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ABACUS

Ontario Mathematics Gazette

OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

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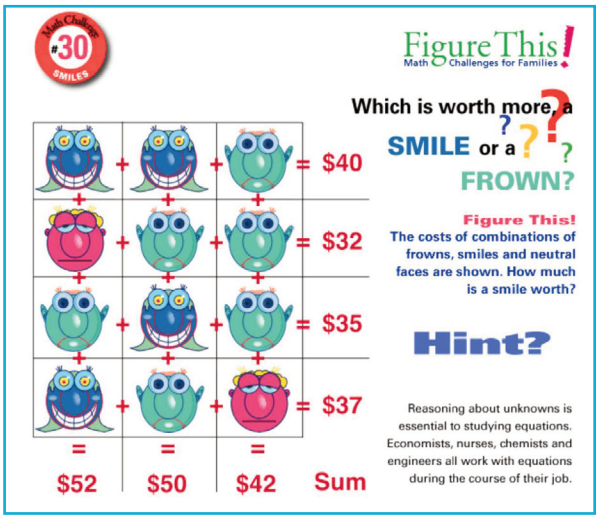
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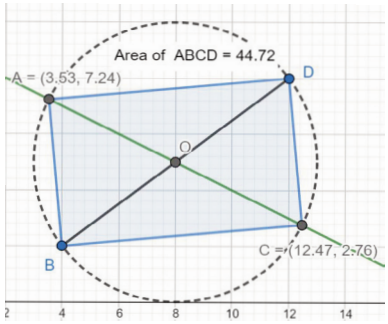
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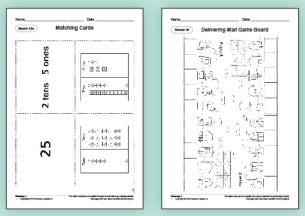




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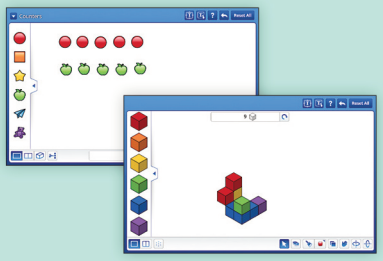


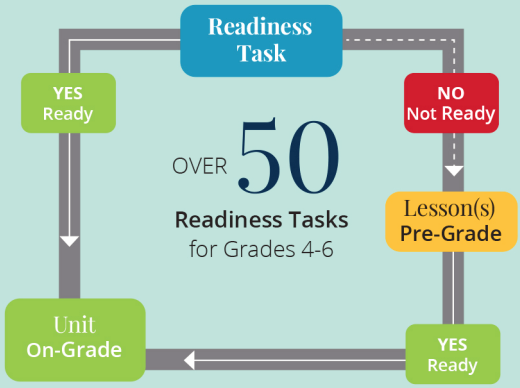
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Submission of Articles

The *Ontario Mathematics Gazette* (OMG) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *Gazette*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles are to be word–processed, MS Word is preferred, and prepared according to the *Publication Manual of the American Psychological Association*, Seventh Edition. However, please use single-line spacing (not double). Articles should not exceed five numbered pages of text, and figures, images, and photographs should be placed in the text close to where they belong, with captions. The photographer's permission is required, and for photos of students under the age of 18, the written permission of a parent or guardian is required.

Please submit your article in one blind file (i.e., identity of author is not evident), and include author names, contact information including email and mailing addresses, photos—head and shoulders, biographies—less than 100 words, and all content removed for blinding in a second file. Please email these two files to Tim Sibbald at gazette@oame.on.ca.

Upon review, you will be notified whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected. The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME/AOEM.

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Submission of Advertisements
Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** by email or at the above address. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be print ready, and colour advertisements should have no bleeds.

Advertising Rates
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▲ Editor's Report



TIM SIBBALD, OCT, PHD
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Tim Sibbald holds the Universities Director position on the OAME Board of Directors. He is a former president of OAME. He is an associate professor in the Schulich School of Education, Nipissing University, with a focus on mathematics education.

> As our spaceship approached Alpha Centauri, the cabin lit up, and every student in the class cheered. At the same time, they held up their educational service devices and got readings of the intensity of light that could compare to an equivalent distance from the Earth's home star….

Virtual reality is coming. How soon, we do not know, but in some ways, it is already here. Consider flight simulators that are used by pilots in their training, or other technologies that are entering classrooms (but hereafter, I will simply refer to flight simulators). There is a general understanding that during the learning process, a crash in a flight simulator is better than the *authentic* experience.

There are many interpretations of what contributes to a scenario being authentic or makes a context realistic. Classic context questions are pencil and paper, text driven, sometimes with an illustration, that references a scenario. I recall a question from the 1930's about the value of a stack of hay, but I also recall pictures of ladders leaning against walls in modern textbooks. Sometimes these are referred to as "real-life" questions—a term that seems set to distinguish from "fake-life" questions such as reading a gauge in a flight simulator, but I digress... The question of how genuine or realistic a scenario needs to be to register as a genuine context is unsolved.



Anne Kiesenhofer, an Austrian mathematician, won Olympic gold in Tokyo in July after winning a 137 km cycling road race.

As an area of research, my initial recognition of the underlying problem occurred when a student asked why I had a cash register in my classroom. They clearly did not connect cash registers with mathematics. About the same time, I had a student complain when I referred to a career-oriented question as "real life." The student rebutted that, for

students, the classroom environment *is* their real life, and the term diminishes the experiences they are having. I asked her what she felt I should call it, and after thinking for much of a class, she came up with "career life." A valid point. Years later, after complaining that a demonstrator slide rule, gathering dust atop a bookshelf in the math office, should be mounted on a classroom wall as an authentic historical mathematics artifact, I was given the slide rule as a parting gift. In both cases, it was a broad interpretation of authenticity in the overall landscape of mathematics education.

For most teachers, the issue connects to decisions about when to use hands-on materials as opposed to an illustration. There are challenges addressing this because teachers have a variety of opinions on what constitutes a math problem that has a context. The research issue is about how students transfer their learning from the classroom to actual events in their daily life and later careers. Beswick (2011) calls this "putting context in context," and she notes that "authenticity of a problem is dependent upon the solver" (p. 369).

If you consider the coming world of increased virtual reality, and even today's steps in that direction, such as flight simulators, it is evident there can and will be opportunities to go beyond simply hands-on in the classroom. As the holidays approach, one might consider which way to park a sleigh on a snowy roof for safety—whimsical for sure, but also not something one is going to pursue as a field trip. It is by no means simple to think about what a *context* is. As Beswick summarizes: "…the impacts of various types of problems needs to take account of all of these factors and of the complexity of contexts in which mathematics is taught and learned, as well as the diversity of learners in them" (Beswick, 2011, p. 388).

At the same time, there seems to be an ongoing need to remind that a picture is a convenience. A flight simulator is a representation that is as good as one can realistically achieve if you want students reading aircraft gauges. And, genuine hands-on with authentic materials is necessary, *in all courses*, at least some of the time.

Distinct, but perhaps related to context, researchers at McMaster University have recently demonstrated that modelling of music downloads can be achieved using an infectious disease model (Rosati et al., 2021). It appears the ideas will lead to new ways of thinking about disease susceptibility and transmission.

Join the Editorial Team?

The *Gazette* is in need of a member (or two) who is (are) interested in joining the editorial team as an associate editor. We are particularly interested in engaging younger members of the organization. The associate editors, as a team, review every contribution to the *Gazette* after the editor. Their focus is clarity of writing, avoiding repetitious writing, and considerations of the way authors are saying things. The role is distinct from the review process and from article proofreading—associate editors work with accepted content just ahead of proofreading. Associate editors also proofread the final version of the *Gazette* prior to publication and, periodically, are called on for opinions by the editor.

The work associate editors do is ongoing, and divided across all associate editors. So, while the preparation of individual *Gazette*s has a busy week each quarter, the level of effort is directly proportional to the number of associate editors. If you would like more details, please contact Tim at gazette@oame.on.ca.

FYMSiC Item

An interesting observation by Andrew Skelton appeared in the *FYMSiC Newsletter* (First Year Mathematics and Statistics in Canada). It has to do with a modular course design and when students complete modules. He noted a specific correlation between the number of proactively completed modules and the final course grade. The graph in Figure 1 represents the data.

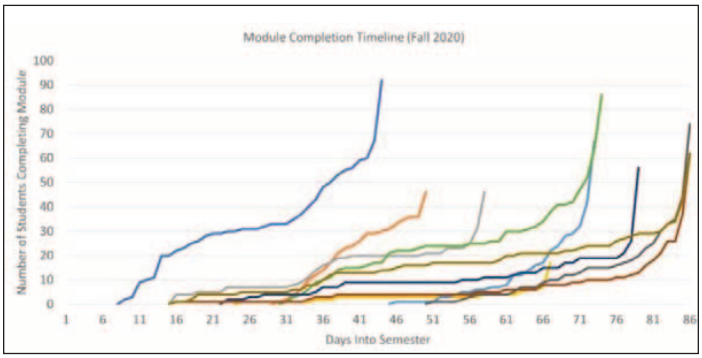


Figure 1: *Timeline for module completion*

Ed? No, the other Ed!

It became known in September that Ed Sheeran has announced his 2022 tour, which is called "The Mathematics Tour." (Maybe someone could write a letter to the editor to fill us in on the details?)

Errata

A sincere apology to Ami Mamolo, whose name was spelled incorrectly in the last issue.

Also, a thank you to Peter Saaramaki for spotting the problematic homonym in my last editorial. The new *Abacus* editors have taken the reins, and I wish them a long reign

with just enough metaphorical rain to keep them prospering.

In This Issue

Our President, **Judy Mendaglio**, responds to the editorial problem brought up in the last issue. The Fields Report, by **Angelica Mendaglio**, focuses on "Teaching Math in Public." **Jacqueline Hill** brings more student reflections about online learning in her NCTM Report. **Heather Theijsmeijer** brings news from OMCA.

Iain Broadie and **Beyza Sezer** interview **Dr. Diane Tepylo** and provide a taster, where the full interview is on OAME Talks. **David Costello** writes about strategies called "writing to learn," as he has just had another book published, *Making Math Stick*. It seems something he is well suited to write about. **Lynda Colgan** gives perspective to elements of the curriculum change—both the process and the end result. **Shawn Godin** has a puzzling rectangle problem that he solves using quadratics and trigonometry. He then leaves us with a tantalizing coin problem. **Jennifer Holm** brings order to operations by considering the details of why the order is what the order is. **Jeff Irvine** considers how technology has changed over the years—the one constant with technology seems to be that it never stays the same. **Carly Ziniuk** draws on her students discovering algebra as a result of a shopping trip! **Jérôme Proulx** elaborates on his research regarding student understandings of perimeter. Lastly, **Ron Lancaster** draws on his love of air travel that will give you some pandemic relief.

Natalie Robinson and **Fred Ferneyhough** explain and justify the use of Computer Algebra Systems for supporting student learning. **Geneviève Lespérance** writes of language learning in late French immersion math classes. Lastly, **Jeff Irvine** draws on a low-floor, high-ceiling problem that emphasizes sequences. **Fareen Kanji**, **Shauna Speare**, and **Jacqueline Hill** share perspectives from different roles in a book review, and lastly, **Tim Sibbald** reviews a book by **David Costello**.

Thank Yous!

John Rodger, **Ralph Connelly**, **Anne Yeager**, **Kyla Kadlec**, **Gitta Berg**, **and Penny Clemens.** What an editorial team! They are *simply the best*! (With a nod to Tina Turner.)

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Rosati, D.P., Woolhouse, M.H., Bolker, B.M., & Earn D.J.D. (2021). Modelling song popularity as a contagious process. *Proceedings of the Royal Society A. 477*, 20210457. doi.org/10.1098/rspa.2021.0457 ▲

▲ OMCA News – Fall 2021

HEATHER THEIJSMEIJER (OMCA PRESIDENT)
OMCAmath@gmail.com

Though once again the school year has started amidst pandemic-induced uncertainty, the Ontario Mathematics Coordinators Association (OMCA) continues to provide space for Math Coordinators, Consultants, and Coaches from across the province to learn and network. This year, all of our monthly members' meetings will be virtual, with the potential exception of our May meeting: should the OAME2022 conference be in person as planned, we will also meet in person in Windsor. Fingers crossed for a road trip!

This past July, OMCA was the first provincial association to respond to the Ontario Ministry of Education's removal of material referencing the subjective nature of mathematics, the role of mathematics in normalizing racism, and the use of mathematics to interrogate social issues from the new Grade 9 destreamed mathematics curriculum. Our full statement appears on our website.

Curious about our monthly members' meetings? Each meeting includes updates from stakeholder groups sharing similar interests as OMCA (such as the EQAO, the Math Knowledge Network, the Ontario Ministry of Education, and more), networking opportunities, resource showcasing, and a guest speaker. Non-members are welcome to attend as guests for the guest speaker component of the meeting for a fee of $30. In October, we welcomed Dr. George Gadanidis, who spoke on improving the quality of mathematics brought to the classroom, with examples that fit the new Grade 9 curriculum.

We are continuing with our literature circles this year. OMCA members can choose one of four books: *Grading for Equity* (Feldman, 2018), *Math Recess: Playful Learning in an Age of Disruption* (Singh & Brownell, 2019), *Indigenous Writes: A Guide to First Nations, Métis, and Inuit Issues in Canada* (Vowel, 2016), or *NCSM Framework for Leadership in Mathematics Education* (2020). Literature circles will meet three times throughout the year for informal conversation around content that resonates with us, as well as how it impacts our practice.

Once again, our annual retreat is going virtual! OMCA will be hosting a digital symposium on Friday, February 11, 2022, featuring a number of presenters speaking to the

continued on page 12

President's Message



JUDY MENDAGLIO
judy.mendaglio@oame.on.ca

Judy Mendaglio is the President of OAME/AOEM. She has worked in education in Ontario in many capacities: college instructor, secondary teacher and department head, university instructor, elementary and secondary consultant, and writer. Currently, Judy devotes most of her time to her many volunteer activities.

Judy Mendaglio est la présidente de l'AOEM/OAME. Elle a exercé de nombreuses fonctions dans le secteur ontarien de l'éducation : formatrice au niveau collégial, professeure et chef de section du secondaire, chargée de cours au niveau universitaire, consultante pour le primaire et le secondaire et écrivaine. Judy se consacre actuellement à de nombreuses activités bénévoles.

I read with interest a little golden nugget that appeared in the September issue of the OAME/AOEM *Gazette*.

What would you do?

In this edition of the *Gazette*, we ran into a small, but important editorial issue. If both sides of $-x > -5$ are multiplied by –1, then $x < 5$. However, what happens to the ">"? Age-appropriate language might be "flipped" or "reversed," but neither is a mathematical operation. "Inverted" was also suggested (Thank you, Stewart Craven), but is the inverse of $x < 2$ properly $x > 2$ or $x \geq 2$?

Is there a better mathematical way to convey this? (gazette@oame.on.ca)

This brought to my mind one of my greatest sorrows about how students experience mathematics. We teachers seem to really want to give students quick rules, seemingly to reduce the pain of thinking mathematically. However, for me, the attraction of mathematics has always been the beauty of its underlying structure. Quick rules circumnavigate the explorations of these beautiful structures that I hold so dear.

How would I answer this wonderful question? I visualize these sorts of "greater than" and "less than" questions as geometric relationships between numbers. In the same way that "he's my brother-in-law" tells us about a relationship between people, it isn't clear what *other* relationships are connected to this one. Is the brother-in-law married to my sister? Is he my husband's brother? Telling us that $-x > -5$ informs us that $-x$ and -5 have a relationship, specifically, that $-x$ lies to the right of -5 on the number line (the geometry). Could we be looking to understand that relationship in a different way, perhaps looking for the underlying relationship that created this relationship, in the same way that we looked for the underlying relationship that created the brother-in-law relationship? Of course we can.

Upon reflection, we understand that multiplying by negative 1 is related geometrically to the reflection transformation. For example, when we reflect this triangle through/across that y-axis (horizontally), we find that we have had to multiply the x-coordinate of each vertex by –1, to determine the vertices of the reflected triangle. We notice that if a vertex was on the left of another, it is now on the

Dans le présent document, le masculin est utilisé sans aucune discrimination et uniquement dans le but d'alléger le texte.

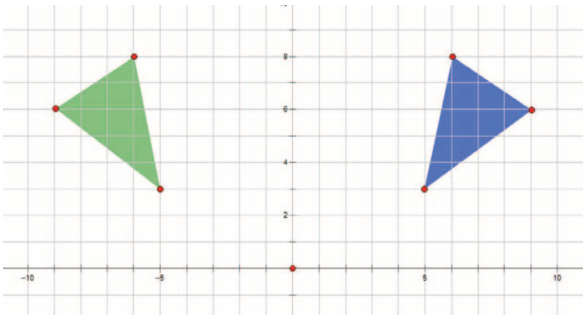
J'ai lu avec intérêt une petite pépite d'orée parue dans le numéro de septembre de la *Gazette* de l'AOEM/OAME.

Cela m'a rappelé l'un de mes plus grandes tristesses quant à la façon dont les élèves vivent les mathématiques. Nous, les enseignants, semblons vraiment vouloir donner aux élèves des règles rapides, apparemment pour réduire les défis d'avoir à penser mathématiquement. Cependant, pour moi, l'attrait des mathématiques a toujours été la beauté de leur structure sous-jacente. Des règles rapides contournent des explorations de ces belles structures qui me sont si chères.

Comment répondrais-je à cette merveilleuse question ? Je visualise ce genre de questions « plus que » et « moins que » comme des relations géométriques entre des nombres de la même façon que je visualise des relations entre les gens, par exemple, « c'est mon beau-frère ». Quelles autres relations sont liées à cet énoncé ? Le beau-frère est-il marié à ma sœur ? Est-il le frère de mon mari ? En outre, nous pouvons dire que $-x > -5$ nous informe que x et -5 ont une relation spécifique, que $-x$ se trouve à droite de -5 sur la droite numérique (la géométrie). Pourrions-nous chercher à comprendre cette relation d'une manière différente ? Bien sûr que nous le pouvons. Peut-être on considère la relation sous-jacente qui a créé la relation entre $-x$ et -5 de la même façon que nous avons établi la relation sous-jacente qui a créé la relation trouver dans le scénario du beau-frère ?

Après avoir réfléchi, nous comprenons que la multiplication par le négatif 1 est liée géométriquement à la transformation de la réflexion. Par exemple, lorsque nous

right. The "less than" relationship is now a "greater than" relationship. If we reflect the triangle across/through the *x*-axis (vertically), the same thing happens, but this time the effect is as if we had multiplied the *y*-coordinates of the vertices by –1.



Taking this two-dimensional relationship between reflection and multiplying by –1 and applying it to the one-dimensional $-x > -5$ question, we see this related relationship between x and 5.

In this way, we can see that the relationship that is created through multiplication by –1, which in this case means geometrically reflecting through zero (horizontally), is $x < 5$. Interestingly, if we horizontally reflect the symbol >, we get <.

This is why mathematics is so beautiful.▲

reflétons ce triangle à travers l'axe *y* (axe numérique horizontal), nous constatons que nous avons dû multiplier la coordonnée *x* de chaque sommet par –1 pour déterminer les sommets du triangle reflété. Nous remarquons que si un sommet était à gauche d'un autre, il est maintenant à droite. La relation « moins que » est maintenant une relation « plus que ». Si nous effectuons des réflexions d'un triangle à travers l'axe *x* (axe numérique vertical), la même chose se produit, mais cette, fois l'effet est comme si nous avions multiplié les coordonnées *y* par des sommets par –1.

En établissant la relation à deux dimensions entre la réflexion et la multiplication par –1, et en l'appliquant à une dimension, $-x > -5$, nous pouvons voir la relation entre x et 5.

De cette façon, nous pouvons voir que la relation qui est créée par multiplication par –1, qui dans ce cas signifie une réflexion géométriquement à travers zéro (axe numérique horizontal), est $x < 5$. Il est intéressant de noter que, si nous reflétons horizontalement le symbole >, nous obtenons <.

C'est pourquoi les mathématiques sont si belles. ▲

Annual Conference News

The SWOAME Planning Committee for OAME Annual 2022 is hard at work shifting the conference from an in person experience to a Virtual Conference. While the decision to make this pivot was not what we were hoping, the uncertainty around re-opening constraints have made it very challenging to plan an effective conference with so many unknowns.

With this recent shift to a Virtual Conference, we are working diligently through the transition phase and are happy to share these details:

The conference will be taking place in the late afternoon and evening, stretching out across the four (4) following dates:
Wednesday, May 4 • Thursday, May 5
Monday, May 9 • Tuesday, May 10

It is anticipated that we will be accepting proposals until mid-December.

The OAME 2022 Planning Committee is eager to offer an exciting and professional learning-filled event that all mathematics educators can participate in from anywhere across Ontario!

Updates as the transition progresses can be found at www.oame2022.ca.



We are happy to be able to announce our confirmed speakers:

Sunil Singh
Peter Liljedahl
Lisa Lunney Borden
Marian Small
David Poras
Vanessa Vakharia
Howie Hua
Cathy Bruce
Shelley Yearley
Tara Flynn

▲ Blast from the Past! The Rise and Fall of Hand-Held Calculators in Math Education



JEFF IRVINE
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Jeff Irvine, PhD, has been a secondary mathematics teacher, department head, and vice-principal. He has taught at three faculties of education and at a community college. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. With over 45 years in education, Jeff is particularly interested in the interplay of student motivation and mathematics achievement.

In 1974, shortly after the birth of the *Gazette*, many math classrooms had a large slide rule hanging on the walls, representing the state-of-the-art technology of the time. At that time, a math teacher friend of mine received a hand-held calculator as a birthday gift from his spouse. The calculator was the size, shape, and weight of a small brick, cost hundreds of dollars, and featured five functions, including square root. Since, at the time we were still teaching the formal method for finding a square root, this calculator represented a great leap forward.

A few years later, in 1978, I was a part of a math department in a new secondary school. The department head, Joe Stein, was a former aeronautical engineer, who had a simple philosophy about technologies such as calculators: Use them because they're there. His department required every student to have a scientific calculator, and bring them to every class, where they had unlimited use (see Blast from the Past, *Ontario Mathematics Gazette* [*OMG*], *59*(1)).

By 1980, scientific calculators were widely used, and the *Gazette* featured a "mini-theme" issue on calculators (*OMG, 19*(2)). The issue included articles on using calculators with high-ability students (Szetela); with students having learning difficulties (Houser); several articles on calculator activities (Lessard; Templeton; Mura et al.); and research on the use of calculators in math classes (Craig; Suydam).

Over the next 15 years, scientific calculators enabled teachers to ignore trigonometry tables and interest rate tables in textbooks; render calculations with logarithms obsolete; feature real-world problems with "messy" data in their teaching; and positively affect students who previously feared or hated extensive arithmetical computations. Over this period, *Gazette* articles focused more on the use of computers in math classrooms (e.g., Doctorow, *OMG 20*(3); Rahim, *OMG 34*(1); Storey, *OMG 36*(3)). There were very few articles focused explicitly on using scientific calculators, although often the availability of calculators in the classroom was assumed.

In 1998, as part of the revised math curriculum, the Ontario Ministry of Education placed a strong emphasis on investigations and real-world problems. To support this stance, the Ministry provided $100 million for learning materials (textbooks and technology). Many math departments used this money to purchase graphing calculators and related technologies. The *Gazette* featured a Technology Corner column (e.g., Fred Ferneyhough, *OMG 39*(3), and later, Mary Bourassa, *OMG 48*(3)), that focused on graphing calculator use, as well as computer software such as Geometer's Sketchpad® (Ferneyhough, *OMG 41(*1)). In 2001, Peter Harrison (*OMG 39*(3)) offered a comparison of hand-held calculators, desktop computers, and what he called "skull-based" technologies in solving geometry problems.

Graphing calculators became regular fixtures in math classrooms and saw extensive use by students in analyzing data. Students learned about correlation and regression analysis, and investigated problems that were of interest to them, while learning the math behind real-world issues. Assessments often featured rich tasks that involved investigation, data collection, and data analysis, with graphing calculators playing a prominent role.

As technology advanced, the *Gazette* also featured a CAS Corner (e.g., Ferneyhough. *OMG 41*(1)) on the use of computer algebra systems. However, as the 2010's approached, most technology articles in the *Gazette* began to discuss computer-based technologies, and articles about graphing calculators waned. Regular columns on technologies such as the Ministry-developed CLIPS and other provincially available technologies (e.g., Isenegger et al., *OMG 54*(4)) became the norm. Technology Corner now featured more articles about sites like Desmos (Bourassa, *OMG 51*(1)) and other Internet-based technologies. Students still had access to scientific and graphing calculators through their smartphones, and the rise of Bring Your Own Device (BYOD) policies. However, the graphing calculators in classrooms began to see much less use.

Graphing calculators do not need Internet access, which may be a plus to some classroom teachers. In addition, the

use of graphing calculators in investigations using motion detectors is still a valuable tool (see, e.g., Irvine, *OMG 57*(3)). Graphing calculator technology may also be a useful tool to teach the new expectations around coding.

The introduction of calculators in math classrooms, first scientific and then graphing, had a profound impact on what we teach and especially on how we teach. Calculators allowed students to deal with real-world problems; investigate problems of interest to them; pose and solve problems, using mathematical techniques that would have been inaccessible without the technology. They removed the drudgery of long calculations and allowed teachers to focus on important mathematics, rather than on basic calculations.

In the year 2000, my math department had 360 graphing calculators, 60 CAS, and 30 business calculators, as well as 6 overhead graphing display tablets, 12 motion sensors, and 2 calculator-based lab systems for collection of data with the use of sensors. However, the story of the graphing calculator illustrates the technology–innovation cycle. New, better, and more accessible technologies rendered the graphing calculators, which resided in classrooms and were not accessible to students 24/7, almost as obsolete as the slide rules of the 1960's and 1970's.

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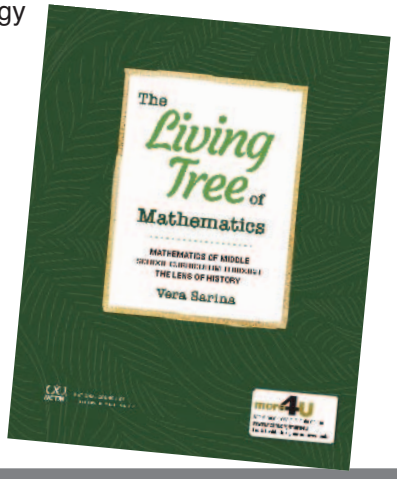
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OAME member publishes book. Congratulations, Vera Serina!

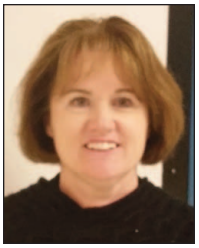
A member of TEAMS, who has been involved with math Olympiads in the TEAMS chapter and provincially. She has presented at chapter and provincial conferences.

Last June, Vera published a book with NCTM. The book is the result of her years of exploration of mathematics as a teacher in a junior high school. Her book explores the history of mathematics as a source for answers to many questions that arise in math classrooms of today. Vera has described the book as providing material for rich, culturally responsive pedagogy with relevant historical vignettes and mathematical approaches.



Vera has indicated she is available to book clubs who would like to discuss her book and as a speaker.

▲ OAME/NCTM Report: Students Giving Students Advice on Destreaming (Part 2 of 3)



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Jacqueline Hill is a Grades 9–12 teacher of mathematics. She is a Past President of OAME and OMCA, as well as the recipient of the award for Exceptional Teaching in Secondary Mathematics. She also wrote the "Director's Dialogue" for the Gazette *for a number of years.*

As you may recall from the "NCTM Report" in the October *Gazette*, my students used Mr. Todd Romiens' article (March 2017 *Gazette*) to reflect on their learning as students. They gave advice directly to students on how, in a destreamed classroom, they could be the best they could be.

This report continues the journey focused on student learning. It looks at how students reflect on the home activities that are given to them. Michael Jordan didn't slam dunk a basketball the first time he went out onto the court. Similarly, Wayne Gretzky didn't perform a hat trick his first time on the ice. Students need to be taught mathematics in a meaningful way and then have time to practise their skills. My students were quick to point out that Mr. Romiens strongly suggested that students should focus on perseverance and practice, as it will help them succeed in their homework and learning. They also suggested that students be given "work days" when no additional home activities are assigned, so that they have a chance to catch up.

When I surveyed my online classroom at the end of last June, students reflected on the fact that the strategy of "highlighting" questions that prove difficult in their assignments is a wonderful way to know where additional help is indicated. They were quick to check "Mathify," as well as their older sibling for clarification. The highlighted questions were ones with which they also requested help during the following class. In addition, they used the highlighted questions as helpful review prior to assessments and evaluations (they understood it would be the questions they could not do well the very first time, which would prove more difficult on evaluations). Over 90 percent of the students felt that home activities were really important to their learning. They were often heard using the classroom catchphrase, "Keep up! Don't catch up!"

Stay tuned for the next *Gazette*, when my students give their thoughts and advice on Note Taking.

At the NCTM Bookstore – New Publications Released

- Chval, K.B., Smith, E., Trigos-Carillo, L., & Pinnow, R.J. (2021). *Teaching Math to Multilingual Students, Grades K–8: Positioning English Learners for Success.* Corwin. NCTM Stock #16053. List Price $32.95
- House, P., Stenglein, S., & Day, R. *Activity Gems for Classroom Series.* NCTM. List Price $36.95
- *Activity Gems for the PK–2 Classroom* (2019) – NCTM Stock #15753
- *Activity Gems for the 3–5 Classroom* (2020) – NCTM Stock #15754
- NEW! *Activity Gems for the 6–8 Classroom* (2021) – NCTM Stock #15755
- COMING LATE 2021! *Activity Gems for the 9–12 Classroom* (2021) – NCTM Stock #15756
- Wills, T. (2020). *Teaching Math at a Distance, Grades K–12: A Practical Guide to Rich Remote Instruction.* Corwin. NCTM Stock #16067. List Price $34.95

NCTM annual conferences provide face-to-face networking.

- New Orleans, Louisiana
  February 2–4, 2022
- Indianapolis, Indiana
  March 16–18, 2022
- Baltimore, Maryland
  November 30–December 2, 2022

Follow NCTM on Twitter, Facebook, Instagram, and LinkedIn to stay up-to-date with all NCTM events. ▲

π

It was announced in August that *pi* has been calculated to 62.8 trillion digits. The computation took 109 days and is a demonstration of the capacity for computing available to be applied to other more practical problems.

π Day

The theme for the International Day of Mathematics, coming on March 14, 2022, is "mathematics unites."

See www.idm314.org for ways you and your students can get involved!

▲ Events at the Fields



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Angelica Mendaglio is an instructional designer at Vretta Inc. in Toronto, Ontario, where she helps to create interactive digital mathematics lessons and activities for middle school students.

The 2021–2022 year of the Fields MathEd Forum kicked off on Saturday, September 25 with an in-depth discussion on "Teaching Mathematics in Public." The theme was influenced by the increased role of digital videos as educational tools, especially over the past year. The Forum took the opportunity to have attendees share and discuss how they have used videos in their classroom, and the opportunities and challenges videos present.

This meeting had a slightly different structure than usual. A typical MathEd Forum has invited speakers (usually teachers and researchers) address Forum attendees on their area of expertise. This time, Forum participants had an opportunity to connect with each other directly and share their experiences and impressions on a variety of topics linked to the theme of public math education and how it informs their practice.

The day started off with a brief talk from Dr. Trefor Bazett, an assistant teaching professor at the University of Victoria, British Columbia. He also has a YouTube channel with 132 000 subscribers, that focuses on helping people learn math. He shared his perspective on why he loves videos, what they can offer the learner, and the biggest challenges he sees with the medium as a teaching tool. In particular, he likes to structure his videos like a story, starting with a hook, introducing tension, and leading viewers to a resolution. He also appreciates the freedom to choose from a variety of different styles, rather than stick with a "chalk-and-talk" lecture format. In particular, Dr. Bazett said that the digital medium enables him to create visualizations and animations to make the content more engaging. His favourite feature of videos as a learning tool is the pause button, allowing learners to progress through the content at their own pace, and struggle with problems on their own before proceeding. The biggest challenge that he sees with videos as a learning tool is that learners can get the impression that simply watching videos is sufficient to understand the topic. Dr. Bazett emphasizes that watching videos is not a complete learning experience, and that learners need to "ask questions, try problems, and do as much math as you can on your own too."

The organizers then introduced the discussion questions, which would be the focal point of the rest of the meeting. Each attendee chose the question that sparked the most interest for them, and breakout rooms facilitated the discussion. The questions were: When are visual intuitions helpful and when are they harmful in the learning process? How can ideas be expressed more fluidly than "chalk-and-talk"? How can one work around the limits of interaction in a digital environment? What makes for a popular video? And, how can teachers with rich on-the-ground experience be empowered to contribute more to the pool of high-quality, widely dispersed content?

After the breakout sessions, all attendees congregated to report back on the key discussion points of their group. One of the common threads in the discussions was that there is a vast number of tools to support the creation of interesting and engaging videos and other digital pedagogical content, but that it is a challenge to select them and learn how to use ones that will fit a specific class best. In particular, a few attendees advocated limiting the number of tools used, so as not to overwhelm students. They advised that each new tool is an additional piece, which both teacher and student need to learn. This can add an extra cognitive load for students in particular, as they will need to learn the tool as well as the mathematical content, which the tool is being used to convey. Some of the tools, which were recommended for creating digital content, included Mathmatize and Mathigon.

Another common theme across the discussion groups was the importance of scaffolding and planning when using digital tools. Dr. Bazett's sentiment that watching videos needs to be accompanied with "doing math" was echoed by other attendees, who stipulated that what students do is what truly determines how much they learn. Carefully planning student activities for before and after watching a video can help to get the most out of the video. Additionally, integrating a video into a lesson by scaffolding it before and after can help bring the teacher's voice back into the lesson. Some attendees voiced concerns that they felt the need to create any videos they used themselves to keep the voice of the teacher present. Other attendees assured that they feel free to use videos they find online, as their voice is carried through what the students actually "do" with the video, which is arguably the most important part. This way, teachers are also able to take advantage of the excellent education-focused videos that are publicly available, including from YouTube channels such as Dr. Trefor Bazett, Numberphile, 3blue1brown, and Veritasium. Dr. Bazett recommended a video by Veritasium titled "The Simplest Math Problem No One Can Solve—Collatz Conjecture." It starts with a very simple problem, but becomes quite advanced, and includes great visualizations and interviews with mathematicians.

The Fields MathEd Forum will convene on the last Saturday in January, February, March, and April. You can sign up to attend remotely through the Fields website, where you can also see the schedule of other upcoming math education events. ▲

▲ What's the Problem? Diagonals and Area



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Shawn is a former mathematics teacher, department head, and consultant. He strongly believes in the central role of problem solving in the mathematics classroom. He continues to be involved in mathematical activities: presenting workshops, writing articles, working on local projects, and helping create mathematics contests.

Welcome back, problem solvers. Last issue, I left you with the following problem:

> *Rectangle* ABCD *has diagonal* BD *with endpoints* B*(4,2) and* D*(12,8). Diagonal* AC *lies on the line with equation* x + 2y − 18 = 0. *Determine the area of* ABCD.

This is team problem #23 from the 2020 Canadian Team Mathematics Contest, run by the Centre for Education in Mathematics and Computing (CEMC) at the University of Waterloo. The competition has several events with competing teams of six secondary school students. The events are the individual problems, team problems, and the math relays. For the team problems, the team works as a group to solve as many problems as possible. For more contests and resources from the CEMC, you can visit their website at www.cemc.uwaterloo.ca.

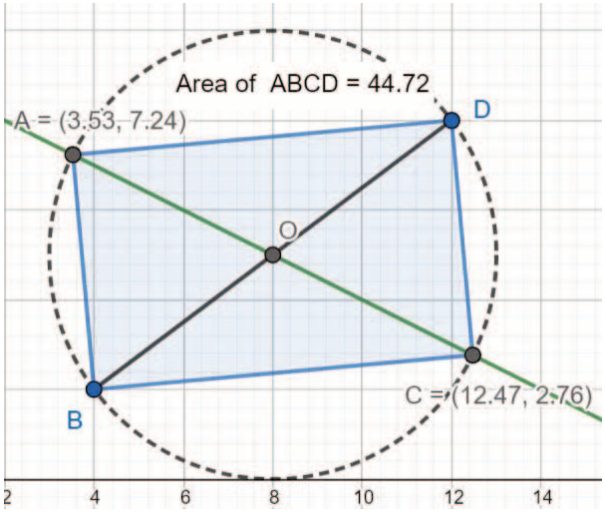


Figure 1: *Our rectangle in GeoGebra*

This problem is a little challenging, so let's start by getting some help from GeoGebra. We can plot the points *B* and *D* and the line. The diagonals intersect at the centre of the rectangle, *O*, which is equidistant from the corners of the rectangle. Thus, if we find the points of intersection of the line and a circle about *O*, we can get the coordinates of the other two vertices, *A* and *C*. Then we can construct rectangle *ABCD* and measure its area, and we are done.

Finding the answer, using dynamic geometry software, isn't very satisfying from a mathematical point of view. However, it is a useful exercise because we may not have considered the circle otherwise. Also, we have the coordinates of *A* and *C*, as well as the area calculated to two decimal places, so we can check our answer.

Doing a little circular reasoning (excuse the math pun), we can easily determine the coordinates of *O*, as it is the midpoint of *BD*, that is,

$$O = \left(\frac{4+12}{2}, \frac{2+8}{2}\right) = (8,5),$$

which agrees with our sketch. Hence the equation of the circle, centred at *O*, is $(x-8)^2 + (y-5)^2 = r^2$, where *r* is the radius of the circle. Substituting the coordinates of *B* or *D* into the equation will allow us to determine that the radius is 5 units. If we rearrange the equation of the line to solve for *x* and substitute it into the equation of the circle, we get $(10-2y)^2 + (y-5)^2 = 25$.

Expanding, simplifying, and dividing out a common factor yields the quadratic equation $y^2 - 10y + 20 = 0$, which has solutions $y = 5 \pm \sqrt{5}$. Substituting these into the equation of the line, we can calculate the *x* coordinates of *A* and *C* so that we end up with $A(8 - 2\sqrt{5}, 5 + \sqrt{5})$ and $C(8 + 2\sqrt{5}, 5 - \sqrt{5})$, which agree with calculations from GeoGebra.

Now, we can just calculate the length and the width, using the distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2},$$

which yields

$$d_{AB} = \sqrt{50 - 10\sqrt{5}}$$

and

$$d_{AD} = \sqrt{50 + 10\sqrt{5}}.$$

Therefore,

$$A = d_{AB}d_{AD} = \sqrt{2500-500} = \sqrt{2000} = 20\sqrt{5},$$

which agrees with the approximation in our sketch.

So, we have solved the problem using, mostly, mathematics from Grade 10 Academic. For those of you who protest, we could adjust our solution slightly by just measuring the distance from *O* to *B*, then setting the distance from *O* to *A* to the same value, and solving (as in the official solution from the CEMC). The problem is difficult because we are using methods from different areas of the

course, and the values that we are working with are not pretty.

Let's look at the problem again, using some different tools. In Figure 1, we see that our rectangle is broken very naturally into four triangles by the diagonals, and triangles scream out for trigonometry.

In Figure 2, we know two sides, *a* and *b*, and the contained angle θ of a triangle. Hence, the height of the triangle, indicated in the diagram, is given by $h = a\,\mathrm{Sin}\theta$, and so the area is $A = \frac{1}{2}\,ab\,\mathrm{Sin}\theta$.

Note that if we had found the height from *A* instead, we still would have the same expression for the area.

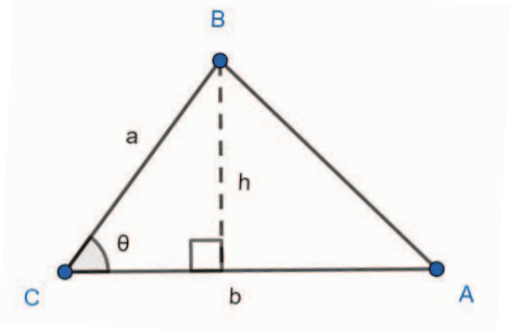


Figure 2: *Triangle with two known sides and contained angle*

Returning to our original problem, we know that we have two pairs of isosceles triangles with equal sides 5 units long, as shown in Figure 3. We also see that the indicated angles, α and β, are supplementary. Hence, from what we know about the sine function, $\mathrm{Sin}\alpha = \mathrm{Sin}\beta$, and so all four triangles have equal area. Thus, the area of the rectangle is $A = 4(\frac{1}{2} \times 5 \times 5\,\mathrm{Sin}\alpha) = 50\,\mathrm{Sin}\alpha$.

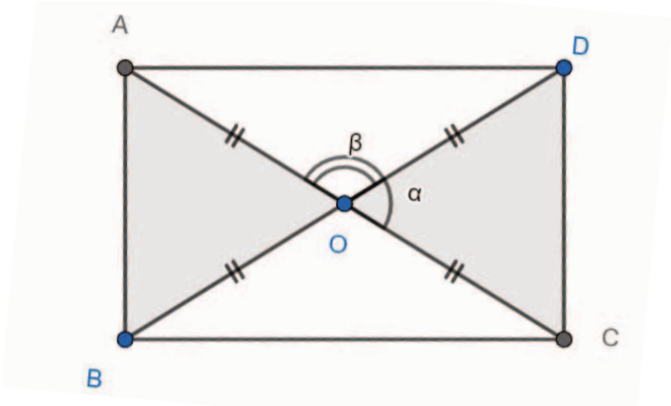


Figure 3: *Two pairs of triangles*

We also know that if *m* is the slope of a line and θ is the angle the line makes with the *x*-axis, then $\mathrm{Tan}\theta = m$. Since we know the coordinates of *B* and *D,* we can calculate $m_{BD} = \frac{3}{4}$. If we rearrange the equation of the line, we get $m_{AC} = -\frac{1}{2}$.

Drawing a line parallel to the *x*-axis through *O*, we see that $\alpha = \theta - \phi$, where the angles are *directed* angles and, in this case, ϕ is negative. Using a trigonometric identity, we get

$$\mathrm{Tan}\alpha = \mathrm{Tan}(\theta - \phi) = \frac{\frac{3}{4} - \left(-\frac{1}{2}\right)}{1 + \left(\frac{3}{4}\right)\left(-\frac{1}{2}\right)} = 2.$$

Then, using the definitions of the trigonometric ratios for angles in standard position, we can determine that $\mathrm{Sin}\alpha = \frac{2\sqrt{5}}{5}$ and hence, we get the same value for the area that we calculated earlier.

Looking for alternate solutions to problems strengthens your understanding of mathematical concepts and often allows one to see connections between different areas of mathematics. In some cases, you can make "discoveries." We can show from our work that if a rectangle has a diagonal of length *d*, and the angle between the diagonals is α, then the area of the rectangle is $A = \frac{d^2}{2}\mathrm{Sin}\alpha$. You could also generalize this problem by writing the area in terms of the diagonal length and the slopes of the diagonals. I will leave that to your amusement.

Now for your homework:

You have 164 coins made up of nickels, dimes, and quarters worth $19.50. When you calculate the value of all the nickels, all the dimes, and all the quarters, you notice that two of the three totals are a whole number of dollars. How many coins of each type do you have?

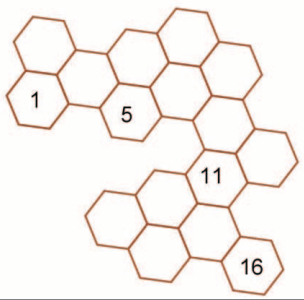
Until next time, happy problem solving. ▲

continued from page 4

theme of ***Rehumanizing Mathematics***. At the time of writing, registration details are still being finalized. Keep an eye on our website for more information.

From Kenora to Cornwall, and from Fort Severn to Fort Erie, new members are always welcome to join OMCA. Contact us directly at OMCAmath@gmail.com, or visit our website at www.omca.website (note the different domain) to learn more about the organization, find out about upcoming events, and connect with us. ▲

Hidato
Fill cells so that neighbouring cells have sequential values.



▲ Linking Literacy and Math: Using Writing to Support Instruction and Learning



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David is a principal, who has held many roles. He taught in the Primary division before assuming roles of numeracy interventionist, numeracy coach, numeracy leader, and curriculum consultant for Prince Edward Island. David has also instructed university courses focused on curriculum, differentiation, mathematics, and language arts. He facilitates professional learning in the areas of mathematics and school development. Dr. David Costello recently published two books: Making Math Stick: Classroom strategies that support the long-term retention of math concepts *and* Using What Works: Strategies for Developing a Literacy-Rich Environment in Math.

Writing can be a powerful instructional and learning strategy. Oftentimes, when math teachers hear suggestions about incorporating writing into daily routines, they respond that they would like to, but are not sure they have time. I want to offer an alternative to writing tasks that are lengthy and obtrusive. In this column, I will suggest two approaches to writing in mathematics that can support student learning and be used to make student thinking visible.

From the perspective of supporting students as learners, writing tasks can be based on the writing-to-learn framework. Writing to learn is an informal, exploratory approach to writing that consists of short tasks that provide students support as they think through concepts (Costello, 2019). Within this strategy, tasks should be low stakes, such that students will not worry about marks assigned to the writing. The goal in writing-to-learn activities is to use writing as a tool to explore one's thinking, delve into concepts, and discover possibilities.

Writing-to-learn tasks provide teachers with opportunities to observe student thinking by making it visible. This observation is not for the purposes of evaluation, but for the purposes of enabling students to explore concepts, and for teachers to have student work samples highlighting how students are engaging with a concept, whether it be correct or filled with misconceptions.

Writing-to-learn tasks can take as little as 3 to 5 minutes. It could be a prompt on an exit slip, asking students to tell what they know about surface area. It could be a warm-up activity, asking students to recall their previous learning associated with circle geometry. It could be a small guided activity, whereby students are to recount a definition of a particular math term. Regardless of what it is, writing-to-learn tasks are meant to support understanding and make thinking visible.

Feedback can make writing-to-learn tasks more impactful in terms of student learning. From the perspective of self-feedback, students can go back to pieces that they have previously written, to answer any questions they asked themselves earlier or to clarify their thinking. As feedback from others, students can share their written pieces with you or their peers as a way to seek clarification on their thinking or to seek ideas for further insight on what they have so far. Just be careful not to provide feedback related to spelling, punctuation, and so on.

Overall, writing-to-learn tasks can be used at any grade level and applied to support the student in developing their understanding of a concept, and the teacher in observing student thinking, facilitating their identifying possible next steps. Depending on the age or individual learning needs of the student, writing-to-learn tasks can be modified by having a student share their thinking orally, while someone scribes for them. Students could also make use of speech-to-text software to record their thinking in words.

For this column, I want to share two similar, but different writing-to-learn tasks: *Free write* and *30 words or less*.

Free Write

During a free write, students are asked to write down anything and everything they know about a concept. Students are given a limited amount of time, perhaps 5, 10, or 15 minutes. The time chosen by the teacher depends on the age of the students, their writing stamina, and the complexity of the concept (Costello, 2021).

The free-write task can be a powerful tool to support students in recalling previously learned concepts. The writing serves as a retrieval exercise that strengthens the pathways to the retrieved information and consolidates their memory of the concept. By having students recall previous learning, they are engaging in retrieval practice, which strengthens the memory and makes it easier to recall (Costello, 2021).

During a free write, students brainstorm about a given topic, without judgment from others. The goal is to generate as much information as possible related to the topic. From this, students organize their thinking and revise their thoughts, all the while strengthening their connections to this knowledge. Within this process, students may informally connect topics as they search for any knowledge related to the focal concept. A free write is a flexible writing-to-learn

tool. Teachers can assign this particular writing task within or immediately following a lesson, a day after the lesson, or after an extended period of time.

The following three examples are free-write samples taken from differing grade levels. In each example, the teacher had modelled the free-write strategy and provided various time frames for the exercise.

Primary Example

What can you tell me about a referent?

- A referent helps when you estimate. You can picture what a small amount of the item looks like. This small amount is one group. You then use that picture to help you see how many groups are in the total amount. You then skip-count the number of groups by the amounts in each group.

Junior Example

What do you remember about discrete data?

- Discrete data are data that you can count. It can be things like the number of people, the number of marbles in a collection, or the number of plants in a garden. It is about whole numbers.

Intermediate Example

What can you tell me about a misleading graph?

- A graph that represents information in a misleading way. For example, if it is a pictograph, it could use different sizes of symbols for different categories. This would make it look like some categories have a bigger or smaller amount than they actually do. Another example is when the scale is broken and starts at a larger number. This may mislead people to think that the differences between categories are large, when actually they are not. Misleading graphs can be seen in the media or at other times, when people are trying to persuade others to see their perspective.

30 Words or Less

Similar to a free write, 30 words or less is a writing task assigned within a limited amount of time. The only difference is that 30 words or less has a cap on the total number of words. By introducing the cap, students must focus on choosing the most crucial information from memory. Along with this, students need to be concise in what they write and to structure the information in an organized format to avoid exceeding the word-count cap (Costello, 2021).

30 words or less can be assigned at any point in time, as it is meant to support students in recalling and strengthening their understanding of a concept. 30 words or less can be assigned before, during, after, or long after initial introduction of a concept. Depending on the age and individual learning needs of students, teachers may wish to adjust the parameters of 30 words or less. Both the cap of words as well as the time could be altered. Similar to a free write, 30 words or less could be done orally and/or using speech-to-text.

The following three samples of 30 words or less are taken from differing grade-level bands. In each example, the teacher had modelled the 30-words-or-less strategy and provided various time frames for the exercise.

Primary Example

In 30 words or less, what can you tell me about increasing patterns?

- Patterns with an increase from one term to another term. The increase can be constant like 3, 6, 9, 12, or a growing increase like 3, 6, 12, 24.

Junior Example

In 30 words or less, what can you tell me about a regular polygon?

- A regular polygon is an enclosed 2D shape that has straight sides. Each side is the same length, and all the angles are equal.

Intermediate Example

In 30 words or less, what can you tell me about the relationship of the sides of a right-angle triangle?

- For a right-angle triangle, the squared length of the longest side is equal to the sum of the squares of the lengths of the other two sides.

Final Thoughts

Writing tasks in mathematics do not have to be time intensive and laborious. By strategically providing students writing-to-learn tasks, students will have opportunities to strengthen their understanding, and they will make their thinking visible. Whether it is an entrance slip, exit ticket, workstation task, and/or small-group activity, writing-to-learn tasks can be a game changer.

In this column, I highlight just two examples of writing-to-learn tasks: *free write* and *30 words or less*. These tasks can be used as an approach to instruction and learning that can support students in recalling previous learning so that the memory is strengthened and easily accessible to retrieve.

I look forward to continuing the discussion of linking literacy and math via my website (www.costellomath.com), email (david@costellomath.com), or Twitter (@dr_costello).

References

Costello, D. (2019). *Using what works: Strategies for developing a literacy-rich environment in math.* Rubicon Publishing Inc.

Costello, D. (2021). *Making math stick: Classroom strategies that support the long-term understanding of math concepts.* Pembroke Publishers Ltd. ▲

OAME/AOEM AWARDS FOR EXCELLENCE IN MATHEMATICS EDUCATION

The purpose of the OAME/AOEM Awards is to formally recognize individuals and educators who demonstrate an outstanding contribution to mathematics education in Ontario. The successful candidate for each award enhances the learning environment for all students, providing opportunities for students to do, see, hear, and touch mathematics in a profound and meaningful way. If you know someone who qualifies for this honour, please nominate her/him/them for one of the following awards.

OAME/AOEM Life Membership Award: This award recognizes an educator who has contributed in a significant way to OAME/AOEM, demonstrates outstanding leadership in mathematics education, and has accumulated 10 or more years of membership in OAME/AOEM.

Award for Exceptional and Creative Teaching in Elementary Mathematics: This award recognizes an exceptional and creative elementary educator, who demonstrates excellence in mathematics education and contributes to the overall development of students.

Award for Exceptional and Creative Teaching in Secondary Mathematics: This award recognizes an exceptional and creative secondary school mathematics educator, who demonstrates excellence in mathematics education and contributes to the overall development of students.

Award for Exceptional and Creative Teaching in Post-Secondary Mathematics: This award recognizes an exceptional and creative post-secondary educator or team of educators who demonstrates excellence in mathematics education, contributes to the overall development of students, and demonstrates leadership in the mathematics education community.

Elementary School Staff Award for Exceptional and Collaborative Mathematics Teaching: This award recognizes an elementary school (a school accommodating any or all of Grades K–8), whose staff fosters collaborative and excellent mathematics teaching, contributes to the overall development of student learning of mathematics, and demonstrates leadership in the mathematics community.

Secondary School Mathematics Staff Award for Exceptional and Collaborative Teaching: This award recognizes a secondary school mathematics staff which fosters collegiality, teamwork, and excellent classroom teaching, contributes to the overall development of students, and demonstrates leadership in the mathematics education community.

Award for Leadership in Mathematics Education: This award recognizes an educator, who has demonstrated leadership by contributing in a significant way to the development of mathematics educators and the enhancement of mathematics education in Ontario through innovation and/or research-related activities.

Award for Outstanding Contribution to OAME/AOEM and Mathematics in Ontario: This award recognizes an individual who, has made a significant contribution to OAME/AOEM and contributed directly or indirectly to the improvement of mathematics education in Ontario, but who is not necessarily an educator or a member of OAME/AOEM.

OAME/AOEM Awards Nomination Process

1. Complete the **Award Nomination Cover Page** appropriate to the specific award, found at www.oame.on.ca (look for the **Awards** tab on the left of the page) or contact the Executive Directors at eds@oame.on.ca for an electronic version. You may include a one-page letter of recommendation.
2. Include with the nomination cover page up to three **letters of support**, of no more than two pages each. Please ensure all letters of support:
   - Address the criteria for the award as outlined in the nomination cover page.
   - Include the name and position of the supporter to the application.
   - Include the nominee's name and the award in the letter.
3. Include the **Curriculum Vitae**, of up to five pages, for award nominees, except for Elementary and Secondary Staff awards.
4. Email or mail all **completed** packages (cover page, supporting letters and CV) by the deadline advertised below to the Executive Directors. Email address and mailing address are included in the nomination cover page.

DEADLINE: Nominations are due by January 31, 2022.

The awards will be presented at the Annual OAME Conference in May 2022.

▲ Mathematical Snapshots: Math Takes a Trip



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Ron Lancaster is an Associate Professor at the University of Toronto, where he teaches mathematics courses for pre-service middle and high school teachers. He has over 20 years of experience teaching Grades 7–12 mathematics. Ron's professional activities include consultations and conference presentations in North America, Asia, England, the Middle East, Africa, India, and Europe. Ron is an author for the NCTM (The Mathematical Lens) and member of the Advisory Board for the Museum of Mathematics in New York. He is the recipient of the 2015 Margaret Sinclair Memorial Award Recognizing Innovation and Excellence in Mathematics Education, awarded by the Fields Institute.

Before COVID, Pearson International Airport in Toronto felt like my second home. I have been fortunate to have travelled a great deal—usually over 100 000 miles a year. Some years ago, I reached the Million Mile milestone with Air Canada (Photograph 1) and am hoping to reach the 2 million mark someday. I am aware that my carbon footprint is large, but my travels have been important to my teaching practice at OISE/UT and to my growth as a teacher. I travel mostly to speak at conferences and for consultant work. This work has provided me with the opportunity to constantly develop new activities and to have teachers try out them out. Everything that I have done with my students at OISE has either been used by me when I was a classroom teacher, or by teachers I have worked with, who provide feedback on my material. Travel has made me a better teacher, it has connected me to teachers around the world, and it has moved me along on my journey to become a global citizen.

All my trips begin at Pearson International Airport. I have become very familiar with every corner of that airport. I love travelling, but I am always happy to come home and be greeted by family and the YYZ sign shown in Photograph 2.



Photograph 1: *Celebrating a million miles for math with Air Canada employees*



Photograph 2: *YYZ sign in arrivals area for passengers on US and international flights to Canada*



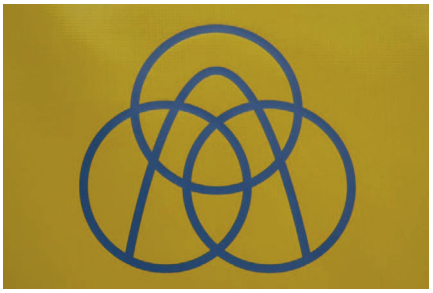
Photograph 3: *"LHR2YYZ" – personalized licence plate noticed at YYZ*



Photograph 4: *"DAVE YYZ" – personalized licence plate noticed at YYZ*



Photograph 5: *Walking speeds to gates for flights within Canada*



Photograph 6: *Logo for Thyssenkrupp, found on sign at YYZ, advising passengers that an escalator was closed for repairs*

Question 1

Photograph 2 shows the sign with three large letters that greets passengers who are arriving at the Pearson International Airport from the United States or other international destinations. As readers might know, YYZ is the airport code for Pearson International Airport, that is assigned by the International Air Transport Association (IATA).

(a) The IATA airport codes for airports located near the capitals of provinces and territories in Canada are given in Table 1. What do you notice about these codes?

Table 1: Airport Codes

Charlottetown Airport, Charlottetown, Prince Edward Island	YYG
Edmonton International Airport, Edmonton, Alberta	YEG
Fredericton International Airport, Fredericton, New Brunswick	YFC
Halifax International Airport, Halifax, Nova Scotia	YHZ
Iqaluit Airport, Iqaluit, Nunavut (Territories)	YFB
Jean Lesage International Airport, Quebec City, Quebec	YQB
Regina International Airport, Regina, Saskatchewan	YQR
St. John's International Airport, St. John's, Newfoundland and Labrador	YYT
Pearson International Airport, Toronto, Ontario	YYZ
Victoria International Airport, Victoria, British Columbia	YYJ
Whitehorse International Airport, Whitehorse, Yukon (Territories)	YXY
James Armstrong Richardson International Airport, Winnipeg, Manitoba	YWG
Yellowknife Airport Yellowknife, Northwest Territories	YZF

(b) Visit the website given below and read about how codes are assigned to airports. Search online for an example of an airport and its code for each of the eight ways mentioned on the website. Your examples must be different from those provided on the website.

8 Ways Airport Codes Are Chosen: Where These 3-Letter Acronyms Come From: www.flytrippers.com/how-airport-codes-are-chosen/

(c) Which of the eight ways describes how codes are assigned to airports in Canada?

(d) How many different codes can be made with three letters? How many airports are there in the world? Do you think that the IATA will run out of codes anytime soon? Why or why not?

Note: To obtain the number of airports in the world, you might want to think of this as a Fermi question, where you find an estimate for the total number.

Question 2

What do you think the people who drive a vehicle with the licence plates shown in Photographs 3 and 4 do for work? Where do you think they work?

Question 3

Suppose that you work for the company that designed the YYZ sign for Pearson International Airport. In the early stages of the design process, you have been asked to take over the design at the stage shown in Figure 1, where the first Y has been laid out along with the base of the second Y and the Z.

(a) Finish the design by adding the second Y and the Z. Determine the coordinates of all points that you add to make the letters.

(b) Determine the area of the Y shown in Figure 1 and the area of your letter Z. Are the areas the same? If not, how would you change the coordinates of your Z to make the areas the same?

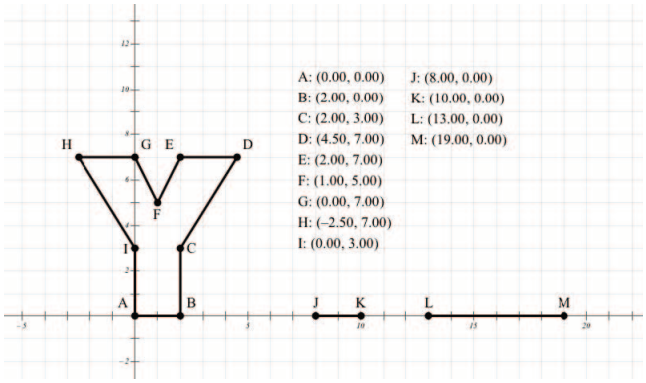


Figure 1: *Design of letters on a grid in progress*

Question 4

Photograph 5 shows the walking times from a central location in the Canadian departures area of YYZ to six different locations near various gates.

(a) Suppose you work for a company that has been hired to verify these walking times and to put up similar signs in the US departures area of the airport. What data would you need and how would you collect it?

(b) Figure 2 shows the walking paths from the central location to the six areas. This figure was constructed on top of Photograph 5, using the Geometer's Sketchpad®. Measure the six walking paths with a ruler. Are these lengths consistent with the times given in Photograph 5? Why or why not?

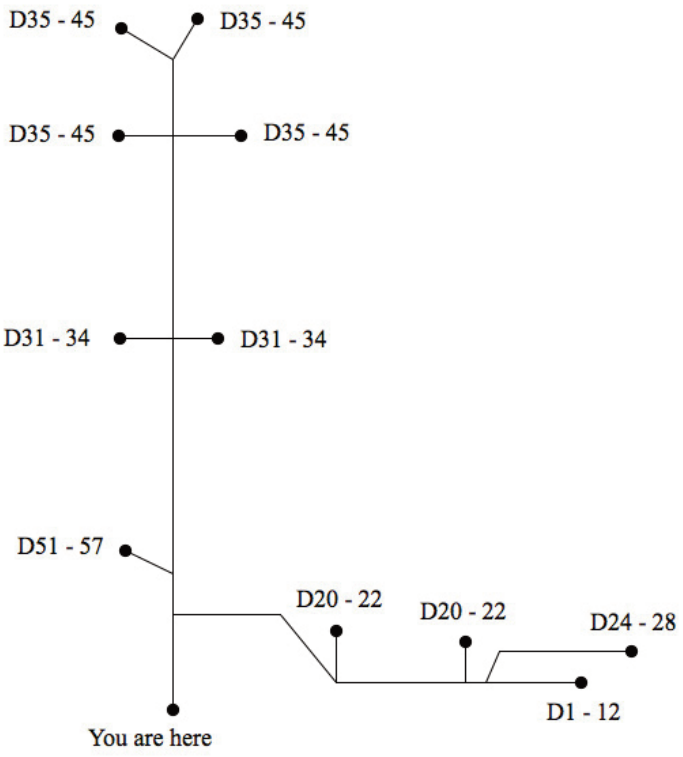


Figure 2: *Walking paths from central location to various locations*

Question 5

Photograph 6 shows the logo of Thyssenkrupp, a company that installs and repairs escalators at YYZ. Suppose that you work for a company that has been hired to create a digital version of the logo. Figure 3 shows the work that has been done so far when you take over the task. The circle whose centre is at A1 passes through the points A2 and A3, the circle whose centre is at B1 passes through the points B2 and B3, and the circle whose centre is at C1 passes through the points C2 and C3. A parabola is to be used for the curve, and it passes through the points D1, D2, and D3.

(a) Determine equations for the three circles and the parabola.

(b) The logo actually consists of three additional circles and a second parabola. The complete logo is formed from the regions between the circles and parabolas. What transformations could be used to find the equations of the three additional circles and the second parabola?

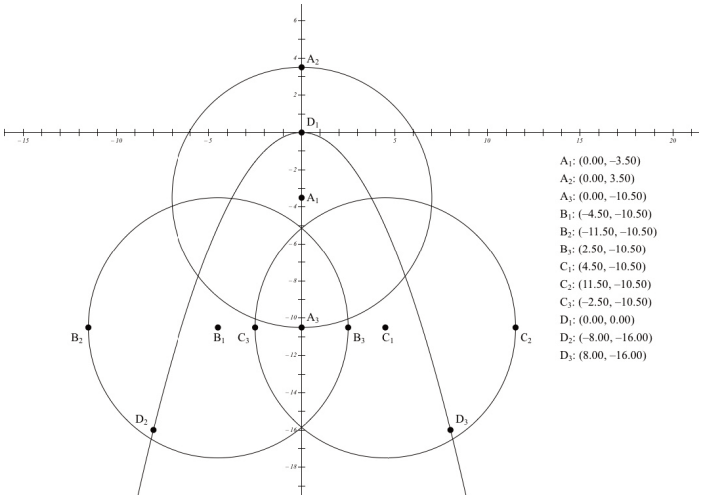


Figure 3: *Construction of the logo*

I am grateful that I have had opportunities around the world to work with teachers, and that my position at OISE/UT has allowed me to travel. As an Associate Professor, I have been expected to do creative professional work that has been up to me to define. I have centred my work on conference presentations, visiting educator positions, and writing articles for publications such as this one. I wish that it were easier for classroom teachers to attend conferences, to visit other schools, and to develop a professional learning community outside of their school. Virtual conferences are helping, but it is not the same as a face-to-face experience. I mentioned the importance of travel earlier. For those who are not able to travel, it is possible to see the world by having meals in restaurants that serve foods from around the world, by seeing international films at a local cinema or at a film festival, by reading newspapers such as the *New York Times*, by listening to public radio stations from around the world, and so on. These things can bring the world to you. ▲

Print version of the *Gazette*?

Photos in Ron's column, among others, have better resolution in the digital version of the *Gazette*. The digital (PDF) version in the members-only area can be enlarged to assist with printing them for classroom use.

▲ In the Middle: Makeup, Emojis, and Algebra



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Carly Ziniuk teaches Grade 9 Mathematics, Grade 12 Data Management, and Advanced Placement Statistics at the Bishop Strachan School in Toronto, Ontario, Canada. She is very active in adopting real-life data to engage her students in solving problems.

A few years ago, three of my students burst into our classroom, telling me, "Our life last night was a math problem. NO JOKE! We got back from Sephora and we literally spent the rest of the night doing algebra with makeup! Not for school, but for actual life!"

Some students remembered the Jon Scieska classic children's book *Math Curse*, where a child is "cursed" by finding math problems in every part of their life. These three students explained their Sephora problem to our class, drawing makeup items instead of variables. We extended this to emojis, and that works much better in this column, instead of the "makeup" manipulatives. As these students are graduating from high school this year, I have been remembering how wonderful it was to see my students and their friends laughing over an algebra problem of their own creation.

Here's the situation as explained to me by my students:

> The Sephora Tax-Free Minis sale allowed you to purchase two different mini-items at a significant discount, with a special decorative promotional bag. At the sale, Jaq bought a Mini-Sephora Waterproof Eye Makeup Remover and a Fresh Soy Face Mini-Cleanser for $34. Hale bought the same Fresh Soy Face Mini-Cleanser and a Smashbox Foundation Primer Mini for $38. Karri liked their purchases and chose the Smashbox Foundation Primer Mini and the Mini-Sephora Waterproof Eye Makeup Remover for $32.

When they got home, they actually decided to redistribute the items so that each of them could get two of the same items, their favourites. At that point, they realized that perhaps it wasn't fair because they hadn't all paid the same amount of money.

How much should each new bag cost? Their real-life math problem!

This problem reminded me of the Gone Fishing Problem #58, presented by *Figure This!: Math Challenges for Families,* by NCTM (https://figurethis.nctm.org). If you have not explored the *Figure This!* collection, check it out! You can assign these online challenges automatically now, using Google Classroom or Canvas.



I have used the Gone Fishing Challenge here with younger students (Grade 4–6), who used guess-and-check to find the solution. Since the numbers were small and integral, it was relatively simple to reason by balancing and redistributing. With this smaller problem, students found a solution that worked for two of the scales, but not the third. Considering then, if the weight of one of the fish goes up by one, the weight of each of the other fish has to each go down by one. Since the fish are sitting on a scale, it extends well to the equation-as-balance thinking. Another way many students solved it was by piling all of the fish onto one scale, then realizing that was just two of every kind of fish.

The Sephora problem, as my students named it, was the Gone Fishing Challenge with higher constants. As we discussed the problem with the actual purchasers, the students explained that they had solved the problem by first putting all the fish into one big pile. The fish on one scale represented the makeup dumped out of the bags and piled together. The students then rearranged the items and divided them into two even piles. The students kept track of the changing prices as they moved the items into different groupings with sticky notes.

Jaq	Hale	Karri
34	38	32

Although the students used the items as manipulatives,

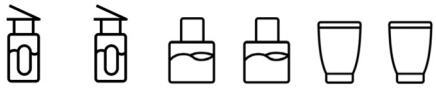
together we made a similar visual here to illustrate their real-life math thinking with all icons from The Noun Project: Andi Nur Abdillah (see www.thenounproject.com/darthpier/collection/beauty-hack-outline/).

All together



34 + 38 + 32 = 104

Then rearranged into matching items



Still 104

Then divided into two even groups



104 ÷ 2 = 52 | 104 ÷ 2 = 52

BUT each student's original purchases are included in that group of three



Jac's purchase + extra primer
34 + Primer = 52

Extra Remover + Hale's purchase
Remover + 38 = 52

Which means we can figure out what is left over in that overall purchase



Primer = 52 – 34
Primer costs $18

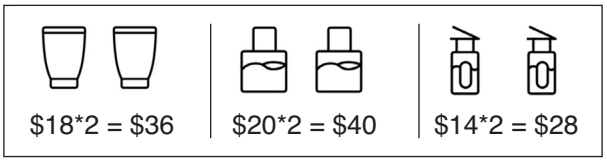
Remover + 38 = 52
Remover costs $14

Figure out how much the Cleanser is by using Hale's original purchase



Cleanser + 18 = 38
Cleanser costs $20

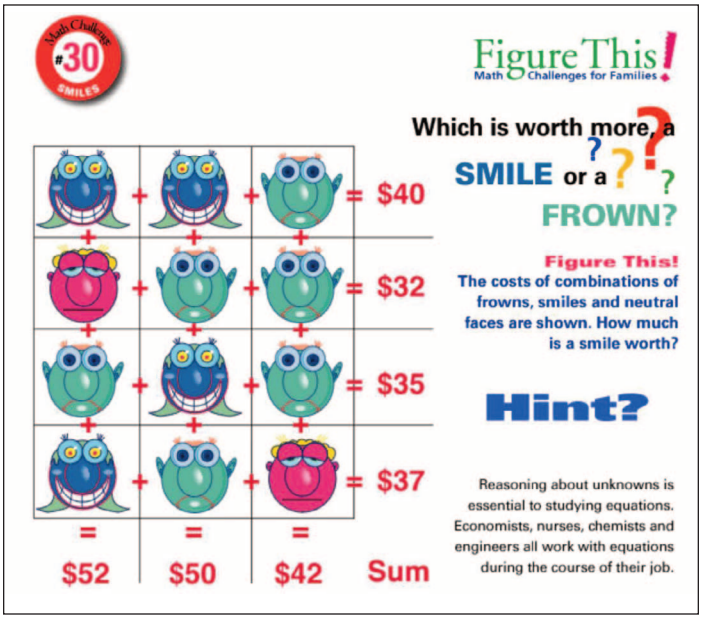
Each new bag then costs each item doubled!



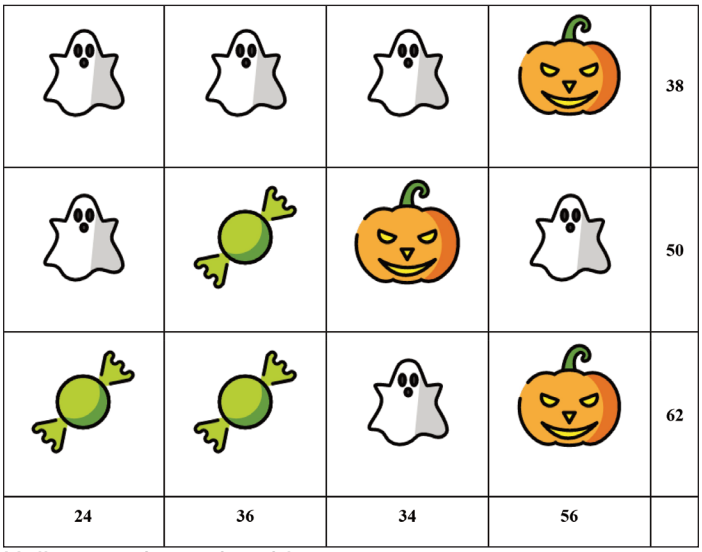
$18*2 = $36 | $20*2 = $40 | $14*2 = $28

The students showed me how they had actually used the items and realized what they had done was algebra, instead of with variables, but with actual Sephora manipulatives and sticky notes. When we made the visual here, we discussed how the mini-images were replacements for variables. As they discussed how emojis had replaced words and even sentences in their lives, these icons here were now replacing variables too! I asked the class to consider the Sephora problem with algebra, using the exact same steps the students showed with emojis. It made a clear indication of the steps and equations.

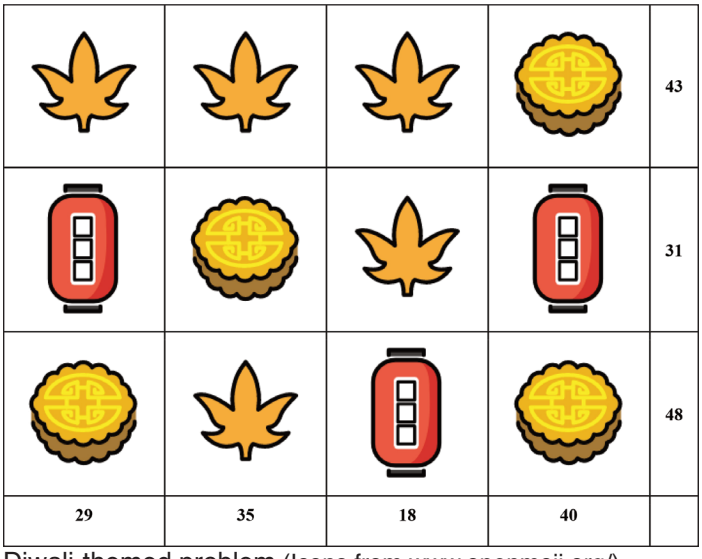
I also asked the students to think of different ways they could reason it. Other students argued that the manipulative or emoji approach the students took, even with algebra, was not the most efficient. In many ways, they were arguing that reasoning could be faster than algebra. When considering alternate approaches, I heard comments like, "I'll pick a reasonable cost for the first item, then figure out what the second and third one should be based on that choice. Once I see what happens during that process, I know how close my first guess was." Many students love adjusting their intuition with real-life prices of items. This approach suggests a good translation to coding and numerical solving, rather than algebraic reasoning. Some students used a table to illustrate their thinking, but could not figure out how to show it on Desmos. I saw their eyes widen when they heard me explain that they would need 3D graphing!



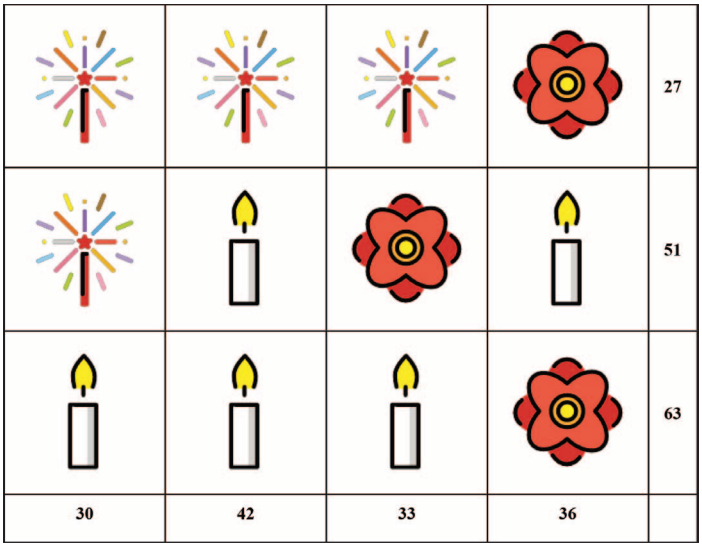
The similar challenge from *Figure This!*, Smiles #30, includes another image as a variable problem. Once I remembered this challenge, I realized I could have students make their own! The students made their own challenges for each other around different holidays and celebrations, using emojis as variables. We discussed how much easier it was to build the answers than it was to solve them. Another question we posed was whether any example they could make could be unsolved with this framework. What dimensions below would make it unsolvable? How many different emojis? Enjoy some of our examples as challenge problems with your students too!



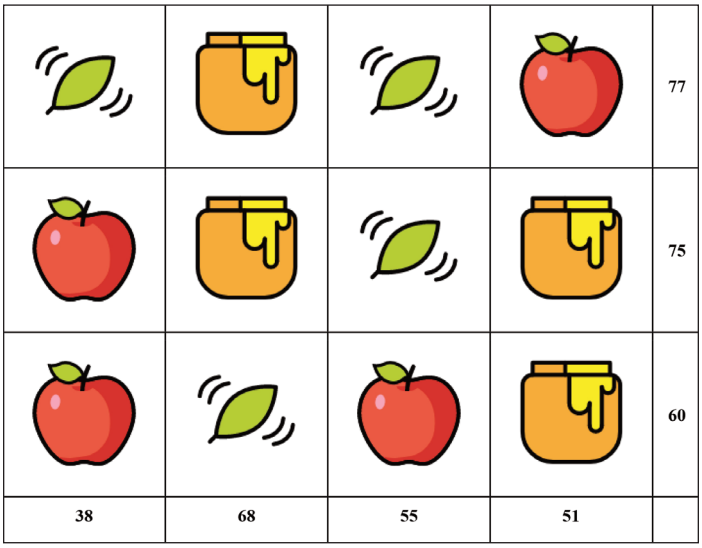
Halloween-themed problem (Icons from www.openmoji.org/)



Diwali-themed problem (Icons from www.openmoji.org/)



Jewish New Year-themed problem (Icons from www.openmoji.org/)



Fall-themed problem (Icons from www.openmoji.org/) ▲

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▲ MB4T (Mathematics by and for Teachers): Order of Operations



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Jennifer Holm is an Assistant Professor at Wilfrid Laurier University and works with Primary/Junior and Junior/Intermediate pre-service teachers, as well as in the field supporting current mathematics teachers. She is interested in developing mathematics knowledge for teaching with both pre-service and in-service teachers. She focuses on the beliefs and opinions that pre-service teachers hold about mathematics and teaching and the connection they have to past experiences. She uses this research to support future teachers in developing beliefs and knowledge that will encourage and support effective teaching practices in mathematics.

In working with future teachers, I often notice some different mathematical habits that make me wonder how we might think about teaching mathematics differently to support learners. One such question I had was related to having pre-service teachers explain why 3 – (–4) is equal to 7. I like to give questions like this early in the semester to make a point about the difference between being a mathematics student and being a mathematics teacher. Knowing that 3 – (–4) = 7 is something that any mathematics student should be able to answer, but this is not enough for a mathematics teacher. A mathematics teacher needs to understand why this answer makes sense and how to help their students see the answer. This is the reason for this column: mathematics for teaching is more complicated than just knowing how to answer mathematics questions.

This question leads into some interesting discussions about integers, which I will save for a later column, but the response to why that I hear quite often is "because of BEDMAS." Initially this statement surprised me, but when pressed on what is meant, the response is usually some variation of, "Well, you have to do the brackets first," so teacher candidates consider –1 × –4 as the first part of the number sentence. This would make the sentence as 3 + 4 afterwards. It was this curiosity about the use of "BEDMAS" that had me recall an article in a National Council of Teachers of Mathematics journal by Ameis (2011) that discussed the order of operations. His article led me to consider another way of teaching the order of operations, which is the focus of this column.

For most mathematics concepts, we would agree that learning why a mathematics concept works through building understandings with models helps to solidify that understanding. When I think about how I learned about the order of operations and how I have heard it talked about in classrooms, I often hear the term "BEDMAS" (or Brackets, Exponents, Division, Multiplication, Addition, Subtraction) to show the order of operations used, and that is the end of it. As I have discussed in previous columns, there can be a danger in giving rules before understanding with students. When I think about my adult learners, I can also see that the rigidity of this "BEDMAS" rule leads to a feeling that there is a right way to answer questions, and they lose sight (or have never known) the purpose of the order of operations. In my pre-service classrooms, I enjoyed using an example inspired by Ameis (2011), where you can violate "BEDMAS" if you pay attention to the relationships between the numbers. You can read about the example in the cited article, or consider the following: $8 – 3 + 2 \times 8 + 7(6 – 4) – 8 + 3 \times 4^2 \div (2 \times 2) + 6 – 8 + 5$. How would you solve this question, using the order of operations as you were taught?

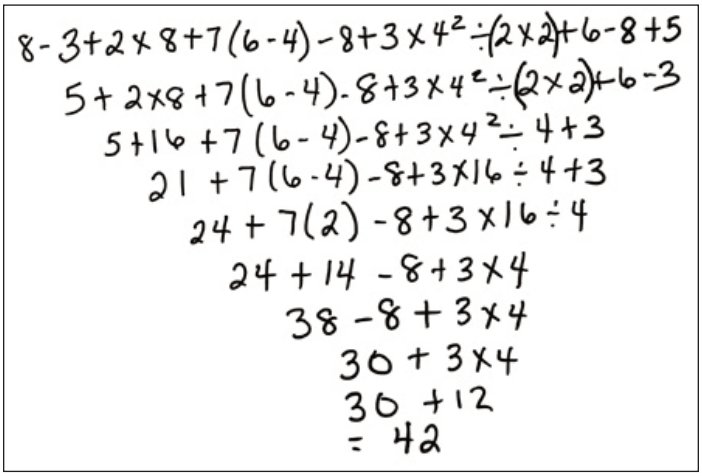


Figure 1: *One method for solving a number sentence*

Consider the method used in Figure 1. Why does this work? It most certainly does not follow the rules of "BEDMAS" exactly. The reason it works is that when you pay attention to the relationships between the numbers, you do not have to follow "BEDMAS." In his article, Ameis (2011) also suggests using a triangle to create an order of operations hierarchy instead of the "BEDMAS" rule and to build this with students. For the remainder of this column, we consider how to use what students know about operations to build a hierarchy, and ways to create situations to help students understand why there might be a particular order to follow. Oftentimes when teaching order of operations, we start by giving number sentences and then

teach the rules. In this column, I advocate using multiple-step word problems to discuss why a certain action needs to happen before another. I am also using a ladder shape so that each step is a next rung down the ladder.

First, we are going to start with the bottom layer of the hierarchy. Consider a word problem such as the following: A bowl of fruit contains 7 pieces of fruit. April adds 2 pieces of fruit, and Devon removes 6 pieces of fruit. How much fruit is in the bowl? Students can draw the actions of the problem and consider, "Does it matter whether we remove the fruit or add the fruit first?" Consider the ways of thinking about the problem in Figure 2. By examining this, it is clear that it does not matter if you add or subtract first, or even what order you put the numbers in the number sentence. If we refer to "BEDMAS," it shows that you need to add before you subtract. By giving some opportunities to play with the numbers in a word problem, students can consider what happens with the numbers and learn more about the relationships. We can now see that addition and subtraction have an equal footing when considering the order of operations because no matter the order, you would arrive at the same solution.

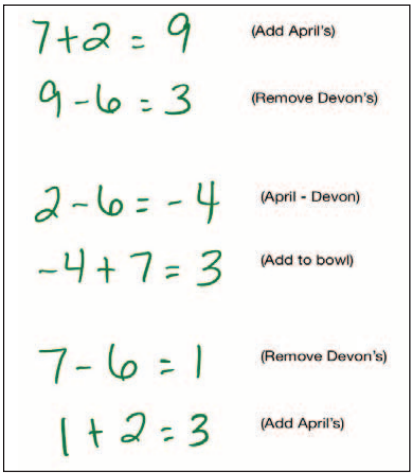


Figure 2: *Solving the bowl of fruit word problem*

If we move on to consider something more complicated, consider the following word problem: Anne is picking apples from 3 apple trees in her garden. If she picks 4 apples from each tree and 5 apples from the ground, how many apples does she have? Figure 3 looks at different ways to consider using the numbers in the problem. Based on looking at these ways, there is a hierarchy between multiplication and addition. Unlike the problem presented in the previous example, there are different answers, depending on the order in which we put the numbers and how we tackle the operations. This is where we can start conversations with students about the purpose behind the "order of operations." The bonus of using word problems allows students to draw out the problem and see what make the most sense in this case. If we look back at the problem and consider what is being said, multiplication has to happen before the addition, as we need to see how many apples are on the trees, and then combine that with the number of apples on the ground. Multiplication would go on the next order level, above addition and subtraction, since it would need to go first in a number sentence such as this one.

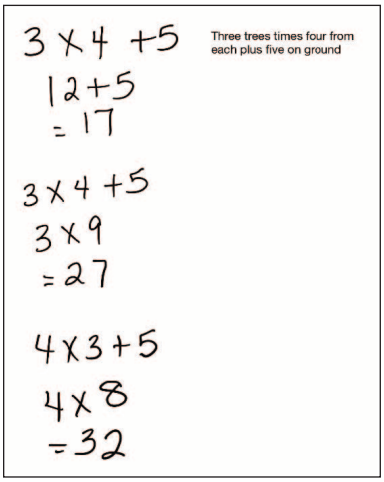


Figure 3: *Applying different orders to a number sentence*

Now consider the following problem: Mondo is packing treat bags for his daughter's sleepover. He needs to divide the cookies evenly into 4 bags so that each child gets the same number. If he buys 2 containers with 12 cookies in each container, how many cookies will each child receive? Figure 4 shows different ways to consider how to work with the numbers in the word problem. Clearly, it does not matter whether you divide or multiply first. If you think about the word problem as written, any of these ways to solve the problem could be correct number sentences. Students could draw pictures or use manipulatives to show that each of these interpretations reveals the correct answer of 6. You could work through other examples with students to show that the order of multiplication and division does not matter, which means they should be placed on the same level of the hierarchy. Note that there is an ambiguity with the divide sign notation in cases like $8 \div 2 \times 2$, which could be avoided as either $(8 \div 2)$ or $8 \div (2 \times 2)$.

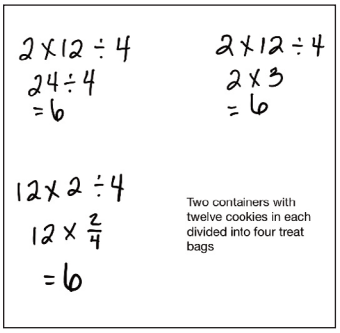


Figure 4: *Using different orders to solve the cookie problem*

At this point, Figure 5 shows the hierarchy that has been created so far. Since we have established that multiplication and division need to be considered before addition and subtraction, we can add another layer of complexity to the discussion.

Consider the following word problem: Traeyvon is creating a tower with sugar cubes. He creates a square, using a side length of 5 cubes. He then puts 3 identical square layers (for a total of 4) on top. How many sugar cubes did he use to make the shape? Using an exponent to represent the "square," Figure 6 shows the number sentence to solve the problem, and two different ways to work with the operations in the problem. A different

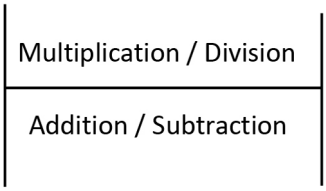


Figure 5: *Hierarchy considering the four basic operations*

answer is achieved, depending on the order in which you deal with the exponent or power. If you multiply the two numbers first or use the exponent first, the answers are quite different. When you return to the word problem and draw it out, the correct answer would be the second one, with 100 sugar cubes. Additionally, the definition of a square $5^2 = 5 \times 5$ implies that $4 \times 5^2 = 4 \times 5 \times 5$. This shows that exponents need to take precedence over multiplication and are above multiplication and division. You could use different examples with students in order for them to try and see if there is any difference when you include addition, subtraction, or division with exponents and how the answers change. Again, tying the ideas to word problems can help students see the reason there is a correct answer and standardized order.

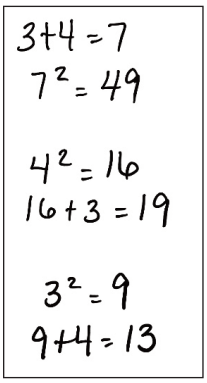


Figure 6: *Examining square numbers*

At this point, the hierarchy is complete, based on the article by Ameis (2011). You may be wondering at this point, since there is another letter to consider in "BEDMAS": that of Brackets. What happens if we consider the following problem: Everly is making 5 art projects for her friends. She puts 3 stickers on each of the projects. She has stickers left over, so she adds another 2 stickers to each. How many stickers did Everly use? If we consider our hierarchy, it would seem that we need to multiply first saying 5×3 first and then adding 2, but if we look at our problem, something different is happening. In this case, we have added brackets to our number sentence, making the questions $5 \times (3 + 2)$. Figure 7 shows different ways of considering the numbers from the problem, showing that there is a correct method for considering the brackets, since we can get two different answers, depending on what we do first. Returning to the word problem, you could rewrite this problem as $5 \times 3 + 5 \times 2$. Ameis (2011) suggests leaving brackets off the hierarchy, since they are not operations; instead, they are "containers" (p. 419), and could be left for the last. In working with elementary students, it is important to understand how the brackets are being used and why—to figure out where they go in the order of operations. By rewriting the number sentence based on the word problem in this example, it is clear both numbers that are added need to be multiplied by 5 to get the correct answer.

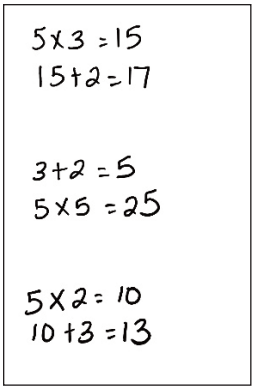


Figure 7: *Using the numbers in the sticker word problem*

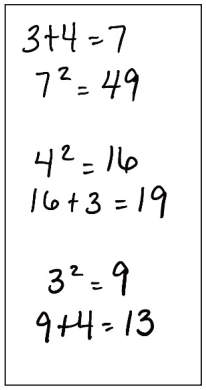


Figure 8: *Breaking $(3 + 4)^2$ into different formats to see the effect of order of operations*

If we consider exponents with brackets, as in Figure 8, we see something different, which shows that the numbers inside the brackets need to be added together before applying the exponents for an example of $(3 + 4)^2$; otherwise, we get two different answers. Again, considering word problems would be helpful to see the connections. In this case, we are adding 3 and 4 before we square the number, so the correct solution would be to add first. If we consider a question like $(2 + 3) – 6$, then the brackets really serve no purpose, and removing them would give the same answer no matter how we solved the number sentence. The removal of brackets is the same with a number sentence that only has multiplication and division. Having students understand the purpose of brackets through different experiences and word problems so that they see how they work within the number sentence helps to increase their understanding. My suggestion is to put the brackets at the top of the hierarchical ladder so that elementary students consider their purpose first and then can solve the number sentences.

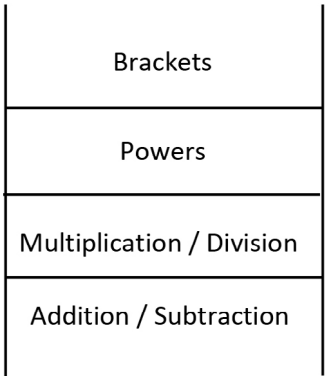


Figure 9: *Final order-of-operations hierarchy*

In summary, Figure 9 shows the final order-of-operations hierarchy proposed in this column. To use the hierarchy, you would start at the top and work down the ladder, checking for each operation at each stage in the number sentence. Operations higher up the hierarchy need to go first. Now, if we return to the example in Figure 1, how else could you solve this without following BEDMAS directly? Since there is no word problem attached to this string of numbers, we must consider how each of the numbers relates to the other ones. If you look at how it was solved, in

Figure 1, the brackets, exponents, and the multiplication and division questions were all left intact so that we could add or subtract first. For example, the 3 + 2 at the start of the number sentence could not be added first because the 2 needed to be multiplied by 8. Keeping this intact means that we preserve the multiplication question in the order of operations. If we considered the hierarchy discussed in this column, Figure 10 shows how you might solve the number sentence using it. In the end, the most important consideration is helping students understand why the rules are in place, since it does make a difference in the answers when looking at number sentences. "BEDMAS" or the hierarchy can be helpful for remembering the "rules," but teaching rules without understanding can lead to misinterpretations, misuse, or even forgetting. Since the goal is to help students know and remember mathematics, spending time developing an understanding can be an incredible payoff for the future of students.

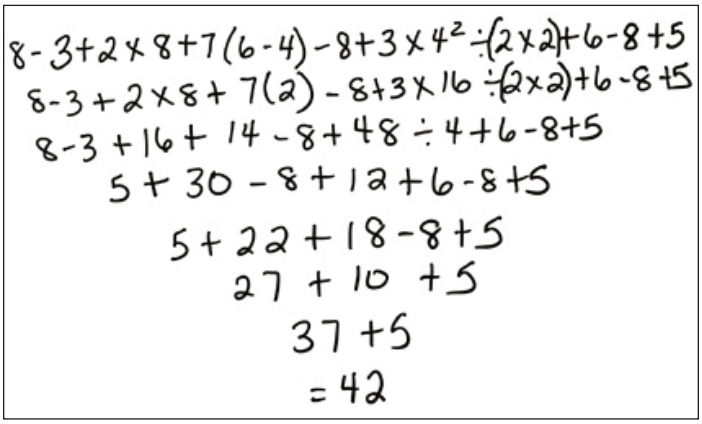


Figure 10: *Solving the number sentence from Figure 1, using the order-of-operations hierarchy*

Reference

Ameis, J.A. (2011). The truth about PEDMAS. *Mathematics Teaching in the Middle School, 16*(7), 414–420. ▲

Match the Equation!

Which equation belongs to which graph?

$x^3 + y^3 = 5$

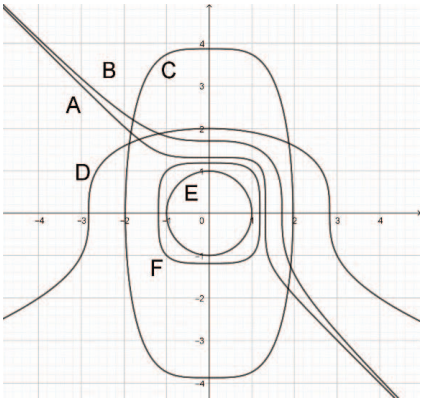
$x^4 + y^4 = 2$

$x^5 + y^5 = 4$

$x^2 + y^2 = 1$

$x^2 + y^3 = 8$

$x^4 + y^2 = 15$



▲ Some Thoughts about Perimeter: Units of Measurement



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In the last column, some conceptual problems related to perimeter and the frequent insistence on calculating a figure's sides were introduced. It presented one possible effect, through Marie and Marco's tendency to keep the figures' sides rigid, with little or no malleability in their consideration of perimeter. This implicitly raised elements concerning the ambiguous status of perimeter as a one-dimensional property of a two-dimensional figure. It showed the need for students to understand the dynamic unfolding of the perimeter to create a line segment, and in return, its refolding to create figures, uncovering a dual one- and two-dimensional interplay sitting at the core of the nature of perimeter.

This dual one- and two-dimensional nature of perimeter can also be seen to play a role in relation to the standard unit selected to measure perimeters. Indeed, the choice of units for measuring the surface or boundary of a figure creates a number of issues. That the perimeter offers a measure in terms of units, and the area in terms of square units, is not always clear from the context of the task at hand. On different images of figures, the thickness of the boundary of the figure can present difficulties in determining what is to be considered for the perimeter and for the area, or simply blurs their difference (Moreira Baltar & Comiti, 1994). As well, when a figure is given on a grid, the perimeter is at times thought to be the surrounding squares (Vighi & Marchini, 2011).

Issues with units of measurement can also take place in relation to what represents the unit with which to measure; for example, 1 cm^2 is often associated with a small square

with a side of 1 cm, used as the referent (Hart & Booth, 1984); where students experience difficulties in associating 1 cm^2 with other referents for measuring, like a rectangle with sides of 2 cm and of ½ cm, a triangle with 4 cm of base and ½ cm of height, etc. Additionally, Douady and Perrin-Glorian (1989) have shown that for a number of students, this association can even lead them to erroneously associate, for example, ½ cm^2 with a square of side lengths of ½ cm. It is this particular issue that is at stake here in this second column, concerning the referent used as the unit for measuring the perimeter.

The Task and the Unit of Reference

In one task, Marco and Marie were asked to first imagine a rectangle, then, to imagine another rectangle, for which the perimeter was exactly doubled. This task was solved easily by Marco and Marie at first, but when it came to explaining how their new rectangle had double the perimeter, things became more difficult.

At first, Marco drew his initial rectangle, then doubled it and doubled this double (Figure 1). He then had a grid of 2 by 2 with four rectangles in it, which gives a big rectangle with four times the area, but indeed twice the perimeter.

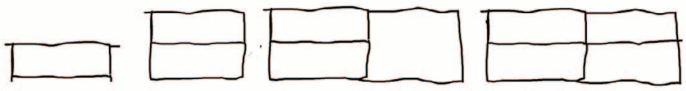


Figure 1: *Marco's steps to double the perimeter of the initial rectangle*

When trying to explain how this new rectangle had a perimeter double the initial perimeter, Marco became confused, seeing the four rectangles, and associated the perimeter with every line of his last drawing. Counting the lines for each rectangle, he concluded that finally, this rectangle had four times the perimeter of the initial rectangle, as well as four times the area (Figure 2).

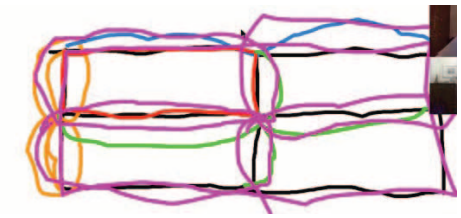


Figure 2: *Marco's subsequent confusion while counting the "lines" inside the resulting figure*

He then suggested that having only two rectangles would make a new rectangle that was double in terms of its perimeter (here was a glimpse of a mix-up between doubling the area, implying doubling the perimeter). In this case, even by giving specific values to the figure's sides (Figure 3), Marco concluded that the perimeter of the first rectangle being 8, the second was then the double, since there were two rectangles with their own lines, giving a perimeter of 8 for each.

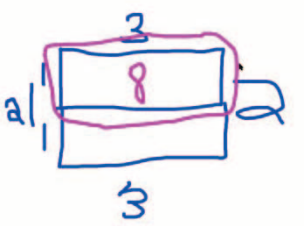


Figure 3: *Values added by Marco on a possible doubling the perimeter figure*

As for Marie, she explained how she also struggled with the task. She explained that while attempting to find a rectangle with double the perimeter, she became trapped with the many lines in her proposition. Because her initial rectangle (on the upper left side of Figure 4) had two long sides, she tried to double it and obtained four of those. She then realized that this would create too large a rectangle and stopped her attempt, saying that it would not work and that she did not know how to continue on with the task. She expressed that "the perimeter comes with its rectangle," to explain how she did not see how it was possible to construct such a rectangle with double the perimeter because too many lines are added to it and intersect one another.

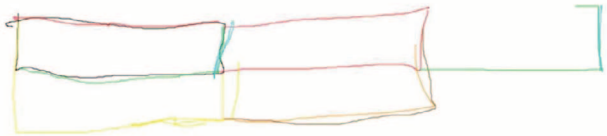


Figure 4: *Marie's "doubling the rectangle" solution* (The initial rectangle is on the upper left side.)

In both Marco and Marie's cases, the perimeter of the rectangle was not really explained as a length, but felt more as the rectangle itself, with all its sides directly coming into play (and being tangled up with the rectangle's area). It was the rectangle itself that had to be duplicated for the perimeter to double, as it represented or embodied its perimeter: they did not seem to be able to separate the perimeter from the shape of the figure itself. In that sense, the preservation of the shape of the rectangle was not only an important attribute of the transformation, but was central to its resolution.

This view became more explicit with another of Marco's strategy to find the perimeter of a punctured square (taken from Danielson, 2005). To solve this task, Marco mistakenly found the area, using one small square (on the left-hand side) to explain that there were 32 of them in the punctured square (placing "1's" in each square).

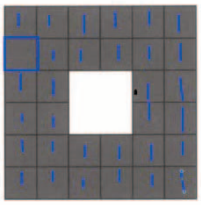


Figure 5: *Marco's strategy to count the area of the punctured square*

Marco quickly realized that his answer was about area, and then started to count the perimeter. He determined the perimeter by going around the figure and counting "lines" all the way around (Figure 6).

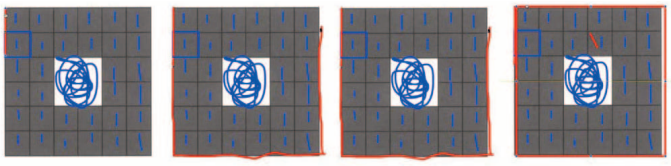


Figure 6: *Marco's counting process for the perimeter of the punctured square*

As he was creating his red lines, he said: "1, 2, 3, 4, 5, 6 lines, 6 lines again, 6 lines again, again 6 lines. This gives a square of 6 x 6. So it is 6 small squares because it is 4 times the lines." Questioned as to what he meant, he replied, "A small square has 4 lines," while drawing the four lines around one of the squares (in the middle area of the square, above the hole in Figure 7).

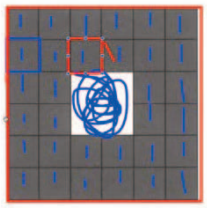


Figure 7: *Marco's selection of a square as the unit for measuring the perimeter*

Marco then added, "With these lines, you have 1, 2, 3, 4; that's one small square, and you have that six times in total." Marco here referred explicitly, and successfully, to the perimeter of one small unit square as the referent to measure up the perimeter of the entire large square. When questioned about the centre "hole" piece in the punctured square, he said, "Oh, yes," and went on to do the same thing: "1, 2, 3, 4, 5, 6, 7, 8, so it is again two small squares." To explain why he would also count the centre piece for the perimeter, he replied, "Because they are also lines," showing how the interplay of one-dimensional lines and two-dimensional figures were used simultaneously to consider the perimeter.

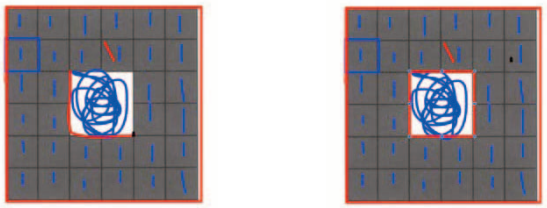


Figure 8: *Marco's counting of the "centre" for the perimeter*

Marie also referred to the perimeter of a square as a measuring unit when confronted with another task. This time, it was about a task, inspired by Barnes and Kieren's (1995) work on fractals, where the question was first to find the maximum possible area and perimeter for the "lake" on the square grid (Figure 9).

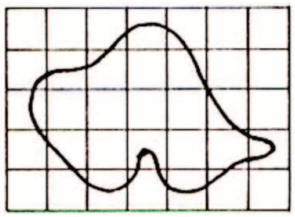


Figure 9: *What is the maximum area of the lake? What is the maximum perimeter?*

To find the maximal area, Marie first counted all the full squares inside the figure. Then, to elaborate, she explained that one could count up each square that touches the lake. In other words, each square through which the curve passes amounts to an additional square to count for maximizing the possible area for the lake.

When then asked about the maximum perimeter for the lake, she looked puzzled. She suggested doing the same thing with area; that is, each time the border touches a square, one could take all four sides into account: "If all small bits of squares were counted, as one full square of perimeter, then all the small squares around, if we unfold them, would give the maximum perimeter" (Figure 10).

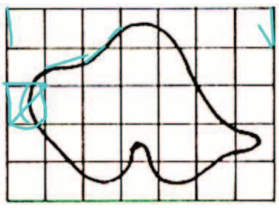


Figure 10: *Marie's way of maximizing the perimeter of the lake*

Here again, the perimeter of a small square is used by Marie as a referent to obtain an assessment of the perimeter of the figure.

Some Thoughts about Perimeter, Units, and Attachments to Two Dimensions

All these solutions about the perimeter of a figure, used as the referent to measure another perimeter, appear analogous to the previous case of students linking an area of 1 cm^2 to a square with 1 cm side lengths, although it did not always create explicit difficulties or wrong solutions. These ways of solving raise the dual nature of the perimeter as a linear one-dimensional property of a two-dimensional figure, where the two-dimensional figure itself is used for considering perimeters: Marco and Marie did not use a line segment to measure the perimeter, but rather, two-dimensional figures like a square or a rectangle. For the last task about the lake, it is not the form of the lake that had an impact, but mostly that the perimeter was considered in relation to a two-dimensional object, here a square that carries with it a certain perimeter that acts as a referent. The perimeter in this case seems strongly attached to a two-dimensional object in order to consider its measure.

The reference to the perimeter of another figure for the unit of measurement points to this dual nature of perimeter as a linear (one-dimensional) property of a (two-dimensional) figure. It points to how attached to figures, hence two-dimensional objects, the consideration of perimeter can be. Lengths, like segments, sides, etc., are measured with rulers, tapes, sticks, etc., that are also length in themselves. However, because the perimeter of a figure is represented on, and constituted by, the two-dimensional object to measure, this blurs the situation. Any unit can obviously be used to measure the perimeter, from a standard centimetre ruler to an inch ruler, to a pencil length to… another perimeter used as a referent unit!

While at times successful, Marco and Marie's ways of referring to a square or rectangle as the unit for measuring perimeter continued to reveal this interplay between one and two dimensions inherent in the nature of perimeter. It appears again to be this dynamic interplay between one and two dimensions that is called for when measuring perimeter: be it whatever unit one works with, measuring the perimeter requires first to unfold the sides of the figure along a one-dimensional segment for then measuring it (as mentioned in the previous column). And when this unfolding appears unconsidered and the perimeter unit is kept as a fixed two-dimensional figure, the perimeter can be more difficult to assess.

As was mentioned in the previous column, this one- and two-dimension interplay, inherent in the nature of perimeter, appears to hold the key to how perimeter can be fruitfully and adequately handled. This also illustrates how perimeter calls beyond the simple calculations of values of sides, and into issues of measurement of a length constituting a figure. Building on these first two columns, the third column will raise questions relative to the conceptualization of perimeter as the "distance around the figure."

Note:

[1] The image was taken from www.warmaths.fr/SCIENCES/unites/AIRES%20SUR/AirSufCurvilign.htm.

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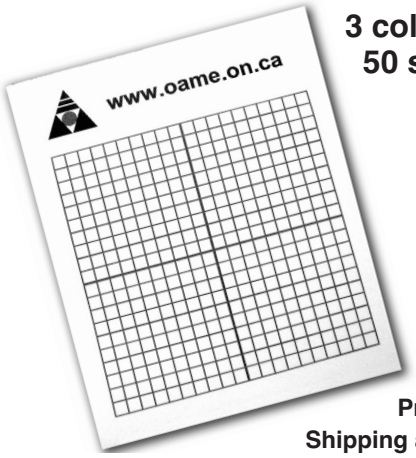
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▲ Elementary Math Matters: The Peacock's Solution to the Mysterious Case of the Disappearing Text in the Summertime



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Lynda Colgan is Professor Emerita, Faculty of Education, Queen's University. Lynda is currently working on a number of research, resource-creation, and teacher-education projects, funded by the Natural Sciences and Engineering Research Council (NSERC), the Social Sciences and Humanities Research Council of Canada (SSHRC), and the Mathematics Knowledge Network (MKN). She will carry on her long-established commitment to STEM outreach by continuing to seek out creative avenues to engage students, parents, and educators in mathematics at home and in the classroom.

> To be effective in building up a civilization that rejects inequity, arrogance, and bigotry, education must give special attention to the redemption of peoples who have been for a long time subordinated, and must give priority to the empowerment of the excluded sectors of societies (D'Ambrosio, 2007, p. 179).

"Summertime… and the livin' is easy." At least that's what jazz great, Billie Holiday, said back in 1936. No matter how much Ontario educators may have wished those would be the words to capture the spirit of July and August of 2021, for many, nothing could have been further from the truth. Summertime 2021 was, for countless educators, the summer of their discontent… a time when unhappiness was, in many ways, at its highest. ("Now is the winter of our discontent" is a line from *Richard III*, by Shakespeare. This means that the time of unhappiness will soon end.)

Why? Ontario's Ministry of Education announced its reformed Grade 9 mathematics curriculum policy document in June—or rather, it air-dropped the *written* curriculum for a new destreamed program that is compulsory for all first-year high school students, except those in locally developed programs.

At the time, I reviewed the document for an OpEd (Colgan, 2021) that I was writing for the *Ottawa Citizen* newspaper, and I found that it was impossible to find fault with the introduction to the document. It spoke to inclusivity, equity, diversity, and indigeneity—respecting the global roots of mathematics, different ways of knowing and showing understanding, and the need for positive attitudinal and strong conceptual development. The front matter acknowledged the ubiquity and applicability of mathematics, underscoring its role in nature, art, sports, commerce, and science. The content was described as a continuation of the mathematics learning trajectory outlined in the 2020 Grades 1–8 curriculum, building on the knowledge, skills, and strategies that are fundamental to number systems, algebra, and financial literacy. Included as well were coding (computer programming) and data literacy—critically assessing the graphs, tables, and charts used for communication purposes.

Nor could I find fault with the philosophy underlying the destreaming ideal—i.e., removing barriers for students who were routinely channeled into course paths that were not consistent with their goals. Grade 9 "Applied" students were often deemed to be "underachieving," for academic, attitudinal, or other reasons, and intentionally directed away from courses that opened more doors. In many cases, disengagement led to dropping out of school or skipping Grades 11 and 12 mathematics—a costly decision. According to *Let's Talk Science* (2013):

> In general, students without Grade 12 math can expect to be excluded from 40 to 75 per cent of program areas… at Canadian universities. Students without Grade 11 or 12 math can also expect to be excluded from half of community college programs (p. 8).

My major criticism back in June was the fact that the curriculum was not available as a downloadable PDF document. It could only be accessed online, and that was a major red flag. When the Grades 1–8 math curriculum was introduced in the summer of 2020, many educators were dismayed to find that the web-based curriculum was routinely altered. There were numerous incidences shared by educators about using a piece of the document for program planning or presentation purposes and then being unable to find the same content the next day—it had simply vanished.

In an email conversation about this, *Gazette* Editor, Tim Sibbald, noted:

> This points to a question of stability regarding the curriculum. Where formerly it could be relied on to remain unchanged for six to eight years, now there may be the prospect that the "living document" approach may lead to confusion and alterations with minimal consultation (Personal communication, June 12, 2021).

Tim's foreboding words became a sinister reality on July 14, when it was discovered that the Ministry of Education had removed all language about racism and colonialism from the front matter originally included in its Grade 9 math curriculum. Originally, the introductory text of the document had said "mathematics has been used to normalize racism and marginalization of non-Eurocentric mathematical knowledge, and a decolonized, anti-racist approach to mathematics education makes visible its historical roots and social constructions."

According to the popular press (*Toronto Sun*, *Toronto Star*), Minister of Education, Stephen Lecce, unilaterally decided that this vital section should be removed from the preamble to the curriculum, along with mentions of "anti-racist and anti-oppressive teaching and learning opportunities," and "the colonial contexts of present-day mathematics education" (Sharp, 2021).

The next day, on July 15, Premier Doug Ford defended the egregious deletions, saying, "In math, let's stick with math. Other social issues, let's talk about it—there's no doubt—but let's not talk about it in math. Talk about it in other courses that the schools offer" (Benzie, 2021).

Amplifying the negativity and insensitivity of these actions and words were the concurrent headlines telling us of more and more unmarked graves of Indigenous children being found in Canada; the establishment of a national day to honour Truth and Reconciliation was announced for September 30; and efforts by educational systems at all levels, to make equity, diversity, inclusion, and Indigeneity priority areas.

It is well known that "timing is everything," which makes the deplorable actions by the Ontario Education Minister, the Ministry of Education, and Ontario's Premier even more stinging. Bad enough to pile salt on the wounds of the authors of the curriculum, but even more callous to demoralize the dedicated group of people who have been working so tirelessly to upend the general White, Eurocentric curriculum experience of most students.

Not surprisingly, one of the first people to voice his outrage was distinguished mathematics educator, Sunil Singh—someone whom I am proud to call my friend. Sunil is recognized widely for his work in anti-racist mathematics and his efforts to make math education more equitable through better content and pedagogy.

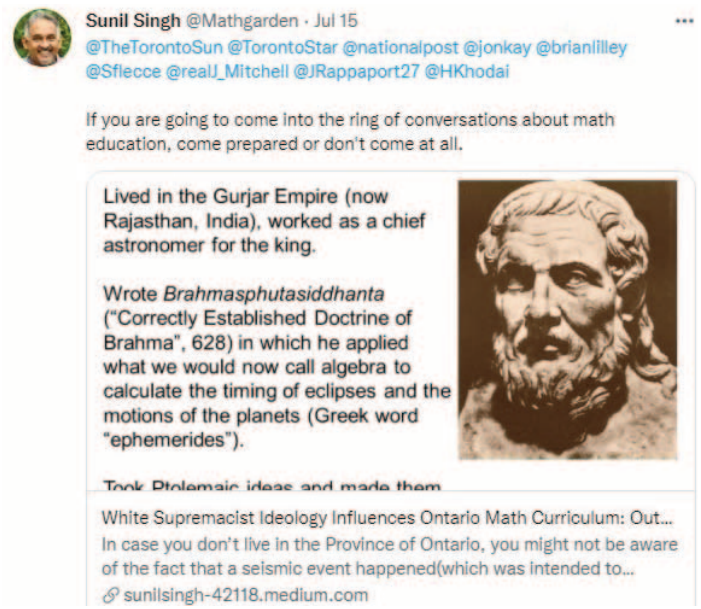


Figure 1: *Tweet on July 15, 2021 by Sunil Singh (@mathgarden)*

The about-turn by Minister Lecce and Premier Ford, removing the statements that truly mattered, rendered Ontario's definition of "equity" impotent. Our current provincial definitions speak of equal access to resources across socio-economic demographics, increased access to more academic courses for BIPOC students, and efforts to improve the enrolment of women and LGBTQ2S+ individuals in mathematics. We ask educators to be aware of their biases about students in these categories. All positive sentiments, but "blue sky" at best.

The original statements in the Grade 9 curriculum document addressed the root of the problem truthfully and boldly, which is why being told by our politicians not to entertain difficult conversations about our Eurocentric curriculum and being advised not to stop the perpetuation of blatant untruths about the origins of mathematical discoveries, theories, and concepts, makes it abundantly clear that equity has never been the true standard for those in positions of power.

Even the most innumerate among us know the formula $a^2 + b^2 = c^2$ and its name: the Pythagorean theorem. The "story on the street" states that the theorem was discovered by Greek mathematician, Pythagoras, while he was staring at the square tiles of Samos' palace, waiting to be received by Polycrates—a hyper-generalized anecdote that is dubious at best and most probably untrue. According to Fields Medal winner and Princeton University Professor Dr. Manjul Bhargava, the Pythagorean theorem:

> should either be an Egyptian theorem if you look at the standard of just having an idea about it, an Indian theorem if you're looking for a complete statement of

it, or a Chinese theorem if you're looking for the proof of it (Rajendran, 2015).

For Sunil and others who share his vision, it is by sharing these stories and hundreds of others about the global history of mathematics, that we, as educators, can provide inspirational and authentic opportunities for students from diverse cultures to see themselves as mathematicians, and, in so doing, move toward the ideals of equity. Sunil provides these examples to emphasize his point and pose the big elephant-in-the-room question:

Every student of mathematics will come across zero and the negative sign. Yet, there is very rarely a mention of Brahmagupta in classes. You don't think that students seeing key historical referencing early on in their math education will support the true trajectory of the equity arc? Calculus was a monumental achievement and deserves to have authorship recognized with Newton. However, what if it wasn't Newton who discovered calculus? What if it was Japanese scholar Seki Takakazu (March 1642–December 5, 1708)? We will never know. But the bigger idea is why couldn't this be a possibility? **Do we not want to be dislodged from the entrenched Western narrative?**



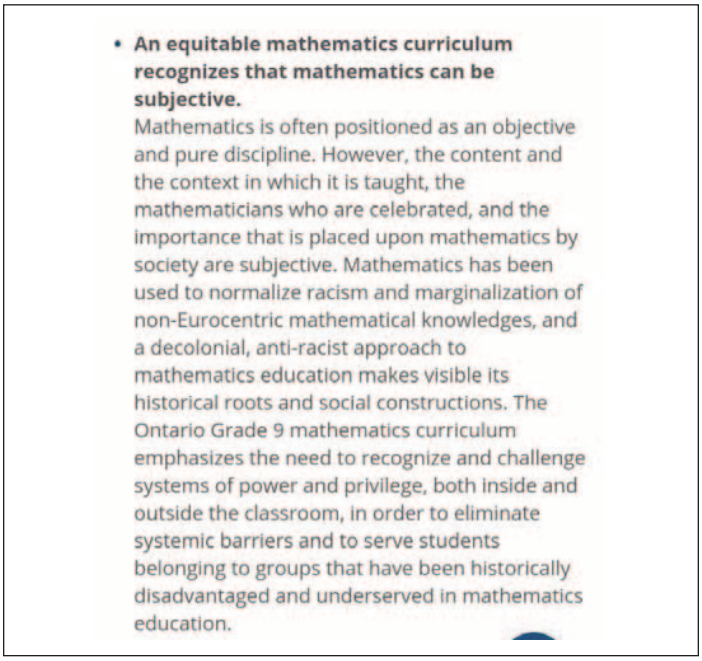
Figure 2: *Brahmagupta (598–668 BCE) was an Indian mathematician and astronomer, who was the first to give rules to compute with zero.*

Sunil's questions serve to raise many others:
Why don't we teach that the word "algebra" comes from a Persian scholar named Muḥammad ibn Mūsā al-Khwārizmī? Why don't we teach about the recent work of Australian scholar Daniel Mansfield, whose study of tablets dating back to the Old Babylonian period about 3700 years ago (first unearthed in 1894 near Istanbul), suggests that trigonometry was in use a thousand years before Pythagoras was even born?



Figure 3: *Plimpton 322, a tablet the size of a postcard, was discovered by Dr. Daniel Mansfield to be the oldest known example of trigonometry written in cuneiform, or wedge-writing (an ancient script once used in the Middle East).*

As Sunil and others point out, inequity is rooted in the content we choose to include in our curricula, but the hope of equity disappeared along with the July 14 deletions from the Grade 9 math curriculum.



- **An equitable mathematics curriculum recognizes that mathematics can be subjective.**
  Mathematics is often positioned as an objective and pure discipline. However, the content and the context in which it is taught, the mathematicians who are celebrated, and the importance that is placed upon mathematics by society are subjective. Mathematics has been used to normalize racism and marginalization of non-Eurocentric mathematical knowledges, and a decolonial, anti-racist approach to mathematics education makes visible its historical roots and social constructions. The Ontario Grade 9 mathematics curriculum emphasizes the need to recognize and challenge systems of power and privilege, both inside and outside the classroom, in order to eliminate systemic barriers and to serve students belonging to groups that have been historically disadvantaged and underserved in mathematics education.

Figure 4: *Screenshot from the Ontario Grade 9 mathematics curriculum, before this section was deleted on July 14, 2021*

Conversations about how our curriculum privileges whiteness could have led to questions about why we celebrate Pythagoras, Euclid, Descartes, Gauss, and Newton, but neither the mathematics of the ancient Egyptians or Andeans, nor the more recent accomplishments of brilliant women like Katherine Johnson or Maryam Mirzakhani. The original front matter not only invited, but advocated for, the difficult, but necessary conversations that could have been happening in our math classes. The enlightened text promoted positive activism: opening the door to spirited dialogue having the potential to confront equity issues head-

on by underscoring the reality that mathematics is not neutral from issues of race, class, or gender.

> Back in June, in my *Ottawa Citizen* OpEd, I wrote:
> If there is one criticism of the curriculum, it would be that it is fundamental, basic even—tied to "real-life" examples (e.g., calculating cellphone plan costs, perimeter and area) that are uninspired, and unlikely to meet the needs of those children who already enjoy and succeed in math.

But there is a way to add some spark to those pedantic examples and build in important conversations that bring together the stories that make it clear to our students that human beings around the globe have been capable of advanced and innovative mathematical thinking since time immemorial. Fascinating anecdotes abound that not only transcend time and space, but personalize and humanize mathematics.

For example, today, most people use Excel® to do accounting, and certainly using electronic spreadsheets would be the norm for a Grade 9 unit on budgeting. But think about how interesting it would be for students to know that accounting has a long and storied history. For example, the ancient Incans used the quipu—a system of knotted, coloured, cotton, or camelid fiber cords for such purposes. While today we keep detailed ledgers, imagine living in ancient Babylonian times and using stone tablets to record the sizes of fields when surveying land or defining boundaries with comparable accuracy.

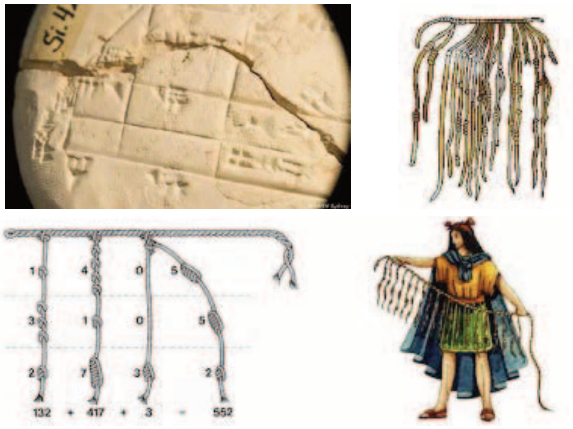


Figure 5: *Left: Si.427, a split clay disc holds legal and geometric details about a piece of land that was subdivided after part of it was sold. It's basically a land-registry document, detailing the exact boundaries and ownership of a plot of land. Right: Incan officials used quipus to record the wealth of the empire—how much gold, how much silver, how many people were in a farming community for tax purposes, and more.*

Imagine a scenario in which we find ourselves without a cellphone and therefore without a calculator, when faced with a multiplicative computation. Then imagine that you are looking to purchase seven bulls for 22 Maria Theresa (Austrian) dollars each in a village in a remote part of Ethiopia in the mid-1900's. To your amazement, in accurately computing the bill, the "local priest" multiplied 7 bulls by $22 per bull, using doubling and halving (and ignoring the remainder). Kolpas (2018) explains what you learn:

> The multiplicative system just demonstrated is very ancient indeed: it is probably the very earliest mathematical system worthy of the name and was doubtless invented, reinvented, and forgotten innumerable times throughout human history. Since it does not require any form of writing and involves only three operations, pairing off, halving, and doubling, which are both easy to carry out and are not troublesome conceptually, the system remained extremely popular with peasants the world over and became known as Russian Multiplication because, until recently, Russia was the European country with by far the largest proportion of innumerate and illiterate peasants.

In my courses for elementary mathematics educators, at Queen's university, I tried to bring such examples in as often as possible—teaching computation on a Soroban to support understanding of the base-10 system, going outside for a lesson on indirect measure to find the height of a tree by using an Indigenous method (www.youtube.com/watch?v=F6fltSqImFM), creating Islamic art tessellations, playing Mancala, paper-folding a Baggi Box to explore the maximum volume of an open-top box, and more.

I was supported in my efforts to engage the pre-service teacher candidates by many people, especially Susan Stuart and Bill Higginson, but my interest was born in 1992, when as a gift at the time of defending my Ph.D., I received from a dear friend, Susan Kelly, a copy of the book *The Crest of the Peacock: Non-European Roots of Mathematics*, by George Gheverghese Joseph.

After all this time, my copy is dog-eared, though it is never far from the stack of books by my side and is often picked up to be leafed through as bedtime reading, I admit that there is much content that I do not understand, but more than ever, in light of recent events in Ontario, I know why this important book about the mathematics of ancient Indigenous peoples, as well as those from Egypt, Mesopotamia, China, India, Africa, and the Middle East, must become required reading for every pre-service and in-service mathematics educator.

If, as educators, we truly believe in equity, and mourn the loss of the language in the front matter of the original Grade 9 mathematics curriculum, then we must learn about, celebrate, and frequently include the magnificent heritage of

non-Western mathematics. It is only by casting off our own constraining European bias, that we, as role models and leaders of change, will have the knowledge, attitude, and skills to empower students to see mathematics and its development as the product of civilizations over millennia from every corner of the planet.

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Etymological Distinction

Average comes from the concept of sharing losses due to damage to freight. It comes from the French *avarie*, Italian *avaria*, Spanish *averia*, and Arabic *awariya*. First mathematical usage was in 1755.

Mean is a place halfway between extremes. It comes from the French *meien* and Latin *medianus*. While used in music in the thirteenth century, the first mathematical use was in the mid-fifteenth century.

Source: www.etymonline.com

Prime News!

In September 2021, it was reported that $3 \times 2^{17748034} - 1$ is prime. In case you're wondering, it has 5.3 million digits. Details can be found at www.primegrid.com/download/321-17748034.pdf.

▲ Coding in the Classroom: Interview with Diane Tepylo



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Iain has been an elementary school educator for 28 years, and is now into a second career, teaching in-service and pre-service teachers. He teaches face-to-face at Ontario Tech University and virtually at Western University. He is the current editor of the Math+Code 'Zine, www.researchideas.ca/mc/.



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Beyza is a former teacher and long-term researcher. She is currently a graduate student in education at Western University, and her research focus is the integration of computational thinking into mathematics education. She is an editor at Math+Code 'Zine, www.researchideas.ca/mc/.

This column presents interviews of Ontario teachers, who use coding in their practice. Beyza and I have a conversation with the teacher to find out how they got started and their favourite pedagogical strategies. In this column, you will find some of these teachers' best ideas about using coding in their classrooms. If you want more, you can also access the complete interviews at OAME Talks to get a fuller sense of how these remarkable teachers have struggled, worked hard, and been successful with coding in their classrooms.

We were fortunate to be able to chat with Dr. Diane Tepylo. Diane has been teaching coding for about 20 years and has witnessed the transition of computational thinking from a secondary school elective, to routine use in elementary schools, and its inclusion as part of the mathematics curricula from Grades 1 to 9.

> Computers came into my high school in Grade 12, and I took my first programming course then, but never ever touched it again for 20 years, until I started teaching it at a high school I had moved to. I wanted a job. They said, "Could you teach this programming class?" I had been good in that one programming class in high school, so I said, "Sure." And then I started learning how to code. So I taught

programming, computer science in high school for about 15 years, and I had lots of questions about math teaching, as I am also a secondary math teacher. I went and did a master's and a PhD in math, science, and tech education. I was just graduating as efforts started about bringing coding into the Ontario curriculum, and so I got excited, and started teaching coding for teachers at Ontario Tech University.

The Importance of Coding

Diane understands the importance of having students in elementary and secondary school learn to code. Asked, "Why code?," the response was:

> We don't all need to be programmers, but we need to know a little bit about it. We don't all become writers when we learn to write, but we all learn to write to meet the needs of a range of jobs. If we're looking at that elementary curriculum, equity point of view, we have research out there that says if students aren't exposed to coding before they're 14, they barely even think about it. So you know, programming jobs are some of the best-paying jobs. There are many jobs that aren't programming jobs, but require a little bit. All sorts of places require programming now, they're high paying, and they are well sought after. So, we want to make sure that all students have that exposure to it, that opportunity to learn, and that it's an enjoyable experience.

The Affordances of the Coding Environment

The work Diane has done at Ontario Tech University for the past five years with pre-service teachers, learning how to teach through coding, and how to think computationally, has led them to a refined sense of how to introduce this pedagogy to diverse learners. All teacher candidates at Ontario Tech take a coding course, whether they are Primary/Junior or Intermediate/Senior candidates, and regardless of their teachability. While learning to code, they are building up their knowledge of their coding environment.

> Yes, more scaffolded at the beginning. So, you know, giving them small challenges so they figure out they can do it. They start getting inspired, "What else could I do?" The other big thing is the social element of these coding environments today. Sharing their computers with the people beside them, walking around the room, they see different people's ideas, and then they think, "Oh, how did you do that?," so much like you share writing, to inspire each other.

Low Floor, High Ceiling, Wide Walls

Diane carefully creates learning challenges that take advantage of coding's low-floor, high-ceiling, wide-walls attribute. This allows for all students to engage in the challenge at some level, and to be able to naturally make their code more complex. One way they do this is to create starter code programs that do something simple, from which the students can learn. Then, with small prepared challenges, the students add to what the program can do, while learning more complex coding operations.

> We have low floors, high ceilings, and wide walls—the idea started with Papert, and then built with George Gadanidis, as to the wide walls. There are very low floors, [so] kids can feel successful, and then the ones who grasp quickly, they have very high ceilings; they can just keep building and building and building. These types of activities are so engaging—it helps people feel successful. And we just need to help them see what things they can do because if you don't know tools exist, you don't even know that you can use them. One of the things that we're trying to do is provide those beginning activities to allow people to play and to explore, but give them some ideas of what they can do and what they want to do.

Debugging = Productive Struggle

Over the years, Diane has found some highly effective practices in helping students learn to code and starting to code to learn. One of these is the practice of debugging. As students become more confident with their ability to use coding as a tool in their learning, they get to a point quite quickly, where they can start to fix their own programs to eliminate bugs, with some prompts from the teacher. The practice of debugging their own programs is one that builds confidence in students, and they seem to learn even more when things do not always go well with their programming efforts.

> It's practice; you learn to code by coding. And I can't stress this enough with teachers. It's great to learn alongside your students. But it really does help to have some of those debugging strategies, and even knowing where people sometimes get frustrated, and where they get stuck. We can't prevent it, but we can help quickly, help them move past it. In math education, the productive struggle is my favourite. If it's too easy, you're not learning. We don't want to take all the struggle away from our students, but we want to be able to ask them questions and provide prompts to help them problem-solve so that they get that joy, that eureka moment. "Yes, I figured it out!"

Outreach Initiatives

Reaching out and helping others is just one of the many parts of Diane that goes toward making her a highly effective and respected member of the education community. Whether it be teaching a retired women's club to learn how to code, using Scratch Jr. so that they can code with their grandchildren, or creating continuous learning courses for teachers to earn a micro-credential in teaching through coding, Diane is frequently as busy outside the Faculty of Education as inside it.

> Right now, we're doing some work for the Durham District School Board, where we're bringing in groups of teachers online, and we're taking them through scaffolded challenges—still thinking, still focused on creativity, and on what do you want to do, but enough scaffolding to get them started. We have groups of five or less, working with some of our previous students, who are amazing. As well, we're going to have a weekly online drop-in at Ontario Tech (https://ontariotechu.zoom.us/j/94212775313) so that if teachers get stuck with a problem, they can drop in and work with one of our students.

There is much more that Diane shared with us during the interview that I encourage you to listen to on OAME Talks. Many ideas and insights were shared, that are valuable for teachers beginning their journey into coding and computational thinking, as well as those who are already on their journeys.

We are actively looking for teachers to interview for future columns and podcasts. If you are, or know, a teacher in Ontario who is doing great things with computational thinking, let Beyza and me know by emailing iainbro1.625@gmail.com/ or hbeyzamercan@gmail.com. ▲

Propeller Symmetry (Bracebridge)



▲ Let's Give Them Something to Talk About: Improving Late French Immersion Students' Communication in Mathematics



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Geneviève is an intermediate French Immersion mathematics teacher with the Limestone District School Board in Kingston, Ontario. She is passionate about mathematics and FSL education. Geneviève loves collecting and playing math games with her students.

The French immersion program in Canada originated in St. Lambert, Quebec in 1965 as a pilot program to promote French-language learning for anglophone students. This was perceived by parents and educators as a means to improve "cross-cultural communications between French and English Quebecers" (Safty, 1991, p. 474). Almost 50 years later, French immersion (FI) has become an increasingly popular program across Canada. FI is an intensive French as a Second Language (FSL) program, whereby students who do not speak French as a first language are placed in a classroom where French is the language of instruction for several subjects. FI is often classified by the entry point of the student into the program: early (Primary grades), middle (Junior grades), or late (Intermediate grades).

Individual school boards in Ontario have the authority to determine which FSL programs are offered, when enrolment may occur, and which subjects will be taught in the second language (L2) (Ontario Ministry of Education, 2020, para. 9). This lends itself to a wide variety of programs across the province, thus making it difficult to compare student achievement, procedures, and teaching strategies.

Throughout my career, I have primarily taught mathematics to Early French Immersion (EFI) and Late French Immersion (LFI) students at the Intermediate level. In my own context, LFI students in Grades 7 and 8 have participated in the Core French program consisting of one daily 40-minute French class, which would provide them approximately 750 hours of FSL instruction between Grade 1 and Grade 6. By contrast, EFI students would have

had at least 6000 hours of FSL instruction by Grade 7.

This significant difference in hours of FSL instruction explains the pronounced differences between EFI and LFI students' confidence and ability to spontaneously converse and write in French. The difference in instructional hours between EFI and LFI programs impacts the mathematics instruction of LFI students and necessitates specialized teaching strategies. Building on my professional practice as an EFI and LFI teacher, I share my experiences supporting these different learners, and the related theories.

LFI and EFI

LFI students are studying mathematics in French for the first time. They do not possess an extensive academic vocabulary in French, as their experience in the Core French program did not expose them to other subjects being taught in the second language. These students have less confidence speaking, and have challenges communicating their thinking orally in groups. However, LFI students are highly motivated by their novel classroom environment. There is a sense of personal "buy-in," as it is frequently the individual students who decided to enroll in the LFI program at the Intermediate level. These are students seeking scholastic challenges and who know that the LFI program will involve additional work and skill development. Students in each cohort are keen for a new start in a new school, which is the case with a decision to enroll in the LFI program. Despite coming from a diverse set of feeder schools, students in LFI classes are united in their common experience of attending a new school and sharing a similar level of L2 proficiency. There is a sense of community in the classroom of approaching challenges together, and a sense that these adolescents find it socially acceptable to ask questions and ask for help.

The contrast in L2 proficiency between EFI and LFI students was shown in my teaching approaches. With EFI students, I adopted a more modern approach to instruction. Students often worked in groups, presented their findings through conversations, and were expected and able to explain their mathematical thinking in detail. In comparison, I found my approach with LFI students to be focused on vocabulary acquisition. I expected students to show their thinking through calculations done in independent settings. I justified using a traditional approach because I had determined that the language proficiency and confidence in the L2 impeded LFI students' ability to effectively participate in communication-based problem solving.

I realized that the enthusiasm inherent with my LFI classes was being ignored. These were students who had demonstrated a willingness to take risks by enrolling in a challenging program at a new school. Focusing on the perceived linguistic gaps of these students was leading me to overlook the initiative and enthusiasm they possessed. My perception of LFI students being somehow "behind" or less capable than EFI students was a damaging mindset that was having me teach in a manner that denied the LFI class access to more innovative and differentiated instructional methods.

Comparison between LFI and English Language Learners

One of the ways for LFI teachers to find strategies for instruction of mathematics for FSL students is by examining resources and research conducted into the experiences of English Language Learners (ELL). ELL and LFI students share several similarities that can help teachers to understand the student experience and target effective teaching strategies. ELL and LFI students are learning a new language through the teaching of mathematics, and both groups have prior experience learning grade-level mathematics in their primary language. ELL and LFI students have been exposed to the new language in a limited way or only for a short time, and both are likely to have parents (including guardians), who may not be fluent in the L2 language of instruction. This lack of parental understanding of the L2 would increase the challenge for parents offering at-home support.

The differences between ELL and LFI students must be considered before adopting all strategies designed for ELL to an LFI mathematics context. In terms of classroom composition, ELL students may be a few individuals within a larger group of English speakers, whereas LFI students are frequently in a single-program class. The role of student choice to learn a L2 differs between ELL and LFI. Some ELL students may not have chosen to learn another language, whereas LFI students have voluntarily opted into this program and its inherent challenge. According to Bournot-Trites and Reeder (2001), the French immersion student choice "could, in turn, motivate them to work harder and to concentrate more during the lessons" (p. 39). The ability of the mathematics teacher to communicate in the student's first language (L1) varies widely. The ELL teacher might not be fluent in the L1 of the ELL student. In these circumstances, the ELL student will only engage with strategies, supports, and interventions in the L2. This is by necessity. The majority of LFI students have English as their L1, and FSL teachers are expected to be fluent in both French and English (Safty, 1991, p. 478). This means that FSL teachers are able to offer support bridging both languages.

Instructional Strategies

Instructional strategies are the means by which teachers present information in the classroom. It can be tempting to address gaps in language understanding by reducing the complexity of mathematical content. The study of LFI students in science, conducted by Turnbull, Cormier, and Bourque (2011), showed that LFI teachers tended to oversimplify subject content, which negatively affected second-language learners' understanding, and lowered communication of complex content as compared to their peers in English programs (p. 194). Instead of simplifying mathematics content, teachers should endeavour to improve LFI students' ability to recognize, understand, and use mathematics concepts being learned.

The development of mathematics vocabulary is a critical first step. Fortunately, several mathematical terms are identical in French and English (e.g., "multiplication," "addition," "fraction," "rectangle"), and some terms are so similar auditorily (e.g., "percentage" and "pourcentage") that they ease LFI students' transition from learning mathematics in L1 to L2. Drawing attention to these linguistic similarities "decreases student anxiety in learning a new language or subject" (Ross, 2011, p. 124). However, teachers should not settle for students knowing the translation of a term—they must understand its meaning. Murrey (2008) recommends that teachers "formally define new vocabulary after students have learned the associated concept and can connect the term with the known concept" (p. 149).



Figure 1: *Student participation in "Living Geometry Word Wall" activity (own photo)*

In my Grade 7 LFI class, the influx of geometric vocabulary invariably caused students to struggle. To address this, I prepared a "Living Geometry Word Wall" activity. Each student had a personal individual glossary they had been updating with newly acquired explanations of geometric terms over the previous weeks. The class was divided into teams of three or four students and given a set of nine words from our classroom word wall. Then they were asked to discover and photograph themselves next to examples of these terms they found throughout the school. Each word was ascribed a point value. Points were awarded for originality of photographed examples of geometric terms, but no points were awarded if two or more teams used the same item for the identical geometric term. This increased challenge level and ensured that students would find a wider variety of photographic representations of the key geometric concepts. Students were given 30 minutes to complete this task. Students then shared the photos, using the classroom projector, and we discussed how each photograph illustrated the term. Students then updated their personal glossaries with examples of what they had seen. The activity engaged LFI students with developing tangible understanding of mathematics vocabulary through written, oral, and visual means.

Another effective strategy for LFI is occasional code-switching—when the teacher seamlessly switches between the L2 and L1 during instruction. This code-switching needs to be a purposeful tool, one that is implemented to translate a few words or clarify a term, while maintaining the integrity and flow of instruction in the L2. In the study by Bournot-Trites and Reeder (2011), LFI students who learned science in L2 with code-switching showed improved understanding and communication of science content compared to those in L2 classes who did not (p. 194). The use of code-switching in the French immersion classroom requires judicious application. Turnbull (2001) states that French should be the target language of instruction, but occasional use of the L1 is acceptable, so long as teachers do not resort to the L1 "to save time, especially when one is tired or when students are particularly agitated" (p. 536). Similarly, Bournot-Trites and Reeder (2011) discourage teachers from overusing the L1 in the immersion classroom, as it undermines the fundamental learning experience of being immersed in a second-language environment (p. 195).

Resource-Based Strategies

The objective for the French immersion program is that "the second language is no longer a language to study, it is a language to be used" (Safty, 1991, p. 479). Therefore, students need to be able to record their learning and later

be able to access knowledge and content. Ross (2011) indicates that graphic organizers are an effective way for students to "organize the information with a limited vocabulary to demonstrate an understanding of a topic" (p. 122). To this end, graphic organizers such as flow charts, Venn diagrams, mind maps, and illustrated examples that are populated by students in the L2 can further be used as self-reminders to aid in mathematics activities. By way of keeping the L2 text succinct, these graphic organizers can highlight the mathematical ideas and essential skills that should be recorded for the benefit of the student.

Language and mathematics are interdependent, since "mathematical thinking, mediated by linguistic processes, is a prerequisite for mathematics achievement" (Anhalf et al., 2007, pp. 19–20). Communication needs to be front and centre in the mathematics classroom, and this applies to far more than mathematics calculations. To improve student L2 communication skills, I needed to consistently encourage and model oral participation. In my LFI class, I recognized that I often was the sole person speaking. As Suurtamm and Vezina (2010) state, teachers need to adopt a "listening orientation," which can only happen if the students are the ones speaking.

To have students be active communicators in group problem-solving situations, the teacher needs to explicitly teach and model how mathematical discussions in French look and sound. Staples and Colonis (2007) break down this communication process as "positioning students for discussion, managing wrong answers, and connecting and linking across ideas" (p. 258). LFI students require knowledge of syntax and sentence structures in the L2 in order to be able to frame their mathematics knowledge and share their ideas with teachers and peers: how to state an idea, question a peer, suggest a strategy, and ask for assistance. This connective communication needs to be explicitly taught to and modelled for students.

The teacher can begin by selecting small-group activities, where students communicate in anticipated structured responses that would be modelled for students before the activity begins. One such group activity I used early in the school year involved finding a mystery number based on multiple clues involving concepts of prime numbers, perfect squares, and factors. Students knew the mathematical vocabulary and concepts from previous lessons. I then modelled the connective sentence structures students needed in order to suggest an idea and agree or disagree with a peer. I found that students were better able to initiate and maintain a discussion in the L2.

Communication aids and prompts are needed to facilitate the development of these connective sentence structures. These could include sentence starters, the sentences that facilitate discussion, simple conceptual vocabulary, and statements for requesting assistance. A list of suggested communication terms is shown in Table 1. This list remains a living document and should be updated by the student as they continue to develop their mathematics communication.

Classroom Environment Strategies

The creation of a classroom environment that promotes communication in the L2 is a deliberate process. Teachers need to help develop LFI students' Willingness to Communicate (WTC) that is "a readiness to enter into discourse at a particular time with a specific person or persons, using a L2" (MacIntyre et al., 1998, p. 547). WTC depends significantly on the student having had positive L2 experiences and the "ability to feel secure in the relationship with the other person" (MacIntyre et al., 2011, p. 90). Students with improved WTC will have the confidence and motivation to volunteer answers and use the L2 with peers and teachers. For LFI students, developing this positive L2

Table 1 *Suggested Communication Terms for Student Use*

Sentence Starters	Sentences That Facilitate Discussion	Simple Conceptual Vocabulary	Sentences to Request Assistance
• I think that – Je pense que	• I agree with – Je suis d'accord	• More than – plus que	• I don't understand – Je ne comprends pas.
• I chose to do – J'ai choisi de faire	• I don't agree with – Je ne suis pas d'accord	• Less than – moins que	• I need more time – J'ai besoin de plus de temps.
• I calculated – J'ai calculé	• …but this doesn't work – … mais ceci ne fonctionne pas	• Greater than – plus grand que	• Could you explain it to me? – Peux-tu me l'expliquer ?
• I found that – J'ai trouvé que	• What if we tried – Pourrions-nous essayer	• Smaller than – plus petit que	• What do I do now? Que dois-je faire maintenant ?
• This answer works – Cette réponse fonctionne	• What do you think? – Que penses-tu ?		• I tried, but I need help. – J'ai essayé, mais j'ai besoin de l'aide.
• I'm looking for – Je cherche	• Could you repeat that? – Peux-tu répéter ?		

experience hinges on feedback received about language and about mathematics. According to MacIntyre et al. (2011), students want to improve their L2 fluency, but are "both willing and unwilling to receive error correction" (p. 86). This places the LFI teacher in a tenuous position. To ignore errors in mathematics and L2 could lead to these errors being compounded or becoming ingrained. To correct every error could erode the relationship between student and teacher, and thereby reduce the student's WTC in the L2 (MacIntyre et al., 2011, p. 93). Therefore, the LFI teacher must decide which errors require immediate intervention and which ones do not.

One way I alleviated this pressure to correct was by valuing student participation in the L2 as its own outcome. I remembered to acknowledge the motivation, risk taking, and participation of LFI students. The more students speak in the mathematics classroom, the more I am able to better determine how the students are expressing their ideas in the L2, and whether certain French or mathematics errors are habitual (requiring intervention) or infrequent. The means of L2 correction given to students must be carefully considered. Corrections should connect the LFI student to existing resources for consultation, or remind them of modelled language or actions they have seen. Making the connection between their error and the resource, or previous experience, encourages the student to self-correct, rather than depending solely on the teacher's rectification.

Learning mathematics in a L2 is challenging for the LFI student. As a teacher, focusing on the linguistic gaps of LFI students overlooks their inherent enthusiasm for learning a L2. By employing instructional strategies to build student capacity in mathematics and French, students can increase their confidence in their own L2 abilities. Students will develop these skills through persistent practice, if they are shown the skills they need and resources they can consult. Developing connective communication structures in the L2 allows students to explain their thinking to others, and opens the door for more collaborative problem-solving situations. Finally, teachers who actively strive to create a classroom environment that makes the L2 learning experience a positive one will see students' communication confidence improve.

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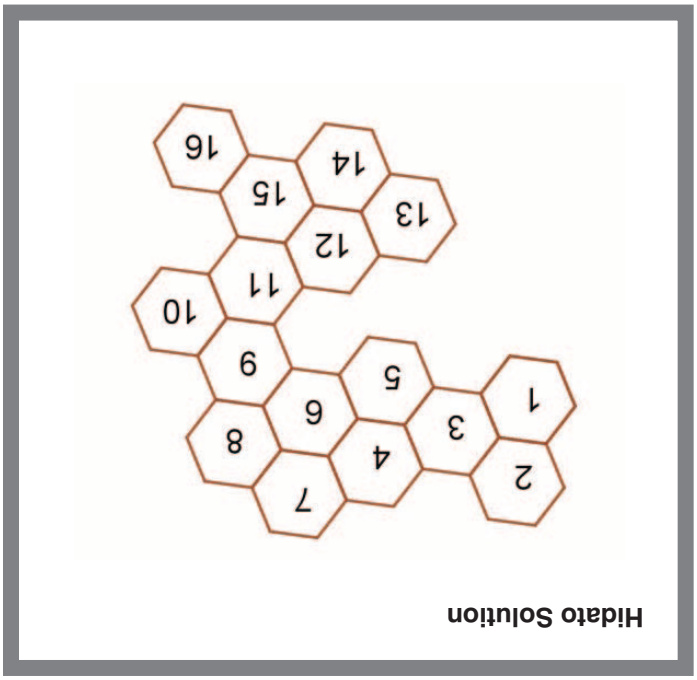
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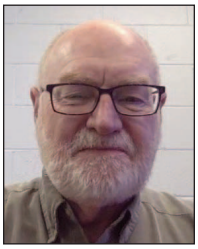
Hidato Solution

▲ Helping All Students Develop Algebraic Thinking



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Natalie Robinson is a secondary math teacher at St. Mother Teresa High School in Ottawa. She has taught all levels of all grades, with a minimum of three Grade 9 courses each year. Natalie has been actively involved in participating in and leading professional development related to technology and innovative instructional practice.



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Fred's opening thoughts...

Since March 2020, Lynda and I have been hearing stories from all over the province. When the pandemic hit, we watched and listened, sometimes in horror, as teachers described to us their situations when teaching resumed. It is safe to say that very few people were experienced at teaching online, but everyone was thrown into the deep end early in April 2020, and we realized that many of you were struggling with the new format.

It is safe to say that the good teaching techniques that you have used in the recent past are almost impossible to implement over an online platform. Somehow, putting students into a breakout room just isn't the same as grouping students together in your classroom.

What I'm hearing now is that, out of necessity due to the principles of social distancing, many teachers have returned to putting students in rows and teaching using the approach that we used when I started teaching in the 1970's. It took me a long time to give up being "the Sage on the Stage" and adopt new methods, so it breaks my heart that so many people are finding that that approach is necessary during this time.

I have also been hearing that administrators are struggling with changing conditions and making decisions that affect classrooms daily. Teachers are being moved from school to school, or from school to online or vice versa, and classes are being collapsed, combined, or created on the fly.

What follows is an interesting story from Natalie.

I began teaching math in the Fall of 2004, which was the year before the current curriculum arrived. As an eager new teacher ready to embrace all that was to come, I was one of the first in my board to be introduced to Computer Algebra Systems (CAS). This arose as part of a lesson study, led by my department head and math consultant, in preparation for the new 2005 curriculum that would require learning algebra: "using a variety of tools (e.g., paper and pencil, manipulatives, CAS)" (AGV 3.4–3.7). The school board provided professional development, and I represented the department, to supplement the board PD for this lesson study, at professional development on a Saturday in Brampton. That was the first time I met Fred. A dedicated group of teachers spent that Saturday at his school, investigating the opportunities this technology could provide for our students. We explored the amazing world of "*apple+apple*" and "*apple•apple*" (see Figure 1). It was awe-inspiring!



Figure 1: *The amazing world of fruit algebra*

For the first five years of my teaching career, I taught Grade 9 Applied and Academic, Grade 10 Applied, and Grade 11 and 12 Essentials (workplace) math. The Grade 9 curriculum was where algebraic reasoning started to take a central role in the curriculum. Students in Applied-, College-, and Essential-level math courses had typically struggled to understand algebra. Suddenly all my teaching focused on

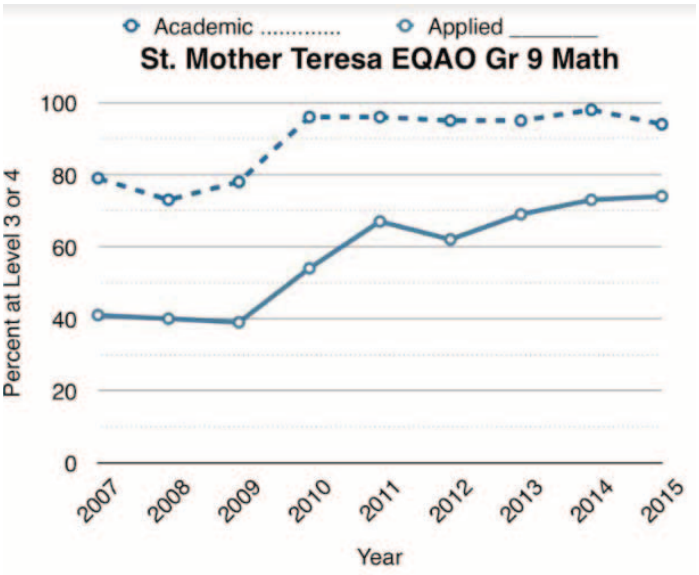
creating an environment that supported *all* students learning and understanding algebra.

The complexity of my teaching assignments was compounded by having varied classroom locations—sometimes in a portable at the back of the school grounds, and other times, bouncing from class to class and floor to floor into any available room. The one tool that I could consistently provide my students, regardless of grade or location, was my class set of TI-89 graphing calculators that had CAS. The learning curve of the CAS component of this technology was very low, and I could count on accessing and using it anytime with any of my classes.

Over the course of those first five years, the composition of my math department changed dramatically. In fact, I was the only one left! The other three teachers had moved on to different positions, and new teachers were added as the school grew. Teachers in the growing department continued to participate in professional development and to learn how best to support our students. We all used CAS to help our students explore important algebraic concepts like variables, collecting like terms, exponent rules, expanding, and of course factoring. We did a lesson study about factoring. We incorporated algebra tiles as area models and CAS as an algebraic tool for students into factoring. CAS gave us the ease to explore factoring, and we uncovered how truly complex teaching and learning factoring continues to be. But we persevered! And we did it together.

We did encounter some scepticism around our use of CAS. Based on our own research and experience, we had confidence that CAS had the potential to help our students. As it turned out, our EQAO scores bore out our belief that CAS could in fact help our students be more successful in algebra. Below is a graphic of what happened in our school the year that CAS technology was integrated into our regular teaching and learning strategies. Prior to 2008, we were using class sets of CAS devices (TI-89's). We then transitioned to student-owned CAS devices (TI-Nspire™ CAS) that could be used at school, at home, and on assessments. The graph below shows the percentage of students from St. Mother Teresa High School achieving at a Level 3 or 4 on the Grade 9 EQAO Assessment of Mathematics for Academic (the dotted line) and Applied (the solid line). It is clear that what we were doing was working for our students!

In March of 2012, my colleague, Maureen, and I attended the Teachers Teaching with Technology (T^3) International Conference in Chicago. We had the privilege of interacting with teachers from all over the world. It was comforting to hear that the challenges we were facing in Ontario were identical to those faced by teachers in Norway, New Zealand, Australia, the United States, Great Britain, etc. We met a kilt-wearing Scot named Nevil Hopley, who shared some amazing TI-Nspire CAS files with us that truly changed the way we approached algebra with our Grade 9 students. Immediately upon our return to school, we tried Nevil's files with our students.



The algebra world was not completely in tune in our Grade 9 classes, but we did see an immediate impact. The vocabulary used by Nevil in Scotland was a bit unfamiliar ("maths class") to our Canadian students. The mathematical content, however, was "spot on!" Nevil had delivered an approach using CAS that required thinking and problem-solving as students worked with algebraic equations.

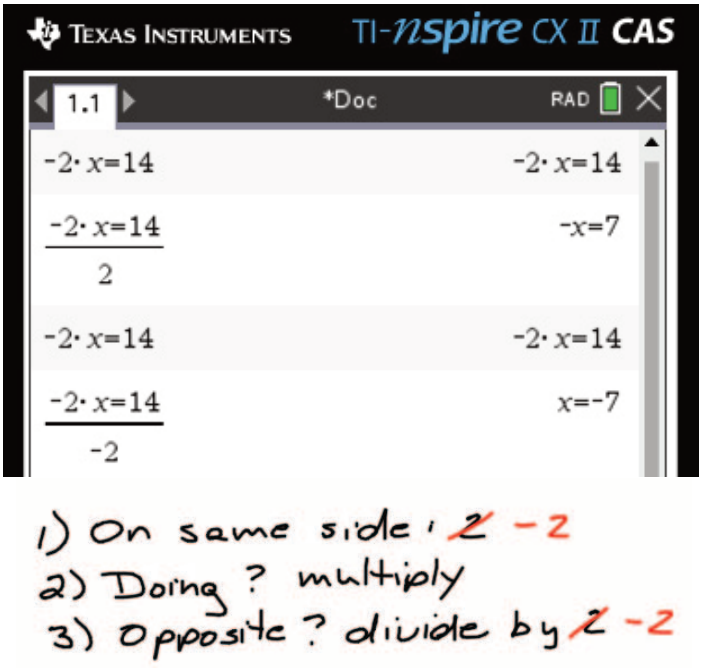


Figure 2: *Example with the sign problem*

It is important to note that during this time, we continued to be involved in additional professional development about general teaching strategies. We worked with teachers from Grades 1 to 12, focused on Jo Boaler's growth mindset work. We used smart cameras to display examples of student work to get a better understanding of their thinking. About five years ago, we dove deeply into the pool of "Building Thinking Classrooms," based on the research of Peter Liljedahl. This is in addition to the professional development focused on math-specific technology. We were truly a department of teachers collaboratively dedicated to learning how to best teach and support our math students.



Growth Mindset PD with Grades 1–12 teachers from our family of schools in 2013. Photo by Natalie Robinson. Used with permission.



Building Thinking Classroom PD in 2015. Photo by Natalie Robinson. Used with permission.

Let's go back to Nevil's awesome contribution to our algebra teaching strategies. We knew that in Grades 7 and 8, students were taught to solve one- and two-step equations (in the Patterning and Algebra strand prior to 2020 and C2.3 in the 2020 curriculum). Maureen and I decided to approach the principle of solving equations based on first principles. We first used algebra tiles to give students a feeling for the balance analogy, and the common view that "what you do to one side, you do to the other." We did not use, or allow students to use, phrases like "move to the other side" or "jump the fence," as those shortcuts become problematic beyond solving linear equations.

It was still too soon to move to paper and pencil, so we would then transition to a CAS environment, where we would have the students type in a simple, one-step equation. We then asked students a series of phrases repeatedly:

1. What number is on the same side of the equal sign as the variable?
2. What math operation is that number currently doing?
3. What is the opposite?

Once the students gave us the answer to the third question, we would then have them "tell" CAS to perform that "opposite operation"—they were communicating their anticipated response from CAS! The formatting CAS used was a little different, but the instant feedback students received was invaluable. This technique required students to analyze the question, identifying what was currently happening, and then choose the best next step. Unlike with paper and pencil, the CAS performed the mathematical operations, but the students did the critical algebraic thinking with instant feedback from the CAS.

The true magic of this technique was the fact that feedback is personal, instantaneous, and non-judgmental. This captured the essence of our growth mindset professional development, as we were creating a safe environment, where students could take risks, make mistakes, and learn and grow from those mistakes. Those magical learning moments happened right at the point when students tested their algebraic thinking for a specific inverse operation move. The calculator does what you tell it to do—no more and no less. If you give the correct "opposite operation" instructions, the next step in the solution appears. If you give the wrong instructions, the equation remains messy or perhaps appears messier. An immediate misconception that CAS helped students uncover was that when they performed the opposite operation to multiplying or dividing a negative number, many students wanted to "drop" the negative (i.e., they erroneously used the absolute value). CAS gave them instantaneous, unexpected feedback when they did this (see Figure 2). It reinforced the initial modelling we had done with algebra tiles, where we constantly reinforced the idea that we need to have a positive variable isolated in the solution.

While students wanted to move on quickly to paper and pencil with more challenging equations, we kept slowing them down. We had them focus on their algebraic thinking to help them solve simple equations with CAS successfully,

so we could then add to the complexity of the equations and keep students in a wonderful *Building Thinking Classroom* state of flow! Eventually we did introduce them to two-step equations using CAS, which made for wonderful discussions about how to decide which algebraic moves to make first. The class was having discussions about following order of operations, and what is the most efficient method to solve equations using the reverse order.

Only after we had "played" with one- and two-step equations using CAS did we progress to the resources from Nevil. The first of his resources had students identifying the inverse operation. Nevil's resource went on to have students identify equivalent algebraic expressions that looked different. The multiple-choice format (see Figure 3) gave students yet another safe environment, where they were getting immediate, personal, and non-judgmental feedback with a "correct" or "incorrect" indicator. (Note: If you would like to try out any of Nevil's resources, please send Natalie an email request.)

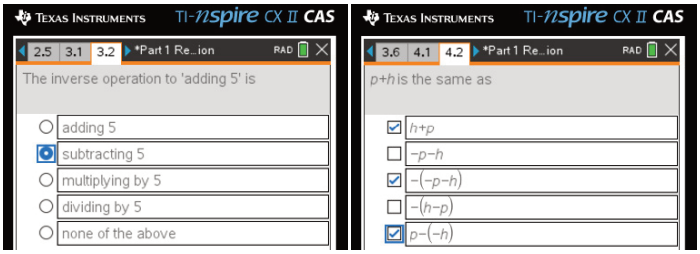
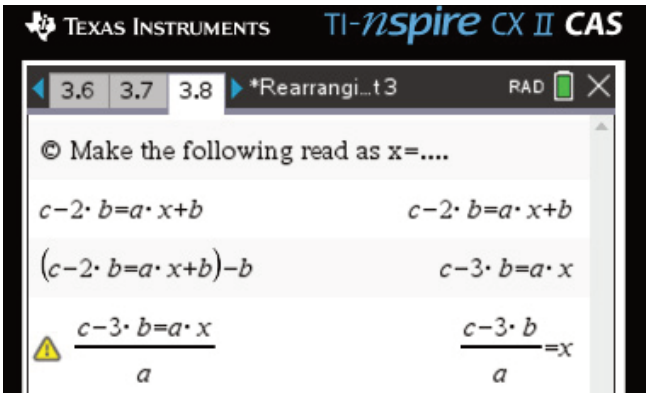


Figure 3: *Multiple-choice approach to equation solving*

Nevil's second resource had numerous equations already loaded, and a progression in degree of difficulty. My favourite part of the file was when students were asked to isolate "x" in equations that were made up of multiple variables. Here was where true understanding and problem solving were developed. Only after students understood the algebraic thinking behind the strategies did we progress to having students record their mathematical operations, using paper and pencil. When we moved to paper and pencil, we continued to emphasize the importance of identifying the progression of the "opposite operation" algebraic moves needed to isolate the variable efficiently and correctly.



This last example was particularly important for the algebraic understanding our science teachers were wanting our students to develop in math class. In science, teachers often had students work with equations with multiple variables, whereas in math, we often only exposed students to equations with one variable. It was our hope that this would build our students' algebraic confidence, and that it would carry over into the algebraic work these students would encounter in other courses, like business, science, and computer science.

In the last two years, we have spiralled, consciously revisiting concepts as the launchpad for new learning, and combining concepts to develop fluidity of knowledge in our Grade 9 classes, both Applied and Academic. CAS became even more important in helping all students learn to solve equations confidently and competently. We started with the one- and two-step equations very early in the course. We then worked on collecting like terms and expanding, using a variety of tools, including manipulatives and CAS. When we spiralled back to solving equations, we built off the initial experience focused on the algebraic thinking behind solving simple equations with algebra tiles and CAS. All the students felt good about their ability to solve simple algebraic equations, but the calm and confidence was even more apparent with students in the Applied classes. We were now able to move into solving more challenging equations that included multiple steps and multiple variables, continuing to use CAS, but with more of a focus on paper and pencil.

Achieving early algebraic mastery, using a variety of tools, was a critical element that led to the improvements in learning. This is reflected in our continued success with EQAO math results in both Academic and Applied. Some may question why, with additional PD, our scores did not continue to rise. Firstly, our results are already well above both the provincial and board averages, and growth is more difficult to achieve. Secondly, our school is an ESL centre, and the number of students in our Grade 9 classes, who do not speak English and/or have significant gaps in their education (due to time spent in refugee camps), has continued to increase as the years have passed. We are proud to have maintained this high level of success.

As we all survive this current, bizarre educational reality, the strategies we developed over the last years have become critical to the success of our students. We cannot work in thinking classrooms with students closely collaborating. We are not supposed to share materials or move freely around the building, so manipulatives and kinaesthetic activities are prohibited. We should not even be close enough to our students to see what they are writing on paper and help them make corrections. So, what should

we do? Many students in our school have either iPads with the calculator app on them, or a CAS handheld device (usually from an older sibling). We have also become creative with sanitizing school iPads and wrapping handhelds in clear plastic bags, which will also allow for sanitizing. This has permitted us to continue working with a technology that provides students with meaningful and immediate feedback. I do not have to be physically near a student to help them!

With the destreamed Grade 9 math curriculum that came last September, our department felt well prepared to meet the challenge of making algebra accessible to all students. Incorporating ideas from resources such as growth mindset and the thinking classroom, as well as a focus on spiralling, coupled with the effective use of powerful thinking and learning tools like CAS into our normal instructional practice, our students are well positioned to continue to thrive and succeed.

Fred's closing thoughts...

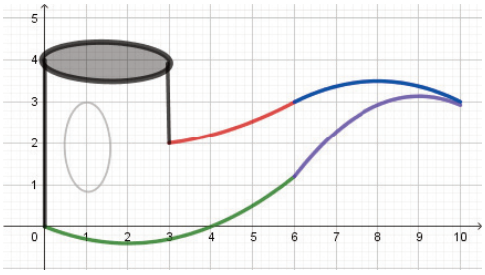
As Natalie has pointed out from her journey, the use of CAS to teach algebraic concepts has been invaluable. Having visited her classroom and her school, I have seen first-hand how well their students have progressed in their understanding of algebra. Their experience is not unique. I did some coaching for the Ontario Ministry of Education, specifically to work with CAS implementation, and saw the same level of understanding in many schools where I visited.

I would point out that Grade 9 is not the only grade where CAS was used. Our current curriculum has many topics in Grades 10 to 12, where the use of CAS helps students learn in topics that can be challenging to many students. This is not just using a calculator, or in this case, CAS, for the sake of using it because we have access to this technology. When we started using technology, we tried out a lot of things and made mistakes, but, over time, we adopted the principle of using any technology because it helped our students learn mathematics.

Much of my experience in using technology comes from my late department head, Ray Nowak, at Bramalea Secondary School. One thing that I loved about Ray's leadership was that he encouraged us to try new approaches. In fact, he started the September meeting of the math department with the exhortation, "Let's go out and make a few mistakes!" His attitude was that, by making mistakes, he knew that we were trying something new and everyone could learn from that.

In 1999, when that new curriculum came out and changed everything that we did, you would not believe how many mistakes I made. One of the first issues that came up was, "What do we do about assessment when students have CAS?" Our department decided to split tests into two parts. The first part was done without any technology, and a focus on the skills from the topic. The second part allowed the students to use their devices, and was intended to address the application and thinking parts of the assessment. That worked well for us.

Each year, at commencement, I asked my students to come back and tell me how they were doing. What I was most concerned about was whether my approaches to using technology hindered or helped them. What I generally heard was that, while the section disallowing technology was sometimes stressful, our approach worked well for them, and they felt confident in their understanding. ▲



René Magritte says, "Ceci n'est pas la page 44."

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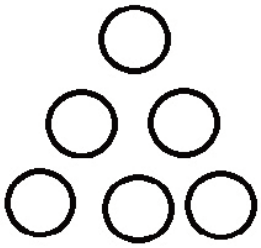
A Little Problem with Big Potential



JEFF IRVINE
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Jeff Irvine, PhD, has been a secondary mathematics teacher, department head, and vice-principal. He has taught at three faculties of education and at a community college. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. With over 45 years in education, Jeff is particularly interested in the role of student motivation in mathematics achievement.

A long time ago, I was shown the following problem by Don Fraser:

> Place the numbers 1, 2, 3, 4, 5, and 6 in the circles so that every side of the triangle has the same sum.
>
> 

I have given this problem to students from Grades 4 through Grade 12, as well as to pre-service teachers and in-service teachers during workshops. It appears to be quite straightforward, and groups quickly find a solution. For example, here is one group's solution (Figure 1).

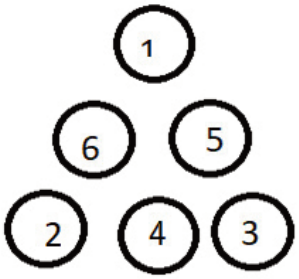


Figure 1: *Solution to the base problem. Total of each row is 9.*

If that were all there was to this problem, it wouldn't deserve much attention. However, in every class, there is a group who says, "Wait a minute. We solved the problem and got a different total." Here is another solution (Figure 2):

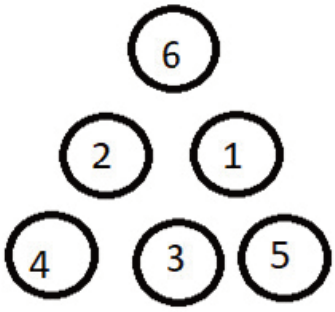


Figure 2: *Another solution to the base problem. Total of each row is 12.*

This may be the first time that students encounter a problem with more than one valid solution. Other groups soon get in on the action, finding valid solutions with row totals of 10 and 11 (Figure 3).

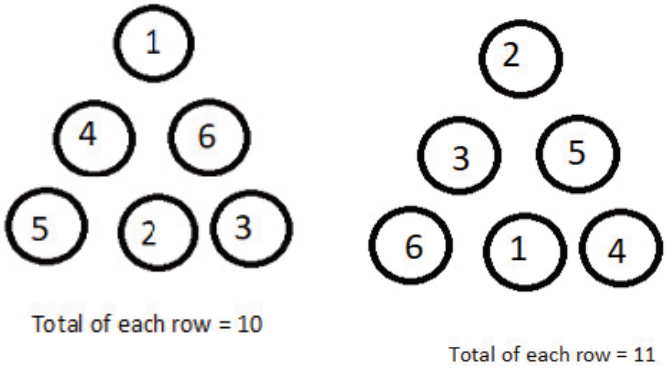


Figure 3: *Valid solutions with row sums of 10 and 11*

So we have one seemingly straightforward little problem with four valid solutions having row totals of 9, 10, 11, and 12. The little triangle problem is what George Gadanidis calls a problem with a *low floor*, but *high ceiling* (Gadanidis, Hughes, Scucuglia, & Tolley, 2009). Every student can access the basic problem, while some students can extend their thinking to algebra and "What if?" scenarios. The problem promotes students' critical-thinking skills in a fun way.

After addressing the initial problem, students are usually eager to check out similar patterns. I do this using a jigsaw approach. Each group is assigned a different sequence of numbers to insert into the triangle template: {2, 3, 4, 5, 6, 7}, {3, 4, 5, 6, 7, 8}, {4, 5, 6, 7, 8, 9}, {9, 10, 11, 12, 13, 14}, and so on. The challenge is to determine whether there are multiple solutions and, if so, find all of them. Are there always exactly four solutions? Do the solutions follow a sequential pattern, since 9, 10, 11, and 12 are sequential?

Some groups resort to the guess-and-check technique

that was used to solve the original problem. Other groups, however, recognize that the four patterns for the base problem can be used as templates, and make one-to-one substitutions, with the smallest number in their sequence replacing the "1" in the base problem, their largest number replacing the "6," and so on. Here is the smallest solution for the sequence {2, 3, 4, 5, 6} (Figure 4).

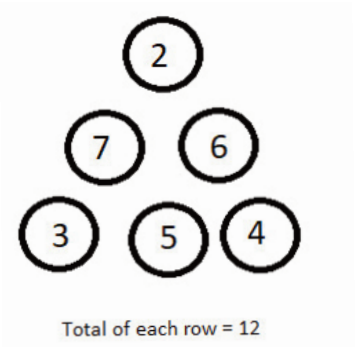


Figure 4: *Minimum solution for sequence {2, 3, 4, 5, 6, 7}*

Using the one-to-one correspondence technique, students quickly find the other three solutions, with totals 13, 14, and 15. Some additional solutions for other sequences are shown in Table 1.

Table 1: *Solutions for some sample number sequences*

Sequence						Triangle row sums of solutions			
1	2	3	4	5	6	9	10	11	12
2	3	4	5	6	7	12	13	14	15
3	4	5	6	7	8	15	16	17	18
4	5	6	7	8	9	18	19	20	21
5	6	7	8	9	10	21	22	23	24
6	7	8	9	10	11	24	25	26	27
7	8	9	10	11	12	27	28	29	30
8	9	10	11	12	13	30	31	32	33
9	10	11	12	13	14	33	34	35	36
10	11	12	13	14	15	36	37	38	39
11	12	13	14	15	16	39	40	41	42
12	13	14	15	16	17	42	43	44	45
13	14	15	16	17	18	45	46	47	48
14	15	16	17	18	19	48	49	50	51

By having students produce a similar spreadsheet in Excel or another spreadsheet program, patterns emerge for further investigation. This can be encouraged by adding a couple of columns (Table 2).

Table 2: *Spreadsheet with added rows. Rich patterns emerge from the data.*

sequence						sum	triangle sums				sum of sums
								1	1	1	
1	2	3	4	5	6	**21**	9	0	1	2	**42**
							1	1	1	1	
2	3	4	5	6	7	**27**	2	3	4	5	**54**
							1	1	1	1	
3	4	5	6	7	8	**33**	5	6	7	8	**66**
							1	1	2	2	
4	5	6	7	8	9	**39**	8	9	0	1	**78**
					1		2	2	2	2	
5	6	7	8	9	0	**45**	1	2	3	4	**90**
				1	1		2	2	2	2	
6	7	8	9	0	1	**51**	4	5	6	7	**102**
				1	1		2	2	2	3	
7	8	9	10	1	2	**57**	7	8	9	0	**114**
		1		1	1		3	3	3	3	
8	9	0	11	2	3	**63**	0	1	2	3	**126**
		1		1	1		3	3	3	3	
9	10	1	12	3	4	**69**	3	4	5	6	**138**
		1		1	1		3	3	3	3	
10	11	2	13	4	5	**75**	6	7	8	9	**150**
		1		1	1		3	4	4	4	
11	12	3	14	5	6	**81**	9	0	1	2	**162**
		1		1	1		4	4	4	4	
12	13	4	15	6	7	**87**	2	3	4	5	**174**
		1		1	1		4	4	4	4	
13	14	5	16	7	8	**93**	5	6	7	8	**186**
		1		1	1		4	4	5	5	
14	15	6	17	8	9	**99**	8	9	0	1	**198**

Having investigated numerically, let's do some algebra with this problem. Since the sequence is consecutive numbers, doing a one-to-one replacement in the first base-problem solution with $\{a, a+1, a+2, a+3, a+4, a+5\}$ results in Figure 5. By doing the same substitution in the other three solutions (Figures 2 and 3), we find that the row totals are $3a + 7$, $3a + 8$, $3a + 9$.

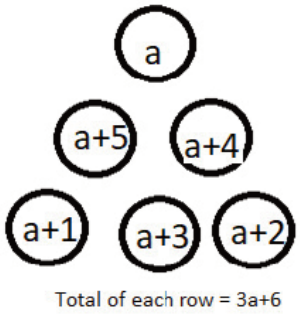


Figure 5: *Algebraic substitution*

The algebra enables us to reverse-engineer the question. For example, to generate a sequence that has largest row total of 330, solve the equation $3a + 9 = 330$, giving $a = 107$. Since the solutions are consecutive numbers, the required sequence is {107, 108, 109, 110, 111, 112}. Desiring a sequence that has smallest row total of 400 means solving $3a + 6 = 400$, which has no integer solution, so such a sequence does not exist. Since $3a + 6$ and $3a + 9$ are divisible by 3, it follows that row totals, either largest or smallest, must also be divisible by 3.

Arithmetic Sequences

Students quickly pose "What if?" questions based on the original question. One question often posed is, "What if the sequence had differences other than 1?" This leads to consideration of arithmetic sequences. For example, suppose the sequence were {4, 7, 10, 13, 16, 19}. Using the one-to-one correspondence replacements, for the minimum row total, this would generate the pattern shown in Figure 6.

Finding the other three solutions gives row totals of 33, 36, and 39., i.e., $3a + 7d$, $3a + 8d$, $3a + 9d$.

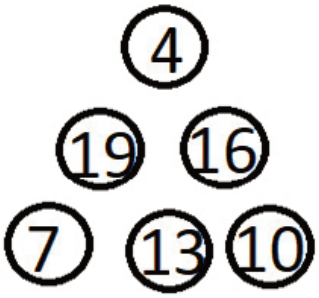


Figure 6: Minimum row total is $3a + 6d = 30$

Other Sequences

Over the years, students have posed other "What if?" questions. These have often included [I have phrased these using mathematical terminology]:

What if another sequence were used, like the Fibonacci sequence?

What if a geometric sequence were used, and the rows had to have the same product?

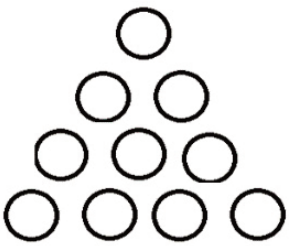


Figure 7: *Triangle with four rows*

What happens if we graph the four totals as families of lines $\{y = 3x + 6k,\ y = 3x + 7k,\ y = 3x + 8k,\ y = 3x + 9k\}$? What if the sequence of numbers is decreasing instead of increasing? What if the numbers are negative? What if the triangle had four rows (Figure 7) and we had a sequence of ten consecutive numbers (or ten terms of an arithmetic sequence)? Is the problem solvable? If so, will it have five solutions (since the triangle with three rows had four solutions)? What about a triangle with five rows? Will it have six solutions?

Before considering "What if?" questions, it is useful to write the terms in the triangle in a more general way, and it will be assumed that the generalization is giving the triangle with the minimum-row sum. The general way is shown in Figure 8 (t_1 = term one, and so on).

This leads to the relations:

$$\begin{aligned} Total_{min} &= t_1 + t_3 + t_5 \\ &= t_1 + t_2 + t_6 \\ &= t_2 + t_3 + t_4 \end{aligned}$$

These can be reduced to:

$$\begin{aligned} t_1 + t_6 &= t_3 + t_4 \\ t_1 + t_5 &= t_2 + t_4 \\ t_3 + t_5 &= t_2 + t_6 \end{aligned}$$

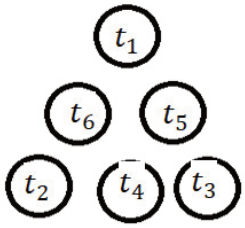


Figure 8: *General pattern for the minimum-row sum solution*

So, any sequence that is a solution must satisfy these relationships. Carrying out the same analysis for the maximum row totals leads to the same relationships. If we assume that the new sequence is increasing, then the one-to-one correspondence method will hold, as will the relations developed above.

Now let's consider another of the "What if?" questions about Fibonacci sequences.

Fibonacci Sequences

The first six terms of the Fibonacci sequence are 1, 1, 2, 3, 5, 8, following the recursion relation $t_n = t_{n-1} + t_{n-2}$. Using the relations developed above, we can see that these six Fibonacci terms do not satisfy the conditions, such as $t_1 + t_6 = t_3 + t_4$.

Therefore, the first six Fibonacci terms do not form a solution to the triangle problem. More generally, consider the terms of a Lucas sequence, which is a generalized Fibonacci sequence that has the same recurrence relationship $t_n = t_{n-1} + t_{n-2}$, but starts with any two numbers. The first six terms of a Lucas sequence are a, b, $a + b$, $a + 2b$, $2a + 3b$, $3a + 5b$. Again, using the relationships from above, like $t_1 + t_6 = t_3 + t_4$, we can see that a Lucas sequence cannot be a solution for the triangle problem.

But just because the first six terms of the Fibonacci sequence do not provide a solution, does it follow that any six Fibonacci sequence cannot be a solution? To answer this question, we need a general term for the Fibonacci sequence, credited to Jacques Binet, a French mathematician who published his formula in 1843. It is called Binet's formula, which is ironic, since Euler published the same formula in 1765, and the formula was known to de Moivre and to Daniel Bernoulli well before Binet was born. Binet's formula is

$$F_n = \frac{(1+\sqrt{5})^n - (1-\sqrt{5})^n}{2^n\sqrt{5}}.$$

This rather amazing formula, involving an irrational number ($\sqrt{5}$), always yields an integer value, which is a term of the Fibonacci sequence. For some six values of the Fibonacci sequence to be a solution to the triangle problem, the terms must satisfy equalities like $t_1 + t_6 = t_3 + t_4$, or $F_n + F_{n+5} = F_{n+2} + F_{n+3}$. Substituting using Binet's formula gives:

$$\frac{(1+\sqrt{5})^n - (1-\sqrt{5})^n}{2^n\sqrt{5}} + \frac{(1+\sqrt{5})^{n+5} - (1-\sqrt{5})^{n+5}}{2^{n+5}\sqrt{5}}$$
$$= \frac{(1+\sqrt{5})^{n+2} - (1-\sqrt{5})^{n+2}}{2^{n+2}\sqrt{5}} + \frac{(1+\sqrt{5})^{n+3} - (1-\sqrt{5})^{n+3}}{2^{n+3}\sqrt{5}}.$$

But this cannot be an equality, since the left side involves the sum of at least first-degree and sixth-degree terms, and the right side is the sum of third- and fourth-degree terms. Therefore, there is no set of six Fibonacci numbers that will be a solution to the triangle problem.

Now let's look at the "What if?" question, "What if a geometric sequence were used, and the rows had to have the same product?"

Geometric Sequences

The general term for a geometric sequence is $t_n = ar^{n-1}$. All rows of the triangle must have the same product. Here is the minimum triangle using a one-to-one replacement, using terms of a geometric sequence (Figure 9).

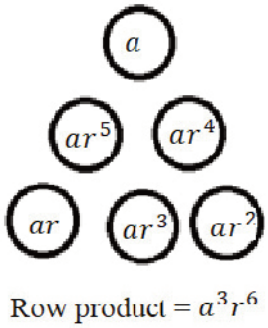


Figure 9: *Geometric sequence with same product for each row*

Using the one-to-one replacement in the other three base-problem solutions, the row products for the other three solutions can be shown to be a^3r^7, a^2r^8, a^3r^9. For example, Figure 10 shows a triangle with row products $2^33^6 = 5832$. It is notable that the four solutions form a geometric sequence.

I will leave it to the reader to consider other "What if?" questions, such as the four-row triangle question posed above, as well as other questions that might be raised related to this "little triangle problem." I found the extension to four or more rows particularly interesting, and accessible to all students.

This interesting low-floor, high-ceiling problem can be related to a number of curriculum expectations. In particular, the question addresses the problem-solving expectation in the mathematical processes, as well as specific expectations in the Grades 1 to 8 curriculum related to patterning and algebra, Grade 9 and 10 expectations in algebra and in graphing, and Grade 11 expectations related to arithmetic and geometric sequences and series. There is a very great upside to this problem, with no downside.

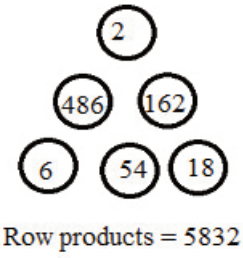


Figure 10: *Geometric sequence with a* = 2 and *r* = 3

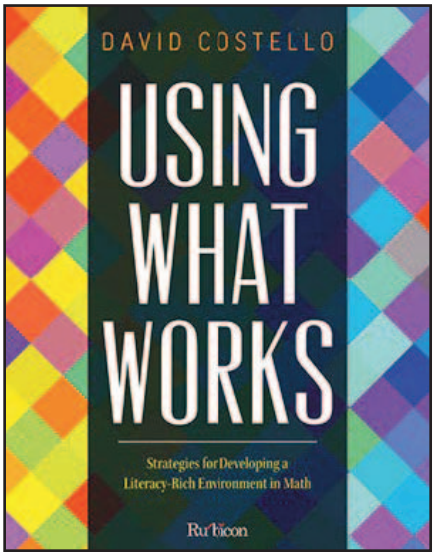
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▲ Book Review: Literacy-Informed Math Teaching

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Costello, D. (2019). *Using what works: Strategies for developing a literacy-rich environment in math.* Rubicon Publishing Inc.



David Costello is well known to regular readers of the *Gazette*. His columns reference his book, and you may think a review is unnecessary. However, books are substantially longer than columns, and provide a different type of opportunity for authors to convey their thinking. In this way, I was curious to see how David developed a comprehensive picture of his thinking beyond his regular column contributions.

The ideas in the book are current and well organized. Three chapters address the instructional space, thinking space, and physical space. The first emphasizes approaches to teaching that includes running discussions, use of groups, and how to include variety. The thinking space is centred on the students and a variety of approaches to engagement that will also reveal student understanding for assessment purposes. Lastly, is the use of different facets of the classroom that addresses technology, but also the use of bulletin boards, arrangement of desks, and other choices about the arrangement of the physical space.

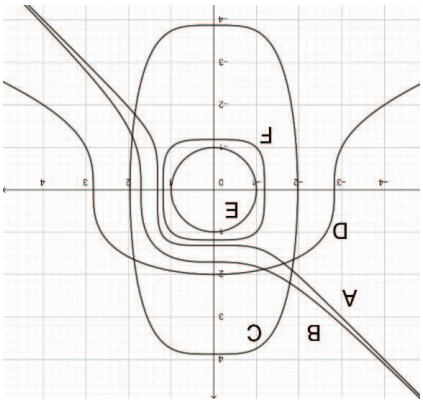
Throughout the book, there are quotations from students and samples of student work. These help to solidify the reader's understanding. In this respect, the book has a balance of practical strategies and concrete details that clarify understanding. While I think the book is likely of more interest to Primary and Junior grades, there is no doubt that it can support Intermediate teachers, and provides some food for thought to Senior teachers.

An aspect of the book that may surprise is the relatively short length that packs in many ideas—busy teachers will benefit from this. However, some techniques are brief to the point that it is not clear that enough is said to assist a teacher who runs into difficulties when trying an idea. I suspect this is simply a compromise to deliver many methods so that teachers find something of interest, rather than making a compendium with so much information that one gets bogged down in the details. Therefore, while it is a challenge for some ideas in the book, it does serve to point out a variety of techniques, and allows one to search for further information, if necessary, to improve their implementation.

The book does deliver where it says it is promoting literacy strategies in support of math learning. This will make the book of particular benefit to teachers who provide instruction in language arts as well as math—they will learn strategies supporting both. It also provides a limit to the scope of the book that it is about the cross-purposing of literacy strategies as opposed to emphasizing nuances within mathematical concepts or how content details fit specific pedagogical approaches.

To get a taste of Dr. Costello's writing, one can consult his column. However, to get an image of the forest instead of the trees, the book is a good solution. The cross-purposing approach is effective and serves to improve math learning, while also supporting student literacy. This constitutes a two-for-one deal that makes the book worthwhile for a broad audience. ▲

Match the Equation! Solution



B: $x^3 + y^3 = 5$

F: $x^4 + y^4 = 2$

A: $x^5 + y^5 = 4$

E: $x^2 + y^2 = 1$

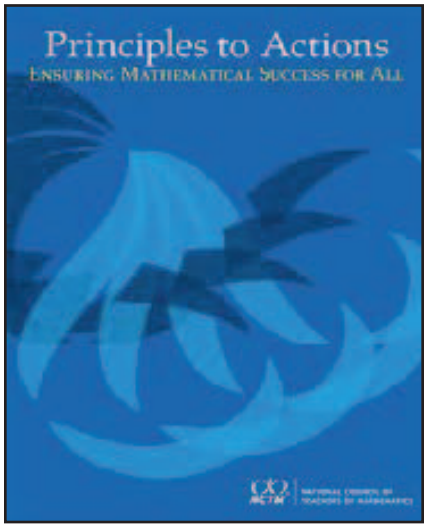
D: $x^2 + y^3 = 8$

C: $x^4 + y^2 = 15$

▲ Book Review: Sharing Perspectives in a Book Group: *Principles to Actions*

FAREEN KANJI, ELEMENTARY TEACHER AND FORMER MATH COACH FOR THE DURHAM DISTRICT SCHOOL BOARD (DDSB); SHAUNA SPEARE, SECONDARY TEACHER, DDSB; JACQUELINE HILL, RETIRED SECONDARY TEACHER AND K–12 MATHEMATICS FACILITATOR FOR THE DDSB.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematics success for all.* Author.



It is wonderful when one has like-minded friends with whom to share things of mathematical interest. As a book group, the three authors of this review explored this book together from each of our educational lenses: elementary, secondary, and board consultant. Each person found value in the book, for their perspective, that emerged through discussion and sharing.

The review takes the three-stars and one-wish approach.

Readability: How the book flows.

The layout of this book made for an easy read. Divided into three sections—*Effective Teaching and Learning, Essential Elements,* and *Taking Action*—allows the reader to see the logical progression of concepts and the connections between the sections.

The *Effective Teaching and Learning* section outlines eight mathematical teaching practices and includes relevant classroom examples. It also includes teacher and student actions. The *Essential Elements* section provides the environment required to implement the eight teaching practices. The book includes charts that help to identify productive beliefs and summarize the points concisely. The *Taking Action* section is geared from Kindergarten all the way through to policy makers. It clearly defines the roles of each of the stakeholders in education, and the measures they can use to implement the essential elements.

Application: Helping teachers move along their own continuum.

Elementary Perspective
Many helpful, practical, and familiar ideas are presented and discussed, with classroom examples. This allows for teachers to feel good about their current practices, at the same time being introduced to new tools (e.g., group work; vertical non-permanent surfaces; Kagan structures such as think/pair/share, Bansho, descriptive feedback, growth mindset, differentiated instruction). The book explains many common-sense approaches to teaching and learning, and allows the teacher to be reflective about their teaching practice.
Elementary Perspective
Two main ideas emerge. Firstly, the book was written to help educators with the notion that a productive struggle in mathematics helps ensure students' understanding of the concepts. Secondly, it helps teachers to refine their questioning techniques. Knowing that teachers ask questions to help students with their learning, this book challenges teachers to ask better questions, which can uncover more and dig deeper in support of their learning.
Board Consultant Perspective
This is a "big ideas" book that can help teachers to see that they are on the right path. Teachers will be able to identify the strategies they are currently using, as well as other strategies that will ensure student success.

Where does this book fit into current mathematical practices?

All three of the reviewers agreed that this title would make an excellent choice for a book club. In that scenario, the sections of the book could be broken down into manageable chunks, which could then be used at all levels (elementary, secondary, and at the board-office level), to help principals lead their teams. It might be good for an OAME/AOEM chapter to pick this title as a book-club choice. As such, it is a read that includes practical examples, but that also makes the readers at the elementary, secondary, and board-office levels think about their current practices.

A wish:

At times, the readability of the book is hampered by the inclusion of a reference at every point. As well, with so many teachers considering a spiralled approach to teaching, it would have been helpful to see the connections among multiple ideas.

In summary, this book, from beginning to end, takes all stakeholders (from the classroom teacher, to the school principal, to a math consultant at the board office) on a self-reflective journey that ultimately helps students. The more in-depth the stakeholders use the approaches and suggestions, the better they will provide for our future generation of students. ▲

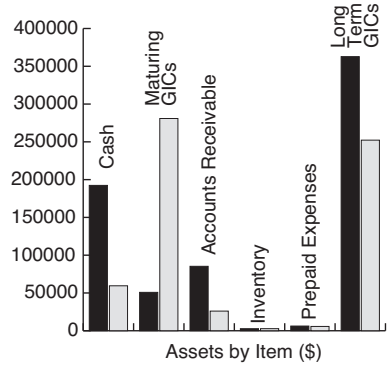
OAME Financial Statement 2020-21

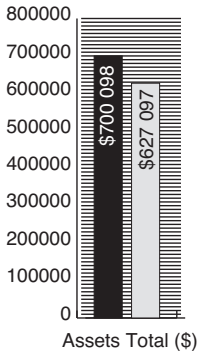
Notice to Readers

On the basis of information provided the organization, we have compiled the statement of financial position of Ontario Association for Mathematics Education (Association Ontarienne pour L'Enseignement des Mathématiques) as of August 31, 2021, and the statements of operations, changes in net assets, and cash flow for the year then ended. We have not performed an audit or a review engagement in respect of these financial statements and, accordingly, we express no assurance thereon. Readers are cautioned that these statements may not be appropriate for their purposes.

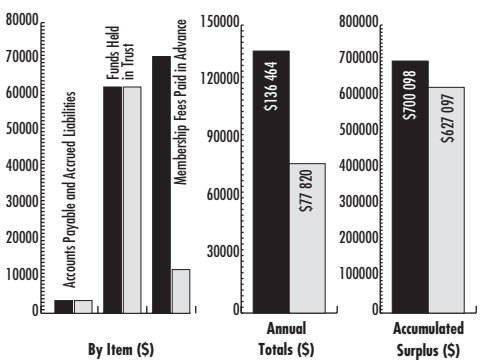
London, Ontario, October 1, 2021, GRANT THORNTON CHARTERED PROFESSIONAL ACCOUNTANTS

Current Assets

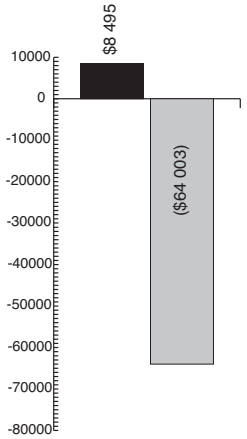




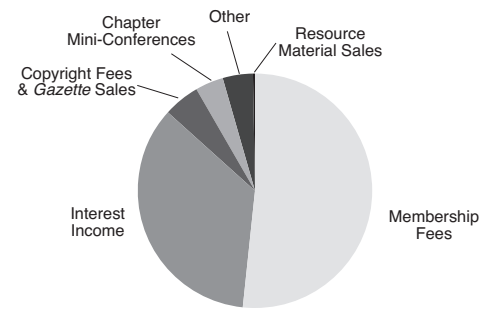
Liabilities and Fund Balance



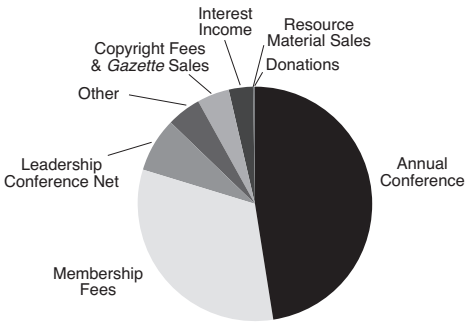
Excess of Revenue Over Expenses



Revenue



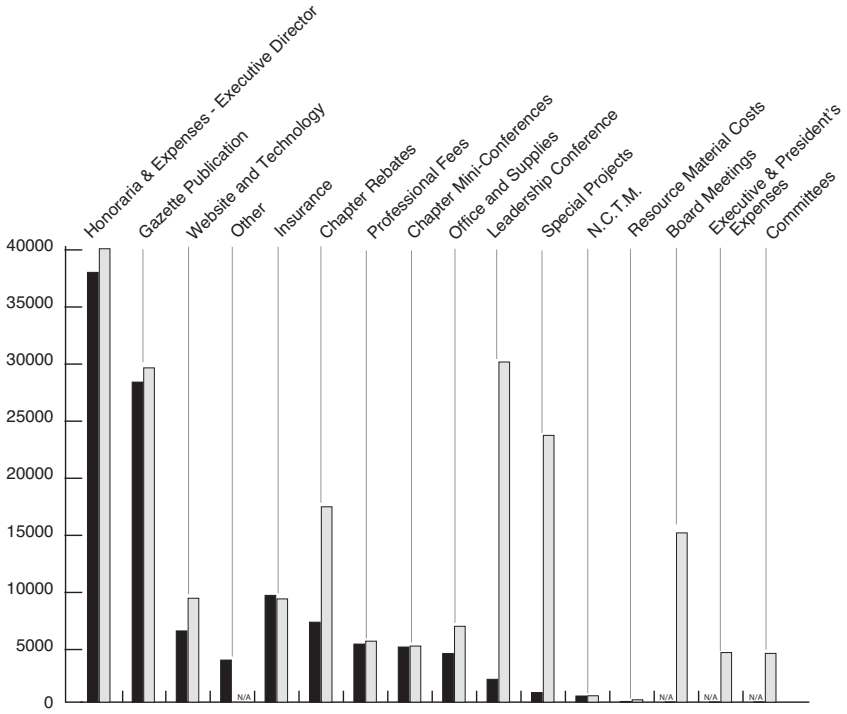
2021 ($117 235)



2020 ($134 462)



Expenditures



2021-2022 OAME/AOEM Board

OAME website
www.oame.on.ca

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NOMINATIONS FOR OAME/AOEM BOARD OF DIRECTORS 2022-2023

Elections are currently being held for a Vice-President (two-year term) and four Directors (three-year term), of the OAME/AOEM.

OAME/AOEM members interested in the position of Director will declare their intent to run for one of Grades JK–6, Grades 7–12, College or University level based on their personal experience, expertise, and interests.

Nominations for these positions are to be submitted electronically by completing an online Nomination Form by 5:00 p.m. on January 14, 2022.

A nominee must be an OAME/AOEM member. By March 2022, the Nomination Committee will publish a slate of candidates, including candidate bios, for consideration by the membership. Voting is open to all OAME/AOEM members. The election results will be released at the Annual Conference of OAME/AOEM in May 2022 and posted on www.oame.on.ca.

An **online nomination form** as well as a summary of the duties of each description is available at http://bit.ly/ OAMEPositions (scroll to the bottom of the screen). Please contact Paul Alves (paul.alves@oame.on.ca) for more information.

Thank you for submitting your nomination!

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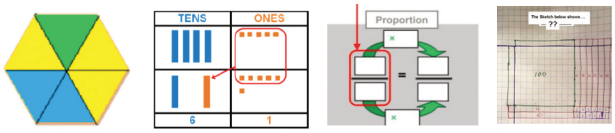
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