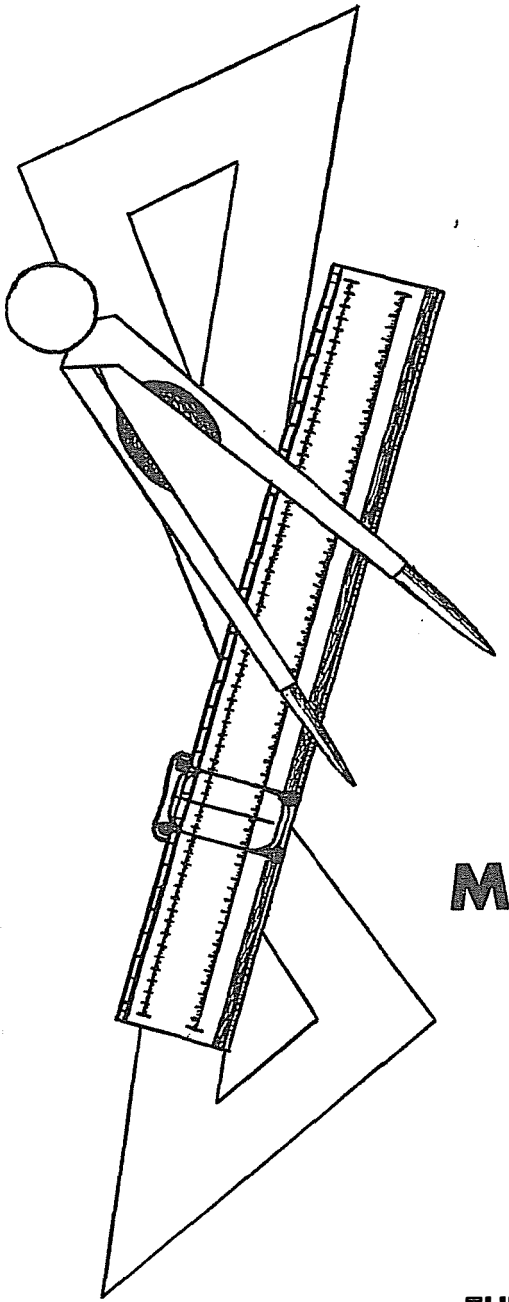


VOLUME 5
NUMBER 1
NOVEMBER 1966

ONTARIO MATHEMATICS GAZETTE

sponsored by
THE SAMUEL BEATTY FUND
THE ONTARIO MATHEMATICS COMMISSION
THE ONTARIO ASSOCIATION OF
TEACHERS OF MATHEMATICS

EDITORIAL

The first wave of reform in mathematics curriculum has reached the Grade 13 level this year, and now it would be well to look back to see what might be following in its wake.

Almost all of the present Grade 9 students in the province have enjoyed the best mathematics background ever provided by our elementary schools, due to the recent revisions in the senior elementary grades. Undoubtedly some modification in the Secondary school curricula will be necessary to capitalize on this improved background, but such changes (representing a second wave) are not likely to be as pronounced as those associated with the first. Indeed, basically, the next few years in Secondary schools may be a mopping up period, during which the new material currently available is digested and methods of presentation are improved.

Two other factors lurk in the background which in several years should affect secondary school education in a major way. Firstly, in the near future the most drastic developments in mathematics and related education are likely to occur in the early years of elementary school. A considerable effort has been launched in that direction and the results could well be spectacular, particularly in the area of intuitive geometric understanding.

Secondly, there is the computer. The computer, everyone agrees, will in the future affect more aspects of our lives in more significant ways, and our education must gear itself to that fact. However, the method is not clear; just how we can best adjust secondary mathematics education to the computer age is certainly not at all obvious. But start we must.

One man who is both professional mathematician and computer expert, and who has given considerable thought to this problem, is Professor Hull of the University of Toronto. Prof. Hull expressed some of his views in a note which was reprinted in the

Bulletin OSSTF, 46, no. 1, Jan. 1966 (27-29). A further paper was presented to the Fifth National Conference of the Computer Society of Canada and is reproduced in this issue of the Gazette.

Also featured in the present issue, is a number of articles and book reviews associated with current mathematics, at the secondary school level, in a few foreign countries.

J.E.H.

THE INTERNATIONAL CURRICULUM CONFERENCE

J.N.C. Sharp
Board of Education, Township of Etobicoke

The International Curriculum Conference was held in Toronto from February 8th to 11th, 1966. Delegates from the United States, England and Canada attended this conference. One section of the conference was devoted to a study of the different curricula that are being introduced in different parts of the world. Delegates from Ontario included J.N.C. Sharp, Chairman, President of the Ontario Mathematics Commission, A.W. Harris, Recording Secretary, Vice-President of the Ontario Mathematics Commission, and Dr. H.A. Elliott, Past President of the Ontario Mathematics Commission.

Mr. Hugh Bradley reported on the African educational program of Educational Services Incorporated. This organization is engaged in curriculum research and development and teaching materials are being produced for a new mathematics program in Ethopia, Kenya, Liberia, Malawi and many other areas. These materials have been developed in co-operation with African administrators and teachers and are being used in Grade 1, Form 1 and teacher training. There are now over 800 classes studying new mathematics under Educational Services Incorporated. The original materials that were tried out in this experiment were based on the S.M.S.G. publications. Since then, the writing groups have been developing their own ideas. In many ways, the African group has benefited from the mistakes of the western countries.

Dr. H.A. Elliott reported on the work of the Kindergarten to Grade 6 Committee on Mathematics. This committee was sponsored by the Ontario Curriculum Institute whose personnel were selected by the Ontario Mathematics Commission. An account

of this summer study can be obtained by reading, "Mathematics, Report of the Committee Kindergarten to 6", published by the Ontario Curriculum Institute.

Dr. Geoffrey Matthews of the Nuffield Project outlined the aims of the joint group for the Nuffield Project and the School Council for primary school children in England. One impression that he left with us after we had observed a film which illustrates the experiment is that quiet in the class room does not necessarily imply concentration. Concentration comes through interest. The Nuffield Project is the first truly national undertaking at the primary level. There are 14 pilot areas involving 1,200 classes with 65 new areas planned for next year. The Schools Council has a large role to play in the setting up of these experimental areas. The two main questions that Dr. Matthews has found for his project area: "What Do We Want?", and "How Do We Test to Determine If We Have It?". His objectives as outlined briefly were: Concept Formation, Discovery, Invention, Persistence, Transfer, Enjoyment, and Team Work. He suggested two films, "I Do and I Understand", and "Math Alive".

Mr. W. Baxter, who is the Chairman of the Secondary Education Committee of the Ontario Mathematics Commission, reported on the four-year study sponsored by the Ontario Curriculum Institute and carried out by members of the Ontario Mathematics Commission. A description of this study can be found in the booklet, "Mathematics, the Four-Year Program", published by the Ontario Curriculum Institute.

Mr. Peter Hilton, Co-Chairman of the Cambridge Conference on School Mathematics was unable to be present, but Mr. Bradley gave an account of the Cambridge Conference in 1964 which resulted in the report, "Goals for School Mathematics". At present, some teachers are experimenting with parts of the proposals.

Dr. Forbes of Perdue University described recent improvements in various programmed methods of presentation. Various

experiments were described in which automation was employed in attempts to provide effectively for varying differences. There still remain a number of problems in using program instructional materials which teachers must solve.

The meetings provoked much discussion and were an excellent opportunity for the interchange of ideas. We still felt that many of the countries beyond our borders look to our evolving curriculum in mathematics for guidance.

NOTES ON THE FAR EAST

D.H. Palmer
Inspector of Schools,
Western Ontario Area, Department of Education

During a recent trip abroad, I was able to visit schools and talk with education officials concerning mathematics teaching in their systems. This article will deal mostly with observations in New Zealand and New South Wales but will mention a few other countries.

The organization of New Zealand's school system bears a number of similarities to that of Ontario. The central Department of Education is responsible for curriculum and appoints subject inspectors to oversee the work. The local area is responsible for staffing the schools. The Department pays 100 per cent of the building costs of the basic high school (and I do mean basic), but the people of the school area provide such frills as gymnasia, swimming pools, cafeterias, and some of the classroom equipment which we consider necessities. There are also in New Zealand departmental examinations to be written by all for certification. More about several of these items.

We were surprised by the hardiness of the teenagers of New Zealand. In the 40 degree weather of Auckland in winter, the high school boys in shorts (and often barefoot outside of school hours) and the girls in school tunics had crimson legs and faces, but they did not seem to notice the cold. Of course, many of the homes are not heated and the schools are cold, by our standards, so that they were not affected by marked external temperature changes. I can still see hundreds of high school students huddled against the outside walls of the school buildings to eat their lunches in the drizzle and not seeming to mind. This incident occurred in a new residential area where the parents had not as yet raised the money for the big extras in the high

school after outfitting the elementary schools. Older schools are often very well equipped, but even in them the parents often take turns staffing the cafeteria. Outdoor physical education all year long is not a great hardship for these students in the North, but in the South it must cause difficulties at times.

The Departmental examinations at the end of the third year are the dominant feature of New Zealand's high school system. Up to this stage almost no pupil fails, but the pass rate on THE exams is fixed between 45 and 55 per cent. As a result the third year classes I saw in September were doing nothing but cramming and working old exam papers for November finals. Many teachers claim the exams should be abolished, but the public and the government want them; a senior department official told me that the teachers want the exams! These are the school leaving exams for most, but a number of the best students return to high school for another year or two to prepare for the advance placement university entrance examinations on which scholarships depend. The mathematics being done by this group is at a more advanced level than our present grade 13 and allows the teachers a chance to really show their scholarship and enthusiasm. I felt that these qualities were stifled in the lower grades by the necessity of trying to get a large proportion through the exams. Since many of the big city schools get 80 per cent passes and better, some schools must be glad to get any passes at all.

So far the "new approach" mathematics has made little impression in the conservative New Zealand system, although there are signs of change. Four young Heads of Mathematics Departments in Christchurch, who were chums in university, grouped together several years ago to write their own material to introduce the new math in the classes they themselves taught. I was told that permission was granted grudgingly and that it was implied that their careers were at stake if their pupils could not pass the REGULAR EXAM. These four

have different personalities, but all are excellent teachers, and their great effort has paid dividents. This year permission has been granted for other teachers in their departments to join the experiment and special exams are being set for their third year students. Since PSSC physics was being introduced two years ago, change is in the air.

Incidentally, in one class which had been studying the parabola a light chain was hung from two nails, and the class challenged to find a simple test as to whether the chain hung in a parabolic shape.

The effects of the New Zealand examinations extend thousands of miles beyond the country's borders. In Fiji, Western Samoa, and Tonga, I saw classes of young Polynesians and Melanesians cramming for the same exams. Imagine youngsters whose native languages contain few words for abstractions and do not have distinguishing names for many of the different plants and fish around them! Give these young people the task of writing the New Zealand exams after only five or six years of studying English, and you see why few qualify for the scholarships to go abroad to university. (I was told that the Malayo-Polynesian languages have no construction at all like our conditional. How would you like to analyze a problem without using "if"?) In school these children do not ask questions because at home young people do not speak unless spoken to, and though they receive affection, the children are not spoken to. Some are almost dumb when they begin school. Most of the high school teachers are sent by New Zealand or are missionaries who spend their first year trying to reach the pupils. They go home after two years. I met only three really effective teachers -- all have stayed on for extra terms; two of these have, in effect, joined the social pattern of the islands. The Roman Catholic missionaries have the edge. These priests go in perhaps for life and learn the language and customs before starting to teach. The Mormons are pouring millions of dollars into island schools and sending their best graduates free to Hawaii.

So far, however, the New Zealand system dominates.

In Australia, where education is a state matter, the only system I could visit before final examinations and summer holidays was that of New South Wales. Here all teachers in public schools are state employees, and all job openings are filled on the basis of seniority. Hence the majority of the experienced teachers who would rather teach in the city are found in Sydney and suburbs while the schools in the remainder of the state are largely staffed by beginners. There are also some excellent private schools which have special attractions for some teachers. State examinations at two levels of accomplishment apparently are not passed by a large proportion of the students. Post secondary education is often of the English type where young technicians and engineers study while they work and do not receive degrees. Assessments of their system by people working in it were so varied that I will not try to comment further.

One facet of the New South Wales system will be of particular interest to mathematics teachers. A few years ago a royal commission there recommended the introduction of the "new approach" mathematics in high school, but, as is the fate of so many royal commission reports, this one sat on the shelf for several years, forgotten by most mathematics teachers who were doing nothing to find out for themselves what the fuss abroad was all about. Then suddenly it was announced at the end of the school year that when school reopened in January, first year high school pupils would be taught the "new math". I was told that there were no in-service training programmes, few instructions, and no textbooks at first. Many teachers were so furious and so sure that the whole idea was nonsense that they did not try very hard to find out what they should be doing. Of course their antipathy or antagonism was transferred to their classes, and several, though certainly not all, of the teachers I saw who had excellent techniques were getting most unsatisfactory results. This situation forms quite a contrast with the introduction of "new approach" mathematics

in Ontario. I cannot help thinking of New South Wales whenever I hear complaints that our Department of Education is not moving quickly enough to make new curricula compulsory.

The problems facing school officials in the Asiatic countries we visited are so immense as to seem almost impossible to handle, but teachers keep trying.

In Malaya, India, and Ceylon the languages which have replaced English as official languages originated in agrarian societies and have no technical vocabulary. While we were in Malaya a government publication listed over 10,000 new words which were being officially added to the language as of that date so that such things as mathematics and automobiles could be discussed without using English. In fact most of these words are corruptions of English words, but Nationalism demands a private language for each country. Ceylon has two official languages, either or both of which may be used in high school. Hence we found in an excellent private school that part of a class receives its science instruction in Tamil, the other part in Singalese, and then all the boys flock to an illegal lesson in English to find out what it was all about.

In several of the Asian countries the Chinese minority are the ones interested in education and provide most of the high school pupils. We met young Canadian and American teachers who had received crash programme instruction in Malay and ready to teach in North Borneo, only to find when they got there that all their pupils were Chinese. In Singapore one university gives instruction in English, the other in Chinese. Many of the young men from Malaya, Thailand, Cambodia, etc. who are being given a year or more of post-high school education in New Zealand and Australia are of Chinese ancestry. Most of them still think of themselves as Chinese after their families have lived for five to ten generations in their new lands.

Hong Kong has its own peculiar disadvantages and advantages for education. Since there are almost no taxes, there can be little government support for education. A very small staff of administrators and inspectors supervises the multitude

of private schools all over the colony. Their job is made more difficult by the fact that many of the schools change rented quarters every eleven months because of the local rent and "key money" customs. The education literature from China is so full of propaganda that the Hong Kong officials have not the time to sift it for useful ideas. Ideas from the West are sometimes not easily adapted. For example, in the use of cuisenaire rods the children here call the number rods by colour and quickly learn a written code involving the word or initial letter of the colour name. But the Chinese characters for colour names are mostly so complicated that they are not learned until at least the third year of school, and of course there are no initials. A completely different code had to be developed. However, the people are so fanatically eager to have their children in school that immense difficulties are overcome.

India, of course, has many excellent schools and universities as well as many extremely substandard schools and great areas entirely without schools. In general the better high schools are private and the government operated schools rather poor. The goal of a majority of high school boys is to get a certificate which will open the door on a secure white collar job in which they can avoid doing any work. Hence one finds certificates in abundance; many of them have the word "failed" stamped in small letters at the bottom. In one school I saw fourteen year old boys doing quite difficult work in differential calculus with an excellent teacher, but some were memorizing without understanding.

The adult illiterates in many areas of India are so limited in culture that millions of people not only do not know how to read and write but actually have no concept of the counting numbers - one, two, and three. Putting battery radios into villages of such people and trying to broadcast instruction is a waste of time because they have so little to begin with. Is it any wonder that one Indian told me,

"Frankly, we have to abandon most of the adults in the villages and concentrate on the children."?

This article is devoid of the mathematical content expected in the Gazette. However, perhaps being aware of some of the problems of the other teachers will make us more appreciative of our own situation.

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The Secondary Education Committee of the Ontario Mathematics Commission has prepared a list of paperback books and pamphlets for use in secondary school mathematics classrooms. These inexpensive books are suitable for teacher and student reference. They may also be used to supply enrichment material for students. A copy of this list was sent to every secondary school principal by the Department of Education. Additional copies may be obtained from:

The Ontario Mathematics Commission
1260 Bay Street,
Toronto 5, Ontario

In order that teachers may study these books before purchase, the OMC is preparing a "roving library". The library will include copies of books on the list that the publishers have provided free. Any group may borrow the library for professional development days, etc. by paying the express charges both ways. Requests for the library should be sent to:

A. W. Harris,
Department of Mathematics,
Althouse College of Education,
1137 Western Road,
London, Ontario.

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SECONDARY SCHOOL MATHEMATICS IN AUSTRALIA

T.G. Room
University of Syndey, N.S.W., Australia.

1. Background information

The Australian Commonwealth is a federation of six States (and some Federal Territories) with an area (omitting New Guinea) about equal to that of the U.S.A. (omitting Alaska) and with roughly the same shape but 1000 miles nearer the Equator.

Population:

Commonwealth: 11.5 million.

State of New South Wales: 4.5 m. with 2.5 m. in the Sydney area, and 3.5 m. within 100 miles of Sydney.

Other States range down to 300,000 in Tasmania.

Education is a State responsibility with (below tertiary level) almost no direct Federal contribution of finance. Each State has its own highly centralized system; at an unofficial level there is reasonable cooperation between States.

The length of the school course and the nature of the curriculum have been under review by each State during the past ten years; some States have already made significant changes, the others are on the verge of doing so.

This report refers to the State of New South Wales, in which the new pattern began with the children who entered Secondary Schools in February 1962.

2. Organization of Secondary Education in N.S.W.

Primary School: six years, $5\frac{1}{2}$ - $11\frac{1}{2}$

Secondary School: six years, $11\frac{1}{2}$ - $17\frac{1}{2}$

Age to which attendance is compulsory: 15.

The school year runs from February to December, with a minimum of 180 full teaching days spread over 40 weeks, with breaks of a fortnight in May, three weeks in August and seven weeks in December - January. Each week consists of 40 periods (35 effective teaching periods after allowing for assembly, sport and so on) each of 35 minutes.

Schools

State High Schools	approx. 65%	of all children attending High School
Independent High Schools, R.C.	" 20%	
Independent High Schools, other	" 15%*	

Approximate numbers in all schools:

First year	60,000
Fourth year	40,000
Fifth year	20,000 .

All recently-built State High Schools are "comprehensive" and older schools are to be converted. The plan of the school is based on 8 to 10 classes (of 30^{+} pupils) for each grade in the earlier years. There is no entrance test, but pupils enter a school with their primary records and are sorted during the first year of the secondary course into three streams for each subject.

The pattern in independent schools is similar, but the boys' and girls' schools are separate, and the schools have the sanction that they are not bound to accept any child as a pupil nor to allow him to continue.

3. Pattern of courses

(a) Course to the School Certificate (Four years $11\frac{1}{2}$ - $15\frac{1}{2}$) Sorting to be carried out in the first year into three levels in each subject: the levels are

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* The proportion in the "other" independent schools of children over 15 is much higher - nearly 30%.

Ordinary: ranked 16% to 50% on some "measure of aptitude",

Ordinary with Credit: 50% to 75%,

Advanced: 75% to 100% .

Each subject is examined externally at the end of the course (one paper for each level). Results are compounded from this examination and the school record.

Subjects and measure of weight in periods per week of 35 periods.

Year 1. Core: English, Social Studies, Mathematics, Science, about 7 units each. Non-examinable courses in art, music, craft, etc.

Years 2, 3, 4

Core: English (7), Social Studies (5), Mathematics (5), Science (5)

Electives (2): Languages etc., Crafts, History, Geography (in place of Social Sciences) (5 each) + non-examinable subjects.

(b) Course to the Higher School Certificate

(Two years, $15\frac{1}{2}$ - $17\frac{1}{2}$)

No entrance requirements except a "pass" in the School Certificate, but syllabuses are written on the following basis

Level III: Pupil is competent in Ordinary-level course

Level II : Pupil is competent in Ordinary-with-Credit Course

Level I : Pupil is competent in Advanced course.

Measure of weights of courses:

All subjects except Mathematics and Science		Mathematics and Science	
III	6 units	III	6 units
II	6	II	Short 6
I	8	II	Full 9
		I	11

Examinations (external) one or two papers each of three hours; only slight reference to school record.

Numbers

School Certificate, 1965			40,000
Entered course for H.S.C., 1966			20,000
Mathematics Level III	3700		
II S	7800		
II F	4900		
I	1000	17,400 .	

(c) University Entrance Requirements

Passes in H.S.C. in 5 subjects of which one must be English.

Mathematics IIF or I + Science IIF or I counts as three subjects.

The normal level of pass required is Level II, but one in Level III will be accepted. (The standard of pass has not yet been laid down.)

The effect of this requirement has been to make Level III non-U, and this accounts for the high number entered for mathematics at Level IIS and the low number entered for Level III.

(d) Construction of syllabuses

Syllabuses are constructed by Committees appointed by the Government through the Secondary Schools Board (for the S.C.) and the Board of Senior School Studies (for the H.S.C.). Each Committee has about 20 members and the two Committees on each subject overlap to some extent. On the S.C. course Committee, the Universities have 2 representatives (the rest of the committee is teachers and officials); on the H.S.C. course Committee the Universities have 7 representatives and the right to nominate the chairman.*

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* There are five Universities in N.S.W., three in Sydney: University of Sydney (16,000 students), University of New South Wales (12,000), Macquarie University (starts 1967), University of New England (300 miles N.W. of Sydney - 2000), University of Newcastle (100 miles up the coast from Sydney - 1000).

Syllabus Committees cannot write text books, and individual members may do so only with explicit permission. The Mathematics Syllabus Committee has written notes to guide teachers over the more unfamiliar territory.

4. Secondary Mathematics courses.

(a) School Certificate course.

The new syllabuses, at three levels, were first taught in 1962 and the first examination was held in November, 1965*. The syllabuses (and the examination papers) were considerably influenced by the Illinois and Stanford programs.

The only Euclidean Geometry is in the Advanced Course. Graphs are used freely at all levels from the beginning, but analytical geometry as such is treated only in the Advanced Course. Discussion of the collection, display and interpretation of numerical data is included in all courses.

One possible point of interest is that in the Advanced Course, the formal set algebra is linked with the algebra of functions which are such that their range is $\{0,1\}$ when the domain of the variables is $\{0,1\}$. This representation of the set algebra was introduced both to show how a system can be defined by two quite different sets of axioms, and to provide the students with practice in algebraic manipulation.

(b) Higher School Certificate Course.

The headings of the various topics (in general in the order in which they might be treated) are listed below. The treatment is intended to be "classical interpreted in the spirit of the School Certificate Course". The syllabus is to be revised in four or five years' time in the light of

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* Professor A. J. Coleman of Queen's University has copies of the syllabus, notes and first examination papers.

experience and developments in Australia and overseas.

Level Three (lowest)

Products and factors -- Quadratic equations -- Logarithms and the slide-rule -- Elementary solid and spherical geometry -- Locus, graph and envelope (the ideas of this section are meant to permeate much of the course) -- Function -- Elementary trigonometry -- Number, number systems and numeration systems (largely review of S.C. work) -- Series -- Sets -- Probability -- Applications of trigonometry to geography and navigation.

Level Two (Full --- the Short course omits items marker "F" and parts of some others.)

The real number system -- The concept of function and its representation in analytical geometry -- Trigonometric ratios -- The linear expression and the straight line -- Series, sequences and the principle of mathematical induction -- Tangent and derivative -- The quadratic polynomial and the parabola -- (F) Change of coordinate systems. Transformations -- Geometrical applications of differentiation -- Integration -- The logarithmic and exponential functions -- The trigonometric functions -- (F) The inverse trigonometric functions -- (F) Analytical geometry in three dimensions -- Applications of the calculus to the physical world -- (F) Further trigonometry and calculus -- (F) Further polynomials -- (F) The binomial theorem -- The Theory of probability. (The last two items may be fitted in anywhere.)

Level One

The first level syllabus is arranged as six main themes: Arithmetic, Algebra, Calculus, Geometric Algebra, Dynamics, Probability. The first four themes are intended to be developed side by side, while the fifth theme may be introduced as soon as the calculus is strong enough, and the sixth may be treated at almost any time. Theoretical arithmetic (The integers, rationals, reals -- sequences and series -- complex numbers). Algebra (Polynomials --

Taylor's theorem -- rational functions). Calculus (The function concept -- continuous functions -- tangents to curves -- the problem of areas -- exponential and logarithmic functions -- area of curves, trigonometric functions -- integration). Geometric algebra (Plane analytical geometry -- linear transformations, matrices (2x2 only but developed as far as Cayley-Hamilton and similarity) -- analytical geometry in three dimensions (later to be developed using matrices)). Elementary dynamics of a particle. Theory of probability.

MODERN MATHEMATICS IN SCOTLAND

A.W. Harris
Aldhouse College of Education

In May of 1965 I used some holidays accumulated during four years with the Department of Education to visit schools in Britain and Denmark. Accompanied by my wife, I spent a week in Scotland two weeks each in England and Denmark, visiting classrooms and interviewing teachers and officials about the teaching of Mathematics.

The School Mathematics Project, supervised by Prof. Bryan Thwaites of Southampton University, and the Midlands Mathematics Experiment, directed by Mr. Cyril Hope of the City of Worcester Training College for Teachers, both let me see important and representative samples of the English move to a modern programme for the schools. The Southampton project especially has had an influence far beyond the schools in the original experiment.

In Denmark a radical revision of the gymnasium programme for Mathematics was completing its second year. Elaborate plans were being worked out for the retraining of the teachers of Mathematics in the elementary schools.

The experimental programmes differ considerably in detail from area to area; however they have many similarities and it is obvious that the meetings and reports sponsored by the Organization for Economic Cooperation and Development[1] have formed a basis for the reforms, to varying degrees. It seems that in all of the countries the teachers now realize that the courses at all different levels of instruction should be closely connected.

The work of the Scottish Mathematics Group was at a very interesting stage of development last year. Partly because of a sentimental interest in Scotland, and partly because, (due to the kindness of Mr. A. G. (Alec) Robertson, Her Majesty's

Inspector in Mathematics), I was able to study the Scottish experiment in more depth, I have decided to restrict this article to the work north of the border.

Secondary education in Scotland begins at age 12, being a five or six year course, leading to the Ordinary Certificate after four years. The curriculum has traditionally been strongly influenced by the state-set external examinations for school leaving. Recently a colleague lent me copies of reports of the Advisory Council on Education in Scotland, which was published by the Scottish Education Department in 1947. These hard-hitting analyses caused a number of changes to be made. In Secondary Education, in the section on Mathematics, the authors stated: "There is agreement that school mathematics in Scotland needs an overhaul, being too formal, divorced from practice and excessively concerned with inner consistency and logical sequence."

The report mentions, among other things, the undue prominence given to formal geometry, the excessive manipulative work in algebra and trigonometry, the compartmentalization of Mathematics. Some recommendations for improvement were greater emphasis on three-dimensional work taught by practical methods, and earlier introduction of logarithms, the slide rule, trigonometry, graphical work, calculus and analytic geometry.

The most recent changes originated in the report of the Committee on Mathematics which was formed in 1962. Their report, "Recent Changes in Honours Courses in Mathematics" mentions set theory, logic, mappings, abstract algebra, groups, rings and fields, vector spaces, topology, among others as topics which have an increasingly important place in pure Mathematics. The Committee recommended that an alternative syllabus for secondary school Mathematics be set up. As a result a Syllabus Committee consisting of the principal teachers of Mathematics for fifteen schools, the principal lecturers in Mathematics at two Colleges of Education, and four H.M.I.'s was formed early in 1963.

The Syllabus Committee made an interim report in April, 1964. In September of that year a complete trial of the alternative

syllabus was begun in the fifteen schools at the first level. Examinations on the Alternative Syllabus will be provided at the Ordinary and Higher levels in 1968 and 1969 respectively. The trial of the new syllabus was widened to 60 schools in September, 1965.

The Alternative Syllabus is contained in a pamphlet published by the Scottish Education Department[2]. At the risk of losing the accuracy that only a detailed statement could ensure, it may be useful to give a brief account of some of the changes. About one-third of the content of the arithmetic syllabus is new, including the laws of operation of the number system, number bases other than ten, an iterative method for square roots, elementary probability and a greater emphasis on statistics. For the remainder, less emphasis is given to computation, and the use of slide rules and calculating machines is encouraged where appropriate; much greater stress is put on approximation and the appropriate limits of accuracy of measurements.

In Algebra again about one-third of the content is new to the school syllabus, but much of the remainder is given a fresh approach or set in a new context. The language of sets, inequations and equations, the meaning of a variable and the idea of a function are stressed throughout. The CAD laws are studied and used widely and a thorough treatment of graphs provides a background for the later development of calculus.

In Geometry, the use of coordinates and transformations is stressed. Geometrical ideas and facts are used with the language of sets and logic. Vectors, as directed line segments and as number pairs, are introduced early.

In Trigonometry, the general definitions of the functions are introduced and their periodic nature is stressed. Coordinate methods and functional notions are brought in from the beginning.

The Syllabus Committee have begun publishing their work under the name of The Scottish Mathematics Group. The title of the series is Modern Mathematics for Schools (Blackie and Chambers). Books 1 and 2 are now available. You will find

them reviewed in the Book Review section.

When I arrived on the scene in early May, 1965, the trial of the Alternative Syllabus with 12 year olds had been going on for the best part of a year. My first port of call was Jordanhill College of Education in Glasgow, which has a fine name for training elementary and secondary school teachers. The principal lecturer in Mathematics is Mr. A.G. Sillitto who has been active in the new developments for several years. His principle interest is Geometry; he was one of the writers of the S.M.P. material.

Mr. Sillitto believes strongly in a programme which brings in group concepts with definitions of geometric forms through motions. The introduction should be intuitive through operations with physical things. In the teaching process he is keen on having involvement of the pupils, freedom for an argument to develop, discovery being what the pupils are ready to discover.

Mr. Sillitto took me along to nearby Jordanhill College School, which educates pupils from infant school to Higher Level. Mr. Finlayson, the principal teacher of Mathematics (department head), one of the members of the Syllabus Committee, stressed the importance of the early introduction of coordinate geometry. In Mr. McDonald's classroom, 12 year old boys and girls had a lesson on ordered pairs and relations, using arrows to show the mapping: e.g. "owns", "is the father of". I was impressed at Jordanhill, as I was at all of the Scottish Schools I visited, with the attention paid to pedagogy, the careful patient questioning.

The following day, back in Edinburgh, I met Mr. A.G. Robertson, H.M.I. at the Education Department, who discussed the Scottish experiment in some detail. Mr. Robertson was an R.A.F. pilot during World War II, having done his training in Canada.

Mr. Robertson feels that in Geometry, Euclid is out. The first introduction will be with shapes, sizes and motions, with initial assumptions that fit. This will give the pupils early

experiences fitting the pattern of particular to general and concrete to abstract. The aim is to build up a set of transformations which will sweep out Euclid along the way. The primary concern in this is two-fold: (1) the pupils' interest in mathematics, (2) useful mathematics.

North Berwick High School is in a resort town on the south side of the Firth of Forth, about 20 miles east of Edinburgh. There, Mr. Mooney, the principal teacher of Mathematics showed me a copy of Mathematics 9 which he had acquired somewhere. Mr. Robertson took over a first year class in order to test them on the knowledge of the experimental work in geometry. One of his questions was "How many ways can a rectangle be fitted back into its shape?". The relationship of this property to the number of axes of symmetry, and for solids, to the planes of symmetry is investigated intuitively.

The North Berwick school is comprehensive, having pupils of widely varying abilities, and all 12 year olds were trying the new materials. The rector, Mr. Davidson, maintains that the pupils in the low ability classes show improved interest in the new approach work and have developed a more mathematical language. Parental complaints about the trials have been negligible.

In Edinburgh itself I had the good fortune to be taken to Sir George Heriot's School, a charitable foundation school, dating back to the 17th century, founded by the King's Jeweller. The school has about 1500 pupils, infant school to sixth form. Many are on scholarships. Fees are about £ 20 per term. In a chat with Messrs. Campbell and Galloway, who supervise the Mathematics in the senior and junior school respectively, I learned that the proposal to extend the reform of mathematics teaching to the early grades is a cause of great concern in Scotland. (I noticed an item in the newspaper about threats of a strike among elementary teachers).

During my brief visits to a handful of Scottish schools, I gained a number of impressions about the educational system. I shall list just a few:

a) The schools have a great deal of local autonomy. The headmaster is largely free to set policy. The curriculum is only an examination syllabus. The external examinations tend to produce some uniformity, being as always an excuse for inertia among the unimaginative. However, the opportunity for creative planning is there.

b) The role of the inspector is largely advisory. The rating of teachers is practically non-existent, being used only when the permanent certificate is given.

c) The programme for gradual introduction of the Alternative Syllabus has been carefully worked out; the canny Scots should be able to avoid some of the excesses and errors of earlier reforms.

d) The schools do not have the financial resources of Ontario; it is clear that physical facilities are no replacement for the dedicated teaching which I believe I saw in Scotland.

e) In Scotland, Euclid seems to be out.

References: 1. (i) New Thinking in School Mathematics
(ii) Synopses for Modern Secondary School Mathematics
(iii) Mathematics Today - a guide for teachers (published by O.E.C.D., available from Queen's Printer, Ottawa).

2. Various Interim Reports on the Mathematics Syllabus, Scottish Education Department, St. Andrew's House, Edinburgh.

A LOOK AT CUBIC EQUATIONS

John Laberge*

Given: $y = ax^3 + bx^2 + cx + d$.

Required: to find the slope of the line joining the maximum and minimum points of a cubic function.

Solution: If $y = ax^3 + bx^2 + cx + d$,
then by using calculus, we know that

$$\frac{dy}{dx} = 3ax^2 + 2bx + c \ .$$

To find the abscissae of the maximum and minimum points, let $\frac{dy}{dx} = 0$.

Therefore, $3ax^2 + 2bx + c = 0$

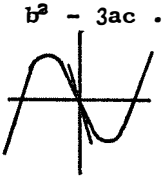
$$x = \frac{-2b \pm \sqrt{4b^2 - 12ac}}{6a}$$

$$= \frac{-b \pm \sqrt{b^2 - 3ac}}{3a}$$

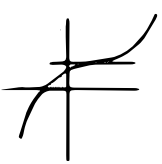
Here, $\sqrt{b^2 - 3ac}$ is a kind of cubic discriminant. Let it be represented by the symbol D .

$$\therefore \quad x = \frac{-b \pm D}{3a}$$

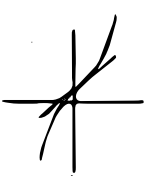
It seems reasonable that the shape of the graph depends on $b^2 - 3ac$.



$b^2 - 3ac > 0$



$b^2 - 3ac = 0$



$b^2 - 3ac < 0$

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* A grade 13 student last year at St. Michael's College School, Toronto, where he discovered for himself an interesting property of cubic equations.

Let $x = \frac{-b + D}{3a}$

$y = ax^3 + bx^2 + cx + d$

$= a(\frac{-b + D}{3a})^3 + b(\frac{-b + D}{3a})^2 + c(\frac{-b + D}{3a}) + d$

$= \frac{D^3 - 3bD^2 + 3b^2D - b^3}{27a^2} + \frac{D^2b - 2Db^2 + b^3}{9a^2}$

$+ \frac{Dc - bc}{3a} + d$

$= \frac{(D^3 - 3D^2b + 3Db^2 - b^3) + (3D^2b - 6Db^2 + 3b^3) + (9acD - 9abc) + 27a^2d}{27a^2}$

$= \frac{D^3 - 3Db^2 + 2b^3 + 9acD - 9abc + 27a^2d}{27a^2}$

$= \frac{D(D^2 - 3b^2 + 9ac) + 2b^3 - 9abc + 27a^2d}{27a^2}$

$= \frac{D[D^2 - 3(b^2 - 3ac)] + 2b^3 - 9abc + 27a^2d}{27a^2}$

$= \frac{D(D^2 - 3D^2) + 2b^3 - 9abc + 27a^2d}{27a^2}$

$= \frac{-2D^3 + 2b^3 - 9abc + 27a^2d}{27a^2}$

And if we let $x = \frac{-b - D}{3a}$

we eventually get $y = \frac{+2D^3 + 2b^3 - 9abc + 27a^2d}{27a^2}$

We now have a set of coordinates representing the maximum and minimum points of the cubic function. The task now is to find the slope of the line joining these two points.

rise = difference of ordinates

$= \frac{-4D^3}{27a^2}$

run = difference of abscissae

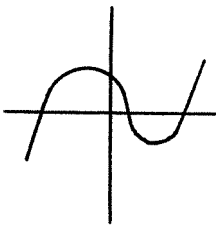
$$= \frac{2D}{3a}$$

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

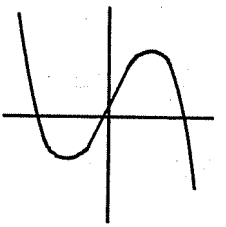
$$= \frac{-2D^2}{9a}$$

$$= \frac{-2(b^2 - 3ac)}{9a}$$

If $b^2 - 3ac > 0$, then the sign of the slope (that is, positive or negative) is opposite to the sign of a. Likewise, if $b^2 - 3ac < 0$, the sign of the slope is the same as a. Thus, the sign of a is most important. This confirms what was stated above, that the shape of the graph depends on $b^2 - 3ac$, and we have found a simple formula to help us.



$a > 0$



$a < 0$

PARAMETRIC EQUATIONS - A NOTE FOR THE CURIOUS

by

Norman Miller

The equations $x = \cos\theta$, $y = \sin\theta$, with parameter θ , are equivalent to the Cartesian equation $x^2 + y^2 = 1$. We are accustomed to the statement that a pair of parametric equations denote the same locus as the Cartesian equation that results from eliminating the parameter. This statement holds for the example just given. We consider now some other examples.

Example 1. $x = \cos\theta$, $y = \cos\theta$. This seems to be just as awkward way of saying that $y = x$, which represents an unlimited straight line. But in the parametric equations x and y are bounded: $|x| \leq 1$, $|y| \leq 1$. Hence these equations represent only that segment of the line that joins the points $(1,1)$ and $(-1,-1)$. (Fig. 1). As θ increases continuously from $\theta = 0$, the point $(\cos\theta , \cos\theta)$ starts at $(1,1)$ and runs back and forth on the line segment. It should be possible to create an animated cartoon showing the point θ running round a circle while the corresponding point (x,y) rushes back and forth on the segment. One could extract more amusement from this example. If θ moves at a uniform speed around its circle, then we find that the corresponding point (x,y) has a limiting speed of zero as it reaches the points $(1,1)$ and $(-1,-1)$ and that the speed increases to a maximum as (x,y) passes through the origin.

Example 2. $x = a\cos 2\theta$, $y = 2a\sin\theta$. Elimination of θ gives $y^2 = -2a(x-a)$. The point (x,y) is confined to this parabola but, since $-a \leq x \leq a$, $-2a \leq y \leq 2a$, its locus is only an arc of this parabola joining $(-a, -2a)$ and $(-a,2a)$ (Fig. 2).

Example 3. $x = e^t$, $y = e^{2t}$, whence $y = x^2$. Since e^t and e^{2t} are always positive, the locus of (x,y) consists of the portion of the parabola that lies within the first quadrant (Fig. 3). The origin is not included. If we think of t as tracing an unlimited t-axis, then the negative half of the t-axis is mapped on the parabolic segment between $(0,0)$ and $(1,1)$, the point $t = 0$ is mapped on $(1,1)$, and the positive half of the t-axis is mapped on the arc above the point $(1,1)$.

The reader may find some interest in graphing the precise loci denoted by the following pairs of parametric equations.

1. $x = \sin t$, $y = b + m \sin t$.
2. $x = \sec t$, $y = \sec t$.
3. $x = x_0 + 1/t$, $y = y_0 + m/t$.
4. $x = \sin^2 \theta$, $y = \cos 2\theta$.
5. $x = \sin \theta$, $y = \sin 3\theta$.
6. $x = \sin \theta$, $y = \cos^2 2\theta$.
7. $x = 2^{-t}$, $y = 2^t$.
8. $x = -2^{-t}$, $y = 2^t$.
9. $x = 2^{-t}$, $y = -2^t$.
10. $x = -2^{-t}$, $y = -2^t$.

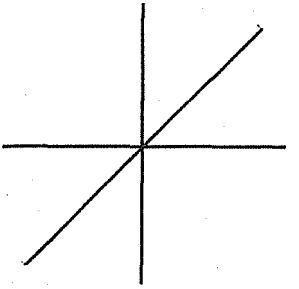


Fig. 1

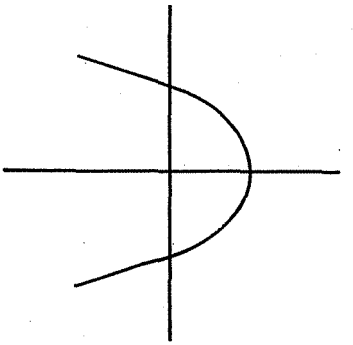


Fig. 2

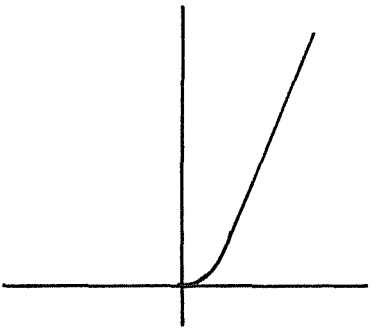


Fig. 3

TEACHERS HONORED BY A.P.E.O.

Two Ontario secondary school teachers, one in Toronto and the other in Niagara Falls, have been honored for their contribution to the teaching of mathematics and science by the Association of Professional Engineers of Ontario.

They are: J. Norman C. Sharp, who is a mathematics consultant for Etobicoke elementary and secondary schools, and also special assistant to the superintendent, Etobicoke Board of Education; and E. Gordon M. Overend, of Niagara Falls, head of the science department, A. N. Myer Secondary School in Niagara Falls.

The award was established in 1958 by the A.P.E.O. as a means of recognizing the contribution made to the engineering profession by the secondary school teachers in maths and science, who, in the opinion of the O.S.S.T.F., had contributed the most to the advancement and teaching of these subjects. All secondary school maths and science teachers in Ontario's 29 districts are eligible for nomination.

Mr. Sharp, a native of Wynyard, Sask., is a graduate of the University of Saskatchewan and received his M.A. degree from the University of Toronto. He has spent 28 years in the teaching profession, and is the co-author of several mathematics texts for secondary schools and is also past president of the Ontario Mathematics Commission.

Mr. Overend, a native of Toronto, graduated from the Ontario Agricultural College, Guelph, in 1937, and has been in the teaching profession for 27 years. He taught science at Niagara Falls' Stamford Collegiate between 1938 and 1957, and then assumed his present position.

Ottawa Journal,
Friday, February 25, 1966.

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COMPUTERS AND THE HIGH SCHOOL CURRICULUM:

SOME NON-NUMERICAL EXAMPLES

By T.E. Hull[1]
Department of Mathematics and Department of Computer Sciences
University of Toronto

The purpose of this paper is to describe example problems which can be introduced, along with the use of computers, into high school courses in mathematics. It appears that there will be a significant change in the kind of problem which can be considered.

In an earlier article (1) I have already described in a general way how I believe the high school curriculum should be affected by computers. I was primarily interested in the academic stream, but similar changes were advocated for the commercial and technical streams. The main feature of these changes is the introduction of algorithms (or procedures, or processes) as objects of careful study. I tried to show how algorithms fitted in very nicely with current developments in the mathematics courses. I maintain that a considerable emphasis should be placed on a study of algorithms, while the computer, along with appropriate compilers, is treated simply as a means of implementing algorithms.

If this point of view is adopted, there remains the question of deciding what kinds of examples should be used. Of course the examples considered will depend on whether the students are

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1 The Proceedings of the Fifth National Conference of the Computer Society of Canada, held at Banff, May 30 - June 1, 1966.

in academic, commercial, or technical streams, and they will depend on whether the subject being treated is mathematics, or science, and so on. In this article I consider only mathematics courses. The examples, and my point of view, are best suited to the academic stream, but they can also be adapted to the interests of students in other streams.

The possibilities

There are three basically different problem areas which might be considered. I will refer to them as numerical, non-numerical (or combinatorial), and simulation problems. The main point I wish to develop is that, although numerical examples may appear to be natural extensions of what is already done in the high schools, there are good reasons for shifting emphasis to the other areas. This point will be illustrated by comparing examples, mainly from the numerical and non-numerical areas.

Numerical examples involving matrices

High school students are now taught how to solve two equations in two unknowns. (The equations are of course linear and algebraic). They also learn to solve three equations in three unknowns. Once a computer becomes available, it is tempting to write a program for solving ten equations in ten unknowns, or for solving twenty equations, or n equations.

It is my contention that it is a mistake to go in this direction, or at least to go very far in this direction. It is true that a high school student can easily understand the basic idea in a procedure for solving n equations in n unknowns. Such procedures involve systematic elimination, in one form or another, and are straightforward generalizations of what he does with two equations, or with three equations.

There are, however, two good reasons for not going in this direction. The first is that high school students do not have enough good sources of real-life problems which lead to such large systems of equations. This is because major sources of these problems are likely to depend on differential

equations, or advanced statistics, for example. The second reason for not proceeding in this direction is that the analysis of error in matrix calculations is very difficult.

I do not advocate that all numerical problems should be omitted. Some of the simpler ones certainly should be considered, such as two equations in two unknowns, or quadratic equations. Since the computer does the tedious arithmetic, the student can spend his time developing carefully constructed algorithms, and making sure that he has provided for every eventuality. Another good example arises in computing approximations to the area under a curve, which is also a good introduction to the calculus.

In order to make comparisons with later examples, let us consider one simple example of what would be needed for a study of large systems of equations. Consider the product of two matrices of order n. If $C = (c_{ij})$ is to be the product of $A = (a_{ij})$ and $B = (b_{ij})$, we have

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{in}b_{nj}$$

for $i,j = 1,2,\ldots,n$.

The part of a computer program for finding such a product would appear in FORTRAN as

```
      DO 1 I = 1,N
      DO 1 J = 1,N
      C(I,J) = 0 .
      DO 1 K = 1,N
    1 C(I,J) = C(I,J) + A(I,K)*A(K,J)
```

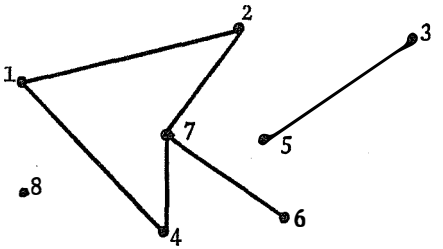
This is to be compared with parts of programs considered in the next section. The point of the comparison will be that quite similar programs are needed in some non-numerical applications. Moreover, as we shall see, the non-numerical problems are better suited to high school courses.

Non-numerical examples involving matrices

A rich variety of non-numerical problems can be described

in terms of matrices. The matrices that arise in this context will sometimes look like the familiar numerical matrices, but it turns out that we treat them in a different, and basically simpler way.

Consider a trivial example to begin with. The following diagram shows several points, numbered 1,2,.... Some pairs of points are joined, and some are not. For example, 1 and 2 are joined, but 1 and 7 are not.



Such a diagram is often called a graph. We can think of the points as being points in a maze, and the joins as tunnels connecting the points. Or the points can be cities, and the joins can be highways connecting the cities. The graph can also represent an electric circuit, or a telephone circuit.

The "connectedness" of any such diagram can be represented by a matrix, usually called an adjacency matrix. If this matrix is $A = (a_{ij})$, then the i-jth component denotes whether or not there is a connection between the ith and jth points. For example, a_{ij} can be 1 if there is a connection, but 0 if not. In FORTRAN IV it is natural to use logical variables, and then A(I,J) can be .TRUE. if there is a connection, but .FALSE. if not. The diagonal elements can be treated either way; we will take them to be 0 , or .FALSE., meaning there is no connecting link between an element and itself.

We are now ready to pose some problems. Suppose we are given the adjacency matrix for a particular graph. Can we then determine whether or not there is a path between two particular points? Or can we determine how many distinct paths there are between two points? Or, if there is at least

one path between two particular points, can we find which one has the smallest number of intermediate points?

In each case we are asking whether or not we can develop an algorithm (or a computer program) which will provide us with an answer to our question. The input to the program is the adjacency matrix, along with the numbers of the two points. The output is "yes" or "no", a number, or a path, depending on which question is being answered.

Some questions of this kind are quite a bit more difficult than others. Nevertheless, it is relatively easy for a high school student to understand the questions. In this respect, such questions are more suitable for high school courses than are those which involve the more usual numerical matrices.

Moreover, manipulations with these new kinds of matrices do not require any complicated error analyses, and are therefore more suitable in this respect as well. We will not try to prove this fact completely, but we can indicate the source of the difference by pointing out that roundoff error does not enter into the basic operations one performs on these new matrices, at least it does not enter in any important way. This is illustrated by considering the following question, which is simpler than those mentioned earlier, but which is clearly a step in the direction of solving some of the earlier questions.

Suppose we are given an adjacency matrix A. We can think of A as telling us which pairs of points are connected by paths consisting of exactly one join, or link. The question is: can we find another matrix, say C, which tells us which pairs of points are connected by paths consisting of exactly two links? The answer is yes. Clearly, the required value of the i-jth component c_{ij} should indicate that there is such a path if a_{i1} and a_{1j} both indicate paths having one link each, or if a_{i2} and a_{2j} both indicate such paths, or if a_{i3} and a_{3j} ... etc. With logical variables, this becomes

$$C_{ij} = (a_{i1} \text{ and } a_{ij}) \text{ or } (a_{i2} \text{ and } a_{2j}) \text{ or } \ldots$$

This is required for $i,j = 1,2,\ldots$

In FORTRAN IV the matrix C can be obtained with the following:

```
      DO 1 I = 1,N
      DO 1 J = 1,N
      C(I,J) = .FALSE.
      DO 1 K = 1,N
    1 C(I,J) = C(I,J) .OR.  (A(I,K) .AND. A(K,J))
```

Of course A and C must both be of logical type. (If logical variables are not available, and the values 0 and 1 are used as mentioned earlier, we can use $c_{ij} = \max(a_{i1}xa_{ij}, a_{i2}xa_{2j}, \ldots)$ in place of the logical expression given above).

The diagonal elements of C need to be interpreted in a special way. They turn out to be .TRUE. (or 1), except for elements which correspond to points which are completely isolated from all other points in the graph. Thus, for example, if A(3,5) is .TRUE., C(3,3) will also be .TRUE., indicating that there is a path of two links connecting point 3 to itself, namely the path from 3 to 5 and back again.

It should now be clear how to derive other matrices to tell us which pairs of points are connected by paths of length 3, of length 4, and so on. Some of our earlier questions can be answered with the help of these matrices. The question about whether or not there is a path is straightforward, and so is the one about which path has the smallest number of intermediate points. But the question about the number of distinct paths is much more challenging.

The approach indicated here is not the only way to tackle these problems, but there is an interesting analogy between some aspects of this approach and the earlier numerical problems. This is evidenced by comparing the two FORTRAN programs.

We will not consider other approaches in any detail, but it is worthwhile giving a brief indication of one other. Suppose one particular point on the graph is taken to be the starting point for a path. Then we can make, with the help of the adjacency matrix, a list of all points which can be

reached from the starting point by paths of length one. Then we can make another list of all further points which can be reached by paths of length two, and so on. Eventually we reach the desired terminal point, or we find it cannot be reached at all. Suppose the terminal point is reached by a path of length s. Then we can trace back through points reached by paths of length $s-1$, $s-2$, etc., until we reach the starting point.

This approach is harder to program than the earlier one, but it would be interesting to compare its relative efficiency.

Related non-numerical examples

The previous example can be modified and extended in a variety of ways. For example, the adjacency matrix can be replaced by a distance matrix, or a cost matrix. Then a_{ij} is the distance between two points, or the cost of moving from the ith point to the jth point. In this context it is likely that a path exists between every pair of points, so that questions about existence of paths, or even about the number of distinct paths, are not the most interesting. The emphasis now would be on finding the shortest path, or the path of minimum cost.

A start in this direction is obtained by finding the matrix C in which the i-jth element is the minimum distance, or minimum cost, for a path of length two. The calculation depends on the following:

$$c_{ij} = \min(a_{i1} + a_{1j},\ a_{i2} + a_{2j},\ \ldots)$$

Diagonal elements need special attention, as before, but the basic calculation is once again analogous to what was required for the usual matrix multiplication.

Another way of interpreting the matrix A is to let a_{ij} denote the capacity of a pipe joining the ith and jth points. This leads to a very difficult question about the maximum flow between two points in a network of pipe lines.

The matrix A need not be symmetric. The i-jth

element can, for example, represent the minimum time required to complete the jth job, once the ith has been completed. From this interpretation we are involved in the so-called "critical path" scheduling problems.

Other non-symmetric situations arise when a_{ij} is a measure of preference for an ith event compared to a jth event. The preference might be determined by the vote of a committee, for example. The elements of A can also be transition probabilities. Further examples can be found in the references (2-5).

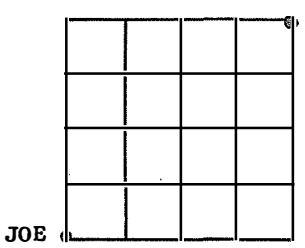
Simulations

So far I have considered only two of the three possible problem areas, the numerical and non-numerical areas, and I have given reasons for expecting the use of computers in high schools to cause a shift in emphasis from the first of these areas to the second.

The third area involves simulations. This area will not be discussed in any detail, but here again I believe we can expect a considerable impact on high school courses. Simple games can be simulated, and so can the formation of queues. The difficulties in this area are quite different from those of the other two areas. The main difficulty now is to interpret the results properly, and to appreciate whether or not they are significant. The main difficulty with numerical problems, on the other hand, is to carry out a proper error analysis. The difficulty with non-numerical problems is usually centred on the development of the algorithm, rather than with interpreting the results, or analysing the numerical errors.

The Blizzardville problem

An interesting problem is used by Professor G.E. Forsythe in his introductory course at Stanford University. It supposes a town, called Blizzardville, whose streets form a square grid as indicated in the following:



The diagram shows five streets each way, but the general problem involves n streets each way. The probability that the section of street between any two adjacent intersections is blocked by snow is $1-p$, where $0 \leq p \leq 1$. The probabilities for different blocks are independent of each other.

Joe lives at the southwest corner of town, while his girl Forlornia lives at the northeast corner. The problem is to determine the probability, as a function of p and n, that Joe can drive to Forlornia's house.

This problem involves simulations, as well as non-numerical or combinatorial considerations.

The problem has its origin in what are called percolation problems (6). For example, suppose we ask whether or not a liquid can penetrate to the centre of a lump of porous material. We can imagine a model of the material consisting of a large number interconnecting pores, some of which are so small that the liquid cannot get through, while others are open.

The analogous situation for Joe would be for him to live at the centre of town, preferably a three dimensional town, and to have a girl at each intersection on the edge of town. Joe has no preferences, and considers his problem solved if he can reach any one of them!

Conclusions and generalizations

A number of examples have been used to indicate, very briefly, how a variety of interesting and worthwhile problems can arise in "non-numerical" situations. We have made a point

of formulating most of these problems in terms of matrices, and we have indicated various ways of manipulating these matrices. This is not the only way of approaching these problems, but it does show how matrices and matrix calculations can arise without running into the difficulties which exist with ordinary matrices, and the usual numerical calculations. The sources of these non-numerical problems are easily understood by high school students, and the methods of solving these problems do not require sophisticated error analyses.

Finally, it should be pointed out that our examples illustrate another general feature of computer applications. The matrices we have introduced are special cases of "data structures". Other examples of data structures include trees, lists, records, and so on. The operations on various kinds of matrices which we have considered are special ways of manipulating data structures. From this point of view, computing becomes essentially a study of data structures, and of techniques for representing and manipulating data structures.

References

(1) T.E. Hull, Computers and the high school curriculum, Quart. Bull. Comp. Soc. Canada, 6, no. 2 (Jan., 1966) 25-28, reprinted in Bull. Ontario Secondary School Teachers' Federation, 46, no. 1 (Jan., 1966) 27-29.

Further references on non-numerical problems

(2) Edwin F. Beckenbach, editor, Applied combinatorial mathematics (John Wiley, 1964)

(3) Robert G. Busacker, and Thomas L. Saaty, Finite graphs and networks (McGraw-Hill, 1965).

(4) Frank Harary, Robert Z. Norman, and Dorwin Cartwright, Structural models, an introduction to the theory of directed graphs (John Wiley, 1965)

(5) Oystein Ore, Graphs and their uses (L.W. Singer Company,

a division of Random House, 1963). A paperback in the New Mathematical Library series.

A reference on percolation problems

(6) J.M. Hammersley and D.C. Hanscomb, Monte Carlo methods (Metheun, also John Wiley, 1964), chapter 11.

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WESTERN ONTARIO REGIONAL MEETING OF ONTARIO ASSOCIATION OF TEACHERS OF MATHEMATICS

Place: Althouse College of Education, London
(Corner of Western Road and Sarnia Gravel Road, just west of U.W.O. campus. Enter from Western Road, parking in rear of the College.)

Time: 9:45 a.m. - 3 p.m. - Saturday, January 14

Programme

9:45 - 10:30 Registration and coffee

10:30 - 11:30 Geometry Through the Grades
Dr. H.A. Elliott, Head of Department of Mathematics, Royal Military College, Kingston

Dr. Elliott is a past president of the Ontario Mathematics Commission and was a member of the committee which prepared the new courses in Grades XI, XII, and XIII Mathematics. He is the co-author of a series of mathematics texts and is currently supervising experimentation in Mathematics from Kindergarten to Grade 6.

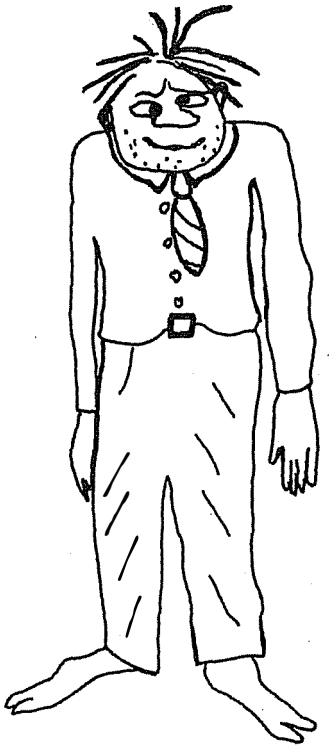
11:30 - 12:30 Seminar discussion groups on the implications of Dr. Elliott's address for each grade from Kindergarten to Grade 12.

There will also be seminars on new Grade XIII courses and Geometry in the 4 year programme.

12:30 - 2:00 Lunch at Althouse College and informal inspection of the building.

2:00 - 3:00 Geometry in the New Grade X.
Professor A.W. Harris, Head of Department of Mathematics, Althouse College.

2:00 - 3:00 Geometry in the Elementary School
Mr. J.R. MacLean, Assistant Superintendent, Curriculum Division, Ontario Department of Education.



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BOOK REVIEWS

Matematik 1 - by Erik Kristensen and Ola Rindung
(G.E.C. Gads Forlag, Kobenhavn, 1962)

Readers who know Danish would enjoy going through this well-written text book. It is reputed to be very faithful to the spirit of the reforms worked out in the seminars and reports of the O.E.C.D.

Prof. Kristensen is on the Faculty of the University of Aarhus, which has a good name for its Matematisk Institut, and H. Rindung is the Danish equivalent of Bob Mitchell, as far as I can make out.

Matematik 1 was written for the new course of study introduced into the first year of the gymnasium in 1963. The gymnasium takes pupils at the age of about 15-16 (compulsory schooling ends at 14). The percentage of the age group in the 3-year course is tragically small; there is great national concern because of the limited number of places in the Universities. This will be alleviated to a certain extent with the opening of two new universities in Odense and Esbjerg. However, at the moment only about 5% of the 15-19 year group will be exposed to Matematik 1, 2 and 3.

The quality of the presentation in this book is perhaps more of the junior college style. However, ideas are developed carefully and intuitively, with numerous examples given. The text book has 325 pages, exclusive of exercise material, which is given in a separate booklet. This is to be taught in a school year of our length, but with a 5-1/2 day week, Mathematics being given 50 minutes in each of 6 days.

The chapter headings are: Sets and Sentences, Sets of Numbers, Vectors, The Analytic Representation of the Straight Line, Mappings, Real Functions, Equations and Inequalities, Equivalence Relations, Induction, Integers, Algebra, Practical Applications of Logarithms, Trigonometric Functions, Geometric Applications of Trigonometric Functions.

The work on sets is written for pupils who have not had it before. There is nothing of our deductive geometry in this book or in books 2 and 3. Students come from the "real" or "middle" school with a great deal of intuitive and construction geometry, but with practically no geometry formally proved.

In the new book the implication symbol is used extensively in the algebra of equations, inequalities, and in the chapter Algebra, where a very rigorous dose of abstract algebra of groups, rings and fields is applied.

You might like to know how they do mathematics in Denmark. Here is a sample, from page 94, which I shall paraphrase.

Theorem: The straight line which goes through the point (x_o,y_o) and p which is perpendicular to the vector $\underline{n} = (a,b,)$ has the equation

$$a(x-x_o) + b(y-y_o) = 0$$

This theorem is stated at the end of the first section in chapter IV. In the section, it is stated that a point P will obviously lie on the line if and only if the vector P_oP is perpendicular to $\underline{n}$, or if P lies at P_o this can be expressed in the condition

$$\underline{n} \cdot \overrightarrow{P_oP} = 0$$

$$\ell = \{P \mid \underline{n} \cdot \overrightarrow{P_oP} = 0\}$$

(There is extensive use of set builder notation in this book; for example a vector is defined as follows:

$$\overrightarrow{AB} = \{\overline{PQ} \mid \overline{PQ} \equiv AB\}^*$$

Then, applying the rule for dot product, we get

$$\ell = \{(x,y) \mid a(x-x_o) + b(y-y_o) = 0\}$$

In an example, set builder notation is dropped and the result only used. For a line through (4,-1) which is

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* $\overline{PQ}$ and $\overline{AB}$ are individual directed line segments.

perpendicular to the vector $(2,3)$, the equation is

$$2(x-4) + 3(y+1) = 0$$

$$\Leftrightarrow 2x + 3y - 5 = 0$$

Last spring I had a talk with H. Rindung about the reception of the new course and his textbooks. It was still stoo early to draw conclusions about the change. Mathematics has always been stiff in the gymnasium. They have hopes for a better elementary programme in a few years which will help, but there are fears among the parents that this new course is more difficult than the old one.

It will be interesting to find out how Denmark made out after the first group graduate from gymnasium.

The Power of Mathematics - by Z.P. Eienes (Hutchinson Educational Press 1964)

In this concise (176 pages), clearly and simply written book, Dr. Dienes, who is at the University of Sherbrooke, P.Q. extends his discussion of the psychology of mathematics teaching, which he began in Building up mathematics in 1960, to the junior secondary school level. His proposals are based on the work of the Geneva School of Jean Piaget, the preliminary findings of the Harvard Center for Cognitive Studies, in which he participated and theoretical work he himself did at Leicester University, England.

While accepting many of the general trends of modern curriculum revisions, and conceding that we have been seriously underestimating the abilities of children to learn mathematical concepts at an early age, the author points out the importance of concrete materials and experiences which make the image of the idea vivid, before moving to the abstract idea and the formalization of the structure.

The chapter headings are: How Mathematics is Learnt, The Study of Powers, Roots and Logarithms, The Mathematical Game, The Mathematical Story, The Power of Mathematical Symbolism,

The Power of Mathematical Reasoning.

The model for a learning situation is made very explicit in the chapter on Powers and roots. We as teachers of a vital subject, need to strive for a more dynamic approach. Dr. Dienes has some ideas for us.

School Mathematics Project - Book T, Edited by A.G. Howson (Cambridge University Press, 1965)

The S.M.P. is under the direction of Prof. Bryan Thwaites of the University of Southampton. Book T is the first of a series to be published, the purpose of which is to provide texts for the full 5 year O level course, which begins at 11 + years. This book was prepared by staff members of a number of public schools (English version), including Sherborne, Winchester, Marlborough College and others. It was used first in pamphlet form in the school year 1964-65. The first O level examination based on this material was written in July 1966.

Book T is intended to break into the series at the level of Book 3, as the pupils of the above mentioned schools commence public school at age 13 + . I suppose T means transition.

This book has 294 pages; it is easy to read and beautifully illustrated, although I miss the use of colour, now that so many of our publishers have gone beyond black and white. The literary style is very good for a Mathematics text but I particularly enjoyed the poems and quotations at the beginning of the main sections.

The chapter headings are: Number, Sets, Inequalities, Simple Transformations, The Slide Rule, Displaying Data, Shearing, Enlargement, Percentage and Proportion, Graphs and Relationships, Algebraic Formulation, Trigonometry, Coordinates and Graphs, Binary Arithmetic, Geometry in three dimensions, Loie.

S.M.P. is an influential example of Mathematics curriculum revision in Great Britain. Book T is a good text to examine, first because it is written at about the equivalent of our Grade 9 level, with pupils who are in a course similar to our

5-year A. & S., and second because of the chapter on Simple Transformations, in which reflection, rotation, symmetry and translation are presented in an interesting, practical manner.

Teachers interested in the meaning of the proposals for Geometry in the new Grade 10 course, I 12D, would be well-advised to get a copy of Book T and read Chapter 4, for background reading, which is written for 13-14 year olds who have not had the motion geometry before. The introduction to chapter 4 needs a bit of study too:

> "Cupid is winged and doth range;
> Her ooountry so my love doth change.
> But change she earth, or change she sky,
> Yet will I love her till I die."

The S.M.P. books are available in Canada from Macmillans.

Practice Mathematics - Book One - H.B. Oikle and G.F. Osborne
Copp Clark. Cloth, 177 pages.

This is the first of four textbooks in mathematics written for students who wish to take a limited program in secondary schools. The four books have been designed to provide a two year programme that is remedial and practical. Much of the material has been used in experimental programs with occupations classes.

Book One consists of five chapters: 1. Introduction to Sets and Numbers (26 pages which includes 14 Exercises)
2. Addition and Subtraction in No (42 pages with 24 exercises)
3. Multiplication and Division in No (37 pages with 32 exercises)
4. Algebra (18 pages with 13 exercises)
5. Geometry (24 pages with 13 exercises) and lastly a table of squares and answers to the exercises.

The first five sections (Lessons) of chapter 1 deal with set, identical and equivalent sets, finite, infinite and null sets, subsets, the union and intersection of sets. The ideas

and exercises here are well within the ability of two year students. The next five exercises deal with the set of natural numbers, the number line, place value, approximation and rounding off. There are two supplementary exercises on Roman Numerals and Numeration Systems with other bases that could be used to provide variety and interest for 2 or 3 lessons. All chapters end with a Test Exercise made up of straightforward questions on the material covered.

Chapter 2 starts with number phrases and sentences, open sentences and variables, and a very simple exercise using n as a variable. Exercise 2 gives practice on the symbols $\neq$, $>$, $<$. Closure, commutative and associative properties of addition are discussed briefly but are not included in exercise work. Exercises 3 to 15 give an excellent review of skills of addition with some special tricks, methods of checking, estimating and word problems leading to addition. Exercises 16-21 deal with subtraction giving drill with this operation as well as methods of estimating and checking subtraction and word problems leading to subtraction. Exercise 23 introduces the rule of performing operations in brackets first and the chapter ends with a Test exercise.

Chapter 3 begins with an explanation of the meaning of multiplication, the closure, commutative and associative properties of multiplication, and the Distributive property. No exercise work is given on these properties and the first exercise is a review of Multiplication Tables. Ideas dealt with in this chapter include factors, primes, multiples, practice in multiplication, checking multiplication, simple examples of powers, square roots of perfect squares using the table, estimating products. These concepts take 17 exercises and division is reviewed in much the same way in the next 11 exercises. Average and order of operations are also studied in this chapter which has 32 exercises including two tests.

Chapter 4 is a short and simple introduction to Algebra which includes substitution, simplifying simple number phrases, collecting like terms, multiplication and division involving

powers with like bases and with unlike bases, simple equations and inequations and solving simple word problems. The explanations and problems are meaningful but are kept at a realistic level of difficulty. There are only 13 revisions in this chapter.

Chapter 5 is a short chapter of 13 exercises mainly on the basic constructions of geometry and the use of ruler and compass. Included are squares, points, lines and planes, angles and their measure, direction, the circle, parts of a circle, measure of angles in a circle, the triangle, construction of triangles, sum of angles in a triangle, and the basic constructions. Exercises are mainly the construction of triangles and other figures with given dimensions and simple experiments such as those showing that the bisectors of angles are concurrent.

In this reviewers' opinion, the book accomplishes the authors' aim of providing the first half-year of a limited two year mathematics program that is remedial, practical and useable and it would be very suitable for terminal or occupations classes.

Modern Mathematics for Schools: Book 1 (1965) and Book 2 (1966) by The Scottish Mathematics Group (Blackie, Glasgow and London) (Chambers, Edinburgh and London)

In my article on Scottish school mathematics I mentioned the published work of the Scottish Mathematics Group which resulted from their labours on the Syllabus Committee. These books are the first two of a series of seven to be based on the Alternative Ordinary Grade Syllabus introduced into sixty secondary schools in 1965.

Books 1 and 2 comprise the two halves of the first year secondary course (12 year olds). In the preface the authors state that each part has been tried out with seven thousand children. It is noted that the pupils should discover for themselves, that equipment is helpful and that calculating machines and slide rules are advocated. The authors also refer to the report on recent changes at colleges and universities

and the international discussions under the auspices of O.E.C.D.

The two books together total 460 pages; each page is 8" x 5-3/4" , somewhat smaller than most of ours: for example Contemporary Mathematics 2 has a 9" x 6-1/4" page. Colour is used effectively throughout. The contents of Book 1 are divided into three units: Algebra, Geometry and Arithmetic, respectively 51, 64 and 77 pages long. The chapters are as follows:

Algebra:

An Introduction to Sets, Mathematical Sentences-Equations, Multiplication using the Commutative, Associate and Distributive Laws, Replacements and Formulae, Mathematical Sentences - Inequations, Revision Exercises

Geometry:

Cube and Cuboid, Rectangle and Square, Coordinates, Right-angled Triangle, The Isosceles and Equilateral Triangles, Some Topics to Explore, Revision Exercises

Arithmetic:

Length, Area and Volume, The System of Whole Numbers, Fractions, Ratios and Percentages, Revision Exercises

In the unit on Algebra, the work on sets goes as far as subsets, Venn diagrams and intersections. Symbols are used for set listing, membership, the empty set, subsets and intersection. One of the problems in the exercise on Venn diagrams reads: "K = {persons under 16 years old} , M = {persons having a driving licence} . Show in a diagram that one must be 16 years old or over to hold a driving licence".

The approach to equations and inequations is through the mathematical sentence; introduction of the variable is with the sentence " $\square$ is a divisor of 12", for which the question of truth or falsity is discussed. The term "open sentence" is used. Some clock arithmetic is introduced in connection with simple equations, solved exclusively by inspection. In one case, a clock face divided into four equal sectors by the numbers 0, 1, 2, and 3 is used to make up an addition table,

and then the following question is asked: "If x is a variable on the set $\{0, 1, 2, 3\}$, use the addition table you have made to solve the following equations:

(i) $x + 2 = 0$ (ii) $x + 1 = 2$ etc. "

Multiplication is reviewed as repeated addition and the variable introduced. e.g. $5 \times x$ = five x's = $x+x+x+x+x$, and $x \cdot 5$ = x fives = $5+5+5+ \ldots$ to x terms. A multiplication table for the set of whole numbers 0 to 6 is used to find solution sets for equations such as $3 \cdot x = 6$ and $x \cdot 4 = 20$, with the commutative law being made explicit.

Space does not allow any more examples to illustrate a simple, pedagogically sound, interesting development of equations and inequations. I was impressed with the clarity, conciseness and simplicity of language. The first sentence of the chapter on inequations may give you an idea: "There is order in numbers."

Ontario readers will be particularly interested in the Geometry unit, since this part of our own syllabus is under review from Kindergarten to Grade 10. The proponents of transformation geometry will find grist for their mill here. The authors begin with familiar three-dimensional objects, discussing walls and bricks, cubes and cuboids. Squared paper is used to assist in diagrams of brick patterns and non perspective drawings of the solids. Skeleton models made of dowel rods or drinking straws are described as well as nets of cuboids. The question of how many ways the object can fit into a box made for its shape is discussed.

The rectangle and square are discussed in the same question, and their properties used to define right angle, right angle being the corner of a square. The nature of the rectangle and rhombus are summarized in statements that they can be used to "tile the plane" and will fit back into the shapes of their outlines in four different ways.

Coordinates, using only the first quadrant, are defined and used to draw tiling diagrams, to assist in studying the properties of the isosceles and equilateral triangles, with

the emphasis throughout on shapes and tilings. The reader will wonder in some cases if this is too juvenile for twelve year olds, but as we have discovered here, it is preferable to start a new approach at the curriculum boundaries, rather than wait for it to work up from infant school.

In the unit on Arithmetic, the chapters on Whole Numbers and Fractions will seem light by our standards; however, there is a skilful blending of drill with a study of properties: operation tables, subtraction as an inverse operation, associativity, L.C.M., prime and composite numbers, H.C.F., division as an inverse operation and so on.

Book 2 repeats the three headings for its units. I am not sure whether this division of a year's work into six units was deliberate or caused by the pressure of time. It looks like a good idea anyway. The chapter headings are as follows:

Algebra:

The Distributive Law, Powers and Indices, Negative Numbers, Mathematical Sentences - Methods of Solving Equations and Inequations, Revision Exercises

Geometry:

Rhombus and Kite, Parallelograms, Triangles and Parallel Lines, Angles-Rotation, A Topic to Explore, Revision Exercises

Arithmetic:

Decimals and the Metric System, Binary Numbers, Introduction to Statistics, Introduction to Probability, Revision Exercises

The sets of numbers are listed on page 32:

(i) the set of whole numbers $W = \{0, 1, 2, 3, \ldots\}$

(ii) the set of natural numbers $N = \{1, 2, 3, \ldots\}$

and so on. Z is used for integers and the rational numbers are indicated by examples, rather than by any formal listing.

The copy of Book 2 I received is labelled 'Teacher's Edition'. The notes are not extensive but seem appropriate and useful; they are inserted in the book at intervals on a

different colour of paper. An example faces page 55; it begins: "The rhombus and kite are important just because of their symmetry; it is not an accident that one or the other of these figures occur in each of the standard straight edge and compasses constructions of bisectors and perpendiculars...."

I am not going to attempt to summarize any more of these well-written books. A group of dedicated teachers have made a good effort to interpret modern mathematics for the pupils of their grades. I believe they have been quite successful. We can learn a lot from their approach. It seems to me now in retrospect that our new Grade 7 and 8 courses of study have not quite hit the mark, particularly in Geometry. When we do further revising, the work of the Scottish Mathematics Group will be well worth studying.

In the meantime, teachers of Grades 7-10 Mathematics should have copies of these excellent books for reference.

A.W.H.

Some Lessons in Mathematics - by Members of the Association of Teachers of Mathematics (England) - edited by T.V. Fletcher, Cambridge University Press - MacMillan of Canada

As the title suggests, this book is a collection of lessons on topics in modern mathematics: The following excerpts from the introduction are indicative of the authors intent:

"We are concerned then with new mathematics, new ways with old mathematics, contemporary applications of the subject, and the psychology of teaching it. Our theme will be developed largely by means of lessons; that is to say by discussions of specific items as much as possible in a way that could be taken into the classroom".

"Mathematics does not start with the finished theorem in the textbooks; it starts from situations. Before the first results are achieved there is a period of discovery, creation, error, discarding and accepting. This period is notoriously difficult to discuss, or even to describe in a convincing way, but in this book we have tried to make this period our concern."

"But it should be remembered that in advocating the study of these controversial ideas we do not imply that we believe them to be correct in every point; we only believe that in confronting this challenge our own ideas may benefit".

The lessons show a variety of styles and approach reflecting the ideas of the twenty authors who shared in writing the book. The topics covered are:

Binary Systems. Finite Arithmetics and Groups. Numerical Methods and Flow Charts. Sets, Logic and Boolean Algebra. Relations and Graphs. Linear Programming. Patterns and Connections. Convexity. Geometry. Vectors. Matrices.

Just as these articles span a broad range of topics so the lessons span a wide range of ability reflecting the authors' views that "it is quite wrong to educate the majority by a course designed for the few". The lessons are often brief and tend to suggest areas for investigation rather than a full treatment of the topic; the book is not a textbook for pupils but is designed for the teacher with explanatory comments in each section.

The chapter on Finite Arithmetic and Groups introduces isomorphism and group structure. This latter idea is introduced through a study of symmetries while modular arithmetics are used to demonstrate the one-to-one correspondence of an isomorphism.

The section on Flow Charts contains material that might be used by teachers introducing a course in Computer Science. This introduction to flow charting is elementary and no mathematics is assumed. The stress is on the logic of order, and iterative processes and the associated idea of looping are demonstrated without reference to mathematics.

There are several rather interesting lessons in the section on Relations and Graphs pertaining to the introductory work on relations. The authors have suggested that an intuitive approach to the concept of relations can be presented at an elementary level and properties such as order and equivalence adequately demonstrated.

Reflections, rotations, translations and dilatations appear in the Geometry section, the suggestion being that school geometry could be different from what it has been and an attempt has been made to indicate possible alternative methods. The introductory lessons on Vectors and Matrices could be considered for the intermediate level of the mathematics curriculum.

This book which is available in a paper back form is a timely edition and is recommended as a reference text for all teachers of mathematics.

G.B.L.

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