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Ontario Mathematics Gazette

OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

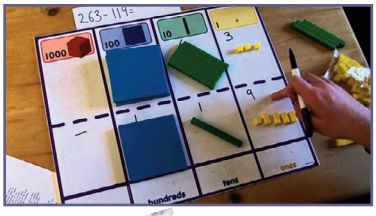
AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

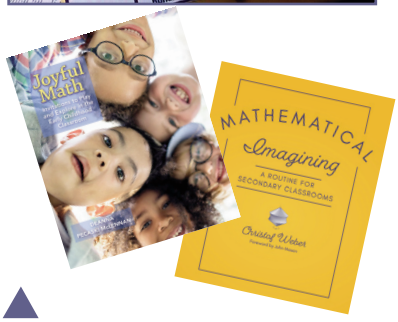
Vol. 59 #4
June 2021

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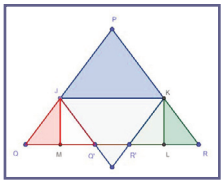
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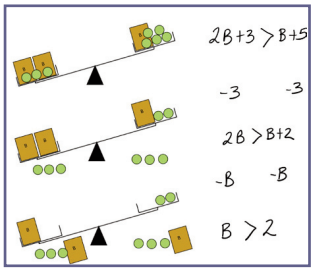




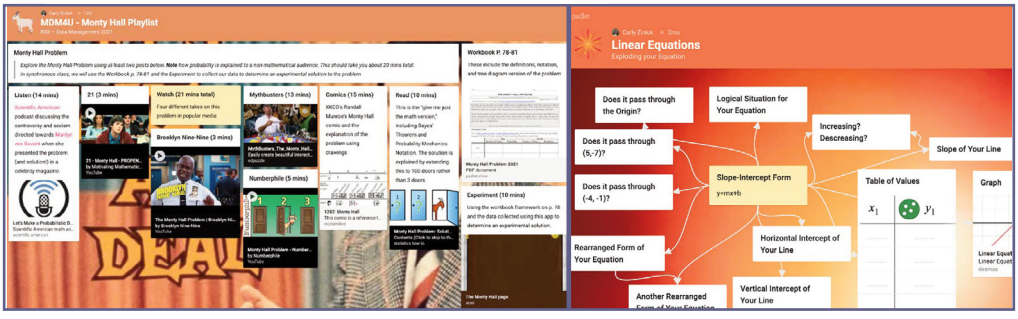
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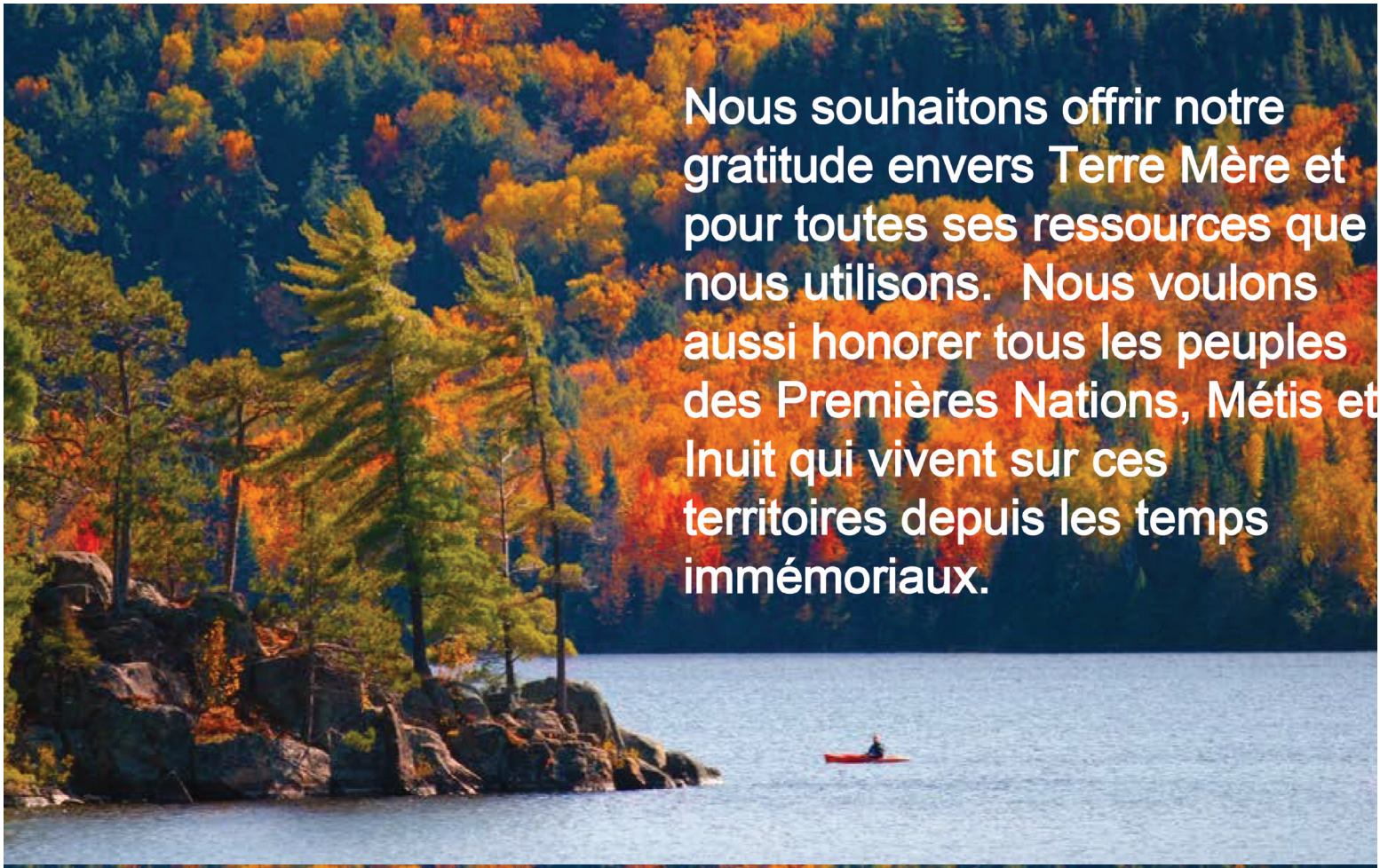


See Mathematical Snapshots: Geometry lives at Georgian!



See Mb4T (Mathematics by and for Teachers): Finding Balance in Algebra




Nous souhaitons offrir notre gratitude envers Terre Mère et pour toutes ses ressources que nous utilisons. Nous voulons aussi honorer tous les peuples des Premières Nations, Métis et Inuit qui vivent sur ces territoires depuis les temps immémoriaux.


We wish to express gratitude to Mother Earth and for the resources we are using, and honour all the First Nation, Métis and Inuit people who have been living on the land since time immemorial.

Submission of Articles

The *Ontario Mathematics Gazette* (OMG) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *Gazette*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles are to be word–processed, MS Word is preferred, and prepared according to the *Publication Manual of the American Psychological Association*, Seventh Edition. However, please use single-line spacing (not double). Articles should not exceed five numbered pages of text, and figures, images, and photographs should be placed in the text close to where they belong, with captions. The photographer's permission is required, and for photos of students under the age of 18, the written permission of a parent or guardian is required.

Please submit your article in one blind file (i.e., identity of author is not evident), and include author names, contact information including email and mailing addresses, photos—head and shoulders, biographies—less than 100 words, and all content removed for blinding in a second file. Please email these two files to Tim Sibbald at gazette@oame.on.ca.

Upon review, you will be notified whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected. The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME/AOEM.

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The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of OAME/AOEM.

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The *Ontario Mathematics Gazette*—ISSN 0030-3011—is indexed in the Canadian Education Index and is published four times per year. Its Canadian Publication Mail Product Sales Agreement Number is 40051074.

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Submission of Advertisements

Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** by email or at the above address. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be print ready, and colour advertisements should have no bleeds.

Advertising Rates

Advertising rates are available by telephoning, emailing, or writing to the Advertising Manager.

▲ Editor's Report



TIM SIBBALD, OCT, PHD
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Tim Sibbald holds the Universities Director position on the OAME Board of Directors. He is a former president of OAME. He is an associate professor in the Schulich School of Education, Nipissing University, with a focus on mathematics education.

Have you noticed during the pandemic that there have been changes in human behaviour? Have you had introspective moments, trying to tease out personal changes from changes made by others? Recently I got caught on this—I suspected as much—and thank you, Anne Yeager, for being the sounding board I needed.

The reason I needed a sounding board is that I have concerns about the sheer number of changes taking place in education and the number of issues that are arising from them. However, while the column was intentionally written in an alarming tone, the weather changed, and as issues around preparing a virtual OAME/AOEM conference presentation, among other things, subsided, the sun began to shine once more.

The challenge of social distancing is that we are social creatures and, as teachers, we know that facilitating social learning opportunities can help support emotional balance for our students. So, what about teachers and the people working around teachers? The social opportunities have waned with the pandemic, and emotional flux seems to be more pronounced.

I think we do some of this to ourselves. Since the pandemic began, I have had more meetings that have run over lunchtime and dinnertime, as if technology has made them available. I point the finger at myself, as well, since I have taken to working weekends and visiting public spaces during the week, when there are fewer people about. Luckily it is beginning to look as if we will get past the pandemic, though I would not want to predict when.

I will share a piece of my original editorial, which is grounded in frustration, for the newest members of the teaching profession. Then I will speak to something more pleasant.

In 2015/2016, pre-service teacher education was moved to a two-year program. Enrolment was halved for a year, and from then on, the number of graduates has been half the

number of teachers compared to previously. Is anyone surprised that we are now having shortages of teachers? It appears to have been intentional, since it is basic regression to predict the number of teachers required to a reasonable level of accuracy.

As of Spring of 2020, all teacher candidates have been required to pass a Math Proficiency Test (MPT) as part of their requirements for OCT certification. The test was suspended after field testing last year because of the pandemic, but not the requirement. Thus, teacher candidates who graduated in 2020 only have conditional and temporary standing as teachers—they cannot be given full standing with OCT until they complete the MPT. The evidence of this is available on the OCT public register. It is unfortunate and demoralizing to our newest teachers, and I will not be surprised if this is later recognized as a contributing factor for many exiting the profession prematurely.

This year, it was announced the MPT would be an online test because of the pandemic. So, in January, at the request of a student, I phoned EQAO to ask when the dates for the MPT would be announced. They advised that students should check every two weeks. I phoned again in late March and got the same response. Apparently, they are waiting on the Ontario Ministry of Education to set the dates. That led me to email my MPP and ask them if they could get any clarification. It has been three weeks, and I have not heard back. Realistically, there is a pandemic occurring, and not hearing could be due to urgent issues.

That said, it is unreasonable that we have teachers in the final stages of their first professional year, who are being treated so poorly. As this goes to press, I did hear from my MPP after a month wait, and a process for the MPT has been announced. However, this has not changed my opinion about poor treatment of the people trying to enter our profession.

On a brighter note, I have a new respect for how well television, films, and movies are made. It was not my intention to appreciate this, but the OAME/AOEM, specifically through the pre-recorded annual conference sessions, caused professional learning in this domain for me. I was supposed to be in Norway during the annual conference, so I made an arrangement to co-operate with a colleague, Miranda Wheatstone, who could attend. All of this was at a time when the pandemic was optimistically expected to be over before the annual conference. As we know, that was not the case, and the conference moved to be virtual. As a result, Miranda and I found ourselves having to develop a pre-recorded session while socially distanced. This entailed professional learning around the tools and options we had for developing a session.

Over the years, I have found the OAME/AOEM a good place for challenges that are reachable. This was no different. Sure, it was a steep learning curve, but it was achieved. Having revisited some of the audio this afternoon, I know I am not a sound engineer, but the end result is better than I could have done a year ago. I am delighted by the potential for new opportunities, given what I have learned from this effort.

While the pandemic has been challenging, I will wonder publicly what we will collectively do over the next five years, given the infusion of professional learning we have all done in response. The challenges have been pervasive, and it will take time to digest what has taken place, but I have no doubt that we have learned. As teachers know, that learning will lead us to aspire to grow, and that is where the next five years will be fascinating to engage with.

In This Issue

Our President, **Judy Mendaglio**, draws attention to significant efforts that have been made to support teachers with the 2020 curriculum change. There is more good news with the **OAME/AOEM Awards** announced in this issue. Congratulations to the recipients and, please, as you read about them, think of colleagues you can nominate this year.

The "Fields Institute MathEd Forum Report" has had a name change and is now "Events at the Fields." It remains in **Angelica Mendaglio**'s capable hands. The change was prompted by wanting an International Women's Day event covered, and the new title reflects the decision that the column should be able to cover such events.



Jacqueline Hill gets ready for summer with an NCTM version of BINGO. You may have noticed an OMCA report in the last issue, and it is continuing. **Heather Theijsmeijer** is taking the reigns and rekindles what used to be a regular item known as the "Coordinator's Report."

My apologies to **Dr. Jeff Irvine** for the absence of his column from the last issue. It was ready to go and quite simply missed in error. I am delighted, however, that Jeff has written a very timely column addressing the history of destreaming. If you teach Grade 9 and are not familiar with the history, then his column is a must-read, though everyone will surely benefit from the reminder.

Shawn Godin draws out a delicious bit of geometry. **Carly Ziniuk** explains how she has found Padlet useful in her instruction. **Jérôme Proulx** shares his research, which highlights some interesting aspects of equation solving.

Lynda Colgan recounts some of her early experiences, which have clearly left their mark for many years. **David Costello** focuses on classroom discourse. **Ron Lancaster** draws inspiration from a visit to Georgian College at a past annual conference.

Lastly, and unfortunately, that word may be a pun, **Mary Bourassa**'s column about coding may be her final one for the *Gazette*. I sincerely appreciate the professionalism Mary has brought to her column. It has always been on time, well written, and informative. Her presentations included easy-to-follow instructions with visual backup. Furthermore, her offerings spanned grade levels, making it accessible to most, if not all, of our readers. While I will welcome her back in a heartbeat if she changes her mind, I also recognize that the quality of her columns hides the amount of effort they entail. Thank you, Mary, for your many columns. We will miss your contributions!

We have a couple of book reviews by **Tim Sibbald**: one is oriented to Primary/Junior and the other to Junior/Intermediate/Senior. As well, we have an article about Fibonacci numbers by **Jeff Irvine**, who once again includes a comic in his column. Some research regarding the Math Proficiency Test is detailed by **Jennifer Holm**, **Ardavan Elzadirad**, and **Steve Sider**. This is followed by **Jeff Irvine** considering how lessons can be planned around student motivation.

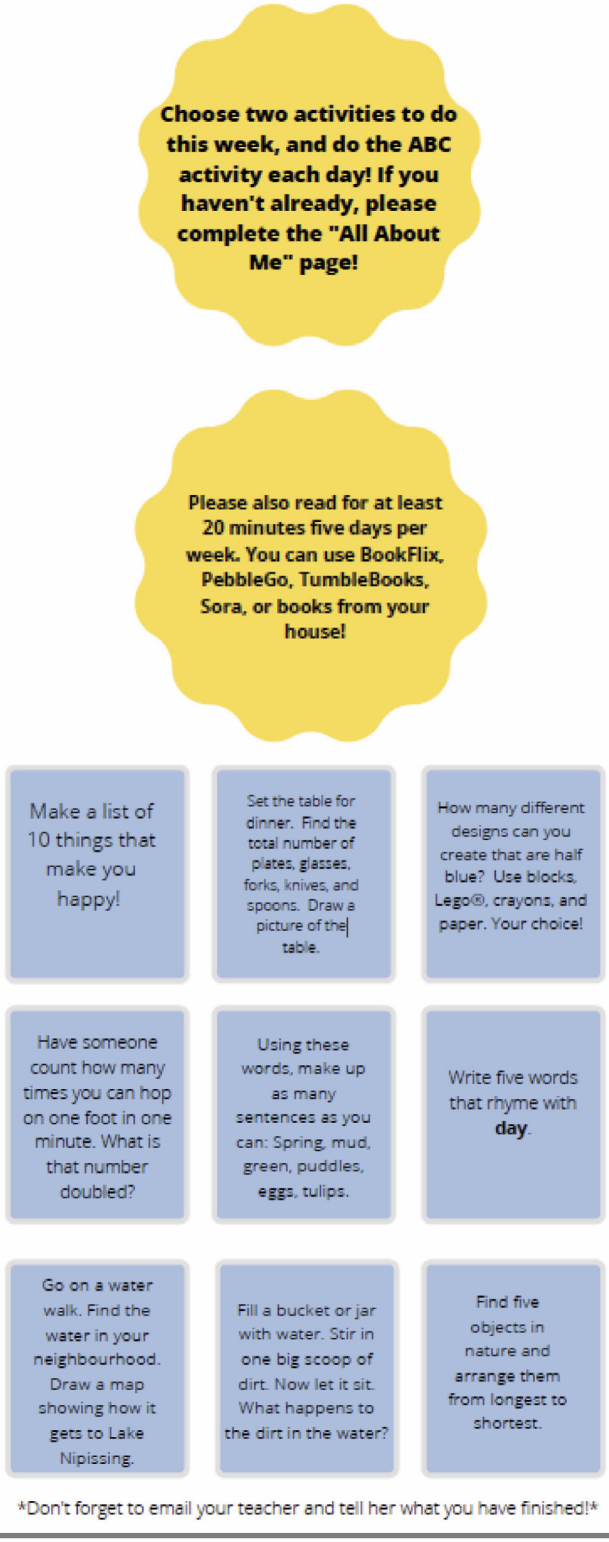
Over the summer months, enjoy the reading, but consider the other side of communication: writing an article. There is a video guide on the website, and you are always welcome to bounce your idea off the editor.

Lastly, I owe many thanks to **Anne Yeager**, who was a tour de force as an associate editor for this issue. Between Anne and **Gitta Berg**, the handling of this issue during a period with a very heavy personal workload was a relief. I also need to thank **Kyla Kadlec** for her oversight in French, particularly in addressing a challenge that Judy brings up in her column. Thank you also to Ralph Connelly, John Rodger, and Anne Roberge, who contributed to proofreading the issue; not to omit **Penny Clemens**, who, as she has for many years, brought everything together. The remarkable aspect of this editorial team is hidden from view, but when I see it in action, I am reminded of the wonderment I feel, looking at the inner workings of a mechanical watch. Thank you, team, for all you do. ▲

Task Choice BINGO?

Lisa Corbett, Sara Byrne, and Jennifer Rogers

A way to offer choice to improve online engagement. Used in Grades 1/2 and 2/3 split classes.



President's Message



JUDY MENDAGLIO
judy.mendaglio@oame.on.ca

Judy Mendaglio is the President of OAME/AOEM. She has worked in education in Ontario in many capacities: college instructor, secondary teacher and department head, university instructor, elementary and secondary consultant, and writer. Currently, Judy devotes most of her time to her many volunteer activities.

This school year, OAME/AOEM partnered with AFEMO in the production of a variety of resources to support the successful implementation of the 2020 revised elementary mathematics curriculum. A team of extraordinary educators from OAME/AOEM and AFEMO were brought together to tackle this rather enormous project. As part of this project, the teams have written 208 lesson and assessment plans, in both English and French, targeting expectations from Grade 1 through Grade 8. Our project also involved the creation of a new public website to house these new lessons. If you have not yet visited Ontario Math Support (www.ontario math.support/), please do. Lesson and assessment plans that can be edited are posted there. They are sorted by grade as well as by curriculum topic, and you will also find additional resources, such as ready-to-use Scratch programs and spreadsheets, and online/hybrid resources such as pre-made Jamboards and Google Slides. Every lesson and assessment plan outlines ways to adapt your lesson to accommodate students learning from a distance. Every lesson and assessment plan identifies opportunities to gather assessment data. Of course, each and every lesson plan also includes learning goals and success criteria, Strand A expectations (SEL and Mathematical Processes) for the lesson, and links connecting to additional resources that will support teachers and students.

As part of this project, the team has also created and delivered 36 webinars on curriculum overview topics such as Program Planning, Mathematical Modelling, and Assessment and Evaluation, as well as grade-specific topics such as Coding in the Primary Grades, Financial Literacy in the Junior Grades, and Number in the Intermediate Grades. Recordings of these webinars are on the website OntarioMath.support (www.ontariomath.

Message de la présidente



JUDY MENDAGLIO
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Judy Mendaglio est la présidente de l'AOEM/OAME. Elle a exercé de nombreuses fonctions dans le secteur ontarien de l'éducation : formatrice au niveau collégial, professeure et chef de section du secondaire, chargée de cours au niveau universitaire, consultante pour le primaire et le secondaire et écrivaine. Judy se consacre actuellement à de nombreuses activités bénévoles.

Dans le présent document, le masculin est utilisé sans aucune discrimination et uniquement dans le but d'alléger le texte.

Cette année scolaire, l'AOEM/OAME s'est associée à l'AFEMO pour produire une variété de ressources afin d'appuyer une réussite de la mise en œuvre du programme de mathématiques élémentaires révisé 2020. Une équipe d'éducateurs extraordinaires, de l'AOEM/OAME et de l'AFEMO, a été réunie pour s'attaquer à ce projet plutôt gigantesque. Dans le cadre de ce projet, les équipes ont rédigé 208 plans de leçon et d'évaluation, en anglais et en français, ciblant les attentes présentées dans le programme-cadre de la 1re à la 8e année. Notre projet comprenait également la création d'un nouveau site Web public pour héberger ces nouvelles leçons. Si vous n'avez pas encore visité le site

« Ontario Math Support » (www.ontariomath.support/?pg=home&lang=FR), veuillez le faire. Affiché sur celui-ci, il y a des plans de leçon et d'évaluation modifiables triés par année d'études ainsi que par thème dans les domaines d'étude du programme-cadre. Vous trouverez également des ressources supplémentaires, telles que des programmes Scratch et des feuilles de calculs prêtes à l'emploi, ainsi que des ressources en ligne/hybrides comme des Jamboards et des diapositives Google préfabriqués. Chaque plan de leçon et d'évaluation décrit des moyens d'adapter votre leçon pour permettre aux élèves d'apprendre à distance. Chaque plan de leçon et d'évaluation identifie des occasions de recueillir des données d'évaluation du rendement des élèves. Bien entendu, chaque plan de leçon comprend également des résultats d'apprentissage et des critères d'évaluation, les attentes du domaine A (Apprentissage socioémotionnel (ASE) et processus mathématiques) pour la leçon, ainsi que des liens vers des ressources supplémentaires qui aideront les enseignants et les élèves.

Dans le cadre de ce projet, l'équipe a également créé et présenté 36 webinaires. L'aperçu de la vision, l'objectif et des domaines d'étude du curriculum est disponible. De plus, il y a des

support/), which you can review at a time that is convenient to you. In addition, the slide decks used in the webinars are provided so that you can share these resources with colleagues in your school, in your school board, or in our local OAME/AOEM Chapter.

I have been very fortunate to be able to work closely with a remarkable and thoroughly dedicated group of educators on this project. Because of my participation in this project, I have found myself re-entering the role that is most rewarding for me—that of learner. I remember when I left my career in the financial services industry to return to school to become a certified teacher, I felt the pull to be in an environment in which learning was an integral component of daily activity. Although I had kept studying "on my own time," completing professional designations, which created personal learning opportunities, the environment in which I spent my days was quite barren of adults who valued learning the way I did. What a delight for me to work on a project that afforded me the opportunity to learn such a variety of new things.

With new areas of content in the revised curriculum, such as mathematical modelling, coding, social–emotional learning, and financial literacy, I often found myself revisiting, revising, and extending my own mathematics learning. Having studied mathematics for many years, I knew a lot about mathematical modelling. It is the most prominent aspect of applied mathematics at the university level as well as in the non-academic world outside the university. What I did not know, however, was how to take this remarkably important area of study and fit it into the understandings, skills, and experiences of students. Since there are no grade-specific expectations for mathematical modelling, other than that it is to be tied to the learning in other content areas, everyone on the project, myself included, needed to learn more about the development of connections to real-world situations. These connections guide us in helping students personally recognize and build on those connections.

When we were brainstorming about how best to introduce financial literacy to each grade level within the mathematics program, I learned a lot about taking into account students' cultural background, personal circumstances, and lived experiences when planning instruction. I was well aware that different cultures represent money differently. I knew that some cultures have important beliefs about money owing and interest charging. What I learned by working with this team was how delicate the discussion of money needs to be. It isn't

webinaires sur la planification des programmes-cadres, la modélisation mathématique, l'apprentissage socioémotionnel en mathématiques, et l'évaluation et communication du rendement des élèves sont accessibles. Les domaines d'étude propres à chaque année d'études tels que le codage au cycle primaire, la littératie financière au cycle moyen et les nombres au cycle intermédiaire sont des webinaires créés pour appuyer les enseignants. Les enregistrements de ces webinaires se trouvent sur le site Web OntarioMath.support (www.ontariomath.support/?pg=home&lang=FR), ce que vous pouvez consulter au moment qui vous convient. De plus, les diapositives utilisées lors des webinaires sont fournies afin que vous puissiez partager ces ressources avec des collègues de votre école, de votre conseil scolaire, ou de notre section locale de l'AOEM/OAME.

Ce projet m'a donné la chance de travailler en collaboration avec un groupe d'éducateurs remarquables et totalement dévoués. Grâce à ma participation à ce projet, je me suis retrouvée dans le rôle le plus gratifiant, celui d'une apprenante. Je m'en rappelle quand j'ai quitté ma carrière, qui était dans le secteur des services financiers, pour retourner à l'école afin de devenir une enseignante certifiée, je ressentais le désir de me trouver dans un environnement où l'apprentissage faisait partie de ma vie quotidienne où je continue à apprendre. En complétant des désignations professionnelles et en collaborant régulièrement avec d'autres éducateurs, je peux honnêtement dire que je suis une vraie apprenante. Quel plaisir de travailler sur un projet qui m'a donné l'occasion d'apprendre une telle variété de nouvelles choses.

Je me suis souvent retrouvée à revoir, réviser et à étendre mon propre apprentissage des mathématiques avec les nouveaux domaines d'étude du programme-cadre de mathématiques tels que la modélisation mathématique, le codage, l'apprentissage socioémotionnel et la littératie financière. Ayant étudié les mathématiques pendant de nombreuses années, j'en connaissais beaucoup sur la modélisation mathématique. Elle s'agit d'aspect le plus important des mathématiques appliquées au niveau universitaire ainsi que dans le monde non académique hors de l'université. Ce que je ne savais pas, cependant, c'était comment appliquer cela dans ma pratique quotidienne pour appuyer le développement de la compréhension, des compétences et des expériences des élèves. Étant donné qu'il n'y a pas d'attente spécifique pour la modélisation mathématique, autre que le fait qu'elle doit être liée à l'apprentissage dans les autres domaines de contenu, tous les participants au projet, moi y comprise, devons en apprendre davantage sur l'établissement de liens avec des situations de la vie quotidienne. Ces liens nous guident sur la façon d'aider les élèves eux-mêmes à reconnaître ces liens et en tirent parti.

Lorsque nous avons fait du remue-méninge sur la meilleure façon d'introduire la littératie financière à chaque niveau du

just about recognizing coins and bills, counting by 10's and 25's, or designing fundraising opportunities to give students "real-world" experiences. The true real world of financial literacy also involves talking about equality of opportunity and recognizing systemic inequities when they occur.

The project team, along with many other teachers in Ontario, learned a lot very quickly about ways in which we can support learning for students who are learning remotely. We learned how to create classroom cultures of collaboration when some students are in the classrooms and others are learning in a virtual classroom. We learned how to harness technologies many of us had never used previously, to provide learning spaces, resources, and opportunities for collaboration for students who were attending using video conferencing. We learned how to look for assessment opportunities, even when we were not able to move from desk to desk or table to table to check in with our students.

Everyone involved in this project began the important process of learning, and the learning is far from over, including the ways in which we take for granted that any resources we create will automatically be accessible to all teachers and students, in spite of what challenges they might face. We have learned about the ways websites are designed to facilitate having technology assist people in navigating between resources posted to a website. We have learned how a screen reader accesses the words, pictures, and diagrams that we put onto our pages and into our slides, necessitating adding certain design considerations to lesson templates,

And now I come to the most surprising learning that I experienced. It did not come from writing for a new math curriculum. It came from working with bilingual teams and listening to the experiences of our colleagues who teach mathematics in French schools. Perhaps it will come as a surprise to you that this curriculum is unique in that the English version and the French version actually have the same expectations. I had known that the previous elementary math curricula were somewhat different, but I had thought that the differences were minor. What I had not appreciated is that the 2005 curriculum represented a decision, and first effort, to align the two curricula, and the current revision completed the process. As we started to design lessons in two languages, it became apparent that we needed to accommodate the different curricula from which the teachers and students would be coming, and that there was previously not just one curriculum that added unexpected challenges to our task.

I also learned that our languages offer different

programme de mathématiques, j'ai beaucoup appris sur la nécessité de tenir compte du contexte culturel, des circonstances personnelles et des expériences vécues des élèves lors de la planification de l'enseignement. J'étais bien consciente que les élèves de différentes cultures représentent l'argent différemment. Je savais que certaines cultures ont des croyances importantes concernant la dette d'argent et la facturation des intérêts. En travaillant en équipe, j'ai réalisé que la discussion sur l'argent doit être délicate. Il ne s'agit pas seulement de reconnaître les pièces de monnaie et les billets, en comptant par 10 et 25, ou de concevoir des occasions de collecte de fonds pour offrir aux élèves des expériences de la « vie quotidienne », mais plutôt c'est la discussion de l'égalité et de reconnaître les inégalités systémiques lorsqu'elles se produisent.

L'équipe du projet, ainsi que de nombreux autres enseignants de l'Ontario, ont appris très rapidement comment appuyer l'apprentissage des élèves qui apprennent à distance. Nous avons appris à développer un environnement qui combine des cultures de collaboration présentielles et virtuelles. De plus, nous avons appris à exploiter des techniques que beaucoup d'entre nous n'avaient jamais utilisées auparavant, pour développer un environnement d'apprentissage, fournir des ressources, et offrir aux élèves des possibilités de collaborer par vidéoconférence. Dernièrement, nous avons appris à rechercher des occasions d'évaluation d'apprentissage lorsque les élèves sont à distance.

Tous ceux qui ont participé à ce projet ont commencé ce processus d'apprentissage important, mais réalisent que cet apprentissage est loin d'être terminé. Nous tenons pour acquis que toutes ressources que nous créons seront automatiquement accessibles à tous les enseignants et élèves, en dépit des défis auxquels ils pourraient faire face. Nous avons appris comment les sites Web doivent être conçus pour que la technologie puisse aider les gens à naviguer entre les ressources publiées sur un site Web. Nous avons appris comment un lecteur d'écran accède aux mots, images et diagrammes que nous mettons sur nos pages et dans nos diapositives, ce qui nécessite l'ajout de certaines considérations de conception aux modèles de leçons.

J'en arrive maintenant à l'apprentissage le plus surprenant que j'ai vécu. On pourrait croire que cela provenait de la participation à la rédaction d'un nouveau programme-cadre de mathématiques, mais ce n'était pas le cas. Mon apprentissage s'est fait en travaillant avec des équipes bilingues et en écoutant les expériences de nos collègues qui enseignent les mathématiques dans les écoles françaises. Alors que le curriculum de 2005 représentait une décision et un premier effort pour aligner les versions anglaise et française, la révision en 2020 a actualisé l'alignement, complétant le processus. Lorsque nous avons commencé à concevoir des leçons dans deux langues, il est devenu évident que nous devions tenir compte des différents résultats d'apprentissage auxquels les enseignants et les élèves

challenges. In English, teachers have become comfortable with the need to distinguish between assessment and evaluation. Since the release of *Growing Success* in 2010, all teachers in Ontario have been navigating through the differences. We now understand that assessment (*as*, *of*, and *for* learning) has a different purpose than does evaluation. However, the French language does not provide a word for assessment that is different from the word for evaluation, so it is only through context that these reasons for gathering evidence are differentiated.

Working on this project has also made it clear that teachers in English Ontario mathematics classrooms have had an abundance of resources available to them, while our counterparts teaching mathematics in French have not. They have come to rely on the *Mathies* tools because they are available in French. However, video clips, mathematical processes resources, financial literacy resources, and even coding resources in French are in short supply. It has been eye- opening to recognize the degree to which English mathematics teachers have been privileged to have many resources at their fingertips, from the original supports that were posted on the former MathGAINS site, to dozens of resources that come, largely in English, from our Ontario math education research departments, to the wealth of resources that are published in the United States and the United Kingdom. We hope that this project, which we expect will be the first of many to come, will allow collaboration between English and French mathematics educators in Ontario, so that we can support each other in ensuring that all students and educators in Ontario have access to the same rich resources and support materials.

Through this journey of learning—that was available to those of us on this team because of the release of a new curriculum—we are all better educators. We understand, in new ways, how mathematics really does connect with everything we do in our lives outside our classrooms, whether we are 8 or 48. We appreciate that we are all in this together, whether we have accessibility challenges, teach remotely, or teach in French. We now know so much more than we did last Spring—about our students and about each other. ▲

étaient habitués. Cela a posé des défis inattendus à notre tâche. Vous serez peut-être surpris d'apprendre que le contenu des programmes d'études d'aujourd'hui aux mêmes attentes.

J'ai également appris que nos langues offrent des défis différents. Depuis la publication de *Growing Success* en 2010, les enseignants anglophones de l'Ontario se sont familiarisés avec la nécessité de faire la distinction entre les termes « assessment and evaluation » et naviguent à travers les différences. Dans le document français *Faire Croître le Succès* en 2010, un terme seulement est utilisé, l'évaluation. Ce n'est donc que par le contexte qu'on vient à comprendre le terme évaluation dans son intégralité. Il est important de reconnaître qu'il y a un processus d'évaluation (l'évaluation au service de l'apprentissage, l'évaluation en tant qu'apprentissage et l'évaluation de l'apprentissage) et que ce processus se différencie de celui de l'évaluation du rendement de l'élève.

Le fait de travailler à ce projet a également montré clairement que les enseignants de mathématiques anglophones de l'Ontario disposaient d'une abondance de ressources, alors que les enseignants francophones en disposaient peu. Ils en sont venus à se fier aux outils *Mathies* parce qu'ils sont disponibles en français. Cependant, les vidéos, les ressources sur les processus mathématiques, les ressources sur la littératie financière et même les ressources sur le codage en français sont rares. Il a été révélateur de reconnaître la mesure dans laquelle les enseignants de mathématiques anglophones ont eu le privilège d'avoir de nombreuses ressources telles que les supports originaux qui ont été affichés sur l'ancien site MathGAINS. Il y avait aussi des douzaines de ressources qui proviennent, en grande partie en anglais, de nos départements de recherche sur l'enseignement des mathématiques en Ontario, et par la richesse des ressources qui sont publiées aux États-Unis et au Royaume-Uni. Nous espérons que ce projet, qui devrait être le premier d'une longue série, permettra la collaboration entre les enseignants de mathématiques anglophones et francophones de l'Ontario. Notre but est de s'assurer que tous les élèves et les enseignants de l'Ontario ont accès aux mêmes ressources et documents de soutien.

Grâce à ce parcours d'apprentissage—qui était à la disposition de ceux d'entre nous qui faisons partie de cette équipe en raison de la publication d'un nouveau programme d'études—nous sommes tous de meilleurs éducateurs. Nous comprenons que les mathématiques sont réellement liées à tout ce que nous faisons hors de nos salles de classe, que nous ayons 8 ou 48 ans. Nous comprenons que nous sommes tous dans le même bateau, que nous ayons des problèmes d'accessibilité, que nous enseignons virtuellement ou nous enseignons en anglais/français. Nous en savons maintenant beaucoup plus que nous ne le savions le printemps dernier—au sujet de nos élèves et les uns des autres. ▲

▲ Blast from the Past! The Common Curriculum and Destreamed Grade 9



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In 1987, journalist George Radwanski issued a report, commissioned by the then Liberal government. The report, called the *Ontario Study of the Relevance of Education, and the Issue of Dropouts*, made a number of recommendations, including:

- early childhood education
- standardized testing
- "destreaming" of high school
- "outcomes-based" education
- a common curriculum to replace the credit system

The impetus for Radwanski's report was concern over student dropouts, and Radwanski found that many students were uninterested in school and were not learning skills and knowledge relevant to a modern economy.

At that time, many educators employed a rule of thumb called "the 25 percent rule": 25 percent of students went on to university, 25 percent went to community colleges, 25 percent went to apprenticeships and the workforce, and 25 percent did not graduate. Radwanski found that that final 25 percent (the dropouts) were disproportionately represented in the General and Basic levels (somewhat synonymous with the Applied and Locally Developed levels today). His proposed solution was destreaming of secondary school and a common curriculum for all students.

Radwanski's solution was a phenomenal confusion of correlation with causation. Being in General- and Basic-level courses was not causing students to drop out. These students usually selected the General- and Basic-level courses for other reasons: a general dislike of school and of mathematics in particular, weak work habits, learning gaps and weak prior knowledge, lack of academic motivation, and "transmission" pedagogy (the dominant pedagogy of the time) being a poor fit for these students' learning modalities. Placing these students together with other students, who were more academic and sometimes gifted, and eliminating the terms "General" and "Basic" levels did nothing to address the characteristics of these students that led them to drop out in higher numbers than more academically inclined students.

Although some schools and some boards of education had piloted destreamed classes in the early 1990s, the Liberal government did not officially implement the Radwanski recommendations prior to their election defeat in 1990 at the hands of the New Democratic Party (NDP). In 1993, the NDP issued a Royal Commission report, *For the Love of Learning*, which reiterated most of Radwanski's recommendations. Based on a working paper published in early 1993, Grade 9 classes were expected to be destreamed as of September 1993. This was followed in early 1995 by *The Common Curriculum, Grades 1–9*, a set of outcomes-based achievement standards for all subjects in Grades 1 to 9. In June 1995, the NDP government was defeated by the Progressive Conservative party led by Mike Harris. However, destreamed Grade 9 was implemented in September 1995 as planned.

As a consequence, mathematics teachers faced mixed-ability classes ranging from Basic level through Gifted students, and were expected to teach all students a common curriculum, and have all students achieve a specified set of outcomes. No textbooks had been produced for this common curriculum, and no pedagogical resources were available from the Ontario Ministry of Education, although some early-adopter boards of education had produced a few resources for their own board's use. The results were decidedly mixed. Some teachers created outstanding math programs, and used extensive differentiated instruction in an attempt to meet the needs of all their students (e.g., Ottawa Board of Education, 1992). Co-operative learning was often employed as an effective instructional strategy. Other teachers "taught to the middle," meeting the needs of what they perceived as "average" students, but doing little for students performing well above or well below that level. There was also academic pushback from educational researchers, who stated that the outcomes-based requirements were incompatible with a constructivist theory of learning, which was beginning to dominate pedagogy in Ontario (e.g., Wien & Dudley-Marling, 1998).

Teachers and administrators began producing resources

to support teachers in implementing this new curriculum. In a large document intended for elementary teachers, Kristensen-Irvine (1996) identified six key foci of the Common Curriculum, and provided instructional and assessment strategies to address these foci: focus on all students, focus on integrated learning, focus on excellence and equity, focus on accountability and standards, focus on collaboration, and focus on learning outcomes. Her document provided a grade-level outline of expected outcomes for every subject at every grade level, with teaching strategies, suggestions for integration and interdisciplinary activities, and assessment strategies.

Articles began to appear in the *Gazette*, suggesting strategies for teaching and assessing a destreamed class. One of the earliest was the June 1993 *Gazette*, which provided an extensive review of the NDP working paper, as well as a large body of sample activities collected from teachers across the province. Jennifer Weiner (1994) provided a sample destreamed Grade 9 final exam. David Haimes (1996) reported on a study involving 45 teachers of destreamed mathematics. He found that the biggest concern was meeting the needs of a wide variety of students, and that teachers principally relied on resources produced by themselves or their colleagues. On a positive note, Haimes' study found an increased variety of assessment tools, such as portfolios, assignments, conferencing, and projects. However, he found limited use of co-operative learning and high demand for supports for exceptional students, such as having special education teachers and peer tutors in classes. High-achieving students were most frequently given enrichment activities. Haimes also found a high level of frustration among the teachers, in attempting to meet the needs of all their students, while still addressing all the learning outcomes specified in the curriculum. As well, comments included reports of very high workload demands in order to teach these classes effectively.

In 1997, the Harris government signalled that a new mathematics curriculum would be rolled out, beginning in 1998. The new curriculum maintained some elements of the Common Curriculum, in particular the list of "expectations." But Grade 9 Mathematics was segregated into Academic and Applied streams (and later, Locally Developed courses) to help address the wide array of student abilities and interests. The Progressive Conservative government, and later the Liberal government, implemented many of Radwanski's other recommendations, including standardized testing (EQAO) and early childhood education (the full-day Kindergarten model).

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▲ Reflections on Solving Equations: Obtaining Readable Forms



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It is a truism that students often come up with surprising ways of solving tasks we give them. Students can at times even come up with strategies or ideas that in turn raise questions or even shed light on mathematical aspects usually taken for granted or kept implicit.

This is particularly the case in the mental mathematics that my research team and I study in classrooms. Within the time constraints, students rarely refer to known procedures because these are often too time-consuming, and hence not economical, to solve in this context (see Butlen & Pézard, 2000). This leads many students to engage with a variety of strategies worth pondering. In particular, we conduct sessions on mental algebra, where the participating students (of various grade levels, from elementary to university-level) are given short amounts of time to solve some equations for an unknown. Questions are posed orally and in writing on the board, without the use of paper and pencil or any other material aids (the equations given are of the usual form $Ax + B = C$, $Ax + B = Cx + D$, $\frac{Ax}{B} = \frac{C}{D}$, $Ax^2 + Bx + C = 0$ and their variants [see Proulx, 2013; Proulx et al., 2017]).

In this column, I share examples of student strategies about equation solving. These strategies, albeit simple, often raise interesting mathematical questions.

The Doubling Strategy

One of the equations given to students was $\frac{2}{5}x = \frac{1}{2}$. One way students solved it was to first double the ½ and thus double the entire equation to $\frac{4}{5}x = 1$. At this point, some students asserted that they knew the answer and that the response gave directly $\frac{5}{4}$ (because it is the value that, multiplied by $\frac{4}{5}$, gives 1).

On Obtaining Readable Forms

What this kind of strategy reveals is that the solution to the equation can be inferred directly from an equation, or simply "read" and not manipulated, as Arcavi (1994) describes it. In the same way, if one is given $2x = 2$ or $x + 1 = 2$, one might be able to infer the value of x directly as being 1, without having to undergo any algebraic manipulations. Other examples also happened, like the following:

- To solve for x in the equation $3x = \frac{3}{4}$, some said that since three times x is equal to three times a quarter, then x is a quarter. Or, if "three-quarters is three x, then x is a quarter."
- To solve $2x + 3 = 5$, some students asserted directly that $x = 1$ because it is like $2 + 3 = 5$, and when a value is substituted for x, it still needs to be equal to 2; hence, x needs to be 1. Others used the same reasoning as above for $2x + 3 = 5$ and erroneously said that x needs to be 0 so that it does not interfere with the equation to give 5, or that x is not needed in $2 + 3 = 5$.
- To solve $\frac{2}{5}x = \frac{1}{2}$, some students said that they wanted to get ½, and that half of 5 is 2.5; however, now it is 2 and not 2.5, so x needs to be 1.25 to get to 2.5. Similarly, other students transformed it to $\frac{4}{10}x = \frac{1}{2}$, and looking for 5, the half of 10, explained x to be 1.25 (because 4 times 1.25 = 5). A similar way was used to transform the equation to $\frac{4}{10}x = \frac{5}{10}$, leading to the use of an erroneous additive strategy, and asserting that x was -1/10.

These strategies, although having some differences or errors, raise the idea that the value of x can be inferred without explicitly manipulating much of the equation itself. This highlights how solving an equation for x is not always about isolating x, nor is it to end up with an equation of the form $x = __$: it is about finding the value of x that satisfies the equation. In other words, isolating x through algebraic manipulations represents only one way of obtaining the value for x, as these strategies show.

Baruk (1995) explains that obtaining a solution to an equation of the form $x = 3$ is mainly to have it in what she calls a *readable* form, so that one can infer that if one replaces x by 3, then the equation is verified. In addition, the strategies used above illustrate how an equation can be "read" directly to get the value of x without being in the form $x = __$. If readability is one intention, then this readability of the equation is thus possible in forms other than $x = __$. Additionally, this readability has been seen as dependent on, or relative to, the reader. The equation $\frac{4}{5}x = 1$ was in a readable form for some students, as $3x = \frac{3}{4}$ or $2x = 2$ can be for others. This notion of readability is reminiscent of what Bednarz and Janvier (1992) call a *global reading strategy* (or what Arcavi (1994) refers to as an *a priori* investigation of the equation). For an equation like $x + \frac{x}{4} = \frac{x}{4} + 6$ to solve for x, they show how some students plunge head first into varied algebraic manipulation to isolate x, and how others take a step back, look at the equation in its whole, and infer that $x = 6$ because $\frac{x}{4}$ is present additively on both sides.

In contrast, attempting to isolate x is not necessarily a panacea to obtain a readable form. The quadratic formula with $x = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$ represents a good example, where the x is isolated, but much more still needs to be done to obtain its value. An equation like $2x = 27^2$ transformed to $x = \frac{27^2}{2}$ is another, being rarely a readable form for solvers. Similarly, arriving at $3 = x$ might not necessarily be a readable form for some students, who might still wonder "what to do next" (P. Liljedal, personal communication).

In this view, solving an equation is related to attempting to obtain a readable form, one that helps infer or read the value that verifies the equation.

On Producing Families of Equivalent Equations

There is more to say about what this strategy entails, and some of it has been initiated in the *Gazette* in Proulx (2021) and is continued here. This first transformation of the equation $\frac{2}{5}x = \frac{1}{2}$ to $\frac{4}{5}x = 1$ is something that a number of students engaged in, most of them explaining that it made the equation easier to work with afterwards. What comes out of this justification is twofold.

A first issue that this strategy highlights is how the focus of algebraic manipulations can be placed somewhere other than on the x when solving. In this case, the manipulation targeted the ½ so as to get rid of it. As one colleague pointed out (P. Liljedahl, personal communication), it makes explicit how we often focus intensely on the x, whereas (young) students also focus on numbers (Filloy & Rojano, 1989, also mention this). An often-heard saying is that one attempts to isolate x in order to know its value, and this is attempted in potentially the least number of steps and as efficiently possible. However, in this case, this first doubling step from $\frac{2}{5}x = \frac{1}{2}$ to $\frac{4}{5}x = 1$ did not aim directly to work on the x, and focused more on the equation itself, its form, its structure, to make it simpler. In this sense, this step might not be seen as that efficient for isolating x.

Combined with this first issue, a second element that this strategy of doubling the ½ raises explicitly is how manipulating an equation amounts to producing equivalent equations. Each algebraic manipulation, if legitimate, produces an equivalent equation— $\frac{2}{5}x = \frac{1}{2}$ to $\frac{4}{5}x = 1$ to and then to $x = 1 \cdot \frac{5}{4}$; e.g., leading to $= \frac{5}{4}$, should this be $x = \frac{5}{4}$?— are all equivalent equations produced when attempting to find the value of x. This transformation of the equation by

doubling it, although may be local and working only for this specific equation, underlines that solving an equation amounts to the establishment of a series of equivalent equations. Even if possibly obvious from one transformation of an equation to another, this also leads to considering that through the various steps taken to solve the equation, a family of equivalent equations is produced. In the case of $\frac{2}{5}x = \frac{1}{2}$, an entire family of equations that shares the same solution of $x = \frac{5}{4}$ is produced; and there would also be many other members, all equivalent to $\frac{2}{5}x = \frac{1}{2}$, like $5 = 4x$, $2x = \frac{5}{2}$, $8\pi x = \frac{30}{3}\pi$, etc., in this family of equivalent equations.

Conceiving the solving of an equation as the production of a family of equivalent equations leads one to consider that one member of that family is also in the form "*x = something*." For this case, it is $x = \frac{5}{4}$. This $x = \frac{5}{4}$ form would often be conceived of as the end point of the solving of $\frac{2}{5}x = \frac{1}{2}$ as the final step that culminates in isolating *x* and finding the value that satisfies it. However, conceived in terms of a family of equivalent equations, $x = \frac{5}{4}$ is merely one member of its entire family, one which could be conceived of as easier to read to find the value of *x*. This makes the pursuit of the value of *x* for $\frac{2}{5}x = \frac{1}{2}$ a pursuit of the simplest form possible from the family of all possible equivalent equations of $\frac{2}{5}x = \frac{1}{2}$. Because $\frac{2}{5}x = \frac{1}{2}$ was possibly too difficult to read for some (but not all, as the previous point highlighted), students transformed it into simpler forms of the same family of equivalent equations from which to infer the value of *x*. Solving an equation is thus conceived as *producing a family of equivalent equations* and searching for the simplest form (that is subjectively readable) in that family, manipulating algebraically until that form is obtained.

Concluding Remarks

In line with the introduction, the issues raised about solving equations offer a salient illustration of how some mathematical insights can be gained from students' ways of solving, even when the strategies engaged with are quite simple. In this case, the doubling strategy opened up reflections about obtaining readable forms to find the values that satisfy the equation. This strategy brought forth a conception that the solving of equations produces not only equivalent equations, but a family of them, all interlinked; and that through its members, some being in a more or less readable form than others, ones easier to work with, at least for the solver himself or herself. This along the way also questioned the notion of "isolating *x*," an usual endeavour when one solves equations in schools.

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In September, 2021, Grade 9 Mathematics is once again being destreamed. OAME/AOEM has been tasked with producing teaching resources to support the destreamed Grade 9, so in one respect this implementation is different than in 1993. However, none of the circumstances surrounding this version of destreaming have substantially changed. Many students who would have chosen Applied Grade 9 Mathematics lack academic motivation; learning gaps and weak prior knowledge remain; lack of engagement persists; perception that the mathematics curriculum is irrelevant to their lives; transmission pedagogy still stubbornly remains in some classrooms. Further, after two years of COVID-disrupted education, for some students learning gaps have been exacerbated. As well, teachers are tired and may be reluctant or unable to make the changes necessary to support the new destreamed mathematics curriculum.

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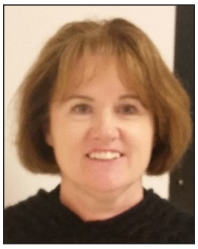
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▲ OAME/NCTM Report: Summertime, Summertime, Glorious Summertime!



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Jacqueline Hill is a Grades 9–12 teacher of mathematics. She is a Past President of OAME and OMCA, as well as the recipient of the award for Exceptional Teaching in Secondary Mathematics. She also authored the "Director's Dialogue" for the Gazette *for a number of years.*

Summer is the time most mathematicians unwind a little bit.

It is the time of year when we sometimes have more time for ourselves.

When we were young, we may have enjoyed hopscotch, Bingo, and even Tic-Tac-Toe!

Well… here is a summertime NCTM variation of those games.

Please feel free to get a line (horizontal, vertical, or diagonal) and then a blackout card as you enjoy some summertime fun! Recharge your batteries, stay safe, and be ever optimistic.

All of the items found in this game can be found on the NCTM website, and you do not need to be a member to access any of them!

Yours in mathematical fun, Jacqueline ▲

N	C	T	M
Take a nap.	Plan to attend the August 5 Idaho Virtual Conference (details found on the NCTM website).	Read a book that truly interests you (and not one for work or class).	Check out the six weeks of back-to-school resources (from manipulatives to *Mathematics Lessons in Today's World*… and all things in between).
Take a peek on the NCTM website at the three *Catalyzing Change* books.	Play the 2021 year game; that is, using only the digits in 2021, try and write the numbers from 1 to 100, e.g., $1 = \left(\frac{2}{2}\right) \cdot 0 + 1$.	Check out the *Classroom Resources* tab (with over ten items to interest even the toughest learner to reach).	Enjoy some time on the water. Maybe just paddle around in a canoe or go fishing.
Mark your calendar for the September 20, 2021 NCTM Annual Meeting & Exposition Atlanta in Atlanta, Georgia.	Reach out to a colleague or friend, with whom you have not been in touch for a while. Just catch up (perhaps a virtual coffee or wine tasting).	Get outside for a walk (listen to the sounds of nature).	Start a gratitude list. Write down one thing a day for which you are grateful.
Get hold of a copy of *Teaching Math at a Distance*, available on the NCTM website.	Under the *Publications* tab on the NCTM website, check out the *Legacy Journals and Blogs*.	Mark your calendar for the *NCTM Research Conference*, which is a virtual event, to be held October 4–8, 2021.	Consider taking out an NCTM membership. Benefits include journals and access to a wide professional-development community.

▲ Events at the Fields



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Angelica Mendaglio is an instructional designer at Vretta Inc. in Toronto, Ontario, where she helps to create interactive digital mathematics lessons and activities for middle school students.

New title, broader scope! This column is typically about the MathEd Forum, a monthly event hosted by the Fields Institute for Research in Mathematical Sciences, featuring talks on a variety of topics in math education. However, the Forum is far from the only public event hosted by the Institute. Going forward, this column will have an expanded scope to report on a variety of offerings from the Institute. It is easier than ever to attend a public Fields event by joining online, or by viewing the recording after the fact on their YouTube channel. You can visit their website to see a schedule of past and upcoming events.

One such public event took place on March 8. To celebrate International Women's Day, the Fields Institute sponsored a talk entitled "Home Advantage: How early supports help girls & women thrive in mathematics." It featured conversations between two sisters, who are both mathematicians. Deirdre Haskell, Fields Institute Deputy Director and Professor of Mathematics at McMaster University, and her sister, Cymra Haskell, Professor of Mathematics at University of Southern California, spoke about growing up in a math-friendly household, which they believe had a positive influence on their eventual independent choices to pursue academic careers in mathematics. Viewers heard about research that has shown that one of the most critical factors that improves the chances for girls to be successful in STEM subjects is support and encouragement at an early age from family at home or from those within early educational environments. The support need only come as interest in what the child is doing, or in asking questions to encourage a conversation to develop. In other words, it is not necessary to become an expert math teacher to see positive benefits for girls.

Deidre and Cymra acknowledged that even with this home advantage, their time as students and as academics had many challenges. They spoke about how, even though they knew that the environment for girls and young women to study mathematics was unfair, they believed there was not much that they could do about it. They attributed their confidence and perseverance to the support they received when they were still very young. The discussion included recognition that progress has been made, and that the mathematical sciences will benefit when the time comes, that all underserved groups have equal access to the field. While we are "still at what feels like the foot of a mountain," with the help of generous and committed role models such as Deidre and Cymra, perhaps we can speed up the change. To view a recording of this conversation, go to the Fields YouTube channel.

Another public offering from the Fields Institute has come from the study of COVID-19 modelling. This has been a key area of research at the Fields Institute for Research in the Mathematical Sciences this year. In fact, the Institute is holding weekly seminars on the mathematical modelling of the COVID-19 virus. While the talks are designed for those doing research in the field, the Fields Institute has also held two public meetings to help explain what this entails. The first, Café Mathématique, hosted a panel of experts at universities across Canada as they answered the question, "What is a model of COVID-19?" Researchers from various fields explained how they use the enormous amounts of data they receive every day to help develop and refine the models they are using for a vast array of purposes—to inform public policy, to assist the medical communities to find effective ways to treat patients, to help hospital administrators manage outbreaks, to name just a few. The second opportunity for the public to hear from the experts at the forefront of modelling the COVID-19 virus engaged members of the Mathematical Modelling of COVID-19 Task Force in a live Town Hall. This was quite different because it focused on answering questions generated by participants, and its goal was to help viewers understand how researchers make sense of the data, and to help clear up some confusion that people were feeling about the information they were hearing through the media. For those interested in watching the panel or the Town Hall meeting, a recording of each is available on the Fields YouTube channel. In the case of both these public offerings, it was fascinating to listen to researchers whose daily work is to interpret the most recent data and look for patterns that can help inform decision makers.

The MathEd Forum in February focused on the work of Dr. Chris Suurtamm, the 2020 winner of the Margaret Sinclair Memorial Award. This award is granted annually to a Canadian educator who has demonstrated excellence and innovation in mathematics education. Each year's award winner presents a talk at the Fields Institute, and it is always one of the most interesting and engaging talks of the year. Dr. Suurtamm's talk, titled "Connecting Research and

Practice: What can we learn about high-quality mathematics teaching?," chronicled four research projects that she conducted in recent years. Rather than bringing the latest research into the classroom, Dr. Suurtamm's work is all about studying effective practice and valuing the critical role of the teacher in research.

The first study investigated the implementation of curriculum in intermediate math courses, which looked into what was actually happening in classrooms to see if the curriculum was working. The second study described the assessment practices of teachers, and the dilemmas faced when implementing new assessment strategies. Another study followed Grade 9 Applied teachers as they created professional learning communities, who determined what they wanted to work on together to enhance their teaching for these students. The final study looked at the values of teachers in mathematics education, and how those values are negotiated in their practice. All of these studies were anchored in observing teacher practice, in many cases, giving teachers the opportunity to work collaboratively to improve their own practice, and often simply noting what came out of those collaborations. It was a very interesting look at how teacher voice and practice is valued in research.

The Fields Institute MathEd Forum has its final meeting of the academic year in April, and will reconvene on the last Saturday in September. You can sign up to attend remotely through the Fields website, where you can also see their schedule of other upcoming math education events. You can also find more information about the Margaret Sinclair Memorial Award and how to nominate an exceptional educator for this award on the Fields website. ▲

▲ OMCA News – Spring 2021

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It has been a busy Spring for Ontario Mathematics Coordinators Association (OMCA) members!

With the new Grade 9 curriculum expected soon, OMCA has been hosting a series of "De-Streaming Networking Opportunities" for its members: a chance to connect with other school board math leaders across the province in conversation around some of the challenges and opportunities that the new curriculum brings.

At our first session on March 1, we heard from Ministry representatives and TDSB educators discussing the "why" and the "how" behind the destreaming initiative. In the follow-up session on April 8, we dove more deeply into specific aspects of implementing a destreamed curriculum, including avoiding racial equity detours; providing multiple entry points for teachers and school teams; calling in (addressing issues with peers in a compassionate way), rather than calling out; examining destreaming beyond Grade 9; and establishing/maintaining the equity, anti-racist, and anti-oppressive rationale for destreaming. On May 6, we had our final networking session, and focused on our potential next steps for implementation. These were hugely valuable sessions, both in terms of content and conversation, but also in helping further our collective understanding and thinking around the impacts of streaming.

Guest speakers at our regular monthly meetings through the Spring have included: Dr. Jennifer Bay-Williams, speaking on what culturally responsive and relevant pedagogy looks like in mathematics education; Octavia Beckles, speaking on centring equity in mathematics teaching and learning; and Dr. Ilana Seidel Horn, speaking on approaching mathematics teaching with an asset-oriented approach of recognizing and building on students' strengths in the classroom. June's meeting will be EdCamp style, with members able to explore topics of their choosing as we close out the school year.

Our final literature circle meeting of the year also occurred in May. Members had the choice of discussing one of three books over the course of the year: *Strength in Numbers: Collaborative Learning in Secondary Mathematics* (Horn, 2011), *Culturally Responsive Teaching & the Brain* (Hammond, 2015), or *Mathematics for Human Flourishing* (Su, 2020).

Finally, on a technological note, our webmaster, Mike Jacobs, has been busy working on the members-only side of the OMCA website, which now includes discussion forums for resources and conversation.

Interested in joining OMCA? New members are always welcome—contact us directly at OMCAmath@gmail.com, or visit our website at www.omca.website/ (note the different domain) to learn more about the organization and connect with us. ▲

▲ OAME/AOEM Awards 2020



Congratulations to **Rebecca Colaiacovo**, who is the recipient of the OAME/AOEM Award for Exceptional and Creative Teaching in Elementary Mathematics. This award recognizes an exceptional and creative elementary educator, who demonstrates excellence in mathematics education and contributes to the overall development of all students. Her character and work ethic truly demonstrate her creativeness in her roles as a math coach and teacher. Her willingness to go above and beyond to help students fosters a love for math. The biggest contribution she has given both the students and the nominator is the notion that math can be viewed and explained from a variety perspectives. Rebecca Colaiacovo has helped teachers in her school incorporate number talks, problems of the week, and math games as ways to foster student engagement. Her focus is on exploring student-led math strategies with hands-on/virtual manipulatives, games, and math websites she has created and/or shared. The students' most treasured activity, created by Rebecca Colaiacovo, was her Harry Potter-themed escape room. The students spent a whole afternoon delving into the Harry Potter universe, completing various math tasks that reviewed and consolidated several months' worth of math expectations. Activities like this, and her wealth of math knowledge, demonstrate her creativeness and exceptionality as a math coach and teacher.

Congratulations also go to the staff at **St. Mother Teresa High School**, namely Natalie Robinson, Jaime de Pippo, Maureen McNeil, and Rita Major, as the recipient of the OAME/AOEM Secondary School Mathematics Staff Award for Exceptional and Collaborative Teaching. This award recognizes a secondary school mathematics staff that fosters collegiality, teamwork, and excellent classroom teaching; contributes to the overall development of students; and demonstrates leadership in the mathematics education community. This close-knit team has struck a fine balance between respecting the individualism of each teacher, while collaborating to ensure a consistently rich learning experience for their students. They have inspired and supported teachers locally, provincially, and internationally for many years by their leadership in spiralling curriculum, developing a thinking classroom, incorporating vertical non-permanent surfaces, and technology integration. Most importantly, learning materials are co-developed and shared with open hands to anyone interested in using them in their own classroom. Most recently, their work in spiralling Grade 9 and the Grade 11 mixed-destination courses have drawn a lot of attention and interest from other teachers in the math community. Their resources have been shared widely and have become the starting point for many other teachers interested in spiralling their curriculum. They have conducted professional development in their board to help teachers learn how they can use the method of spiralling in their own classrooms. In these sessions, every expectation is debated and carefully considered for the developmental appropriateness of the students before being placed somewhere within the course and then put into practice. This most deserving team demonstrates the qualities of the Secondary School Mathematics Staff Award for Exceptional and Collaborative Teaching. ▲



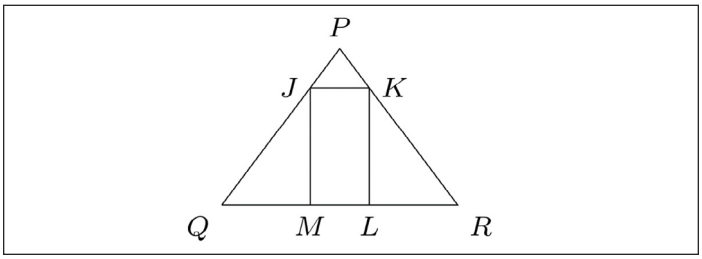
▲ What's the Problem? Maximizing Rectangles



SHAWN GODIN
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Shawn is a former mathematics teacher, department head, and consultant. He strongly believes in the central role of problem solving in the mathematics classroom. He continues to be involved in mathematical activities: presenting workshops, writing articles, working on local projects, and helping create mathematics contests.

Welcome back, problem solvers. In the last issue, I left you with the following problem:



In the diagram, triangle PQR *is isosceles with* PQ = PR = 55 cm *and* QR = 66 cm*. Rectangle* JKLM *is drawn such that* J *is on* PQ*,* K *is on* PR*, and* JK *is parallel to* QR*. What portion of the triangle does the largest such rectangle cover?*

The question was inspired by problem 2 from the 2020 Fryer contest for Grade 9 students. The contest is run by the Centre for Education in Mathematics and Computing at the University of Waterloo. You can check out past contests from all the CEMC contests, as well as other resources for teachers and students on their website, www.cemc.uwaterloo.ca.

This problem is a nice example of taking a contest problem and converting it into an exploration for students. The original problem was computational in nature. It led students to discover the important elements of the problem and to find the fraction of the triangle covered by a specific rectangle. The mathematician wonders: "What is special about this case?," "How does this situation behave?," "Is it related to other things we know?," "Can I extend or generalize this idea?" Reflection of this type is important not only in the mathematics or science classes, but in other subject areas and in life in general. We should encourage our students to ask and explore these types of question, and give them situations where these questions arise naturally.

As always, a good place to start is with play related to the problem, such as with a scale diagram. Since we are dealing with an isosceles triangle, we can use a compass and ruler to construct our triangle. Over the years, the amount of geometry in the curriculum has slowly declined. This is a shame, as geometry allows various levels of abstract thinking, which is missing from other parts of the curriculum. The compass and straightedge constructions allow a natural way to use manipulatives to understand the relationships within the geometry. Dynamic geometry software, like *The Geometer's Sketchpad*® and *GeoGebra*, allow students to explore geometric properties in a dynamic setting. Understanding compass and straightedge constructions allows students to use dynamic geometry software more effectively in their mathematical explorations. I will get down from my soapbox and continue with the problem.

We can start by scaling our side lengths to 5 cm and 6 cm, rather than 55 cm and 66 cm. If we draw lines of length 5 cm and 6 cm, we can use the compass to "mark" a 5-cm distance, as in Figure 1.

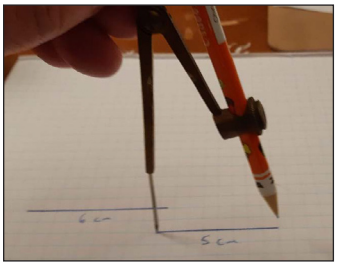


Figure 1: *Marking 5 cm with a compass*

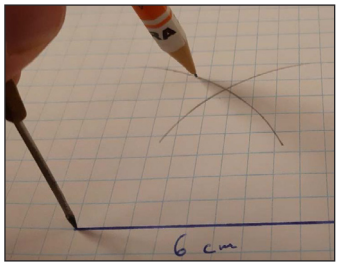


Figure 2: *Constructing the third point of our triangle*

Using a compass, two arcs can be marked off above the centre of the 6-cm line segment. Each arc has radius 5 cm and is centred on one of the end points of the 6-cm segment. The point where the two arcs cross is the point that is 5 cm from each end point of the 6-cm segment, that is, the third point of our triangle, as shown in Figure 2.

We could also use some geometric properties to find the third point another way. Since the third point is equidistant from the two end points, it is on the perpendicular bisector of the 6-cm segment. There is a method, similar to the method used in Figure 2, for finding the perpendicular bisector of any segment. Using any radius (greater than half the length of the base), if two circles are drawn with this radius having centres at the end points of our segment, then the two points of intersection of the circles are on the perpendicular bisector. The bisector can be created by connecting these two points. Alternatively, you can use a protractor and ruler to construct it. Once constructed, you can use a ruler to measure the point on the perpendicular bisector that is 5 cm from one of the end points, as in Figure 3.

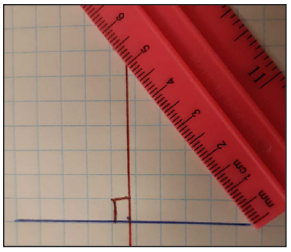


Figure 3: *Locating the third point on the perpendicular bisector*

Once we have our triangle drawn, we can draw several rectangles satisfying the condition of the problem, calculate their areas, and consider what happens to the area as you vary the position of the rectangle.

To make the exploration more manageable, dynamic geometry software can be used. In Figure 4, GeoGebra was used to set up the construction of our triangle. First, point *A* was drawn and then two circles, one with radius 6 and the other with radius 5, were constructed using the *Circle: Center and Radius* option. Next, point *B* was constructed on the larger circle, using the *Point on Object* option. A third circle was constructed with centre *B* and radius 5, and finally, point *C* was constructed at one of the points of intersection of the two smaller circles, using the *Intersect* option. This guarantees that triangle *ABC* has side lengths 6, 5, and 5. Points *A*, *B*, and *C* can then be joined by segments and relabelled *Q, R*, and *P*, respectively. The circles can be hidden, the triangle repositioned or rotated, as desired, and we are set to go.

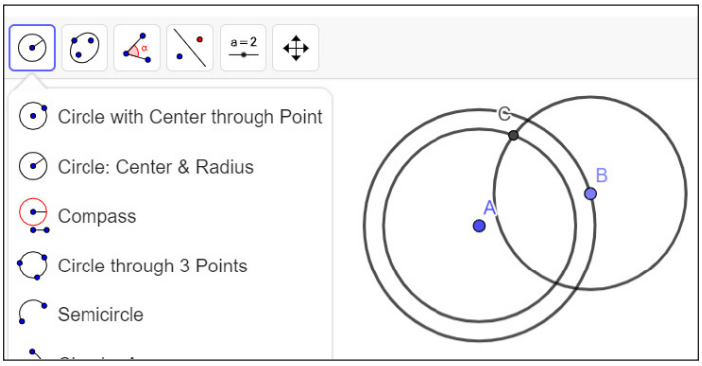


Figure 4: *Constructing the triangle in GeoGebra*

Next, we can construct point *J* on *PQ*, and using the Perpendicular Line option, we can construct rectangle *JKLM*. We can then measure the area of the rectangle and see how it varies as *J* is moved on *PQ*. Doing so, you will discover that the area seems to be optimized when *J* is the midpoint of *PQ*. Note that since our side lengths are scaled, any measurements of length in GeoGebra will have to be multiplied by 11, and any areas will need to be multiplied by $11^2 = 121$. The measurements displayed in Figure 5 were determined using these calculations.

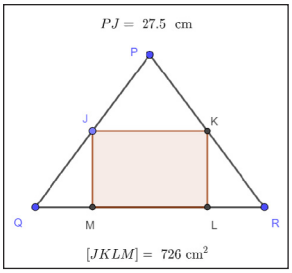


Figure 5: *The optimal rectangle*

We can use the dynamic geometry software to help us "see" why the area should be optimized at the midpoint. In our triangle, we have three triangular regions outside the rectangle. If we reflect each of these over the side that they share with the rectangle, we can see how the area of the rectangle compares to the three triangles. Looking at the diagram on the left in Figure 6, we see that if *J* is closer to *P*, then the images of the two bottom triangles overlap inside the rectangle, with the upper triangle filling in the missing space. Hence, the uncovered area is larger than the area covered by the rectangle. Similarly, the picture on the right in Figure 6 shows that if *J* is closer to *Q*, the image of the upper triangle extends beyond the rectangle, with the images of the side triangles filling the empty space, so again the uncovered area is larger than the area of the rectangle. Placing *J* at the midpoint, we would see that the images of the triangle just cover the rectangle, so the largest rectangle covers half of the triangle.

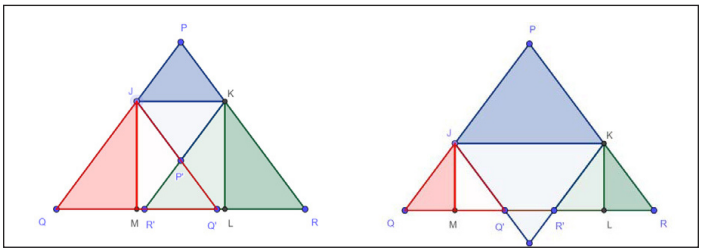


Figure 6: *The relationship between the rectangle and the three inner triangles*

It might be nice to prove that the largest rectangle covers half of the triangle. If we drop a perpendicular from *P* to *QR*, we can calculate the height of the triangle to be 44 cm, using the Pythagorean theorem. Labelling the height of the rectangle *h*, as in Figure 7, we will see that the height of the big triangle creates three pairs of similar triangles.

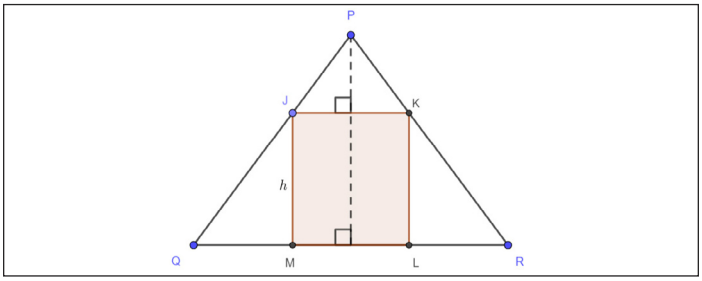


Figure 7: *Similar triangles*

Since the bottom triangles are similar to half the original triangle, we get $\frac{h}{44} = \frac{QM}{33}$, which yields $QM = \frac{3}{4}h$. From that,

we can calculate $ML = 66 - 2QM = 66 - \frac{3}{2}h$, so the area of the rectangle is

$$A = h\left(66 - \frac{3}{2}h\right) = \frac{3}{2}h(44 - h)$$

From this, we can tell that the area is a quadratic function of the height of the rectangle. Since our equation is in factored form, we can deduce that the zeros of the function are 0 and 44, so the optimal value, in this case a maximum, occurs when h = 22. Since this is half the height of the original triangle, we can deduce in this case that J would be the midpoint of PQ. A quick calculation will show that the rectangle does indeed have half the area of the triangle.

At this point, a valid question would be, "Does that largest inscribed rectangle always cover half the area of the triangle?" To answer this question, we will note that any triangle can be placed on a coordinate grid, such that two of the vertices lie on the x-axis, one on the positive side and one on the negative side, and the third vertex lies on the y-axis. Labelling the vertices with coordinates $A(-a, 0)$, $B(b, 0)$, $C(0, c)$, where $a, b, c > 0$, we get the area of the triangle is $A = \frac{1}{2}(a+b)c$. Let our rectangle be constructed with sides parallel to the axes, such that the side that is parallel to the x-axis has y-intercept h. Then, either using linear relations or similar triangles, we can determine the base of the rectangle to be

$$a + b - \frac{h}{c}(a+b) = (a+b)\left(1 - \frac{h}{c}\right) = \frac{(a+b)(c-h)}{c}$$

hence, the area of the rectangle is $A = \left(\frac{a+b}{c}\right)h(c-h)$.

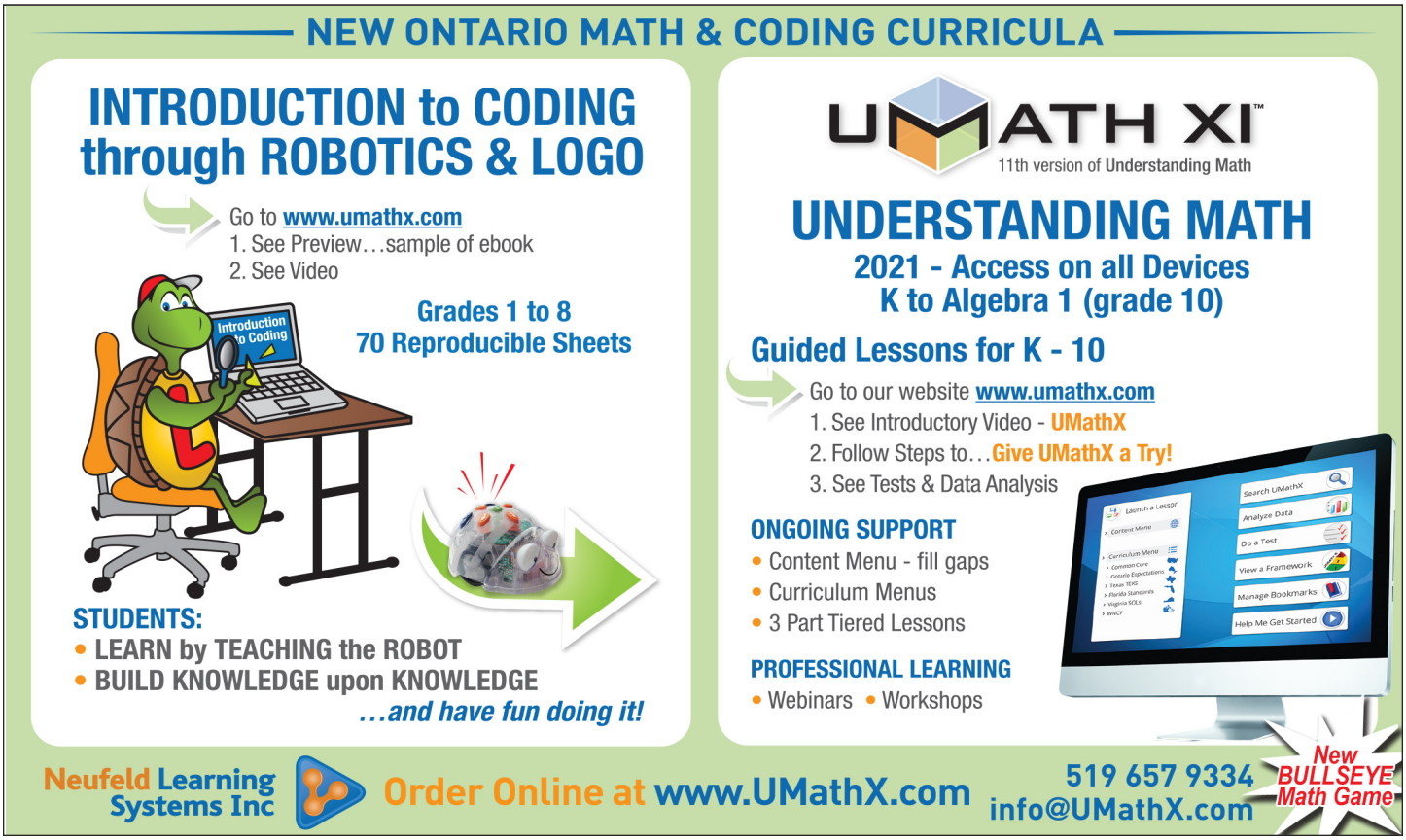
As in our previous case, we can see that the area is quadratic in h and the conclusion follows. I will leave the details to the reader for their amusement.

There are many different ways we can get at this general idea. It is important that when the question has been "answered," we reflect on the reasonableness of the answer and the validity of our argument. It is important to ask yourself "Now what?" because sometimes the answer to that question is more illuminating than the one you originally considered.

Now time for your homework.

In quadrilateral PQRS, *diagonals* PR *and* SQ *intersect at* O *inside* PQRS, SP = SQ = SR = 1, *and* ∠QSR = 2∠QSP. *Marc determines the measure of the twelve angles that are the interior angles of* △POS, △POQ, △ROS, *and* △ROQ. *The measure of each of these angles, in degrees, is a positive integer, and exactly six of these integers are prime numbers. How many different quadrilaterals have these properties and are not rotations or translations of each other?*

Until next time, happy problem solving! ▲



Technology Corner: Coding and Math



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Mary Bourassa teaches mathematics at West Carleton Secondary School in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She has presented workshops internationally, authored mathematics resources, is a past VP of OAME, and a Past President of COMA. An award-winning teacher, Mary continually strives to learn new and better ways of helping students learn and love mathematics.

Coding has emerged as an important skill for all of our students. It is part of the 2020 elementary curriculum, starting in Grade 1, and may be featured in the new destreamed Grade 9 math curriculum. There is good reason for this movement—coding helps students develop fundamental skills in unique ways. Coding provides a fun and creative environment for young children to develop problem-solving skills. It inspires curiosity and "What if?" questions that they can explore. It allows them to try new things and, when the results are not what they planned, to determine what to change. As students progress, they learn to use variables in real contexts, without the level of abstraction generally found in algebra. They observe patterns and learn how to generalize them. By the end of Grade 8, they will be using subprograms, loops, and evaluating complex expressions in their code.

There are many resources available for coding, one of which is the annual *Hour of Code* campaign, organized by Code.org, which endeavours to motivate young women and underrepresented groups to learn about computer science. Their website, www.hourofcode.com/ca, provides activities for pre-readers to Grade 9 and beyond. You can pick a topic, the technology used, and the length of the task. The Code.org website (www.code.org/) also provides many resources for teachers of all grades and levels of coding.

Much of the coding done by children uses Scratch, a free, block-based programming language from MIT, available at www.scratch.mit.edu/. There are detailed tutorials that can be used as a starting point. Users can create a story by adding a backdrop and images called "sprites." What sprites say and do can be controlled with scripts (code).

Let's work through a Code Club project called "Boat Race" from a student perspective, which can be found at https://projects.raspberrypi.org/en/projects/boat-race. This project is also available through the hourofcode.com site. Intended for Grades 2 and up, Boat Race, which uses Scratch, is a rich activity for both students of all ages and teachers. As a teacher, it is important to learn the structures involved in Scratch, and this project starts simply and assists in developing a basic understanding and many different skills.

Students start by trying out the final product, as seen in Figure 1, so that they understand the goal and can see the features of this boat race in action. Students must use their mouse or finger to determine the direction of the boat, and the goal is to reach the island (bottom right corner), without hitting any of the wooden barriers (shown in brown in the diagram).



Figure 1: *The Boat Race*

Students are able to see the skills they will learn through this project, as shown in Figure 2. Along the way, they will also learn numerous soft skills.

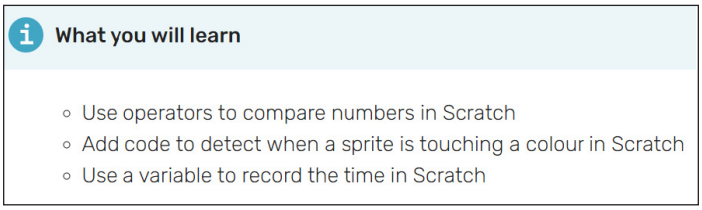


Figure 2: *Sample skills list*

Students begin by opening the starter project, which has the boat sprite and racecourse backdrop. The view, in Scratch, is shown in Figure 3. Notice that the location of the sprite is given in terms of x and y coordinates, with (0,0) being the centre of the screen. The coordinates are integers that are supported by the directions of movement and, while Scratch uses them starting in Grade 1, I do not see this as a problem, given the practical interpretation. I predict that the leap to a Cartesian grid will become seamless for these students.

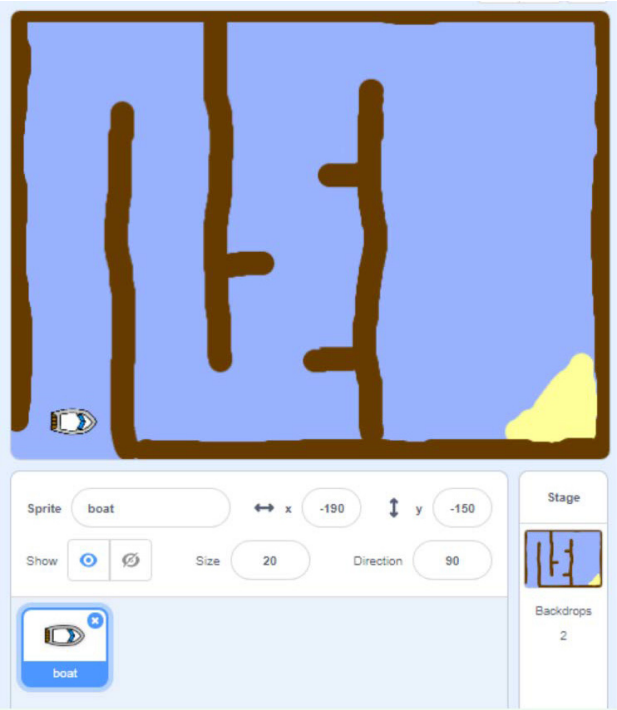


Figure 3: *How the Boat Race appears in Scratch*

The instructions are colour-coded, depending on type, as shown in Figure 4, making it easier for students to find what they need, while helping them learn to classify instructions. (Note that if you have students who are colourblind, this help may be diminished.)



Figure 4: *Colour-coded instructions*

Students begin coding with the instructions that will control the boat. Each instruction is a block that they pull from the menu and edit, as required. Once students have entered all the code to make the boat move, as shown in Figure 5, they can test their code.

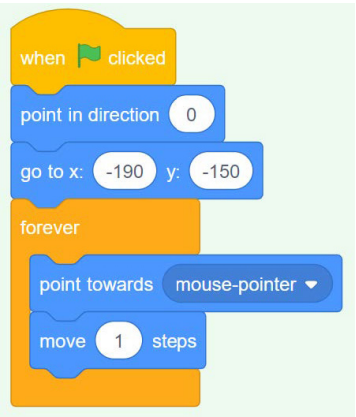


Figure 5: *Code to make the boat move*

If all the code works as intended, the boat will now move based on the location of the mouse. However, the boat never stops and, currently, will even go right through the wooden barriers! Another issue is that if the mouse pointer is on the boat, the boat seems to vibrate in that location. Fixing this will be the first enhancement. This time, the whole code block is not provided. Instead, a nudge in the right direction is given, with the possibility of further help via progressively more detailed hints, as shown in Figure 6, Figure 7, and Figure 8. These hints are given in words with colour-coding, then in code blocks, and finally the entire code block is shown.

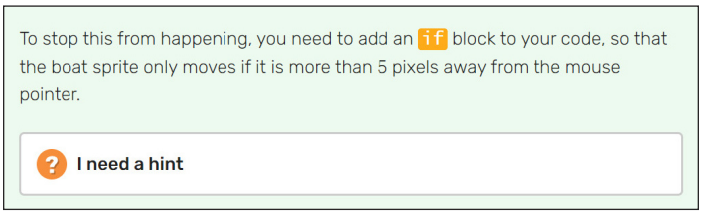


Figure 6: *A nudge in the right direction*

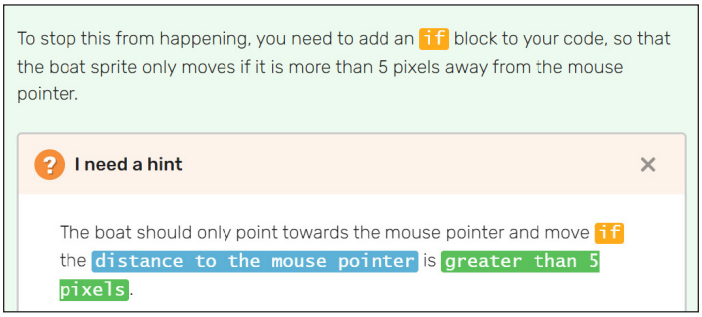


Figure 7: *Example of a hint*

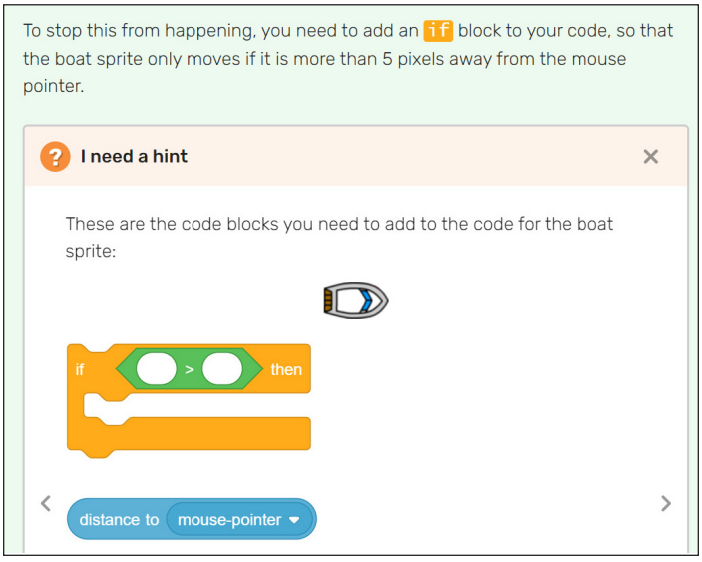


Figure 8: *Additional details, if needed*

The code block needed after the second step is shown in Figure 9. Students should once again test their code to ensure that it is running correctly and fix anything that didn't work the way they thought it would.

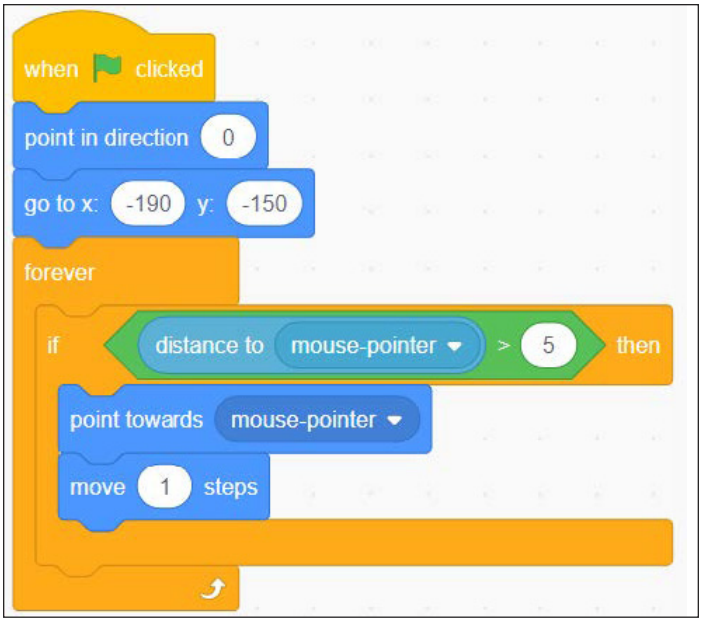


Figure 9: *The code block after the second step*

Next, students will fix the crashing issue, or rather, that the boat currently does not crash when it hits a wooden barrier. They create a new "costume" for their boat to make it look as if it has crashed, then add the code so that the boat dons its new costume when it hits any wooden barrier. The instructions are again given in words, and then hints are provided with progressively more specific, colour-coded statements or actual code blocks. Figure 10 shows the full code, and Figure 11 shows what happens when the boat hits the wooden barrier. I should note that finding the right shade of brown was somewhat challenging, and that students might need help with that line of code.

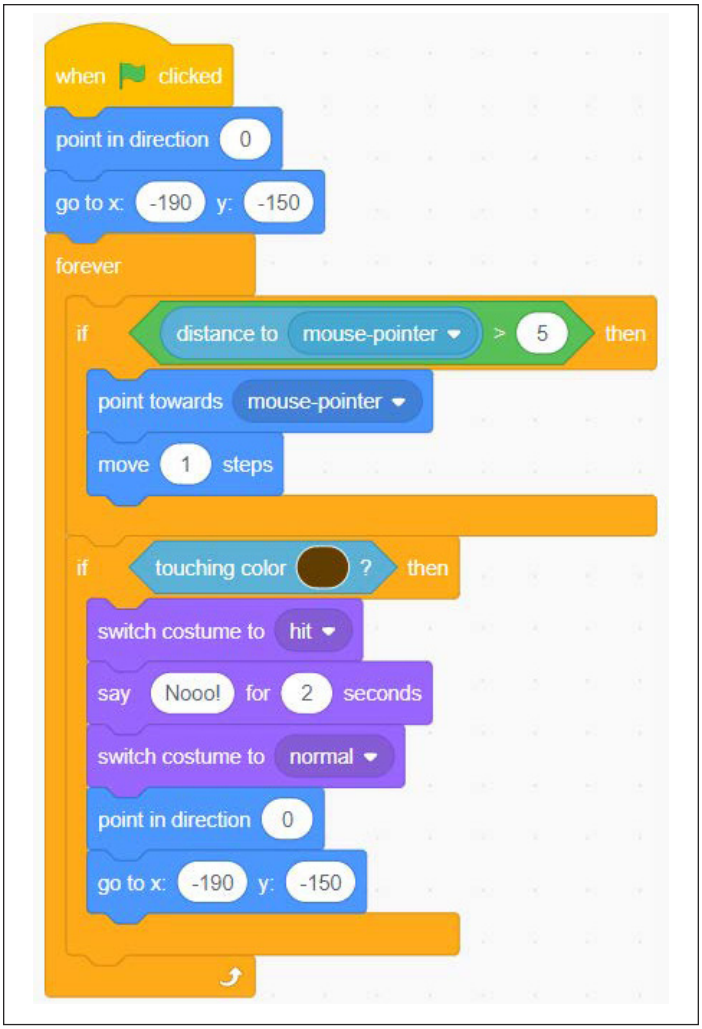


Figure 10: *The code*

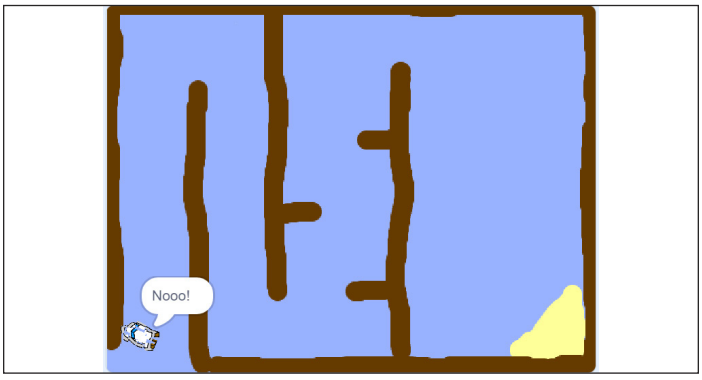


Figure 11: *The collision costume*

This process continues with students controlling how much help they need at each step. They can work at their own pace, and each student may reach a different stage of the challenge, yet all will feel successful. Figure 12 shows all the steps, including the many extensions.

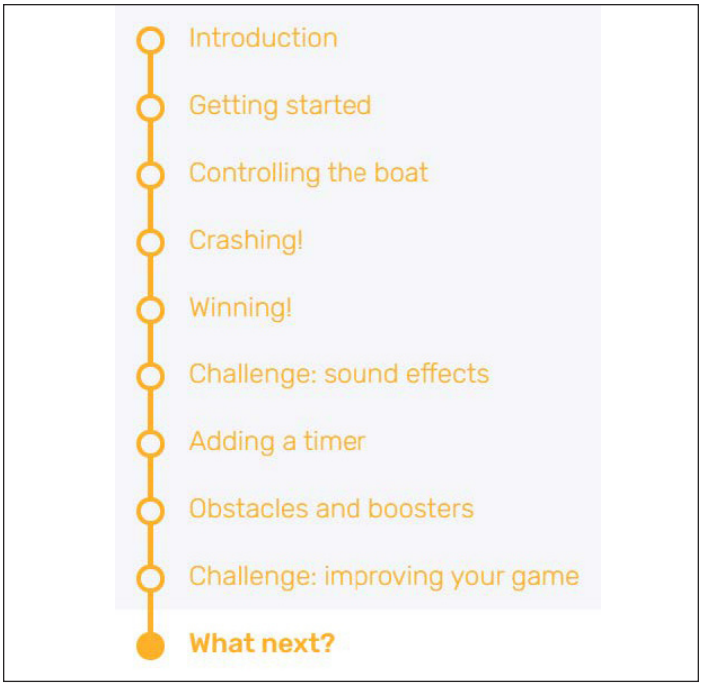


Figure 12: *The steps and extensions*

No matter the level of these ready-to-go projects, there is built-in differentiation, as each task has a low floor and high ceiling. When students feel confident enough to begin their own projects, they will have the coding skills and problem-solving strategies to reach their goals, with the support of their teachers.

This demonstration of Scratch shows that it is a powerful tool that allows even very young students to learn many important structures in computer science. It is not only for small children, though. Look at the Mandelbrot set, created using Scratch, by Mike Larson in Figure 13!

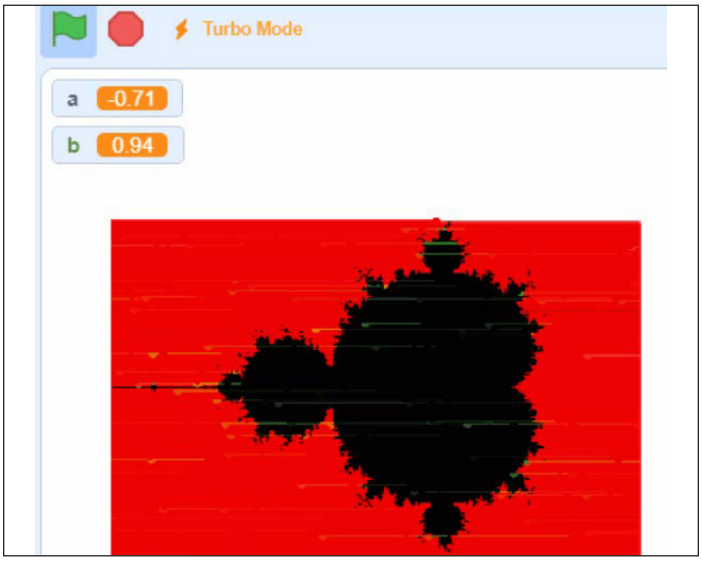


Figure 13: *The Mandelbrot set*

This is one of several projects shared on the CSandMath website (www.csandmath.org/). Their tag line is, "Sense-making math meets career-ready coding." The site was created by two mathematics educators: Ashley Goetz and Mike Larson. They offer free math lessons using Scratch, which are intended to help students learn about mathematics through coding.

This site also offers a tab called "#MTBoS Problem Structures" (#MTBoS refers to the Math-Twitter-Blog-o-Sphere, a hashtag used on Twitter by many mathematics educators), which showcases unique and interesting ways of using code to engage students in classroom discussions. Figure 14 shows three pieces of code and their respective outputs, and asks students what they notice and what they wonder.

Figure 15 shows "Which one doesn't belong?" with four blocks of code relating to probability simulations. Students must choose which of the four does not belong with the other three in the set. Every code block has a reason for not belonging, so this structure encourages all students to share their reasoning and see others' viewpoints.

These twists on using code in the classroom demonstrate innovative strategies to have students think more deeply about code that others have written, and to promote discussion in the mathematics classroom.

If you are looking for more Ontario resources, check out Renfrew County District School Board's site, https://code.rcdsb.on.ca/, dedicated to coding; the French equivalent from CEPEO, www.sites.google.com/cepeo.on.ca/codage-cepeo/accueil; or the Ministry of Education site, www.dcp.edu.gov.on.ca/en/curriculum/elementary-mathematics. Other resources are available from EduGAINS at www.edugains.ca/resourcesTELO/CE/CodingInterface/interface.html, and the Elementary Math Curriculum Resource Project at www.ontariomath.support/?pg=results&type=subject&lang=EN&subject=Coding.

There is no doubt that mathematics and coding go hand in hand. As *The Ontario Curriculum, Grades 1–8: Mathematics 2020* states, "Coding can support students in developing a deeper understanding of mathematical concepts." Through coding, we can help develop children who approach a challenge with a positive mindset, who understand that accomplishing a goal requires many iterations of failure or partial successes leading to success, and who will be able to translate these skills to mathematics classes and beyond.

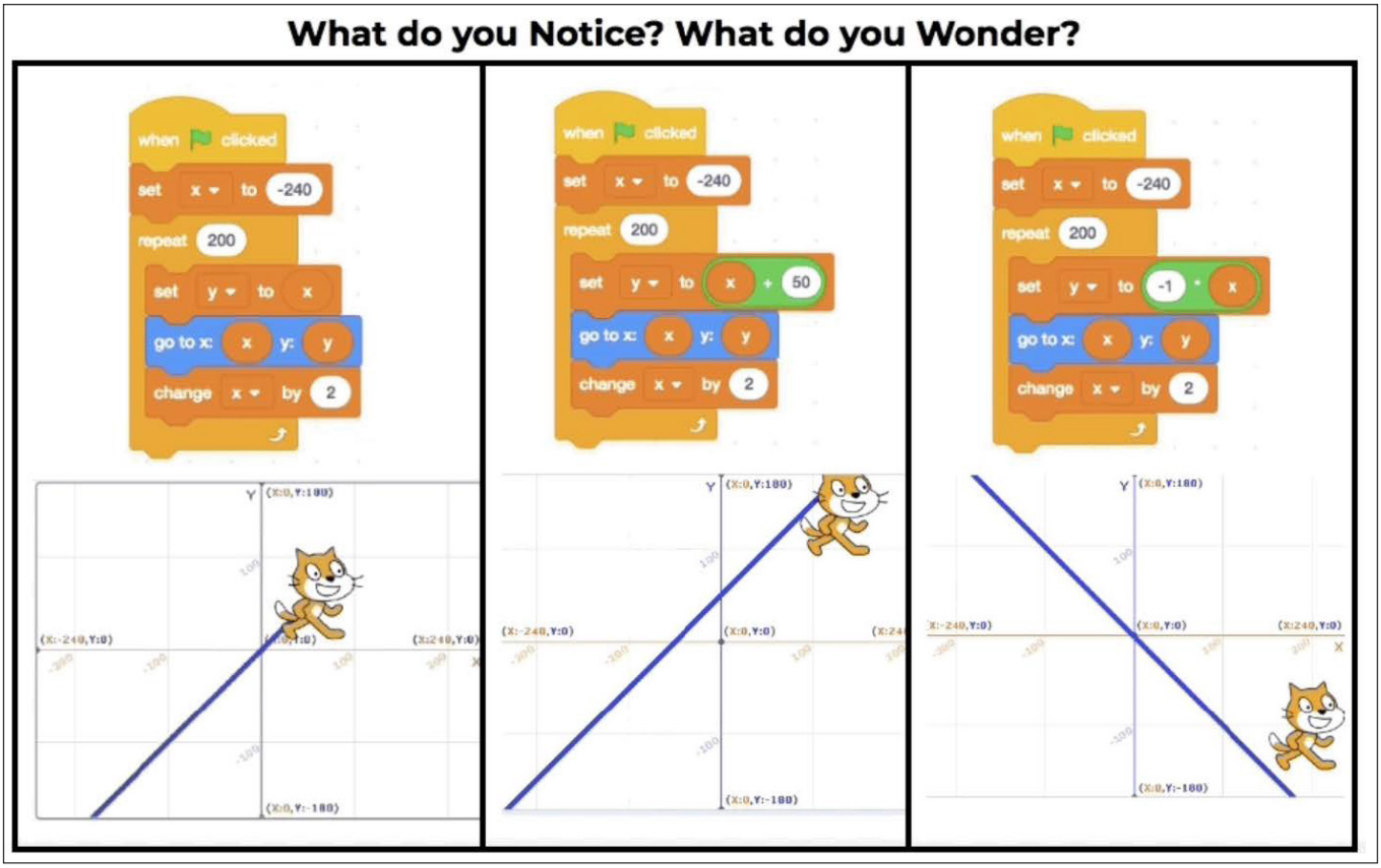


Figure 14: *Notice and wonder, using three pieces of code*



Figure 15: *Which code does not belong?*

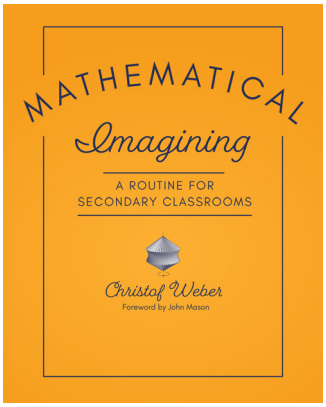
▲ Book Review: Imagining Math with Imagination



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Tim Sibbald has held a variety of positions within the OAME/AOEM. He is a former president, vice-president, and director. He is an associate professor in the Schulich School of Education, Nipissing University, with a focus on mathematics education. He has edited a book on Interdisciplinary Math and two about the experiences of academics (with a third underway).

Weber, C. (2020). *Mathematical imagining: A routine for secondary classrooms.* Stenhouse Publishers. (Distributed in Canada by Pembroke Publishing. See www.pembrokepublishers.com—the fastest way to receive it is to order through them.)



This book was translated from German in 2010, which speaks to it having stood the test of time. That it can be immediately used in the classroom becomes clear when one begins to explore the book. It has explicit instructions that a teacher can read to explore the possibility of using imagining in the classroom. The book as a whole provides more detail and sets teachers up to work independently of the book because it provides professional advice on the making of effective scripts.

An introductory example has one visualizing having your arms stretch out to the sides. You imagine you are standing on the edge of a triangle demarked by cloth on a large field. You are facing the interior of the triangle. You then imagine shuffling sideways, to your right, until you reach a corner, and rotating so that your left arm sweeps the interior angle of the triangle. You shuffle sideways up the new side, and at the next vertex, imagine rotating so your right arm sweeps that interior angle. Then you shuffle sideways down the third side until you reach the third vertex, where you rotate with your left arm sweeping out the angle. Then shuffle back to where you began. What happened? If you visualize that you ended up turned around and facing the exterior of the triangle, then you have just explored your imagination to find that the three angles add to a half turn, or 180 degrees.

While I may have some details a little different from those in the book, this portrayal of the idea demonstrates that, from memory, I could take the idea straight into the classroom. This is the striking part of the book—the ideas are relatively straightforward to implement, but the book really does add to the idea. Specifically, the book has worthwhile advice on how to set up and run imagining exercises. The fact that I specified the triangle was on a field was not an accident. The way to make a contextual picture that facilitates the imagining process is not difficult, but is very well described and leads to a feeling of confidence that one is being set up for success.

The book contains many examples that are purposefully organized. There are imagination tasks that visualize constructions, problem-solving tasks, reasoning exercises, and paradoxes.

Each is set up with instructions that can be used in the classroom. There is also an explanation for the teacher about the mathematics within each activity. The paradoxes are an amusing twist that set students up with some problems where what one imagines seems to defy alternative conceptions.

If the book has a shortcoming, it is that imagining has limits. After conducting the sum of angles in a triangle exercise, I wondered if it would obviously fail for a triangle drawn on a sphere. I was not convinced that it would clearly fail. However, to my surprise, later in the book, there is a task to do with triangles on spheres, and I had to revisit my own imagining. The limitation of the book is that it develops a single counter-example, where I had been trying to imagine a general case.

Much of the mathematics in the book is suited to the Ontario curriculum. Some topics go beyond the curriculum, for example, the spherical triangle activity just mentioned, and another to do with the Hilbert hotel (fitting customers into a hotel with an infinite number of rooms). These are beneficial for enrichment and show the scope of opportunities. The examples that do fit with the curriculum, and the description of how to make an effective imagining task, will give teachers many opportunities to use the ideas of the book throughout the school year.

Mathematically, I felt this book addresses an area of need within math teaching. Informal surveys I have done of teacher candidates suggest that only half can readily visualize a die and actively rotate the mental image. The issue is not that people cannot create mental images; it is that the discipline is rarely called on in mathematics classrooms. This suggests that there can be a benefit to giving opportunities to explore the use of mental imagining in mathematics. ▲

Linking Literacy and Math: Classroom Discourse



DAVID COSTELLO
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Dr. David Costello has recently published two books Making Math Stick: Classroom Strategies That Support the Long-Term Understanding of Math Concepts *and* Using What Works: Strategies for Developing a Literacy-Rich Environment in Math. *David is a principal who has previously taught in the primary division, been a numeracy interventionist, numeracy coach, numeracy leader, and curriculum consultant for Prince Edward Island. David has also instructed university courses focused on curriculum, differentiation, mathematics, and language arts. His research situates mathematics within a literacy perspective and examines leveraging literacy-based instruction in mathematics.*

The National Council of Teachers of Mathematics (NCTM, 2000) emphasized the significance of communication within mathematics. NCTM's *Principles and Standards for School Mathematics* identified communication as one of the five processes and stated that students should be able to:

- use communication to organize and consolidate mathematical thinking
- communicate their mathematical understanding clearly to others
- analyze and evaluate the mathematical understanding of others
- use mathematical language to accurately express mathematical ideas

Along with NCTM's focus on the importance of communication, many provincial math curricula situate communication as a process through which students learn mathematics. In fact, the Ontario Ministry of Education highlighted communication as one of the high-impact instruction practices in mathematics to support the new math curriculum in Kindergarten through Grade 8 (Ontario Ministry of Education [OME], 2020).

We know how important communication in the form of talking and sharing ideas is in language arts classrooms. Students are regularly asked to communicate their understandings, connections, questions, and opinions with their classmates and their teacher. We provide time for students and teachers to discuss a book during interactive read-alouds, provide time for students to read with others and discuss the text, and provide time for students to share their writing with classmates and teachers. Through talking and writing, students are able to frame their thoughts and consolidate their understanding.

Through communication, teachers can gather solid assessment data on students' learning, and can use this information to identify next steps for both instruction and learning. We want this communication, often referred to as classroom discourse, to be thriving in the learning environment. If we are committed to meeting students where they are in their learning, we have to provide them with opportunities to communicate where they are (Costello, 2019).

More Than Questioning

In many mathematics classrooms, teachers rely primarily on questioning as a means to assess student understanding—the teacher asks a question, students respond, and then the teacher evaluates this response (Fisher et al., 2008). It is important to consider classroom talk to be more than just a means of checking student comprehension. Instead, talk can be a tool to develop student thinking.

When students talk with others, they share their thinking, make connections with the thinking of others, and refine their stance throughout this process. The opportunity to communicate meaningfully in the classroom is particularly important for English Language Learners, who often have limited opportunities to engage in meaningful classroom discourse, and for students struggling with language for other reasons.

West and Cameron (n.d.) stressed the importance of dialogue as an avenue for learning. Such discourse, West and Cameron stated, develops language and the capacity to reason in a content area. The connection with mathematical reasoning, particularly in problem solving, is apparent. Smith and Stein (2011) identified mathematical discourse as a high-leverage instructional strategy. The authors stressed the importance of having students communicate in the classroom, and the important role that the teacher plays as facilitator in this exchange.

Accountable Talk

In mathematics, it is expected students are active participants, who make meaning as they engage with a problem. Being active participants requires students to communicate their thinking and work through the problem. It also means, however, that communication goes two ways:

speaking and listening. Students need to pay attention when others speak.

Encouraging discussion is important, but the discussion needs to be purposeful and structured so that student learning actually occurs and can move forward. Purposeful talk is also called accountable talk. Accountable talk consists of three aspects (Wolf et al., 2006):

- accountability to the learning community
- accountability to accurate knowledge
- accountability to rigorous thinking

Accountability to the learning community emphasizes that students contribute to the understanding and overall learning of all group members. Accountability to accurate knowledge means that students ensure all group members are communicating correct information. Accountability to rigorous thinking means students share thinking that is logical, reasonable, and able to be followed by those listening. Accountable talk provides the necessary framework for discussions to support learning.

Classroom Discourse Involves Everybody

Van de Walle et al. (2014) defined classroom discourse as the interactions among all people in the classroom. This interaction can be in the form of whole-class, small-group, pair, and teacher discussions. Within this emphasis on classroom discourse, Van de Walle et al. stated that the purpose of discourse is to engage all students with a discussion and representation of ideas to promote learning, not to simply state a response. Effective mathematics instruction and learning provides numerous opportunities for students to participate in meaningful math conversation (OME, 2020).

From the perspective of the teacher, classroom discourse should create an environment of inquiry and discovery that is based on the students' curiosity of new ideas. Students engaging with their natural curiosity about ideas and concepts results in a class filled with wonder. Classroom discussion can help generate this wonder by probing students' prior knowledge and provoking them to want to know more.

Classroom discourse can be developed by encouraging relevant questioning of concepts and then discussing topics in which students show the most curiosity (Colyer et al., 2018). To craft an environment full of wonder, teachers need to take on a role that challenges students to find creative ways of approaching problems. Teachers introduce students to concepts that interest them and offer potential for inquiry. It is important for teachers to build a culture in which ideas are cultivated and classmates discuss each other's discoveries (Literacy and Numeracy Secretariat, 2013).

Classroom Discourse Makes Student Thinking Visible

Classroom discourse makes student thinking visible. When students talk or write, teachers can see what students are thinking, what misconceptions they may have, and what next steps are required in the instructional and learning processes. We can express our thoughts through oral language, symbols, and pictures—all of which make learning and thinking visible. Through speaking with their classmates, students communicate their understanding and read or listen to the understanding of their classmates, all the while synthesizing new information with their prior knowledge. Teachers often have students write their thoughts with the purpose of using the writing as a springboard for sharing. In this way, student writing can strengthen the mathematical discourse in the classroom and generate a community of learners.

Manipulatives can also help students express their thoughts. Concrete representations are a very important part of discourse in math because they can make student thinking visible as students work with manipulatives to communicate their thoughts. This concrete representation can be used to respond to the questions of others, to share ideas, and to make connections with another's work. It might be relevant to point out here that the Math Processes of Representation and Connecting are mentioned here, and that these processes clearly connect to the Process of Communication,

Final Thoughts

Classroom discourse is a significant part of mathematics, both in terms of instruction and learning. It is embedded in everything we do and everything we want to do. It is about sharing our learning and using the learning of others to support the learning of all.

Recognizing the importance of classroom discourse, I wanted to take the opportunity in this column to pause and reflect on some of its criteria. Too often, I think we may go straight to sharing strategies, without reflecting on the importance of the topic. So, to that end, this is part one of a two-part column. In the next column, I will continue to build upon the idea of classroom discourse and will share strategies that will support instruction and learning reflecting this topic.

I look forward to continuing the discussion of linking literacy and math via my website (www.costellomath.com), email (david@costellomath.com), or Twitter (@dr_costello).

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▲ Mathematical Snapshots: Geometry lives at Georgian!



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*Ron Lancaster is an Associate Professor at the University of Toronto, where he teaches mathematics courses for pre-service middle and high school teachers. He has over 20 years of experience teaching Grades 7–12 mathematics. Ron's professional activities include consultations and conference presentations in North America, Asia, England, the Middle East, Africa, India, and Europe. Ron is an author for the NCTM (*The Mathematical Lens*) and member of the Advisory Board for the Museum of Mathematics in New York. He is the recipient of the 2015 Margaret Sinclair Memorial Award Recognizing Innovation and Excellence in Mathematics Education, awarded by the Fields Institute.*

In 2016, I gave an IGNITE talk, a featured talk, and a workshop at the OAME Annual Conference held at Georgian College in Barrie. In the absence of face-to-face conferences, I have been reflecting on the importance, for me anyway, of in-person conferences. I always enjoy the OAME Annual Conferences—they are well organized, the program has something for everyone, and I treasure the opportunity to see old friends and meet new people. The 2016 conference really stands out for me. The lineup of keynote speakers was off the scale and included Robert Lang, Steven Strogatz, and Sarah Greenwald. What I remember most about the conference was the outstanding support I received from folks such as Lyn Vause, Bruce McKay, Greg Clarke, as well as Victoria and Jim Baumgart, who helped me with my presentations.

My workshop involved a math walk on the beautiful campus of Georgian College, and some of the photos that I took around the college are featured in this column.

Before looking at the questions that follow, discuss what you notice in the photographs, and suggest reasons for why I took these photos. When you look at the photos, what do you notice? What are you wondering about?



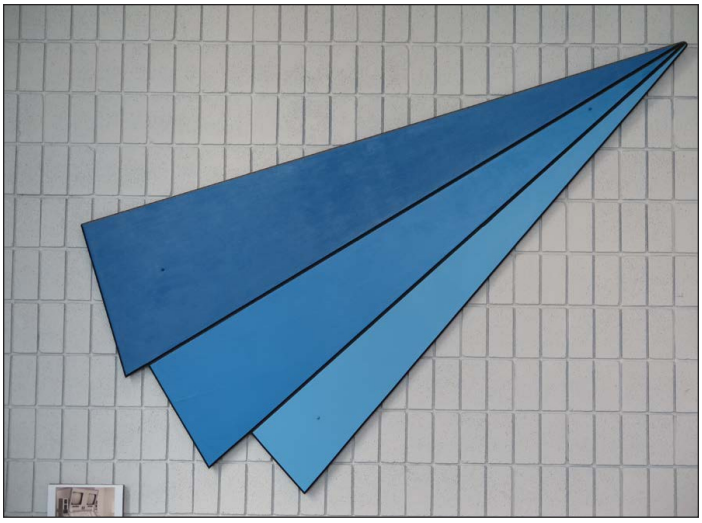
Photograph 1: *Chair on the campus of Georgian College in Barrie*



Photograph 2: *Tessellation on the campus of Georgian College in Barrie*



Photograph 3: *Sadlon Centre for Health and Wellness at Georgian College*



Photograph 4: *Artwork at the Canadian Aviation Institute, Georgian College*



Photograph 5: *Logo of the Georgian College Student's Association*

Question 1

(a) The small hole in the middle of the chair is surrounded by a ring of holes (let's call this Ring 1), which in turn is surrounded by a larger ring of holes (Ring 2). What is the purpose of these holes?

(b) What possible benefits or drawbacks would there be if all of these holes were larger? Is there a limit to how big the holes could be?

(c) Count the number of holes in these two rings and record your answers in Table 1.

Table1		
Ring 1		
Ring 2		

(d) Suppose that a third ring of holes, surrounding Ring 2, were added to the chair. Although there is very little data to go on, suggest several possibilities for the number of holes in the third ring (Ring 3).

Question 2

Photograph 2 shows a tessellation made up of octagons and squares that can be found on the ground near the front entrance to the college. Figure 1 shows a geometrical diagram, drawn to scale, of the tessellation.

(a) Without making any measurements in Figure 1, decide which of the following is closest to the ratio of the length of the side of the square to the length of the shortest side of the octagon. After you have made your estimate, measure the sides and check the closeness of your estimate.

(i) 1:1

(ii) 2:1

(iii) 3:1

(iv) 1:2

(v) 1:3

(vi) 1:4

(b) Without making any measurements in Figure 1, decide which of the following is closest to the ratio of the area of the octagon to the area of the square. After you have made your estimate, measure the sides, calculate the area of the octagon and square, and check the closeness of your estimate.

(i) 1:1

(ii) 2:1

(iii) 3:1

(iv) 1:2

(v) 1:3

(vi) 1:4

(c) Without making any measurements in Figure 1, decide which of the following is closest to the ratio of the area of the octagon to the area of the square. After you have made your estimate, measure the sides, calculate the area of the octagon and square, and check the closeness of your estimate.

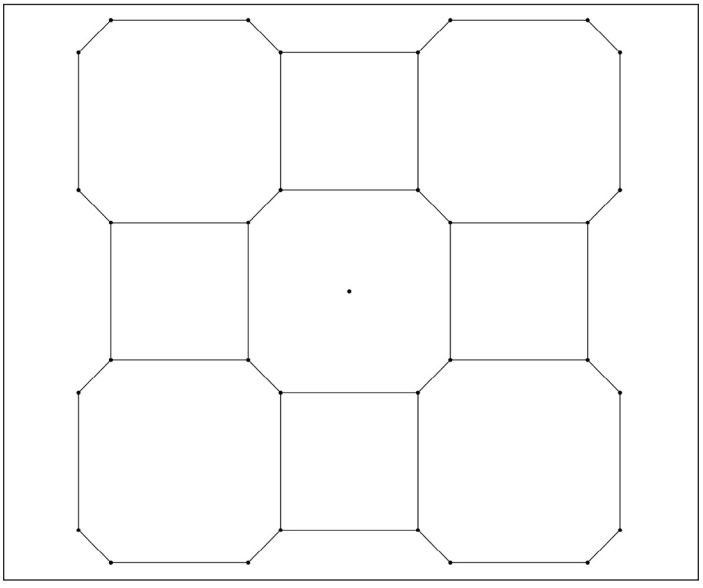


Figure 1: *Scale diagram of the tesselation*

Question 3

Figure 2, created with the Geometer's Sketchpad®, shows a geometrical diagram, drawn to scale, of the nine trapezoidal windows that appear on the side of the Sadlon Centre for Health and Wellness at Georgian College, shown in Photograph 3.

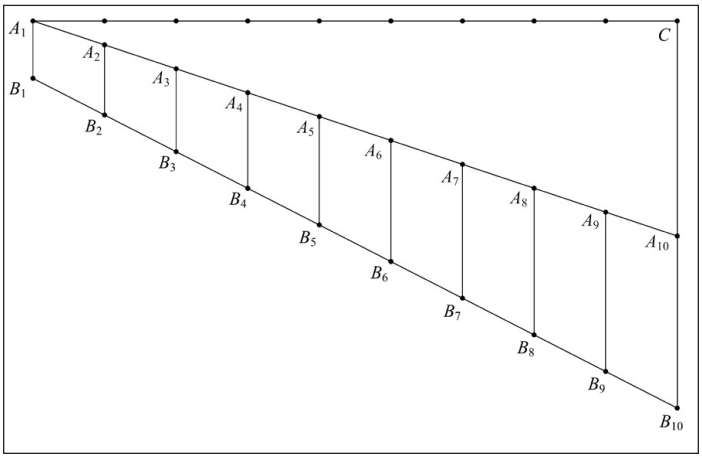


Figure 2: *Scale diagram of windows*

(a) The measurements provided in Table 2 were obtained using the Geometer's Sketchpad's measuring tool. Graph the length of the left side of the window versus the window number. Is this linear or non-linear data? How do you know?

Table 2

Window number (counting from left to right)	Label of the left side of window	Length of the left side of window (cm)
1	A_1B_1	2
2	A_2B_2	2.44
3	A_3B_3	2.89
4	A_4B_4	3.33
5	A_5B_5	3.78
6	A_6B_6	4.22
7	A_7B_7	4.67
8	A_8B_8	5.11
9	A_9B_9	5.56

(b) Determine a mathematical model for the graph in part (a). Suggest how this model might be used by the company that designed these windows.

(c) Use the measurements given below to calculate the area of the nine windows. Record your answers in Table 3. Graph the area of the window versus the window number. Is this linear or non-linear data? How do you know?

$A_{10}B_{10} = 6$ cm, $CA_{10} = 7.5$ cm, $A_1C = 22.5$ cm

Table 3		
Window number (counting from left to right)	**Label of the left side of window**	**Area of the left side of window (cm^2)**
1	A_1B_1	
2	A_2B_2	
3	A_3B_3	
4	A_4B_4	
5	A_5B_5	
6	A_6B_6	
7	A_7B_7	
8	A_8B_8	
9	A_9B_9	

(d) Determine a mathematical model for the graph in part (c). Suggest how this model might be used by the company that designed these windows.

(e) In part (c), three measurements were given. Is it necessary to have all three measurements in order to calculate the area of the windows? Could the areas be calculated with less than three measurements? Which ones would not be needed?

(f) Explain how the area of the rectangles in Table 2 can be used to determine the real-life areas of the nine windows on the side of the Sadlon Centre. What information would be needed to determine the cost of the windows, excluding labour?

Question 4

Photograph 4 shows a work of art on the wall near the entrance to the Canadian Aviation Institute at Georgian College. Figure 3 shows a geometrical diagram, drawn to scale, of the three triangles in the artwork.

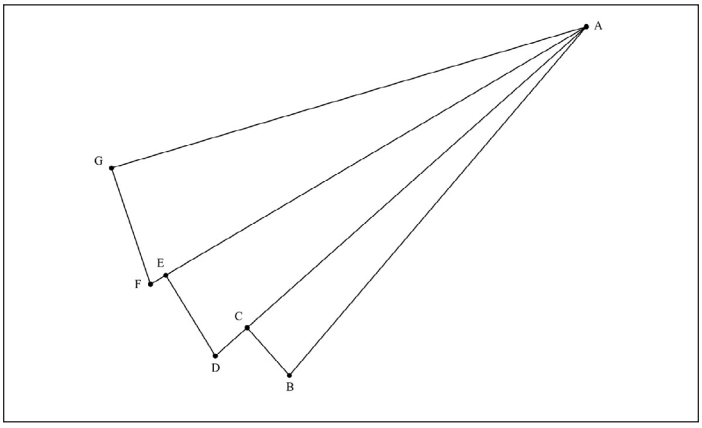


Figure 3: *Scale diagram of geometrical artwork*

(a) The following measurements of the angles in Figure 3 were obtained using the Geometer's Sketchpad's measuring tool. Note that both sets of angles form an arithmetic sequence.

$$\angle CAB = 8^\circ,\ \angle EAD = 11^\circ,\ \angle GAF = 14^\circ,$$

$$\angle CBA = 82^\circ,\ \angle EDA = 80^\circ,\ \angle GFA = 78^\circ$$

Figure 4 shows a modified version, and an arguably more pleasing one, of the artwork, in which the lengths of the sides AB, AD, and AF also form an arithmetic sequence.

AB = 10 units, AD = 12 units, AF = 14 units

Determine the area of each of these three triangles. Do the areas form an arithmetic sequence? How do you know?

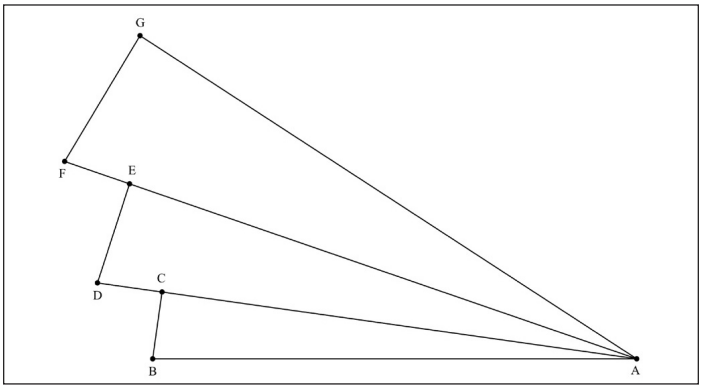


Figure 4: *An alternative geometric pattern*

(b) The nine triangles shown in Figure 5 were created by continuing the arithmetic sequences of angles and sides up to and including the ninth terms. Ron Lancaster was quite surprised that BAS was a straight line—he was not expecting that the triangles would loop around in this manner. He verified it was a straight line by find the sum of the arithmetic series given below, and sure enough, it was equal to 180°.

$8^\circ + 11^\circ + 14^\circ + 17^\circ + 20^\circ + 23^\circ + 26^\circ + 29^\circ + 32^\circ = 180^\circ$

Ron wondered if there were other arithmetic sequences that would lead to the same result using the first nine terms. Are there? How do you know? How many are there?

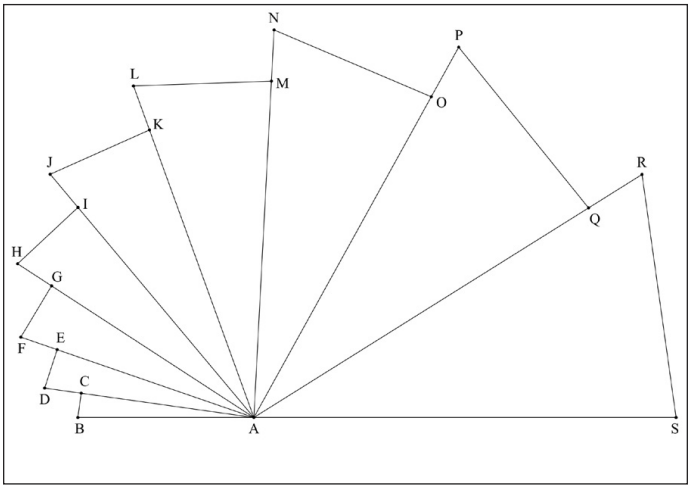


Figure 5: *Nine triangles making BAS straight*

Question 5

Figure 6 shows a geometrical diagram of the logo for the Georgian College Student's Association, shown in Photograph 5. In the figure, arcs of circles were used to make the rounded edges in the upper left and lower right corners of the logo.

(a) Suppose that parabolas were used to make the rounded edges instead of circles. Find the equations of the parabolas for both corners.

(b) Suppose that functions of the form $y = x^n$ were used to make the rounded edges. Describe how the logo would change as n tends to infinity.

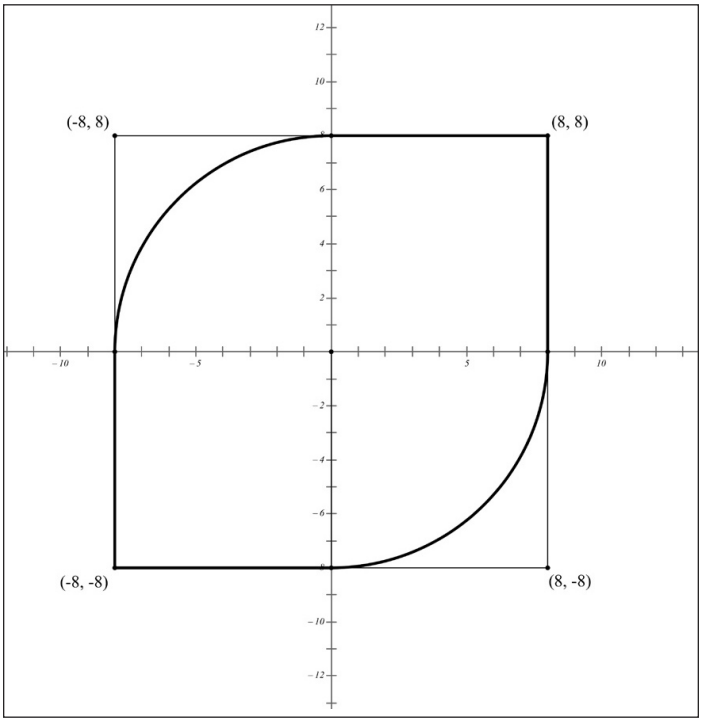


Figure 6: *Geometric diagram of logo*

I hope you take inspiration from these ideas and use your camera in the summer to capture your own images. Using them for your students to notice and wonder is a good way to focus your students and encourage them to see math in the world around themselves. ▲

What do you think?

If the *Gazette* were to have a make-over and change up its appearance and layout, what would you like kept and what would you change? Are there features we should leave alone because they work for you? Are there things that can be improved or simply changed to give a fresh look?

Share your thoughts with the editor!
gazette@ oame.on.ca

▲ In the Middle: Playing with Padlets



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Carly Ziniuk teaches Grade 9 Mathematics, Grade 12 Data Management, and Advanced Placement Statistics at the Bishop Strachan School in Toronto, Ontario, Canada. She is very active in adopting real-life data to engage her students in solving problems.

As we began online teaching and learning last year, I stumbled upon Padlet. At first glance, Padlet is an online bulletin board, often called a collaborative canvas. My students love working with Padlet, and the learning curve for everyone is shallow enough to move quickly to demonstrating math, rather than just learning a new technology tool. This column illustrates the many different ways I have found Padlet to be useful to encourage collaboration and make student learning visible.

While Padlet is appropriate to the middle grades, the need for real examples of teaching materials and student work has required I use Padlets from Data Management (MDM4U) that I am currently teaching. The approaches are easily adapted for middle grade learners, and my Grade 9 students readily employed Padlet in the past when given free rein on tools. Similar approaches could be adapted to collaborative whiteboard tools such as Jamboard, Stormboard, or Lucidspark. Padlet leans heavily into its customizable creative options, ease of use for younger students, and engaging presentation. My students and I, for example, continue to be charmed by the whimsical titles and descriptions that come up when beginning a new Padlet (Figure 1).

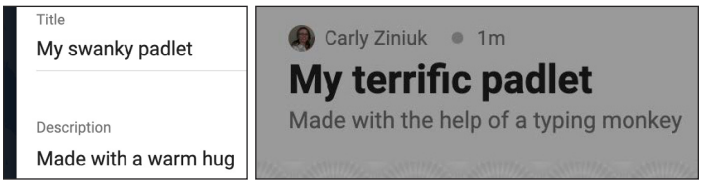


Figure 1: *Examples of the generated titles and descriptions*

Padlet.com runs fully within a browser, offering three Padlets with a free account (your students can also get free accounts). Additional free Padlets can be gained from customer referrals, or you can recycle the three free Padlets. The app on a phone or tablet resizes Padlets appropriately,

but can awkwardly shift the layout, if designed for horizontal viewing. Sharing with students is seamless with a QR code or link. The sharing and privacy options (Figure 2) are more varied and appropriate for educational settings than other collaborative whiteboards. If you use Padlet within an organizational account called Padlet Briefcase, the "Org wide" option is also available.

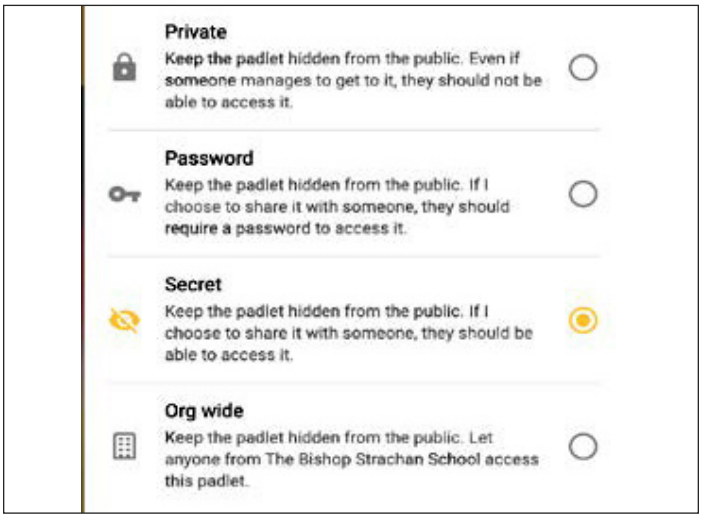


Figure 2: *Sharing settings*

Users can upload existing images, text, or links (including direct search of YouTube and Spotify), or create their own drawings, audio, video, or screen recordings right within their Padlet (Figure 3). Dragging an image, including a Desmos graph or table, imports it directly onto a post on the Padlet, which is automatically hyperlinked.

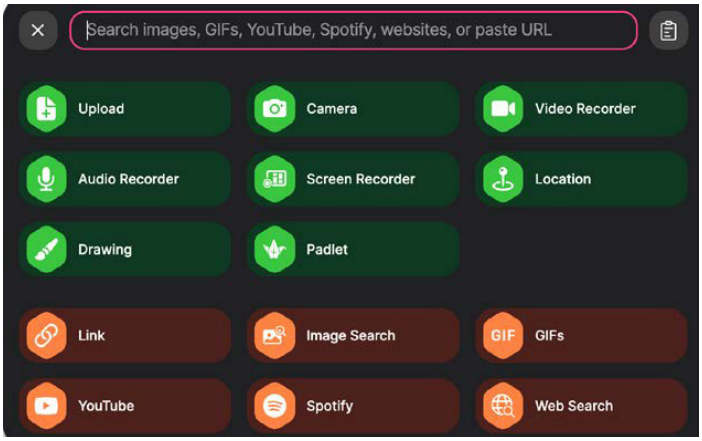


Figure 3: *Importing, searching for, and creating items*

Students can create various kinds of content within the single platform and do not have to open up another tab or a separate program. This feature makes it easy for students to post something on the Padlet with a supporting audio or video recording, for example, explaining their steps, describing a graph, or justifying a choice between two items.

Online whiteboards work well in classrooms for brainstorming a topic, collaboratively responding to a bank of questions, posting responses to a poll, or adding to a timeline or sequence. I have been using Padlet regularly to determine what students already know about a topic or to consolidate their thinking. Figure 4 shows the Project Zero (Harvard University) Thinking Routine, "Word–Phrase–Sentence," with students responding to the question, "How does social distancing affect the Normal Curve, using Mathematical Language?," as an exit ticket. Padlet's Shelf Format naturally organizes postings under headings, like many of the Thinking Routines incorporate. This example also shows how anyone, including the teacher, can respond to a student post.

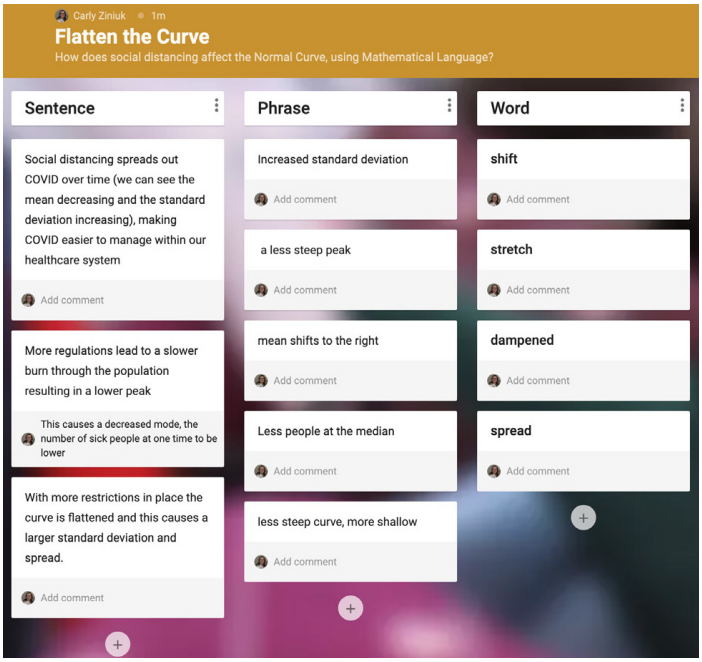
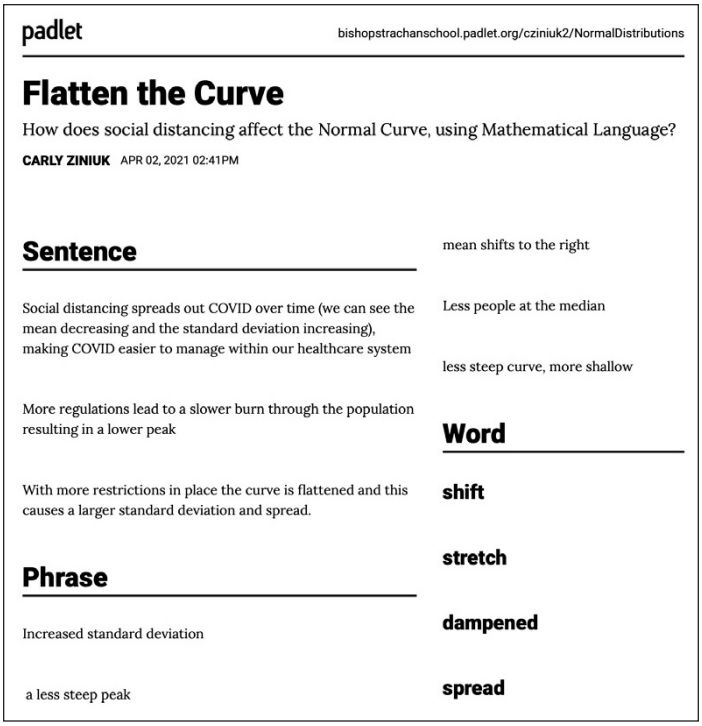


Figure 4: *Shelf format used for the Sentence–Phrase–Word Thinking Routine*

Before an evaluation, posting the Learning Goals at the top of a Padlet and then inviting students to post sample questions, success criteria, content videos, and possible solutions is a quick collaborative review activity. Without any prompting, my older students have been adapting their old review techniques (which previously were sticky notes or cue cards) to instead use Padlets for group studying or to prepare summary notes together. As you can see in Figure 5, any Padlet can be downloaded and printed as student notes, a feature my students use for their own notes. The export includes the web link at the top and headers for each column, providing scaffolding for students to organize and summarize and add their own thinking afterward.



padlet bishopstrachanschool.padlet.org/cziniuk2/NormalDistributions

Flatten the Curve

How does social distancing affect the Normal Curve, using Mathematical Language?

CARLY ZINIUK APR 02, 2021 02:41PM

Sentence

Social distancing spreads out COVID over time (we can see the mean decreasing and the standard deviation increasing), making COVID easier to manage within our healthcare system

More regulations lead to a slower burn through the population resulting in a lower peak

With more restrictions in place the curve is flattened and this causes a larger standard deviation and spread.

Phrase

Increased standard deviation

a less steep peak

mean shifts to the right

Less people at the median

less steep curve, more shallow

Word

shift

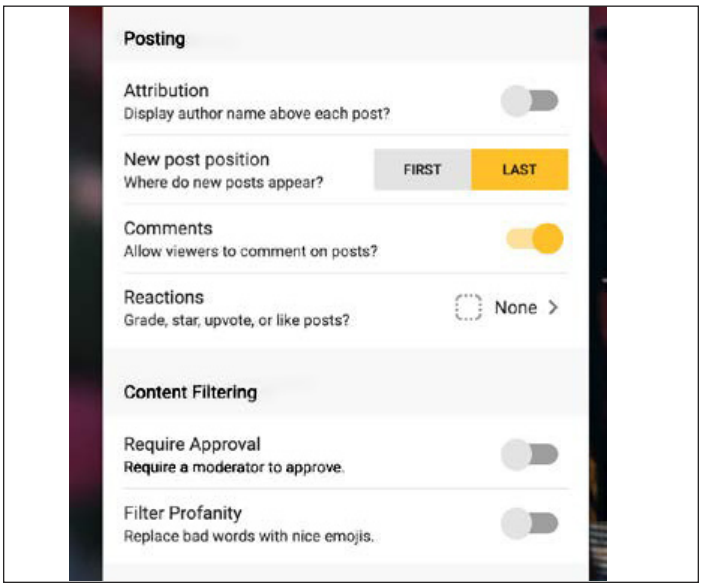
stretch

dampened

spread

Figure 5: *Exported version of the Padlet in Figure 4, to create a series of study notes*

Teachers who establish a collaborative Padlet for their class can refine the features for Posting and for Content (Figure 6). The Padlet's creator decides whether the post will require author attribution, or allow anonymous posting. Reactions and peer review can be included as well, with or without attribution. During online learning in real time, the Require Approval option can be used, which allows the teacher to approve every item.



Posting

Attribution
Display author name above each post?

New post position
Where do new posts appear? FIRST LAST

Comments
Allow viewers to comment on posts?

Reactions
Grade, star, upvote, or like posts? None >

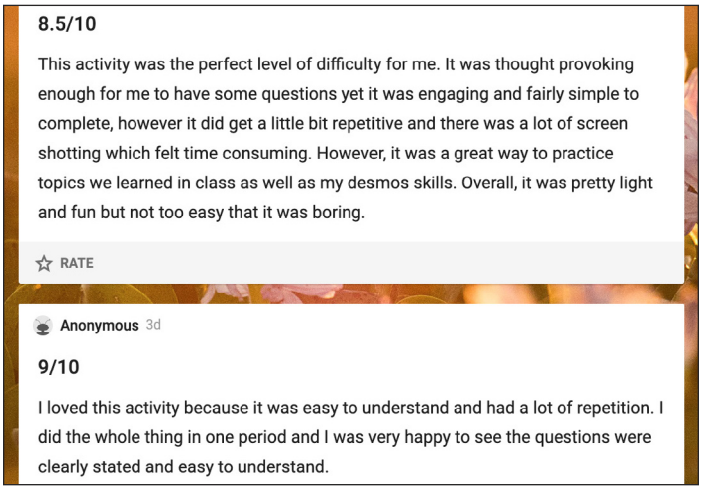
Content Filtering

Require Approval
Require a moderator to approve.

Filter Profanity
Replace bad words with nice emojis.

Figure 6: *Settings for Posting and Content Moderation*

With my Grade 9 students, I have used Choice Boards to have them complete activities at their own pace and sequencing (as I discussed in my last column). Padlets allowed students to provide reviews anonymously, and rate the activities that they had chosen, offering suggestions to their peers (Figure 7).



8.5/10

This activity was the perfect level of difficulty for me. It was thought provoking enough for me to have some questions yet it was engaging and fairly simple to complete, however it did get a little bit repetitive and there was a lot of screen shotting which felt time consuming. However, it was a great way to practice topics we learned in class as well as my desmos skills. Overall, it was pretty light and fun but not too easy that it was boring.

☆ RATE

Anonymous 3d

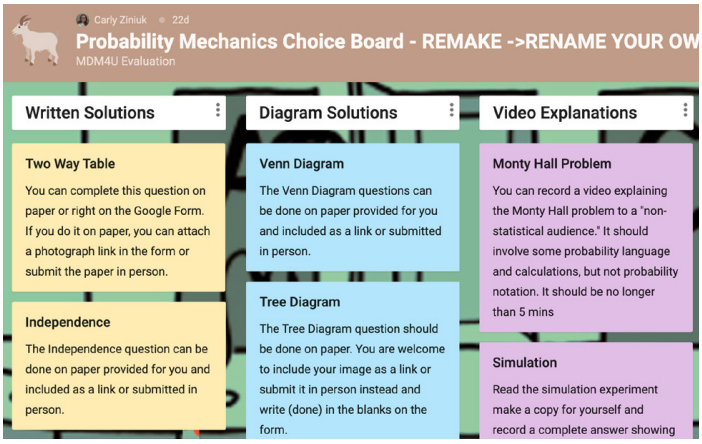
9/10

I loved this activity because it was easy to understand and had a lot of repetition. I did the whole thing in one period and I was very happy to see the questions were clearly stated and easy to understand.

Figure 7: *Anonymously posted reviews on class Padlet, designed using the Stream format*

For the Choice Boards, I create a template that the students copy using the "Remake" option.

When the students "Remake" the Padlet, they can choose to mimic the design and/or posts. Once they have (re)made their individual Padlet, they customize it with their own personal components, or adapt the design elements. Figure 8 shows a Choice Board using the Shelf format, where students select one item from each shelf to demonstrate and delete the rest. They then post their own completed work underneath the same shelf.



Carly Ziniuk · 22d

Probability Mechanics Choice Board - REMAKE ->RENAME YOUR OW

MDM4U Evaluation

Written Solutions

Two Way Table
You can complete this question on paper or right on the Google Form. If you do it on paper, you can attach a photograph link in the form or submit the paper in person.

Independence
The Independence question can be done on paper provided for you and included as a link or submitted in person.

Diagram Solutions

Venn Diagram
The Venn Diagram questions can be done on paper provided for you and included as a link or submitted in person.

Tree Diagram
The Tree Diagram question should be done on paper. You are welcome to include your image as a link or submit it in person instead and write (done) in the blanks on the form.

Video Explanations

Monty Hall Problem
You can record a video explaining the Monty Hall problem to a "non-statistical audience." It should involve some probability language and calculations, but not probability notation. It should be no longer than 5 mins

Simulation
Read the simulation experiment make a copy for yourself and record a complete answer showing

Figure 8: *Choice Board for student choices and submissions of work*

The Canvas feature of Padlet allows you to visualize relationships and make connections between different features of the learning, including processes, mind maps, and flow charts. I created an "exploded" line template for my students to "remake." Each student received a unique linear

equation and replaced the components of the Padlet template with the details of their own line. Figures 9 to 12 show the original template and three Padlets presenting each student's own thinking within the same base design. Their Padlets allowed online and in-person students to peer-review each other's work and to compare the features of different lines.

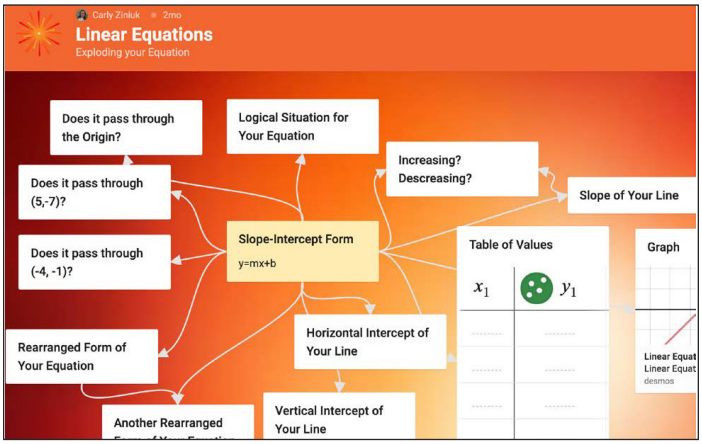


Figure 9: *Original template for Exploding Linear Equations*

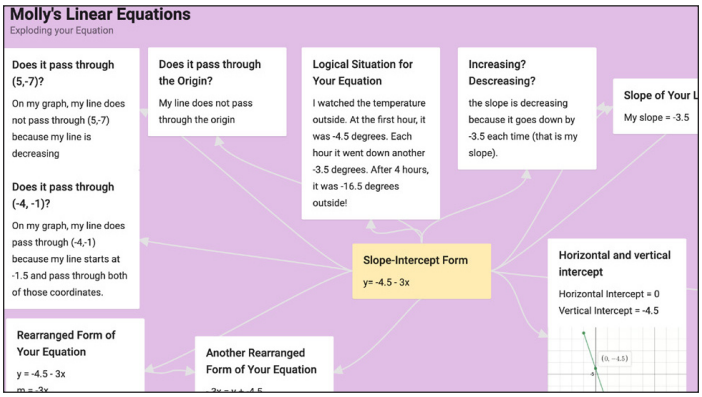


Figure 10: *Molly's version of her Linear Equation Padlet*

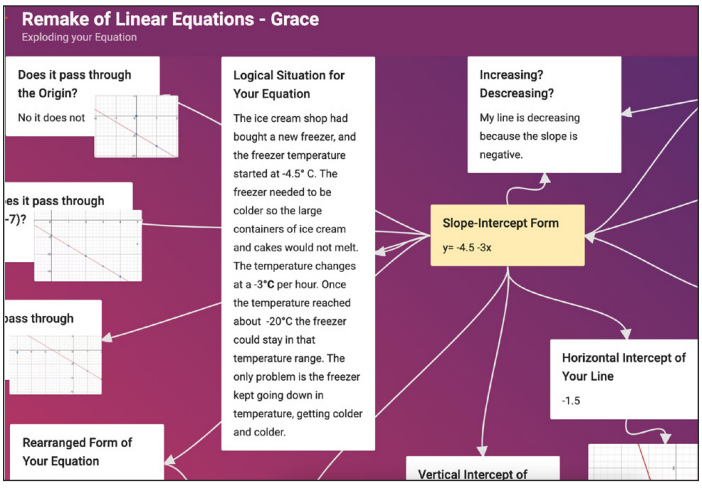


Figure 11: *Grace's version of her Linear Equation Padlet*

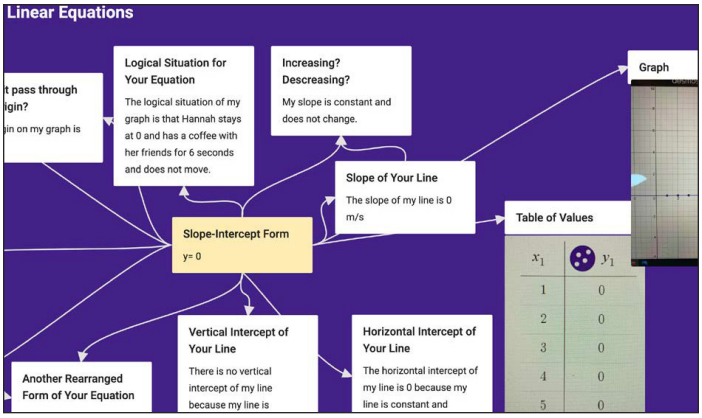


Figure 12: *Hannah's version of her Linear Equation Padlet*

For flipped lessons, I have previously used Playlists in a bulleted text document with a series of videos or web pages to review before class. If I added more options or gave students a choice for how to learn the content ahead of time, the Playlist looked overly long and seemed daunting. Students would often do the first or second option, regardless of what modality they might have actually preferred. Since then, I have found Padlet's Grid distributed presentation and the ability to post varied media directly on the page more inviting. It also allowed me to show the instructions across the top and include approximate times required, making the flipped classroom learning more productive.

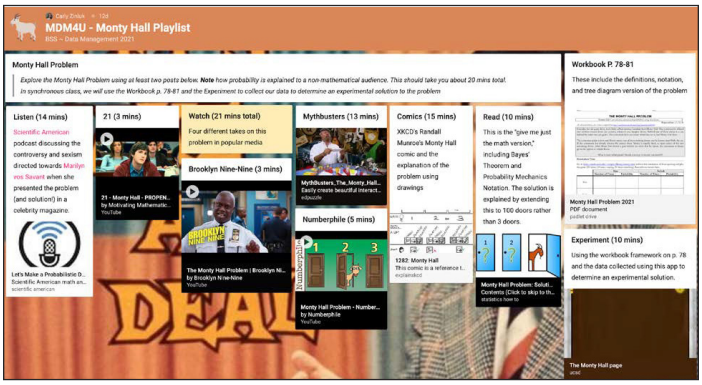


Figure 13: *Padlet as Playlist, using Grid format*

Figure 13 shows a Monty Hall Problem playlist, where students learn about the Monty Hall Problem in varied modalities, including a podcast, pop culture videos, comics, and simulations. I also include the corresponding classroom materials (labelled "Workbook"), which will lead to the in-class learning. Each of the videos, links, and audio can be used right from the Padlet, and do not need to be opened in another tab. Although the overall image here is small, you can see the overall layout of a full-screen arrangement on Padlet, and as such, looks very different on mobile devices. When you use the Grid content arrangement, I recommend checking how it adjusts to various window sizes. Embedded

Padlets, such as on Google Sites, need attention when the layout design is important on different screen arrangements.

I have always posted a Word Wall in my classroom, and had students develop an ongoing Glossary with definitions and diagrams in their own notes. Padlet has allowed students to recreate that Word Wall online by collaborating in a Glossary, which now can just as easily include videos and examples. Students can order the posts by dragging and dropping them into the Grid, to create an evolving understanding of a topic. Figure 14 shows the student-curated Glossary including videos, sample questions, and links to other Glossary Words.

Figure 15 shows a Padlet that replaces a paper formula sheet. It can now include videos demonstrating how to use them, and links to web pages with practice questions.

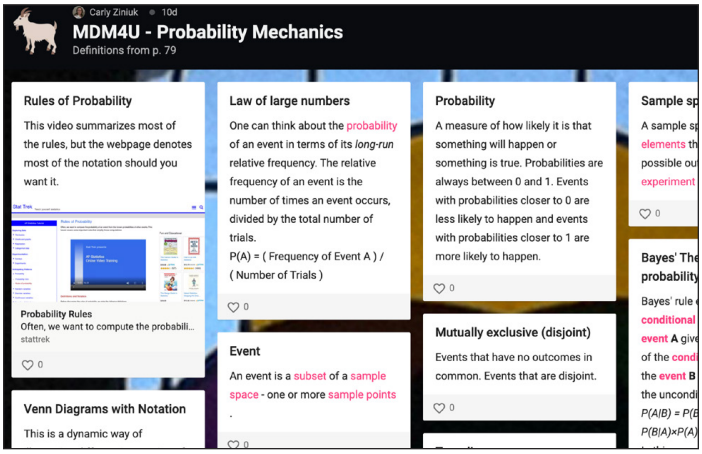


Figure 14: *Example of Glossary (or Word Wall) Padlet using the Grid format*

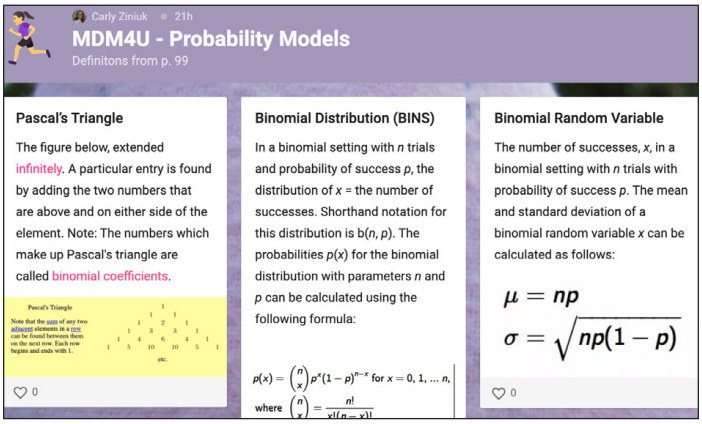


Figure 15: *Padlet Glossary also used as a collaboratively developed formula sheet*

The entire Course Glossary is a Padlet of Padlets, all linked with the terminology and definitions connecting to other Padlets (Figure 16). I used the Timeline feature previously, but students liked the stacked version, in Grid format, as they did not have to scroll to see all of the chapters. As more elements get added to the interior Padlets, the thumbnail images change and help to remind the student of key visuals or anchoring concepts.

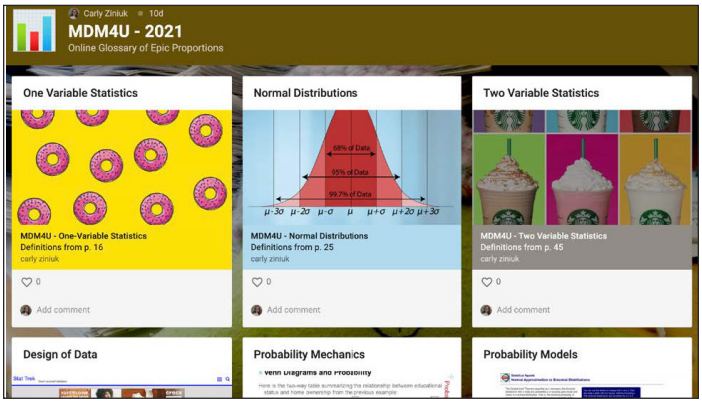


Figure 16: *Padlet of Padlets, Course Glossary connecting all chapters*

I have also been experimenting with the Padlet Mini Chrome extension to organize web pages, bookmarks, short recordings, and existing Padlets. This has facilitated sharing them with other teachers. The collaboration using Padlet has not just been with my students!

This year, my typical approaches for observing student learning and having "casual" conversations required much more mindful planning and choices. These choices couldn't be limited to what I had used previously, but had to be efficient. I needed tools that quickly showed student progress, and that students enjoyed and could familiarize themselves with quickly. I have incorporated Padlet in these different ways to allow online and offline students to collaborate, synchronously and asynchronously, and to demonstrate to everyone the stages of the process and the final product. As we continue to adapt our approaches for collaboration, I encourage you to experiment with some of these different uses of Padlet for making students' learning visible. ▲

Continued from page 39

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▲ MB4T (Mathematics by and for Teachers): Finding Balance in Algebra



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Jennifer Holm is an Assistant Professor at Wilfrid Laurier University and works with primary/junior and junior/intermediate pre-service teachers, as well as in the field supporting current mathematics teachers. She is interested in developing mathematics knowledge for teaching with both pre-service and in-service teachers. She focuses on the beliefs and opinions that pre-service teachers hold about mathematics and teaching and the connection they have to past experiences. She uses this research to support future teachers in developing beliefs and knowledge that will encourage and support effective teaching practices in mathematics.

The Algebra strand of the Ontario curriculum has undergone many changes from the 2005 curriculum to the current one. The focus of this column is the solving of equations. According to the Ontario Ministry of Education (2020) *Key Changes* document, the 2005 curriculum introduced variables in equations in Grade 5 and had no expectation of solving inequalities, whereas the 2020 curriculum has students working with variables in Grades 1 to 8 and solving inequalities in Grades 4 to 8. The foundational understandings discussed in this column with equations can create a strong foundation for the skills needed to solve inequalities. With the focus on conceptual understandings, equations and inequalities that do not include working with negative numbers are an important starting point.

Within the Algebra strand, there is some vocabulary that, during my career, I have noticed being used loosely and that might lead to difficulties with students. A first example is the difference between an *expression* and an *equation*. In my experience, many learners use these words interchangeably, which becomes problematic when interpreting the curriculum or looking for a strong understanding of algebra. An expression is "numbers, symbols and operators (such as + and ×) grouped together that show the value of something" (*Math Is Fun*, 2018b). An equation, on the other hand, "says that two things are equal" (*Math Is Fun*, 2018a). Two expressions could be used to make an equation, such as $8 + x = 5x$, where both "$8 + x$" and "$5x$" are expressions, and together, with the equality, they make an equation. Another vocabulary example to consider is that of the equal sign. Based on the ways that we tend to use the equal sign in common language, students often misinterpret it to mean that the answer is on the other side. This leads to difficulty when solving $3 + 7 = \square + 4$, and students interpret that the box has to be 10, since that is the answer to 3 + 7. Instead, we want students to focus on the fact that the equal sign needs to show two equivalent expressions, or something that is balanced. Consider the image in Figure 1 from Carpenter et al. (2003), and how the equal sign is used in each of these cases.

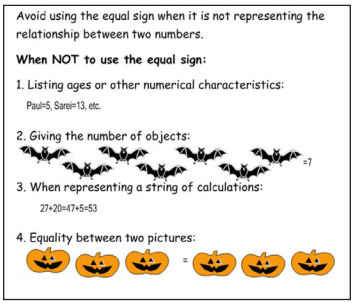


Figure 1: *From Carpenter, Franke, and Levi, 2003*

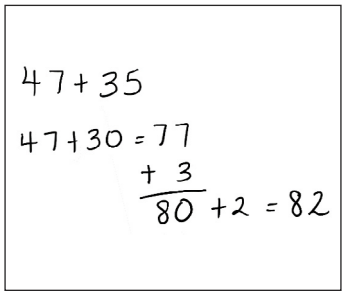


Figure 2: *Method for solving 47 + 35, watching the equal signs*

In the third example, the equal sign is used to mean "the answer to," since 27 + 20 does equal 47, so the idea would lend itself to the misconception discussed. If we want students to focus on how the equal sign means equality and that the amounts are balanced, then the first part of the expression must be equal to the second part. This is something to think about in number talks and how you show the steps a student is following: how are you using the equal sign? Consider instead Figure 2 as a way to show a string of calculations, where the equal signs each show things that are actually equivalent. In the options for 1, 2, and 4 in Figure 1, the equal sign is used to mean "the same as," which becomes problematic when we move into algebra. Carpenter et al. (2003) suggest avoiding each of these ways to use the equal sign in classrooms so that students understand that the equal sign is intended to show "relationships" between numbers and expressions.

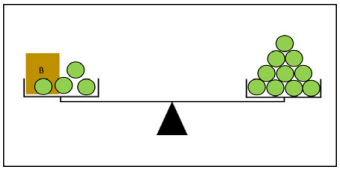


Figure 3: *Pan balance image for* B + 4 = 10

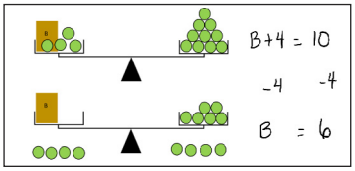


Figure 4: *Example of how to show the physical model with a procedure*

Now that we have discussed some of the small ways we can work with vocabulary to support an understanding of algebra, the rest of the column focuses on a conceptual way to build an understanding of the "undo" method for solving for the value of an unknown variable. Small (2017) suggests not using the letter "*x*" with students as they are first working with algebra, since it looks like a multiplication sign and can be confusing. I have found that when printing on the board, the letter "*t*" also becomes problematic, since it resembles an addition sign. For the remainder of this column, we are going to use the letter "*B*" to represent the bags that we are putting on the balances.

When students are first learning to solve for the unknown, they often use a guess-and-check method by putting any number into the blank and then adjusting until the two amounts are equal. Although this is good way to start, it becomes problematic when moving on to equations with an unknown value that is large, a decimal, or a fractional amount. Since the goal of the 2020 math curriculum is to help students understand the concept and then to work efficiently, another way to solve for the unknown is needed. Rules are often taught to students—such as "you need to undo the operation" or "what you do to one side, you need to do to the other"—which can be effective tools to help students remember, once they understand the concept. This column, as well as many others in the *Gazette*, have focused on supporting student understanding so that they are not blindly following rules they do not understand. One way to help develop these concepts is to use a balance. I suggest creating initial activities that actually use the balance so that students can manipulate them on their own and see what happens. Consider the image in Figure 3, where the bag, or "*B*" (illustrated by the rectangle in the image), has an unknown number of balls. How might a student determine the number of balls in the bag, without opening it?

Initially students may remove everything from the balance, then put the bag on one side, and add balls until it balances on the other pan. This resembles the guess-and-check method, and shows the student understands that they need to get the bag alone on one side. They have "isolated the variable" and are now solving, which works here, but does not work when they only have equations. Careful prompting can help a student move past this idea and into something that pushes toward the idea of "isolating the variable" by undoing the operations. Students could be asked to consider what happens when they just remove the "4" balls on the same side as the bag. What happens to the balance? Placing the balls in front of the balance helps students to recall how many were there. A subsequent prompt could be to ask how one could make this balanced again. Hopefully, students will realize that they can remove the same number of balls on the right side, and again the sides are balanced. If not, you ask which side is heavier and how it could be lightened. Students could suggest removing balls from the heavier side, one at a time. Again, if the balls are placed on the right side in front of the balance, students can observe that they have subtracted the same number of balls from both sides. Figure 4 shows a way to illustrate what happened on the board to begin developing a procedural understanding of the process.

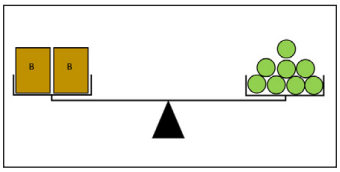


Figure 5: *Pan balance image for 2*B *= 8*

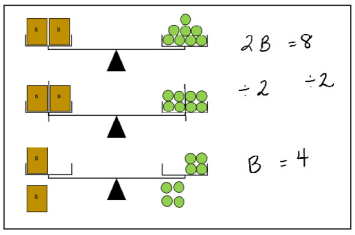


Figure 6: Example of how to use the model to solve for B *in 2*B *= 8*

The balance clarifies the concept of subtraction "undoing" addition, and can lead to discussion about what might happen if there were subtraction in the question, and how students can use the pattern from these questions to figure it out. Unfortunately, the physical balance is limiting on being able to illustrate this concretely. Illuminations provides an online pan balance (www.nctm.org/Classroom-Resources/Illuminations/Interactives/Pan-Balance----Numbers/) applet that allows for experimentation, while showing how addition can be used to "undo" subtraction. After students have practice with similar questions to Figure 3, where the bag should also be placed on the right at times, a problem like the one shown in Figure 5 could be attempted. How might a student solve this concretely?

The model is not as intuitive to use to solve the equation in Figure 5 as the one in Figure 3, since students cannot just remove the same something on both sides to keep it balanced. In this example, the idea is to determine how to separate the balls into the same number of "sets" or equal groups as the bags. Again, students may start by just removing one bag, and then removing balls until it balances again. They may think that they have subtracted in order to figure out the answer. Since the bags are being multiplied ($2 \times B$), you can lead students to think about what operation works with multiplication. Some students may need to see the equation as $2 \times B = 8$ so that they can understand what is happening before introducing the convention of $2B$. In the example in Figure 5, they may need a second step, where you look at splitting the bags (perhaps using two scales, placing one bag on each, and splitting the balls between the two trays on the other side) and see there are two groups,

and then look at how to split the balls into two groups. Students will equate the "splitting into groups" as a division procedure. Figure 6 shows the steps again with a written representation to help build an understanding of the procedure.

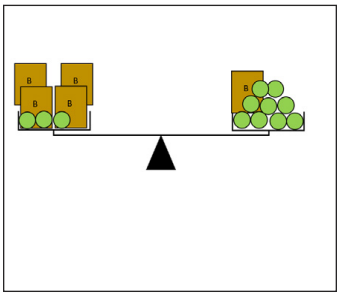


Figure 7: *Pan balance image for 4*B *+ 3 =* B *+ 9*

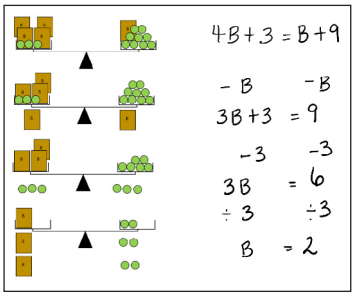


Figure 8: *Possible solution method for solving using the concrete representation of the pan balance*

Now that students have a basis of understanding of the operations individually, they can be combined together, as illustrated in Figure 7. How might a student use the balance to solve for *B*, using a concrete model?

Again, for this example, students may benefit from initially seeing or writing the equation as $4 \times B + 3 = B + 9$ to help build an understanding. With this equation, there are multiple ways that students could start to isolate a bag on the pan balance. I have intentionally set up the example so that the bag will be alone on the left if you keep with positive numbers, since students may default to wanting it to read as $B = ?$. Students may start with the balls, or they may start with the bags, but hopefully they understand that they need to have the bags alone on one side (in this case, the left), and the balls on the other. Figure 8 illustrates a sequence of steps that could be taken to reveal the solving process. In this case, one bag was initially removed from each side to clear the bags off of the right. Then three balls are removed from each side. Again, after each step, the removed items are placed in front of the balance to show the equality of what was removed. To keep the image clear, the items placed in front of the balance each time are removed when moving to the next step in the process. However, students may wish to leave them there as a record of their steps. Finally, both sides need to be divided into three equal groups to isolate the bag, so that students can determine $B = 2$.

Once students have an understanding of using the balance model for equations, they can move into an examination of inequalities, with the model to use as a basis. Since inequalities are referring to an unbalanced scale with the model, they would not be able to refer to keeping the scale level in order to work. In this case, they are now looking at keeping what they remove or add balanced instead of the scales itself. Consider the inequality of $2B + 3 > B + 5$, as shown in Figure 9. If you tie or tape the balance so that it stays in the "greater than" position, this enforces the idea that the purpose is to get the scale to balance.

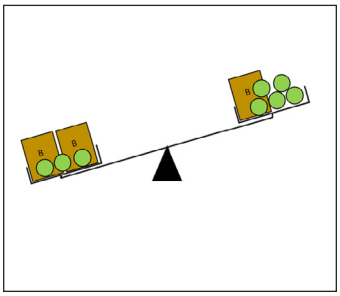


Figure 9: *Concrete image for 2*B *+ 3 >* B *+ 5*

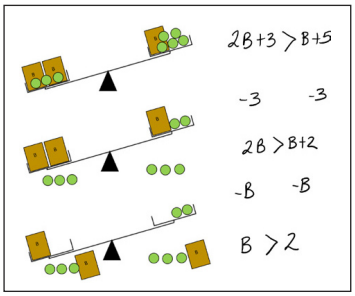


Figure 10: *Example of merging the concrete and the abstract method for solving an inequality*

Conversations with this example would revolve around drawing comparisons to the techniques and understandings that have already been developed, and how we could apply them to this example. In the end, students would still need to use the inequality sign, but the process would still be the same, as provided in all the previous examples. In this case, students may decide to remove the three balls from both sides, keeping what is removed balanced, but the physical scale itself would still not be. Next, they may remove one bag, or *"B,"* from each side, again maintaining balance with what is removed. In the end, they could determine that $B > 2$, as shown in Figure 10.

Online programs, like the Illuminations site, or having labelled bags set out for tasks, allows students to continue using the balance to practise until they are ready to work independently. Students could also use a pictorial representation of the pan balance to help keep track of their steps until they are ready for the abstract representation of solving equations. Once students are able to solve on their own, they can leave the pan balances and can keep track of their steps in an abstract representation. Small (2017), among others, advocates for the educational trajectory in mathematics concepts to move from the concrete, to the pictorial, and then to the abstract or symbolic representation. Allowing students time to use the concrete representations until they are ready to work with the abstract through purposeful practice, can ensure that the foundational understanding of the topic will allow them to solve more complicated problems in the future.

Continued on page 36

▲ Elementary Math Matters: Not so easy, eh?



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Lynda Colgan is a Professor at the Faculty of Education at Queen's University. As a lifelong educator, Dr. Colgan constantly searches for creative avenues to engage students, parents, and educators in mathematics at home and in the classroom. She is inspired to do so by Seymour Papert, who once called upon his audience of "computers in education consultants" and mathematics teachers to bring about change by saying: "Instead of trying to make children love the math they hate, make a math they'll love."

Like every university student who came before me, and every one that will come afterward, each year of my B.Sc., I sought out that one special course believed to be an "Easy A." So, as a teacher candidate at the University of Toronto many years ago, I trusted my "best guess for success" and enrolled in a course called *Teaching Elementary Mathematics* with Frank Ebos.

My reasoning was this: How difficult could it be to teach 1 + 1 = 2 and distinguish between a circle and a square? It would be a breeze to design lesson plans about multiplication; I had all the "carry the one" rules firmly planted in my brain, ready to share. I was, after all, a math major, and this "kids' stuff" could hardly be a challenge when juxtaposed beside abstract algebra, vectors, and calculus.

How wrong I was about these and countless other assumptions pertaining to the course. I credit Frank, as well as my peers in those sessions, for laying a strong foundation in understanding the wondrous complexity of young learners and the elaborate trajectory of their conceptual development.

I remain grateful today, some 40 years hence, for the growth and discovery initiated by the experiences and opportunities that Frank and my peers provided. As a tribute to that impactful course, now as a B.Ed. instructor myself, I begin the first Primary/Junior math curriculum classes of the year with a lesson about the equal sign. Simple? As I learned then, and my students learn now, not so much.

Frank's classes were in stark contrast to my other education course in secondary school mathematics teaching, which, as I recall, were highly competitive and focused on content, never pedagogy and the learner. It was in Frank's "ahead-of-their-time" classes that I found that teaching and learning math could and should be fun. His secret was to transform us into the students we would one day teach by giving us fresh contexts and tools for seeing, doing, and making sense of the math skills we used efficiently and correctly, but without understanding.

Each week, Frank encouraged us to "play" with mathematics by introducing us to such things as *Dienes Blocks*, and teaching that included games, stories, and dance, true to the spirit of Zoltan Dienes' philosophy.

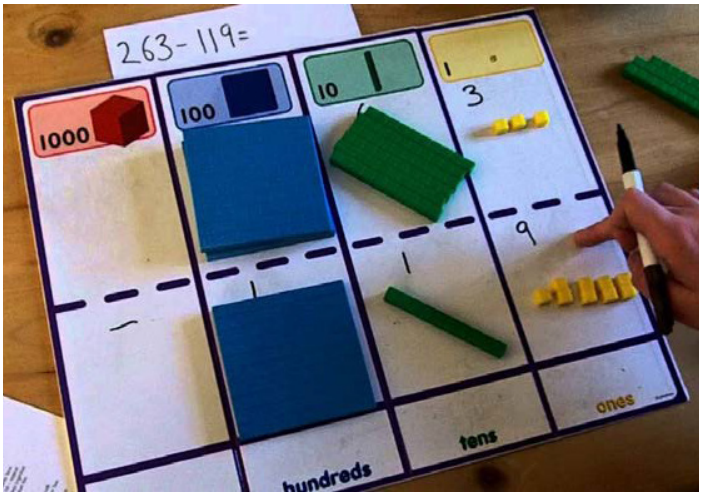


Figure 1: *An example showing how to subtract using Dienes Blocks, also known as Base-10 Blocks*

At the time that I was a student teacher—back in 1978—Zoltan Dienes (1916–2014) was simultaneously famous and infamous: known as the "maverick mathematician" (also the title of his autobiography). The father of the new field of *psychomathematics*, he was radical at the time because he believed in the power of learning mathematics, using manipulatives (i.e., embodied knowledge), and that children needed time for exploration before structured learning and practice (Dienes, 1973).

As my classmates and I "played" with his invention—Dienes Blocks—we constructed and compared numbers; performed calculations; and modelled decimals in what were, for us, uncharted and uncertain ways. But, like the children for whom the manipulatives were intended, my fellow student teachers and I experienced, first-hand, increased enjoyment and motivation—feelings attributable to… blocks?

While we played card and board games (like *Snakes and Ladders*), we were challenged to look beyond the smiles and laughter to observe and appreciate the many math skills and strategies unfolding in our midst—mental computations, trial-and-error strategies, and recognizing the value of learning from mistakes (Dienes, 1963).

Dienes left a lasting impact, not only on my classmates and me, but on the field of elementary mathematics. His influential theories and hands-on inventions underpin much of what we do in math classes today… some 40 years later.

We were introduced to other psychologists, mathematics educators, theorists, and innovators from that era in the course, most of whom also held firm to the tenet that children do not have to reach a certain developmental stage to experience the joy of learning and doing mathematics.

In Frank's course, we were subdued by the intricacies of young children's cognitive development as we learned about Jean Piaget's stages, and were simultaneously intrigued by its possible actualization through the knowledge-mobilization work of his student, Constance Kamii—author of *Physical Knowledge in Preschool Education* (1978). Kamii would later write a number of important books that supported the use of games and play when working with children, to develop their own sense of numbers and operations.

Like Dienes, Kamii was ahead of her time. She believed that math was taught incorrectly (1982; Kamii & Dominick, 2009), leading to anxiety. She believed that teaching mathematics as a set of rules, with little application to real life, is not the same as children learning mathematics. She was an early proponent of multi-digit addition using decomposition and number bonds, adding from left to right, mathematical communication, manipulatives, and constructivist ideals.

In addition to the work of Dienes, Piaget, and Kamii, my course with Frank Ebos introduced me to the research of Herbert Ginsberg. At the time, my classmates and I read his 1977 book, *Children's Arithmetic: The Learning Process.* Like Dienes and Kamii, Ginsburg was a student of and expert on Piaget, writing several books about him on the subject of children's intellectual and mathematical development.

The book remains unforgettable after all these years because Ginsberg (1977) included fascinating transcripts of conversations among students and teachers. The lively transcripts captured a child's mental steps while learning or mis-learning basic arithmetic operations. The purpose of those transcripts, according to Ginsberg, was to equip educators to understand and diagnose errors and misconceptions in order to resolve children's difficulties. His book was my introduction to qualitative evaluations of mathematics learning—the true enactment of the Latin root of the word assess, *assidere*, which means "to sit beside."

It was Ginsberg's work that resonated with me most deeply all those years ago. And, as I wrote to him in a recent email, I remain "his biggest fan," having built a career that shares some modest overlap in terms of authoring children's books (print and interactive, online), contributing to educational television programs, and working to involve parents, guardians, and family members as teaching partners, particularly in low SES demographic areas.[2]

My reminiscences about a long-ago course, the people it featured, and their chance impact on my professional life were recently evoked at the first *Canadian Early Mathematics Education Conference,* March 1–4, 2021. This virtual conference, sponsored by the Mathematics Knowledge Network (Critical Transitions, Community of Practice), the Social Sciences and Humanities Research Council of Canada, and the Mathematics, Science, Technology Education Group at Queen's Faculty of Education, was provided free of charge to educators across the country. Over 1200 people registered for the event, and daily attendance ranged from 250 to 600 links (with many teachers reporting "watch parties" at their school on the big-screen TV).

Organized by myself and Dr. Sandy Youmans, our goal was to break down the "silos" associated with studies about early mathematics teaching and learning by hosting speakers from disparate fields: psychology, cognitive science, early childhood education, mathematics curriculum and assessment, and classroom-based action research.

As I attended the sessions daily, and reflected back on the presentations, I was delighted to note more similarities than differences. Dr. Daniel Ansari's young son was playing with blocks on a coffee table, while working on his understanding of "two-ness." Dr. Natalie Sinclair was guiding us through an experience using *embodied cognition*, i.e., using our fingers and gestures to learn math, using the touch-based interfaces of digital devices. PhD candidate, Hanna Wickstrom discussed the continuum of play, concretizing the nuanced message of poet David Kresh, who once said: "The understanding of atomic physics is child's play compared with the understanding of child's play." Dr. Martina Metz demonstrated research-based activities to structure mathematical "sense making" about even and odd numbers in a way that supports all learners in engaging ways with conceptually sound mathematical ideas.

As I tried new tools, investigated new challenges, and played new games, I was struck by the rich history that they shared with the seemingly "futuristic" scholars, who were instrumental in my own teacher education.

Seeing the burgeoning interest in early mathematics is rewarding, but it would serve us well to remember that the field is not new—it has deep roots that have lain dormant

as the pendulum has swung in rhythm with new mathematics education priorities and reforms.

In our eagerness to embrace the field of early mathematics education, let us not forget to acknowledge the early pioneers in the field like Dienes, Kamii, and Ginsberg. Their seminal work and "big ideas" have influenced curriculum, instruction, and assessment in obvious ways, from valuing socio-emotional considerations that may lead to the development of a child's productive disposition and perceptions of mathematics as sensible, useful, and worthwhile (maybe even joyful), to nurturing children's procedural flexibility, strategic ideation, and capacity to communicate computationally (Kilpatrick et al., 2001).

And while we are at it, let us not be so quick to dismiss Piaget as being wrong or irrelevant to mathematics education today. As John Crossland (2015, p. 30) writes:

> Piaget's theories of the structure of knowledge, constructivist learning, and stages of development in thinking have been a cornerstone of cognitive psychology and teacher education for half a century. More recently, his ideas about stages of cognitive development have received criticism from many quarters, including from neurological research. In Piaget's defense, in common with all theories, his are a product of their time based upon the evidence available… Piaget is not wrong… when his ideas are reviewed with reference to new evidence from brain research, one can see that some of his basic principles are still relevant to teaching today. His constructivist views fit very neatly with the undeniable flexibility of the brain to respond and grow as the result of experience. Although Piaget's ideas about stages of development have been challenged with new evidence, research shows clearly that there are periods of optimal performance and spurts of growth, and that these are in agreement.

In closing, I would like to say, without reservation, that: (1) because of the first Canadian Early Mathematics Education Conference, I am looking forward to future research about and advancements in early mathematics education; and (2) because of Frank Ebos and his introductions to the unsung innovators in the field, I am feeling most grateful for the past.

Notes

[1] I was fortunate, some years later, to attend sessions led by Constance Kamii at several NCTM Annual Conferences, to learn about early mathematics learning—memorable experiences that helped me to understand the egregious misconceptions developed by children in response to algorithmic teaching. Sadly, in 2000, Kamii abandoned her efforts to reform the teaching of number in the elementary math curriculum because too many parents were teaching "carrying" and "borrowing" at home, resulting in confusion for children and leaving educators open to unrelenting criticism for their non-standard teaching methods.

[2] It was my great privilege and honour to meet Dr. Ginsberg in 2016 at The Library of Congress in Washington, DC, where he presented me with the Mathical Honor Book Award for my children's book, *Mathemagic: Number Tricks*, on behalf of the Mathematical Sciences Research Institute (MSRI), in partnership with the National Council of Teachers of English (NCTE) and the National Council of Teachers of Mathematics (NCTM), and in coordination with the Children's Book Council (CBC).

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Arithmetic Fun With Fibonacci!

What is the consequence of the bold values for each Fibonacci style row? Can you follow the pattern? A few values are given to get you started.

1	**1**	**2**	**3**	**5**	**8**	**13**	**21**	**34**
6	–2	4	2	**6**	**8**	14	22	
		6	1	**7**	**8**	15		
				8	**8**			
				9	**8**			
				10	**8**			
				11	**8**			

▲ Leonardo of Pisa: Bunnies, Bees, and the Golden Ratio



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Jeff Irvine, PhD, has been a secondary mathematics teacher, department head, and vice-principal. He has taught at three faculties of education and at a community college. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. With over 45 years in education, Jeff is particularly interested in the interplay of student motivation and mathematics achievement.

It is incumbent upon us to stimulate our students' interest and engagement in mathematics. I have found that student interest can be enhanced through vignettes and curious facts that relate mathematics to historical figures. In this article, I begin with the story of Leonardo of Pisa, also known as Fibonacci, leading to his numbers, and how those connect to the golden ratio. The sequence of vignettes is an opportunity to relate mathematics to nature as well as to history. At the intermediate level, all three topics can stimulate interest and address expectations around patterning; at the junior level, his story speaks to place value and the Hindu–Arabic numeration system; at the intermediate/senior level, Leonardo's story provides interesting ways to relate quadratic equations to other areas of mathematics.

Leonardo of Pisa

Leonardo of Pisa (c.1170–c.1250) has been called "the most talented Western mathematician of the Middle Ages" (Eves, 1969, p. 209). Historical records of the period are sparse and sometimes contradictory. For example, we do not know Leonardo's year of birth, year of death, or what he looked like. Surprisingly, if you do an Internet search, you will find dozens of images of him, mostly copies of the same image, and there is a statue of him in Pisa, Italy, even though there are no historical records of his image.

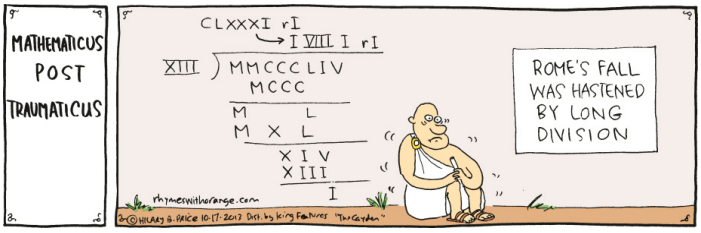


Figure 1: *Rhymes With Orange © 2013 Hilary B. Price. Dist. By King Features Syndicate Inc.*

Leonardo was the son of a merchant trader named Bonacci. The Latin word for son is *filius*, abbreviated *fi*, so Leonardo was known as *Fibonacci* (son of Bonacci) in the same way that more modern surnames like Anderson and MacPherson indicated lineage in other cultures. Fibonacci worked as a bookkeeper for his merchant father, computing using Roman numerals. On a voyage on one of his father's ships, Fibonacci landed in Arabia and saw an amazing new computational system, called the Hindu–Arabic numeration system, using the digits 0 through 9, and a place value system with columns for 1's, 10's, 100's, and so on (the numeration system that we use today). Fibonacci was so impressed that in 1202, he published a book (Latin *liber*) called *Liber Abaci* (Book of Numbers) that introduced the Hindu–Arabic numeration system to Europe. [The singular form of *abaci* is *abacus*, which you may recognize.] In an ironic twist, a copy of *Liber Abaci* found its way to Arabia, where it was translated into Arabic, with the title *al jebr* (Book of Numbers). [Hopefully, you recognize this name as well.] Fibonacci did not invent algebra, but interesting anecdotes such as this one can engage students to pursue further study of a topic. The story of Fibonacci addresses curriculum expectations around the importance of place value, as well as recognizing the role of Hindu and Arabian mathematicians in the development of our current numeration system.

The Fibonacci Sequence

Also in *Liber Abaci* was the sequence 1,1,2,3,5,8,13,…, which came to be known as the Fibonacci sequence, even though the sequence was known to Indian mathematicians in the sixth century. The formation rule for the Fibonacci sequence is

$$F_1 = F_2 = 1,\ F_n = F_{n-1} + F_{n-2}$$

This sequence has a large number of interesting patterns and relationships among its terms. Interested students could investigate these patterns, some of which are related to Pascal's triangle. The Fibonacci sequence addresses curriculum expectations in patterning, sequences, and recursion relations.

Classroom Resource

There is a Fibonacci resource at https://mathiest.wordpress.com that has a lot of scaffolded approaches that overlap with concepts in this part of the article. This resource was authored by OAME member Iain Brodie.

In *Liber Abaci*, Fibonacci generated the sequence, using an idealized pattern of rabbit reproduction (Figure 1). A (male, female) pair of rabbits is born. They spend one month maturing, and then reproduce one (male, female) pair every month thereafter. The newborn pair mature for one month, and then also reproduce one (male, female) pair each month. The pattern continues. No rabbits die. The total number of pairs in each month are the terms of the Fibonacci sequence.

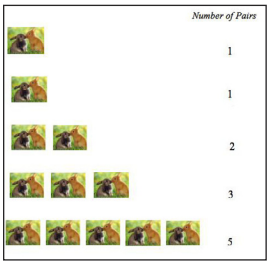


Figure 1: *Fibonnacci's rabbit pattern*

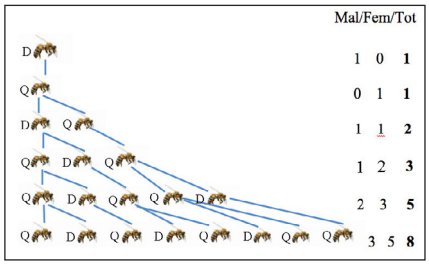


Figure 2: *Family tree of the drone bee*

A similar pattern can be observed in the family tree of the drone bee. Drone bees are male bees whose only function is to fertilize the eggs of the queen bee. Drone bees are unique in nature, since they hatch from unfertilized eggs. Fertilized eggs produce females, either worker bees or queens. Since drones do no work in the hive, when winter comes, drones are pushed out of the hive to freeze to death. The family tree of the drone bee is shown in Figure 2.

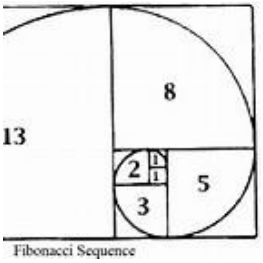
Both the above situations stimulate interest in students who enjoy biology. Some of my students have chosen to investigate the reality of rabbit reproduction or bees, and compare reality to the idealized Fibonacci results.

A general term for the Fibonacci sequence is denoted Binet's formula, credited to Jacques Binet, a French mathematician, who published his formula in 1843. Ironically, Euler published the same formula in 1765, and the formula was known to de Moivre and to Daniel Bernoulli well before Binet was born. Binet's formula is

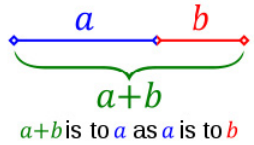
$$F_n = \frac{(1+\sqrt{5})^n - (1-\sqrt{5})^n}{2^n\sqrt{5}}.$$

Amazingly, this formula, involving an irrational number ($\sqrt{5}$), always yields an integer value, which is a term of the Fibonacci sequence.

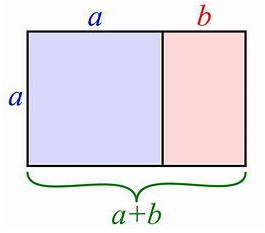
Fibonacci sequences are also found in geometry (e.g., the Fibonacci spiral); in music; and in a wide variety of instances in nature, such as leaf arrangements on stalks, and petal arrangements of flowers. This is a rich area for student investigations and presentations. Curriculum expectations involving algebra can be enhanced by having students investigate the Brahmagupta–Fibonacci identity, which involves multiple ways to express sums of squares. The identity was first published by Diophantus in the third century AD, was then rediscovered by the Indian mathematician Brahmagupta (598–668), and then published by Fibonacci in 1225. Computer science expectations on sorting techniques can include discussion of the Fibonacci search technique. There are many other interesting relationships involving Fibonacci. Among these is the relationship for the Fibonacci sequence to the golden ratio.



The Golden Ratio $\frac{a+b}{a} = \frac{a}{b}$



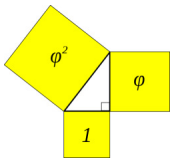
$a+b$ is to a as a is to b



The golden ratio was studied by the Greeks, including Pythagoras and Euclid. This presents an opportunity to relate curriculum expectations on ratio and proportion to geometry. Students of art can be engaged with the golden rectangle.

In addition to constructing golden rectangles, art students will find that the proportions of the golden rectangle were aesthetically considered especially pleasing, and that the dimensions are approximated in many architectural structures, such as the Greek Parthenon. The astronomer Johannes Kepler (1571–1630) even found a relationship between the golden ratio and the Pythagorean theorem.

The Value of Φ



There are a number of ways to generate a numerical value for Φ (the golden ratio). One way involving curriculum expectations for the quadratic formula is to evaluate the infinite continued fraction

$$x = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1+\ldots}}}}.$$

This can be evaluated by recognizing that the equation is equivalent to $x = 1 + \frac{1}{x}$. Simplifying gives the quadratic equation $x^2 - x - 1 = 0$.

Solving using the quadratic formula gives $x = \frac{1+\sqrt{5}}{2}$ or $x = \frac{1-\sqrt{5}}{2}$, two irrational numbers, with values approximately 1.618 and –0.618. The first of these is

denoted Φ and the second Φ^{-1}. So, the "pleasing" proportion identified by the Greeks is approximately 1.6:1. Note that you have seen the exact values before, in Binet's formula for the terms of the Fibonacci sequence, which students find to be an interesting result. Another interesting relationship that can be verified by students is that Φ and Φ^{-1} are negative reciprocals, i.e., $\Phi\ \Phi^{-1} = -1$.

Another way of generating the value of Φ is to let $= \frac{a}{b}$. Then the golden ratio $\frac{a+b}{a} = \frac{a}{b}$ becomes $1 + \frac{1}{x} = x$, a quadratic equation that is then solved as above.

Fibonacci and the golden ratio can be linked with calculus expectations involving limits. Students can show, using successive approximations of Fibonacci numbers, that

$$\lim_{n \to \infty} \frac{F_{n+1}}{F_n} = \Phi.$$

This provides another interesting connection. This result can also be found using limits with Binet's formula, but requires somewhat deeper thinking by students.

Finding ways to engage students and stimulate interest is an important dimension of teaching mathematics. We need to communicate content, and encourage students to become inquisitive, lifelong learners. The books in the reference list are all accessible by students, and provide a plethora of ideas related to the topics in this article. Copies can often be found in used bookstores, and reprints can sometimes be found online. More recent resources, such as the Internet, also provide opportunities in a student-friendly way. We must keep students interested and engaged in their learning of mathematics if we hope to inculcate the joy of mathematics that many of us found in our own studies and our own classrooms.

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Note: *In addition to print references, the Internet provides a myriad of sources involving Fibonacci and the golden ratio. A simple Internet search for the term "Fibonacci" yields 8.4 million results.*

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▲ Exploring the Experiences of Teacher Candidates with the Math Proficiency Test: Recommendations for Ontario Educational Systems



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research grants examining inclusive leadership practices of Canadian school principals. He travels regularly to Haiti, Ghana, and Egypt, where he is involved in school leadership and special education training and research. Prior to his work in the Faculty of Education, Dr. Sider was a school administrator, special education teacher, and classroom teacher for 15 years.

A major change occurred this past school year for teachers graduating from an Ontario Faculty of Education with the implementation of the Mathematics Proficiency Test (MPT) as a new certification requirement in Ontario. The news was met with a lot of anxiety and stress, not just about the test, but with the lack of information, the changing dates, and the slow rollout of details. When the dates were finally scheduled for the MPT, an additional challenge was presented for graduating pre-service teacher candidates. In order to understand how our pre-service teachers were feeling, we conducted a survey to gather some initial information about the experiences of teacher candidates who wrote the pilot MPT in February and March of 2020. Given the rise of the COVID pandemic and the change of dates of requirement for successfully passing the MPT to August 31, 2021, we felt that it was important to share this information with practising teachers and school boards. We felt that the MPT may impact the dynamic of working with new teachers, since not all of them were able to even take, let alone pass, the MPT during the pilot period. We begin by discussing the current context of the MPT and its place in the certification process and requirements, then talk about the survey results and information that have potential implications for the field of education.

The MPT was announced in summer of 2019 (e.g., Ontario Ministry of Education, 2019) as a certification requirement for all internationally educated and newly certified teachers in Ontario (Ontario College of Teachers [OCT], 2020). The initial announcements noted that anyone applying for certification in Ontario on or after March 31, 2020 would need to successfully pass the newly designed MPT before certification (internationally educated teachers have two years to pass). At the time, any currently certified teachers could volunteer to take the MPT. After the announcement, a pilot of the MPT was announced for February and March of 2020. As of the writing of this article, this is the only opportunity that anyone has had to take the MPT. Those who were successful in the pilot would not have to take the MPT again and would be fully certified. With the COVID pandemic, the OCT has since changed its requirements, now stating for all Ontario graduates who have applied for certification: "An Ontario applicant who is required to pass the Mathematics Proficiency Test can become certified with a condition to pass the test by August 31, 2021. If they do not pass the test by August 31, 2021, their certificate will expire and there will be no extensions. They will be required to pass the test and meet all other applicable certification requirements before re-certification" (OCT, 2020, para. 4). As such, any graduates from the 2018–2020 cohort of Ontario teachers may or may not have passed, or even taken, the MPT, and will need to do so if they want to keep their certification past August 31, 2021. Due to this requirement, we felt it was important to share some of the information we learned from the study we completed.

At the time that some teacher candidates wrote the pilot MPT, the newly launched *Ontario Mathematics Proficiency Test for Certification* website (www.mathproficiencytest.ca/ #/mpt-launch) provided some information for candidates about the MPT. The Assessment Blueprint (EQAO, 2020) document noted that, "The MPT has 75 items. Each unique test (test form) contains 71 items that count towards the teacher applicant's score. These are 50 mathematics content items (70% of the test) and 21 pedagogy items (30% of the test)" (p. 6). On the site, there were a minimal number of practice questions (less than 20) that those who were taking the MPT could review prior to the first writing. At the time of writing of this article, there is now an entire "practice" MPT posted to the website to allow those who will write the MPT a chance to examine the entire test (www.math proficiencytest.ca/#/en/applicant/learn/prepare: sample_questions). It is noted that successful passing of the MPT is a requirement for OCT certification, but not a graduation requirement for Bachelor of Education programs in Ontario.

We created and conducted the survey at Wilfrid Laurier University with our teacher candidates from May 5 to June 3 of 2020. We had 50 of the 130 teacher candidates agree to participate in the research study representing a 38 percent response rate. Based on those who gave the information, our sample was divided between 2020 graduates (27 percent of the cohort) and 2021 graduates (46 percent of the cohort). Within the sample, 20 identified as being part of the Junior/Intermediate program, and 28 from the Primary/Junior program. There were 21 questions on the survey. All of the participants who participated in the study wrote the MPT in the pilot period in February or March of 2020. The survey was administered through Qualtrics (i.e., online survey software) and consisted of 3 questions related to their graduation year and program of study, 12 open-ended questions, and 5 closed questions.

In looking at the available locations for the pilot test centres for writing of the MPT and comparing them to the cities where there were Faculties of Education, it was difficult for many teacher candidates across the province to conveniently access a test location. Although Waterloo is a geographically central location in the province, our teacher candidates shared that they had to drive long distances to reach an available testing centre.

Most of the participants (82 percent) had to travel more than 50 km to write the MPT, with 44 percent having to travel more than 100 km. Distance to testing centres was a potential difficulty, so school boards may need to provide some support subsidies in allowing new teachers the time to be able to travel to a site or offset the costs in order to take the test, if the testing centre locations remain the same when the MPT is offered a second time. In addition, 70 percent were not able to register for their preferred location. Since not all teacher candidates had the means to travel far to write the MPT, it is likely that some teachers who started their teaching careers in the Fall of 2020 will not have completed the MPT. A portion of this year's new teachers have not taken or potentially passed the MPT; therefore, support from those in the field might be helpful to be included in new teacher mentoring programs.

The remainder of this article considers questions from our study related to how the participants felt about the MPT, what they feel is needed to be successful mathematics teachers, and what supports were helpful for preparing for the MPT. Whenever possible, participants' quotations are used to allow their own words to illustrate key messages. Any identifying information has been removed to ensure the anonymity of the participants.

In describing their feelings about having to take the MPT prior to the test itself, many teacher candidates used words that are commonly associated with negative emotions. Words used to describe their anticipation of writing the MPT were anxiety, nervousness, doubt, worry, stress, distraction, fear, uncertainty, panic, fear of failure, avoidance, apathy, frustration, annoyance, and the feeling of being overwhelmed. The most frequently stated words by teacher candidates were nervous, anxious, and stressed. As one participant noted, "I was extremely anxious and had no clue what to expect. All information about the test was very vague, and I was going to a completely new campus that I was very unfamiliar with, which added to the anxiety." Participants expressed a lack of information about the testing centres and locations . Given the frustration and anxiety that participants claimed, there is a possibility that the test heightened feelings of insecurity and impacted their results in the MPT. Some participants felt that they would be fine with the mathematics portion of the MPT, but the pedagogy questions raised concerns. There was a consistent response of a need to simply memorize the three documents, the math curriculum, *Learning for All*, and *Growing Success*, that were being tested because of the very specific pedagogical questions.

Most of the participants felt the MPT was not helpful in becoming better math teachers, with 76 percent (34 of 45) of the written responses saying unequivocally, "No, it would not help." (Others were not as strong in their response, but still said it was not effective). In the responses that did not say no, most of them noted that the MPT only taught them what not to do in mathematics or to remind them how these types of tests are inappropriate to use with students. As one participant said: "Potentially, because it made me realize the negative effects of large tests on students." Others noted that the test forced them to look at mathematics again, and that having an understanding of mathematics would help in teaching. Instead, participants offered suggestions about what they felt could be helpful in supporting their ability to be effective mathematics teachers. For example, one participant stated, "Qualified teachers and teaching candidates alike need to be better supported in learning math concepts before being expected to teach math to students. Money should be invested in helping teachers understand math concepts, not a test." As seen in the example, participants focused on larger-scale changes to make a lasting difference in mathematics teaching and learning. Participants also pointed to needing a culture shift in mathematics to allow all students to understand that anyone can do mathematics, focusing on growth mindset (Boaler, 2015). Others pointed to the need for more pre-service or in-service courses that focus primarily on the mathematics they need to teach and the skills that would make them more effective teachers. As one participant noted, "I think having specialized math teachers might be a good idea to improve math scores in schools. I don't think this MPT helps, as existing teachers don't have to take it. It might also help to change the overall perception or attitude about math in schools. Anyone can be good at math. Maybe teachers should take a course on that instead of creating more math anxiety in future teachers because they have to pass the MPT." The participants pointed to the knowledge that there is a specialized understanding of mathematics that is needed to be effective teachers that goes beyond simply being able to answer rote mathematics questions (cf. Ball et al., 2008; Silverman & Thompson, 2008). This specialized content knowledge is the focus of a regular column in the *Ontario Mathematics Gazette: Mb4T (Mathematics by and for Teachers).* Overall the general feeling was that more professional development dedicated to mathematics content for teaching that is specific to the areas they will be teaching would be more beneficial and create a long-term effect on mathematics students in the province.

The participants provided a variety of thoughtful pieces of advice for future teacher candidates anticipating the test. Some of the suggestions may be helpful to add to new teacher education programs or in schools to support those in the field who still need to pass the MPT. Most participants emphasized the importance of focused studying in order to be fully prepared for the MPT, using online resources such as EQAO practice problems, and studying with their peers. As one explained, "It is very different than anything you've done in the B.Ed. It is more like going back into undergraduate studies and taking an exam for a course you didn't take. Study all the materials provided in

depth. Pay special attention to terminology. In fact, I found studying and memorizing the glossaries very helpful. Study math problems with a friend and solve them together." Giving new teachers chances to work with more seasoned colleagues with a deeper understanding of mathematics for teaching may have a lasting impact on their deep understanding of mathematics, which would benefit students (Baumert et al., 2010).

Most participants noted that it was important to prepare (e.g., do practice questions), but they stressed reading the pedagogy documents, particularly the math curriculum documents, *Learning for All*, and *Growing Success*. They noted that it was important for them to know the pedagogy documents, but that there was a general consensus that the only strategy that was necessary for the MPT was to simply memorize all of the information in them. A further suggestion provided by participants in our study was to have teacher candidates work with a mentor teacher prior to the MPT, thus allowing them to see the documents in action and facilitate retaining important information for the test.

Many participants focused on encouraging a positive mindset and staying calm in preparation for the MPT. As one stated it: "You are a TEC [teacher education candidate] because of your desire to teach and love for education. This test result does not define you and your ability to teach inside and outside of the classroom. In life, sometimes we need more than one opportunity in order to grow and learn. Do your best. Prepare in a way that meets your needs, whether it be small study groups, reading about math concepts before bed, or completing math sheet after math sheet. You got this!"

In the end, the recommendations provided by the participants in the pilot of the MPT were clear: the test was not a helpful way to help them feel more confident and comfortable in teaching mathematics. Instead, what is suggested is to work within school boards to help develop effective supportive mentors in mathematics. This would positively facilitate both the practices of new teachers and those who are in the field by extending and strengthening previous learning in the Bachelor of Education programs. A strong focus placed on developing and expanding the specialized mathematics knowledge for teaching (Ball et al., 2008; Silverman & Thompson, 2008) would allow new teachers to effectively support students through professional development or changing the focus of the Mathematics AQs through a direct focus on mathematics for teaching and not just mathematics pedagogy. As one participant summarized recommendations for the future: "Focus on teacher professional development and support in schools. Reduce class sizes, provide greater small-group math support, and make more essential resources for emergent math [teachers] available. All classrooms should have math manipulatives, access to relevant mathematics technology and software, and funding for substitute teachers so that classroom teachers may experience ongoing professional development. Provide materials, training, and funding to support teachers in the classroom: these are paramount." New teachers require support in teaching mathematics more than ever after their experience with the MPT, and especially for those who are still anxiously waiting for the next opportunity to take the MPT in order to keep their teaching certificate.

Overall, based on the emerging findings from our study, greater investment in professional and pedagogical training for existing and new teachers is an important way forward for supporting effective mathematics teaching in Ontario.

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▲ Research into Practice: A Framework for Lesson Planning That Considers Student Motivation



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Designing mathematics lessons that are motivating for students can be challenging, yet there is abundant research indicating that motivation plays a large role in student achievement (Smith & Star, 2007), as well as other positive student behaviours like persistence, resilience, risk taking, and creativity in mathematics (Duckworth & Gross, 2014).

One way of designing lessons that support student motivation is through considering the acronym TARGETTS (Gilbert & Musu, 2008; Gilbert & Musu-Gilette, 2011). The acronym itself is not the important part, although it is easy to remember. What is important is that teachers consider the components of TARGETTS when planning lessons, with student attitudes in mind.

Tasks: Selecting tasks that generate student interest and/or illustrate the value to the students of the mathematics content.

Autonomy: Opportunities for students to take responsibility for their learning, for example, providing student choices in some aspects of the task in content, process, or product.

Recognition: Why students are to receive recognition, for example, recognizing effort toward a goal, exhibiting a growth mindset (Dweck, 2006), specific praise with respect to progress toward a mastery goal.

Grouping: Grouping students in ways that de-emphasize social comparisons. Liljedahl (2018) found that visibly random student groupings had a significant impact on achievement.,

Evaluation: Criterion-referenced and ipsative assessment (assessing students' personal growth, rather than assessment against an absolute standard (Hughes, 2014)) is preferred over norm-referenced evaluations (comparisons against other students). This encourages mastery goal orientations in students, in which students desire to master the material for internal reasons (Tumposky, 2003). Mastery goals are associated with positive student behaviours such as effort, persistence, and intrinsic motivation (learning motivated by interest and internal desire to learn) (Hurlington, 2018; Midgley et al., 2011).

Time: The time dimension reflects the need to allocate time strategically, to emphasize important content and big ideas through the use of rich tasks and interesting scenarios.

Teacher expectations: Set clear and consistent expectations for all students. This should be done early in the year and reinforced continually (Patrick et al., 2003).

Social interactions: Facilitate classroom discourse involving all students by communicating social norms that permit respectful, productive mathematical discussion. One way to do this is to involve students in determining norms, through methods such as Post-it® Pileup (Irvine, 2020) or Google Jamboard.

Research has shown that considering the dimensions of TARGETTS when planning a lesson supports the development of adaptive motivation in students (Gilbert & Musu, 2008; Gilbert & Musu-Gilette, 2011). Adaptive motivation is a collective term for a group of desirable student attributes, all of which have extensive research support as being positive for student achievement. These include intrinsic motivation (Ryan & Deci, 2000), self-confidence (Vandecandelaere et al., 2012), interest (Hidi & Renninger, 2006), mastery goal orientation (Midgley et al., 2001), and perceived usefulness (Eccles & Wigfield, 1995).

Intrinsic motivation is motivation that comes from the student's desire to learn for the sake of learning, based on interest in the subject matter (Ryan & Deci, 2000). Self-confidence is a dimension of the student's self-image, and is related to self-efficacy, which is the student's perception of his or her ability to successfully complete a task (Bandura, 1997; Vandecandelaere et al., 2012). A mastery goal orientation is the student's goal to master material based on a desire to learn, rather than a desire to be perceived as better than their peers (Midgley et al., 2001; Turner & Patrick, 2004). All of these traits are desirable in students, and all can be encouraged through the use of well-planned lessons that take student motivational dimensions into account. TARGETTS is one framework that allows teachers to consider the multiple dimensions of student motivation in an organized manner.

Gilbert and Musu-Gilette (2011) suggest that TARGETTS can be used in two ways. The framework can be used to plan a lesson; or the framework can be used to analyze a lesson that has already been constructed, to look for ways to modify the lesson to better support student motivation. They give examples of both uses of TARGETTS in their chapter in the 73rd NCTM yearbook (2011). Teachers beginning to consider ways to support student motivation may find the analysis of an existing lesson a good place to start, since using the framework will illustrate how to see a math lesson from the viewpoint of students.

A productive use for TARGETTS is to have a grade team or course team complete Table 1. This encourages all teachers to think about the goals of their lessons related to student motivation. I have found that when using Table 1 with teachers, they sometimes wish to switch column 2 (*What should be*) with column 3 (*What could be*), using *What should* be as the ultimate goal, and *What could be* as a more pragmatic, intermediate goal. It is often encouraging for teachers to find that *What is* can also be the goals of *What could be* and *What should be*, i.e., that their current practices are already supporting students' adaptive motivation. For example, tasks that challenge students and generate interest, visibly random groupings, or explicitly identifying expectations are already part of instructional practices, so focus can shift to other dimensions of TARGETTS.

Table 1 *TARGETTS Analysis Sheet*

	What Is	What Should Be	What Could Be
Tasks			
Autonomy			
Recognition			
Grouping			
Evaluation			
Time			
Teacher expectations			
Social interaction			
Resources*			
**School-wide focus*			

If Table 1 is used on a whole-school basis, or if school administration is involved at the grade or course level, another R (Resources) may be added to Table 1, identifying that if significant changes are determined to be needed, additional resources may be needed to enact those changes (Maehr & Anderman, 1993). These resources frequently will involve professional learning opportunities, mentoring, or time release for collaborative lesson development using TARGETTS.

Recognizing that student motivation is a desirable goal of mathematics teaching, since it can be related to both student achievement and broader student self-development goals, has led to an explosion of research into student affective dimensions and their impact on learning. TARGETTS is one framework that recognizes that the planning of a math lesson needs to take into account more than just mathematical content, and students as learners need to be supported in developing productive mathematical dispositions (Kilpatrick et al., 2011).

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▲ Book Review: Joyful Reading About Early Math Education



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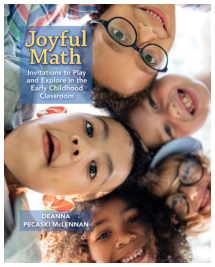
Tim Sibbald has held a variety of positions within the OAME/AOEM. He is a former president, vice-president, and director. He is an associate professor in the Schulich School of Education, Nipissing University, with a focus on mathematics education. He has edited a book on Interdisciplinary Math and two about the experiences of academics (with a third underway).

McLennan, D.P. (2020). *Joyful math: Invitations to play and explore in the early childhood classroom.* Stenhouse Publishers. (Distributed in Canada by Pembroke Publishing. See www.pembroke publishers.com—the fastest way to receive it is to order through them.)

Joyful Math is a wonderful reflection about instruction in the early years. At first glance, the book is inviting, with colour throughout. As one engages the book, the lack of specific ties to any particular curriculum will become obvious. However, the author is from Ontario, and the ideas she shares are clearly classroom tested. The strength of the book is the provision of a holistic view of the richness of opportunities available for exploration in the early years. This is the strength of the book—it brings about the joy in young minds by encouraging their explorations.

The book as a whole draws on vignettes that illustrate the roles of the teacher, students, parents, and the resources brought into instruction. The details are well written and communicated. It gives the sense that one is being encouraged to learn a new dance that has a variety of intricacies, but rather than take it apart and examine the pieces, the book offers different perspectives and leaves the reader to interpret how the roles fit together. This results in the reader seeing a coherent play-based approach that engages students.

The book is readable, with a few supporting references that can allow one to investigate particular ideas, if one wishes. In this respect, the book focuses on portraying an overall practical approach to play-based learning and gives only a few connections to theoretical underpinnings. This is by no means a fault of the book; it is an intentional focus that maintains the reader's reflection on what young students can achieve if circumstances are conducive to learning.



There are six chapters in the book that draw the reader's attention to discovery: the instructional environment, art with math, literacy and math, use of the outdoors, and documentation. The organization begins with the largest scale where discovery could apply to many areas of study. This gives a sense that the book is broadly applicable, but subsequent chapters reign this in, and one gets more sense that there is a holistic approach, where subject silos hinder many play-based opportunities. The middle chapters cross subject boundaries with purpose, and an instructional richness develops. The final chapter caps the book by addressing techniques for documenting when developing the richness.

As a whole, the book is readable and worthwhile reading for Kindergarten teachers and Early Childhood Educators. It will also be of interest to Grades 1 and 2 teachers. While I have indicated the book is not tied to any particular curriculum, the author is from Ontario, and the ideas she shares are clearly classroom tested. In my opinion, everything I read is consistent with the Ontario curriculum.

Note: Local OAME chapters may be interested in having the author at an event. The contact for making such arrangements is Alison Parker at Pembroke Publishing. She can be reached at alison@pembrokepublishers.com. ▲

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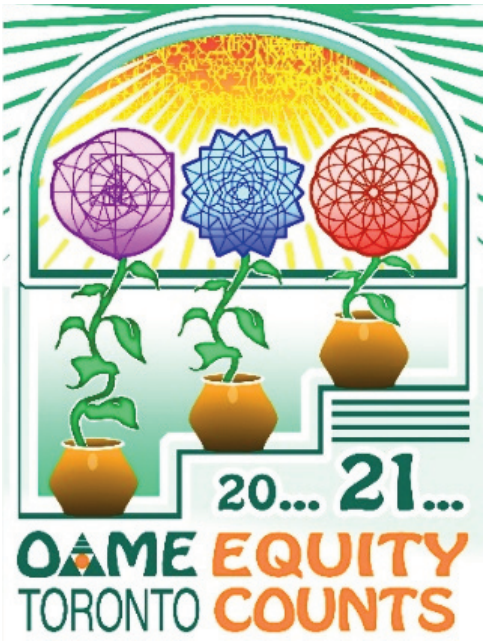
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A GREAT BIG THANK YOU!

OAME/AOEM wishes to extend its thanks to the TEAMS chapter, who worked tirelessly for over two years to bring the 2021 conference to its fruition. Due to COVID-19, the conference had to transition to a virtual format, and the TEAMS team came together, with hard work and determination, to make this transition possible.

Congratulations on an outstanding virtual conference.

The conference couldn't have come off without the support of many others. We wish to thank the following groups for their support:

- The volunteers, who acted as technical hosts and moderators during the sessions. Teacher candidates, high school students, classroom teachers, and retirees from across Ontario donated their time to support the presenters before and during their sessions.
- The Keynote and Featured Speakers, along with all the Breakout Session Presenters. Preparing for a virtual conference can be significantly more challenging than for an in-person event. Thank you for sharing your expertise and knowledge.
- The sponsors, who supported professional learning for our attendees, both financially and with breakout sessions of their own. We hope you took advantage of the virtual trade show and virtual swag bag to learn more about our sponsors' products and services.
- The many school boards that signed up for group codes, to allow centralized payment of fees for their educators.
- And finally, the attendees. Not having organized a fee-based virtual conference previously, it was an unknown risk of whether people would attend. Well, you did! OAME/AOEM 2021 had well over 2300 delegates from across Ontario, the rest of Canada, and around the world. Thank you for attending. We hope you enjoyed the conference.

We hope that next year's conference will be an in-person event. We look forward to seeing many of you in Windsor on May 5-6, 2022.

