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Ontario Mathematics Gazette

OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

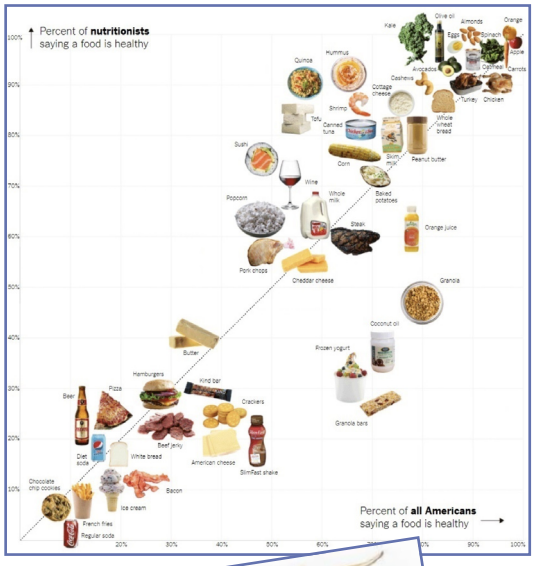
AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

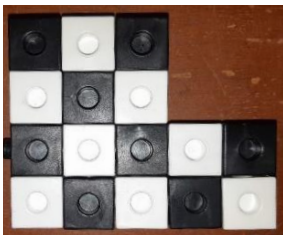
Vol. 59 #2
DEC 2020

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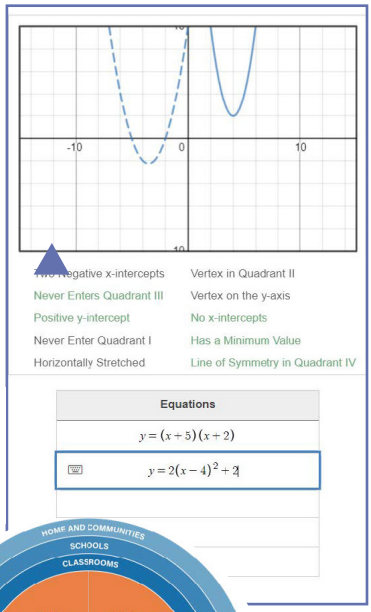
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See What's the Problem? Rearranging Rectangles



See Technology Corner: Menu Math



See Elementary Math Matters: If Math Were an Animal



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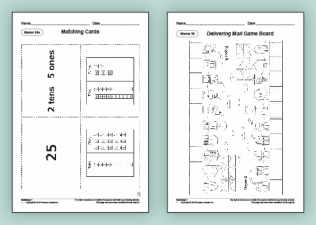




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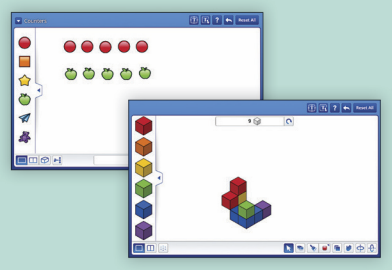


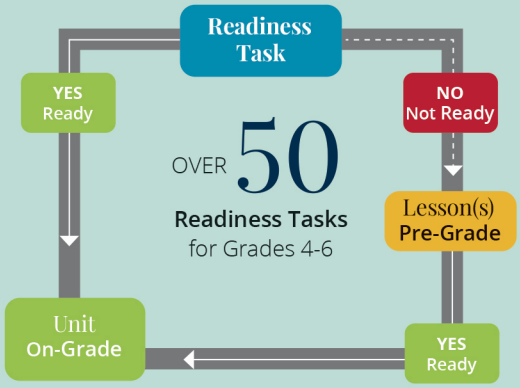
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Submission of Articles

The *Ontario Mathematics Gazette* (OMG) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *Ontario Mathematics Gazette*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles are to be word–processed, MS Word is preferred, and prepared according to the *Publication Manual of the American Psychological Association*, Seventh Edition. However, please use single-line spacing (not double). Articles should not exceed five numbered pages of text, and figures, images, and photographs should be placed in the text close to where they belong, with captions. The photographer's permission is required, and for photos of students under the age of 18, the written permission of a parent or guardian is required.

Please submit your article in one blind file (i.e., identity of author is not evident), and include author names, contact information including email and mailing addresses, photos—head and shoulders, biographies—less than 100 words, and all content removed for blinding in a second file. Please email these two files to Tim Sibbald at gazette@oame.on.ca.

Upon review, you will be notified whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected. The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME/AOEM.

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The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of OAME/AOEM.

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Editor
▲ **Tim Sibbald**
gazette@oame.on.ca

Associate Editors
▲ **Anne Yeager**
anne.yeager@rogers.com
▲ **Jacqueline Foster**
jacquie.a.foster@gmail.com

Associate Editor (French articles) /Rédactrice adjointe (articles français)
▲ **Kyla Kadlec**
Kyla.kadlec@oame.on.ca

***Abacus* Co-Editors**
▲ **Mary Lou Kestell**
marylou.kestell@oame.on.ca
▲ **Kathy Kubota-Zarivnij**
kkz@oame.on.ca

Copy Editing
▲ **Gitta Berg**
gitta.berg@sympatico.ca

Design and Production
▲ **Penny Clemens,** Graphic Designer
pennyydezign@gmail.com

Printing & Binding
▲ **Pole Printing,** Box 69, 1-89 King Street East
Forest, ON N0N 1J0 (519) 786-5112
poles@xcelco.on.ca

Advertising Manager
▲ **Robert Sherk**
4366 Snider Rd., Verona, ON K0H 2W0
Home (613) 374-1515
robert.sherk@oame.on.ca

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Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** by email or at the above address. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be print ready, and colour advertisements should have no bleeds.

Advertising Rates
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▲ Editor's Report



TIMOTHY SIBBALD, OCT, PhD
gazette@oame.on.ca

Tim Sibbald has held a variety of positions within the OAME/AOEM. He is a former president, vice-president, and director. He is an associate professor in the Schulich School of Education, Nipissing University, with a focus on mathematics education. He has edited a book on Interdisciplinary Math and two about the experiences of academics (with a third underway).

One of the presentations in the Virtual NCTM Leadership Conference identified ways that members engage with their organization. During the time I was engaged with the conference, I was also working on a project to improve the *Ontario Mathematics Gazette* index. The coincidence of these two events was profound. It was suggested in the leadership conference that many members begin with small engagement, encouraged by invitations from their colleagues. This often progresses as they realize the benefits of their engagement with the organization. In some cases, they progress to writing articles, though there are many other paths for increased engagement. What I noticed through the indexing work was the depth of the connection.

In the *Gazette* archives, we can see members who we now know to have had years of engagement, but we can see them through the archives in their younger roles. You can see their progression and find surprise connections—I have found professors I had and even the high school I attended mentioned. This sort of connection gives a deep perspective on the longevity of the *Gazette* and the relationship many authors have had with it over the years.

Through a leadership lens, the *Gazette* is one of many opportunities—and make no mistake, I applaud those who engage with their chapters, local conferences, annual conferences, creating resources, and many other roles within the organization. The point of the leadership conference was focused on the proverbial thin end of the wedge. It can be a period of time for many teachers, where they ask what they get for the membership fee. The surprising answer for many is an opportunity to engage with the highest-functioning mathematics educators, the catch being that getting the maximum benefit from membership entails participation!

With participation in mind, I would like to point out a variety of opportunities with the *Gazette* that may be of interest to some of the readership:

- Write a letter and share an opinion or something you have noticed.
- Contribute to the effort to celebrate the 60th anniversary of the *Gazette*.
- Serve as a reviewer for the *Gazette*.
- Contribute small items that help to fill gaps at the end of articles. This is well suited to poems, puzzles, cartoons, etc. (If you look back over the last few years, you will find examples such as word searches, patterns, photos, and games.)
- Write a book review. The editor has a guide and can explain the process to be reimbursed for the cost of the book, if you ask.
- Work with your students to generate student opportunities. An example is that I have expressed my interest in student-generated math cartoons and math artwork, and those invitations continue.
- Initiate something new—there is a push to facilitate the use of French; the index is being worked on; a resource for using the *Gazette* to help train the next generation of teachers needs updating. Perhaps start a *Gazette* discussion group in your local OAME/AOEM chapter.
- Write an article based on your own practice and fresh ideas. A video about this process was produced for the last annual conference and is available on the website.

Why should I belong to the Ontario Association of Teachers of Mathematics and Physics?

This question has been asked many times by Ontario mathematics teachers. For the teacher who is concerned only with direct personal benefits from membership, any answer that can be given will be unsatisfactory; but for the teacher who is seriously concerned about the state of mathematics education in the province, there are numerous reasons why the Association deserves [their] support.…(So begins an article by Garth Kaye, *Gazette 2*(2), p. 24.)

These are the opportunities that come to mind, but discussion of other possibilities is always welcome. In all cases, if you wish more detail, contact the editor (gazette@oame.on.ca).

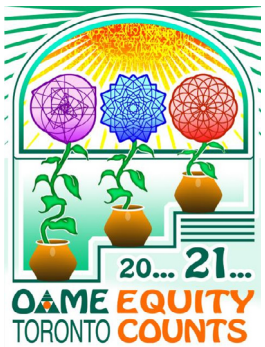
I call engagement with the organization the thin end of the wedge, whether through any of the listed *Gazette* opportunities or other forms of participation in the organization, such as nominating someone or letting your name stand for nomination (see details on nominations for the Board of Directors in this issue). It is a wedge in the sense that when you engage, you find the professional learning is powerful, and in many cases, suitable for one's resumé, summarizing progress in your career. In saying that, I place the emphasis on *your* growth as opposed to your career growth—they happen simultaneously through participation, but the reason why all the subject associations (and the OAME/AOEM is one among many in Ontario) are so important is because personal growth develops our professional skills and efficiency, which improves instruction and generates better outcomes for our students.

This mirrors our own students' growth. Student engagement facilitates students who need or want an opportunity to learn more. However, this is not a matter of students simply answering more homework questions; it entails variety in the forms of engagement that they experience. This is why diversity in enrichment needs to be sought. It is the same for teachers, where professional learning needs to go beyond further finessing of lesson plans. It must have variety, and the OAME/AOEM, as an independent long-established community, addresses that need.

That is why, in my humble opinion, our members should be encouraging their colleagues to look on the OAME/AOEM as a community that is worthwhile for the relatively low (not-for-profit) membership cost. It should not simply be a community that is there when you attend the annual conference. Instead it should be a two-way engagement with a recognition that you don't build community through sporadic involvement. I hope each member who reads this will tap a colleague on the shoulder and help clarify the role of the OAME/AOEM as a community that has been supporting teachers for nearly five decades (over a century, if you look to the organizations from which the OAME/AOEM has grown).

I encourage you to think about your participation, talk to a colleague, and see if you can help the community grow, while finding new opportunities for your own growth. While it goes without saying, I feel obliged to point out that teachers are more effective when they share and work together, and the OAME/AOEM is exactly that opportunity.

OAME/AOEM 20... 21…
Equity Counts: Addressing Equity in Mathematics Education and Best Classroom Practices



The primary focus of the conference is to address equity in mathematics education and

promote best classroom practices. This means to strive to have students attain proficiency in mathematics, regardless of race, gender, socio-economic status, or learning style. The conference will be held on Thursday, May 20, and Friday, May 21, 2021 at Beanfield Centre, Exhibition Place, Toronto.

Keynote speakers are Robert Q. Berry III, Dan Meyer, and Eugenia Cheng. Featured speakers are Brian Aspinall, Gail Burrill, Graham Fletcher, Nancy Kawaja, Alex Lawson, Lisa Lunney Borden, Karen Murray, Marian Small, and Christine Suurtamm.

During the OAME/AOEM 2021 Conference, all necessary measures will be taken to protect against COVID-19, including supplying hand sanitizer, making masks available, promoting safe food handling, and separating chairs to promote physical distancing. If, under the direction of Toronto's Medical Officer of Health, in-person conferences are not permitted due to COVID-19, the OAME/AOEM conference will transition to a virtual, online format. It will be complete with keynote and featured speakers, as well as breakout sessions. We would refund the portion of the fee (TBA) over and above our fixed expenses (e.g., speaker fees), with an option to request a full refund and full cancellation of your registration.

For further information, please visit the conference website at www.oame2021.ca. If you would like to join the organizing team, please contact us at inquiries2021 @oame.on.ca.

In This Issue

On the inside of the front cover is the land acknowledgement adopted by the OAME/AOEM. It is included as the *Gazette*'s acknowledgement and as a matter of documenting a feature of the organization.

Our president, **Judy Mendaglio**, gives a fascinating portrait of challenges migrating from French to English while learning math. A feature article introduces us to **OntarioMath**, which is a new OAME/AOEM website to support curriculum change. We have two features regarding Social–Emotional Learning (SEL). The first, by **Renee Billiau**, provides a historical perspective, while **Jeff Irvine**, in the first of a two-part article, provides an in-depth look at current thinking about SEL.

Bill Morrison, as a long-standing member of OAME/AOEM, provides an invited historical perspective. **Fred Ferneyhough**, in a letter to the editor, looks at a long trend in technology and suggests destreaming and CAS might fit well together.

Jacqueline Hill suggests using NCTM resources to address SEL. Later, Jacqueline and I (**Tim Sibbald**) report on the NCTM Virtual Leadership Conference. **Angelica Mendaglio** reports on the Field's Institute MathEd Forum, focusing on equity.

Carly Ziniuk brings many online resources together and shows how they can be integrated for effective teaching. **Lynda Colgan** brings several strategies for exploring SEL with students. **David Costello** draws on literacy strategies to address some aspects of SEL. **Dr. Rochelle Gutiérrez** is interviewed by **Ann Arden** and **Idil Abdulkadir**. This is Ann's final interview column. Thank you, Ann, for the interesting interviews you have conducted! **Ron Lancaster** gets to the core of using photos from an orchard. **Jennifer Holm** considers how data science fits with our curriculum. **Jérôme Proulx** examines details of domain that often get glossed over when solving equations. **Shawn Godin** rearranges rectangles to meet a perimeter requirement. **Mary Bourassa** brings Menu Math to life and shows how it can be connected to Desmos. Last, but not least, **Jeff Irvine** writes about the forward thinking of **W.W. Sawyer**.

The editorial team has a new Associate Editor, Kyla Kadlec, who is primarily providing support with French content. She has been involved in all French content over the last year and I hope she will continue for many more.

As always, thank you to the associate editors Anne Yeager and Jacqueline Foster, as well as reviewers who have been working behind the scenes. The 56 pages of this issue show that they have diligently responded as the need to address contributions has increased. Thank you!

Thank you to all our contributors for their efforts, and to the editorial team. The fact that the Gazette *has managed to continue to do what it does best in spite of the times speaks to a collective effort to overcome obstacles and persevere. Speaking as editor, but likely for the entire readership, thank you!*

Erratum: In the September issue on page 48, a formula appears with the equal sign and half in the wrong order. It should be $AC = \frac{1}{2}\left(\sqrt{d^2 - a^2} - \sqrt{d^2 - b^2}\right)$. Apologies for the error. ▲

In the news…

Western News is reporting that Donna Kotsopoulos will be the next dean of the Faculty of Education at Western University. Readers may know of Dr. Kotsopoulos through her role with the Mathematics Knowledge Network or as a professor at Wilfrid Laurier University, where she was cross-appointed between education and the mathematics department. (https://news.westernu.ca/2020/11/donna-kotsopoulos-to-become-new-dean-of-westerns-faculty-of-education/)

President's Message



JUDY MENDAGLIO
judy.mendaglio@oame.on.ca

Judy Mendaglio is the President of OAME/AOEM. She has worked in education in Ontario in many capacities: college instructor, secondary teacher and department head, university instructor, elementary and secondary consultant, and writer. Currently, Judy devotes most of her time to her many volunteer activities.

Hello again, everyone. Bonjour.

As I am writing this, we are now into the school year and things are pretty chaotic. Many schools are facing yet another reorganization as parents (including guardians) elect to have their children learning remotely because of rising regional COVID-19 numbers. Other schools are facing yet another reorganization because parents, seeing low infection rates in their regions, are deciding to send their children back into the classroom. I cannot recall a time when teachers were faced with this level of uncertainty—almost daily! All we can do is support each other as best we can, so that we have the energy to support our students, who are also feeling this uncertainty, and who may be experiencing levels of stress that surpass those of a "normal" year. Remember that, whatever subjects or courses you may be teaching, that teaching will only result in learning if students are able to feel safe, valued, and included.

I promised in my first column that I would tell you a little bit about myself in columns to come, and so, here we are. In that President's Message, I said that growing up in Montreal had a profound effect on my outlook in many ways, and that having worked in a field outside of education impacted who I became as a teacher.

First—Growing up in Montreal

From the moment that my husband and I arrived in London, Ontario from Montreal, Quebec, to attend graduate school, we felt like strangers in a strange land. Even though we were just a seven-hour drive down Highway 401 from where we had grown up, we found everything different. We often described the "culture shock" we experienced. We could not find foods we were used to (and, yes, the legendary Montreal bagels were one such food). In stores, price signs were weird—$6.49 rather than 6,49 $. We found the learning environment at Western very staid and stodgy

Message de la présidente



JUDY MENDAGLIO
judy.mendaglio@oame.on.ca

Judy Mendaglio est la présidente de l'AOEM/OAME. Elle a exercé de nombreuses fonctions dans le secteur ontarien de l'éducation : formatrice au niveau collégial, professeure et chef de section du secondaire, chargée de cours au niveau universitaire, consultante pour le primaire et le secondaire et écrivaine. Judy se consacre actuellement à de nombreuses activités bénévoles.

Bonjour à tous. Hello.

Dans le présent message, le masculin est utilisé sans aucune discrimination et uniquement dans le but d'alléger le texte.

Au moment où j'écris ces lignes, nous sommes maintenant dans l'année scolaire et les choses sont bien chaotiques. De nombreuses écoles sont confrontées à une nouvelle réorganisation, car les parents (y compris les tuteurs) choisissent que leurs enfants apprennent à distance en raison de l'augmentation du nombre de COVID-19 au niveau régional. D'autres écoles sont confrontées à une autre réorganisation car les parents, constatent de faibles taux d'infection dans leur région, décident de renvoyer leurs enfants dans la salle de classe. Je ne me souviens pas d'une époque où les enseignants étaient confrontés à ce niveau d'incertitude—presque quotidiennement ! Tout ce que nous pouvons faire, c'est nous soutenir mutuellement du mieux que nous pouvons, afin d'avoir l'énergie nécessaire pour soutenir nos élèves, qui ressentent eux aussi cette incertitude et qui peuvent connaître des niveaux de stress supérieurs à ceux d'une année « normale ». N'oubliez pas que, quels que soient les sujets ou les cours que vous enseignez, cet enseignement n'aboutira à un apprentissage que si les élèves se sentent en sécurité, valorisés et inclus.

J'ai promis dans ma première chronique que je vous parlerais un peu de moi dans des colonnes à venir, et voilà, nous y sommes. Dans ce message de la présidente, j'ai dit que le fait d'avoir grandi à Montréal avait eu un effet profond sur mes perspectives à bien des égards, et que le fait d'avoir travaillé dans un domaine autre que l'éducation avait eu un impact sur celui que je suis devenue en tant qu'enseignante.

D'abord—grandir à Montréal

Dès l'instant où mon mari et moi sommes arrivés à

(certainly compared to the informal "everyone heads to the pub after class"—profs and students together—that we had had in Montreal). We were not used to so much English. So much English! And *we* were English! We had come from a much more ethnically diverse, religiously diverse, culturally diverse city. Everything felt familiar (we spoke English, we knew winters), but at the same time, so foreign.

Many years later, when I decided to go back to school to pursue a professional future in education, I encountered another strange land. Now, recognize that I had studied mathematics, at school or at university, for 18 years. But, when I started my B.Ed. program, wow, what was all this stuff everyone was talking about? When we left elementary school in Montreal, there was no middle school, we started high school. Until Grade 7, we had Arithmetic on our report cards. Not math. "Math" began in Grade 8. Then there were the acronyms: BEDMAS, CAST, FOIL, SOHCAHTOA. I had never seen any of these before, yet everyone seemed to be completely familiar with them. I felt as though I should know these things too, and I was too uncomfortable to actually ask anyone what these things were, so I didn't, I did not even know they were acronyms until I saw them written down. I thought *semesters* were only in university and was shocked to find that students did not study math all year long. There was "March Break." What is that?

There were also the weird student-centred practices I needed to learn about: electives and spares, marks given for quizzes and tests, and marks accumulating all year long? Marks for doing homework? Really? Not assignments that were handed in, mind you, but homework. What is a Guidance Counsellor? And those super weird, non-descriptive course names. (Grade 10 Advanced? What happened to the Geometry course? The Algebra course? That's what I took.) When Grade 13 became OAC and then finally disappeared, I could not understand other teachers' consternation. We left high school after Grade 11 and, I suppose because we knew from the start that that was what was going to happen, we were "ready." I always felt that it was weird telling a 20-year-old student not to bring their coffee into class.

Studying to become a teacher in Ontario required not just that I learn about the Education Act and classroom-management techniques. No. I had to learn a new language, a new culture, and make some serious adjustments to what I perceived to be the natural, right way of *doing* math and *teaching* math.

I use this experience and these feelings as a lens through which to reflect on issues of culture, inclusivity, and equity in education. My experience of moving was really difficult for me, but it was minor compared to the experiences of many

London (Ontario) en provenance de Montréal (Québec) pour suivre des études supérieures, nous nous sommes sentis comme des étrangers dans un pays étranger. Même si nous n'étions qu'à sept heures de route de la 401 où nous avions grandi, nous trouvions que tout était différent. Nous avons souvent décrit le « choc culturel » que nous avons vécu. Nous ne trouvions pas les aliments auxquels nous étions habitués (et, oui, les bagels légendaires de Montréal en étaient un de ces aliments). Dans les magasins, les affiches de prix étaient bizarres : $6.49 au lieu de 6,49 $. Nous avons trouvé l'environnement d'apprentissage chez Western très statique et bourratif (certainement comparé à l'ambiance informelle « tout le monde va au pub après les cours »—les profs et les étudiants ensemble—que nous avions à Montréal). Nous n'étions pas habitués à autant d'anglais. Tant d'anglais ! Et nous étions anglais ! Nous venions d'une ville beaucoup plus diversifiée sur le plan ethnique, religieux et culturel. Tout nous semblait familier (nous parlions anglais, nous connaissions les hivers), mais en même temps, tellement étranger.

De nombreuses années plus tard, lorsque j'ai décidé de retourner à l'école pour poursuivre un avenir professionnel dans l'éducation, j'ai rencontré un autre pays étranger. Maintenant, reconnaissez que j'ai étudié les mathématiques, à l'école ou à l'université, pendant 18 ans. Mais, quand j'ai commencé mon B. Ed., wow, de quoi parlait tout le monde? Quand nous avons quitté l'école primaire, à Montréal, il n'y avait pas d'école intermédiaire, d'abord nous avions commencé l'école secondaire. Jusqu'à la 7e année, nous avions l'arithmétique sur nos bulletins scolaires. Pas les maths. Les « maths » ont commencé en 8e année.

Puis il y avait les acronymes : PEDMAS, CAST, FOIL, SOHCAHTOA. Je n'avais jamais vu aucun de ces acronymes auparavant, pourtant tout le monde semblait les connaître parfaitement. J'avais l'impression que je devrais connaître ces choses aussi, et j'étais trop mal à l'aise pour demander à quelqu'un ce que c'était, donc je ne l'ai pas fait. Je ne savais même pas qu'il s'agissait d'acronymes jusqu'à ce que je les aie vus par écrit. Je pensais que les *semestres* n'étaient qu'à l'université et j'ai été choquée de constater que les étudiants n'étudiaient pas les mathématiques toute l'année. Il y avait les « vacances de mars ». Qu'est-ce que c'est ? Puis il y avait les pratiques bizarres centrées sur les étudiants dont je devais me renseigner : les cours optionnels et les périodes libres, les notes attribuées pour les tests et les quiz, et les notes qui s'accumulent tout au long de l'année ? Les notes attribuées pour les devoirs ? Vraiment ? Pas de devoirs à être remis, mais des devoirs à faire à la maison. Qu'est-ce qu'un conseiller d'orientation ? Et ces noms de cours super bizarres et non descriptifs. (10e année

of our students. I know how easy it is to assume that everyone has shared our experiences and shares our beliefs. I still do it. But I am getting better. We are all getting better. We ask. We listen. That is the place to start. As we want our students to learn about mathematics, so too, our students want us to learn about them and see them as people, distinct from one another, in all their glorious differences and complexities. Our students are not like reproductions of an old master, hung side-by-side, indistinguishable one from another. They are each works of art, and they create a wonderful gallery experience for us, their teachers. ▲

▲ Letter to the Editor

It has been announced that Grade 9 will be destreamed when the next secondary curriculum revision is released, which I assume will be next year. Destreamed Grade 9 is a return to what we tried in the early 1990's, and I wish to offer an opinion. This letter reflects my passion for teaching mathematics—it is a personal opinion and does not necessarily reflect the opinions of OAME/AOEM, even if I happen to be an Executive Director of OAME/AOEM.

For Lynda and I, our teaching careers have spanned almost five decades—we've seen a few changes. Teaching math used to be mostly about teaching algebraic skills, and as teachers, we valued the acquisition of manipulative skills, but, back then, there wasn't a lot of technology being used in mathematics. Lynda and I bought a scientific calculator in 1976 that was "just amazing." We no longer needed logarithm tables or trigonometry tables, and doing square roots by hand became a thing of the past. Now, if only my department head would have allowed me to use them back then, I could have done a lot of marvelous things with students. Instead, I was relegated to using a six-foot-long slide rule mounted at the top of my chalkboard. Don't laugh! I was good at using that thing and so were my top students.

When the curriculum changed in 1985, the use of scientific calculators was encouraged in the new documents. My old department head retired and our new department head allowed us to move ahead with using scientific calculators with our students. Then, one day in 1987, two of my students showed up with something called a graphing calculator, on which they had drawn the cartoon characters Gumby and Pokey—because they knew that I didn't like those characters. And, believe it or not, in 1987, I was involved in Peel's first attempt at using something called electronic mail. I could not see that catching on back then.

continued on page 8

avancée ? Qu'est-il arrivé au cours de géométrie ? Le cours d'algèbre ? Ce sont des sujets que j'ai suivis.) Quand la 13e année est devenue CPO et finalement a disparu, je ne pouvais pas comprendre la consternation des autres enseignants. Nous avons quitté l'école secondaire après la 11e année. Je suppose que nous étions « prêts car nous nous savions dès le départ que c'était ce qui allait se passer ». J'ai toujours trouvé bizarre de dire à un élève de 20 ans de ne pas apporter son café en classe.

Pour devenir enseignant en Ontario, je n'ai pas seulement dû me familiariser avec la loi sur l'éducation et les techniques de gestion de classe. Non ! J'ai dû apprendre une nouvelle langue, une nouvelle culture, et faire des ajustements sérieux à ce que je considérais comme la façon naturelle et correcte de *faire* et *d'enseigner* les mathématiques.

J'utilise cette expérience et ces sentiments comme une lentille à travers laquelle je réfléchis aux questions de culture, d'inclusion et d'équité dans l'éducation. Mon expérience du déménagement a été très difficile pour moi, mais elle était mineure par rapport à celle de beaucoup de nos étudiants. Je sais à quel point il est facile de supposer que tout le monde a partagé nos expériences et nos convictions. Je le fais encore. Mais je m'améliore. Nous nous améliorons tous. Nous demandons. Nous écoutons. C'est par là qu'il faut commencer. Tout comme nous voulons que nos élèves apprennent les mathématiques, nos élèves veulent aussi que nous les apprenions et que nous les considérions comme des personnes, distinctes les unes des autres, dans toutes leurs différences et complexités glorieuses. Nos élèves ne sont pas comme des reproductions d'un vieux maître, accrochés côte à côte, indiscernables les uns des autres. Ils sont tous des œuvres d'art, et ils créent une merveilleuse expérience de galerie pour nous, leurs enseignants. ▲

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Letter to the Editor, continued from page 7

In 1992, Lynda and I took a leave of absence and we taught in the United Arab Emirates for three years. We taught in networked computer labs, using an integrated software package called DERIVE!, and that experience completely changed our perspective on using technology.

So, fast-forward to the late 90's. Our curriculum caught up to the NCTM Standards, and our teaching changed radically. The approach was revamped in a manner with which most teachers were not familiar. We were starting to use rubrics for assessment, and we had technology that was either licensed for computers or handheld devices. The Ontario Ministry of Education came up with large sums of money to invest in computer labs, where we could use software such as dynamic geometry, dynamic statistics, and graphing, along with the old standby, the spreadsheet. The handheld devices that we were using allowed students to visualize the mathematics that we were presenting in class. Many of the curriculum objectives strongly emphasized the need to use graphing calculators. We also had access to virtual manipulative tools.

When the curriculum was modified in 2005–2006, a change came along that bears some comments. The use of Computer Algebra Systems (CAS) was suggested as a tool, with the idea that its use could be advantageous in teaching algebraic concepts for students who struggle with algebra. It was not mandated, but it was listed as a tool that teachers should consider.

The use of a CAS could turn out to be a huge advantage if we return to destreaming in Grade 9 mathematics next year. Many teachers see the use of a CAS device as just a black box that allows students to avoid learning the algebraic skills. Our experience has been anything but that. We see CAS as a teaching tool, and if you are an OAME/AOEM member, check out the *Gazette* archive and you will find several articles related to using CAS to teach mathematics.

I believe that the rate at which children learn mathematical concepts varies greatly. Our top students sometimes learn in spite of our efforts, but our weaker students are going to show up in Grade 9 mathematics with those stronger students. If we only teach to the best students, we're going to leave others in the dust. We need tools that help all children move forward with as many opportunities as possible. The use of technology speaks to our current students in a way that traditional methods can't. I'd encourage the use of CAS to help teach algebraic concepts to all students in a destreamed mathematics class.

Sincerely, Fred Ferneyhough

▲

▲ Blast from the Past! W.W. Sawyer



JEFF IRVINE
dr.jeffrey.irvine@bell.net

Jeff Irvine, PhD, has been a secondary mathematics teacher, department head, and vice-principal. He has taught at three faculties of education and at a community college. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. With over 45 years in education, Jeff is particularly interested in the interplay of student motivation and mathematics achievement.

"If you have two minutes to solve a problem, the first ninety seconds should be spent on thought."

Dr. Walter Warwick Sawyer (1911–2008) was a mathematics education professor emeritus at the University of Toronto. Dr. Sawyer was passionate about the need to teach mathematics in context. He argued that students, especially young students, needed hands-on, real-world applications of the arithmetic and mathematics that they were learning, and manipulatives, often ordinary household items, were vital in their understanding of mathematical concepts. Dr. Sawyer believed that students needed to develop both conceptual understanding and procedural fluency if they were to succeed in mathematics and in later life. In this, Dr. Sawyer's views were consistent with the current philosophy of mathematics education in Ontario.

At the time, in the age of New Math and beforehand, his views were seen as somewhat radical. New Math focused on students learning abstract overarching systems. Using set theory and logic, the teaching was devoid of real-world applications. Dr. Sawyer believed that it was important for students to be interested in their learning and to develop (or retain) positive attitudes toward mathematics.

Dr. Sawyer thought deeply about the learning of mathematics, and he was very eloquent in his speech. As a consequence, he was a very quotable math educator. The quotation at the beginning of this article was Dr. Sawyer's paraphrasing of a quotation attributed to Albert Einstein. There are many other Sawyer quotations extant. The website www.wwsawyer.org has an entire section devoted to his quotations.

There were six articles published in the *Gazette* authored by Dr. Sawyer or about his work. Below are brief comments

on these articles, many of which provide really interesting reading, as they reflect to a large extent views that are held in math education today.

In the second issue of the *Gazette*, John McKnight (1962) summarized a speech given by Dr. Sawyer at the 1962 annual conference of the Ontario Association of Teachers of Mathematics and Physics (forerunner of OAME). The topic of the speech was mathematics of rocketry, a timely topic given the space race between the United States and the then Soviet Union. Also in that *Gazette* was a letter signed by 75 professors, including Dr. Sawyer, George Pólya, and others, pointing out their concerns with the New Math's lack of connections to real life, and what they saw as the extreme abstractions of set theory and logic being taught beginning in Grade 1.

In 1965, the *Gazette* published a book review of Dr. Sawyer's book, *What Is Calculus About?*. The book explains in straightforward terms the basic ideas of calculus, related to real-life applications. A 1966 article by Dr. Sawyer, titled *Missing Links*, advocates for hands-on learning of arithmetic using manipulatives, and the need to stress understanding in addition to building operational fluency. He points out that teachers should not assume that all students function at the same Piagetian level; some students will take longer to acquire concepts; and wrong answers should be used to diagnose misconceptions, not just for evaluation on a pass/fail basis.

"To master anything from football to relativity requires effort. But it does not require unpleasant efforts, drudgery. The main task of any teacher is to make a subject interesting." (www.wwsawyer.org/sawyer-quotes.html)

In an untitled talk published in the *Gazette* in 1970, Dr. Sawyer discusses the Ontario implications of the Hall Dennis Report, *Living and Learning* (1967), and he criticizes the lockstep approach to learning that characterized math education at that time as having devastating impacts on students' attitudes toward mathematics. Dr. Sawyer argued for evidence-based change, and derided the prevailing "This math is important to learn" with the simple question "Important to whom?"

A 1972 article by Dr. Sawyer examined elementary school mathematics education, and found that student attitudes were often negatively impacted by the assumption that all students learn at the same rate. "All evidence indicates that young children enjoy arithmetic, but that this interest is progressively destroyed by the adult population" (*Gazette 11*(1), p. 52). Dr. Sawyer again argued for the use of concrete materials, stating that their use was essential, and that young children needed hands-on, real-world relationships to the mathematics being taught them. He decried the notion that all mathematics was paper-and-pencil activities. He also felt that teachers should use student responses to diagnose misconceptions and remediate them. "The main job of a teacher is not to talk, but to listen" (*Gazette* 11(1), p. 58).

Dr. Sawyer's final article in the *Gazette* (1974) discussed the strengths and weaknesses of the Ontario New Math curriculum, which was then coming under pressure from a "back to basics" movement. The positive aspects that he identified included the addition of Calculus to the curriculum, which had been advocated since the 1930's; the addition of matrices and linear algebra; and the updating of some areas of the mathematics curriculum that had remained virtually unchanged for over 50 years. He objected to the view that mathematics is the study of abstract systems throughout the Kindergarten to Grade 13 system; the extensive use of the language of set theory; and finally, the view that everything before the New Math was obsolete and valueless.

Dr. Sawyer worked in a wide variety of countries around the world, including England, Scotland, Ghana, New Zealand, the United States, and Canada. His tenure in Canada was his longest appointment, from 1965 until his retirement in 1976. After his death, the University of Toronto established a scholarship in his name, awarded annually to an outstanding student who had demonstrated exceptional achievement in Mathematics. The Navnirmiti Society in Pune, India established a website, www.wwsawyer.org, to celebrate his life and work. Dr. Sawyer's book, *Mathematician's Delight*, is considered one of the best-selling mathematics books in history. His work has been translated into over ten languages and his books continue to sell today.

"Complete success would mean that every individual felt, 'I enjoyed the mathematics that I had time to learn. If I ever need or want to learn some more, I shall not be afraid to do so.'" (www.wwsawyer.org/sawyer-quotes.html)

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▲ Introducing a New Member of the OAME/AOEM Family!!

A Message from the OAME Elementary Mathematics Project Team

OAME/AOEM is very happy to welcome OntarioMath.support to its family of sites that are here to serve and support the math teachers in Ontario.

OntarioMath.support (Figure 1) joins our beloved matriarch, oame.on.ca (Figure 2), and her children, math4thenines.ca (Figure 3) and mathies.ca (Figure 4).

So what is this new Ontario math website all about? This wonderful new site is the place to be this season! It has all the latest news about the revised elementary Mathematics curriculum (2020), and it is available in English and in French! Designed by an amazing team of Ontario educators, this is where you will find webinars, lesson and assessment plans, support materials, and links to Ministry resources—all in one place!

Back in the spring and summer of 2020, before the revised elementary math curriculum was even finalized, members of OAME/AOEM and her sister math subject association, AFEMO (l'Association Francophone pour l'Enseignement des Mathématiques en Ontario), began working together to define a project that would support teachers during the first year of implementation of the revised curriculum and even beyond. All the components of the project would be developed by teams of English and French educators, creating resources in both languages, by working together. Well, the project is live!

The key components of the project are as follows:

- A series of five webinars, presented live in English and in French, introducing a different component of the Curriculum Context (what we once called the "Front Matter"), so that teachers in Ontario can better understand the intent of the curriculum. While the expectations might tell us what students are to learn, the curriculum context explores why and how they are learning it. Each webinar is offered on several dates, to give educators many opportunities to attend a live session, and then a recorded version is posted on OntarioMath.support.
- A set of Lesson and Assessment Plans for:
  - Number (with a focus on fractions)
  - Coding
  - Mathematical Modelling
  - Financial Literacy
- Thirteen unique lesson and assessment plans will be created for each of Grades 1 through 8, each available in both languages.

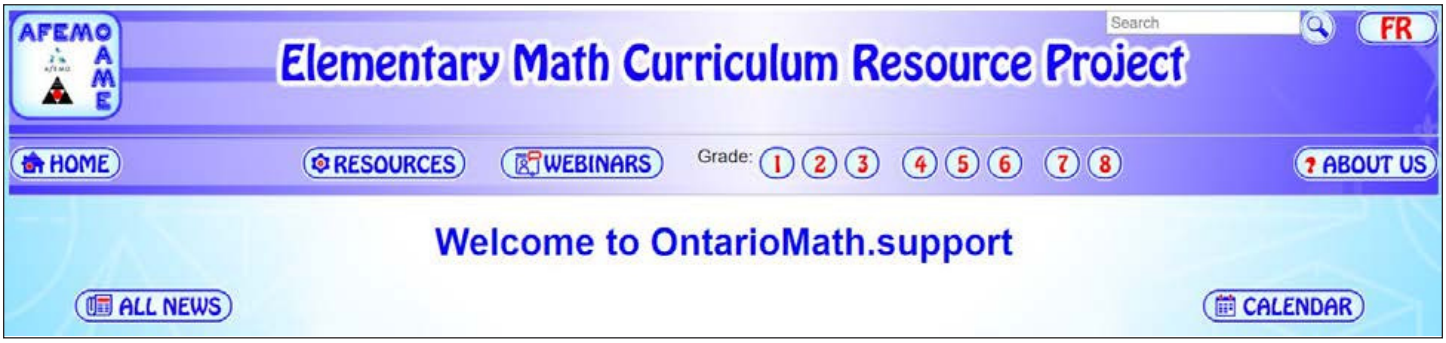


Figure 1: *OntarioMath.support*



Figure 2: *oame.on.ca*

- A series of webinars, again presented live in English and in French, will be introducing the lesson and assessment plans. Webinars will be provided for each division and will be specific to one content area. For example, for Coding, there will be a webinar that introduces the Grades 1, 2, and 3 lesson and assessment plans; a different webinar to introduce the Grades 4, 5, and 6 lesson and assessment plans; and a third webinar to introduce the Grades 7 and 8 lesson and assessment plans. Again, each webinar will be offered in English and in French, and will be repeated live on several different dates, and then a recorded version will be posted on OntarioMath.support.
- OntarioMath.support is the website where anyone can register for a webinar, pick up one of the lesson and assessment plans, check out the additional supports, or just hang out for a while.

Already, recordings of webinars have been added to the site. Slide decks are there too, as are some of the resources and supports attendees have asked for in their feedback.

Please consider signing up for the newsletter so you don't miss out on getting just the right resource or professional learning you were wanting. We are very proud of the work we have done so far, and there is more to come!



Figure 3: *math4thenines.ca*

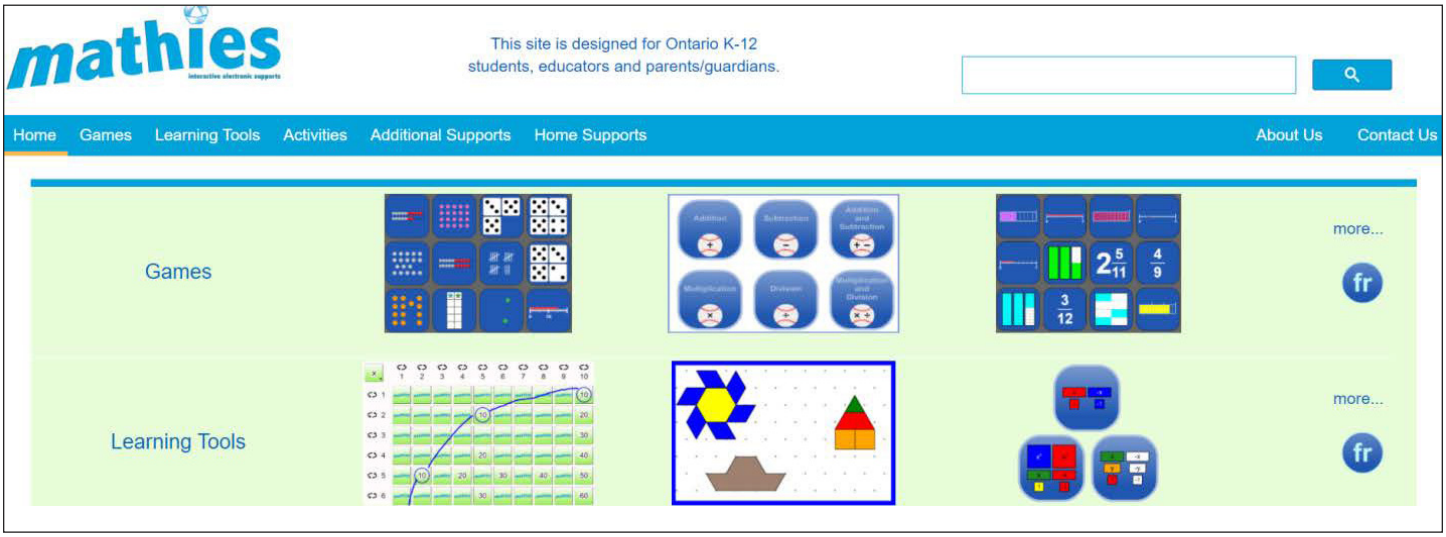


Figure 4: *mathies.ca*

▲ Technology Corner: Menu Math



MARY BOURASSA
mary.bourassa@ocdsb.ca

Mary Bourassa teaches mathematics at West Carleton Secondary School in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She has presented workshops internationally, authored mathematics resources, is a past VP of OAME, and a Past President of COMA (now O34ME). An award-winning teacher, Mary continually strives to learn new and better ways of helping students learn and love mathematics.

Some of the most useful and versatile mathematical tasks are ones that provide a structure that can then be adapted to a variety of content. *Menu Math* is just such a structure. It was the brainchild of Nat Banting (@NatBanting), and the collection can be found here at www.natbanting.com/menu-math/.

In Nat's words, "A Math Menu is a collection of constraints that appear as an unordered list, generally about 6–10 constraints long. Each menu prescribes a type of mathematical object that needs to be designed to satisfy these constraints. That object could be a certain type of function, an expression, a shape, a number, etc. The key to a Menu Math task is this: Each constraint must be satisfied at least once, and students try to complete this goal *using as few mathematical objects as possible*."

The menus that have been curated on the site have topics that range from shapes to characteristics of numbers to data sets and calculus. They span elementary and secondary grades and are available in both English and French.

These tasks turn the usual questions we ask upside down. Instead of having students pull out a characteristic given a function/expression/shape/number/graph, students will have to create the object that has that characteristic and other characteristics that are part of the menu. For example, instead of asking for the slope of a linear equation, students will be asked to create a linear equation that has a positive slope. They will then have to decide whether the equation they created also meets other constraints, such as a negative *x*-intercept, or that the graph does not go through a particular quadrant. The decision making required to complete these tasks inherently leads to a higher level of thinking, but the openness of the structure provides the opportunity for every student to access the task. There is also no single correct answer, but many ways in which to create objects that meet all constraints.

Let's look at some examples of Menu Math tasks on the site. The first, shown in Figure 1, was created by Chris Hunter (@ChrisHunter36).

Characteristics of Numbers Menu Task:

Build as *few* numbers as possible to satisfy each constraint at least once.

A.	Is even	B.	Is a multiple of 3
C.	Is a perfect cube	D.	Is prime
E.	Is a factor of 72	F.	Is a perfect square
G.	Has exactly 4 factors	H.	Is odd
I.	Is composite	J.	Has both 4 and 6 as factors

Which constraints pair nicely?
Which constraints cannot be paired?
Is it possible to solve in 2, 3, or 4 numbers?

Describe how and why you built each number.
Be sure to identify which numbers satisfy which constraints.

Figure 1: *Example of a characteristic number menu task*

As a starting point, students could build one number to satisfy each constraint. This would demonstrate that they understand each constraint; however, the goal is to build as few numbers as possible. One strategy involves determining whether there are constraints that cannot be paired. These could be separated into groups, and then students can determine which constraints work nicely together within each group. In the example from Figure 1, constraints A (is even) and H (is odd) cannot both be met using the same number, so at least two numbers are required to satisfy all constraints.

Another strategy is to choose a number that meets one constraint and grow from there. If the number, being a perfect cube, were the starting point, then the student could determine which other constraints that number also met. In the first example, the number 27 satisfies constraints C (is a perfect cube), B (is a multiple of 3), G (has exactly 4 factors), H (is odd), and I (is composite). Next, students will have to find another number that satisfies all remaining constraints, or determine that they need more than two numbers to meet all constraints.

A third strategy could be to address the most difficult constraint first. This would often lead to other constraints also being met. This strategy could be particularly helpful to get a student who may feel overwhelmed by the task going forward.

The second example addresses advanced functions or calculus. It is shown in Figure 2 and deals with rational functions.

Rational Functions Menu Task:

Build as *few* rational functions as possible to satisfy each constraint at least once.

A.	Has a hole at x = 4
B.	Has a double root at x = -3
C.	Vertical asymptote at x = -2
D.	Has a point of inflection at (1 , 0)
E.	Has a horizontal asymptote y = 2
F.	Has an oblique asymptote at y = x + 2
G.	Has a single root at the origin
H.	Has a y-intercept at -4

Which constraints pair nicely?
Which constraints cannot be paired?
Is it possible to solve in 2, 3, or 4 rational functions?

Describe how and why you built each rational function. Be sure to identify which rational functions satisfy which constraints.

Figure 2: *Example of a rational function menu task*

The constraints that do not pair nicely may not be obvious, but should become apparent as a student works through this task. As constraints are then combined, students need to determine whether each new combination has repercussions on previously used constraints. For example, adding a factor to create a vertical asymptote will change the *y*-intercept of the graph. The combinations that students will try, and the adjustments that ensue, make this task both interesting and challenging. Menu Math highlights the need to understand the relationship between mathematical ideas.

Menu Math tasks adapt well to in-person or virtual group work. If students are working remotely, using a collaborative whiteboard application such as Jamboard allows students to work together to find one or more solutions. Alternatively, these tasks can be done individually as practice, working on a particular topic or even as an evaluation. Every student can start a Menu Math task, but it takes more skill and thought to complete the challenge with as few objects as possible. The addition of choice increases the metacognitive demand required to successfully complete the task.

It is my experience that students need to practise working with this structure before completing one as an assessment. Using Menu Math tasks as review of previous work allows students to become familiar with the requirements of the task, while strengthening the connections for that topic. For example, at the secondary level, students could work on a fractions Menu Math task before doing one that is specific to their course.

A few Menus have been turned into Desmos activities. They are also linked on the Menu Math page. Figure 3 shows a parabola Menu Math activity. The constraints are listed below the grid.

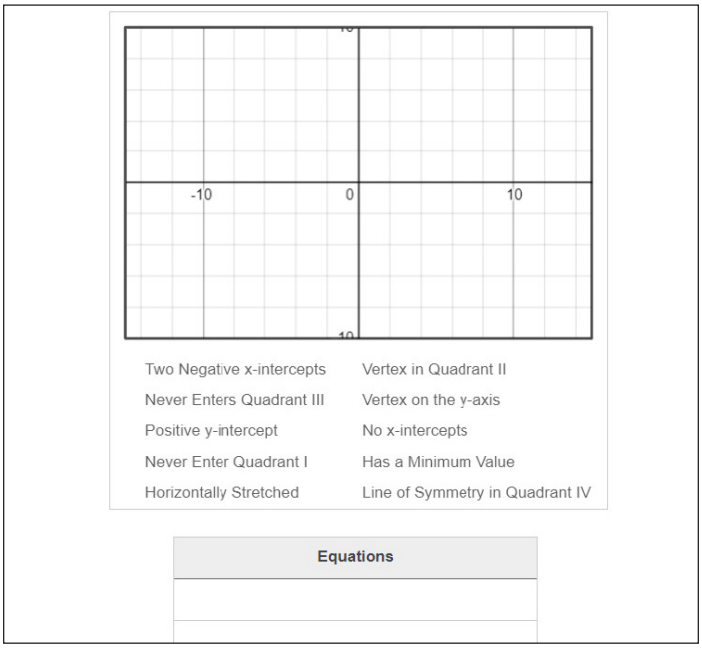


Figure 3: *Parabola Menu Math activity*

When an equation is entered, the graph is shown, and the font for the constraints that are met by that equation changes to green, as shown in Figures 4 and 5.

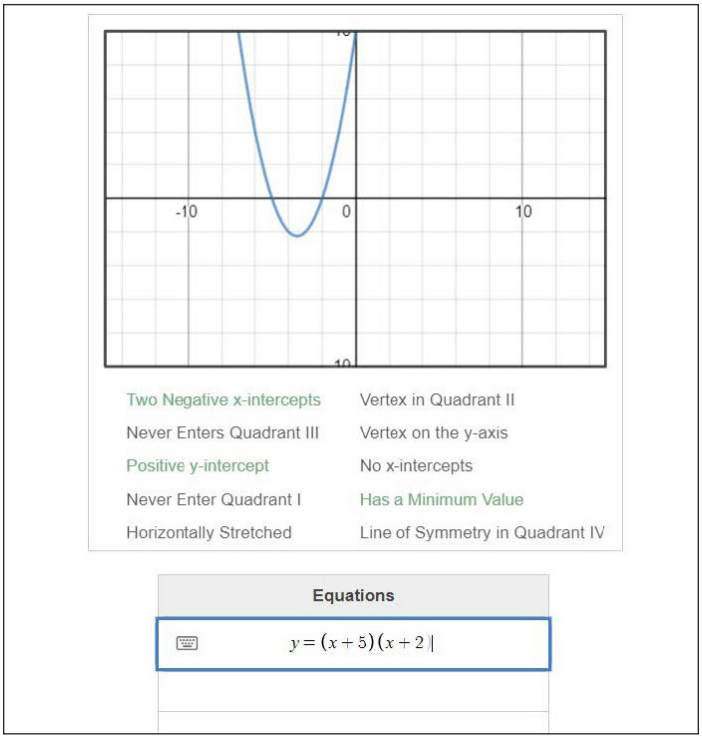


Figure 4: *Example function showing which constraints are met*

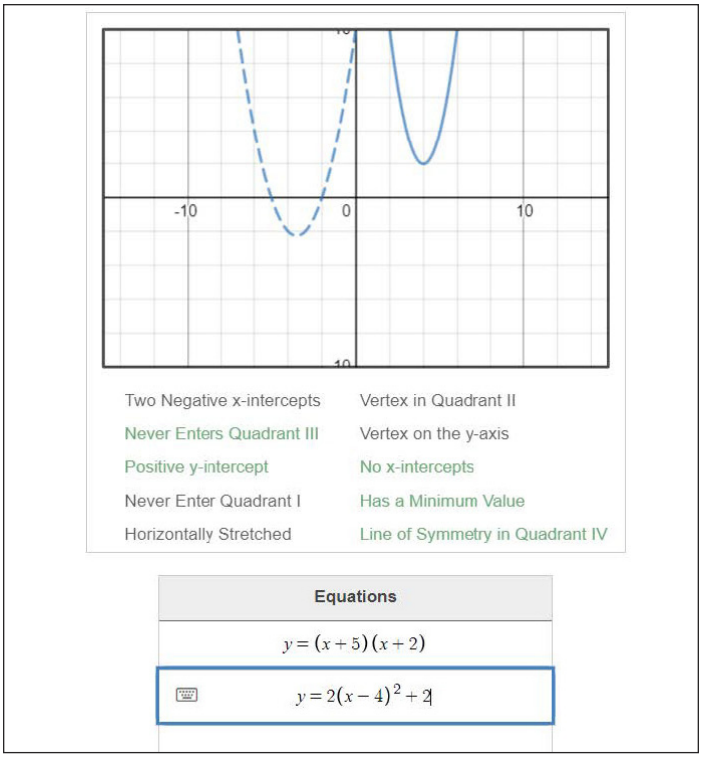


Figure 5: *Adding a second function*

As more equations are entered, students have to keep track of which constraints have been met, and again, make adjustments to try to meet all constraints with the fewest equations possible.

The visual aspect of doing Menu Math with Desmos can help strengthen students' understanding around functions and their graphs. As they create an equation to meet a particular constraint, they immediately get feedback on whether it was successful and can adjust as needed.

I invite you to explore the possibilities with Menu Math and work through a few yourself from the many that have been shared. Nat Banting's site gives you insight into the student experience. This helps to better anticipate issues that your students may have, and allows you to prepare questions that might help nudge student thinking forward. These are delightful tasks, and I hope you enjoy them as much as I do. The list of Menu Math tasks continues to grow, so be sure to share what you create and add to the collection at www.natbanting.com/menu-math/. ▲

Did you know …

Roger Penrose who shared the 2020 Nobel Prize in physics lived in London, Ontario, between the ages of 8 and 14. This was during world war II.

▲ What's the Problem? Rearranging Rectangles

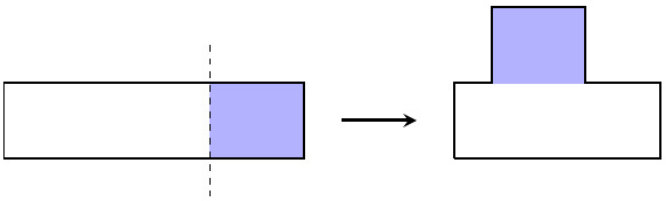


SHAWN GODIN
shawn.godin@ocdsb.ca

Shawn teaches at Cairine Wilson Secondary School in Orleans. He strongly believes in the central role of problem solving in the mathematics classroom. He continues to be involved in mathematical activities: presenting workshops, writing articles, working on local projects, and helping create mathematics contests.

Welcome back, problem solvers. I hope your year is going well. In this year, unlike any other, we are all being forced to teach under new conditions. As Monty Python says, "Always look on the bright side of life!" This gives you a perfect opportunity to try out some new things in your classroom, perhaps having students playing with some different types of tasks. For example, last time, I left you with the following problem:

> *A rectangle with perimeter 20 and area 16 is cut parallel to its shortest side to create two unequal rectangles. The smaller of these rectangles is then translated so that it abuts the top of the larger, as shown below.*



> *If the perimeter of this new figure is 19, what is the area of the smaller rectangle?*

This problem appeared as question #8 in the team questions from game 1 of the 2018–2019 Nova Scotia Math League (NSML). The NSML was started in 2002 and based on the Newfoundland Math League. Three times a year, teams of four students meet at regional centres to compete in a three-hour competition. The competition is made up of team problems and two separate relays. The top teams at the end of the season meet for the provincial championships. You can learn more about the math league and sample some of the problems at their website: http://cs.smu.ca/ ~nsml.

When faced with a mathematical *problem*, that is, a situation that you have never encountered before, it is

important to keep in mind that our first strategy for solving the problem may not yield the desired result. In cases where our technique fails, it is useful to reflect on what we did to see if we have learned something that might prove useful moving forward. It's always good to keep in mind the mathematical processes from the curriculum documents. They really outline the essence of being a mathematical thinker.

For example, we might decide to solve the problem, using a physical model. If we are using manipulatives, like linking cubes, then the only rectangles we can build that have area 16 square units are the 1 by 16, 2 by 8, and 4 by 4 rectangles. We can then determine that the 2 by 8 rectangle has area 16 and perimeter 20 as desired.

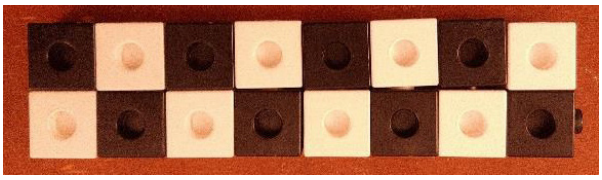


Figure 1: *A 2 by 8 rectangle*

We can proceed by breaking larger and larger rectangles off the end and putting them on top. In Figure 2, we see what happens when we break off pieces of sizes 1, 2, and 3. Calculating the perimeters of these figures, we get 22, 20, and 18. The next step would leave us with a 4 by 4 square with perimeter 16 units.

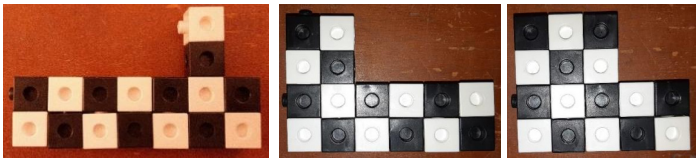


Figure 2: *Cutting of pieces of sizes 1, 2, and 3*

As such, our method does not yield the solution, but it wasn't all for naught. Since we wanted the perimeter to be 19, which is halfway between 18 and 20, we may conjecture that the solution we want is halfway between these two cases—that is, we should cut off a piece of size 2.5 by 2.

Another thing we may have noticed along the way is that it does not seem to matter where we place the cut piece along the top. For example, in the leftmost picture in Figure 2, the 1 by 2 piece that was cut off is placed, giving the perimeter as:

$$7 + 2 + 1 + 2 + 1 + 2 + 5 + 2 = 22$$

Alternately, if we moved the piece a little further down, as in Figure 3, we would have:

$$7 + 2 + 2.5 + 2 + 1 + 2 + 3.5 + 2 = 22$$

If we experimented with the 1 by 2 piece placed in other locations, we would find that the perimeter is unchanged. We might then conjecture that the location of the block doesn't matter. We can then put it wherever we feel is the most convenient, like right at the leftmost edge, as in the second and third pictures in Figure 2.

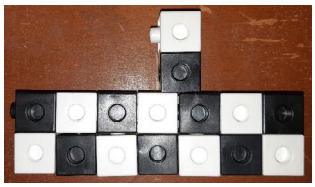


Figure 3: *A different configuration for cut of size 1*

It's one thing to make a discovery about a property and another to understand why it works. In Figure 4, we see what happens when the piece is cut from our rectangle and added to the top. The two blue sides were originally part of the perimeter, and they continue to be part of the perimeter. The dashed line (red) was not part of the perimeter, but now shows up in the perimeter twice. The dotted line (green) was part of the perimeter and now is not, and it covers up another segment of the original perimeter of the same size. Thus, if we let x represent the length of the piece we cut off (green), we get:

$$P_{new} = P_{old} + 2 \times red - 2 \times green = 20 + 2 \times 2 - 2x = 24 - 2x$$

Since we want the perimeter to be 19, we get the equation $19 = 24 - 2x$, which yields $x = 2.5$, as conjectured in our earlier work. Hence, the area of the smaller rectangle is $2 \times 2.5 = 5$ square units.

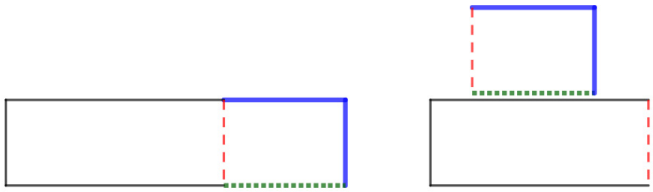


Figure 4: *Examining the relationship*

We can generalize the ideas from our last solution to a rectangle with length l and width w. Still using x for the size of the cut, then the perimeter of the new shape would be:

$$P = 2l + 2w + 2w - 2x = 2l + 2w + 2(w - x)$$

We can see from the equation that when we cut a square off the end so that $x = w$, the perimeter is unchanged. If the cut is less than the width, the new perimeter is longer, and if the cut is longer than the width, the new perimeter is shorter. We can also see the new perimeter is linear in the cut size.

Let's not stop here. We can search for a purely algebraic solution to our problem. Using l, w, x, as in the last solution, the original problem can be translated into the following system of equations:

$$2l + 2w = 20 \text{ (original perimeter)}$$
$$lw = 16 \text{ (original and new area)}$$
$$2l + 4w - 2x = 19 \text{ (new perimeter)}$$

Subtracting the third equation from the first yields $2x - 2w = 1$ or $w = x - \frac{1}{2}$. Substituting into the first equation and solving for l gives $l = \frac{21}{2} - x$. Finally, substituting

into the second equation gives the quadratic equation $\left(\frac{21}{2}-x\right)\left(x-\frac{1}{2}\right) = 16$. Multiplying both sides of this equation by – 4, expanding, and gathering terms together, yields $4x^2 - 44x + 85 = 0$, with solutions $x = \frac{5}{2}$ and $x = \frac{17}{2}$.

We get the same solution as before, but it seems we have another. A little checking shows that the second one is impossible. It is clear from our set-up of the problem that we must have $x < l$ which, using our expression for l in terms of x, yields $x < \frac{21}{4}$. Hence, the second solution is not possible for our situation, as it falls outside of the domain of the problem. It is always good to check the reasonableness of our solutions because, in this case, $x = \frac{17}{2}$ leads to a valid solution to the system of equations. It just doesn't make sense for what we are trying to model with those equations. It is also a good idea to consider all restrictions to the variables that we use so that we can more easily spot "false solutions."

You may want to investigate this problem further. For example, what happens if we have a rectangular prism and cut a chunk off the end and move it to another side? How does the surface area of the new figure relate to the old surface as well as the size of the cut?

Now it is time for your homework:

Alice has 24 apples. In how many ways can she share them with Becky and Chris so that each of the people has at least 2 apples?

Until next time, happy problem solving. ▲

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▲ Reflections on Solving Equations: Inverse Operations and Square Roots



JÉRÔME PROULX
proulx.jerome@uqam.ca

Jérôme Proulx is a professor of mathematics education (www.profmath.uqam.ca/~jproulx), and runs the Laboratoire Épistémologie et Activité Mathématique *(www.leam.uqam.ca). His research focuses on epistemological issues related to cognition and/in mathematics. His current research program is dedicated to mental mathematics and school mathematics content development.* (Photo by Nathalie St-Pierre of Université du Québec à Montréal)

It is a truism that students often come up with surprising ways of solving tasks we give them. Students can at times even come up with strategies or ideas that in turn raise questions or even shed light on mathematical aspects usually taken for granted or kept implicit.

This is particularly the case in the mental mathematics studies that my research team and I conduct in classrooms. Within the time constraints, students rarely refer to known procedures because these are often too time-consuming and hence not economical to solve in this context (see Butlen & Pézard, 2000). This leads many students to engage with a variety of strategies worth pondering. In particular, we conducted sessions on mental algebra, where the participating students (of various grade levels, from elementary to university level) are given short amounts of time to solve some equations for x, posed orally and on the board, without the use of paper and pencil or any other material aids (the equations given are of the usual form $Ax + B = C$, $Ax + B = Cx + D$, $\frac{Ax}{B} = \frac{C}{D}$, $Ax^2 + Bx + C = 0$ and their variants—see Proulx, 2013; Proulx et al., 2017).

In this second column, I am sharing other examples of these strategies about equation solving that students engaged with. These strategies, albeit simple, often raise interesting mathematical questions.

The Inverse Operation Strategy

One equation given in the form $Ax^2 + B = C$ was $x^2 - 4 = 5$, to solve for x. Out of the many strategies engaged with, one solver mentioned obtaining $x^2 = 9$ and doing the square root

of 9 to get 3. Then, realizing that he was in mental mathematics and had probably gone too fast, he checked his answer through the equation $x^2 = 9$ and realized that both +3 and –3 could give 9 when squared. His solution then became ±3.

From Squaring to Square Root

One first thing this strategy reminds us of is the difference between the solutions of $\sqrt{9}$ and $x^2 = 9$. The operation of taking the square root always gives a positive value, thus $\sqrt{9}$ gives 3, whereas $x^2 = 9$ has two solutions, that is, +3 and –3 (i.e., ±3). This distinction directly reminds us that the *square root is not the inverse operation of squaring*: $\sqrt{x^2}$ does not equal x. There are two numbers that give 9 when squared: +3 and –3. However, there is only one answer to $\sqrt{9}$ and it is 3. Hence, both operations are not the inverse of one another. And, when it is said that doing $\sqrt{x}$ is looking for the number that when squared gives x, then it works only if we are dealing with natural numbers. With integers, this assertion about $\sqrt{x}$ does not work because not all numbers squared can be accepted as solutions (negative numbers).

This raises the question about equivalence, and whether taking the square root of the equation $x^2 = 9$ produces an equivalent equation. The question is to know what happens to $x^2 = 9$ when we take the square root. Although not completely wrong, the answer $x = 3$ initially obtained by the student is not complete, neither is the subsequent validation to $x = \pm 3$. Simply said, applying the square root to $x^2 = 9$ does not lead to $x = 3$, since the degrees of $x^2 = 9$ and $x = 3$ are not the same. Additionally, the difference in the degree accounts for the loss of one solution from ±3 for $x^2 = 9$ to only 3.

The Insertion of the Absolute Value

An element that appears implicitly forgotten along the way and that this strategy raises to the fore is the use of the absolute value concept. It is the absolute value that plays a determining role in obtaining ±3 as an answer. The usual steps one would find for solving $x^2 = 9$ are the following:

(1) $x^2 = 9$

(2) $x = \pm\sqrt{9}$

(3) $x = \pm 3$

These steps are not necessarily wrong, but hide important elements, mainly related to the absolute value. One way of unearthing these details is to analyze each step in detail in order to draw out possibly hidden sub-steps in between.

The passage from (1) to (2) reports applying the square root. However, to produce an equivalent equation, the same operation has to be applied on both sides of the $x^2 = 9$ equation. Going from $x^2 = 9$ to $x = \pm\sqrt{x}$ hides two issues of significance that, if left unspoken, may wrongfully associate the square root and the ±. First, the square root of 9 is not ±3 nor $+\sqrt{9}$. Its answer is only 3, the positive root. Second, because of that, as mentioned, the square root is not the inverse of squaring; hence, square rooting does not enable us to "take out" or "eliminate" the exponent of 2 in x^2. Simply said, $\sqrt{x^2}$ does not equal x. The passage from (1) to (2) has thus to be adapted, as it hides something important. From (1) $x^2 = 9$, applying the square root on both sides would give the following sub-step (1.5) $\sqrt{x^2} = \sqrt{9}$. Again, here, no ± sign appears. The next step is where the absolute value comes into play. $\sqrt{x^2}$ is not equal to x, but to $|x|$. Indeed, as we know, x^2 is positive, but we do not know if x itself is positive or negative. And, "taking out" the square root of x^2 also needs to give a positive number. That positive number can only be $|x|$, and not x, since we do not know its initial sign. Thus, $\sqrt{x^2} = |x|$.

Because $\sqrt{9}$ is positive, the dual possibilities for ± are not in $\sqrt{9}$, but in the indeterminate value of x itself in $x^2 = 9$, which is encapsulated in $|x|$ after having taken out the square root in $\sqrt{x^2}$. Hence, another sub-step (1.75) is $|x| = \sqrt{9}$. Then, if one wants to eliminate the absolute value, one has to consider both possible values for x. If x is positive, its value is $\sqrt{9}$. If x is negative, then its value is $-\sqrt{9}$. Therefore, the ± comes from "eliminating" the absolute value, or from passing from $|x|$ to x, and not from the square root of 9, which only gives one value, the positive or primary root 3. The next step then is (2) $x = \pm\sqrt{9}$, leading to (3) $x = \pm 3$. The answer 3 is obtained directly from $\sqrt{9}$, and the ± from the absolute value is taken out. In the end, the following steps would be needed or be made explicit:

(1)	$x^2 = 9$		(1)	$x^2 = 9$
(1.5)	$\sqrt{x^2} = \sqrt{9}$	or	(1.5)	$\sqrt{x^2} = \sqrt{9}$
(1.75)	$\lvert x\rvert = \sqrt{9}$		(2)	$\lvert x\rvert = 3$
(2)	$x = \pm\sqrt{9}$		(3)	$x = \pm 3$
(3)	$x = \pm 3$			

Therefore, coming back to the original question about equivalencies, one can say that "yes, all these equations are equivalent," since the solution set is kept unchanged to ±3. One can move from $x^2 = 9$ to $|x| = 3$ or from $|x| = 3$ to $x^2 = 9$ without modifying the solution set.

What would be your answer to these?
Here are a couple of expressions to simplify:

$\sqrt{3^2} = ?$

$\sqrt{(-3)^2} = ?$

$\sqrt{(x-3)^2} = ?$

$\sqrt{(x-y)^2} = ?$

(Inspired by Cai & Nie, 2007)

The Presence/Absence of Absolute Value

The presence, or reinsertion, of the absolute value when working with square roots is of significance, in order to truly maintain equivalent equations. This is so, particularly because with the initial (1), (2), and (3) steps, the equivalence was fragile, albeit somehow respected in terms of the solution obtained. The addition of the sub-steps (1.5) and (1.75) makes the process more transparent and more robust mathematically. It also shows that some unwanted shortcuts have been taken when working with squares and square roots, and that our use of or reference to the absolute value has somehow "suffered" along the way. Even if we know that this absence of the absolute value is something of concern, we hardly notice when solving equations, since it seldom creates problems when we do solve these. However, this "absence" of the absolute value is permeating most solving where squares and square roots come into play.

For example, take the formula for the quadratic discriminant, obtained through a series of algebraic manipulations that are synthesized in the following:

(1) $ax^2 + bx + c = 0$

(2) $x^2 + \frac{b}{a}x = -\frac{c}{a}$

(3) $x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$

(4) $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$

(5) $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$

(6) $\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$

(7) $x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$

(8) $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

(9) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

At first, nothing appears problematic in these algebraic manipulations. Going from step (6) to step (7) does, however, leave in the background the absolute value, again, which is the reason why the ± sign appears. But there is more. Passage from step (6) to (7) hides also what is being done with $4a^2$, or with $\sqrt{4a^2}$, being is reduced to $2a$ without any recourse to the absolute value, nor the ± sign that unfolds from it. In other words, $\sqrt{4a^2}$ becomes equal to $2a$! Obviously, one can argue that the ± that would accompany $2a$ is already present in the expression $\sqrt{\frac{b^2 - 4ac}{4a^2}}$ and that this would not change a thing in the formula, but this is exactly the point! It shows how taking the square root leaves unnoticed the place and role that the absolute value has, and almost creates an unfortunate direct link between taking the square root and the appearance of a ± sign. It creates an implicit link between them, which is mathematically questionable, and keeps the absolute value hidden in the dark.

There are other numerous examples where the place of the absolute value is kept hidden. Another example lies in this algebraic transformation (found with a quick search on the Web):

(1) $\frac{x^2}{a^2} = 1$

(2) $x^2 = a^2$

(3) $x = \pm a$

In this one, passing from step (2) to step (3) does not give a wrong answer, but is an oversimplification. The step $x^2 = a^2$ could lead to $\sqrt{x^2} = \sqrt{a^2}$, which could lead to $|x| = |a|$. The $|x| = |a|$ could lead to another step of the type $\pm x = \pm a$. This last step, $\pm x = \pm a$, could lead to $x = \pm a$, as step (3) shows. However, the deletion of other possible cases with $\pm x = \pm a$ also remain hidden: $x = a$, $^{-}x = {}^{-}a$, $x = {}^{-}a$, $^{-}x = a$. In that sense, only asserting $x = \pm a$ appears to be reducing the process significantly.

An issue to ponder: $\sqrt{x}$ or $x^{½}$

Raising issues about square roots can also raise issues about fractional exponents. The following is taken from a book on short mathematical enigmas (Mazza, 2007, p. 123), in which issues of square roots and of fractional exponents are put to the test. Where is the problem? And where would the concept absolute value come into play to settle it?

Are all real numbers positive?

For all real x $\quad x^2 \geq 0$

For all real x $\quad (x^2)^{\frac{1}{2}} \geq 0$

For all real x $\quad x^{\left(2 \times \frac{1}{2}\right)} \geq 0$

For all real x $\quad x^1 \geq 0$

For all real x $\quad x \geq 0$

Concluding Remarks

In line with the introduction, the issues raised above about solving equations offer a salient illustration of how some mathematical insights can be gained from students' ways of solving, even if the strategy adopted for ±3 is quite simple. In this case, the (wrongly conceived) inverse operating between squaring and square root shows how taking the square root has been implicitly associated with adding the ± sign. This illustrates how the absolute value has been kept hidden in these algebraic manipulations of

squaring and square rooting, in favour of this implicit association between the square root and the ± sign. Although not necessarily problematic in relation to the answers obtained, this association remains troublesome and, as mentioned, hides an important concept, the absolute value, which would gain from being put forward in attempts to make sense of squaring and square rooting.

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▲ MB4T (Mathematics by and for Teachers): Bringing Data Science into the Elementary Classroom



JENNIFER HOLM
jholm@wlu.ca

Jennifer Holm is an Assistant Professor at Wilfrid Laurier University and works with primary/junior and junior/intermediate pre-service teachers, as well as in the field supporting current mathematics teachers. She is interested in developing mathematics knowledge for teaching with both pre-service and in-service teachers. She focuses on the beliefs and opinions that pre-service teachers hold about mathematics and teaching and the connection they have to past experiences. She uses this research to support future teachers in developing beliefs and knowledge that will encourage and support effective teaching practices.

This column is inspired by a talk that Dr. Jo Boaler (2020a) gave during the summer through OAME/AOEM. In case you missed it, you can watch it in the members-only section of the OAME/AOEM website. The talk highlights her thoughts on Data Science as a new, equitable pathway in mathematics. Following the talk, I signed up for the 21st Century Learning Course (Boaler, 2020b) that was offered through Stanford University and youcubed® to further explore the place of data science in the lives of learners. In honour of these new ideas, and the new Ontario curriculum, I decided to focus on how data science fits into our elementary curriculum by putting together various thoughts about data science for Ontario teachers. To begin, I will explore the benefits of data science and what it is; then I will look at how it can fit into the current elementary curriculum; and finally, we will explore some activities that address both data science tenets and the Ontario curriculum.

To start, we will explore what data science is and why it is important for our learners. Davenport and Patil (2012) note that a data scientist is "a high-ranking professional with the training and curiosity to make discoveries in the world of big data" (para. 6). They go on to describe how data scientists help companies make sense of the data they collect, and then make decisions to help businesses move forward. As Boaler notes in her talk, as well as the course, "90% of all data has been collected in the last two years" (Boaler, 2020a, 2020b). This is the information age of data, so it is important that students learn more about data sciences so

that they can navigate this world. “Students should be learning to understand why methods are used and how they can be interpreted. That’s important, not doing long calculations by hand!” (Boaler, 2020b, section 2.3). Boaler also notes that the current pathway of a calculus-driven curriculum has proven to be very inequitable for different groups of students. Data science gives a way to start a pathway or career that allows for equity for all students, since it is not just relegated to mathematics. Data science can be integrated into many other subject areas simply by bringing in data about the subject area. For example, data about reading could be brought into a literacy class, or information about scientific studies to a science classroom. Boaler (2020a) does caution that data science is not the same as a statistics course. In her view, statistics is taught as a set of methods or procedures, and data science is meant to be more open and exploratory, as you will see in the discussion below.

In general, data science is about seeking out relationships between variables or ideas, asking questions while looking for patterns, considering sampling and probability, and then being creative in communicating thoughts. The “start of data science is a free exploration where you can ask any questions of data and explore patterns” (Boaler, 2020b, lesson 3.5). It is important that a discussion about data science starts with real data that allows students to explore and ask questions. Students are then charged with working on answering their questions and then telling what they have learned by using the data they have gathered. Boaler notes four stages for students to go through: question asking, pattern seeking, meaning making, and communication. In the first stage, Boaler emphasizes that the big idea in data science is that students need to ask questions and be curious about the data. By presenting them with data that is relevant to their lives, students can learn to ask questions and explore topics of interest. Once they have explored the initial piece of data, then it is important to allow them to “explore patterns” and ask their own questions to find or collect more data to answer the questions. Afterwards, they need to communicate their results in different ways.

In thinking about these tenets of data science, it is clear that this fits strongly into both the data sections of the curriculum and the new modelling expectation. Within the Algebra strand in all Grades 1 to 8 classrooms, the modelling expectation is that students will “apply the process of mathematical modelling to represent, analyze, make predictions, and provide insight into real-life situations” (Ontario Ministry of Education [OME], 2020). The expectation goes further to explain that “mathematical modelling is an iterative and interconnected process that is applied to various contexts, allowing students to bring in learning from other strands. Students’ demonstration of the process of mathematical modelling, as they apply concepts and skills learned in other strands, is assessed and evaluated” (OME, 2020). The general idea is to present students with a problem, have them ask questions and determine what they need to solve the problem, and then communicate findings, all in an iterative process as they continue to cycle through asking questions and finding new information. The problem in this case could be a graph or set of data that would allow students to ask questions and learn more about a topic. In this way, the modelling would be connected to the data strand of the new curriculum. In the new curriculum, data science also fits nicely into the overall expectation named “data literacy,” “manage, analyze, and use data to make convincing arguments and informed decisions, in various contexts drawn from real life” (OME, 2020). At every grade level, there are specific expectations related to “Data Collection and Organization,” “Data Visualization,” and “Data Analysis,” which are all major components of the Data Science area as they were discussed above.

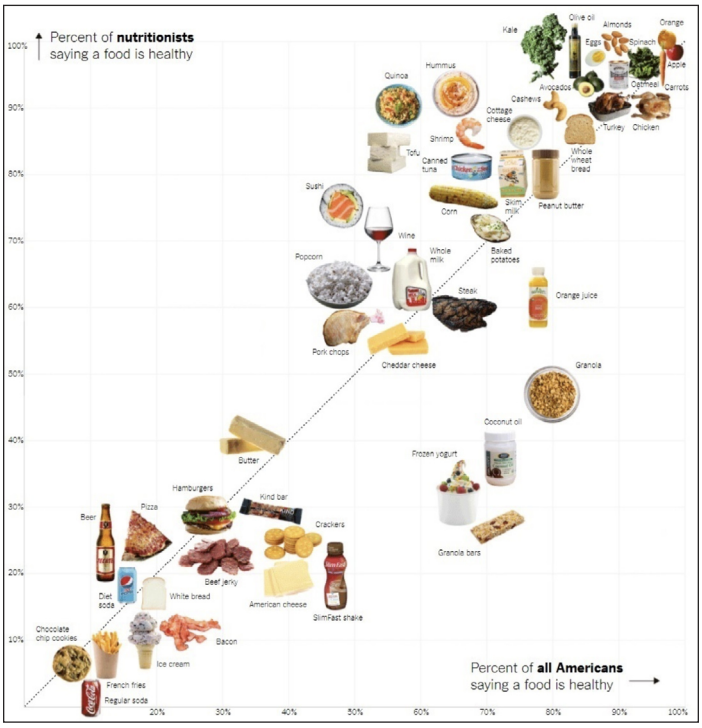


Figure 1: *A graph by The Learning Network from* New York Times *column, “What’s going on in this graph?”* www.nytimes.com/2017/10/09/learning/whats-going-on-in-this-graph-oct-10-2017.html

Now that I have established how this fits into the Ontario curriculum, I will work through an example of a data science set of lessons, using an actual graph. Figure 1 shows a

graph from the "What's going on in this graph?" website of the *New York Times*. This site provides many interesting graphs to provoke conversations about data science in the classroom. It allows students to interact with the website, including allowing them to input their notice-and-wonder thoughts when given a graph and headline. There are also many interactive graphs that can be explored on the website, such as www.nytimes.com/interactive/2018/05/03/learning/08W GOITGraphLN.html, where students can ask questions and explore the possibility that temperatures really are rising. Since the interactive nature does not translate well to this column, I have included a different graph in Figure 1.

The first part of the lesson would have students exploring the graph and keeping track of what they notice and wonder about what they are seeing. As the students become more familiar with talking about data science, they should start to include questions or statements about the types of graphs as well as its features. Hopefully students will notice that the *x*-axis has graphed the percentage of the American people polled who think that the foods pictured are healthy, and the *y*-axis has graphed the percentage of nutritionists who think the foods are healthy. Once students are curious, the hope in data science is that students will look for patterns. Based on this graph, students may surmise that foods along the dotted lines are where both of the groups agree. They might ask questions or wonder about the foods where there are disagreements. In this case, students may observe that there are some foods that many Americans think are healthy, such as frozen yogurt, coconut oil, and granola bars, but fewer nutritionists rated them that way.

Once the graph has been discussed, questions have been asked, ideas have been shared, and students can start their own explorations. What do they want to learn more about? An interesting question may be, "Why do people think that frozen yogurt, coconut oil, and granola bars are healthy, but nutritionists do not?" Students could collect information from health organizations, polls from peers and parents, packaging, or continue to read more about the graph from the website. Throughout the process, students are encouraged to ask questions, deciding what data they need to answer the questions, collecting the data, and refining their process (all parts of the modelling expectation as well as data science). Finally, students would create an infographic or some sort of report to share their findings with a group or the class. What did they learn from their work, and what process did they follow in order to find the answers?

As students become more familiar with graphs and data, the Concord Consortium (https://learn.concord.org/) provides many interesting data sets that students can explore. What makes this unique is that once the data set is assigned, students can choose which variables they want to explore as they look for relationships between their ideas and trends in the data. By giving choice and freedom to explore, students can be led in many different ways, depending on which variables they chose, how they went about exploring, and the patterns that arise from those explorations. Meaningful conversations can then occur between groups of students who explored different variables or in different ways. A group may be able to address some of the other groups' questions or research. In the end, students can gain a deeper understanding of the phenomenon studied through the data.

In conclusion, data science is an essential area of the knowledge that students gain in schools because data is all around them. We need students who are critical thinkers and can explore what is being presented to them. As mentioned in a previous column (Holm, 2020), we need students who can see when graphs are being misleading or when people are intentionally obscuring the meaning of data. Data scientists are expected to be one of the big careers of the future, as well, so helping students learn more in elementary school can help prepare them for this career pathway. In the end, data science has the potential to open new pathways for students as well as give them skills that will help them be successful adults in the twenty-first century.

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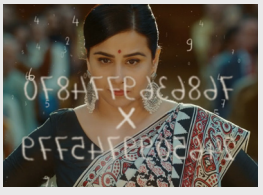
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In the news…



Amazon Prime has a Hindi-language movie (with subtitles), titled, and about, Shakuntala Devi, who is famous for mental calculations including "… taking the 23rd root of a 201-digit number in under a minute, and multiplying two 13-digit numbers in under 30 seconds." (https://www.sciencenews.org/article/math-documentary-bollywood-film-women-devi-mirzakhani)

▲ Mathematical Snapshots: Apple for the Teacher



RON LANCASTER
ron2718@nas.net

*Ron Lancaster is an Associate Professor at the University of Toronto, where he teaches mathematics courses for pre-service middle and high school teachers. He has over 20 years of experience teaching Grades 7–12 mathematics. Ron's professional activities include consultations and conference presentations in North America, Asia, England, the Middle East, Africa, India, and Europe. Ron is an author for the NCTM (*The Mathematical Lens*) and member of the Advisory Board for the Museum of Mathematics in New York. He is the recipient of the 2015 Margaret Sinclair Memorial Award Recognizing Innovation and Excellence in Mathematics Education, awarded by the Fields Institute.*

In October 2020, my spouse, a family physician, worked at a clinic for migrant workers at Schuyler Farms in Townsend, Ontario. I drove her to the clinic, and while she worked, I photographed the farm equipment and apple orchards. I was struck by how much mathematics I found growing at the farm, and there was so much to notice and wonder about. Although I should have asked permission, I ate several Honeycrisp apples that were absolutely outstanding. That variety is one of my favourite apples.

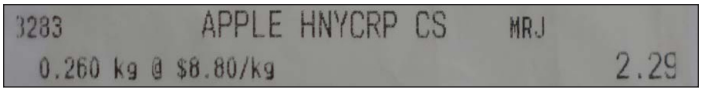
Before looking at the questions that follow, discuss what you notice in the photographs, and suggest reasons as to why I took these photos. When you look at the photos, what do you notice? What are you wondering about? These are core habits of mind that we all should practise.



Photograph 1: *Honeycrisp apples at Schuyler Farms*



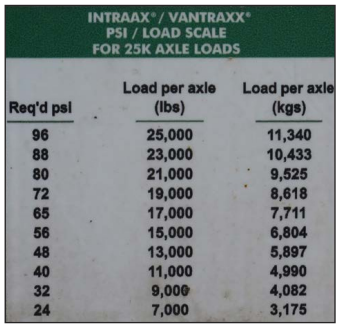
Photograph 2: *Price of Honeycrisp apples at a store in Hamilton on October 11, 2020*



Scanned Image 1: *Store receipt for purchase of one Honeycrisp apple on October 11, 2020*

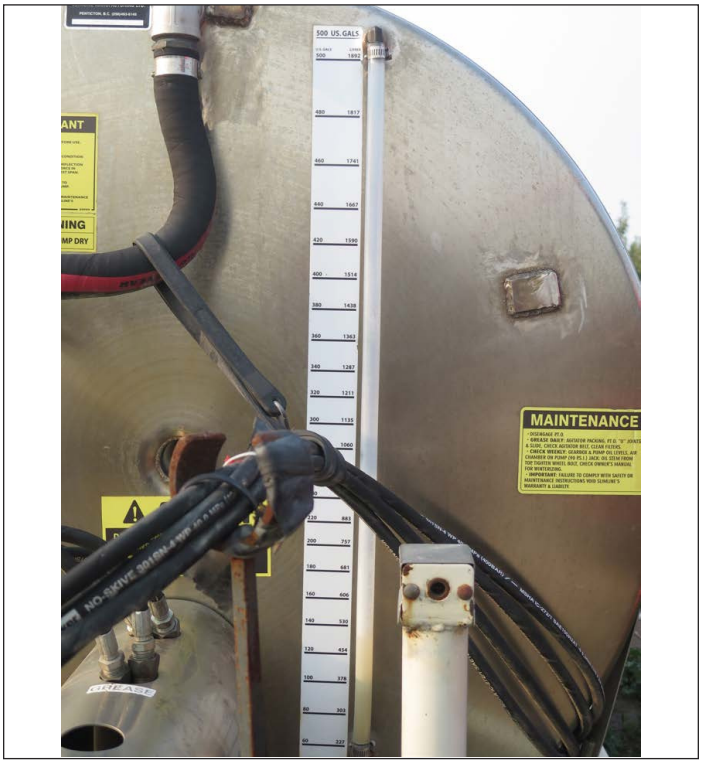


Photograph 3: *Shipping container at Schuyler Farms*



Req'd psi	Load per axle (lbs)	Load per axle (kgs)
96	25,000	11,340
88	23,000	10,433
80	21,000	9,525
72	19,000	8,618
65	17,000	7,711
56	15,000	6,804
48	13,000	5,897
40	11,000	4,990
32	9,000	4,082
24	7,000	3,175

Photograph 4: *Sign on a transport truck trailer at Schuyler Farms*



Photograph 5: *Scale on farm equipment at Schuyler Farms*

Question 1

(a) Estimate the number of Honeycrisp apples on the tree shown to the left in Photograph 1.

(b) According to the website given below, Schuyler Farms has 500 acres of apple orchards. What is an acre?

Future looks bright for young Ontario apple farmer:
www.onapples.com/meet-grower.php?id=p9bee6104fy2

(c) Using the information provided at the website given

below, estimate the number of Honeycrisp apple trees in a square plot of land with an area equal to one acre.

How to plant honeycrisp apples:

https://homeguides.sfgate.com/plant-honeycrisp-apples-39252.html

(d) Estimate the total number of apples on the trees at Schuyler Farms. What assumptions have you made?

Question 2

(a) The cost of purchasing a Honeycrisp apple is shown in Scanned Image 1. How was this cost calculated? Is it correct? Is the cost consistent with the information shown in Photograph 2?

(b) How would you estimate how much it would cost to buy all the apples on the trees at Schuyler Farms? How much of this money do you think goes to the farm?

Question 3

Photograph 3 shows the door of a shipping container that can be used for shipping apples.

(a) By simply looking at the numbers, suggest a relationship among the three numbers listed for the MAX. GROSS, TARE, and NET.

(b) Calculate the ratio of the measurement in pounds to the measurement in kilograms for the MAX. GROSS, TARE, and NET. Are the ratios the same? Should they be the same? Why or why not?

(c) The website given below provides descriptions of all the quantities that appear on the shipping container in Photograph 3. Also included is information about the unique serial number assigned to every shipping container and its check digit. Read the instructions for calculating the check digit, and write a summary that could be posted on a blog about the applications of mathematics that is written for the general public.

What is tare, net, max. gross, CU. CAP, and the prefix?:

www.alconet-containers.com/what-is-tare-net-max-gross-cu-cap-and-the-prefix/

Question 4

(a) Graph the data for the *Req'd psi* versus the *Load per axle (kg)* shown in Photograph 4. Calculate the first and second differences for this data. Is the relationship between the Req'd psi versus the Load per axle linear or quadratic? How do you know?

(b) Determine a mathematical model for the data in part (a). Determine the Req'd psi for a trailer whose Load per axle is 12 000 kg. Describe how the model could be used to determine the Req'd psi if the Load per axle is 27 000 lbs.

(c) Search online for definitions of *Req'd* and *psi*. What do these abbreviations stand for? Are there limitations on the values of these quantities? For example, could the Req'd psi be 5? Could the Load per axle be 100 000 kg?

Question 5

Figure 1 shows a cropped and rotated view of a scale that appears on a piece of farm equipment at Schuyler Farms (Photograph 5). This figure was created by pasting Photograph 5 into a Geometer's Sketchpad® sketch and then adding points and line segments to match up with the markings on the scale.

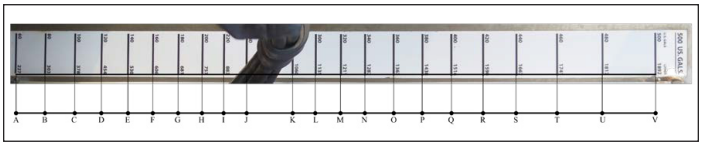


Figure 1: *Cropped and rotated view of a scale on a piece of farm equipment at Schuyler Farms*

Table 1 contains the length of each line segment in the sketch, starting with AB, then BC, CD, and so on, ending with UV. How do these lengths change from AB to UV? How can you use the circular cross-section of the drum to make sense of how these lengths change?

Table 1

	line segment	length of linesegment (cm)
1	AB	2.26
2	BC	2.33
3	CD	2.08
4	DE	2.08
5	EF	1.94
6	FG	1.98
7	GH	1.87
8	HI	1.69
9	IJ	1.80
10	JK	3.60
11	KL	1.76
12	LM	1.98
13	MN	1.90
14	NO	2.26
15	OP	2.22
16	PQ	2.29
17	QR	2.47
18	RS	2.58
19	ST	3.21
20	TU	3.53
21	UV	4.17

Enjoy this *Apple for the Teacher*. I am sure it will have "a-peel" with your students. As you get out and about, remember that your camera can capture lots of classroom resources! Perhaps ask your students what they wonder, and see what questions arise. ▲

▲ Linking Literacy and Math: Going Beyond Surface Level When Problem-Solving (Part Two)



DAVID COSTELLO
dacostello@edu.pe.ca

Dr. David Costello is an author who has recently published Using What Works: Strategies for Developing a Literacy-Rich Environment in Math. *David is a vice-principal, who has held many roles. He taught in the primary division before assuming roles of numeracy interventionist, numeracy coach, numeracy leader, and curriculum consultant for Prince Edward Island. David has also instructed university courses focused on curriculum, differentiation, mathematics, and language arts. He facilitates professional learning sessions in the areas of mathematics and school development. His research situates mathematics within a literacy perspective and examines leveraging literacy-based instruction in mathematics.*

I want to begin this column by reiterating my perspective on problem solving. Problem solving is a verb, not a noun. What I mean is that problem solving is a process that students experience through engagement. It is how they frame, support, and consolidate their thinking when working through a question or task that is novel, with no immediate recognition of the solution. It is not practice; it is productive struggle that encompasses cognitive dissonance (having to move beyond your expected practice when faced with a novel situation).

Teachers are expected to teach through problem-solving (Ontario Ministry of Education [OME], 2020). Teachers provide a context or reason for the learning task, with the necessary procedures being drawn out from this purpose. Students are given a problem, they must decide on a strategy to solve this problem, and then decide how to carry out the chosen strategy. Problem solving, particularly when connected to real contexts and situations such as those found in financial literacy, is powerful for student learning.

Building on this approach of teaching through problem-solving, the OME (2020) has identified teaching about problem solving as a high-impact instructional practice. What this means is that when we teach students about the process of problem-solving, the thinking required in it becomes explicit.

What I attempt to do in this article is share how we, as teachers, can strengthen student problem-solving strategies by teaching about problem solving. I will begin by explaining what teaching about problem solving entails. From there, I will share three instructional strategies that support students in becoming problem solvers, and in making meaning as they engage in this process.

What Is Teaching about Problem Solving?

Teaching about problem solving highlights and emphasizes characteristics of problem solving, and provides opportunities for students to *see* it, *experience* it, and *reflect* on it. It approaches problem solving from a perspective of complexity that goes beyond a superficial level. What do I mean by this? Teaching about problem solving underscores the importance of fluidity in thinking. It goes beyond approaching problem solving from a simplistic stance of following a prescribed model, to one of productive struggle, making mistakes and adapting one's reasoning throughout the problem. It highlights the fallacy of teaching students to use keywords to solve problems. It is about the recognition that students need instruction regarding how to problem-solve, and the multiple aspects that they will encounter when working toward a solution. In summation, teaching about problem solving is simply about making implicit, unconscious thinking explicit.

Importance of Student Independence in Teaching about Problem Solving

Student independence is the goal of most teachers. It is perhaps one of the foundational components of classroom structures and management. In terms of reading, teachers spend considerable time identifying the reading level of a student so that a student can read a text independently and find it is neither too easy, nor too difficult (Fountas & Pinnell, 2006). The premise supporting this match is that students should have an appropriately levelled text so that they can effectively use reading strategies independently. With this independence, teachers can monitor students' use of strategies, without having to spend time constantly focusing on problems in the classroom and addressing questions such as, "What should I do?," "Can you help me?," and the common "I don't get it."

Similar to reading instruction, the goal of mathematics instruction is to have students engaged with a problem or task independent of the teacher, and to be able to work through it to a meaningful solution. Although students are independent of the teacher, there is still an expectation that they collaborate with one another. Student collaboration that is independent of teacher instruction provides opportunities

for students to learn from another classmate's thinking. While teachers play a significant facilitating role in the classroom, it is important to let students work through problems. It is not possible to have an effective mathematics environment if students are always in need of a teacher to support them as they solve problems.

Recognizing the significance of student independence in problem solving, I will provide an overview of three instructional strategies—modelling productive struggle, self-regulation checklists, and co-constructing anchor charts—that provide leverage when teaching about problem solving. Each of these three strategies provides students with an understanding of the importance of independence when problem solving.

Teaching about Problem-Solving Strategy #1: Modelling Productive Struggle

By recognizing productive struggle as part of problem solving, students will understand that this is a natural part of working through a problem, and will not easily give up when they encounter a stumbling block. This modelling can be demonstrated by the teacher or by other students. With modelling, the teacher should take time to highlight instances of productive struggle, and show how it was recognized, worked through, and how progress was monitored throughout the process.

For students to be independent, they need to have a clear understanding of the learning goal. They must understand (Costello, 2019):

- the teacher's expectation
- what successful problem-solving looks like
- how a growth mindset helps them learn from their experiences
- how to clearly communicate their thinking
- what it means to have finished the task

Teachers' modelling of productive struggle when working through a task is important (Teague, 2015). If teachers always demonstrate success when problem-solving, without apparent struggle, students will assume that struggle is not part of solving problems. What this may imply to students is that if they encounter stumbling blocks when problem-solving, they are doing something wrong, which may lead them to give up. Students need to observe what happens when a plan does not lead to instantaneous results, and see what teachers and other students do when they hit a misstep. By observing such a misstep, students will recognize and understand how to look for progress when they are engaged in problem-solving. Teague (2015) emphasizes that if our goal for students is independence, then it is crucial they see teachers (or their classmates) engage in productive struggle. Students will come to appreciate that such a struggle is inherent in the problem-solving process and not a sign to withdraw from the process. The following provides teachers with some points for consideration when planning for modelling productive struggle (Costello, 2019).

Addressing the following questions should be considered when modelling productive struggle for students:

- Does the problem I am working with provide opportunities for authentic productive struggle?
- Will this productive struggle be highlighted in the modelling process?
- How will I communicate when I determine I am encountering a stumbling block?
- What will I do to work backwards from the stumbling block?
- What strategies do I rely upon to redirect my path to work through the stumbling block?
- What strategies do I rely upon to monitor progress toward a solution?
- What learning will I take from this experience?

Teaching about Problem-Solving Strategy #2: Student Self-Regulation Checklists

Self-regulation supports students in working through a problem. By having supports that serve to assist students in self-regulating, students' independence as mathematicians is promoted and encouraged. Checklists serve as one kind of support.

Providing students with a self-assessment checklist not only supports students by serving as an organizer for sequencing their work, but it also supports them in creating a plan and carrying out this plan with problem solving. Such supports will encourage students to continue to engage with a problem, as it provides suggestions in moving forward without relying on teacher support.

The following checklist is an example from an upper-elementary classroom that was co-constructed by the teacher and students (Costello, 2019). The list grew over time as students gained experience solving problems and added items based on their experiences. Students had this checklist printed on a piece of paper in their math binder and would often rely on it when problem-solving. Some students benefited from using a digital version of the checklist. As a digital version, it was easier for teachers to check in with students' progress and offer support as required. The checklist may be differentiated based on an individual student's learning. For example, a step may be omitted for some, or additional steps may be added for others. The third column was added near the end of the year, as the teacher and students thought it was important to have a space to add any questions and/or comments at certain points in the process.

Step	Check if completed	Notes
I have a plan (Plan A).		
I will try my plan (Plan A).		
If this plan does not work, I will think of another plan (Plan B).		
I will try this new plan (Plan B).		
If this plan does not work, I will think of a question I need to ask to help me with my learning.		
I will ask this question to another student.		
If another student cannot help me, I will ask my teacher.		
After this question, I will examine my work for mistakes.		
If I find mistakes, I will address these in my attempt to solve the problem.		
If I cannot find a mistake, I will come back to this problem later.		

The following excerpt is taken from a conversation with an elementary-aged student, who used the student self-assessment checklist strategy (Costello, 2019).

A Student's Voice

I like using this checklist. It helps me keep track of my thinking when I work on the problems. Instead of waiting for the teacher to come and help me, I know what my next step is. Right now, I'm working on the Plan B part of my checklist. I sometimes find it hard to pick a new plan.

Teaching about Problem-Solving Strategy #3: Anchor Charts

Anchor charts can be used in different ways to help students become independent problem solvers. The traditional anchor chart is usually created on a large sheet of paper and posted in the classroom. Teachers and students usually work together to write out points, terms, or questions to benefit student learning. From there, these charts support students when they encounter stumbling blocks during problem solving, assist students in recognizing and calling upon the different attributes of problem solvers, and provide a scaffold throughout the problem-solving process. In summation, anchor charts make the process of problem solving explicit by providing students with reminders and supports of important aspects of problem solving.

Digital anchor charts have a similar purpose, but they are primarily accessed online. Having a digital copy of an anchor chart makes it more accessible for students when they are outside of the classroom. The digital copy offers support for homework and can help with self-regulation at times when students are away from their desks. Students having access to individual digital anchor charts, where they can add specific words or example problems, is helpful for their own learning. They can also share these charts online with other students, who might have similar questions.

A group of teachers from a variety of grade levels co-constructed anchor charts that reflected what independence would look like at three grade levels: Kindergarten, Grade 3, and Grade 6. Independence closely relates to perseverance, since it is students who are active in the learning process. By being active learners, students are taking charge as they problem-solve. These anchor charts served as visual supports for students when they were working through a problem. Students used the anchor charts as reminders of steps to work through, as necessary, when engaged with a problem. It is important to note that the charts were not static; instead, the charts were living documents that were constantly being revised in the classroom. The following is a summation of what the teachers and students co-constructed when asked what working independently looks like in the classroom (Costello, 2019).

In Kindergarten, working independently looks like students ...

- playing individually or in groups
- using things suggested by the teacher
- meeting with the teacher for check-ins
- talking about the activity and the learning that took place
- self-selecting different stations

In Grade 3, working independently looks like students ...

- asking three friends before the teacher
- working individually or in groups
- deciding on the objects to use (teachers may list suggestions)
- reading the problem after the teacher reads the problem first
- making a plan
- using self-monitoring strategies (can be prompted by the teacher)
- sharing their work in small groups (questions provided by the teacher)

In Grade 6, working independently looks like students ...

- asking at least three others for help before the teacher
- working individually or in groups
- deciding on the manipulatives to use
- reading the problem more than once if needed
- making a plan
- using self-monitoring strategies
- reflecting on their work individually or in small groups

Final Thoughts

Problem solving is a process and an experience. When working through a problem, it is inevitable that students will experience productive struggle. It is important that we, as teachers, recognize this, and that we support students in

working through the stumbling blocks. This underlines the significance of teaching about problem solving and why it is considered a high-impact instructional practice.

In this column, I shared three instructional strategies that support teachers when teaching about problem solving. By leveraging these strategies, we go beyond the surface level and address important characteristics of problem solving that students must master to be successful in reaching a solution.

In the next issue, I will continue to focus on problem solving, this time focusing on the importance of metacognition when solving problems. I look forward to continuing the discussion of linking literacy and math via Twitter (@dr_costello) or email.

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▲ In the Middle: Representing and Reinforcing Mathematical Processes Amid Uncertainty



CARLY ZINIUK
carlyziniuk@gmail.com

Carly Ziniuk teaches Grade 9 Mathematics, Grade 12 Data Management, and Advanced Placement Statistics at the Bishop Strachan School in Toronto, Ontario, Canada. She is very active in adopting real-life data to engage her students in solving problems.

Last issue, I discussed strategies for interleaving practice that I was using at the turbulent end of the last school year. The recent curriculum revision includes a new strand of Social–Emotional Learning Skills, including Mathematical Processes. This revision reinforced my commitment to increasing student confidence through skill retention and regular communication of mathematical processes. This

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column will share the strategies that I am actively employing and adapting for varied delivery models, wide-ranging social and emotional student needs, and mathematical backgrounds, many impacted by the current uncertainty. These strategies include Google Quizzes (Forms) with specific targeted feedback; observation of student work and form, using Desmos Teacher Mode; and the use of written Mathematical Language Routines in journal writing.

Asynchronous Google Quizzes are an effective interleaving practice strategy, since it can be created at one point in the year as a check-in of current learning, and then repeated as a review opportunity. You can also easily combine quizzes or make new quizzes with a combination of questions from previous ones. Google Quizzes also respond to the need for the variety of delivery modalities. Both in face-to-face learning and during remote learning, Google Quizzes have allowed me to include direct feedback for correct and incorrect answers (Figure 1).

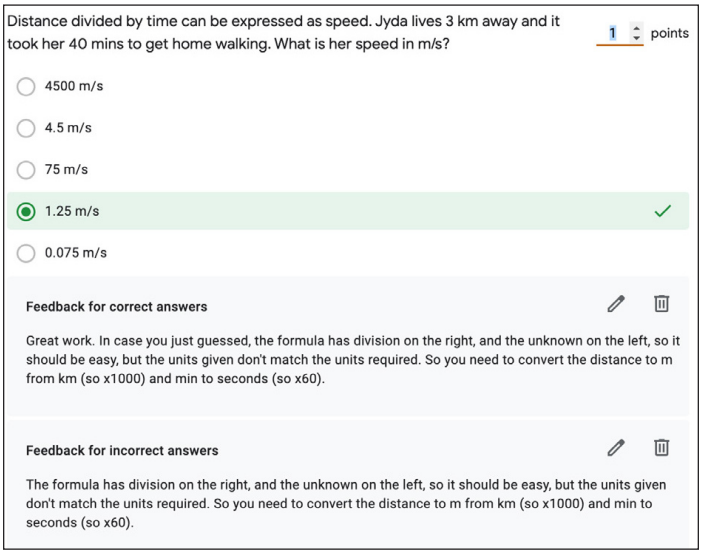


Figure 1: *Example of feedback for correct and incorrect answers in Google Quiz*

Students with good intuition or developing mechanical skills may choose the correct answer from multiple-choice options on a Google Quiz, yet still need work on producing correct algebraic form. Modelling appropriate mathematical conventions has been challenging with social distancing, asynchronous homework, and less observation of their individual work. In response, I am including more fulsome descriptions in the model solutions as well (Figure 2).

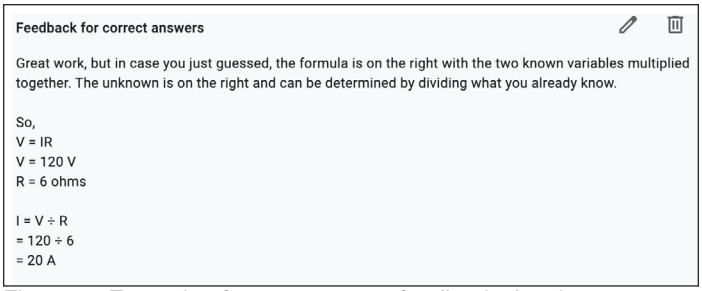


Figure 2: *Example of correct answer feedback showing proper algebraic form and modelling self-talk for step-by-step thinking*

Within the Google Quiz feedback, I also include external videos like those from MooMooMath (Figure 3), to provide short reminder lessons. However, when I want to solve the specific problems or highlight common misconceptions among my students, I created short personal videos demonstrating the step-by-step correct form to include in the feedback. Students can improve their understanding by using the collection of quizzes to move at their own pace. Early in the year, the quizzes were short, and I was able to create multiple versions, especially using the EquatIO add-on (https://equatio.texthelp.com/) for math formatting within Google Forms.

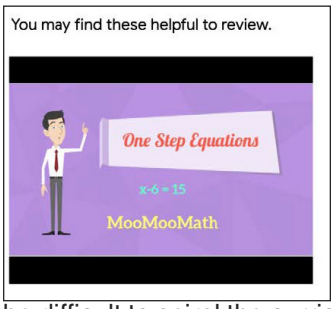


Figure 3: *Example of online video from MooMooMath and Science included as Feedback*

Since Google Quizzes allow you to import questions from other Forms (Figure 4), it is easy to interleave old and new material. Where it might be difficult to spiral the curricular topics, or to find a practical problem involving disparate skills, I created the routine of having questions from previous topics within the Google Quizzes. This embedded review makes students more comfortable, and lowers anxiety about having to do a quiz.



Figure 4: *Google Quiz's Import question option*

For example, as we moved on to linear systems, I layered exponent law questions throughout (Figures 5 and 6). Reusing and adapting questions that were previously problematic reminded the class that we need to take time regularly to address misunderstandings and maintain skills.

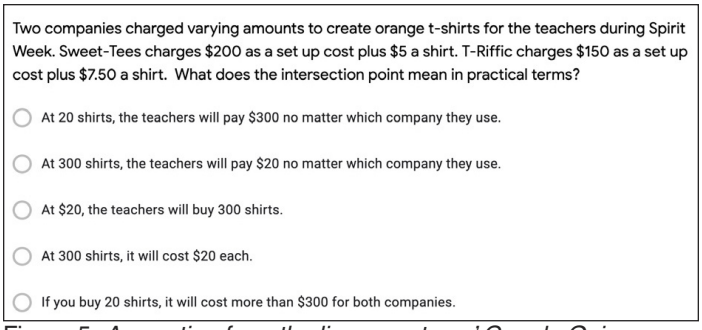


Figure 5: *A question from the linear systems' Google Quiz*

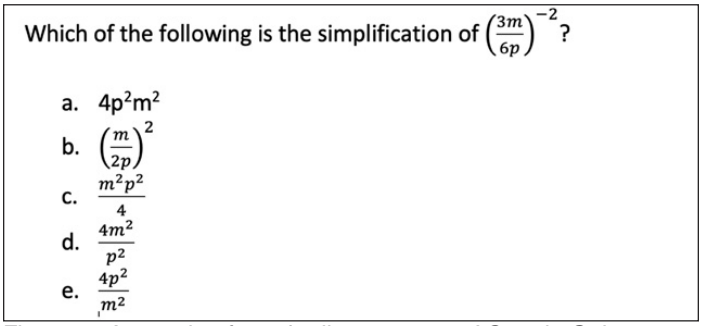


Figure 6: *A question from the linear systems' Google Quiz*

Within each Google Quiz, you can choose in Settings to randomize the order of the questions (Figure 7), which facilitates interleaving practice without the students feeling the questions are tacked onto the end, or that they have an intentional order.

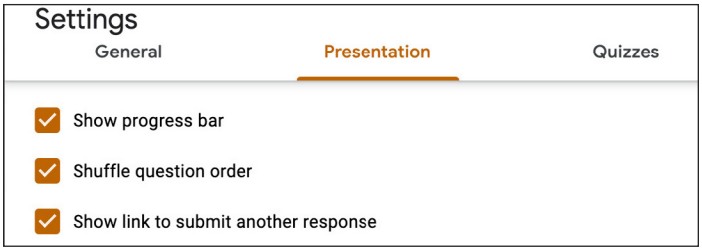


Figure7: *Google Quiz's option to shuffle (randomize) the questions*

CEMC Courseware has additional online practice questions and videos to use for reinforcement. With exponent laws, for example, CEMC (Figure 8) provides questions with increasing difficulty on the same topic, with videos explaining detailed examples. This supports students' agency to make independent choices in their exponent review, which is well suited to asynchronous reinforcement.

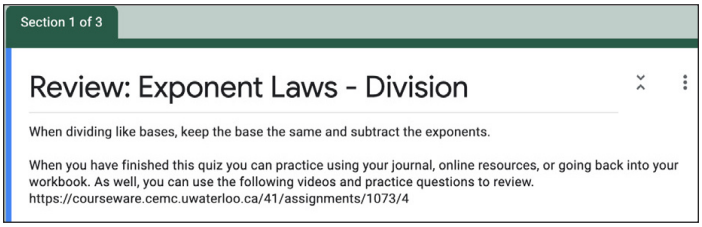


Figure 8: *Including CEMC videos and practice questions*

In my face-to-face classes for many years, I have had students create a physical math journal. As part of their daily routine with these journals, my students note the key findings/processes and document an example, independently, often as their end-of-class task ("exit ticket"). They often also use their journals to note recurring errors, with corrected examples from class or evaluations, as well as homework. I have extended this practice by including the final question of each asynchronous Google Quiz, by asking students to write out the process for one problem in their journal (Figure 9).

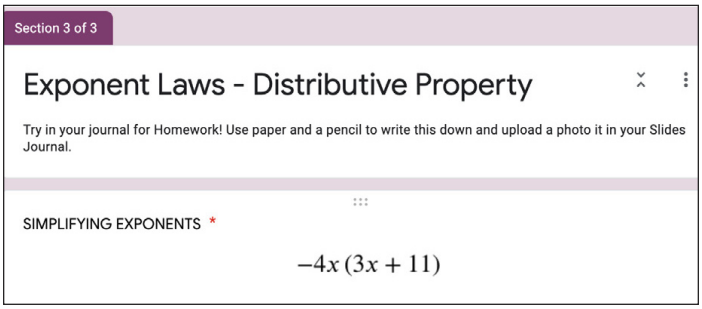


Figure 9: *Example of a journal prompt from a Google Quiz*

Asking students to record sample solutions in their journals has also prompted them to use the correct answer feedback, and supports the development of correct form. I encourage students to use their journals to "talk to their future selves" and provide examples that they might forget. In an end-of-year reflection exercise, students last year noted that these journal entries helped with their interleaving practice and made remembering easier. One student wrote that despite the upheaval, "having my journal to fall back on helps me feel better about math next year."

I often use journal prompts that require the students to draw diagrams on their own and make further connections, like this question concluding the Pythagorean theorem quiz (Figure 10).

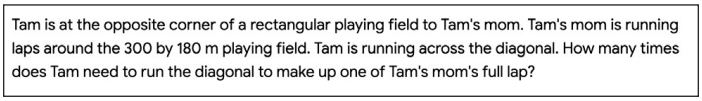


Figure 10: *Example of a journal prompt that required creating a diagram*

Providing these interleaving challenges asynchronously gives students an opportunity to embrace difficulty and engage in productive struggle. Since it is more difficult to ask for help from their peers with distanced seating, I am working with different online collaboration tools. Recording comments in Voice Thread to explain a photo of their work, for example, has supported collaboration asynchronously. Both my students and I are becoming more comfortable with recording our voices explaining our thinking shown in a

written solution. I also post solutions at the end of the week in our Google Classroom, and encourage them to check their journal with the posted solution, something I had not previously expected students to do independently.

This year, with the intention to have fewer shared physical items, I am using online journals and portfolios, with Google Slides, for my students to post pictures from their journal as public communication of their learning. In doing so, they recognize what makes their solutions different and what they have in common with those of others (Figure 11, with many thanks to my colleague, Dora Bellos, for her work on setting up this engaging and clear template). Students choose to explain their journal choices with the use of photos, video or audio recordings, including their Voice Thread explanations (Figure 12).

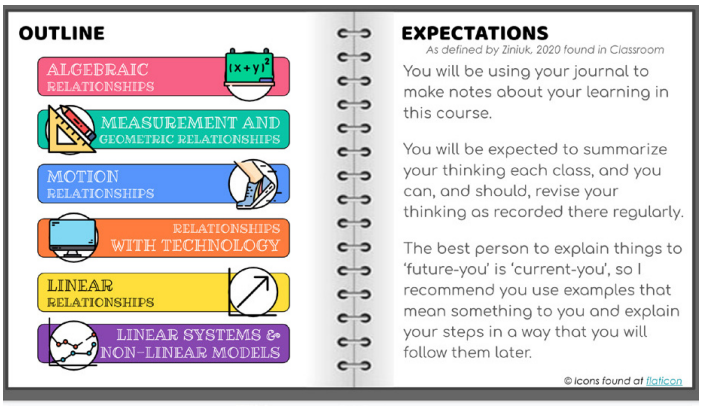


Figure 11: *Journal template in Google Slides, as a portfolio of work*

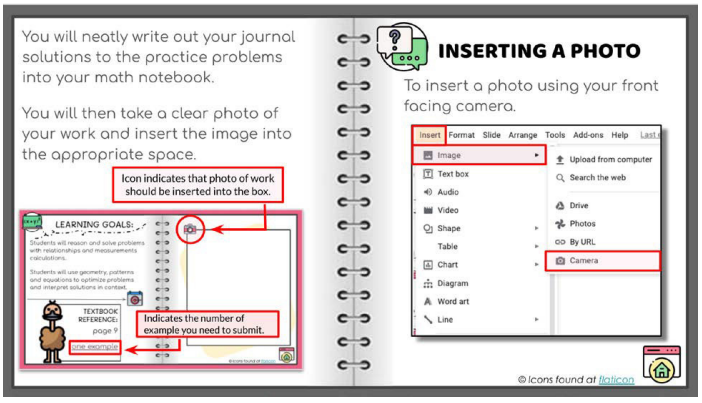


Figure 12: *Journal submissions to an online portfolio*

Achieve the Core (www.achievethecore.org/) presents Mathematical Language Routines, such as Compare and Connect and Collect and Display, as opportunities for students to use multi-modal ways of learning and expressing their math understanding. The Compare and Connect routine works well with journal submissions. While I have used this routine previously in collaborative learning and small-group discussions, I have adjusted it recently to compare written form and connect, using online tools. Students compare their journal solutions and then connect those solutions on Google Docs, using mathematical language. Similarly, we have been using Jamboard (in the Google Suite) to collect and display varied solutions to multi-step problems (Figure 13).

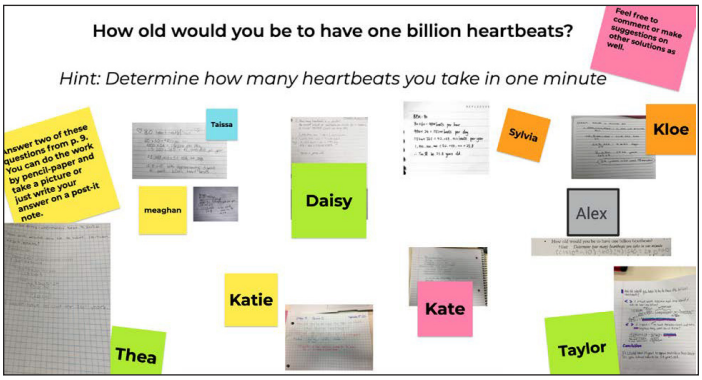


Figure 13: *Jamboard using the Collect and Display Mathematical Language Routine*

I am also incorporating the following online strategies with interleaving practice, circling back to topics from last year and from current learning. These are occurring with students in face-to-face class together and with students who are at home. Since some of the skills from last year were not fully practised or form was not well refined, I have been using Desmos Teacher Mode to have them practise older skills, but focus on their form as well. Desmos Teacher Mode (www.teacher.desmos.com/) allows students to work concurrently on their own individual document, and the teacher can watch all of the screens at the same time. Students who are away can work on it asynchronously. You can also choose the anonymize option in order to demonstrate student work, without worry of embarrassment. For extra extension, the anonymous names are famous mathematicians (Figures 14 and 15).

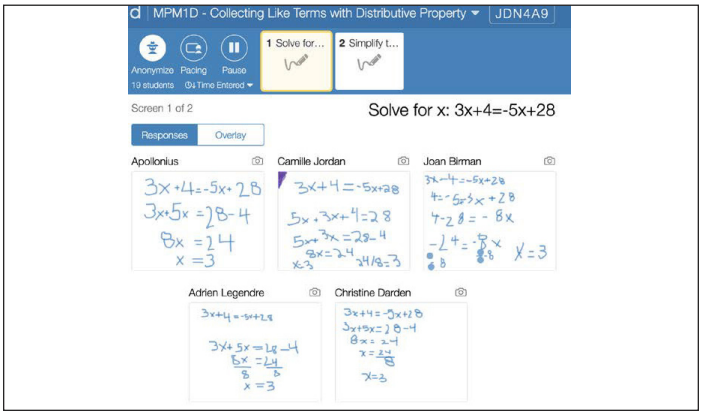


Figure14: *Desmos Teacher Mode illustrating viewing all of the students' work, with anonymized names*

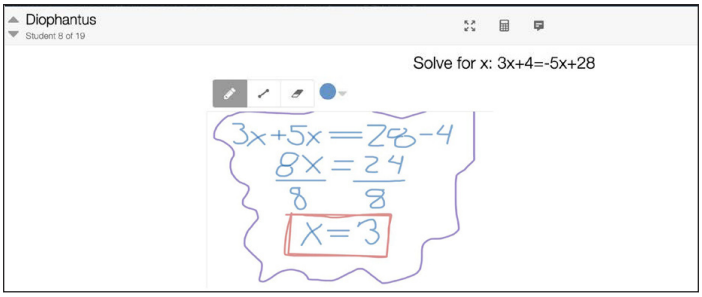


Figure 15: *Using the camera in the corner of each to take a photo of the work or zoom into each individual's work*

Using Desmos Teacher Mode with the entire class, during class and asynchronously, has allowed me to showcase different examples to highlight how the problems differed (Figure 16), for error finding, and for discussion of form. This is important to identify some misconceptions that were not identified in Google Quizzes, multiple choice, but does so faster than reading through journal posts. It more quickly reproduces the observations gleaned from looking over students' shoulders and checking in as they move through algebraic questions.

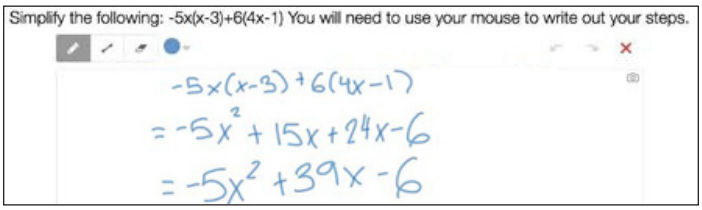


Figure 16: *Example of two different student solutions in the same problem set on Desmos Teacher Mode*

Using Desmos' Card Sort and Polygraph with many mixed topics in the bank of "cards" (Figure 17) provides an opportunity for students to work independently and together. An example of students working together appears in Figure 18, and an astute teacher will realize a need to follow up on some communication skills.

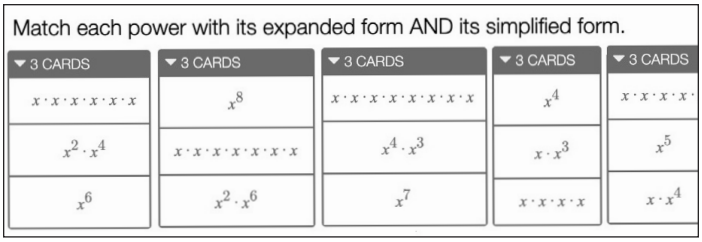


Figure 17: *Card Sort to develop the product law for exponents*

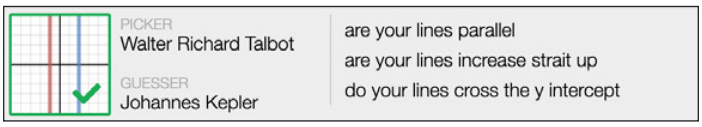


Figure 18: *Polygraph showing one student asking yes/no questions to determine the correct graph*

Interleaving practice helps provide students with the skills to break complex processes down into smaller well-remembered components. Google Quizzes and Desmos Teacher Mode allow me to check on those processes in repeated ways, quickly provide feedback on their form, and maintain physical distancing. The use of the journal with the Mathematical Language Routines helps students to feel confident that they are creators of their own understanding and responsible for recording it. With so many changes and uncertainty in this current school year, I continue to look for flexible strategies that also allow me, as *Learning for All* (2013) recommends, to collect information on each student's individual needs and the collective class requirements. ▲

OCMA Ontario Colleges Math Association

▲ OCMA News

MARIA SAIROGLOU
maria.sairoglou@senecacollege.ca

The first ever OCMA Virtual Symposium took place from October 27–29, 2020 and featured 14 presentations around online learning, strategies and ideas, and helpful resources for teaching mathematics. The Symposium was organized as an alternative to the annual conference usually held in May, which had to be cancelled due to COVID-19.

The purpose was to allow attendees to discover ways to help build algebra and numeracy skills, expand our math teaching footprint in new vocational and cultural directions, and share successes and challenges in teaching math during a global pandemic. Never before has connecting with our colleagues and sharing best practices been as crucial as it is during these challenging times.

There were 18 presenters from colleges across Ontario, with a few also joining from Alberta. Over 120 registrants participated in the interactive presentations, and there was opportunity for great discussions to be had. At the end of each day, there was a "Social Hour" that allowed registrants and presenters to connect and share lighthearted stories of how everyone has been these past months.

The presentation recordings and materials will be posted to the OCMA website and made available to all members. Visit www.theocma.org for more details.

The OCMA looks forward to continuing our proud tradition of bringing college math faculty together for over 40 years, to connect and collaborate around curriculum and delivery of mathematics. ▲

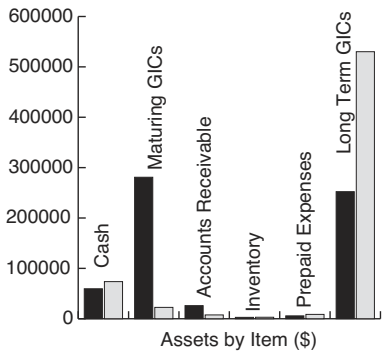
▲ OAME Financial Statement 2019-20

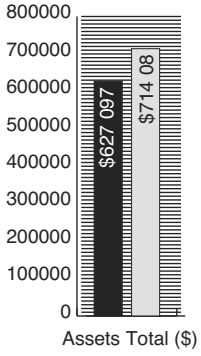
Notice to Readers

On the basis of information provided by management, we have compiled the statement of financial position of Ontario Association for Mathematics Education (Association Ontarienne pour L'Enseignement des Mathématiques) as of August 31, 2020, and the statement of operations, changes in net assets, and cash flow for the year then ended. We have not performed an audit or a review engagement in respect of these financial statements and, accordingly, we express no assurance thereon. Readers are cautioned that these financial statements may not be appropriate for their purposes.

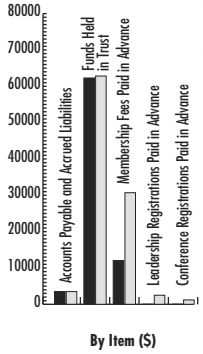
London, Ontario, October 29, 2020, GRANT THORTON CHARTERED PROFESSIONAL ACCOUNTANTS

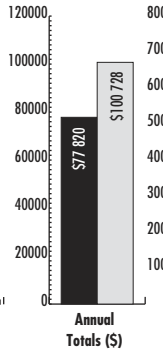
Current Assets

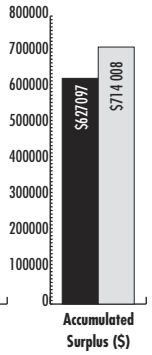




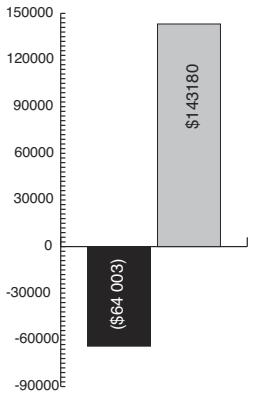
Liabilities and Fund Balance



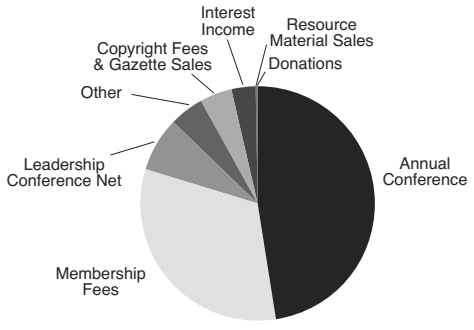




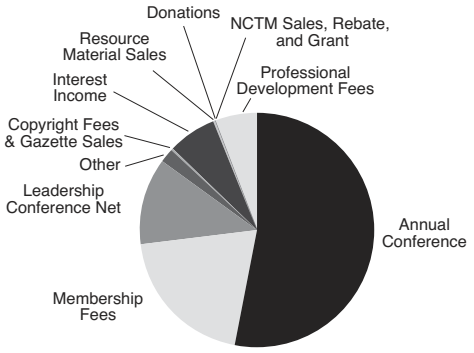
Excess of Revenue Over Expenses



Revenue



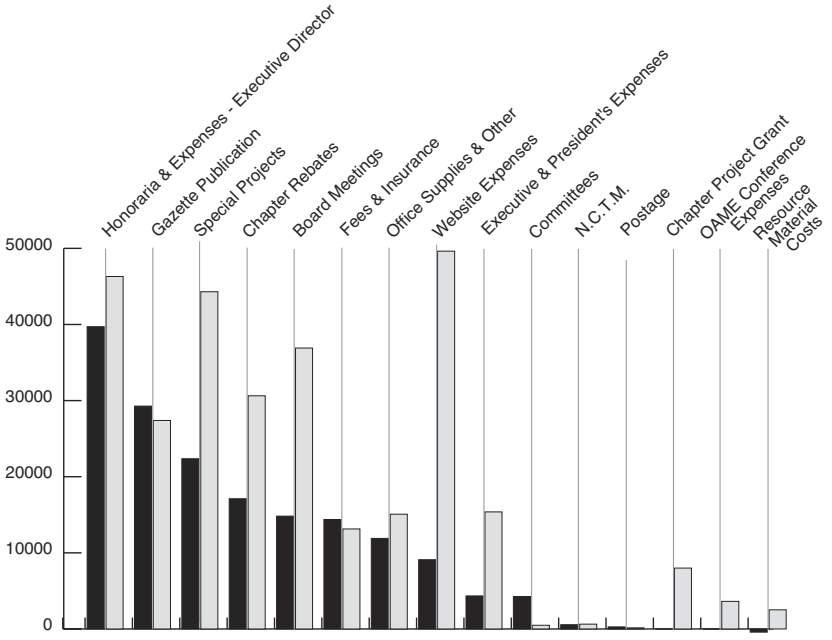
2020 ($134 462)



2019 ($519 898)

2020 2019

Expenditures



Expenses by Item ($)

▲ Fields Institute MathEd Forum Report



ANGELICA MENDAGLIO
angelicamendaglio@gmail.com

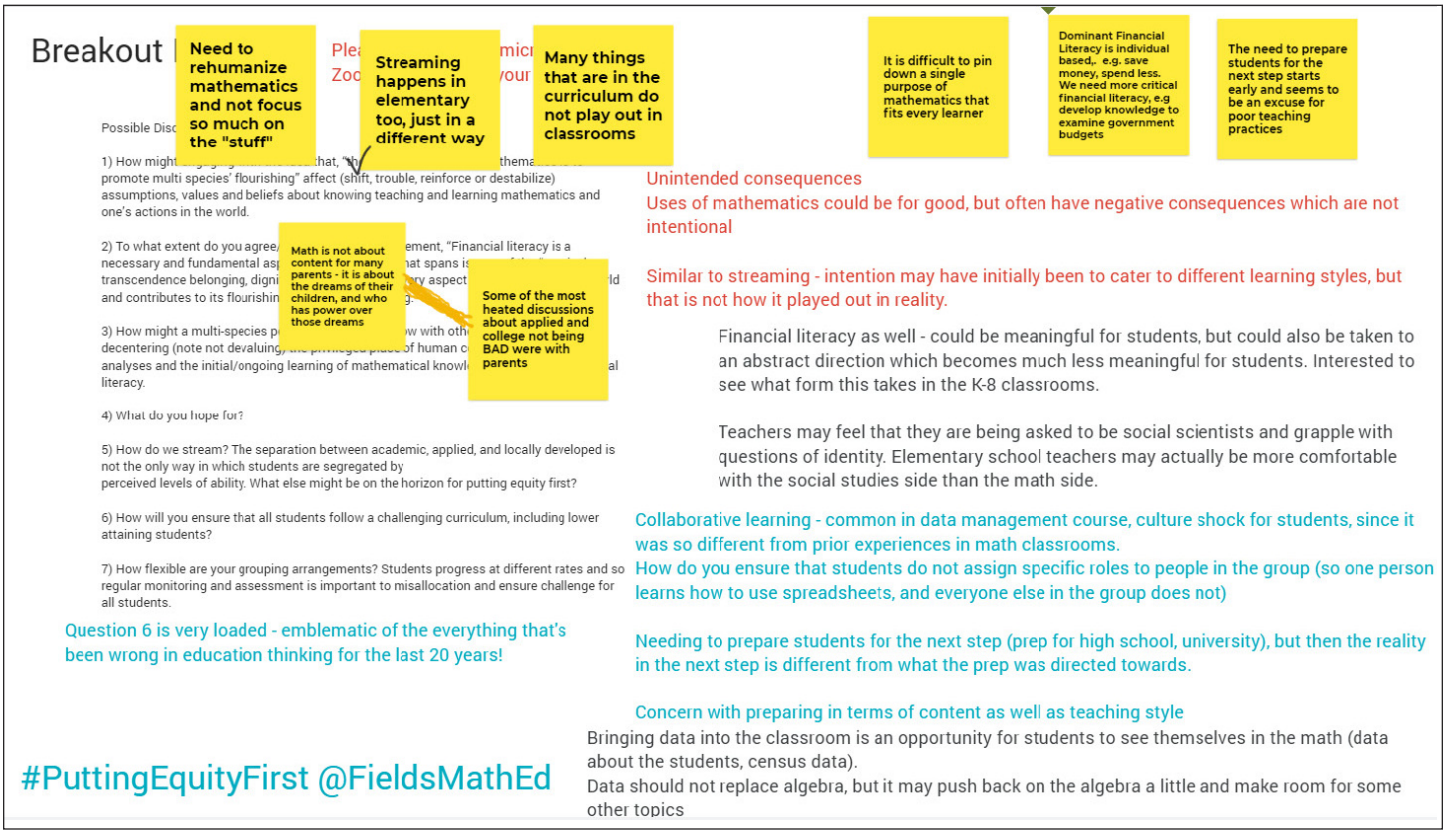
Angelica Mendaglio is an instructional designer at Vretta Inc. in Toronto, Ontario, where she helps to create interactive digital mathematics lessons and activities for middle school students.

The Fields Institute MathEd Forum kick-started the 2020–2021 year with an online meeting titled *Putting Equity First*. While it was an unusual format for the Forum, the organizers were able to not only invite interesting speakers to present on this crucial topic, but also maintain the collaboration that is a central part of the Forum for many attendees.

As was outlined in the Forum's agenda for the September meeting, two upcoming or current paradigm shifts are occurring in Ontario, which present an opportunity for improving equity for students—the move to online learning, and destreaming of Grade 9 math. This session was hosted on Zoom, with some 60 people in attendance. The Forum invited three speakers to present to session attendees different perspectives on the challenges and opportunities presented by these shifts. Dr. Steven Khan from Brock University spoke about *Equity and Multispecies' Flourishing: A Beginning and an End for Mathematics Education.* He was followed by Stacy A. Costa, a doctoral candidate from OISE, who gave her talk, titled *Mathematics During the Information Age: Conversations Surrounding Complex Issues and How to Shift Landscapes.* Finally, Dr. Peter Taylor, from Queen's University, spoke to attendees about *Streaming: What It Is Supposed to Celebrate, Why It Is Broke, and How We Might Fix It.*

After the morning sessions, attendees were assigned to random small groups through the Zoom meeting structure. Each group was provided with a link to a collaborative Jamboard, which included a series of discussion prompts from the morning's presentations. My own group had members from diverse professional backgrounds, and so it was very interesting to hear different perspectives on this important topic.

The first discussion prompt centred around "the purpose of learning mathematics," and members of my group remarked that it is difficult to pin down a single purpose of mathematics that fits every learner. This idea was echoed for me later in the month when I attended the OAME/AOEM webinar on *Program Planning* for the revised elementary math curriculum. One of the core ideas in that webinar was creating a classroom experience that is both math-centric and student-centric. By selecting the teaching techniques and styles that best support



My group's Jamboard at the end of the hour-long breakout session

the learning outcomes of individual lessons, the presenters posited that teachers would naturally create a classroom that also supports students with diverse learning needs. This resonated with what my breakout group discussed at the Forum—a single purpose of mathematics will not fit every learner, just as a single teaching style will not. Recognizing this, and adjusting teaching to meet the needs of individual students, would help to support a more equitable classroom.

After the breakout sessions, the full Forum gathered in the afternoon to participate in a panel discussion. The panelists were Steven Khan, Joe Restoule General (learning resource teacher at Jamieson Elementary School), Phiona Lloyd-Henry (vice-principal in the Peel District School Board), and Dr. Beyhan Farhadi (secondary teacher in the Toronto District School Board). Forum attendees also had access to the Jamboards from the other breakout rooms, so we could peruse the discussions had by the other groups. The panelists spoke at length about financial literacy in particular, and its inclusion in the new elementary curriculum. This linked with quite a few points on the Jamboards, which spoke about "the idea of privilege and to what degree financial literacy curricula are written by (and perhaps imposed by) people who hold a position of privilege."

All of the presentations, as well as the panel discussion from the September meeting, are now available on the Fields Institute YouTube channel. However, two aspects from the day, which cannot be accessed through this recording, are the breakout discussion sessions, and the chat which was occurring throughout the day among Forum attendees. One benefit to the new digital platform for the Forum is, it is easier for the Forum attendees to share their own resources with other attendees. Among the resources mentioned was *Maththatmatters: A Teacher Resource Linking Math and Social Justice* (2006), a book by Toronto teacher David Stocker, which includes math activities for middle school students, with social justice themes. Forum attendees shared many links to websites they have used in the past to form the basis of mathematics lessons or activities with links to real-world social justice issues.

The move to a digital platform is shaping up to be an inclusive transition for the Forum. At the time of writing this, the plan is for the remainder of the Forum meetings to take place online. Given the excellent speakers, the preserved opportunities for discussions among attendees, the ability to include people who could not travel to the Fields Institute in Toronto, and the ease of access to resources recommended by other attendees, this 2020–2021 season at the Fields Institute MathEd Forum will surely be a very interesting one.

The Fields Institute MathEd Forum will be convening on the last Saturday in January, February, March, and April. Visit the Fields Institute website to register. ▲

▲ Elementary Math Matters: If Math Were an Animal



LYNDA COLGAN
lynda.colgan@queensu.ca

Lynda Colgan is a Professor at the Faculty of Education at Queen's University. As a lifelong educator, Dr. Colgan constantly searches for creative avenues to engage students, parents, and educators in mathematics at home and in the classroom. She is inspired to do so by Seymour Papert, who once called upon his audience of "computers in education consultants" and mathematics teachers to bring about change by saying: "Instead of trying to make children love the math they hate, make a math they'll love."

I will be forever grateful to my colleague and friend, Dr. Nancy Dalgarno, for introducing me to an elementary mathematics curriculum course icebreaker many years ago, soon after I arrived at Queen's.

Nancy had recently enrolled in the doctoral (i.e., PhD) program at Queen's and I was her supervisor. To round out her experience in all three areas in academia—research, teaching, and service—Nancy was hired as a Graduate Teaching Fellow, and it was my privilege and pleasure to mentor her as she became a pre-service instructor for the first time. We had sections of the same course and met regularly to co-plan and debrief our classes, sharing the responsibility for the preparation of teaching materials: slides, tasks, assessments, rubrics, and student feedback opportunities. For me, the experience was energizing, stimulating, and fun.

Nancy brought innovative, creative ideas to me and for the courses. Her frank "*Why* do you teach about {fill in the blank} like that?" questions prompted reflection on my part; i.e., *Why* did I use a particular task? She forced me to confront my assumptions and biases; rethink my priorities; acknowledge my strengths and own my weaknesses as an instructor; and, most importantly, take risks by trying new things.

During the first year of our teaching partnership, Nancy suggested that we begin the first class of the year-long course with an art-based task: asking our pre-service students to draw the animal (real or imaginary) that they believed best represented mathematics, and then describe the rationale for their choice.

At the time, I was wary of Nancy's suggestion. I had always started the elementary math program with what I

considered to be a "fun" math task—using origami to fold Baggi Boxes to use, first as nameplates, and ultimately, for what I believed to be more important… a manipulative for a geometric/algebraic inquiry.

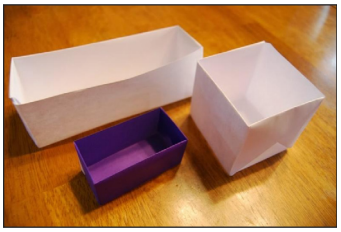


Figure 1: *The open boxes were made by using the origami instructions attributed to Guiseppe Baggi. For folding instructions and full lesson, see Higginson and Colgan (2001).*

Nancy suggested that although the activity might seem "fun" to me, it had a very mathematical "feel" and might be intimidating to incoming Primary/Junior pre-service students, who could be anxious about their mathematics education course.

Fortunately, for me, and for my students, I trusted Nancy's assessment of one of my favourite inquiry activities/tools, and followed her lead, walking into my mathematics education class with blank sheets of 11 x 17 paper, brightly coloured markers, and a tummy full of butterflies.

That class changed my life and set the path for all future pre-service courses that I taught. The responses were eye-opening and most revelatory in terms of the attitudes and dispositions of our teacher candidates. As Nancy and I compared our findings, we noted many similarities.

Many students suggested that mathematics is like a bee: a creature with clear value for society as a pollinator, yet potentially painful when handled. A large number suggested that mathematics is a wolf in sheep's clothing, i.e., looking innocent, but always primed to attack and eat vulnerable victims. Several students suggested that mathematics is like an owl "because owls stay up all night, just like students who are trying to do math." One suggested that mathematics is like a giraffe, in that it was "way over my head." Among the other "real" animals: mosquitoes, porcupines, spiders, octopi, angler fish, and snakes—all of which carry negative societal images—and a few votes for the mysterious Great Sphinx of Giza, the poser of impossible riddles.

Happily, there were some exceptions, including a limited number of students who suggested that mathematics is like a butterfly, something that could transform into a beautiful life form over time, and in the case of mathematics, as one learned skills and strategies.

The *"If math were an animal"* challenge wasn't the only such exercise to which Nancy introduced me. Another was "The Blob Tree" (Figure 2), originally developed by behavioural psychologist, Pip Wilson, to help children discuss their feelings. The "Blobs" are genderless characters, with each blob figure displaying both a mood and position on the tree.[1] For this activity, students were asked to answer the questions, "Which 'blob' best represents where you see yourself at this time with respect to teaching elementary mathematics? Why?"

Again, after comparing our classes' responses, we noted that a significant number of education students identified as Blob #1 or #7: hanging on for one's life. Many identified as #3 or #6: just beginning the learning curve to climb up the tree of mathematics education. We were pleased to note that others identified as being in the #16, #17, #18 cluster: they felt that they would be supported by peers and professors and ultimately arrive at a safe spot, pretty high up on the tree.

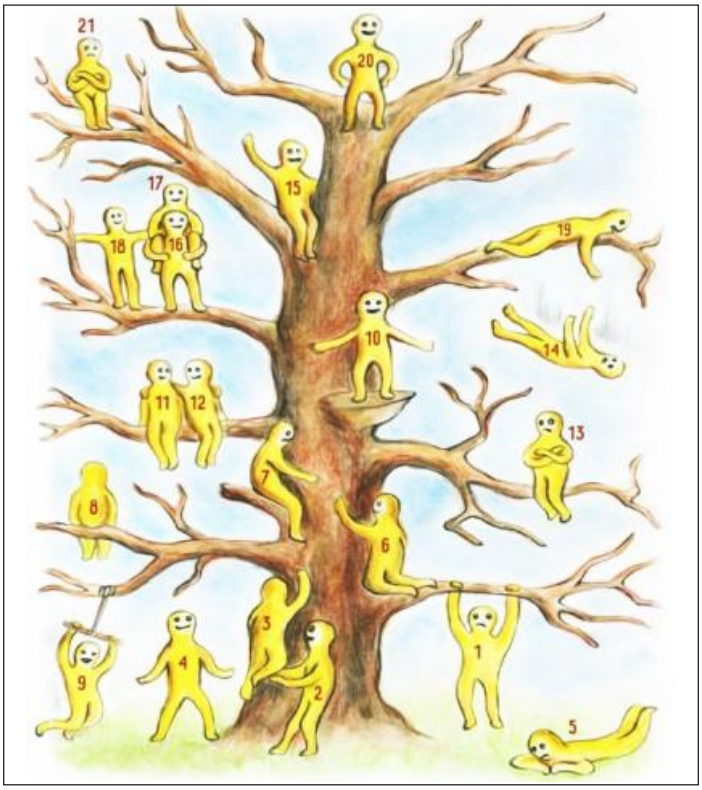


Figure 2: *A Blob Tree example. The Blob Tree is a self-reflection tool in that it challenges individuals to observe the blob characters on the tree and select which one expresses how they currently feel.*

Of course, through these exercises and others, what Nancy had introduced to me were instruments to gather much needed informal data about my students and their socio-emotional position. Like other instructors at the time (the era of a new curriculum and sweeping pedagogical reforms), I was so busy being concerned about mathematics content, processes, skills, and strategies, that I had ignored the critical attitudinal and dispositional side of the coin. I credit Nancy with teaching me the power of collecting my own data and applying that information daily to support

students' affective growth, rather than following the norm of that era, i.e., relying on academic papers about mathematics phobia and women's mathematics anxiety, and then teaching a few classes on those topics.

Much has changed for me, and the world of mathematics education since then, and those changes have been positively progressive as well as long overdue. As a broad community, we have been inundated with conclusive research findings about, and knowledge mobilization strategies to nurture, growth mindsets—for ourselves and in our students—and after a very long time, it has become widely accepted that the learning process is 50 percent social–emotional and 50 percent cognitive (Brown et al., 2012).

Happily, the new Ontario mathematics curriculum for Grades 1–8 concretizes these beliefs and changes. It is, without a doubt, visionary in its explicit inclusion of Social–Emotional Learning (SEL) skills as a priority for our classrooms and students (and by extension, we, as educators and role models).

The policy document states that educators must help students to develop confidence, cope with challenges, and think critically by learning how to manage their stress, stay optimistic, and persevere. Since those self-regulation and emotional literacy competencies are at the core of student well-being and skill-building, but are not intrinsic, educators must model and demonstrate those behaviours to show them what they look like in practice

As we do when introducing other skills, we must be explicit in our instruction, and use successful teaching strategies such as "think alouds." When we make a mistake at the white- or blackboard, we need to "own" it, examine and learn from it, i.e., *What went wrong, why, and how can it be handled differently next time?* If we are asked a question to which we do not know the answer, we must model resourcefulness; i.e., *Does anyone know the answer? Who has a suggestion for how to find the answer?* When the solution to a problem is a challenge and frustration is building, we must become a manager of negative emotions, i.e., *taking a break; figuring out what we do understand; breaking the problem down into "chunks"; and referring to notes, earlier tasks, and other helpful resources, such as textbooks or the Internet.* When we are confronted with the inevitable "When are we ever going to use this in real life?" question, we respond by making connections among mathematics and everyday life, the home, and community; and, most importantly, routinely, we give students authentic reasons about why we feel happy and/or optimistic for them and their future, emphasizing in those comments, optimism about their ability to succeed in a career in mathematics and encouraging conversations about how they feel learning mathematics will contribute to their success in the future.

In a blog post from *Teacher Vision* (www.teachervision.com/mathematical-dispositions), it is suggested that in order to become the type of teacher who can mentor students in constructive ways to adopt positive dispositions and improve achievement, Faculties of Education must train pre-service teachers who:

- demonstrate a keen personal interest in learning and growing
- hold the view that all students can learn, but that each student is an individual learner
- are reflective practitioners
- have high, but realistic, expectations of learners
- show flexibility in adapting instruction, materials, and assessment to varying contexts
- have the ability to create a positive, yet goal-oriented, classroom environment

Indeed, graduates with these overall beliefs and skills will be central to the implementation of the socio-emotional expectations infused throughout our new Ontario mathematics curriculum.

With these qualities in mind, I will instruct future elementary classroom mathematics educators. With these starting points in mind, along with Nancy's "Math as an Animal" and "Blob Tree," I will strive to acculturate a secure classroom environment, where my teacher candidates and I can, together, do mathematics and think mathematically. By so doing, I am confident that they will be primed for the development of a powerful, essential, and complementary suite of important skills, foundational to teaching mathematics, i.e.:

- genuine interest in mathematical concepts and connections
- persistence with finding solutions to problems
- the willingness to consider multiple processes or multiple solutions to the same problem
- an appreciation for mathematics-related applications such as those in music, art, architecture, geography, demographics, or technology (www.teachervision.com/mathematical-dispositions)

As a reminder to my students and me, I will open each class with the words of Karl Subban, former school principal (and ultimate hockey Dad) on being a role model to children in the classroom and at home:

> *You are their idol. They look up to you. They don't do a good job listening to you, but they do a fantastic job imitating you.*

Note

[1] Pip Wilson designed this as a two-part psycho-emotional test. The first part was to find the blob figure that resembles you the most, and after that, find the one that you would like to resemble. Dr. Dalgarno and I did Part 1 only. Please note that Wilson and Long (2018) state in their books that the images therein can be reproduced for instructional purposes.

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Higginson, W., & Colgan, L. (2001). Algebraic thinking through origami. *Mathematics Teaching in the Middle School, 6*(6), 343–349.

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NOMINATIONS FOR OAME/AOEM BOARD OF DIRECTORS 2021-2022

Elections are currently being held for a President-Elect (four-year term: one year as President-Elect, two years as President and one year as Past President) a Vice President (two-year term) and two Directors (three-year term), of the OAME/AOEM.

OAME/AOEM members interested in the position of Director will declare their intent to run for either Grades JK-6 or Grades 7-12, based on their personal experience, expertise, and interests.

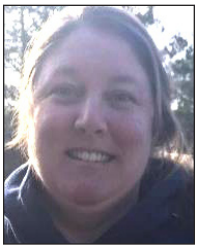
Nominations for these positions are to be submitted electronically by completing an online Nomination Form by 5:00 p.m. on January 14th, 2021.

A nominee must be an OAME/AOEM member. By March 2021, the Nomination Committee will publish a slate of candidates, including candidate bios, for consideration by the membership. Voting is open to all OAME/AOEM members. The election results will be released at the Annual Conference of OAME/AOEM in May 2021 and posted on www.oame.on.ca.

Online nomination form as well as a summary of the duties of each description is available at http://bit.ly/OAMEPositions (scroll to the bottom of the screen). Please contact Paul Alves (paul.alves@oame.on.ca) for more information.

Thank you for submitting your nomination!

▲ Interview with Dr. Rochelle Gutiérrez



ANN ARDEN
ann.arden@ocdsb.ca

Ann Arden is a vice-principal in the Ottawa-Carleton District School Board (OCDSB). Prior to that, she was a math teacher and instructional coach supporting students, teachers, and administrators in the OCDSB.



IDIL ABDULKADIR
idil.abdulkadir@ocdsb.ca

Idil Abdulkadir is a math teacher with the Ottawa-Carleton District School Board. She has been thinking about, studying, and working to improve the experiences of racialized and Muslim students in Ontario schools since 2010. Idil was a Desmos Fellow in 2019 (Cohort 4).



In this column, Ann and Idil interview **Rochelle Gutiérrez**, who is a Professor of Education at the University of Illinois at Urbana-Champaign. Her research interrogates the unearned privilege that mathematics holds in society, and the roles that race, class, language, and gender play in teaching and learning mathematics.

Ann: Can you tell us a bit about your own experiences with school mathematics (K–12) and in university as a student?

Dr. Gutiérrez: I was in a program in public school in the early 1970's, when there was a lot of progressive education. There were a lot of really creative, experimental ideas. In my home city of San José, California, they decided to create a non-graded education program. Unfortunately, it was only for the so-called "gifted and talented" kids. The way we learned mathematics was not your traditional way, just like the way we learned lots of things that was not traditional. In science class, we were making ricotta cheese and lotion, and we were going to operas. In math class, we were doing all kinds of things, like solving logic problems, doing point-perspective drawings, pre-algebra stuff when we were in second grade and third grade. We were multiplying and adding in other bases. Just all kinds of stuff that nobody told us we couldn't do, so we didn't know we couldn't. Because

the program was non-graded, we had Grades 1–3 together and Grades 4–6 together. They wanted us to get three years together with a single teacher with the same group of people, where some are moving out and some new people are moving in. You start to play different roles as a learner and as a member of that community. That part of it, for me, felt very different. At the time, the big thing was Bloom's taxonomy, with these six different levels of learning, where the lowest level of learning was *memorization*, the next level was *comprehension*, and it moved up to *analysis* and *synthesis*. *Creation* was the highest level of this kind of work. Every time we did any kind of work, whether it was math or not, we had to justify what level of thinking we thought was manifested in that work that we turned in. So we had to say, "I think this is comprehension because I didn't just use this formula. I had to think about what I understood about the concept before I figured out what I was going to do" or, "This was analysis because I was comparing and contrasting two different ways of thinking, and I had to choose between them and decide which one of these was the better one for this case for this particular problem." We did that for everything, for every single thing we did—we had to analyze our own thinking and justify where we thought we fell. So I think that both, as a result of how that affected me as a learner, and what I even thought mathematics was about, had changed, right? But, also my role in the classroom and my commitment to other people was also changed by that because of the non-graded part. For teachers, I think they must have had to be thinking, "When are we going to be creating mathematics?" To be honest, I don't remember anything in particular that I justified to be at the level of creation, but I do know there was a lot that we did that was at the level of synthesis and analysis. We got really good at distinguishing between those small differences.

Idil: That's a lot of metacognition at a young age. That's fantastic.

Ann: That's so interesting. You're talking about different ways of interacting with each other in addition to different kinds of mathematics that you were learning, too.

Idil: For me, it clarifies some things around your work. In a lot of your work, you're able to imagine ways of doing mathematics, and what mathematics can look like. If you're educated in a traditional classroom, it can be difficult because you've never seen it modelled. So, for me, it clarifies how you're able to have such an expansive view and look for different things.

Dr. Gutiérrez: The only thing I would say is, "Why was that held back for only the so-called 'gifted' kids?" Right? I don't think there was anything about us that was particularly gifted.

Ann: Did your experience in that kind of setting go from Grade 1 to Grade 12, or did you end up in a more traditional high school math program?

Dr. Gutiérrez: It went from Grade 1 to 6 in my elementary school. In fact, in Grade 6, my parents were worried about the kind of potential elitism that was getting ingrained in us as kids. My two older sisters were not in the gifted program, but my brother was in the program after my parents kind of understood how you navigate the system. My mom and dad pulled me out of that program and put me into "regular" sixth grade. The teachers realized I was bored, and I was also kind of making trouble in the class because I was trying to say, "When are we going to justify what kind of thinking we are doing?" or "When are we going to do these whatever?," and they were probably just like, "Get this kid out of my class because I don't want to do that level, or I don't know how to do that level!" Like you said, how do you prepare to do something that you yourself haven't experienced? How do you teach people in ways that are different, if you yourself have either not been convinced that that's something good, or you haven't actually gone through it yourself. So, in sixth grade, my parents pulled me out, and basically what ended up happening was the school put me back into the gifted program for math, science, and English. I think I was only in the so-called "regular" sixth grade for PE and art, and maybe social studies. In seventh and eighth grades, we had a junior high school program, and I was in the gifted track, so I was put into special classes. In high school, it was the same. Even if it wasn't called the so-called "gifted" class, there's the de-facto tracking, right? By being in the honours math class and in band, and by being in National Honor Society and whatever else, there are only so many other permutations of courses, so you get slotted with all the other kids taking those things. I took AB calculus and BC calculus [Advanced Placement courses] while I was in high school, and participated in the math team, but it was definitely a much more traditional program at the high school. In junior high school, my parents moved me out of a seventh-grade class because the teacher put permanent marker on my forehead. He was trying to get the class not to say "point"—instead of saying "3 point 2," you were supposed to say "3 and two-tenths." If anyone used the word "point," he took a permanent marker and put a mark on their head, almost like a *bindi*. It was ridiculous. So, my parents pulled me out of his class and put me into a different class.

Idil: Wow... speaking of a dehumanizing way to learn mathematics! Just stepping back a little bit, I'm thinking about your gifted program and your de facto tracking as gifted. What was the makeup of your classes—were there Black, Indigenous, and Latinx kids, or were you one of a

few? What did it look like?

Dr. Gutiérrez: This creates a lot of emotion for me because it was a very difficult process to be in that gifted program. It was "gifted" in a sense that I got all this really great enrichment from the class, and in an academic sense. But I didn't belong. I think there were about 26 to 30 of us, who were chosen [to be] in that program. There was me. There was Linda Horio, who was Japanese, and there was Kevin Lofton, who was Black. And everyone else was White. It affected who got invited to birthday parties. It affected who wanted to work with you on things. It affected a lot of stuff. It just got worse. I ended up staying with that same group of kids until I graduated high school, just because of the way the tracking system works. I was almost always the only girl, and I was almost always the only person of colour in those spaces. It was a challenge. Yes, it was a challenge.

Idil: Thank you for sharing with us. Thank you for responding so honestly and for accessing the memories in a way that was very real. I appreciate that, and I think that your story highlights the problem with these kinds of segregated gifted classes and pulling kids out. Obviously, there are so many problems with the "pipeline" of who gets selected and the subjectivity of that and the tests, but there's also something that we don't often think about: the well-being of high-achieving students of colour. We assume that kind of academic success will wash away any kind of social or internal understanding of yourself and belonging and development.

Dr. Gutiérrez: Absolutely.

Idil: It's an incredibly important conversation to be having. Often the conversation around Black, Indigenous, and Latinx students is implicitly about struggling students. It's about students who are at the lower half of your class, and there's kind of an unspoken understanding that "rehumanization" is for the kids who are "underperforming." I think your story really highlights the fact that at every level, there are challenges for Black, Latinx, students of colour, and Indigenous students. Academic success is not the answer. A specialized class is not the answer.

Dr. Gutiérrez: I think even when people feel they are addressing equity, or even when they think that they are being on the students' side, it's not just about access and achievement. People would argue that I was a winner in the system. I was somebody who ended up being in the STEM field. I was somebody who took BC calculus and I was on the math team. I went to Stanford, where nobody from my high school had ever been accepted. I was supposedly the success case, but what is the damage that gets done to people by way of their identity? In my four dimensions of equity [achievement, access, identity, and power] (Gutiérrez, 2012), identity and power are both really important because I know what it's like to have people say to me, "Wasn't that great you got this amazing experience?" I always think it was great and, "Why was it only great for me?" How come I was one of a few people who got that experience? Why is it that we can't imagine that would be the experience for most everyone? And, why was I expected to park my identity at the door to be able to be seen as a legitimate participant in those spaces? Why did I stand out when I chose to go as an Indian instead of a Pilgrim when we had Thanksgiving celebrations? Why did I have to stand out when we were creating our autobiographies? When I brought a picture of myself to the class, which everybody was required to do, somebody took that picture and scratched it up with a pin—it was my very first baby picture—and wrote "Mexican" on it at the bottom. That was also part of my education in the gifted program. We use mathematics to crunch numbers in order to say, "We've closed the achievement gap" or "We've gotten this many students to go on into the STEM field" or "This many students are in our calculus classes now." We have to ask, "But is it a healthy space for people?" Is it a space where people can be authentic and be their best selves, and not just try to be the kind of person somebody else wants them to be? When do we get to use mathematics to ask real questions about our lives, to be able to imagine a different future from the one in which we're currently participating? So, for me, it's not equitable. It's not humanizing to have a version where you just get access to this more rigorous version of mathematics' or more creative version of mathematics, than what's come before you, if in the end, you don't get to be whole.

Idil: You use the idea of math to transform realities in emancipatory ways. Francis Su has published his book, *Mathematics for Human Flourishing* (2020), so in the context of your math experience and these new ideas about flourishing and emancipation, can you tell us what you mean by "emancipation" and how you envision mathematics playing a role in that?

Dr. Gutiérrez: Part of advancing, or being successful, or being grounded as a society, is being able to be whole, being able to be vulnerable and many of the things like you said that Francis Su has brought up in his book. We need to flourish. We need to thrive as people. We've done that in the past, so it's not like it's something new that we have to do. Whether the word is "emancipatory" or "liberatory" or just "toward helping us thrive," I think there are ways that we can use the power that mathematics has held in society to leverage it toward things that we want to happen. Also, for me, mathematics is just the science of patterns. How do we think about what counts as a pattern? What kind of patterns do we want to pay attention to in the world around us? And

to whom do those patterns matter? Whom do those patterns benefit? I think that what we've done in the past is we've tried to make it seem as if the notion of patterning is somehow universalistic, and that we need to think about always finding the most general case, that the general case then can tell us something about all other cases. I think we fool ourselves into hiding behind the power of abstraction because abstraction is occluding us from the original context and meaning of any variable or any kind of data that's driving the notion of pattern. I think we've convinced ourselves that somehow the more abstract we become, the less we actually have to pay attention. But, doing that, in some ways, is a form of erasure. It's like having erased people from society. It's the way we're continually erasing Indigenous people in particular from society. I think one of the ways that mathematics can help is to be thinking seriously about the level of abstraction that we are privileging, and also thinking about what we "count" as pattern. Who gets to decide what pattern is and for what purposes we want pattern? Do we only think about patterns that have come before? Why do we put so much emphasis on symmetry? Why do we not put as much emphasis on asymmetry when we know, in the natural world, that it's a really important phenomenon biologically? In terms of plants and protein folding, asymmetry is actually what helps the world move forward. So, we've made these choices, and I'd say these are aesthetic choices. We've made choices about what counts in mathematics. We can really start to question, "Are those the kind of choices that are going to fuel us to be able to thrive or flourish or to liberate ourselves from a kind of straightjacket that mathematics has been and that has propelled forward warfare, and economics, and profit, and Indigenous erasure?"

Idil: I want to ask about "neutrality" because in so much of what you're saying, you use the words "choice" and "decision." You speak about "rehumanizing," so something is happening already, and it is "dehumanizing." In particular, you identify Black, Indigenous, and Latinx students. What I understand from your work is that you challenge this idea of "neutrality" in the undercurrent of mathematics and science. You challenge the idea that there is an "objective" thing that we're measuring, and that there are no decisions being made along the way. Could you reflect with us about "neutrality" and what that has meant for you? At what point, if at all, did you believe that math was "neutral?" At what point did you decide maybe it's not neutral?

Dr. Gutiérrez: I think that idea of "neutrality" kind of gets thrown out the window, once we recognize that mathematics is a human practice. It's always going to bring with it all of the power dynamics and all of the identity issues. I'm with people who are practising mathematics. I think we fool ourselves into thinking mathematics is apolitical or acultural. When we do that, we remove it from the human aspect. When I use words like "choice" or "aesthetics," or when I'm arguing a version that is reflecting all our relations that are with us in this world—like I do when talking about living mathematx—I'm saying that by virtue of it being human, it is not devoid of politics or of any kind of culture because of the decisions that have been made. We could have made different ones. I think that's the most exciting and most emancipatory aspect—recognizing we've made particular decisions. I think it may feel very overwhelming if we have no control, if mathematics were not a human endeavour and we were constrained by the mathematics we currently have. Because we know that the mathematics we currently have is one we've created, we participated in creating. That opens the door for us to say, "We can create something different!" We can value something different. If we valued certain judgments and decisions that have been made, we can value other ones. We could have another kind of mathematics. Once you realize that the version that we have is, in some ways, based on decisions that real people have made, and hasn't just been kind of sent down from the heavens or something, then that's what is exciting, thinking things could be different. It might support a sustainability of our planet that might produce more interconnected relationships between humans and other-than-humans that are here with us. I guess that's what gives me hope.

Idil: In my reading of this edition (Goffney, Gutiérrez, & Boston, 2018) , my main questions that I didn't know how to ask until now were, "How do you find hope? How do you create it? How do you maintain it?"

Dr. Gutiérrez: How do you create hope in your classrooms? How do you create hope for yourself as a math teacher? How do you create hope for your students? What drives you to do the work that you're doing? Which of these are you wanting me to answer?

Idil: It's passing on to students that life is not happening to *you*. You are an active participant in your life, your learning, and your growth. It's just interesting that I did not extend that to math. We are all participants in this active thing called "mathematics." We are the drivers of it and it is of our making.

Ann: I connect to a part of your opening chapter (Gutiérrez, 2018) in your NCTM text. You write, "Partly because it departs from a Western view of math, rehumanizing mathematics seeks to not only decouple mathematics from wealth, domination, and compliance; it also recouples it with connection, joy, and belonging" (Gutiérrez, 2018, p. 4). That seems very hopeful. When I

think about students I work with, I very much connect with what you wrote there about connection, joy, and belonging, and that sense of math being part of us. I find hope when students listen to each other and respond to each other, rather than being filtered through me as the teacher. When students are conversing or arguing and they are intensely in this thing called "mathematics," I find that really hopeful and inspiring.

Idil: For someone who is just getting introduced to "rehumanizing" mathematics, particularly for Black, Indigenous, and Latinx students, how would you explain it to them?

Dr. Gutiérrez: I put a lot of emphasis on the "re" in rehumanizing. I didn't choose to talk about this as "humanizing" mathematics. It's "rehumanizing" mathematics for me because it really is recognizing and honouring that we have been doing mathematics historically. There are a lot of people outside of school, who are not in "sanctioned" mathematical spaces. On one hand, it's kind of exciting and hopeful that we don't have to invent something new. Rather, we just have to kind of bring back that which is getting erased through school. I think that for me, that notion of "rehumanizing" is really bringing back the parts that are human. What are the parts that are human? One part is our cultures and histories. There's living practice. There's how we participate. How we position ourselves with respect to mathematics. All of the eight dimensions that I use in *Rehumanizing Mathematics* (Goffney, Gutiérrez, & Boston, 2018) are attending to the ways that students and teachers have told me in the past, that mathematics has felt dehumanizing [the eight dimensions are: participation/positions, cultures/histories, windows/mirrors, living practice, creation, broadening mathematics, body/emotions, and ownership]. I didn't kind of invent this "thing" out of nowhere, and in some ways, I'm not even offering anything super new. I think that what I'm offering is a mapping space for people to be able to say, "How are we doing on these different dimensions?" because these are the dimensions that have been reported as being dehumanizing. If we want to not just capture or note what has been dehumanizing, we want to kind of flip that and say, "How do we rehumanize that aspect or that dimension, what would that look like?" For me, creating those eight dimensions and naming them gives them a space. It allows people to kind of map our practice and say, "If I'm a math teacher and I'm looking at these eight dimensions, in what ways am I already potentially successful in some of these dimensions?" and "In what ways am I not necessarily attending to certain things?"; and, "By not attending to those certain things, are there consequences for my students?" We alone, as teachers, don't get to decide that we are rehumanizing mathematics. We actually have to have evidence from the very communities that we say we're humanizing mathematics with, and I say "with," not "for," because then we actually know the work that we're doing has the effect that we would like. It's one thing to feel "this is my intention," but for students, this is how it feels. It's not a kind of, "Here's how you should teach." It's not, "Here is culturally relevant math," or anything like that. It's really more: "Here's some dimensions to pay attention to if we want people to be able to flourish or thrive."

Idil: Reading your work, for me, has been a very powerful personal experience. I have a similar experience in being "the gifted Brown kid." I've been so inspired by you bringing ideas of hope and emancipation to mathematics because growing up as someone who was inclined to math and science, the neutrality of it, as damaging as it was, was the only place purporting to be neutral. And, so in other things I loved, like history and English, they were saying, "Yeah, we're humans, and it is humans who do this, but here is the type of human who does it." So, the sad reality was that the only place I felt like I was not actively being dehumanized was a place where there were no humans—you're just a brain here. It's such a shallow thing to aspire to: that the only place you can belong is where there are no people. And, every place where there are people, the definition is so narrow that it's not for you. I'm grateful for you, for the work you're doing pushing the boundary and saying you can want more, and that you can want a human mathematics for Black, Indigenous, and Latinx students, and for everyone. You can want a mathematics, and we deserve a mathematics, and a way of knowing, and a field and knowledge that is deeply human in all of the ways that every human brings to it. The emancipatory element, the human element, the questions of culture were really powerful for me and so, thank you.

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OAME/AOEM AWARDS FOR EXCELLENCE IN MATHEMATICS EDUCATION

The purpose of the OAME/AOEM Awards is to formally recognize individuals and educators who demonstrate an outstanding contribution to mathematics education in Ontario. The successful candidate for each award enhances the learning environment for all students, providing opportunities for students to do, see, hear, and touch mathematics in a profound and meaningful way. If you know someone who qualifies for this honour, please nominate her/him/them for one of the following awards.

OAME/AOEM Life Membership Award: This award recognizes an educator who has contributed in a significant way to OAME/AOEM, demonstrates outstanding leadership in mathematics education, and has accumulated 10 or more years of membership in OAME/AOEM.

Award for Exceptional and Creative Teaching in Elementary Mathematics: This award recognizes an exceptional and creative elementary educator who demonstrates excellence in mathematics education and contributes to the overall development of students.

Award for Exceptional and Creative Teaching in Secondary Mathematics: This award recognizes an exceptional and creative secondary school mathematics educator who demonstrates excellence in mathematics education and contributes to the overall development of students.

Elementary School Staff Award for Exceptional and Collaborative Mathematics Teaching: This award recognizes an elementary school (a school accommodating any or all of grades K-8) whose staff fosters collaborative and excellent mathematics teaching, contributes to the overall development of student learning of mathematics, and demonstrates leadership in the mathematics community.

Secondary School Mathematics Staff Award for Exceptional and Collaborative Teaching: This award recognizes a secondary school mathematics staff which fosters collegiality, team work, and excellent classroom teaching, contributes to the overall development of students, and demonstrates leadership in the mathematics education community.

Award for Leadership in Mathematics Education: This award recognizes an educator who has demonstrated leadership by contributing in a significant way to the development of mathematics educators and the enhancement of mathematics education in Ontario through innovation and/or research related activities.

Award for Outstanding Contribution to OAME/AOEM and Mathematics in Ontario: This award recognizes an individual who has made a significant contribution to OAME/AOEM and contributed directly or indirectly to the improvement of mathematics education in Ontario but who is not necessarily an educator or a member of OAME/AOEM.

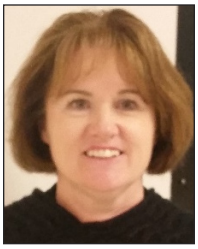
OAME/AOEM Awards Nomination Process

1. Complete the **Award Nomination Cover Page** appropriate to the specific award, found at www.oame.on.ca (look for the **Awards** tab on the left of the page) or contact the Executive Directors at eds@oame.on.ca for an electronic version. You may include a one page letter of recommendation.
2. Include with the nomination cover page up to three **letters of support**, of no more than two pages each. Please ensure all letters of support:
   - Address the criteria for the award as outlined in the nomination cover page.
   - Include the name and position of the supporter to the application.
   - Include the nominee's name and the award in the letter.
3. Include the **Curriculum Vitae**, of up to five pages, for award nominees except for Elementary and Secondary Staff awards.
4. Email or mail all **completed** packages (cover page, supporting letters and CV) by the deadline advertised below to the Executive Directors. Email address and mailing address are included in the nomination cover page.

DEADLINE: Nominations are due by January 31, 2021.

The awards will be presented at the Annual OAME Conference in May 2021.

▲ OAME/NCTM Report: SEL from an NCTM Perspective



JACQUELINE HILL
jacqueline.hill@bell.net

Jacqueline Hill is a Grades 9–12 teacher of mathematics, as well as an online instructor for York University. She is a Past President of OAME and OMCA, as well as the recipient of the award for Exceptional Teaching in Secondary Mathematics. She also wrote the "Director's Dialogue" for the Gazette for a number of years.

In education, generally, there is periodic motion, a back and forth like a pendulum, that investigates and emphasizes change on different aspects of mathematics education. A current focus prompted by the change in the elementary curriculum is SEL, or Social–Emotional Learning, and the process expectations—that are now receiving much more attention. It is also anticipated that the weaving of SEL and the process expectation in the elementary curriculum is a tapestry that sets the stage for an expected update of the high school curriculum.

The NCTM Leadership Conference this summer had many references to SEL parts and pieces. The bottom line was to "Believe in the brilliance of all students." Well… that is just a swing of the pendulum back to the Targeted Implementation and Planning Supports (TIPS) developed in the early 2000's by the Ontario Ministry of Education. The "Necessary for some, good for all!" approach of the in-services by Anne and Rod Yeager got to the heart of the learner.

As we look to the release of a new/revised high school curriculum (which starts with destreaming in the Fall of 2021), we know that SEL will be woven in nicely. With respect to the pendulum… we took a one-year swing back to destreamed Grade 9's in the mid-1990's. As a teacher of the Grade 9 course, I truly understand why it only survived one year. There was no way to reach every student in your classroom due to the diverse levels of understanding and ability. The 2007 revised math curriculum has a wonderful path for students who enter the 9 Applied stream (for that hands-on approach to high school mathematics) and decide later in their mathematics education (after they have had a good taste of success) to change streams to enable them to go to the University stream. The three streams of mathematics enable all students to shine in the destination of their choice.

But I digress… As an online teacher, this year, I have more time in my course to get to know students in a meaningful way. With the online environment, I have a conversation with each and every one of them every day. I believe if you get to know your students, they will be better able to identify and manage their emotions. It doesn't matter what courses I am teaching in a semester, we all have "naptime" and "story time" (mathematical stories, of course) each Friday. Educators, collectively, have recognized for hundreds of years that Fridays sometimes bring stress, and the best way to cope with the stress is to breathe/relax/close your eyes/AKA nap for a moment to recognize what is stressing them, and then to develop coping strategies.

There are five aspects of SEL. As a classroom teacher/educator/coach/parent/etc., we all need to be aware of how students learn, what stresses them, and how to help them be the best people they can be. By being a member of NCTM, you can use their extensive library of research to help you understand the five aspects of SEL.

I believe that SEL is really talking about the anonymous quotation, "It is not the math you teach them, but the size of your heart that they will remember." We need to reach each and every student in a meaningful way and help them to grow and thrive so that they can be the leaders the world needs tomorrow.

Keep your nctm.org web page as a favourite for cutting-edge research in mathematics education.

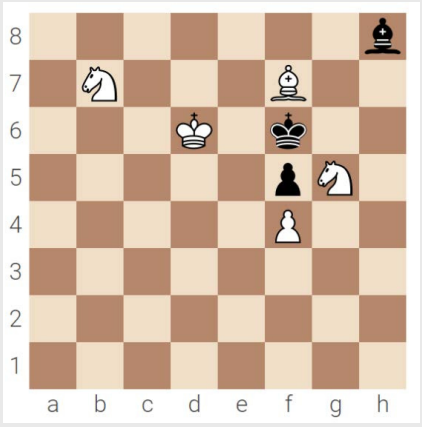
Yours in mathematical fun,

Jacqueline Hill ▲

White to move and mate in three.

From Netflix mini-series *Queen's Gambit*, episode 6.

See www.chessinstitute.ca/blog/2020/10/27/queens-gambit-mate-in-3-problem/ for a solution.



The New Social–Emotional Learning Strand in Elementary Mathematics—Part One



DR. JEFF IRVINE
dr.jeffrey.irvine@bell.net

Jeff Irvine, PhD, has been a secondary mathematics teacher, department head, and vice-principal. He has taught at three faculties of education and at a community college. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. With over 45 years in education, Jeff is particularly interested in the interplay of student motivation and mathematics achievement.

Would you like the students in your math classroom to be confidently engaged in their learning, persevere when they encounter difficulties, work co-operatively and respectfully with you and their peers, search for alternative routes to success in problem solving, develop strong work habits, and see themselves as productive students and lifelong learners? These are the goals of the new Social–Emotional Learning strand in elementary mathematics. This strand recognizes that positive attitudes toward mathematics and learning are essential for success, not only in mathematics, but also in life beyond the classroom.

The six expectations in this strand are: identify and manage emotions; recognize sources of stress and cope with challenges; maintain positive motivation and perseverance; build relationships and communicate effectively; develop self-awareness and sense of identity; and think critically and creatively. Who would argue that these are not important overarching goals of education? However, for some math teachers, these goals may fall outside their personal comfort zones, for two reasons. First, they say nothing about math content or how they are interrelated. Second, similar to the mathematical processes, they are not stand-alone expectations, but must be interwoven with content expectations, and so influence both what we teach and the way that we teach it.

Research related to student motivation has important implications for teachers (Tumposky, 2003). This article provides descriptions of research that teachers can use to support student motivation in mathematics. Student motivation and related constructs are recognized as important correlates with student achievement. In Ontario, in a study that explicitly examined issues involving student motivation of over 90 000 students, who wrote the EQAO Grade 9 Assessments in 2012, Pang and Rogers (2014) found that positive student attitudes and self-confidence were associated with higher student achievement, as well as increased effort, engagement, and homework completion. This same study emphasized the reinforcing nature of attitude and achievement, finding that students who had previously been unsuccessful on EQAO assessments had negative attitudes toward mathematics, as well as lower self-confidence and lower overall motivation. These negative attitudes were reflected in lower achievement on subsequent EQAO assessments.

Smith and Star (2007) found that traditional teaching methods that focus on content and ignore student emotions, attitudes, and motivation have performed very poorly, both for student achievement and for student affective development. Shernoff et al. (2003) found very low levels of engagement in mathematics classes, where they reported that the classes had very high cognitive intensity, but very low motivation. Students reported being more negative about mathematics and less engaged than any other subject. This is not just an Ontario problem. To date, globally there has been limited success in addressing the impact of attitude and motivation on student achievement (Clarkson, 2013). Therefore, it is imperative to identify ways to positively influence student attitudes toward, and motivation to engage in, mathematics.

Motivation declines over students' time in school. Almost every child begins school with very high levels of intrinsic motivation. Intrinsic motivation is motivation based on interest and the desire to learn for the sake of learning. This begins to decline the very first day of school, and continues to decline until about age 16, where it reaches a minimum and never gets any better (Middleton & Spanias, 1999; Steinmayr & Spinath, 2009). The inclusion of social–emotional learning in the curriculum responds to these findings.

The expectations in this strand are all interconnected. Thus, strategies that support one expectation also support other expectations. The expectations are all part of what Kilpatrick et al. (2001) call productive mathematical disposition, "the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics" (p. 116).

General Strategies to Support Student Social–Emotional Learning

- Know your students: This enables differentiated instruction, knowledge of how students prefer to be recognized, frustration levels, and more.
- Co-operative learning: This promotes respective social interactions, the opportunity for students to become experts in some area, and builds communication skills. For details of the structure of co-operative learning strategies such as jigsaw, and conditions under which it can be applied to mathematics instruction, see Irvine (2019b) and Irvine (in press-a).
- Recognition, praise, and feedback: General praise such as "good work" is ineffective. Specific praise and feedback that celebrate effort can be very effective.
- Recognize mistakes as learning opportunities.
- Establish a learning environment that allows students to consider alternative strategies, multiple solutions, multiple representations, and non-standard algorithms.
- Utilize non-linguistic representations such as graphs, diagrams, and graphic organizers.
- Utilize flexible groupings and whole-class instruction.
- Employ high-yield instructional strategies as outlined in *Some Considerations for Program Planning in Mathematics* in the new mathematics curriculum document (Ontario Ministry of Education [OME], 2020).

Below is a discussion of each of the six expectations of the Social–Emotional Learning strand. The discussion includes the goal for each expectation, as indicated in the curriculum policy documents (OME, 2020); why this expectation is important; and some possible strategies that teachers can employ to address the expectation.

Identify and Manage Emotions

Express and manage their feelings, and show understanding of the feelings of others, as they engage positively in mathematics activities (OME, 2020).



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This expectation certainly pertains to more than just the mathematics curriculum. On a pragmatic basis, teaching students to manage their emotions results in fewer emotional outbursts, less bullying behaviour, and more consistent classroom management. However, emotions also occupy an important place in students' development of belief systems. Emotions are the building blocks of attitudes. Accumulations of emotions lead to the development of attitudes; accumulations of attitudes result in beliefs; and accumulations of beliefs develop values.

DeBellis and Golding (2006), expanding on the seminal work by McLeod (1992), identified a taxonomy of affect, shown in Figure 1. Emotions play a large role in the learning of mathematics. In the NCTM *Handbook of Teaching and Learning Mathematics*, William McLeod states, "Learning mathematics is entirely about emotions" (McLeod, 1992). Some examples of the affective hierarchy for mathematics are shown in Table 1.

Emotions can be positive or negative. An extreme example of negative emotions in mathematics is math anxiety. Math anxiety can be debilitating for students, resulting in persistent emotions of fear, tension, apprehension, and anxiety that interfere with performance involving numerical tasks or mathematical problem solving in both ordinary life and in school (Irvine, 2019a). Math anxiety may produce attitudes that reduce mathematics self-efficacy (belief in one's ability to do something); a lack of persistence; and learned helplessness with respect to mathematics (Irvine, 2019a). Math anxiety may also influence student values by reducing intrinsic motivation with respect to mathematics.

A positive emotional dimension generates and sustains student interest in their learning. Hidi and Renninger (2006) structured a four-phase interest model that states that initial interest is triggered by a situation or topic (*Triggered Situational Interest*), which may be fleeting, and may be positive or negative. If interest in the situation becomes sustained (*Maintained Situational Interest*), students have a positive focus and more persistence with the material. If the student develops *Emerging Individual Interest*, they will independently re-engage with the material and express curiosity by asking questions, building their knowledge, and valuing the material. Finally, at the *Well-Developed Individual Interest* stage, the student willingly re-engages with the content, self-regulating to reframe questions, and seeks answers. This level is characterized by positive feelings toward the material, perseverance through frustration and challenges, and actively seeking feedback on his or her learning. The four-phase model has abundant research evidence supporting it. Teachers can use their understanding of the four-phase interest model to identify situations that may stimulate student interest and generate

positive student emotions toward math.

Table 1: *An affective hierarchy for mathematics*

Values	Learning is useful to me. I am a good student. Cheating is wrong.
Beliefs	I am good at algebra. Math is based on rules. Learning is competitive.
Attitudes	I enjoy investigations. I dislike proofs. I hate problem solving.
Emotions	Joy when I get it right. Frustration in problem solving. Pleasure in the beauty of math.

Note: Reproduced with permission from Irvine (2014)

It is important for teachers to realize that they serve as role models for their students (Ediger, 2012). If teachers genuinely display interest, enthusiasm, and joy in the learning of mathematics, students are very likely to mirror these same emotions (Palardy & Rumberger, 2008). Hattie (2003) found that teachers paying attention to student affective factors and providing feedback on student emotions was one of five factors that distinguished expert teachers from merely experienced teachers. Teacher attitudes have a significant effect on both student attitudes and student achievement.

Teaching Strategies to Support This Expectation

On a pragmatic basis, teaching students to manage their emotions results in fewer emotional outbursts, fewer classroom disruptions, less bullying behaviour, and steadier classroom management. The curriculum documents suggest using strategies such as feelings charts and emotional thermometers to help students identify and monitor their emotions. These strategies are consistent with the mathematical process of representing, and presenting information in non-linguistic formats.

A very important role for teachers is affecting a balance of challenge and skill level for students because if this balance is not achieved, negative emotions can result. Figure 2 illustrates that the ideal level of challenge matches the skill level of the student. For example, low skill and high challenge results in anxiety; high skill and low challenge results in boredom; low skill and low challenge results in student apathy; high skill and high challenge results in flow, characterized as absorption in the activity, enjoyment, and intrinsic motivation (Csikszentmihalyi, 1997). Therefore, when a teacher knows their students, they are better able to develop activities that match their students' skill levels with appropriately challenging activities, resulting in the development of positive emotions. This research also points out that a teacher needs to address the impact of activities on students with low skill levels—it is not enough to provide activities with low challenge, since this results in student apathy. Providing activities with multiple entry points, open questions, or parallel tasks (Small, 2009) allows all students to participate while honouring their current developmental level. Parallel tasks are differentiated by students' developmental levels, but address the same big ideas, and are sufficiently close in context to allow all students to participate in whole-class discussions. Dr. Marian Small's book, *Good Questions: Great Ways to Differentiate Mathematics Instruction* (2009), provides some excellent examples of open questions and parallel tasks for elementary mathematics classes.

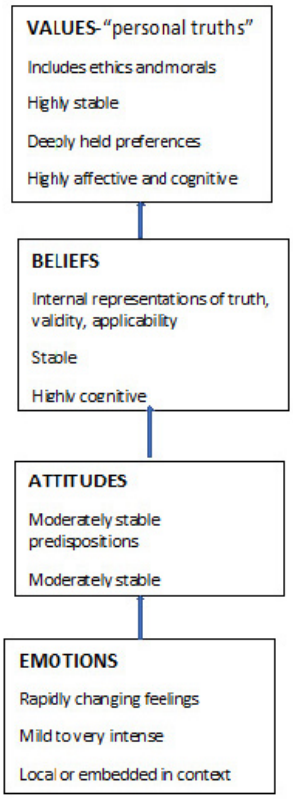


Figure 1: *Hierarchy of Affect. Based on DeBellis & Golding (2006)*

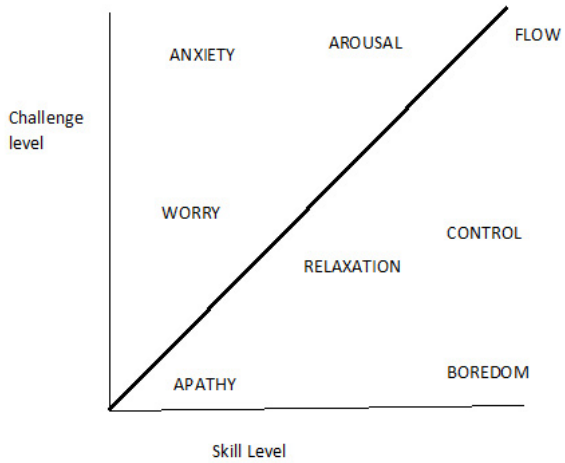


Figure 2: *Matching skill level with challenge level 1, based on Csikszentmihalyi (1997)*

One way to assess student emotions is through brief surveys. I have found that short anonymous surveys provided me with information on students' feelings toward mathematics and specific lessons or strategies, while

maintaining student privacy. Keeping the surveys anonymous provided more honest responses from students. A sample survey is shown in Figure 3.

Circle the emoji that best represents your feelings for today's class.

How interested were you in today's class?



How confident are you about what you learned in today's class?



How useful do you think what you learned today is to you?



Are you looking forward to tomorrow's math class?

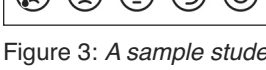


Figure 3: *A sample student survey, reproduced with permission from Irvine (2020b)*

Research with Canadian adults found that 80 percent were math uncomfortable, math averse, or math phobic (C. Bruce, personal communication, April 12, 2011). Positively influencing student emotions is an important dimension of teaching mathematics. This can be difficult if teachers themselves are uncomfortable with mathematics. So social–emotional intelligence is not just a student dimension, but also applies to teachers. Teachers may need to build their own mathematics self-efficacy in order to genuinely express positive affect with respect to mathematics. One way to do this is to recognize that the teacher does not need to always know all the answers in advance. Saying, "I will check and get back to you on that," or recognizing that there is not always only one right answer or correct method for solving a problem, goes a long way to supporting student attitudes. Using open questions and encouraging students to investigate their own conjectures helps empower students and build students' self-images as mathematics learners.

Recognize Sources of Stress and Cope with Challenges

> *Work through challenging math problems, understanding that their resourcefulness in using various strategies to respond to stress is helping them build personal resilience (OME, 2020).*



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Stress is a major factor for students in mathematics. Shernoff et al. (2003) found mathematics to be the most stressful of all academic subjects due to the high cognitive demand and sometimes inadequate prior knowledge of math concepts that students bring. Stress can have a debilitating impact on student performance. We are all familiar with students who "freeze" or "blank" on evaluations. This can be caused by the brain enzyme cortisol, which is produced in response to high levels of stress. Cortisol causes "extraneous" functions such as problem solving to shut down so the body can focus on reacting to the perceived stressors (Marx & Gilon, 2014). Alternatively, when the body does not perceive itself under stress, the enzyme serotonin is produced. Serotonin enhances certain brain functions such as critical thinking and problem solving as well as spatial reasoning, and produces positive mood (Dayan & Huys, 2008). As teachers, we do not have direct impact on enzymes, but understanding these effects in the classroom context can allow them to be addressed.

We all want our students to face challenges, persevere in addressing them, and celebrate and seek them out. Liljedahl (2018) reinforced that the challenges a student faces need to match their skill level; otherwise, negative affective consequences will result. Leikin (2018) has proposed a four-factor model for mathematical challenges:

- *Conceptual characteristics of a problem or task:* consisting of the mathematics topic, conceptual density, length of solution, openness, and insight required
- *Instructional setting:* individual problem solving, collaborative problem solving, heterogeneity of groups, availability of manipulatives, and technological tools
- *Socio-mathematical norms:* preciseness required, justifying, explaining, exploring, and questioning
- *Individual characteristics of participants:* mathematical readiness and proficiency, general intelligence, affect (global and local), self-esteem and persistence, personality (flexibility/openness/risk taking)

Leikin's model is useful for teachers, since it emphasizes that all four factors need to be considered when structuring a task in math classrooms. Of particular note is that the model re-emphasizes the need to know your students' interests, learning preferences, prior knowledge, and

attitudes toward learning mathematics, since challenges are personal and dependent on the characteristics of the learner.

Teaching Strategies to Support This Expectation

An important way that teachers can reduce student stress is to provide clear learning goals and success criteria (OME, 2010). Co-creating success criteria provides students some control over their own learning, which in turn reduces stress. One technique for co-creating success criteria is Post-It® Pileup (Figure 3). It is also possible to construct similar activities for virtual teaching, using Google Jamboard or similar synchronous discussion technologies.

Post-It Pileup

What Is It?

- A collaborative strategy involving evaluation

How Does It Work?

- Individually, students respond to a question or series of questions (e.g., "What does success look like for this learning goal?").
- Students place ideas on sticky notes, one idea per note.
- In groups, students then group ideas with common themes.
- The teacher may provide the themes, or they may be student generated.
- Collated piles of sticky notes are then displayed by the group.
- A gallery walk allows class sharing.

Where Can You Use It?

- Co-constructing success criteria
- Consensus for class norms
- Cause–effect links
- Brainstorming
- Real-world applications of a topic

Figure 3: *Post-it Pileup*

To assist in constructing success criteria, exemplars can be provided. Student-generated exemplars have the most impact.

Other teacher behaviours that help reduce student stress include scaffolding of activities, descriptive feedback, allowing students some level of choice in their learning, and appropriate use of technologies and manipulatives to support student-learning growth (Irvine, 2020a). The curriculum policy documents also suggest strategies such as guided imagery, deep breathing, stretching, and pausing for reflection during learning. Other possible strategies include Brain Gym® and yoga as stress reducers.

A significant way for teachers to reduce student stress is to ensure student understanding of concepts. One strategy to promote understanding is the use of Frayer models (Figures 4 & 5: OME, 2004). In a Frayer model, the concept is placed in the centre rectangle, and students complete the remaining rectangles. Alternatively, the student can be given a Frayer model with the outer rectangles completed and asked to identify the concept and place it in the centre rectangle.

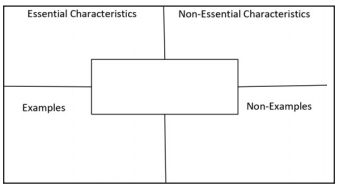


Figure 4: *Frayer model, version 1*

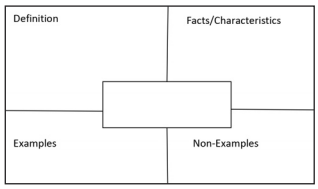


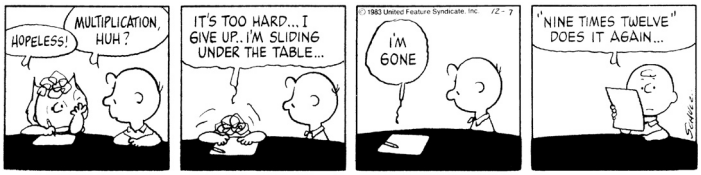
Figure 5: *Frayer model, version 2*

Other strategies to ensure student understanding include interviews, conferencing, journal writing, and Math Talk Learning Communities (MTLC), discussed in more detail later in this article.

Another strategy to reduce student stress involves providing student choice in content, format, strategy, level of difficulty, or product. Instructional strategies such as RAFT (Role-Audience-Format-Topic), menus, tiered assignments, or choice boards allow students some flexibility in requirements, which reduces stress associated with stringent or inflexible academic demands (Irvine, 2013). For examples of these strategies, see www.edugains.ca/newsite/di/dilearningexamples.html. Other approaches include open questions and open-routed questions (Small, 2009). Open questions have multiple possible answers and multiple routes to solution. Open-routed questions have a single correct answer, but allow multiple routes to that answer. Dr. Small's book, *Good Questions: Great Ways to Differentiate Mathematics Instruction* (2009), provides numerous examples of classroom-tested questions in both categories.

Maintain Positive Motivation and Perseverance

Recognize that testing out different approaches to problems and learning from mistakes is an important part of the learning process, and is aided by a sense of optimism and hope (OME, 2020).



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Bobis et al. (2011) found that there is a large body of research indicating that many teaching strategies designed to increase student motivation also improve learning outcomes, especially in mathematics. Below is a discussion of four such theories: expectancy-value, grit, mindset, and goal theory.

A major theory of motivation is expectancy-value theory (Wigfield & Eccles, 1999). Expectancy-value theory states that students' choice of tasks, persistence, and achievement depends on two factors: beliefs the students hold about their probability of success on the task, and the value that the students place on the task (Eccles, 2005). Students choose a task based on degree of difficulty, and the cost associated with that choice (Eccles & Wigfield, 2002). Knowing this theory allows teachers to structure lessons that address both the value of the activity, and build the self-confidence and self-efficacy of their students. Emphasizing value may be done through linking the mathematical content to real-world applications, or demonstrating the usefulness of the mathematics to students. Increasing expectancy of success is linked to students' mathematical self-image, discussed later in this article.

Another concept related to motivation is grit, defined as *grit* = *passion* + *perseverance* (Duckworth & Gross, 2014). Grit has been shown in some studies to be related to persistent deliberate practice, academic achievement, and lifelong learning (Credé et al., 2016). Developing grit in our students should be a goal of all teachers, but is especially important in mathematics since perseverance in the face of errors is vital to both student learning and development of a student's self-image as a capable mathematics learner. Hurlington (2018) found that classrooms that promote student resilience and persistence have learning environments where students are encouraged to explore and try different strategies. Some strategies to increase student grit are outlined below.

A third motivational concept is mindset, sometimes referred to as theories of intelligence. There are two dominant mindsets. One is that intelligence is essentially fixed at birth, and cannot be substantially changed. This is referred to as the *entity* or *fixed* view of intelligence. The second view is that intelligence is not fixed and can be improved through effort. This view is referred to as an effort, incremental, or *growth* mindset (Dweck, 2006). A student's view of intelligence, their *mindset*, has significant implications for behaviour. If intelligence is fixed, then there is no point in making an effort to learn. The outcomes of learning are predetermined by fixed intelligence, and will not change significantly, no matter how much effort is exerted. Thus, there is no motivation to engage, and no motivation to learn. We see this mindset in someone who states, "I was never good at math." On the other hand, if intelligence can be grown by effort, students will not be limited by their assumptions about their own level of intelligence, and will be motivated to make an effort, engage, and learn. Dweck (1999) states that entity theory (fixed mindset) predisposes students to see failure as a direct indicator of their low ability, which is distressing and disengaging, whereas incremental theory (growth mindset) makes students use failure as a cue to try new strategies and to try harder. Suh et al. (2011) found that students who develop a growth mindset are more likely to become persistent and flexible problem solvers in mathematics.

In addition, teachers should know about achievement goal theory, since they can then tailor activities toward productive student goal setting. Goal theory postulates two types of goals: mastery goals, where students want to learn material because they want to master the material for internal reasons; and performance goals, which are related to how students want to be perceived by others (Maehr & Anderman, 1993). Performance goals can be further subdivided into performance-approach goals (wanting to do well with respect to peers) and performance-avoidance goals (not wanting to look bad in front of peers). There is abundant research that mastery goals are related to effort, persistence, and intrinsic motivation (Midgley et al., 2001), while performance-avoidance goals relate to maladaptive behaviours such as acting out and lack of effort. The research related to performance-approach goals is mixed (Midgley et al., 2001). If the goal can be associated with a student-perceived valuable outcome, such as attaining a high grade to obtain acceptance into a desired program of study, these goals can be positively motivating. However, if the goal is simply to look good in front of others, these goals tend to lead to short-term effort, memorization instead of actual understanding, and sometimes to negative social behaviours such as denigrating others (Midgley et al., 2001).

There are numerous theories related to motivation, but the four theories outlined above provide teachers with deeper understanding of student motivational dimensions, and can be used to shape instruction toward positive motivational stances by students. Teachers can encourage growth mindsets, mastery goals, development of grit, and

enhance both students' expectancy of success (related to self-efficacy), as well as explicitly indicate the value of the mathematics being learned.

Teaching Strategies to Support This Expectation

One way to do this is to organize lessons to transparently address student motivational issues. A strategy that has worked well in some schools is identified by the acronym MUSIC (Irvine, 2018):

- eMpowerment
- Useful
- Successful
- Interesting
- Caring

When teachers design lessons using this acronym, they are able to view the lesson from the students' point of view. *Empowerment* means that the student has some control over their learning environment. This doesn't mean that students make all the decisions; rather, that they have some choices in some aspects of their learning. *Useful* to a student is: they can use it outside of school, now; they can use it in another class; someone they know uses it (a relative, friend); there are examples of people in the real world using it. *Successful* is self-explanatory, but it also involves scaffolding large tasks so that students have success on the smaller chunks, leading to success on the larger task. *Interesting* is very personal to the student, and teachers will structure lessons that interest segments of their students at different times. *Caring* is a critical component. "Students don't care how much you know until they know how much you care" (based on a quotation by Theodore Roosevelt). The greater the number of elements of the MUSIC acronym that are addressed, the higher the probability that students will be motivated and engaged.

The second part of this article will examine the final three expectations in this strand.

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▲ Our Strength Is People

BILL MORRISON
bill_morrison@sympatico.ca

An invited opinion from a long-standing member of OAME/AOEM.



I spent five years in Hamilton while attending McMaster University and the Ontario College of Education in Toronto. At that time, one of the dominant companies in the city was the Dofasco Steel Company. Their motto—"Our product is steel, but our strength is people"—has always stuck with me because I have seen so many examples of it in my life and career. I believe their motto is also applicable to mathematics education in Ontario.

Classroom teachers are the heart of the system. What they do every day makes the most difference in the mathematics education of students. Their supporting framework includes department heads, consultants, and administrators. Another very important part of it is provided by the OAME/AOEM and its local chapters through conferences and publications like the *Gazette* and *Abacus*. Each issue is filled with creative ideas, curriculum-related articles, and information for teachers of mathematics at all grade levels. Tremendous online resources and information are provided by the association's outstanding website. Kudos to the many people who provide these supports. As a long-time member of OAME/AOEM, I have been motivated and inspired by their knowledge and ideas. I can think of many people, some of them no longer with us, to whom I am very grateful. Each of you probably have your own list of people in your mind. They truly are part of what makes the organization so strong. ▲

▲ Discovering the New (2020) Math Strand: "Social–Emotional Learning Skills and the Mathematical Processes"



RENEE BILLIAU
renee_billiau@msn.com

Renee Billiau is a Primary French Immersion teacher with the Waterloo Regional District School Board, where she enjoys teaching primarily Math and French to Grades 2 and 3 students. Having focused on creating a collaborative math approach with students, Renee enjoys hearing students explain their thoughts, reasoning, and solutions in French. Renee has taught in three provinces: Saskatchewan, Quebec, and for the last 19 years, in Ontario. She has taught in three different countries—Japan, France, and Canada—throughout the last 26 years teaching.

The Ford government announced that the new math curriculum (Ontario Ministry of Education [OME], 2020) would get back to the basics. Depending on the research regarding approaches to teaching math, some studies show how rote learning and experiential learning complement each other positively. Some teachers in primary grades will view this change as a welcome opportunity to build math fluency. Think learning number bonds (5 and 2 are parts, and 7 is the whole) and the memorization of number facts. In the opinion of some folks, this is what is holding students back when solving math problems. With a quiet flourish, a document was delivered (many really didn't think it would happen so soon). Included in the curriculum was something timely and relatively new to Ontario curriculums: a strand called SEL—Social–Emotional Learning.

For educators, the key innovation in the new curriculum involves teaching and reporting on "Social–Emotional Learning Skills and the Mathematical Processes" (www.dcp.edu.gov.on.ca/en/curriculum/elementary-mathematics/grades/g6-math). As Mary Reid, an assistant professor of math education at the Ontario Institute for Studies in Education put it, "The social–emotional learning component is critically important to the new curriculum and will help kids tremendously." For teachers, this means we will be reporting on how students apply SEL skills while working through math concepts and challenging problems.

Let's take a closer look at this new inclusion in the 2020 Ontario math curriculum, specifically:

- Where does the SEL come from?
- What do we know about it?
- How and where has it been implemented?
- What are the results?
- How will this pertain to Ontario math classes and school climate?

Before diving into SEL, it's important to make a connection to the much-needed growth mindset required during the past year. We can acknowledge that this strange and memorable school year of 2019–2020 has had an emotional toll on students, educators, and parents (including guardians) from the beginning. The year started with the ominous threat of job action and painful contract negotiations. Then at the height of what was proving to be bitter negotiations and intense job action, the bigger, more worrying issue of the COVID-19 pandemic hit. Both sides briskly resolved their issues. COVID-19 brought a flurry of changes to school as we had known it; distance learning became the only mode of reaching students. Never has it been more apparent that our students require the collaborative support of school boards, teachers, and parents. Finally, the year ended with distance education in all corners of the province. The end happened ever so quietly. In fact, the school year came to a close at different times for different people with the "fade to black" of their computer screens. No solid goodbyes and "wish you wells." Just the push of a button.

However, not to be distracted by the unique end of a school year, teachers obtained the link to the new 2020 math curriculum, and circulated access to the document to everyone who wanted to take a look. The promise of a revamped, "back to basics" curriculum is, however, hard to spot as you peruse the document, as a recent CBC article by Mike Crawly (2020) points out. There is no mistaking it. This curriculum is not going back in time! It is in fact quite progressive and sets out objectives that will prepare the students for the twenty-first century.

As it turns out, this concept can be dated as far back as Plato, who wrote in *The Republic* that education's role was to produce knowledgeable and morally sound citizens of good character (Anonymous, 2011). The article goes on to describe how two schools from New Haven, Connecticut, struggling with attendance and achievement, saw vast improvements in the 1980's, using an SEL approach. Change in school culture was achieved by a highly collaborative team that consisted of teachers, parents, principals, and social workers. In a unified approach to

address behaviour, many changes were brought about to reconcile the contrasts between home life and school. These schools eventually exceeded national standards.

Interest in SEL grows, with New Haven being the hub of research into the early 1990's. At this point, an organization called CASEL (Collaborative for Academic, Social, and Emotional Learning, www.casel.org/what-is-sel/) propelled SEL into the public consciousness through what is roughly acknowledged as Emotional Intelligence. Through its research and by developing frameworks and protocols, CASEL would help schools boards across America implement five core competencies: self-awareness, self-management, social awareness, relationship skills, and responsible decision making. An example of an SEL program would be the "Tribes" model (www.tribes.com/), which has been around for about 50 years.

With floundering scores on an international level, and large discrepancies from one region to another, it is easy to understand the need for better programming. "Among the 35 members of the Organization for Economic Cooperation and Development, which sponsors the PISA initiative, the U.S. ranked 30th in math and 19th in science" (Desilver, 2017). Perhaps, in an effort to improve education, in 2011, the 112th American Congress legislated the implementation of SEL into its Elementary and Secondary Education Act (www.congress.gov/bill/112th-congress/house-bill/2437). This amendment provides for funding "...among the activities funded under the Teacher and Principal Training and Recruiting Fund program." A large emphasis for the success of this program is being touted in the training of principals, teachers, and parents. According to the Chicago District Public Schools, the results are very encouraging, as noted below:

- In 2012 the graduation rate was 59.3% and steadily increased to 77.5% in 2017.
- Out-of-school suspension has declined 76% since the 2012–2013 school year.
- In-school suspension has declined 41% since the 2012–2013 school year.
- Expulsions have dropped by 59%, and police notifications have dropped 50% since 2012–2013.

www. casel.org/partner-districts/chicago-public-schools/

Whereas the American school districts are implementing SEL as an overall learning expectation for schools, in Ontario, our first exposure to SEL principles comes to us in the context of the math curriculum. Teachers have all witnessed occasions where students fail to perform at their best. Take, for example, SEL skill 3: "Maintain positive motivation and perseverance in order to recognize that testing out different approaches to problems and learning from mistakes is an important part of the learning process, and is aided by a sense of optimism and hope" (OME, 2020).

SEL skills would take a situation I have encountered many times from a student feeling defeated:

Student: Smacks the paper down in front of the teacher with a pout.

Teacher: "Janelle, I see on your paper that you tried to solve the problem. It also looks like you know that your answer doesn't fit."

Student: "Yeah, I tried. It doesn't work." (Signalling this is the end of this particular interaction!)

To a positive growth mindset, which I have also encountered, such as:

Student: Smacks the paper down in front of the teacher with a smile.

Teacher: "Janelle, I see on your paper that you tried to solve the problem. It also looks like you crossed out a solution that didn't work and tried again."

Student: "Yeah, I tried to tell myself that I now know what doesn't work, so that gets me closer to something that will work."

And even if the problem does not have a solution that works, this student understands the power of "yet": "I don't have it yet, but I am working on it!"

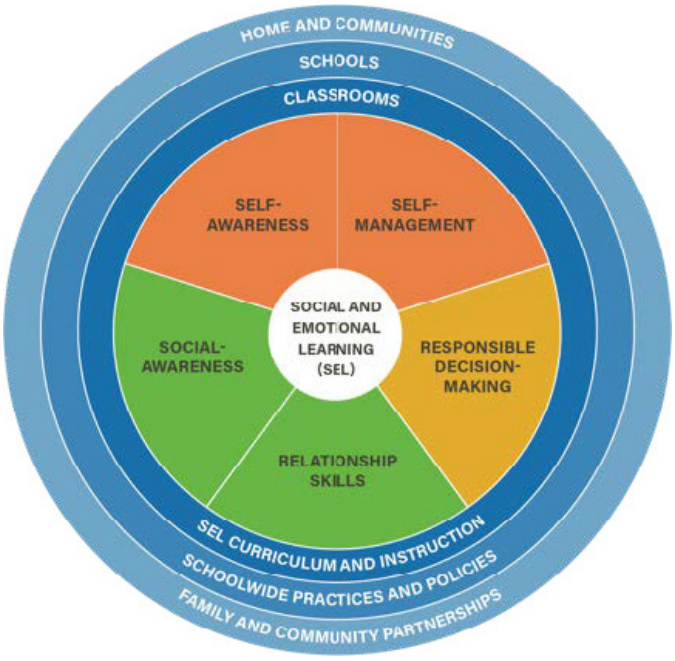


Figure 1: *CASEL framework (from www.casel.org/what-is-sel)*

Often, we can attribute a lack of performance to students feeling overwhelmed or inadequate and not having the mindset that would allow them to persist with the task and try other solutions. Giving students tools to manage their feelings, be resourceful, and respond to challenges is something teachers can encourage. With "math anxiety"

being a term used in math circles (see www.youtube.com/watch?v=592WKZ0eKP0), it surely is a good beginning to address positive mindset attitudes as it pertains to math.

If we look at the six key SEL skills in the new 2020 math curriculum, five of the expectations are closely linked to the five CASEL framework competencies:

1. Identify and manage emotions.
2. Recognize sources of stress and cope with challenges.
3. Maintain positive motivation and perseverance.
4. Build relationships and communicate effectively.
5. Develop self-awareness and sense of identity.
6. Think critically and creatively.

The past year has been challenging for teachers, parents, and students. The implementation of this new curriculum and instructional practices would benefit students, and teachers certainly welcome the new tools to encourage growth mindset. As it is now, we might have to wait and use this time to reflect and put well-being in the forefront as we look for ways to make the learning environment manageable and non-intimidating. Here's to 2021!

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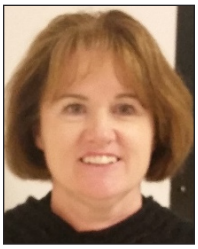
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▲ NCTM Virtual Leadership Report



JACQUELINE HILL
jacqueline.hill@bell.net

Jacqueline Hill is a Grades 9–12 teacher of mathematics. She is a Past President of OAME and OMCA, as well as the recipient of the award for Exceptional Teaching in Secondary Mathematics. She also wrote the "Director's Dialogue" for the Gazette *for a number of years.*



TIM SIBBALD
timothys@nipissingu.ca

Tim Sibbald has held a variety of positions within the OAME/AOEM. He is a former president, vice-president, and director. He is an associate professor in the Schulich School of Education, Nipissing University, with a focus on mathematics education. He has edited a book on Interdisciplinary Math and two about the experiences of academics (with a third underway).

The Virtual NCTM Leadership Conference ran from July 13–30, 2020, with synchronous presentations that were subsequently provided as recordings. Registration was straightforward through the NCTM website, and access was provided through MyNCTM.

The program for the conference provided a full range of leadership topics. The conference emphasized the current NCTM focus on *equity and advocacy*. It was good to see Canadian content, with Ruth Beaty presenting about Algonquin mathematics. The use of beading as a form of patterning in elementary mathematics is quite interesting, but equally important for leaders was the emphasis on a consultative process that included Indigenous representatives who helped provide cultural perspective.

There was an interesting vignette-driven presentation that supported teacher thinking about equity details, such as, "Differences are not an issue; assigning value to difference is." The Past President of NCTM, Robert Berry, presented on the theoretical distinctions between Culturally Relevant Pedagogy and Culturally Responsive Teaching. Both of these theories are well established, but challenging to implement, making his presentation useful as ongoing support for turning the theory into practice.

The *leadership* program strand had a mix of presentations that addressed all levels of leadership development. The President of NCTM, Trena Wilkerson, spoke quite well on the wide range of activities with which NCTM is engaged. Her talk included promotion of the *Catalyzing Change* books. She mentioned the February 2014 (volume 20, issue 6) article on Teacher Leadership and how you must do four things with intention:

1) Assess lots.
2) Empower students to make sense of mathematics.
3) Be a catalyst for change (that is, initiate conversations that make a difference).
4) Make a difference for students.

Other presentations were product driven and, therefore, less useful. A talk given about team dynamics would be beneficial for high-level thinking by people pursuing leadership. Similarly, there was a presentation about growing leadership from within that is reflected in the editorial. It was oriented to supporting affiliates, but really applies in almost any leadership role. Another presentation in this strand was "Taking on Almost Impossible Problems," presented by Zachary Herrmann. He spoke about Obama's 2016 adviser, who said, "It doesn't make it to the desk of the President unless it is virtually impossible." The presentation went on to look at how that quotation applies to education. Hermann's book, *Teaching a Future President* (2019), looks at leadership, collaboration, and justice through the lenses of knowledge, skills, and mindsets. He spoke about being intentional in the teaching you do. Every child has the potential to be the next President (or Prime Minister). What do they need to know from your course to help them run the country?

The program strand on *advocacy and social justice* included several presentations about addressing the pandemic. This was clearly an effort to be responsive and supportive to the needs of leaders of the day. There was also a presentation focused on equity within evaluation. Angela Towers and Ho Nguyen provided a presentation on De-Tracking: Believing in the Brilliance of All Students. They discussed math being a web (that is content and how students learn), not a ladder. They went on to add that if we just allow more ways to be smart (that have foundation in the cultures of our students), then we will find that all students are mathematically brilliant. They discussed core values for instruction, including: group work/collaboration, learning is social, work to get rid of perceived hierarchies (and note that we are all smart), relationships matter. They concluded by providing their research-based evidence that de-tracking helps with racism, whether it be race, gender, or ESL learners.

A theme on affiliate planning included a roundtable discussion for affiliates. The affiliates had a hearty discussion about engaging unengaged board members, and once an organization has members, they need to treat them well (support them and provide freebies/swag). It was noted to make certain you shadow and co-chair committees. Be certain to bring programs and professional development to your meetings, and to honour new membership and outgoing members. ▲

▲ OMCA Goes Virtual!

HEATHER THEIJSMEIJER
NOMArep@oame.on.ca

It has been a year of unprecedented changes, but the Ontario Mathematics Coordinators Association (OMCA) is rising to the challenge, and taking the opportunity to reach out to new members across the province.

The new executive, consisting of President Rhonda Hewer (Waterloo Region DSB), Past President Livia Covino (Peel DSB), Vice-President Heather Theijsmeijer (Rainbow DSB), Treasurer Vincent Arseneau (Peel DSB), and Secretary Allison Lawrence (Brant Haldimand Norfolk Catholic DSB), has shifted the association's monthly meetings to a fully virtual platform, allowing more members to connect, share resources, and engage in topical conversation on a regular basis.

Guest speakers at the monthly meetings so far this school year have included Dr. Chris Suurtamm speaking on the new elementary curriculum, Dr. George Gadanidis speaking on coding as a mathematical modelling and learning tool, and Dr. Cathy Marks Krpan on social–emotional learning as it is reflected in the math curriculum.

In a twist on a typically analog activity, members have also been invited to take part in a digital literature circle through the winter months. Members have the choice of collectively diving into *Strength in Numbers: Collaborative Learning in Secondary Mathematics* (Horn, 2011), *Culturally Responsive Teaching & The Brain* (Hammond, 2015), or *Mathematics for Human Flourishing* (Su, 2020).

Even the annual retreat is going virtual! Instead of an in-person event, OMCA will be hosting a digital symposium in February 2021, featuring a number of presenters speaking to the theme of Learning in a Virtual World. The symposium will be open to all, with registration fees for non-members to include membership to OMCA for the remainder of the year.

With the switch to a completely virtual forum, events are now even more accessible to mathematics coordinators, coaches, and consultants from Kenora to Cornwall, and from Fort Severn to Fort Erie. New members are always welcome to join–contact us directly at OMCAmath@gmail.com, or visit our website at www.omca.website (note the different domain) to learn more about the organization. See the poster for the upcoming symposium and connect with us.

References

Hammond, Z. (2015). *Culturally responsive teaching & the brain.* Corwin.

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DIRECT GENERAL INQUIRIES TO: EXECUTIVE DIRECTORS

Fred and Lynda Ferneyhough
4751 Mack Ave.
Plympton-Wyoming, ON N0N 1J6
Business & Fax: (519) 899–4203
eds@oame.on.ca

PRESIDENT

Judy Mendaglio
Mississauga, ON
judy.mendaglio@oame.on.ca

PAST-PRESIDENT

Paul Alves
Acton, ON
paul.alves@oame.on.ca

VICE-PRESIDENTS

Sandra Jean Price
Oshawa, ON
sandra.jean.price@oame.on.ca

Connie Quadrini
Stouffville, ON
connie.quadrini@oame.on.ca

EMERITUS

Bill Otto
Stillwater Lake, NS
bill.otto@oame.on.ca

CHAPTERS & REPRESENTATIVES

CHAMP: Kassandra Kobelak
Etobicoke, ON
kassandra.kobelak@oame.on.ca

GOLDEN: TBD

GVMA: Glen McMillan
Guelph, ON
glen.mcmillan@oame.on.ca

ISOMA: Cal Armstrong
Burlington, ON
cal.armstrong@oame.on.ca

MAC²: Paulene Washington
Barrie, ON
paulene.washington@oame.on.ca

NOMA: Heather Theijsmeijer
Kagawong, ON
heather.theijsmeijer@oame.on.ca

NWOAME: Corrine Russell-Pritoula
Thunder Bay, ON
corrine.russell-pritoula@oame.on.ca

$O_3$4ME: Jimmy Pai
Ottawa, ON
jimmy.pai@oame.on.ca

PRMA: Lori Yee
Pickering, ON
lori.yee@oame.on.ca

QSLMA: Siobhain Broekhoven
Kingston, ON
siobhain.broekhoven@oame.on.ca

SAME: Franca Genco
Scarborough, ON
franca.genco@oame.on.ca

SWOAME: Elizabeth Pearsall
Kingsville, ON
elizabeth.pearsall@oame.on.ca

TEAMS: Alvarine Aldridge
Toronto, ON
alvarine.aldridge@oame.on.ca

WOMA: TBD

Y⁴MA: Gina Micomonaco
Richmond Hill, ON
gina.micomonaco@oame.on.ca

DIRECTORS – TERM EXPIRING 2021

Freda Liu
Toronto, ON
freda.liu@oame.on.ca

Melissa Black
Kanata, ON
melissa.black@oame.on.ca

DIRECTORS – TERM EXPIRING 2022

Jane Silva
North York, ON
jane.silva@oame.on.ca

Liisa Suurtamm
St. Catharines, ON
liisa.suurtamm@oame.on.ca

Victoria Maystruk
Peterborough, ON
victoria.maystruk@oame.on.ca

Timothy Sibbald
North Bay, ON
tim.sibbald@oame.on.ca

NCTM REPRESENTATIVE

Jacqueline Hill
Lindsay, ON
jacqueline.hill@oame.on.ca

WEBSITE COORDINATOR

Greg Clarke
Orillia, ON
web@oame.on.ca

GAZETTE EDITOR

Timothy Sibbald
North Bay, ON
gazette@oame.on.ca

ABACUS CO-EDITORS

Mary Lou Kestell
Toronto, ON
marylou.kestell@oame.on.ca

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kkz@oame.on.ca

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