8-PAGE INSERT ELEMENTARY MATH
ABACUS

Ontario Mathematics Gazette

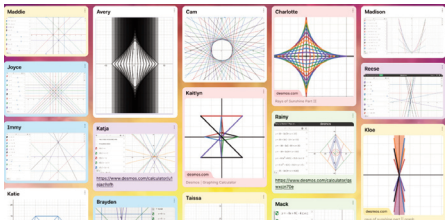
OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

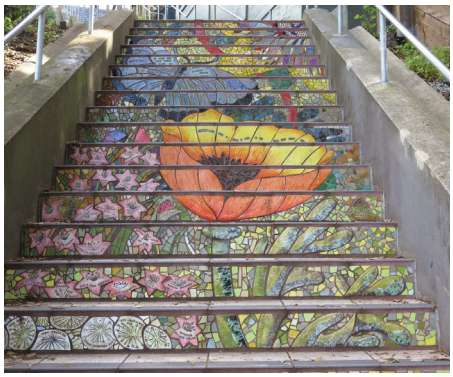
Vol. 60#4
June 2022

IN THIS ISSUE

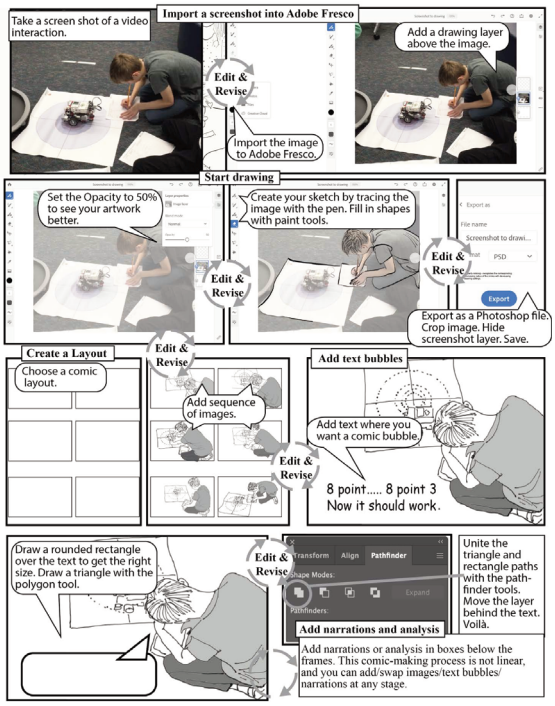
- Editor's Report
- President's Message
- Blast from the Past! Assessment Through the Ages (of the *Gazette*)
- OAME/NCTM Report: April Showers May Bring May Flowers! And June Brings a Much-Needed Recharging of Your Batteries!
- Events at the Fields
- What's the Problem? Counting Coins
- OMCA News: June 2022
- Linking Literacy and Math: Reconsidering Our Perspective on Learning
- Mathematical Snapshots: Stepping aside, Step up!
- In the Middle: Boole, Bézier, and Beauty
- Mb4T (Mathematics by and for Teachers): Using Concrete Tools, Exploration, and Reasoning to Develop Factoring
- Some Thoughts about Perimeter: Vocabulary and Metaphors
- Elementary Math Matters: Paying Privilege Forward: The Second Annual Early Mathematics Conference
- Coding in the Classroom: Interview with Amanda Deneau
- Promoting Action with Math: A Review of *Activity Counts*
- Book Review: A Taste of Mathematical History
- The 2021 Annual Meeting of the Canadian Mathematics Education Study Group (CMESG)
- Book Review: Math Routines Can Meet Academic and Play-Based Learning Goals
- Destreaming/De-tracking Math Position Statement
- Book Review: Planning "Effortful" and Periodic Retrieval in Mathematics Learning
- Caribou Math: International Math Contests Made in Ontario
- Translating Video to Comics for Sharing Classroom Practice



See In the Middle: Boole, Bézier, and Beauty

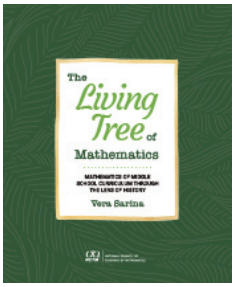


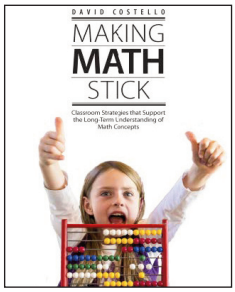
See Mathematical Snapshots: Stepping aside, Step up!

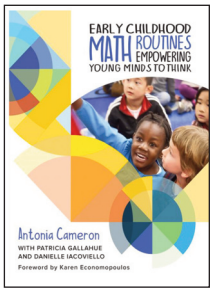


See Translating Video to Comics for Sharing Classroom Practice









See Promoting Action with Math: A Review of *Activity Counts*; Book Review: A Taste of Mathematical History; Book Review: Planning "Effortful" and Periodic Retrieval In Mathematics Learning ; Book Review: Math Routines Can Meet Academic and Play-Based Learning Goals

Submission of Articles

The *Ontario Mathematics Gazette* (*OMG*) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *Gazette*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles are to be word–processed, MS Word is preferred, and prepared according to the *Publication Manual of the American Psychological Association*, Seventh Edition. However, please use single-line spacing (not double). Articles should not exceed five numbered pages of text, and figures, images, and photographs should be placed in the text close to where they belong, with captions. The photographer's permission is required, and for photos of students under the age of 18, the written permission of a parent or guardian is required.

Please submit your article in one blind file (i.e., identity of author is not evident), and include author names, contact information including email and mailing addresses, photos—head and shoulders, biographies—less than 100 words, and all content removed for blinding in a second file. Please email these two files to Tim Sibbald at gazette@oame.on.ca.

Upon review, you will be notified whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected. The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME/AOEM.

PERMISSION TO REPRINT: No part of this publication may be reproduced, distributed, or transmitted in any form or by any means, including photocopying, recording, or other electronic or mechanical methods, without the prior written permission of the publisher, except in the case of brief quotations embodied in critical reviews and certain other non-commercial uses permitted by copyright law. Full credit must be given to the author and to the *Ontario Mathematics Gazette*. For permission requests, contact the Editor of the *Ontario Mathematics Gazette*.

The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of OAME/AOEM.

▲ Table of Contents

Up Front

▲ Editor's Report by Timothy Sibbald 2
▲ President's Message/Message de la présidente by Judy Mendaglio 4

Regular Features

▲ The *Abacus* . . . . . 8-Page Insert
▲ Blast from the Past! Assessment Through the Ages (of the *Gazette*) by Jeff Irvine 7
▲ OAME/NCTM Report: April Showers May Bring May Flowers! And June Brings a Much-Needed Recharging of Your Batteries! by Jacqueline Hill 9
▲ Events at the Fields by Angelica Mendaglio 10
▲ What's the Problem? Counting Coins by Shawn Godin . . . 11
▲ OMCA News: June 2022 by Heather Theijsmeijer 15
▲ Linking Literacy and Math: Reconsidering Our Perspective on Learning by David Costello 18
▲ Mathematical Snapshots: Stepping aside, Step up! by Ron Lancaster 21
▲ In the Middle: Boole, Bézier, and Beauty by Carly Ziniuk . . 24
▲ Mb4T (Mathematics by and for Teachers): Using Concrete Tools, Exploration, and Reasoning to Develop Factoring by Ann Kajander 30
▲ Some Thoughts about Perimeter: Vocabulary and Metaphors by Jérôme Proulx 32
▲ Elementary Math Matters: Paying Privilege Forward: The Second Annual Early Mathematics Conference by Linda Colgan 34
▲ Coding in the Classroom: Interview with Amanda Deneau by Iain Brodie and Hatice Beyza Sezer 38

Special Features

▲ Promoting Action with Math: A Review of *Activity Counts* by Tim Sibbald 8
▲ Book Review: A Taste of Mathematical History by Tim Sibbald 15
▲ The 2021 Annual Meeting of the Canadian Mathematics Education Study Group (CMESG) by Jennifer Holm 16
▲ Book Review: Math Routines Can Meet Academic and Play-Based Learning Goals by Edward Schroeter 27
▲ Destreaming/De-tracking Math Position Statement 41
▲ Book Review: Planning "Effortful" and Periodic Retrieval in Mathematics Learning by Edward Schroeter 42

Articles

▲ Caribou Math: International Math Contests Made in Ontario by Jeff Irvine 44
▲ Translating Video to Comics for Sharing Classroom Practice by Krista Francis and Steven Khan 47

Notices

▲ OAME Sticky Graph Pads 6
▲ OAME Board Members 52

Advertisements

▲ OAME 2023 . . . . . Inside Front Cover
▲ Neufeld Learning Systems Inc 6
▲ Teaching Math during the Pandemic: Lessons Learned and Best Practices. 29
▲ CMS . . . . . Inside Back Cover
▲ Texas Instruments . . . . . Back Cover

▲ About the *Ontario Mathematics Gazette*

The *Ontario Mathematics Gazette*—ISSN 0030-3011—is indexed in the Canadian Education Index and is published four times per year. Its Canadian Publication Mail Product Sales Agreement Number is 40051074.

Editor
▲ **Tim Sibbald**
gazette@oame.on.ca

Associate Editor
▲ **Anne Yeager**
anne.yeager@rogers.com
▲ **Peter Saaramaki**
saarimaki@rogers.com

Associate Editor (French articles)
/Rédactrice adjointe (articles français)
▲ **Kyla Kadlec**
kyla.kadlec@oame.on.ca

***Abacus* Co-Editors**
▲ **Marc Husband**
marc.husbandl@oame.on.ca
▲ **Tina Rapke**
tina.rapke@oame.on.ca

Copy Editor
▲ **Gitta Berg**
gitta.berg@sympatico.ca

Design and Production
▲ **Penny Clemens,** Graphic Designer
pennyydezign@gmail.com

Printing & Binding
▲ **Pole Printing,** Box 69, 1-89 King Street East
Forest, ON N0N 1J0 (519) 786-5112
poles@xcelco.on.ca

Advertising Manager
▲ **Robert Sherk**
4366 Snider Rd., Verona, ON K0H 2W0
Home (613) 374-1515
robert.sherk@oame.on.ca

Submission of Advertisements
Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** by email or at the above address. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be print ready, and colour advertisements should have no bleeds.

Advertising Rates
Advertising rates are available by telephoning, emailing, or writing to the Advertising Manager.

▲ Editor's Report



TIM SIBBALD, OCT, Ph.D.
*Gazette***@oame.on.ca**

Tim Sibbald holds the Universities Director position on the OAME Board of Directors. He is a former president of OAME. He is an associate professor in the Schulich School of Education, Nipissing University, with a focus on mathematics education.

I am going to share a SEL moment—I am tired. This seems to be something that everyone has experienced with the pandemic, but it has arisen for different people and their roles at different times. My time has come. I am confident it will pass in due course. However, that does not change the fact that I have multiple tasks coinciding with time constraints and have the social–emotional challenges of the pandemic like everyone else. Part of the solution, in this instance, is that the editorial will be brief.

All of us, in my humble opinion, need to change gears and start cautiously emerging out from under the pandemic rock. I hope that in the coming months, you find time to recharge in a way that suits you. We all need to collectively take a deep breath and restore ourselves. I wish the readership good health and hope that you find time for yourselves—you have surely earned it!

In This Issue

Judy Mendaglio gives us a reflective look as she once again moves from being President to being Past President. I would like to take a moment to acknowledge the strain that Judy has endured, seeing her aspirations for her term come under the cloud of the pandemic. She has been an important leader through the cloudy period, and I wish to offer my personal thanks for her efforts.

Angelica Mendaglio reports on the talk given by **Dr. Cathy Marks Krpan**, who is the winner of the Margaret Sinclair Award. **Jacqueline Hill** challenges us to a game of NCTM-style bingo over the summer. **Heather Theijsmeijer** reports on what has been taking place with the OMCA.

Carly Ziniuk avoids getting tangled while curve-stitching! **Lynda Colgan** brings out the challenges of teaching position to early learners. **Iain Brodie** and **Hatice Beyza Sezer** interview **Amanda Deneau**, who has a STEAM focus. **David Costello** considers a model of teaching with a memory perspective. **Jeff Irvine** has a historical peek at assessment. **Ann Kajander** takes **Jennifer Holm's** place for Mb4T, and

brings manipulatives to factoring. **Jérôme Proulx** continues his careful look at perimeter and area, and **Shawn Godin** investigates a problem to do with making change.

Lastly, and unfortunately, that is a pun, **Ron Lancaster** has decided this is to be the last issue with "Mathematical Snapshots." Ron was first published in the *Gazette* in December of 1988, and this column is the 71st time he has appeared in the *Gazette*. Ron is a Past President (1993–1994) of OAME/AOEM, and has presented at 40 consecutive OAME annual conferences. Thank you, Ron, for you support of the *Gazette* and math educators.

This issue brings another OAME/AOEM position paper. The focus of this one is destreaming and detracking.

Krista Francis and **Steven Khan** explain how classroom events can be rendered as comics for an instructional benefit. **Jeff Irvine** takes a look at the Caribou Math contest.

We also have some book reviews to inspire your summer reading. **Ed Schroeter** reviews *Making Math Stick* by columnist **David Costello**. **Tim Sibbald** reviews *The Living Tree* by **Vera Sarina**. A review of an OPHEA resource by **Tim Sibbald** has been waiting for the pandemic to subside.

I wish to express a special thanks to **Nancy Louis** for translating the President's Message. It is also a pleasure to thank **Peter Saarimaki** for joining the editorial team as an associate editor. He joins the expert company of **Anne Yeager**, **Gitta Berg**, and **Penny Clemens**. Thank you all for making the process of building this issue such smooth sailing!

News: Congratulations to Alice Aspinall, a member of OAME/AOEM, who has won a Prime Minister award for math teaching. For details, see www.windsorstar.com/news/local-news/walkerville-math-teacher-honoured-with-prime-ministers-award.

Erratum: Apologies to Dr. Jeff Irvine where, in the December issue of the *Gazette*, the formatting of some numbers in Table 2 on page 46 is not right. Some two-digit numbers in the table became vertically aligned, which is awkward for reading.

Letters: *(Good to see several!)*

Conference Ring? Response to the ring question on page 70 of the last issue.

My daughter-in-law made some math jewellery and buttons for me. I asked her to include the spiral that was the 2018 logo in rings. I gave one to Jill.

There were also infinity, pi, Fibonacci, etc.

Lynda Ferneyhough

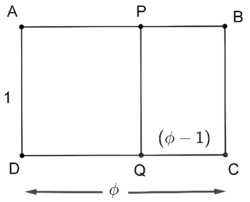
Problem Contributions

Shawn Godin, who has been a columnist for the *Gazette* for many years, is a co-editor of a publication, *MathemAttic*, focused on problems for pre-university students and their teachers. The publication is interested in contributions of problems, but also people, who might be interested in helping with reviewing and editorial support. Please contact Shawn for details: GodinMathStuff@gmail.com.

Binet's Formula

In the December 2021 issue of the *Gazette*, the article by Jeff Irvine states, without derivation, Binet's formula for Fibonacci numbers. The accompanying derivation incorporates the golden ratio, as well as Binet and Fibonacci, for students with knowledge of the quadratic formula.

Rectangle *ABCD* and *BCQP* are similar:



$$\therefore \frac{\phi - 1}{1} = \frac{1}{\phi}$$

$$\therefore \phi^2 - \phi = 1 \ (Eq. 1)$$

$$\therefore \phi^2 - \phi - 1 = 0$$

$$\therefore \phi = (1 \pm \sqrt{1+4}) / 2$$

Let $\phi = (1+\sqrt{5}) / 2$ and $\tau = (1-\sqrt{5}) / 2$

Using Eq. 1 $\phi^2 = \phi + 1$:

$$\phi^3 = \phi^2 \cdot \phi = (\phi + 1)\phi = \phi^2 + \phi = \phi + 1 + \phi = \phi 2 + 1$$

$$\phi^4 = \phi^3 \cdot \phi = (2\phi + 1)\ \phi = 2\phi^2 + \phi = 2(\phi + 1) + \phi = \phi\ 3 + 2$$

$$\phi^5 = \phi^4 \cdot \phi = (3\phi + 2)\ \phi = 3\phi^2 + 2\phi = 3(\phi + 1) + 2\phi = 5\phi + 3$$

$$\phi^6 = \phi^5 \cdot \phi = (5\phi + 3)\ \phi = 5\phi^2 + 3\phi = 5(\phi + 1) + 3\phi = 8\phi + 5$$

Coefficients of ϕ and constants are (1, 1), (2, 1), (3, 2), etc. These are the Fibonacci numbers F_n:

F_1	F_2	F_3	F_4	F_5	F_6	...
1	1	2	3	5	8	...

From the above pattern:

$$\phi^n = F_n\phi + F_{n-1} \text{ and similarly, } \tau^n = F_n\tau + F_{n-1}$$

$$\therefore \phi^n - \tau^n = F_n(\phi - \tau)$$

$$\phi - \tau = (1+\sqrt{5}) / 2 - (1-\sqrt{5}) / 2 = \sqrt{5}$$

$$\therefore F_n = (\phi^n - \tau^n) / \sqrt{5}$$

$$\therefore F_n = \frac{1}{\sqrt{5}} \left(\left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right) \text{ Binet's Formula}$$

Corollary $\frac{F_n}{F_{n-1}} \to \phi$ as $n \to \infty$

I really like this one, as it ties in golden ratio and, I think, has a real "Wow!" factor.

Tom Griffiths ▲

President's Message



JUDY MENDAGLIO
judy.mendaglio@oame.on.ca

Judy Mendaglio is the President of OAME/AOEM. She has worked in education in Ontario in many capacities: college instructor, secondary teacher and department head, university instructor, elementary and secondary consultant, and writer. Currently, Judy devotes most of her time to her many volunteer activities.

As unbelievable as it feels to me, I have served as the OAME/AOEM President for a total of three years, and now, as the term of my second presidency is coming to an end, I am writing my final (of 12) President's Message. As I reflect upon my incredible good fortune to have had the opportunity to lead this amazing association (twice!), I am feeling so very grateful to the many people who have offered me guidance, encouragement, and even the occasional forceful nudge, all of whom were instrumental in any successes I have had.

I was extremely fortunate to have had two innovative and forward-thinking department heads, Jeff Irvine and John Rodger, both of whom were very involved (and continue to be) in OAME/AOEM as well as their regional chapter, CHAMP (no surprises here). These two mathematics educators provided the fuel that launched my voyage to the wonderful world of OAME/AOEM.

It was Jeff's encouragement and open-mindedness that led to the development of an innovative, individualized, five-day summative, culminating in a one-on-one role-play interview (long before *Growing Success*), and it was his nudge that got me to submit my first OAME Annual Conference session proposal. A participant at that session was someone from NOMA, and this connection led to an invitation to travel to Sudbury to speak at their regional conference. This was the first time I really had a sense of the chapter structure of OAME/AOEM, and the targeted and wonderful work that chapter members do to address the needs of teachers in their regionals.

John Rodger was heavily involved in organizing CHAMP's Math Olympics, and because he recruited me to help, I began a relationship with the wonderful people at CHAMP, which culminated in my assuming the role of President of CHAMP. John Rodger was also a forward-thinking math educator, and he introduced me to new ways of thinking about instruction and assessment, especially with

Message de la présidente



JUDY MENDAGLIO
judy.mendaglio@oame.on.ca

Judy Mendaglio est la présidente de l'AOEM/OAME. Elle a exercé de nombreuses fonctions dans le secteur ontarien de l'éducation : formatrice au niveau collégial, professeure et chef de section du secondaire, chargée de cours au niveau universitaire, consultante pour le primaire et le secondaire et écrivaine. Judy se consacre actuellement à de nombreuses activités bénévoles.

Aussi incroyable que cela me paraisse, j'ai été présidente de l'AOEM/OAME pendant un total de trois ans, et maintenant, alors que le mandat de ma deuxième présidence touche à sa fin, j'écris mon dernier (sur 12) message de la président. Alors que je réfléchis à mon incroyable chance d'avoir eu l'occasion de diriger cette incroyable association (deux fois !), je me sens très reconnaissante envers les nombreuses personnes qui m'ont offert des conseils, des encouragements et même un coup de pouce énergique occasionnel, tous qui ont contribué à tous les succès que j'ai eus.

J'ai eu la chance d'avoir deux chefs de département innovants et avant-gardistes, Jeff Irvine et John Rodger, qui étaient tous deux (et continuent d'être) très impliqués dans l'AOEM/OAME, ainsi que dans leur chapitre régional, CHAMP (pas de surprises ici). Ces deux éducateurs de mathématiques m'ont fourni l'énergie qui a lancé mon voyage au merveilleux monde de l'AOEM/OAME.

Ce sont les encouragements et l'ouverture d'esprit de Jeff qui ont conduit à l'élaboration d'un sommatif innovant et individualisé de cinq jours, aboutissant à une entrevue de jeu de rôle en tête-à-tête (bien avant de faire croître le succès). C'était lui aussi qui m'a encouragé à soumettre ma première proposition d'une session à la conférence annuelle de l'AOEM/OAME. Un participant de cette session était une personne de NOMA, et ce lien m'a mené à une invitation de voyager à Sudbury comme présentateur dans leur conférence régionale. C'était la première fois que j'avais vraiment compris le sens de structure de chapitres de l'AOEM/OAME, ainsi que les magnifiques travaux dirigés que les membres de chaque chapitre font pour répondre aux besoins des enseignants et enseignantes dans leurs régions.

respect to the teaching and evaluation of students' mathematical thinking. Many of the assessments we created under his leadership were the subject of OAME/AOEM Annual Conference sessions. In fact, I only missed presenting at one OAME/AOEM Annual Conference from that first one, up until COVID struck. As a result of giving sessions at the annual conferences, I have many, many new math friends, and I continue to learn from and with them. John Rodger also generously recommended me to work on a writing project associated with the 2007 Grades 11 and 12 curriculum revision. It was there I first saw the power of dynamic geometry software (thank you, Greg Clarke!), the creativity that university professors can bring to secondary math topics (thank you, Walter Whiteley!), and the importance of sharing resources in an open and accessible way (thank you, Myrna Ingalls!).

My next major step on the path to the presidency of OAME/AOEM came along with the opportunity to help plan and implement Math GAINS' Math CAMPPP, and then to take a mini-version of that program out to London as a Summer Institute. It was during these two projects that I had the extreme pleasure of working with Anne Yeager, and it was Anne, who first suggested that I should run for a seat on the OAME/AOEM Board of Directors. I honestly scoffed when she first brought this up—me? I was just a teacher. But she was persistent, and lo and behold, when I put my name in for Director, I was elected. No one could have been more surprised than me. But I am forever grateful to Anne for giving me that encouragement and for bolstering my confidence, both of which I really needed. Because of her belief that I belonged in the OAME/AOEM family, I have built relationships with people who have propped me up on many occasions when I felt the ground slide out from under me.

I have sat on the Board of Directors under eight different Presidents. As President, I have had on the "3P Team" (the team of Past-President, President, and President-Elect), the extraordinary group including Tim Sibbald, Jill Lazarus, David Petro, Paul Alves, and Sandra Jean Price. What I have learned from these amazing mathematics educators and wonderful individuals could not fit into a single volume. Nothing happens within OAME/AOEM without the support of the 3P Team. Put these heads together and you know wonderful things will happen. Thank you to each of you for all you have given me.

Finally, I am grateful for the continuous leadership provided by our Executive Directors, Lynda Ferneyhough and Fred Ferneyhough, and for the wisdom and knowledge that resides in the illustrious Bill Otto. These people have been the constants during my time on the Executive Committee, and their strength and patience have always

John Rodger a été fortement impliqué dans l'organisation des Olympiques de mathématiques de CHAMP, et c'est grâce à lui, quand il m'a recruté pour l'aider, que j'ai commencé une relation avec les personnes magnifiques de CHAMP. Cette relation m'a mené à prendre le rôle de présidente de CHAMP. John Rodger était aussi un éducateur visionnaire de mathématiques, et il m'a présenté de nouvelles façons de penser à l'enseignement et à l'évaluation, en particulier en ce qui concerne l'enseignement et l'évaluation de la pensée mathématique des élèves. Bon nombre des évaluations que nous avons créés sous sa direction ont fait l'object de sessions de la conférence annuelle de l'AOEM/OAME. En fait, je n'ai manqué de présenter qu'à une seule conférence annuelle de l'AOEM/OAME à partir de ma première, jusqu'à ce que COVID frappe. En donnant des séances aux conférences annuelles, j'ai beaucoup, beaucoup de nouveaux amis en mathématiques, et je continue d'apprendre d'eux et avec eux. John Rodger m'a aussi généreusement recommandé de travailler sur un projet d'écriture associé à la révision du curriculum de 2007 des 11e et 12e années. C'est là que j'ai vu pour la première fois la puissance des logiciels de géométrie dynamique (merci, Greg Clarke!), la créativité que les professeurs de l'université peuvent apporter aux domaines de mathématiques de secondaire (merci, Walter Whiteley !), et l'importance de partager les ressources dans une façon ouverte et accessible (merci, Myrna Ingalls !).

Ma prochaine étape majeure sur la voie de la présidence de l'AOEM/OAME s'est accompagnée de l'opportunité d'aider à planifier et à mettre en œuvre Math GAINS' Math CAMPPP et ensuite prendre une mini-version de ce programme à l'Institut d'été à London. C'est lors de ces deux projets que j'ai eu l'extrême plaisir de travailler avec Anne Yeager, et c'est Anne qui, la première, m'a suggéré de me présenter pour un siège au conseil d'administration de l'AOEM/OAME. Honnêtement, je me suis moquée quand elle a évoqué cela pour la première fois. « Moi ? Je n'étais qu'une enseignante ! » Mais elle était persistante. Et voilà, quand j'ai mis mon nom pour la directrice, j'ai été élue. Personne n'aurait pu être plus surpris que moi. Mais je serai éternellement reconnaissant à Anne de m'avoir donné cet encouragement et d'avoir renforcé ma confiance, dont j'avais vraiment besoin. En raison de sa conviction que j'appartenais à la famille AOEM/OAME, j'ai noué des relations avec des personnes qui m'ont soutenu à de nombreuses reprises lorsque j'ai senti le sol glisser sous moi.

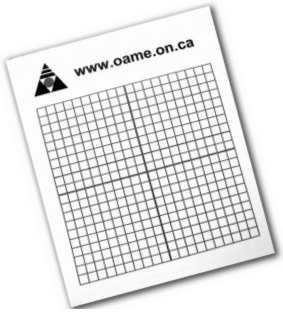
J'ai siégé au conseil d'administration sous huit

been there to support me when my own has waned.

As I say goodbye as President, I hope that everyone who reads this column will think about the people you may never have thanked and make the time to thank them. I hope that I have provided other math educators opportunities for new challenges, as well as the encouragement and support they may have needed to take on new challenges and test new directions. I hope that you, the reader, will look for those Judy Mendaglios around you, who need your encouragement and support, and convince them that they are indeed "the right people." I know that I would be in a very different place if no one had done that for me. ▲

OAME Sticky Graph Pads

Great for Student Use!



3 colours, 2.5" x 3"
50 sheets per pad

$10.00
for 10 pads

Price includes taxes.
Shipping and handling extra.

ORDER BY MAIL, FAX, OR ONLINE:
OAME, 4751 Mack Ave., Town of Plympton-Wyoming, ON N0N 1J6
Phone/Fax: (519) 899-4203, or order online at www.oame.on.ca
Do not send cash or cheque. You will be invoiced, with shipping and handling costs added.

présidents différents. Comme présidente, j'avais « l'équipe 3P extraordinaire » (l'équipe de président-antérieur, président, et président-élu) : Tim Sibbald, Jill Lazarus, David Petro, Paul Alves, et Sandra Jean Price. Ce que j'ai appris de ces incroyables professeurs de mathématiques et de ces personnes merveilleuses ne pouvait tenir dans un seul volume. Rien ne se passe à l'AOEM/OAME sans l'appuis de l'équipe 3P. Mettez ces trois ensemble et vous savez que des choses merveilleuses se produiront. Merci à chacun de vous pour tout ce que vous m'avez donné.

Enfin, je suis reconnaissante pour le leadership continu fourni par nos directeurs exécutifs, Lynda Ferneyhough et Fred Ferneyhough, et pour la sagesse et les connaissances qui resident dans l'illustre Bill Otto. Ces personnes ont été des constantes pendant mon mandat au sein du comité exécutif, et leur force et leur patience ont toujours été là pour me soutenir lorsque la mienne s'est affaiblie.

Alors que je dis au revoir en tant que présidente, j'espère que tous ceux qui liront cet article penseront aux personnes que vous n'avez peut-être jamais remerciées et prendront le temps de les remercier. J'espère avoir fourni à d'autres professeurs de mathématiques des occasions de relever de nouveaux défis, ainsi que les encouragements et le soutien dont ils auraient pu avoir besoin pour relever de nouveaux défis et tester de nouvelles orientations. J'espère que vous, le lecteur, chercherez ces Judy Mendaglios autour de vous, qui ont besoins de vos encouragements et de votre soutien, et les convaincrez qu'ils/qu'elles sont en effet « les bonnes personnes ». Je sais que je serais dans un endroit très différent si personne ne l'avait fait pour moi. ▲

MATH
CODING
BULLSEYE
...oh my!

from Neufeld Learning Systems Inc
since 1999
FOR HOME AND SCHOOL

Congratulations OAME for the 60th Anniversary of *The Gazette*

UMATHXI™
Guided Lessons for K to 10
COUNTING
OPERATIONS
COMPARING & ORDERING
PLACE VALUE
FRACTIONS
EQUATIONS
ALGEBRA
WHOLE NUMBERS & INTEGERS
PERCENT
PROBABILITY
EXPONENTS
GRAPHING
MEASUREMENT & GEOMETRY



Samples and Order Online at
www.umathx.com | info@umathx.com

▲ Blast from the Past! Assessment Through the Ages (of the *Gazette*)



JEFF IRVINE
dr.jeffrey.irvine@bell.net

Jeff Irvine, Ph.D., has been a secondary mathematics teacher, department head, and vice-principal. He has taught at three faculties of education and at a community college. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. With over 45 years in education, Jeff is particularly interested in the interplay of student motivation and mathematics achievement.

Dr. Christine Suurtamm, now Professor Emerita at the University of Ottawa, provided an excellent overview of changes in assessment practices in Ontario (2012). In this column, I look at how these changes in assessment played out in the pages of the *Ontario Mathematics Gazette* (*OMG*).

In the beginning (*OMG* 1(1), 1962), assessment and evaluation focused on evaluation, with sample questions from Grade 13 departmental exams. I found this interesting because they often presented multiple solutions to questions, sometimes as many as four different solutions. It was not clear whether these different solutions were student generated or produced by the posers of the exams. The distinction between assessment and evaluation was blurred, with the terms being used interchangeably. Although assessment refers to the collection of data, and evaluation denotes making a value judgment about that data, the distinction was not referenced. For example, two articles, Bramwell (1978) and McKenna (1978), both had titles with the word "evaluation" in them, but both were really about assessment. After Grade 13 departmental exams ended in 1967, articles on evaluation and assessment were rare over the next 21 years. It was not until 1988 that the word "assessment" appeared again in an article title (Traub et al., 1988).

In the late 1980's and early 1990's, there was recognition that the traditional quiz-test-exam sequence was inappropriate for assessing the problem-solving emphasis in the curriculum and pedagogy. However, the move away from the paramount position of examinations also brought concerns around consistency in reporting of student achievement (e.g., Traub et al., 1989). In addition, views of assessment broadened to include a more fulsome picture of student achievement. This represented a significant paradigm shift in assessment practices, as evidence-based assessment came to the forefront.

In many cases, this new view of assessment requires a shift in thinking that encompasses new understandings about the nature of mathematics, mathematics instruction, and mathematics learning. New assessment techniques also assume a shift in underlying assumptions about the nature and purpose of assessment, learning and teaching (Suurtamm, 2003).

The emphasis on evidence is illustrated by Suurtamm's rubric for teachers: What is the student doing well? How do you know?; What is the student struggling with? How do you know?; What should be the student's next steps? (Suurtamm, 2003).

Articles began appearing, outlining different assessment techniques, such as portfolios (Hall et al., 1996; Suurtamm, 1997, 2004); journals (Lim, 2004); and others, such as student conferencing, using assignments and presentations, as well as peer and self-assessment. A broadening of assessment as data gathering to include questioning was illustrated by Suurtamm (2008). A major contribution to assessment changes was the "Assessment Corner" column, written by Dr. Christine Suurtamm from 2001 to 2007. Suurtamm wrote knowledgeably about numerous assessment issues and introduced terms like *formative* and *summative assessment* to many math teachers. Later, the "Ask Assessment Abby" column ran from 2014 to 2019, and dealt with assessment issues by responding to teacher-submitted questions.

The need for consistency in assessment was also addressed through the years. The *Gazette* featured articles on moderated marking (Traub et al., 1989), as well as rubrics and "checkbrics" (Suurtamm, 2003). As teacher understanding of the new assessment strategies grew, articles began appearing about classroom-tested assessments, such as math trails, as a cumulative assessment task (Hill, 2001) and a polyhedral summative task (Sibbald, 2003).

The links among instruction, assessment, and learning were further enhanced with the advent of Assessment *for* Learning (A*f*L), Assessment *of* Learning (A*o*L), and Assessment *as* Learning (A*a*L). The linkage between new assessment practices and new pedagogy, such as a focus on problem solving, were explored in a *Gazette* article by Hoerath (2011). The importance of student metacognitive learning embedded in A*a*L was recognized to be a major

driver of student engagement (Ciobano, 2014). There were also articles discussing how student assessment could be analyzed to reflect higher-order thinking skills, using research-affirmed structures such as taxonomies (Colgan & Stephen, 2009). More and more articles in the *Gazette* began to reflect classroom practices influenced by research like Dweck's mindset research (Assessment Abby, 2014) and research on the role of engagement and attitudes in mathematics achievement (Irvine, 2020).

Assessment practices in mathematics have come a long way since the era of departmental exams as the ultimate measure of student achievement. We now assess much more than just student performance on a timed, written exam. We are able to integrate assessment with instruction and student learning, and use backward design (Wiggins & McTighe, 2004) to begin with the end in mind, resulting in more useful evidence-based assessments and increased student success.

References

Assessment Abby. (2014). Ask Assessment Abby: Assessing students' mindsets. *Ontario Mathematics Gazette, 53*(1), 32–33.

Bramwell, J. (1978). Evaluation: Close encounters of the fourth kind. *Ontario Mathematics Gazette, 17*(1), 9–10.

Ciobanu, M. (2014). In the middle—Whose learning is it anyway? Increasing students' engagement through assessment as learning techniques. *Ontario Mathematics Gazette, 53*(2), 16–21.

Colgan, L., & Stephen, E. (2009). Hey, it's elementary! Using Bloom's taxonomy to prepare an assessment blueprint, using the Ontario mathematics grade 3 curriculum: A look at the "thinking" distribution of the expectations. *Ontario Mathematics Gazette, 47*(3), 12–20.

Hall, J., Ritchie, L., Morrison, B., & Zimmer, D. (1996). Problem solving and portfolios. *Ontario Mathematics Gazette, 34*(3), 29–32.

Hill, J. (2001). Math trail: Cumulative assessment for grade 10 applied students (proportions, similar triangles, and trigonometry. *Ontario Mathematics Gazette, 40*(2), 8.

Hoerath, E. (2011). Assessment for learning in a problem-solving context. *Ontario Mathematics Gazette, 49*(4), 39–40.

Irvine, J. (2020). Some alternatives to using tests and quizzes for assessment. *Ontario Mathematics Gazette, 58*(3), 10–15.

Lim, L. (2004). Implementing journal writing in grade 9 applied mathematics. *Ontario Mathematics Gazette, 42*(3), 14–18.

McKenna, M. (1978). Evaluation—A general schema. *Ontario Mathematics Gazette, 58*(3), 27–31.

Sibbald, T. (2003). Polyhedra summative task for MAP4C. *Ontario Mathematics Gazette, 41*(3), 17–21.

Suurtamm, C. (1997). Examining portfolio assessment: What it is and how it measures up. *Ontario Mathematics Gazette, 35*(3), 21–29.

Suurtamm, C. (2003). Assessment corner: Using assessment to promote learning. *Ontario Mathematics Gazette, 42*(1), 19–22.

Suurtamm, C. (2004). Assessment corner: Using portfolios in the mathematics classroom. *Ontario Mathematics Gazette, 42*(3), 20–21.

Suurtamm, C. (2008). Formative assessment: Questioning, listening, and responding. *Ontario Mathematics Gazette, 46*(4), 20–24.

Suurtamm, C. (2012). Ontario: Evolution of assessment ideas and practices. *Ontario Mathematics Gazette, 50*(4), 37–41.

Traub, R., Nagy, P., MacRury, K., & Klaiman, R. (1988). Teacher assessment practices in high school calculus: Differences in class time, assignment and test content, and grading practices. *Ontario Mathematics Gazette, 27*(2) 30–35.

Traub, R., Nagy, P., MacRury, K., & Klaiman, R. (1989). Teacher assessment practices in high school calculus: Differences in response to a common marking task. *Ontario Mathematics Gazette, 27*(3) 17–21.

Wiggins, G., & McTighe, J. (2004). *Understanding by design.* ASCD.

▲

▲ Promoting Action with Math: A Review of *Activity Counts*

TIM SIBBALD
Tim.sibbald@oame.on.ca

OPHEA. (2020). *Activity counts: Introducing fundamental math concepts and skills to students (grades 1–6) in a fun, active way!* ISBN 978-1-77184-045-3. Order details available at www.ophea.net/product/activity-counts-.Xmaj_C2-LaY

The Ontario Physical and Health Education Association (OPHEA), like OAME/AOEM, is a subject association in Ontario. It is a registered charity that supplied the *Activity Counts* resource to the *Gazette* without charge.

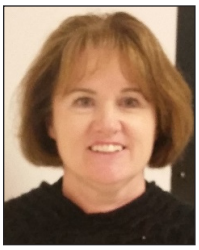


Activity Counts is a collection of activity cards that are labelled according to division, Primary or Junior, and by strand of the mathematics curriculum. Each card is 5½ by 8 inches and provided on card stock that will prevent undue wear and tear. They seem well suited to mobility that the activities encourage. I was surprised, however, that the collection is loose and there is no obvious way to hold the collection together. This issue can be easily addressed with a small binder clip or perhaps by punching a hole through the collection and adding a ring clip.

The content is varied. There are 11 Number Sense and Numeration activities, 3 for Measurement, 6 Geometry and Spatial Sense, 3 for Patterning and Algebra, and 4 for Data Management and Probability. In this respect, it provides enough activities that they can be used, or reused, through the school year on a regular basis, without too much repetition. In many cases, students will be aware of the embedded mathematics activity, but are likely to find motivation in the active engagement, and are likely to want to play each activity more than once. The timing of activities can be relatively short. However, the time can be lengthened using options identified in the activity (such as

Continued on page 14

▲ OAME/NCTM Report: April Showers May Bring May Flowers! And June Brings a Much-Needed Recharging of Your Batteries!



JACQUELINE HILL
jacqueline.hill@bell.net

Jacqueline Hill is a Grades 9–12 teacher of mathematics. She is a Past President of OAME and OMCA, as well as the recipient of the award for Exceptional Teaching in Secondary Mathematics. She also wrote the "Director's Dialogue" for the Gazette *for a number of years.*

The days are longer, the days are warmer, there is a wee joy springing in our hearts. Summer is a time for us. For us to do what we want. Not what our profession requires, but truly what we want to do. Sleep late or get up early. Our time is somewhat our own!

The past five June *Gazettes* have suggested books you might want to read in the summer (a book tasting of sorts), fuel in your tank (with NCTM conferences available), what the next 25 years of mathematics might look like, and finally, a tic-tac-toe of self-reflection and recharging of your batteries. I am changing it up a bit, but keeping to the same themes of things just for you. Just for you, to keep you calm and grounded, happy, and content!

Please feel free to get a line (horizontal, vertical, or diagonal) and then a black-out card as you enjoy some summertime fun! Recharge your batteries, stay safe, and be ever optimistic :)

ALL of the items found in this game can be found on the NCTM website (and you don't even need to be a member to access any of them).

2022 NCTM Annual Meeting & Exposition in Los Angeles, #NCTMLA22

September 28–October 1, 2022

Join the National Council of Teachers of Mathematics, the public voice of mathematics education, in Los Angeles at the LA Convention Center for the NCTM Annual Meeting & Exposition. It is the premier professional development event for mathematics educators, from Pre-K–12 and teachers to university professors. This event will feature hundreds of education sessions, and provide a full range of program content, including learning opportunities, networking, and collaboration.

Yours in mathematical fun,
Jacqueline ▲

N	C	T	M
Set a timer for 5 minutes to accomplish having one clean flat surface in your office/ workroom.	Plan to gather details on the 2022 NCTM Annual Meeting & Exposition in Los Angeles, Sept. 28–Oct. 1, 2022.	Read a book that truly interests you (and not one for work or class).	Take a nap.
Reach out to a colleague or friend, with whom you have not been in touch for a while. Just catch up (perhaps a virtual coffee or wine tasting).	Take some time to enjoy some sunshine.	Check out the "Notice and Wonder" section of the NCTM website. This helps students become active doers of mathematics.	Enjoy some time on the water. Maybe just paddle around in a canoe or go fishing.
Don't spend any money for one day.	Check out the "Problem of the Week" section on the NCTM website. The questions have many entry-level places.	Watch a show that makes you laugh.	Start a gratitude list. Write down one thing a day for which you are grateful.
Do an exercise you enjoy.	Check out some of the 13 different subheadings under the "Classroom Resources" tab on the NCTM website.	Dig out a craft/hobby you would like to spend some time on (but just can't quite get to during the school year).	Consider taking out an NCTM membership. Benefits include journals and access to a wide professional development community.

▲ Events at the Fields



ANGELICA MENDAGLIO
angelicamendaglio@gmail.com

Angelica Mendaglio is an instructional designer at Vretta Inc. in Toronto, Ontario, where she helps to create interactive digital mathematics lessons and activities for middle school students.

The Margaret Sinclair Award is given annually to a Canadian educator, who has demonstrated excellence and innovation in mathematics education. Each year, the winner presents a talk at the Fields Institute, and it always proves to be one of the most engaging talks of the year. This year, the winner was Dr. Cathy Marks Krpan, a professor of math education at the Ontario Institute for Studies in Education in Toronto, and author of several math education books. The announcement of the award stated that Cathy has "a prolific body of work aimed at helping students overcome their fear of mathematics." This was the prevailing theme of her address to the Fields Institute, entitled "Empowering All Learners to Become Deep Thinkers of Mathematics."

Cathy focused on valuing student voices. She told the attendees that she learned a lot about this practice while working with First Nations communities—that valuing student voice is crucial, creating spaces for students to share their voices in safety is imperative, and that part of valuing student voice is "quieting our own voices and listening to theirs." She says these were not practices that she was told about by the teachers she was working with, but rather, that they were embedded practices that she observed in their classrooms. She highlighted that it can be very difficult to quiet our own voices as teachers because we often want to "save" the students who are struggling. However, in doing so, we deprive students of the opportunity to express themselves as mathematical thinkers, and have those expressions be honoured and valued, even if (and perhaps *especially* if) they reveal misconceptions.

The first section of Cathy's talk centred around early explorations with writing in math. She worked to see how we can better support students with communicating in math and encourage them to share their thinking. Even with a more conversational question, such as, "If someone asked you what algebra is, what would you say?," you can gain insight into how students think, and reveal misconceptions, even among the students who are doing well. Cathy emphasized the importance of having these types of discussions to honour all viewpoints, even those that are negative, to foster that safe space, where students can express themselves and know that their viewpoints will be respected and valued. One example Cathy provided was a student who responded to the question, "Is this correct or incorrect? Why? 4 + 1 = 7" with "It is wrong because the teacher said it is wrong." Cathy argued this is an example of how students can attribute the role of mathematical thinker to someone else, and encouraged working to find ways of empowering students to see themselves as capable mathematical thinkers.

The next section of Cathy's lecture focused on mathematical argumentation as a means of accessing student thinking. An analogy she used was interviewing an artist about their art—you will gain a much deeper insight into the artist's thinking and intention by listening to the artist describe their work directly than by looking at the work and interpreting it yourself. Cathy gave a great example of working with proofs in a Kindergarten classroom, where the students were so proud of their proofs at the end that they all wanted their pictures taken with their work. It was an excellent example of how students can be empowered to take ownership over their mathematical thinking.

Another area of research that Cathy spoke about was her work with English Language Learners (ELLs). She interviewed a number of ELLs about their experience in math class. One sentiment that was expressed by many of the students she interviewed was that their teacher will think that they are struggling because they are quiet, and that when they are given "the easier math" as a result, they think that the teacher is telling them (and the rest of the class) that the teacher doesn't have confidence in their ability. Here, Cathy circled back to the idea of teachers wanting to "save" struggling students, and that while it comes from a good place, it can be harmful to a student's self-image as a mathematical thinker. She argued that a student having another language to draw on should be seen as an advantage, not a disadvantage, and that we as teachers should help students make connections between the language they are learning in math class and the language they use at home.

Cathy also spoke about her research on self-talk in math classrooms. She started by describing the *think-aloud* strategy in language arts, where teachers vocalize their internal voice when reading, to model how they gain

understanding while reading. One benefit to using this type of exercise in a math class that Cathy highlighted is that it can dispel many of the most common misconceptions about what it means to be a "good mathematical thinker"—in particular, the idea that good mathematical thinkers arrive at the answer quickly, without needing to take time to understand the question, consider strategies, or make mistakes. When modelling mathematical thinking with a *think-aloud*, Cathy emphasized that it is important to vocalize the struggle, planning, consideration of different options, and corrections to earlier ideas. The key is that students see that thoughts can appear that are not directly leading to the question's answer, and that this is not only okay, but an important part of mathematical thinking. Cathy argued that *think-alouds* can be very revealing for students, and can encourage self-talk practices that help students in their development and self-regulation.

After the conclusion of the talk, Cathy responded to questions from the attendees, one of which was how she recommends dealing with those who disagree with this style of teaching. Cathy's response was that the best way to handle these kinds of questions is through experience. She has found that if she takes parents/guardians through a number talk and is able to reveal the different ways that people can think about a question and arrive at an answer, it can really help to demystify what is happening in the classroom. She points out that the curriculum does not specify that students should be able to use a specific strategy, and so allows for many different access points. She also thinks it is important to clarify that the more traditional ways are not being discarded, as she finds that people can become defensive if they think that the way they were taught is being delegitimized. The key is that all ways of thinking are being embraced, the traditional ways included.

I highly recommend watching Cathy's lecture. The recording can be found on the Fields Institute YouTube channel, in a video titled *MathEd Forum: February 26, 2022* (www.youtube.com/watch?v=i3etgGa_Hd4). The talk was loaded with examples of her experience in classrooms, and Cathy is an engaging and enthusiastic speaker.

The Fields MathEd Forum will convene on the last Saturday in September, October, and November. You can sign up to attend remotely through the Fields website, where you can also see the schedule of other upcoming math education events. ▲

▲ What's the Problem? Counting Coins



SHAWN GODIN
GodinMathStuff@gmail.com

Shawn is a former mathematics teacher, department head, and consultant. He strongly believes in the central role of problem solving in the mathematics classroom. He continues to be involved in mathematical activities: presenting workshops, writing articles, working on local projects, and helping create mathematics contests.

Welcome back, problem solvers. Two issues ago, I left you with the following problem:

> *You have 164 coins made up of nickels, dimes, and quarters worth $19.50. When you calculate the value of all the nickels, all the dimes, and all the quarters, you notice that two of the three totals are a whole number of dollars. How many coins of each type do you have?*

This problem was inspired by a problem from the *About Problem Solving* column in the *Ontario Mathematics Gazette*. The problem was introduced in McLoughlin, 1995, and was discussed in more detail in McLoughlin, 1996.

When we are faced with a problem that we are not sure how to attack, it is often fruitful to "play" with it. This can involve some trial and error, as well as altering the conditions of the problem to consider a similar problem that might yield a solution more easily.

Let's start by creating a spreadsheet (see Figure 1), where the number of nickels, dimes, and quarters can be entered, while the total number of coins and the total amount of money will be calculated for us. We could then play with the numbers of coins of each type to search for our result. A discussion of how to create the spreadsheet used in this article, as well as how to use it to solve our problem, can be found at https://youtu.be/-794oO2vnKE.

We have a couple of choices of how to proceed. Our first solution will focus on a total of 164 coins. To start, let's make things easy and pick 100, 60, and 4, as in Figure 1. When those numbers are entered, we have the correct number of coins, but we don't have enough money. We can replace some coins of one type with another, while keeping the number of coins the same. For example, replacing a nickel with a dime increases the amount of money we have, while replacing a quarter with a dime reduces the amount of money. Since we need to increase the amount of money by quite a bit, we will

replace some nickels with quarters to get the biggest change (ha, ha… change… get it?). In the next line, 10 dimes have been replaced by 10 quarters, and the total amount of money went up by $2. We are on the right track, but still off by a bit. Next, we replace 30 nickels with 30 quarters, and we shoot up to $20. We are now too high, but close. To decrease in value, we change 10 dimes for 10 nickels, and we find a solution!

	A	B	C	D	E	F	G	H
1	Nickels	Dimes	Quarters	# coins	$	N$	D$	Q$
2	100	60	4	164	$12.00	$5.00	$6.00	$1.00
3	90	60	14	164	$14.00	$4.50	$6.00	$3.50
4	60	60	44	164	$20.00	$3.00	$6.00	$11.00
5	70	50	44	164	$19.50	$3.50	$5.00	$11.00

Figure 1: *A spreadsheet solution*

There was nothing that made us focus on the number of coins, so we could have focused on the money. An easy starting point is 195 dimes, which give us the desired $19.50, but that is too many coins. We can decrease the number of coins, while keeping the money the same by replacing a collection of dimes with a collection of quarters worth the same amount. For example, 5 dimes and 2 quarters are both worth $0.50, but has fewer coins using quarters. As you can see in Figure 2, my next step was to replace $5 of dimes with an equal value of quarters, and this brought us close. However, when working with just dimes and quarters, we surpass our mark of 164 coins, so we must bring nickels into the mix to get a solution. Unfortunately, this "solution" doesn't satisfy a condition of the problem, namely having two of the totals as whole numbers.

	A	B	C	D	E	F	G	H
1	Nickels	Dimes	Quarters	# coins	$	N$	D$	Q$
2	0	195	0	195	$19.50	$0.00	$19.50	$0.00
3	0	145	20	165	$19.50	$0.00	$14.50	$5.00
4	0	140	22	162	$19.50	$0.00	$14.00	$5.50
5	2	139	22	163	$19.50	$0.10	$13.90	$5.50
6	4	138	22	164	$19.50	$0.20	$13.80	$5.50

Figure 2: *Focusing on $19.50*

It isn't necessary to focus on keeping either of the amounts from the problem constant through our search. We can manipulate our choices to approach both goals simultaneously, as shown in Figure 3. In this case, again, the "solution" we found fails to satisfy all conditions of the problem.

	A	B	C	D	E	F	G	H
1	Nickels	Dimes	Quarters	# coins	$	N$	D$	Q$
2	50	50	50	150	$20.00	$2.50	$5.00	$12.50
3	80	50	40	170	$19.00	$4.00	$5.00	$10.00
4	85	40	40	165	$18.25	$4.25	$4.00	$10.00
5	85	35	44	164	$18.75	$4.25	$3.50	$11.00
6	80	35	49	164	$19.75	$4.00	$3.50	$12.25
7	85	30	49	164	$19.50	$4.25	$3.00	$12.25

Figure 3: *Working both numbers at once*

Students employing spreadsheets would use variations of one of the three strategies above and probably a few of their own invention.

At this point, it is fair to ask if the result found in Figure 1 is the only solution. Can we continue to play with the "close" solutions in Figures 2 and 3 to construct other valid solutions? To answer these questions, we need to focus on the relationship among the three different types of coins. Consider this with a simpler problem by reducing our total value to $0.50 and ignoring all other conditions. This way, we can focus on relationships among the numbers of the three types of coin involved.

What can we tell right away? Since we want the total value to be $0.50, we could have, at most, 10 nickels, and that would only happen if we only used nickels. Similarly, we could have at most 5 dimes or 2 quarters. This corresponds to 198 different possibilities (11 x 6 x 3, since we could have 0, 1, 2, 3,…, 9, 10 nickels; 0, 1, 2, 3, 4, 5 dimes; and 0, 1, 2 quarters). Not all of these yield our desired $0.50. To get solutions with the correct value, we could use coding and write a program to try all cases and print those that work. However, we will proceed by hand working systematically.

Since quarters have the largest value, we will focus on them first, then dimes, then nickels. Hence, to make $0.50, we could use 2 quarters. If we use 1 quarter, we have the remaining $0.25 to be made up of nickels and dimes. If we focus on dimes, we could use 0, 1, or 2. If we use 2, we have $0.05 and we must use 1 nickel. Continuing systematically like this, we get the results in Figure 4.

Nickels	Dimes	Quarters	Total Money	Coins Used
0	0	2	$0.50	2
1	2	1	$0.50	4
3	1	1	$0.50	5
5	0	1	$0.50	6
0	5	0	$0.50	5
2	4	0	$0.50	6
4	3	0	$0.50	7
6	2	0	$0.50	8
8	1	0	$0.50	9
10	0	0	$0.50	10

Figure 4: *Representing $0.50 with nickels, dimes, and quarters*

There are ten different combinations of coins that give us $0.50, using anywhere from 2 to 10 coins almost inclusively (only a 3-coin solution is missing). This suggests that there will be many ways to represent $19.50, using different combinations of coins. Another interesting thing to notice is that there are two pairs of combinations that use the same number of coins. That is, 3 nickels, 1 dime, and 1 quarter, and 5 dimes, are different 5-coin combinations that yield $0.50. Similarly, 5 nickels and 1 quarter, and 2 nickels and 4 dimes, are different 6-coin combinations that yield $0.50.

What can we learn from this? Upon closer inspection, the two 5-coin combinations both have dimes. If we remove the common dimes, we get 3 nickels and 1 quarter, and 4 dimes, which are two different 4-coin combinations that yield $0.40.

Similarly, reducing the two 6-coin combinations give the same two 4-coin combinations of $0.40. What does that mean? It means that if we have any combination of coins and we replace 3 nickels and 1 quarter with 4 dimes, or vice versa, we will have a different combination of coins, yet the number of coins and amount of money are both unchanged. Thus, if we return to the situations presented in Figures 1, 2, and 3, we can use our new-found relationship to "step" between situations that have the correct number of coins and the correct amount of money to look for any that satisfy the condition that two of coins yield an integer number of dollars. I will leave this search to interested readers.

Is this our only possible solution? The problem seems to be screaming out for variables, so let's look at an algebraic representation. If we let n, d, and q represent the number of nickels, dimes, and quarters, respectively, we can represent the numerical part of the information with the following system of equations:

$$\begin{aligned} 5n + 10d + 25q &= 1950 \\ n + d + q &= 164 \end{aligned}$$

However, we haven't accounted for *every* constraint in the problem. The problem states that two of the three coins total an integer number of dollars each. That means, the "extra" $0.50 must be made up by one type of coin. From Figure 1, we know there is a solution where nickels make up the extra $0.50, so let's model that here by *removing* 10 nickels from the collection. Now *all* coins yield an integer number of dollars. We can do this removing by assigning a new variable n', such that $n = n' + 10$, that is n' is the number of nickels after we have removed the extra $0.50. We can now rewrite our system of equations, using the new variable:

$$\begin{aligned} 5n' + 10d + 25q &= 1900 \\ n' + d + q &= 154 \end{aligned}$$

Since we have a whole number of dollars for each of these variables, we know n' must be a multiple of 20, d must be a multiple of 10, and q must be a multiple of 4 (why?). Therefore, we can introduce three new variables—N, D, and Q—that satisfy $n' = 20N$, $d = 10D$, and $q = 4Q$. Thus N, D, and Q represent the number of dollars we have in nickels, dimes, and quarters after the extra $0.50 was removed. Rewriting our system in terms of N, D, and Q:

$$\begin{aligned} 100N + 100D + 100Q &= 1900 \\ 20N + 10D + 4Q &= 154, \end{aligned}$$

then dividing out common factors leaves us with:

$$\begin{aligned} N + D + Q &= 19 \\ 10N + 5D + 2Q &= 77 \end{aligned}$$

Subtracting twice the first equation from the second yields $8N + 3D = 39$. Since we are after non-negative integer solutions, we could just solve by trial and error. However, 39 is a multiple of 3, so we could rewrite this equation as:

$$8N = 3(13 - D).$$

This tells us that since 8 and 3 share no common factors, N *must* be a multiple of 3, which narrows our search quite a bit. We can also argue that $13 - D$ must be a multiple of 8, which has only two solutions: $D = 5$ and $D = 13$. Working backwards through our variable definitions shows that $D = 5$ corresponds to the proper solution that we found in Figure 1. Similarly, $D = 13$ yields a new solution: 10 nickels, 130 dimes, and 24 quarters.

We can now go back and do a similar analysis of the cases where the extra $0.50 is either dimes or quarters. I will leave this as an exercise for interested readers.

Let's try another tactic. We will attack the problem head on to see if the extra $0.50 takes care of itself. Thus, if we go back to our first system of equations and divide out the common factor of 5 in the first equation, we are left with:

$$\begin{aligned} n + 2d + 5q &= 390 \\ n + d + q &= 164 \end{aligned}$$

Subtracting these equations yields $d + 4q = 266$. Thinking in terms of divisibility again, we see that 4 and 226 are both even, so d *must* be even as well. However, if the extra $0.50 were coming from the dimes, we would have to have either 5, 15, 25, 35,… dimes, which is impossible if d is even. Hence, the extras must come from nickels (which we solved above) or quarters. If this expression is rearranged to $d = 266 - 4q$, we can substitute it into the second equation and rearrange to get $n = 3q - 62$. Those of you who know a little about vectors will recognize the equations in our system as the equations of non-parallel planes in R^3, which must intersect in a line. Our result:

$$\begin{aligned} n &= 3q - 62, \\ d &= 226 - 4q, \\ q &= q, \end{aligned}$$

when q is an arbitrary real number, are the parametric equations of the line of intersection of the two planes. Each value of q generates values of n and d so that the three values is a solution to the system. In our case, we are interested in all three of our variables being non-negative integers. This means we must have $3q - 62 \geq 0$, $226 - 4q \geq 0$, and $q \geq 0$, with q being an integer. Solving these inequalities leads to:

$$21 \leq q \leq 56,$$

which means that there are only 36 triples of non-negative integers (n, d, q) that satisfy the system. We could work them all out by hand, or with technology, and find all the ones we want.

However, if we think a bit more, we can reduce our work. By the conditions of the problem, each coin will yield either a whole number of dollars, or a whole number of dollars with an extra $0.50. This means that each coin must yield a value that is a multiple of $0.50. It turns out this condition is a bit

weaker than the condition in our problem (why?). Consequently, if we use this idea in our solution, we will have to check any results to make sure they satisfy the stronger conditions in the problem.

How does this help? To begin with, since the total value of quarters must be a multiple of \$0.50, q must be a multiple of 2. Hence, we can rewrite our system as:

$$n = 6Q - 62,$$
$$d = 226 - 8Q,$$
$$q = 2Q,$$

where Q is a non-negative integer that must satisfy:

$$11 \le Q \le 28.$$

We have already reduced the number of items we have to check to 18.

Continuing in this way, if we consider that the money in dimes is also a multiple of \$0.50, d must be a multiple of 5. Hence, we can write $226 - 8Q = 5k$ for some non-negative integer k. If we get creative, we can rewrite this as $(225 + 1) - (10Q - 2Q) = 5k$ which can be rearranged to $2Q + 1 = 5(k + 2Q - 45) = 5l$ for some non-negative integer l. Thus, $2Q = 5l - 1$, which means that l must be odd (why?), so we can write $l = 2L - 1$ for some non-negative integer L.

That's a lot of new variables. How does this help? If we work our way backwards, we can rewrite the parametric equations in terms of the new variable L to get:

$$n = 30L - 80,$$
$$d = 250 - 40L,$$
$$q = 10L - 6,$$

where L is a non-negative integer that must satisfy:

$$3 \le Q \le 6.$$

Now the system has exactly four solutions, each of them yielding a valid solution to the problem. I will leave it to the reader to verify these solutions.

We started with a problem that could be understood and tackled or explored by students of a fairly young age (especially if we reduced the numbers in the problem). We saw that we could use spreadsheets to aid our computations. Along the way, we discovered some patterns (notice in the solution, we are "stepping" between solutions by replacing 40 dimes with 10 quarters and 30 nickels; where have we seen that before?) that helped us use more and more sophisticated techniques to solve the problem. Those of you familiar with modular arithmetic could rewrite the last part of the solution in a more compact and concise way.

This is an example of *Diophantine equations*, that is, equations where variables are restricted to either integer values, or non-negative integers. It is interesting how "simplifying" the problem by moving from the real numbers to the integers can, in some cases, increase its difficulty.

Now for your homework.

Create seven mathematical expressions, whose answer is 7, using as many operations as possible.

Until next time, happy problem solving.

References

McLoughlin, J.G. (1995). About problem solving: Making math with 1995. *Ontario Mathematics Gazette, 33*(3), 27–28.

McLoughlin, J.G. (1996). About problem solving: A curious sequence. *Ontario Mathematics Gazette, 34*(3), 17–18. ▲

Book Review continued from page 8

specifying a different operation) or modifications provided with each activity card.

In addition to the set-up and instructions, each card identifies the purpose, equipment, modifications, and questions for checking student understanding. Some sample activities are described briefly below.

Twister Movement – The combination of a spinner to select a body part and a pair of cards to identify a Cartesian coordinate sounds like a fun way to learn about coordinates. The activity card reminds to discuss "physical and emotional safety, since students will be moving in close proximity to each other." I was a little unsure about the set-up requirement for two quadrants, and wondered if they meant two axes; however, I can picture this activity being extended to use integers.

Forehead Fun – Using numbered cards from a deck, a group of three students has a "mathematician" and two "solvers." Each of the two solvers picks a random card and, without looking at it, holds it on their forehead. The mathematician selects whether the sum or product is to be used, and announces the result based on seeing what cards both solvers are holding up. Each solver then has to determine what card they are holding up. (In the same category, *Move to the Pattern* has clearer activity, but *Forehead Fun* can be set up to entail physical movement.)

Rock-Paper-Scissors Probability – This activity aims to connect theoretical and experimental probability. Each round entails a full-body physical action (e.g., crouch in a ball for a rock). Students tally the outcomes of a number of trials.

Among the other activities, I'll give an honourable mention to *Order Up*, which caught my eye as a Geometry and Spatial Sense activity that could also be used to draw on logical thinking skills. It again struck me as an activity that was described for Primary that has potential for other divisions. Similar remarks also apply to *Bank Bonanza*.

All this is to say that OPHEA has generated a resource that will be a useful addition to many classrooms. It provides (pun intended) "wiggle room" and encourages physical activity that supports both mathematical skills and being active. ▲

▲ OMCA News: June 2022

HEATHER THEIJSMEIJER (OMCA PRESIDENT)
OMCAmath@gmail.com

The Ontario Mathematics Coordinators Association (OMCA) continued to offer exciting opportunities for mathematics coordinators, consultants, and coaches throughout the winter and spring.

Recent members' meetings have included guest speakers Dr. Saskia Van Viegen on "Addressing Multilingualism in Math Teaching, Learning, and Assessment Practice," Dr. Gurpreet Sahmbi on "How the Other Side Lives: Sharing Insights on the Transition from Secondary to Tertiary Mathematics," Dr. Marian Small on "Thinking about What Matters Is Central to Inclusion," and Ron Lancaster on "Patterning, Coding, and Chika's Test."

New this year, we hosted a series of "Armchair Sessions"—opportunities for members to join experts in the field for a deep dive into one aspect of our practice. Our first session was with a team of colleagues at EQAO, and focused on the implementation of the new digital assessment for Grade 9. There was also the opportunity for boards who had already experienced the assessment to share implementation tips and tricks with boards who had yet to do so. A second armchair session was held with EQAO in April about the Primary and Junior assessments.

OMCA's virtual symposium on February 11 saw over 300 delegates come together to learn more about the day's theme of "Rehumanizing Mathematics." It was a full day of presentations by Dr. Rochelle Gutiérrez, Sunil Singh, Dr. Lisa Lunney Borden, Georgina Rivera, and Dr. Paul Gorski. We also tried something new this year by hosting the event in a virtual "town"! Every delegate had the opportunity to create an avatar, and then move around the virtual conference hall to visit the vendor booths, view the sessions, or sit down for a chat with other delegates, all in real time.

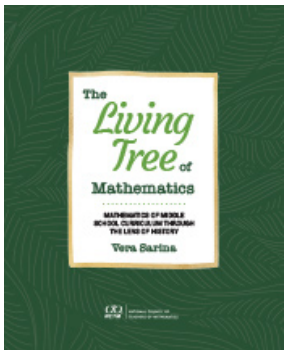
Finally, our literature circles wrapped up in April. OMCA members were able to choose one of four books: *Grading for Equity* (Feldman, 2018), *Math Recess: Playful Learning in an Age of Disruption* (Singh & Brownell, 2019), *Indigenous Writes: A Guide to First Nations, Métis and Inuit Issues in Canada* (Vowel, 2016), or *NCSM Framework for Leadership in Mathematics Education* (2020). We met three times over the course of the year for informal conversation around content that resonated with us, as well as how it impacts our practice.

From Mississauga to Moosonee, and from Atikokan to Arnprior, new members are always welcome to join OMCA. Contact us directly at OMCAmath@gmail.com, or visit our website at www.omca.website (note the different domain), to learn more about the organization, find out about upcoming events, and to connect with us. ▲

▲ Book Review: A Taste of Mathematical History

BY TIM SIBBALD

Sarina, V. (2021). *The living tree of mathematics. Mathematics of middle school curriculum through the lens of history.* National Council of Teachers of Mathematics.



The Living Tree is an apt description of the history of mathematics. There are different branches where much is known, yet parts of the tree are obscured underground. Sarina has taken on the task of looking at the major thematic aspects of the Ontario and Common Core curricula with respect to history. It is timely, given the renewal of history components that the 2020 curriculum includes.

This book is 146 pages and covers a wide swath of mathematics. In this sense, it is a survey of math topics alongside historical snippets, as opposed to examining topics or history in depth. It does avoid the Eurocentric challenges that many math history books have had and opens the door to further investigation of areas of particular interest to individual teachers. This is exemplified by the introduction of the Ishango bone on the second page. Notably, a photo of the Ishango bone and many other artifacts are included in the book. The book is brief regarding details and does not dwell on the conflicting interpretations of the markings on the bone—as this is not the intent of the book.

A topic addressed early in the book is the use of variables, which is followed by different typographic expressions for numbers. This leads into discussion of math as language and language for mathematics. It is rounded out with remarks about different representations within mathematics. Each is provided with historical connections.

In some cases, the book diverges from the historical lens, such as when it discusses the partitive and quotative interpretations of division. Similarly, order of operations is explained without any historical connection. The section regarding measurement begins with a strong historical connection that is followed by an overview of instructional approaches to teaching the underlying concepts of several

Continued on page 41

▲ The 2021 Annual Meeting of the Canadian Mathematics Education Study Group (CMESG)



JENNIFER HOLM
jholm@wlu.ca

Jennifer Holm is an Assistant Professor at Wilfrid Laurier University and works with Primary/Junior and Junior/Intermediate pre-service teachers, as well as in the field supporting current mathematics teachers. She is interested in developing mathematics knowledge for teaching with both pre-service and in-service teachers. She focuses on the beliefs and opinions that pre-service teachers hold about mathematics and teaching, and the connection they have to past experiences. She uses this research to support future teachers in developing beliefs and knowledge that will encourage and support effective teaching practices in mathematics.

The 2021 Meeting of the Canadian Mathematics Education Study Group (CMESG) marked an interesting new frontier, with the entire meeting being held virtually due to concerns with the pandemic. The meeting itself was held June 11–13, 2021, with a pre-event panel held jointly with the Canadian Mathematics Society (CMS) on June 7. CMESG included a plenary, featured five working groups and thirteen new Ph.D. talks, as well as the social events that are an integral part of this conference. Although the entire conference had a different modality in a virtual space, the organizers kept the essence of what makes CMESG such a special event. For me, this is one academic event I look forward to each year, knowing I will interact with my academic peers and hear what is new about the world of mathematics education, both from mathematicians and mathematics educators. In this report, I am giving my own thoughts and summaries of the different activities, but you are invited to read the full reports and write-ups of each of the activities in the CMESG proceedings that will be published on the website before the next annual meeting (Holm & Megroureche, forthcoming).

The panel, moderated by Patrick Reynolds, University of New Brunswick, kicked off the CMESG annual meeting with an examination of data literacy in the classroom, while focusing on the use of real-world data. Four panelists were invited to share their differing perspectives on the topic. Nat Banting, classroom mathematics teacher and mathematics education lecturer in Saskatoon (Saskatchewan), provided a video that detailed his thoughts about engaging secondary students in working with real-world data for their mathematics careers and their roles in society. He spoke about the need to create opportunities for students to engage with data in order to be critical consumers, and not just be shown data-rich activities. Kseniya Graraschuk, University of the Fraser Valley (British Columbia), focused on ideas that she has used in presenting data-literacy activities in her university-level courses. She discussed using planned activities in her course, but also pulling in ideas that were current in the media to provide data-rich opportunities for discussion and interaction with the mathematics. Minnie Liu, from Vancouver, British Columbia, presented ideas based on her experiences as a secondary mathematics teacher that improve the data literacy of her students. She spoke about creating a culture in the classroom that included an emphasis on critical thinking and how data literacy activities provided rich contexts for this discussion. Ryan Lukeman, St. Francis Xavier University (Nova Scotia), provided ideas of problems relating to data literacy with undergraduate mathematics students. He discussed the use of MATLAB (2020) as the environment for activities in his classroom. Each panelist provided opportunities to consider how data-literacy activities using real-world data could be incorporated into a mathematics classroom, as well as the reasons why data literacy is so important in today's society.

The annual meeting plenary was given by Sarah Mayes-Tang, University of Toronto, entitled "Teaching on Empty: Trauma, Achievement, and What's Next in Our Math Education Community." Her opening session focused on personal stories related to living and working in mathematics during the COVID pandemic.

Each working group held three separate sessions with participants, and featured a wide range of ideas. I will now give a very brief summary of some of the discussion points or themes from the conversations over the entire annual meeting.

Working Group A was led by Joyce Mgombelo, Brock University (St. Catharines, Ontario), and David A. Reid, University of Agder (Norway), and was entitled *Learning Theories / Théories (de l') apprenant*. This working group used Lakatos' Methodology of Scientific Research Programs (Lakatos, 1978) to begin their discussions. The participants and organizers grappled with many different nuances about learning theories and ended with many more questions about the discussion than conclusions. The group was organized around the idea of looking at global learning theories and centred their initial discussions around

examining a list of different theories: enactivism, behaviourism, la théorie des situations didactiques, Indigenous knowledge systems, embodied cognition, commognition, interdisciplinary approaches, discourse analysis, and the theory of developmental instruction. Themes from the session included looking at ways of seeing, metaphors, and the meaning of the word "learning" as they discussed theories.

Richelle Marynowski, University of Lethbridge (Alberta), and Jhonel Morvan, Brock University, led Working Group B, entitled *Pour ou contre les tests : Est-ce la bonne question ? / To Test or Not to Test: Is This the Question?* The group framed their discussions around questions related to tests in mathematics, and the purposes and limitations of this method of gathering data. They also explored alternatives to testing. Their initial discussion group divided into two subgroups: one focused on the testing of mathematics teachers and the other on the testing of mathematics students. The working group grappled with issues related to the definition of assessment and how testing fits into the broader landscape of mathematics education. This led to participants discussing alternatives to tests for both mathematics teachers and classes, and considerations of equity, systems of oppression, and other harmful factors that can be inherent in testing.

The Rewards and Challenges of Video in the Field of Mathematics Education: Looking Back in Order to Prepare for the Future / Les apports et défis de la vidéo pour (la formation à) l'enseignement-apprentissage des mathématiques : Regard du passé pour préparer le futur was the title of Working Group C, led by Mathieu Thibault, Université du Québec en Outaouais, and Scosha Merovitz, Bishop's University (Quebec). This group began discussions by examining participants' current experiences with video and both the highlights and challenges of this medium. Different technological tools suitable for working with mathematics students were discussed, as well as websites known to the group that could aid in the conversation and advancement of the use of video in mathematics. Conversations focused on both the teacher and student use of video, with the latter also including the possibilities of having students create videos in mathematics classes.

Working Group D was led by France Caron, Université de Montréal, and Wes Maciejewski, San Jose State University (California), and was entitled *How can we be creative with large classes?* The group examined issues related to engaging in productive mathematics instruction with larger class sizes, including large first-year university math courses or problem-solving classes with 60 students. They discussed the challenges presented when class size is not ideal for the content of the mathematics courses, and solutions that could be used to help make these mathematics classes enjoyable for learning or to teach. They divided their sessions into orienting themselves to the challenges of large class sizes, focusing on the teaching and learning that can and has been used, and then a focus on assessment in these courses. The use of technology and online resources was highlighted by the group as being important and effective for learning.

Lastly, Working Group E was entitled *Returning to Our Roots: Exploring Collaborative Possibilities for Research and Teaching in Mathematics and Mathematics Education*, which was led by Sarah Mathieu-Soucy, Russian School of Mathematics (Massachusetts), Kathy Nolan, University of Regina (Saskatchewan), and Tara Taylor, St. Francis Xavier University (Nova Scotia). This working group organized around the desire to consider collaborations among mathematicians who are teaching and those who work in mathematics education. Through the discussions, the group considered the benefits and struggles, as well collaboration possibilities that could be initiated by the mathematics community. This working group laid the groundwork for continuing discussions among participants to build bridges between the two groups, and engage in productive research for the benefit of all members of the mathematics community.

Thirteen talks, given by people who had recently earned their doctoral degrees, spanned many different areas of mathematics education. One new aspect of this CMESG was that "new Ph.D.s" submitted a video of their talk, to be watched by participants prior to the scheduled time. The talks were then assigned in groups of two for breakout rooms so that conversations and questions could be had between the presenters and their participants. The authors and the titles of their presentations are listed here, and I encourage you to explore the proceedings to read the full papers about the work.

- Marc Husband, *Making Connections to Deepen New Teachers' Understanding of Elementary Mathematics*
- Colette Lemieux, *The Use of Story-Based Tasks in Post-Secondary Students' Learning of Statistics*
- Jimmy Avoseh, *Engaging Multiple Representations in Grade Eight: Exploring Mathematics Teachers' Perspectives and Instructional Practices in Canada and Nigeria*
- Nicolas Boileau, *An Investigation of the Relationship Between Two Norms of the Instructional Situation of Geometric Calculations with Algebra in U.S. High School Geometry*
- John G. Bosica, *Mixed-Methods Study on Mathematics*

Anxiety, Mathematics Teacher Efficacy, and Mathematics Teaching Anxiety on Elementary Preservice Teachers in Ontario

- Laura Broley, *The Development of (Non-)Mathematical Practices Through Paths of Activities and Students' Positioning: The Case of Real Analysis*
- Alexandre Cavalcante, *The Financial Numeracy Afforded in Secondary Mathematics: A Study on the Textbooks, Perceptions and Practices of Teachers in Quebec, Canada*
- Christopher Charles, *Comparing the Effects of Two Inquiry-Based Teaching Strategies on Secondary Students' Conceptual Understanding and Achievement in Mathematics: A Mixed-Methods Approach*
- Caroline Damboise, *Fostering the Development of Reasoning Using Dynamic Geometry Software in Future High School Mathematics Teachers*
- Shana Graham, *Disrupting Euro-Western Onto-Epistemologies: (Re)imagining Possibilities for Mathematics Education through/with Indigenous Knowledges and Complex Conversations*
- Oyemolade (Molade) Osibodu, *Critical Mathematics Education Towards Epistemic Freedom: Exploring Njo's Journey*
- Gurpreet Sahmbi, *A Tale of Two Universities: Investigating Factors Affecting the Secondary to Tertiary Transition into Calculus for Students in STEM Disciplines*
- Xiong Wang, *Understanding Math Teachers' Participation in a PLN*

It was wonderful to see so many CMESG community members give their "new Ph.D." talks and speak about their research and findings.

In the end, although the annual meeting of CMESG was not the traditional model of in-person discussions and engagement, the move to a virtual space still afforded the opportunities to connect, learn, and debate ideas. As with most CMESGs, there were more questions at the end than the beginning, but that hopefully challenged participants to continue their thinking and research into the future.

References

Holm, J., & Megroureche, C. (Eds.). (forthcoming). *Proceedings / actes 2021 annual meeting / rencontre annuelle of the Canadian Mathematics Education Study Group / Groupe canadien d'étude en didactique des mathématiques.* CMESG/GCEDM. www.cmesg.org/past-proceedings/

Lakatos, I. (1978). Introduction: Science and pseudoscience. In J. Worrall & G. Currie (Eds.), *The methodology of scientific research programmes* (pp. 1–7). Cambridge University Press. doi.org/10.1017/CBO9780511621123.002

MATLAB. (2020). *Version 9.9.0* (R2020b). The MathWorks, Inc. ▲

▲ Linking Literacy and Math: Reconsidering Our Perspective on Learning



DAVID COSTELLO
david@costellomath.com

David is a principal, who has held many roles. He taught in the Primary division before assuming roles of numeracy interventionist, numeracy coach, numeracy leader, and curriculum consultant for Prince Edward Island. David has also instructed university courses focused on curriculum, differentiation, mathematics, and language arts. He facilitates professional learning in the areas of mathematics and school development. Dr. David Costello recently published two books; Making Math Stick: Classroom Strategies That Support the Long-Term Retention of Math Concepts *and* Using What Works: Strategies for Developing a Literacy-Rich Environment in Math.

There is beauty in mathematics. Within this beauty is a tapestry that is represented through the interconnectedness of mathematical concepts. However, when considering instruction, far too often this beauty is threatened. Concepts have been compartmentalized in an approach to make instruction *manageable*.

Before moving forward, let me first contextualize what I mean by *manageable*. When using the term *manageable*, I am referring to *coverage*. *Coverage* refers to the instructional focus that ensures teachers and students encounter each of the curricular outcomes. It is, unfortunately, too often a teach, test, and move-on approach to instruction and learning. When the focus is on *coverage*, attention shifts away from learning.

There are factors that contribute to why mathematics has been compartmentalized. First, as mentioned above, teachers are responsible for ensuring students receive instruction and experiences that address all the grade-level curricula. Teachers feel pressure to move through the curriculum at a brisk pace to ensure that they cover all necessary content. Such a pace does not allow for delving deeply into concepts. Second, classroom assessments often focus on the content that has just been covered. This assesses student knowledge and understanding on a short-term basis.

Consider year plans and lessons within units. Although there has been marked progress in the last few years (i.e., spiralled curriculum), much of the instructional sequence students encounter is that of blocked practice (Costello,

2021). Concepts are treated in isolation (in terms of a year plan) and then are further compartmentalized into sub-units. Let's consider measurement. Measurement tends to be treated in isolation in many year plans. Then, when it is time for measurement, lessons are further compartmentalized into sub-units (e.g., length, perimeter, area, volume), an approach to planning that is referred to as blocked practice (Costello, 2021). Blocked practice occurs when students encounter one particular concept, and then all practice questions within the session are focused on this one sub-unit. Within this approach, students are not needing to consider which concept to apply to the problem, as all problems are based on the same concept. Instead of focusing on the problem and strategy selection, the student can focus solely on procedure. Is this meeting our instructional goals?

Within this approach to mathematics, the focus is not so much on learning as it is on short-term performance. Students are presented a curricular outcome, then explore this particular outcome, and are assessed on this very outcome. For too long, there has been the notion that success on assessments is indicative of student understanding. However, students may simply refresh their short-term memory long enough to accomplish the evaluation, and there are only limited connections to long-term memory. The evaluation measures the ability of students to recall concepts that students immediately focused on.

What Do We Mean by Learning?

How do you define learning?

It is not uncommon to see teachers whose practices are consistent with a definition of learning that emphasizes students encountering new knowledge and skills. When the focus is on coverage, the underlying message is that instruction is about transmitting knowledge onto students. This is not what many of us are wanting for our students. This is not what many of us see as mathematical understanding within a literate environment (Costello, 2019).

There is an alternative to the narrow definition, and it is that learning is the acquisition of knowledge and skills, but it is also inclusive of having these readily available from memory to support future meaning-making opportunities. More specifically, there are three stages of learning: encoding, consolidation, and retrieval (Brown et al., 2014).

- **Encoding** means getting knowledge into our heads. During this stage, students encounter a mathematical concept, whether it is through explicit instruction or exploration.
- **Consolidation** means assigning meaning to that knowledge, which gets it into our short-term memory. Connections are established between recently encountered knowledge and knowledge already stored in long-term memory.
- **Retrieval** means getting knowledge out of our heads. Retrieval is the process of reaching back and bringing something we previously learned into mind.

When thinking about how to teach, it is important to consider all three stages of learning (Costello, 2021). Each of the stages plays a considerable role in the learning process. Let's look at the three stages of learning in relation to the make-ten strategy.

- **The encoding stage.** Students, whether through explicit instruction or exploratory activities, realize that they can apply the make-ten strategy to fact learning. Through this process, students encode the make-ten strategy into their short-term memory.
- **The consolidation stage.** Not until students consolidate their new learning with other relevant and interconnected previous learning will the make-ten strategy move from short-term memory to long-term memory. An example of this would be when students make connections between the make-ten strategy, sums to 10, and part–part–whole. By connecting these three strategies, students can make ten and then add on the remaining part of the addend.
- **The retrieval stage.** When students recall and apply this make-ten strategy, they are in the retrieval stage of learning. It is during retrieval that they look for the knowledge in their memory and apply it to a given situation.

It is important to note that our memory generally stores information based on relevance and use. The more we use a strategy, the more accessible it becomes. The less we use it, the more difficult it is to access; hence the necessity to retrieve new information from memory multiple times over a period of time. The more students retrieve the new information, the more securely it sits in long-term memory. This retrieval could take the form of drills, practice, or other challenges. One caveat is that students must understand the strategy before moving to practice.

By focusing on the three-part learning process of encoding, consolidation, and retrieval, you can develop an instructional approach highlighting learning, as opposed to only activating short-term memory (Costello, 2021). By working through the three stages of learning, students can strengthen their use of strategies as well as their strategy-selection skills. With multiple retrieval opportunities, students' recall improves. Students thereby develop automaticity with the facts. Here we have the difference between memorizing (putting isolated facts into short-term

memory) and developing memory (by understanding and repeatedly retrieving).

Why Encoding Is Not Enough

In the introduction of this column, I stated how, when teachers are focused on coverage, we remove the beauty of mathematics. Within such an approach to instruction and learning, it is encoding that takes centre stage. When focusing on the encoding stage of learning, attention is paid to short-term performance.

In classrooms, the teacher instructs students (or helps them to explore) a specific concept. The teacher immediately provides practice questions and tasks. At the completion of a unit, the teacher assesses student understanding of the concept, possibly with a test. Students may demonstrate little to no difficulty with the assigned tasks and tests. Both the teacher and these students then conclude that the students have mastered the relevant math concept. This belief may be inaccurate because students have not learned to solve questions or tasks without knowing the strategy in advance. This fallacy is referred to as illusion of mastery (also illusion of fluency and illusion of knowing) (Agarwal & Bain, 2019; Brown et al., 2014).

Illusion of mastery is problematic for both the teacher and students. When all they can see is short-term performance, both teachers and students assume that comfort using a skill or strategy in the short term equates with effectively using it in the long term. Teachers and students both conclude that students who have demonstrated understanding of a concept once will be able to apply it moving forward. This fallacy in perspective results in teachers and students favouring learning strategies that promote short-term retention instead of learning strategies that are effective for both the short term *and* long term.

When we focus solely on encoding, we are not providing students with opportunities to consolidate new learning with previous learning, retrieve learning from previous experiences, or work through the important step of strategy selection. While students would perform well on immediate assessments, they may not demonstrate this same success in the long term.

We need to recognize the importance of all three stages of learning, as opposed to only encoding. There has been a shift in mathematics to recognize the importance of consolidation. It is through consolidation that students make meaning with knowledge and connect it to previous learning. This connection strengthens the process of remembering and adds additional layers of detail.

We now need to continue our work as teachers to support student learning. We need to recognize the importance of retrieval and how this stage of learning can aid our students in being able to recall and apply previous learning to solve novel problems. While we are beginning to do this more and more through various initiatives, there is considerable work yet to be done.

Reaching Our Goal

Independent learners are those who can apply what they learn to new situations, and who can recognize the knowledge they have and the areas that need additional focus. Our goal, as educators, is to support our students becoming independent learners. This requires students to have accurate metacognition. For students to have an accurate metacognition, we must move beyond an instructional focus of *coverage*, that rests solely on the encoding stage of learning. We must continue our work on consolidation, and we must address and expand our attention to include the retrieval stage of learning. Without including all three stages of learning, we run the risk of an illusion of mastery.

Final Thoughts

How we define learning shapes our instructional practices and the learning environment of our students. To move beyond short-term performance and to encourage students to be independent learners, we must broaden our perspective of learning to include opportunities for retrieval. By including retrieval practice in the classroom, students will strengthen their understanding of mathematics and be able to recall previous learning to solve problems. Retrieval practice enables students to revisit concepts throughout the year, and through this revisiting, students have opportunities to clarify their understanding.

In my next column, I will share learning strategies that can applied to any classroom that will support the three stages of learning. And, through these strategies, students will have an accurate metacognition and will move toward our goal of students becoming independent learners.

I look forward to continuing the discussion of linking literacy and math via my website (www.costellomath.com), email (david@costellomath.com), or Twitter (@dr_costello).

References

Agarwal, P.K., & Bain, P.M. (2019). *Powerful teaching: Unleash the science of learning.* Jossey-Bass.

Brown, P.C., Roediger, H.L. III, & McDaniel, M.A. (2014). *Make it stick: The science of successful learning.* The Belknap Press of Harvard University Press.

Costello, D. (2019). *Using what works: Strategies for developing a literacy-rich environment in math.* Rubicon Publishing Inc.

Costello, D. (2021). *Making math stick: Classroom strategies that support the long-term understanding of math concepts.* Pembroke Publishers Ltd. ▲

▲ Mathematical Snapshots: Stepping aside, Step up!



RON LANCASTER
ron2718@nas.net

*Ron Lancaster is an Associate Professor at the University of Toronto, where he teaches mathematics courses for pre-service middle and high school teachers. He has over 20 years of experience teaching Grades 7–12 mathematics. Ron's professional activities include consultations and conference presentations in North America, Asia, England, the Middle East, Africa, India, and Europe. Ron is an author for the NCTM (*The Mathematical Lens*) and member of the Advisory Board for the Museum of Mathematics in New York City. He is the recipient of the 2015 Margaret Sinclair Memorial Award Recognizing Innovation and Excellence in Mathematics Education, awarded by the Fields Institute.*

This is going to be my last "Mathematical Snapshots" column. It is time for me to *step aside* and give someone else a chance to *step up* and take the column in a new direction.

All of the photographs in the "Mathematical Snapshots" columns that have been published to date were taken by myself in various locations around Ontario. For a departure (literally), I decided to take readers on a field trip to some of my favourite places outside of Ontario. The photographs for this column are from my collection of photographs of steps, as I have taken steps to find where math lives.

Question 1

In September 2000, I was a visiting educator/consultant at the Bandung International School in Bandung, Indonesia. I have fond memories of working with teachers and students at the school, staying at the Chedi Hotel, and seeing students walk and hop up and down the steps shown in Photograph 1. This was 20 years before it became commonplace to ask students to notice and wonder, yet that is precisely what I did at the base of the steps with students. I have used this photograph with teachers for many years, and the questions given below are some of the ones that teachers work on in my workshops. Readers are encouraged to come up with their own questions and to consider what other numbers could be put on the steps. Better yet, work with your students and put numbers on a set of steps at the school where you teach.



Photograph 1: *Numbers on steps at the Bandung International School*

(a) How many steps are there? What patterns do you see in the numbers on the steps? What numbers would you put on the steps if there were a total of 20 steps?

(b) Find the sum of the numbers on each step. What do you notice about the sums?

(c) Find the product of the numbers on each step. What do you notice about the products?

(d) Starting at the top, let a and b be the numbers on the left and right sides of the nth step, respectively. Determine mathematical models for a and b as a function of n.

(e) Let S and P be the sum and product of the pair of numbers on the nth step, respectively. Determine mathematical models for S and P as a function of n.

(f) Use the mathematical model for P as a function of n to determine the value of n for which P is a maximum.

Question 2

I am fortunate to have travelled to Hong Kong many times. It is one of my favourite places in the world to visit, and I have enjoyed a number of long walks all over the island. I often see staircases that have information or drawings on the steps, like the one shown in Photograph 2.



Photograph 2: *Steps with numbers and names of cities in Hong Kong*

(a) What patterns do you see in the numbers on the steps?

What patterns do you see in the names of the cities on the steps? How is the name of each city connected to the number that appears beside it?

(b) The staircase shown in Photograph 2 is situated in between other staircases that have approximately the same number of steps and continue the pattern of numbers and city names. How many staircases are there? What is the number and city name on the first step of the bottom staircase? What is the number and city name on the last step of the top staircase?

(c) Determine the sum of the 11 numbers that appear on the risers of the steps in Photograph 2 in at least two different ways.

Question 3

I took the photograph of the roped-off staircase near a John Fluevog store on Water Street in Vancouver. I remember passing by this store while I was on a long walk that ended at Vij's, one of my favourite Indian restaurants, that has a reputation for being one of the best in the world.



Photograph 3: *Steps with names of cities in Vancouver*

(a) What patterns do you see in the names of the cities and their order on the steps?

(b) There is a second flight of stairs after the one shown in Photograph 3. The names of the cities on the second flight of stairs are slightly visible. Is Halifax one of these cities? How do you know?

(c) What products are available at a John Fluevog store? Who is John Fluevog?

Question 4

During the academic year 2017–2018, I was fortunate to have a year-long research and study leave (sabbatical) from OISE/UT. I spent a month in Manhattan and tried to walk at least 20 000 steps per day. On one of those days, I walked from West 72nd Street to West 215th Street, to see a collection of flights of stairs known as the West 215th Street Steps (Photographs 4 and 5). There are 110 steps in all, that connect Broadway to Park Street Terrace. Without these steps, people would have to take an indirect route, that would take much longer to get from one of these streets to the other.



Photograph 4: *First flight of stairs at the bottom of the West 215th Street Steps in Manhattan; text at the top of this section of the staircase reads 11 Steps/+6.3'*



Photograph 5: *Second flight of stairs near the bottom of the West 215th Street Steps in Manhattan; text at the top of this section of the staircase reads 22 Steps/+12.6'*

(a) There is some text that appears at the top of each section of the staircase. For example, the text 11 Steps/+6.3' appears on the first flight of stairs, and the text 22 Steps/+12.6' appears on the second flight. How many flights of stairs are there all together? What

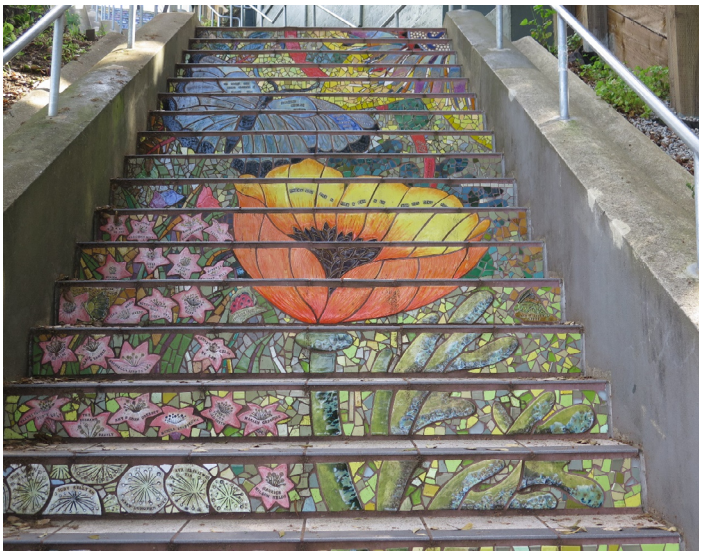
text appears on each flight of stairs? How do you know?

(b) Let *a* steps/+*b*.*c*' be the text that appears on the *x*th flight of stairs. Determine mathematical models for *a*, *b*, and *c* as a function of *x*.

(c) What is the difference in elevation between Broadway and Park Street Terrace?

Question 5

In 2016, I attended and gave a workshop at the NCTM Annual Conference in San Francisco. A friend of mine gave me directions from the Moscone Convention Center to an undisclosed location. Curious, I followed the instructions on foot and via public transit and ended on a street where there seemed to be nothing out of the ordinary. As I looked around, I suddenly came across a sign for the Hidden Steps and discovered a series of staircases that took me way up to a place, where I had stunning views of San Francisco. Two of the staircases are shown in Photographs 6 and 7.



Photograph 6: *The Hidden Steps, San Francisco*



Photograph 7: *The Hidden Steps, San Francisco*

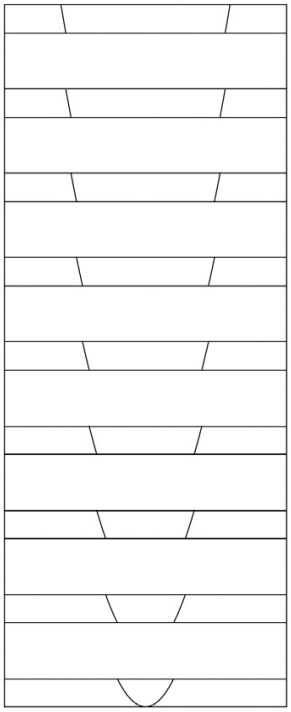


Figure 1: *A quadratic on the risers*

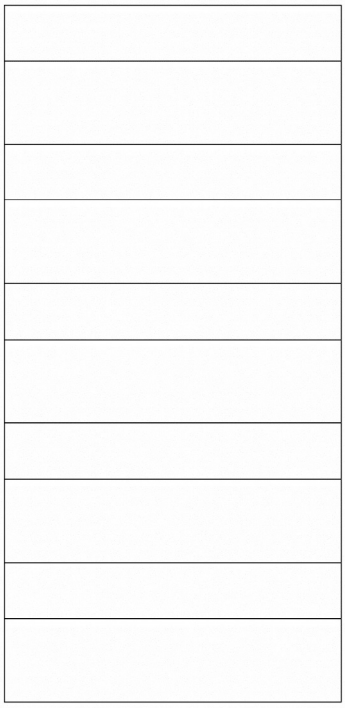


Figure 2: *Staircase template*

Cut out the model in Figure 1, which was made using the Geometer's Sketchpad®, and fold it to make a set of stairs. If you look at it the same way the photographer looked at Photograph 7, you should see a graph of $y = x^2$. How would you modify the drawings on the risers to reveal another graph, such as $y = x^3$.

Using the stair template in Figure 2, make your own version of the Hidden Garden Steps that reveals an image of a geometrical shape.

As I step away from this column, my wish for all readers is that you will find your own mathematical snapshots, one step at a time. Be well, take care, and walk good (a Jamaican expression of good wishes). ▲

"… the traditional view of the medieval Islamic world as a mere custodian of Greek learning and passive transmitter of knowledge is both a partial view and a distorted one. We have seen how original were the Islamic scholars' contributions to algebra and trigonometry, and how crucial was the role they played in bringing together two different mathematical strands—the algebraic and arithmetic traditions so evident in the mathematical cultures of Mesopotamia, India, and China; and the geometric traditions of Greece and the Hellenistic world." (*The Crest of the Peacock* by George Joseph, p. 508)

▲ In the Middle: Boole, Bézier, and Beauty



CARLY ZINIUK
carlyziniuk@gmail.com

Carly Ziniuk teaches Grade 9 Mathematics, Grade 12 Data Management, and Advanced Placement Statistics at the Bishop Strachan School in Toronto, Ontario, Canada. She is very active in adopting real-life data to engage her students in solving problems.

The new MTH1W course encourages students to view mathematics from a variety of perspectives. Approximating curves by using lines and line segments is fundamental to the understanding of Calculus, yet middle school students can explore this concept through both historical contexts and modern applications. This column will show how curve approximations using lines can be revealed to these younger students with a variety of perspectives. These include Mary Everest Boole's *curve stitching*, Eli Hess's nature art, coding with Scratch and Desmos, and Bézier curves from architecture and computer graphics.

Curve stitching, which will be explained, is an elementary application for younger children to create curves with line segments that formally originated with Mary Everest Boole. Mary Everest Boole, the wife of the founder of Boolean algebra, George Boole, and niece of Mount Everest namesake George Everest, introduced a playful approach to mathematics as a teacher, mother, and librarian. As an educator, she incorporated sticks, stones, and other natural materials, what we now would call manipulatives, to help students understand mathematical concepts. As a self-taught mathematician, she believed that mathematical thinking arose from interaction with the natural world.

Even though she wrote her educational philosophy in the early 1900's, middle school classrooms today still reflect her pedagogical theories. She suggested co-operative learning and that students create their own formulas. As early as 1904, she asserted that children understand mathematics through questioning and recommended journal writing. Akin to our current understanding of Social–Emotional Learning and math anxiety, she sought to reduce "nerve storms" for young children during calculations with discovery, emphasizing beauty and patterning. Mary Everest Boole encouraged the use of mathematical imagination with critical thinking and creativity, using "sewing card" algorithms.

Following her work with her husband on the development of Boolean algebra, she began to explore ways for children to experience algorithmic thinking. In doing so, she created sewing cards revealing mathematical patterns, now referred to as *curve stitching*. Curve stitching is also referred to as *string art* outside of mathematics. As an arts-and-crafts project, string art was popularized in the 1960's, aligning with the trend toward geometric abstraction and the return to natural materials. An example by Eli Hess, an artist known for nature-inspired public art, is shown in Figure 1. He created this rope-and-tree sculpture.



Figure 1: *Rope-and-tree sculpture*

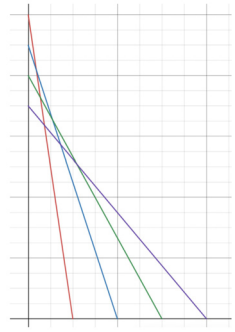


Figure 2: *Example of curve stitching*

Mathematically, the basic curve-stitching design presents a shape or figure with straight-line pieces. The diagram in Figure 2 begins with a right angle formed by the first quadrant's axes. Then, in corresponding pairs, subdivide the lines and connect the points with adjacent lines. This is easily done using graph paper, with the axes as obvious subdivided lines, or through measurement, as Hess does on the tree. Madison Children's Museum's YouTube video, *Holiday Crafts Kit: String-Art Plaque*, is an appropriate approach for very young children, without formal graph paper or obvious mathematical notation. (The video is available at www.youtube.com/watch?v=5P3UBCWBuGM).

The diagrams in Figure 3 shows a less tangible creation, but equally creative in its discovery of curves made with straight-line pieces. The curve stitching, as with Figure 2, begins in the first quadrant, using Desmos to model what could easily be done on graph paper, creating a line segment joining (0, 10) to (1, 0), then (0, 9) to (2, 0), and so on. Figure 3 continues with additional illustrations showing that the more subdivisions that

are used, the smoother the curve stitching becomes.

In the MTH1W course, drawing a conclusion about where those coordinate points are, and how the slopes and equations change, is a creative next step. Reflecting how this can be generalized is an opportunity for less formal coding algorithmic thinking, as Mary Everest Boole suggested. This diagram also shows the use of lists in Desmos to create an array-like structure within a series of equations, without knowledge of any programming language syntax.

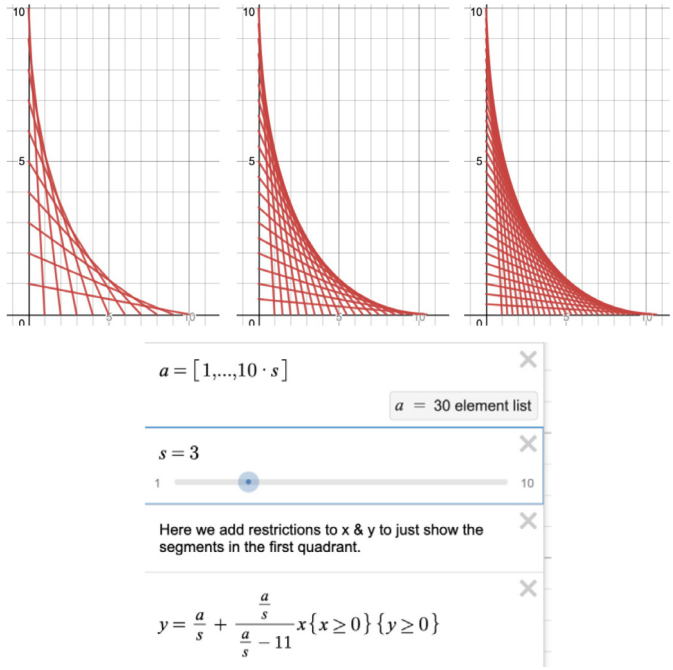


Figure 3: *Curve stitching, using technology*

Ken Valente writes that Everest Boole used her "sewing cards" to describe similar behaviours in nature. "The spiral behaviour of a whirlwind, mathematically describable in terms of components that act tangentially and perpendicularly to its motion, compelled Mary to consider the powerful result of forces that act orthogonally" (Valente, 2010, p. 56).

Applying perpendicular lines to show this whirlwind motion, the equations of negative reciprocals produce beautiful pictures on Desmos as well (see Figure 4).

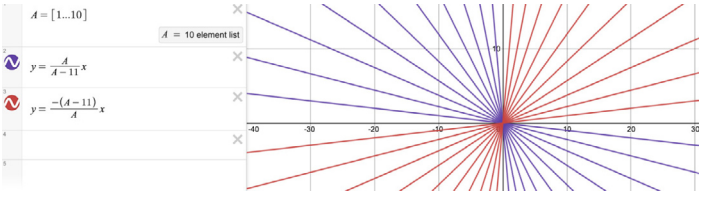


Figure 4: *Lines perpendicular to the curve-stitching line segments*

A series of related programs written in Scratch by "estebistec" have been adapted by several students to create extensions of string art into reusable blocks at www.scratch.mit.edu/projects/154933958/. Examples are shown in Figure 5.

Other Scratch users have extended these blocks to include colour and to increase the number of subdivisions.

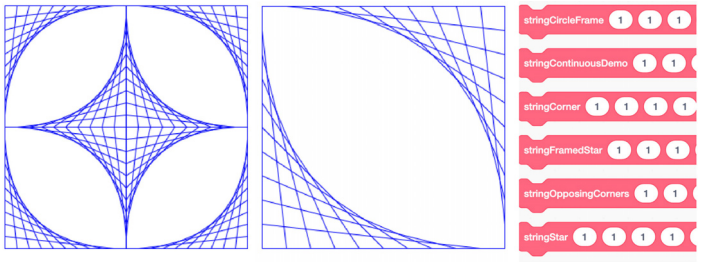


Figure 5: *Examples developed using Scratch*

We can also use lists on Desmos to generate similar images—see Figure 6. Note the changing restrictions and altered signs in the equations and the slopes.

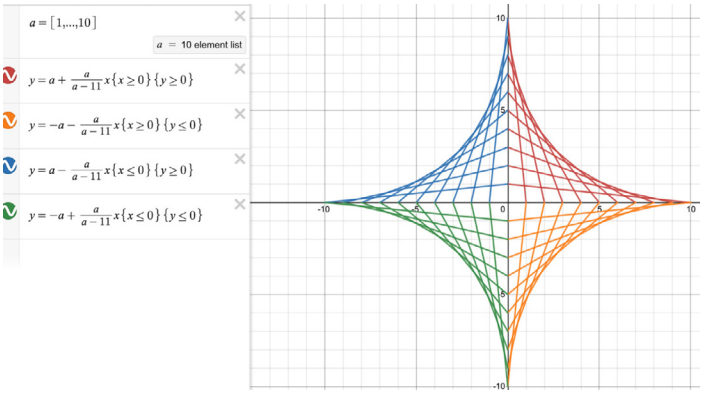


Figure 6: *Demonstrating the process, using Desmos*

The size of the set can be combined with a slider and animated, as shown in Figure 7. This coding alternative can make the curve approximations more exciting and beautiful!

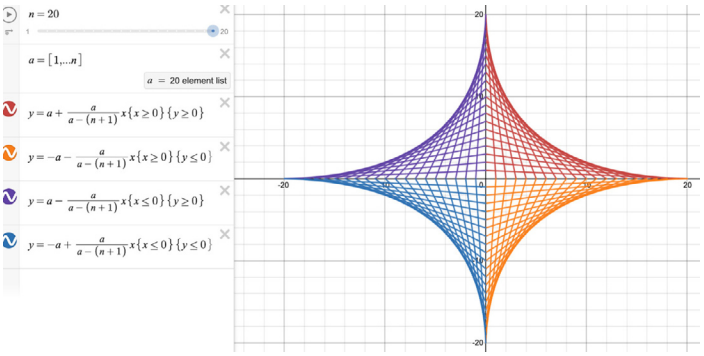


Figure 7: *Using a slider to alter the size of the set*

While I was developing the curve-stitching ideas, Ron Lancaster sent me a *Toronto Star* article featuring the Bézier Curve House in Toronto (see Figure 8). This architectural design has won a series of awards and is well documented for its use of the Bézier curve. French engineer Pierre Bézier did not actually create his namesake curve, but rather, developed the consistent notation for the property used to create the curve, and he popularized it in the 1960's for use

with the Renault automobile company. Similar use of this Bézier curve is the scene in the Jerusalem Chord Bridge (Figure 9), which presents a parabolic shape three-dimensionally. TrueType fonts use quadratic Bézier curves to create smooth curves on the letters, while SVG graphics use cubic Bézier curves to create smooth shapes.



Figure 8: *Bézier Curve House in Toronto*



Figure 9: *Jerusalem Chord Bridge*

Bézier curves, simply explained, are smooth curves created with many control points. Mike Kamerman's interactive online book, *A Primer on Bézier Curves*, shows Figure 10, which illustrates the extension beyond Boole's string art.

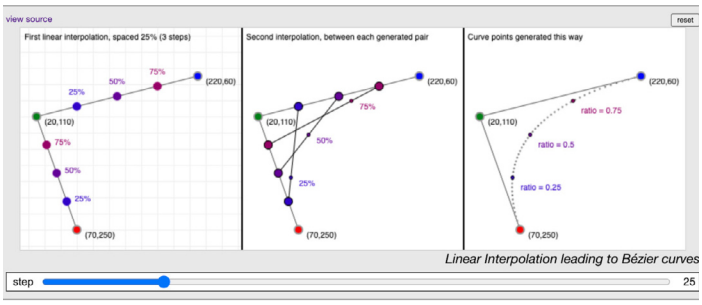
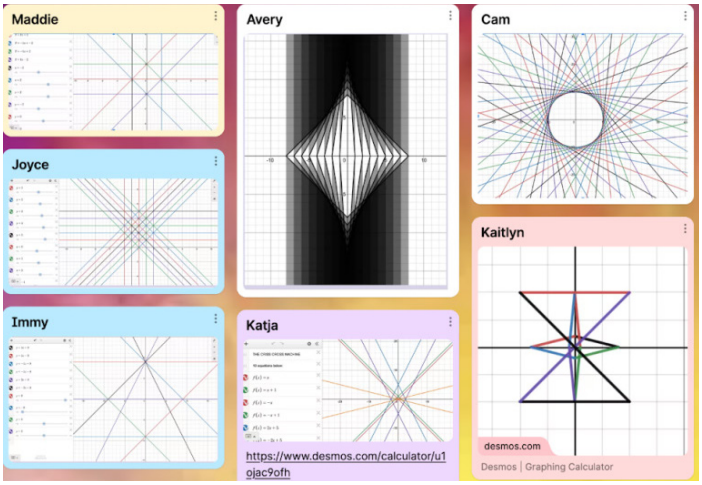


Figure 10: *Illustration from Kamerman's book*

Jason Davies (2010) has animated these Bézier curves simply for students to explore and understand. The American Mathematical Society (2008, 2013, 2020) has explanations accessible to high school students, explaining their use in Computer Graphics and presenting activities for students to try. Generating where these points would be on graph paper and then translating to creating an algorithm with Desmos or Scratch can challenge middle school students to link data and geometry. Although Bézier curves are usually best described parametrically, pre-made blocks on Scratch can be implemented. The Scratch Wiki about Bézier curves, at www.en.scratch-wiki.info/wiki/Bézier_Curve, details two straightforward methods and reproducible blocks.



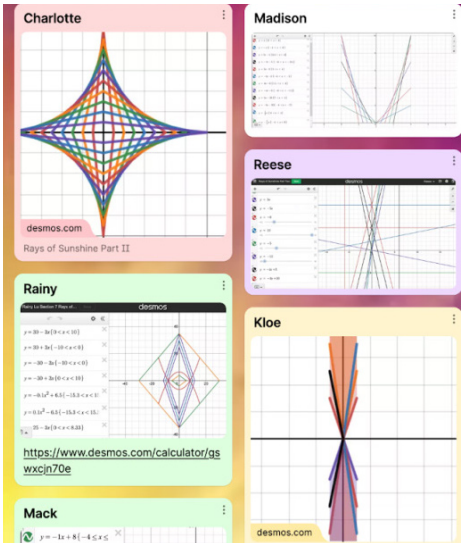


Figure 11: *"Rays of Sunshine" Padlet*

Math for America (MfA, mathforamerica.org) has run a *Mathematical Construction* mini-course for teachers, with George Hart using curve stitching with Graph Theory and the Mandelbrot set. This mini-course, as Mary Everest Boole might have, builds activities with cardboard, string, and bicycle wheels, then incorporates technology. In 2013, I attended a Math Monday program at New York City's Museum of Mathematics (MoMath), building Bézier curves with very young children, using string, Spirographs®, and

3-D printers. Glen Whitney (2019) documented some of the participants' creation in his 2019 Math Monday column. These children presented different perspectives and curves, even when given varied materials. Both the MoMath and MfA programs have online documentation and virtual programming for teacher professional development that I highly recommend if you are interested in incorporating these kinds of activities into your middle school classes.

When I was at the MoMath, I watched young children building many beautiful designs and creatively exploring mathematical patterning. I have seen the same thing happen with my students with Desmos, using the list slider features, revealing what can be accomplished creatively. Last year, my students created a "Rays of Sunshine" Padlet (see Figure 11), many using abstract art design principles and curve stitching. As Everest Boole proposed, many students noted that creating their art pieces helped them to understand linear equations fully.

Mary Everest Boole's 1909 work was entitled *Philosophy & Fun of Algebra*. The creativity of these beautiful and geometrical creations by children allows them to explore their own algebraic fun, all while developing their own perspectives on linear pieces to create a curve, while I continue to use their learning to add to my own understanding of the new MTH1W course and its philosophy on mathematical viewpoints.

References

Binks, G. (2021, July 13). Toronto home shows off big curves. *Toronto Star*. www.thestar.com/life/homes/2021/07/13/toronto-home-shows-off-big-curves.html

Davies, J. (2010). *Animated Bézier curves.* www.jasondavies.com/animated-bezier/

Frost, M. (2022). Mary Everest Boole. *Biographies of Women Mathematicians*. www.mathwomen.agnesscott.org/women/boole.htm

Gross, R. (1970, March 5). *Bridges, string art and Bézier curves.* Plus Maths. www.plus.maths.org/content/bridges-string-art-and-Bézier-curves

Kamerman, M. (2022, January 8). *A primer on Bézier curves.* https://pomax.github.io/bezierinfo/

Poole, C. (2017, February 6). *How to create parabolic curves using straight lines.* WonderHowTo. https://mathcraft.wonderhowto.com/how-to/create-parabolic-curves-using-straight-lines-0131301/

Valente, K. (2010). Giving wings to logic: Mary Everest Boole's propagation and fulfilment of a legacy. *The British Journal for the History of Science, 43*(1), 49–74. doi.org/10.1017/S0007087409990380

Whitney, G. (2019). *Math Monday—Beginning of a string.* National Museum of Mathematics. www.momath.org/home/math-monday-beginning-of-a-string/ ▲

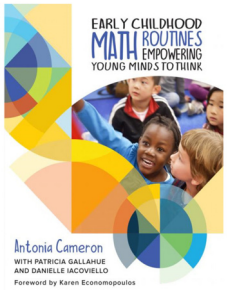
▲ Book Review: Math Routines Can Meet Academic and Play-Based Learning Goals



EDWARD SCHROETER
edschroeter@outlook.com

Ed Schroeter, B.J., B.Ed., OCT, is a retired elementary teacher, having taught for 30 years, 18 of them in Kindergarten. He was the Grade 1 Lead Writer for the Ontario Mathematics Curriculum Resource Project (2020–2021)*. He led three* Teacher Learning and Leadership Projects (TLLPs) *lesson studies. From 2009 to 2019, he taught two ETFO Summer Academy courses and presented more than a dozen workshops on inquiry learning around Ontario as part of the* Presenters on the Road Program*. He is the author of Chapter 9 of* Teaching for Deep Understanding: An ETFO Curriculum Learning Resource Compilation.

Cameron, A. (2020). *Early childhood math Empowering young minds to think.* Stenhouse Publishers. (Available through Pembroke Publishing)



One of the most difficult challenges facing Kindergarten to Grade 2 teachers and registered Kindergarten early childhood educators (RECEs) is meeting the mathematics curriculum while also creating effective learning activities that are engaging. As a retired Ontario educator with 18 years of experience teaching Kindergarten, I can attest to the fact that the mathematics standards of the Ontario Ministry of Education's *The Kindergarten Program* (2016) and *The Ontario Curriculum, Grades 1–8: Mathematics* (2020) challenge teachers to use engaging and developmentally appropriate activities that honour young children's playfulness and curiosity.

The problem is not new and has haunted Ontario early years educators for 140 years. Challenges have been documented, such as the difficulty that free play on its own is unlikely to promote solid mathematics learning, given class sizes, time constraints, limitations of elementary teacher mathematics knowledge, particularly in recognizing the mathematics embedded in young children's play; and the difficulty of planning enriching learning activities. Shumway (2011) described how number sense routines, particularly her

5- to 15-minute warm-ups, can shoulder some of the burden of building playfulness into early years math programs, while developing mathematical understanding.

Now a new book on the topic, *Early Childhood Math Routines: Empowering Young Minds to Think*, by Antonia (Toni) Cameron, proposes an improved approach to playful early math routines. Her goal is to help teachers replace typical and possibly stale math routines. She proposes versions that focus on important mathematical ideas in early childhood, as well as provide solution-reconciling curriculum standards with "the need for play." Helping early years educators explicitly find the balance between play-based and academic learning is admirable and overdue. I experienced a moment of concern when I read her comment on page one, that Kindergarten programs need "play at their centre." It caused me to wonder whether her approach might sacrifice effective, developmentally appropriate math learning for play. Fortunately, my worries were unfounded.

Cameron takes a detailed look at five familiar math routines typical of Ontario Kindergarten to Grade 2 classrooms: calendar routines (days at school), attendance routines, the "Is it fair?" routine, fluency and flexibility, and quick images for developing reasoning. She examines the range of mathematical concepts, skills, and dispositions that can be taught with each routine; provides a rationale, research notes, educator scripts, diagrams, practical classroom tips and alternatives; and she profiles three variations of each routine, as well as the mathematics they can teach. In the process of outlining the math that can be taught in these routines, Cameron clearly explains the big ideas in early childhood mathematics. She also shows how the activities can assist teachers to engage young children in meaningful math thinking, and to build problem-solving communities through stories, classroom examples, and resources.

The book includes a handy, day-by-day planning guide for introducing and developing each routine in the classroom. There are also additional details in five appendices that cover mathematical research about using the calendar as a teaching tool, important big ideas in early math, a math routines planning sheet, a notice-and-wonder routine based on the impressionist painting *Drina House in the Morning Mist* by Eliza Donovan, and a detailed chart that explains the fundamentals of assessment in the early years. Colour text boxes illustrate concepts and support educator planning and facilitation by providing expanded definitions, research notes, lesson scripts, student–teacher dialogues, and teaching analysis.

Purchasing the book entitles the owner access to a robust companion website that provides a wealth of resources. They include downloadable and printable cards and game boards, more than 30 slide decks for facilitating routines, additional practice routines, supplemental readings, and a place-value assessment.

I was pleasantly surprised by how much experienced teachers can learn from *Early Childhood Math Routines*! For example, Chapter 3, "Making Attendance Routines Matter," offers four variations on attendance routines that develop important mathematical ideas, such as one-to-one correspondence, magnitude, equal groups, part–whole relations, missing addend and removal models of subtraction, equivalence, and the associative property. Using an attendance routine—counting students who are present and absent—to model and assess understanding of counting principles, quantity concepts, addition and subtraction is well known to early math educators. Cameron, however, explains how to teach a greater variety of math skills through simple modifications and the use of underutilized resources. For example, she shows how to teach magnitude as well as the missing addend (e.g., 22 + ? = 24), and the removal models of subtraction (24 – 2 = ?), simply by employing Unifix® Cubes to represent student attendees.

The book states that educators "can get their math program started right away with these new routines," but I am not convinced. Cameron's routines are carefully thought out, thorough, require consideration and planning, and can help keep teachers' all-important formative assessment on track. They cannot be implemented mindlessly on educator autopilot. For those reasons, this book may be better suited to a summer reading project, when there is plenty of time to digest the contents and map out how to integrate them into one's teaching practice.

Cameron's book is highly recommended for mid- and late-career elementary teachers from Kindergarten to Grade 2, who are looking to learn more deeply about the potential math embedded in their mathematics routines, and to make them more meaningful learning activities for their students. In my opinion, educators should tackle changing their mathematics routines, such as attendance or calendar, one at a time, using the book.

Early years educators who begin adapting their math routines should also follow Cameron's advice to add their own playful dimensions to her suggestions. She cites research that indicates that teacher spontaneity and silliness are positively related to higher levels of child playfulness and satisfaction with learning. Cameron points out in Chapter 7 of her book that her routines encourage "intellectual play" and contribute to an overall playful learning environment, but it is up to educators to personalize the routines and make them truly engaging, fun-filled, and playful.

Reference

Shumway, J.F. (2011). *Number sense routines: Building numerical literacy every day.* Stenhouse Publishing. ▲

Teaching Math during the Pandemic: Lessons Learned and Best Practices

You are invited to participate in a survey to identify the impact of teaching math during COVID, and what we have learned.

The survey is completely anonymous. To access the survey, click on the link below, or copy and paste the link into a browser. www.surveymonkey.com/r/32L3LKJ

Please consider completing this survey, and please share the survey link with math teachers you know. The more teachers who complete the survey, the more we can identify what worked (and what didn't work).

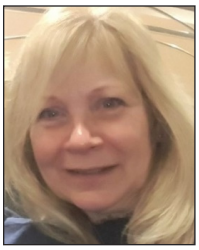
Thank you for considering this request.

Jeff Irvine, Ph.D., OCT

Disclaimer: This research has no connections to OAME, any university, or government body.

Any questions about this survey can be sent to dr.jeffrey.irvine@bell.net.

▲ MB4T (Mathematics by and for Teachers): Using Concrete Tools, Exploration, and Reasoning to Develop Factoring



ANN KAJANDER
akajande@lakeheadu.ca

Ann Kajander is a Professor at Lakehead University, working with pre-service elementary and secondary teachers of mathematics, as well as graduate students. She is particularly interested in studying and designing the kinds of supports that are helpful to teachers to both understand and teach mathematics in a more visual, conceptual, and exploratory way. The use of representations and reasoning in mathematics learning, including at the secondary level, is a current focus.

A quadratic function can be generated by multiplying two varying terms together. The term relates to the word *quadrilateral*; think "rectangles."

Quadratics may be informally described as functions with an x^2 term, but this doesn't give a sense of what their important characteristics are. We can think of quadratics with the analogy of a two-dimensional area, where two varying amounts are multiplied. For example, for a rectangle with perimeter of 24, if we define x as one dimension, then the other dimension must be $(12 - x)$. (You might think: *The two sides add to half the perimeter, that's 12. If we use up* x *of the 12, then the other dimension must be (12 –* x*).*) So, the area of any rectangle with perimeter 24 has the form $x(12 - x)$. In this way, a quadratic can represent the product of two varying quantities.

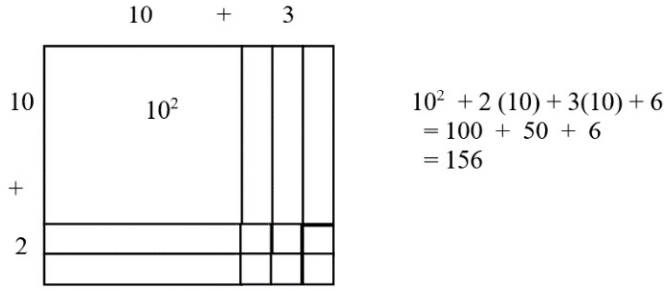


Figure 1: *Area model for 13 × 12*

Thinking about quadratics as a product brings to mind the area model. For example, to quickly review the idea of the area model, it is a useful way to model and calculate a product such as 13 × 12. A two-digit number such as 13 can be thought of as 10 + 3. When we construct an area model for 13 × 12, it can be modelled as (10 + 3) × (10 + 2), which generates four distinct regions (see Figure 1).

These important ideas can now be used to think about algebraic expressions, and algebra tiles are an important tool here. Algebra tiles are similar to base-ten blocks, but include representations of lengths and areas that can change. To use them, we need to agree on the assumption that the *x*-tiles can *vary* in length, even though they may be represented in packages of manipulatives as fixed-length plastic pieces. Virtual algebra tiles, like those found on online sites such as Desmos, are also an insightful way to visualize this.

For example, the expression $x + 3$ could be illustrated with algebra tiles by combining one *x*-tile and 3 unit tiles, as shown in Figure 2.

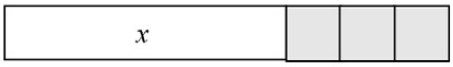


Figure 2: x *+ 3 in algebra tiles*

The expression $2(x + 3)$ could be shown as two sets of the above representation, either separately or placed so the two sets are one above the other, creating a rectangle. This latter arrangement is an area model, with height 2 and width $x + 3$. Without telling students anything, it becomes clear when examining the area model (and counting the distinct pieces), why $2(x + 3)$ is $2x + 6$.

To continue on, it would be very helpful to have algebra tile representations at hand. There may be plastic ones available to you, or you may wish to use virtual ones online. As well as the unit tiles and *x*-tiles, you will notice some tiles that are larger squares, with each dimension the same as the length of the *x*-tiles. These represent x^2. It is important to remember that the x and x^2 tiles cannot be measured by lining them up with unit tiles because they represent amounts that can vary. The *x*-tiles actually have dimensions x and 1, so unit tiles can be arranged along the side that is 1, but not the varying, x, side.

Algebra tiles are an excellent way for students to construct and understand the methods of factoring simple quadratics. Imagine the following classroom activity, which I have successfully done many times with Grades 9 and 10 students.

The teacher prepares, in advance, some bags of algebra tiles representing quadratic functions. Some are factorable, and some are not. This is a low-floor activity in that students do not need to know much about quadratics to be successful doing this. Students are asked, for each bag, to attempt to

arrange the pieces to make a rectangle. As they work, they are further asked to explore if this process is arbitrary, or if there is a way to predict, in advance, if the pieces will make a rectangle, and what the dimensions would be. You might try this activity, imagining the following combinations of pieces in the five sample bags:

Bag 1 has an x^2, 5 of the x's, and 6 units

Bag 2 has an x^2, 7 of the x's, and 12 units

Bag 3 has an x^2, 5 of the x's, and 13 units

Bag 4 has an x^2, 3 of the x's, and 2 units

Bag 5 has an x^2, 6 of the x's, and 9 units

You likely noticed that Bag 1 is equivalent to a representation of $x^2 + 5x + 6$, Bag 2 is $x^2 + 7x + 12$, and so on. For now, it can be explained to students that positive quantities will be the starting point.

The process will be illustrated with Bag 2. A little experimentation shows how this collection of tiles can be arranged as a rectangle (see Figure 3).

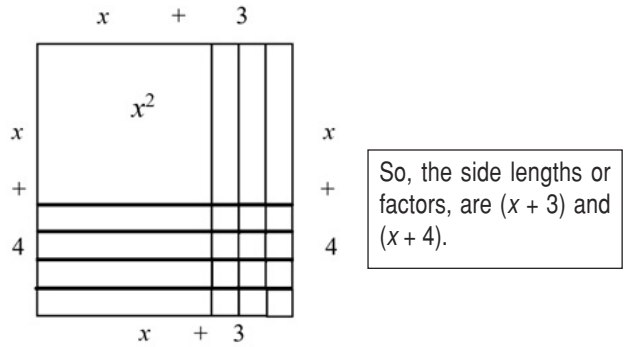


Figure 3: *Rectangle created from* $x^2 + 7x + 12$

Let's keep our eyes open for a pattern. It does not take students very many examples to figure out the key idea in factoring—namely that the x-tiles need to be split into two groups, in a way that allows the unit tiles to fill in the remaining space exactly. Specifically, the ends of the x-tiles form the sides of the sub-rectangle of unit tiles. So, in the expression $x^2 + 7x + 12$, shown above, the $7x$ is split into a group of $3x$ and a group of $4x$, and the ends of these groups of x-tiles (a group of 3 and a group of 4) form the 3 by 4 rectangle exactly filled by the 12 unit tiles. Similarly in the expression $x^2 + 6x + 9$ (Bag 5), the $6x$ is split into two groups of 3 x-tiles, and the 3 by 3 region is exactly filled by the 9 available units (Figure 4). In this case, the rectangle is a square rectangle. The $6x$ term is split into two equal parts, resulting in the 3 by 3 section of unit tiles. Using algebra tiles, the representation of $x^2 + 6x + 9$ represents a square, and so the expression is called a perfect square. The key idea in identifying a perfect square is that the x-term is split into two *equal* groups—resulting in a rectangle with two equal sides—a square.

The example $x^2 + 5x + 13$ (Bag 3), however, cannot be factored because there is no way to split $5x$ into two amounts, where the amounts multiply to give 13, since the only pairs of numbers adding to 5, are 1 and 4, 2 and 3, and 0 and 5. Since the physical pieces in the bag represent positive numbers of quantities, only the whole numbers need to be considered. None of these multiply to 13. (Students might also notice that for primes such as 13, the only factorable quadratic has 14 x-tiles, since 1×13 is the only possible way to factor the 13.) It's important to include at least one example of a quadratic that *cannot* be factored, so that students can begin to form the ability to look for what is needed for a factorable expression.

Beginning to Generalize

The question of how and when to encourage students to generalize from cases that can be modelled with positive algebra tiles, to more general cases, is relevant, as it brings up a more general question: when is it appropriate to move away from concrete models to more abstract or procedural methods? Probably the best answer to that question is "whenever the student is ready to see that the reasoning from the concrete case applies more generally."

The key idea here is that to factor a quadratic in the form $x^2 + bx + c$, the number of x-terms (b, above) is split into two groups (let's call them b_1 and b_2), and the number in each group forms the sides of the rectangle with c units. So $b_1 \times b_2 = c$ (Figure 5).

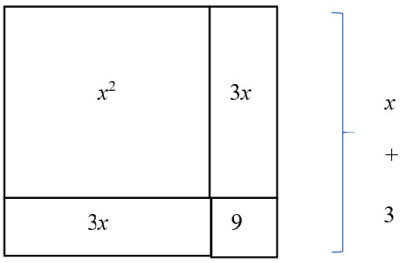


Figure 5: *Generalizing factoring with algebra tiles*

High school students working with algebra tiles are generally able to construct this key idea of the relationship between b and c themselves, with little teacher direction. The idea that the number of each of the two groups of x-tiles (here, b_1 and b_2), also multiply to give c, is the central idea, and this idea generalizes. Students can then explore some further examples to test this conjecture. Here are some examples.

1. $x^2 + x + ¼$ (Hint: Since the two subgroups of x add to 1, and multiply to ¼, they may be fractions.)
2. $x^2 - 2x - 15$ (Think: "What are two numbers that add to -2 and multiply to -15?")

3. $x^2 - 16$ (Think: "What are two numbers that add to 0 and multiply to -16?")

The reasoning above can no longer be modelled directly, with concrete algebra tiles representing whole numbers, but the *reasoning* used in manipulating the algebra tiles is purposely maintained. While there are algebra tiles that represent integer quantities by using another colour, I have found these rather confusing for students to use, and there is also a tendency to revert to a lot of teacher instructions about lining up colours, potentially causing confusion.

This process of starting with concrete examples, developing reasoning, and then using the same reasoning to generalize is a fundamental and very helpful one in developing this type of mathematical thinking and learning.

To summarize, first, a conceptual understanding is developed, usually with a concrete model. Then, the important ideas and reasoning are *generalized*. The difference between such a process and simply "telling" students a rule or procedure are the crucial ingredients of conceptual understanding, reasoning, and generalizing. Such deep understanding means that, if a student loses track of the process or method, they can also return to a concrete example and remind themselves of the reasoning. ▲

Dreamland Mathematics

As night falls and starts to slumber,
The mind plays and starts to fidget.
Tinkering with a large number,
Exploring details of a digit.

Exploring, as one is unwinding,
Patterns emerge that delight.
As rest merges with a finding,
Inspiration runs in the night.

Discovering the strange,
As fantasy plays a part.
As number digits rearrange,
A discovery gives a start.

But great thinking, in the dawn,
Hides the inspiration with a yawn.

▲ Some Thoughts about Perimeter: Vocabulary and Metaphors



JÉRÔME PROULX
proulx.jerome@uqam.ca

Jérôme Proulx is a professor of mathematics education (www.profmath.uqam.ca/~jproulx), and runs the Laboratoire Épistémologie et Activité Mathématique (www.leam.uqam.ca). His research focuses on epistemological issues related to cognition and/in mathematics. His current research program is dedicated to mental mathematics and school mathematics content development.

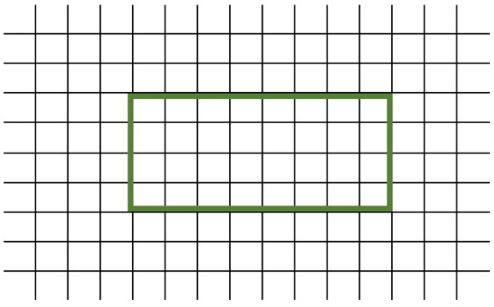
(Photo by Nathalie St-Pierre of Université du Québec à Montréal)

Words and expressions in mathematics often play an important role in the ways we convey concepts, be it when attempting to name them (*tri*angle, *ir*rationals, etc.), as well as when explaining them through metaphors (negative numbers represent debts, slope is rise over run, etc.). The same is true for perimeter, often distinguished from area by presenting it as the distance around the figure, whereas area is the covered surface; or perimeter as being a fence around a field and area as the field itself.

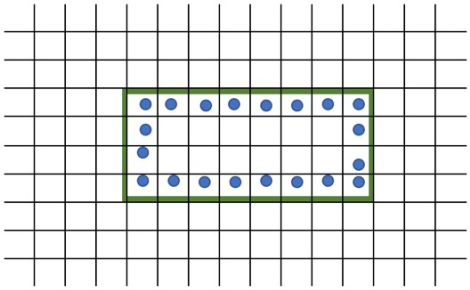
For this third column, we leave Marco and Marie and plunge directly into a classroom with Grade 5 students like Paul, Rose, and Julia. Their ways of solving a task on perimeter raise interesting mathematical questions concerning ways of talking about and making sense of perimeter.

The Task and the Three Perimeters

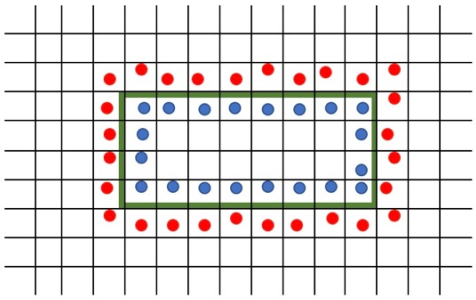
The Grade 5 students had to find the perimeter of a rectangle with an area of 32 cm^2, with dimensions of 8 cm and 4 cm drawn on the front board.



Right away, Paul suggested “20” as the perimeter, placing a dot in each square inside the rectangle to keep count and go around the figure.



Not all students agreed with this answer. Some talked about a field that one has to go around fully, or even a prison, where one has to check the corners. Another student, Rose, then suggested considering the fence around the field by counting the green border of the rectangle. To go fully around, she took the side of 8, the one of 4, the other of 8, and the other of 4, giving her a perimeter of “24.” Some students then appeared to agree with this, mentioning that this is usually what they do in class. Others continued to disagree. Then Julia asserted being able to close the debate by truly going around the rectangle. She counted all the squares outside the rectangle, also putting a dot in each, and obtaining “28” as the perimeter.



Julia explained that this really goes around and closes the fence, adding along the way that if this rectangle is a house, you would never put the fence *on* the house, but genuinely around it. Doing this, she added that by counting the corners, all is completely closed, and prisoners would not be able to escape the prison.

Some Thoughts about… Perimeter, Metaphors, and Boundary

In terms of “going around” the figure, all three answers appear viable. Each describes a strategy for going around the rectangle: inside-around of 20, border-around of 24, and outside-around of 28. That said, none of these answers was given with units, raising the issue of one-dimensional (1D) and two-dimensional (2D) units discussed in the previous column (December). It is not clear if these perimeters were considered as surfaces, as lengths, or simply as numbers obtained from a calculation. Although viable in terms of the concept of “going around,” all three answers cannot be representative of the perimeter: they are mostly the symptoms of how it is usually conceived in schooling.

The “fence” metaphor seemed to have played an important role in the three possible answers. Each student attempted to find the fence in their own way, encircling the rectangle for finding the perimeter; and each endeavouring to make sure that the prisoners could not escape. The quest for finding the “best” fence subsided when Julia mentioned that it could not be placed *on* the building, but *around* it; concluding the quest, but not without the cost of possibly adding to the confusion around the perimeter concept.

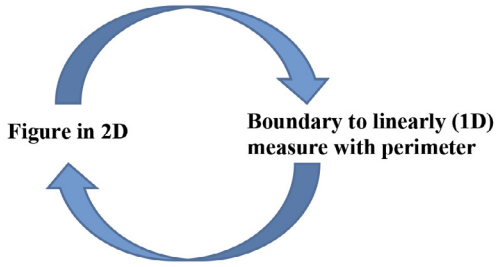
There is no doubt that vocabulary played a part in these answers. The language issue was beyond a mere technical level of labelling and suggests a conceptual issue. The three answers raise questions regarding the idea of “going around” or of “placing a fence.” It questions these metaphors for conceiving of the perimeter, since all three answers satisfied it as a criterion. In addition, if the idea of a fence directly calls on a one-dimensional idea, the notion of “going around” does not. In other words, the notion of “going around” is not strictly perceived as a one-dimension length. This raises issues in relation to how perimeter is talked about, particularly the notion of “distance around.”

To help conceptualize the 1D and 2D interplay, as well as avoid confusion raised by the notion of “distance around the figure,” Danielson’s (2005) suggestion to address the perimeter in terms of the boundary of a figure is of interest. In mathematics, a *boundary* is a general concept that relates to the points that delimit a region and that can be approached either from the inside or from the outside. In this sense, the concept of boundary applies to any dimension: the boundary of a segment is represented by its points at the extremities, and the boundary of a solid is represented by its lateral faces. In the case of a figure, its boundary is the collection of points that create the closed curve delimiting the figure. The perimeter then becomes the measure of this boundary.

Beyond any formal definition of a boundary, it is the way it gives meaning to the concept of perimeter that is significant: in a mathematical sense, a boundary only delimits two regions. The boundary constitutes the region and gets constituted by it; without the boundary, there is no region, but without a region, there is no boundary either. In topology, one talks about the Jordan curve, that is, a closed curve that enables one to delimit a surface. Jordan’s

theorem stipulates that if a curve is traced back to its starting points without crossing itself, this curve is said to separate the plane in two regions: an inside part, that is, the figure bounded by the curve, and an outside non-bounded region. The Jordan curve is the common boundary to both regions created; the perimeter of the figure, the bounded region, then becomes the length of this boundary curve.

This boundary conception offers an intuitive way to address the perimeter. It makes salient the interplay between 1D and 2D inherent in the perimeter nature, which aims to measure linearly the boundary of a 2D figure. The perimeter then measures the 1D length of an object, its boundary, which in turn delimits another object, the 2D figure. The following offers a view of this circular view of this 1D and 2D interplay in perimeter.



The perimeter finds itself to be as much relative to the 1D length of the boundary as the 2D nature of the figure. As the other previous two columns illustrated, the perimeter cannot be uniquely defined in terms of 1D, as a length, since it is also relative and belonging to a 2D figure, nor is it simply a "going around": it is the measure of the boundary of the 2D object. Its core meaning takes shape in the dynamical (un)folding from 1D to 2D and 2D to 1D. This is where the concept of boundary is fruitful, and where the perimeter as an object of study is situated, in considering the interplay between 1D and 2D at the same time. It is this sense that the concept of boundary contributes to enrich this conceptualization of perimeter, to conceive of it as a dynamic object through the 1D and 2D interplay.

The next and last column builds on these ideas and illustrates some potential of the boundary conception of perimeter, through the work of Marco and Marie.

References

Butlen, D., & Pézard, M. (2000). Calcul mental et résolution de problèmes numériques au début du collège. *Repères IREM n° 41*, 5–24.

Danielson, C. (2005). Perimeter in the curriculum. *For the Learning of Mathematics, 25*(1), 30–33. www.flm-journal.org/Articles/3895AB59AD73E9F04CD21AD9201DFB2.pdf ▲

▲ Elementary Math Matters: Paying Privilege Forward: The Second Annual Early Mathematics Conference



LYNDA COLGAN
lynda.colgan@queensu.ca

Lynda Colgan is Professor Emerita, Faculty of Education, Queen's University. Lynda is currently working on a number of research, resource-creation, and teacher-education projects, funded by the National Sciences and Engineering Research Council of Canada (NSERC), the Social Sciences & Humanities Research Council (SSHRC), and the Mathematics Knowledge Network (MKN). She will carry on her long-established commitment to STEM outreach by continuing to seek out creative avenues to engage students, parents, and educators in mathematics at home and in the classroom.

The word *privilege* comes from the Latin *privilegium*, meaning a benefit enjoyed by an individual or group beyond what is available to others. In today's lexicon, *privilege* is commonly used in social-justice contexts to mean "an unearned advantage that a dominant group has over marginalized groups."

For the purposes of this column, I would like to define *privilege* as a special right or advantage that most people do not have, for it is in this way that I have been *privileged* over a long career in education. My *privilege* as a leader in mathematics education afforded me a certain level of acceptance, inclusion, and respect, which allowed me to operate within a wide comfort zone.

My *privilege* opened doors, allowing me to meet, interact with, and learn from educators who spanned age, background, pedagogical stance, and demographics. Because of my unique position in the mathematics education community, I know that I am fortunate indeed to have been a multiple beneficiary. I am enormously grateful to the individuals who invited me into their sphere to gain a better understanding of content, and concepts or contexts. Their teachings have allowed me to learn, reflect, investigate, grow, and develop as an educator and a person.

Over the years, I hope that I have been successful in sharing, rather than appropriating, knowledge gifted to me by many benefactors, with others, including *Gazette* readers.

I have been thinking about this a lot while planning the

second annual Canadian Early Years Mathematics Education Conference for pre-service and in-service educators with Dr. Sandy Youmans. This includes reviewing thoughtful workshop descriptions and practical resource materials prepared by researchers and practitioners. Their work is based on the unique lessons taught to them by young children in operational classrooms. The conference is sponsored by the Mathematics Knowledge Network's special focus group, Critical Transitions in Early Mathematics. The one-day conference is slated for August 22, 2022 at the Donald Gordon Centre in Kingston, Ontario. The goal of the conference (as with all our initiatives) is to support current and future classroom teachers by sharing trustworthy, practical resources for teaching mathematics to young learners in Kindergarten and Grade 1—an area that has been woefully neglected for years as an area of research, specialization, and curriculum development.

The mission statement by which Sandy and I live (based on a line from Liping Ma, in *Knowing and Teaching Elementary Mathematics*) is that *teaching elementary mathematics is anything but elementary*. Nowhere is this truth more evident than in the Kindergarten classroom—a key chapter in the crucial foundation building upon which much of later mathematics learning is layered.

As co-leaders of the Early Years group, Sandy and I have been buoyed by the overwhelming support for our projects. This has included conferences (in-person and virtual), workshops, and resources for home and the classroom. These have been provided by classroom educators, Faculty of Education and Community College instructors, and parents (including guardians). Attendance numbers, positive feedback from participants, and uptake of online resources confirms the need for more professional development and content of this kind.

Our conference program is based on the needs of educators. It is guided by data obtained anecdotally through conversations with our school-district partners and colleagues, and features hands-on, participatory presentations by and with researchers and practitioners. We include individuals we have personally had the privilege to learn with: Dr. Deanna Pecaski McLennan (Greater Essex District School Board); Hannah Wickstrom (Dr. Eric Jackman Institute of Child Study); and award-winning children's book author-illustrator, Peggy Collins, who has more than 35 titles to her name, including *Harley the Hero*.

Sandy and I will also be presenting with two current collaborators: gifted Kindergarten educator, Melanie Martin (Millbrook-South Cavan Public School), and accomplished, retired Kindergarten teacher/education writer, Edward Schroeter. Together, we have been working on the development of a suite of lessons, *The Roots of Coding*. The idea for the resource was born when coding was introduced

Spatial Concepts Curriculum Synthesis	K 2016 17.0 Describe the location and movement of objects through investigation	Grade 1 2020 E Spatial Sense E1. Geometric and Spatial Reasoning
1. Describing position 2. Identifying location 3. Finding objects, using landmarks (relational understanding) 4. Using positional language 5. Representing static location 6. Perspective-taking relative to other objects 7. Giving directions, using distance and direction	We can use positional language to **describe** an object's **location**. 17.2 understand basic spatial relationships - use terms [**positional language**] such as "above/below," "in/out" - **use visualization, perspective**	Location E1.4 **describe the relative locations** of objects or people **using positional language** Key concepts: - Words and phrases such as a*bove, below, to the left, to the right, behind,* and *in front of* **describe the [static] position of one object in relation to another**. Numbers can describe the distance of one object from another. - Positional language often includes **direction** and **distance** to **describe the location of one object in relation to another**.
1. Giving directions, using distance and direction 2. Perspective-taking relative to other objects	When an object **changes its position in space**, or when we change our **perspective** on an object, it may look different, but it is still the same object. 17.2 understand basic spatial relationships - use terms such as "**forward**/**backward**" - use **movements** [flips/reflections, slides/translations, and turns/rotations] - **use visualization, perspective**	Movement E1.5 **give and follow directions** for moving from one location to another Key concepts: - Movement encompasses **distance and direction**. - A combination of words and numbers [**giving directions**] can **describe a path to move from one location to another**. The order of the steps taken on this path is often important. - Words or phrases [**giving directions**] such as *above, below, to the left, to the right, behind,* or *in front of* describe the direction of one object in relation to another. Numbers can **describe the distance of one object from another**.

Figure 1: *Spatial concepts mapped onto Kindergarten and Grade 1 Ontario curriculum*

formally into the Grades 1–8 Mathematics curriculum in 2020.

At the time that the curriculum was mandated, many educators and parents alike told us that *coding* is a somewhat daunting and unfamiliar word. Most equated *coding* with *computer programming* and had no conception of how it could be included in an elementary classroom. They explained that, to them, *coding* was the domain of high school computer science students. Even more alien was the intimidating phrase *computational thinking* that peppered the curriculum, and the media reports stating that teachers will *teach coding or computer programming skills starting in Grade 1 to improve problem solving and fluency with technology [and] to prepare students for jobs of the future*.

To dispel the anxiety for educators and families, we shared activities through social media. Here we focused on familiar geometry tasks to reinforce that we have been doing "unplugged coding activities" for many years. These support the spatial reasoning expectations in the Geometry strand, e.g., identifying location; representing static location; giving directions, using distance and direction; finding objects, using landmarks; and perspective-taking relative to other objects.

As requests for more materials flooded in, the plan for an extensive *Roots of Coding* resource ramped up, with the goal of laying early, strong foundations in spatial thinking and visual understanding.

We began by examining the spatial concepts mapped onto the 2016 Kindergarten document and the 2020 Geometric and Spatial Reasoning strand. The chart in Figure 1, by Edward Schroeter and myself, synthesizes the curriculum expectations for each grade, mapped onto the nine major spatial concepts in Geometry, as defined by Ginsberg and Oppenzato (2022): describing position; identifying location; giving directions, using distance and direction; finding objects, using landmarks (relational understanding); using positional language; representing static location; and perspective-taking relative to other objects

We decided that our first lessons would be about the concept of *position*. But before beginning our own development process, we completed an extensive search of commercial and online resources about *position*. That resulted in innumerable worksheets and matching games focused on the same limited vocabulary, i.e., *below*, *above*, *beside*, *between*, *inside*, *outside*, *in front of*, and *behind*.

Where were the nuanced position tasks that required children to describe relative position in horizontal (before, behind) and vertical (above, below) contexts? Or, illustrated spatial, positional opposites; above and under, front and back, top and bottom, and inside and outside? Where were the tasks that focused on *complex positions* and *dual/multiple relations* (e.g., my nose is **above** my mouth and **below** my eyes in the **centre** of my face)?

Research tells us that children who acquire a strong understanding of space and spatial language tend to demonstrate higher math achievement than students who do not achieve such mastery (Wolfgang et al., 2001). Research also tells us that while children have an informal knowledge about space on which early math education can build, they are quite capable of learning more, deepening their understanding of position, and learning the correct mathematical words for talking about it (Bruce et al., 2012).

For these reasons, we took on the challenge of filling in the gaps by developing five lessons about complex positions and dual/multiple relations. I will discuss one that I contributed to the project, *The Sandwich Shop* (see Figure 2), because it proved to be a challenge for the children.

As with all lesson ideas presented in *The Roots of Coding*, Melanie and (often) Ed field-tested the lesson in Melanie's Kindergarten classroom. Through a lesson-study process that included field testing and video-recording *The Sandwich Shop* with the children in Melanie's class, we learned that describing complex positions, using the precise vocabulary associated with traditional positional language teaching (e.g., *inside*, *on*, *under*), is decidedly not elementary. Repeatedly, the children described the position of, say, the tomato slice in their sandwich as being *in the middle of the pickle and lettuce*, or *in between the top and bottom of the bun*. Unfortunately, many other contents, such as cheese slices and ham, were also *in the middle of or in between* other sandwich fixings.

Dr. Herbert Ginsburg (Ginsburg & Oppenzato, 2022), a pioneer in research about young children's mathematical development, explains the difficulty with describing complex positions this way: "Because children tend to be egocentric, that is, to see things and their relations only from one point of view, they find it difficult to deal with dual, or more generally, multiple relations."

As Ginsburg emphasizes in his article, and Melanie, Ed, Sandy, and I learned through our lesson study, teachers play a key role by modelling with manipulatives (e.g., building sandwiches). They also provide specific language (e.g., the white cheese is *under* the lettuce and *above* the ham) to help children become increasingly skilled at using positional language to describe two or more different perspectives at the same time.

Multiple positions required a dive into geometry by the children that was far deeper than Melanie, Ed, or I had anticipated when I wrote the lesson plan and they implemented it. We concluded that part of the challenge rests in the fact that positions and positional language feature prominently in our

SANDWICH SHOP

Skills: More Complex Positional Language

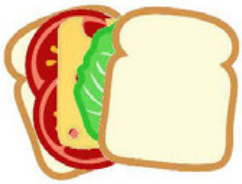
Activity

This activity is about using language and concrete materials to describe and understand spatial ideas that emerge in everyday experiences and use relative positions and locations, e.g., the pickle in the middle of a sandwich is at the same time **above** the bottom piece of bread and **below** the top piece of bread.

1. Introduce this activity by reading *Carla's Sandwich*. To help students to visualize the sandwiches, use paper cut-outs to build Carla's *Olive, Pickle and Green Bean Sandwich* or her *Liver, Potato Chips, and Cucumber Sub* as you read the description aloud.
2. Prompt the students to identify a sandwich component that is simultaneously above and below another ingredient.
3. Change the order of the ingredients and ask if the statement is still true.

Introduce the paper cut-outs that will be used to represent the parts of a sandwich, e.g., bread, salami, cheese, lettuce. Explain that one student will be the sandwich maker and the other student will be the customer. They are separated by a barrier (the counter of the shop). The customer orders from the list of available ingredients. The customer and the sandwich maker each have a set of ingredient cut-outs but they are not visible to each other. The customer requests the ingredients (e.g., two slices of Swiss cheese, four pieces of salami) and specifies where they are placed (e.g., one slice of cheese is above the salami and the other is below the salami, the lettuce is below the top slice of bread and above the tomato slices) and each one builds the sandwich. When the customer is finished, the sandwich maker serves the sandwich and compares it to the customer's sandwich. A set of position words can be used as prompts to describe the placement of the ingredients.

Once the children are familiar with the activity it can become a centre.



Look Fors

You might see...

- Students using positional language with varying degrees of precision.
- Students limiting their descriptions to one position word.
- Students sharing and taking turns.
- Students rephrasing their requests if it seems their partner didn't understand.

You might hear...

- "I said to put two onions above the cheese and two on the bottom of the sandwich. That means there are two onions on the bread." (rephrasing)
- "Our sandwiches look different. What happened?" (self-assessment, reasoning)
- "How do keep track of the steps? I forgot what Azza said." (students asking for clarification)

Reflection/Next Steps

What did students find easy? Frustrating?
What skill requires further/explicit instruction (e.g., Positional vocabulary, verbalizing instructions, understanding instructions, etc.)?

Next Steps:

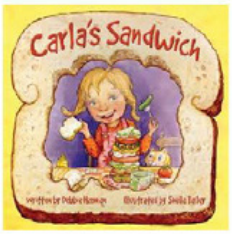
If students found this challenging:

- Demonstrate the skill that is challenging, modelling giving or receiving instructions while speaking your thought process.
- Ask the student to use linking blocks to build a vertical tower. Then ask the student to point to a block that is below the red block but above the blue block. Is there more than answer? Can we use other words to find the exact block (e.g., on top of the black block, in the middle of the two green blocks).

To extend learning, have students try to:

- Describe the sequence for building a hamburger and then label their burger design using position words.
- Ask students to use the artwork from *Everything on It* and use three or more position descriptors to identify the location of one specific item in the stack.

Materials



Carla's Sandwich by Debbie Herman is available on Epic!

- ✓ Cut-outs of Carla's Sandwich ingredients.
- ✓ Click here to view a video of this activity.
- ✓ Cut-out sandwich components for the Sandwich Shop.
- ✓ Position word cards.

Extension and Integration Ideas

Read the poem *Everything on It* by Shel Silverstein (first few lines):

I asked for a hot dog
With everything on it,
And that was my big mistake,
'Cause it came with a parrot,
A bee in a bonnet,
A wristwatch, a wrench, and a rake.

Display Silverstein's drawing on the Smart Board and ask students to use position words to describe something that is on top of the basket of apples but below the boot. Repeat with other objects. Next, ask students to guess an object by describing it as on the chair and above the plant. Switch roles, asking students to describe the location of an object and give position clues for an object.

Challenge students to make up their own "everything on it" non-sense poem using another context, e.g., making a bed and tidying their room, folding the laundry, or decorating a cake. Ask them to illustrate their poem and then make up some questions for classmates to locate objects in the stack by using position clues.

Figure 2: The Sandwich Shop *lesson*

everyday experiences, knowledge and colloquial language, lacking the precision and replicability of mathematical communication (e.g., describing how to make an exact copy of a towering sandwich with multiple layers).

Through the exercise, we hoped to provide a classroom-ready learning activity that illuminated one of the stages in the sequence to the development of the mathematical understanding of space (what we call *The Roots of Coding*). We quickly concluded that we needed to build in explicit instructional examples to support the teacher as Melanie negotiated the children's cognitive wrestling matches around multiple positions. Mathematizing and elaborating upon children's intuitive, passive understanding of space and position required "just in time" teaching prompts. In this way, it reduced the cognitive dissonance caused by an object that is simultaneously above one landmark (i.e., something that helps define the location) and below another.

The Sandwich Shop—on the surface, nothing more than a fun positional language activity—represents a critical step in children's achievement of one of the central tenets of *coding*. Coding consists of understanding an *algorithm*, which is basically sequencing/completing a task in a certain order or putting a set of instructions in precise order.

Our team is excited to be part of the August 22 program—we are "paying it forward" by offering the fruits of what we have had the privilege to learn from research, children, and each other. The other presenters feel the same way. Keep an eye on the Math Knowledge Network website (www.mkn-rcm.ca) for details as they become available. Hope to see you there.

References

Bruce, C., Flynn, T., & Moss, J. (2012, June 25). *Mathematics for young children: Literature review.* Comprehensive literature review submitted to the Literacy and Numeracy Secretariat, Ontario Ministry of Education.

Ginsburg, H., & Oppenzato, C. (2022). *What children know and need to learn about shape and space* [Stanford University, DREME Network]. https://prek-math-te.stanford.edu/spatial-relations/what-children-know-and-need-learn-about-shape-and-space

Ontario Ministry of Education. (2020). *The Ontario curriculum, grades 1–8: Mathematics.* Queen's Printer for Ontario. www.dcp.edu.gov.on.ca/en/curriculum/elementary-mathematics

Ontario Ministry of Education. (2016). T*he kindergarten program.* Queen's Printer for Ontario. https://files.ontario.ca/books/kindergarten-program-en.pdf?_ga=2.171028712.993688128.1650408882-1661041846.1650408882

Wolfgang, C.H., Stannard, L.L., & Jones, I. (2001). Block play performance among preschoolers as a predictor of later school achievement in mathematics. *Journal of Research in Childhood Education, 15*(2), 173–180. doi.org/10.1080/02568540109594958 ▲

▲ Coding in the Classroom: Interview with Amanda Deneau



IAIN BRODIE
IAINBRO1.625@GMAIL.COM

Iain has been an elementary school educator for 28 years, and is now into a second career, teaching in-service and pre-service teachers. He teaches face-to-face at Ontario Tech University and virtually at Western University. He is the current editor of the Math+Code 'Zine, www.researchideas.ca/mc/.



HATICE BEYZA SEZER
HSEZER2@UWO.CA

Beyza is a former teacher and long-term researcher. She is currently a graduate student in education at Western University, and her research focus is the integration of computational thinking into mathematics education. She is an editor at Math+Code 'Zine, www.researchideas.ca/mc/.

This is a column that interviews Ontario teachers who use coding in their practice. Iain and I have a conversation with the teacher to find out how they got started and their favourite pedagogical strategies. In these columns, you will find some of these teachers' best ideas about using coding in their classrooms, but if you want more, you can also access the complete interviews at OAME Talks to get a fuller sense of how these remarkable teachers have struggled and been successful with coding in their classrooms.

(Quotations within this column may have been lightly edited to reflect the difference between oral and written language.)

We were delighted to have Amanda Deneau talk about coding in her classroom and in her role as an instructional leader. In addition to being a great mum, Amanda Deneau is an innovative education consultant and both the creator and owner of We STEAM-ED Innovations, as well as a basketball coach and musician. She has been an Ontario Certified Teacher (OCT) since 2008, with experience teaching students from Grades K–12. Amanda has been

training teachers and working with innovation companies in Canada since 2015.

> I'm a mum, number one. And number two, I'm a teacher. I teach Grades 7 and 8 STEAM. It's new this year, a program we're building for science and technology in the arts program and math. I've been an instructional leader and consultant for my board. I have a side business called We STEAM-ED (www.westeamedinnovations.com.westeamedinnovations.com/home). The main goal of this business is to bring coding, computational thinking, and robotics to my small town, for my children to have some robotics and activity—nobody was offering it, so I thought, "Let's do it."

Coding, as a Manipulative and a Tool

Amanda shared her perspective on teaching coding as an everyday tool with which to learn. Her primary focus is teaching computational thinking, while she employs coding as a manipulative and a tool; that perspective resonates with the curriculum expectations.

> I think we've always really been teaching computational thinking; I think it's kind of innate and just being a little more identified because coding definitely opens the doors to many job possibilities in the future and all those fun things, but the process of computational thinking applies everywhere. So, even if you don't go into coding, this computational thinking needs to happen. We've got technology everywhere we are. And so, if we can learn to think like that, then we can learn to solve that.
>
> I'm teaching, and [coding] is another tool. So, learning block coding and Scratch, we're not focusing [on] that, [that] wasn't the project, like our project this week, for example, was teaching the seven sacred teachings in relation to orange shirt day, and we started an interactive poster. So, they've got micro bits and Makey Makey, and the Scratch component was coding. But the goal of the project was an interactive poster. Yes. So, we used coding as a manipulative and a tool.

Curriculum Expectations on Coding

Amanda summarized the pedagogy of coding in the math curriculum expectations, which is to read code, alter code, describe the effects of the change to the code, and make it in your own way. As she indicated, curriculum expectations in the math curriculum fit everywhere. Therefore, there is no need to limit the scope and sequence of coding with the math subject.

> Some teachers were asking questions during the webinars, like "What program? What coding platform or project is related to the curriculum?" The thing is, the curriculum is designed from what it looks like. I mean… it's to read code, alter code, describe the effects of the code of the changes. And could you make it better? What did it do? So, your curriculum expectations in the math curriculum fit everywhere.

Not Problem Solving, But Problem Identifying

Amanda highlighted a very important point that is commonly missed.

> The big thing for me a long time ago is everyone's talking problem solving, and I was like, no, we can't just talk about problem solving because we've been talking about that for a long time. It has to be problem identifying. And that's what I'm trying to teach my kids at home too. I feel that a lot of times, we're just used to having someone tell us what the problem is. It's a lot easier to fix a problem when it's pointed out to you when we're handing out these questions to the kids.
>
> She explained the way to let kids identify a problem.
>
> I'm trying to get the kids to identify problems. So yes, you know, like teaching the kids to change a tire. Where's the problem? Where's the sound coming from? And so, when you're using computational thinking strategies, which is, you know, identifying, finding the patterns, where is it not working? How do I fix it? We're starting to teach little kids that, and I've got students in class asking me questions before even trying to find the problem, and I won't answer.
>
> So what we're doing right now is troubleshooting. We have to troubleshoot because it makes you better problem solvers at the end, when you can identify where it broke down.

Coding Has to Be a Fun Thing, But Not a Hard Thing

As an educator trainer, Amanda pointed out that one of the big misconceptions that teachers have is the idea that one must be an expert in coding to teach it. She mentioned that educators do not have to be experts or [be] better than students. On the contrary, students are the ones who need to be better than teachers.

> You don't have to be better than the students [at coding]. In fact, if at the end of the year, they're not more advanced than me, we're in trouble because I'm not going to be the one in the future...

they have to be better than us.... My biggest message is to show your vulnerability and learn together. That's why I try to reassure everyone that this doesn't have to be a hard thing. It just has to be a fun thing.

She mentioned that one of the common questions coming from educators is how often coding should be done?

How often should they [students] do it [coding]? Well… how often do they write? It's not a question of how often they do it, it's just an extra tool. Let's teach coding as another language.

She also added a significant point about the assessment of coding that the original program and modifications done by students are where the real learning is going on. She indicated one of the best ways of doing an assessment.

What I recommend to teachers is when you are tinkering, ask them [students] what they've done to modify it [the coding]. It's all through the conversation. The best assessment for all of this is conversation.

She provided an insightful example for assessment through conversation:

Teacher: What is this?

Student: Well, it's a fishing rod.

Teacher: Why did you make a fishing rod?

Student: Well, I've altered the end of it to pick up plastic, rather than catch fish.

Teacher: So, it's like you bring it on your fishing boat, but you can also pull the plastic out of the water in relation to the global goal underwater.

The Importance of Playing in Coding

Amanda indicated the importance of playing, and mentioned that robotics such as Sphero and game-based tools like Minecraft are the most engaging ones.

In a Grade 3 classroom, there was a student with severe difficulty, an IEP [Individual Education Plan], behavioural, motivation, all of it. And he's isolated, of course, at his desk, and I introduced just code.org. It was the Hour of Code of some sort, just a quick lesson, and he started to do the Minecraft puzzle, and it started to make sense. And you can see, he was very excited about it. So we did it again the next week, and then he's standing over his teacher, pointing to the screen and explaining how this works. And the role reversal was super cool. And I was like, oh, because he was talking coordinates. And he didn't know he was talking coordinates.

Coding Boosts the Confidence of Students

Amanda emphasized how excitement leads to students' confidence boost. When they are given something that interests them, they push themselves out of their comfort zones.

It's the excitement when it works at the end that's giving them that confidence boost. I had a boy who's a little reserved, has an IEP, and all the things. And he told me this week that he wants to be an engineer and he wants to go to MIT (Massachusetts Institute of Technology). So, because of Scratch, he says, "I wanted to be in the military, but now I think I want to go to MIT."

The Gender Gap is Getting Smaller through Coding

As one of the important issues, coding is an important tool to support equity and eliminate the gap between genders in education, as Amanda said.

I've done some work with Hacker Gal, and I'm in communication with that organization. And I do have an all-female class. It's the dance academy, and they're all female. Oh, wow. I've got 26 girls in one class, and they're killing it. They're good. They're loving it. But again, it's more on the artistic side, where the stereotype of math for boys, language for girls, it's really gaps like this, yes, this is them not even realizing what's happening. And the boys are doing more of the language stuff. Like it's kind of like the gap is getting smaller.

The Size of the Class Is a Challenge

Amanda mentioned that her biggest challenge is implementing coding and using maker spaces in a large class. That challenge comes out due to a lack of material and engagement.

I have a class of 32, who comes into my maker space. And I can definitely affirm that it is way harder to reach them than the 25. It makes a big difference in the organization and planning and the actual facilitating of that many bodies. It is my biggest challenge… as much as… they're engaged, they're loving it. It's not about students, but it's about the organization having the material and being able to help them with the material and troubleshooting.

Coding for a Cause: UN Global Goals

From beginning to end, Amanda emphasizes coding for a cause.

> Everything that we're doing in coding and science is connecting one of the United Nations global goals, [such as] sustainable development goals… With coding for a cause, we're going to also make "making" for a cause, like we're making stuff. This lets them just go be inventors and create, and they feel good about themselves.

Amanda shared with us significant information and insight from her multiple perspectives as an educator, educator trainer, and innovator. We have highlighted some of her points of view, but there are more than these, which include practices for teachers and teacher candidates who promote coding at school and beyond. Therefore, we encourage you to listen to OAME Talks with Amanda Deneau.

We are actively looking for teachers to interview for future columns and podcasts. If you are, or know, a teacher in Ontario, who is doing great things with computational thinking, let Iain and me know. ▲

Book Review continued from page 15

geometric formulas. The Grade 9 Academic formula sheet is referenced making a connection for Ontario teachers. Throughout these sections, as per the title of the book, the focus is on the middle school curriculum that contributes to the general overview of relevant content.

The algebra section has the best connection between history and instruction. It is sprinkled with historical connections that can serve to enrich classroom instruction. Similarly, historic graphs including bar charts from 1482 and 1786 speak volumes. This treatment is food for thought for Intermediate teachers.

One small aspect of the book that does not do justice to the topic is the section that mentions the cubic formula. This seems out of place in the book, though it is used to clarify why graphs might be used to estimate solutions. The historical details of the cubic formula are passed over rapidly to advance Abel's discovery (which I will leave for the book to speak of). Yet, this particular topic has a very rich history.

Overall, the book is a good summer read for Intermediate, and possibly Junior, teachers, who will benefit from an overview and taste of historical connections. The various components of the book are sure to help build a focus that aligns with the curricular need to bring math history into the classroom. For teachers who are aware of some historical details, the book may help build a well-rounded knowledge; however, it will not provide in-depth details. ▲

▲ Destreaming/De-tracking Math Position Statement

The goal of each OAME/AOEM Position Statement is to outline OAME/AOEM's beliefs on issues surrounding math education by providing transparency and guidance to those involved with mathematics education in Ontario. Any clarification regarding a position, or how to proceed in accordance with it, can be brought to the attention of the OAME/AOEM Executive Committee.

Focal Question

How do we as a community of educators support effective implementations of practices for a destreamed mathematics classroom?

Summary

OAME/AOEM promotes, supports, and advocates for excellence in mathematics education for all students. There is a body of research which has demonstrated that academic streaming based on perceived mathematical ability has negatively impacted Black students, Indigenous students, students from lower socio-economic backgrounds, and students receiving special education supports. As stated in the OAME/AOEM position statement on Access, Equity, and Inclusion, "All students should have equitable access to a high-quality, research-informed mathematics education that meets their diverse needs and is provided in a safe and inclusive environment." As such, OAME/AOEM supports mathematics educators in the dismantling of formal and informal structures rooted in bias that lead to academic streaming.

Academic streaming (or tracking) is the separation of students by perceived ability and/or previous academic achievement. Destreaming (or de-tracking) refers to the dismantling of academic streaming systems, leading to academically heterogeneous or diverse student groupings that are consistent within a classroom and between classrooms. Academic streaming and destreaming may be occurring explicitly (visible structures that noticeably separate students by perceived ability) and implicitly (invisible structures and practices that separate students by perceived ability).

Why is destreaming important?

Research has shown that separating students into academic pathways too early in their schooling widens achievement gaps and inequities, particularly for Black

students, Indigenous students, students from low socio-economic backgrounds, and students receiving special education supports (e.g., OECD, 2012; Ontario Ministry of Education, 2017). An educator who is effectively implementing a mathematics curriculum in a destreamed setting might address these inequities by:

- fostering a healthy and positive relationship with mathematics for all students
- fostering active self-reflections on potential educator biases
- valuing and affirming diverse knowledge, experiences, and contributions from all students
- combatting existing systemic and societal presumptions about students and about mathematics

Effective Practices for a Destreamed Mathematics Classroom

OAME/AOEM is committed to creating, supporting, and promoting opportunities for educators to learn and collaborate on strategies to effectively teach in a destreamed mathematics classroom. As an organization, we advocate for equitable and inclusive practices that provide all students with high-quality mathematics learning experiences and empower them to see themselves as mathematicians.

Destreamed instructional practices should:

- allow for student voices and choices while learning mathematics (e.g., Universal Design for Learning (UDL), differentiated instruction)
- reflect students' diverse backgrounds and experiences, allowing them to see themselves in the content (e.g., Culturally Responsive and Relevant Practices (CRRP), using mathematics as a tool to investigate topics of social justice)
- foster a safe learning environment, where students feel valued and comfortable enough to share their thinking (e.g., building an inclusive classroom culture and community, thinking-classroom practices, productive struggle, rough-draft thinking)
- recognize the diversity of ways that students demonstrate their learning (e.g., triangulating evidence of learning, open and parallel tasks, providing multiple opportunities to demonstrate learning over time)
- support all students to see that mathematics includes a collaborative human story across cultures (e.g., examining historical development of mathematics from all cultures beyond the currently dominant Eurocentric perspective)

Continued on page 51

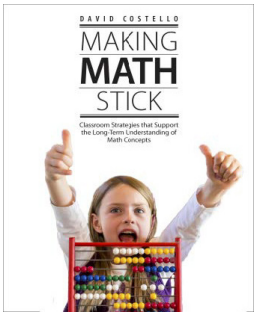
▲ Book Review: Planning "Effortful" and Periodic Retrieval in Mathematics Learning



EDWARD SCHROETER
edschroeter@outlook.com

Ed Schroeter, B.J., B.Ed., OCT, is a retired elementary teacher, having taught for 30 years, 18 of them in Kindergarten. He was the Grade 1 Lead Writer for the Ontario Mathematics Curriculum Resource Project (2020–2021)*. He led three* Teacher Learning and Leadership Projects (TLLP) *lesson studies. From 2009 to 2019, he taught two ETFO Summer Academy courses and presented more than a dozen workshops on inquiry learning around Ontario as part of the* Presenters on the Road Program*. He is the author of Chapter 9 of* Teaching for Deep Understanding: An ETFO Curriculum Learning Resource Compilation.

Costello, D. (2021). *Making math stick: Classroom strategies that support the long-term understanding of math concepts.* Pembroke Publishers Limited.



In his 2021 professional book for elementary and secondary math educators, *Making Math Stick*, Dr. David Costello identifies a problem to which most classroom teachers can relate: students seem to master math concepts and skills, only to forget them in the weeks that follow. Describing the prevalence of retrieval problems across classes, grades, and schools as a "serious educational issue," the author immediately captures the reader's attention when he conjures an image of an academic epidemic: the proverbial sea of blank student expressions that greets teachers when asking their class to recall a concept or skill is sure to resonate with many educators.

Dr. Costello turns to cognitive science for a solution to the perennial challenge of student learning loss. It describes three phases of learning: encoding (get knowledge into our heads), consolidation (assigning meaning to the knowledge in our heads), and retrieval (bringing something previously learned to mind and applying it). Retrieval is considered to be the most powerful form of learning. The author

convincingly argues the case for intentionally embedding retrieval of learning in long-range plans, and for calling upon students to retrieve and apply previously learned concepts and skills repeatedly throughout the school year.

The author explains his belief that mandated provincial curriculum and standardized assessments have tended to promote an instructional climate that focuses on encoding information, without demanding retrieval. The effect of this is to put knowledge and "understanding of the concept" into relatively fleeting short-term memory, instead of locking it into long-term memory.

Costello goes on to demonstrate how the practice of retrieving knowledge multiple times in a variety of situations strengthens learning and embeds it more strongly in long-term memory. He points out that each time a memory is retrieved, the student must think about it again and reconstruct it in terms of previous knowledge and the current experience. He cites research that supports his thesis that "every time a person interacts with a memory, they bring new understanding and experiences to add further structure to the memory." In his view: "A common misconception is a belief that if students can recall knowledge once, the learning is complete" (p. 13).

Kindergarten to Grade 8 and secondary math educators can benefit from studying this book. Costello's general approach is to combine Jerome Bruner's 1960 spiral curriculum with unit planning in annual long-range plans. As he elaborates his methodology, Costello takes us on a journey that explains the function of spaced and mixed practice, carefully planned homework, low-stakes testing, cumulative mixed testing, and fundamental key concepts in the improvement of retention and learning. He defines foundational key concepts as the highest-priority curriculum outcomes, which support learning many other standards and concepts—the big ideas in math. With that definition he suggests front-end loading instruction at the beginning of the year with foundational key concepts.

After explaining how the aforementioned approaches enhance memory, Costello fully demonstrates how to integrate these features into long-range plans. As Costello explains on page 100: "Following a spiralled curriculum year plan, you constantly review previously taught material instead of always broaching new concepts in sequential order...." Costello solidifies his intuitive argument that his approach supports long-term retention by citing research such as Henry Roediger and Jeffery Karpicke (2006).

Costello also emphasizes the importance of "effortful" retrieval for retaining long-term learning: "Learning must be developed over time through meaningful, purposeful practice—and it must be hard!" (p. 13). It must be accomplished through "desirable difficulty," also known as "productive struggle," and it must push right up against the far edge of Lev Vygotsky's well-known Zone of Proximal Development (ZPD).

The book is divided into two key main sections: learning strategies students can use to improve retention, and instructional strategies that educators can employ with their classes. As one might hope from a book about enhancing learning, the organization of the book's key content is an effective technique for readers. In the first section, Costello outlines how to use student self-assessment strategies (e.g., quizzes, flash cards, self-monitoring), building a network of memory (e.g., questions, dual coding, concrete examples, comparing and contrasting), learning by figuring out problems (e.g., analyzing errors, testing a list of strategies on the same problem, working backward), learning by picturing solutions (e.g., graphic organizers, diagramming and visualizing), learning by writing (e.g., paraphrasing, doing a free write, using a plus–minus chart), and using awareness strategies (e.g., using a learning journal, reflecting on experiences, recording next steps). In the second section, Costello presents whole-class instructional strategies for teachers to adopt, which foster long-term retention, including focusing on key concepts, annual plans, low-stakes testing, and review methods.

Costello also provides excellent support for teacher understanding with summaries, plain language, his clear writing style and generous use of text features. He elaborates concepts and ideas with classroom examples, photographs, testimonials, diagrams, textboxes, key points in marginal notes, charts, section previews, and chapter summaries. He also includes technology suggestions called "Digital Solutions."

The other good news for experienced teachers is that many of Costello's retention-building strategies are familiar, with only slight twists. For example, many teachers will recognize and have used ideas such as the strategy of free sketching to understand equivalency, but may not have thought to ask their students to conjure a mental image of a hundreds chart to help them remember skip counting by 2.

Adventurous teachers might pick a chapter, any chapter, skim its content and summary, jump into the classroom and begin experimenting, analyze the results, and revise their approach as they read in more detail. However, most teachers will want to begin by digging into one chapter at a time, studying it, and then testing it in their classrooms. That's also Costello's advice (p. 139). Teachers should start with a chapter that appeals to them. For example, Intermediate teachers might want to begin with "crafting homework that gives meaningful practice opportunities."

One appealing feature of Costello's approach is that it does not require significant extra work. Most teachers are already using long-range plans. The work involved in changing teaching practice is limited to adding retention strategies to existing documents. Costello stops short of doing the actual work of creating annual spiralled plans for us. It would be impractical for Costello and his publisher to generate these in a book intended for Kindergarten to secondary school teachers in a North American market. Teachers are autonomous, and it is far better if they develop their own personalized plans.

The lack of long-term plans aside, this book is long overdue—at least in the elementary panel. It is a sensible, in-depth remedy to a longstanding issue in elementary mathematics education—the trend in Ontario away from repetitious learning and revisiting content already learned. Somewhere in the struggle to ensure coverage of Ontario's crowded elementary math curriculum, it appears that many teachers switched to unit planning and lost sight of the value of spiralling the curriculum. Unit planning may ensure coverage, but coverage does not guarantee learning. Costello describes some unit-planning methods as blocked practice and a "teach-test-move-on" pedagogy.

This book should be received as welcome relief for teachers who have been perplexed about why mathematical understanding doesn't seem to "stick" with their students, and should be seen as an important resource for new teachers to start them off on the right foot. I wish that this book had been on my own shelf when I was a classroom teacher.

Reference

Roediger, H.L., & Karpicke, J.D. (2006). Test-enhanced learning: Taking memory tests improves long-term retention. *Psychological Science,17*(3), 249–255. doi.org/10.1111/j.1467-9280.2006.01693.x ▲

"Europe was exposed only to the Greek vision as represented by various translations into Latin and other languages from Arabic texts. In a search for their roots, Europeans bypassed their Islamic and non-European heritage and homed in on Greece and Rome. Greece thereby became the fount of their intellectual and cultural heritage, while for their religious roots they looked toward Rome and the Byzantium." (*The Crest of the Peacock* by George Joseph, p. 509)

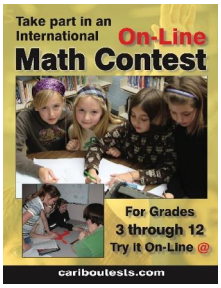
▲ Caribou Math: International Math Contests Made in Ontario



DR. JEFF IRVINE
dr.jeffrey.irvine@bell.net

Jeff Irvine, Ph.D., has been a secondary mathematics teacher, department head, and vice-principal. He has taught at three faculties of education and at a community college. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. With over 45 years in education, Jeff is particularly interested in the interplay of student motivation and mathematics achievement.

Editor note: The author has no financial connection to the Caribou Math contests.



The *Caribou Math Contests* ("Caribou") were created by Dr. Thomas Wolf at Brock University in 2009. In the first contest, 600 students participated, although the program crashed midway through the first contest. This year, there are now over 50 230 participants from 34 countries, from Australia and Azerbaijan to Vietnam and Uganda. Caribou Math offers six contests per year, now for students in Grades 2 through 12. The contests are answered online during the school day, and typically have up to 18 questions for each one-hour contest. Some sample problems are shown throughout this article. Student results are available online immediately following the contest date. Students can find out their standing and print personal certificates within one minute of the close of the contest.

Dr. Wolf identifies the goals of Caribou Math as:

- showing that mathematical puzzles can be fun, and that competing in math contests with interactive questions is exciting
- improving students' problem-solving techniques by providing a wide variety of challenging math activities suitable for all students
- motivating students to utilize their school math when a question needs a technique or formula they have learned in school

Questions are created by Dr. Wolf and his team, often inspired by problems found on the Internet, and are programmed by students at Brock University. Dr. Wolf's team has also written computer programs to generate unlimited problems of specific types. The contests are currently offered in four languages: English, French, Persian, and Mandarin. Schools can register for the contests at www.cariboutests.com/. The first contest each year is free, as are all contests for Grade 2. The Grades 3/4 contest fee is $9 per student for the full year, and $15 per student in Grades 5 to 12 for the entire year of six contests. Alternatively, a school may purchase a school-wide pass for $280, allowing an unlimited number of participants for the entire year. These fees cover the costs of creating the questions and programming for online access by the students.

A rectangular table has a length three times as long as its width. If it were 1.5 m shorter and 1.5 m wider, it would be a square. What are the dimensions of the rectangular table?

- 6 m x 3 m
- 7.5 m by 2.5 m
- 8 m x 5 m
- 3.5 m x 11.5 m
- 4.5 m x 1.5 m
- None of these options are correct.



Figure 1: *A sample problem*

Today's Calcrostic Puzzle

a	×	bc	=	dcc
+		/		/
bc	/	e	=	a
=		=		=
ba	-	a	=	bc

Figure 2: *Letters* a–e *are unique digits from 1–9. Determine which letter goes with which digit.*

Each contest includes an interactive question; for example, every second contest includes a Calcrostic—a calculation-based acrostic-style puzzle. Some contests include a history of math question. The topics for the interactive questions and history questions are announced two weeks in advance to allow for practice. There is also a daily Calcrostic available on the website. These puzzles require letters to be replaced with single-digit numbers to make equations in two directions work. In Figure 2, $axbc = dcc$ must be true; for example, it is true when $a = 6$, $b = 2$, $c = 4$, $d = 1$, making 6 x 24 = 144. Simultaneously, $a + bc = ba$ is required in the first vertical column. Using the values identified, is it true that 6 + 24 = 26? No, so the example values are not quite right.

Some participating schools ask for student volunteers to participate. Other schools identify groups of students based on enhanced classes, strong interest in math, or other criteria such as demonstrated ability. All students who have participated indicated that they had fun, learned a lot, and felt challenged by the contest questions.

Dave Yaciuk is principal of a middle school of over 400 students in Grades 6 to 8. Dave got his school involved with Caribou Math because of his mentor principal, who was an enthusiastic supporter of the contests. Dave sees Caribou Math as an excellent extension for his enhanced students, as well as a motivator for strong math students in his other classes. Preparation sessions were built into his enhanced classes' regular math periods. Approximately 60 of his students participate in the Grades 5/6 and 7/8 contests. He found that the contests increase student interest and engagement, foster a desire to improve, and promote ownership of learning in mathematics. While he found that the best part of contest participation was providing a challenge for his stronger students and reducing student boredom, he also found that word of mouth about the contests and their content encouraged teachers to "step up their game" with respect to building their own Content Knowledge for Teaching Mathematics (CKTM; Ball & Bass, 2005).

Kyra Kristensen-Irvine, former principal of a large Kindergarten to Grade 5 school, used the Caribou Math contests as enrichment and motivation for her students. "I had run other math contests when I was an administrator in other schools, but they were usually targeted at older students, and often were one-offs (only one contest per year). Caribou Math, with six contests per year, provided the opportunity to work with students for an entire year and build their problem-solving skills." Teachers were asked to identify students who might be interested in participating. Other potential contest students were identified using their Canadian Cognitive Abilities Test (CCAT) scores, which are an indication of ability rather than achievement. Students who scored in the 90th percentile or above on the CCAT were invited to the preparation sessions and then to write the Caribou Math contests. Ironically, some of these students were not performing at a high level in their regular math classes, perhaps due to boredom or lack of self-esteem. Across Grades 3, 4, and 5, over 100 students participated each year.

Preparation sessions were held during the school day at least once a week and often twice a week throughout the year. Kyra led the Grade 5 sessions, while her two vice-principals led the Grade 3 and Grade 4 sessions. Kyra developed a large book of previous contest questions for each grade, sorted by type of question. The sessions involved students working together in groups to solve problems, using the

heuristic "What do they want to know?; what do we know?; how can we solve this?" Kyra and her vice-principals functioned as guides and supports, not providing answers, but furnishing scaffolded questions when groups got stuck. The work paid off. In one year, one of her students placed first in the entire international Grades 5/6 contest, and in the same year, her school had four of the top ten students in the Grades 5/6 contest internationally, even though the students were all in Grade 5. Top students were recognized at a board-wide celebration at the end of the school year.

There were two interesting outcomes of her students' participation. First, the contests and associated preparation sessions were extremely popular with parents and guardians, whose children sometimes felt a lack of challenge in their regular math classes. This increased parental involvement and pride in the school. Second, Kyra had the opportunity to engage in some gap closing with female students. She found that some very bright female students lacked self-confidence, but really blossomed when placed in sessions with a collaborative atmosphere, where they were seen as equals to strong male math students.

Four classmates are discussing events and their associated likelihoods:

Jasmine: In a bag with one blue ball and one green ball, I randomly selected the blue one. I put it back. When repeating this process four times, I randomly selected the blue ball every time. My next pick is very likely to also be the blue ball.

Kendra: I watched Jasmine. In her fifth try, she will more likely pick the green ball next because it is very unlikely to get five blue balls in a row.

Tim: I also picked a ball randomly six times. I got blue, green, blue, green, blue, green. Because my sequence of alternating pattern already lasts longer than Jasmine's sequence, my next ball is more likely than hers to be blue.

Linda: When it starts snowing in Caribouville (30 km east of my town), it starts snowing here a few hours later. It just started snowing in Caribouville; therefore, it will probably snow here soon.

Whose statement is most reasonable?

- Jasmine's
- Kendra's
- Tim's
- Linda's
- Jasmine and Kendra's
- Kendra and Tom's
- All four of their statements are equally reasonable.

Kyra also served as an ambassador for the Caribou Math contests, enthusiastically acquainting her fellow principals with the contests, and even allowing students from other schools to write the contests at her school on contest days. Sometimes, there were over 30 students from nearby schools writing the contests at Kyra's school, in addition to the 100 students from her own school.

Some of the teachers at Kyra's school recognized the value of the Caribou Math contests and found their participating students to be more engaged in their regular math classes. The enthusiasm and excitement of the Caribou Math contest participants was contagious in the regular math classrooms.

The sum of the lengths of three sides of a rectangle is S cm, where S is a positive whole number. Each side is a whole number of cm in length. The difference in length of a longer and a shorter side is 8 cm.

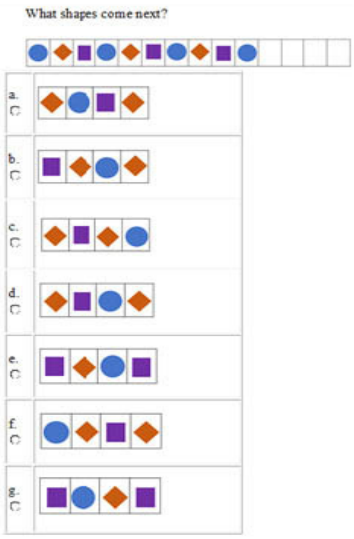
Let k be any positive whole number. Which option gives a formula that could never be the value for S, no matter what S is and no matter which whole number k is?

- $3k + 1$
- $4k$
- $3k + 2$
- $3k$
- $4k + 1$
- $4k + 2$
- $2k + 1$

Since the contest is international in scope, contest questions are not explicitly linked to Ontario math curriculum expectations. However, the contest questions certainly address the problem-solving elements of the math processes, and in most cases, questions can be identified with specific Ontario mathematics expectations. For example, the sample questions shown in this article can be linked to expectations in algebra, data management, and probability, and, in the case of the last question, to the patterning expectations in Grade 2.

Additional features of the Caribou Math website include a YouTube channel, where students demonstrate solutions to over 250 questions: a selection of interactive games for students to play, a daily Calcrostic, online mini-courses, and questions from past contests with solutions.

The Caribou Math contests present a low-cost opportunity to engage and challenge math students on an ongoing basis. Preparation sessions provide students with the chance to develop their problem-solving skills as well as their collaborative and social skills. The burgeoning growth in student participation in Caribou Math is a testament to the



efficacy of these contests in engaging our students in a fun and interactive way. An anecdote related by Dr. Wolf speaks to the popularity of the Caribou Math contests. “On the first day of school in September, a student came to the school office to ask the principal when the Caribou Math contests would start. She was eager to get started on the contests for the year.”



Reference

Ball, D.L., & Bass, H. (2003). Toward a practice-based theory of mathematical knowledge for teaching. In E. Simmt & B. Davis (Eds.), *Proceedings of the 2002 annual meeting of the Canadian Mathematics Education Study Group* (pp. 3–14). https://files.eric.ed.gov/fulltext/ED529557.pdf ▲

▲ Translating Video to Comics for Sharing Classroom Practice



KRISTA FRANCIS
kfrancis@ucalgary.ca

Dr. Krista Francis is an Associate Professor of STEM Education at the Werklund School of Education at the University of Calgary in Canada. Her research is about the teaching and learning of mathematics, robotics, and spatial reasoning.



STEVEN KHAN
skhan6@brocku.ca

I am a recent settler-immigrant from Trinidad, whose identity and ancestry tell a story of multiple migrations, intersections, and collisions in the colonial history of the Caribbean. I am currently an Assistant Professor at Brock University, St. Catharines, in the Department of Educational Studies, with foci/interests in Mathematics/Education for Multispecies’ Flourishing, teacher education, ethnomathematics, coding and computational thinking, open education resources, representations of mathematics and mathematicians, enactivism, complexity thinking, variation theory, mythopoetic curriculum studies, maroon theory, and teaching/learning as curating identity. I am grateful for the opportunities I have to live and work on these lands with my many relatives.

Have you ever had an insightful classroom incident that perhaps you captured on video (or wish you had captured on videos) and then wanted to share with colleagues or a broader audience? Maybe you are concerned about revealing too much about students’ identities or the classroom context in video clips or photographs? Or perhaps you’ve been exploring incorporating sketchnoting (Duckworth, 2016; Wheeler, 2018), photo essays as pedagogical documentation, or just enjoy sites like *Math with Bad Drawings* (Orlin, 2020).

In this article, we demonstrate how one can represent video of students working in classrooms with comics. In mathematics education, graphic narratives and sequential art are used as a form of research presentation and dissemination (e.g., Parker, 2017; Tschida et. al., 2019). Video and re-enactments are useful for communicating

findings in professional settings such as classrooms, conferences, and research reports. We have both used, and continue to use, video in our own research and pedagogical practices.

The study of comics has come into its own as a diverse and respected field of serious interdisciplinary scholarship (Aldama, 2018). Comics have been, historically, and are increasingly, used for communicating complex technical information (Munroe, 2015). The Beast Academy series (AoPS, 2020) is an excellent example of using the comic form to communicate mathematical ideas. McClain (2018) describes several examples of the use of comics for teaching mathematics. Wolczuk et al. (2018) have recently received funding to explore comics as pedagogical tools in an undergraduate mathematics course at the University of Waterloo. Gadanidis (n.d.) also uses compelling visual illustration and comic storytelling techniques to teach mathematics. Concept cartoons have been used in mathematics education for over a decade now (Dabell et al., 2008). In this article, we attempt to open up the ability to create comics from video data recorded in classrooms by teachers or researchers by discussing one approach we have found successful.

Affordances of Using Comics for Illustrating Classroom Practice

Non-Linear Sense Making

One of the main affordances of a medium like comics is the ability to quickly move forward and backwards over a set of panels to make sense of the scene and to connect disparate parts. This is different from watching video, where quickly and repeatedly moving across moments in time is difficult and time-consuming, and extraneous elements can be distracting. This is also different from reading text alone, where the scene must be imagined to include relative positioning and gesture. Thoughtfully constructed comic panels and sequences can serve as multimodal informational artifacts. It is worth pointing out that picture books, which are also a form of sequential narrative art, differ from comics in the strategies required for reading and interpretation. For example, there is less frequent "gutter"[1] work in making meaning in some mathematical picture books. In working with complex data, i.e., where there are multiple things happening either simultaneously or across different moments in time (in quick succession or across larger time frames), the comic format, through its decomposition into individually linked scenes, helps to highlight aspects for sense making.

Reducing Working Memory and Focusing Attention

A main consideration in choosing to use comics to represent video data is the way the form consolidates complex information in the juxtaposed sequences, thus making them more accessible to readers due to reduced working memory requirements. The comics we use attempt to focus attention on what we believe is relevant as evidence of learning. They reduce the amount of distracting information in a scene to allow a focusing of attention. For example, when looking at raw video data, there is the background, chairs, other movement, other individuals, and the elements of the sonic landscape. Reducing the complexity of a scene allows us to focus in on the same information and discuss salient ideas as they relate to the argument we are developing.

We find that the comic form allows for a juxtaposition in a small space of two times, that is, the time of the events in the data, and the time of our analysis/interpretation. We believe this allows readers to more closely experience and understand our analysis as they are not moving back and forth between watching a video or a direct capture of the whole scene and reading the analysis. Readers can thus engage both in close proximity and together, with the opportunity for non-linear sense making, develop a clearer understanding of what we are trying to communicate.

Performing Knowledge Mobilization as Public Pedagogy

Teaching is a complex endeavour. This complexity is challenging to communicate to diverse audiences, including students. In teaching–learning situations, there are multiple and simultaneous interactions to which a teacher has to attend. The use of comics has helped us to reduce the extraneous elements within the video data, include relevant multimodal information, and intentionally focus in on and attend to what is important to the frame we are using as an interpretive lens for our imagined peers and students. They are easy to incorporate into classrooms, presentations, posters, and discussions with family and friends who want to know, "What are you working on?" For teachers, a comic of classroom video data could be useful for taking an event from one class and presenting it to future classes for pedagogical reasons, or to create contrasting cases for class discussions and formative assessment purposes. Using comics in this way allows students to notice and comment upon different strategies identifying relative merits and weaknesses as part of consolidating their understanding.

For instance, in my classes (Krista), I aspire for pre-service teachers to attend to how programming robots to move can represent number-as-measurement. As students program the robot to move specific distances, they use

decimal numbers. We have found that robotics provides an enactive–iconic–symbolic (Bruner, 1964) representation of number-as-measurement that is very useful for students to learn rational numbers. The comic format enables us to focus on the student–robot–number interactions and highlight the student's engagement with rational numbers. Teachers could use similar comics as an instructional tool for teaching how to start/complete the task. They could also use the comics to highlight how decimal numbers are used for programming the robot to move specific distances. Similar comics could be developed for any concept, including solving an equation, or graphing a line from slope intercept form of an equation.

I (Steven) am developing an assignment where I will invite pre-service teachers to convert video clips from previously curated online sources into comics. These clips illustrate students' growth in mathematical understanding and associated teacher moves. The comics will be assembled into an openly licensed education resource. We will also be attempting to create comic cases for deepening student consolidation of understanding and assessment. This type of work could also be undertaken by district and school-level professional learning communities.

We credit one of the reviewers for pointing out that teachers could also open this process up to students in terms of sketchnoting their learning and communicating their understanding and connections of mathematical concepts through comics.

In this paper, we are attempting to show how video-based research/classroom data can be worked with and represented through ideas from comics.

Connecting Communities and Respecting Privacy

In this work, we see the development of a bridge connecting research and practitioner communities. In many schools, rules (or legislation) regarding student and teacher privacy makes showing classrooms and students' faces impossible. At the same time, teachers and researchers require ways to share what they are doing with others, while respecting these (legal and moral) rights to privacy. We see comics as one means of achieving these goals of sharing what is happening in classrooms in respectful ways. We acknowledge that we are drawing attention to what we have chosen to attend to in these scenes, which is influenced by our purpose of understanding mathematical thinking and learning.

The section below provides a step-by-step description of the process, apps used, and images of intermediary steps (or a video). To create the comics from video captures, we used the following: iPad Pro with stylus pen, Adobe Illustrator, Photoshop, Fresco (mobile app), and video data.

Steps for Creating Sketches of Video Screenshots

Initially when we started this work on this article, we had several pages of panels with descriptive text alongside images. One of us suggested that we self-referentially try to use the format and genre we were arguing for to represent the idea itself (Figure 1, p. 50). Figure 1 condenses several pages of technical instructions and information into one comic panel and, we believe, is more useful to a broader community of readers and educators.

We want readers to appreciate that the choice of video scenes to convert into comics is intentional, and there is a rigour and ethical consideration to these decisions. As a researcher, I (Krista) likely collect more video than most classroom teachers. However, there are some days when I or a teacher anticipate that there will be important and interesting actions and discussions. Capturing these moments on video can provide a fruitful resource for communicating these important and interesting actions. Typically, I am interested in engagement in spatial reasoning and/or mathematics.

Once I have made my final scene selection, I transcribe and analyze the video according to the theoretical framing I am using. In a recent study, we were looking at growth in mathematical understandings, using the Pirie and Kieren model (1994) as a Grade 5 student was learning how the *move steering programming block* of the Lego EV3 robot worked. From my final scenes, I looked for a portion that captured some of the key aspects of the interaction. Then I took six to eight screen captures of that portion. As I saved them, I numbered them so I could remember the sequence. I also took notes of the time-stamp on the transcriptions. This helped me find the narration later, when I assembled the images into a comic format.

The sequence of steps I use are listed below and illustrated in a comic format in Figure 1. I included a cycle arrow to indicate the iterative nature of many of the steps. This iconic element serves to add another dimension of time and depth to the two-dimensional image.

Step 1: Import a screenshot into Adobe Fresco

Step 2: Start drawing

Step 3: Crop the image

Step 4: Create a Layout

Step 5: Add text bubbles

Step 6: Add narration and analysis

Step 7: Edit and revise

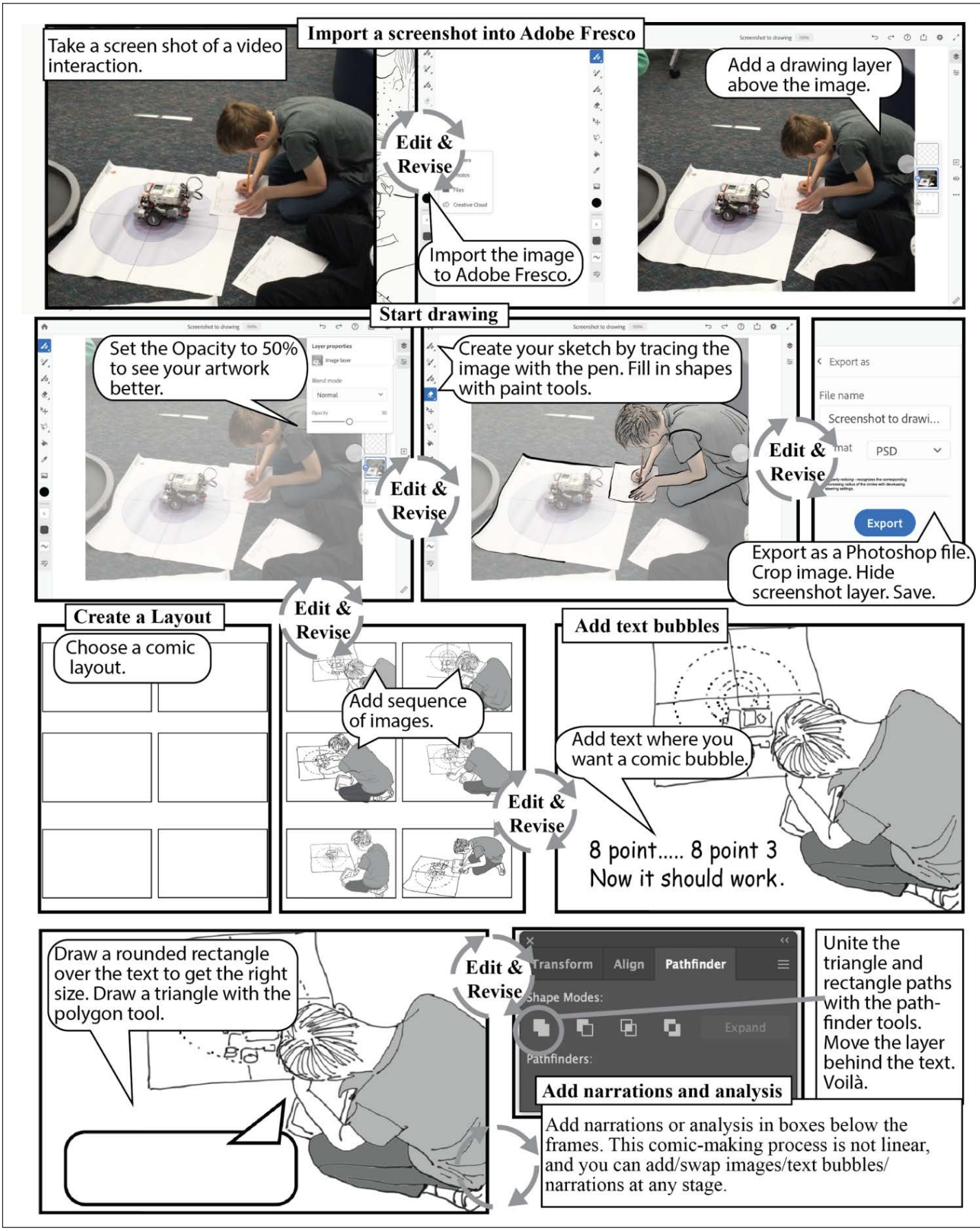


Figure 1: *Comic illustraton of how to create comics from image captures of video data*

Conclusion

We hope that researchers and teachers involved in classroom action research, using video, will try this approach and experiment with the medium of comics for themselves. We think it may be a meaningful way to begin to communicate data and findings to a wider audience. We would like to imagine that one day, we might see more work presented in this genre, and hope to inspire the next generation of students to consider the medium for their scholarly work.

In this paper, we have not engaged as deeply with the critical dimension of comic studies around issues such as the ethics and politics of representation, power, identity, and privilege, as our purpose was to communicate and demonstrate technical aspects related to a specific set of tools. We do feel this is an area for serious study, and wish to encourage researchers and their students to take the form seriously. We note that these critical issues will likely emerge for each researcher differently in their own context as they try to do this work, as they have for us. We intend to address these issues in future communication.

Note

[1] The gutter is the space between panels in a comic. As McCloud (1993) argues, "...in the limbo of the gutter, human imagination takes two separate images and transforms them into a single idea... comics panels fracture both time and space, offering a jagged staccato rhythm of unconnected moments. But closure allows us to connect these moments and mentally construct a continuous unified reality" (pp. 66–67).

References

Aldama, F.L. (Ed.). (2018). *Comics studies here and now.* Routledge.

AoPS. (2020). *Beast Academy*. www.beastacademy.com/

Bruner, J.S. (1964). The course of cognitive growth. *American Psychologist, 19*(1), 1–15. doi.org/10.1037/h0044160

Daball, J., Keogh, B., & Naylor, S. (2008). *Concept cartoons in mathematics education*. Millgate House Education.

Duckworth, S. (2016). *Sketchnotes for educators: 100 inspiring illustrations for lifelong learners*. Elevate Books.

Gadanidis, G. (n.d.). *What did you do in math today?* www.researchideas.ca/wmt/c5.html

McClain, J. (2018). Using comics to teach mathematics. *Pop culture classroom.* https://classroom.popcultureclassroom.org/blog/using-comics-to-teach-math/

McCloud, S. (1993). *Understanding comics: The invisible art.* HarperCollins Publishers Inc.

Munroe, R. (2015). *Thing explainer: Complicated stuff in simple words.* Houghton Mifflin Harcourt.

Orlin, B. (2020). *Math with bad drawings.* https://mathwithbaddrawings.com/

Parker, M. (2017). *Art teacher in process: An illustrated exploration of art, education and what matters* [Unpublished master's thesis]. Simon Fraser University Institutional Repository. https://summit.sfu.ca/item/17806

Pirie, S., & Kieren, T. (1994). Growth in mathematical understanding: How can we characterise it and how can we represent it? *Educational Studies in Mathematics, 26*(2/3), 165–190. www.jstor.org.ezproxy.acsmc.talonline.ca/stable/ 3482783

Tschida, C., Gallagher, J.L., Anderson, K.L., Ryan, C.L., Stapleton, J.N., & Jones, K.D. (2019). Using video capture and annotation technology to strengthen reflective practices and feedback in educator preparation. In T.L. Heafner, R. Hartshorne, & R. Thripp, *Handbook of research on emerging practices and methods for K–12 online and blended learning* (pp. 314–332). IGI Global. doi.org/10.4018/978-1-5225-8009-6.ch015

Wheeler, L. (2018). *Sketchnoting: What is it & how can I get started?* www.mslwheeler.wordpress.com/2018/01/05/sketchnoting/

Wolczuk, D., Garcia, A., Goh, J. & Sellaroli, G. (2018). *Evaluating comics as pedagogical tools in an undergraduate mathematics course.* www.uwaterloo.ca/centre-for-teaching-excellence/descriptions-funded-lite-grant-projects/evaluating-comics-pedagogical-tools-undergraduate ▲

Continued from page 42

Important Considerations

There are formal and informal structures rooted in systemic discrimination and educator bias that lead to academic streaming, and there is no one-size-fits-all solution. OAME/AOEM believes that, as a community of educators, it is imperative for us to continue to learn about systemic racism, classism, ableism, and other forms of discrimination in education. We must also reflect continuously in order to mitigate biases and the negative impacts of our instructional decisions. These decisions must consider the diversity of lived student experiences, and that this diversity may vary from year to year, grade to grade, and course to course. OAME/AOEM supports mathematics educators' ongoing commitment to removing barriers to equitable mathematics education and in creating learning environments that support **all** students.

Resources

CASE (Coalition for Alternatives to Streaming in Education). (2021). *CASE fact sheet and recommendations.* Author. https://static1.squarespace.com/static/5ee2aedfd4916c68b6bb2c26/t/60e384cfad15b92d09754e47/1625523434220/CASE_Reccomendations_2021.pdf

Hattie, J., Fisher, D., Frey, N., Gojak, L.M., Moore, S.D., & Mellman, W. (2016). *Visible learning for mathematics, grades k–12: What works best to optimize student learning.* Corwin.

OAME/AOEM. (2019). *Access, equity, and inclusion position statement*. Author. www.oame.on.ca/main/files/posStmts/EquityPositionPaper-2019-06-08.pdf

OECD. (2012). *Equity and quality in education: Supporting disadvantaged students and schools.* Author. doi.org/10.1787/9789264130852-en

Ontario Ministry of Education. (2013). *Capacity building series K–12: Culturally responsive pedagogy: Towards equity and inclusivity in Ontario schools* (Secretariat Special Edition #35). Queen's Printer for Ontario. www.thelearningexchange.ca/wp-content/uploads/2017/02/CBS_ResponsivePedagogy.pdf

Ontario Ministry of Education. (2017). *Ontario's education equity action plan.* Queen's Printer for Ontario. www.edu.gov.on.ca/eng/about/education_equity_plan_en.pdf ▲

▲ 2021-2022 OAME/AOEM Board

OAME website
www.oame.on.ca

DIRECT GENERAL INQUIRIES TO: EXECUTIVE DIRECTORS

Fred and Lynda Ferneyhough
4751 Mack Ave.
Plympton-Wyoming, ON N0N 1J6
Business & Fax: (519) 899–4203
eds@oame.on.ca

PRESIDENT

Judy Mendaglio
Mississauga, ON
judy.mendaglio@oame.on.ca

PRESIDENT ELECT

Sandra Jean Price
Oshawa, ON
sandra.jean.price@oame.on.ca

VICE-PRESIDENTS

Melissa Black
Kanata, ON
melissa.black@oame.on.ca

Paul Alves
Georgetown, ON
paul.alves@oame.on.ca

EMERITUS

Bill Otto
Stillwater Lake, NS
bill.otto@oame.on.ca

CHAPTERS & REPRESENTATIVES

CHAMP: Kassandra Kobelak
Etobicoke, ON
kassandra.kobelak@oame.on.ca

GOLDEN: TBD

GVMA: Glen McMillan
Guelph, ON
glen.mcmillan@oame.on.ca

ISOMA: Cal Armstrong
Burlington, ON
cal.armstrong@oame.on.ca

MAC²: Paulene Washington
Barrie, ON
paulene.washington@oame.on.ca

NOMA: Heather Theijsmeijer
Kagawong, ON
heather.theijsmeijer@oame.on.ca

NWOAME: Corrine Russell-Pritoula
Thunder Bay, ON
corrine.russell-pritoula@oame.on.ca

$O_3$4ME: Suhana Kadoura
Ottawa, ON
suhana.kadoura@oame.on.ca

PRMA: Jennifer Clark
Bowmanville, ON
jennifer.clark@oame.on.ca

QSLMA: Siobhain Broekhoven
Kingston, ON
siobhain.broekhoven@oame.on.ca

SAME: Franca Genco
Scarborough, ON
franca.genco@oame.on.ca

SWOAME: Elizabeth Pearsall
Kingsville, ON
elizabeth.pearsall@oame.on.ca

TEAMS: Alvarine Aldridge
Toronto, ON
alvarine.aldridge@oame.on.ca

WOMA: TBD

Y⁴MA: Gina Micomonaco
Richmond Hill, ON
gina.micomonaco@oame.on.ca

DIRECTORS – TERM EXPIRING 2022

Jane Silva
North York, ON
jane.silva@oame.on.ca

Melissa Baker-Cox
Inverary, ON
melissa.baker-cox@oame.on.ca

Victoria Maystruk
Peterborough, ON
victoria.maystruk@oame.on.ca

Timothy Sibbald
North Bay, ON
tim.sibbald@oame.on.ca

DIRECTORS – TERM EXPIRING 2023

Bart Vanslack
Oakville, ON
bart.vanslack@oame.on.ca

Sam Garrison
Toronto, ON
sam.garrison@oame.on.ca

DIRECTORS – TERM EXPIRING 2024

Lori Yee
Pickering, ON
lori.yee@oame.on.ca

Jimmy Pai
Ottawa, ON
jimmy.pai@oame.on.ca

NCTM REPRESENTATIVE

Jacqueline Hill
Lindsay, ON
jacqueline.hill@oame.on.ca

WEBSITE COORDINATOR

Greg Clarke
Orillia, ON
web@oame.on.ca

GAZETTE EDITOR

Timothy Sibbald
North Bay, ON
gazette@oame.on.ca

ABACUS CO-EDITORS

Marc Husband
Toronto, ON
marc.husband@oame.on.ca

Tina Rapke
Toronto, ON
tina.rapke@oame.on.ca

50TH ANNUAL OAME CONFERENCE

@OAMELEARNS
OAME2023.CA

OAME2023

BEANFIELD CENTRE
EXHIBITION PLACE
TORONTO
APRIL 27-28, 2023

PRE-REGISTRATION
NOV 1, 2022
FULL REGISTRATION
FEB 1, 2023

EMPOWERING ME
TORONTO

KEYNOTE SPEAKERS:

ROCHELLE GUTIERREZ



https://education.illinois.edu/faculty/rochelle-gutierrez

DAN FINKEL



https://mathforlove.com/-dan-finkel

DEBORAH LOEWENBERG-BALL



https://soe.umich.edu/directory/faculty-staff/deborah-loewenberg-ball

FEATURED SPEAKERS:

OCTAVIA BECKLES
ROBERT DUROCHER
LINDSEY GOLD
HEMA KHODAI
CATHY MARKS KRPAN
ANTHONY MELI & JASON TO
CHAD MOWBRAY
MARIAN SMALL
HEATHER WARK

HOSTED BY NOMA
AND TEAMS

NOMA
NORTHERN ONTARIO MATHEMATICS ASSOCIATION

TEAMS
Toronto Educators Association for MathematicS

design by Drew Ferneyhough bedowynn@gmail.com

Canadian Mathematical Society
Société mathématique du Canada

CMS COMPETITIONS This Fall

Canada's most prestigious competition

Canadian Open Mathematics Challenge (COMC)

OCTOBER 27

COMC.MATH.CA



Compete to be part of Math Team Canada!

Fall competition for younger students!

Canada Jay Mathematical Competition (CJMC)

NOVEMBER 17

CJMC.MATH.CA

AGES

CJMC: grade 5-8 curriculum

COMC: Students must be under 19 years of age as of June 30, 2022