons that we expected (see Figure 4): two five-sided figures second from the left and five two-sided figures on the right. Three (and seven) is not a factor of 10, so when we move by steps of three, we cycle through all the points before returning to the start. However, something else is happening. Four is not a factor of 10, yet 4 and 10 share a common factor of 2. Notice that when we move by fours, we get two disjoint five-sided figures, but these are themselves stars!



Figure 4: *Stars with 10 starting points*

Our investigation is causing us to refine our conjecture. It seems we have three different types of stars, depending on the number of points being used, p, and the step size, s.

If s is a factor of p, we get a star that is made up of s disjoint regular (p ÷ s)-gons.

- If s and p do not share any factors greater than 1, then our star is created by one complete path that involves all points. This type of star could be drawn without lifting our pencil. When $s = 1$ our "star" is a regular p-gon.
- If s and p share a common factor greater than one, but s is not a factor of p, it seems we get several disjoint stars. We will need to investigate this further to understand what is going on.

To validate our second point, let's look at stars made from 11 points. Since 11 is prime, it will not share any common factors, greater than one, with our step size. Figure 5 seems to validate our conjecture. Each of the figures is created by one connected path. Visually we can see how, as our step size increases, the star becomes narrower and pointier.

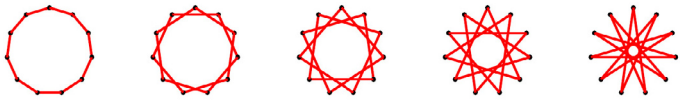


Figure 5: *Stars with 11 starting points*

To get a handle on the last point, we will look at stars starting with 14 points. Since 14 = 2 × 7, the factors of 14 are 1, 2, 7, and 14. Thus, we would expect two heptagons for step size 2 and seven "2-gons" for step size 7. On the other hand, 4(10) and 6(8) share a common factor with 14, and we are hoping to see disjoint stars similar to what arose when we stepped by 4 with 10 points. Figure 6 shows the stars created with steps of 2(12), 3(11), 4(10), 5(9), and 6(8). We have left the step sizes of 7 and 1 off, but they do create the 7 2-gons and a regular 14-gon. I encourage you to verify this for yourself.

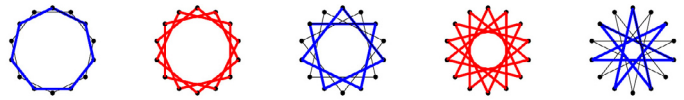


Figure 6: *Stars with 14 starting points*

We see from Figure 6 that in the cases of 3(11) and 5(9), which share no common factors with 14 that are greater than 1, we get the connected path stars like the ones in Figure 5. For step sizes of 2 and 7, we get our expected disjoint regular polygons. It seems that stepping by 4(10) or 6(8) gives us the disjoint stars as expected. In both cases, we get 2 disjoint stars made up of 7 points. In Figure 7, we see the 2 stars starting with 7 points and stepping 2(5) or 3(4). These 2 stars match with our bold copies in Figure 6.

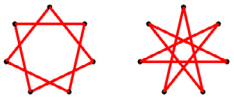


Figure 7: *Stars with 7 starting points*

Let's number the 14 points 0, 1, 2, …, 13 as in Figure 8. Point 0 represents our starting point. When we step by 4 = 2 × 2, since the step size and the number of points are

both multiples of 2, we will only step on even numbers (highlighted). That is, we will only be visiting 7 points. Since we are stepping by 4 on the original 14 points, this corresponds to stepping by 2 on the 7 points. In Figure 7, the leftmost star is created by stepping by 2. In Figure 6, the middle figure corresponds to stepping by 4 with 14 points, which is clearly two copies of the star from Figure 7. I encourage you to see the connections between the rightmost stars in Figure 6 and Figure 7.

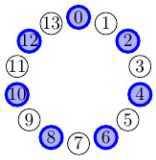


Figure 8: *Numbering the 14 starting points*

Mathematics is wonderfully interconnected. Mathematicians regularly discover connections between seemingly unrelated topics. Seeing these connections and trying to understand them helps to strengthen our understanding of the topics at hand. Help your students see these connections whenever possible.

Also give your students the chance to explore whenever possible. I have only explored one possible idea for the given problem. Notice the wording of the original problem said, "How many stars can you create by connecting ***some*** of the dots?" What happens if we don't use all the dots? What angles are formed in our stars? How are the four 3-gons and the three 4-gons in Figure 3 related? Continue the investigation by posing your own problems.

If we ask the question, "What happens if we are allowed to step by different amounts each time (including no step at all)?," we are led to a whole other vast area of mathematics—permutations. There are many fascinating investigations you can do looking at permutations, and the diagram adds another way we can study these things. Lots of things to play with this month!

Now it is time for your homework:

A sequence is defined by $t_1 = 58$, $t_2 = 4$ and $t_n = \dfrac{1+t_{n-1}}{t_{n-2}}$ for $n > 2$. Determine t_{2020}.

Until next time, happy problem solving.

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Kleiner, I. (1981). Modular arithmetic (congruences). *Ontario Mathematics Gazette, 20*(1), 32–36.

Ridout, D. (2019). Arithmetic of remainders. *Crux Mathematicorum, 45*(3), 118–121. ▲

▲ MB4T (MATHEMATICS BY AND FOR TEACHERS): FRACTION CONCEPTS VIRTUALLY



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Jennifer Holm is an Assistant Professor at Wilfrid Laurier University and works with primary/junior and junior/intermediate pre-service teachers, as well as in the field supporting current mathematics teachers. She is interested in developing mathematics knowledge for teaching with both pre-service and in-service teachers. She focuses on the beliefs and opinions that pre-service teachers hold about mathematics and teaching and the connection they have to past experiences. She uses this research to support future teachers in developing beliefs and knowledge that will encourage and support effective teaching practices.

In the last weeks, I have been thinking about how best to teach pre-service classes online. In the last weeks, I have been thinking about how best to teach pre-service classes online. Now was the perfect opportunity to think about the mathematical understandings that are developed in the classroom, and how these can be similarly created while using virtual tools and manipulatives. For this column, the focus is on generating knowledge of fractions with students, and discussing the mathematics resulting from some virtual manipulatives and tools available free online. The specific focus is the part–whole definition of a fraction.

In a unit on fractions, students need to comprehend how the fractional part relates to the whole, and using only fraction bars or circles sometimes limits student understanding. In a previous column, I discussed building fraction kits and some games that might not work well with students working at a distance. Something different needs to be used that will allow deeper conceptions of fractions to be built. Pattern blocks seemed like a good place to start. The online site *Cool Math 4 Kids* has a web-based set of pattern blocks (www.coolmath4kids.com/manipulatives/pattern-blocks) to create activities for discussing fractions. If you consider that the yellow hexagon is equivalent to "one whole," what other fractional amounts can be made by building hexagons out of the other pieces? The orange square and tan rhombus, shown on the web page, are not helpful shapes because they cannot be used to build a regular hexagon. Figure 1 shows the different ways that you

can use the other shapes individually to make a hexagon (black lines have been added for readability). Looking at Figure 1, you can see that six green triangles make one hexagon, so each triangle is $\frac{1}{6}$ of the hexagon. Each blue rhombus is $\frac{1}{3}$ of the hexagon, and each red trapezoid is $\frac{1}{2}$ of the hexagon.

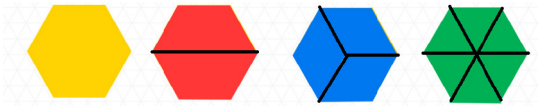


Figure 1: *Fractional images of the hexagon, using* Cool Math 4 Kids

The image in Figure 2 sparks debate with students who are still struggling with their understanding of fractions, who may say that each piece is $\frac{1}{3}$ of the hexagon, since there are three pieces in the image. This is where the definition that "the denominator tells… how many *equal* parts a whole measure is partitioned into" (emphasis added, Small, 2017, p. 258) becomes critical for student understanding. Just because there are three pieces, since they are not equal sizes, we cannot just assign them as "thirds." We have to look at how they each individually relate to the whole. Since it would still take two red trapezoids to fill the hexagon, the red trapezoid is still of the whole. As such, the blue rhombus is still $\frac{1}{3}$ of the hexagon, and the green triangle is still $\frac{1}{6}$ of the hexagon, even though they are combined into the same shape. Figure 2 is therefore showing that $1\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=1$.



Figure 2: *Patterns blocks showing another way to make one whole hexagon*

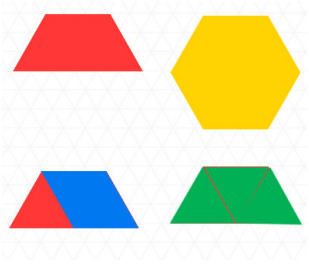


Figure 3: *Images using the red hexagon as one whole*

Since there are no fractional amounts listed on the pattern blocks, it becomes an interesting activity to change the whole and allow for observations to be made about what happens to the fractional amounts of the blocks.

If we now change the definition of a whole from the yellow hexagon to the red trapezoid, so that the red trapezoid is equal to "one whole," what other fractions can we make? With the change, the hexagon represents two wholes, the blue rhombus is $\frac{2}{3}$ of the trapezoid, and each green triangle is $\frac{1}{3}$ of the trapezoid (see Figure 3). By making the whole half as big (red trapezoid instead of the yellow hexagon), each of the fractional amounts takes up double of the original space. Another interesting follow-up scenario arises if the hexagon is defined as one-half of a whole. What other fractions can then be made? Changing the definition of the whole and seeing how the same shapes change in their relative size can lead to rich discussions about fractions.

Once students have knowledge of how the fractional amounts relate back to the whole and the relationships between the shapes, the Fraction Talks website (www.fractiontalks.com/) has some interesting new images to experiment with (or you can build your own) in the manipulatives section. For example, Figure 4 is from the website, and students can use their understandings to decide what fraction of the entire shape is a red trapezoid.

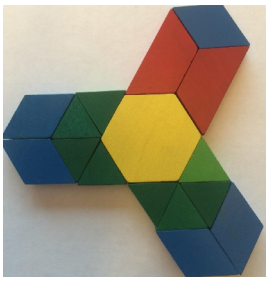


Figure 4: *Image from www.fractiontalks.com/*

An online Lego® builder, such as www.publishyourdesign.com/design, can also serve as a tool to support development of fractional understandings. The different bricks can be assigned a fractional amount, and then have students determine what the whole would look like. For example, if the image in Figure 5 is $\frac{1}{4}$ of the total image, what would one whole look like? Students could then use the program to experiment with building the different shapes to determine what the "whole" could be if you know what a certain part is. The image in Figure 6 is one example of what the whole could look like. To make things a bit more difficult, if the building block in Figure 7 is $\frac{3}{4}$, what would one whole look like? Students can use the site to experiment with the blocks to "build" the whole by understanding that they know what a single portion looks like.

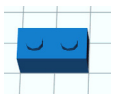


Figure 5: *If this is $\frac{1}{4}$ of the image, what would be one whole?*

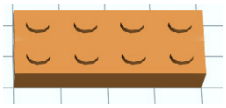


Figure 6: *If Figure 5 is $\frac{1}{4}$, this could be an example of one whole.*

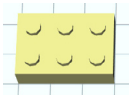


Figure 7: *If this is $\frac{3}{4}$ of the image, what would be one whole?*

Sets are another important way that students need to understand fractions. In working with future teachers, most are usually very familiar with activities based on the part of a whole definition of a fraction, using area models, but they are less familiar with thinking about fractions in terms of sets. An online tool to aid in the conceptual development of this definition of fractions could be Colour Tiles from Mathies (www.mathclips.ca/). Consider the following problem that is

suited to this resource: If you have 16 blue disks, how many disks would be $\frac{1}{4}$ of the set? If you look at Figure 8, you can see one way to determine that there would be four blue disks in one-quarter of the collection of 16 disks. Similarly, consider the following problem: If you know that $\frac{2}{3}$ of the set of disks is six, how many disks are there in a complete set? Figure 9 shows that $\frac{1}{3}$ is three disks, so the total set would be nine.

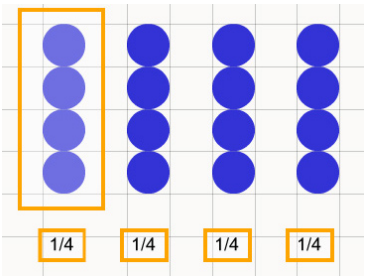


Figure 8: *Determining $\frac{1}{4}$ of 16 disks*

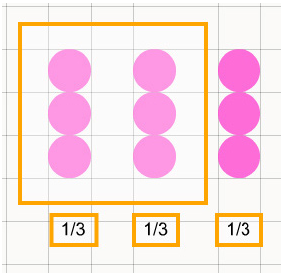


Figure 9: *Using Mathies to solve a fraction problem*

In conclusion, many different online resources are available to help students comprehend fractions even if they cannot physically touch and feel the manipulatives. With a little creativity, there are ways to support the development of an understanding of mathematics through these online resources by encouraging experimentation and discussion.

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Sodoku

1	2	3						9
4	5	6	1					2
7	8	9			4			
			6	9	3			5
	7		5	8	2	1		
			4	7	1			
				1		9	8	7
				2		6	5	4
9						3	2	1

▲ Blast from the Past: The Birth of the *Gazette*



JEFF IRVINE
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Jeff Irvine, PhD, has been a secondary mathematics teacher, department head, and vice-principal. He has taught at three faculties of education and at a community college. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. With over 45 years in education, Jeff is particularly interested in the interplay of student motivation and mathematics achievement.

The *Gazette* was born in 1962, ten years before OAME existed. At the time, math teachers were part of the Ontario Association of Teachers of Mathematics and Physics (OATMP), and the *Ontario Mathematics Gazette* (or *Gazette* for short) replaced that association's newsletter. The first editor of the *Gazette* was Dr. Ralph Stanton, who went on to found the Faculty of Mathematics at the University of Waterloo.

This new column, *Blast from the Past*, will examine issues in Ontario mathematics education through a historical lens, as reflected through previous issues of the Gazette. We have seen immense changes in mathematics in Ontario, in both curriculum and pedagogy. Curriculum changes included the "new math" of the early 1960's; OS:IS and the elimination of Grade 13; destreamed Grade 9 in the mid-1990's; the "new curriculum" of 1998–2002; and there is ongoing talk of curriculum revisions with concerns having been raised by Premier Doug Ford about discovery math.

Pedagogical changes across this history have been a massive paradigm shift from transmission teaching to student-centred learning; introduction of new technologies; the impact of the Internet and social media; new understanding of how students learn, and the role of motivation in student learning. This column will place these changes in perspective by examining how articles in the *Gazette* have mirrored the changes, and in some cases, driven the changes.

I have been fortunate to live through most of these changes, either as a student or as a teacher. We have come a long way since the birth of the *Gazette*. But it is important

not to forget where we came from.

Dr. Stanton's first editorial is available through the *Gazette* archives in the members-only area of the website. It is reprinted below, and I invite you to compare it to more recent *Gazette* issues and editorials. Have we achieved the goals set out by Dr. Stanton in 1962?

EDITORIAL

It is with some trepidation that I look forward to the task of prefacing each issue of the Ontario Mathematics Gazette with an editorial. However, the first editorial is really an easy one to write; I shall simply tell you what the Ontario Mathematics Gazette is, and indicate what it aims to accomplish.

This first issue of the Ontario Mathematics Gazette is made possible through the financial assistance of the Ontario Mathematics Commission (through its Advisory Committee on Research and Publications) and the Ontario Association of Teachers of Mathematics and Physics. Indeed, the Gazette includes among its functions those previously performed by the OATMP Newsletter; it can be considered as a successor, in amplified form, of the Newsletter.

Among the aims of the Gazette will be that of increasing the liaison between secondary school teachers and students on the one hand and university professors on the other hand. It is hoped that portions of the Gazette will attract student readers; perhaps this aim could best be achieved if mathematics teachers would induce their boards to buy bulk lots of future Gazettes, since most boards probably have funds set aside for purchase of educational literature and materials.

The actual production of the Gazette will be made easier if teachers will be generous with suggestions for future features or articles. What would you like to see in the Gazette? We welcome articles, problems, correspondence; do not be shy about offering help. If we can't use your first submission, perhaps we can use your third! Any articles to be considered for publication should be sent to the Editor-in-Chief, who will pass them on to a suitable referee for judgment; correspondence on other matters should be addressed to the appropriate editor. In particular, any "newsy" material should be sent to the Newsletter Editor.

One question we would like to see answered is the following: how should the Gazette be financed? We hope to put out three or four issues per year once we get rolling. Would you favour a subscription of, say, one dollar per annum? Do you think school boards could be encouraged to subscribe for their teachers? Here is one area where we would welcome correspondence.

And now, before concluding, let me make the usual disclaimer. Any views expressed in articles are the views of the particular authors, and do not necessarily reflect the views of the Ontario Mathematics Commission, the Ontario Association of Teachers of Mathematics and Physics, or the Editorial Staff of the Ontario Mathematics Gazette.

This first issue has had little precedent to rely on. We are trying to develop a typically Canadian magazine which will differ from corresponding American publications in two respects: the percentage of articles on mathematics, rather than on "education", will be higher; the articles on purely American themes will be lacking. Naturally, we won't achieve perfection in one effort! But we count on you to help us; how do you like our first effort, and what suggestions have you for the future?

With best regards to present and future friends.

Ralph G. Stanton

Editor-in-Chief

▲

▲ What is: OAME/AOEM Outreach Committee

BY THE OUTREACH 2019/2020 COMMITTEE



Top Row: Melissa Black, Siobhain Broekhoven, Lori Yee, Kerri Evershed, Jane Silva, Freda Liu
Front Row: Paul Alves, Sandra Jean Price, Franca Genco

There are a variety of responsibilities that fall under the umbrella of the Outreach Committee. Primary among these is soliciting proposals for hosting the annual conference. The committee asks chapter representatives to meet with their local executives and determine if they want the opportunity and the responsibility of hosting the conference in their area. The committee tries to ensure that the annual conference has hosts for at least the next three years. Recently, the committee took on the responsibility of defining the role of the *conference venue planner* and advertising for possible candidates. This move is in response to a shift in having the conference hosted in a regular rotation of venues that are still planned by local chapters, but held consistently in the same venues that rotate on a three-year cycle.

The Outreach Committee is also responsible for the OAME/AOEM awards. The committee prepares a package defining awards that are available. The package is shared with the membership through the website, the *Gazette*/*Abacus*, and social media. After the application deadline, the committee deliberates using the nominations and details about the candidates to determine the winners of each award. Often the calibre of the nominees necessitates care in the adjudicating process.

Lastly, the committee reaches out (!) by creating promotional materials and information that are shared at the chapter level, with faculties of education, and with other organizations to spread the mission and vision of OAME/AOEM and promote membership. ▲

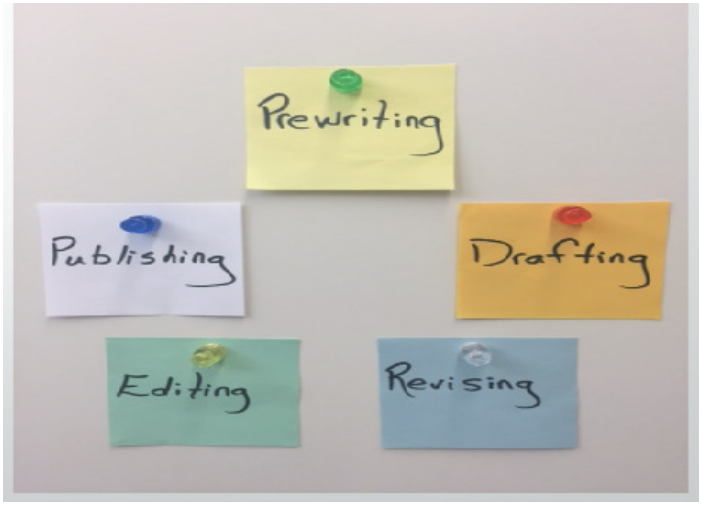
▲ Linking Literacy and Math: Strategies to Support a Writing Process in Mathematics



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Dr. David Costello is an author who has recently published Using What Works: Strategies for Developing a Literacy-Rich Environment in Math*. David is a vice-principal, who has held many roles. He taught in the primary division before assuming roles of numeracy interventionist, numeracy coach, numeracy leader, and curriculum consultant for Prince Edward Island. David has also instructed university courses focused on curriculum, differentiation, mathematics, and language arts. He facilitates professional learning sessions in the areas of mathematics and school development. His research situates mathematics within a literacy perspective and examines leveraging literacy-based instruction in mathematics.*

In my previous column, I proposed that the five-step writing process can be applied to mathematics as a way for students to frame, support, and consolidate mathematical understanding. The writing process consists of five phases: pre-writing (planning), drafting, revising, editing, and publishing (Calkins, 1994).



When one thinks about writing in mathematics, it tends to be what is viewed as writing to demonstrate knowledge (Scardamalia & Bereiter, 1987). Within this approach to writing, students write for a specific audience with a specific purpose. This writing is focused on students showing what they now understand after engaging with a concept or idea. Students provide evidence of their understanding.

While there is value in writing to demonstrate knowledge, what I am proposing when emphasizing the writing process in math is the inclusion of writing to support learning in math classrooms. *Writing to learn* involves writing short, informal pieces to support thinking through concepts and ideas (Knipper & Duggan, 2006; Scardamalia & Bereiter, 1987). This type of writing is meant to provide students with an exploration into their thinking, the concepts, and ideas being explored. Instead, it is how students are working through a concept and idea that is of interest. *Writing-to-learn* pieces make student's thinking visible (Knipper & Duggan, 2006).

By applying a writing process to mathematics, students are supported as they work through a concept and/or strategy. There are multiple opportunities built into the writing process for students to stop, take stock of their thinking, and plan their next step. This approach to thinking, metacognition (Keene & Zimmermann, 2007), is what we want of our students: thinking about our thinking. It is not enough to "do math"; rather, we want students to be constantly thinking about their understanding of the problem, strategy selection, what they will look for to determine if they are on the "right track" to solve the problem, and whether they have addressed what was being asked of them.

What I will do next is provide a brief overview of each of the five phases of the writing process, and highlight how these can be applied to students' mathematical understanding. While I will describe the five phases of the writing process in isolation, it is important to note that there is fluidity between the phases. As writers write, they move through most, if not all of these five phases, sometimes going back-and-forth between the phases. I say most because sometimes, depending on the purpose, the writer may choose not to edit or publish the text. Supports for each of the phases can be found in more detail in previous columns I have written for this publication. I will not go in-depth into each strategy, as that is not the intent of this particular column. Instead, I am sharing a brief overview of the strategies to highlight how the writing process can be applied to mathematics.

Pre-Writing (Planning)

The first phase of the writing process is planning. Within this phase, students are to think about the task that they are ready to undertake, and craft a plan as to how to solve this task. It is important for the student to consider all ideas related to the task and the underlying mathematical concept. It is this phase that often gets "rushed through." Some will only invest minimal time planning; instead, they will rush into solving. What tends to happen is that students return to this

phase later on in the process, as their thinking is refined. Within the planning phase, three strategies can provide students with support: graphic organizers, think/write/share, and paraphrasing.

Graphic organizers support students throughout the problem-solving process. Such organizers assist students in planning their approach and carrying it out to completion by allowing students to organize their thinking within a supported structure. Graphic organizers can have many forms and functions, and are designed to support students in many ways, such as working through a problem, sequencing their thoughts, and/or noticing similarities and differences.

One strength of a graphic organizer is the visual support it provides for students. Students can refer to checkpoints within the graphic organizer while working through a problem. Graphic organizers can help to categorize key concepts while removing distractions and/or irrelevant information. It is important for students to experience a variety of supports, as one organizer will not necessarily assist in solving all problems.

The think/write/share strategy involves three stages of communication: thinking, writing, and sharing. Before expecting students to record their thinking in written form, students are afforded the time to think and reflect on the concept and/or strategy. This then shapes the written product that will be shared with others. Within this three-part strategy, students can have multiple chances to think about the prompt and then evolve it throughout the process.

The process involved in think/write/share provides a tiered approach to communicating. By providing students time to think about the concept and/or strategy, they are afforded opportunities to clarify their thinking. Then, writing it provides a step whereby students can refine their ideas. The third part, sharing, allows students to share their thinking with others, hear the thinking of others, and then use the two to revise their thinking. This is a useful support for students as they plan their approach to a problem.

The ability to paraphrase is a beneficial planning strategy, as it focuses on the student's ability to digest the content of the task and articulate it in their own words. Being able to reword a task is a clear indicator of whether the task was understood by the student. Entailed in paraphrasing is the ability to review the content of the task, identify the highlights of the task, and then to rephrase this in your own words.

A fundamental question is: "How can you be expected to understand a task and be able to solve it if you cannot state it in your own words?" This, summed up into one question, can be identified as the purpose of paraphrasing. What is the point of students moving forward with the problem if they are unable to paraphrase the context of the problem? What will they use as a barometer of success when working through the problem? What will they use to monitor comprehension as they work through the problem?

Drafting

Once planning is complete, the student begins preparing a draft of their work. It is during this phase that the student composes their approach to solving the task. The student will apply what was generated during the planning phase to guide their work. This work can be thought of as a rough draft because the focus isn't on neatness, but instead, it emphasizes getting their ideas onto paper as they work through the problem. Within the drafting phase, two strategies can provide students with support: eraser free, and margin notes.

The eraser-free strategy can provide the students an opportunity to draft their work without losing the train of thought when erasing misconceptions. Instead of students erasing their work, students will cross it out and move forward. What this provides is documentary evidence of thinking throughout the problem, instead of focusing only on their final response. Students can use a pencil, with the understanding that they cannot erase their work; students can also use a pen or marker to work so that erasing is not an option.

When student work is erased, so is the record of their thinking to that point. It is vital that their changes in thoughts and points of clarification are maintained so that they can review their thinking throughout, and at the conclusion of the problem-solving process. Also, it provides the teacher with an incredible source of information about student thinking. By removing the option to erase, it is the student thinking that is emphasized instead of the final response. Using an eraser-free approach to drafting, the focus is getting thoughts onto paper instead of preparing it for a good copy.

A second strategy that can be applied to the drafting phase is that of margin notes. Margin notes provide students the opportunity to record their thinking throughout the problem. It is a strategy that is typically applied to reading. When a student reaches various points in the book, they note it symbolically. When the book leads to a question, they record a question mark ("?"). If there are details a student connects with, they record an asterisk ("*"). When they reach a point that is a stumbling block, they record an exclamation mark ("!"). Teachers may add additional symbols as they see a need. This form of note taking tracks student thinking. In mathematics, margin notes provide students the opportunity to record when they encounter a stumbling block and points

of clarification. When using symbols for stumbling blocks and points of clarification, students are asked to include a few words about their understanding.

The purpose of margin notes is two-fold. First, when students use margin notes as they work through a problem, they record their thinking, which can be used to support their understanding of the problem and assist them in understanding their strategies. Second, if students leave the problem and need to re-enter it later, they can use the margin notes as a guide when "beginning" the problem again. Margin notes provide opportunities for students to make "brief, key notes" about their thinking, without losing the momentum of drafting.

Revising

Within revising, the third phase of the writing process, students examine the content of their work for the purpose of clarity and craft. Students will add, remove, and change elements of their drafted work as they review it. During revision, students consider changes that will strengthen their work in moving toward a solution: What parts are unnecessary? What parts could be strengthened? And, overall, are there flaws in the plan they applied to solve the problem?

Within the revising phase, two strategies can provide students with support: eraser free (which has already been explained) and bookmark,

Similar to a physical bookmark used when reading, the bookmark writing strategy serves as a place to re-enter the problem. Consider, that after drafting your work, it may be a struggle to re-enter the problem when it comes time to revise one's thinking and subsequent written work. When students leave notes for themselves so that they can re-enter the problem at another point in time, they do not have to start from scratch. Instead of stressing (implicitly or explicitly) the need to solve a problem in one sitting, bookmarks provide students the option to leave a problem when they encounter a stumbling block, when they have exhausted all strategies available to them, or when time has run out in the class. Bookmarks require students to write a brief note about what they have tried, what questions they have that need to be answered before re-entering the problem, and/or a possible strategy to try upon re-entry.

A benefit of the bookmark strategy is that students strengthen their resolve to complete a problem and that they understand that a problem does not need to be solved within one sitting. Using the bookmark strategy, students use metacognition in their problem solving when they think and record what they have attempted thus far and what they may need to do upon re-entry.

Editing

Now that revising is completed, the student moves to the fourth phase of the writing process: editing. It is during the editing phase that students attend to correctness of their work. It is not intended to be as broad or comprehensive as revising when students are to examine their approach to the problem; rather, it is at the micro level, whereby students examine their work for calculation errors and misprints. Within the editing phase, one strategy that can provide students support is sticky notes.

An important part of mathematics is the ability to examine concepts and/or strategies and then communicate your understanding. In the sticky-note strategy, students will examine their work for any errors. When doing this, they will then record their responses on sticky notes as a way to identify the error and keep track of this change. It is a process of articulating what they see and don't see in their work.

The purpose of the sticky-note strategy is to provide students with the opportunity to examine their work and articulate their thinking in a succinct, targeted manner. By having students record their thoughts on a sticky note, it forces them to be direct and purposeful. Students will need to first think about their response and then refine it to fit the parameters of the task.

Publishing

The fifth phase of the writing process, publishing, entails the final preparation of a piece of writing for the intended audience, whether the audience is a teacher, another student, or themselves. It may involve preparing a copy of the final draft. Within this published copy, the revisions and edits made to the rough draft have been made and reviewed by the student. It is this published piece that will demonstrate the student's knowledge of the concept and its application. Within the publishing phase, two strategies can provide students with support: summary and reflection.

Students summarize their thinking involved in working through a problem. This does not mean that students are to record each single point of their thinking. Instead, like summarizing as a reading strategy, students are to identify the highlights of their work and record these in a succinct manner. What this does is clarify their thinking throughout the problem and have them think about it from the lens of what was necessary in solving the problem. Summarizing one's work is to be done at the conclusion of the problem-solving process.

It is not enough that students just list all of their work when articulating their problem-solving process. It is

imperative that students review their work and summarize it in a way that addresses the highlights of their thinking. It is a process of determining importance and then communicating it to others.

Reflection is something done in language arts classrooms from Kindergarten through Grade 12. It can be used in mathematics for that grade-level range as well. When reflecting, students are to record their experience with the problem, whether the experience was negative or positive. It is a place for students to comment on their successes, struggles, questions, and impression of both the problem and of their thinking. Reflecting on a problem usually occurs at the end of the problem-solving process, whether it is after solving the problem, or moving onto another problem before the first is solved.

Reflections can be a cumulative record of the problem-solving experiences of a student. They become a record of the student's journey as a problem solver, and how this journey has shaped them as a problem solver. What is included are experiences that have impacted their ability to problem-solve, lessons that they have had along the way, and a source of support when working with future problems that may be similar. Reflections are highly individual pieces that reflect the student and what they feel is important for them in moving forward.

It is important that students have an opportunity to record their emotions, successes, and/or struggles while engaged in a problem. Reflecting on their cumulative experiences and lessons they have learned along the way helps shape them as problem solvers. Too often the focus in mathematics classrooms tends to be on the solution instead of the affective. Providing students an opportunity to write about their encounter with a problem, how it made them feel, and what they will take away from the experience can support them in building self-esteem and perseverance.

Final Thoughts

We need to move beyond the answer in mathematics. We need to understand and appreciate the thinking of students as they work toward this answer. By applying a writing process to mathematics, both students and teachers see the benefit. Let's consider students first. As students work through a problem and are engaged in a five-phase writing process, they are actively thinking about their thinking (metacognition) in that they plan an approach, draft their work as a rough draft, revise their work by asking themselves questions and determining whether their approach was effective, edit their work by scrutinizing it for any procedural errors, and then making a published copy for the purpose of sharing. Within this process, students are actively engaged in thinking throughout the problem-solving process.

For teachers, having students engaged in the writing process when problem-solving provides a detailed, documented account of the thinking that supported the final response. It is much more than checking to see if students are correct; rather, it is examining student thinking from the beginning of the problem-solving process through to the conclusion.

I look forward to continuing the discussion of linking literacy and math via Twitter (@dr_costello) or email.

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▲ Using Surveys to Assess Student Attitudes Toward Mathematics



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Jeff Irvine, PhD, has been a secondary mathematics teacher, department head, and vice-principal. He has taught at three faculties of education and at a community college. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. With over 45 years in education, Jeff is particularly interested in the interplay of student motivation and mathematics achievement.

No math teacher would be surprised that a subset of their students dislikes math. On the other hand, a different subset of their students loves math. This article is about ways to minimize the former subset and maximize the latter one. We need to do this because researchers have found correlations between motivation and achievement in mathematics (Bodovski & Farkas, 2007; Moller, Stearns, Mickelson, Bottia, & Banerjee, 2014). Shernoff, Csikszentmihalyi, Schneider, and Shernoff (2003) found very low levels of engagement in mathematics classes, where they reported that the classes had very high cognitive intensity, but very low motivation. In their study, students reported being more negative about mathematics and less engaged than in any other subject.

How interested were you in today's class?
How confident are you that you understand the material in today's class?
How useful do you think what you learned in today's class is to you?
Do you want to learn more about this topic in tomorrow's math class?

Dgespc / 8 ? qrsbcl r_rrgsbcq qsptcw

As part of a larger project, I developed a series of brief surveys to assess students' attitude toward, interest in, and engagement in their math classes. A sample survey is shown in Figure 1. Surveys were completed anonymously, usually at the end of class. By keeping the responses anonymous, social desirability bias (Miller, 2012) was minimized—my students told me the truth about how they feel. Social desirability bias is a problem when identities are required in surveys and interviews. It causes some responders to tell you what they think you want to hear. Anonymity minimizes this problem, but may result in brutal honesty. However, sometimes I am also pleasantly surprised when a majority of students tell me that the lesson was interesting, or useful, or fun. These surveys are the basis for action, on the part of both the teacher and the students. For example, if a particular class was found to be uninteresting, the teacher could plan to use more interest-generating activities in the near future, such as open questions, student choice, or real-world connections. If the students respond that they lack confidence about today's topic, skill consolidation or alternate explanations could be utilized. Student action plans could involve detailed reviews, asking more questions during class, attending extra help sessions, or reviewing homework to identify areas of strength and areas in need of attention.

In a similar way, I have found it important to vary the style of student surveys, both to minimize boredom and repetition, but also to acquire additional knowledge about my students and my teaching. By examining the majority of responses, I can tell if a lesson needs revising, reworking, or replacing. While it might have been useful to know which students responded in a specific way, I have found that responses with anonymity give a more accurate reading on how the class felt.

The first few times I ask students' opinions, I keep the responses limited to a five-point opinion scale (Figure 2) or a ten-point laddered scale (Figure 3). While the ten-point ladder scale may seem excessive, I believe that my students provided more thoughtful responses, judging from the number of surveys where students had crossed out their initial responses and repositioned their answers elsewhere on the scales.

Circle how you feel about what you learned in math class today.				
Today's class was interesting.				
Strongly agree	Agree	Neutral	Disagree	Strongly disagree
Today's class was useful to me.				
Strongly agree	Agree	Neutral	Disagree	Strongly disagree
I feel confident that I understand the material from today's class.				
Strongly agree	Agree	Neutral	Disagree	Strongly disagree
I will complete all the assigned homework for today's class.				
Strongly agree	Agree	Neutral	Disagree	Strongly disagree
I would like to learn more about the ideas from today's class.				
Strongly agree	Agree	Neutral	Disagree	Strongly disagree

Figure 2: *Student attitudes survey with a question that encourages student action*

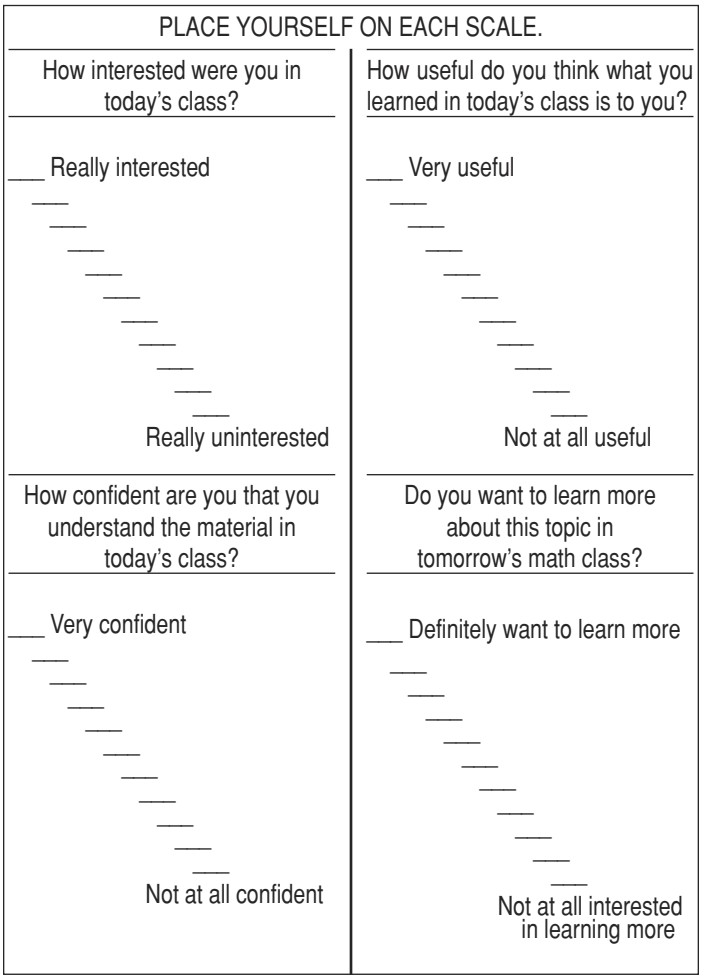


Figure 3: A student attitudes survey with multiple gradations for each question

These surveys are typically completed rapidly, taking less than one minute of class time, and the responses have been useful in modifying my teaching. An added bonus is that just by asking my students for their opinions, it appears that there have been positive influences on student attitudes. As Lina (all student names in this article are pseudonyms) stated, "I liked a lot of your activities. I also liked the fact that you're interested in what the students think. I feel like sometimes our opinion gets overlooked and we kind of just get passed along to Grade 12." Other students provided similar responses. As the students become more comfortable with being asked their opinions, the surveys can move toward asking why (Figure 4).

The reasons behind students' responses are important information for the teacher and can lead to improved lessons and increased student engagement. Some students respond with very specific reasons for their affective reactions. For example, Rob stated, "Today we did stuff that was already covered in Grade 9. We spent a whole class on it, and I was really bored because I already knew it all." This comment pointed the teacher (me) to finding better, more differentiated ways to review prior knowledge, tailoring classes to focus on how the prior knowledge was going to be used in the current grade.

How do you feel about [e.g., solving quadratic equations]?		
Circle a word in each row that shows how you feel about _____ [e.g., solving quadratic equations.]		
• Like	Dislike	_____
• Excited	Bored	_____
• Afraid	Comfortable	_____
• Worried	Calm	_____
• Clear	Confused	_____
• Happy	Sad	_____
• Love	Hate	_____
• Confident	Anxious	_____
Now fill in a reason for each of your choices on the blank lines.		

Figure 4: *Student attitudes survey asking for reasons behind the responses*

Student opinions may be solicited on a particular topic (Figure 4) or on a complete unit (Figure 5). For example, if a majority of students indicate that they are confident in solving quadratic equations, it is probably safe to move on to new material. Alternatively, if a majority indicate some trepidation about a new unit, the teacher can introduce the unit with some hands-on and fun activities to build student interest and confidence. These activities might involve a hands-on pattern-building activity, an Internet search for examples of the topic being used in the real world, or active strategies during instruction, such as a placemat, think/pair/share, timed retell, or concept attainment. For teachers unfamiliar with any of the above strategies, a wealth of information is available on the Internet.

THIS NEW UNIT IN MATH				
When I think of starting this new unit in math:				
I feel				
very confident	confident	okay	not confident	really not confident
I feel				
really excited	excited	neutral	not excited	really not excited
I feel that the new unit will be				
really interesting	interesting	okay	not interesting	really not interesting
I feel that the new unit will be				
really useful to me	useful to me	neutral	not useful to me	really not useful to me
I feel motivated to do my best on this unit				
strongly agree	agree	so-so	disagree	strongly disagree
What one word sums up your feelings about this unit? _____				

Figure 5: *Student attitudes survey prior to starting a new unit*

The surveys may ask general feelings about the lesson, about a new unit, or about a particular skill embedded in the lesson. Ultimately, I would like students to form a plan of action based on their survey responses (Figure 6). Initially, students need to be taught how to build a plan of action. For example, "try harder" is not useful. Action plans need to be "SMART" (Specific, Measurable, Achievable, Results-oriented, and have a Time limit (Mindtools.com., n.d.). As an example, Alicia gave her action plan for learning when to use factoring and when to use the quadratic equation as: "By reviewing my notes and the examples we have completed, I will look for clues about when I can use factoring, which is faster, and when I need to use the quadratic equation. I already found one pattern: when all the signs are positive and c is bigger than b^2, then the equation won't factor, so I won't spend time trying to do that. I need to figure this all out before our midterm test in two weeks."

How confident are you that you can convert among the different forms of a quadratic relation?				
Very confident	Confident	Not sure	Not that confident	Really not confident
How interesting do you find these conversions?				
Very interesting	Interesting	Neutral	Not interesting	Really not interesting
Given your answers to the questions above, what action will you take regarding these conversions? _____				

Figure 6: *Student attitudes survey on a specific skill, asking students to plan a personal course of action*

These quick surveys, which usually take a few minutes, serve two functions: they inform me how my lesson was perceived by my students, and more importantly, they give students a voice in their learning. By being asked their opinions, and being listened to by their teacher, student buy-in is generated, and students take additional ownership of their own learning. By surveying student opinions on a regular basis (once a month or at appropriate times during the unit), teachers can get a picture of not only where student emotions lie, but also how their emotions are changing over time.

At the end of some weeks, I asked students for a summary of their feelings about the week (Figure 7). This can be done online, which tends to add a perceived additional level of anonymity. Responses can be used to modify the week's lessons for the next time the course is taught.

Student Weekly Reflection				
Choose the response that best matches your feelings about math this week.				
I enjoyed this week in math.				
Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
I worked hard this week in math.				
Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
I would like more math classes to be like this week's math classes.				
Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
Comments (optional)_____				

Figure 7: *Weekly survey of student attitudes*

I have found that these surveys raise student engagement and have the potential to positively increase student attitudes toward mathematics. These are a quick way to check motivation, confidence, and interest. These are critical affective variables that we need to address.

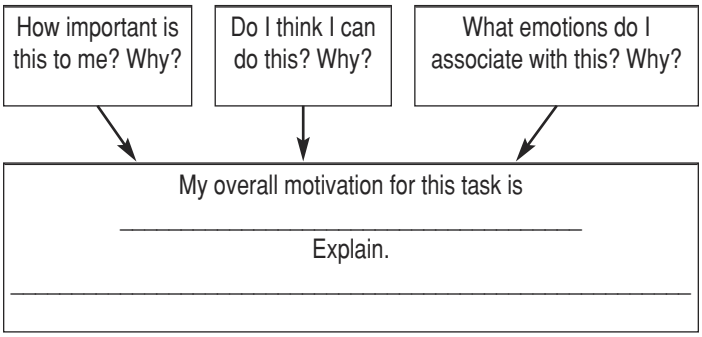


Figure 8: *Asking for students to provide reasons for their responses and to draw conclusions about their overall motivation*

For attitude surveys that may require more thought or more time, the surveys can be taken home (Figure 8). If you really want to get a snapshot of the attitudes in your class, I recommend the *Attitudes Towards Mathematics Inventory*© (ATMI: Tapia & Marsh, 2005). This is a 40-item survey with extensive research behind it. The ATMI is copyrighted, but

Dr. Tapia is very generous in allowing teachers and researchers to use it. When I used the ATMI, I used an online survey program like SurveyMonkey™ and had my students complete the ATMI outside of class time. Once the teacher sets up the ATMI online, it only takes about 15 minutes for students to complete it. I got an extensive picture of my class. The ATMI has four subscales: enjoyment, value, self-confidence, and motivation (to take more math classes). I found that the individual subscales were the most useful, since a class may have only one area (such as self-confidence) that is a concern. I found that using the ATMI at the beginning of the semester was the most valuable, since it provides' focal areas to address over the term. I only used the ATMI once per semester, since it is quite extensive.

Improving student attitudes toward mathematics is an important dimension of what we do as teachers. This article outlines some ways of asking student opinions, gathering a more fulsome picture of our students, and at the same time, increasing student buy-in, engagement, and positive attitudes about mathematics. You may feel that these surveys take up too much class time, but you can pick and choose which surveys to use in your own classrooms. I often used all eight surveys described in this article, and it took less than 10 minutes overall of class time.

I have found that very few teachers have asked students their opinions on what they learn, especially in mathematics. This is tragic, in my opinion, and has influenced the common perception that mathematics is uninteresting, boring, difficult, and only for the few. We have to change those perceptions. "It's not about the math, it's about the attitude" (Author, July 23, 2013, 3:55 a.m.).

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▲

▲ Supporting Early Geometry and Spatial Reasoning in Kindergarten: A Collaborative Inquiry



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There is growing awareness about the importance of early math education in North America. For example, research suggests that early math abilities are a key predictor of future math and academic success in school (Duncan et al., 2007; Jordan et al., 2009; Romano et al., 2010). However, a recent survey of 130 early childhood educators, Kindergarten teachers, and Grade 1 teachers revealed that only 30 percent of these educators learned about early math development and education in their training (Youmans, Coombs, & Colgan, 2018). Given the importance of early math education, there is a need for educators to be equipped to effectively teach and support early math learning.

A team of 11 Kindergarten educators (six early childhood educators and five Kindergarten teachers) in Ontario became aware of the importance of spatial reasoning in young children through a Kindergarten Additional Qualification course offered by the Elementary Teachers' Federation of Ontario (ETFO). While mapping the Ontario Kindergarten Program (2016) mathematics expectations and content in children's play, they noticed connections between children's block and puzzle play with geometry, measurement, patterning, algebra, and number. Furthermore, they learned that children's Kindergarten math skills are the strongest predictor of later school achievement, regardless of gender or socio-economic status (Duncan et al., 2007; Claessens, Duncan, & Engel, 2009; Claessens & Engel, 2011). The team also become aware of how spatial reasoning skills enhance success in early mathematics performance (Cheng & Mix, 2014) and school readiness (Verdine, Michnick Golinkoff, Hirsh-Pasek, & Newcombe, 2014), and that such skills can be improved with practice across all age groups, socio-economic levels, and gender (Uttal et al., 2013). For example, strong spatial skills can be developed in young children through guided play, facilitated by knowledgeable adults, with materials such as blocks, puzzles, and shapes (Verdine et al., 2014), In response to their new learning, and in recognition of their limited training in mathematics, the team of educators decided to deepen their understanding of spatial reasoning and age-appropriate instructional practices through a collaborative inquiry (CI) funded through the Ontario Ministry of Education's Teacher Learning and Leadership Program (TLLP) with support from the Ontario Teachers' Federation (OTF).



Figure 1: *The 2017–2018 Kindergarten TLLP Spatial Reasoning & Geometry CI. (Left to right) Back row: Margaret Warner, Helen Stock, Linda Hillier, Wilma Armstrong, Rebecca Costello. Middle Row: Kayla Ingram, Michelle McKeown, Victoria (Powell) Manley, Edward Schroeter. Front: Kristine Swift. Absent (upper right): Katherine Benz.*



Figure 2: *Research suggests that when adults insert themselves in children's play, materials such as blocks, puzzles, and shapes can help improve children's spatial skills, which undergird early math development.*

The strengths of a CI approach to Professional Development (PD) are that it is educator driven, embedded in classroom practice, and involves collaboration and reflection with colleagues (Ontario Ministry of Education [OME], 2014a). Research suggests that this type of PD is associated with positive educator and student outcomes because it incorporates active and authentic teacher learning experiences and time for teachers to reflect on their learning (Dunst, Bruder, & Hamby, 2015).

The Collaborative Inquiry

The team of educators developed professional learning goals and a theory of action for their early geometry and spatial reasoning CI. The professional learning goals were

threefold: 1) to improve the geometry and spatial skills teaching and learning in their Kindergarten classrooms by developing a deeper understanding of age-appropriate early geometry and spatial reasoning skills, 2) to consolidate their learning by teaching each other and aligning their learning with the Kindergarten curriculum and reporting processes, and 3) to document and share their collective learning about effective early geometry and spatial reasoning assessment and instructional strategies. To help the team achieve these goals, they developed an action plan that included the following activities: a book study on mathematics learning trajectories (i.e., *Learning and Teaching Early Math: The Learning Trajectories Approach*, by Clements & Sarama, 2009); a book study of 32 field-tested and research-based activities designed to appeal to young children (i.e., *Taking Shape: Activities to Develop Geometric and Spatial Thinking,* by Moss, Bruce, Caswell, Flynn, & Hawes, 2016); participation in workshops on early geometry and spatial sense; a site visit to a laboratory school to see the implementation of early geometry and spatial sense activities; the application of geometry and spatial sense activities in their own classrooms, using the student work study method; and times of team learning to consolidate and reflect on the CI.

Over the course of the school year, the team met six times to work on their CI. The team's key CI learnings about early geometry and spatial reasoning were that learning trajectories are a powerful tool for Early Math Education; geometry and spatial reasoning are best developed in Kindergarten students through hands-on activities; and guided play helps students acquire new math skills. A description of each of these key learnings follows.

Learning trajectories are a powerful tool for Early Math Education. The use of early math learning trajectories and their companion website (see www.learningtrajectories.org) is an effective tool to help educators: 1) understand key early years mathematics concepts and vocabulary, 2) assess individual students' developmental levels, and 3) plan appropriate learning activities. They align with the Ontario Kindergarten curriculum expectations (OME, 2016) and provide guidance for developing mathematical proficiency in young children.



Figure 3: *In October, a Year 1 Kindergarten student chooses to work on and independently complete an easy pattern-block puzzle.*

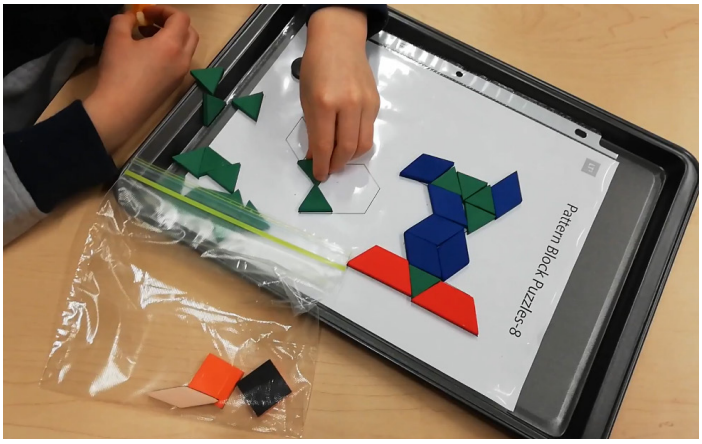


Figure 4: *In February, a Year 2 Kindergarten student completes a moderately difficult pattern-block puzzle, but with constraints imposed by a teacher: all hexagons have been removed.*

Geometry and spatial reasoning are best developed in Kindergarten students through hands-on activities. The educator professional resource book, *Taking Shape*, by Moss et al. (2016) is an accessible entry point for educators to deepen their understanding of spatial and geometric hands-on instructional activities. Essentially, providing a variety of geometric and spatial reasoning materials (e.g., puzzles, pattern blocks, pentominoes, loose parts, and building blocks) and activities (e.g., identifying shapes based on what they feel like without seeing them and identifying as many unique structures as possible that can be made with four cubes) for students facilitates their growth in geometry and spatial thinking, creativity, engagement, and positive attitude to mathematics.

Guided play helps students acquire new math skills. Free-play opportunities can be used to reinforce geometry and spatial reasoning skills, but educators should guide new learning through vocabulary and skill development

(especially positional and directional language and gestures) in playful settings. For example, giving students design challenges in geometry and spatial reasoning can be used to help them develop planning skills and structure the learning (e.g., in reflective symmetry, children can be given half a pattern block image and be asked to create the other half). No matter how technologically proficient young children become, they still need to develop foundational spatial skills through structured play (e.g., games, and play that contains spatial movements).

The Impact of the Collaborative Inquiry

To document the impact of the collaborative inquiry, team members completed a pre- and post-test of the van Hiele Geometry Test (used with the permission of the University of Chicago School Mathematics Project [UCSMP]) and responded to an online survey about their experiences at the end of the school year. All team members completed the tests and survey. The average number of years that team members had worked in education was 16, with an average of ten as a Kindergarten educator.

The van Hiele Geometry Test has six levels of performance, ranging from 0 (lacks basic understanding) to 5 (rigour – geometry is understood at the level of a mathematician). Many adults, including teachers, remain at Level 2 all of their lives (Mayberry, 1983), but elementary educators should be at a Level 3 to teach geometry effectively (Clements & Sarama, 2011). At the beginning of the CI, the average van Hiele Geometry Test score for the team was 1.7, and this increased to 2.4 at the end of the CI (see Table 1).

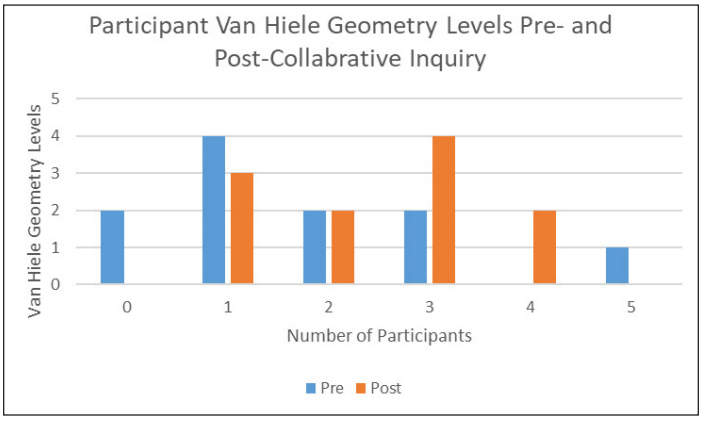


Table 1: *Participant Pre- and Post-Test Levels*

After the CI, six educators showed an increase in their van Hiele Geometry levels (ranging between 1 to 3 levels), three educators stayed the same, and two educators dropped levels (see Table 2).

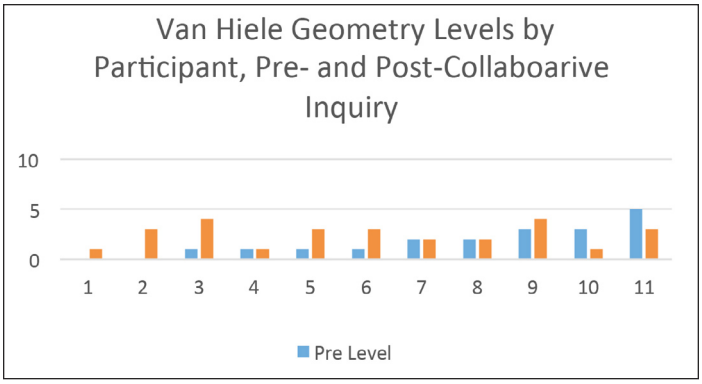


Table 2: *Pre- and Post-Test Levels by Participant*

The online survey gave educators an opportunity to share their collaborative inquiry experiences. It included a combination of closed-ended and open-ended questions. Where educators responded similarly to open-ended questions, responses were grouped and tallied. When asked to describe their personal learning(s) related to the CI, educators reported that the CI:

- helped them understand how to develop richer learning experiences for students in geometry and spatial reasoning (eight educators)
- improved their personal understanding of geometry and spatial reasoning (six educators)
- facilitated professional development through collaboration (four educators)
- helped them develop a better understanding of early math development/learning trajectories (three educators)
- increased their confidence and willingness to take risks in math teaching (three educators)

When asked what educators were doing differently in their classroom, they responded that they were:

- implementing more hands-on math learning experiences (nine educators)
- providing differentiated instruction for students who need it (two educators)
- naming the learning for students (two educators)
- using math vocabulary more intentionally (one educator)
- documenting student learning through notes, photos, and videos (one educator)

Team members were asked to rate the effectiveness of the collaborative inquiry in supporting several math-related areas. Responses indicated that the vast majority of educators believe the CI was either effective or extremely effective for supporting: the importance of early math (100 percent), self-confidence in early math instruction

(100 percent), trying new strategies (100 percent), early math advocacy (100 percent), understanding math development (90.91 percent), collaboration (90.91 percent), and math development trajectories advocacy (90.91 percent) (see Table 3).

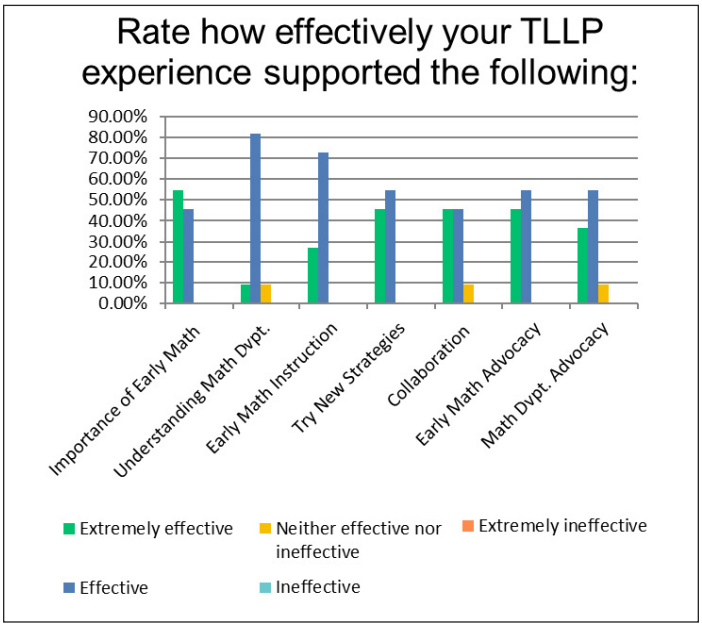


Table 3: *The Effectiveness of the CI in Supporting Math-Related Areas*

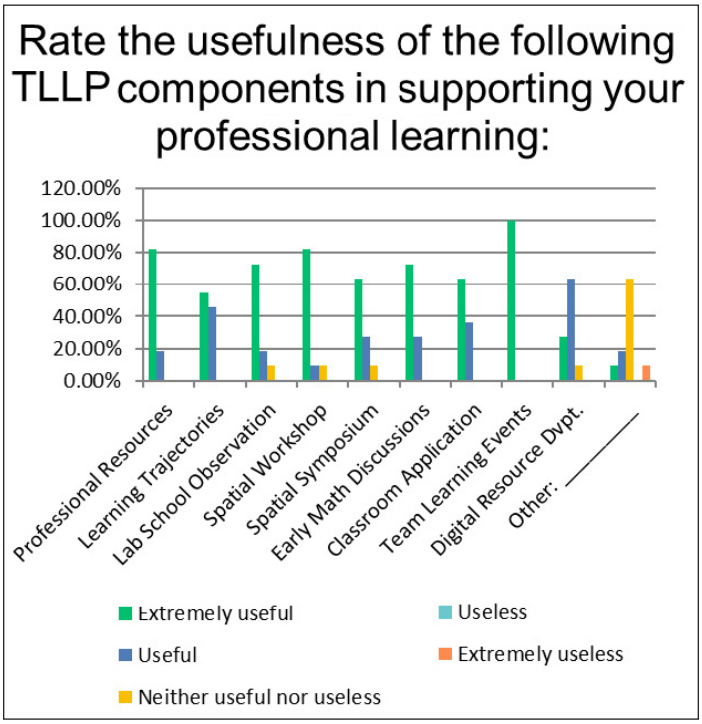


Table 4: *The Usefulness of CI Components in Supporting Educators' Professional Learning*

Educators were also given the opportunity to rate the usefulness of the CI components. All educators rated the team learning events as extremely useful. Other components rated as extremely useful or useful by a large majority of educators included: professional learning resources (100 percent), learning trajectories (100 percent), early math discussions (100 percent), classroom application (100 percent), lab school observation (90.91 percent), spatial workshops (90.1 percent), mini-symposium on spatial reasoning (90.91 percent), and digital resource development (90.91 percent) (see Table 4).

Discussion

The educators involved in this collaborative inquiry were motivated to improve their early math practices in geometry and spatial reasoning when they became aware of how these skills are related to students' early math abilities (see Cheng & Mix, 2014) and school readiness (see Verdine et al., 2014). This CI, initiated and implemented by a team of Kindergarten educators, is an example of effective professional development because it was embedded in classroom practice, enabled educators to reflect on their learning, and promoted collaboration (Dunst, Bruder, & Hamby, 2015). It familiarized educators with the age-based developmental sequence of geometry and spatial reasoning skills in young children (i.e., learning trajectories), helped them understand how to develop richer geometry and spatial reasoning learning experiences, and improved their personal understanding of geometry and spatial reasoning. It is interesting to note that educators' van Hiele Geometry levels increased without formal training, possibly due to their use of Clements' and Sarama's math learning trajectories, which embed accurate mathematics content and a research-based curriculum. However, it may be important to include formal content knowledge training and mentoring/coaching (see Sarama & Clements, 2013; Clements & Sarama, 2020) in similar future CIs, given that geometry and spatial reasoning appear to be underdeveloped skills in the majority of adults (Mayberry, 1983). While student outcomes were not systematically collected for this CI, it is encouraging to note that educators reported providing students with more opportunities to engage with hands-on, open-ended geometry and spatial reasoning activities that promote conceptual understanding as a result of the collaborative inquiry. It is recommended that similar CIs collect student-level data to help evaluate their effectiveness in regard to student learning outcomes. All educators in this CI rated team learning as extremely useful; educators highly value collaboration. PD opportunities that incorporate collaboration promote shared accountability and allow educators to learn from, build on, and reflect on each other's ideas.

Conclusion

The findings of the CI suggest that educator-directed professional learning is an effective means to develop instructional and assessment practices that support early geometry and spatial reasoning, the development of which is of vital importance to young children and is an often-neglected area in early mathematics programming. Given that teacher knowledge of mathematics content, teaching strategies, learning activities, and assessment methods are fundamental to improved student learning, it would seem prudent for Ontario mathematics education leaders to provide early math professional development to Kindergarten, primary grade teachers, and early childhood educators on geometry and spatial reasoning. It is recommended that such PD be delivered in a format marked by high-quality, collaborative, hands-on, job-embedded learning and mentoring. Moreover, given gaps in Early Math Education training in Ontario, a coordinated provincial Early Math Education Strategy would be extremely beneficial. Such a strategy could be modelled after the successful Adult Education Strategy (OME, 2014b) that has provided multi-year funding to establish seven regional partnerships with specific mandates to promote regional collaboration, capacity, and learner outcomes.

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Jumble Solution

Something they have in common?

1. rotate
2. pyramids
3. table
4. percent
5. rational

Answer: Penta-Star

▲ Fundamental Math Skills and Elementary Math Students in Ontario

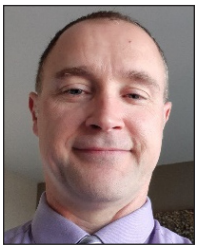
EQAO is a government agency that supports student learning in Ontario. Our data, research, and resources empower parents, guardians, educators, policymakers, and others with information to determine how best to strengthen student outcomes across the province.



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MAXIME GENDRON
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ASSESSMENT AND REPORTING

In summer 2018, the Ontario Ministry of Education released a teacher's guide called *Focusing on the Fundamentals of Math*, designed to help teachers build students' knowledge and skills in mathematics (Ontario Ministry of Education, 2018). The guide highlights the importance of fundamental math concepts and skills, and provides information on how to support students in improving in these areas. The guide also outlines examples of the fundamental concepts and skills considered foundational for students in Grades 1 to 8 in publicly funded school boards.

In response to the Ministry of Education's focus, data was generated and examined from large-scale assessments that aligned with the fundamental math skills outlined by the government. The primary objective of the analysis was to determine the overall success rate associated with these math skills in light of attention being placed on fundamentals.

Method and Measures

Data included in the analysis were gathered in the 2015–16, 2016–17, and 2017–18 school years from students at the end of the primary and junior elementary years (i.e., Grades 3 and 6). Curriculum specialists identified items from the large-scale assessments that aligned with the expectations outlined in *Focusing on the Fundamentals of Math*. This process yielded 78 items for Grade 3 and 68 items for Grade 6 (see endnote 1), resulting in 146 items included in the analysis.

The items were further identified as Multiple-Choice (MC) (116 questions over three years: 63 from Grade 3; 53 from Grade 6), and Open Response (OR) (30 questions over three years: 15 from Grade 3; 15 from Grade 6).

Each item was mapped to one of three categories: Knowledge and Understanding (KU), Application (AP), and Thinking (TH). Given the low number of open-response items (i.e., n = 30), analysis of the individual categories was not examined. Descriptions of the three categories and the corresponding number of items in the categories are below.

Knowledge and Understanding (KU): "Students must demonstrate only subject-specific content (knowledge) or comprehension of its meaning and significance (understanding), or both, in order to answer the question" (EQAO, 2017, p. 3). Of the 116 multiple-choice items, 39 were categorized as Knowledge and Understanding (24 from Grade 3; 15 from Grade 6).

Application (AP): "Students must select the appropriate tool or get the necessary information and "fit" it to the problem" (EQAO, 2017, p. 3). Of the 116 multiple-choice items, 51 were categorized as Application (30 from Grade 3; 21from Grade 6).

Thinking (TH): "Questions that require students to either select and sequence a variety of tools or to demonstrate a critical-thinking process are mapped to the category Thinking" (EQAO, 2017, p. 3). Of the 116 multiple-choice items, 26 were categorized as Thinking (19 from Grade 3; 7 from Grade 6).

For multiple-choice formatted items, the unit of measure was success on the individual items; for open-response formatted items, the unit of measure was achieving or exceeding expectations. For the purpose of the current

analysis, item difficulty was not included.

Results

The results are based on over 400 000 students at each of Grades 3 and 6. See endnote 2 for a detailed breakdown of the number of students and which EQAO assessment.

The percentage of students who were successful on each MC question, along with the percentage of students who met or exceeded the Level 3 provincial standard for OR questions, were first generated separately for English- and French-language students, examined, and combined by averaging across the two languages of study.

The combined percentage of student success rates is presented in Table 1. For MC, the percentage reflects the percent of students getting the item correct; for OR, the percentage reflects students achieving or exceeding expectations (i.e., Level 3 or above).

Panel	MC-Overall	MC-KU	MC-AP	MC-TH	OR
Primary (Grade 3)	71%	81.5%	68%	58%	54%
Junior (Grade 6)	66%	72.5%	65%	58%	59%

Table 1: *Percentage of Students Successful on Fundamental Math Skill Items*

Note: MC – Multiple-Choice; KU – Knowledge and Understanding; AP – Application; TH – Thinking; OR – Open Response.

Discussion

This study offers a preliminary examination of student achievement on fundamental skills in mathematics. The results show that students are better able to demonstrate their skills in the multiple-choice format than on open-response items. It is also clear that Ontario students in Grades 3 and 6 have stronger skills in Knowledge and Understanding of fundamental math than is the case with their ability to apply their skills and to think critically about them. The challenge with mathematics in Ontario may be less about students "knowing" math and more about their ability to apply math knowledge and to engage in related critical thinking.

This analysis serves as a baseline toward continuous improvement as educators focus on the fundamentals of mathematics in Ontario schools. The results highlight that student achievement in fundamental mathematics skills varies between knowledge of math facts, applying math facts, and developing metacognitive skills in critical math thinking. As Ontario focuses on improvements in mathematics, development of skills in the application of math facts, and associated critical thinking are key components for success. Instructional leaders who bring strategies for math applications and who foster opportunities for critical thinking about math will support favourable student outcomes.

As has been noted, this is a preliminary study of mathematics fundamentals in response to a renewed focus; there is additional research required. A broader understanding of the varying results between multiple-choice and open-response items is likely to yield an enhanced understanding of the findings. Similarly, an in-depth examination of the open-response items may well point to specific skills for focus and growth, particularly for students who have not yet demonstrated provincial expectations.

Endnote 1: Item selection was across English- and French-language assessments. Note that items for French- and English-language students were combined, as the unit of measure was success on items identified as fundamental.

Endnote 2: The results presented are based on the assessments of 417 080 Grade 3 students (133 843 in 2015–16; 141 791 in 2016–17; and 141 446 in 2017–18) and 410 443 Grade 6 students (131 073 in 2015–16; 138 502 in 2016–17; and 140 868 in 2017–18).

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Sodoku Solution

9	6	7	8	4	5	3	2	1
8	3	1	9	2	7	6	5	4
5	4	2	3	1	6	9	8	7
6	9	5	4	7	1	2	3	8
3	7	4	5	8	2	1	9	6
2	1	8	6	9	3	7	4	5
7	8	9	2	6	4	5	1	3
4	5	6	1	3	9	8	7	2
1	2	3	7	5	8	4	6	9

▲ Challenging Students to Generalize



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Jenna O'Dell is an assistant professor at Bemidji State University in Minnesota in the Mathematics and Computer Science Department with a specialization in elementary and middle school mathematics. Her research is helping children develop an understanding of what mathematics truly is through engaging them in famous unsolved mathematics problems. Prior to working at Bemidji State, she taught elementary school for six years. Jenna is passionate about getting girls involved in STEM and helping them find a love for science and mathematics.



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Todd Frauenholtz is a professor in the Mathematics and Computer Science Department at Bemidji State University in Minnesota. His main focus is preparing future and current teachers to teach mathematics through problem solving and productive struggle. His favourite teaching area is helping students develop algebra skills through patterns. In his free time, he enjoys spending time with his wife and children.

In this article, we present a lesson that will provide students with an opportunity to generalize a mathematical solution. We demonstrate how students can be engaged in the exploration of a graph-theory problem, the Graceful Tree Conjecture, as they work on problem solving, conjecture, and generalization. The lesson has worked well with students in Grades 4 through 8. We found that with different opportunities, students were able to advance their level of reasoning and creations of justifications.

Currently mathematics education in elementary school focuses on procedural and conceptual understanding of mathematical content (Bieda, Ji, Drwencke, & Picard, 2013). This lesson challenges students' conception of what mathematics is, and provides students mathematical experiences that broaden their understanding of the subject as the extensive discipline mathematicians know it to be. The discrete mathematics problem we will describe below allows you to focus on the Mathematical Processes including: problem solving, reasoning and proving, and representation. Also, this problem allows teachers to make connections to the content expectations, such as, generating patterns, investigating variables as unknown quantities, describing pattern rules, representing patterns algebraically, and using algebraic expressions.

Introduction

We like to begin with a class discussion centred on the question "What is mathematics?" Most students respond with addition, subtraction, multiplication, and division. We steer the discussion to include ideas such as patterns, conjecture, generalization,… to let students know that the discipline of mathematics is much broader than the arithmetic they have explored in the primary grades. We then give the students the definition of mathematics as the study of patterns, and explain that our work with them will be focused on finding patterns and creating justifications for those patterns. This could be a new idea for many students, as engaging in mathematical generalization or justification is not characteristically presented until middle school (Lin & Tsia, 2016) or even high school (Stylianides, 2007). Furthermore, Carpenter, Franke, and Levi (2003) stated that elementary students do not typically complete generalizable arguments, but they should be encouraged to do so and understand that just giving an example restricts their argument.

Following the discussion on what is mathematics, we explain how we will be examining a topic in graph theory focused on a specific type: a tree graph. A tree graph is an undirected graph that does not contain any cycles. The tree graph is made of nodes and edges. Readers familiar with graph theory may be more familiar with the terminology vertex and edge graphs; however, in our work, we found it to be easier for primary and middle-level students to use node and nodes, rather than vertex and vertices, when discussing plural situations.

The Graceful Tree Conjecture

Tree graphs do not contain a cycle; in other words, if you trace the edges in a tree graph, you will never come back to a node you have already visited without retracing an edge.

After looking at some examples and counter-examples of tree graphs, students have discovered that all tree graphs always contain one fewer edge than nodes, we label the nodes one through n, with n being the number of nodes on the graph. We next define the values of an edge as being the absolute value of the difference of the nodes. A graceful labelling is one in which the nodes are labelled one through n, and the edges are one through $n - 1$.

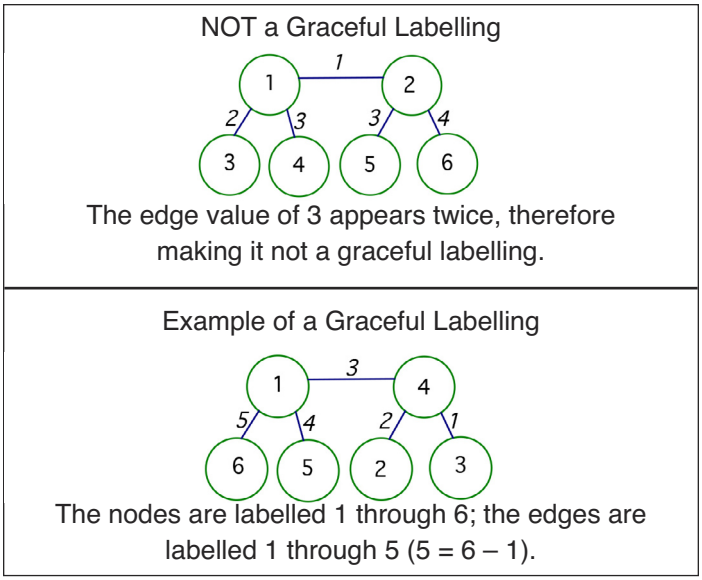


Figure 1: *Examples of Not Graceful and Graceful labels*

Finally, we describe the Graceful Tree conjecture. This conjecture states that all tree graphs can be labelled gracefully. This is a conjecture because it is thought to be true, but has not yet been proven.

Exploration

We start the exploration with the class of tree graphs known as Star Graphs, where the centre node connects to all other nodes (see Figure 2).

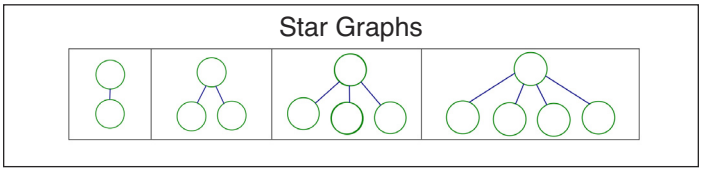


Figure 2: *The first four distinct graphs in the Star Class*

For this first exercise, we ask students to consider a basic star graph with two nodes and one edge. Could they find a graceful labelling? They find two different ways to label the graphs (these are revealed, but take time to consider it). Next, we proceed to a star graph with three nodes, then four, and then five nodes. During this time, we encourage students to look for patterns, and if they put a one or the largest number in a place on one graph, it might be a good idea to do it on the next graph. After they have found a graceful labelling for the first four distinct graphs in the Star Class, we ask them to draw the next graph in the Star Class and find a graceful labelling. While doing this, we again focus on the idea of patterns. Next, we ask students to write about the pattern they found. As they progress to this part, we ask students if they could label a Star Graph with 50 nodes gracefully? One hundred nodes? One million nodes? Or a Star Graph that has a large unknown number of nodes? This leads to the students creating a justification for the Star Class. If students are struggling to find a pattern, we and other students help with ideas of where the biggest, smallest, and middle numbers go (see Figure 3 for Star Graphs not labelled gracefully).

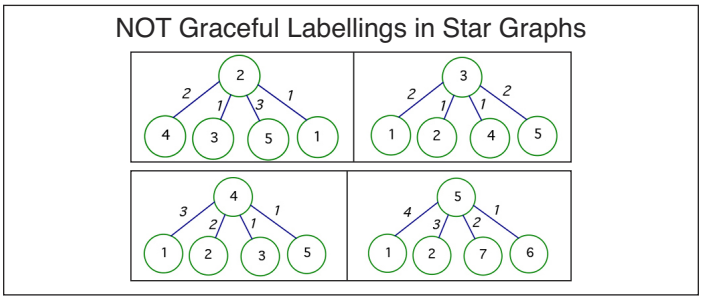


Figure 3: *Examples of Star Graphs with ungraceful labellings*

Through working with multiple primary and middle-level grades, we found that students create two different patterns for Star Graphs. They tend to put the smallest number (1) or the largest number (*n*) in the middle (top) node (see Figure 4 for examples of generalized patterns our students have found).

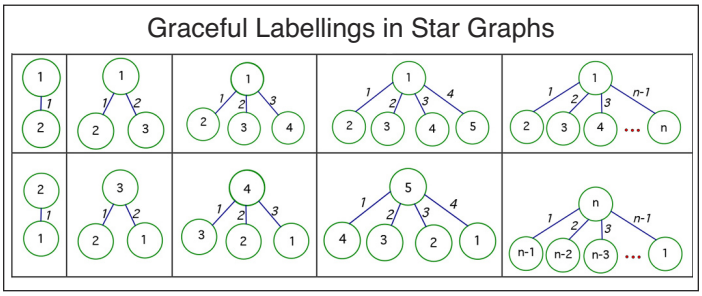


Figure 4: *Examples of the graceful labellings of the first four distinct Star Graphs and the generalized solution*

We try to gather all the possible ways to gracefully label the Star Graphs and then ask students to answer our big question: Can every Star Graph be labelled gracefully?

To aid students in their work of finding patterns, we gave them enlarged copies of the first four distinct graphs for each of the classes with which students engaged. Rather than have them repeatedly write and erase on the paper, we gave them circular chips with numbers written on them for labelling the nodes, and square chips with numbers on them to label the edges. This helped to reduce student frustration with continuously writing and erasing.

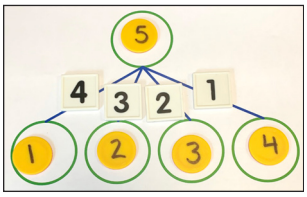


Figure 5: *Image of an enlarged Star Graph with the circle and square numbered chips*

Our experiences with elementary-grade students showed they could find the patterns and would describe them with verbiage similar to, "The biggest number goes here, the second-biggest number there,…," whereas upper middle-grade students could quickly jump to the use of a variable and put the numbers in order from 1 to *x*, using *x* to describe the largest number and *x*–1 to be the second-biggest number. This took a conversation with the younger middle grades because they wanted to use a second variable for the second-smallest number, and we had to tell them they could only use one variable. Further graphs (Path Graphs) require students to consider different patterns when there are odd or even numbers of nodes and also look at where in the tree graph the middle number will land (Caterpillar Graphs). We followed the same outline for each class of graphs as described for Star Graphs.

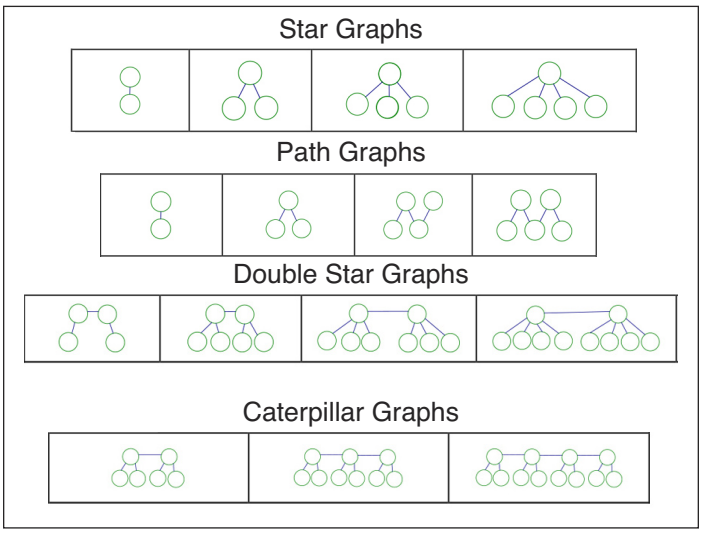


Figure 6: *The four different classes of graphs we had students engage with and the first four distinct graphs for each class*

Further Explorations

Students were quite interested in the class of graphs known as Star Graphs, and we reviewed our initial discussions about what mathematics is, and worked on other classes of graphs, following the same line of exploration, pattern seeking, and description, then justification. Again, we tasked students to not just create a graceful labelling for each graph, but to find patterns, and to create justifications and generalizations that any graph in the given category could be labelled gracefully. The different types of graphs used are presented below.

All of the primary and middle-grade students were able to make progress on each class of tree graph. They were able to find graceful labels for the first four distinct graphs in each of the first three classes of tree graphs: Stars, Path, and Double Star Graphs. Fourth-grade students struggled more to find patterns and create justifications for the different graphs than eighth-grade students, but they were able to make progress and experience the frustrations and joys of real mathematics.

Summary

Past researchers (Ball, 1993; Keith, 2006; Lin & Tsai, 2016) have documented that elementary students are able to make conjectures and develop a justification. Many researchers have recommended all elementary students engage in argumentation, generalization, and making mathematical justifications in all mathematics content areas (e.g., Ball & Bass, 2003; Carpenter et al., 2003; NCTM, 2014; Stylianides, 2007). Bieda, Ji, Drwencke, and Picard (2013) argued that students' difficulties with learning to construct formal proofs in geometry high school classes and post-secondary-level mathematics could be because of the lack of experiences that elementary and middle school students have had with justifying, reasoning, and proving.

This activity provides students with opportunities to learn that mathematics is more than arithmetic. They experience first-hand the frustration and joy that come with problem-solving, identifying, describing, and quantifying patterns, which are at the heart of mathematics. Students will also be building a foundation of mathematical concepts of conjecture, reasoning, justification, and proof.

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The Application of Information and Communication Technology (ICT) in Elementary Mathematics Instruction



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Note regarding this article: The author of this article is an Apple Teacher, Microsoft Innovative Educator, Minecraft for Education Educator, and Google Certified for Education Trainer; however, he is not employed nor paid by Apple, Microsoft, or Google, and does own shares in these companies, including Reflex. The views expressed in this article are based on his classroom experience using the technologies and does not constitute endorsement or advocacy for these specific technologies.

How can we engage our learners? How can we evidence their progression in mathematics? How can we create opportunities for children to apply their mathematical understanding? These are some of the questions that I often ask myself as I am planning math lessons. I think that one way to answer these questions is to personalize the learning of mathematics so that children's learning preferences and needs are addressed, while creating fun and engaging learning experiences through the use of information and communication technology (ICT).[1]

Case Study 1: Instant Feedback, Auto-Marking, Statistical Analysis, Using *Google Forms*

As a Grade 4 teacher (ages 9–10 years old) at a private not-for-profit independent school in Bangkok, Thailand, I taught a class of 25 English Language Learners (ELLs). Since English is a second language, our paper-based weekly English mental math assessments arguably favour logical–mathematical and verbal linguistic learners, since they are administered orally and require reading and writing[2], so I trialed digitizing the mental math tests by converting them into *Google Forms* quizzes. This fostered a more inclusive environment to better cater to the needs of visual learners by allowing the addition of pictures, YouTube videos, and website links.

Google Forms quizzes also provide immediate feedback to students because the test is automatically marked by the software, and calculates the range, mean, median, and mode. The software then transfers the results into a *Google Sheets* spreadsheet, which I can use to analyze the class results and evidence progression. Consequently, I can use this evidence to monitor my own teaching because I can import multiple test results into a single *Google Forms* spreadsheet. In addition, *Google Sheets* has an "add-on" software feature called *Flubaroo*. This add-on enables the customization of *Google Sheets* data by showing which questions were the most difficult, based on correct and incorrect responses, and see which students were below the median and average to target specific areas for reteaching.

Consequently, I was able to transform a paper assessment into a digital assessment. The former is primarily concerned with gathering data, while the latter provides immediate feedback and statistics (mean, median, mode). The availability of that data allows me to use limited instructional time to target key mathematical misconceptions.

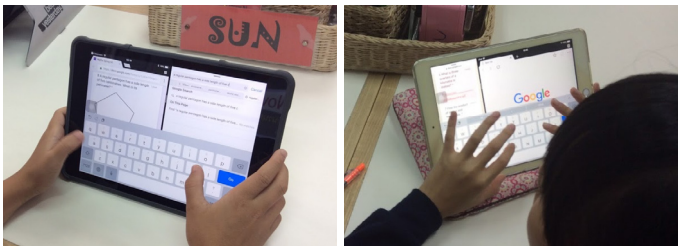
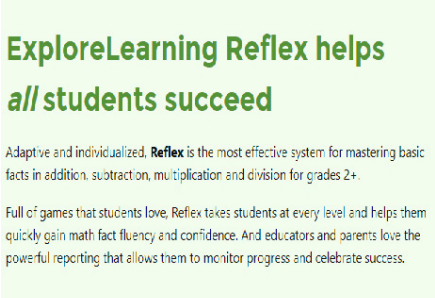
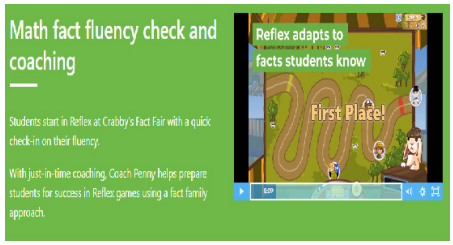
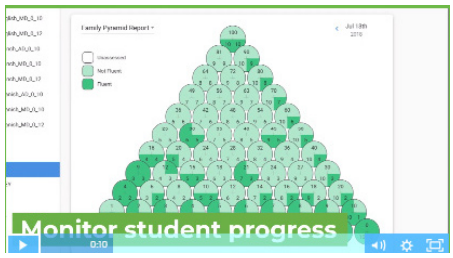


Figure 1: *Students can then use pictorial representations in their* Google Forms *assessment in order to solve problems. Since they know what areas they are weak in, they can also use split-screen view on the iPad and conduct a Google Search for terminology or concepts that they do not remember.*

Case Study 2: Adaptive Online Visual Learning (Reflex Mathematics), Collaboration, and Teacher Judgment

My colleagues, Catherine Ann Frayn from The Columbus School in Medellin, Colombia, and Erin Huddleston from Colegio Jorge Washington in Cartagena, Colombia, introduced me to the use of *Reflex Mathematics*. This is a web-based "system for mastering basic facts in addition, subtraction, multiplication, and division for Grades 2+" (www.reflexmath.com). The software makes use of Jerome Bruner's *Concrete, Pictorial, Abstract*

Concrete	Pictorial	Abstract
"Concrete is the 'doing' stage... every abstract concept is first introduced using physical, interactive concrete materials." (Mathsnoproblem)	"Pictorial is the 'seeing' stage. Here, visual representations of concrete objects are used to model problems. This stage encourages children to make a mental connection between the physical object they just handled and the abstract pictures, diagrams, or models that represent the objects from the problem."	"Abstract is the 'symbolic' stage, where children use abstract symbols to model problems."
 The computer and iPad are physical materials. Students can interact with math by moving objects or inputting data.	 Students can "see" the math by "gameifying" the mathematics as visual representations replace abstract concepts.  Students can see their progress throughout the course of their math learning, and self-assess in order to review areas where they need to develop.	 Abstract symbols are put into a real-life context, an ice cream shop, so that children can apply their knowledge of math.

(CPA) model[3] in order to provide personalized mathematics lessons to students (see table).

Reflex Mathematics provides a Monitor Student Progress Pyramid, whereby shades of green indicate how proficient students are in a particular mathematical strand. The Pyramid student monitor was quite effective as a form of self-assessment so that students would know what they needed to work on in order to improve. As Catherine Ann Frayn stated: "I really do like *Reflex Math*, as I think it is helpful in developing their basic fluency of facts," as well as the importance of fostering positive teacher–student relationship: "I have a pre-teaching class that I offer to four or five students twice a week. They receive exposure to the math concepts in a small group before I teach to the whole class." This small-group conferencing approach can also be used within the class. For instance, Erin Huddleston uses one-to-one in class conferencing to micro-target specific individual student needs, while the rest of the class works on *Reflex Math*.

Inspired by the above approaches, I implemented the following:

Step 1: Pre-teach to a select group of students by looking at the data to better predict math concepts that might be difficult for students to grasp.

Step 2: Use math software, e.g., *Minecraft for Education*, to foster student collaboration and application of mathematical concepts (Grades 4–6); *Google Forms* quizzes, to gather data and collate using *Google Sheets* in order to micro-target for teaching specific areas, both whole class and individuals (Grade 4); and Khan Academy videos that enable students to independently learn math concepts (Grade 6).

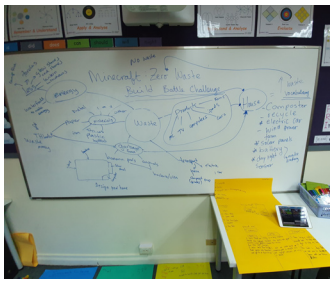
Step 3: Use quantitative data and formative assessment in order to inform individual conferencing to achieve greater personalized learning.

The result has been a more engaged classroom atmosphere, where the students have taken greater ownership of their learning. They ask me precise questions regarding specific areas so that I can provide more explicit feedback and instruction. Therefore, it is not the software that matters, but how the information from the software can be used to "help inform instructional decisions, promote student engagement, and improve student learning" (Ontario Ministry of Education, 2010, p. 6).

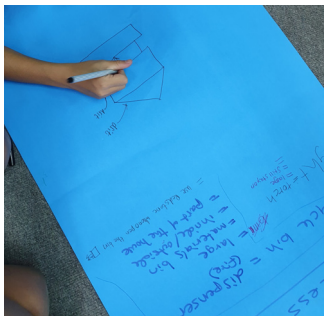
Case Study 3: Collaborative Virtual Reality (Microsoft: Minecraft for Education) and Papert's Constructionism Framework in Mathematics

While personalized formative (*Reflex Math*) and summative (*Google Forms*) assessments are an important component to mastering mathematics, I also think that creativity, collaboration, critical thinking, and communication are also important. In other words, *Reflex Math* and *Google Forms* quizzes can be boring because students either answer questions that have been created by the teacher (*Reflex Math*), or they have to answer questions that are created by the software (*Google Forms*).

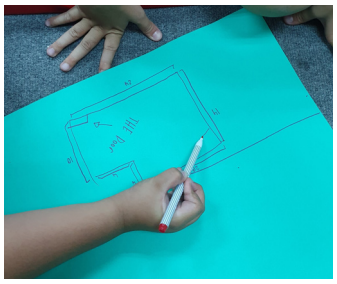
What was needed was an open-ended problem-solving platform so that students could apply their mathematical knowledge in a constructive and fun manner. I noticed that students were highly engaged with online games and Minecraft was a favourite. According to Microsoft, *Minecraft for Education* is "a game-based learning platform that promotes creativity, collaboration, and problem-solving in an immersive digital environment. Educators in more than 115 countries are using Minecraft: Education Edition across the curriculum!" (https://education.minecraft.net/) This description is in keeping with Seymour Papert's *Constructionism Theory of Learning*, i.e., "The central tenet of his Constructionist theory of learning is that people build knowledge most effectively when they are actively engaged in constructing things in the world" (http://news.mit.edu/2016/seymour-papert-pioneer-of-constructionist-learning-dies-0801). In other words, students learn best when they create things and communicate their understanding with peers because they potentially become an active *agent* in their own learning as opposed to being acted upon by an external actor, i.e., the teacher. Indeed, this was the lesson goal for my mixed Grades 4–6 (8- to 11-year-olds) after-school coding club:



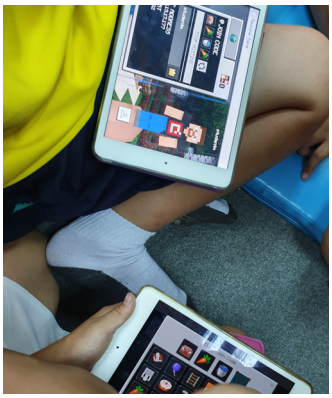
Step 1: Discuss the parameters to define our zero-waste home.



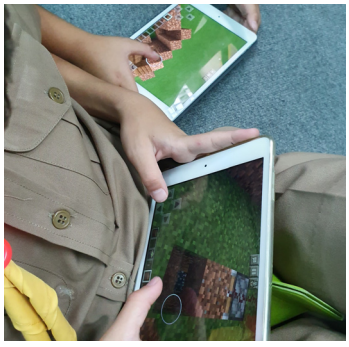
Step 2: Design home based on student-generated success criteria.



Step 3: Apply mathematical concepts of perimeter and area to decide how many blocks will be used.



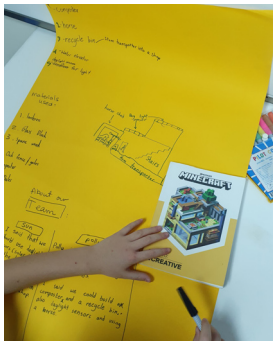
Step 4: Join peers in a virtual online world in order to collaborate on building the house.



Step 5: Decide on type of blocks to employ based on cost, aesthetic, and environmental considerations.



Step 6: Discuss how to improve and modify build by applying knowledge of fractions, e.g., replace ½ of the wall with X block.



Step 7: Self-assess by listening to what their individual contribution was to the group project learning activity.

In the Zero Waste learning challenge, students projected their inner feelings and ideas through discussion and written planning. They shared their understanding by co-operating in an online virtual simulation. In that simulation, they could as Ackerman (2001) posits: "invent for themselves the tools [(voxel blocks)] and mediations [(division of labour, mathematical concepts, reflections, success criteria)] that best support the exploration [(whether their choices would in fact produce a realistic zero-waste home)] of what they most care about." In other words, they were working collaboratively with their friends to accomplish a meaningful, self-directed, and fun learning opportunity.

Whether students apply mathematical concepts or use Minecraft for Education (or another technology), what matters most are the principles established by Papert's Constructionism, namely that we learn by doing, collaborating, and discussing with each other. If this is true, then any technology that adheres to Constructionist principles should reproduce similar results. Consequently, it is teacher judgment and the careful use of ICT, more than a specific brand-name technology, that are crucially important.

Conclusion: Personal Reflection

There are many different technologies that can be used: *Google Forms, Reflex Mathematics,* and *Microsoft Minecraft for Education* are just some of the options. What is important is that we have a clear framework, such as Papert's *Constructionist Theory*, for how these technologies can be used to enhance teaching and learning.

- Do we want to create a more inclusive learning environment (*Google Forms*)?
- Do we want to personalize students' assessment (*Reflex Math*)? or
- Do we want to foster twenty-first-century skills, creativity, communication, collaboration, and critical thinking (*Minecraft for Education*)?

Ultimately, technology is a tool, or a *how*, to achieve something, but it cannot supplant or undermine teacher judgment or relationship with students. For instance, Catherine Ann Frayn organized pre-teaching small groups; Erin Huddleston used individual conferencing; and I used a Constructionist and inquiry-based model in order to foster engagement so that students could master, not the technology *per se*, but the mathematical concepts and, more importantly, themselves! By discussing misconceptions and working collaboratively, students were able to begin to understand how mathematics is a language. It is a language because it has rules and requires someone outside of oneself in order to truly construct meaning. This is something that requires a designer and facilitator of co-constructing engaging, fun, and creative learning experiences that meet the needs of all learners. A technology that simulates online virtual environments (*Minecraft for Education*), adapts to students' responses (*Reflex Math*) or gathers data (*Google Forms*) simply will not suffice, since it cannot replace human relationships (student–student, teacher–student), which are critical for learning.

Notes

1 According to Deloitte (2016), "getting to do assignments that are fun" and "having the chance to learn new things" are the primary motivators for learning outside of school for Grades 4–8 students.

2 For a discussion on *Howard Gardner's Theory of Multiple Intelligences*, see www.niu.edu/facdev/_pdf/guide/learning/howard_gardner_theory_multiple_intelligences.pdf; www.verywellmind.com/gardners-theory-of-multiple-intelligences-2795161; and www.institute4learning.com/resources/articles/multiple-intelligences/.

3 "According to Bruner (1964, 2006), conceptual learning begins with an experience from actions undertaken (enactive) that was subsequently translated into images of the experience formed (iconic). With an accumulation of enactments and their corresponding iconic representations, links are formed to connect some of the representations into a collective structure. The criteria for selection into the collective structure are governed by a certain rule derived from organizing common attributes found embedded in those qualified representations. This rule is the piece of information that informs what one representation and those in the collective structure mean. Eventually this rule ascends above the enactive and iconic representations to stand exclusively by itself and is denoted by a symbol" (Bruner & Kenney, 1965).

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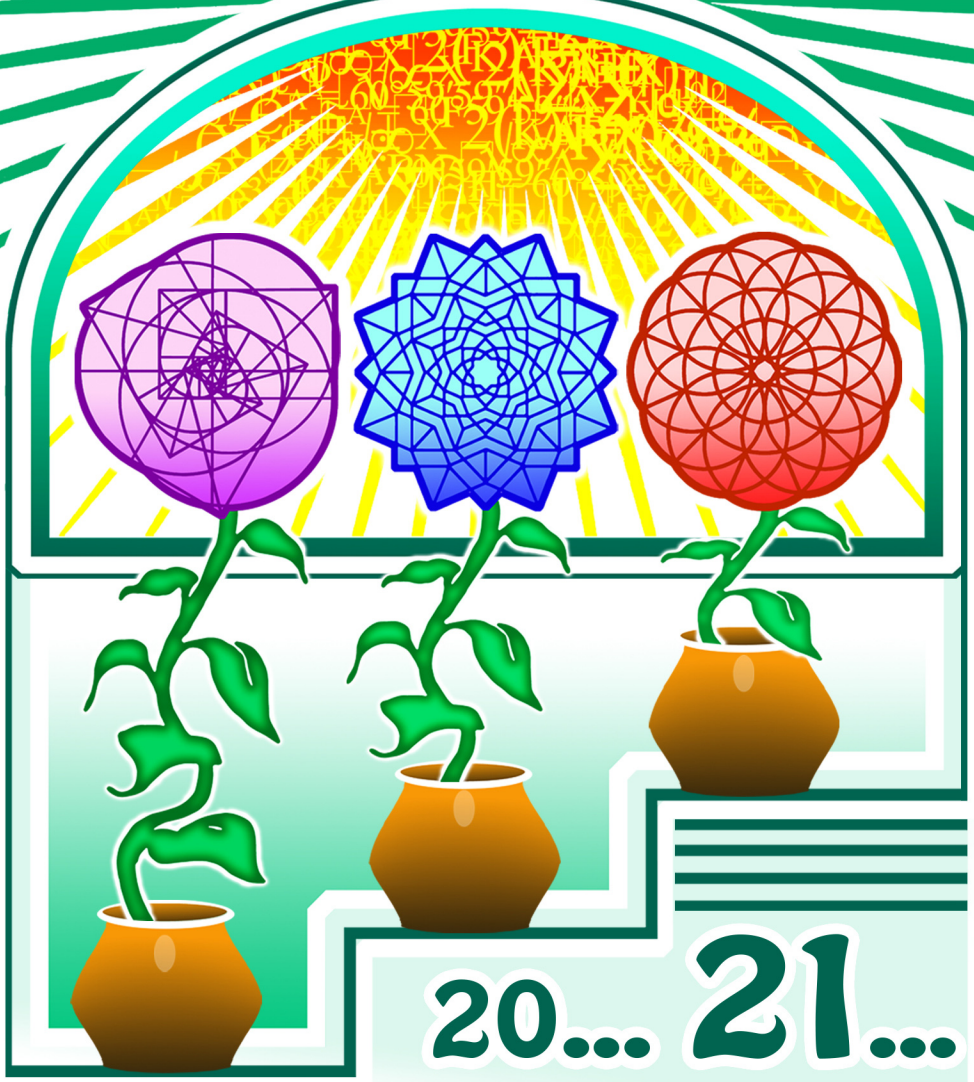
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