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Ontario Mathematics Gazette

OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

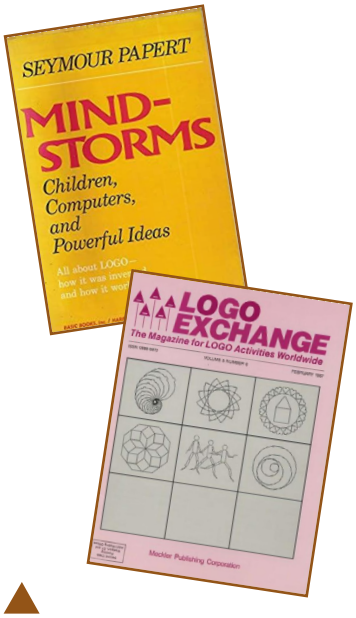
Vol. 59 #1
Sept 2020

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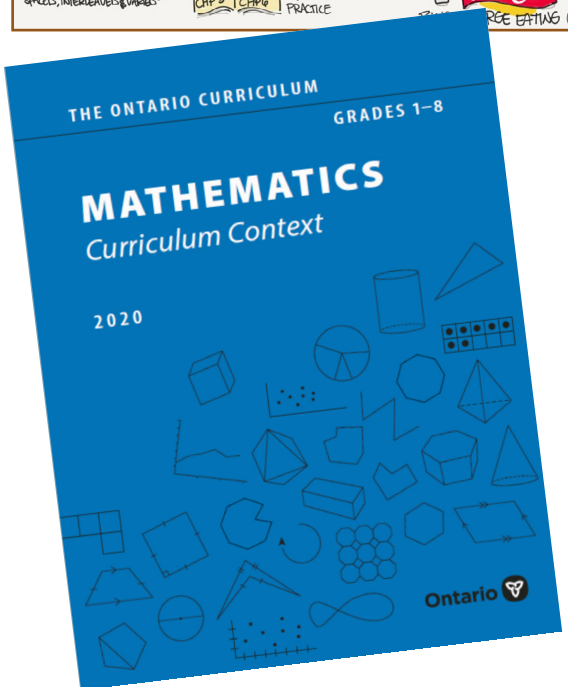
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- Seven Classroom Strategies to Challenge "Rapid Graspers" to Think Deeply

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▼ See Mathematical Snapshots: Visit to Thunder Bay



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mathology

Why are districts and schools adopting Mathology as their go-to family of math resources to encourage the love of math in students?



Teacher uses Mathology to help students meet expectations, gain math confidence, and more.

By Wendy McMahon

"Mrs. Drotos, I did it!" Those words are music to Mrs. Droto's ears during a math lesson. And she hears them from her grade one students increasingly often since her school, St. Lawrence Catholic School, adopted Pearson's Mathology Family of Math Resources.

A teacher for 22 years, Mrs. Drotos was searching for a Math Resource that provided more support for math instruction than past tools. She wanted support for differentiation, assessment, and hands-on learning—along with tools to help grow students' math vocabulary and build confidence in their math skills.

"As soon as I saw the Mathology program, I thought 'this meets so many of our requirements and supports what we're being asked to do in terms of how we teach math and how we think about math now.' It just kept ticking off all the boxes," explains Mrs. Drotos.

Pearson

Just ask Christine Drotos

Grade 1 teacher at Ontario St. Lawrence Catholic School

Hands-on learning

With a school-wide goal to bring more hands-on learning to her math lessons, Mrs. Drotos says one of her favourite aspects of the Mathology program is the activities.

Support for assessment and differentiation

Moving more students from a level two to level three (the expectation for grade one student achievement in math) was a major goal for Mrs. Drotos's district. And Mathology's assessment and differentiation tools supported her in working toward this goal in her classroom.

Growing math vocabulary

Another goal for teachers at Mrs. Drotos's school was improving students' use and understanding of math vocabulary. With the help of Mathology's Little Books and vocabulary suggestions included in lessons and activities, students were encouraged to use math language and explain their thinking.

To learn more, visit
pearsoncanada.ca/mathology

PearsonK12 @PearsonK12

Submission of Articles

The *Ontario Mathematics Gazette* (OMG) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *Gazette*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles are to be word–processed, MS Word is preferred, and prepared according to the *Publication Manual of the American Psychological Association*, Seventh Edition. However, please use single-line spacing (not double). Articles should not exceed five numbered pages of text, and figures, images, and photographs should be placed in the text close to where they belong, with captions. The photographer's permission is required, and for photos of students under the age of 18, the written permission of a parent or guardian is required.

Please submit your article in one blind file (i.e., identity of author is not evident), and include author names, contact information including email and mailing addresses, photos—head and shoulders, biographies—less than 100 words, and all content removed for blinding in a second file. Please email these two files to Tim Sibbald at gazette@oame.on.ca.

Upon review, you will be notified whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected. The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME.

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Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** by email or at the above address. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be print ready, and colour advertisements should have no bleeds.

Advertising Rates
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▲ Editor's Report



TIMOTHY SIBBALD, OCT, PhD
Gazette@oame.on.ca

Tim Sibbald holds the Universities Director position on the OAME Board of Directors. He is a former President of OAME. He is an associate professor in the Schulich School of Education, Nipissing University, with a focus on mathematics education.

I suspect for many people, the pandemic experience was akin to being in Bill Murray's shoes in the movie *Groundhog Day*—a sense that day after day, the pandemic continued causing a blurring of days that lacked distinct qualities. As with the movie, there is a range of psychological outcomes and loss of focus that can generate a sense that day in and day out is the same. Care needs to be taken to maintain invisible boundaries between work and home, even if they largely transpire in the same space. We also have to take care of our own wellness—for me, that entailed imposing no scheduled work on Sundays (though I allowed choice, if I wanted to).

Second to the pandemic is the new curriculum for Grades 1–8. While one can debate the merits of the timing of the release, given the increased necessity of changing the medium—it is in many ways a non-starter because the curriculum document is a consensus document. It represents a collective voice that emerged after a myriad of consultations. Sure, there are challenges alongside beneficial changes, but let's keep our proverbial stick on the ice and recognize that even under ideal circumstances, curriculum change takes place on a scale of years. In the words of a former assistant superintendent of curriculum for the Ontario Department of Education: "School systems are much like dinosaurs—it takes a long time for a message from the brain to get the whole organism moving" (MacLean, 1967, p. 3). In that respect, having a navigator work out the updated course (you can choose whether the pun was intended or not), while knowing it takes time to steer a ship or aircraft as large as education may make sense, even if it may add significant challenge to the changes already required.

With the curriculum change, it is advisable for all teachers of Grades 9 and 10 to apprise themselves of the changes that are coming, though they have more time to prepare. The changes will impact the way they teach and really should not simply wait until the secondary curriculum is updated. Knowing what skills students may be arriving with is important, as is recognizing that curriculum implementation takes time, making

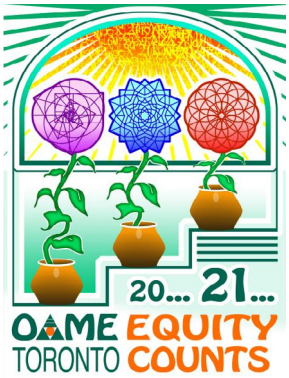
assessment for teaching increasingly important during the transition. If I can only cause you to consider one detail for your students, I would ask you to consider whether students actually need to know that they are being assessed. Energy directed at assessing and instruction is surely more useful than being expended explaining to students how things have changed.

All teachers will be challenged this Fall to continue with instruction that includes online components. Rather than hope for face-to-face teaching, it is necessary to be prepared for a second wave or another pandemic. (In my reading, I discovered I have apparently lived through several pandemics, ignorant of the level of impact they could have had.) We, as a collective group of educators, need to ensure that we maintain the importance of human instructors and avoid the pitfalls of pretending computer software can replace what we do. That, I think, is the most significant challenge we face—the feeling that our value as instructors as opposed to machines is potentially being put to the test by the pandemic. In this case, I am betting ole John Henry will show the steam engine its due—I simply do not believe that good mathematics educators can be replaced by apps, games, or software.

On that note, I will acknowledge the challenges we all face in our instruction and wish you every success with your students.

News

The 2021 OAME/AOEM Annual Conference has had a change of venue and date. It will now be held at the Beanfield Centre at Exhibition Place, Toronto, on May 20–21, 2021 (I wonder if Wayne purposefully alliterated the date?).



In This Issue

We have a change of some significance, with the *President's Message* now being printed in both official languages! Thank you to both Kyla Kadlec and Judy Mendaglio for their efforts in making this a reality! In keeping with the changes that Judy articulates in her message, you will find short invited articles by Dr. Chris Suurtamm and Anne Yeager that provide two perspectives on the new curriculum. Related to this, Dr. Lynda Colgan's new column, *Elementary Math Matters*, gives a historical perspective about coding.

September, as always, features the OAME/AOEM Award winners—I hope this inspires readers to begin thinking about nominating their colleagues later this Fall. Jacqueline Hill provides the *OAME/NCTM Report*, with a focus on the lengthy relationship that has existed between NCTM and OAME/AOEM. Last, but certainly not least, is a report on the Virtual OAME/AOEM Annual Conference.

Jérôme Proulx has joined the *Gazette* as a columnist with *Reflections on Solving Equations*. Mary Bourassa returns with a focus on building community in *Technology Corner*. In *What's the Problem?*, Shawn Godin explains periodic sequences. Carly Ziniuk takes *In the Middle* into the realm of interleaving instruction. Ron Lancaster uses *Mathematical Snapshots* to recount a visit to Thunder Bay. Dr. Jeff Irvine has a *Blast from the Past*, portraying the influence of one teacher who had been an Avro Arrow engineer. Dr. David Costello is *Linking Literacy and Math* in the context of problem solving. Dr. Jennifer Holm focuses on *Math by and for Teachers*, with a spotlight on using the number line as a model while teaching operations.

Articles in this issue include Jeff Irvine asking, "Whose history is it?" Gabriel Flores returns with strategies for addressing "rapid graspers." Nancy Louis brings a perspective about French immersion. Lastly, Greg and Oleksandr Akulov bring a problem-solving perspective to a flower garden design.

Lastly, I would be remiss if I did not take time to thank the Editorial Team. It is always a pleasure working with professionals who bring so much skill to the tasks at hand. The team includes Anne Yeager, Jacquie Foster, Robert Sherk, Gitta Berg, and Penny Clemens. However, behind the "double blind" reviewing screen, we also need to thank the people who have reviewed articles over the last year (and I will apologize in advance if I miss anyone):

Ann Arden
Nat Banting
Iain Broadie
Trevor Brown
Cathy Chaput
Ralph Connelly
Andrew Dmytriw
Tara Duncan
Sonia Ellison
Jacqueline Hill
Jennifer Holm
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Judy Mendaglio
Kameal Mina
Eric Muller
Jimmy Pai
Jamie Pyper
Anne Roberge
Reginald Robson
John Rodger
Peter Saarimaki
Barb Seaton
Robert Sherk
Christopher Stewart

I sincerely appreciate being able to request reviewers, knowing they will step up to the task! Anyone who is interested in being a reviewer can email the editor, and after answering a couple of questions, I will be happy to add you to the list. I will leave you with a note of encouragement for all readers to endeavour to write an article for the *Gazette* as a rather interesting professional learning opportunity!

Reference

MacLean, J.R. (1967). Where are we—in mathematics education? *Ontario Mathematics Gazette, 6*(0), 3–5. ▲

President's Message



JUDY MENDAGLIO
judy.mendaglio@oame.on.ca

Judy Mendaglio is the President of OAME/AOEM. She has worked in education in Ontario in many capacities: college instructor, secondary teacher and department head, university instructor, elementary and secondary consultant, and writer. Currently, Judy devotes most of her time to her many volunteer activities.

The world's spinning madly.
It drifts in the dark.
Swings through a hollow of haze.
A race around the stars.
A journey through the universe ablaze.

With changes Excerpt from *Changes*, by Phil Ochs

Welcome back, everyone.

The lyrics you see above come from a song written by Phil Ochs, an American singer-songwriter known for his activism. His songs were often referred to as "protest songs." This lyric is one of two passages I chose for my high school yearbook entry. It came to my mind recently, and I could not help but feel that his words continue to resonate with meaning. Timeless.

The process of writing the *President's Message* harkens back to a time that feels as though it is slipping away. We live in a world of increasing immediacy. News of a world event is shared via social media minutes after it happens, suddenly meetings are being arranged in the moment via text messages and take place without moving from where you are using meeting platforms, and people may feel guilty when they have not checked their emails for 20 minutes. The *President's Message*, however, is written months before publication; it is being written in early July, during an unusually long period of extreme heat warnings. Outbreaks of COVID-19 are rising again in too many jurisdictions. Anti-Black racism rallies are happening in cities all over the world. Educators from Kindergarten through to graduate school are trying to navigate the world of remote learning. And, not to forget, Ontario elementary teachers are returning to schools with a new mathematics curriculum. Changes.

Change. As of September 1, 2020, OAME/AOEM has a new President. Me. I am Judy Mendaglio, President of

Message de la présidente



JUDY MENDAGLIO
judy.mendaglio@oame.on.ca

Judy Mendaglio est la présidente de l'AOEM/OAME. Elle a exercé de nombreuses fonctions dans le secteur ontarien de l'éducation : formatrice au niveau collégial, professeure et chef de section du secondaire, chargée de cours au niveau universitaire, consultante pour le primaire et le secondaire et écrivaine. Judy se consacre actuellement à de nombreuses activités bénévoles.

« Le monde tourne à la folie.
Il dérive dans l'obscurité.
Il bascule dans un tourbillon de nuages
Une course autour des étoiles
Un voyage à travers l'univers de feu
Des métamorphoses. »

Extrait [traduction libre] de la chanson *Changes* de Phil Ochs

Je me félicite de vous voir tous de retour.

Les paroles citées ci-dessus sont tirées d'une chanson écrite par l'auteur-compositeur-interprète américain Phil Ochs, bien connu pour son engagement militant. Son œuvre appartient au genre de la chanson protestataire. La citation est extraite de l'un des deux passages que j'avais choisis pour servir d'épigraphe à l'annuaire de mon école secondaire. J'y ai pensé récemment et je n'ai pas pu m'empêcher de constater que ses mots continuaient à avoir du sens. Intemporel.

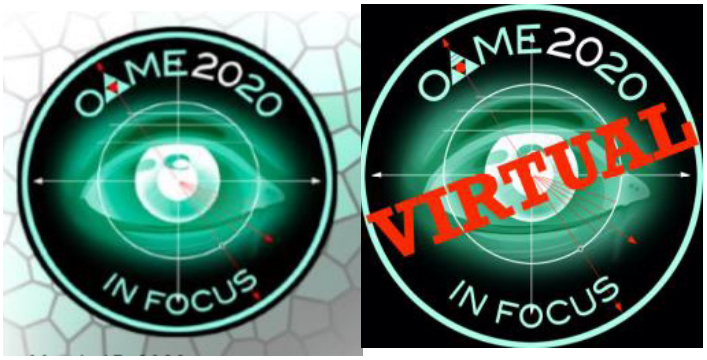
Le processus de rédaction du message du président nous ramène à une époque sur le point de disparaître. Nous vivons aujourd'hui dans un monde marqué par une instantanéité croissante. Les nouvelles du monde sont diffusées par les réseaux sociaux quelques minutes à peine après qu'elles se sont produites ; nous organisons des réunions impromptues par SMS sans avoir à nous déplacer grâce aux services de téléconférence ; et un sentiment de culpabilité nous envahit si nous ne vérifions pas nos courriels toutes les 20 minutes. À l'inverse, le message du président est rédigé des mois avant sa publication ; celui-ci a été écrit au début du mois de juillet, une période marquée par de longue d'alertes à la canicule. L'épidémie de COVID-19 a aussi connu une recrudescence dans de nombreuses circonscriptions. Des rassemblements contre le racisme envers les noirs ont eu lieu dans les villes

OAME/AOEM. If you have read Paul Alves' *President's Message* in the June 2020 *Gazette*, you will know that I am entering the first year of a two-year presidency—the result of restructuring that has been in the works for a couple of years. Changes.

I will only tell you a little bit about myself in this column. We will be together for two years, so you will have lots of time to get to know me. I was born and raised in Montreal, Quebec. I migrated to Ontario for graduate school. After completing a Masters in Math, I worked in the financial services sector. I left the business world to seek "personal fulfillment," which I believed I would find in education. I wasn't wrong. Being a teacher was exhausting, and much more difficult and demanding than working in a financial institution, but the cliché is true—it didn't feel like work, so earning far less money was okay. Being surrounded by learners and mathematics was where I wanted to be.

I believe that growing up in Montreal had a profound effect on my outlook in many ways. I also believe that having worked in a field outside of education impacted who I became as a teacher. We are complex creatures, we humans, and we bring our experiences with us into the classroom in strange and often unpredictable ways. More about these in future issues.

Last Spring, then-President (now Past President) Paul Alves' *President's Message* talked about the difficult decision that the OAME/AOEM 2020 Organizing Committee and the OAME/AOEM Executive Committee had to make when it was decided to cancel the annual conference. At that time, he announced that we were in the process of planning a virtual conference. A small group of volunteer organizers was formed. These people worked at a remarkable pace to make the virtual conference a reality. A very big change indeed. The OAME/AOEM 2020 Virtual Conference was held, and it was a huge success. We are math people so, of course, we had to gather data! We had 21 pre-recorded sessions and 30 live sessions on offer, and we had 5364 registrants. With data comes the opportunity to wonder about what the numbers can tell you.



du monde entier. Le corps enseignant, de la maternelle à l'université, tente de se frayer un chemin dans l'univers de la formation à distance. Sans oublier les enseignants et enseignantes des écoles élémentaires qui doivent retourner dans les écoles pour appliquer le nouveau programme-cadre des mathématiques. Changements.

Changement. Depuis le 1er septembre 2020, l'AOEM/OAME a un nouveau président. C'est-à-dire moi-même. Je m'appelle Judy Mendaglio et je suis la présidente de l'AOEM/OAME. Pour ceux qui ont lu le message du président Paul Alves, publié dans le numéro de juin 2020 de la *Gazette* (en anglais), vous savez déjà que j'entame la première année d'une présidence de deux ans à la suite la restructuration en cours depuis déjà quelques années. Changements.

Je vous parlerai très peu de moi dans cette rubrique. Nous allons passer deux ans ensemble et vous aurez donc beaucoup de temps pour apprendre à me connaître. Je suis née et j'ai grandi à Montréal au Québec. J'ai déménagé en Ontario pour y suivre mes études supérieures. Après avoir obtenu ma maîtrise en mathématiques, j'ai travaillé dans le secteur des services financiers. J'ai quitté le monde des affaires à la recherche d'un « épanouissement personnel » que je pensais trouver dans l'enseignement. Je ne m'étais pas trompée. Le métier d'enseignant est épuisant et beaucoup plus difficile et exigeant que celui d'employé d'une institution financière, mais le cliché est vrai : je n'avais pas l'impression de travailler et j'ai accepté de gagner beaucoup moins d'argent. Ce que je voulais avant tout, c'était la compagnie des apprenants et apprenantes et des mathématiques.

Le fait d'avoir grandi à Montréal a influencé profondément mes attentes à bien des égards. Mon travail dans un autre domaine que celui de l'éducation a aussi joué un rôle décisif dans mon métier d'enseignante. Nous, les humains, nous sommes des créatures complexes et nous apportons nos expériences dans la salle de classe de manière étrange et souvent imprévisible. J'en dirai plus à ce sujet dans les prochains numéros.

Au printemps dernier, le message du président de l'époque, M. Paul Alves, évoquait la décision difficile prise par le Comité organisateur de l'AOEM/OAME 2020 et le Comité de direction de l'AOEM/OAME d'annuler la conférence annuelle. Il avait alors annoncé l'organisation d'une conférence virtuelle. Il a fallu former un petit groupe d'organisateurs bénévoles. Ces personnes ont travaillé à une vitesse remarquable pour faire de cette conférence virtuelle une réalité. C'était vraiment un grand changement. La conférence virtuelle AOEM/OAME 2020 a bien eu lieu et fut un énorme succès. Nous sommes des mathématiciens et mathématiciennes et nous avons donc rassemblé toutes nos

The graph in Figure 1 shows the number of conference registrations by date, beginning on April 21 and ending on May 11.

Question 1: Describe how you might use this graph to estimate the average number of registrations per day.

Question 2: Indicate on the image where the rate of change is greatest. How do you know?

Question 3: What type of relation/function do you think best fits the data?

During one particular session timeslot, we had 2908 attendees. We had concerns about reliability and access.

Question: What would be the total difference in bandwidth and data usage if the attendees all muted their microphones and turned off their cameras, rather than having both "On"? (How might students conduct an experiment to estimate an answer?)

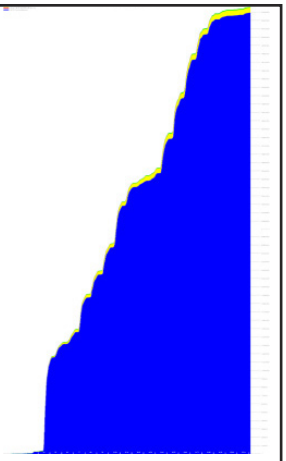


Figure 1: *A graph of the number of Virtual Conference registrations by date*

Graphique du nombre d'inscriptions à la conférence virtuelle par date

Attendees came from ten different countries (see Figure 2). At one session, David Costello was presenting from Prince Edward Island, and he welcomed several participants attending from Saskatchewan. What a wonderful opportunity the virtual platform afforded us to collaborate with educators from across the country and across the world.



Figure 2: *Graph showing the number of attendees by country*

Graphique montrant le nombre de participants par pays

Question 1: Who "travelled" the farthest to attend? What assumptions do you have to make in order to answer this question?

Question 2: On Saturday morning, there was a session at 10:00 a.m. EDT. There were participants from British Columbia, Texas, and the Netherlands. What time was it where they were?

In May 2020, many were still novice users of Zoom. Some participants needed extra support during the live sessions (e.g., accessing audio, viewing the chat, posting in the Q&A), and the meeting hosts had to learn where to find things on different devices quickly (see Figure 3) (e.g., "Are you on an iPad? You are? Okay, in the top right corner,….")

Question 1: What implications does this have for teachers who are teaching in an online environment?

Question 2: What could be the implications of this data

données. Nous avons organisé 21 séances de travail pré-enregistrées et 30 séances en direct avec 5364 personnes inscrites. Ce résultat nous donne l'occasion de nous interroger sur la pertinence de certains chiffres.

Le graphique de la Figure 1 indique le nombre d'inscriptions à la conférence du 21 avril au 11 mai.

Question 1 : Comment peut-on utiliser ce graphique pour estimer le nombre moyen d'inscriptions par jour ?

Question 2 : Quel est le taux de changement le plus élevé sur le graphique ? Comment le sait-on ?

Question 3 : Selon vous, quel est le type de relation/fonction qui correspond le mieux aux données ?

Nous avons enregistré la participation de 2908 personnes sur une seule tranche horaire. Nous avons eu quelques inquiétudes quant à la fiabilité du système et les accès à la téléconférence.

Question : Quelle aurait été la consommation totale de bande passante et de données si tous les participants et participantes avaient coupé leur microphone et leurs caméras au lieu de les laisser allumés ? (Des étudiants pourraient-ils faire une expérience pour faire une estimation ?)

Les participants et participantes venaient de dix pays différents (voir Figure 2). Lors d'une séance, David Costello a présenté ses travaux depuis l'Île-du-Prince-Édouard devant plusieurs participants et participantes de la Saskatchewan. Cette conférence virtuelle nous a offert une merveilleuse occasion de collaborer avec des enseignants et enseignantes des quatre coins du pays et du monde.

Question 1 : Quelle personne a « voyagé » le plus pour participer à cette conférence ? Quelles sont les hypothèses de travail pour répondre à cette question ?

Question 2 : Le samedi matin, nous avons organisé une séance à 10 h (heure avancée de l'Est). Il y avait des participants et participantes de la Colombie-Britannique, du Texas et des Pays-Bas. Quelle heure était-il dans ces différents pays ?

En mai 2020, de nombreux participants et participantes

for an organization such as OAME/AOEM, which offers webinars (see OAME Talks), as well as mathies virtual manipulatives?

Needless to say, apart from the Virtual Conference, the pandemic, physical and social isolation, and teaching online generated a lot of discussion within our Board of Directors and Executive Committee. How could we follow our Guiding Principles within this new environment? The Curriculum Committee took the challenge to heart and quickly established a framework for teachers to meet and work collaboratively to surface issues and solve problems related to teaching in general and teaching in an online environment specifically. Thus, two OAME Clubhouses were established, the Elementary Clubhouse and the 7–12+ Clubhouse. In each, a group of volunteer facilitators plan Clubhouse sessions to meet the needs of participants. Within each Clubhouse, the group drives the topics discussed, the timing, and frequency of the meetings, and the direction of future gatherings. Each of the two Clubhouses had their inaugural meetings in the Spring and continued to meet into the summer. Those who are engaging are enjoying the opportunity to connect with other Ontario math educators and talk about a variety of topics, including assessment and evaluation, feedback, student engagement, and sharing experiences with online resources. As participants look toward an uncertain Fall (Will we be back in physical classrooms? fully online? a hybrid?), the communities that are being formed within the Clubhouses will continue to be valuable and rewarding. The focus of the Clubhouse meetings will continue to be responsive to new and better information of what the future will bring.

On June 8, OAME/AOEM released a statement in response to events in the United States that sparked a global movement to surface and combat systemic anti-Black racism. In that statement, the three presidents (past-president, current, and president-elect) of OAME/AOEM reaffirmed OAME/AOEM's position paper on Access, Equity, and Inclusion (which can be found in the September 2019 issue of the *Gazette*, as well as on the OAME/AOEM website.) We believe that teachers play a critical role in creating a positive change in the lives of all students.

On June 23, the Ontario Ministry of Education released a long-awaited revised elementary math curriculum, the first curriculum revision for math in Ontario since 2007, when

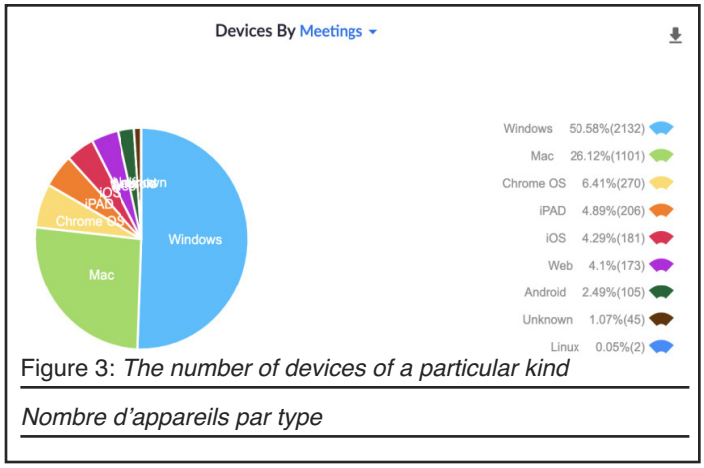


Figure 3: *The number of devices of a particular kind*

Nombre d'appareils par type

étaient encore des utilisateurs débutants de Zoom. Nous avons dû fournir une assistance supplémentaire pendant les séances en direct (par exemple des conseils pour l'utilisation de l'audio, la consultation des chats, la publication des questions-réponses). De leur côté, les organisateurs des séances de travail ont dû trouver rapidement des solutions pour les différents appareils (voir Figure 3) : « Vous êtes sur un iPad ? C'est cela ? OK, alors dans le coin supérieur droit.... »

Question 1 : Quelles sont les implications de cette expérience pour les enseignants et enseignantes qui doivent utiliser une plateforme de formation en ligne ?

Question 2 : Quelles sont les implications de ces résultats pour une organisation comme l'AOEM/OAME qui propose des webinaires (voir OAME Talks), ainsi que des outils d'apprentissage comme mathies ?

Inutile de dire que la pandémie, les mesures d'isolement social et physique et l'enseignement en ligne, sans oublier la conférence virtuelle, ont été au cœur de nombreuses discussions au sein de notre Conseil d'administration et de notre Comité de direction. Comment allions-nous mettre en pratique nos principes directeurs (en anglais) dans ce nouvel environnement ? La commission des études a relevé le défi et a rapidement mis en place un cadre permettant aux enseignants et enseignantes de se réunir et de collaborer pour faire remonter les problèmes et résoudre les questions liées à l'enseignement en général et l'enseignement en ligne. Nous avons créé deux Clubhouses de l'AOEM : l'Elementary Clubhouse et le 7–12+ Clubhouse. Dans chacun de ces Clubhouses, un groupe de bénévoles anime des séances qui visent à répondre aux besoins des participants et participantes. Chaque groupe décide des thèmes, du calendrier et de la fréquence des réunions et de l'orientation des prochaines rencontres. Chaque Clubhouse tient une séance inaugurale au printemps et continue à se réunir durant l'été. Les participants et participantes profitent ainsi de l'occasion pour rencontrer d'autres enseignants et enseignantes de mathématiques de l'Ontario et pour discuter d'une variété de sujets, notamment de l'évaluation et de la communication du rendement, de la rétroaction, l'engagement des élèves et le partage d'expériences avec des ressources en ligne. Dans le contexte incertain de la rentrée cet automne (serons-nous de retour dans les salles de classe ? Les cours feront-ils entièrement en ligne ? Une solution hybride ?), les

Grades 11 and 12 courses were revised. While the press focused on the addition of coding and financial literacy, those who have been deeply engaged with the content of the new curriculum were very happy to see within the all important "front matter," the portion of the curriculum document that describes the intent of the curriculum. The section on Human Rights, Equity, and Inclusive Education in Mathematics includes a section on Culturally Relevant and Responsive Pedagogy in Mathematics and Implementing Principles of Inclusive Education. Also in our elementary curriculum is the strand Social–Emotional Learning Skills in Mathematics and the Mathematical Processes. From the new curriculum document: "When students understand the influence of thoughts and emotions on behaviour, they can improve the quality of their interactions." We all need to work together, not just to ensure that our students no longer feel they live in the margins of society, but also to have conversations with our students to define and identify privilege and the implications of privilege. We are able to know about inequality and injustice in our world because of mathematics. Mathematics has the power to help us determine and analyze (and so to understand) patterns of oppression and privilege in our society and communities. We can, and we must, look at these patterns with our students if we are to change the future to make it better and safer for everyone.

This brings me to the most recent OAME/AOEM webinar, "21st-Century Teaching & Learning," with Dr. Jo Boaler. Dr. Boaler is a very influential mathematics education researcher at Stanford University and cofounder of *youcubed*. She is currently a member of a group of people who are spearheading an American movement to include data science offered as an alternate pathway for students in mathematics. She argues that the "Calculus pathway" is inequitable in that it has been designed to be available to only some students. Teachers in Ontario will attest that it is a difficult path for an Ontario student who begins secondary school in Grade 9 Applied Math to complete Calculus and Vectors. There may be systemic barriers that limit a student's access to Calculus when the student's course selections are made in Grade 8. Perhaps this will be another change we will see in the future. Some believe that the mathematics of the twenty-first century is not aligned with the mathematics we teach in our schools.

Changes just keep coming. Yesterday, the Minister of Education announced that Ontario will discontinue the process of academic streaming in Grade 9. In the announcement, the minister described streaming as a "systemic, racist, discriminatory" practice (Rushowy, 2020). While this will be a change to the curriculum and probably a change for most teachers, there are also some teachers who

groupes formés au sein des Clubhouses apporteront une aide précieuse et profitable aux participants et participantes. La priorité des réunions des Clubhouses sera de réagir aux événements et de relayer les meilleures informations sur ce que l'avenir nous réserve.

Le 8 juin, l'AOEM/OAME a publié une déclaration (en anglais) en réponse aux événements survenus aux États-Unis qui ont déclenché un mouvement mondial pour dénoncer et combattre le racisme systémique envers les noirs. Dans cette déclaration, les trois présidents (ancien, actuel et élu) de l'AOEM/OAME ont réaffirmé la position de l'AOEM/OAME sur l'accès à l'éducation, l'équité et l'inclusion (cette déclaration se trouve dans le numéro de septembre 2019 de la *Gazette* (en anglais), ainsi que sur le site web de l'AOEM/OAME). Nous sommes convaincus que les enseignants et enseignantes contribuent largement à créer des changements positifs dans la vie des étudiants.

Le 23 juin, le ministère ontarien de l'Éducation a publié le nouveau programme-cadre de mathématiques pour les élèves des écoles élémentaires, tant attendu, la première refonte du curriculum en Ontario depuis 2007, année de la révision des cours de 11e et de 12e année. Alors que la presse se concentrait sur l'ajout du codage et de la littératie financière, ceux qui étaient impliqués dans le contenu du nouveau programme-cadre étaient très heureux de voir figurer une description des intentions du programme d'études dans les textes préliminaires. La section sur les droits de la personne, équité et éducation inclusive des mathématiques comprend un passage sur la pédagogie culturellement significative, la pédagogie adaptée aux communautés et la mise en œuvre des principes de l'inclusion scolaire. Notre programme-cadre de l'enseignement élémentaire comporte également un domaine sur l'apprentissage socioémotionnel en mathématiques et processus mathématiques. Extrait du nouveau programme-cadre : « Les élèves voient la qualité de leurs interactions s'améliorer lorsqu'elles et ils comprennent l'effet qu'ont les pensées et les émotions sur le comportement. » Nous devons tous travailler ensemble, non seulement pour nous assurer que nos élèves n'ont plus l'impression de vivre en marge de la société, mais aussi pour avoir des conversations avec nos élèves afin de définir et de cerner les privilèges et leurs répercussions. Les mathématiques nous permettent de mieux connaître les inégalités et les injustices dans le monde. Les mathématiques peuvent nous aider à déterminer et à analyser (et donc à comprendre) les modèles de l'oppression et des privilèges dans notre société et nos communautés. Nous pouvons et nous devons examiner ces modèles avec nos élèves si nous souhaitons changer l'avenir et le rendre meilleur et plus sûr pour tous.

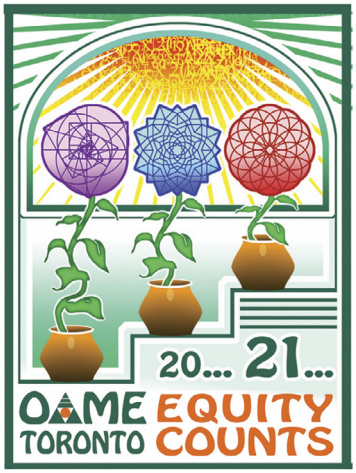
Cela m'amène à vous parler du dernier webinaire de

have felt that our Grade 9 students could have a more inclusive mathematics experience. Some of these teachers have been teaching with a "destreamed" mindset for Grade 9 math by having most students enroll in the Academic Grade 9 course. Listen to this webinar, given by co-presenters Michelle Defilippis and Jason To during the OAME 2020 Virtual Conference, to learn about the experiences at the Toronto District School Board.

Change can often be difficult. Sometimes change happens because it is unavoidable. Sometimes change happens because it is time. OAME/AOEM will continue to be here for you as you navigate these changes and whatever changes await us around the corner. Keep checking our website and our Twitter feed for news about upcoming opportunities.

Reference

Rushowy, K. (2020, July 6). Ontario to end streaming in grade 9 and change other 'racist, discriminatory' practices. *The Star*. www.thestar.com/politics/provincial/2020/07/06/ontario-to-end-streaming-in-grade-9-and-change-other-racist-discriminatory-practices.htm ▲



OAME 2021 Conference Update

Note: Due to complications arising from the COVID-19 pandemic we have had to change the date and the location for the **OAME 2021 Conference**. The new date and the new location are both listed below. Please **"Save the Date"** and consider staying in Toronto for the long weekend! Thank you!

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l'AOEM/OAME « 21st-Century Teaching & Learning » avec la docteure Jo Boaler. Dre Boaler est une chercheuse de premier plan dans le domaine de l'enseignement des mathématiques à l'Université de Stanford. Elle est la cofondatrice du site *youcubed*. Elle fait partie d'un groupe de personnes qui milite pour l'inclusion de la science des données dans les programmes d'enseignement (en anglais) des mathématiques. Elle affirme que la filière « le parcours de calcul » est inéquitable, car elle est conçue pour être accessible à certains élèves seulement. Les enseignants et enseignantes de l'Ontario attesteront qu'il est difficile pour un élève de l'Ontario qui commence ses études secondaires en 9e année en mathématiques appliquées de réussir le calcul et les vecteurs. C'est possible qu'il existe des obstacles systémiques qui limitent l'accès d'un élève au calcul lorsque les cours choisis par l'élève sont faits en 8e année d'études. Peut-être assisterons-nous à un changement de cap à l'avenir. Certains pensent que les mathématiques que nous enseignons à l'école ne correspondent plus aux mathématiques du XXIe siècle.

Les changements se poursuivent. Hier, le ministre de l'Éducation a annoncé que l'Ontario mettra fin à la diffusion en continu en 9e année. Dans son communiqué, le ministre décrit la diffusion comme une pratique systémique, raciste et discriminatoire (Rushowy, 2020). Il s'agit d'une modification du programme-cadre et probablement d'un changement pour la plupart des enseignants et enseignantes, mais plusieurs d'entre eux ont estimé que l'apprentissage des mathématiques serait moins discriminatoire pour les élèves de 9e année. Certains de ces enseignants et enseignantes ont enseigné avec un état d'esprit « sans division » pour les mathématiques de 9e année de façon que la plupart des élèves suivent la filière académique. À ce titre, écoutez ce webinaire (en anglais), animé conjointement par Michelle Defilippis et Jason To à l'occasion de la conférence virtuelle AOEM 2020, pour en savoir plus sur les expériences du Conseil scolaire de district de Toronto.

Le changement est souvent difficile. Parfois, il se produit, car il est inévitable. Parfois, il se produit, car l'heure du changement est venue. L'AOEM/OAME se tiendra à vos côtés pour vous aider à surmonter ces changements et tous ceux qui nous attendent à l'avenir. Continuez à consulter notre site web et notre fil Twitter pour plus d'informations sur les nouvelles opportunités.

Référence

Rushowy, K. (6 juillet 2020). Ontario to end streaming in grade 9 and change other 'racist, discriminatory' practices. *The Star*. www.thestar.com/politics/provincial/2020/07/06/ontario-to-end-streaming-in-grade-9-and-change-other-racist-discriminatory-practices.html ▲

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION
ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

Science, Technology, Engineering, and Mathematics (STEM) OAME/AOEM Position Statement (June 2020)

The goal of each OAME/AOEM Position Statement is to outline OAME/AOEM's beliefs on issues surrounding math education by providing transparency and guidance to those involved with mathematics education in Ontario. Any clarification regarding a position, or how to proceed in accordance with it, can be brought to the attention of the OAME/AOEM Executive Committee.

The term STEM in this position statement acts as a placeholder for a wide variety of definitions that may include The Arts, Business Studies, and other subject areas along with Science, Technology, Engineering, and Mathematics.

Focal Question

How do we as a community of educators regard STEM, while taking into consideration mathematics education in Ontario JK–Grade 12?

Summary

OAME/AOEM supports the continuing development and integration of existing goals and expectations for STEM programs. OAME/AOEM maintains that the creation of interdisciplinary activities, projects, courses, and programs must support the theoretical foundations of the study of mathematics sufficiently to ensure the integrity of mathematics concepts. Integration of these goals and expectations must simultaneously fulfill the Ontario Ministry of Education Mathematics Curriculum, Grades JK–12.

Increasingly, Ontario students of all ages are involved in STEM activities, projects, courses, and locally designed programs, both inside and beyond the classroom. The key role mathematics plays now and in the future lives of our students is undeniable. The mathematical understandings, ideas, concepts, and related skills are the foundational building blocks of the other STEM subjects.

OAME/AOEM will continue promoting and supporting the following:

- Excellence for our students by developing activities, projects, and courses for STEM programs that are consistent with the Ontario Mathematics Curriculum in terms of content, overall goals, individual expectations, and instructional time for all grades
- Ontario mathematics educators in creating, developing, and producing activities, courses, and programs that maintain the prescribed mathematical teaching and learning outcomes, while integrating knowledge from other STEM subjects
- STEM activities/programs that provide opportunities for students to continue to refine their mathematics learning and extend their deeper understanding of related concepts, so that students can readily and successfully transfer their math knowledge into new contexts
- The development of activities that meet Ontario Curriculum expectations and locally developed programs that successfully use mathematical concepts to enhance the validity, quality, and real-world authenticity of STEM activities/programs for students

OAME/AOEM is committed to the following objectives:

- Aiding the professional growth of mathematics educators within STEM instruction by supporting the creation, development, and use of program design that maintains the integrity of mathematics education
- Promoting professional learning that supports quality mathematics education in Ontario, as well as the development of STEM activities/programs, that fluidly and accurately upholds the learning of various subjects and authentically supports the Ontario Ministry of Education Mathematics Curriculum
- Promoting and effectively communicating with professional educational organizations and other stakeholder groups the vital role mathematics plays in the implementation of successful STEM opportunities

OAME/AOEM is committed to the following actions:

- Establishing a grant system for OAME/AOEM members to develop STEM activities that feature and support mathematical concepts as their foundation, and making the activities available to our members
- Visibly highlighting mathematics in STEM through various venues and professional learning opportunities, for example, by designating a portion of the OAME/AOEM annual conference to emphasize the role of math in STEM; encouraging submissions to the OAME/AOEM *Ontario Mathematics Gazette* and *The Abacus*, which feature math within STEM topics; informing the community with news articles and social media about the significance of math within STEM
- Sharing a leadership role while working in partnership with other professional educational organizations and stakeholder groups, to develop a common definition, vision, program, and organizational pathway regarding JK–Grade 12 STEM education that precisely focuses on mathematics. In addition, the OAME/AOEM Board of Directors will designate a STEM representative, who will report to the Board of Directors.

Mathematical Modelling: A New Addition to the Ontario Mathematics Curriculum for Grades 1–8



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Dr. Chris Suurtamm is a Professor of Mathematics Education at the University of Ottawa. Her research focuses on the complexity of mathematics teaching, and the use of formative assessment to improve student learning. She has been involved in many large-scale research projects and is dedicated to connecting research and practice. She has been the Canadian Representative on NCTM, Co-Chair of Assessment Topic Study Groups at International Congresses for Mathematics Education, and a member of the Ontario Ministry of Education Curriculum Council.

The new Ontario mathematics curriculum for elementary students has recently been released. A first step for those responsible for mathematics education in Grades 1–8 is to examine this curriculum to consider what might be new and different. There are some obvious differences that will be noticed on first inspection, such as the social–emotional learning skills at the start of the curriculum that emphasize students developing a sense of themselves as mathematics learners and establishing their mathematical well-being. At a surface level, you will notice the strand name changes, the combining of some strands, and the addition of the Financial Literacy strand. In digging deeper, one will notice such things as some differences in the approach to fractions, changes in developing algebraic thinking, an emphasis on helping students navigate representations of data in addition to creating their own representations of data, the explicit introduction of coding, an integration of geometry, location, movement, and measurement in developing spatial reasoning, etc. Another addition to the curriculum is mathematical modelling, which is the focus of this article.

There is one overall expectation that spans all Grades 1–8 related to mathematical modelling in the Ontario curriculum. Furthermore, there are no specific curriculum expectations associated with it. I suspect that the reason that the overall expectation is the same across all grades, is that students at all grade levels can engage in the mathematical modelling process. The tasks or issues that students will be modelling may differ across the grades or, in some cases, the same task could be used across several grades, with students developing different approaches, depending on their mathematical toolkit. In terms of having no specific expectations, the modelling process is iterative and cyclical and cannot necessarily be broken down into distinct components or skills, but must be viewed as a whole, hence the overall expectation keeps the process intact. So, let's talk about what this mathematical process looks like, and what tasks across the grades might make sense.

Given the current pandemic, many people have become familiar with the terms "mathematical modelling" or "mathematical models" as they view various graphs (e.g., bar graphs, linear, and logarithmic graphs) that are used to make predictions about the possible pandemic scenarios. The development of a mathematical model considers information or data that has been gathered over time, and also builds in assumptions or educated guesses about human behaviour and other variables to develop a model that will help to make predictions. As more data becomes available or more information is obtained, changes are made to the model so that the model can be a better predictor of future events (Hirsch & Roth McDuffie, 2016).

Mathematical modelling links mathematics and authentic real-world questions (Cirillo et al., 2016) or even imagined contexts that appear authentic to students (Wernet et al., 2016). Mathematical modelling can highlight the importance and relevance of mathematics, and can strengthen the role of mathematics within a STEM focus (Garfunkel & Montgomery, 2019). Mathematical modelling starts with a situation in the world, possibly a messy situation or context, that you would like to better understand. The process begins by trying to understand the context. This requires that students engage in making sense of "the context and the underlying mathematics which sets it apart from traditional story problems and application problems" (Wernet et al., 2016, p. 304). Students pick out the most important features of the situation and decide how to investigate them mathematically. They then work with these mathematical ideas and create a mathematical model or method of solution. The creation of the model is not the end of the process, as next comes analysis of the model to provide insights into the phenomenon and make predictions. The process also requires that the model be revisited to see how well it fits the situation, whether revisions need to be made, and whether the predictions make sense based on the outcomes (Felton-Koestler, 2016; Hirsch & Roth McDuffie, 2016).

Mathematical modelling is used by many businesses or industries. For instance, airlines use a mathematical model

to determine how many seats should be sold for a particular flight. As we all know, it is not as simple as selling the same number of airline tickets as the number of seats on the plane. Rather, more tickets are sold than are on the plane, and the number to be sold is determined by a mathematical model that takes into account such things as passengers not showing up due to delayed connections, and the need to have some flexibility to accommodate passengers (particularly the airline's elite passengers) who perhaps missed connections. Hence, the mathematical model considers human behaviour and makes predictions based on those behaviours. As we know, it sometimes gets it wrong.

Mathematical modelling problems at the elementary level might include gathering data and considering all of the variables related to setting up a lemonade stand, and then determining the best price and time for selling the lemonade, the amount of each ingredient required, and the location. This would include topics in financial literacy, spatial sense, and working with data. Or, perhaps junior or intermediate students might work on setting up a school newspaper to be sold to students for a small charge to raise money for a trip. They need to consider many things. For instance, as part of this project, students may be determining the cost of selling advertising to local businesses. They might determine the cost of each ad by analyzing the ratios of cost to the area of the advertising space, and also consider what they think advertisers are willing to spend. This modelling task would include number sense, working with ratios and proportion, and determining areas, to name a few. Another modelling task might rest on the question of which uses less water: taking a shower or a bath (Felton et al., 2015). This would certainly take into account variables in human behaviour as well as the collection of data, volume formulas, etc. Each of these tasks also has human decision making, such as projecting when people are most thirsty, or how much someone might be willing to pay for lemonade or a newspaper advertisement. As can be seen from these examples[1], although mathematical modelling rests in the Algebra strand, it actually ties in many concepts across strands and, as mathematics educators suggest, is not a topic to be taught in isolation, as it should help to better represent the complexity of mathematics and how mathematics might be used in the world (Felton-Koestler, 2016; Wernet et al., 2016).

[1] For more ideas for modelling tasks, see the February 2015 Focus Issue of *Mathematics Teaching in the Middle School* (NCTM), titled "Mathematical Modeling".

Continued on page 24

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▲ Big-Picture View of *The Ontario Curriculum, Grades 1–8: Mathematics, 2020*



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Anne Yeager taught secondary mathematics for 30 years in Ontario. She was seconded to the Ministry of Education for 2 years beginning in 2004, where she was lead developer for the 2005 Mathematics Curriculum for Grades 9 and 10, and assisted with aspects of the math curriculum for Grades K–8 and Grades 11 and 12. For the following 10 years, Anne provided professional learning supports of her own design to educators as they implemented the 2005 curricula. As a member of the OAME Board of Directors and Associate Editor of the Gazette *since 2006, she remains involved in the math community. Anne is the inaugural winner of the OAME Leadership in Mathematics Award.*

The Ontario Curriculum, Grades 1–8: Mathematics, 2020 was released on June 23, 2020. There was some understanding among math educators that its development was underway, without details. Its "arrival" was a surprise to some, including myself. I was involved in the development of the 2005 Mathematics Curriculum, and a member of the Curriculum Committee of OAME; however, I was not involved in any aspect of the development of the 2020 curriculum.

Like many educators, I was curious about what the 2020 curriculum would include. What would be different? What would be the same? Would the thoughtful efforts poured into the 2005 curriculum still exist?

An overview and comparison of the "front matter" of The *Ontario Curriculum, Grades 1–8: Mathematics, 2020* to some aspects of the 2005 math curriculum has revealed two general themes. In particular, it is evident that the language has been updated and some emphases have changed. These adaptations reflect advances in technology, current issues in education, and educational research of the past 15 years. There are of course changes, yet the intent and objectives of many sections in the front matter have remained similar. For example, the 2005 section titled "Some Considerations for Program Planning in Mathematics" is now titled "Considerations for program planning" (www.dcp.edu.gov.on.ca/en/program-planning/considerations-for-program-planning).

Table 1 shows subsection headings that further demonstrate some of the parallels and updating.

The Ontario Curriculum, Grades 1–8: Mathematics, 2020 –"Considerations for program planning"	**The Ontario Curriculum, Grades 1–8: Mathematics, 2005 –"Program Planning in Mathematics"**
Instructional Approaches	Teaching Approaches
Human Rights, Equity, and Inclusive Education in Mathematics	Anti-Discrimination Education in Mathematics
The Role of Information and Communication Technology in Mathematics	The Role of Technology in Mathematics
Planning Mathematics Programs for Students with Special Education Needs	Planning Mathematics Programs for Exceptional Students

Table 1: *Comparing "Program Planning" headings*

Further similarities are evident in the section "Assessment and Evaluation" (www.dcp.edu.gov.on.ca/en/assessment-evaluation/introduction), previously titled "Assessment and Evaluation of Student Achievement." In 2010, the Ontario Ministry of Education released *Growing Success: Assessment, Evaluation, and Reporting in Ontario Schools.* A summary of some aspects of this publication is included within the 2020 Curriculum, and thus, this section is more fulsome. Several subsection headings are compared and contrasted in Table 2.

The Ontario Curriculum, Grades 1–8: Mathematics, 2020 – "Assessment and Evaluation"	**The Ontario Curriculum, Grades 1–8: Mathematics, 2005 – "Assessment and Evaluation of Student Achievement"**
Fundamental Principles	Basic Considerations
Categories of Knowledge and Skills (Categories remain as: Knowledge and Understanding, Thinking, Communication, Application) • Criteria and Descriptors	Categories of Knowledge and Skills (Categories: Knowledge and Understanding, Thinking, Communication, Application) • Criteria • Descriptors • Qualifiers
Sample Achievement Charts for Science and Technology, and The Arts, Grades 1-8 **Includes description of categories and levels of achievement**	The Achievement Chart for MathematicsIncludes descriptions of categories and levels of achievement

Table 2: *Comparing and contrasting heading for "Assessment and Evaluation"*

Regarding assessment and evaluation, the following is the same as per 2005: "All curriculum expectations must be accounted for in instruction and assessment, but evaluation

focuses on students' achievement of the overall expectations. Each student's achievement of the overall expectations is evaluated on the basis of the student's achievement of related specific expectations" (www.dcp.edu.gov.on.ca/en/assessment-evaluation/evaluation).

Some media releases indicate that this is a new "back to basics" elementary math curriculum (see, for example, www.cbc.ca/news/canada/toronto/ontario-math-curriculum-announcement-1.5623777). The basic principles of understanding and developing mathematical concepts, procedures, and skills imbedded in the 2005 curriculum remain in 2020, with some language modification and updating. Table 3 compares and contrasts some specific language identified while reviewing the front matter. Although there is often not a clear one-to-one relationship, the similarities become apparent when taken as a whole.

The Ontario Curriculum, Grades 1–8: Mathematics, 2020	The Ontario Curriculum, Grades 1–8: Mathematics, 2005
Refers to: "foundational mathematics concepts and skills" and "fundamental concepts, skills, and processes"	**Refers to:** "mathematical concepts **and** procedures" and "requires student to learn concepts and procedures, acquire skills, and learn and apply the "math processes"
"overall program integrates and balances concept development and skill development, including social–emotional learning skills, as well as the use of mathematical processes and real-life applications" "development of conceptual understanding and procedural fluency, skill development, communication, and problem-solving skills"	"teacher is expected to weave together related expectations from different strands to create an overall program that integrates and balances concept development, skill acquisition…" "developing the ability to perform mental computations, and to estimate, is an important aspect of student learning in mathematics"
"Practice is best when it is purposeful and spaced over time. It must always follow understanding."	Refers to "mathematical concepts and procedures" and "meaningful practice"

Table 3: *Comparing and contrasting specific language in the front matter*

The seven Mathematical Process Expectations remain the same in the 2020 curriculum, with some variations in their descriptions (www.dcp.edu.gov.on.ca/en/curriculum/elementary-mathematics/context/the-mathematical-processes). This is particularly affirming to the developers of the 2005 curriculum, as there was significant debate about whether the Math Processes would be included in the 2005 document. The Math Processes are now linked with Social–Emotional Learning. The association is evident in Strand A, where, for each grade, the strand is titled: Social–Emotional Learning (SEL) Skills in Mathematics and the Mathematical Processes. Below is an example of how one of the six SEL Skills and the Math Processes are correlated:

"Students will… learn to maintain positive motivation and perseverance (SEL skill) as they apply the mathematical processes so they can recognize that testing out different approaches to problems and learning from mistakes is an important part of the learning process, and is aided by a sense of optimism and hope" (www.dcp.edu.gov.on.ca/en/curriculum/elementary-mathematics/grades/g1-math).

The names of most of the other strands are similar to those used in 2005, as shown in Table 4.

STRANDS for: The Ontario Curriculum, Grades 1–8: Mathematics, 2020	STRANDS for: The Ontario Curriculum, Grades 1–8: Mathematics, 2005
Strand A: Social–Emotional Learning and the Mathematical Processes (noted is a greater emphasis on student mental health)	Mathematical Processes: These preceded the strands of each course and were to be applied as students worked toward meeting the expectations in all the strands.
Strand B: Number	Number Sense and Numeration
Strand C: Algebra (includes coding)	Patterning and Algebra
Strand D: Data	Data Management and Probability
Strand E: Spatial Sense	Geometry and Spatial Sense Measurement
Strand F: Financial Literacy (new as a strand, and beginning in Grade 1)	From Grade 4, Financial Literacy was to be incorporated into math strands and other courses as appropriate, as per the 2016 curriculum document, *Financial Literacy: Scope and Sequence* (www.edu.gov.on.ca/eng/document/policy/FinLitGr4to8.pdf).

Table 4: *Comparing the strand names*

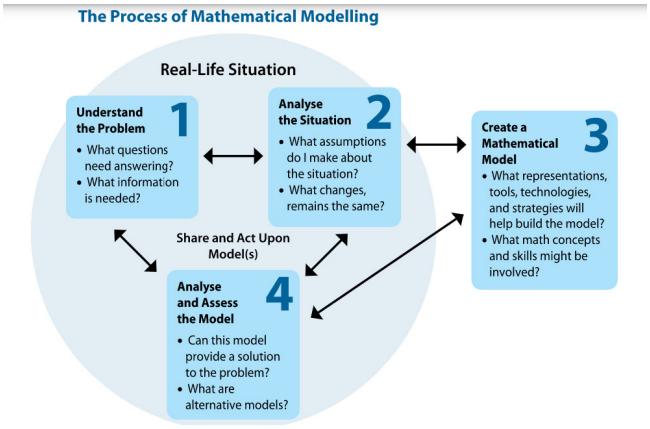
Details regarding the strands, including the overall and specific expectations, can be found at www.dcp.edu.gov.on.ca/en/curriculum/elementary-mathematics/context/the-strands-in-the-mathematics-curriculum. As per the 2005 curriculum, specific expectations are accompanied by [updated] examples and/or sample tasks for clarification purposes.

The organization of the 2020 Mathematics Curriculum is somewhat different from 2005, and has some aspects that are helpful to educators and parents/guardians. A few are described below:

- Navigating online has links to various sections of the

document, as well as links to other Ontario Ministry of Education publications (e.g., *Growing Success*).

- There are some visuals/charts to describe connections and ideas being communicated (an example follows) at www.dcp.edu.gov.on.ca/en/curriculum/elementary-mathematics/context/the-strands-in-the-mathematics-curriculum.



- The strands, overall expectations, and specific expectations are identified and connected with appropriate alphanumerics, which simplifies their navigation and identification. The strands are identified as A, B, C, D, E, F, followed by the overall expectations for that strand (e.g., overall expectations B1, B2… for Strand B). These are followed by the specific expectations for each overall expectation (e.g., specific expectations B1.1, B1.2… for overall expectation B1).
- There are some supports for parents and guardians; for example: Each strand has a link to, "Why is my child learning this?," whereby a short paragraph pops up, describing briefly what the child will be learning within this strand. This is followed by, "For informational purposes only, not part of official issued curriculum."

To obtain a deeper understanding of the 2020 curriculum and to better realize detailed changes, including a one-to-one comparison of expectations, please review the document itself and interpret the information in its full context.

In summary, this article is intended to provide an overview and description of some of the similarities and differences between the 2005 and 2020 Math Curricula. In general, the language and content are updated to respond to current issues and to incorporate new technologies and research, as well as reorganizing and renaming some sections. As per all Ontario Ministry of Education publications, study is required to fully understand its content and subtleties. It is hoped that educators will access professional learning to acquire the tools and strategies needed to implement *The Ontario Curriculum, Grades 1–8: Mathematics, 2020* for the benefit of our students. ▲

▲ Technology Corner: Building Community with Desmos



MARY BOURASSA
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Mary Bourassa teaches mathematics at West Carleton Secondary School in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She has presented workshops internationally, authored mathematics resources, is a past VP of OAME, and a Past President of COMA. An award-winning teacher, Mary continually strives to learn new and better ways of helping students learn and love mathematics.

Desmos is a free suite of math software tools, including an online graphing calculator and digital classroom activities. If you haven't visited www.desmos.com lately, you should take a look at the updated landing page and the amazing finalists and winners of the Global Math Art Contest. The imagination, creativity, talent, and remarkable mathematics behind these masterpieces is something to behold. What can be created with this powerful tool seems endless.

September 2012 was the first time that Desmos was featured in a technology column in the *Gazette*. Since then, Desmos has continued to grow and evolve, always valuing the voices of educators as they suggested ways that Desmos could help them better serve students. Desmos' guiding principles, which are written on the wall in their office for all to see, are: 1. Do not harm, 2. Trust teachers, 3. Design for real classrooms, 4. Design for delight, and 5. Works every time. As the company has grown, they have been intentional about their choices, from how they create an activity to who represents the company. It is important to know the values of a company, and the Desmos Equity Principles are posted on the Des-Blog. They are committed to helping all students learn and love learning math. They are also committed to seeking to know which communities they are not serving, or are harming, and continue to work toward recognizing their biases and reducing them.

As schools closed this past March, when teachers had to quickly provide emergency distance learning, Desmos assembled a group of senior members of their team to focus on providing community, professional learning, activity clinics, and support.

Professional Learning

Desmos started holding webinars to help provide teachers with helpful professional learning opportunities. The recordings continue to be available on their website at https://learn.desmos.com/webinars and offer teachers numerous topics including assessment, the *computation layer* (described later), and a beginner's guide to building an activity. This is in addition to the existing short video tutorials that are available at learn.desmos.com.

They also started a Desmos Educators group on Facebook, www.facebook.com/groups/DesmosEducators/, which provides a platform for trading ideas with other math educators and engaging in a community. In addition, Desmos continues to be incredibly responsive to the math community on Twitter.

Starter Screens

Checking in with our students has always been important, but became more challenging when we stopped seeing them face-to-face. Enter Starter Screens. As shown in Figure 1, this collection has screens for checking in, screens for checking understanding, as well as other popular and useful screens to edit and use with your own classes. Each of the screens within this collection can be copied and pasted into any existing or new activity.

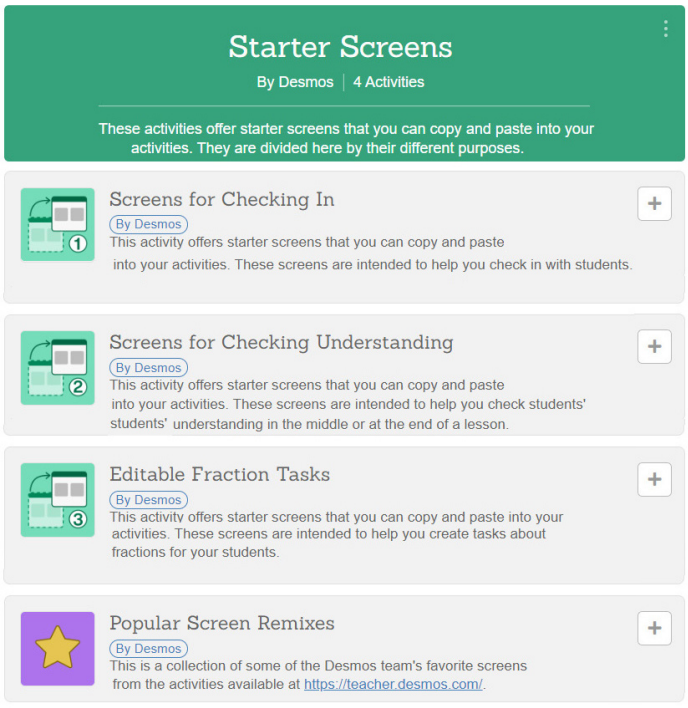


Figure 1: *Desmos starter screens*

The screens for checking in, shown in Figure 2, allow students to let you know how they are doing. These work very well at the beginning of an activity and let students know that you care about their well-being, rather than simply jumping into the mathematics.

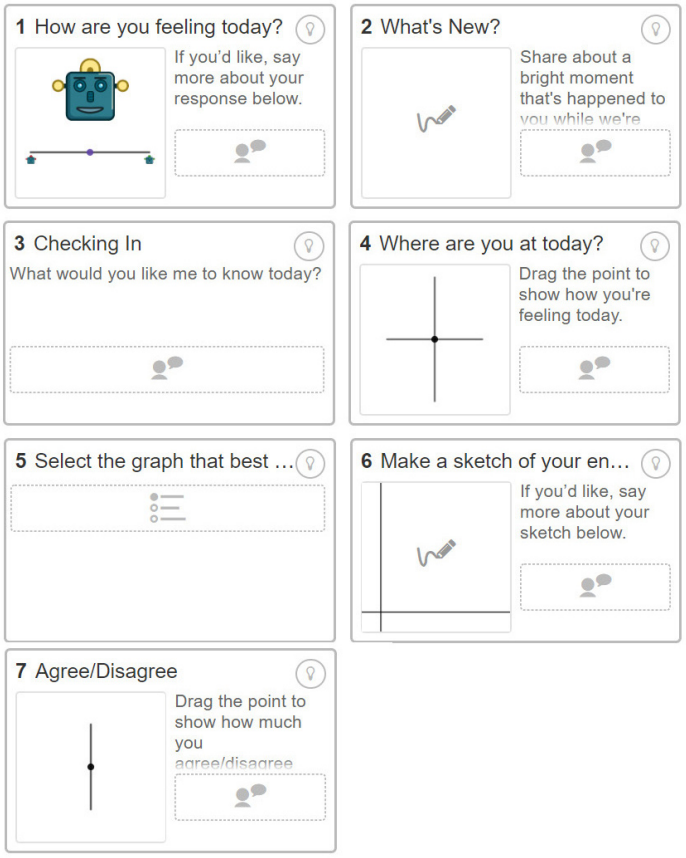


Figure 2: *Screens for checking in*

Figure 3 shows the robot from the first check-in screen expressing various emotions based on where the student places the slider. Students can also tell you more in the input box, which can help you be more responsive to them and their needs.

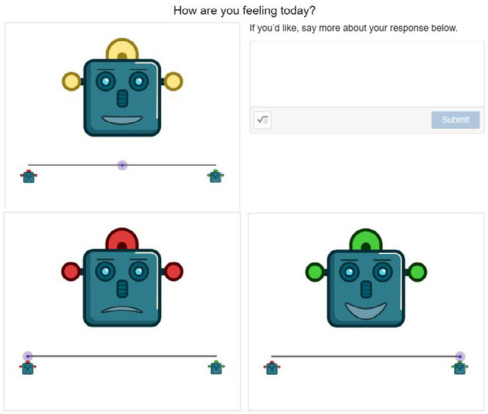


Figure 3: *Check-in screen robot*

The screens that check for understanding, shown in Figure 4, work extremely well at the end of an activity, as

they allow students to reflect upon what they have learned and give you insight into what they continue to struggle with. This is extremely helpful, as it guides your next steps in helping your students learn, and provides a starting point for conversation with a student who is confused about the material.

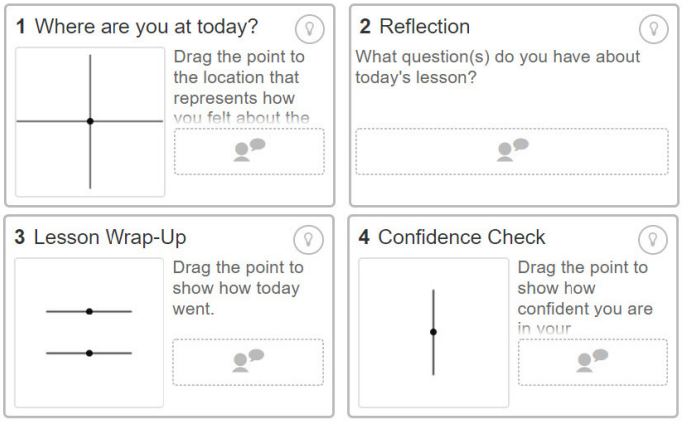


Figure 4: *Screens for checking understanding*

Figure 5 shows the Lesson Wrap-Up screen, which provides a very nice visual of student understanding, along with the opportunity for students to tell you more.

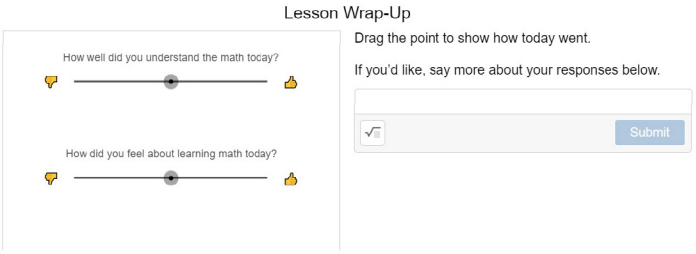


Figure 5: *Lesson wrap-up screen*

The overlay feature, shown in Figure 6, allows you to see how well your class understood the math. This feature is available in the teacher dashboard.

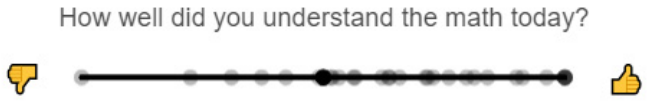


Figure 6: *Overlay feature*

It is very powerful to be able to provide students the opportunity to check in with you at the end of an activity/lesson or to reflect on their work. Giving students the opportunity to think about what they understood well, and where they may have a more tenuous grasp of the material, is important, as it helps them develop metacognitive skills.

If you need help copying and pasting Starter Screens, head to https://support.desmos.com/, or watch the YouTube video at www.youtube.com/watch?v=b2qMrFpKpoM.

Written Feedback

One of the most significant feature requests that Desmos received was the ability to provide students written feedback within an activity. This became paramount in March and Desmos delivered. This is currently still a beta feature, so you will need to activate it—sign in, go to https://teacher.desmos.com/labs, and check the Written Feedback option, as shown in Figure 7.

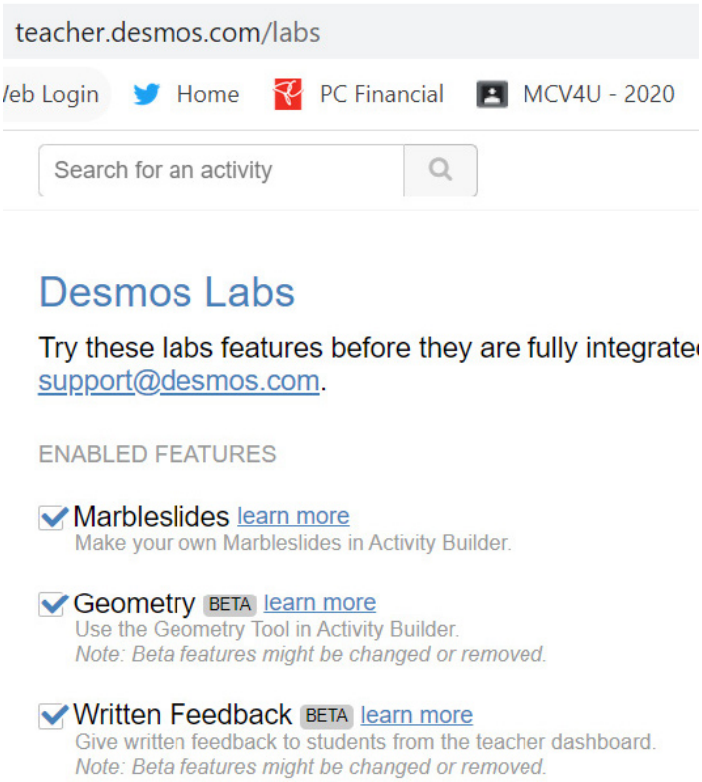


Figure 7: *Written feedback option*

When you click to see a student's screen from the dashboard, you will see a message icon at the top of the page, which will allow you to type feedback to that student. See Figures 8 and 9.

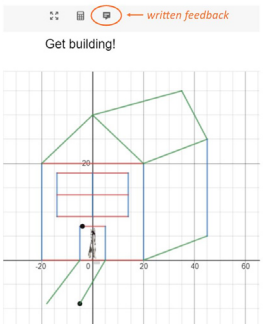


Figure 8: *Message icon that allows for student feedback*

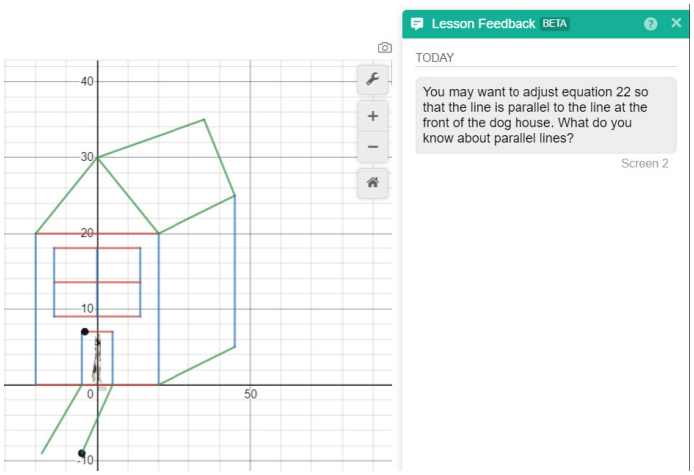


Figure 9: *Lesson feedback screen*

Students will receive a notification if they are in the activity when you send the feedback, or when they log back into the activity later. This will allow them to adjust their work and makes the activity workspace more dynamic.

Co-Teachers

Desmos allows co-teachers to collaborate. After adding a co-teacher to your account by entering their email address, you can send them the dashboard link to any activity you will be using with your class by clicking on the icon shown in Figure 10. The teacher who created the class code is the only one who can pause and pace the activity; however, both teachers can leave feedback and take snapshots of student work.

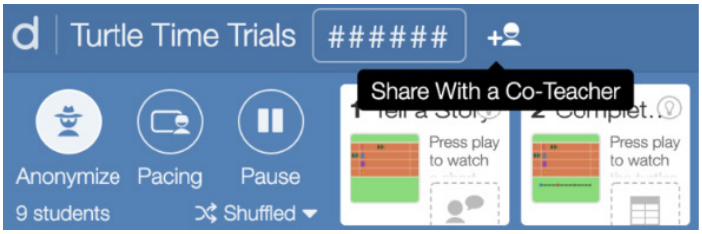


Figure 10: *Ability to collaborate with another teacher and allow for additional feedback*

Distance Learning Collections

Desmos has curated collections of activities that work well for asynchronous distance learning. They range from (US) Grade 6 to calculus. You can edit these activities to include starter screens or screens that check for understanding. If you haven't used many Desmos activities, this is a starting point. Simply type "distance learning" in the search box at teacher.desmos.com to see the various collections.

Activity Builder

In addition to the amazing activities created by the teaching team at Desmos, you can create your own activities or edit activities created by others. You can add components from the list shown in Figure 11 to create a sequence of interactive, dynamic screens to help students observe, predict, question, learn, and reflect about mathematics. Students can show you their thinking and provide explanations. They can match graphs and equations, or anything else you choose, using a *Card Sort*. They can order items in a list. They can use their knowledge of graphs to complete *Marbleslides* challenges. If working on paper makes more sense, you can have students upload a picture of their work on a blank graph screen.

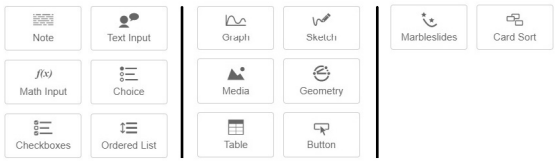


Figure 11: *Activity-builder components*

If you get stuck, don't worry. Support is only a click away. The Desmos team will be happy to answer any questions you send via Twitter, Facebook, or email.

As you begin to edit and build your own activities, you may notice that certain components and screens are connected to each other. These interactions are powered by a scripting language called Computation Layer (CL). When creating your own Desmos activities, you can add a little CL magic so that students can know whether they are correct or not at each step, and can get tailored feedback based on their responses. Learning CL is an undertaking, but it gives you the ability to customize the experience for your students. You can create assessments, using Desmos, that allow students to show their mathematical thinking through algebraic steps, sketching, constructing geometrical objects, graphing, and so much more. You can use CL to use random numbers in your assessment questions, or even randomly choose one of a selection of questions. These are all possible with Desmos. For more on how to build with the Desmos CL, visit https://cl.desmos.com/.

Community

Desmos helps you create a community of learners in your classroom, whether in person or remotely, and the entire Desmos community is there to support you. Editing existing activities to include starter screens and checking for understanding screens allows you to ensure that every student has a voice in your classroom. Your next moves can be intentional and focus on what your students need. You can anonymize your students so that sharing work comes without judgment, and helps everyone learn. I need this community for my classroom, and I welcome you to join the greater Desmos community. They will welcome you and value you the way we value each of our students. ▲

Linking Literacy and Math: Going Beyond Surface Level When Problem-Solving



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Dr. David Costello is an author, who has recently published Using What Works: Strategies for Developing a Literacy-Rich Environment in Math. *David is a vice-principal, who has held many roles. He taught in the primary division before assuming roles of numeracy interventionist, numeracy coach, numeracy leader, and curriculum consultant for Prince Edward Island. David has also instructed university courses focused on curriculum, differentiation, mathematics, and language arts. He facilitates professional learning sessions in the areas of mathematics and school development. His research situates mathematics within a literacy perspective and examines leveraging literacy-based instruction in mathematics.*

Problem solving is not a new focal point for mathematics education. It has been highlighted as the goal of mathematics instruction for decades. So, while Ontario is doing reasonably well on problem solving, as indicated in recent PISA testing, it is important that we remain attentive to problem solving, as current revisions to curriculum continue to emphasize the significance it has in both instruction and learning. Regardless of the province or school board, educators stress that their students should strengthen their ability to problem-solve when introduced to novel situations. We have seen numerous resources flood the market that stress the silver-bullet approach to teaching through problem solving. However, what I will emphasize in this column is that much of the approach taken to problem solving is only surface level. Questions are not enough. We need to dig deeper into instruction and learning to support educators and students in the area of problem solving.

What have we seen in literacy? We see students who self-monitor comprehension when engaged with a text. We see students who are able to communicate their understandings, stumbling blocks, questions, and connections with text. We see students who are able to make meaning as they work through a text. Overall, we see independence in how students engage with text, both as readers and writers (Clay, 2006).

Now, let's consider what we have seen in mathematics. Too often, it is self-doubt. Think about when you assign students a task. How many students will ask you if their response is correct? Students are relying on you, the teacher, to be the determiner of whether they are on the "right track" or not. This is not acceptable. Students need to self-monitor their work, as they do when they read. If students can self-monitor in literacy, they can self-monitor in mathematics.

To dig deeper than the surface level, students have to adopt problem solving from a metacognitive approach. Metacognition consists of students thinking about their thinking as they engage in problem solving, and being able to clarify their thoughts so that they can move forward toward a solution. As described in the *The Ontario Curriculum, Grades 1–8: Mathematics* (Ontario Ministry of Education [OME], 2020), the process of reflecting is a metacognitive process that occurs through the problem-solving experience. Students pause, look back, and self-monitor thinking to clarify understanding. What I attempt to do in this article is to define problem solving, define students as problem solvers, and highlight how to dig deeper. Teachers should be confident in the process of problem solving and how students apply strategies to make meaning.

What Is Problem Solving?

Problem solving in mathematics can be seen as analogous to critical thinking in reading. In language arts, students are often given texts with the instructional expectation that they read between the lines, question the author's motives, think about how they as readers are positioned, and decide how this text connects to their experiences inside and outside of school. Similarly in math, students should be flexible and think critically about the problems with which they are engaged. Regardless of the subject area, students should access a variety of strategies to solve problems, adjust their approach to a problem when faced with a stumbling block, recognize whether their response is reasonable, and self-monitor (reflect) as they work through a problem.

For mathematics, teachers are expected to teach through problem-solving (OME, 2020). Teachers provide a context or reason for the learning, with the necessary procedures being drawn out from this purpose. Students are given a problem, they have to decide on a strategy that is required to solve this problem, and then decide how to carry out the chosen strategy. Problem solving, particularly when connected to real contexts and situations such as are found in financial literacy, is powerful for student learning.

When teaching through problem-solving, we encourage students to focus on ideas and meaning making instead of thinking of solving problems in terms of speed; the focus is on developing understanding, not time. Instead of decoding a process, simply going through steps without fully

understanding them, it is important that students engage in mathematical thinking: knowing what to do, why they are doing it, and how to do it.

So, what can we take from this overview of problem solving? First, problem solving is a verb, not a noun. What I mean is that problem solving is a process and that students experience through engagement. It is how they frame, support, and consolidate their thinking when working through a question or task that is novel and there is no immediate recognition of the solution. It is not practice; it is productive struggle that encompasses cognitive dissonance (having to move beyond your expected practice when faced with a novel situation).

Too many times, when considering problem solving, educators focus on finding a problem-solving question. Such an approach positions problem solving as a noun, as a thing that will ensure students are problem-solving. However, we have all seen that this is not always the best way to approach instruction and learning. What may be a problem-solving experience for one student may only be practice for others. We cannot ensure that students will problem-solve just by assigning a "problem-solving question"; we must ensure that we foster a willingness to engage in a process of seeking a solution, as those questions don't exist in and of themselves. It is not so much about the question as it is about fostering a willingness to engage in a process of seeking a solution. Why does focus tend to be on finding "problem-solving questions" instead of approaching it from a thinking perspective?

Students as Problem Solvers

Students should learn to know when problem solving is completed. A good sign is if students are able to draw reasonable conclusions about concepts with which they have been working. At this stage, students can reflect on the decisions they made while problem-solving and assess the effectiveness of those choices. Students can move into consolidating their learning and identifying trends, patterns, and ideas. Connections should be made between students' ideas and the evidence that has been found during the problem-solving process. When drawing conclusions, students should be filling in the blanks of their understanding, considering not only aspects of successful attempts at finding a solution, but also those attempts that may have taken them to a dead end or a blind alley.

When some elementary school students were asked, "What counts in effective problem solving?," they generated the following list:

- Don't give up — persevere!
- Make connections to previous learning.
- Be open minded.
- Be flexible.
- Be able to defend your thinking.
- Know when to try something else.
- Understand that you can arrive at a solution in multiple ways.
- Believe in yourself and your ability to solve problems.

As we can see from the above list, how students characterized problem solving did not include aspects such as time; instead, what students shared were characteristics of the process of problem solving. It wasn't about the question or task provided; it was about how students approached the problem, how they worked through this experience, and how they consolidated their thinking. Within this list, students' strategies will lead them beyond surface level. This is what our instruction should strive for.

The following are two important considerations for problem solving: process and strategies. I will highlight each of these considerations individually and will provide an overview that goes beyond the surface level that we may tend to see.

Problem-Solving Process

While problem solving cannot be simplified by following a memorized process, there are general steps involved in the process that students and teachers can follow when working through a problem. The work of Pólya (1945) presented a four-step process that can be applied to problem solving. While the reference to Pólya's work may not be present in the revised math curriculum (OME, 2020), the four steps still play a significant role in problem solving within revised curricular expectations, and many educators are familiar with Pólya's approach. These four steps are broad enough to allow students to individualize their problem-solving process. Such an approach to problem solving provides students with a structure to rely upon during instances when they feel overwhelmed. It also provides the framework for teachers and students to use when discussing problem solving.

Table 1 outlines the four-step process developed by Pólya (1945), as well as questions students can ask when engaged in each step (Costello, 2019).

Steps	Questions
Understand the Problem	Can I restate the problem in my own words? What is the problem asking of me? What part of the problem is not necessary?
Craft a Plan	Given what I know from the problem, what strategy makes sense? Why did I select the strategy? What are other strategies that I can use if my first strategy does not get me to a solution?
Carry Out the Plan	Am I using the strategy accurately? Is this strategy still the right one to solve the problem? What trends am I seeing? Am I making progress?
Review	Does my response make sense? Did I answer the question in the problem? How can this help me with future problems?

Table 1: *Pólya's four-step process*

Strategy	Brief Description
Paraphrase the Problem	Students need to understand the problem before reaching a solution. Being able to paraphrase the problem allows students to restate the problem in their own words while maintaining the integrity of the problem.
Guess and Check	Guess and check is an important part of developing algebraic reasoning. Conjecturing and testing hypotheses is significant in the work of mathematicians. When undertaking the guess-and-check strategy, it is important that students maintain a record of their thinking. They should not erase guesses that didn't work.
Look for Patterns	Looking for patterns involves examining all the information available (pictures, numbers, tables, words, etc.) for patterns. Such careful examination can help students decide upon the pattern's next element.
Draw a Picture	Drawing a picture is a problem-solving strategy that can aid in understanding the parameters of a problem. Students can rely on visualizing as a process when working through a problem. Drawing a picture provides students with a way to document these visualizations.
Make a Table	Making a table of information presented in the problem provides students with an opportunity to organize their thinking. Students need to ask themselves what needs to be included in the table and how to organize this information. With a table, the identification of patterns and trends can be more apparent to students.
Solve a Simpler Problem	Solving a simpler problem is something that mathematicians do all the time and that students can adapt. Mathematicians identify ways to make a problem simpler (e.g., simplifying the numbers, removing restrictions) and apply what they learned from the simpler problem to give them insight into whatever they are trying to figure out.
Work Backwards	Sometimes it is not enough to begin at the beginning and work through a problem. Instead, students have to work backwards. The key to this particular strategy is to ask this question: "What must have happened immediately before that?" By working backwards, students can segment the work to highlight aspects that would have caused that to happen and then move on from there.
Act Out the Problem	It may not be enough to work through a problem with pencil and paper or manipulatives. Instead, students can rely on acting out a problem. This is a good way for students to embed themselves into the problem and to represent their thinking in a non-traditional way.
Get Unstuck	Students will hit stumbling blocks. What is important is that students monitor their work and have strategies available to get unstuck. Students need to have strategies that they know they can rely upon to get back on track and continue to make progress in solving the problem.

Table 2: *Problem-solving strategies*

The arrows to the left of the diagram above are not intended to be confusing. However, the complex pattern of arrows moving back and forth through the process illustrates the non-linear nature of problem solving. As students encounter stumbling blocks, discover new connections, or have "aha moments," they may move forward in the process or revert back to an earlier phase of the process to refine or redefine their thinking.

Problem-Solving Strategies

Problem solving is not about following a specific strategy all the time. Instead, students can generate strategies and/or rely on strategies that they have experienced both inside and outside school. Having a toolbox of strategies provides students with options as they encounter problems. Not all students need to master every strategy; instead, students will gravitate to strategies that they understand, have used, and see the benefit of using. These strategies can be applied flexibly by students as they encounter problems.

Table 2 (Costello, 2019) presents a list of problem-solving strategies and a brief description of each strategy. In a classroom, keeping a running list of strategies ensures that students remember the strategies that they used and that they identified as being beneficial for future problem-solving opportunities.

The strategies listed in Table 2 are important considerations for both the teacher and student. It is through these strategies that students frame, support, and consolidate their thinking when problem-solving. However, I want to emphasize that these strategies are not a list that students are expected to master. Instead, the strategies are to be considered as tools in a toolbox that students can explore and experiment with when working toward a solution. The teacher can highlight these strategies from student work, and use this learning as a way to broaden students' problem solving experiences.

Final Thoughts

Problem solving is a process and an experience. It is not a question. While a question may provide problem-solving opportunities for some students, it may only be practice for others. Although this was stated earlier in this column, it is something of which we must remain cognizant.

We need to go beyond thinking of problem solving as a question, as a linear process, as more than being, "What strategy did you use? Why did you use that strategy? Are you correct?"; instead, we need, "How did you select a strategy? Why did you suspect the strategy would work? How did you know that the strategy didn't work? What was plan B? How did trying a strategy and not having it work help you move into Plan B? What were your 'aha moments'? How did you self-monitor comprehension?"

In the next issue, my column will continue to build upon the idea of going beyond surface level and digging further into problem solving. It's important to take time to consider how we approach problem solving in the classroom, from the perspective of both instruction and learning. I look forward to continuing the discussion of linking literacy and math via Twitter (@dr_costello) or email.

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▲ Mathematical Snapshots: Visit to Thunder Bay



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Ron Lancaster is an Associate Professor at the University of Toronto, where he teaches mathematics courses for pre-service middle and high school teachers. He has over 20 years of experience teaching Grades 7–12 mathematics. Ron's professional activities include consultations and conference presentations in North America, Asia, England, the Middle East, Africa, India, and Europe. Ron is an author for the NCTM (The Mathematical Lens) and member of the Advisory Board for the Museum of Mathematics in New York. He is the recipient of the 2015 Margaret Sinclair Memorial Award Recognizing Innovation and Excellence in Mathematics Education, awarded by the Fields Institute.

I was in Thunder Bay in November 2010 to present workshops for teachers at a PD event organized by the Thunder Bay Catholic District School Board and Apple Canada. I had some free time to explore the city, see the Sleeping Giant, take math-related photos, and enjoy a dinner at the 5 Forks restaurant.

Before looking at the questions that follow, discuss what you notice in the photographs, and suggest reasons as to why I stopped and took these photos. When you look at the photos, what are you wondering about? What questions do you have?



Photograph 1: *Centennial Shelter in Connaught Square, Thunder Bay, Ontario*



Photograph 2: *The Picture Store and Framing Centre & GamesPark Cafe, Thunder Bay, Ontario*



Photograph 3: *Close-up view of sign for the GamesPark Cafe, Thunder Bay, Ontario*



Photograph 4: Scrabble® *game that I played by myself, using the nine words on the sign for the GamesPark Cafe, Thunder Bay, Ontario*



Photograph 5: *Entrance to Gateway Casino, Thunder Bay, Ontario*



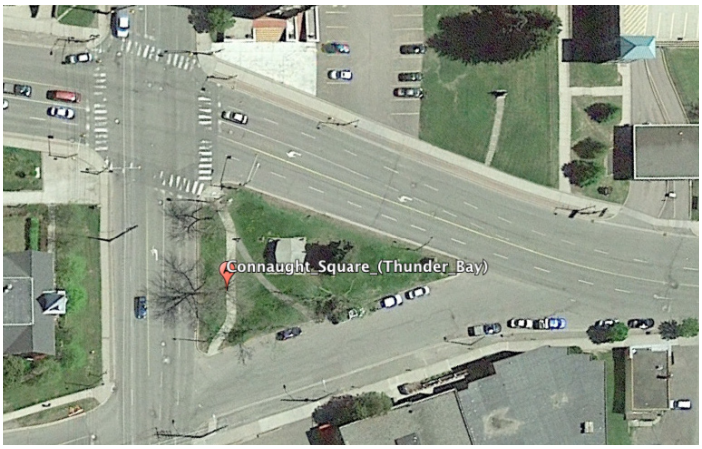
Photograph 6: *No Smoking sign, Thunder Bay, Ontario*

Question 1

(a) Photograph 1 shows the Centennial Shelter located in Connaught Square, a public park in the north end of Thunder Bay. How many sections are there in the roof of the structure? What type of shapes are the sections? Are the sections all the same?

(b) The shape of Connaught Square is actually triangular (Screen capture 1)! It is bounded by Algoma and Waverly Streets and Red River Road.

Figure 1 shows isosceles triangle ABC constructed on top of a cropped version of Screen capture 1. Figure 1 was created using the Geometer's Sketchpad®. As shown in the sketch, angle ABC is equal to 38˚. According to Wikipedia, the area of the park is 1000 square metres. Find the dimensions of the sides of the triangular park. How could you check the reasonableness of your answers if you were able to visit the park? How could you check without being there?



Screen capture 1: *Connaught Square, Thunder Bay, Ontario*
www.google.com/maps/@48.4365763,-89.2239218,124m/data=!3m1!1e3



Figure 1: *Screen capture 1, cropped and pasted into a Geometer's Sketchpad file*

Question 2

Photograph 2 shows how the letters of the store GamesPark Cafe, have been spelled out using tiles from the game Scrabble® (very clever, given that Scrabble is one of the games played there).

(a) There are 100 tiles in the standard game of Scrabble. The value of each letter and the number of them is given in Table 1. What do you notice about the distribution of the letters and their value? What are you wondering about? What questions do you have?

Letter	Value	Number	Letter	Value	Number
E	1	12	M	3	2
A	1	9	P	3	2
I	1	9	F	4	2
O	1	8	H	4	2
N	1	6	V	4	2
R	1	6	W	4	2
T	1	6	Y	4	2
L	1	4	K	5	1
S	1	4	J	8	1
U	1	4	X	8	1
D	2	4	Q	10	1
G	2	3	Z	10	1
B	3	2	blank	0	2
C	3	2			

Table 1: *Value and Number of Scrabble Letters*
Source: https://en.wikipedia.org/wiki/Scrabble_letter_distributions

(b) Photograph 3 shows the nine words that appear on the sign for the GamesPark Cafe (I consider GamesPark to be two words: games and park). Photograph 4 shows a Scrabble game that I played by myself. I limited myself

to the nine words on the sign, and I did not allow any other words to be formed in the process of making these words. I also let myself see all the tiles and never played more than seven at a time. Aside from this, I followed the standard rules of Scrabble.

It is your turn. Pick any seven letters from those that remain, and make a word that gives you the highest score. Now play for me and make sure I get the highest score. Keep going until the end of the game. If we don't count the first nine words, what are our scores? Who wins?

(c) Find a different way that I could have played the game by myself.

Question 3

Framing stores, such as the one shown in Photograph 2, often provide a PLAK-IT® service that provides customers with an alternative to framing posters, photographs, and artwork. Table 1 shows the PLAK-IT price list for the most popular sizes at Ferrante Picture Framing in St. Catharines, Ontario. The Picture Store and Framing Centre in Thunder Bay is closed due to COVID-19, and I was not able to obtain a price list for them online.

		Perimeter	Area
5x7	$11.95		
8x10	$15.95		
11x14	$24.95		
16x20	$39.95		
18x24	$49.95		
24x36	$69.95		

Table 2: *PLAK-IT Price List (most popular sizes)*
Source: *Ferrante Picture Framing, St. Catharines*
www.ferranteframing.com/index.php?p=Plak-It

(a) Calculate the perimeter and area for each of the sizes in Table 2.

(b) Graph the perimeter versus the price for the sizes in Table 1. Determine a mathematical model for this data.

(c) Graph the area versus the price for the sizes in Table 2. Determine a mathematical model for this data.

(d) Which model is a good choice for calculating the price for sizes not listed in Table 1? Why did you choose this model? Would a combination of the two models be better?

Question 4

Figure 2 shows a geometrical diagram of one of the doors that leads into the casino in Thunder Bay. The diagram is drawn to scale and very accurate.

(a) Which of the following is closest to the percentage of space taken up by the six circles on the door?

(i) 10%

(ii) 20%

(iii) 30%

(iv) 40%

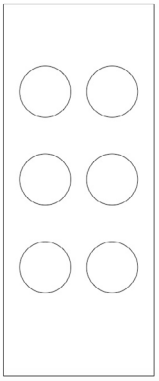


Figure 2

b) I created Figure 2, using the Geometer's Sketchpad. Using the measuring tools, I found that the base of the rectangle in the diagram is 9.3 cm, the height is 23.07 cm, and the radius is 1.58 cm. Calculate the combined area of the six circles and the area of the rectangle. Compare these areas. Is the choice you made in part (a) consistent with the calculation?

(c) Figure 3 shows a variation of the same door, with 14 circles the same size as the ones in Figure 2. Estimate and then calculate the percentage of space that these circles take up on the door.

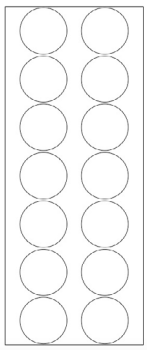


Figure 3

Question 5

(a) Look up the no-smoking bylaws for other cities in Ontario. How do the distances from a front entrance and the fines from other cities compare with the ones in Thunder Bay (Photograph 6)? Are they the same for all cities in Ontario?

(b) Is there a uniform non-smoking law in other provinces, or does it vary from city to city? ▲

Continued from page12

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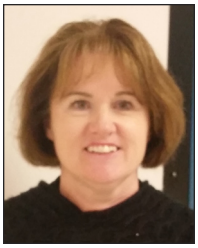
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▲ OAME/NCTM Report: How are NCTM and OAME/AOEM Connected? A Brief History…



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Jacqueline Hill is a Grades 9–12 teacher of mathematics. She is a Past President of OAME and OMCA, as well as the recipient of the award for Exceptional Teaching in Secondary Mathematics. She also wrote the "Director's Dialogue" for the Gazette *for a number of years.*

NCTM and OAME/AOEM have a 70-year relationship. This would be our "platinum" anniversary, and I am certain as a member of OAME/AOEM you can use the occasion to treat yourself.

NCTM was first formed in 1920 and is currently celebrating its centennial—100 years of advocacy for mathematics education. The NCTM is the world's largest mathematics organization! It has affiliates all across Canada. It publishes five math journals each year. Its strategic framework consists of:

- **Teaching and Learning:** NCTM provides guidance and resources for the implementation of research-informed and high-quality teaching that supports the learning of each and every student in equitable environments.
- **Access, Equity, and Empowerment:** NCTM advances a culture of equity, where each and every person has access to high-quality teaching, empowered by the opportunities mathematics affords.
- **Building Member Value:** NCTM provides community and resources to engage and listen to members in order to improve the teaching and learning of mathematics.
- **Advocacy:** NCTM engages in advocacy to focus, raise awareness, and influence decision makers and the public on issues concerning high-quality mathematics teaching and learning.

Did you know that OAME/AOEM history has been uninterrupted since 1891? It wasn't until 1918 that the first Teachers' Federation was formed (OSSTF), so the predecessors to OAME/AOEM were the earliest associations with the voice of education in Ontario.

It was in 1950 that Bob Rourke convinced the Association to become an affiliate of NCTM. There was a presentation on "Field Work in Math" and an informal dinner to mark the occasion. In 1951, there was a name change to "Ontario Association of Teachers of Mathematics and Physics (OATMP)." And it wasn't until 1973 that OAME arose from the merging of OATMP and the Ontario Math Commission (OMC). David Alexander was the first OAME President. NCTM continues to give OAME members benefits of our long-standing affiliate relationship as this column routinely reports. (Note that if you join NCTM, indicate you are a member of OAME!)

If you are still looking for exciting and innovative professional development, you can tap into the recoded webinars from the 100 days of professional development at the NCTM website, www.nctm.org. You can register for the professional development opportunities that suit you best. Webinars are recorded and made available at nctm.org/100.

NCTM is dedicated and devoted to helping mathematics educators delve deeper into their craft. Whether you are a classroom teacher, retired teacher, university professor, or person in a position of responsibility at a school board or the Ministry of Education, NCTM is ready to help you make the most of your mathematical adventures.

PS… a quick shoutout to Tim Sibbald, our *Gazette* editor, for alerting me to this 70-year relationship!

Yours in mathematical fun,
Jacqueline

Resources

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Spiral Honeycomb?

From www.smithsonianmag.com/smart-news/stingless-bees-build-spiral-honeycombs-grow-crystals-180975405/

▲ In the Middle: Interleaving Practice—The Constant of Change



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The only constant in life is change. This often-recited quotation from Greek philosopher Heraclitus has once again found new relevance. Our everyday experiences have changed fundamentally, yet we remain anchored in one place. In the final months of last year, school was no longer the place where we saw our students and colleagues, or where we actually did our teaching. Daily, I reflected on what I knew was constant amidst the changes: my understanding of my students; how they learned; where we still needed to practise; and that building on their mathematical competencies mattered, even with social distances and through a screen.

The strategy of interleaving practice itself encapsulates Heraclitus' constant of change. I first learned about the strategy from *Make It Stick: The Science of Successful Learning* (Brown et al., 2014). The book summarizes research showing that when consecutive tasks are arranged without direct connections from one to the other, students remember what they have learned longer and more accurately than if they specialize in one skill at a time. Interleaving practice in math is often presented as an arrangement of practice problems that require varied strategies and techniques, as well as having more than one concept presented at different times and in different ways throughout a course.

Before adopting interleaving practice, my students learned one exponent law at a time, becoming experts in a single law each lesson. My intent was to have the students truly understand how each rule worked, and it was supported by practice—applying the rule over and over with increasing difficulty. This is called *blocked*, or *massed*, practice, and is characterized by moving from one skill to the next, class after class. On each isolated topic, students practise each skill to proficiency. At the end of the unit, however, when we put the laws together, my students often could not remember the rules correctly, or apply multiple rules correctly, without reviewing all of the rules repeatedly. Once the test was over, the rules were often forgotten until the exam, at which point, as Figure 1 emphasizes, the students needed to cram: massed practice led to rapid forgetting. A year later, they had forgotten how to use exponents and were haphazardly applying them in common factoring. Rapid forgetting seemed to be the one constant component of our exponent law learning.



Figure 1: *Interleaving practice chapter 3 visual from www.retrievalpractice.org/make-it-stick*

Interleaving practice is often incorporated into warm-ups or application/extension questions, but *Make It Stick* presents it as an overall instructional strategy. For the exponents unit, I intentionally incorporated interleaving practice into a revised unit design. I began by teaching all of the exponent laws together. Then I adapted the creation of an exponents foldable (Figure 2) that serves as a memory aid for the different laws. Previously, the foldable had been used as a consolidation opportunity, but with interleaving, creating the foldable was a note-taking exercise, with the students writing the laws in their own words.

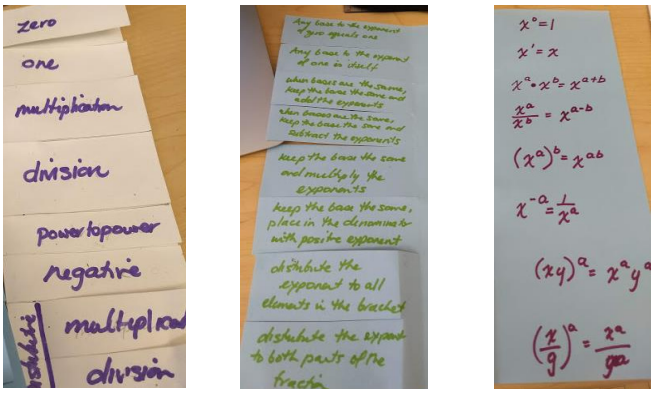


Figure 2: *Exponent foldable outside and inside, left and right*

Mixed problem sets were used, rather than concentrating on one rule at a time. Finally, students chose problems and

the level of difficulty they want to do. The following instructions appeared on each page of interleaved practice, which was divided into about a third of each emoji type (Figure 3).

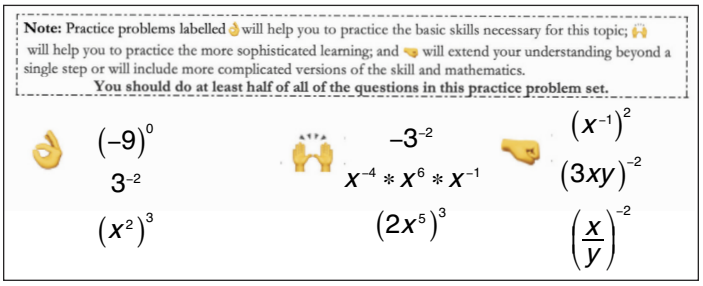


Figure 3: *An example of explaining the emoji labelling of questions for the same topics*

Using the strategies from *Make It Stick*, strategies presented by Rohrer, Dedrick, and Agarwal (2017), and classroom experimentations, the following strategies seemed to work best for my students:

- Students getting to make a choice about which strategy to use, rather than the teacher chunking or ordering the questions (e.g., mixing exponent law questions into one problem set)
- Having some problems look similar, but that require different approaches (e.g., some of the exponent problems required using negative exponent laws, while others looked like they, did but just required using the power to power law first)
- Making the skills conceptually or stylistically related at first (e.g., our first interleaving practice set focused on exponent mechanics, while the second set asked students to find errors from multiple algebraic concepts mixed together)
- Mixing old skills with new skills (e.g., we incorporated some exponent problems involving bases, which were numbers, that students could do with their calculators, with some that had variable bases)

I was direct with my students about why we were "mixing it up," and explained the concept of interleaving, using the video www.youtube.com/watch?v=AX_lCOjLCTo from MythBusters, where two interleaved phone books can't be ripped apart by two forklifts (also amusing to discuss why every home used to have a phone book). *Scientific American* presents an interleaving experiment you can try with sticky notes: www.scientificamerican.com/article/phone-book-friction/. The harder you pull, the more difficult it is to take the pads apart. This interleaving strength is a powerful connection to developing learning sturdiness.

Initially students commented that they felt like they were learning slowly, a more gradual accomplishment, rather than confident in their expertise. Our practice problem sets began to expand the breadth of topics covered and we were interleaving more topics. When the content became more significant, students realized the number of topics they were reviewing in daily practice. Students were getting used to it, and they were finding it helpful. One student said, "We never get to forget anything in this class. You have to be on your toes. You never know when something is going to come back at you."

The real strength of interleaving practice revealed itself to us when we went exclusively online. We found that although occasional blocked practice did make the students calmer, initially, especially when we didn't have as much face-to-face contact, when they realized how much less reviewing of previous learning had to occur on their own, because they had reviewed regularly, their nervousness decreased. It was evident that it was going to be easier for students to solve practice questions on their own or in small study groups than it was to use direct instruction.

By the end of the year, most of the students were earning better scores on the mixed problems than they had earned on the simpler blocked practice, and better than previous years, despite the turbulent times and less contact. Using Google Quizzes gave us the opportunity to measure and compare that quickly and confirm that interleaving practice was working to help students remember. We also noted that students were more likely to come to our meets with a specific question, almost as though they were expecting to be challenged by some problem and were more willing to admit they couldn't remember or got it wrong, when it was content we had not recently covered.

We knew our students needed the constancy of school, despite the changing environment and the unknowns we were all living with at the end of the school year. We needed to figure out how to reach them, something that seemed a constant refrain, despite all of these changes. I have found the interleaving practice, throughout the learning, and especially at the end of this school year, provided both the calmness of constancy and the novelty of change.

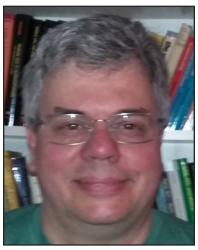
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▲ What's the Problem? Sneaky Sequences



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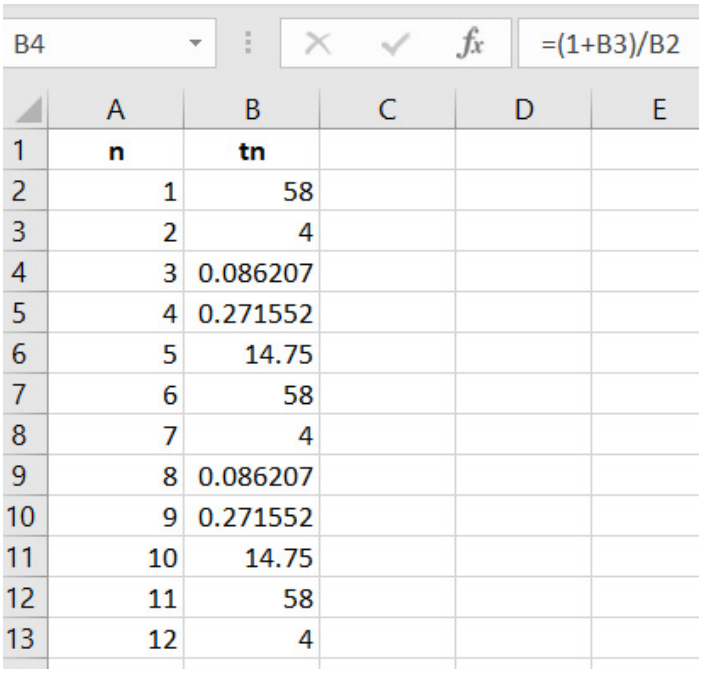
Shawn teaches and is a department head at Cairine Wilson Secondary School in Orleans. He strongly believes in the central role of problem solving in the mathematics classroom. He continues to be involved in mathematical activities: presenting workshops, writing articles, working on local projects, and helping create mathematics contests.

Welcome back, problem solvers. I hope you had a restful summer after the unprecedented semester we had in the Spring. As I am writing this in late June, there are still some unknowns for September. With adversity comes opportunity, and I encourage you to use this year as an excuse to work some new methods into your practice.

Last time, I left you with the following problem:
A sequence is defined by $t_1 = 58$, $t_2 = 4$ and $t_n = \frac{1+t_{n-1}}{t_{n-2}}$ for $n > 2$. Determine t_{2020}.

This problem was inspired by problem MA2 from MathemAttic. MathemAttic is the high school section of *Crux Mathematicorum*, the problem-solving journal published by the Canadian Mathematical Society (CMS). A few years ago, *Crux* became a fully online, free publication. All issues (currently it is in its 46th year) can be accessed at its website, www.cms.math.ca/publications/crux/. The problem in question appeared on page 4 of volume 45, issue 1, and the solution appeared on page 308 in issue 6. I strongly suggest you check out *Crux*. Although much of the material is beyond the high school level, you will find it contains some gems for you and your students. As an add-on, the CMS is in the process of creating a new math contest for elementary students. It should premiere sometime this Fall, and is so new, at the time I am writing this, it still does not have a name. Grades 6–8 teachers can look for it soon.

OK, let's deal with this problem. As it stands, the recursive formula does not look familiar to me. The strategy I chose is to calculate some values to see if I see any patterns. Putting the information into a spreadsheet, I notice something interesting. In Figure 1, you can see that the terms begin to repeat with period 5.



B4 =(1+B3)/B2

	A	B	C	D	E
1	n	tn			
2	1	58			
3	2	4			
4	3	0.086207			
5	4	0.271552			
6	5	14.75			
7	6	58			
8	7	4			
9	8	0.086207			
10	9	0.271552			
11	10	14.75			
12	11	58			
13	12	4			

Figure 1: *The first 12 terms of the sequence*

One advantage of using a spreadsheet in this case is that I can very easily change the initial two terms and see what happens with the sequence. Every initial value (unless $t_1 = t_2 = \frac{1 \pm \sqrt{5}}{2}$) ends up with a sequence of period 5. Hence, for our problem, $t_{2020} = t_5 = \frac{59}{4} = 14.75$.

As always, we will dig a little deeper to see if we can understand *why* the sequence is periodic. If we assign variables to the first two terms, $t_1 = a$ and $t_2 = b$, and do some substitutions, we get

$$t_3 = \frac{1+b}{a}$$
$$t_4 = \frac{1+a+b}{ab}$$
$$t_5 = \frac{1+a}{b},$$

after which $t_6 = a$, $t_7 = b$, and the sequence repeats.

Are there any other sequences that are periodic? Consider the following: $t_1 = 3$, $t_2 = 7$, $t_n = t_{n-1} - t_{n-2}$, $n \geq 3$.

I usually present this one to my MCR3U classes and ask them to find the 100th or 1000th term. A little calculation shows the sequence goes 3,7,4,–3,–7,–4,3,7,4,… and is periodic with period 6. Employing the same method as with the last problem, we see that this is true for all starting values (unless $t_1 = t_2 = 0$. Where do these periodic sequences come from?

Let's do a very rough sketch of why this last case worked. If we have a *linear recursive sequence*, that is a sequence

with recursive formula of the form

$$t_n = a_1 t_{n-1} + a_2 t_{n-2} + \cdots + a_k t_{n-k},$$

then assuming the general term is of the form $t_n = ar^n$, this leads to the equation

$$r^k = a_1 r^{k-1} + a_2 r^{k-2} + \ldots + a_k$$

with solutions $r_1, r_2, \ldots, r_k$ (not necessarily unique, and some may be complex numbers). This leads to the explicit formula for the general term of the form

$$t_n = A_1 r_1^n + A_3 r_2^n + \ldots + A_k r_k^n.$$

Famously the Fibonacci sequence: 1,1,2,3,5,8,12, … has general term (known as Binet's formula)

$$F_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right).$$

If the solutions $r_1, r_2, \ldots, r_k$ are all complex kth roots of 1, the sequence is periodic. For the sequence 3,7,4,–3,–7, –4,3,7,4, …, the corresponding equation is $r^2 = r - 1$ with solutions

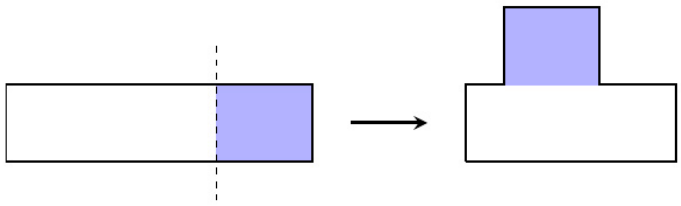
$$r = \frac{1 \pm \sqrt{-3}}{2}$$

which both satisfy $r^6 = 1$ (complex 6th roots of 1).

I had planned to do a presentation at our local professional development day for high school teachers in April. This was going to centre on ideas outside the curriculum that are intricately linked to the curriculum. One of the ideas I was going to talk about was linear recursive sequences. I was looking forward to doing the presentation, but when schools shut down, and PD days were cancelled, I decided to do it anyway, using video. The resulting video was made available to math teachers by my board. If you are interested in additional details about this topic, you can access my talk at www.drive.google.com/drive/folders/1zsaQ1gtGunhObrIjVMk6mLgrcWLaCQyZ?usp=sharing.

And now, time for your homework:

A rectangle with perimeter 20 and area 16 is cut parallel to its shortest side to create two unequal rectangles. The smaller of these rectangles is translated so that it abuts the top of the larger, as shown below.



If the perimeter of this new figure is 19, what is the area of the smaller rectangle?

Until next time, happy problem solving. ▲

▲ MB4T (Mathematics by and for Teachers): It's All about That Number Line



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Jennifer Holm is an Assistant Professor at Wilfrid Laurier University and works with primary/junior and junior/intermediate pre-service teachers, as well as in the field supporting current mathematics teachers. She is interested in developing mathematics knowledge for teaching with both pre-service and in-service teachers. She focuses on the beliefs and opinions that pre-service teachers hold about mathematics and teaching and the connection they have to past experiences. She uses this research to support future teachers in developing beliefs and knowledge that will encourage and support effective teaching practices.

If you have not had the opportunity to explore the mathematics vlog and website of the "Recovering Traditionalist" called Build Math Minds (www.there coveringtraditionalist.com/), I recommend taking some time to explore the different topics that Christina Tondevold discusses there. Although she is in the United States and therefore talks about the Common Core a lot, she presents some interesting ideas and workshops around teaching mathematics to children, without relying on the direct teaching method or "drill and kill" ideas. The basis of this column was inspired by a vlog of hers entitled, "The Difference Between Math Strategies and Math Models," which discusses how sometimes there is a tendency to conflate what a strategy is and what a model in mathematics is. This column will focus on different strategies that students may use in clarifying operations, and how teachers can notate or model them, using a number line.

For each of the sections in this column, different strategies that students may present for the operations in different contexts are explained and then shown, as they might be during a number talk, with the number line model.

Addition

There are many different strategies that students can use when thinking about how to add two numbers. For example, the numbers 139 and 85 could be added together by thinking about them in relation to 140 and 85. 140 and 60 makes 200, and then adding the last 25, for a total of 225, but then you have to remove the 1 that was added in the beginning,

making the sum 224. Figure 1 shows one way to model this idea, using a number line as a student explains their thinking. This number line could also help show students the strategy that if you add 1 to 139 and then subtract 1 from 85, you can add 140 and 84 in order to get the correct answer.

Another way of adding can be illustrated with 2.8 and 3.6. If you add 4 to 2.8, you get 6.8. From there, you can remove the 0.4 to end up with 6.4. The number line in Figure 2 can show a way of modelling this strategy.

Numbers can be also added in parts. For example, consider adding $1\frac{3}{4}$ and $\frac{3}{8}$. If you start with the $1\frac{3}{4}$, you can add $\frac{1}{4}$ to make 2, and then the remaining $\frac{1}{8}$, for a total of $2\frac{1}{8}$. The number line in Figure 3 shows one way of modelling this process.

Subtraction

Subtraction suits modelling using a number line for many different reasons. The number line can also help students to see how their strategies make sense and how they differ from addition strategies.

The first subtraction strategy can be illustrated using 142 and subtracting 79 from it. If you make 79 into 80 by adding 1, you can now subtract it from 142: first taking away 40 to get 102, and then another 40 to 62, but then you have to deal with the 1 you added, and add it back in to make 63 the answer. The number line in Figure 4 illustrates this strategy. This number line can be useful for helping students to see why you add the 1 back in at the final stage of this process instead of subtracting it.

When using 6.8 and subtracting 3.9, a student can start with 3.9 and add up to 6.8. By adding 0.1, you get to 4, and then another 2.8 would get to 6.8. By combining the two amounts you added up (0.1 and 2.8), you can get the answer of 2.9. Figure 5 shows a number line to illustrate the process.

One strategy that sometimes gets missed in talking about subtraction is to look at the difference between two numbers, such



Figure 1: *Adding 139 and 85*

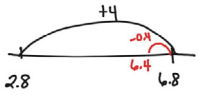


Figure 2: *Adding 2.8 and 3.6*

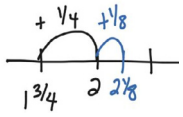


Figure 3: *Adding 1 3/4 to 3/8*

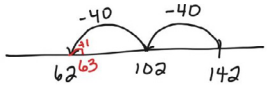


Figure 4: *Subtracting 79 from 142*

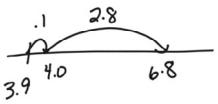


Figure 5: *Subtracting 3.9 from 6.8*

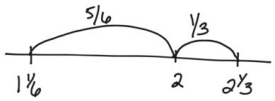


Figure 6: *Subtracting 1 1/6 from 2 1/3*

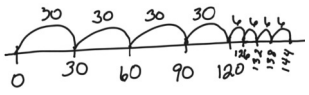


Figure 7: *Multiplying 36 and 4*

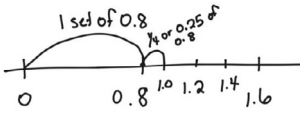


Figure 8: *Multiplying 1.25 and 0.8*

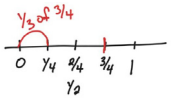


Figure 9: *Multiplying 1/3 an d 3/ 4*

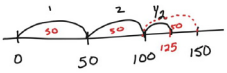


Figure 10: *Dividing 125 by 50*

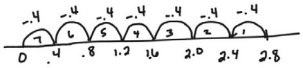


Figure 11: *Dividing 2.8 by 0.4*

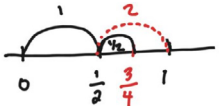


Figure 12 : *Dividing 3/4 by 1/2*

as with the numbers of $2\frac{1}{3}$ and $1\frac{1}{6}$. You need $\frac{5}{6}$ to get from $1\frac{1}{6}$ to 2, and $\frac{1}{3}$ to get from 2 to $2\frac{1}{3}$. The difference between the two numbers would be the sum of the two steps, $\frac{5}{6}$ and $\frac{1}{3}$, or $1\frac{1}{6}$ total. The number line in Figure 6 illustrates the strategy.

Multiplication

Multiplication can be represented as "sets of" a number, where a number line can be used to illustrate this strategy. With 36 multiplied by 4, you can start with 4 sets of 30 for 120, and then 4 sets of 6 for 24, giving a total of 144. The number line in Figure 7 shows the strategy, but you can also use the number line to show that the total is the same as 4 sets of 36.

With decimals, the operation of multiplication can also be used in parts, like with whole numbers. For 1.25 multiplied by 0.8, a student may see this as two operations: 1 × 0.8 and 0.25 × 0.8. For the first part, you would need 1 set of 0.8, which is 0.8, and then using an understanding of fractions, 1/4 of the 0.8, which is 0.2. To find the total, add 0.8 and 0.2, for a solution of 1.0. Figure 8 shows a model of this strategy by dividing the number line into 2 sets of 0.8 and then splitting the last set into 4 equal parts.

A similar strategy is used with the fractions $\frac{1}{3}$ and $\frac{3}{4}$. If you split $\frac{3}{4}$ into 3 equal parts, $\frac{1}{3}$ of this would be $\frac{1}{4}$. The number line in Figure 9 illustrates this strategy using a model.

Division

To divide 125 by 50, you could count the sets of 50 in 125. By counting out sets of 50, a student could determine there are 2 full sets and then 25 left, which is $\frac{1}{2}$ of the next set for a solution of $2\frac{1}{2}$. Figure 10 shows one illustration of this strategy using a number line.

Division can also be interpreted as repeated subtraction. A student can repeatedly subtract 0.4 from 2.8 to determine the answer. By counting

backwards by 0.4 (i.e., 2.4, 2.0, 1.6, 1.2, 0.8, 0.4, 0), a student could see the solution is 7 (i.e., 7 steps are taken). The number line in Figure 11 models how this strategy could be used with these numbers.

To divide $\frac{3}{4}$ by $\frac{1}{2}$, a student could determine the solution by seeing how many sets of $\frac{1}{2}$ are in $\frac{3}{4}$. There is 1 full set, but then $\frac{1}{4}$ left, which is $\frac{1}{2}$ of the next set. The total would be $\frac{1}{2}$ fits into $\frac{3}{4}$ exactly $1\frac{1}{2}$ times. Figure 12 shows the number line to model this strategy.

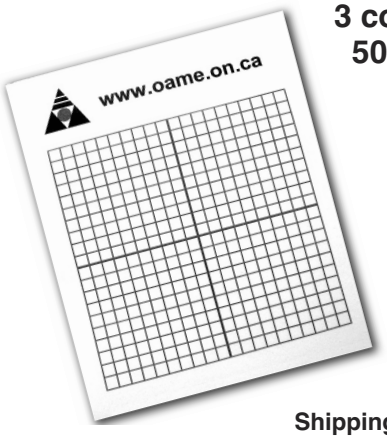
Why is the number line important for modelling the various strategies? The number line gives students a way to think about numbers, and a way to track their process as they work with more complicated numbers. In remembering that number lines are models, it is clear that when students are thinking about the numbers and creating strategies, they likely will not invent a number line. By using the number during number talks, it can help students to see the value in modelling their strategies this way. This will then give students another tool to use when the mathematics becomes more difficult. This is something that fits with what the students are already seeing with the numbers, and a tool they can use in an "organizational" and flexible way.

Reference

Tondevold, C. (2020). *The difference between math strategies and math models.* www.therecoveringtraditionalist.com/math-strategies-math-models/ ▲

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▲ Blast from the Past: Joe Stein – The Class Problem and Just-in-Time Teaching



JEFF IRVINE
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Jeff Irvine, PhD, has been a secondary mathematics teacher, department head, and vice-principal. He has taught at three faculties of education and at a community college. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. With over 45 years in education, Jeff is particularly interested in the interplay of student motivation and mathematics achievement.

In February 1959, the Diefenbaker government cancelled the Avro Arrow jet fighter project, throwing almost 30 000 people out of work. One of the aeronautical engineers was Joe Stein, who chose to become a math teacher to support his wife and eight children. Avro's loss was math education's gain. Joe's engineering background meant that his math classes had a strong application orientation, using models and manipulatives to illustrate and explain math concepts. In the 1960's, this was rare, as the push to "new math" was focused on set theory, logic, and the underlying structures of mathematics.



Figure 1: *Joe Stein, seated in the middle, and his math department. (North Park Secondary School Yearbook, 1978)*

In 1978, Joe had the opportunity to become department head at a brand new school that was being constructed. He chose his first department members by observing their classes and seeking buy-in for an application-oriented philosophy to engage students in the learning of mathematics.

Each and every class began with a "class problem" (CP) that immediately demonstrated to students the utility of the

math they would be learning. Examples of these class problems included: break-even analysis, to teach the intersection of two lines; the LORAN navigation system, to introduce hyperbolas; Mach numbers, to introduce exponents; the Mormon Tabernacle (an ellipsoid), to introduce ellipses—the choir is at one focus of the ellipse, the audience is at the other focus, so all sound reaches the audience at the same time. As the class tackled the problem, they would identify some needed mathematics. At that point, "just-in-time" teaching (Irvine, 2015) would occur. The teacher would develop the mathematics needed to continue with the problem solution, and teach a mini-lesson to either the whole class or the group of students who identified the need. The class would then continue solving the original class problem. In this way, the utility of the mathematics was obvious to the students, avoiding the "trust me—you'll use this later" philosophy that dominated math teaching at the time.

As founding members of Joe's department, we spent long hours generating CPs for every topic that we taught. Some topics leant themselves to CPs, while others, such as polynomials, were much harder to embed in a realistic situation. However, the entire department of six people enthusiastically embraced Joe's philosophy.

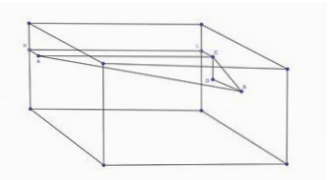


Figure 2: *The spider and fly*

Joe's engineering background was evident in his construction of various models to explain math concepts and solve math problems. In one example, he constructed a hinged Plexiglass® box to explain the "fly" problem. The fly problem involves a housefly that is sitting on the wall of a room, and that wants to crawl to the diametrically opposite wall. The question is to find the shortest route for the fly to take. Sometimes, adding a spider sitting on the opposite wall enhances the problem. In this version, the spider wants the shortest distance to the fly. Joe's collapsible box illustrated that the shortest route can be found using the Pythagorean theorem (Figure 2).

Another pillar of Joe's philosophy was the use of technology, with his simple rationale "because it's there." This, to me, was the beginning of the important tenet that students should not have to "power down" when they entered school. The technology of the time was scientific calculators. We required every student to purchase a scientific calculator and bring it to class daily. Joe made arrangements for bulk purchases from suppliers, and we subsidized (or simply provided) calculators for students in financial need. Having access to scientific calculators was extremely freeing. No longer did we need to "fudge" interest calculations to fit tables in the back of the textbook; no longer did we use logarithms for messy calculations; trigonometric tables were a thing of the past. Instead of teaching onerous calculation techniques, we focused on estimation and reasonableness of answers. This move to technology was not without its issues, and Joe spent many hours talking to unhappy parents, whose arguments were usually versions of "I didn't learn math this way."

Joe also identified the need for structure around student assessment. At the beginning of the semester, we issued master calendars for major tests. The tests occurred on this regular schedule, covering whatever material we had addressed in class up to that point. The school produced a master timetable for major tests in all subjects, so that no student faced more than one major evaluation on the same day. In mathematics, every test was cumulative; 50 percent of the test covered current work, while the other 50 percent addressed work done earlier in the semester. We taught study skills, how to summarize, and allowed students to bring a single page summary to every major test.

Joe Stein was ahead of his time in many ways. His emphasis on engaging students, while still teaching a rigorous mathematics curriculum, foreshadowed much of the current emphasis on student engagement, modelling, and problem-solving. His ready acceptance of technology and application-followed-by-theory way of teaching made him a pioneer in Ontario mathematics education.

In memorium: Joseph Stein (May 1, 1930–February 6, 2014): visionary, leader, teacher, aeronautical engineer, and father.

Reference

Irvine, J. (2015). Problem solving as motivation in mathematics: Just-in-time teaching. *Journal of Mathematical Sciences, 2*, 106–117. ▲

Help wanted!

There is an effort underway to improve the indexing of the *Gazette* archive. Members are needed, who are willing to carefully scan through at least three issues and add markers when they see items fitting a list of keywords that is provided.

The goal is to do this in an orderly way so that many hands make light work!

If you are willing, email gazette@oame.on.ca.

▲ Reflections on Solving Equations: Equivalent Equations and Algebraic Transformations



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Jérôme Proulx is a professor of mathematics education (www.profmath.uqam.ca/~jproulx), and runs the Laboratoire Épistémologie et Activité Mathématique *(www.leam.uqam.ca). His research focuses on epistemological issues related to cognition and in mathematics. His current research program is dedicated to mental mathematics and school mathematics content development. (Photo by Nathalie St-Pierre of UQAM)*

It is a truism that students often come up with surprising ways of solving tasks we give them. Students can at times even come up with strategies or ideas that in turn raise questions or even shed light on mathematical aspects usually taken for granted or kept implicit.

This is particularly the case in the mental mathematics studies my research team and I conduct in classrooms. Within the time constraints, students rarely refer to known procedures because these are often too time-consuming and hence not economical to solve in this context (see Butlen & Pézard, 2000). This leads many students to engage with a variety of strategies worth pondering. In particular, we conducted sessions on mental algebra, where the participating students (of various grade levels, from elementary to university-level) are given short amounts of time to solve some equations for x, posed orally and on the board, without the use of paper and pencil or any other material aids. (The equations given are of the usual form $Ax + B = C$, $Ax + B = Cx + D$, $\frac{Ax}{B} = \frac{C}{D}$, $Ax^2 + Bx + C = 0$, and their variants [see Proulx, 2013; Proulx et al., 2017].)

In this column, I am sharing examples of these strategies that students engaged with in order to solve these equations. These strategies, albeit simple, often raise interesting mathematical questions.

The Inversing Strategy

One kind of equation given in the form $\frac{Ax}{B} = \frac{C}{D}$ and its variants are $\frac{6}{x} = \frac{3}{5}$ and $\frac{1}{3x} = \frac{2}{5}$. One transformation that some students engaged to solve it was to first "flip the equation upside down," so that the x would be the numerator or, in their words that it was "on top." This inversion transformed the above equations to $\frac{x}{6} = \frac{5}{3}$ and $\frac{3x}{1} = \frac{5}{2}$, and the main argument given was that it felt easier to work with the equation when it was in this form. This strategy led students afterwards to operate, in various ways, on the equation to end up with $x = 10$ and $x = \frac{5}{6}$, respectively.

On Equivalent Equations

This initial transformation for solving the equation raises a number of issues in relation to equivalent equations. It raises the question how one knows if these two sets of equations ($\frac{6}{x} = \frac{3}{5}$ and $\frac{x}{6} = \frac{5}{x}$; $\frac{1}{3x} = \frac{2}{5}$ and $\frac{3x}{1} = \frac{5}{2}$) are indeed equivalent equations. Since both and $\frac{6}{x} = \frac{3}{5}$ and $\frac{1}{3x} = \frac{2}{5}$ have the restriction that $x \neq 0$, this inversion changes the conditions for which these equations can be verified. For example, from $\frac{6}{x} = \frac{3}{5}$ and $\frac{x}{6} = \frac{5}{3}$, the first $\frac{6}{x} = \frac{3}{5}$ possesses a supplementary condition that is absent from the second $\frac{x}{6} = \frac{5}{3}$. One needs to exclude $x = 0$ as a possible solution for $\frac{6}{x} = \frac{3}{5}$ *before* looking for values of x that can verify it. In other words, solving $\frac{6}{x} = \frac{3}{5}$ for x implies two conditions: x has to be different from 0 and, additionally, when 6 is divided by x, it gives $\frac{3}{5}$. In $\frac{x}{6} = \frac{5}{3}$, only the second condition is required ($x \neq 0$ is not needed). Hence, even if both $\frac{6}{x} = \frac{3}{5}$ and $\frac{x}{6} = \frac{5}{3}$ share the same solution, that is $x = 10$, what Whitehead (1958, p. 14) calls their field for x is different: one has $x = 0$ as a possibility, and the other does not. The field of possibilities for x of $\frac{x}{6} = \frac{5}{3}$ is different from the one for $\frac{6}{x} = \frac{3}{5}$. Therefore, even if this inversing transformation preserves the solution set, it does not seem possible to assert that these two equations are equivalent. The same is to be said for $\frac{1}{3x} = \frac{2}{5}$ and $\frac{3x}{1} = \frac{5}{2}$.

This raises questions about what it means to have equivalent equations. The standard definition given for equivalent equations is that they share the same solution set (Baruk, 1995). However, students' transformation evokes that an implicit element in the usual definition needs to be brought forth, that is, that the field of possibility for the solution set also needs to be the same in both equations after any transformation. Equations thus need to satisfy both conditions to be considered equivalent: (1) having the same solution set, and (2) sharing the same field of possibilities for that solution set.

The consideration of an extended definition of equivalent equations, through this double condition, raises important questions in relation to solving strategies. For example, one might wonder if the students' inversing transformation is legitimate or not. Specific examples can help to see the issues more clearly. Take the following two equations: $\frac{6}{x} = 0$ and $\frac{x}{6} = 0$. These two equations do not share the same solution set, even if they seem to be obtained through the

inversing transformation. Another example is with $\frac{6}{x} = \frac{5}{x}$ and $\frac{x}{6} = \frac{x}{5}$, also obtained through the inversion transformation. Issues emerge, since $x \neq 0$ represents a necessary restriction for $\frac{6}{x} = \frac{5}{x}$, even if this equation has no solution. In $\frac{x}{6} = \frac{x}{5}$, $x = 0$ is not only part of the field of possibilities for x, but is also the solution. Hence, the transformation from $\frac{6}{x} = \frac{5}{x}$ to $\frac{x}{6} = \frac{x}{5}$ changes the field of possibilities for the solution set, and it changes the solution set itself. What these two examples suggest is that this inversing transformation, albeit seemingly harmless, works in some instances and becomes problematic in others: to the extent of modifying the solution set at times.

This issue becomes even more salient when reflecting on the usual strategy of cross-multiplying, one often referred to by students for solving these equations. With $\frac{6}{x} = \frac{3}{5}$ and $\frac{1}{3x} = \frac{2}{5}$, some students transformed it into $6 \cdot 5 = 3 \cdot x$ or $1 \cdot 5 = 2 \cdot 3x$. This transformation produces equations that share the same solution set, but again for which the field of possibilities is transformed (because $x = 0$ does not need to be excluded for the new equations). The same applies to other equations like $\frac{6}{x-1} = \frac{3}{x}$, with restriction being $x \neq 1$ and $x \neq 0$, which would be transformed through cross-multiplication to $3x - 3 = 6x$, and for which $x \neq 1$ and $x \neq 0$ are no longer restrictions. As well, with the equations $\frac{6}{x} = \frac{5}{x}$ and $\frac{x}{6} = \frac{x}{5}$, using cross-multiplication results in $6x = 5x$ for both equations, for which the solution is $x = 0$. This is problematic, since $x \neq 0$ represents a restriction for $\frac{6}{x} = \frac{5}{x}$ and cannot be its solution; cross-multiplication appears, surprisingly, to be an illegitimate transformation.

Regular and Non-Regular Transformations

These questions offer a way to consider what Deledicq (2004) refers to as regular transformations, that is, transformations that enable the writing of an equation equivalent to another (e.g., $x = 3$ transformed to $x^2 = 9$ is not regular, nor is the other way around, as it changes the solution set). The double-conditioned equation equivalency means that to be regular, the transformation satisfies both conditions of equivalent equations, that is: (1) to maintain the solution set, and (2) to maintain the field of possibilities of this solution set. Therefore, transformations that do not maintain the field of possibilities, like this inversing one, could be termed non-regular, hence not producing equivalent equations, even if the solution set is maintained. The following table summarizes this.

<table>
<tr><td colspan="2">Legitimate Transformations</td><td>Illegitimate Transformations</td></tr>
<tr><td>Regular</td><td>Non-Regular</td><td rowspan="3">Different solution set
Can also change the field of possibilities</td></tr>
<tr><td>Same solution set</td><td>Same solution set</td></tr>
<tr><td>Same field of possibilities</td><td>Not the same field of possibilities</td></tr>
<tr><td colspan="2">Equivalent equations</td><td>Non-equivalent equations</td></tr>
</table>

These distinctions point to the fact that the same cross-multiplication can, in some instances, be legitimate and regular ($\frac{x}{6} = \frac{5}{3} \rightarrow 3 \cdot x = 5 \cdot 6$); legitimate and non-regular ($\frac{6}{x} = \frac{3}{5} \rightarrow 6 \cdot 5 = 3 \cdot x$); or again, simply non-legitimate $\frac{6}{x} = \frac{5}{x} \rightarrow 6x = 5x$. What this offers is that the judgment of legitimacy and of regularity of transformations is not an inherent property of the transformation itself (e.g., inversing, cross-multiplying), but is mostly in relation to its impact on the solution set and its field of possibilities. The legitimacy of transformations on equations becomes contingent on, and relative to, the equations themselves.[1]

Concluding Remarks

In line with the introduction, the issues raised above about solving equations offer a salient illustration of how some mathematical insights can be gained from students' ways of solving, even if the strategy is quite simple. In this case, the inversion transformation strategy, albeit relatively harmless at first sight, opened up the consideration of mathematical dimensions relative to equivalent equations and algebraic transformations. It brought to the fore an implicit double-condition about equivalent equations, where they both need to: (1) have the same solution set, and (2) share the same field of possibilities for that solution set. This double-condition for equivalency leads to questions about the legitimacy of usual transformations when solving equations, bringing forth the consideration of regular and non-regular transformations when solving equations and its relativity to the equation at hand.

Note

[1] Of course, history has shown us that the legitimacy of transformations is also time dependent. One can think of transformations that lend negative or complex solutions to equations that were once considered as atrocities or extraneous and needed to be discarded (see Struik, 1987), but are now considered important.

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▲ Elementary Math Matters: LOGO—Coding to Learn, Not Learning to Code



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Lynda Colgan is a Professor at the Faculty of Education at Queen's University. As a lifelong educator, Dr. Colgan constantly searches for creative avenues to engage students, parents, and educators in mathematics at home and in the classroom. She is inspired to do so by Seymour Papert, who once called upon his audience of "computers in education consultants" and mathematics teachers to bring about change by saying: "Instead of trying to make children love the math they hate, make a math they'll love."

By now, it is "old news" that the Government of Ontario introduced a new Grades 1 to 8 math curriculum, but for a few days in June, 2020, social media was abuzz with commentaries and judgments by pundits of all stripes and degrees of veracity. The old debates about "back to basics" and "discovery math" resurfaced along with reflections on the inclusion of "financial literacy."

The day after the press conference announcing the new policy document, I received a message from a former collaborator, Peter Skillen. He had "tweeted" his negative reaction to the claim that for the first time in the province, computer programming would be included in the curriculum1, and I "liked" Peter's comment. The exchange that followed was the catalyst for this column in that it evoked many memories of our work together as coding pioneers in this province.

Peter was a "Computers in Education" consultant with the former North York Board of Education at the same time that I held a parallel position with the Board of Education for the City of Scarborough. Peter and I were early adopters of LOGO—the first programming language developed for children who wrote the commands to control the movements of a "turtle"—either in the form of a small mechanical robot or a graphic object on the computer screen.

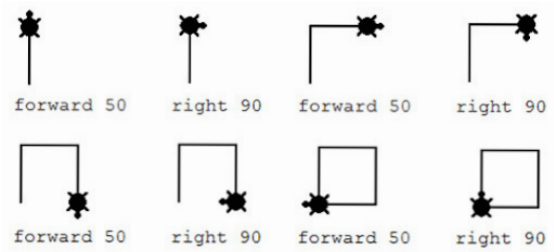


Figure 1: *The sequence of steps (or code) required to teach the turtle "to square"*



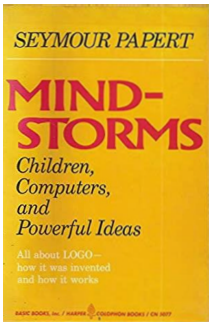
Figure 2: *Members of the 1986–87 ECOO Board of Directors, including Peter and me*

Peter and I were deeply entrenched in the LOGO community in Ontario and held leadership roles with the Educational Computing Organization of Ontario (ECOO), spearheading a special-interest group in LOGO for many years.

As a result, we also enjoyed many experiences of a lifetime because of collaborations with provincial leaders, including Drs. William Higginson, Dale Burnett, Ronald Ragsdale, and Gary Flewelling. Among the most memorable was the chance to meet (for what would be the first of many times), the venerable Seymour Papert, who was in Kingston for a launch celebration of an important 1985 provincial report about the implementation of LOGO in Ontario: *Computers, children and classrooms: A multisite evaluation of the creative use of microcomputers by elementary school children: Final Report.*

Known as the "father of educational computing," Dr. Seymour Papert (a colleague of Piaget and the founder of constructionism—a way to learn and solve problems by trying, testing, and playing around) was among the first to recognize the revolutionary potential of computers in education. To that end, he, along with Wally Feurzeig and Cynthia Solomon, designed LOGO as an educational computing program for children in 1967.

In his seminal book, *Mindstorms: Children, Computers and Powerful Ideas* (1980), Papert argued against "the computer being used to program the child." He suggested a radical alternative approach, in which "the child programs the computer and, in doing so, both acquires a sense of mastery over a piece of the most modern and powerful technology and establishes an intimate contact with some of the deepest ideas from science, from mathematics, and from the art of intellectual model building" (p. 5).



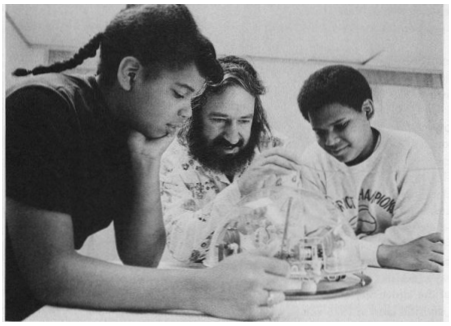


Figure 3: Mindstorms *and a photo of Papert with two students, problem- solving with the robot turtle featured in the book*

In those early, heady days of the first microcomputers (the development of which is also attributed to Papert) and "Turtle Geometry," LOGO was studied in depth as a vehicle for the development of mathematical thinking, spatial understanding, and problem solving. Familiar names like Douglas Clements (who today studies young children's math learning trajectories) were revered for their research into the cognitive and developmental power of computer programming with young children.

LOGO was the focus of provincial, national, and international mathematics and computing conferences, and Peter and I were fortunate enough to travel widely to attend academic meetings and deliver keynote addresses about our own implementation efforts in classrooms, as well as sharing our informal findings through professional journals including MIT's *Logo Exchange*.

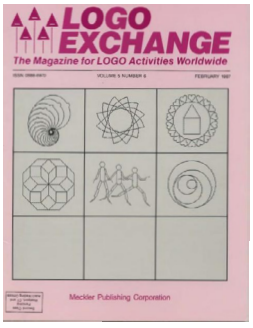


Figure 4: *Issues of the national and international* Logo Exchange*, dating back to 1982, available at https://el.media.mit.edu/logo-foundation/resources/nlx/index.html*

A highlight of that time for us was the international conference, *Look to the Learner: A Celebration of Process*, that we hosted at York University in 1986. A close look at the poster for that event, designed by renowned graphic designer Giles Talbot Kelly, captured the spirit of the day and the era. While some attention was paid to the "how" of creating the code for spinning squares, the real focus of any LOGO seminar or article was about the process: the way LOGO taught children how to think and, more importantly, how to try and fail (and try and fail), and ultimately learn from that recursive path.

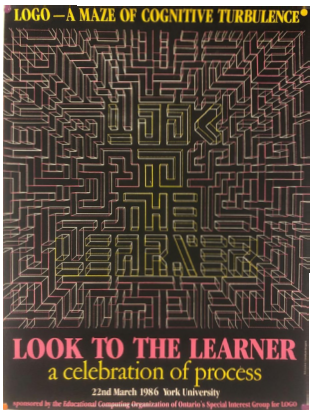


Figure 5: *The poster and cover of the proceedings from the 1986 SIG-LOGO conference*

As LOGO's worldwide reach exploded, new tools and resources were invented to make Papert's vision of concrete "thinker tools," that create a visual and immersive "MathWorld" for children, a reality. In fact, it was Seymour Papert himself, along with his former student and longtime collaborator, Mitchell Resnick (now the *LEGO Papert Professor of Learning Research* and Director of the Lifelong Kindergarten Lab at the Massachussetts Institute of Technology), who partnered with LEGO® to create the first programmable bricks. This expanded LEGO's capabilities for learning and play by inventing tools that allowed children not only to build, but control (through programming), their creations: LEGO/LOGO. Those first programmable LEGO bricks evolved into LEGO Mindstorms Robotics Kits—resources that were launched in the late 1980's and continue to be available today, some 40 years hence. It was also the enormous influence of LOGO that has allowed for the development of tools such as Scratch and ScratchJr, which are making the powerful ideas of computation accessible even to our youngest learners.

Which brings us to 2020. It explains the disappointment Peter and I voiced in response to the announcement that "coding" is "new" to Ontario. The word "coding" and the phrase "computational thinking" may indeed be novel, unfamiliar, or vague to many current educators and parents, but the concepts and deep pedagogical roots upon which they are built carry history and an enormous longitudinal body of empirical research that must not be ignored.

Although my life's trajectory took a different direction and I pursued other research interests after completing my PhD in the 1990's, Peter has never diverted from his attempts to actualize Papert's vision to make learning active, relevant, and personal by using technology as a creative medium for the expression and correction of ideas and understanding.

In an interview after co-winning an International Society for Technology in Education Award, Peter emphasized that he sees computers as "cognitive partners," but is quick to

note that "it is not at all about the technology. It is about a philosophy of learning and teaching." To that end, Peter has invested many hours as Project and Curriculum Leader for *Can Code to Learn* (a federally funded CanCode initiative) with collaborators Brenda Sherry and Michael Quinn, to bring "free, made in Canada, multilingual, created-by-Papert software that keeps coding focused robustly on math" (*LYNXcoding.club*) to educators and students. In fact, OTF/FEO's *Coding Connections* and other computational thinking projects, such as Dr. Ruth Beatty's *Indigenous Ways of Knowing Mathematics*, are actually chosen and funded by CanCodeToLearn.

It was Peter's tweets that served as a reminder that LOGO, created in the late 1960s and spread to classrooms across the world by the 1980s, is still alive and well. LOGO has disappeared neither in name nor philosophical foundation, though it may not be front and centre on the lips of current educators touting "coding" as the "next best thing."

It is simply untrue to state that with the announcement of the revised 2020 elementary mathematics curriculum, for the first time, computer programming would be included in the Ontario curriculum. It is but the latest chapter in the nearly 60-year story of LOGO—the little programming language that is so much more than code.

Let us hope that LOGO's deep pedagogical roots and complex cognitive branches are traced and climbed as we enact the current curricular coding in Ontario classrooms. To ignore either is to doom Ontario's children to learn to code, not code to learn.

Note

The claim can be found at https://news.ontario.ca/opo/en/2020/06/ontario-introduces-new-math-curriculum-for-elementary-students.html.

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▲ Annual Conference Report



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Paul Alves, current OAME/AOEM Past President, is department head of math at Fletcher's Meadow Secondary School in the Peel District School Board.

The OAME/AOEM 2020 In Focus Conference was to be held at the Ontario Tech University in Oshawa, hosted by the Pine Ridge Math Association (PRMA) and the Scarborough Association for Mathematics Education (SAME). Of course, the prevailing circumstances that emerged in the Spring precipitated the cancelling of that event. That decision was extremely difficult for all involved, and it is important to recognize the work of the committee members who invested close to two years of their time planning that conference. Countless hours go into organizing a provincial conference. To then be faced with the possibility that it will not happen is very difficult to accept. Once the decision was made, a message went out to the prospective presenters for the conference, gauging their interest in participating in an online event shortly after the cancellation. Again, this underlines the debt of gratitude we owe to the OAME/AOEM 2020 group. They gathered, reviewed and accepted the presentations that were to be part of the conference program. None of what followed would have been possible without their work.

The message that went out to presenters determined if they were willing to present a live session or a recorded talk. The response was overwhelmingly positive and spurred an organizing committee of about a dozen volunteers to put in place what became the *OAME/AOEM 2020 Virtual Conference*. Providing access to professional learning digitally is not new ground for OAME/AOEM. The 2014 Annual Conference included an e-conference option for attendees. This option was intended to provide access to the high-quality program available at the conference to attendees that couldn't make it to the conference physically. This was truly a groundbreaking innovation for any subject association. The success of that initial foray into digital learning has expanded over time. Subsequent conferences have endeavoured to maintain some sort of digital offering

for delegates, and it informed the virtual conference.

When the group tasked with organizing the OAME/AOEM 2020 Virtual Conference set upon the work to be done, there were many more questions than answers including when the conference would occur, what format it would follow, how much it would cost, what platform would be used, and many more. The dates of the conference that the group decided on overlapped with the dates of the planned *In Focus Conference*—May 6–9. This gave the group just over six weeks from the initial meeting to the start of the conference. In those six weeks, the organizing committee put into place the framework and structure for the conference. This involved many meetings online for this group, many of whom were also juggling commitments to their classrooms and families during the pandemic. A big public *thank you* to the group pictured for their commitment to providing professional learning during a challenging period.



There are some others who should not be neglected in our thanks. Special thanks go to all of the **presenters** who stepped up and provided either a live session or a pre-recorded session. Presenting at the best of times in front of a room of peers can be an intimidating experience. Doing that online and trying to ensure that you provide engaging learning adds to the level of anxiety and intimidation. All of our speakers were courageous in their willingness to share their passion for teaching and we all benefited.

Thanks as well to those who volunteered to serve as **discussants** during the conference. The discussants acted as hosts and allowed the presenters to focus on their presentation, but they also directed audience questions to the presenter at the appropriate time. We heard from several presenters how valuable it was to have that extra set of eyes in the "room" to help manage all that was happening.

Thank you to our **partners** and **supporters** in the educational community, who provided door prizes and other forms of support. This group of educational partners in our province continually supports OAME/AOEM and teachers in classrooms across the province. Special thanks go to Texas Instruments for sponsoring the conference and the infrastructure needed to support online learning.

The total number registered for the virtual conference reached 5329—far exceeding our expectations. The scale of the conference is something that we didn't expect. There were over 25 000 hits on the individual session pages. The peak attendance for the live sessions during a time slot was close to 3000. The recordings of the live sessions were accessed nearly 4500 times before Monday at midnight. And the conference reached beyond the borders of Ontario. We had educators from across Canada, the United States, and countries oceans away, such as Singapore, Portugal, India, Vietnam, the United Kingdom, and Brazil, to name a few. If you were unable to attend the virtual conference, all of the videos from the pre-recorded and live sessions are available in the members-only section. Visit the OAME/AOEM website and login to access all the videos plus the Google Docs, which contain a wealth of extra resources and comments from participants and presenters related to each session. The videos have also been closed-captioned in an effort to make our digital resources more accessible. During these extraordinary times, even when we cannot connect in person, connecting as a community of learners online reminds us of the excitement we feel when we hear, watch, or do something we hadn't thought of before, and say, "I can't wait to do that in my own classroom." Whether it is through face-to-face learning opportunities or continuing to explore virtual learning opportunities, OAME/AOEM remains committed to serving its membership by providing access to professional learning that engages, challenges, and enriches what we do for all students in Ontario. ▲

▲ OAME/AOEM Awards 2020

OAME/AOEM Award for Secondary School Mathematics Staff Award for Exceptional and Collaborative Teaching



The OAME/AOEM Secondary School Math Staff Award was presented to a unique group this year. It was awarded to a team of math educators composed of one Numeracy Coach from each of seven secondary schools within the Durham Catholic District School Board. The teachers making up this team are: Trish Maecker (Father Austin Catholic High School), Leanne Oliver (Msgr. Pereyma Catholic High School), Lisa Salamone (St. Mary Catholic High School), Laura Auld (Msgr. Paul Dwyer Catholic High School), Lynn Hollingworth (Dennis O'Connor Catholic High School), Colleen Morgulis (All Saints Catholic High School), and Joanna Ooms (Notre Dame Catholic High School).

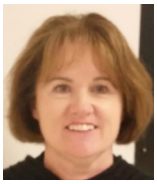
This group of teachers "fosters collegiality, teamwork, and excellent classroom teaching, contributes to the overall development of students, and demonstrates leadership in the mathematics education community." Willing to take risks in the wake of research-based pedagogies to improve student learning, they worked to break down barriers and open their classrooms to any teacher in the board. They also inspired, encouraged, and challenged all math teachers in the board to work at, learn from, and share their best practices. This approach matched teachers with common goals and interests from different schools to work collaboratively to make positive changes in their classrooms, for the benefiit of their students.

The group was involved in projects and presentations including:

- Quest Conference, to share Unlimited Potential Math Program (2016, see www.dcdsb.ca/en/programs-services/ultimate-potential-math-program.aspx)
- organizing and facilitating learning sessions for math teachers in our board (2017–2019)
- membership in the board's numeracy team and numeracy leadership team (2016–2019)
- meeting with teachers from other boards across the province to share Unlimited Potential Math Program and assessment practices (2016–2019)
- OAME/AOEM Annual Conference (2017, 2018, 2019)
- participation in Teacher Learning and Leadership Program (TLLP) projects (2018–2019)
- leading workshops regarding Thinking Classrooms, Spiralling, and Assessment during PA days (2018–2019)

It is with pleasure that OAME recognizes this group of teachers for their outstanding contribution to mathematics education in Durham Region and throughout Ontario.

Lifetime Membership Award



Jacqueline Hill is this year's OAME/AOEM Lifetime Membership Award winner. Over the last two decades, she has been an instrumental force in the area of mathematics education in our region. She has held various roles on the Pine Ridge Math Association (PRMA) executive, including multiple years as chapter President. Throughout this time, she spearheaded various professional development opportunities for the teachers in our chapter. From organizing mini-conferences to publishing newsletters to willingly sharing her teaching strategies and ideas, Jacqueline has strived to spread OAME/AOEM's vision of mathematics into our local classrooms.

Her impact on mathematics education has not been limited to our chapter. She has served as President of both OAME/AOEM and OMCA, been a chair of the OAME/AOEM Annual Conference, and has coordinated multiple Ontario Math Olympics. In these positions, she has demonstrated and shared her passionate and spirited approach to mathematics with colleagues from across the province. She is currently serving as OAME/AOEM's NCTM representative, as well as chair of the Strategic Planning and Renewal Committee.

Jacqueline has also utilized her expertise throughout Ontario with her work on the TIPS and CLIPS resources, the Think Literacy documents, and various Nelson and McGraw-Hill textbooks. Her contributions to these projects helped to ensure that students are exposed to quality resources that allowed them to think mathematically and experience the learning that comes with investigation and exploration.

It is with pleasure that OAME/AOEM awards Jacqueline with a much-deserved Lifetime Membership. ▲

▲ The History of Mathematics—Whose history is it?



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Jeff Irvine, PhD, has been a secondary mathematics teacher, department head, and vice-principal. He has taught at three faculties of education and at a community college. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. With over 45 years in education, Jeff is particularly interested in the interplay of student motivation and mathematics achievement.

Everyone is familiar with the Pythagorean theorem, $a^2 + b^2 = c^2$. But how many students know that forms of this relationship were known to the surveyors of ancient Egypt, the *rope-stretchers*, who used a rope with 12 knots to re-establish field corners after the annual spring flooding of the Nile river delta? They used the 3–4–5 triangle to form right angles, around 3150 BCE, long before Pythagoras (c.570–c.495 BCE) was born. A Babylonian tablet, called Plimpton 322, written around 1800 BCE, contains a long list of Pythagorean triples. The theorem is also mentioned in Indian manuscripts written between 800 BCE and 400 BCE. In spite of this history, many teachers interpret Pythagoras being Greek as fitting with later European developments in mathematics. The Eurocentric view of math history that this leads to has resulted in students thinking that everything they learn in mathematics was discovered by "old dead white men" from Europe. This is far from the actual history of math, and we do our students a disservice if we perpetuate this myth.

Another example of the Eurocentric view of history is Pascal's triangle. Blaise Pascal (1623–1662) published the triangle in 1654. However, the triangle was known to the Indian mathematician Pingala in the second century BCE; to Omar Khayyam (1048–1131), an Arabian mathematician, whose work was referred to as Khayyam's triangle; and in China, the Yang Hui triangle, named for the mathematician Yang Hui (1238–1298). Often, geographic distances prevented the dissemination of ideas across cultures. That is not to say that Pythagoras and Pascal did not add to the theory and development of ideas. However, to have relationships known for over 3000 years named for later European developers seems not to be circumspect, and contributes to the Eurocentric focus of much of the history of mathematics for the content that we teach.

The importance of communicating a global view of mathematics is reflected in the newest mathematics curriculum (Ontario Ministry of Education [OME], 2020).

Teaching about diverse mathematical figures in history and from different global contexts enables students not only to see themselves reflected in mathematical learning – a key factor in developing students' sense of self – but also to learn about others, and the multiple ways mathematics exists in all aspects of the world around them (OME, 2020, p. 48).

Development of Mathematical Ideas

In many math classes, ideas are presented in a manner to suggest that the concepts were born fully formed, rather than developed across various civilizations over time, sometimes over a great deal of time. An example of this is the quadratic formula. The story of the quadratic formula begins in ancient Egypt around 1500 BCE, where engineers knew of the relationship, although in written descriptions and tables, rather than the symbolic algebra of today. By 400 BCE, Babylonian and Chinese mathematicians had developed solutions by completing the square. Persian mathematician Al-Khwarizmi (780–850 BCE) also explained completing the square in his algebra text, published in about 823 BCE. By 300 BCE, Pythagoras and Euclid had developed geometric solutions to quadratic equations. By 650 CE, Indian mathematician Brahmagupta (598–668 CE) had presented the general solution of the quadratic equation. Then, in 1637, René Descartes (1596–1650) published the quadratic formula in the form that we know today. Interestingly, no European mathematician's name is associated with the formula, a rarity in the history of mathematics.

Recognizing and communicating the history behind math concepts can enrich students' learning, and can positively affect student motivation. This is especially true in our modern diverse classrooms, where recognition of other cultures' contributions to mathematics can humanize the face of mathematics. Math concepts don't always have to be ideas from "old dead white men from Europe."

Women in Mathematics

Before the turn of the twentieth century, women's contributions to mathematics was often unrecognized. This was primarily due to the societal norms of earlier periods, when women were often seen as strictly caregivers and housekeepers, and were often excluded from higher education. There were notable exceptions, some of whom

are described below.

Hypatia of Alexandria (370–415). A renowned mathematician, philosopher, astronomer, and was also the head of the Neoplatonic Scholl in Alexandria Egypt. Hypatia was killed by a Christian mob in 415.

Maria Gaetana Agnisi (1718–1799). Eldest of 21 children, Agnisi was the first woman appointed as a university professor of mathematics (University of Bologna). Best known for the witch of Agnisi (a curve, not a follower of Wicca), which has the equations $y = \frac{8a^3}{x^2 + 4a^2}$, or, when $a = \frac{1}{2}$, $y = \frac{1}{1 + x^2}$.

Sophie Germain (1776–1830). Germain studied geometry to escape boredom while secluded to avoid the French Revolution. She did important work on Fermat's Last Theorem (which was ultimately proven in 1995).

Mary Somerville (1780–1872). Known as the "Queen of Nineteenth-Century Science," she studied mathematics despite the opposition of her family. She published work in mathematics and science and produced the first geography textbook in England.

Ada Lovelace (1815–1852). Daughter of poet Lord Byron, she is credited with the first computer software program. Lovelace died at age 36 of uterine cancer.

Sofia Kovalevskaya (1850–1891). First woman to obtain a doctorate in mathematics; first woman in Northern Europe to be appointed a full professor; first woman to be an editor of a scientific journal.

Shakuntala Devi (1929–2013). Most famous female Indian mathematician, known as "the human computer," she competed against a computer to find the cube root of 188138517 (and won!).

Emmy Noether (1882–1935). Called by Albert Einstein and others the most significant creative mathematical genius since women were admitted to higher education. She escaped Nazi Germany and emigrated to the United States. She did important work in abstract algebra and theoretical physics.

After the turn of the twentieth century, there was a burgeoning number of female mathematicians (a list on Wikipedia, https://en.wikipedia.org/wiki/List_of_women_in_mathematics, runs over 50 pages). There are opportunities here to engage students by involving them in research on a female mathematician of their choice. While much of the mathematics is beyond high school, this is a chance to humanize a subject often seen as uninteresting and stale, while addressing a gender bias that is an unintended consequence in many math history books.

Vignettes of Mathematicians in History

Student interest can also be created by referencing lives beyond mathematics. Here are some factoids that may create interest and spur further study.

Isaac Newton (1642–1727), English, and Gottfried Leibniz (1646–1716), German. The calculus wars. Both these prolific mathematicians claimed to be the first inventors of calculus. In their era, mathematicians worked in isolation and then published their discoveries as challenges to other mathematicians.



Evariste Galois (1811–1832), French. Solved a 350-year-old problem in algebra while a teenager; specifically, that there is nothing equivalent to the quadratic formula for fifth-degree polynomials or in general. Killed in a duel over a woman, age 21.

William Rowan Hamilton (1805–1865), Irish. While walking, he discovered quaternions and inscribed the definition in a bridge because he had nothing to write with. Killed in a duel over a woman at the age of 60.

Brahmagupta (c.598–c.668). Indian mathematician who introduced the concept of zero and provided rules for computing with zero.

Leonhard Euler (1707–1783), Swiss. Called the most prolific mathematician in history, his works filled 92 volumes. He could compute the sum of the cubes of the first 1000 whole numbers in his head. Richard Feynman called Euler's formula $e^{\pi i} + 1 = 0$ the most elegant formula in mathematics.

Goro Shimura (1930–2019). Japanese mathematician who posed the Taniyama-Shimura conjecture, which led to the proof of Fermat's Last Theorem.

Karl Friedrich Gauss (1777–1855), German. His method for adding arithmetic series, reputed to be demonstrated at age 6, is a staple of high school finite mathematics.

John von Neumann (1903–1957), German. Escaped Nazi Germany and emigrated to the United States. He was the father of game theory and one of the leads on the Manhattan Project, which developed the atomic bomb during World War II.

Shen Kuo (1031–1095), Chinese. He was a mathematician, astronomer, meteorologist, entomologist, anatomist, climatologist, zoologist, botanist, pharmacologist, medical scientist, agronomist, archaeologist, ethnographer, cartographer, geographer, geophysicist, mineralogist, military general, diplomat, hydraulic engineer, economist, inventor, encyclopedist, academy chancellor, finance minister, governmental state inspector, philosopher, art critic, poet, and musician.

Elbert Frank Cox (1895–1969), American. First Black person in the world to earn a doctorate in mathematics (1925).

Ernő Rubik (1944–), Hungarian. Inventor of Rubik's Cube.

Georg Pólya (1887–1985), Hungarian. His work on problem solving was in the front matter of every Ontario mathematics curriculum policy document until the most recent update.

Anna Pell Wheeler (1883–1966), American. Married former Russian double agent Alexander Wheeler. Faced significant hostility when attempting to find a teaching position in the United States.

Marjorie Rice (1923–2017), American. With only a high school education, discovered four new types of tessellating pentagons and over 60 distinct ways of tessellating with pentagons.

Cathleen Synge Morawetz (1923–2017), Canadian. Worked in partial differential equations related to fluid flow. Was named Outstanding Woman Scientist of 1993.

Srinivasa Ramanujan (1887–1920), Indian. Brilliant self-taught scholar, who compiled over 3900 results. Instantly determined that 1729 is the smallest number that can be written as the sum of two cubes in two different ways. One way is $1729 = 12^3 + 1^3$. I leave it to the reader (or a student) to find the other way.

These are only a few of the interesting stories behind the history of mathematics. This is a rich area from which student interest can be enhanced. Putting a human face on the mathematics we teach is one way to generate student engagement and positively impact student attitudes in our math classes. With a little research, it is possible to give both female and male students a personal connection to the mathematics they study, and to minimize the Eurocentric view that permeates many of the history of mathematics extant.

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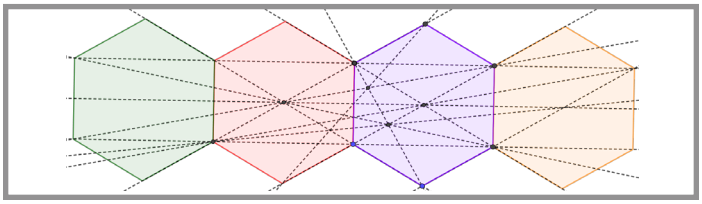
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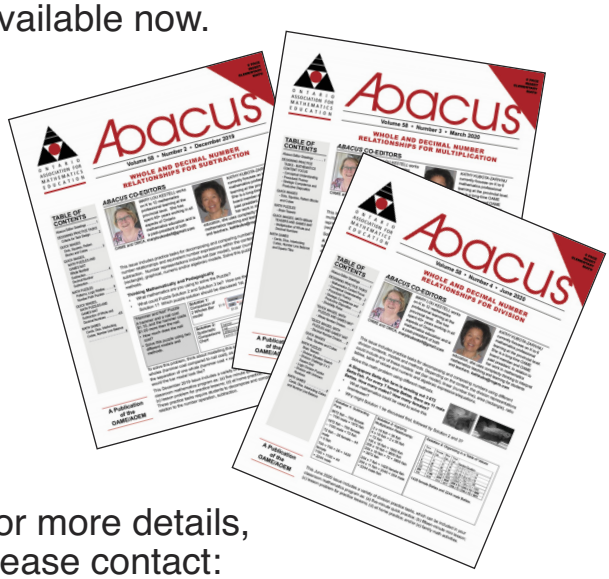
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The Cecropia Catepillar

What is a pattern you see relating to the Cecropia caterpillar?

Can you estimate the length of the Cecropia caterpillar?

How could you estimate its volume?



Seven Classroom Strategies to Challenge "Rapid Graspers" to Think Deeply



GABRIEL FLORES, OCT
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Gabriel Flores is a Primary–Junior Ontario Certified Teacher, UK Qualified Teacher, and an IB PYP Educator. He began his professional teaching career in London, England before moving to Bangkok, Thailand, where he is currently a primary teacher in a UK–Thai bilingual school. Prior to becoming a qualified teacher, he worked as a policy analyst for the Government of Canada in his hometown of Ottawa, Ontario. He holds a MA (Carleton University), BA (University of Ottawa), and B.Ed. (Queen's University).

I. What do we do with students who finish the work before most of their peers?

As an elementary classroom teacher, I plan lessons that are differentiated to meet the needs of all learners; however, sometimes our students can achieve the learning objective sooner than expected. These "rapid graspers," or students who pick up the concept being taught more quickly than their peers, require careful consideration. The term "rapid grasper" is "not a rewording of 'high attainers' or 'gifted and talented' because… no attempt is made to identify which particular children will be a rapid grasper on any particular day or topic, recognizing that students' aptitudes vary between topics and over time" (NCETM, 2018).

As a teacher, I often struggled to know how to challenge these rapid graspers, who seem to "get it." Initially, I thought that I should just give them more difficult work, maybe accelerate. However, "acceleration encourages only a shallow mastery of the subject, and so promotes procedural learning at the expense of deep understanding" (ACME, 2012, p. 2). Furthermore, if we are to meet the goals of *Focusing on the Fundamentals of Math:* "A strong foundation in the concepts and skills emphasized here will prepare students for success in high school, and ensure that they have a set of essential skills for employment and responsible citizenship in the future" (Ontario Ministry of Education, 2018, p. 1), then a deep understanding of the structure and relationships inherent in mathematical concepts is arguably better than a shallow understanding of the subject. In fact, mastery learning, which will be explained in a moment, has a clear positive and significant impact on student achievement. Dr. John Hattie (2013, p. 268) found that mastery learning is ranked 31 out of 150 influences related to achievement and can have considerable impact. *But what exactly is mastery learning in mathematics?*

II. What is mastery learning? Thinking deeply in order to understand mathematical relationships

Mastery learning encourages students to think deeply about mathematical relationships in order to deeply understand how and why math works. Indeed, several jurisdictions have recognized that students learn best when they strive to understand mathematical concepts, relationships, structures, and behaviour in how math works. For example, in the United Kingdom, the National Centre for Excellence in the Teaching of Mathematics (NCETM) states: "Achieving mastery means acquiring a solid enough understanding of the maths that's been taught to enable pupils to move on to more advanced material" (NCETM, 2016). *What does this practice look like?*

The following table shows how the United Kingdom and Ontario approach emphasize the need for application of general concepts, mental math, discussion, collaboration, efficient recall, and understanding relationships for effective mathematical learning:

Both jurisdictions seem to recognize that students should strive to understand how and why math "works" as opposed to just memorizing math facts; this is likely to occur if students are encouraged to ask questions, discuss, explore, and discover why or how math makes sense (see Section III for exemplars). Using the table at the top of the next page as a framework, I have implemented the following seven strategies to challenge rapid graspers.

III. Seven Strategies to Challenge Rapid Graspers

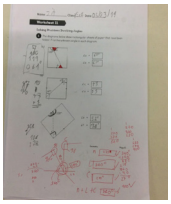
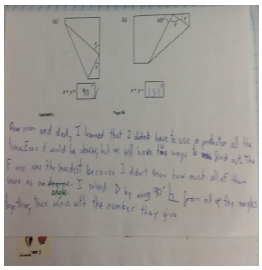
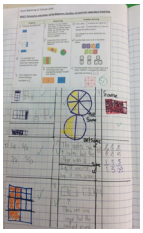
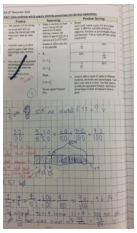
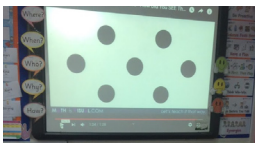
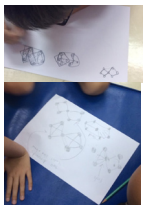
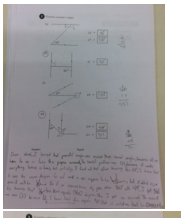
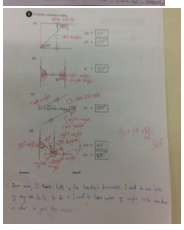
1. Create your own problems.
2. Write a letter to your parent or guardian, explaining how you solved the problem.
3. Teach another peer how to solve a problem.
4. Don't ask the teacher. Research the problem first.
5. Reflect deeply on your choices by looking at different possible solutions.
6. Show your thinking with diagrams, words, number sentences, and manipulatives, not just mental math.
7. Find solutions, using both logical reasoning and tools such as a protractor.

United Kingdom Big Ideas in Teaching for Mastery: Key Messages	Ontario Ministry of Education (MOE) Focusing on the Fundamentals of Math: A Teacher's Guide	Author Insights
Coherence Lessons are broken down into small connected steps that gradually unfold the concept, providing access for all children and leading to a generalization of the concept and the ability to apply the concept to a range of contexts.	**Developing Proficiency with Operations** Performing calculations with ease, precision, and consistency, and with a general understanding of number and operations, number properties, and their appropriate application in problem solving.	The end goal is not memorization or rote learning, but the application of mathematics to solve problems.
Representation and Structure Representations used in lessons expose the mathematical structure being taught, the goal being that students can do the maths without recourse to the representation.	**Developing Mental Math Skills** Doing calculations in the mind, with little or no use of paper and pencil or calculator.	The Ontario approach suggests that thinking flexibly about mathematical relationships mentally is important as opposed to just a written approach that might lead to rote learning. Mental math activates skills such as listening for understanding and mental visual representations of concepts to solve problems.
Mathematical Thinking If taught ideas are to be understood deeply, they must not merely be passively received, but must be worked on by the student: thought about, reasoned with, and discussed with others.	**Working with Numbers** Understanding and using numbers (e.g., being able to read, represent, count, order, estimate, compare, compose, decompose, and recompose numbers).	Understanding mathematics and applying that understanding, as demonstrated through discussion, analysis, and evaluation.
Fluency Quick and efficient recall of facts and procedures, and the flexibility to move between different contexts and representations of mathematics.	**Mastering Math Facts** Understanding and recalling math facts, using a variety of strategies.	Both the UK and Ontario place emphasis on recalling math facts "efficiently" and the "use of variety of strategies." Again, the emphasis is not on memorization, but mental flexibility.
Variation Variation is twofold. It is firstly about how the teacher represents the concept being taught, often in more than one way, to draw attention to critical aspects, and to develop deep and holistic understanding. It is also about the sequencing of the episodes, activities, and exercises used within a lesson and follow-up practice, paying attention to what is kept the same and what changes, to connect the mathematics, and draw attention to mathematical relationships and structure.	**Recognizing and Applying Understanding of Number Properties** Understanding how numbers behave in operations, and drawing on that understanding to master math facts and perform calculations.	In order to "understand how numbers behave" (MOE), it might be beneficial to students to "draw attention to mathematical relationships and structure" (UK).

IV. Discussion

In developing the seven strategies for challenging rapid graspers, I discovered that I did not need to change the content or accelerate students. Instead, I had to change the behaviour of both the teacher and student. As a teacher, I had to adapt: "These rapid graspers do not need more 'advanced' work per se. They need to deepen their understanding and communicate their thinking with others." For rapid graspers, we had to move from, "I know the answer; let me do something different" toward, "I know the answer; now let me share my learning with teachers, peers, and parents, or let me figure out another way to know the answer!" This is intellectually challenging because explaining something that you know very well to someone who is a non-expert is really challenging (it is one of the hallmarks of teaching). The challenge stems from discovering that your solution might not "make sense" to another person, and so you need to think of another way to explain the answer, perhaps by using a manipulative, bar graph, online research, discussion or drawings! When this occurs, the rapid grasper comes to the realization that the answer is only the beginning of knowing something because solutions can be experienced and understood conceptually, pictorially, or abstractly, and each presents a unique way to communicate and understand mathematical concepts.

Furthermore, I observed that when rapid graspers were asked to share their understanding, in writing or in person; students had to collaborate with other students, thus helping to foster friendships, which according to Abraham Maslow,

Strategy	Exemplar	Observation
1. Create your own problems. "What is angle L if angles C and N are less than a reflex angle?" (Oral word problem and written solution)		The student has applied his understanding of one problem (black) in order to create another more challenging problem (red). In addition, the student has provided the solution by showing his understanding of how knowing one angle can lead to knowing another, using geometrical relationships.
2. Write a letter to a parent or guardian, explaining how you solved the problem. "Dear mom and dad, I learned I didn't have to use a protractor all the time. Even if it is solver, but we will have two ways to find out. F one was the hardest because I didn't know how much all of them were as degrees. I solved D by using 90 degrees angle from all of the angles together, then minus with the number they give."		The student recalls a series of logical sequential steps and math facts in order to explain how the problem was solved. By doing so, the student can relate how column method, knowledge of angles, addition, and subtraction are all interrelated to solve a problem. As such, place value, number sense, geometry, and logical thinking are not separate from each other, but necessary in order to understand why the solution works.
3. Teach another peer how to solve a problem. The child uses a manipulative in order to explain how he solved a problem. The manipulative helps him organize his thoughts.		The student collaborates with a peer to show that his mathematical thinking and knowledge of angles can lead to solutions that "work." In this case, the student selects a manipulative to help with his reasoning in order to support the understanding of another peer.
4. Research online for an answer instead of asking the teacher. The child performs a Google search to gather information related to an unknown math concept.		The student investigates unknown facts and independently conducts online research. After researching multiple sources, the student makes sense of new knowledge by changing or adapting his mental models, subsequently applying these new representations to solve the problem.
5. Show your thinking with diagrams, pictures, and words.	 The child uses fractional circles and fraction bars in order to justify answer choice. The child uses bar method and multiplication in order to solve and justify answer choice. 	The student uses the bar model to represent and understand the whole–part relationship and its relationship with fraction notation. Indeed, the student can "see" how fraction notation and the denominator and numerator are related to the bar model.
6. Reflect deeply on your choices by looking at different possible answers. Boaler, J. (2015). The dot challenge. How do you see the dots? 	The child names the different patterns: "Fish," "Two-diamond: looks like a bow," "hand." The child relates his experience to something other than math in order to accommodate this new experience. 	The student understands the different ways that the dots could be arranged or "seen." The student realizes that there are multiple possible answers to this problem. As such, it is not whether he gets an answer, but how many answers he can get that is truly important. This also requires that several mental models be represented.
7. Find solutions, using both logical reasoning and tools such as a protractor.	The child diagrams and uses knowledge of different angles (acute, reflex, right) in order to solve for missing angles without the use of a protractor. The child uses observation, "I solved it by looking at it for a long time, then I saw that I can break it into a straight angle…." The child then proceeds to develop an argument based on additional factual information.  	The student reflects that the problem was the hardest, but after "finding out" that one angle is related to another because of parallel lines, the student reasons that he has found another angle, and then, coupled with his knowledge of reflex angles, solves the problem.

fulfills the psychological need for belonging and esteem. For example, as a teacher, I recognized and praised their effort and positive behaviour in helping others learn. The effect was that other children began to see them not as "the smart kids," but as friends, who could help them develop their own thinking, creating a win-win situation for everyone.

Moreover, the seven strategies can be integrated seamlessly within lessons in order to deepen and challenge rapid graspers within lessons. Arguably, this can be more responsive to the actual student needs and achievement, as the strategies are adapted based on student needs as opposed to categorizations such as "Gifted" and "Talented" or "High Attainers." In other words, I do not prejudge or assume that a set of children will be the rapid graspers during the lesson. Rather, I provided a space, where multiple ways of knowing a math concept could be explored. I noted that visual and kinesthetic learners achieved a better understanding of the concept than I anticipated. Consequently, this recognizes that multiple types of learners can be addressed and developed, and, as educators, we need to enter each lesson with an open mind and ready to adapt to our students' changing needs and interests.

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Non-university math beyond Grade 10?

"In this way, a solid four year mathematics programme could be introduced which would provide a wide background for four year students heading for educational institutions after completing grade 12." (1966, *Gazette, 4(*2), p. 13)

▲ Mathematics in French Immersion: A Teacher's Perspective



NANCY LOUIS
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Nancy is a primary French Immersion teacher with the Simcoe County District School Board in Barrie, Ontario. She is currently a Grade 1 teacher. Nancy is also a DELF, Diploma in French Language Studies Examiner and Corrector. She shares her passion for French and math with her students, colleagues, and school community.

School boards that offer French Immersion programs often debate if mathematics should be taught in English or French. Among the many reasons is the assumption that French Immersion students don't perform as well as their peers in the English programs because math is not taught in their first language. The assumption that French Immersion students lack the language proficiency required to succeed in math is not necessarily true.

Evidence from immersion programs suggests that students can achieve similar scores in mathematics to those of students enrolled in regular English programs (Turnbull, Hart, & Lapkin, 2003; Swain & Lapkin, 2005; Turnbull, Lapkin, & Hart, 2001). In some cases, students in immersion programs outperform students in regular English programs in mathematics. Furthermore, French Immersion students learning math in French achieve higher scores on standardized tests of mathematical ability than those of French Immersion students taught math in English (Bournot-Trites & Reeder, 2001). Finally, students in Early French Immersion, which provides intensity at the beginning of the program, quickly achieve sufficient second-language proficiency to understand math and other subject content taught in French (Bournot-Trites & Reeder, 2001; Netten, 2007; Germain, Netten, & Movassat, 2004).

Bournot-Trites and Reeder (2001) looked at the French Immersion students' performance on mathematics tests administered in English. The study followed two groups of students from Grades 4 to 7. One group received 80 percent of the instructional time, including math, in French. The other group received 50 percent of the instructional time in French, and 50 percent, including math, in English. At the end of Grade 6, students' math scores were reviewed. The results indicated that students who received 80 percent of the

instructional time, including math, in French showed an advantage in math compared to the group who only received 50 percent of the instructional time in French.

Effective Practices

To achieve the results mentioned, it is necessary for French Immersion programs to provide an effective mathematics learning environment in French, and for teachers to use proven instructional strategies. Some of these strategies are the use of visual cues, manipulatives, pictures and diagrams, and the use of familiar and simplified language. In many French Immersion classrooms, including mine, students often refer to a math wall, where they find specific math words associated with picture cues to use when needed. It's essential to pre-teach the math vocabulary and allow wait time for students' responses. I find math talks very helpful for students to orally communicate their mathematical understanding, while enabling teachers to model the correct math terms. It's also necessary to connect symbols with words, pose problems in familiar context, and model different methods of problem solving along with the strategic use of students' first language. On many occasions, I have witnessed my students communicating in French to each other while solving math problems. Their ability to share their math learning, using simple and correct French language, increases dramatically year after year.

The *Report of the Expert Panel on Mathematics* states that, "Teachers can make effective use of problem-oriented methods with specific modifications to better address the needs of students from different cultures or languages" (Ontario Ministry of Education, 2004, p. 34), which also applies to the French Immersion programs. It goes on to say, "The same techniques used to support literacy skills can be used in the development of mathematics in a second language classroom" (p. 34). I have been teaching mathematics in French for Primary French Immersion students since 2014. Throughout these years, different initiatives and mind shifts have influenced and may have changed the way math is taught in many classrooms. Amid all of these changes, one element remained unchanged: the way my students thrive in math class compared to any other subject.

Conclusion

Effective strategies for teaching mathematics in French Immersion classrooms are essential for students' achievement, which matches or surpasses student performance in English programs. French Immersion students are taught to be problem solvers; creative, critical thinkers; communicators; and risk takers. These qualities are the core of the four categories in the mathematics achievement chart.

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MP^3 2020 = The Mathematics Post-secondary Preparation Package 2020

Short message from Jack Weiner (Former Gazette editor and well respected math professor.)

MP^3 is not only available as a free pdf from http://mathematicssurvivalkit.ca/ or www.lifegears.org/math-prep. It has also been transformed into an interactive online package by wizedemy.com, again free, including online tutoring.

▲ Some from London, Some from Rome, Plus Some Classroom Math from Home

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Olivia likes flowers, math, and design. Every year, when snowy winter in Ontario is over, she draws a new colourful geometric pattern for a semicircular flowerbed adjacent to her house.

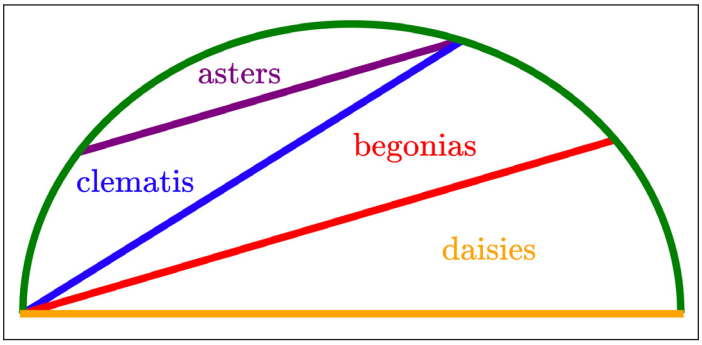


Diagram (not to scale)

This Spring, her flowers are divided into four asymmetric sections, separated by three chords, two of which are parallel. The parallel chords and diameter of the semicircle have the lengths $a = 1.6$m (asters), $b = 5.6$m (begonias), and $d = 6.5$ (daisies). In order to determine the length c (clematis) of the third chord, Olivia uses *arc midpoint computation*, which she learned at the Garden Math Camp on the flowery Vancouver Island last summer,

$$2c = \sqrt{8.1\times 12.1} - \sqrt{4.9\times 0.9} = 7.8,$$

and finds the exact answer: $c = 3.9$m.

1. Verify the answer, using alternative approaches. Compare solutions.
2. Show that a, b, c, and d satisfy

$$2c = \sqrt{(d+a)(d+b)} - \sqrt{(d-a)(d-b)}.$$

An arc midpoint computation solving: Some from London, Some from Rome, Plus Some Classroom Math from Home

When the chords of the same length a, b, and c are redrawn so that they are parallel to diameter d, the point C will bisect the arc AB. Let the semicircle be an origin-centred one, then x-values of points A, B, and C are $\frac{a}{2}, \frac{b}{2}$, and $\frac{c}{2}$, respectively.

The arc midpoint computation result is:

$$2\left(\frac{c}{2}\right) = \sqrt{\left(\frac{d}{2}+\frac{a}{2}\right)\left(\frac{d}{2}+\frac{b}{2}\right)} - \sqrt{\left(\frac{d}{2}-\frac{a}{2}\right)\left(\frac{d}{2}-\frac{b}{2}\right)}.$$

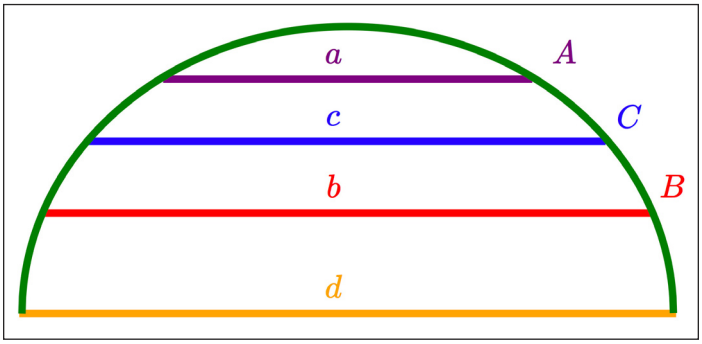


Diagram (not to scale)

Another Solution to: Some from London, Some from Rome, Plus Some Classroom Math from Home

Let's redraw chords of the same length a, b, and c so that the first chord (a) and third one (b) are parallel to diameter (d). Draw AC perpendicular to the third chord and find $BC = \frac{b}{2}+\frac{a}{2} = \frac{1}{2}(a+b)$. Using the Pythagorean theorem two times, determine the distances from the midpoint of each chord to the centre of the circle (midpoint of d), resulting in $\sqrt{\frac{d^2}{4}-\frac{a^2}{4}}$ and $\sqrt{\frac{d^2}{4}-\frac{b^2}{4}}$. Hence, find the distance between parallel chords, $AC\,\frac{1}{2} = \left(\sqrt{d^2-a^2} - \sqrt{d^2-b^2}\right)$. Use the Pythagorean theorem a third time for ΔABC. to find:

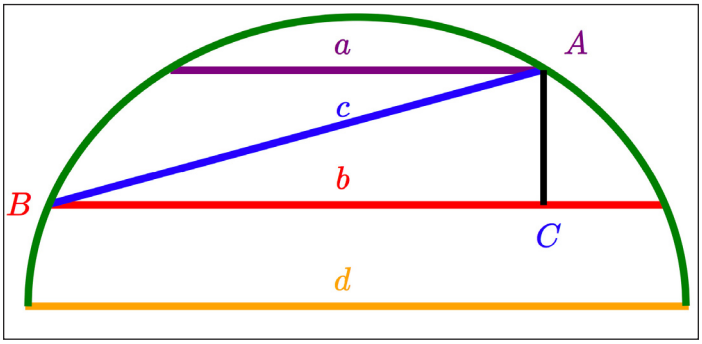


Diagram (not to scale)

$$AB = c = \sqrt{BC^2+AC^2} = \frac{1}{2}\left(\sqrt{(a+b)^2 + \left(\sqrt{d^2-a^2} - \sqrt{d^2-b^2}\right)^2}\right.$$

Transform result $2c = \sqrt{2ab+2d^2-2\sqrt{(d^2-a^2)(d^2-b^2)}}$

$$= \sqrt{\left(\sqrt{(d+a)(d+b)} - \sqrt{(d-a)(d-b)}\right)^2}$$

$$= \sqrt{(d+a)(d+b)} - \sqrt{(d-a)(d-b)}.$$

▲

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