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ABACUS

Ontario Mathematics Gazette

OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

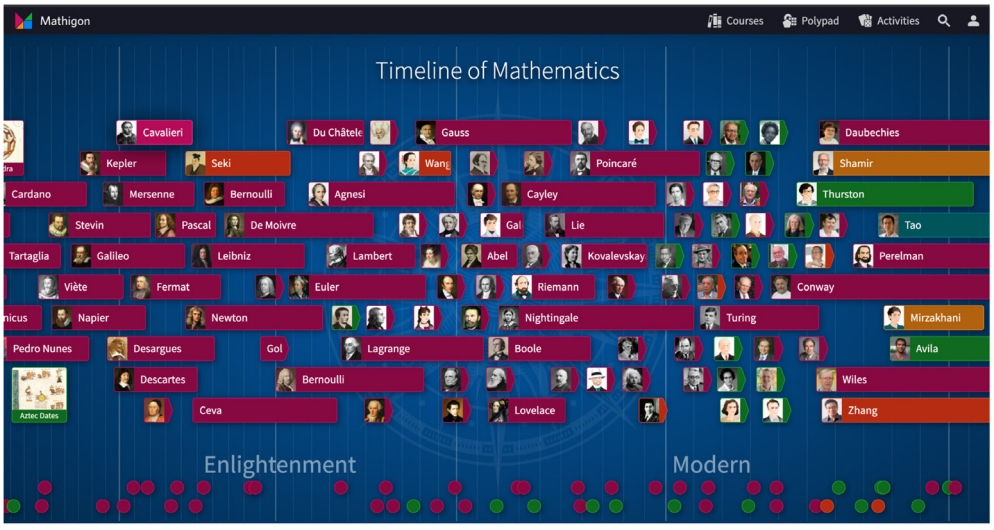
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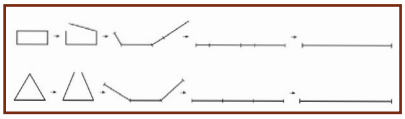
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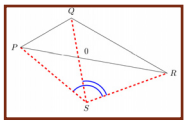


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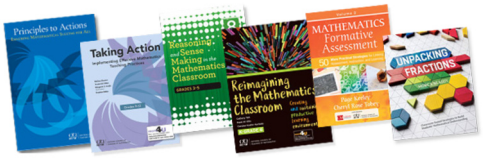
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▲ See OAME/NCTM Report: Students Giving Students Advice on Destreaming

Submission of Articles

The *Ontario Mathematics Gazette* (OMG) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *Gazette*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles are to be word–processed, MS Word is preferred, and prepared according to the *Publication Manual of the American Psychological Association*, Seventh Edition. However, please use single-line spacing (not double). Articles should not exceed five numbered pages of text, and figures, images, and photographs should be placed in the text close to where they belong, with captions. The photographer's permission is required, and for photos of students under the age of 18, the written permission of a parent or guardian is required.

Please submit your article in one blind file (i.e., identity of author is not evident), and include author names, contact information including email and mailing addresses, photos—head and shoulders, biographies—less than 100 words, and all content removed for blinding in a second file. Please email these two files to Tim Sibbald at gazette@oame.on.ca.

Upon review, you will be notified whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected. The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME/AOEM.

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The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of OAME/AOEM.

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Submission of Advertisements
Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** by email or at the above address. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be print ready, and colour advertisements should have no bleeds.

Advertising Rates
Advertising rates are available by telephoning, emailing, or writing to the Advertising Manager.

▲ Editor's Report



TIM SIBBALD, OCT, PHD
gazette@oame.on.ca

Tim Sibbald holds the Universities Director position on the OAME Board of Directors. He is a former president of OAME. He is an associate professor in the Schulich School of Education, Nipissing University, with a focus on mathematics education.

Imagine for a moment that, while eating breakfast, you could choose the type of school day you would have. The choice is constrained and you can only choose between being criticized, ignored, or acknowledged by a peer. What would you choose?

Sure, this is a fictional choice, but all too often, we do not find time to explicitly acknowledge our peers. We know that with students, the use of strategic compliments helps encourage them to push their learning further—"You catch more flies with honey." So, why not our peers? Perhaps, as teachers, we feel we are too wise for the tactics we apply to our students? Or is it that the word "tactic" implies that we dismiss such acknowledgements as simply being part of the teacher toolbox? It is, after all, easier to engage in small talk and avoid talking about teaching.

Yet, we all know that, over breakfast, the pick would be for a credible acknowledgement by peers. It is a rare kind of event, which recognizes the beneficial aspects of our teaching days, but what would be acknowledged and what makes it authentic? Since teaching is a multi-faceted endeavour, it is often our efforts to enact change or to provide unwavering support for student learning that come to mind. Yet, we all know that it can be subtle and hidden in classrooms, where the teacher is isolated from their teaching peers. And that is the challenge….

Authenticity is not as simple as having clear evidence. It uses professional judgment to consider an ongoing collection of small details that create a portrait that is deserving of acknowledgment. Perhaps it is challenging to put the puzzle of evidence together when it transpires over many busy days, months, or years. It is also important to have an acknowledgement that fits the details and does so in a professional manner. Many of us have surprised a colleague with a coffee or donut, but credibility in the professional judgment sense, needs a professional acknowledgement.

Now turn this around. When was the last time you

What would you do?

In this edition of the *Gazette*, we ran into a small, but important editorial issue. If both sides of $-x > -5$ are multiplied by –1, then $x < 5$. However, what happens to the ">"? Age-appropriate language might be "flipped" or "reversed," but neither is a mathematical operation. "Inverted" was also suggested (Thank you, Stewart Craven), but is the inverse of $x < 2$ properly $x > 2$ or $x \geq 2$?

Is there a better mathematical way to convey this? (gazette@oame.on.ca)

acknowledged a colleague? Sure you have, and it may take some thought to recall a moment of positive support. You will, however, because teachers are "all in it together" and they support each other routinely. The difficulty thinking of such moments is that they are often small and informal. Genuine, sure, but subtle to the point of being private and constrained. Perhaps not the sort of acknowledgement you thought you might get at breakfast.

What you can do about this is "be the change you would like to see." Rather than pick the type of day for yourself, why don't you pick someone else's day—by nominating them for an award? But wait, what is the evidence? Look for the small details and develop that portrait. But, wait, will it be complicated and take a lot of time? Not as much as you might think. In fact, one of the biggest hang-ups around awards is a sense that they have to be a complete surprise—often it is more effective to talk to the person about your intention and ask them to help with supporting details.

In my experience, the most significant hindrance is the concern that the nominee will "lose." Yet, there are no losers when it comes to awards. I speak from experience, having been nominated for four awards and having not received any of them—being nominated is an acknowledgement, regardless of the outcome! To have peers take the time to express their sense that you are a candidate for an award is the acknowledgement. Sure, one would like a nomination to lead to the award, but perhaps as a nominator, the focus should be on recognizing a colleague.

The OAME/AOEM awards are described on page 11 of this issue and on the website in the prominent second button down the left side. I will not reiterate the details, but I will encourage consideration of details—mull over the actions of your colleagues in light of words like "creative" and give it its due. The evidence is likely there, even if it is not immediately obvious. Consider the awards and ask yourself, over breakfast, whom you should acknowledge today.

On another note, I want to take a moment to wish you every success going into the new school year. The effort to recover from the pandemic period is going to be challenging. As Anne Yeager has commented during the editing of this issue, instructional strategies such as differentiating social–emotional learning are new territory with few resources. As always, I am confident that professionals who read the *Gazette* will do their best, and I hope that, as a community, we continue supporting each other. And, on that note, please consider encouraging your colleagues to engage with the OAME/AOEM, as it is through strong membership that we have the strongest support.

Curriculum News

After being released in June, a change in the Grade 9 curriculum took place around July 14. For more details, see what has been reported in the *Toronto Sun* (www.torontosun.com/news/provincial/ford-government-drops-curriculum-language-that-called-math-racist).

Abacus Changes!

I would like to express a warm welcome to our new *Abacus* editors! **Marc Husband** and **Tina Rapke** have taken the reigns, and their first issue comes out with this *Gazette* issue. Welcome aboard!

With that, I would be remiss if I did not acknowledge the years of effort by **Mary Lou Kestell** and **Kathy Kubota-Zarinvnij**. They have been a class act, and I say that, knowing that editing is a strange business, where making it look effortless belies the ups and downs that go on behind the scenes.

Thank Yous

It continues to astonish me how varied the circumstances of *Gazette* preparation are. This is hidden by the professionalism of the team who puts the *Gazette* together. Thank you to the dedicated and hard-working team: **Anne Yeager, Kyla Kadlec, Ralph Connelly, John Rodger, Gitta Berg,** and **Penny Clemens**.

Additionally, thank you to article reviewers over the last year:

Ann Arden
Lynda Colgan
Andrew Dmytriw
Tara Duncan-Smith
Sonia Ellison
Kathleen Goslin
Jacqueline Hill
Nancy Louis
Nilofar Mani
Kameal Mina
David Petro
Anne Roberge
Reginald Robson
John Rodger
Peter Saarimaki
Barb Seaton
Robert Sherk
Chris Stewart
Miranda Wheatstone

Apologies if I have inadvertently missed anyone. Any OAME/AOEM member who is interested in being a reviewer can contact the editor at gazette@oame.on.ca for details.

Do you have the *write* answer?

Articles, letters, short pieces, are always welcome. Share your best practices to help out your OAME/AOEM colleagues! If you want to talk about it, ask the editor—gazette@oame.on.ca

In This Issue

Judy Mendaglio outlines the many facets of OAME/AOEM that will be useful to new members, but is also beneficial for established members, to make sure they do not miss opportunities. **Jacqueline Hill** addresses destreaming with advice from students. **Heather Theijsmeijer** brings us news from the OMCA.

Lynda Colgan writes of butterflies while speaking of social–emotional learning. **Jeff Irvine** speaks about a historical period called "The New Math," which is a very important historical lesson in curriculum development. **David Costello** speaks about supporting classroom discourse. **Jennifer Holm** addresses inequalities within equation solving. **Carly Ziniuk** introduces a game, where one endeavours to work out the timing of historical events—this will be particularly useful for addressing math historical elements in the new curriculum. **Ron Lancaster** pulls mathematical ideas from a variety of photos. **Jérôme Proulx** addresses student thinking about perimeter and area, and **Shawn Godin** shows us various angles for solving an angle problem.

We have a new column by **Iain Brodie** and **Hatice Beyza Sezer**. They have taken an interview approach to addressing coding, but with a modern twist, their article gives a taste, and the interview is available as a podcast through OAME Talks!

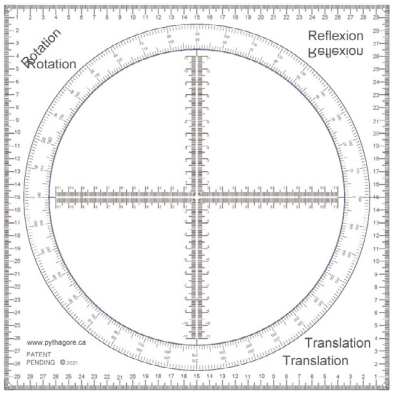
The articles in this issue address a variety of topics. **Ami Mamalo** and **Nkechi Ibeh** draw on infographics and consider remixing them. **Jeff Irvine** shows us the money in economics in an article that provides teachers insight into financial thought and opportunities for using economic ideas in the classroom. Lastly, **Nicolaas Lau** works with readers on understanding collaborative mathematics classrooms.

Erratum

In **Jeff Irvine**'s article about Fibonacci numbers, on page 45, it says "let = a/b," but should read "let $x = a/b$." Apologies for the omission. ▲

▲ Letter to the Editor

DOING GEOMETRY WITH THE COOL TOOL FOR GEOMETRY



This is an innovation in math and particularly in geometry. It shows us how to teach and learn geometry differently, gaining some advantages with the *Cool Tool for Geometry.*

This Cool Tool was invented and conceived by Alexandro Mancini, a long-experienced math teacher in Québec. The main purpose to bring this cool tool into schools and into the hands of students and teachers is for teaching geometrical transformations such as translation, rotation, and reflection.

With the Cool Tool for Geometry, you can carry out more benefits than these geometrical transformations. The Cool Tool for Geometry is a multifunctional instrument. It plays the roles of compass, protractor, square, triangle, and ruler. You can do many activities in geometry, such as drawing parallel lines; measuring and drawing angles; making circles, quadrilaterals, polygons; putting and reading points in the Cartesian plan; drawing the altitude of a triangle; making pie charts; and many other things rapidly.

The best way to get a sense of the Cool Tool is to search YouTube for "Cool Tool for Geometry" (or go to www.youtube.com/channel/UCZDXxvRFRpzuJwBvikjcqpA). Some videos in English and French explain how it can be used.

I hope that the Cool Tool for Geometry helps you in your daily tasks and brings success to your students. I am available, if you have questions and need samples or a visit to your school to show and explain the Cool Tool for Geometry.

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▲

President's Message



JUDY MENDAGLIO
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Judy Mendaglio is the President of OAME/AOEM. She has worked in education in Ontario in many capacities: college instructor, secondary teacher and department head, university instructor, elementary and secondary consultant, and writer. Currently, Judy devotes most of her time to her many volunteer activities.

A new school year is beginning. Hopefully, you have had a refreshing rest during the summer break. We have no idea what the year will hold, but most of us are hoping that our endless hours in front of screens, trying to teach from a distance, will be behind us. However, we know that we will be facing other challenges this year:

- Year 2 of a new elementary curriculum, which teachers are still learning about and planning for
- A new destreamed Grade 9 curriculum, with its emphasis on anti-racism, equity, inclusion, culturally responsive and relevant pedagogy, and student voice
- The as yet unknown consequences of COVID-related school closures on students' learning, academic achievement, and social development
- The degree to which school closures and other COVID-related factors have disproportionately affected vulnerable populations

This will probably be another year, during which educators will face challenges that they may never have faced before, but which, as we saw these past two years, they will navigate successfully. What we need to keep reminding ourselves is that educators in Ontario are a community of learners. We are very lucky to have the range of excellent resources that are available to us.

Since you are reading this column, there is an excellent chance that you are a member of OAME/AOEM. The membership of this wonderful organization comprises a community of mathematics educators, who support one another, always, but especially in times of unusual challenges. Unfortunately, many members are unaware of just how much members of OAME/AOEM contribute to the resources available to support math educators in Ontario. Most people know that we run provincial conferences: the Spring Annual Conference and the Fall Leadership

Judy Mendaglio est la présidente de l'AOEM/OAME. Elle a exercé de nombreuses fonctions dans le secteur ontarien de l'éducation : formatrice au niveau collégial, professeure et chef de section du secondaire, chargée de cours au niveau universitaire, consultante pour le primaire et le secondaire et écrivaine. Judy se consacre actuellement à de nombreuses activités bénévoles.

Dans le présent document, le masculin est utilisé sans aucune discrimination et uniquement dans le but d'alléger le texte.

Une nouvelle année scolaire commence. J'espère que vous avez eu l'occasion de vous reposer pendant les vacances d'été. Nous n'avons aucune idée de ce qui nous attend en cette année scolaire, mais la plupart d'entre nous espèrent que nos heures interminables devant les écrans, à essayer d'enseigner à distance, seront derrière nous. Cependant, nous savons que nous ferons face à d'autres défis cette année :

- La deuxième année d'un nouveau programme-cadre élémentaire que les enseignants sont encore en train d'apprendre et de planifier
- Un nouveau programme-cadre de 9e année décloisonné, axé sur l'antiracisme, l'équité, l'inclusion, une pédagogie culturellement adaptée et pertinente, et la voix des élèves
- Les conséquences encore inconnues des fermetures d'écoles liées à la COVID-19 sur l'apprentissage, le rendement scolaire et le développement social des élèves
- La mesure dans laquelle les fermetures d'écoles et d'autres facteurs liés à la COVID-19 ont affecté de manière disproportionnée les populations vulnérables

Ce sera probablement une autre année pendant laquelle les éducateurs seront confrontés à des défis qu'ils n'ont peut-être jamais rencontrés auparavant, mais qu'ils réussiront à surmonter, comme nous l'avons vu au cours des deux dernières années. Ce que nous devons garder à l'esprit, c'est que les éducateurs de l'Ontario sont vraiment une communauté d'apprenants. Nous sommes très chanceux d'avoir la gamme d'excellentes ressources qui sont à notre disposition.

Puisque vous lisez ce message de la présidente, il y a de fortes chances que vous soyez membre de l'AOEM/OAME. Les membres de cette merveilleuse organisation comprennent une communauté d'enseignants en mathématiques qui se soutiennent les uns les autres, toujours, mais surtout en période de défis inhabituels. Malheureusement, de nombreux membres ne savent pas à quel point les membres de l'AOEM/OAME contribuent aux ressources disponibles pour soutenir les enseignants en mathématiques en Ontario. La plupart savent, probablement, que nous organisons des

Conference. In fact, many of you will have become members as a benefit of attending or presenting at one of our provincial conferences. However, membership to OAME/AOEM provides so much more, and I would like to tell you more about the supports you can find on the OAME/AOEM website, www.oame.on.ca.

OAME/AOEM is a provincial subject association, but we are structured a bit differently than some other subject associations in Ontario in that we have organized the province into 14 regional chapters, such as TEAMS (Toronto Educators Association for Mathematics), PRMA (Pine Ridge Math Association), and NWOAME (Northwestern Ontario Association for Mathematics). A 15th chapter is ISOMA (Independent Schools of Ontario Mathematics Association), which is not a regional chapter, but covers all of Ontario. The 15 chapters of OAME/AOEM are there to support you. You probably know that when you became a member of OAME/AOEM, you automatically became a member of the chapter of OAME/AOEM that you selected (if you are not sure, you can check by logging into the OAME/AOEM website). So, what might that mean to you? Well, the chapters conduct their own conferences, often called mini-conferences. For example, in July, the Quinte St. Lawrence Mathematics Association (QSLMA) held a one-day virtual conference, "Reaching All Learners Together," which featured speakers Chris Suurtamm, Peter Liljedahl, and Nat Banting, along with breakout speakers Cathy Marks Krpan, George Gadanidis, Octavia Beckles, and Daniel Ansari, among others. All members of OAME/AOEM were welcomed to attend this amazing conference. What a wonderful opportunity for the 500 or so people who came out to this conference (from the comfort of their homes)! To find out more about what your chapter, and others, are doing, visit the OAME/AOEM Conference page (www.oame.on.ca/main/index.php?lang=en&code=events).

In 2021–2022, OAME/AOEM will be offering a new series of podcasts on OAME Talks: www.sites.google.com/oame.on.ca/oame-talks/home?authuser=0. In addition to announcements about this season's podcasts, you will find recordings of conversations with Marian Small (Talk 2: Teaching with intention: Focusing on what matters), Fawn Nguyen (Talk 3: What if we've been teaching mathematics all wrong?), Kaneka Turner (Talk 11: Beyond the invitation: Lesson structure and routines that foster agency and equity), and Tara Flynn and Shelley Yearley (Talk 20: Unit fractions unlock learning), among others.

If you follow the Members-only link, you will also find recordings of OAME/AOEM presentations. For example, all the presentations from the 2020 virtual conference can be found here, as well as recordings of the featured speakers

conférences provinciales : la conférence annuelle du printemps et la conférence de leadership de l'automne. En fait, plusieurs d'entre vous seront devenus membres pour assister ou présenter à l'une de nos conférences provinciales. Cependant, l'adhésion à l'AOEM/OAME offre bien plus, et j'aimerais vous en dire plus sur les mesures de soutiens que vous pouvez trouver sur le site Web de l'AOEM/OAME (www.oame.on.ca).

L'AOEM/OAME est une association provinciale spécialisée en mathématiques, mais sa structure est légèrement différente de celles de certaines autres associations spécialisées en Ontario en ce sens que nous avons organisé la province en 14 chapitres régionaux, par exemples : TEAMS (Toronto Educators Association for Mathematics), PRMA (Pine Ridge Math Association), et NWOAME (Northwestern Ontario Association for Mathematics). Une 15ème chapitre est ISOMA (Independent Schools of Ontario Mathematics Association) qui n'est pas un chapitre régional, mais qui couvre tout l'Ontario. Ces 15 chapitres de l'AOEM/OAME sont là pour vous soutenir. Lorsque vous êtes devenu membre de l'AOEM/OAME, vous êtes automatiquement devenu membre du chapitre de l'AOEM/OAME que vous avez sélectionné. Si vous n'êtes pas certains de lequel des chapitres dont vous appartenez, vous pouvez vérifier sur le site Web de l'AOEM/OAME. Alors, qu'est-ce que cela peut signifier pour vous ? Eh bien, les chapitres organisent leurs propres conférences, souvent appelées miniconférences, dont vous pouvez en participer. Par exemple, en juillet, la Quinte St. Lawrence Mathematics Association (QSLMA) a organisé une miniconférence virtuelle d'une journée, « Reaching All Learners Together », à laquelle ont participé les conférenciers Chris Suurtamm, Peter Liljedahl et Nat Banting, et les conférenciers Cathy Marks Krpan, George Gadanidis, Octavia Beckles et Daniel Ansari, entre autres. Tous les membres de l'AOEM/OAME ont été invités à participer à cette incroyable miniconférence organisée par QSLMA. Quelle opportunité incroyable pour les 500 personnes qui ont participé à cette miniconférence dans le confort de leur foyer ! Pour en savoir plus sur ce que font vos chapitres, veuillez visiter la page Conférence de l'AOEM/OAME (www.oame.on.ca/main/index.php? lang=en&code=events).

En 2021–2022, l'AOEM/OAME proposera une nouvelle série de podcasts OAME Talks à www.sites.google.com/oame.on.ca/oame-talks/home?authuser=0. En plus des annonces sur les podcasts à venir, vous trouverez également ici des enregistrements de conversations avec Marian Small (Talk 2: Teaching with intention: Focusing on what matters), Fawn Nguyen (Talk 3: What if we've been teaching mathematics all wrong?), Kaneka Turner (Talk 11: Beyond the invitation: Lesson structure and routines that foster agency and equity) et Tara Flynn et Shelley Yearley (Talk 20: Unit fractions unlock learning), entre autres. À noter : Les discussions sont

from OAME 2019 and Jo Boaler's webinar (June 2020), "21st-Century Teaching and Learning."

Of course, also in the Members section are the current issues of the *Ontario Mathematics Gazette* and *Abacus* (which you are reading now)! But, were you aware that the *Gazette* and *Abacus* archives go back to volume 1, issue 1, from February 1962? Imagine the wealth of knowledge that can be retrieved from these pages!

OAME/AOEM also supports the effective use of technology in mathematics classrooms with our mathies tools (www.mathies.ca/learningTools.php#gsc.tab=0). If you have not recently visited the mathies site, please take a half hour sometime and play. With the new elementary and Grade 9 curricula, you may see new ways of incorporating the mathies tools into your math program planning. These tools are designed by Ontario teachers to support the Ontario math curriculum, and are one of the few tools available also to French and French-immersion math teachers and students. And, they are free!

Additionally, since the summer of 2020, OAME/AOEM has been partnered with AFEMO (l'Association francophone pour l'enseignement des mathématiques en Ontario) in the creation of webinars and lessons, as well as assessment plans, to support the implementation of *The Ontario Curriculum Grades 1–8: Mathematics, 2020* and the 2021 Grade 9 destreamed curriculum. These resources are free to all educators and are found at our project website, OntarioMath.support (www.ontariomath.support/).

The members of OAME/AOEM try very hard to create meaningful learning experiences and useful resources for the mathematics educators of Ontario. I hope that this stroll through our association has provided you with some new places for you to find what you need to support your own challenges in the 2021–2022 year.

I wish everyone a wonderful school year! ▲

No One Knows (Open Problem)

Can one pack all $\frac{1}{k} \times \frac{1}{k+1}$ rectangles (k = 1, 2, 3, …) into a square of side 1?

(Brass, Moser, & Pach, 2005, p. 121)

Brass, P., Moser, W., & Pach, J. (2005). *Research problems in discrete geometry.* Springer-Verlag.



seulement en anglais.

Si vous suivez le lien réservé aux membres, vous trouverez également des enregistrements de présentations AOEM/OAME. Par exemple, toutes les présentations de la conférence virtuelle 2020 se trouvent ici, ainsi que les enregistrements des conférenciers vedettes de l'AOEM/OAME 2019 et du webinaire de Jo Boaler (juin 2020), « 21st-Century Teaching and Learning ».

Bien entendu, vous trouverez également dans la section réservée aux membres les numéros actuels de la *Gazette* et de l'*Abacus* (que vous lisez maintenant) ! Mais saviez-vous que les archives de *Gazette* et *Abacus* remontent au volume 1, numéro 1, de février 1962 ? Imaginez la richesse des connaissances qui peuvent être récupérées à partir de ces pages !

L'AOEM/OAME soutient également l'utilisation efficace de la technologie dans les salles de classe de mathématiques avec nos outils mathématiques mathies (www.mathies.ca). Si vous n'avez pas récemment visité le site de mathies, s'il vous plaît veuillez prendre une demi-heure de temps en temps et jouer. Avec les nouveaux curriculums mathématiques du palier élémentaire et de 9e année, vous pourriez voir de nouvelles façons d'intégrer les outils de mathies dans la planification de votre programme de mathématiques. Ces outils sont conçus par des enseignants de l'Ontario pour soutenir le programme-cadre de mathématiques de l'Ontario et sont l'un des rares outils disponibles pour les enseignants et les élèves de mathématiques en français et en immersion française. Ils sont gratuits !

De plus, depuis l'été 2020, l'AOEM/OAME est partenaire de l'AFEMO (l'Association francophone pour l'enseignement des mathématiques en Ontario), dans la création de webinaires et de leçons, ainsi que de plans d'évaluation pour appuyer la mise en œuvre du programme-cadre, l'édition révisée en 2020, palier élémentaire, ainsi que le programme décloisonné de mathématiques de la 9e année de 2021. Ces ressources sont gratuites pour tous les éducateurs et peuvent être consultées sur le site Web de notre projet, OntarioMath.support (www.ontariomath.support/?pg=home&lang=FR).

Les membres de l'AOEM/OAME s'efforcent de créer des expériences d'apprentissage significatives et des ressources utiles pour les enseignants de mathématiques de l'Ontario. J'espère que cette promenade au sein de notre association vous a fourni de nouveaux endroits pour trouver ce dont vous avez besoin pour relever vos propres défis dans l'année 2021–2022.

Je souhaite à tous une merveilleuse année scolaire ! ▲

▲ Blast from the Past! The New Math

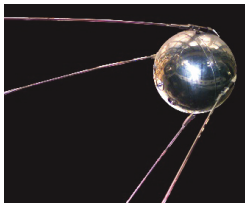


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Jeff Irvine, PhD, has been a secondary mathematics teacher, department head, and vice-principal. He has taught at three faculties of education and at a community college. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. With over 45 years in education, Jeff is particularly interested in the interplay of student motivation and mathematics achievement.

This column is about the "old" new math, circa 1960, as opposed to the "new" new math of the 1980's and 1990's. The new math (1960's version) was a supernova in mathematics education; it burned hotly, brightly—and briefly. I was a student in a "guinea pig" class in Grade 9, piloting the new math in Ontario. Less than ten years later, when I began teaching, the new math was mostly gone.

In this article, all references, except Phillips (2016), are to *Gazette* articles and can be found in the online *Gazette* archives in the members-only area.



The advent of the new math is usually attributed to the launch of Soviet satellite Sputnik 1, and the beginning of the space race between the United States and the USSR. However, like virtually all reforms in mathematics, the new math was founded much earlier, based largely on the work of the Bourbaki group in the 1930's (Phillips, 2016). It grew out of dissatisfaction with the then-current mathematics curriculum, which Coleman (1965b) derisively called "memorization and regurgitation," and which had remained largely unchanged for over 50 years. Pressman (1975) cites as an example his Grade 5 math textbook, which contained over 1000 long division questions, all of which he was required to complete.

There was a call to have students understand the concepts of mathematics, rather than simply memorize facts and procedures. The new math sought to have students see the big picture, the structure of mathematical systems, as advocated by Jean Dieudonné, a key member of the Bourbaki group, in a 1975 *Gazette* article. No doubt Dieudonné endeared himself to many mathematics teachers by saying in the article:

> …one should not be extremely surprised that high school teachers, who have no possibility of grasping the deep motivation behind the new introduction of the new mathematical concepts, should adhere to the letter of the new curriculum instead of understanding the spirit which made it necessary (p. 138).

Clearly his perception was that high school teachers could not understand the purpose of the curriculum and, in his view, were going to have their hands full, learning how to teach the content contained in it. Personally, I find that attitude extremely insulting and one that implies understanding mathematics was deemed the exclusionary domain of elite mathematicians.

In Ontario, curriculum changes at that time included introducing topics such as set theory, formal logic, matrices, calculus, and the binomial theorem. Set theory was the centrepiece, and links to applications were downplayed or removed from the curriculum entirely (Coleman, 1965a). This aligns with Dieudonne's influential view within Bourbaki that algebraic and logical derivation from axioms was the primary defining feature of mathematics as a field.

Into this milieu, in 1962, the *Gazette* was born. In the very first issue, George Pólya expressed reservations about the direction of mathematics reform. "There is virtually unanimous agreement that the high schools should teach more mathematics and should teach it better. Yet it is much less clear how this goal should be attained" (p. 25). Early *Gazette* articles provided information to teachers on topics such as vectors, which were new to the curriculum (Berman & Fryer, 1964).

The new math was already coming under significant criticism. Parents lamented that their children were not learning math the same way that they had learned it. Elementary teachers were frustrated that they didn't understand the math they were supposed to teach. Proponents of the new math claimed that it taught students to think logically and appreciate the overarching concepts of mathematical structures (Coleman, 1965b), but even some university mathematics professors were disillusioned with the outcomes (Thom,1974). Articles began to appear in the *Gazette*, advocating against the new math curriculum, although very few wanted a return to the old curriculum, which Coleman (1970) claimed had resulted in 98 percent of adults admitting that they didn't understand the math that they had supposedly been taught. Coleman (1975, 1978), a Queen's University mathematics professor, was an ardent

supporter of the new math, and advocated that it be extended. Most authors argued for a balanced approach, keeping the positive aspects of the new math, but dropping some features (Pressman, 1975; Usiskin, 1974).

Dr. W.W. Sawyer (1974) published a *Gazette* article that clearly identified both the good and the bad of the new math. In his article, Sawyer advocated for retaining calculus, matrices, and proof by induction, but deleting or radically downsizing the set theory aspects, especially the set theory notation. Throughout his career, Sawyer was a champion of linking mathematics to the real world, and he saw the lack of applications and connections in the new math curriculum as a major flaw.

The 1970's saw pushback against the new math, although often based on specious evidence, resulting in a major "back to basics" thrust. The March 1975 issue of the *Gazette* was particularly on point with respect to the new math. In addition to Dieudonné's defense of new math, the issue contained articles by Wheeler, in which he compared teaching math to teaching art, and emphasized the need for student creativity, rather than what he called "learn-by-proxy"; May, who stated that incoming university freshmen lacked basic math skills; and MacNeil, who argued that the mathematics curriculum should serve all students, and not just the select few who would go on to what we now call STEM courses.

In 1989, Allan Pitman published an excellent article in the *Gazette* that compared the "old" new math with the "new" new math of the 1980's and 1990's, which placed problem solving at the centre of curriculum reform. Pittman listed five features of the "old" new math: it was designed by mathematics professors, not educators; it was a formal system based on strict assumptions about formal logic and mathematical structure; it was born in the Cold War era, with demands to train more scientists and engineers (largely for military defence purposes); it featured top-down implementation; it was mediated by educators in schools, many of whom lacked academic background in the curriculum; and it largely divorced mathematics from applications. The contrasts between the "old" new math and the "new" new math will be explored in a future column.

The goal of the new math was to move mathematics education away from rote memorization and toward a system that reflected the modernity of the era. However, the changes in curriculum were not accompanied by concomitant changes in pedagogy, which remained largely the same as those in the previous century, featuring transmission and Socratic lessons. The new math resulted in many new math textbooks, seen as the sole resource for teaching. However, in many cases, the textbooks were rushed into production and were not accompanied by professional development or teacher training on how to use these new resources. It is informative to compare the new math with other mathematics reforms that followed. It is disappointing to see that many potentially worthwhile reforms suffered from the same implementation problems.

There are numerous parallels to today's calls for back to basics. The current student-centred approach, with problem solving at its heart, in many ways resembles the 1960's move away from rote memorization and regurgitation. However, there are significant differences, principally centred on a curriculum developed by educators, rather than academic mathematicians and concomitant changes in pedagogy. However, the political motivation behind curriculum changes remains, and we may see more "back to basics" movements before the pendulum swings back in favour of students as learners.

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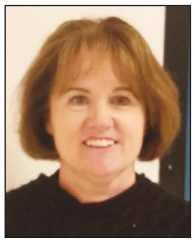
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▲ OAME/NCTM Report: Students Giving Students Advice on Destreaming



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Jacqueline Hill is a Grades 9–12 teacher of mathematics. She is a Past President of OAME and OMCA, as well as the recipient of the award for Exceptional Teaching in Secondary Mathematics. She also wrote the "Director's Dialogue" for the Gazette *for a number of years.*

With this year being the 60th anniversary of the *Gazette*, I thought a blast from the past was in order! This year, my students did work related to journaling. One such assignment took them on a tour of the OAME and NCTM websites. They looked at the sites through an intermediate mathematical lens. They then took a dive into the March 2017 *Gazette* at Todd Romien's OAME/NCTM report, which looked at the key issue of homework. Both north and south of the Canada/United States border, we deliver curriculum to the same audience. We try our best to help students move from where they are today, to where they can be in mathematics tomorrow. Three parts of that movement toward understanding are conversations around homework, note taking, and being the best students they can be. The blast from the past is that in 2017, I gave my students a very similar assignment (to first summarize the main points of Mr. Romien's article, then reflect on the three parts of movement toward understanding). This column will briefly describe what my 2021 students gave as "Advice for Destreamed Students."

Last year was like no other in the history of teaching. My students were more "profile icons" than cameras on (as my rural Internet would not regularly handle all of the video required). They more commonly asked questions in the chat box and in breakout rooms, then in front of the other students during whole-class sessions. And yet, I truly believe I got to know each and every one of the students a little bit more each day. We had great adventures: virtual trips to Canada's Wonderland for some distance-versus-time graphs; an eyes-in-the-sky visit to a chicken coop to discuss rural-versus-urban life; and to many, a piano concert performed by students in class to discuss the connections between math and music. My best advice to teachers is… get to know your students (know what they like, know their interests, know their pets' names). I would start each day, 10 minutes before class, with "would you rather" questions. One would usually be quite silly (such as, "Would you rather live in the sky or under the sea"?), while others would be starting points for conversations—such as, "Would you rather give advice or get advice"—and that led to our journal question related to "Advice for Destreamed Students."

The Key Ideas My Students Shared

NEVER—Never be afraid to ask for help and or clarification. If you don't understand something that is being taught, there are at least five others who do not as well. Be BRAVE!! Ask during the lesson.

KNOW YOURSELF—If you know you are the type of person who gets distracted easily, be certain to tell your teacher so they can have you where there are the least distractions.

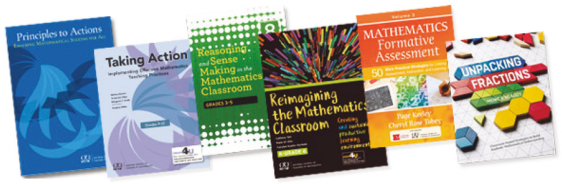
KEEP UP, DON'T CATCH UP—Get an agenda to keep yourself organized. Always try the homework right away (in case you need to go in for extra help). Use the *TVO mathify* website if you need help right away that can't wait until your teacher is available.

BE PREPARED—Be prepared with your questions. If you know you need help, find an example that you get stuck on, and highlight exactly where you get stuck. Try to never use the phrase, "I don't get any of this." You need to be PROACTIVE in asking for help.

TAKE TIME—Take time to try and have a balanced Grade 9 year. Take time for yourself. Schedule in homework and assignment time each week.

Stay tuned for our next series of columns, when my students give their thoughts on homework and note taking.

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OAME/AOEM AWARDS FOR EXCELLENCE IN MATHEMATICS EDUCATION

The purpose of the OAME/AOEM Awards is to formally recognize individuals and educators who demonstrate an outstanding contribution to mathematics education in Ontario. The successful candidate for each award enhances the learning environment for all students, providing opportunities for students to do, see, hear, and touch mathematics in a profound and meaningful way. If you know someone who qualifies for this honour, please nominate her/him/them for one of the following awards.

OAME/AOEM Life Membership Award: This award recognizes an educator who has contributed in a significant way to OAME/AOEM, demonstrates outstanding leadership in mathematics education, and has accumulated 10 or more years of membership in OAME/AOEM.

Award for Exceptional and Creative Teaching in Elementary Mathematics: This award recognizes an exceptional and creative elementary educator, who demonstrates excellence in mathematics education and contributes to the overall development of students.

Award for Exceptional and Creative Teaching in Secondary Mathematics: This award recognizes an exceptional and creative secondary school mathematics educator, who demonstrates excellence in mathematics education and contributes to the overall development of students.

Elementary School Staff Award for Exceptional and Collaborative Mathematics Teaching: This award recognizes an elementary school (a school accommodating any or all of Grades K–8), whose staff fosters collaborative and excellent mathematics teaching, contributes to the overall development of student learning of mathematics, and demonstrates leadership in the mathematics community.

Secondary School Mathematics Staff Award for Exceptional and Collaborative Teaching: This award recognizes a secondary school mathematics staff which fosters collegiality, teamwork, and excellent classroom teaching, contributes to the overall development of students, and demonstrates leadership in the mathematics education community.

Award for Leadership in Mathematics Education: This award recognizes an educator, who has demonstrated leadership by contributing in a significant way to the development of mathematics educators and the enhancement of mathematics education in Ontario through innovation and/or research-related activities.

Award for Outstanding Contribution to OAME/AOEM and Mathematics in Ontario: This award recognizes an individual who, has made a significant contribution to OAME/AOEM and contributed directly or indirectly to the improvement of mathematics education in Ontario, but who is not necessarily an educator or a member of OAME/AOEM.

OAME/AOEM Awards Nomination Process

1. Complete the **Award Nomination Cover Page** appropriate to the specific award, found at www.oame.on.ca (look for the **Awards** tab on the left of the page) or contact the Executive Directors at eds@oame.on.ca for an electronic version. You may include a one-page letter of recommendation.
2. Include with the nomination cover page up to three **letters of support**, of no more than two pages each. Please ensure all letters of support:
   - Address the criteria for the award as outlined in the nomination cover page.
   - Include the name and position of the supporter to the application.
   - Include the nominee's name and the award in the letter.
3. Include the **Curriculum Vitae**, of up to five pages, for award nominees, except for Elementary and Secondary Staff awards.
4. Email or mail all **completed** packages (cover page, supporting letters and CV) by the deadline advertised below to the Executive Directors. Email address and mailing address are included in the nomination cover page.

DEADLINE: Nominations are due by January 31, 2022.

The awards will be presented at the Annual OAME Conference in May 2022.

▲ What's the Problem? Chasing Angles



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Shawn is a former mathematics teacher, department head, and consultant. He strongly believes in the central role of problem solving in the mathematics classroom. He continues to be involved in mathematical activities: presenting workshops, writing articles, working on local projects, and helping create mathematics contests.

Welcome back, problem solvers. As I write this, a second school year with remote learning is coming to a close. The number of people vaccinated is increasing, so I hope that the new school year will see things moving back toward normal. Last issue, I left you with the following problem:

> *In quadrilateral* PQRS*, diagonals* PR *and* SQ *intersect at* O *inside* PQRS*,* SP = SQ = SR = 1*, and* ∠QSR = 2∠QSP*. Marc determines the measure of the twelve angles that are the interior angles of* ⊿POS*,* ⊿POQ*,* ⊿ROS*, and* ⊿ROQ*. The measure of each of these angles, in degrees, is a positive integer, and exactly six of these integers are prime numbers. How many different quadrilaterals have these properties and are not rotations or translations of each other?*

This was the last problem, #25, from the 2019 Gauss Contest for Grade 8 students. The contest is run by the Centre for Education in Mathematics and Computing (CEMC) at the University of Waterloo. The CEMC runs many mathematics and computing contests for elementary and high school students. They also have other resources for teachers and students. For more information, check out their website, www.cemc.uwaterloo.ca.

A good strategy for many problems, and essential for most geometric problems, is drawing a well-labelled diagram. Figure 1 shows the information with the sides of equal length noted in red dashes. The arcs on angles $\angle QSR$ and $\angle QSP$ not only indicate that they are different, but in this case, give us a visual reminder that $\angle QSR = 2\angle QSP$.

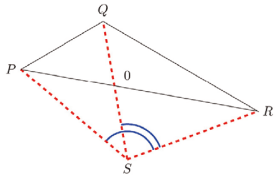


Figure 1: *Quadrilateral* PQRS *with diagonals*

In this case, it might be helpful to have some other diagrams. Since we have those three segments that are equal in length, our diagram contains three isosceles triangles: $\triangle PQS$, $\triangle QRS$, and $\triangle PRS$, as shown in Figure 2. The three arcs in the third diagram remind us that $\angle QSR = 2\angle QSP$ implies that $\angle PSR = 3\angle QSP$. Even though this is not the usual convention for marking angles, we can always decide what the representation means *to us* in the problem-solving process. Later, when we present the solution, we can decide if our markings are necessary for the reader of your solution.

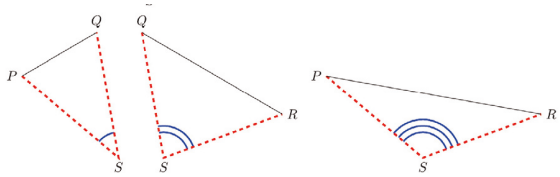


Figure 2: *Three isosceles triangles*

Before diving too deeply into the problem, let us get a sense of the relationships within our original diagram by looking at a specific case. For example, let $\angle QSP = 40^\circ$ and hence, $\angle QSR = 80^\circ$ and $\angle PSR = 120^\circ$. Since $SP = SQ$, then $\triangle PQS$ is isosceles and $\angle SPQ = \angle SQP$. As the angles in a triangle add to 180°, we can deduce that $\angle SPQ + \angle SQP + \angle QSP = 2\angle SPQ + 40^\circ = 180^\circ$ and hence, $\angle SPQ = \angle SQP = 70^\circ$. Using a similar line of reasoning on the other two triangles yields $\angle SQR = \angle SRQ = 50^\circ$ and $\angle SPR = \angle SRP = 30^\circ$. We can put these back into the original diagram and calculate the rest of the angles, as in Figure 3. Angles $\angle OPQ$ and $\angle ORQ$ can be determined first from the fact that we know $\angle SPQ = 70^\circ$, $\angle SRQ = 50^\circ$, $\angle SPR = \angle SRP = 30^\circ$ and PR cuts angles $\angle SPQ$ and $\angle SRQ$ into pieces, one of which is the aforementioned 30°. The four angles at O can then be deduced by looking at the small triangles and noting that the angles in a triangle add to 180° (or by doing it once and using opposite angles and straight angles).

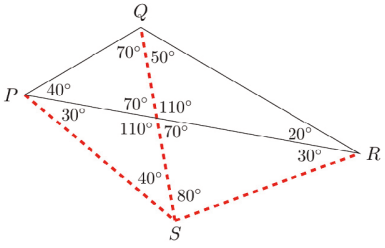
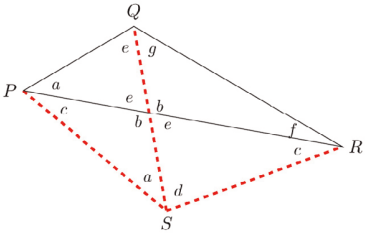


Figure 3: *The angles when* ∠QSP = *40°*

Looking at our result, we note that there are only seven unique angle values, none of which are prime. This may be a result of our choice of angle, or it might be a property of this figure under the given conditions. To solve the original

problem, we could just keep picking values for one of the angles and solve everything else, but that does not seem efficient or that much fun. We will have to search for another "angle" to attack the problem.

It makes sense to introduce a variable into our problem. If we let $\angle QSP = x^\circ$ and proceed as we did previously, we can determine, since $\triangle PQS$ is isosceles, that $\angle SPQ = \angle SQP = \frac{180^\circ - x^\circ}{2} = 90^\circ - \frac{x^\circ}{2}$. We could go through the same process as we did for the starting angle of 40° and get all other angles in terms of x, but we can make a simplification if we recall the wording of the problem. It said that all the angles were positive integers, which means that $90^\circ - \frac{x^\circ}{2}$ must be an integer, so x must be *even*. Hence, we can introduce *another* variable, $y^\circ = \frac{x^\circ}{2}$ or $x^\circ = 2y^\circ$. Following the same analysis as we did with the starting angle of 40°, we get the results in Figure 4. I will leave the details as an exercise.



$a = 2y^\circ$
$b = (90 + y)^\circ$
$c = (90 - 3y)^\circ$
$d = 4y^\circ$
$e = (90 - y)^\circ$
$f = y^\circ$
$g = (90 - 2y)^\circ$

Figure 4: *The relationship among the 12 angles*

Our calculations reveal that, in many cases, there are going to be seven unique angle measurements. It is possible that some of these angles match up. For example, if $y = 15$, then $4y = 60 = 90 - 2y$, in which case, there would be only six unique angles. You may wish to explore all possible ways that we could have fewer than seven unique measurements later. Starting with a specific value for one of the angles allows us to go through the calculations with concrete values before we model the problem algebraically. It also allows us to check our calculations. Our concrete case corresponds to $x = 40$, or $y = 20$. Substituting into the expressions gives us the same as our earlier result, which gives us a bit of confidence in both.

Now the fun begins! We need to determine which values of y yield exactly six prime angle measurements. Since we have formulas for each of the angles in terms of a variable y, we can program them into a spreadsheet, as shown in Figure 5. I started by marking the formulas across the top and labelled them with the number of repetitions. Then, under y in cell A2, I entered 1, and in cell A3, I entered the formula =A2 + 1 (note the = tells the spreadsheet to interpret the rest as a formula to be calculated). Once the formula has been entered, the result, 2, is displayed. I then clicked on cell A3, grabbed the small square that appears in the bottom right corner, and pulled it down. Doing so "fills down" the selection with the formula. The spreadsheet knows to "update" things at each line, so since I put =A2 + 1 in A3, in A4, it puts =A3 + 1, and so on. I can then fill in the rest of row 2 with formulas in terms of A2 (which is y). Note, in Figure 5, cell F2 is highlighted, it is recorded in the top left corner, and you can see that the formula =90 – A2 (corresponding to 90 – y) is in the input area. Once B2 through G2 have their formulas entered, I highlighted the section and did the fill-down process described above, calculating the values of the other angles. I then went through the numbers, looking for primes. When I found one, I marked the cell green. When I found a case where we had exactly six primes, the first occurring when $y = 7$ (with three 83's and two 97's), I wrote "yes" in the satisfied column. A spreadsheet is a powerful tool for doing mathematical investigations. Since there is easy access to spreadsheets, it is worthwhile for students, and their teachers, to be familiar with some of the features.

F2 =90-A2

	A	B	C	D	E	F	G	H
1	*y*	*2y (x2)*	*4y*	*90-3y (x2)*	*90-2y*	*90-y (x3)*	*90+y (x2)*	*Satisfies?*
2	1	2	4	87	88	89	91	
3	2	4	8	84	86	88	92	
4	3	6	12	81	84	87	93	
5	4	8	16	78	82	86	94	
6	5	10	20	75	80	85	95	
7	6	12	24	72	78	84	96	
8	7	14	28	69	76	83	97	yes
9	8	16	32	66	74	82	98	
10	9	18	36	63	72	81	99	
11	10	20	40	60	70	80	100	
12	11	22	44	57	68	79	101	yes

Figure 5: *Using a spreadsheet to find solutions*

We could continue with our spreadsheet and find all solutions, but it would be more satisfying to deduce them. First, since all the formulas represent angles in a triangle, we know they must be positive and less than 180. Since $4y$ is larger than y and $2y$ for all positive values, we can just look at it and we get $0 < y < 45$. For all these values, the rest of the angles are less than 180°, but some of them will not be positive for all those values. The smallest of the remaining angles is $90 - 3y$, and if we subject it to the conditions for an angle in a triangle, we get $-30 < y < 30$; hence, since y must make all angles a possible angle for a triangle, we get $0 < y < 30$. Checking your spreadsheet, you would find for $y = 30$, we get 0 in the $90 - 3y$ column, and for larger values, it becomes negative, which is not allowed as an angle measure.

Since we are looking for prime values, it might be easier to identify non-primes. For example, $4y$ can *never* be prime.

On the other hand, if $y = 1$, $2y = 2$, which is a prime. For all other values of y, $2y$ is not prime. Checking $y = 1$, we get only five angles with prime measure, as seen in Figure 5 (1, **2**, **2**, 4, 87, 87, 88, **89**, **89**, **89**, 91, 91—primes are bolded).

Notice that $90 - 3y = 3(30 - y)$, which is only a prime if $30 - y = 1$, that is $y = 29$. If you check, you will find that we have the desired six angles with prime measure (**29**, 58, 58, 116, **3**, **3**, 32, **61**, **61**, **61**, 119, 119). If we similarly look at $90 - 2y = (45 - y)$, we see it would be prime only when $y = 44$, but that is outside the domain of the problem.

Thus, the only other possible way that we can get a solution is if y, $90 - y$, and $90 + y$ are all prime. Checking the ten possible primes less than 30, we get five more solutions corresponding to $y \in \{7, 11, 17, 19, 23\}$, giving six solutions in total.

There are many ways to use this problem with a class, depending on the group. As an exercise in angle relationships, you could give a measurement of a specific angle and have students calculate all other angle measurements (I would suggest giving $\angle PRQ$, rather than $\angle PSQ$ or $\angle QSR$ to make it more challenging). Those who work through it quickly could be given another value to try, and then could look for patterns between the two results. If they notice groups of equal angles between the two cases, you could encourage students further, to try to discover if it is always true. The problem could be given as is, and as students get stuck, you could hint them toward the part of the presented solution to which they are closest. That is, you could either encourage them to pick a value for one of the angles and work things out, or encourage them to introduce a variable. Even when they get "the answer," you could encourage them to see if they could come up with a different strategy. You could even give students the set-up of the problem, and ask them to find a case where as many of the angles have prime measure as possible. This could lead very naturally to the original problem.

For teachers, it is a good exercise to work out some problems that would be suitable for your students and think about how to modify them, how the students will approach them, and how to support and aid them, without giving it all away.

Now it's time for your homework:

> *Rectangle* ABCD *has diagonal* BD *with endpoints* B*(4,2) and* D*(12,8). Diagonal* AC *lies on the line with equation* $x + 2y - 18 = 0$. *Determine the area of* ABCD.

Until next time, have a great year, and happy problem solving. ▲

▲ OMCA News – September 2021

HEATHER THEIJSMEIJER (OMCA PRESIDENT)
OMCAmath@gmail.com

The wrap-up of the 2020–2021 school year brought with it the celebration of another year of fantastic learning at the Ontario Mathematics Coordinators Association (OMCA).

Just before the new Grade 9 math curriculum was released on June 9, OMCA hosted a fourth and final destreaming networking session for members on Thursday, June 3. At this session, we heard from three more boards about their destreaming initiatives. The meeting included informal conversations on topics related to the implementation of a destreamed math curriculum. Thank you to all who attended these networking sessions throughout the year! We look forward to continuing the learning in the Fall.

This year's recipients of the 2021 OMCA Award for excellence and innovation in mathematics teaching are Marilu Deal (Ottawa-Carleton District School Board) and Jason To (Toronto District School Board). This award is given annually to OMCA members, based on nominations by their peers, in recognition of excellence and innovation related to any areas of mathematics education (e.g., integration of technology, equity, implementation of curriculum, motivation of teachers, assessment and evaluation, professional development ideas). Congratulations, Marilu and Jason!

OMCA members Hema Khodai (Peel District School Board), Ashley Kozak (Upper Grand District School Board), and Sarah Taylor (Peterborough Victoria Northumberland and Clarington Catholic District School Board) participated in a Council of Ontario Directors of Education (CODE) project this Spring, to produce a support guide for the CODE summer learning program. Entitled *Grade 7 and 8 Mathematics Summer Learning Program: Guide for Numeracy Component*, it is a fantastic resource for learning, instructional, and assessment strategies for Grades 7 and 8 teachers. Visit www.ontariosummerlearning.org/ for more information.

At the final members' meeting of the year on Monday, June 14, outgoing president Rhonda Hewer (Waterloo Region District School Board) passed the gavel to incoming president, Heather Theijsmeijer (Rainbow District School Board), and welcomed our new Executive for the Fall. The Executive will be comprised of Rhonda Hewer (Past

President), Heather Theijsmeijer (President), Jason To (Toronto District School Board, Vice-President), Allison Lawrence (Brant Haldimand Norfolk Catholic District School Board, Secretary), and Marilu Deal (Ottawa-Carleton District School Board, Treasurer).

Are you a math leader in your board? Interested in joining OMCA? New members are always welcome. Contact us directly at OMCAmath@gmail.com, or visit our website at www.omca.website (note the different domain), to learn more about the organization and connect with us. Our first meeting of the 2021–2022 school year will be Monday, September 13. We hope to see you there! ▲

(Continued from page 48)

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(Continued on page 34)

▲ Linking Literacy and Math: Classroom Discourse (Part 2)



DAVID COSTELLO
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David is a principal, who has held many roles. He taught in the primary division before assuming roles of numeracy interventionist, numeracy coach, numeracy leader, and curriculum consultant for Prince Edward Island. David has also instructed university courses focused on curriculum, differentiation, mathematics, and language arts. He facilitates professional learning in the areas of mathematics and school development. Dr. David Costello recently published two books: Making Math Stick: Classroom Strategies That Support the Long-Term Retention of Math Concepts *and* Using What Works: Strategies for Developing a Literacy-Rich Environment in Math.

Classroom discourse is an important part of the mathematically literate environment. In my previous column, I delved into the meaning of classroom discourse and how it plays a role in student learning. In this column, I want to continue the focus on classroom discourse and offer strategies that can be applied to support it in the classroom.

The National Council of Teachers of Mathematics (NCTM, 2000) emphasized the significance of communication within mathematics. NCTM's *Principles and Standards for School Mathematics* identified communication as one of the five processes and stated that students should be able to:

- use communication to organize and consolidate mathematical thinking
- communicate their mathematical understanding clearly to others
- analyze and evaluate the mathematical understanding of others
- use mathematical language to accurately express mathematical ideas

Corresponding with NCTM's focus on communication, many provincial math curricula set communication as one of the mathematical processes through which students learn. The Ontario Ministry of Education highlighted math conversations as one of the high-impact instructional practices in mathematics to support the new math curriculum in Kindergarten through Grade 8 (Ontario Ministry of

Education, 2020).

What follows are four strategies that can be applied to support classroom discourse. While teachers may have experience with these strategies, it is important to take note of how we can strengthen classroom discourse and have it as a support for student learning and a means for students to share and make visible their mathematical understanding.

Classroom Discourse Strategy #1: Facilitating Discussion

Facilitating classroom discussions is a skill that continues to be explored in both literacy and mathematics. Although not a new skill, proficiency in facilitating classroom discussion can be impactful in supporting instruction and learning. Van de Walle et al. (2014) offer four points that assist teachers in facilitating purposeful discussion. Use of the four points, along with questions and comments, ensures productive discussions.

Points	Notes
Allow time for students to share their work and thinking.	Provide wait time after asking for volunteers. Ensure that you have a time for sharing that will be uninterrupted. Don't rush to get as many students as possible to share.
Before sharing with the class yourself, take a step back and ask students to explain their ideas.	Don't say something yourself if a student could say it. Provide opportunities for students to share similarities and/or differences in their thinking.
Question student responses instead of answering student questions.	A few question stems: • What led you to think that? • Were there other strategies that you could have selected? • How does this relate to other problems we have solved? • How does this relate to what _____ shared?
Encourage student-to-student dialogue as a form of learning.	Provide opportunities for students to work in small groups with assigned roles. Each role has a clear focus and can support discussion (cooperative learning). Provide sentence stems to students when engaged in dialogue: • _____'s idea is like my thinking because _____. • I like how you chose to _____. • Why did you choose to solve the problem that way? • What made you think of using that particular strategy?

Accompanying the chart above, the following suggestions can be utilized by the teacher to guide discussion.

Repeat.
Ask students to repeat what another student has said.

- Do this without criticism or judgment.

As the teacher, repeat what a student has said.

- The emphasis is on repeating what was shared, not commenting.

Paraphrase.
Ask students to paraphrase what has been discussed to this point.

As the teacher, paraphrase the discussion to this point.

Add on.
Ask students to add on to another student's response.

- Take the initial response one step further.

As the teacher, add on to the conversation.

- Use this as a guide to shift the discussion, if necessary.

Provide wait time.
Provide students with an appropriate amount of wait time to construct a thought and/or to build upon another student's thinking.

As the teacher, provide wait time before asking students to respond to the question.

Classroom Discourse Strategy #2: Classroom Conversations

Conversation is a significant part of classroom discourse. Discussion usually takes place in a larger group about a specific topic, while conversations occur between two individuals with a variety of ideas. Emphasizing the importance of conversation for students should not be understated. Clearly communicating expectations for student participation in conversations in the classroom will assist in strengthening such engagement. The following is a sample of conversational expectations that were established in a Grade 6 classroom (Costello, 2019). This list was under continuous revision, as students and the teacher would add to the list when opportunities arose for further supports to encourage purposeful classroom conversations.

- Acknowledge all contributions positively.
- Encourage the idea that learning stems from making mistakes.
- Use language that encourages problem solving.
- Promote collaboration among students in the form of table talk, small-group discussions, and think/pair/share activities.
- Value various problem-solving approaches.
- Encourage students to vocalize the problem they are working on, and their thinking at various points of the problem-solving process.
- Take time to write about your thinking—where are you at and where are you going?

Classroom Discourse Strategy #3: Think/Pair/Share

The think/pair/share strategy is often used in reading and writing instruction; however, there is clearly a place for it in mathematics (West & Cameron, 2021). In mathematics, this particular strategy provides students with time to think about the problem, time to share their thoughts with a partner to strengthen or refine their thinking, and then time to share their thinking with the whole group. After the sharing, and after hearing other students' ideas, students then have one more opportunity to refine their thinking. Throughout this process, students share their thinking and build on their understanding of the concept by examining the thinking of others and using this to expand their thinking. The following provides considerations when using the think/pair/share strategy (Costello, 2019).

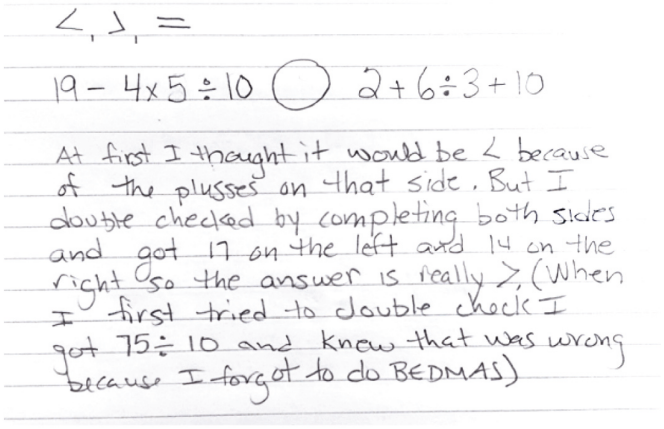
- Provide a task that is open enough to generate multiple strategies to get to a solution.
- Provide sufficient wait time so that students can develop a response to the task.
- Determine how students will share with a partner:
  - Will students be able to select their own partner?
  - Will students be grouped by the teacher based on like ability (homogeneous) or on mixed ability (heterogeneous)?
- After students share with a partner, will they share with another set of partners?
- What will the format be for sharing? Will all groups be provided a chance to share their strategy with the whole class? Will groups volunteer to share their strategy? Will the teacher select groups to share based on the strategies selected by the groups? After the sharing portion, will students have an opportunity to reflect on how their thinking progressed or changed during this activity?

Classroom Discourse Strategy #4: Written Work

When considering classroom discourse, teachers should not overlook written work. Encouraging students to write about their thinking or read the writing of others is an effective way to strengthen the mathematical discourse within the classroom. Journals, entrance/exit slips, free writes, and margin notes are but a few examples of writing activities that can support mathematical discourse.

Journals can be used to support mathematical discourse in the classroom in two ways. First, having students record their mathematical thinking in the journal provides them with the opportunity to reflect on their thinking, and it provides teachers with documented evidence of student learning. This record could include the questions students had while working through a problem, a list of strategies students applied to attempt to solve the problem, any stumbling blocks students encountered, and/or a detailed documentation of student work.

Second, the journal can be a springboard for further conversations. Students can share their journal entry with others in the classroom as a way to create a dialogue. What follows from this sharing can be written responses to the original journal entry or oral conversation. Using written texts as the springboard for conversation provides students with time to think about their thinking and to think about how they want to document it. The following example is from a math journal, and it was used to initiate conversation between two students. The journal writer read this entry to a classmate, and then the classmate responded to the points within the journal entry (Costello, 2019).



Final Thoughts

Classroom discourse is about sharing our learning and using the learning of others to support the learning of all. It is not something to be added on, but it is at the core of

instruction and learning.

It was important to examine classroom discourse over two columns. I didn't want to share strategies before stressing the importance of classroom discourse. My hope, now at the end of the second column, is that you can appreciate the need to strengthen classroom discourse as part of learning and to have a few strategies in your toolbox to make this happen.

I look forward to continuing the discussion of linking literacy and math via my website (www.costellomath.com), email (david@costellomath.com), or Twitter (@dr_costello).

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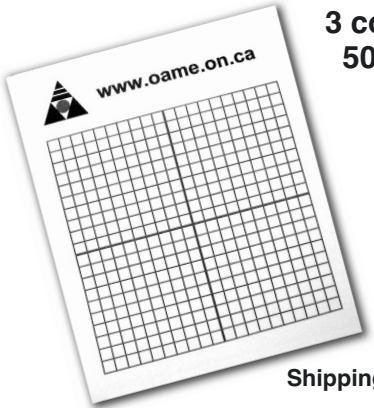
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▲ Mathematical Snapshots: A Math Walk at Richmond Green Secondary School



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*Ron Lancaster is an Associate Professor, Teaching Stream Emeritus at the University of Toronto, where he taught mathematics courses for pre-service middle and high school teachers. He has over 20 years of experience teaching Grades 7–12 mathematics. Ron's professional activities include consultations and conference presentations in North America, Asia, England, the Middle East, Africa, India, and Europe. Ron is an author for the NCTM (*The Mathematical Lens*) and member of the Advisory Board for the Museum of Mathematics in New York. He is the recipient of the 2015 Margaret Sinclair Memorial Award Recognizing Innovation and Excellence in Mathematics Education, awarded by the Fields Institute.*

On Saturday, September 14, 2019, I facilitated a workshop titled "Learning Mathematics Beyond the Classroom Walls Through a Math Walk" for teachers who attended the "Infinite Possibilities: A Math Getaway, K–12" Conference, held at Richmond Green Secondary School and a nearby park. I was not able to visit the site ahead of time, but Marcel te Bokkel, the conference chair, took a long walk on the site with several colleagues and sent me photographs and a map to get me started. Marcel is the Curriculum Consultant for Mathematics for the York Region District School Board and I appreciated his support.

On the day of the conference, I arrived at the site several hours ahead of time, followed the route that Marcel suggested, and took several hundred photos. A few of them appear in this column.

Several days later, Dr. Louis Lim, a Vice-Principal at Richmond Green Secondary School, contacted me and heard that I was with teachers working on mathematical questions related to the window of his office. This was a small-world moment. In the 1990's, I served on the search team at St. Mildred's-Lightbourn School that hired Louis for his first teaching position. Louis assisted me with my very first math walk/math trail at St. Mildred's. How fitting that 25 years later, I created a math walk while looking into his office!

Before looking at the questions that follow, discuss what you notice in the photographs, and suggest reasons why I

selected these photos. When you look at the photos, what do you notice? What are you wondering about?



Photograph 1: *Office window at Richmond Green Secondary School*



Photograph 2: *Circular cover for a maintenance hole near Richmond Green Secondary School*



Photograph 3: *Canada 150 flag in park near Richmond Green Secondary School*



Photograph 4: *Shed in park near Richmond Green Secondary School*



Photograph 5: *Licence plate spotted on car in parking lot near Richmond Green Secondary School*

Question 1

Photograph 1 shows windows for an office on the main floor at Richmond Green Secondary School.

(a) Suppose the area of the small window in the lower left corner is one square unit. Estimate the area of the

entire collection of eight windows.

(b) Figure 1 shows a geometric diagram that closely approximates the office window with all frames removed. In the diagram:

AB = BC = CD = DE

AF = FG = GH

AB = 2 x AF

Determine the ratio of the area of the area of rectangle AEKH to the area of rectangle ABLF. Is this result consistent with the estimation you made in part (a)?

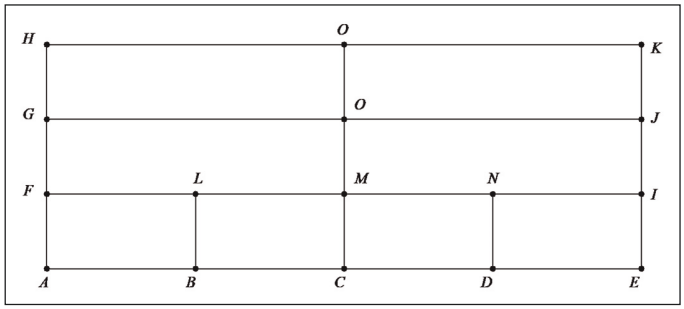


Figure 1: *Geometric diagram that closely approximates the office window with all frames removed*

(c) How many rectangles, regardless of dimensions, are there in Figure 1?

Question 2

(a) Suggest three different ways of counting the number of square holes in the circular maintenance hole cover (Photograph 2).

(b) Estimate the ratio of the area of the holes to the area of the circular cover. Express your answer as a percentage. Would it be a problem if this percentage were above, say, 75 percent? Why or why not?

(c) Figure 2 shows a geometric diagram created using the Geometer's Sketchpad®, which closely approximates the circular cover and one of the small square holes. In the diagram (not the actual cover), the radius of the circles is 10 cm, and the length of the side of the square is 0.7 cm. Calculate the ratio of the area of the holes to the area of the circular cover. Express your answer as a percentage. Is your calculation consistent with your estimate in part (b)?

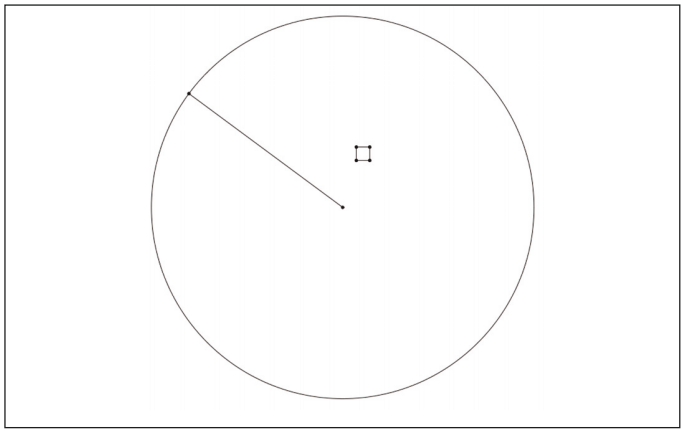


Figure 2: *Geometric diagram of circular cover*

Question 3

Read the following information about the official emblem of Canada 150, shown in Photograph 3.

The official emblem of Canada 150 is a stylized maple leaf with 13 diamonds representing provinces and territories. The four red diamonds represent provinces at confederation. (Source: https://en.wikipedia.org/wiki/150th_anniversary_of_Canada)

(a) Where are the 13 diamonds in the emblem?

(b) What is the mathematical definition of a diamond and a rhombus. Is a rhombus a diamond? Is a diamond a rhombus?

(c) In Figure 3, a model of the emblem is provided. Let p and q be the length of the diagonals HA and MT, respectively, of rhombus MATH. Let A be the area of the rhombus, and let r be the inradius (the radius of a circle inscribed in the rhombus). Show that:

$$A = \frac{pq}{2} \qquad r = \frac{pq}{2\sqrt{p^2 + q^2}}$$

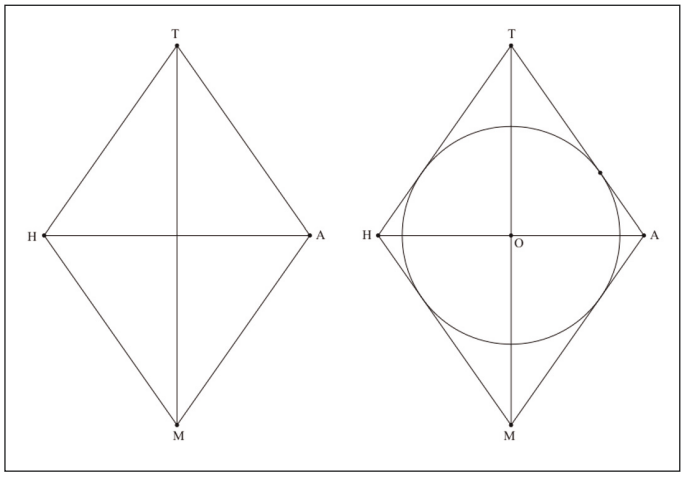


Figure 3: *Diagonals, area, and inradius of a rhombus*

Question 4

(a) List all the things in Photograph 4 that need to be changed so that the entire image is perfectly symmetrical about the vertical line that passes through the photograph.

(b) In Figure 4, let a and b be the width and the height of the doors on the left and right sides of the building. Estimate a formula for each of the following quantities as a function of a and/or b. You might find it helpful to have a printed copy of Figure 4 for these questions.

(i) width of the building

(ii) height of the building

(c) Which of the following fractions is closest to the slope of the roof?

$$\frac{b}{a} \quad \frac{b}{2a} \quad \frac{b}{3a} \quad \frac{b}{4a} \quad \frac{b}{5a} \quad \frac{b}{6a} \quad \frac{b}{7a} \quad \frac{b}{8a} \quad \frac{b}{9a} \quad \frac{b}{10a}$$

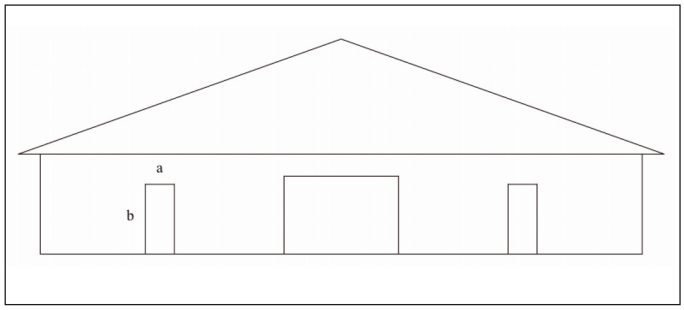


Figure 4: *Geometric diagram of building*

Question 5

The following questions will require some research online.

(a) In Ontario, what is a "green vehicle"? Is the definition of a green vehicle the same everywhere in Canada?

(b) How many different licence plates (Photograph 5) are available in Ontario that begin with the letters GV (for Green Vehicle)?

(c) How many licenced vehicles are there in Ontario? What percentage of them are green vehicles? Do you think this percentage will go up or down in the future? Why?

(d) If the number of green vehicles in Ontario continues to go up, will there be a time when there are not enough licence plates that begin with GV? When might this happen?

Perhaps students can team up and develop a set of photos and related questions that they can share with the other groups of students (or other classes). ▲

▲ In the Middle: *Timeline*, the Card Game



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Frederic Henry created the card-based family game *Timeline*. It is a card game where each card shows a single historical event or invention. It can be played competitively or co-operatively. The goal of the game is to place your cards in a *Timeline*, showing the correct historical sequence. In addition to the original game, there are expansion packs (Figure 1), distributed by Asmodee Editions, and I will talk about making one's own card sets.

The game can be readily used in a social studies class, but I have found it provides some unique opportunities for math learners and their curious teachers. This column presents some ways I have used the game, along with some adaptations, and I plan to incorporate it into future math classes, especially the new MTH1W course.



Figure 1: *Expansion* Timeline *games*

To play the game, the players look at the front of the cards, with just the events shown (Figure 2), sort them, and then, when they think they are correctly ordered, flip them over to check (Figure 3). The game can be played solo, with small or large groups, or even as a whole class. When I have played this with my family, we often spent about 15 minutes arranging a few handfuls of cards, and then at least as much time investigating and discussing many related historical events, falling down the inevitable rabbit holes of "Where was the X-ray invented?" or "How could the Pythagorean theorem be older than the catapult?"



Figure 2: *Sample* Timeline *cards*



Figure 3: *Flipped* Timeline *cards*

At school, I have often pulled out this game on the last day before holidays, encouraging the students to note any math-themed cards. While teaching online this Spring, I used these as starters, asking: "When do you think this happened?" I also used older or newer comparisons: "Is the next card in the pack older or newer than this?" My sons recently curated a custom *Timeline*—mathematical edition, with the express purpose of seeing if they could stump their mom. In that pack, the most perplexing cards were about the invention of mathematical symbols and notational conventions. The notation for basic arithmetic operations captivated my entire family, and the development and adoption as standard symbols was an evening of quarantine entertainment. From this new set, I used a few small sequences for a miniature collaborative game to pull students back together and, on other occasions, to

1. Can the *Gazette's* clever readers put these five probability-themed cards (Figure 4) in order?

A	B	C	D	E
				
The invention of playing cards	The invention of the die	The invention of fractions	The theory of probability	The invention of currency

Figure 4: *Probability-themed cards*

2. Thinking about showing our work and making errors, what is the order of the cards in Figure 5?

a	b	c	d	e
				
The invention of the pencil sharpener	The invention of paper	The invention of the rubber eraser	The introduction of writing	The invention of the pencil

Figure 5: *Paper–pencil cards*

3. Is it easier to order modern items such as the technology creations shown in Figure 6?

I	II	III	IV	V
				
Public access to the Internet	The invention of ENIAC	The invention of the calculating machine	The invention of the computer mouse	The invention of the laptop computer

Figure 6: *Technology cards*

encourage student discussion in breakout rooms.

You may have noticed that the BCE cards are noted using negative signs, which has an obvious mathematical extension for middle school (i.e., 300 BCE is given as –300). If you don't have the cards, sticky notes or recipe cards work very well for a similar DIY activity. I have created these sample cards, using icons from the Noun Project.

A small sequence can have many possible problems, including integer and fraction operations. Some additional ways to use a sequence mathematically include the following:

- When three cards are placed in a sequence, which card is in the middle, and would the median date of the three cards be older or newer than the date of the middle card?
- How many years separate any two cards?
- Which two adjacent cards in the set are the closest or the furthest part?
- What is the time span of the entire set?
- When the items are ordered on a *Timeline*, what would a scatterplot of year versus ordinal number (i.e., sequence number) look like?
- With a larger set of cards, a histogram can be constructed. What are the appropriate horizontal bin widths? How would you determine that? How might it change if the number of cards or the theme of the cards were different?
- With a larger set, how else might you want to arrange the cards into groupings?

As I used the game with my Grade 9 class, I began to consider the historical connections to the content in my Grade 12 Data Management and Statistics classes. As I did, I grew conflicted about some difficult contexts, reflecting our changing cultural examinations in areas outside of mathematics. I spoke frankly with my older students about my discomfort with referring to our Galton Board. We watched a section of PBS's *American Experience: The Eugenics Crusade*, about Francis Galton together. In the end, we agreed to rename it our "Plinko Board." Similar discussions occurred around Karl Pearson and Pearson's coefficient. There is no avoiding the connection between foundational statistics and its problematic legacy, where the use of statistics perpetuated and often engendered institutional bias with immigration, anti-Semitism, class, and race in the early 1900's. In the year-end surveys of my students, many commented on how much those conversations affected their understanding of statistics as a complicated discipline, and how they found those discussions meaningful.

Culturally responsive pedagogy brings the experiences of students into their math classes, and connects to their cultures and experiences. An extension to *Timeline* considers where these inventions and discoveries happen, not only in time, but also where those mathematical discoveries occurred and how different cultures responded. India had the development of the concept of a placeholder (zero) and the use of negative numbers, for example, which show the region's rich mathematical inheritance. The well-documented triangular notation for coefficients in a binomial expansion by Chinese mathematician Jia Xian occurred in the eleventh century, before Pascal, for whom the triangle is often named. This provided an opportunity to consider historical accuracy. When there was no historical name, I began to encourage my students to name their quick tricks or their clever solutions with their own names. The Jenny-Hannah method of how to remember negative reciprocals with perpendicular slopes, for example, ended up being a class favourite.

The *Timeline* game has an overreliance on European discoveries that reflects common English sources. The newest Diversity pack attempts to include a greater variety of inventions. Many non-Western cultures have an extensive history of mathematical discoveries that are rarely represented in classrooms, and are similarly ignored by the original *Timeline* game. This is also true for discoveries by women. Asking our students for contributions to a custom "pack" of cards invites us to listen to their curiosity and their culture. Some examples of concepts that we could consider in building a new set for our own *Timeline* are:

- Mayan organization of a calendar
- Chinese use of decimal representation and place value
- Florence Nightingale's creation of an infographic
- Origin of the word "Algebra"
- Ishango bone: possibly the oldest recording of a prime number table

In connection with my previous column, Padlet also has a *Timeline* format that students can use to make their own relevant cultural mathematical investigations (Figure 7). Mathigon also has a *Timeline* that is linked to other content on their website, displaying cultures by colour bands and themes visually (Figure 8). Either style could also be adapted for a classroom bulletin board or individual poster project creating a custom timeline.

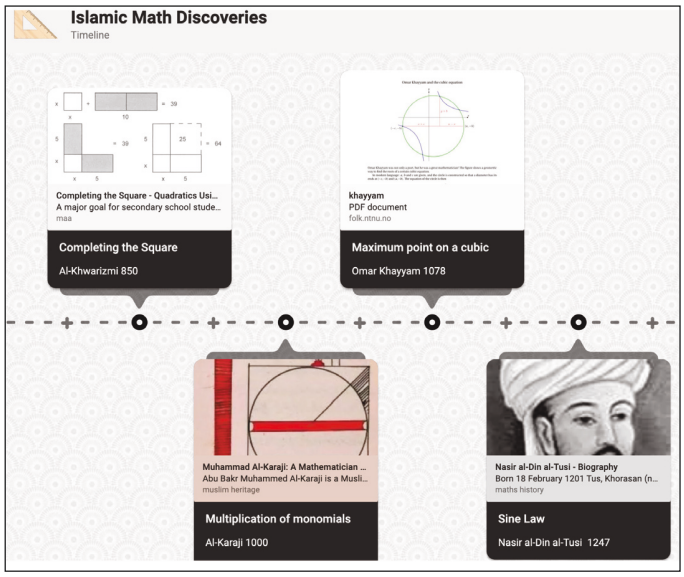


Figure 7: *Sample padlet* Timeline

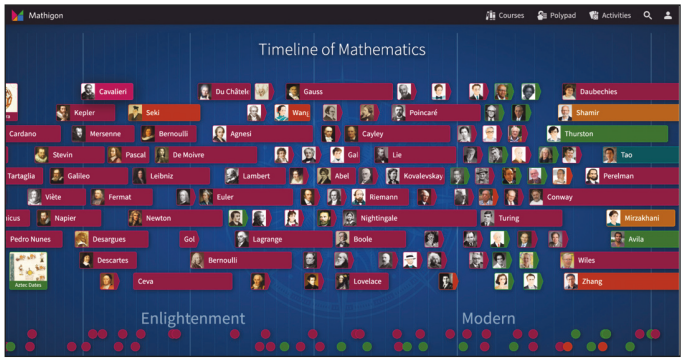


Figure 8: *Mathigon* Timeline *of Mathematics*

The new MTH1W curriculum includes the specific expectation to research a concept and tell a story about its development, relevance, and connections. This language is used within the strands of Number, Algebra, Geometry, and Measurement. Using *Timeline* or an adaptation like those discussed here can fulfill this expectation repeatedly throughout the course. With different topics, students can choose concepts and add them to a personal *Timeline* poster, collaborate on a small-group padlet, or develop a classroom bulletin-board display, like a word wall.

I have added a *Timeline* section onto the bottom of activities (Figure 9) that uses concepts, ideas, notations, or discoveries connected to the learning that has taken place or the particular section of the course. By preparing such sections ahead of time, I plan to be a co-learner of mathematical history with my students. It also reinforces meeting that expectation regularly, and integrates and enriches the coverage of the concepts and topics.

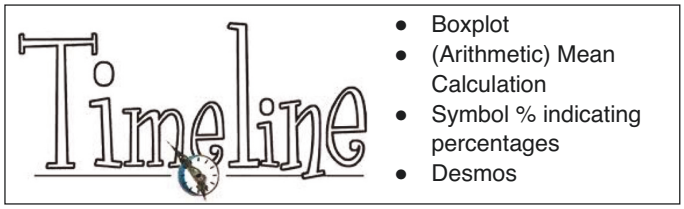


Figure 9: *Example of the* Timeline *topics in a MTH1W one-variable statistics activity*

The suggested uses have students anchor mathematical processes as the work of real people, who are learners and creators of mathematical thinking just like they are. It connects well with the Social–Emotional Learning strand that runs through the elementary and the MTH1W curriculum. When students identify why and when concepts were developed, we help them to see that the distributive property, for example, was not dug up from the ground by only one explorer in *one* place in the *exact way* we use it now. What students are learning is the product of many people handing down a legacy of development. Investigating varied historical contexts reinforces that the discipline of mathematics is constantly adapted and refined, and is continuing to evolve. Our students can see themselves as further creators of mathematics. For me, that emphasizes the tension between standard forms, conventions, or rules, and creative expressions of mathematical thinking. It gives us a chance to discuss how we are all learning and working as we try to communicate mathematics to each other. *Timeline*, along with the adaptations described in this column, are tools to connect mathematical processes, social–emotional learning, along with history and culture. Thinking of themselves (or ourselves!) as part of a long history of mathematical creators is very empowering for all students, and one I am excited to share and learn with them. ▲

Technology Cards – Answers

III Calculating machine (1642, Blaise Pascal)
II The invention of ENIAC (1944, John P. Eckert and John Mauchly)
IV The invention of the computer mouse (1963, Douglas Englebart)
V The invention of the laptop computer (1981, Adam Osborne)
I Public access to the Internet (1990)

Paper–Pencil Cards – Answers

d The introduction of writing (-3400)
b The invention of paper (-200)
c The invention of the rubber eraser (1770, Edward Naime)
e The invention of the pencil (1795, Nicolas-Jacques Conté)
a The invention of the pencil sharpener (1828, Bernard Lassimone)

Probability-Themed Cards – Answers

C The invention of fractions (Ahmes, -1650)
B The invention of the die (-1250)
E The invention of currency (-680)
A The invention of the playing card (900)
D The theory of probability (Blaise Pascal and Pierre Fermat, 1654)

▲ What OAME Has Meant to Me

MARYLOU KESTELL
PRESIDENT, 1990



My career in mathematics education began at the University of Waterloo, where I graduated in 1973 from the Co-op Teaching program with a degree in Mathematics and a Teaching degree from Althouse College of Education. I had a specialist degree in Mathematics and Computer Science. In my second year in the classroom, I became a member of OAME/AOEM and attended the annual conference, missing only two in all my years from 1974 to the present, 2021. At my first conference, in Kingston, I heard Bill Higginson present a talk entitled "Mathematics, a Human Endeavour," and I was hooked. I appreciated the learning I gained from interacting with like-minded math educators from across the province and abroad. By about 1980, I became a member of the OAME Council and lived for the inspiration I gathered at Council meetings and mini-conferences held by chapters of the organization.

My high school classrooms looked like most others at the time, with students in single-file desks and me presenting a lesson, using a blackboard and chalk. Then I would move around the room, answering questions from individuals and gathering small groups together to clarify concepts as required. I cared that all students be successful and hated it when students failed.

In 1989, as an executive member of OAME/AOEM, I had the opportunity to attend the National Council for Teachers of Mathematics (NCTM) sessions on "Leading Mathematics into the 21st Century." That training set me up to present numerous sessions to math educators, administrators, and superintendents, explaining why math education needed to change. Math instruction needed to be more inclusive; more student centred; focused on their thinking, rather than the teacher's. Instead of being a sieve to eliminate most students, we wanted math to become a funnel capturing student engagement.

Throughout the 80's, I learned to listen to students' thinking, and when I became a math consultant in 1993, I learned to listen more carefully to teachers of Grades 3–8, with whom I was now working. Sadly, I found that most math lessons were teacher centred and comprised of activities to keep students busy and not really aligned to a series of learning goals.

At the time, I was vocally opposed to large-scale assessment, but when my board administered the Canadian Achievement Tests (CAT), I noticed the teachers paid attention. My thoughts began to change. In 1995, reports from the Royal Commission on Learning, which was established to "ensure that Ontario's youth are well-prepared for the challenges of the twenty-first century," resulted in the creation of the Education Quality and Accountability Office (EQAO). As a member of OAME, I was called to review and provide input on the first-census Grade 3 assessment, a review that was a surprising ten days long. It assessed numeracy and literacy and modelled the way we wanted mathematics integrated with the world around the student. I came to be a supporter of EQAO assessments because they reported on what students know and can do. They suggested next steps for improvement. They assessed the expectations in the policy documents from the Ministry, and for the first time, elementary teachers had scope and sequence charts that outlined a trajectory for instruction.

Eventually I worked on the Grade 6 assessments and developed the Grade 9 assessments by setting up writing teams from across the province, using OAME/AOEM leaders as advisers for each group. Thus, we had assessments that were built by Ontario teachers and marked by Ontario teachers. The professional learning on the marking sites was matched by none, not because of what EQAO did, but because teachers got together to talk with other teachers and establish assessment norms.

Tests are, however, only useful if they light a way for action. So, in 1999, I moved to the Literacy and Numeracy Secretariat. The Premier at the time, Dalton McGuinty, promised to help teachers like they had never been helped before. We visited classrooms and worked with teachers to make student thinking explicit. We developed Collaborative Inquiry for Learning – Mathematics for Teaching (CIL-M). Michael Fullan, as an adviser to the Premier, stated that professional learning is inversely proportional to its distance from the classroom door. We went into classrooms across the province—with teachers, administrators, superintendents, and Ministry staff—to listen to student thinking and act upon it, just in time, to improve instruction and student achievement.

After retirement in 2009, I continued to co-write the *Abacus* with esteemed colleagues, Kathy Kubota-Zarivnij and Pat Margerm, and worked as a Provincial Math Facilitator in Ontario classrooms. About eight years ago, I met some very talented young men working on a project to improve mathematics education. I called them my "Whiz Kids." That project has turned into a very successful educational technology company called Knowledgehook, which is thriving during the pandemic.

OAME/AOEM has offered a guiding light to me throughout my almost 50 years as a math educator, and I am grateful for all those who have made it so. OAME/AOEM associations made me the life-long learner I have become. ▲

▲ MB4T (Mathematics by and for Teachers): Not All Inequalities Are Created Equal



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Jennifer Holm is an Assistant Professor at Wilfrid Laurier University and works with primary/junior and junior/intermediate pre-service teachers, as well as in the field, supporting current mathematics teachers. She is interested in developing mathematics knowledge for teaching with both pre-service and in-service teachers. She focuses on the beliefs and opinions that pre-service teachers hold about mathematics and teaching, and the connection they have to past experiences. She uses this research to support future teachers in developing beliefs and knowledge that will encourage and support effective teaching practices in mathematics.

The focus of this column is an expansion of the ideas from my June column (Holm, 2021) relating to inequalities. In focusing on inequalities, we can see that the Ontario curriculum has changed from the 2005 curriculum to the current one. According to the Ontario Ministry of Education (2020) *Key Changes* document, the 2005 curriculum had no expectation of solving inequalities, whereas the 2020 curriculum has students working with solving inequalities in Grades 4 to 8. This column focuses on the skills needed to solve inequalities through understanding.

Focus on effective language is one of the key skills of support for students to understand inequalities. Sometimes when we first discuss inequalities with students, we ask them to draw teeth on the inequality so that the animal "eats the bigger number." Although this can be a helpful way to remember which way the sign points, it can also lead to students focusing too much on distractedly drawing the animals and diminishing the point of the sign. A focus on effective language can help to eliminate some of the distraction of drawing, and bring the emphasis on understanding what the signs mean. When students start operating with the signs, this understanding will be vital, as discussed further in this column. Having students read the inequalities correctly can help to bring the focus on the meaning: $14 > 3$ as "14 is greater than 3."

In my last column, the focus was on the balance-beam model for solving equations, and it included a single example of inequalities to draw comparisons. In this column, we will first look at a few more examples, with a focus on the language and process for understanding inequalities, and then focus on the understandings of why there are certain processes that are followed in working with inequalities.

To start, if we build on the balance-beam model to understand inequalities, it is important that we choose examples where the unknown is on the left and is positive (which it needs to be with this particular model). I am using the letter n in this example, since it does not look like an operation sign, as Small (2017) has raised concerns about previously. This ensures that students are focusing on the process, which is initially identical to the one with equalities. Consider the example of $n + 2 < 10$ in Figure 1. Once the inequality is placed on the balance beam, a student could use the knowledge they have from using this representation to solve equalities in order to solve this inequality. To isolate the n value on the left, both sides should have 2 subtracted to keep the inequality balanced. In the end, the solution shows that $n < 8$, which can also be demonstrated using the scale.

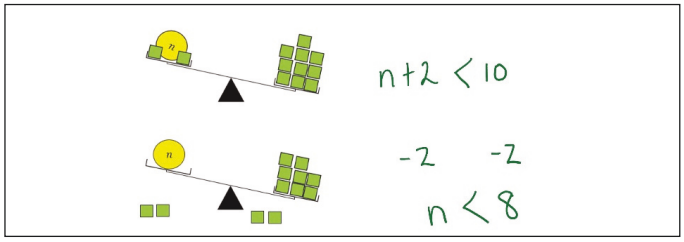


Figure 1: *Using a balance beam to solve* n *+ 2 < 10*

When starting to make solving inequalities more complicated, we want to continue to make sure the larger n value is on the left, and that there will be a positive n when exploring initial models with inequalities. This will allow students to still apply the knowledge they have gained from working with equations to gain a foundational understanding of inequalities. Consider how a student may work through the example $3n + 4 > n + 6$. In making the comparison to equation solving, they will likely start by removing 4 from each side. Then the student could remove n from each side, leaving $2n > 2$ as shown in Figure 2. Dividing each side by 2 would show that the solution is $n > 1$.

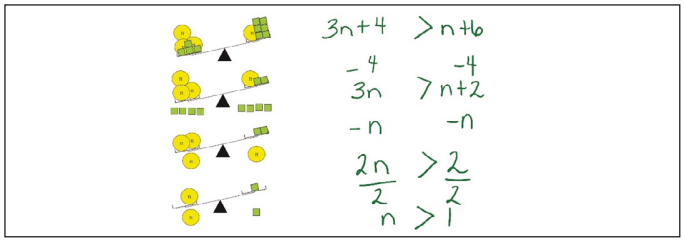


Figure 2: *Using the balance method to solve* 3n *+ 4 >* n *+ 6*

By considering exercises and examples like the ones above, students can begin to understand the process of working with inequalities. By again focusing on language and having students read the answers as "*n* is less than 8" in the first example and "*n* is greater than 1" in the second, students can continue to conceptualize what the symbol means as they work with concrete examples.

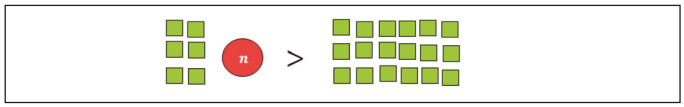


Figure 3: *An image representation for 6 – n > 18*

Now consider what would happen if we give an example like 6 – *n* > 18. Now, a balance beam would not be an overly effective model to use concretely, since you have a negative *n* value, but let's consider an image where the red ball with the *n* is negative in Figure 3. How might a student consider solving this inequality?

If students consider what they have learned about working with equations, they will likely solve the example, as shown in Figure 4, by removing 6 from both sides and then dividing by -1 to have *n* isolated on the left. Although the steps are correct, the answer is not. Now we need to consider why this is not the correct answer.

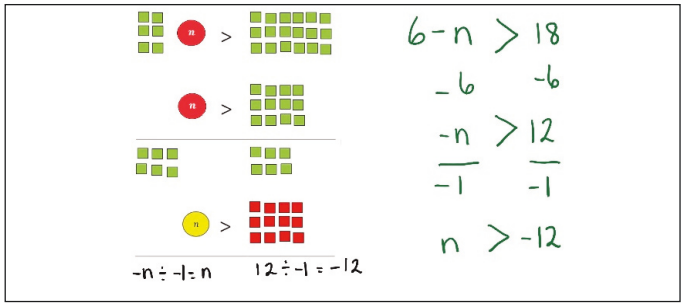


Figure 4: A partially correct solution for solving the inequality *6 – n > 18*

Consider the inequality 7 > 4. We know this is correct, as 7 is greater than 4 (and it can be demonstrated using a balance). What happens when we divide both sides by -1? We now have –7 > –4, which is not correct. If we have students focus on the number line in Figure 5, we can see that when we divide by a negative, we have altered our inequality, and in order to make it true, we need to invert the inequality symbol. Some teachers and students might prefer the use of "flip" or "reverse," as well as using "invert." In this example, –7 < –4 (negative 7 is less than negative 4). Note that discussion regarding the language is invited in the editorial.

If we return to the example in Figure 4, we can solve it the same way, but look at the difference in the last step shown in Figure 6. When we divide by a negative, we need to invert the inequality symbol, since we are changing sides of the number line. Therefore, the solution in this case would be *n* < –12.

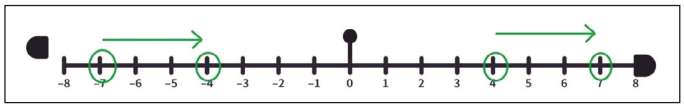


Figure 5: *Number line from Mathigon polypad at www.mathigon.org/polypad*

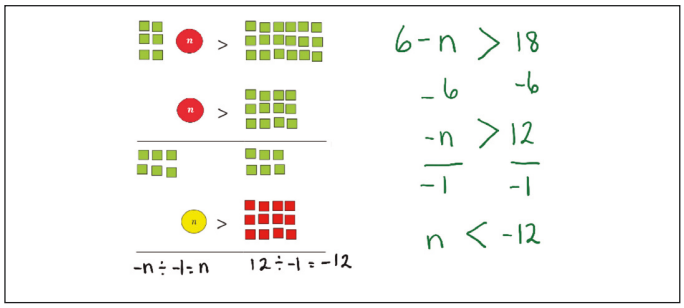


Figure 6: *Completely correct solution for 6 – n > 18*

Using numbers, we can have students perform operations on each side, and see if and when we alter the inequality sign. Students can then internalize this concept with their own sets of "rules" for performing operations with inequalities. We can use the example of 8 > 2 and then have students perform experiments, using the four operations to see when the inequality is no longer true. Having an understanding of equations and equality will help a student to see that as long as they perform the same operation on both sides, they keep the same balance, as we do with equalities. Starting with addition, we could pose the question, "If we add any value to both sides, does it alter the inequality?" Students could add the same positive and negative integers to both sides and see that, no, it does not alter the inequality, and it remains true.

This process can be repeated, with a series of positive and negative numbers, for subtraction, multiplication, and division. See Figure 7 for an example of each that could be used. From this experience, students can determine that when we add and subtract, we do not need to change the direction of the inequality symbol. This makes rational sense when we consider that if someone has more cookies than their friend, no matter how many cookies we add or subtract (when we do the same to both), that person is always going to have more cookies than their friend. When we multiply or divide by positive numbers, we also do not need to change the inequality symbol; however, when we multiply or divide by negative numbers, we need to change the direction of the inequality symbol. As a note, when using *greater than* or *equal* and *less than* or *equal* symbols, there can be exceptions about having to change the direction of the

inequality symbols.

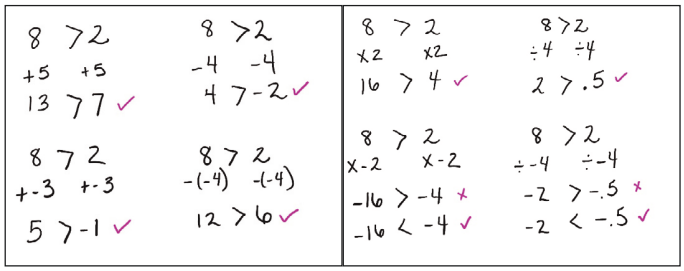


Figure 7: *Examples of experiments to perform on inequalities to determine when the inequality symbol needs to be inverted*

One final exercise to try is what happens when the *n* is isolated on the right-hand side? With equations, we can just reverse the statement (i.e., 5 = *n* and *n* = 5), but can we do the same thing with inequalities? Consider the example 16 < *n* + 5 in Figure 8. Here is where language becomes so important for students. If they read the answer as "11 is less than *n*," it makes it clear that if we move the *n* to the left, we also have to alter the inequality symbol because the statement ends up different: "*n* is greater than 11" would be correct. In effect, one is writing the statement right to left.

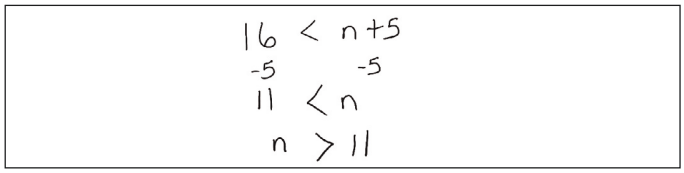


Figure 8: *Solution for 16 <* n *+ 5*

Inequalities share a lot of similarities to working with equations. When discussing them, it is important to take advantage of those similarities while building an understanding and the language for working with them. As the level of difficulty increases, making sure students have a thorough understanding of integers and the meaning of the symbols becomes key. Performing experiments where students can compare to a number line can help increase their understanding of when they have to worry about changing the orientation of the inequality symbol in their work.

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▲ Some Thoughts about Perimeter: Rigidity of Sides



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Students often come up with surprising ways of solving tasks we give them. At times they even come up with strategies or ideas that, in turn, raise questions or shed light on mathematical aspects usually taken for granted or kept implicit. This is particularly the case in the mental mathematics that my research team and I study in classrooms. When faced with time constraints, students rarely refer to known procedures because these are often too time-consuming and hence not economical to solve in this context (see Butlen & Pézard, 2000). This leads many students to engage in a variety of strategies that are worth pondering.

For the last two years, our research team has been working with young students, from 10 to 13 years old, on various tasks about perimeter (and area) in a mental mathematics context. Perimeter is often considered a simple concept. It is distinguished from area, as a measure of surface, while perimeter is usually defined as the measure of the distance around a figure. Frequently conceived, and measured, through using a piece of string wrapped around a figure, perimeter can also be trivialized as the addition, one by one, of the measures of the sides of this figure. Indeed, most textbooks and lexicons complement its presentation through ways of calculating it, with assorted examples. However, beyond its measure or simple computation, perimeter is not such a simple concept, and is much richer than it appears at first glance.

This represents the theme that runs through this year's series of columns, where I share examples of various students' ways of solving tasks about perimeter. Their

strategies and solutions raise a number of questions regarding the nature of perimeter itself and the meaning often given to it. This first column focuses on issues about ways of computing and considering perimeter.

Tasks Regarding Equal Perimeters

Two of the students we worked with were Marco, a 10-year-old boy, and Marie, a 12-year-old girl (both pseudonyms). Both Marco and Marie were given Figure 1 and asked if and how the shapes A and B could have the same perimeter. (The tasks and their figures discussed here were inspired by a number of sources, in particular the work of N. Krikorian and C. Leclerc (pedagogical advisers at the *Commission scolaire de Montréal*).)

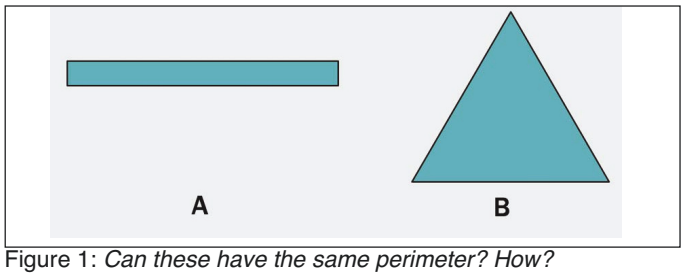


Figure 1: *Can these have the same perimeter? How?*

Both students associated each side of A with sides of B. First, the longer sides of A were matched with two of the "upper" sides of B. Then, the shorter sides of A were grouped to see if they were both as long as the third side of B (see Figure 2). This ended up with Marco and Marie each saying that it was not possible for A and B to share the same perimeter, since the combined length of the short sides of A was not as long as the third side of B.

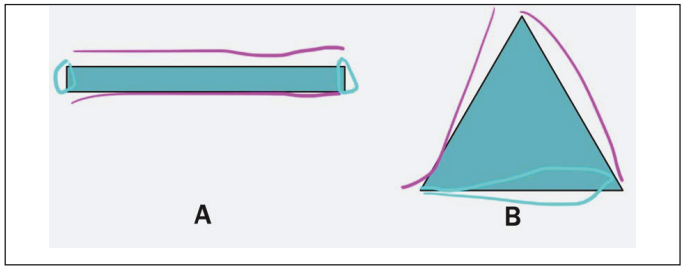


Figure 2: *Correspondence and grouping of lengths to obtain equal perimeters*

When questioned if there could be a way for the perimeters to be equal, Marie suggested that if both short sides of A were doubled (see Figure 3 on the left), then, if combined, they might be of equal length with the third side of B.

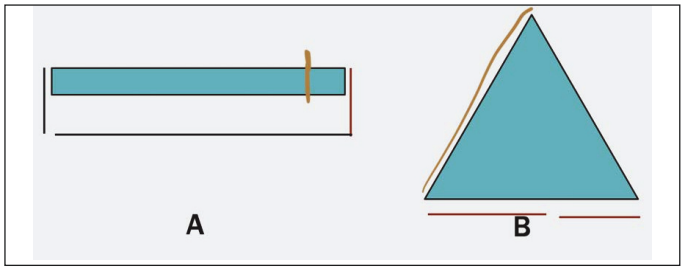


Figure 3: *Marie's suggested alterations to generate equal perimeters*

It thus appeared impossible for Marco and Marie to, for example, cut the longer side of A at a place where parts of it would be taken and distributed to cover another part of the perimeter of B (as shown by the brown line in Figure 2). In short, each side of the figures was presumed to have to remain uncut, and could not be broken into parts or aligned, for example, in a long string of equivalent length to be wrapped up along B (or the opposite, where all sides of B would be aligned as one long length and then wrapped up around A). Each side was somehow conceived as rigid, as a stick, and not as a malleable object.

As a way to explain this impossibility, Marie referred to Jenga blocks she had handy. Placing them in a way to recreate figures A and B, she showed how they would have to be organized for them to share the same perimeter (Figure 4 reproduces her suggestion).

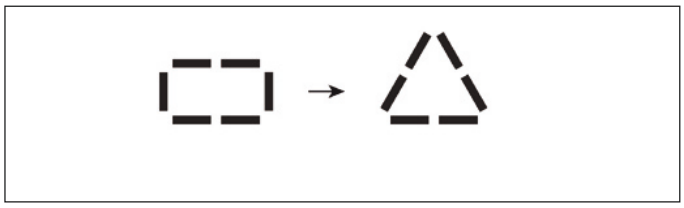


Figure 4: *Marie's simulation with Jenga blocks to obtain equal perimeters*

The perimeter appeared to be rigid relative to the sides of the figure (and the discrete blocks); thus, she felt stuck with these sides. Marco completed Marie's idea by creating a bigger rectangle with the Jenga blocks as well, by showing that there would be pieces missing to complete the perimeter of the triangle (Figure 5 reproduces his idea).

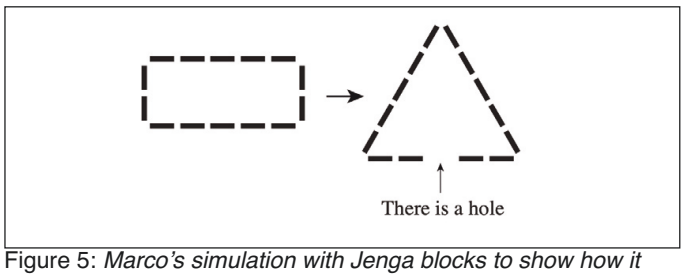


Figure 5: *Marco's simulation with Jenga blocks to show how it does not work*

The sides of the figure, even when rendered in small blocks, remained rigid. This obviously is linked with the nature of the material at hand (i.e., the Jenga blocks), but it was first linked to the figures of rectangle and triangle, and how their sides had to be kept fixed. Somehow, the perimeter was not detached from the figure; the nature of the figure and its constituent sides were maintained fixed, rigid, when considering the perimeter. The same sort of explanation was later offered for a similar task, again asking the two students whether the following figures A and B could have the same perimeter (Figure 6).

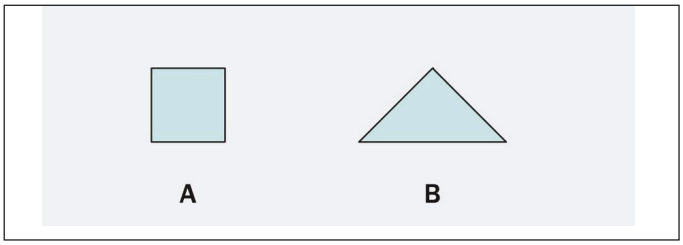


Figure 6: *Can these have the same perimeter? How?*

As with the previous task, they attempted to match sides of each figure as they were, without folding or breaking them up to constitute the perimeter of the other. They also insisted on the fact that one side of square A was not long enough to cover one of the upper sides of triangle B, and additionally that two sides of square A were "not half of this [long] side" of triangle B. Hence, here as well they concluded that it was simply not possible for A and B to have the same perimeter. Again, the sides were kept unchanged, unaltered, as if they had to rigidly remain exactly as they were.

Solutions for both tasks show how the perimeter appeared in direct relation to the sides of the figure. These sides influenced what had to be considered in the assessment of the figure's perimeter: their length and position played a direct role in what could be done. The maintenance of attributes of the figures refrained Marco and Marie's actions in their comparisons and assessment of the perimeter of these figures.

Some Thoughts about… Perimeter, Formulas, and (Un)folding

It is possible that Marco and Marie initially thought that the question required that figures A and B be kept rigid for both tasks, and that they could not be transformed much. Or, they might have thought that these figures were built with a particular ratio in mind that had to be maintained. This could explain their ways of dealing with the sides of the figures as if rigid (even though the Jenga blocks made them a little malleable). But in all cases, what Marco and Marie both did in relation to perimeter points to how formulas can take an important role in influencing the understanding of perimeter, in particular, for calculating (not measuring) the perimeter. As mentioned, in many textbooks, as soon as the perimeter is presented, be it as the distance around the figure, as a line delimiting it, as a fence or piece of string to wrap around it, and so forth, a way to calculate it, often by "adding its lengths," is offered. This recurring attachment of formulas to perimeter can impact the meaning drawn about it. Without implying a causal connection, for many students the perimeter can *become* its formulas (Vighi & Marchini, 2011) or be conceived as a "multiplication" or an "addition" exercise (Douady & Perrin-Glorian, 1989).

That the measure of perimeter is obtained through calculations is not necessarily problematic in itself; we indeed do this regularly as adults. What is at stake is mostly that conceptual issues and difficulties may emerge from this constant presence of formulas for determining the perimeter. Marco and Marie's ways of keeping the form rigid, static, and not very malleable points to some limits of insisting on summing the value of each side, one by one, to calculate a figure's perimeter. The focus on the formula, through the figure's sides, creates a shortcut that somehow hides an important initial step when assessing the perimeter: *it is not sides that are important, nor adding them up, but mostly what they represent together, as a whole, all aligned.* Said differently, measuring perimeter asks that the figure be unfolded as a line segment, and it is this line segment that is to be measured to obtain the perimeter of the figure. Producing this unfolded segment for the figure can transform the assessment of the perimeter from a simple computational exercise toward a measuring one (Figure 7).

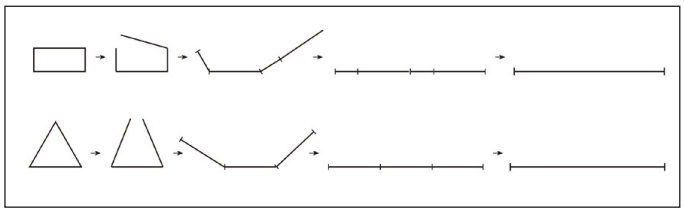


Figure 7: *Unfolding figures*

It is when a figure is unfolded, when the sides are all aligned, that one can measure this complete segment, the total perimeter, or compare it with another unfolded segment representing the perimeter of another figure. This has little to do with summing up the value of each side of the figure; indeed, there are no formulas to measure a line segment, except with the use of a ruler! In these unfolded segments, the creases initially delimiting each side of the figure disappear: the interest is in the total length of the perimeter, not its sub-parts that are the sides of the figure. It is to some extent the entire piece of string that got wrapped around the figure, and then laid flat, that is to be measured.

In return, it is from this unfolded and uncreased segment

that the figure can be reconstituted, be it in the initial figure or in constituting another figure through a different folding. In this specific case, the unfolded segment does not need to reproduce the initial creases that created the rectangle or square, and can lead to other figures of different shapes that share the same perimeter, be they other rectangles, squares, triangles, rhombus, etc., which are all isoperimetric figures, as Figure 8 shows.

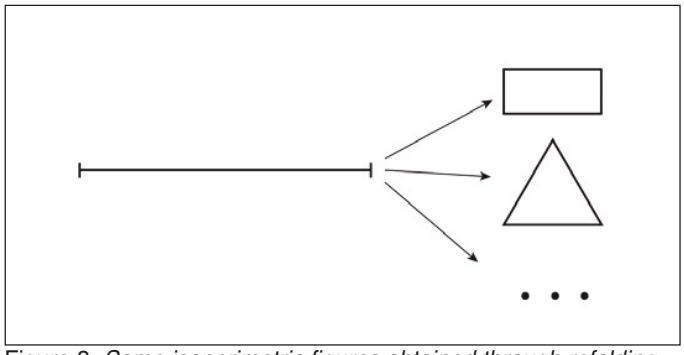


Figure 8: *Some isoperimetric figures obtained through refolding the same line segment*

This folding–unfolding describes a dynamic process, one that enables the perimeter to gain significant meaning. It offers a view of perimeter that comes from a figure, but that does not need to be rigidly attached to it, as it is a unidimensional measure (1D) of a length belonging to and describing a two-dimensional (2D) figure. Conceiving perimeter in this way goes a long way from presenting it merely as the addition of a figure's sides or a distance around the figure. It makes it *something* to measure, a length in this case, which constitutes the figure itself, but does not need to be rigidly attached to it.

Continuing on with these ideas, the next column raises questions relative to units of reference when measuring perimeter.

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Lynda Colgan is Professor Emerita, Faculty of Education, Queen's University. Lynda is currently working on a number of research, resource creation, and teacher education projects, funded by the Natural Sciences and Engineering Research Council of Canada (NSERC*),* the Social Sciences and Humanities Research Council of Canada (SSHRC), *and the Mathematics Knowledge Network (MKN). She will carry on her long-established commitment to STEM outreach by continuing to seek out creative avenues to engage students, parents, and educators in mathematics at home and in the classroom.*

Editor note: This column was written with the sense that we will be teaching face-to-face in the Fall. Whether we are or not, it has an important message about moving forward as the pandemic diminishes.

Even after decades as an educator, I could never sleep on the Monday night of the Labour Day long weekend. Year after year, on "New School Year's Eve," I had a tummy full of butterflies that would not let me rest. Some were flying excitedly in anticipation of all the wonderful learning adventures that lay ahead. Following a flight path fashioned after a fanciful route from Dr. Suess' *Oh, the Places You'll Go!*, my butterflies soared high in the sky and then plummeted deep down into valleys, circling back to revisit special spots, then ascending to catch a serendipitous wind that would carry them to a new and even more exciting destination. But many of my butterflies sat still, with wings wide open as if they had forgotten how to fly. Was it foreboding? Had I forgotten how to teach?

Even now, when confronted with butterflies in my stomach, I recall the words of Peter Skillen, my friend and colleague, who would remind me in advance of every workshop or presentation that it was good to have butterflies—they were a marker of how deeply one cared. The secret, said Peter, was to get them to fly in formation—the ascent of orderly butterflies meant that the workshop would take wing! The butterflies would follow my lead if I transformed my negative anxiety into positive enthusiasm.

Science tells us that butterflies are part of the "fight or flight" reaction that manifests when we confront something that is frightening. When danger is imminent, our body releases adrenalin and cortisol, causing the heart rate to become rapid and blood to be shunted quickly to our heart and leg muscles. At the same time, the blood flow is dramatically decreased away from our digestive system, causing the muscles of the stomach and intestines to constrict, thereby creating the sensation of "butterflies" spiralling in flight and hurtling into the abdominal wall.

I expect that on Monday, September 6, 2021, this science will contribute to a sleepless, butterfly-filled night for most Ontario educators. After all, their worries are real and copious after 18 months of living through the challenges of a global pandemic with its innumerable personal and professional consequences, and even more uncertainties.

It would be nearly impossible to get the butterflies to "fly in formation" this "New School Year's Eve," in spite of fond memories of past beaming faces and pristine new backpacks; distant echoes from high-pitched squeals of delight at the sight of long unseen friends; and, auditory imprints of the clanging school bell, along with whispered, yet earnest promises to "see you at recess."

For most Ontario educators, the shiny promise of a new school year during the pandemic would have been dulled by the daunting specter of a harsh reality. Since March 2020, Kindergarten to Grade 12 students in Ontario have been in their classrooms for approximately 5 of a possible 18 months. According to many media sources, there are few places in the world where children have been out of school more than Ontario, and no other Canadian province had school closures as long.

While parents and educators have been consistently vocal about their angst in response to virtual classrooms and online learning (e.g., technical difficulties, broadband issues, competing work and childcare demands), they have since raised proverbial "alarm bells" about the inevitable decline in their children's or students' academic achievement, especially in language and mathematics.

There is no question that most children lost much to remote learning, including the "normalcy" of time spent in sports, clubs, and socializing with friends. Some children suffered loss in more impactful ways because their "safe place" was taken away, leaving them vulnerable to physical, verbal, or emotional abuse, and food insecurity.

While there will be invisible disparities on school reopening, we must remember that when all children return to classrooms, there are some inviolable truths. *All children will be two grades past where they were when the pandemic began.*

While some educational researchers argue that it is not possible to quantify general learning loss over this unprecedented time in our history, one study out of the Netherlands (Engzell, Frey, & Verhagen, 2021) claims that even in contexts with short lockdowns, equitable school funding, and high levels of broadband access, *students made little or no progress while learning from home*, and learning loss was most pronounced among students from disadvantaged homes. In fact, those same education scientists report as much as *5.6 weeks of learning delay over an eight-week school closure*.

Whether it is big or small, learning gaps will exist, but they are not unique to Ontario—more than 800 million students globally have shared in the experience of online learning for extended periods of time.

Yet, the advice from the OECD (2020) is encouraging. There are research-validated ways to remediate, support, and move forward; however, there is a process that must unfold over time. The first, and let's just focus on that for now, is that the process of returning to school should not be a scramble to evaluate, diagnose, and test children—to do so would only exacerbate problems and widen gaps.

September is *not* the time to fret about children's recall of common denominators and the characteristics of a rectangular pyramid. The primary focus of the start to the 2021–2022 school year must be a focus on *strengthening student well-being*, through either improving students' non-cognitive skills or mindset, or ensuring a positive learning climate (Agasisti et al., 2018). In other words, our math lessons in September must set aside content and focus on the six skill categories within the critically important affective strand in the mathematics curriculum, i.e., Socio-Emotional Learning (SEL):

- Identifying and managing emotions
- Recognizing sources of stress and coping with challenges
- Maintaining positive motivation and perseverance
- Building relationships and communicating effectively
- Developing self-awareness and a sense of identity
- Thinking critically and creatively

For many mathematics educators, this will be a foray into uncharted territory. To those educators, mathematics is dispassionate, certain, scientific, and analytical. Discussions about difficult and sensitive topics are not part of their mathematics education equation, and facilitating discussions beyond comparing visual methods for proving the Pythagorean theorem is a skill that may not be in their wheelhouse.

As children come back to the classroom in September with their own butterflies—anxieties about whether they have forgotten everything they learned and the work will be too difficult—they bring with them opportunities for teachers, not only to recognize the symmetry between their struggles and those of their students, but to implement effective SEL lessons that will be mutually beneficial. Just as it is with any content, sufficient time and attention must be devoted to teach carefully designed SEL tasks for learning to occur about how social and emotional skills, attitudes, and behaviours impact academic performance, especially in mathematics. In the absence of resources for, or teacher education about, SEL from the Ontario Ministry of Education, it is only natural to look to our peers for strong grassroots-level professional support.

In her blog, *The Chrysalis Chronicles*, Chelsea Cleveland, a Mathematics Specialist and middle school math educator at the Ottawa Jewish Community School, has a three-part series on Socio-Emotional Learning that I recommend as a self-study or group study for educators. By posing Chelsea's questions to oneself and to one's students, September may yield the answers needed to move forward in identifying and addressing learning gaps more effectively than any diagnostic test yet developed.

> Think back to when you failed at something, and ask yourself these questions: How did that make you feel? Did you want to try again or did you give up? Did you ask for help? Were you able to share your concerns and areas of struggle with another person? Did you get angry and upset at yourself? Did you blame someone else? Did it bother you that you wanted to figure it out, or did you forget about it, pack it away, and never looked at it again? Did it stay with you, and if it came around again, would you worry or think you couldn't do it, knowing you failed at it before?

By reflecting on and answering these questions, we are confronting our individual fears and preparing mentally for the teaching and learning challenges ahead—building academic resilience and positive attitudes in ourselves and our students. A slow and steady September focus on socio-emotional learning is just the transformative chrysalis we need to set everyone's butterflies free. As educators, we will be better equipped to identify, address, and be sensitive to learning difficulties in ourselves and others. And more importantly, our students will be empowered to become the drivers of their own learning, recognizing that there are no gaps, only lessons to be learned and new opportunities to welcome… very hungry caterpillars indeed.

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▲ Coding in the Classroom



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This is a new column that interviews Ontario teachers, who use coding in their classrooms and schools. Beyza and I have a conversation with the teacher to find out how they got started with coding and their favourite pedagogical strategies. In addition to the column below, you can access the audio of the interview at OAME Talks to get a fuller sense of how these remarkable teachers have struggled, worked hard, and been successful in using coding in their classrooms to enhance student learning.



Our first interviewee is Pekka Reinio, who has been teaching for well over two decades. He is a teacher-librarian in the Simcoe County District School Board.

An Interactive Way of Teaching Coding

Pekka first started coding after meeting Mitch Reznick at a conference organized by the Educating Computing Organization of Ontario, where Mitch was presenting his new coding platform, Scratch. He immediately started to use Scratch in his classrooms, and for many years, struggled to figure how to best use this new pedagogical support. After continually refining his approach, Pekka hit upon a successful approach: have students create interactive games in Scratch. Examples of his game programs can be found at www.youtube.com/channel/UCqLQh_HovpmwKGOBkUQyhbw.

Game making is a creative activity that can easily incorporate storytelling. What is not immediately evident is the amount of mathematics and science used to make these games "realistic." Diane Tepylo, from the Faculty of Education at Ontario Tech University, tells teachers to watch for the mathematics that inevitably results from creating games. It comes in the form of math strands in *Number*, *Spatial Sense*, *Algebra*, and *Data*, as well as physics that is doable, but beyond grade level.

> My job as teacher-librarian is to partner with classes, and one of the things that teachers have enjoyed having me come in to do is coding with their students, and especially this last year, not only because of the pandemic, but because coding is now part of the math curriculum. And so the teachers are eager to see how to teach coding in the classroom, beyond things like code.org (www.code.org) or something like that. They want better coding experiences for their kids.

In creating their own games, Pekka has found that his students are highly motivated to make their games work. They start with a barebones version that Pekka has created for them, and then they take over and create the full game. "Not only does it teach problem solving and logical thinking, but it incorporates many of the expectations from all of the strands." The process that Pekka has gone through over many years is to constantly refine his practice, using feedback from his students in order to present code that makes sense to them.

Learning in Small Bites

Pekka makes sure to structure his coding lessons into bite-sized chunks. He will teach a new concept, then have the students work on it. When they return to the classroom, he will show them a trick, give them a way to refine their code, or give them a mini-lesson, leaving these ideas on a whiteboard for everyone to refer to within a one-hour lesson.

> If you overwhelm them, they will give up as soon as they get stuck. I have a structure so that it progresses in a very easily understood way so that everything leads into the next thing, and they can see the connection to those in three ways.

This is a type of semi-open problem that Pekka designs for his students. There are parameters, but the ultimate decisions are left to the students. "If you don't have some parameters there and you just leave it open to do whatever, they won't have any satisfaction when we play their games."

Wide Opportunities through Coding

One really important process the students learn in coding is debugging. The curriculum includes the expectation that students will write and rewrite code, looking at the effects of the changes. Pekka notes:

> In rewriting code, what students are doing is they're looking for errors and ways to make their code more efficient, or more elegant, right? So some kids will write code that takes up 20 to 30 lines. Or some kids do write the same and get the same results in 10 or 15 lines of code. And they learn that to be able to write code much more efficiently is better in the long run and makes their games run smoother, and it just enhances that logical thinking and collaborative problem solving.

One thing Pekka has noted is the collaborative aspect of coding, especially during debugging. Students work with each other and seek or give advice to each other freely.

Another thing that Pekka has found is that coding provides access to all students, where they are able to see themselves as successful. This links directly to the social–emotional learning skills and mathematical processes of strand A of the new elementary math curriculum.

> Well, cognitively for sure, they've been able to apply a range of math expectations through coding, and really learned how those expectations work and what they mean, so there's no question there. They enjoy it, they like coming to school to do coding and math. Often at the end of the year, they will say that that's part of one of their most favourite things that they've done throughout the year.
>
> I can remember a student I had in Grade 6 about three or four years ago. He couldn't do paper–pencil work. He couldn't stay focused on a task. He often was in trouble at school for talking out or other behaviours. But he discovered that he was a coding genius! When he came to coding class, everyone saw him as the expert, and so that really helped his self-esteem, and it made him see himself in a different light. When in the regular classroom, kids didn't think he was a very good student, and he was often troubled. When he came to the coding class, this picture completely changed—he was the smartest one there. He was the most on task; he would gladly go around and help other students. And I remember that year being very important to my thinking about how valuable coding is.
>
> I like to do things that don't always get a lot of attention from the teachers and that some students who are left out can benefit from. Coding is one of those things, although now I teach code and get 100 percent buy-in. It's been really nice to see all of the kids be able to have fun, and successfully build their own games while coding with Scratch.

Pekka has brought up a really fascinating aspect of coding with students that we will explore in future columns. Coding allows those who are not necessarily the most successful students in other ways to thrive. Is it the coding realm in and of itself or is it something more? We will explore this and other concepts in future columns. ▲

▲ Infographics: A Remixing Approach



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Ami is an Associate Professor at Ontario Tech University, who researches educational mathematics. Her work explores how to foster and elicit reasoning that can disrupt misguided and ingrained preconceptions about mathematics content, learning, and teaching. She's especially interested in how creative and multi-modal approaches to math teaching and learning can be networked to encourage conceptual growth, meaningful engagement, and enjoyment with mathematics. Ami currently serves as co-chair of the Fields Institute Math Education Forum.



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Nkechi is a Master of Arts candidate in the Faculty of Education at Ontario Tech University. She holds Bachelor and Master degrees in Chemical Engineering from Sofia, Bulgaria, and her current research focuses on the importance of math beyond the classroom and the power of visualization for reaching diverse learners. Nkechi has taught STEM students, especially math learners of all ages, in both formal and informal settings, and has a passion for instilling in students the love for learning.

In this article, we share some examples, questions, and resources for learning with infographics and data visualization in middle school math, with connections to key ideas in number, proportion, and data. We take a *remixing approach* to infographics that fosters robust mathematical understanding, critical thinking about real-world issues, and opportunities for learners to be creative with mathematics. *Remixing* is a term we borrow from recording music to describe when an artist might alter a song by adding, removing, or changing aspects of it, so as to create a new piece of media that can offer new insight, express new voices, reach new audiences, and open up new possibilities. Analogously, when we refer to "remixing" infographics, we mean the activity of altering existing infographics so as to create a new piece of media that can offer new insight, express new voices, reach new audiences, and open up new possibilities.

The objective of this article is to offer teachers ideas and strategies that can be enacted while addressing the new Ontario math curriculum, and we highlight connections to curriculum expectations for Grades 7 and 8. We organize ideas around two main examples of infographics: data visualization to represent a single data set, and graphic design that integrates multiple data sets. In both examples, data is used to tell a story. The questions we pose in our examples focus on unpacking the data and its representations, with an eye toward developing a critical understanding of the mathematical content and its uses for understanding and communicating stories about the world. At the end of this article, we provide a list of resources to help implement some of these ideas.

Why Infographics? What Infographics?

What are infographics? What do they look like? What will comprise an infographic? At the core, infographics are simply artifacts that combine data with artistic and graphic design to concisely communicate a story (Islamoglu et al., 2015; Newson & Haynes, 2004; Smiciklas, 2012). The visual design element, the connection to real-world issues, and the activity of creating all contribute to making infographics a powerful, engaging, and stimulating learning tool. In particular, the combination of verbal, pictorial, and numerical representation of ideas enhances the possible connections that students can make in their learning, and allows them various ways of expressing their ideas. According to Whiteley and Mamolo (2014; also Mamolo, Ruttenberg-Rozen, & Whiteley, 2015), visual representations of numeric data can help learners "see" important trends and relationships in the data that were previously obscured. Further, they suggest that the ability to switch back and forth between representations is an important aspect of mathematical literacy, and artifacts that capture and track both numerical and pictorial data offer powerful teaching tools. They also open the door for important conversations around which representations are best suited for a particular purpose, and why.

How Infographics? Which infographics?

A rich infographic captures a lot, and takes time to create, especially if it involves issues of real importance to students. In line with Flewelling and Higginson's (2001) *Rich Learning Tasks*, our approaches, using infographics for developing or reinforcing mathematical ideas and processes, are intended to give students opportunities to explore and challenge ideas; pose and answer their own questions; formulate, discuss, and defend their ideas; identify, select, and apply math ideas; and justify, summarize, and communicate

On a scale from 1 to 5, rate each infographic in the following categories:							
		1	2	3	4	5	
Appeal (visual and verbal)	Presentation and format are weak; infographic lacks balance, is cluttered, confusing, or dull						Presentation and format are visually and verbally appealing; balanced and purposeful appearance
Accuracy	Represents mathematical information inaccurately; uses incorrect terminology, diagrams, visuals, or principles						Represents mathematical information accurately; depicts relationships and facts correctly, precisely, and in context
Connections	Poor or misleading connections between mathematical and non-mathematical information						Meaningful connections between mathematical and non-mathematical information
Representations	Mathematical representations are poorly chosen, misleading, or confusing						Appropriate and helpful representations are used to depict mathematical facts and relationships
Audience	Unclear what audience is targeted, or poorly suited for intended audience						Presents concepts in a way that appeals to a range of audiences and abilities
"Wow" Factor	Contains little or nothing beyond the expected; has no surprises						Offers layers of richness or interest beyond the predictable or expected
What about the infographic helped you decide your ratings? What aspects of the infographic stood out the most?							

Table 1: *Rating infographics for their math content*

conclusions. These opportunities tie into the mathematical processes described in the new Social–Emotional Learning strand of the Ontario math curriculum (2020). Applying such processes supports students' positive and productive engagement in mathematics, while also enhancing their sense of self as collaborative, resourceful, and independent mathematical thinkers.

When deciding how to incorporate infographics into the classroom, taking a "modular" approach that scaffolds different aspects of infographic creation can help keep lessons well defined and targeted to specific learning goals. The use of learning stations or centres could be a useful aid to this process, and the sample questions and ideas we present in this article highlight concepts around which we have organized learning stations. We take a *remixing approach* to the introduction of infographics that includes establishing a class set of clear criteria for what makes a "good" infographic, critically unpacking and challenging the mathematical and non-mathematical content, and taking action to make change and improvements.

A tool for rating infographics

Comparing and contrasting infographics, as well as rating them based on pre-defined categories, can help direct students' attention to key features and ideas. The tool shown in Table 1 can be adapted and applied for introducing infographics and establishing criteria for what your class will consider "good."

There are many freely available infographics that can be rated, and it is good for students to encounter a variety. Adapting the rating scale by using –2, –1, 0, 1, 2 might suit grades learning to work with integers. Choosing trustworthy examples to unpack and discuss is important—real and reliable data exists out there, but you cannot trust everything you find. The following examples come from two of our favourite sources for reliable and relevant infographics: Information Is Beautiful (www.informationisbeautiful.net) and Statistics Canada (www150.statcan.gc.ca/n1/pub/11-627-m/index-eng.htm).

Example 1: The best things in life really are free

This data is from Beautiful News Daily, a resource dedicated to collecting "good news, positive trends, uplifting statistics and facts—all beautifully visualized" (www.informationisbeautiful.net). The website contains a variety of data displays across a number of different themes, and includes links to spreadsheets of the numeric data used to populate the images. This resource includes a variety of standard and non-standard data displays that can be unpacked and analyzed, with an accessible entry point (a low floor), a variety of strategies (wide walls), and opportunities to extend further (a high ceiling). There are opportunities to

connect to other subject areas and discuss important issues that will contextualize the math and resonate with your students, though not all of the issues are appropriate for adolescents. One of the most impressive features of this resource is the artistry of the mathematical representations, which can inspire new and creative ideas for how to present and communicate abstract math concepts.

Sample Questions for Figure 1:

A. Is this diagram to scale? What needs to be true if it is? How could we check to find out?

B. If 200 people were polled in this survey, how many people would be represented in the "nature" category?

C. If you wanted to create this image so that it fills as much of an 8.5x11" piece of paper as possible, then how big would each rectangle need to be?

D. What would this picture look like if we used triangles instead of rectangles to depict the data?

E. Imagine there were an error with the data, and that the correct percentages are 44% for affection, and 24% for achievement. How does the picture need to change to represent these new figures?

F. How different would the picture look if we used data from the class/school instead of the data provided? What questions would we need to ask to collect our own data?

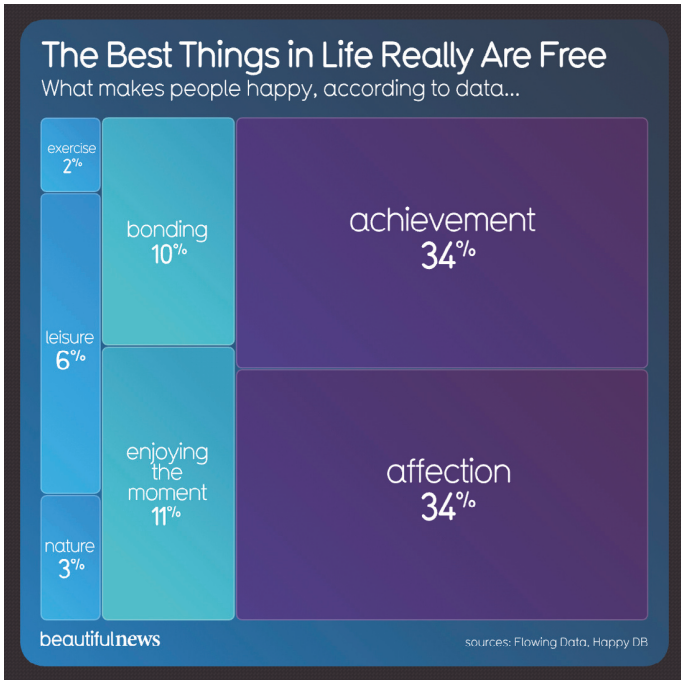


Figure 1: *The Best Things in Life Really Are Free (www.informationisbeautiful.net/beautifulnews/5 75-best-things-free/)*

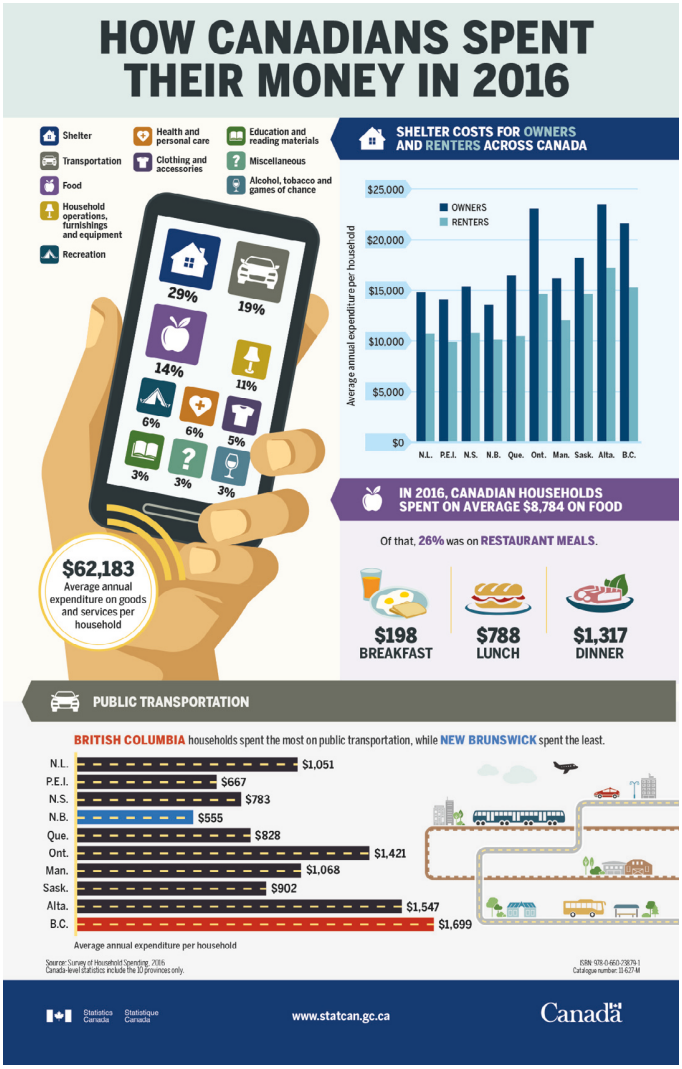


Figure 2: *How Canadians spent their money (www150.statcan.gc.ca/n1/en/pub/11-627-m/11-627-m2021006-eng.pdf?st=NPMK6yS9)*

Example 2: How Canadians spent their money

Statistics Canada publishes infographics on a variety of issues based on their survey data. These can be useful ones to unpack and extend because they have familiar representations that can be reproduced with most freely available infographic software. Most tools and software (see the resources section for links) for creating infographics are fairly comparable and user-friendly, with built-in templates and basic design tools. Though, as with all technology, there is a learning curve and the mathematical functionality is quite limited. Creating or remixing infographics by applying the technology you already use with your students can be a more powerful and efficient approach.

Unpacking the math in an infographic can lead to many interesting discussions, and there is no shortage of questions that can be posed while interpreting an

infographic. Including broad as well as focused questions will help students navigate among the specific issues presented and the overall "story" being told. Simply asking students what they might be skeptical about when interpreting an infographic can be a wonderful way to begin an exploration.

Sample Questions for Figure 2:

A. What are your first impressions of this infographic? What stands out to you?

B. What questions do you have about this infographic?

C. Is there anything in this infographic that we should be sceptical about?

D. What math concepts/representations are used in this infographic? Why do you suppose the designers chose these representations?

E. Take a closer look at the data pictured in the smartphone. What's wrong with this picture?

F. How does this picture need to change so that the sizes of the icons are proportional to the percentage they are meant to represent?

This first set of questions sets the stage for subsequent "remixing" that could adapt the visuals of the infographic, could reimagine the scope or relevance of the issue, or extend into new directions.

Extension Questions for Figure 2:

A. Take a closer look at the smartphone. What will the smartphone look like with proportional icons for this data? Draw your own version of this smartphone with icons that accurately represent the percentages.

B. What are some different ways to represent this data? Generating a variety of representations from the class can allow for a Bansho display (OME, 2011).

C. If you had designed this infographic, what would you have done differently?

D. Which kinds of graphs could we use to represent…, and why? Which kinds of graphs would be inappropriate, and why?

E. What additional data could we collect to better understand this issue?

F. Whom else could we poll, and what questions could we ask?

Summary

Remixing infographics allows students to engage in critically unpacking and understanding the mathematical concepts and representations used, as well as the issues and trends represented. It offers students an opportunity to use their voices to add new insight, new dimensions, and new relevance to world issues. It also helps teachers tackle expectations from the new Ontario math curriculum in connection to other key and familiar ideas in number, proportion, and data. The table below links each of the above questions and examples with their related curriculum expectations for Grades 7 and 8.

Activities	Sample Questions	Grade 7 Curriculum Expectation	Grade 8 Curriculum Expectation	Curricular Themes
Figure 1	A & D	E1	E1	Geometric Reasoning
	B & C	B2	B2	Rational Number Sense
	E	B2, E1	B2, E1	Rational Number Sense; Geometric Reasoning
	F	A1, D1	A1, D1	Social–Emotional Learning; Data Collection & Organization
Figure 2	A	A1, B1, D1, E1, E2	A1, B1, D1, E1, E2	Social–Emotional Learning; Rational Number Sense; Data Collection & Organization; Geometric Reasoning & Measurement
	B	A1, D1	A1, D1	Social–Emotional Learning; Data Collection & Organization
	C	A1, B1, B2, D1, E1, E2	A1, B1, B2, D1, E1, E2	Social–Emotional Learning; Rational Number Sense; Data Collection & Organization; Geometric Reasoning & Measurement
	D, E, & F	A1, B2, E1, E2	A1, B2, E1, E2	Social–Emotional Learning; Rational Number Sense; Geometric Reasoning
Figure 2 (extension questions)	A	E1, E2	E1, E2	Geometric Reasoning & Measurement
	B & D	A1, B1, B2, E1, E2	A1, B1, B2, E1, E2	Social–Emotional Learning; Rational Number Sense; Geometric Reasoning
		A1, B2	A1, B2	Social–Emotional Learning; Rational Number Sense
	E & F	A1, D1	A1, D1	Social–Emotional Learning; Data Collection & Organization

Links to online resources:

- Information Is Beautiful: www.informationisbeautiful.net/
- Running the Numbers: www.chrisjordan.com/gallery
- Create online Bansho board with Padlet: www.padlet.com
- Create digital mosaics: https://mosaically.com/
- Create infographics: www.piktochart.com

- Statistics Canada Infographics: www150.statcan.gc.ca/n1/pub/11-627-m/index-eng.htm
- Population Education Infographics: www.populationeducation.org/infographics/
- Investigate dynamic trends from UN data: www.gapminder.org/for-teachers/
- Practise interpreting quantitative information and constructing graphs: http://datanuggets.org/about-nuggets-2/what-teachers-need-to-know-about-nuggets/
- Explore and plot real-time data: www.nationalgeographic.org/education/resource-library/?q=data&page=1&per_page=25&content_type_category=Activity&content_type_category=Lesson

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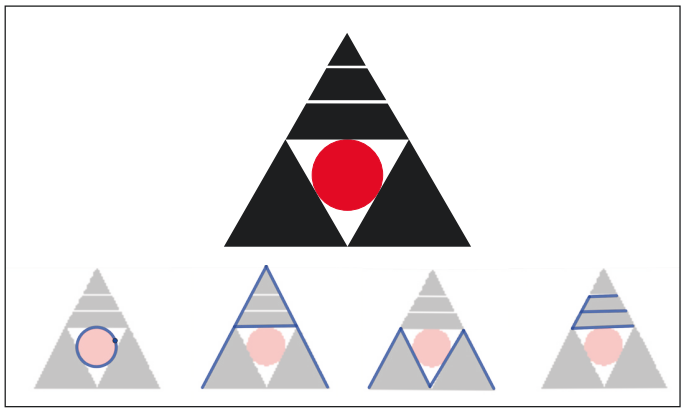
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▲ Linking Mathematics Concepts to Economics



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Jeff Irvine, MA, MBA, PhD, CFP, OCT, has been a secondary mathematics teacher, department head, and vice-principal. He has taught at three faculties of education and at a community college. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. With over 45 years in education, Jeff is particularly interested in the interplay of student motivation and mathematics achievement.

We frequently attempt to link what we teach to the *real world* to our mathematics instruction. Real world for students can take several forms: students could use the mathematics immediately, for example, in their part-time jobs, budgeting, or sports; students could use the mathematics in another subject, in the near term, such as in science, geography, technical shops, economics, family studies; someone close to the student could or did use the math content, such as a family member, relative, adult acquaintance; there were examples in the real world of people using the mathematics; and the mathematics often flows from investigations, experiments, or models that students are involved with (Irvine, 2015).

This article looks at ways that our math content can be related to what our students are studying in economics courses. Not all our students will be studying Economics, so these linkages provide a way to differentiate content for some (not all) of our students. As a math teacher, you don't need to know a lot of economics; rather, this presents an opportunity to liaise with the economics teachers in your school, and to demonstrate to students that there are economic uses for what we are teaching them. So, as a math teacher, you don't need to know the differences between Gross National Product and Gross Domestic Product. Your economics teachers can explain these concepts to both you and your students. Alternatively, students can look up economic definitions and terms online. However, for ease of reading this article, some basic economic information is included.

The examples below are separated into macroeconomic concepts (relating to the entire economy of a country) and microeconomics (relating to individual businesses).

Macroeconomics

There is a wealth of data available about our economy, particularly from the Statistics Canada database (www.statcan.gc.ca/eng/). Examples of this data are inflation, unemployment, Gross Domestic Product, and much more. Much of this data is provided as time series, i.e., annual or monthly values over multiple years or months.

Time Series

A time series is a collection of data over several periods of time. Table 1 shows a time series for annual cellphone usage in Canada for the period 2012–2019. Row 2 shows number of cellphone users in Canada, in millions. Time series are often converted to indices to facilitate comparisons, and this is seen in rows 3 and 4. Row 3 shows the cellphone data as a Paasche index, where all values are expressed as percentages of the current year (2019). Row 4 shows the data expressed using a Laspeyres index, as percentages of a base year (in this case, 2012). Both types of indices are useful for comparison purposes, although the Laspeyres index facilitates percentage growth comparisons.

Cellphone Users in Canada 2012–2019								
Year	2012	2013	2014	2015	2016	2017	2018	2019
Millions of phones	25.7	26.8	27.7	28.2	29.2	29.8	30.1	30.6
% of 2019 phones	83.99	87.58	90.52	92.16	95.42	97.39	98.37	100.00
% of 2012 phones	100.00	104.28	107.78	109.73	113.62	115.95	117.12	119.07

Table 1: *Time series for cellphone usage in Canada*

Students can be encouraged to search for data that is of interest, and then to analyze that data, using indices and regression analysis. Regression analysis provides opportunities for discussion of the appropriateness of various models, and extrapolation with relevant ranges.

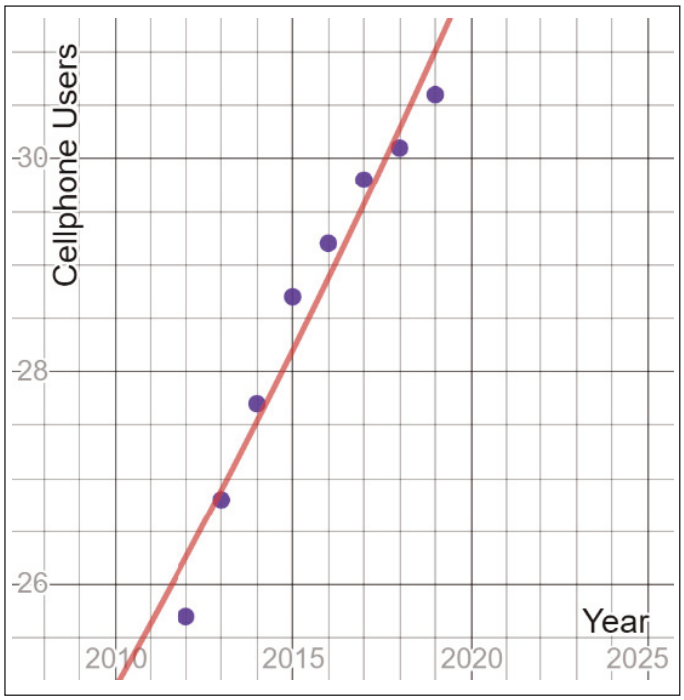


Figure 1: *Linear regression for cellphone usage,* r *= 0.9811,* $r^2 = 0.9626$

As shown in Figure 1, a linear model is a very good fit for the given data (correlation coefficient $r = 0.9811$). However, students should be encouraged to question assumptions and discuss whether this model is appropriate for extrapolation. Some of their comments could include the following:

- The population of Canada is about 37 million, and as of 2019, over 30 million people already had cellphones, so this level of growth cannot continue forever.
- Anecdotally, "everyone I know already has a cellphone, so the market is getting saturated already."
- Of Canada's population, over 3 million are children under age 6, so the limit will be reached even sooner.
- Some people have a cellphone for personal use and another one for business, so the market is larger than it appears.
- Some large businesses buy blocks of cellphones for their employees to use, so the market is larger than it appears.

Students were encouraged to hypothesize different models for the data and examine the consequences of these models.

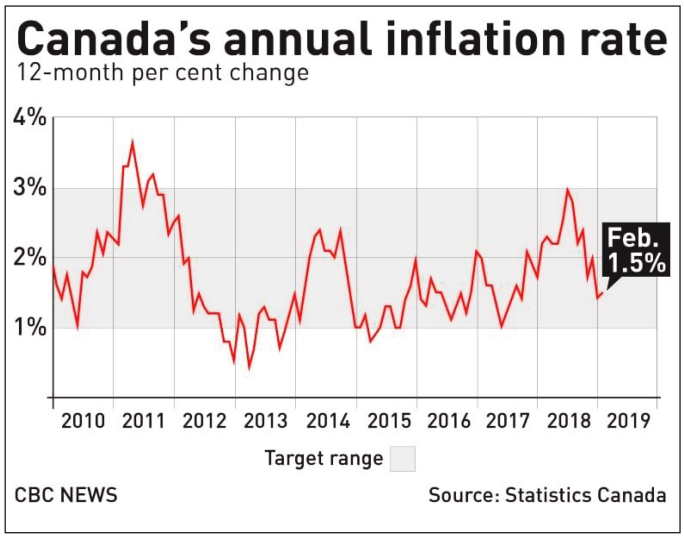


Figure 2: *Inflation rates 2010 to 2019*

Laspeyres indices are frequently used to illustrate Consumer Price Index data, and from there, to calculate annual inflation rates. Figure 2 shows Canada's inflation rates from 2010 to 2019. Figure 3 shows the same data as a Laspeyres index.

Relation between Inflation and Unemployment Rates

A simple linear model can be used to hypothesize a relationship between inflation rates and unemployment rates. A sample student assignment is shown below, after a brief description of the types of unemployment.

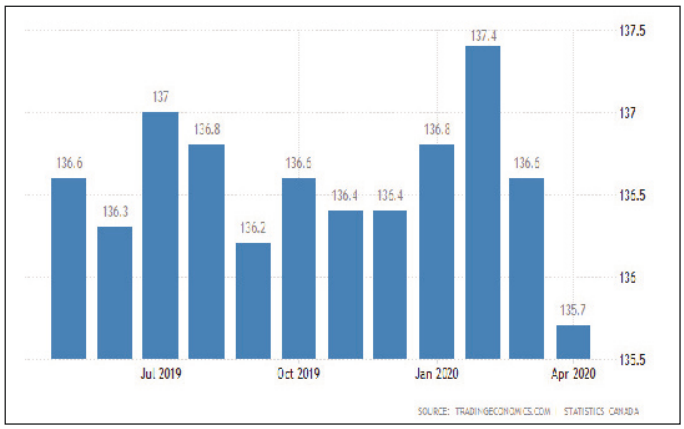


Figure 3: *Consumer Price Index, year 2000 = 100*

Types of Unemployment

Frictional: A person has changed jobs, has left their previous job, but has not yet started their new job; this person is technically unemployed (for a short period of time).

Seasonal: Job losses due to the seasons of the year. Farming is a typical example.

Structural: A major problem for the economy. Jobs lost because the industry has been replaced or greatly reduced. Manufacturing jobs over the last few decades in Canada is an example, as is loss of retail jobs due to the shift to online shopping.

Cyclical: Jobs lost due to downturns in the business cycle (discussed below).

In this assignment, students are asked to investigate the relationship between inflation and unemployment, using a linear model $U + I = k$, where k is a constant, which the students select based on historical data. For example, currently, inflation averages approximately 2% per year, and unemployment rates have been around 5% per year. So, the equation would be $U + I = 7\%$. Suppose the government wants to reduce unemployment to 3% per year. Then our model predicts that inflation would rise to 4% per year.

An Economic Model Relating Unemployment and Inflation

Economists use mathematical models to predict the effects on the economy of government policies. One model connects Unemployment (U) and Inflation (I), using a linear equation:

$$U + I = k \text{ (a constant)}$$

Your task is to investigate the impact of changes in government policy, using this equation.

1. How do economists measure inflation? What are the differences among the various measures? Decide which one you will use for your investigation and justify your choice.
2. There are also different types of unemployment, such as "frictional," "seasonal," "cyclical," and "structural." Decide which one you will use for your investigation and justify your choice.
3. Use Statistics Canada data for the period from the year you were born until now, to estimate the value of the constant k.
4. Find out the current level of inflation. Use your equation to predict the current level of unemployment. Then check your prediction against the published current unemployment rate.
5. Use your equation to predict the impact of a "tight" government monetary policy. If the policy results in lower inflation, what do you predict will happen to unemployment?
6. Write a report outlining your findings. Be sure to include reasons why the equation represents a reasonable model relating inflation and unemployment.

The Business Cycle

All countries' economies go through a cycle of growth and recession, usually with a period of four to five years (see Figure 4). As an economy grows, employment grows, but demand for goods begins to exceed supply, resulting in inflation. Eventually, this inflationary pressure causes the growth to slow and peak. This is followed by a period in which the economy begins to shrink as demand drops; this shrinkage bottoms out in a depression, after which the economy begins to grow again. Governments employ monetary and fiscal policies to slow the growth of inflation, and to reduce the severity of the recessionary periods.

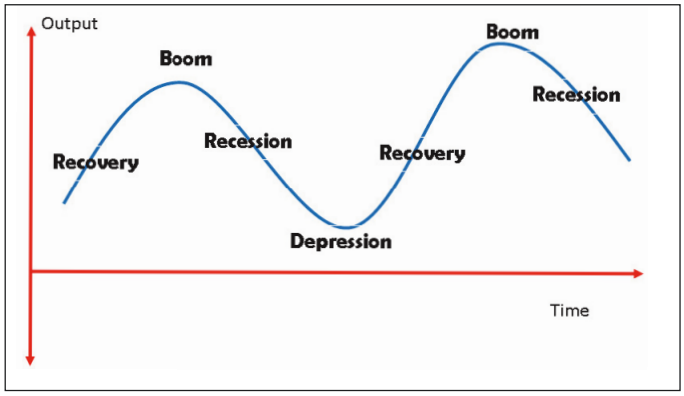


Figure 4: *Business cycle*

How is economic growth measured?

There are two ways that economists measure a country's economic output:

- *Gross National Product (GNP):* The value of all goods and services produced by Canadian citizens and Canadian companies. This means the output of a Canadian-owned gold mine in Australia would be counted in GNP, but the output of a foreign-owned mine in Sudbury would not count.
- *Gross Domestic Product (GDP):* The value of all goods and services produced in Canada, no matter who owns the source of the production. The output of the Sudbury mine would count in GDP, but the Canadian-owned mine in Australia would not count.

Canadian GDP is shown in Figure 5 below. This contrasts with the business cycle shown in Figure 4, which is idealized and, generally, on a smaller time scale.

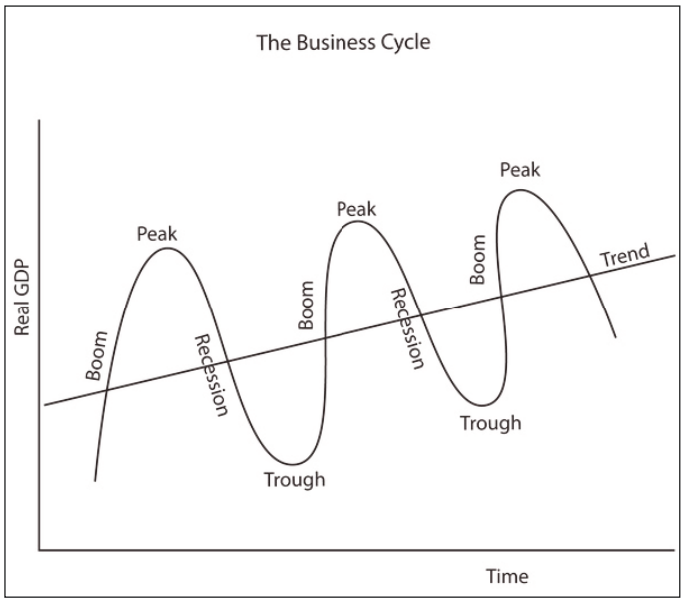


Figure 5: *The growth of GDP over time*

I have had students download GDP data from the Statistics Canada website and then perform sinusoidal regressions to construct a model of the business cycle, ignoring inflation. As can be seen from Figure 5, the business cycle is only approximately sinusoidal, and the periods for the cycles vary, sometimes significantly. This provides an opportunity to discuss the limits of mathematical models and the assumptions behind them. An additional issue is that GDP is growing over time, so the business cycle is really closer to Figure 6.

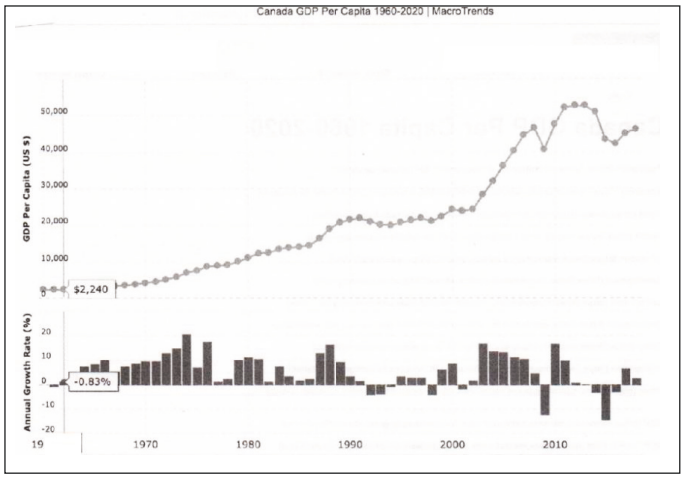


Figure 6: *Canada GDP and GDP per capita 1960 to 2020*

Students then discuss how to model this feature, such as a sinusoidal function combined with a linear growth term (for short-term modelling); they then fit their models to the Statistics Canada data and identify goodness of fit for their models.

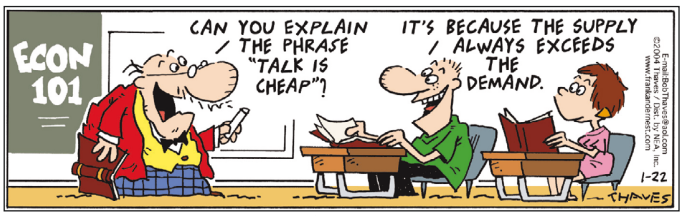


Figure 7: FRANK & ERNEST © *2004 Thaves. Used By permission of ANDREWS MCMEEL*

The interaction of supply and demand is fundamental to microeconomics. Supply and demand curves can be modelled mathematically, equilibrium points (where supply is equal to demand) found, and the consequences can be discussed. Figure 8 gives a typical equilibrium point problem, where *x* is the number of units of the product.

Equilibrium Point

- Find the equilibrium point if the supply function is $S(x) = x^2 + 6x$
- and the demand function is $D(x) = 16 + 20x - x^2$
- What does this point tell you?

Figure 8: *Finding the equilibrium point for supply and demand*

There are also some calculus concepts that can be applied to economics, particularly with respect to elasticity of demand. This is illustrated in Figure 9. Elasticity is a measure of how much a dependent quantity changes when there is a change in the independent variable. Examples include price elasticity of demand, elasticity of cost with respect to changes in the cost of inputs, and price elasticity of supply. Discussing elasticity provides the opportunity to examine ideas about price elasticity (Figure 10). Something is said to be price elastic if, when its price changes, the change in demand is greater than the change in price. For example, if the price of butter increases, demand will fall, and the magnitude of that percentage decrease is likely to be greater than the percentage increase in price, since ready substitutes are available, such as margarine, and people will choose to switch, rather than pay more for butter. On the other hand, gasoline is price inelastic; changes in price typically do not greatly influence demand, since gasoline is a commodity and necessary for travel.

A somewhat related concept in economics is indifference curves. Indifference curves relate two different goods, where consumers wish to have at least some of both goods. For example, consumers want both food and clothing. However, when consumers have a lot of food, they will be unwilling or "indifferent" to give up much clothing to get more food, and conversely.

Elasticity of Demand

- ***Elasticity of demand*** gives the percentage change in demand corresponding to a 1% increase in price.

$$E(x) = D'(x) \times \frac{x}{D(x)} \times 100$$

- Find and interpret the elasticity of demand for the demand function $D(x) = 16 + 20x - x^2$, when $x = 12$

Figure 9: *Elasticity of demand*

Elastic and Inelastic Goods

- If the change in demand is greater than the change in price, a good is called *elastic*.
- Give some examples of elastic goods.
- If the change in demand is less than the change in price, a good is called *inelastic*.
- Give some examples of inelastic goods.

Figure 10: *Elastic and inelastic goods*

This leads to the concept of marginal rate of substitution, which indicates how much of one good a person is willing to give up to gain one unit of another good.

Economics is a senior-year course in most schools. Thus, the concepts discussed in this article are of most benefit to senior-year students, and I would caution teachers against attempting to use this material in junior-level math courses, where a substantial amount of explanation would be required, and where the material may not be as motivating for students. The content of this article outlines only a few areas in which mathematics and economics can overlap. For classes in which a significant number of students are concurrently taking economics, these concepts suggest ways that both the mathematics teacher and the economics teacher can make students' learning more relevant. This is also an opportunity to enhance collaboration among your school's departments, and demonstrate to students that learning is not really "siloed," but is intermingled throughout their time in school.

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Creating the Collaborative Mathematics Classroom of the Future



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This article investigates progressive teaching philosophies. Progressive teaching is reconstructing a teacher-centred (traditional) framework as a student-centred model. Traditional teaching encompasses the old methodologies of teaching: students sit in uniform desk arrangements, silent, teacher-centred, and with a large emphasis on memorization of lessons (McParland, Noble, & Livingston, 2004). However, the world of today does not fit this style. Teachers are preparing students for an environment and technologies that do not exist yet (Kelly, Ohler, & McCain, 2019). Progressive teaching needs activities that are collaborative, engaging, questioning, and demonstrating knowledge in multiple facets (Sherman, 2009). Teacher-centred to student-centred education techniques, such as the two mentioned above, aid in creating a collaborative mathematics classroom environment for students.

Two pedagogical approaches to enhance student collaboration will be discussed and implemented: first, with the use of non-permanent vertical surfaces; and second, through collaborative seating, which is multifaceted. Once analyzed, these approaches will be applied to a hypothetical Grade 12 classroom, while taking pedagogical practices from numerous grade levels.

Many facets must be considered in this process to accurately define useful pedagogical approaches for promoting collaboration among students. The pedagogical approaches will be evaluated with respect to their impacts on student persistence, discussion/participation, and knowledge mobility within the classroom, with the latter being of the greatest interest.

In short, as mathematics is a challenging subject, persistence becomes a defining factor. Persistence can be regarded as the continuation in a task, despite said task's difficulty. Student participation utilizes knowledge mobility in the sense that it observes items that are evaluated outside of regular assessments. This includes student engagement in class discussions, engagement in online discussions, and student behaviour in group settings (Poole, 2000). Lastly, knowledge mobility is the interaction between students in pairs or groups (Liljedahl, 2016a). This takes place while discussing answers, or reading and reacting to one another's written work, and suggesting improvements around the classroom (Jones, 2007; Liljedahl, 2014). Coined by Liljedahl (2014), it is this need "to borrow an idea" that prompts students to move throughout the classroom. The ease with which students share knowledge decreases their reliance on the teacher as the source of information, thus creating a student-centred classroom (Garrett, 2008).

Collaborative Seating

Visibly Random Groupings

Collaborative seating is a combination of desk arrangements, random grouping, and flexible seating to enhance the collaboration of the students of the class. One such article by Liljedahl (2014) demonstrates the importance of random groupings in progressive classrooms. When placed in groups, students will likely have different goals from their teammates. Thus, the implementation of visibly random groups becomes a possible strategy to promote work among companions. Tinto (1997) supports this notion, as groups that are made randomly in class are more involved over several consecutive class days. This is done through informal group meetings and study groups, promoting the notion of knowledge mobility and persistence. Liljedahl (2014) also states that despite the class period time limit, the boundary that is enforced is temporary or non-existent in nature. This allows students to interact with multiple members of the class, extending the knowledge mobility in the classroom (Liljedahl, 2014).

Visibly random groups allow students to become more agreeable, as well as eliminate social barriers between students (Roberson, 2019). The elimination of social barriers cannot be understated. Tinto, Goodsell, and Russo (1993) suggest it is those students who are involved socially, through shared academic learning experiences, who will invest the time and energy (persistence) necessary to succeed. This persistence allows students to bridge the academic–social divide and create friendships, while learning at the same time (Tinto, 1997). Furthermore, utilization of visible random groupings decreases reliance on teachers for answers, while a reliance on co-constructed intra- and inter-group answers increases (Roberson, 2019). This intra- and inter-group community is highlighted by Liljedalh (2014), who describes three key developments that transpire:

- Members of a group go out to other groups to "borrow an

idea" to bring back to their group.

- Members of a group go out to compare their answer to other answers.
- Two (or more) groups come together to debate different solutions (p. 14).

Thus, visibly random groupings give rise to an environment that is conducive to knowledge mobility within the classroom (Roberson, 2019; Liljedahl, 2014). It is important to note that teachers must ensure that the discussion among students is procedurally correct, and step in when misconceptions are identified.

Rarely thought of are the benefits provided by the physical movement that occurs when groups interact. Dennison et al. (1994) indicate that movement promotes collaboration, while aiding in developing neural connections in the brain. Furthermore, Jensen (2005) suggests numerous benefits that come with movement, as well as flexible seating that is described later. Jensen (2005) states that movement can "strengthen learning, improve memory and retrieval, and enhance a learner's motivation and morale." These benefits further indicate the need of movement within a classroom, whether it be through knowledge mobility or other strategy.

Flexible Seating

Flexible seating is a variety of alternative seating forms that allow student choice. This creates a classroom that decreases off-task behaviours, which in turn increases student attentiveness (Erz, 2018). Educators today have a plethora of flexible seating options from which to choose. These include rockers, wobble stools, carrels, adjustable-height chairs on wheels, freestanding desks, jigsaw tables, recliners, therapy balls, and bean bags (Breithecker, 2006; Mann, 2019). Burgeson (2017) suggests that students can identify the types of seats that help them stay engaged in the classroom. Through these alternative seating options, all students, including those with disabilities such as ADHD and autism, have the freedom to stay on task, while also manipulating their bodies in their ideal ways (Havig, 2017). Osborne (2016) describes that an innovative learning environment that utilizes flexible seating can increase mathematics achievement by as much as 19 percent as compared to traditional seating environments. Although not as obvious as common physical activity, flexible seating provides students with the opportunity to be active, by not being obliged to sit still. Active children have demonstrated a greater attention span, faster cognitive processing speed, and improved performance on standardized academic tests compared to children who are less active (Sattelmair & Ratey, 2009). This increased performance bodes positively for students' persistence, as academic success is a crucial indicator of motivation and persistence.

This ideology is further supported by arguably the largest company in the world, Google. Stewart (2013) explains that the core of Google, its software engineers, have the ability to design their own unique desks. This freedom coincides with the aforementioned point of choice, as Mann (2019) describes it allows Google's employees to be more creative and productive while at their workstations. Thus, the ability for students to sit with freedom, and have seating choice, offers a classroom that may bring about the fundamental principles of a young "Googler."

Desk Arrangement

Traditional teaching continues to use rows as the primary desk arrangement. However, students need an environment that enables ongoing discussion. Marx, Fuhrer, and Hartig (1999) investigated the relationship between classroom seating arrangements and question-asking among Grade 4 students. Results demonstrated that a semicircular arrangement was superior to the traditional row–column seating as a means to promote student question-asking. This methodology is also supported for group activities where semicircle or circle arrangements have been demonstrated to be the most effective (Rosenfield et al., 1985; Wannarka & Ruhl, 2008).

As classrooms have changed, so have the individuals within them. Students come from varying backgrounds and languages. Tanahashi (2007) describes three possible arrangements that can be utilized to promote collaboration among English Language Learners (ELLs): U-shaped seating, modular (pod) seating, and fishbowl seating. U-shaped seating allows students to have constant visual contact with one another, which aids group discussions. Pod seating offers autonomy from the rest of the class and encourages student–student and teacher–student interaction. While in a fishbowl arrangement, students participate in conversations more often, allowing knowledge mobility to take shape (Tanahashi, 2007). Having only chairs and no desks in the classroom is conducive for non-permanent vertical surfaces—discussed next.

Non-Permanent Vertical Surfaces

Vertical whiteboards allow for greater "thinking classroom" behaviour. Liljedahl (2016a) describes increased persistence, discussion, participation, and knowledge mobility of students who partake in math-based use of non-permanent vertical surfaces. As well, these surfaces increase engagement among students, while promoting problem-solving fundamentals (Lijedahl, 2019). The use of these surfaces visibly demonstrates all work being done, not just to the teacher, but to the entire class as well (Liljedahl, 2016a).

The benefits are further demonstrated while using small individual handheld whiteboards. These handheld whiteboards

provide opportunities for every student to share their answers, give feedback, and have a voice that is valued in the classroom (William & Leahy, 2015; Swan, 2006). Swan (2006) suggests that the non-permanence of these surfaces allows students to take risks during learning, and to continually edit their written work to deliver their best possible answer. Additionally, whiteboards foster an alternative representation of problems, allowing students to display work as sketches, graphs, system maps, motion diagrams, pie charts, equations, or other forms (MacIsaac, & Falconer, 2000). Forrester et al. (2017) noted that students using whiteboards in a first-year undergraduate calculus course generated far more work in tutorials as compared to their counterparts (paper), indicating an increase in persistence. Moreover, these very same students were more engaged with the work, and increased their collaboration skills by analyzing each other's work, asking for help, and sharing possible solutions (Forrester, 2017).

Once desks are in an optimal position for using whiteboards, Henry et al. (2006) suggest three possible methods for students to share their ideas with the class: museum walk (gallery walk), "the circle" (everyone facing in a circle for discussion), and presentations. Each is unique, but allows students to see all of their classmates' ideas in a larger font than on standard paper. This is conducive to knowledge mobility in the sense that teachers may opt to allow students to reflect and re-plan their final products with their brand-new ideas/views of the problem from their peers (Henry et al., 2006). Thus, using whiteboards creates a collaborative classroom by decreasing wait time, by prompting students to aid one another through knowledge mobility, allowing a shift to a student-centred progressive classroom.

Creating the Collaborative Classroom

Costa and Kallick (2010) emphasize the importance of active and engaging learning. The constructivist model describes the brain's way of making sense of the world, and this is through active learning (Xie et al., 2018). Thus, implementation of non-permanent vertical surfaces must be conducive to these fundamental principles. Liljedahl (2016a) coined the term "thinking classroom" as a non-traditional, progressive classroom. Liljedahl (2016b) states that to build a thinking classroom, the space must allow for collective thinking, learning together, and enhance knowledge and understanding through activity and discussion. With these aspects in mind, the design will be utilizing the previously mentioned research, as well as the points above to create a collaborative classroom.

The Design

As an example, consider a classroom that revolves around a Grade 12 (MEL4E) mathematics classroom, unit of budget and taxes. After careful considerations personally/professionally, and through research of various desk formations, pod seating is the methodology of choice for this classroom, owing to its impacts on student persistence, discussion/participation, and knowledge mobility within the classroom. Pod seating offers autonomy from the rest of the class and encourages student–student interaction, increasing discussion and participation. Moreover, with less off-task behaviour in pod seating, students demonstrate a high degree of persistence, as they sense that they are competing to generate an answer versus other pods. This competitive atmosphere amplifies as each pod competes against the other pods for the duration of the unit, before being visibly and randomly placed into other groupings. As students are completing budgets to figure out how much it costs to live away from home, knowledge mobility is observed. By discussing ideas and solutions, and reacting to others' work, groups can share knowledge around income and expenses needed to live on their own. Students reflect on the process in an ongoing manner, collaboratively and individually.

The chosen non-permanent vertical surfaces include wipebooks, chalkboard/whiteboards, and handheld whiteboards. While using these non-permanent surfaces, students communicate openly by writing words such as "groceries" and "rent" (and "estimating prices"), which allows students to visibly see factors that influence their living. Using words or drawings to describe what students desire increases through promoting their ideal depiction of their knowledge (examples above). Thus, as these are surfaces that are easily visible to all, students can take time to reflect and compare their own work to that of their classmates.

Many of the flexible seating options are not common for this age group (i.e., Grade 12). However, critical evaluation of research and professional/personal experience suggest a benefit of having the classroom composed of wobble stools, adjustable-height chairs on wheels, freestanding desks, therapy balls, as well as regular and beanbag chairs. Even older students become fidgety after long durations in any class, and these options reduce off-task behaviour, while being conducive to the environment of a progressive classroom for teenage students.

When creating groups for the budgets and taxes unit, the key is choosing an approach that can be done easily in front of the students, as well as in minimal time, to keep pedagogy time at a premium. Possible sorting options for the class year include wooden sticks with names on them, which the teacher picks; students choosing cards that associate to a group; online platforms; and others. One method that has worked very well is Excel coding. Students are all assigned a number and put into a code within Excel, which is displayed on the front screen.

As the teacher hits enter to run the code, all students are witness to the events and cannot argue the results. Although many may be displeased, students know each event is as likely, and must move forward to complete the unit tasks as a team.

The Classroom

The formation of this example classroom will utilize the design components mentioned above and be in line with the ideology as well. Desks will be arranged to face one another, spread across the classroom. Using a class size of 28, there will be seven pods scattered throughout. Each pod will have access to a wipebook and handheld whiteboards at their pod, while also being granted access to a certain location of the chalkboard/whiteboard (in close proximity to the pod). Next to that, different-coloured markers/chalk will be available for use. At the beginning of each class, students have the opportunity to choose their seat from the variety of seating options within the classroom. This amplifies students' desire to arrive early, as "first come, first served" seating is an ideal motivator. Prior to the bell, students put their seats back in a designated location for the next class coming in, as high school students travel between rooms for each class. Each unit will spell the beginning and end for pods. It is at this time that visibly random groupings are established, with former groups disbanded and new collaborations formed. To add an element of competition to the classroom, groups name themselves at this time. Providing students with the ability to name their team under a heading implemented by the teacher (e.g., Houses of Harry Potter) gives rise to student empowerment and voice, and provides an idea of student interests within the class. This pedagogical approach requires students to collaborate, and requires that the task be open-ended, full of meaningful discussion, and answered in a variety of ways.

An example of a collaborative question that can be used in an MEL4E course is a multi-step Fermi problem, where students develop estimation skills through collaboration. The Ontario Ministry of Education (2007, p. 21) math curriculum states that "[k]nowing how to estimate and recognizing when it is useful to estimate and when it is necessary to have an exact answer are important mathematical skills." Furthermore, the curriculum states that, "[e]stimation should not be taught as an isolated skill or a set of isolated rules and techniques. Recognizing calculations that are easy to perform and developing fluency in performing basic operations contribute to successful estimation" (p. 21). The question states: "How many heartbeats have occurred in London's Grade 12 classrooms since 8 a.m. this morning?"

Throughout the entirety of the problem, teachers must guide students through the process with prompts. These prompts must spark curiosity and questions, promote opportunities for various thinking/solutions, and endeavour to develop the student's mathematical knowledge. Prompts include: "What does... mean?" "Why is…?" "Or would it be… if…?" Students may write openly about factors such as, "How many students take gym class to affect heartbeat?" or "How many students walk to school?" Students are able to bounce ideas off one another to generate the most accurate estimation possible.

It is through these prompts, as well as open communication, where progressive teaching becomes the main principle. Students are the owners of their knowledge, and the teacher is the promoter of reflection and critical evaluation. Through intertwining these variables, students' collaboration increases as the number of interruptions decreases. This occurs while increasing the community within the classroom that remains focused on curricular/learning outcomes of that day, month, and school year. That is why the approaches are viewed as supporting student learning.

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NOMINATIONS FOR OAME/AOEM BOARD OF DIRECTORS 2022-2023

Elections are currently being held for a President-Elect (four-year term: one year as President-Elect, two years as President and one year as Past President) a Vice President (two-year term) and two Directors (three-year term), of the OAME/AOEM.

OAME/AOEM members interested in the position of Director will declare their intent to run for either Grades JK-6, Grades 7-12, University, or College, based on their personal experience, expertise, and interests.

Nominations for these positions are to be submitted electronically by completing an online Nomination Form by 5:00 p.m. on January 31st, 2022.

A nominee must be an OAME/AOEM member. By March 2021, the Nomination Committee will publish a slate of candidates, including candidate bios, for consideration by the membership. Voting is open to all OAME/AOEM members. The election results will be released at the Annual Conference of OAME/AOEM in May 2021 and posted on www.oame.on.ca.

Online nomination form as well as a summary of the duties of each description is available at http://bit.ly/ OAMEPositions (scroll to the bottom of the screen). Please contact Paul Alves (paul.alves@oame.on.ca) for more information.

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