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ABACUS

Ontario Mathematics Gazette

OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

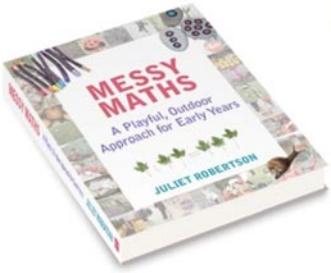
AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

Vol. 61#1
September 2022

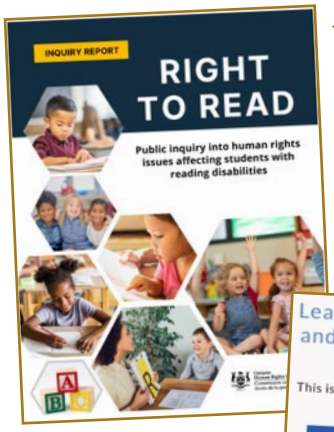
IN THIS ISSUE

- Editor's Report
- President's Message
- OMCA News: September 2022
- Coding in the Classroom: Computational Thinking, a Norwegian View
- Blast from the Past! (New)^2 Math—The "New" New Math
- OAME/NCTM Report: Happy Fresh New School Year!
- What's the Problem? Seven Sevens
- In the Middle: How Teacher Talk Meets the Thinking Classroom
- Mb4T (Mathematics by and for Teachers): Using Models and Reasoning to Develop Factoring by Decomposition
- Some Thoughts about Perimeter: String Perspective
- Elementary Math Matters: It's Time for Another Public Inquiry: The Right to [Do] Math
- Collaborative Learning in a Diverse Classroom and Its Impact on Students' Achievement in Mathematics: A Lesson Learned from the Treisman Model
- A Brief History of TEAMS: Our Spelling and Our Logo
- Annual OAME/AOEM Awards 2022
- Book Review: Math in the Great Outdoors
- Mathematically Engaging Students in Their World
- Business Applications of Calculus
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- A Look at the Transformations of Power Functions

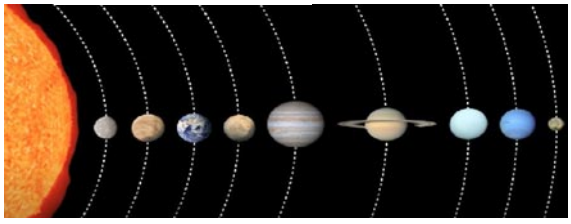
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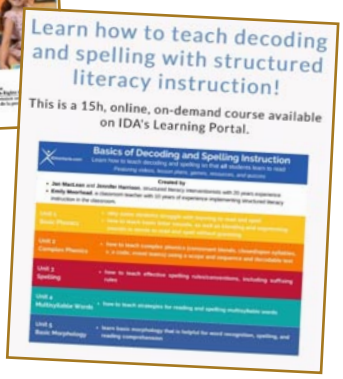


◀ See Elementary Math Matters: It's Time for Another Public Inquiry: The Right to [Do] Math



▼ See Mathematically Engaging Students in Their World





▼ See Annual OAME/AOEM Awards 2022



Life Membership Award, **Ron Lancaster**



Award for Exceptional and Creative Teaching in Elementary Mathematics, **Catherine Koehler**



Award for Exceptional and Creative Teaching in Secondary Mathematics **Pushpa Gunawardena**



Award for Outstanding Contribution to OAME/AOEM and Mathematics Education in Ontario, **Thomas Steinke**



Award for Leadership in Mathematics Education, **Elizabeth Pattison**



Award for Leadership in Mathematics Education, **Jason To**

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LINDSEY GOLD
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CHAD MOWBRAY
MARIAN SMALL
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NORTHERN ONTARIO MATHEMATICS ASSOCIATION

Toronto Educators Association for Mathematics
TEAMS

design by Drew Ferneyhough bedowynn@gmail.com

Submission of Articles

The *Ontario Mathematics Gazette* (*OMG*) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *Gazette*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles are to be word–processed, MS Word is preferred, and prepared according to the *Publication Manual of the American Psychological Association*, Seventh Edition. However, please use single-line spacing (not double). Articles should not exceed five numbered pages of text, and figures, images, and photographs should be placed in the text close to where they belong, with captions. The photographer's permission is required, and for photos of students under the age of 18, the written permission of a parent or guardian is required.

Please submit your article in one blind file (i.e., identity of author is not evident), and include author names, contact information including email and mailing addresses, photos—head and shoulders, biographies—less than 100 words, and all content removed for blinding in a second file. Please email these two files to Tim Sibbald at gazette@oame.on.ca.

Upon review, you will be notified whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected. The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME/AOEM.

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The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of OAME/AOEM.

▲ Table of Contents

Up Front

▲ Editor's Report
by Timothy Sibbald 2

▲ President's Message
by Sandra Jean Price 5

Regular Features

▲ The *Abacus* . . . . . 8-Page Insert

▲ OMCA News: September 2022
by Heather Theijsmeijer 4

▲ Coding in the Classroom: Computational Thinking, a Norwegian View
by Iain Brodie and Hatice Beyza Sezer 7

▲ Blast from the Past! $(New)^2$ Math—The "New" New Math
by Jeff Irvine 10

▲ OAME/NCTM Report: Happy Fresh New School Year!
by Jacqueline Hill 12

▲ What's the Problem? Seven Sevens
by Shawn Godin 13

▲ In the Middle: How Teacher Talk Meets the Thinking Classroom
by Carly Ziniuk 15

▲ Mb4T (Mathematics by and for Teachers): Using Models and Reasoning to Develop Factoring by Decomposition
by Ann Kajander 17

▲ Some Thoughts about Perimeter: String Perspective
by Jérôme Proulx 19

▲ Elementary Math Matters: It's Time for Another Public Inquiry: The Right to [Do] Math
by Lynda Colgan 21

▲ Collaborative Learning in a Diverse Classroom and Its Impact on Students' Achievement in Mathematics: A Lesson Learned from the Treisman Model
by Nancy Louis 46

Special Features

▲ A Brief History of TEAMS: Our Spelling and Our Logo
by Peter Saarimaki 6

▲ Annual OAME/AOEM Awards 2022 8

▲ Book Review: Math in the Great Outdoors
by Edward Schroeter 40

Articles

▲ Mathematically Engaging Students in Their World
by Dr. Jeff Irvine 25

▲ Business Applications of Calculus
by Dr. Jeff Irvine 28

▲ Contexts for Learning Integers: Integrating Hockey into the Intermediate Mathematics Curriculum
by Christine Green and Ann Kajander 35

▲ A Look at the Transformations of Power Functions
by Ali Hamour 42

Notices

▲ OAME Sticky Graph Pads 12

▲ OAME Board Members 2022-2023 48

Advertisements

▲ OAME 2023 . . . . . Inside Front Cover

▲ Jeff Irvine - Survey 34

▲ CMS Competitions . . . . . Inside Back Cover

▲ Texas Instruments . . . . . Back Cover

▲ About the *Ontario Mathematics Gazette*

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Submission of Advertisements
Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** by email or at the above address. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be print ready, and colour advertisements should have no bleeds.

Advertising Rates
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▲ Editor's Report



TIM SIBBALD, OCT, PH.D.
Gazette@oame.on.ca

Tim Sibbald is a professor in the Schulich School of Education, Nipissing University, with a focus that includes mathematics education. He is a former President of OAME/AOEM and has held a variety of other OAME/AOEM posts over the years.

As Editor, I avoid writing about myself. However, having written that I was feeling tired in my last editorial, and the need to crawl out from under the pandemic rock, I would like to acknowledge that I personally did achieve both and had some much-needed downtime. Your Editor is no longer feeling tired. I would like to thank Jacqueline Hill, who surprised me before that editorial was printed by mailing me a hat. The design was made by Marlene Griffiths, appears in the *Gazette* archive (Griffiths, 1998), and Tom Griffiths recounted Marlene's handywork when the Past Presidents get together in conjunction with the annual conference. Jacqueline's gesture speaks to the support that runs between Directors in the OAME/AOEM, and I sincerely appreciate her reaching out when I was feeling tired.



An issue we face in education was evident in news reporting after the Rogers networks succumbed to severely limited capacity on July 8. The issue for education can be interpreted in an immediate sense of what should teachers do if the Internet or other technology were to abruptly not



work in their classrooms? Most teachers make a contingency plan or have activities that they can draw upon at a moment's notice. However, there is a much larger issue for education.

The larger issue is akin to the expression, *"You don't know what you don't know."* It goes well beyond simple technologies. The issue, in a nutshell, is that teachers, schools, and students tend to be localized and make normative assumptions about the place where they live. It is no surprise then that the news emphasized people in Toronto who have migrated away from using cash. As the network issues developed, their ability to buy things was diminished. What is also important to realize is that, while it is rare, this type of disruption has occurred previously, and while I cannot give a date, I recall Interac being down on a Friday headed into a long weekend in the early 2000's. The issue then was not store-based payments; it was accessing funds from ATMs.

But how does this relate to local circumstances? It speaks to assumptions in teaching. If you teach in a large urban centre, then you might well mention paying with a phone. However, in some geographical areas, this is far less likely. Yes, you can pay with a phone in most places (including North Bay), but if I hop onto my bicycle and travel a little over 5 km from my house, while still in North Bay, there is no cellphone access. The ubiquitous cellphone becomes useless.

At issue here is not the value of phones, but the assumptions teachers and their students live by in their classrooms. The normality of your place is what is typically talked about and assumed as normal in your classroom, unless you make strides to educate your students beyond that normality. Do you want to educate your students for dependence on a device, or do you teach them how to thrive, even when one device, for whatever reason, is not available?

In my case, part of my downtime this summer included a canoe trip, where there was no cellphone option. Yes, I could have acquired a satellite phone, and had the duration of the trip been long or the risk higher, I might have. I also had it suggested that I use downloaded maps on a cellphone, which failed, as GPS did not seem to locate me correctly *for whatever mysterious technological reason* (a phrase that seems to emerge regularly with technology). The point, however, is that no matter how much you buy into the modern world, there is a need to remember that it limits students' options if you only teach them about the local circumstance.

The problem for many technologies seems to be the numerous details in the fine print. While travelling in the United States and headed to Scarborough, I suggested to my spouse that we call the people we were meeting, to give a prediction of our arrival time. However, the cellphone we were using had a plan that only worked in Canada. Yes, I hear some say to pay more money and get rid of the issue, but economics suggests weighing the risk versus reward. I also recall an exchange student, who arrived to discover their bank card would not work due to particular details that were only found after significant digging. It seems fairly regular that the limitations of technology are discovered only after one has established the need, and that implies a need for being able to develop contingencies as opposed to assuming technology is a panacea.

So, whether it is Internet outages, Interac interruptions, or whatever other technological blips we shall see going forward, it is incumbent upon all teachers to prepare our students for whatever circumstances in which they may find themselves. Emphasis of lessons might well match the local community; that is reasonable, but we need to be inclusive of all circumstances that students may experience, whether they remain in that local community or move to new communities. And an important part of that, in my humble opinion, is to regularly set aside all technologies and engage students in the human condition of making do without all the perks of technology.

A Post-Pandemic Note

As we begin recovering and rebuilding from the pandemic, please consider sending the *Gazette* photos of your events. It would be good for the entire OAME/AOEM community to see smiling face-to-face events, and the *Gazette* wants to help make that a reality—but we can't do it without you!

An example is the OAME2023 annual conference logo. It is a message of empowerment, of shedding light on teaching matters, and it invites us all to come together as a community. I hope to see you there!



In This Issue…

Sandra Jean Price brings her thoughts as the new President. Also, with OAME/AOEM in mind, the annual **award winners** are presented, and everyone is encouraged to consider nominating colleagues at the appropriate time this year (see the website for details). **Jacqueline Hill** and **Heather Theijsmeijer** bring us the NCTM and OMCA reports, respectively.

Iain Brodie and **Hattice Beyzer Sezer** interview **Siri Krogh Nordby** to get a Norwegian perspective about computational thinking in Ontario. **Ann Kajander** explains an interesting way to tackle quadratic factoring with algebra tiles. The method she explains is effective when the highest-order coefficient is not one! **Carly Ziniuk** examines the intersection of teacher talk and the thinking classroom. **Shawn Godin** brings a problem that inspired his interest in mathematics many years ago. **Jérôme Proulx** takes a close look at perimeter, but doesn't string us along. **Lynda Colgan** makes a call for a public inquiry that is thought-provoking. **Jeff Irvine** considers the historical progression from "new math" to the "new new math" and continues to the "New… New math." "Events at the Fields" and "Linking Literacy and Math" will both return in the next issue.

We have more articles than usual and, hopefully, they inspire others to share their ideas by writing an article for the future. **Christine Green** and **Anne Kajander** consider how hockey can infuse the intermediate math class. **Ali Hamour** considers how completing the square can be extended to higher order polynomials. Nancy Louis explains how the Treisman model can be beneficial to instruction. **Jeff Irvine** considers the challenge of engaging students and adding connections that add depth to instruction. In a second article, **Jeff Irvine** draws on his business experience and provides an in depth look at business connections that will be of interest to senior math teachers. **Ed Schroeter** reviews *Messy Math* with its ideas about teaching math outside.

Thank Yous

The editorial team of Anne Yeager, Peter Saarimaki, Gitta Berg, and Penny Clemens have once again brought excellence to the *Ontario Mathematics Gazette*. Their efforts are exactly why the publication is 60 years strong! Thank you.

Lastly, an apology to Peter, whose last name, Saarimaki, appeared in a misspelled way in the last issue.

Reference

Griffiths, M. (1998). Math toque. *Ontario Mathematics Gazette, 37*(1), 12–13. ▲

▲ OMCA News: September 2022

HEATHER THEIJSMEIJER (OMCA PAST PRESIDENT)
OMCAmath@gmail.com

In fine fashion, the Ontario Mathematics Coordinators Association (OMCA) capped off another school year in June with a meeting of mathematics coordinators, consultants, and coaches from across the province.

This year's recipients of the 2022 OMCA award for excellence and innovation in mathematics teaching are Earl Totten (Toronto Catholic District School Board), and the Waterloo Regional District School Board (WRDSB) team of Grade 9 Math Itinerant Coaches: Priya Aggarawal, Laura Kingston, Aleda Klassen, Angela Shaefer, and Andrea Teichroeb. These awards are given annually to OMCA members, based on nominations by their peers, in recognition of excellence and innovation related to any areas of mathematics education (integration of technology, equity, implementation of curriculum, motivation of teachers, assessment and evaluation, professional development ideas, etc). Congratulations, Earl and WRDSB!

Recent meetings have included guest speakers Tara Flynn and Shelley Yearley on "Rethinking Fractions," Dr. Juan-Carlos Chavez and Douglas Sinclair on "Indigenous ways of knowing and doing mathematics: The Journey," and Dr. Francis Su on "Building Virtues, Not Just Skills." This rounded out a year of incredible guest speakers, who spoke about everything from coding to multilingualism, and from how our youngest students learn, to how post-secondary mathematics education is evolving.

Recognizing that many boards were creating resources to support the new Grades 1–8 and Grade 9 curricula, a new standing item was added to meetings this past year. Members were encouraged to share links to board documents, folders, and websites in a centralized spreadsheet, and if they chose to, briefly speak to the resources during meetings. Shared resources included lesson plans, MTH1W course outlines, coding resources, puzzles and games, assessment resources, a model for board-wide facilitator updates, and equity resources, among others. This became an excellent forum for sparking new ideas among boards.

At the final members' meeting of the year on Monday, June 6, outgoing President, Heather Theijsmeijer (Rainbow

Continued on page 19

▲ President's Message



SANDRA JEAN PRICE
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Sandra Jean has been on the Board of Directors of OAME/AOEM for almost 20 years. She is currently teaching in the Durham District School Board and has more than 25 years of teaching experience. She has had many roles, including classroom teacher, Math Coach, and Instructional Leader of online Math Additional Basic Qualifications (ABQs). Sandra Jean has contributed to a variety of Ministry projects and was a coordinator of the most recent OAME/AEMO writing projects. Sandra was the recipient of the 2008 OAME Award for Exceptional and Creative Teaching in Elementary Mathematics for her innovative instructional practices, integration of technology, and belief in success for all.

As we embark on a new school year, I am beginning a new journey as the President of OAME/AOEM. I am incredibly humbled by this opportunity to give back to the association that has supported me throughout my career. I am hopeful that with the new school year and newly elected members on the Board of Directors, OAME/AOEM will continue to serve mathematics teachers of Ontario in reaching their goals.

On a more personal level, I am also intimidated. There have been many exemplary educators who have filled this role in the past. They have helped to steer this organization through times of smooth sailing and those that included some rough waters. Regardless of the conditions, I believe that we, as an organization, have always emerged each year stronger than the one before. So why am I intimidated? I often reflect on my own strengths and areas for growth, and wonder if I really have what it takes to lead an organization that is, truly, an association full of leaders.

We live in a world that has presented many challenges over the past few years. Equity has rightfully claimed the stage around the world. Human rights and protection under the Charter of Rights and Freedoms have been highlighted. Individuals are speaking up against injustice, prioritizing human rights, even in the face of adversity. In Ontario, there has been a new elementary mathematics curriculum released recently, and a new Grade 9 destreamed mathematics curriculum. Resources that used "Flash" software (i.e., Adobe Flash) have become unusable, forcing educators to find alternative resources. And then there is COVID-19, which has altered forever our view of the world, including education.

We have suffered losses, but while we grieve for a world that will never quite be the same, we have an opportunity to begin again. I am hopeful that this school year will be one that is closer to normal, albeit a new normal. Staff members and students who were engaging online may now return to physical classrooms. In-person school-wide events may become the norm again. But most importantly, we can refocus on our pursuit of a positive, inclusive environment focused on community, caring, and learning for all.

Through the OAME/AOEM *Gazette* and *Abacus* publications, OAME Talks podcasts, the OAME/AOEM website, the Ontario math support website (www.ontariomath.support/), where revised curriculum resources are housed, and our face-to-face annual conference, "Empowering Me in 2023," in Toronto, our organization will continue to work on equity and inclusivity as we embrace change and build connections within our organization and beyond. A critical step in this process is to revisit our position papers written during the last few years, including those on equity and destreaming, and create an action plan to ensure our organization is meeting the needs of teachers of mathematics in the province. We will continue to support our local chapters, which have always been integral to our organization, as they may be able to meet face-to-face again. We will continue to support teachers through events that are relevant and meaningful to the teachers and students in each region.

Again, on a more personal note, the last few years have allowed me to recognize my own strengths and areas of growth, while also learning much about my personal history through the creation of a personal land acknowledgment to recognize the place of Indigenous cultures in our country, something I encourage everyone to do. It is important to continue to confront our biases as we strive to do better, and to be better. Through such experiences that allow us to learn and grow, we have the opportunity to re-establish that we are not only teachers of mathematics—we are teachers of students. Am I still filled with self-doubt in terms of my leadership ability? Absolutely! But I am ready to listen, and I am ready to ask for help while adapting to this new role and this new world. And perhaps it is that self-doubt and willingness to admit not knowing how to do everything that is what makes me ready to embark on this journey.

We need to embrace the doubt, the fear, the mistakes, and the uncertainty as we move into a post-pandemic world. After all, we have a fresh start every school year. And we have a fresh start every day.

"Be willing to be a beginner every single morning."
—Meister Eckhart

▲ A Brief History of TEAMS: Our Spelling and Our Logo

BY PETER SAARIMAKI

Before 1992, teachers in the City of Toronto did not have an OAME/AOEM chapter to represent them. There was a short time in the late 1980's when TORNY (TORonto North York) provided workshops once or twice a year for teachers from Toronto and North York. A representative reported to OAME/AOEM and the *Gazette*, but it was not a recognized chapter.





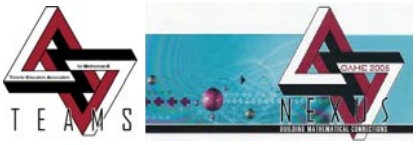
In early 1992, as an elected councillor for OAME/AOEM, I was asked by several OAME executive members to see about starting a chapter for Toronto teachers. In the GTA, i.e., Metro Toronto, Scarborough public teachers could join SAME (Scarborough Association for Mathematics Education, which evolved from SEYMA Scarborough, East York Math Association, representing Scarborough and East York), North York public teachers had Y⁴MA, and Etobicoke public teachers had CHAMP (Credit Humber Association for Mathematics Promotion).

I realized that besides Toronto, there were unrepresented teachers in East York and the City of York, as well as those working for the Metro Separate School Board (MSSB). Since the MSSB encompassed a large number of teachers, I talked to Brian McCudden, the board's Math Coordinator, and together we gathered interested teachers and our consultants! From the beginning, we wanted the chapter to have a close connection to the curriculum departments of our boards, to be able to facilitate communications at a board level more easily.

In June of **1992**, we brought together consultants and teachers from Toronto, East York, (City of) York, and the MSSB. We combined the initials and got **TEY²MS**, which we pronounced "teams." Other name considerations recognized that our geographic area fit into the shoreline of the glacial Lake Iroquois, and suggested we become **LIME** (Lake Iroquois Math Educators), but we were concerned we might be thought of as old fossils!! Another suggestion was the Toronto Association of Mathematics Educators, but we also did not want to be known as a **TAME** group!!

We started small, with an afternoon workshop with my good friend, Don Fraser. We hosted some mini-conferences and eventually were able to get finances and a constitution together to apply for provisional chapter status with OAME/AOEM in 1995. We became an official chapter in 1996... and then turned around and ran the 1997 Annual Conference—no slouches us!!

In 1998, the executive members of CHAMP kept telling me the Etobicoke people "were not participating" and didn't feel like they belonged there, so we invited them to join us, becoming **TE²Y²MS**, with the first extra "E" for Etobicoke.





2017 *OMO*

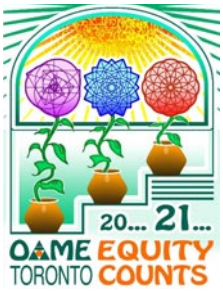
We continued vounteering, running mini-conferences and local Math Olympics and hosting the provincial OMO (Ontario Mathematics Olympics) in 1999. Then the government decided to combine all the boards in Metro Toronto into just Toronto, and all our little boards became the the Toronto District School Board (TDSB). (pronounced "tedious board"), while the MSSB became the Toronto Catholic District School Board (TCDSB). Since we couldn't refer to any of the former boards, in 2002, we began using our homonym name: **TEAMS**, standing for **Toronto Educators Association for MathematicS**, with the last "S" capitalized, just because we could, and because we were many, not just one.



2013 40th Annual (Think Big)



We experimented with several logos over the years, highlighting Toronto, especially the CN Tower, but mostly used a simple collection of our letters.



2021 Annual (Equity Counts)

It was not till 2005, when we hosted the annual conference again, that we developed a logo we felt we could use for all activities. The 2005 "NEXUS" Annual Conference logo eventually morphed into the TEAMS logo. Since the type in the logo is very small, we also use a spread-out version for our letterhead. ▲



▲ Coding in the Classroom: Computational Thinking, a Norwegian View



IAIN BRODIE
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Iain was an elementary school educator for 28 years, and is now into a second career, teaching in-service and pre-service teachers. He teaches face-to-face at Ontario Tech University and virtually at Western University. He is the current editor of the Math+Code 'Zine, *www.researchideas.ca/mc/.*



HATICE BEYZA SEZER
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Beyza is a former teacher and long-term researcher. She is currently a research associate in education at Western University, and her research focus is the integration of computational thinking into mathematics education. She is an editor at Math+Code 'Zine*, www.researchideas.ca/mc/.*

Quotations may have been lightly edited to reflect the difference between spoken and written language.



Siri Krogh Nordby is a former secondary teacher (nine years) and is currently on leave from her position as a teacher educator at Oslo Metropolitan University. She is a second-year Ph.D. student in the Faculty of Education and International Studies at Oslo Metropolitan University.

The implementation of computational thinking in mathematics and science classrooms across Ontario has some very real and practical challenges. There are some extensive resources created for the Grades 1–9 mathematics classrooms, like those produced by OAME/AOEM and AFEMO (Association francophone pour l'enseignement des mathématiques en Ontario) at www.ontariomath.support, but schools, school boards, and teachers have been mainly left to their own devices in order to figure out how to code in the classroom. That being said, our guest for this issue is here to tell us that Ontario is ahead of the game. We still have a lot of work to do, implementing computational thinking in our mathematics classrooms, but we do have some good things going on.

This is what brought Siri Krogh Nordby to Ontario from Norway. Siri is a Ph.D. candidate and teacher educator at Oslo Metropolitan University. She taught mathematics, science, and the arts in high school before joining the Faculty of Education and International Studies at Oslo Metropolitan University. Her research focus is a longitudinal study of four primary teachers as they learn how to implement a new computational thinking curriculum in Norway, and integrate it with their mathematics teaching. Her paper, *Primary Mathematics Teachers' Understanding of Computational Thinking*, tweaked our interest, so we reached out to see if we could interview her. Siri stated:

As a former teacher myself, I struggled to see how computational thinking could benefit mathematics learning, and I'm a mathematics teacher. I didn't know anything about coding before my Ph.D. project. So, I was really into, okay, how is this going to work? That's the reason I did this review that you referred to, because I wanted to know more about what research outside Norway is.

What Siri found in ten examples of classroom coding is a majority of examples that kept the learning of computational thinking and the learning of mathematics separate. Her response was:

I found that they [teachers] are mainly concentrating on computational concepts, and that was worrying me a bit. That's why I wanted to emphasize that it's only three [teachers] in my paper, who were emphasizing how it can be used to teach mathematical concepts.

One of the teachers in her paper, who completely integrated computational and mathematics concepts into the learning, was Western University's George Gadanidis (www.imaginethis.ca). He has designed and developed many lessons, where even our youngest students can engage with complex mathematical ideas, while playfully learning grade-level mathematics, using the power of coding a computer to help them. This is what brought Siri from Norway.

I'm actually so glad that I could come here to see how you connect the curricula together with these computational and mathematical concepts because I think you do it very well here. You're really emphasizing how you can use these skills in learning mathematically.

Siri has found that this is something that is difficult for teachers to implement because of the general lack of resources or guidance as to what computational thinking is and how it can be used to learn mathematics. There persists a common belief that we can teach computational thinking as a separate subject or unit. There are boards of education in Ontario that have separate coding units in their scope and sequence for mathematics. While this is not the intent of the mathematics curriculum, it is easy to see why this is happening. Siri described the current situation as follows:

The mathematics is not in the learning [and teaching of coding]. It's not there at all, actually. It's a new part of teaching that's not connected to mathematics. And again, it worries me because then, maybe they [teachers] just do it in lessons because they don't see the mathematics in the materials they found. So, they just use the two or three math lessons.

One of the things we talked about was the different conceptions of what students are doing when engaged in coding. While it is called coding in the curriculum, this is a very simple way of looking at what students are actively doing and thinking. Some refer to this act as computational thinking—thinking analytically about how to get a computer to do mathematics through code. Others prefer the idea of computational modelling as coding a program to model mathematical situations and concepts, to assist in building greater understanding. This view is helpful, as all programs are models and allow us to play and explore mathematical ideas that are either too difficult or too boring to do by hand. Imagine flipping a coin 10 000 times. Frightfully boring to do physically, but it is easy to create a computational model to do it for you. These different conceptions were evident to Siri from her research, and brought her to the idea of computational literacy, as expressed by Andrea diSessa. Siri stated:

I was struggling when I was reading the research on computational thinking, I found it difficult to see how it can empower the domain in itself. It's widely expressed, so I'm struggling. When I found this computational literacy, it made more sense to me using that, because it's a bigger picture. How can you use this computational thinking in each domain? And how can it empower learning? And in what way? And does it have any blind spots? Or does it have any? So that was the reason, I think, to get a bigger picture of it.

Andrea diSessa theorizes that computation might become a literacy in the same way that algebra is a mathematical literacy that fundamentally changed the way we do mathematics. However, unlike algebra, there are fewer resources for teachers to use to learn or to use in their classrooms with their students. This is one of the projects that Siri will be tackling soon.

That's actually going to be my fourth article, I'm now investigating these research resources. I'm trying to write an article about which [computational] resources are giving students a conceptual understanding of mathematics. I think we have 42 different resources, and then we are going to analyze them.

This will be a very good start to help teachers, not only in Ontario, but around the world, as more and more jurisdictions include coding in their curricula.

As always, the full interview can be found at OAME Talks. We also continue to welcome ideas and suggestions of other people whom readers would like to see interviewed. If you have someone in mind, let us know. ▲

▲ Annual OAME/AOEM Awards 2022

Life Membership Award
Ron Lancaster
Ontario Institute for Studies in Education, University of Toronto



Ron Lancaster is this year's Life Membership Award winner. Over the last four decades, he has been influencing mathematics education in Ontario. Through his work as a high school teacher and university professor, he has helped numerous students develop a love for math. He strives to ensure that all learners reach their full mathematical potential to become insightful thinkers and problem solvers.

Ron has also made significant contributions to OAME/AOEM as a Past President and a regular presenter at over 35 annual conferences, in addition to chapter events. He is a strong believer in sharing his mathematical ideas, and has written over 65 articles for the *Ontario Mathematics Gazette* and over 160 articles for NCTM publications.

Award for Exceptional and Creative Teaching in Elementary Mathematics



Catherine Koehler
Catholic District School Board of Eastern Ontario

Catherine is a passionate and creative teacher, who promotes a growth mindset and constantly challenges her students to take risks. Her kind manner and encouragement push even the most hesitant learner to try. Catherine builds a community with her students, where they can make mistakes, learn from mistakes and each other, all while celebrating their successes. She consistently looks for the positives in student work. Her

knowledge, experiences, and passion for mathematics is contagious. Catherine is genuine in her care for others, always willing to help her colleagues, and she strives to make math more accessible and friendly to others.

Award for Exceptional and Creative Teaching in Secondary Mathematics



Pushpa Gunawardena
Ottawa-Carleton District School Board

The Award for Exceptional and Creative Teaching in Secondary Mathematics has been awarded to Pushpa Gunawardena, who exemplifies the criteria of contributing to the overall development of students. Many students identify Pushpa as a caring individual, who not only expands their capacity for and enjoyment of mathematics, but also instills in them the skills necessary to succeed in life. They credit her with building up their math understanding and self-confidence through her effective and creative teaching style. Initially students place their confidence in Pushpa, but ultimately grow to have confidence in their own skills and strategies. She always adjusts her methods not only to the needs, but also to the interests, of her students.

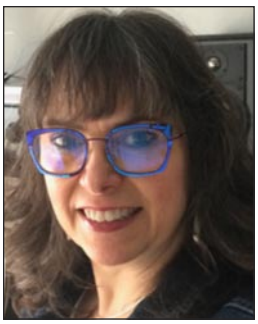
Award for Outstanding Contribution to OAME/AOEM and Mathematics Education in Ontario



Thomas Steinke
Texas Instruments

Throughout his teaching and consulting career, Thomas has been actively involved in finding and promoting engaging and innovative math instruction. He has worked as a teacher; department head; consultant; T3 National Instructor; curriculum and resource developer; provincial, national, and international workshop developer and leader. Tom has worked to build a community of like-minded math educators that can support teachers in mathematics instruction across the country. He has forged partnerships with OAME/AOEM, sponsoring the annual conference and making significant contributions to the O34ME chapter mini-conference. Thomas has strong connections and has had a lasting influence on many math educators in the Ottawa area.

Award for Leadership in Mathematics Education



Elizabeth Pattison
District School Board of Niagara

Elizabeth has been extensively involved with OAME/AOEM for over 30 years. She has been the President of the Golden Section Chapter since 1991, and has organized countless events and chapter conferences, including the Annual Provincial Conference in 2009. She was also an OAME/AOEM Director from 2003–2007. Elizabeth has contributed to the mathematics community in her school board through her work as a teacher, department head, and math consultant. She has also had a hand in the pre-service education of students at the Faculty of Education at Brock University, and has worked as an Instructor for Texas Instruments. In all her positions, Elizabeth has been a strong advocate for her colleagues. She listens to their needs, and plans professional learning to support those needs. Elizabeth also works to find the strengths in each of her colleagues, and encourages them to share and co-lead the learning with her. In this way, she supports her colleagues and builds leadership capacity at the same time.

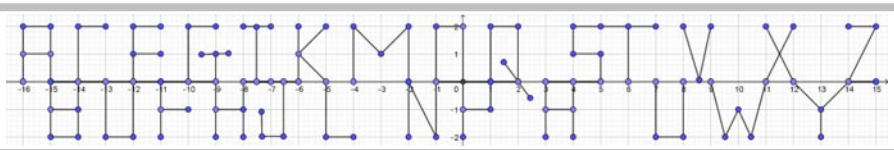
Award for Leadership in Mathematics Education



Jason To
Toronto District School Board

Jason To is a Coordinator of Secondary Mathematics and Academic Pathways with the Toronto District School Board (TDSB). He is perhaps best known for his work in equitable practices in math education, particularly within the context of destreaming Grade 9. He has demonstrated outstanding leadership within his board and across the province, with a strong lens on equity, anti-racism, and anti-oppression. Jason's commitment to addressing systemic discrimination to break down barriers for students has been pivotal in ensuring all students have an equal opportunity to succeed, thrive, and reach their full potential.

In addition to working with TDSB teachers, he has shared his knowledge and passion with educators from other school boards in the province. In addition to OAME/AOEM, Jason is actively involved with the Ontario Mathematics Coordinators Association. ▲



▲ Blast from the Past! (New)² Math—The "New" New Math



JEFF IRVINE
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Jeff Irvine, Ph.D., has been a secondary mathematics teacher, department head, and vice-principal. He has taught at three faculties of education and at a community college. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. With over 45 years in education, Jeff is particularly interested in the interplay of student motivation and mathematics achievement.

In 1998, the Ontario Ministry of Education (OME) issued curriculum policy documents for mathematics that radically shifted the paradigms under which teachers operated (OME, 1998, 2000). These policy documents differed in three significant ways from previous Ministry guidelines. First, they contained lists of expectations that were expected to be taught, whereas previous Ministry guidelines provided guidance to teachers, but were not required. Secondly, the 1998 revisions integrated curriculum, pedagogy, and assessment into a cohesive whole-student strategy. Third, the curriculum centred around problem solving and investigations, turning mathematics into a "laboratory science" (Pitman, 1989, p. 34) featuring investigations, hypothesizing, and experimentation. Fourth, the curriculum and supports were developed in conjunction with the Fields Institute and OAME/AOEM, resulting in a curriculum emphasizing a classroom focus. This was the "new" new math, differing in multiple ways from the "old" new math of the 1960's and 1970's (Irvine, 2021). Allan Pitman (1989) provided an excellent comparison of this "new" new math compared to the "old" new math. Key among the "new" new math was a focus on problem solving; the advent of student-centred mathematics; coherence with math reforms across the world; and advocacy for reform, generated by practitioners, rather than academics.

Like most changes in mathematics education, the 1998 documents were a culmination of thinking about math education that began in the late 1970's, perhaps as a reaction to the brief "back to basics" movement that superseded the "old" new math, which was very abstract and based on logic and set theory. The *Ontario Mathematics Gazette* (*OMG*) had issues focused on problem solving in both 1980 (*OMG*, 18(3)) and 1981 (*OMG*, 20(2)). Problem solving was still seen as the solution of "mathematical problems," typically of three types: traditional word problems, which were practice exercises with words added (Ridge, 1981); mathematical puzzles (McLoughlin, 1994, 1995; Rodrigues, 1980; Rubenstein, 1993); and algebra exercises, with the word "exercises" replaced with "problems" (Nobel, 1985; Norrie, 1988). Some authors began featuring "real world" problems, with the numbers manipulated to give "easy" calculations (e.g., Campbell, 1980).

Articles began to appear in the *Gazette*, identifying Pólya's four-step heuristic (understand the problem, make a plan, do the plan, reflect/look back) as a valuable approach to problems for which a solution method was not immediately obvious (Crealock, 1981; LeBlanc, 1980; Walsh, 1980). There was some confusion around these new types of problems. For example, one author (who shall remain unnamed) stated that students needed to "identify all unknown assumptions"; no suggestions on exactly how to do this were given.

In 1985, the OME issued *Ontario Schools: Intermediate and Senior Division*s (*OS:IS*) (OME, 1985a, 1985b). The major changes in these guidelines were the abolition of Grade 13 and the creation of Ontario Academic Credits (OACs). In the mathematics guidelines, problem solving was identified as an important route to student understanding that should be a major feature of mathematics instruction. It is important to recognize that these were guidelines, not compulsory policy documents. This was in keeping with previous Ministry curriculum publications. The guidelines were in line with mathematics reform thinking around the world (e.g., NCTM Standards, 1989) which identified problem solving as central to the learning of mathematics. Various *Gazette* articles (e.g., Onslow & Reesor, 1989; Reesor, 1988) examined the benefits to students of employing a problem-solving pedagogy.

In the early 1990's, advances in technology, such as graphing calculators and computer spreadsheets, led to an emphasis on student investigations to generate primary data for analysis (Clark, 1994). A wealth of articles on using technology appeared in the *Gazette* (e.g., de Bruyn, 2003), and technology articles have continued to be a major feature of the *Gazette*.

Combs (1999) pointed out that for a problem to have real meaning for a student, the problem must have a personal connection to the student. One way of creating this personal connection is to emphasize problem posing as well as

problem solving (Giganti, 2008; Irvine, 2017). Asking students to create their own problems, or to modify existing problems by changing wording or parameters, also encourages students to utilize higher-order thinking skills and take ownership of the problem (Irvine, 2019).

The *Gazette* featured columns on problem solving, such as the "About Problem Solving" column, which ran from 1994 (*OMG*, 32(2)) to 1999 (*OMG*, 38(1)), as well as other articles identifying the value of multiple solution methods (e.g., Giganti, 2008; Godin, 2006) and the impact of a problem-solving focus on student engagement (McKenzie, 1989; Todd, 2011). There continued to be confusion between actual problem solving and algebraic exercises that might motivate students (e.g., Todd, 2011). I was struck by the parallels between the 2011 advocacy of beginning a class with a problem and the work of Joe Stein and his department (Irvine, 2020) of the late 1970's.

Once the 1998 curriculum was published, some articles appeared, critiquing the curriculum, such as Nicholls (1998a; 1998b). Much of this analysis centred on the elimination of the OAC credits. Very little focused on the integration of curriculum, pedagogy and assessment, or on the problem-solving approach that dominated the new curriculum.

Beginning in 2011, the *Gazette* has had a "What's the Problem" column (Godin, *OMG*, 49(3)). However, this column features "mathematical problems" that usually are not congruent with the laboratory science thrust of the "new" new math. The last *Gazette* article on problem solving also appeared in 2011 (Todd, 2011). There had already been curriculum revisions (OME, 2005a, 2005b, 2007), which moved the focus to overarching big ideas such as the mathematical processes (which I will call the "new new new math" (New)^3 in a future BFTP column). However, the integration of curriculum, pedagogy, and assessment that was pioneered by the "new" new math of 1998 remains in mathematics education in Ontario today, as do the collaborations between the OME, OAME/AOEM, and the Fields Institute. The problem-solving approach also foreshadowed the current curriculum changes (OME, 2020, 2021) that focus on mathematical modelling (the (New)^4 math?), which also features the introduction of coding in the curriculum and consideration of students' social–emotional learning.

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Continued on page 14

▲ OAME/NCTM Report: Happy Fresh New School Year!



JACQUELINE HILL
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Jacqueline Hill is a Grades 9–12 teacher of mathematics. She is a Past President of OAME and OMCA, as well as the recipient of the award for Exceptional Teaching in Secondary Mathematics. She also wrote the "Director's Dialogue" for the Gazette *for a number of years.*

If you have never visited the NCTM website, I would encourage you to do so NOW! OAME/AOEM has a primary focus on math education content for the Province of Ontario, whereas the NCTM has resources diverse enough for the entire USA! Their website is a treasure trove of amazing learning opportunities and materials.

Each fresh new school year, I like to think about what I can do for myself and for my students that will make our year more rewarding. NCTM can help with your new initiatives. Here are my top three initiatives for a new set of students.

1) Student Engagement

When I did a search of the NCTM website with just the words "Student Engagement," it came up with 293 journal articles. Since technology, whether on student phones or their calculators, is here to stay, the key is to harness that technology for good. One such article is about the use of "The Into Math Graph Tool." This tool gathers data on how "into" mathematics students are. The data is gathered over time. The tool can help teachers converse with students around engagement in mathematics.

2) Meaningful Downtime for Students

Not every moment of every class can be jam-packed full of content for every student. Meaningful downtime is very important. On the main page of the NCTM website, there is a classroom resources section. Under that tab, there are 13 tabs to explore! The content of the tabs ranges from "The Year Game," where students use the digits found in the current year, and use mathematical operations (read the rules carefully) to obtain the answers from 1 to 100 (see www.nctm.org/Classroom-Resources/Year-Game/The-Year-Game/). This game reminds me of the "Four Fours" version that uses the same operations and only four fours to obtain answers from 1 to 100 (see Shawn Godin's column for seven sevens!). Other tabs contain problems of the week, problems to ponder, and a tab entitled "Rigor and Coherence."

All of these activities can be given to students to explore and think about critically. Many a wonderful conversation or group discussion could be about one of the classroom resources listed.

3) My Own Professional Learning

I used the search engine on the website and found 956 results on professional learning.

Try the NCTM online materials for "100 Days of Professional Learning," found at www.nctm.org/onlineevents/, as a starting point to your professional learning.

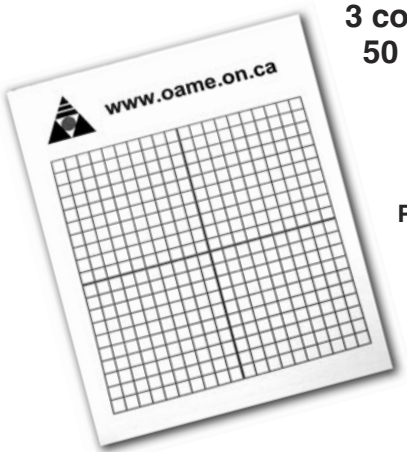
Each new school year is a fresh opportunity for a new beginning! The NCTM website can be your one-stop experience—beyond the OAME/AOEM.

The next NCTM Regional Conference will take place in Baltimore, Maryland from November 30 to December 2, 2022.

Yours in mathematical fun,
Jacqueline ▲

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▲ What's the Problem? Seven Sevens



SHAWN GODIN
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Shawn is a former mathematics teacher, department head, and consultant. He strongly believes in the central role of problem solving in the mathematics classroom. He continues to be involved in mathematical activities: presenting workshops, writing articles, working on local projects, and helping create mathematics contests.

Welcome back, problem solvers. We're doing something a little different this year. Last year, I wrote a piece for the anniversary edition of the *Gazette*, which didn't match the theme of the issue. I went back and wrote a new one, more in the flavour of what they were planning, and that is what appeared in the March issue. The original piece was looking at problem solving in the Ontario mathematics classroom over the last 50 (or so) years from a personal perspective. What I plan to do is save much of that original work and turn it into the four columns for this year. The OAME/AOEM and the *Gazette* have been a big part of my professional life, as we will see as the story unfolds.

I have always loved mathematics. Reflecting on my early experiences, I suspect this was because I was "good" at it. However, as a younger student, I was more drawn toward science because there were many more resources where I could explore the subject outside the school environment. The mathematics classrooms that I experienced throughout elementary and secondary school were very traditional. A typical class was homework take-up, lesson and examples, then practice problems. Next day, rinse and repeat.

I didn't experience very many genuine problem-solving activities. I know this must be true because a couple of events stick in my mind. I suspect they stick there because they were so different from the usual day-to-day activities. The first of these experiences was in Grade 5. The teacher gave us a problem similar to your homework from last issue that appears below.

> *Create seven mathematical expressions whose answer is seven, using as many operations as possible.*

This was fascinating to me: a problem where there wasn't a "right way" to do it and no one true correct "answer." I lost myself in the problem. At some point, one of my friends asked me how many I had so far. He had four, while I was well into my second page. How can one get lost in such a simple problem?

Keep in mind that since I was in Grade 5, we had yet to work with negative numbers, although we would have worked with some decimals and fractions. Let's keep things easy and start with the whole numbers. If we only consider addition, we get four expressions; eight if we ignored the commutative property of addition and treated 2 + 5 and 5 + 2 as different expressions. As seven is prime, using multiplication, we only add one more expression (or two).

However, something interesting happens when we look at subtraction and division. It seems that we have expressions aplenty!

7 – 0	7 ÷ 1
8 – 1	14 ÷ 2
9 – 2	21 ÷ 3
10 – 3	28 ÷ 4

We have discovered two *infinite families* of expressions, where each answer is 7.

Examining the subtractions, several interesting patterns appear. First, as we go from one example to the next, it appears that both numbers in our expression are increasing by one each time. This works no matter which two numbers we are subtracting. In Figure 1, below, a spreadsheet is set up. In cell C2, the formula =A2–B2 was set up so that column C is always the difference between column A and column B. In cells A2 and B2 are arbitrary numbers, in this case, 13 and 9. In cell A3, the formula =A2+1 was used, and in cell B3, we used the formula =B2+1. Now all the formulas are "filled down" to fill the spreadsheet. If we went back and changed A2 and B2, we would still get a family of subtraction questions with the same answer (although the answer for the family depends on your original choice).

	A	B	C
1	**a**	**b**	**a-b**
2	13	9	4
3	14	10	4
4	15	11	4
5	16	12	4
6	17	13	4
7	18	14	4
8	19	15	4
9	20	16	4
10	21	17	4

Figure 1: *A spreadsheet subtraction pattern*

We can create a similar spreadsheet for division. However, column A cannot just go up by 1 each time. Can you figure out how to create a spreadsheet that starts with 7 and 1, and continues to give results that when you divide them, you get 7? It is more difficult to create a spreadsheet such as in Figure 1, where A2 and B2 can be anything, and whatever the answer is for the division problem follows through. I will leave this to you to play with. I will explore it a bit on my YouTube channel, *Mathematical MishMash*.

Let's introduce some algebra. If we start with 7 – 0 and increase each of the two values by some unknown, x, then the new numbers are $(7 + x)$ and $(0 + x)$. It is straightforward to see that our result is just going to be 7, no matter what value we assign x. The nice thing is that we don't need to confine ourselves to the whole numbers: $x = 12.8$ yields 19.8 – 12.8; $x = \frac{13}{7}$; yields $\frac{62}{7} - \frac{13}{7}$; and $x = -4$ yields $3 - (-4)$ —each having an answer of 7.

Taking the algebraic approach a bit further, if we look at the addition problem and let x and y be our two numbers, then $x + y = 7$ or $y = 7 - x$, and the resulting graph is a picture of the infinite family of solutions. On top of that, we see that in our addition problem, a subtraction is necessary to represent the relationship between the two numbers we are adding. We are seeing the relationship between addition and subtraction. For the addition problem, we can express it in words as: if two numbers *add* to 7, and we know one of the numbers, the other number is the result of subtracting the first number from 7.

When I had tackled the problem in Grade 5, even though I didn't have the language or experience to realize it, I had created several algebraic identities. For example, if you consider $2 \times (n + 4) - (6 \times n + 9) \div 3 + 2$, then each value of n yields a result of 7. In my notebook, it would have looked something like the examples below:

$$2 \times 5 - 15 \div 3 + 2 = 7$$
$$2 \times 6 - 21 \div 3 + 2 = 7$$
$$2 \times 7 - 27 \div 3 + 2 = 7$$
$$\vdots$$

I got a taste of what mathematics really was, and I liked it! This one activity really helped boost my own love of mathematics.

If students don't go in this direction, how can we gently nudge them toward deeper thinking? Since the problem wanted us to use as many operations as possible, we could go from one operation to two, or more. The following activity could give them the nudge they need.

On four pieces of paper, have students write +, –, ×, and ÷, fold them up, and put them into a container. On a sheet of paper, have students write:

__ □ __ □ __ = __

Students then pick two of the operations at random from the container and write them in the squares *in the order they picked them*. Next, they pick numbers for the first three blanks and work out the answer. The challenge is to find more sets of numbers that, when used with the same operations, give the same answer. This becomes a bit more complex because we have more wiggle room in the problem. It will be interesting to see what students come up with and how they do it.

I hope you enjoyed playing with the problem from my early years. I will make up a video on my YouTube channel, showing how to do some of the things I talked about with the spreadsheet. The channel contains videos that I have done for students and teachers for $O_3$4ME, as well as supplementary materials for this column and a similar one I do for *The Variable* (published by the Saskatchewan Math Teacher's Society). I will create videos exploring past columns on the channel, so you can watch for those as well.

Now, for your homework, a problem from later in my math career:

If $90! = 90 \times 89 \times 88 \times \ldots \times 2 \times 1$*, then what is the exponent of the highest power of 2 which will divide 90!?*

Until next time, happy problem solving! ▲

Continued from page 11

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▲ In the Middle: How Teacher Talk Meets the Thinking Classroom



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Carly Ziniuk teaches Grade 9 Mathematics, Grade 12 Data Management, and Advanced Placement Statistics at the Bishop Strachan School in Toronto, Ontario, Canada. She is very active in adopting real-life data to engage her students in solving problems.

In March 2022, *tes magazine*'s "3 Teacher Phrases I Must Stop Using," Scottish teacher Jamie Thom wrote about the repeated phrases he believes are pointless, after posting a similar prompt on Twitter at @teachgratitude1. Thom's motivation to explore his "toolbox of verbal and non-verbal skills" led him to resign his use of:

- Shhhhhhhh
- Do you all understand? or Are we all clear?
- Is everyone paying attention/listening?

Thom describes choices in the classroom to make volume levels explicit, asking students to verbalize their understanding, and effective use of the teachers' pause. His conclusion to the article reads, "(teachers) talk endlessly. Giving that talk some of the attention it deserves can ultimately improve the learning that happens in our classrooms."

I have been working with Liljedahl's Thinking Classroom (www.buildingthinkingclassrooms.com/14-practices/) frameworks for the past few years, and Thom's attention to the improvement of learning caused me to examine my talking. Developing specific classroom routines based on a few of Liljedahl's 14 Practices for a Thinking Classroom has been my focus for the past few years. The examples here demonstrate ways I hope to improve the learning that happens there, with attention to my own talking.

Starting every class with a quick-thinking task allows my students to know how each class will begin. Liljedahl suggests giving each task orally, with students in a loose formation around the teacher. However, for this to work effectively, especially without the "shhh-ing," the task needs to be a highly engaging problem and easily understood orally. For middle-level learners, I have found that short, one-sentence questions work most effectively. An application problem for distributive property and algebra is an example:

"What happens to the area of a rectangle if its length and width are doubled?"

Transitioning from this initial problem, the question connects to several similar prompts:

- What happens to the area of a rectangle if only one of its dimensions is doubled?
- What happens to the area of a rectangle if both its length and width are halved?
- What happens to the area of a rectangle if its length and width are multiplied by a factor of n?
- What happens to the area of a rectangle if its length and width are divided by a factor of n?
- What happens to the volume of a cube if its side lengths are doubled?
- What happens to the surface area of a cube if its side lengths are doubled?
- What happens to the volume and surface area of a cube if its side lengths are multiplied by a factor of n?
- What happens to the volume and surface area of a cube if its side lengths are halved?
- What happens to the volume and surface area of a cube if its side lengths are divided by a factor of n?

Although given orally, once the students are given a prompt, I present the same question on a slide or reveal it on a board or poster paper. Intentionally waiting until they have begun working, this acts as visual space to park their learning. Students can then record their findings on this larger space and, as a class, discuss the common discoveries.

Additional classroom starter questions can be found at CEMC's Contest Preparation and Courseware (www.cemc.uwaterloo.ca), Transum's Starter of the Day (www.transum.org), and rich tasks from University of Cambridge's NRICH Project (nrich.maths.org). Considering questions that are easily adaptable with varied ratios or values facilitates students generalizing and extending their thinking naturally. Consider the following example of a ratio swap from the NRICH project:

"In a group of 48 people, the ratio of adults to children is 3:5. How many adults need to join the group to make the ratio of adults to children 5:3?"

A prompt that goes along with this is asking students what is special about the ratios 3:5 and 5:3 when the starting value is 48.

Another example from Transum asks for a series with the same sum:

"Find an arithmetic series and a geometric series that both have 155 as the sum of their first five terms."

This can be followed by asking students if it works with any number? What is special about 155?

In 2009, at a conference, Carmel Schettino discussed intentional teacher design and sequencing of problems like these to move students forward in a spiral curriculum with Problem-Based Learning. Another suggestion Schettino offered was to situate the learning of skills and knowledge in context. The examples presented above could be included during lessons on area and perimeter, ratios, sequences and series, and distributive property. Liljedahl recommends using non-curricular tasks as a provocation for engagement and learning, and a spiral-designed curriculum made of connected questions, with activities that ask students to review previous understandings en route to solve new problems.

For non-curricular engagement, I have found curious images successful for provocation, without much teacher intervention. The images from Dan Meyer's "101questions" (www.101qs.com/) work to motivate students. Similarly, I am a long-time admirer of Ron Lancaster's column in the *Gazette*, and have found that photos from his column, and similar photos found on my local travels, inspire students. Using images with a personal story or examining the archive of many photos from Ron's "Mathematical Snapshots," I have a lot more to discuss mathematically with my students beyond "Does everyone understand?"

Some examples of photos I have used include a sizing chart I found at The Shoe Company (Figure 1 left) and a pyramid of jars (Figure 1 right) the students made during a recent sand art crafting morning.

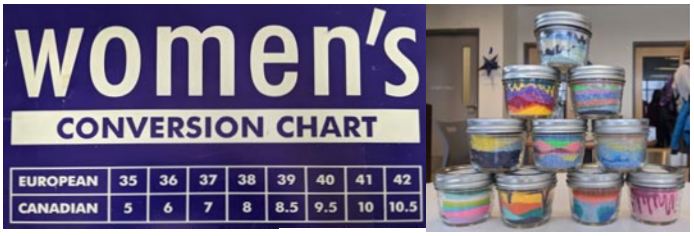


Figure 1: *Math from local photos*

In attempting to improve upon these types of questions, David Nelson's question starters, found through www.equitymaps.com, provide examples of probing question stems, which include:

- What is your hunch about…?
- What was your intention when…?
- What would have to change in order for…?

These kinds of stems allow me to keep the flow of learning moving forward, especially when the different classroom groups veer in different directions. Consider, for example:

What would change for a men's conversion chart sign that starts with European 40 equal to Canadian 7?

What is your hunch about how many complete layers the sand jar pyramid would make with 100 jars?

Another way to keep flow and still provide student autonomy is by actively giving warm and cool feedback to student ideas. The warm feedback provides a supportive response to student action, noting for them what is working well. Cool feedback offers different ways the student can think about the work, or suggests a change in approach. Nelson also provides examples of warm and cool feedback stems like:

- (warm) "The choice to _____ is very effective because _____."
- (cool) "I'm thinking about _____ and I wonder _____. How might….?"

When given the question below, many students initially were challenged because they did not know the price and were thrown off, worrying that they had to use algebra and decimals. Experimenting with a contextual problem allows students to discover the commutativity of (percentage) multiplication, without simply stating this property.

"During a recent scratch-and-win sale, a customer scratched three different discounts, 20%, 10%, and 5%. If the customer wants to maximize their savings on three separate purchases at the sale, in what order should they apply the discounts?"

Clarifying questions, such as "Let me see if I understand…," can help students to restate and reframe their thinking, e.g., asking them whether they could choose a given purchase of $10 or $100, and whether that might make things easier to calculate. How might starting with a number help students engage and act as a stepping stone to move to algebra after that?

I loved Liljedahl's rebranding of homework as check-your-understanding questions, but I knew I had to be strategic in my choices and dramatically reduce the number of questions if that rebrand was actually going to have credibility. I worried whether my students would trust that it wasn't just a change in name. I tried two different approaches to the conclusion of the perimeter, area, and volume investigation presented above. The first was to use a more constrained, but related, problem as an "entrance," rather than "exit," ticket for the next class. These exit/entrance tickets also can serve as the starter problem, as well as for class discussion.

To find the length of a particular rectangle, you must double the width and add 10.

How would you express the perimeter of this rectangle?

Many students saw this straightforward problem as an opportunity to check that they had learned something from the lesson. It also became clear to students when they had lost focus or had been listening, rather than participating with intention. After watching what the students had done both in class and at home after our collaborative discussions, I am preparing to give the different exit/entrance tickets to all students, but varying components like double/half, add/subtract, perimeter/area. I think having students all working on different versions of the same problem further provides accountability without the volume of work I would have typically given for homework on the same lesson. Another idea I had was to change the value of 10 and note how this changes student responses, or have students work with a variable for the multiplying factor or a variable for the added value. Using a Desmos slider or adapting an existing Scratch program with an input factor variable can be an opportunity for students to check their understanding.

Much of our work on Social–Emotional Learning serves to reduce anxiety in the math classroom, and to identify with students the source of what feels overwhelming. Taking the time to make sure they understand the reasoning and logic behind their work and collaborate with their peers has truly reduced the stress I see each day. Focusing only a few specifically selected check-your-understanding questions, or extending the problem using technology at home, has allowed students to be more independent and more confident in their learning, both in and out of the classroom. I am able to discuss mathematics more directly with them each day by using my in-class teacher talk time more effectively. I regularly look forward to observing mathematical learning, rather than shushing them, or checking if they are paying attention or even asking if they understand. Developing these routines continues to allow me to spend more of my verbal and non-verbal teaching time with my students as thinking, confident, metacognitive learners. For me, that is a much better use of my time than endless talking.

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OAME/AOEM Award in the News
www.manitoulin.com/manitoulin-woman-named-ontarios-top-elementary-math-instructor/

▲ MB4T (Mathematics by and for Teachers): Using Models and Reasoning to Develop Factoring by Decomposition



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Ann Kajander is a Professor at Lakehead University, working with pre-service elementary and secondary teachers of mathematics, as well as graduate students. She is particularly interested in studying and designing the kinds of supports that are helpful to teachers to both understand and teach mathematics in a more visual, conceptual, and exploratory way. The use of representations and reasoning in mathematics learning, including at the secondary level, is a current focus.

In the previous column, the area model of multiplication was used as a helpful representation to explore factoring and for simplifying of quadratic expressions. In particular, the set-up for a lesson based on an algebra-tiles exploration by students for factoring simple quadratics was offered. I have used this lesson multiple times with Grade 9 students, and I find students are readily able to independently determine the sum-product method for factoring simple quadratics such as $x^2 + 5x + 6$. After a few trial-and-error attempts, they soon see that the number of pieces representing the "*bx*" term (such as $5x$) needs to be split into two groups, which form the sides of the rectangle with area *c* (such as an area of 6 square units). This visual method exactly aligns with the procedural method, but in a meaningful way.

For many years, however, I have felt that I hit a wall in terms of using problem-based learning with representations and reasoning to develop the more complicated factoring method of factoring by decomposition. The steps in the procedure are still both commonly and formally taught as a skill in Grades 10 and 11. At best, I have seen the formal teaching of the method accompanied by models to "show" what is happening, alongside the algebraic steps. Ideally, however, students should develop and make sense of the method themselves, rather than still require someone to "tell" them the "steps" first or along the way.

For several years, I have offered this problem to pre-service secondary mathematics teachers, and collectively, the best we have achieved is modelling the quadratic by

moving the algebra tiles around to show what is happening in the procedural method—but not independently develop the method directly with the representations. Until this year.

Recently, a pre-service teacher, Michael Nelsons, developed a new way (new to me at least, and not one I have seen taught in Ontario high school classrooms) to explore factoring general quadratic expressions by *using a transformation* to make the second-degree term a square. This temporary transformation subsequently allowed the use of the previous sum-product method! This is mathematical problem solving at its best, in my view. The key idea here was noticing what makes the sum-product method possible, namely that the x^2 part of the area model for a quadratic of the form $x^2 + bx + c$ is a *square*. So, Michael simply generalized this idea to the general quadratic of the form $ax^2 + bx + c$ by using a transformation (temporarily) to make the ax^2 term also into a square. Can you see how? Yes, all we need to do is multiply the expression by the constant a. The "square" part now has the form $(ax)^2$, or $(ax)(ax)$. (Of course, later, the final factored form must be transformed back by dividing each term by a again.) From here, the previous sum-product method can be directly applied—we don't need another method at all, such as decomposition. We can either "remember" that the variable is now (ax), rather than x, or alternatively, temporarily set $w = ax$ and proceed from there. Either works fine.

Examples of the Approach

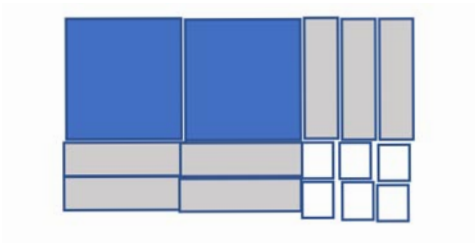
Modelling a quadratic such as $2x^2 + 7x + 6$ with algebra tiles allows the difference between this case and an example such as $x^2 + 5x + 6$ to be visually apparent. In the latter case, the second-degree term is already a square. The key idea of exploiting the need for a square second-degree term is what generated the idea of making the $2x^2$ term into a square as well, by multiplying the (entire) expression by 2. In sharing the idea with peers, Michael did have to explain this starting point, but from there, we were all able to model the expression and complete the factoring method as before. In fact, the method at this stage was basically a known one, albeit with a slightly more complicated second-degree term of $(ax)^2$, rather than x^2.

Perhaps at this stage, you would like to try to use a model to factor $2x^2 + 7x + 6$ yourself, before reading on. It is suggested you first model $2x^2 + 7x + 6$ with algebra tiles, and then generate the square second-degree term by multiplying the model by 2, (thus requiring double the number of tiles in the representation of course) and rearranging like parts. In effect, your model now will show $4x^2 + 14x + 12$. Try it!

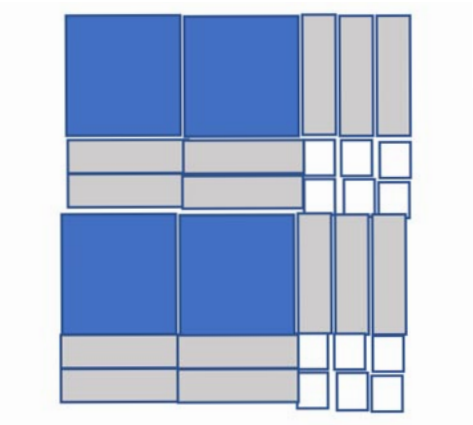
To follow is one possible set of representations showing the process. Another option you may wish to try is to substitute $w = 2x$ and proceed with a model using w.

Perhaps you had a set of models similar to one or both of the following.

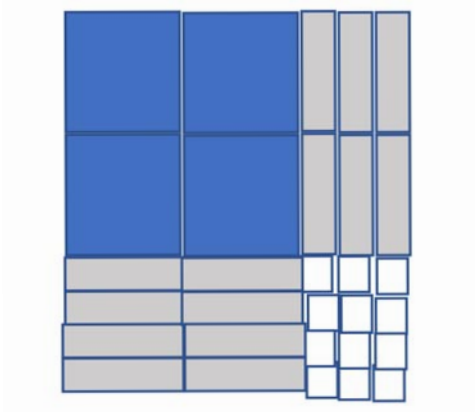
Here is a model of $2x^2 + 7x + 6$:



Visually we see the change needed to make the second-degree term a square:



And rearranging like parts:



From here, the factors can be seen as $(2x + 3)(2x + 4)$. Using the expression only, and writing $4x^2 + 14x + 12$ as $(2x)(2x) + 7(2x) + 12$, we can also now see and apply the sum-product method procedurally, as developed in the previous column ("two numbers that add to 7 and multiply to 12, which are 3 and 4"), to give $(2x + 3)(2x + 4)$ as the factored form. These factors can exactly be found in the model above. Of course, to find the factors of the *original* quadratic, $2x^2 + 7x + 6$, we need to divide the model by a, in this case 2. This process yields the actual factors of $2x^2 +$

$7x + 6$, namely $(2x + 3)(x + 2)$, which you can see in the first model.

Another version of the process is to make the substitution of $w = ax$ after multiplying by a. So instead of using $2x$ as the variable: , we now use w: .

Visually we can see how these are equivalent.

Once the transformation to w is done, the quadratic becomes $w^2 + 7w + 12$. It can be factored with the sum-product method, either with algebra tiles, or the procedural method, or both, and then the transformation back to x requires substitution of $w = 2x$, followed by division by 2 (i.e., the a value).

I have tried this method on a number of quadratics, including ones where the second-degree term is a rectangle with both sides greater than 1, and have not found one for which the new method does not factor a (factorable) quadratic. If there are problematic cases, I hope readers will share them in letters to the editor. Also, if you have a chance to try these methods with your students, please share their responses! ▲

Continued from page 4

District School Board), passed the gavel to incoming President, Jason To (Toronto District School Board), and welcomed our new Executive for the Fall. The Executive will be comprised of Heather Theijsmeijer (Past President), Jason To (President), Jackie Decker (Limestone District School Board, Vice President), Allison Lawrence (Brant Haldiman Norfolk Catholic District School Board, Secretary), and Marilu Deal (Ottawa Carleton District School Board, Treasurer).

Are you a math leader in your board? Interested in joining OMCA? New members are always welcome—contact us directly at OMCAmath@gmail.com, or visit our website at www.omca.website (note the different domain), to learn more about the organization and connect with us. Our first meeting of the 2022–2023 school year will be Monday, September 19. We hope to see you there! ▲



The theme this year is "Mathematics for Everyone."

See www.idm314.org for details.

▲ Some Thoughts about Perimeter: String Perspective



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Jérôme Proulx is a professor of mathematics education (www.profmath.uqam.ca/~jproulx), and runs the Laboratoire Épistémologie et Activité Mathématique (www.leam.uqam.ca). His research focuses on epistemological issues related to cognition and mathematics. His current research program is dedicated to mental mathematics and school mathematics content development. (Photo by Nathalie St-Pierre of Université du Québec à Montréal)

Students often come up with surprising ways of solving the tasks we give them. They can at times even come up with strategies or ideas that, in turn, raise questions or shed light on mathematical aspects usually taken for granted or kept implicit. This is particularly the case in the mental mathematics that my research team and I study in classrooms.

Within time constraints, students rarely refer to known procedures because these are often too time-consuming and hence, not economical for solving in this context (see Butlen & Pézard, 2000). This leads many students to engage with a variety of strategies worth pondering. This column follows on previous columns with a sharing of examples of students' ways of solving tasks about perimeter and figures in mental mathematics contexts.

Perimeter is often considered as a simple concept, almost trivial. Distinguished from area as a measure of surface, perimeter is often presented, in textbooks and various lexicons, as the (measure of the) distance around the outside of a figure, requiring in return the addition, one by one, of the measures of the sides of the figure. However, beyond its measure or computation, perimeter might not be as simple a concept, and is much richer than it appears at first glance.

Recently, our research team worked with two young students, Marco (10 years old) and Marie (12 years old), while they solved various perimeter and area tasks in a mental-mathematics context. Their ways of solving raised a number of questions regarding the nature of perimeter itself and the meaning often given to it. This fourth column, with the perimeter focus, draws on a number of tasks solved and the issues it raises about images used to conceive of perimeter.

The Task and the Long Curve

The previous columns discussed perimeter in terms of a boundary that forms, and is formed by, the figure. In the task about the lake, mentioned in the December (2021) column, Marco and Marie were asked to find the maximum area for the lake on a square grid, shown in Figure 1, and its maximum perimeter. When asked to find the maximum area, Marco mentioned that one would simply have to stretch each side to the limits of the grid and count the total squares because "this is where the sheet stops." This gave him a 5×7 region with an area of 35 squares.

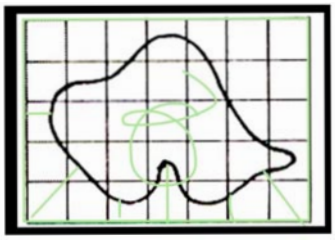


Figure 1: *A lake on a square grid (Image reproduced from www.warmaths.fr/SCIENCES/unites/AIRES%20SUR/AirSufCurvilign.htm)*

When asked about the maximum perimeter, he smiled and appeared relieved. He said, "Well, easy, you just draw a long line like this, for example." As he made sure to connect each end at the end of his drawing, he added, "There are lots of zig-zags."

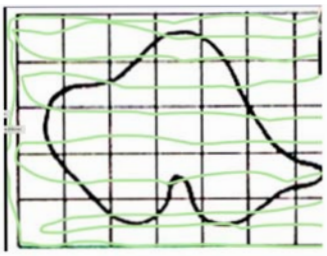


Figure 2: *Finding the maximum perimeter*

The perimeter appeared here to be a long one-dimensional curve, not rigidly attached to a specific two-dimensional shape. A similar way of thinking was also apparent in a subsequent task. Marco and Marie were both shown a piece of string, told it had a length of 60 cm, and that it was used to create the largest possible rectangle there is (inspired by Woodward & Byrd, 1983). They were asked what the perimeter would be for that largest rectangle. Right away, in a matter of an instant, both Marie and Marco laughed. Marie said that it would simply be the string. Marco said it would be 60 cm because it is the same string for every rectangle. In fact, when shown with hands a variety of rectangles that could be formed by the string (e.g., approximating 10×20, 15×15, 25×5 rectangles), both Marco and Marie individually said that the shape did not matter, and that each of the rectangles shown had the same perimeter of 60 cm. This task, simply put, did not take much time to be solved. It contrasted both in time and in matters of justification with tasks mentioned earlier in the September (2021) column, that asked to match both perimeters of rectangle A and triangle B, as well as square A and triangle B.

Hence, when not directly or rigidly attached to a figure or a two-dimensional shape, the consideration of the perimeter appeared altered and easier to consider by both Marco and Marie. This points to the idea that when given the perimeter in a form that is already "unfolded" (or disconnected from a specific shape), the consideration of the perimeter does not appear to be as rigidly connected to the figure or mixed up in two-dimensional considerations. It felt that working from an already one-dimensional length eased the consideration of questions about the perimeter (to assess it, to create a shape with it, etc.).

Some Thoughts about… Perimeter, Boundary, and the String

Marco and Marie's ways of reasoning for these two tasks reminds one of the string metaphor for perimeter. In this case, the string is wrapped up around the figure. This view has, however, been criticized by some, like Danielson (2005), because a string does not inherently preserve the perimeter when taken in itself: the string preserves the length of the perimeter, but does not preserve the two-dimensional figure that allows one to talk about the perimeter *of a figure*. As well, string does not necessarily bound a region (and if not closed, the curve could even be said to not be related to the perimeter of a figure at all). In this view, the string could be seen to be telling only one part of the story, that is, the linear one-dimensional part of perimeter.

However, the string conception can enable one to visualize dynamically and flexibly the notion of perimeter of a figure, easing the interplay from one-dimensional to two-dimensional, which somehow contrasts with the rigid view of sides of figures mentioned in the September (2021) column. The string can represent as much the folding as the unfolding of the perimeter. In that way, it is possible to conceive of the string as wrapping around the two-dimensional figure and tail along its boundary, and vice-versa as unwrapped and constituting a one-dimensional length.

It is thus the malleability of string, as a physical tool or representation, that makes it conceptually attractive. A string is easily folded and unfolded to create a figure and a length, tapping directly into the dynamic interplay between one-dimensional and two-dimensional to create the boundary of the figure and considering its length. And it is this interplay, through the formation of the boundary of the figure and thus the figure itself, that gives the concept of perimeter its meaning. (On this point, and relative to the notion of boundary, as mentioned in the June (2022) column, the Jordan curve as a closed curve is also often called a "simple string.")

The string metaphor can also be seen as offering a way of obtaining a first measure of the perimeter of the figure. The measure of the boundary of the figure becomes the amount of string used as its measure, its perimeter being the length of

string (see Figure 3). Conceived that way, the string metaphor offers a way of measuring the perimeter: the length of string used *is* the measure of the perimeter. And then, if needed, it is even possible to re-measure this initial string measure, that is, to measure the string through other means, such as with a ruler or a square grid. By linearly outlining the string, all straight, the string used can measure 22 units or cm, for example.

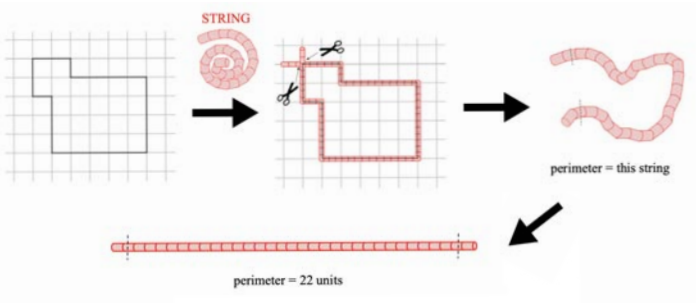


Figure 3: *The malleability of, and measuring with, the string representation*

The physical flexibility that comes with the string metaphor, and the conception of perimeter as a boundary, combine to ground the dynamic interplay of one-dimensional and two-dimensional aspects inherent in the concept of perimeter. The perimeter can then be conceived as the measure of the boundary of a figure, that is, the total one-dimensional length obtained if all the sides of the two-dimensional figure were unfolded and aligned one after the other, which in turn can be refolded into the figure by producing its boundary.

It is in this (un)folding from two dimensions to one dimension to measure the boundary, and vice-versa from one dimension to two dimensions to create the figure, that the concept of perimeter takes its meaning as a linear or one-dimensional measure of an object (the boundary) that delimits a two-dimensional object (the figure). Inspired by work with students, be they Marco and Marie or the Grade 5 classroom students from the June (2022) column, this view concludes the collection of four columns on perimeter.

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▲ Elementary Math Matters: It's Time for Another Public Inquiry: The Right to [Do] Math



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Elementary education has been hit with a tsunami. With the Spring 2022 release of the Ontario Human Rights Commission's (OHRC) long-awaited report, *Right to Read: Public Inquiry into Human Rights Issues Affecting Students with Reading Disabilities* (OHRC, 2022a), the storm hit hard and fast—not only is Ontario's public education system failing students with reading disabilities (e.g., dyslexia) by not using evidence-based approaches to teach children to read, the inquiry found that the province is not fulfilling its obligations to meet *ALL* students' right to read.

Boldly reiterating a 2012 decision by the Supreme Court of Canada, the report underscored the fact that *learning to read is not a privilege, but a basic and essential human right* that goes far beyond reading: reading represents an equal right to a future. The inquiry report shone a glaring floodlight on many systemic impediments to student success in reading, including misguided curriculum documents, ineffective instructional methods, the dearth of available early screening tools and reading-intervention programs, as well as inadequate teacher preparation in and knowledge about the *science of reading*.[1]

The inquiry, which began in 2019, was led by two experts in reading development and reading disabilities, Dr. Linda Siegel and Dr. Jamie Metsala. Its purpose was to analyze significant information obtained from a representative sample of eight English-language public school boards; all 13 Ontario English-language public faculties of education; Ontario First Nations, Métis, and Inuit communities; and the

Ontario Ministry of Education (OME). In addition, the inquiry heard from thousands of students, parents/guardians, organizations, educators, and other professionals through surveys, public hearings, community meetings, and telephone calls.

The report emphasizes that if science-based approaches to reading instruction were used in classrooms, and early screening and intervention programs were available, the province would expect to see only about 5 percent of students still below grade-level expectations on word-reading accuracy and fluency. In sharp contrast, the provincial results for the last EQAO test, i.e., pre-pandemic, 2018–2019, found that 26 percent of all Ontario Grade 3 students and 53 percent of Grade 3 students with special education needs (students who have an Individual Education Plan (IEP), excluding students whose sole identified exceptionality is giftedness) were not meeting the provincial standard. By Grade 6, 19 percent of all students and 47 percent of students with special education needs did not meet the provincial standard.

This excerpt from the findings powerfully captures the authors' conclusions:

> If classroom instruction is based on an evidence-based core curriculum, most students (80–90%) will learn to read words accurately and efficiently, and few students will need more intensive instruction or intervention. Decades of multidisciplinary research has shown that the best way to teach all students to read words is through direct, explicit, systematic instruction in foundational word-reading skills. Beginning in Kindergarten, this includes explicit instruction in phonemic awareness [the ability to identify and manipulate individual sounds (or phonemes) in spoken words], phonics (which teaches letter-sound associations, also known as grapheme-phoneme correspondences and using these to "sound-out" words and to spell words). From about Grade 2, explicit instruction focuses on more advanced knowledge and skills, such as increased study of word structures and patterns (for example, prefixes, word roots, and suffixes), and how word spellings relate to one another. From beginning to teach these decoding skills, students also practise reading words in stories to build word-reading accuracy and speed. (OHRC, 2022a, pp. 20-21)

Unfortunately, the current Ontario Curriculum, Language, Grades 1–8, 2006 (Ontario Language curriculum) and teacher education in Ontario's faculties does not promote these highly effective approaches to early word-reading instruction. Instead, with few exceptions, the main approaches in Ontario are teaching word-solving skills with the three-cueing system and balanced literacy. The three-cueing system encourages students to guess or predict words, using cues or clues from the context and their prior knowledge. In balanced literacy (or comprehensive balanced literacy), teachers "gradually release responsibility" by first modelling text reading, sharing text reading, then guiding students' text reading, with the eventual goal of the student reading texts independently. These approaches for word reading are rooted in a whole-language philosophy, which suggests that by immersing children in spoken and written language, they will discover how to read. Given this philosophy, many of the other important literacy outcomes beyond word-reading skills may also not receive adequate explicit, evidence-based instruction (OHRC, 2022a, pp. 20–21).

These paragraphs paint a pejorative indictment of the current state of the Language curriculum; widespread, entrenched, and ineffective instructional practices; and out-of-touch teacher education programs. To address these and many other issues (e.g., the very long wait times for formal diagnosis by psychometricians and expensive private reading-clinic programs), the report details 157 highly actionable recommendations directed to the OME, school boards, and faculties of education on how to address the systemic issues that are to blame for myriad students' inability to read.

First out of the gate in response to the report was the OME's response to the OHRC report (OHRC, 2022b), which was reactive and specific, stating explicit promises to:

- revise the elementary Language curriculum and the Grade 9 English course with scientific, evidence-based

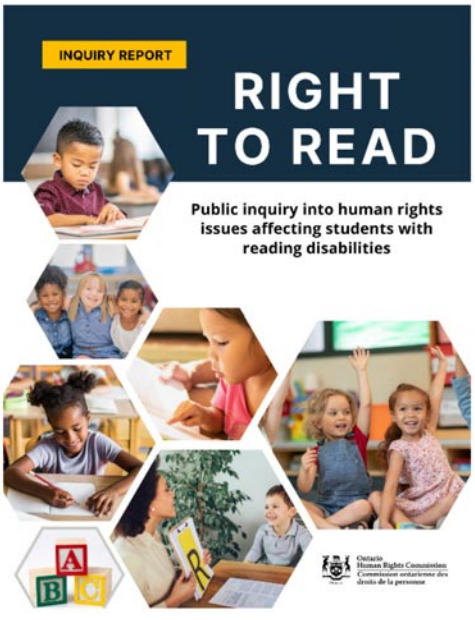


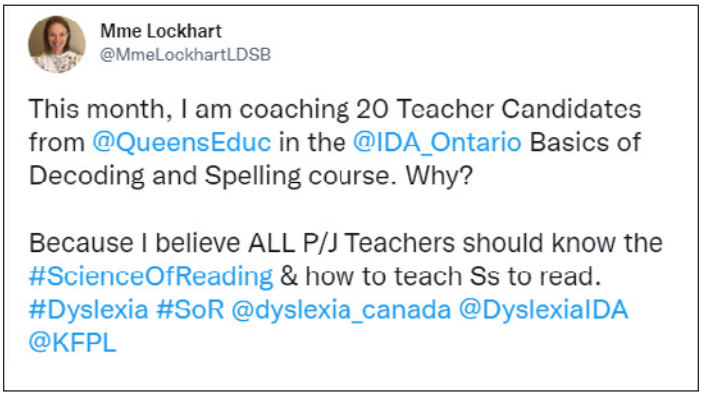
Figure 1: *The cover of the* Right to Read *report, released on February 22, 2022 by the Ontario Human Rights Commission. Many of the 157 recommendations in the report have application to early mathematics teaching and learning.*

approaches that emphasize direct, explicit, and systematic instruction, and removing references to unscientific discovery and inquiry-based learning, including the three-cueing system, by 2023

- release a science-based guide for educators in Spring 2022 that will support effective early reading instruction
- collaborate with partners, including faculties of education, on professional development for educators to ensure they are learning science-based reading-instruction methods

Also notable in the OME response were references to the ways in which the Ontario College of Teachers and faculties of education needed to re-align their programs (pre-service, in-service, and teacher induction) to equip educators with evidence-based approaches to reading instruction, diagnostic assessment/evaluation of reading, and intervention.

Some school boards are following suit, and we are seeing evidence of productive collaborations forming among educators with Orton-Gillingham[2] certification, school councils, public library systems, and faculties of education. In the Limestone District School Board, educator Kimberly Lockhart (@MmeLockhartLDSB; www.twitter.com/MmeLockhartLDSB) is taking the lead by coordinating Family Literacy Nights, Professional Book Clubs, and PA Day activities, as well as teaching formal courses for parents/guardians, pre-service and in-service teachers, and administrators in structured literacy instruction. This tweet shows but one example:



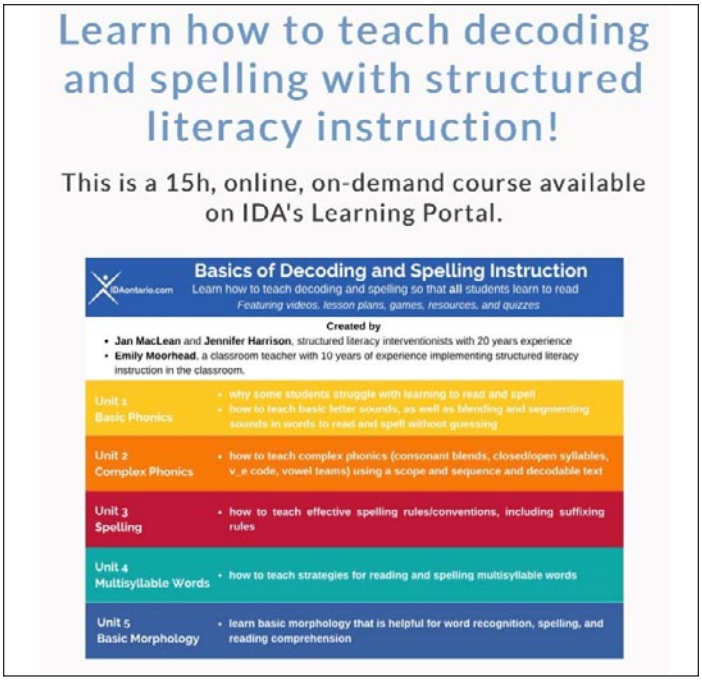


Figure 2: *An example of a local initiative to act upon the recommendations from* Right to Read

I am hopeful that the publication and extensive media and school district attention to the reading report will be the catalyst for a parallel "deep dive" into early mathematics education, since early mathematics consists of important and fundamental math skills, and like reading, should be within reach of most students. In fact, some would argue that it is more important, since it is achievement in Kindergarten math that is the best predictor of both reading and math in Grade 5 (den Braak et al., 2022).

As a field of study, there is no question that mathematics learning has received far less attention than its close cousin, literacy learning. However, according to Dr. Jo-Anne Lefevre, Director of the Institute of Cognitive Science and Centre for Applied Cognitive Research (which includes *The Math Lab*) at Carleton University, fundamental research on the foundational knowledge and skills that support mathematics learning has accumulated to the point that we can articulate a foundational model of the cognitive predictors and the relevant skills.

In addition to developing the *Pathways to Mathematics* model (Di Lonardo Burr et al., 2021; LeFevre et al., 2010; Sowinski et al., 2015; Träff et al., 2017), Dr. LeFevre, in collaboration with Dr. Heather Douglas, created the Early Math Assessment (EMA@School) for Alberta Education, that had, as of October 15, 2021, been field-tested with more than 30 000 Grades 2 and 3 students. It is currently being refined in response to empirical data. Dr. LeFevre is also collaborating with Dr. Helena Osana (Concordia University) on the further development of the Numeracy Kit for

Kindergarten [5-year-olds], or "NyKK-5."

Under Dr. LeFevre's leadership, the EMA@School program and the NyKK-5 will be expanding into Ontario and Manitoba. Through a SSHRC partnership grant, cognitive science researchers, mathematics education researchers, numeracy facilitators, classroom teachers, and mathematics consultants will work to create evidence-based professional development materials. These include: (1) simple, but powerful assessment and intervention tools for early mathematics, and (2) resources to support teacher knowledge about mathematical development. The ultimate goal of the partnership is to help all students achieve success in mathematics.

We know that the many disruptions to education caused by the pandemic over the past two years has contributed to learning loss in mathematics, but we cannot yet quantify that loss. We know that most pre-service educators do not receive targeted teacher preparation for Kindergarten and early primary education (Youmans et al., 2018), nor have most in-service educators participated in professional development about the challenging details involved in cognitive-development pathways and learning trajectories. We know that the learning expectations in the *Early Learning for Every Child Today: A Framework for Ontario Early Childhood Settings* (OME, 2006) curriculum for preschool and Kindergarten, *The Kindergarten Curriculum* (2016), and *The Ontario Curriculum, Grades 1–8: Mathematics* (2021) do not align in terms of content (Colgan et al., in press) and are philosophically and pedagogically akin to the corresponding reading curriculum.

Never has there been a greater need for an OHRC inquiry regarding early mathematics. I vote for Dr. Jo-Anne LeFevre to lead the inquiry, ably assisted by her impressive team of researchers, including Drs. Zachary Hawes, Helena Osana, and Heather Douglas, her Canada-wide school district partners (including the Eastern Ontario Staff Development Network), and OME officials. It is hard not to notice that her team, already in place, mirrors the complement that had to be assembled before the reading inquiry could begin, so why not open the starting gate?

In closing, I thank the reading community for its leadership in bringing *Right to Read* to fruition. *May the Right to [Do] Math* honour your brave precedent.

Notes

[1] The *Right to Read* report uses terms like the "science of reading," "reading science," "research-based," "evidence-based," and "science-based" to refer to the vast body of scientific research that has studied how reading skills develop and how to ensure the highest degree of success in teaching all children to read. The science of reading includes results from thousands of peer-reviewed studies and meta-analyses that use rigorous scientific methods. The science of reading is based on expertise from many fields including education, special education, developmental psychology, educational psychology, cognitive science, and more (OHRC, 2022a, p. 3).

[2] The Orton-Gillingham Approach is one long-established s*cience of reading* method. It has been used by trained educators to plan structured and individualized programs for students with reading difficulties for more than 80 years.

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▲ Mathematically Engaging Students in Their World



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A long time ago, I took a Master's-level course from the eminent Canadian mathematics educator John Mason. Dr. Mason encouraged us to see everything in the world around us as a potential mathematical problem. This world view had a profound impact on me, and, consequentially on my philosophy of teaching. Dr. Ron Lancaster's column in the *Gazette*, "Photomath," is an example of this type of world view. There is significant potential to engage and motivate our students by relating the mathematics that we teach to students' lives and interests. Additional examples of this stance can be found in some of my activities published in previous *Gazette* articles, like "The Justified World of Geometry" (*Gazette*, 58(3), p. 29) and the "How Much Hot Chocolate" problem-solving activity (*Gazette*, 57(4), p. 34).

Possible topics for mathematical investigations are limitless. Some are big topics, like climate change, pollution, inflation, unemployment, traffic lights, resource depletion, price of oil/gold/gas, urban sprawl, agriculture, modification of crops, and nutrition. Other topics, including sports, cellphones, social media, fashion, nature-related concepts, like tree rings indicating growth, and bee populations, art, and music, are often more personal to a specific student or group of students. The interesting thing is that components related to all of these situations can be modelled mathematically, sometimes with fairly straightforward models.

Of course, students need to be taught to see their world through a mathematical lens. For example, when driving to school, how are traffic lights synchronized so that if one drives the speed limit on a busy street, they generally encounter mostly green lights? How is the number of revolutions of a bicycle tire related to the speed of the bike and/or the diameter of the tire? How much fabric is needed to produce a pair of blue jeans of a specific size? How long does a pet live, and how much do choices impact it? The possibilities are endless.

In this article, I provide some dynamic examples of how everyday situations can be modelled mathematically, and relate these to expectations in the Ontario curriculum. I have found that using examples like these is very motivating for students, especially when they can pose and solve their own problems related to the phenomena. Note that the single frames included in this article do not do justice to the dynamic nature of the videos.

Event Math

One way to engage students is to link events that are happening right now to the mathematics that our students are studying. I call this "event math." Some examples of this, such as sporting events and earthquakes, are discussed below. It is possible for teachers to "stockpile" resources for these kinds of events, so that they can be readily available for use when the time comes. However, note my comments under "Video Tips" pertaining to the need to be sensitive to student emotions.

Earthquakes

There are two videos that I like for introducing the Richter scale (a logarithmic scale). The first video has no math in it, but illustrates ten earthquakes of various magnitudes. The earthquake in Nepal, where a screen picture shown in Figure 1 does not do justice to the scope of the video, is from www.youtube.com/watch?v=NT_fVQ1LdQY, and is 7:41 minutes in length. The second video, www.youtube.com/watch?v=NaNw9LHq9dc, 4:56 minutes in length, is a bit wordy, but fairly engaging. I have found this an interesting way to introduce logarithmic scales in Grade 12 Advanced Functions.



Figure 1: *Earthquake in Nepal*

Sports

There are literally millions of videos and data on sporting events available online. When major sporting events occur, such as the Olympic Games, there are excellent opportunities for our students to explore related mathematics. As an example, students can model the decreasing times for the 100m race. Currently, men's times are faster than women's times. Both men's and women's times are decreasing; however, women's times are decreasing at a much faster rate than men's times. So, a question that can be posed is, "Will women eventually run faster than men in the 100m?" Historical data are readily available online (e.g., www.statathlon.com/historical-predictive-analysis-for-the-100m-sprint-race/), and this is an opportunity to explore different models of the data to draw conclusions. This modelling activity fits Grade 9 (linear), Grade 10 (quadratic), as well as Grade 12 (exponential) modelling expectations. If your students are more interested in swimming events, a similar analysis can be done for men's and women's swimming.

Mathematical Modelling

There is a series of seven videos that outline the process of mathematical modelling, starting with an overview of mathematical modelling, followed by "defining the problem" and progressing through "making assumptions," "defining variables," "getting a solution," "analysis," and, finally, "reporting the results." The videos feature students (probably university students) discussing the steps in constructing a mathematical model, testing, revising, and reflecting on the results. The URLs for these short videos, ranging from 2:42 to 3:41 minutes in length, are given in the Appendix. The videos can be used in any grade where mathematical modelling is an expectation. We see illustrated in these videos the steps in Georg Pólya's modelling heuristic: understand the problem, make a plan, do the plan, and look back. Pólya's steps are included in the front matter of every mathematics curriculum policy document in Ontario. For another example of the mathematical modelling process applied to a classroom situation, see Irvine et al. (2016).

Population Growth

Some examples of natural phenomena that engage students are not "events," but are ongoing, such as population growth, pollution, climate change, and agriculture.

These concepts fit naturally into some of our math courses as introductions, motivators, or consolidations. One video that I really like for introducing population growth and exponential models is www.youtube.com/watch?v=PUwmA3Q0_OE (Figure 2 shows a single frame).



Figure 2: *Population growth as exponential growth*

At 6:24 minutes, it has the advantage of showing a dynamic exponential curve at the bottom of the screen. By taking screenshots of the curve, students can fit exponential models to the curve and then make predictions. One disadvantage of this video is that the graphics illustrating various events through history, like the Indian Golden Era, are sometimes difficult to read, especially for someone (like me) who is colour blind. This is certainly not the only video on population growth. For example, www.youtube.com/watch?v=neAzeAAq0hc illustrates population growth over time for the most populous countries, which might appeal to students who have emigrated to Canada from other parts of the world, although with a length of 10:24 minutes, it is a bit long and sometimes slow moving. Both of these videos are an excellent fit for the exponential modelling expectations in various Grade 12 courses.

Should Pluto Be a Planet?

In 2007, the International Astronomical Union demoted Pluto from planetary status. Why did this happen? What factors were considered in this decision? The activity below asks students to investigate this decision, using Kepler's Third Law of planetary motion. The investigation illustrates one way of mathematically analyzing Pluto's status. This investigation can be used in Grade 9 for expectations involving scatterplots and linear modelling, as well as in higher grades and in the Space strand of Science courses.

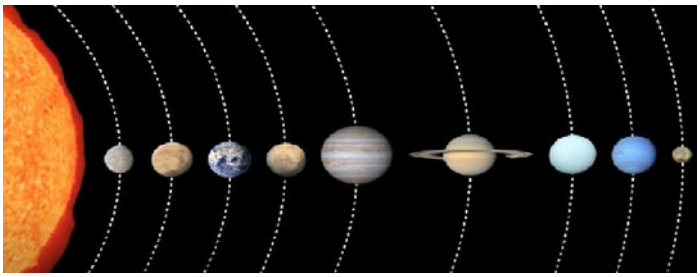


Figure 3: *The solar system in 2005 (from www.youtube.com/watch?v=Z_2gbGXzFbs)*

Should Pluto Be a Planet?

Johannes Kepler (1571–1630) was a German astronomer, who noticed a pattern in the table below (these were all the known planets when Kepler was alive):

Planet	Radius of Orbit (AU)*	Period of Orbit (earth days)
Mercury	0.389	87.77
Venus	0.724	224.70
Earth	1.0	365.25
Mars	1.524	686.98
Jupiter	5.200	4332.62
Saturn	9.150	10759.20

*AU (called an Astronomical Unit) is the average distance from the earth to the sun, 1.49x108 km.

(a) Kepler conjectured that (period)^2 divided by (radius)^3 is a constant. Copy the table above with another column added, and compute this value for each planet. Then average the values to get Kepler's constant.

(b) Write a formula for Kepler's relationship. [This is called Kepler's Third Law.]

(c) In 1781, William Herschel discovered the planet Uranus, which has a period of 30 588.70 days. Use Kepler's Third Law to determine the radius of Uranus's orbit.

(d) In 1846, the planet Neptune was discovered. This planet's orbital radius is 30 AU. Use Kepler's Third Law to find the orbital period of Neptune.

(e) Pluto, discovered in 1930, has an orbital radius of 39.5 AU and a period of 90 588 days. Does Pluto satisfy Kepler's Third Law? Explain your answer.

(f) In 2007, astronomers decided that Pluto is not a planet. Discuss this decision based on your results.

(g) Construct a scatterplot of (period)^2 versus (radius)^3. Find the equation of the line of best fit. The slope of this line is Kepler's constant. Explain any differences between this value and the value you obtained in part (a).

(h) In 2002, an object called Quaoar was discovered. Use research to find its orbital period and radius, and conclude whether Quaoar should be a planet. Justify your reasoning.

(i) What other factors besides Kepler's Third Law might be considered when deciding whether an object should be classified as a planet? Brainstorm with your group.

(j) Conduct research on what factors were considered in the decision to demote Pluto from planetary status. Watch the video at www.youtube.com/watch?v=n6dbT9StGCE. Write a brief report on your findings.

(k) Investigate Kepler's other two laws of planetary motion. Summarize the three laws in your journal.

Source for data (all astronomy questions): http://nssdc.gsfc.nasa.gov/planetary/factsheet

Detroit Airport Parabolic Fountains

At the Detroit airport, there are some amazing fountains that feature parabolic streams and bursts of water. The URL for one video of the fountains is shown in Figure 4. By taking screenshots of the fountains, students can fit quadratic models to the streams of water, and pose and answer questions such as whether all the parabolas are congruent. Watching this short (2:24 minutes) video has never failed to engage almost all my Grade 10 students, whether in Academic or Applied classes. Students who are interested in engineering can watch www.youtube.com/watch?v=y7Hyc3MRKno, which explains the concept of laminar flow, the process that allowed the construction of the parabolic fountains. The fountain video is an excellent introduction to quadratics in Grade 10, as well as an opportunity for students to engage in mathematical modelling.



Figure 4: *Parabolic fountains at Detroit airport (from www.youtube.com/watch?v=xPugzTVILGA)*

Video-Use Tips

I have some suggestions about using videos in your math classes. Generally, if the video does something that you could do in class, don't use it. Do use videos that illustrate situations that cannot easily be replicated in class, especially dynamic, changing scenarios.

- Be sensitive to your students' feelings. For this reason, I suggest avoiding videos related to COVID-19. There are plenty of other situations that can be modelled mathematically. For the same reason, before using an earthquake or other natural disaster video, ensure that you don't have students who come from that region of the world or may have relatives there.

- Keep the videos short (I suggest under 5 or 6 minutes). Longer videos can be assigned for students to watch outside of class time.
- Avoid "talking heads."
- Avoid videos in which someone writes repeatedly on a chalkboard/whiteboard/smartboard; you are already able to do this yourself.
- Avoid "death by PowerPoint" videos.
- Use multiple dynamic videos to illustrate different facets of a phenomenon.
- Be very careful of the video source. There are millions of videos online, most of which are not vetted in any meaningful way. Services like www.youtube.com/ have policies on content that is posted, so be sure to read those policies where available, and act accordingly.
- Some videos carry a political or ideological message within the content. Be sure to preview any video that you are considering, and ensure that the messages contained in it are appropriate for your classroom.

In the Appendix to this article (available online in the members-only portion of the website), I have listed videos that I have examined for classroom use, with length and brief annotations. Dynamic videos like these have the potential to engage our students in mathematics and mathematical modelling, motivate them to study mathematics, and help answer that age-old question, "When are we ever going to use this in the real world?"

Ironically, when I met Dr. Mason at a conference a few years ago, he had no recollection of me (not surprising), the course that he taught, or even the year that he spent as a visiting professor at my university. Considering the profound impact that his philosophy had on my teaching career, I was a bit flabbergasted, but not totally surprised, given the length of time that had passed. This was an example of the impact that a teacher can have on their students, sometimes long after the course that they taught has faded into memory.

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▲ Business Applications of Calculus



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One of the problems that I have encountered when teaching calculus with an applications orientation is that I found most available problems to be on rather random topics and small in number, especially if the textbook is being used as the primary resource. Another difficulty is that many problems require a relatively large amount of prior knowledge in the area, such as calculus applications to physics. Yet applications can be a powerful way to motivate students in calculus, especially if the problems reflect an area of interest to students.

Of course, not all students are motivated by the same applications. Students who are interested in kinesiology may have little or no interest in economics, or in physics. What I did with my calculus classes was to offer a broad range of problems and allow students to choose problems in an area in which they were interested or might even be considering for post-secondary study: kinesiology, business, physics, economics, science, technology, design. This required accumulating a large array of calculus problems and applications, which I sorted into topic areas. By providing enough areas of choice, I could usually provide applications in at least one area of interest for the majority of my students. Students were free to choose any area or areas in which they already had an interest, or which they wanted to explore. Sometimes they chose a subject that they were currently studying in another class, and students were often pleasantly surprised that what we studied in calculus class actually could help them in another subject.

Since no one is an expert in all subject areas beyond mathematics, I enlisted the help of teachers from other departments to generate lists of topics that might be modelled mathematically, and especially that might provide applications of calculus to those other subject areas.

A caution with considering this instructional strategy is that assessments and evaluations also required choice, in order to accommodate all the areas of application focus. The upside of this consequence is that there is a lot of research that has shown that student choice is a powerful motivator for students (e.g., Deci & Ryan, 2008). Appendix A shows a sample evaluation with a focus on business applications. The names in the evaluation were all students in my class, who had chosen to focus on business applications of calculus. This allowed my students to see themselves in the role of business analysts. Students who chose other foci would receive assessments that targeted their area of interest. This approach can be as simple as providing a large set of questions of roughly equivalent difficulty and allowing the students to choose ten of them to do, or the teacher may choose to have students go further into a topic with independent research.

The examples in this article are only starting points. They were selected to demonstrate the range of calculus applications to business. Teachers may choose to extend or modify these questions to better suit the needs of their students, as well as to address specific expectations in the calculus course. I suggest focusing on the business implications of results, such as increased sales, appropriateness of mathematical models, or real-world limitations of assumptions.

I used a lot of assignments, investigations, and open-ended questions without time constraints, which allowed students to go further into their area of interest and learn about additional calculus applications in that field. One particularly engaging strategy was to have students construct case studies: present a real-life situation, propose a mathematical model for the situation (involving calculus), justify the choice of model, and discuss the implications of the particular model. Groups made whole-class presentations of their case studies, role-playing executives making their case for decision makers, consisting of the rest of the class. Teacher, peer, and self-evaluations were used to evaluate the effectiveness of the presentations.

It takes quite a while to accumulate sufficient problems and applications to employ this strategy. This current article provides a range of calculus applications in the area of business, all of which have been classroom tested. No prior knowledge of business concepts is needed, either for the teacher or the students, although the assistance of business teachers was valuable in finding situations for which mathematical models were appropriate. The questions can be used in any order, and may provide jumping-off points for students to go deeper into an area of interest.

Maximizing Revenue, Maximizing Profit, Minimizing Costs

This is the most common use of calculus in business, and often questions are found in calculus texts on this application. However, it can also be approached from a business perspective, using the concepts of marginal cost, marginal revenue, and marginal profit (Figure 1). Marginal cost is the cost of producing one additional unit of product; marginal revenue is the revenue from that one additional unit; and marginal profit is the profit from that one additional unit (marginal revenue minus marginal cost). The mathematical models can be simple polynomials (Figure 2) or more complex functions involving exponential and logarithmic functions. Students were always asked to consider the implications of the model presented.

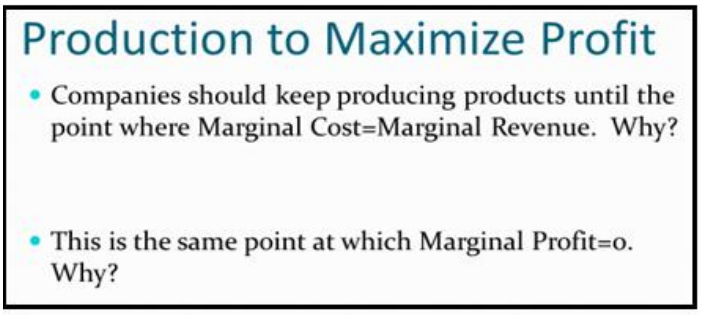


Figure 1: *Marginal cost and marginal revenue approach to maximization*

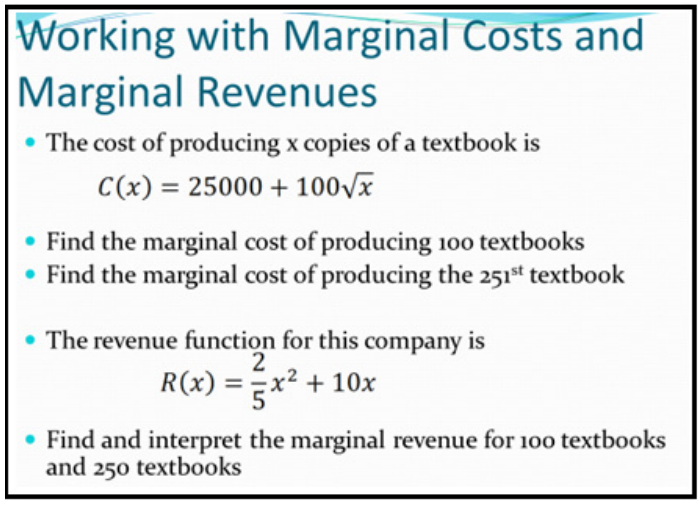


Figure 2: *Marginal cost and marginal revenue computations. Students would then be asked to maximize profit.*

Product pricing

- A theatre charges $12 per ticket, and typically gets 300 customers. The theatre ownership is considering raising ticket prices. However, a market researcher has found that for every 50 cent increase in ticket prices, the theatre will lose 25 customers.
- What price should the theatre charge to maximize their revenue?
- What is the maximum revenue?

Figure 3: *The classic theatre problem*

Some additional problems involving the marginal profit approach to maximization are given in Appendix B.

Figure 3 is the classic "theatre problem," which can be solved either with calculus or by completing the square. This type of problem can be expanded to consider the pricing of transit tickets, sporting events, or music concerts. Students were encouraged to make reasonable assumptions and explicitly list the assumptions that applied to their models.

As students' knowledge of derivatives grew, the same problem could be presented using different models, and students were encouraged to consider which model was more appropriate (Figures 4a and 4b).

Cost function

- A company's cost function for a particular product is

$$C(x) = e^{0.1x} + 4000$$

- Find the marginal cost of the 101th product
- What does this number tell you?

Sales decay

- Sales of a certain product x months after an advertising campaign ends are given by $S = 50000e^{-0.8x}$
- What is the marginal rate of change of sales 3 months after the ad campaign ended?
- What does the answer tell you?

Figures 4a and 4b: *Same problem with different models*

Sales and Marketing

Calculus can be applied to other concepts in business, such as advertising, sales forecasting, and market share (Figure 5). This is also an opportunity to introduce or revisit some basic business concepts that fit well with business classes that students may be taking or considering taking. Occasionally some basic business terminology can be blended into the application, for example, the Noted, Seen–Associated, and Read Most terminology described in Figure 6. These connections between math class and business classes reinforce that mathematics and calculus can be viewed as tools for solving real-world problems.

Cost function

- A company's cost function for a particular product is

$$C(x) = 400\ln(x + 10) + 200$$

- What is the marginal cost of producing the 101th item?
- What information does this tell you?

Figure 5: *Modelling the impact of advertising*

Advertising awareness

- For print ads, market analysts use three measures (as %) to determine the penetration of the ad.
- ***Noted***: the percentage of readers who remember having seen the ad.
- ***Seen-Associated***: the percentage who saw any part of the ad that clearly indicated the brand, service, or product.
- ***Read Most***: the percentage who read half or more of the copy.

Figure 6: *Terminology related to advertising*

Exponential models with a maximum limit

- A company launches an advertising campaign in a town of 30,000 people. The number of people y who have heard of the product x days after the campaign begins is given by

$$y = \frac{30000}{1 + e^{-.2(x-15)}}$$

- How many people have heard of the product 1 week after the campaign began?
- How many people have heard of the product 1 month after the campaign began?
- What is the maximum number of people who could hear about the product?

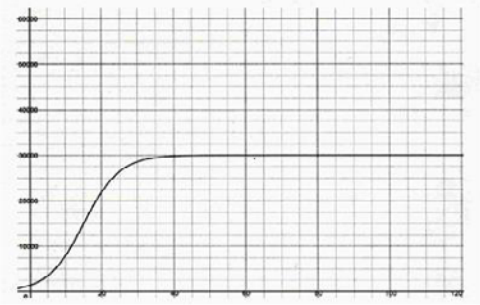


Figure 7: *Modelling a situation with an upper limit*

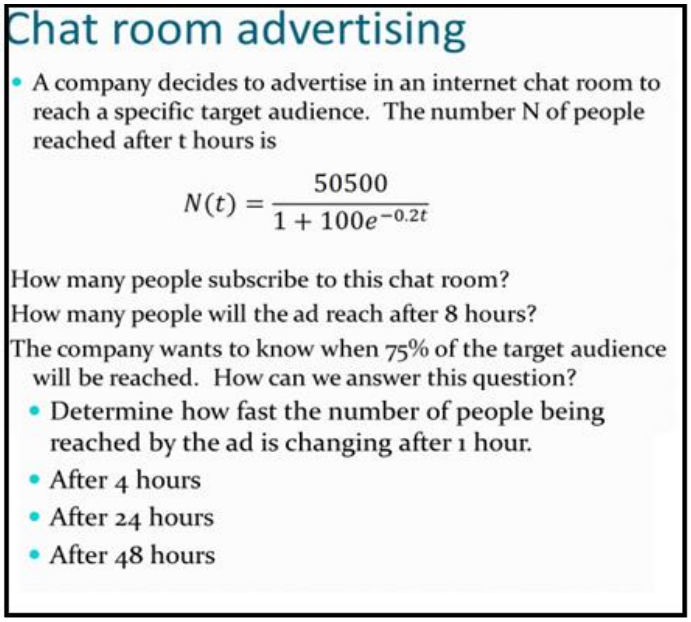


Figure 8: *An advertising problem with an upper limit*

As noted earlier, using an array of different functions encourages students to consider which models most fit the real-world situations that they encounter in business. For example, the next problem looks at a growth model with an upper limit. This is the logistic function, which is not an expectation of either the Advanced Functions or the Calculus courses, but in my teaching, I haven't seen the need to name the function; rather, just present the model and apply calculus to the function (Figures 7 and 8).

Mathematical models can be as simple or as complex as desired, as illustrated by the video game example in Figure 9. Appendix A provides examples of calculus applied to other business concepts, such as depreciation, organizational growth, and product exchange functions.

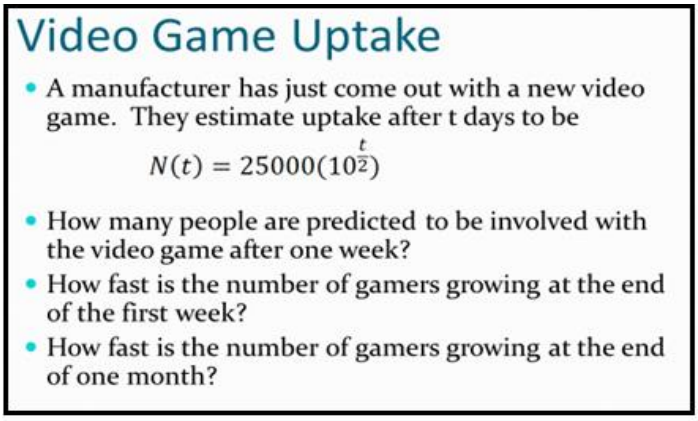


Figure 9: *Mathematical modelling with more complex functions*

By the teacher initially providing models of business applications, students began to see that calculus helps solve real problems outside the math classroom. Students could go beyond the mathematics to consider implications of the model presented, relevant range for a model, justification for choosing a particular function as a model of the business phenomenon, potential other models that might be a better fit for the phenomenon.

It is important that we demonstrate to students that what we teach can be applied to areas beyond the walls of our classrooms. Applications to business is only one of the areas that students can explore. By providing students with some level of choice in their mathematics classes, we encourage them to take ownership of their own learning and to see mathematics as more than just an academic exercise.

Reference

Deci, E., & Ryan, R. (2008). Self-determination theory: A macrotheory of human motivation, development, and health. *Canadian Psychology, 49*(3), 182–185. www.dx.doi:10.1037/a0012801

Appendix A: A Sample Assessment for Calculus Applications to Business

Give complete solutions to any ten of the following questions. Be sure to clearly indicate the question number for each question you do. If you do more than ten questions, the first ten solutions will be marked, unless you indicate otherwise. Enjoy yourself! This assignment is due at the beginning of class tomorrow.

1 Davina just bought a new motorcycle for $10 000. The value of the motorcycle depreciates over time according to the model $V(t) = 10000e^{-\frac{t}{4}}$, where t is the time in years.

(a) At what rate is the value of the motorcycle depreciating the instant Davina drives it off the dealer's lot?

(b) Davina decides that she will stop insurance coverage for collision once the motorcycle has depreciated to $\frac{1}{4}$ of its initial value. When will she drop the insurance coverage?

(c) At what rate is the motorcycle depreciating when Davina drops the insurance coverage?

2. As manager of an 80-unit apartment complex, Stefan is trying to decide what to charge his tenants for rent. He knows from past experience that at a rent of $1000, all the units will be full. However, one additional unit will remain vacant for every $50 increase in rent. What rent should Stefan charge to maximize revenue from the apartment complex? What is that maximum revenue?

3. As company accountant, Youseff determined that the cost to produce x thousand items can be modelled by $C(x) = 1000 + x^2\frac{16000}{x}$.

(a) How many items should the company produce to minimize costs?

(b) What is the marginal cost of producing the 101st item?

(c) How fast is the marginal cost changing at that point?

(d) How can your answer to (c) be used to make a business decision?

4. Marec's Internet company sales this year are $50 000. Five years from now, sales are forecast to be $180 000.

(a) Find an exponential model for Marec's sales in the form $S(t) = S_0 e^{kt}$.

(b) Use your model to forecast sales for Marec's company ten years from today.

(c) Is an exponential model realistic for Marec? Explain.

5. Geraldine's business began making a profit the first year it opened. Her profit function (in $ thousands) when selling x thousand units is $P(x) = \frac{4x}{3x^2 + 243}$.

(a) What is Geraldine's marginal profit when she sells 500 units?

(b) What level of sales will result in maximum profit?

(c) What is her maximum profit?

(d) How fast is Geraldine's marginal profit changing when she sells 10 000 units?

(e) What does your answer to (d) tell us?

6. The percentage of television viewers who remember what product is regularly advertised on the television show *Survivor* is $N(x) = 70 - 30e^{-.2x}$, where x is the number of times the viewer has seen the show.

(a) What percentage of viewers remember the product if they have seen the show five times?

(b) How fast is the percentage of viewers who remember the product changing for viewers who have seen the show six times?

(c) The advertiser wants to know how many times viewers must see the show to result in 50 percent of the viewers remembering the product. Answer their question for them.

7. Nadine's company produces email advertising flyers and sends them to targeted market segments. She estimates their advertising "reach," i.e., the number of people in the target market segment who will read the email before trashing it, to be $N(x) = \frac{75000}{1 + e^{-0.2x}}$, where x is the number of repeats of the same email message.

(a) What is the number of people in the target market segment?

(b) How many people will be reached if the email is repeated eight times?

(c) What is the marginal increase in targeted people reached after the email is repeated five times?

(d) Do you expect this marginal rate to increase or decrease as the number of email repeats grows very large? Why?

8. Tumic estimates her business's profit function (in $ thousands) to be $P(x) = x^3 - 3x^2 - 4x + 12$, where x is thousands of units. Her factory's capacity is 10 000 units.

(a) What levels of production are breakeven points for Tumic?

(b) For what intervals of production is Tumic making a profit?

(c) What level of production gives Tumic her maximum profit?

(d) What is her maximum profit?

9. Steve's honey company costs are given by $C(x) = -2000x + 18000x^{\frac{2}{3}}$, where x is the number of kilograms of honey produced, in thousands.

(a) Find the production level that gives Steve minimum cost.

(b) What is Steve's minimum cost?

(c) What is Steve's marginal cost of production at a production level of 3000 kg of honey?

(d) What level of production will give a marginal cost of $4?

10. Martine has a fast-growing company that sells advertising space on mobile media, like balloons, T-shirts, and portable banners. She estimates her profit function to be $P(x) = x^2 e^{0.1x} + 10x^{\frac{3}{4}}$, where x is thousands of units sold.

(a) What is her profit when she sells 10 000 units?

(b) What is her marginal profit when she sells 10 000 units?

(c) How fast is her marginal profit growing when she sells 10 000 units?

(d) Comment on whether this is a realistic profit model for Geraldine's business.

11. Sala has decided to buy a fish farm. He will raise fish and sell them to supermarkets and restaurants when they are large enough. The length (cm) of a certain species of fish, x months after it hatches, is given by $L(x) = 3x + 12\ln(5x)$.

(a) How long is a fish after two months?

(b) How fast is the fish growing after three months?

(c) Sala plans to harvest and sell the fish when they reach a length of at least 50 cm. When can Sala harvest the fish?

(d) Suppose Sala decides to wait to harvest his fish until the fish's growth rate is 1 cm/month. Explain why this is a poor decision.

12. Rachel's new fashion company has a profit function (in $ thousands) given by $P(x) = \frac{2000}{x} + \frac{25x^2}{\sqrt{(5x+100)}}$, where x is sales in thousands of units.

(a) What is Rachel's marginal profit when her company sells 10 000 units?

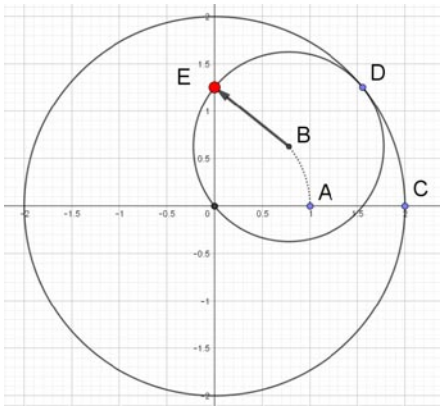
(b) At what rate is Rachel's marginal profit changing when her company sells 10 000 units?

(c) What is Rachel's average profit when she sells 10 000 units?

(d) Comment on this profit model. Does this profit function predict that Rachel's company will be a success or a failure? Why?

Tusi Couple

The Tusi couple was invented by Muhammad ibn Muhammad ibn al-Hasan al-Tūsī, who is also known as Nasir al-Din al-Tusi. He lived between 1201 and 1274. The couple uses two circles to produce linear motion. The Tusi couple can be considered as a math problem. The diameter of the inner circle is the radius of the outer circle. Prove that, as the inner circle rolls along the inside of the outer circle, point E remains on the y-axis. That is, as the inner circle sweeps out, CD, the centre of the inner circle, sweeps along arc AB, and E moves along the y-axis from the origin to the position shown.



Appendix B: Additional Calculus Applications to Business

More on Revenue Functions

- A forecasting model of the revenue function of a company over time ($000) is
$$R(t) = 150(t^2 + 30)(t + 8)$$
- where t is the time in years from now.
- What is the company's current revenue?
- What will be the company's marginal revenue in 3 years?
- What will be the company's marginal revenue in 5 years?
- Should you invest in this company? Why?

Market share for a new product

- When a new product is introduced, it will take m months for the market share to rise to s%
$$m = 20ln\left(\frac{40}{40 - s}\right)$$
- How many months will it take for the market share to reach 35%?
- What is the marginal change per month when the market share is 15%?
- What is the marginal change per month when the market share is 30%?

Profitability

- A company's profit function is
$$P(x) = -x^2 + 50x + 5000$$
- For what range of production is the company profitable?
- Another company's profit function is
$$P(x) = x^3 - 9x^2 + 2x + 48$$
- For what range of production is this company profitable?

Market share

- A certain company's market share y(%) t months after the introduction of a new product is given by
$$y = 40 - 40e^{-0.05t}$$
- Find the company's market share after 3 months.
- What is the maximum market share predicted by this model?

Depreciation

- The value of a machine t years after it was purchased is given by $V(t) = \frac{50000 + 6t}{1 + 0.4t}$
- What was the purchase price of the machine?
- What is the rate of depreciation in the value of the machine 3 years after purchase?
- What is the rate of depreciation in the value of the machine 10 years after purchase?

Sales decay

- After an advertising campaign ends, sales of a particular product decay according to
$$S(t) = 25000e^{-0.1t}$$
- where t is weeks after the end of the campaign.
- The company plans to launch a new campaign when sales drop to 50% of the original level.
- When should the company start the new campaign?

Product Exchange Functions

- Product exchange functions give the relationship between quantities of two items that can be produced by the same machine or factory. For example, a refinery can produce gasoline or heating oil or a combination of both.

Oil Refinery product exchange function

- The product exchange function for gasoline (y) and heating oil (x) , in thousands of litres per day is given by
$$y = \frac{125000 - 25x}{125 + 2x}$$
- Estimate the maximum quantities of each product that can be produced per day.

Business models using e

- A company forecasts sales growth using the model
$$S_x = S_0 e^{0.03x}$$
- Where S_x is the sales forecast in year x (in $000)
- S_0 is the sales in year 0 (current)
- If current sales are $400,000 predict the sales 1 year from now.
- Predict the sales 5 years from now.
- How fast will sales be increasing 5 years from now?
- How fast will sales be increasing 10 years from now?

Organizational growth

- The number of employees of an organization after t years is given by $N(t) = 2000(0.6)^{0.5t}$
- How many employees did the company have when they started operations?
- How many employees will they have after 1 year?
- How long will it take before the company has 1800 employees?
- How might you evaluate the marginal change in employees at the end of the 2nd year?

▲

Teaching Math during the Pandemic: Lessons Learned and Best Practices

You are invited to participate in a survey to identify the impact of teaching math during COVID, and what we have learned.

The survey is completely anonymous. To access the survey, click on the link below, or copy and paste the link into a browser. www.surveymonkey.com/r/32L3LKJ

Please consider completing this survey, and please share the survey link with math teachers you know. The more teachers who complete the survey, the more we can identify what worked (and what didn't work).

Thank you for considering this request.

Jeff Irvine, Ph.D., OCT

Disclaimer: This research has no connections to OAME, any university, or government body.

Any questions about this survey can be sent to dr.jeffrey.irvine@bell.net.

▲ Contexts for Learning Integers: Integrating Hockey into the Intermediate Mathematics Curriculum



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Engaging students in authentic contexts for learning mathematics is critically important for engagement and learning. *The Ontario Curriculum, Grades 1–8: Mathematics, 2020* specifically encourages teachers to provide relevant, authentic, real-life contexts that are inspired by the "lived realities of students" (Ontario Ministry of Education [OME], 2020a, p. 39). This can take many different forms and can be interpreted in a multitude of ways. In mathematics, this is typically interpreted as using real-life situations to frame a question within a test or assignment. Sometimes it means the use of real-life context as an example during a lesson. Yet some topics remain harder than others to situate in contexts relevant to students.

In Grade 7 mathematics (OME, 2020b, p. 397), expectation B2.4 states that students are to "use objects, diagrams, and equations to represent, describe, and solve situations involving addition and subtraction of integers." These skills become vital as students continue in their mathematics education. Integers, however, are an abstract concept for many students, as they arise in mathematics more often than in the "real" world. How then does one integrate authentic real-life situations that will resonate with students learning integers?

When teaching integers, one option is to use weather and temperature to achieve these mandates of relevance and authenticity. Some teachers also use financial math connections to teach integers, which has been shown to be effective. In this article, we will propose another potential option for teaching integers, using real-life contexts.

Throughout Ontario, sports are often a part of students' lives, and even if sports are not part of a student's life, students are typically able to relate sports to their lives and prior understanding of the world. Thus, the integration of sports into the curriculum may make it more accessible and engaging to students. This article will focus on the integration of hockey into the curriculum, but if the majority of your students do not relate to hockey, other sports will be suggested.

The purpose of this article is to share our work using the context of an iconic Canadian pastime to teach integers. The lessons to be described have been field-tested in regular classrooms as well as in one-on-one tutoring situations. The authors welcome additional comments and suggestions from other teachers who use these activities.

Curriculum Context

Problem solving within contexts continues to be a mainstay of learning mathematics, which supports other mathematical processes as well. The *Curriculum Context* document states that:

> It is an effective practice to use a problem, carefully chosen by the teacher or students, to introduce, clarify, or apply a concept or skill. This practice provides opportunities for students to demonstrate their agency by representing, connecting, and justifying their thinking. Students communicate and reason with one another and generate ideas that the teacher connects in order to highlight important concepts, refine existing understanding, eliminate unfruitful strategies, and advance learning (OME, 2020a, p. 40).

Engaging contexts also support meaningful collaboration, which is further described in the *Curriculum Context*, as follows: "In mathematics, students and educators engage with others productively, respectfully, and critically in order to better understand ideas and problems, generate solutions, and refine their thinking" (OME, 2020a, p. 53). When students are engaged in, and invested in, the

outcomes of their work, communication has more meaning. The curriculum further describes this process:

> In mathematics, students and educators use the tools and language of mathematics to describe their thinking and to understand the world. They use mathematical vocabulary, symbols, conventions, and representations to make meaning, express a point of view, and make convincing and compelling arguments in a variety of ways, including multimodally, for example, using combinations of oral, visual, textual, and gestural communication (OME, 2020a, p. 53).

The exploration and introduction of specific contexts that may be engaging to students may or may not be familiar to teachers. In this article, we share the required background information as well as a sample lesson plan, developed with the above goals in mind.

Background

A little hockey background is required for the uninitiated. There are two statistics in hockey that require the adding and subtracting of integers: player plus/minus scores and team goal differentials. Before getting into how these statistics can be used in a Grade 7 mathematics classroom, it is important to understand what the statistics are and how they are tracked.

The following is a brief summary of the context. A hockey player's plus/minus statistic in a game is calculated as follows: the player receives a score of plus one if they are on the ice when the player's team scores an even-strength or shorthanded goal; the player receives a score of negative one if they are on the ice when the opposing team scores an even-strength or shorthanded goal. Any time that a goal is scored when there are more skaters from the scoring team on the ice, it does not count toward the statistic. This means that if the other team has a penalty when the goal is scored, no values are recorded for the statistic. For example, if a player is on the ice when the opposing team scores two even-strength goals (–2) and a power-play goal (+0), as well as their team scoring two even-strength goals (+2) and one shorthanded goal (+1), the player would have a plus/minus of (–2+0+2+1) positive one. The ideal situation is for a player's plus/minus statistic to be as high as possible. A player's plus/minus is also tracked over a season to reflect a player's defensive and offensive abilities. This statistic does come with some heavy criticism, but is still reported this way.

Similarly, goal differential is a team statistic that tracks the difference between the number of goals a team scores (called "goals for") and the number of goals the opposing teams score against the team (called "goals allowed" or "goals against"). The ideal goal differential is a high positive number. This statistic can also be used to determine whether a team advances to the playoffs. This statistic can be important to hockey teams, as it compares the team's offensive production with its defensive abilities.

Connecting with Your Local Context

The statistics described above are the main values needed to integrate a hockey context into the mathematics curriculum to teach integer addition and subtraction. How a teacher finds and uses these statistics will depend on the availability of resources in the area. Of course, hypothetical data can be constructed. However, if the teacher can get access to the local junior team's highlights and statistics, or those of the closest university hockey team, which are generally publicly available on the Web, it is recommended to use these data, as this creates a locality and connection to a student's immediate area that can allow students to better relate to the material. One note of caution: we do not suggest using the team from your students' age group, to avoid unnecessary commentary on your students' abilities in your class. Remember, when looking for junior teams to be used, there are women's junior teams who are available as well. Make sure to include both whenever possible, as your female students should be permitted to see themselves represented in the task just as much as the boys.

#	PLAYER	GP	G	A	TP	PIM	+/-
	OHL						
1.	Zackary Trott (F)	62	25	41	**66**	36	10
2.	Rory Kerins (C/LW)	64	30	29	**59**	30	2
3.	Tye Kartye (C)	64	25	28	**53**	28	14
4.	Cole Mackay (RW)	64	25	26	**51**	42	5
5.	Jaromir Pytlik (C)	56	22	28	**50**	36	5
6.	Robert Calisti (D)	64	18	32	**50**	64	-14
7.	Billy Constantinou (D)	52	8	39	**47**	39	-8
8.	Jaden Peca (RW)	63	15	28	**43**	69	-10
9.	Joe Carroll (LW)	52	25	15	**40**	33	3
10.	Tanner Dickinson (C)	64	9	31	**40**	22	-12
11.	Ryan O'Rourke (D)	54	7	30	**37**	79	16
12.	Jacob LeGuerrier (D)	61	6	25	**31**	74	4
13.	Alex Johnston (C)	64	12	14	**26**	22	1
14.	Holden Wale (D)	36	3	10	**13**	13	1
15.	Marc Boudreau (LW)	59	4	8	**12**	51	-2
16.	Cullen McLean (C/LW)	57	5	5	**10**	18	-13
17.	Jacob Holmes (D)	57	3	6	**9**	12	9

Figure 1: *Sample statistics page taken from Elite Prospects*

It is also important to note that if hockey is not what is most relevant to your students, you can use other sports to connect with your students. Several suggestions are mentioned in the conclusion, which would be worthwhile to explore with your students as well.

Lesson Topics

A couple of ways you can use your local team's player statistics include having your students order players based on plus/minus scores, finding the difference between the highest and lowest scores (integer subtraction), and using highlight reels to track a player's plus/minus score in a game and find the total plus/minus score throughout all games for a player, using league data (integer addition). In each of these situations, students can be provided with different tools for working with integers (e.g., number line, two-colour counters). These activities also allow you to teach and assess comparing, ordering, adding, and subtracting integers. Lessons can be framed as an activity to assist the coach or as a learning task to prepare students for analyzing an NHL/WNHL or other professional teams, such as the Olympic Hockey Teams.

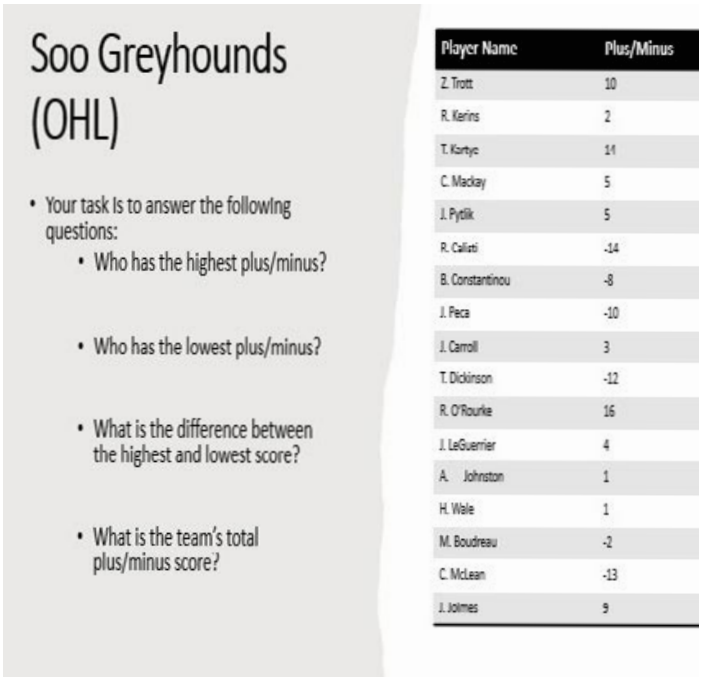


Figure 2: *Sample slide from lesson using Soo Greyhounds plus/minus statistics*

In using professional teams of both genders, such as NHL, WNHL, national, or Olympic teams, an opportunity for students to look at their favourite teams and players, or interact with major world events, using a mathematics lens, is created within the classroom. This furthers the objectives of authenticity and relevance, and allows for work with larger numbers within the same framework as above. In the NHL, for instance, each team normally plays 82 games per season; thus, plus/minus statistics are likely to have higher absolute values, as are goal differential statistics. Students can work with just recent games or full seasons. Students can be asked to place teams on a scale of best to worst goal differential, and compare this scale with where teams are in the standings. This can lead to discussions of why the two scales are the same or different, and why a higher or lower goal differential is better. These contexts help students to look at the meaning behind the numbers, instead of just seeing abstractions and meaningless values, thus allowing them to better engage and more effectively learn the content.

Assessment

As mentioned above, you can use a combination of these ideas to create a pair of tasks, often called a *learning task* and an *assessment task*. The idea of this task pair is to provide students with an opportunity to explore the ideas during the learning task, and receive substantive feedback, which they can then apply to the assessment task (see Atiya, Luca, & Kajander, 2018, for further details). This supports students to make the maximum use of positive constructive feedback and apply it for evaluation, while engaging with the material and continuing to learn, during their assessments. This system of assessment also allows the teacher to assess more than the Knowledge/Understanding category of the Achievement Chart by also assessing students' use of the Process Expectations (OME, 2020b). Tasks of this nature also permit students to see why their learning matters, given the real-life applications of the material. Engaging contexts allow students to be passionate and be motivated to participate in discussions around the topic, thereby concurrently supporting the strand of Social–Emotional Learning Skills (OME, 2020c). It is important that during these discussions, the teacher ensure students are supporting their claims, using appropriate mathematical concepts, and that students are expressing their opinions and thoughts respectfully. The concept of using such embedded learning is supported in the *Curriculum Context,* provided by the OME (2020a), which states that using a problem to introduce, clarify, or apply a concept or skill is an effective practice that "provides opportunities for students to express their agency by representing, connecting, and justifying their thinking" (p. 40).

Communicating Mathematically

Use of such tasks can also lend themselves to presentations and group work. For example, the class can

be split into "scouting teams," where each group takes a team and breaks down the team based on the goal differential, the players with the best plus/minus, and the players with the worst plus/minus. Students can also compare the goal differential of their team with their divisional rivals or the rest of their conference, and make assessments of their performance based on these calculations. When doing activities like this, it is very important that you avoid gender comparison. We need to ensure that our students are not making generalizations about the skills of different genders within sport based on our activities. Students can also compare plus/minus scores with point production, or player positions as a further point of discussion. One possible way of using the data is to have students explain the information, how they obtained it, and present the information to the class. This allows students to practise public speaking, sharing different methods of completing the task, and engaging in effective co-operative efforts, thus further supporting the new expectations of the Social–Emotional Learning Skills strand of the curriculum. This type of activity can also be used for cross-curricular learning with Language Arts, as it ties into the Oral, Writing, and Media strands of the curriculum.

Classroom Observations

When used (by the first author) in a Grade 7 classroom, where many of the students were disengaged with mathematics, this lesson topic provided a context that led to participation by almost every student. The teacher used this method within the context of hockey rivalries in Thunder Bay. Students vary in their fan support, but primarily favour either the Winnipeg Jets, Toronto Maple Leafs, or Minnesota Wild. The students compared data from all three teams, and players from each team. This allowed students to not only build connections within mathematics, but also with each other. The results of this activity appeared to include increased engagement, participation, and understanding. For instance, one student who had been particularly dispassionate about math and loved to tell the teacher that math was stupid and had no part to play in real life, became extremely involved in the lesson on integers. This student went so far as to start a debate in class on the merits of each of the statistics, and wanted to do more work with the data he was given. Another student, who was typically much quieter during class, was having mathematically based discussions with her peers, where she voiced her opinions—a rarity for this student. She was willing to speak in front of the class, as well as in small groups, regardless of whether

Curriculum Expectations

The Ontario Curriculum, Grades 1–8: Mathematics, 2020

Strand	Overall	Specific
A. Social–Emotional Learning (SEL) Skills in Mathematics and the Mathematical Processes • apply, to the best of their ability, a variety of social–emotional learning skills to support their use of the mathematical processes and their learning in connection with the expectations in the other five strands of the mathematics curriculum	**A1.** apply, to the best of their ability, a variety of social–emotional learning skills to support their use of the mathematical processes and their learning in connection with the expectations in the other five strands of the mathematics curriculum	**A1.2.** recognize sources of stress and cope with challenges so students can work through challenging math problems, understanding that their resourcefulness in using various strategies to respond to stress is helping them build personal resilience **A1.4.** build relationships and communicate effectively so that students can work collaboratively on math problems—expressing their thinking, listening to the thinking of others, and practising inclusivity—and in that way, fostering healthy relationships **A1.5.** develop self-awareness and sense of identity so that students can see themselves as capable math learners, and strengthen their sense of ownership of their learning, as part of their emerging sense of identity and belonging **A1.6.** think critically and creatively so that students can make connections between math and everyday contexts, to help them make informed judgments and decisions
B. Number **B2.** Operations • use knowledge of numbers and operations to solve mathematical problems encountered in everyday life	**B2.** use knowledge of numbers and operations to solve mathematical problems encountered in everyday life	**B2.4** use objects, diagrams, and equations to represent, describe, and solve situations involving addition and subtraction of integers

Sample Lesson Plan (Grade 7 Integer Addition/Subtraction)

Resources Needed: • Chart paper, markers (if desired) • Two-colour counters • Number lines • Statistics and highlights from a hockey team • Access to the Internet • Projector	**Learning Goals:** I can… • use my knowledge of integers to order players based on their statistics • use integer subtraction to determine the difference between two players' scores • use integer addition to determine player and team statistics over a given period of time
Minds On 15 minutes	**Discussion (whole class/small groups):** Scenario: The coach of the Thunder Bay Kings has asked us to analyze some of his team's stats: 1. He wants us to order his players based on plus/minus. 2. What is the difference between the best and worst plus/minus on the team? ⟶ You may wish to use a number line for this. 3. In the most recent game: • Track and find players' plus/minus. ⟶ You may wish to use two-colour counters for this. 4. Given plus/minus scores for a couple of games, compute the player's total plus/minus for that stretch of games. • Rank players for that stretch of games. 5. Given the team record for the season, determine their goal differential.
Action 15 minutes You can have students complete this on chart paper as a group or individually.	**Task (independent/small group):** Pick a WHNL, NHL, Olympic, or professional ice hockey team. Remember, there are national women's teams too! (Try to avoid repeats.) • Using scores from the last ten games, determine their team's goal differential. • Select two players and determine their plus/minus from the past ten games. Who has a better plus/minus? What is the difference in their plus/minus scores? • Looking at the statistics, which player has the highest plus/minus? The worst? What is the difference?
Consolidation 10 minutes	**Discussion (whole class):** • Have students discuss in groups what they found out, and how they answered the questions. • If you have students who are willing, have them demonstrate their methods for adding and subtracting integers within the context of hockey. Try to ensure you get several methods, such as two-colour counters and number lines. • If time permits, discuss with students which player they think might have a better plus/minus: a player who plays forward or defence?

they agreed with her perspective. Her confidence in her mathematics skills appeared to be rising, based on her familiarity with the content. As a whole, the class was engaged and excited to participate in activities and assessments following this activity. A couple of students came in the next day with statistics from the previous night's hockey games, and were comparing their data at recess. This activity helped students to become enthusiastic about integers in a way that was beyond what the teacher had expected.

Summary

Hockey can be used to involve students with integers in a way that is potentially more engaging than the weather, and such contexts can also promote teamwork, co-operation, and critical thinking off the ice. Hockey contexts can also be used in several other areas of the Grade 7 curriculum, such as fractions, decimals, and percent (goalie save percentage), averages and data management (goals-against average, points per game, shots per game, etc.); however, the topic of integers is a great place to start for those less familiar with hockey.

By integrating hockey into the mathematics classroom, we have a chance to encourage students to connect with the world around them in a mathematically sound way. If your students do not relate to hockey, you can use golf (shots against par), football (passing, rushing, and lost yards), or any other sport you can find that uses integers. All of the listed sports have a strong use of integers in their real-life applications. For example, if one wanted to use golf, one could use the latest Professional Golfers' Association (PGA) Tour event and look at the shots above par (positive integers) and below par (negative integers). This would provide similar opportunities to the ones used for hockey in this article. Similarly, in football, students can track yards gained and lost by quarterbacks, team defence and offence, and more. In using sports, students are given the chance to become passionate about their work in the mathematics classroom. We have all seen the difference that engagement can make in the classroom; why not use something that most Ontario students are familiar with and passionate about, to capitalize on learning mathematics?

The following is a sample lesson plan, together with the curriculum Expectations addressed, which has been used effectively by Christine (first author) with Grade 7 students in the Thunder Bay region. The authors welcome comments or the sharing of additional lessons focused on the teams in their own region.

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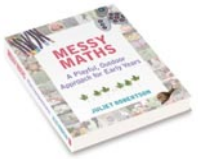
▲ Book Review: Math in the Great Outdoors



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Ed Schroeter, B.J., B.Ed., OCT, is a retired elementary teacher, having taught for 30 years, 18 of them in Kindergarten. He was the Grade 1 Lead Writer for the Ontario Mathematics Curriculum Resource Project (2020–2021)*. He led three* Teacher Learning and Leadership Projects (TLLP) *lesson studies. From 2009 to 2019, he taught two ETFO Summer Academy courses and presented more than a dozen workshops on inquiry learning around Ontario as part of the* Presenters on the Road Program*. He is the author of Chapter 9 of* Teaching for Deep Understanding: An ETFO Curriculum Learning Resource Compilation.

Robertson, J. (2017). *Messy maths: A playful, outdoor approach for early years.* Crown House Publishing.



A variety of professional resources have been published in the last decade to support educators with outdoor education programming, but only a handful have directly addressed mathematics. Few have received as much attention as *Messy Maths: A Playful, Outdoor Approach for Early Years*, by United Kingdom educator Juliet Robertson. Intended for early years math educators, who work with children aged 3 to 6, the book has won at least five awards since its publication in 2017.

To set this in context, readers should be aware that the Ontario education system has a 50-year or longer history of outdoor and environmental education. School boards have been allowed to purchase land and operate environmental field centres to address social concerns about human impact on the natural environment since 1965 (Passmore, 1972). The Ontario Ministry of Education (OME) renewed its commitment to environmental education in 2009, mandating educators to work with local communities to offer outdoor education experiences that foster an understanding of humans' place in ecosystems.

In *Messy Maths*, Robertson demonstrates that the world is a mathematical landscape that, for the careful observer, is full of opportunities to explore concepts of quantity, pattern, geometry, and attributes such as textures and colour. All of these are foundational to early math learning and can be used for learning opportunities.

The book is a resource of 271 outdoor activities, which shows early math educators how to harness engaging, hands-on, outdoor mathematical experiences that will deepen their understanding of math. Robertson, who specializes in outdoor learning, believes that being outside appeals to young children. Math outdoors makes math real for students by putting it into familiar contexts, making connections between their hands and head, and laying the foundation for more complex work. Her definition of outdoor education is quite broad. It includes any place outside the classroom, including playgrounds, parks, wild spaces, and in the community, whether on public transportation, in libraries, or in stores.

Two of Robertson's stated goals are to enable children to have lots of informal, child-initiated and child-led, spontaneous mathematical experiences, and to provide a range of open-ended ideas that can be adapted to children's interests as part of a playful approach to learning math concepts. Most of the activities rely heavily on educators and support staff involvement to "make the most of teachable moments" to develop vocabulary, specific skills, and build upon mathematics that children are already doing during play. Robertson's objective is to integrate math with play, as opposed to using such activities to augment math programs.

A former head teacher at three schools, Robertson is well qualified and up to the task of meeting her goals. However, Ontario early years educators should bear in mind that the book is not a "how to" teachers' guide, but instead, a handy reference work filled with ready-to-use games, activities, field trips, and open-ended ideas. Robertson highlights this point herself on page 2 of her book.

The author also generously maintains a 500-entry education blog, *I'm a teacher, get me OUTSIDE here!*, at www.creativestarlearning.co.uk, which illustrates many of the ideas in her book in greater depth.

Messy Maths is organized into 13 chapters, 9 of which are devoted to math topics: exploring number; number functions and fractions; measurement; money; time; pattern; shape and symmetry; position, direction, and movement; and data handling. There is also a chapter about how to embed math into everyday routines, and another one about how to develop and use a "mathematical garden" for developing math skills and concepts. Many of the activities are illustrated by full-colour photographs that facilitate imagining and planning outdoor learning opportunities.

Each chapter is also prefaced by a brief, but valuable introduction that outlines the relevant key concepts and topic-specific vocabulary, including ambiguous expressions related to each area of study. For example, in Chapter 5, *Money*, Robertson makes the point that learning how to handle coins and bills in our card-based society is the starting point of financial literacy for young children. She provides a list of 19 important words and expressions such as *less than*, *more than*, *how much*, *bargain*, *price*, *change*, and *cheap*, as well as colloquialisms such as "money doesn't grow on trees," that children need to learn to prevent confusion in their young minds.

Robertson provides just enough detail about key concepts at the beginning of each chapter to enable Ontario educators to relate the content to their own math curriculum expectations, and she urges all teachers to balance children's cognitive development with their social, emotional, and physical needs. For example, in Chapter 6, "Measurement," tipping water-filled containers and buckets upside down to see which makes the biggest "splat," can be used to develop the concept of capacity. In an Ontario Kindergarten classroom, this would clearly fall under Overall Expectation 16, Measurement: "compare objects, materials, and spaces in terms of their length, mass, capacity, area, and temperature... through inquiry and play-based learning" (OME, 2016, p. 226).

Splatting containers of water aside, the title of the book might conjure images of other messy math content, from mud pies to mud puddles. However, it also refers to the cognitive messiness of recognizing the early math concepts embedded in outdoor play. The author refers to this challenge on page 11 of Chapter 1, explaining that it may require considerable practice to be able to spot the math in outdoor play—almost a retraining of perspective.

Educators should also be aware that some of the suggested mathematical activities such as tree climbing (recommended to develop measurement and estimation skills) could make their blood run cold, raising fears about legal action should anyone be hurt. Robertson alerts readers about this potential issue in her book's preamble and offers some sage advice. However, there are only a few such risky activities in the book. The majority are very safe.

The fact that the book is written by a British educator and published in Wales is no hindrance. Readers will quickly become accustomed to the United Kingdom term "maths" instead of mathematics, and some vocabulary, particularly noticeable in the chapter on money (e.g., pence, pounds, and quid).

My only unfulfilled wish for this book, and admittedly it was not the author's intent to provide it, would be more guidance to support early years teachers to accurately unpack mathematical concepts and increase their mathematics knowledge for teaching (Ball et al., 2008, p. 400).

At last count, *Messy Maths* was named the Gold Winner in the Mathematics category of the Nursery World Equipment & Resources Awards 2019, the Silver Winner in the Outdoor Play category, and the Professional Books (and

authors) category of the Nursery World Equipment & Resources Awards 2019, a Gold Winner in the IBPA Benjamin Franklin Awards 2018 in the Education category, a finalist in the 2018 Education Resources Awards in the Educational Book Award category, and a 2017 Foreword INDIES Finalist in the Education category. These distinctions are merited and well deserved.

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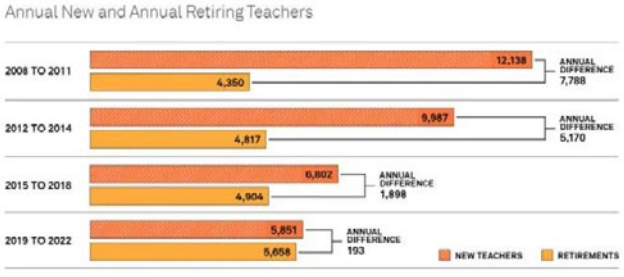
New Teacher Employment

CBC reports issues with initial teacher certification. See www.cbc.ca/news/canada/ottawa/ontario-college-of-teachers-delays-shortage-schools-1.6534156, while OTF President Chris Cowley states, "The ongoing shortage of occasional teachers is another area where we are staring into a vacuum of leadership" (OTF Interaction, 24(3), p. 1, see www.otffeo.on.ca/en/wp-content/uploads/sites/2/2022/04/Interaction-volume-24-3.pdf).

OCT reports "first-year Ontario education graduate unemployment fall to four per cent. Just one per cent of those in career years two through five report they are unemployed." However, OCT also notes the level of concern about teaching jobs is beginning to rise:

> Despite labour market improvements for new teachers in 2021, availability of permanent contracts receded for the second year in a row. Just 17 per cent of first-year teachers obtained these most secure contracts in 2021, compared with 22 per cent back in 2019. Staffing uncertainties given the changing and unclear future requirements for in-person, virtual and hybrid teaching over the past two years may have delayed permanent hiring decisions and account for this reported decline.

See www.oct.ca/becoming-a-teacher/transition-to-teaching/previous-reports/2021-survey for details.



According to a 2020 report from the Ontario College of Teachers, early-career teachers are needed now more than ever to replace those who are retiring. (Ontario College of Teachers, Transition to Teaching report)

▲ A Look at the Transformations of Power Functions



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The mathematics curriculum in Ontario gives significant importance to the study of the transformations of many basic functions. The topic is introduced in Grade 10 and is expanded further in Grades 11 and 12. The emphasis is initially on the study of the transformations of the quadratic function through the introduction of its vertex form. The next step is to teach the student how to change the representation from the quadratic function, in its standard form, to its vertex counterpart by completing the square. The student learns that this technique is applicable to any quadratic function which, of course, is the case.

In Grade 12, the technique is generalized to include polynomial functions of the form $y = a[k(x - h)]^n + d$, with emphasis on cubic and quartic types. The students start making a parallel with the quadratic functions. They try to transfer all their acquired knowledge of quadratic functions to the polynomial functions of the third and fourth degrees, which does not always work. In many instances, the teacher is caught off guard, since the textbooks do not mention which polynomial functions, given in their standard form, can be transformed to the desired form given above. This work responds to this need and answers some of these questions.

In the present study, we will derive simple conditions that will allow a student to tell which polynomial function can be studied in the same way as the quadratic one. The formulas for the parameters h and d are also derived. We will regularly point out the link with Pascal's triangle, since it is a topic included in the curriculum of many secondary-level mathematics courses.

Back to the Quadratic Function

We will start with the vertex form of the quadratic function

as it is taught at the secondary level:

$$y = a[k(x - h)]^2 + d$$

Although this equation seems to have four parameters (a, k, h, d), it depends on three. Indeed, we can write it as $y = A(x - h)^2 + d$, where we have set $A = ak^2$, which in turn shows the link between the vertical and the horizontal stretches.

The above equation is a form of the polynomial function $P_2(x) = Ax^2 + Bx + C$. Assuming the coefficient A is non-zero, we can rearrange the above polynomials as:

$$\frac{y}{A} = (x - h)^2 + \frac{d}{A} = x^2 + 2(-h)x + (-h)^2 + \frac{d}{A}$$

$$\frac{P_2(x)}{A} = x^2 + \frac{B}{A}x + \frac{C}{A}$$

Identifying the corresponding coefficients, we get:

$$h = -\frac{B}{2A} \text{ and } d = C - Ah^2$$

For a given polynomial, the coefficients A, B, and C are known. The parameters of its vertex form can be determined uniquely from the above relations. We end up with the well-known fact that any second-order polynomial has a corresponding vertex form.

The Case of the Cubic Function

The cubic function, and higher-degree polynomials, is a different matter. The form of interest, as an extension of the previous function, is given by:

$$Y = a[k(x - h)]^3 + d$$

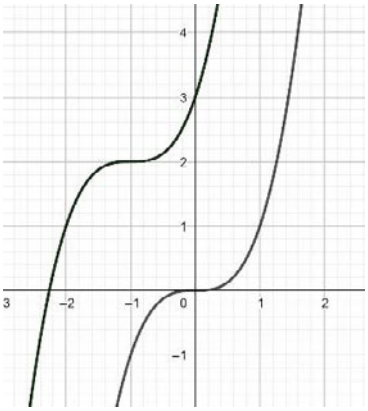


Figure 1: *Graphs for the cubic function example*

Again, this function depends on three parameters—A, h, and d—where A is defined as $A = ak^3$. The corresponding standard polynomial function is: $P_3(x) = Ax^3 + Bx^2 + Cx + D$. In this case, we have four coefficients: A, B, C, and D. Clearly, only a proper subset of third-degree polynomials can be reduced to the above form. We need to find an extra relation between the coefficients that will characterize this subset of polynomials. Using the binomial expansion, or the entries found on the third line of Pascal's triangle, the above relations are rewritten as:

$$\frac{y}{A} = (x - h)^3 + \frac{d}{A} = x^3 + 3(-h)x^2 + 3(-h)^2 x + (-h)^3 + \frac{d}{A}$$

$$\frac{P_3(x)}{A} = x^3 + \frac{B}{A}x^2 + \frac{C}{A}x + \frac{D}{A}$$

Identifying the corresponding coefficients between the two algebraic expressions, we get:

$$h = -\frac{B}{3A},\ h^2 = \frac{C}{3A} \text{ and } d = D + Ah^3$$

There is no need to consider the trivial case ($h = 0$), where the given polynomial function is already in the desired form. We assume therefore that all the coefficients are non-zero, except the last one, which is arbitrary.

We can see that, for the first two relations to be compatible, we need to set the condition:

$$B^2 = 3AC$$

We conclude that only the subset of third-order polynomials that meet this condition are suitable for the technique that we may call "completing the cube." One can also see that there is no constraint on the coefficient D.

As an example of application, let's consider the polynomial function $P_3(x) = x^3 + 3x^2 + 3x + 3$, with $A = 1$, $B = 3$, $C = 3$, and $D = 3$. The condition $B^2 = 3AC$ is met, and from the above results, we have $h = -1$ and $d = 2$. Thus, the completion of the cube, without any tedious steps, gives the final form:

$$y = (x + 1)^3 + 2$$

The graph of this function can be obtained from the one of $y = x^3$, with a horizontal translation of one unit to the left and a vertical translation of two units upwards (see Figure 1).

The Case of the Quartic Function

We will now consider the completion process as it applies to quartic functions. We will do that explicitly, since this type of function is addressed in high school. The desired function form is given by $y = a[k(x - h)]^4 + d$.

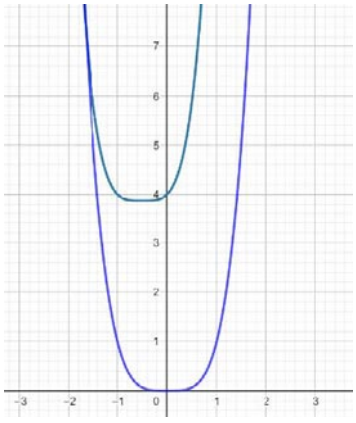


Figure 2: *Graphs for the quartic function example*

Again, this function depends on three parameters—A, h, and d—where A is defined as $A = ak^4$. The corresponding polynomial function is $P_4(x) = Ax^4 + Bx^3 + Cx^2 + Dx + E$. In this case, we have five coefficients: A, B, C, D, and E. Clearly, only a proper subset of fourth-degree polynomials can be reduced to the quartic vertex form. We

need to find two extra relations between the coefficients that will characterize this set of polynomials. Using the binomial expansion or the entries found on the fourth line of Pascal's triangle, the above relations are rewritten as:

$$\frac{y}{A} = (x-h)^4 + \frac{d}{A} = x^4 + 4(-h)x^3 + 6(-h)^2x^2 + 4(-h)^3x + (-h)^4x + \frac{d}{A}$$

$$\frac{P_4(x)}{A} = x^4 + \frac{B}{A}x^3 + \frac{C}{A}x^2 + \frac{D}{A}x + \frac{E}{A}$$

Identifying the corresponding coefficients between the two algebraic expressions, we get:

$$h = -\frac{B}{4A},\ h^2 = \frac{C}{6A},\ h^3 = -\frac{D}{4A} \text{ and } d = E - Ah^4$$

For the first three relations to be compatible, we need to set the two conditions:

$$3B^2 = 8AC \text{ and } 4C^2 = 9BD$$

We conclude that only the subset of fourth-order polynomials that meet these two conditions can be reduced to the desired function form through a completion process. As in the cubic function case, we can see that there is no constraint on the final coefficient *E*.

The conditions above are not unique. We could have chosen two different conditions for the same purpose. However, all the alternative conditions can be deduced from the above. The two conditions have been selected in such a way as to relate only three coefficients, as in the cubic function case. This choice is clearly reflected in Table 1. It is also worth noting that the sign constraints implied by the *h*, h^2, and h^3 relations on the coefficients *A*, *B*, *C*, and *D* are carried over to the conditions.

As an example, let's consider the polynomial function $P_4(x) = 2x^4 + 4x^3 + 3x^2 + x + 4$, with $A = 2$, $B = 4$, $C = 3$, $D = 1$, and $E = 4$. The two pertinent conditions are met, and from the above results, we have:

$$h = -\frac{1}{2} \text{ and } d = \frac{31}{8}$$

The resulting vertex form for $P_4(x)$ is:

$$y = 2\left(x + \frac{1}{2}\right)^4 + \frac{31}{8}$$

The graph of this polynomial function can then be obtained from the basic one of $y = x_4$, with a vertical stretch of factor 2, a horizontal translation of $\frac{1}{2}$ unit to the left, and a vertical translation of $\frac{31}{8}$ units upwards. In Figure 2, the graphs of $P_4(x)$, its vertex equation and the basic power function are shown. We can see that the graphs of the given polynomial function $P_4(x)$ and its vertex counterpart coincide perfectly.

Higher-Degree Polynomials

Following the same procedure, we can generalize the results to include polynomial functions of any degree. Avoiding a heavy notation, we can write:

$$\frac{y}{A} = (x-h)^n + \frac{d}{A} = C_n^0x^n + C_n^1(-h)x^{n-1} + C_n^2(-h)^2x^{n-2} \cdots + C_n^n(-h)^n + \frac{d}{A}$$

$$\frac{P_n(x)}{A} = x^n + \frac{B}{A}x^{n-1} + \frac{C}{A}x^{n-2} + \cdots + \frac{Z}{A}$$

In the second expression, we have noted the constant term of the polynomial as *Z* to convey the idea of it being the last term. Of course, the letter may change, depending on the degree of the polynomial and thus, on the number of terms. Identifying the corresponding coefficients between the two expressions, we infer:

$$h = -\frac{B}{nA},\ h^2 = \frac{1\cdot 2}{n(n-1)}\frac{C}{A},\ h^3 = -\frac{1\cdot 2\cdot 3}{n(n-1)(n-2)}\frac{D}{A}, \cdots \text{ and } d = Z - A(-h)^n$$

The compatibility between the different powers of the parameter *h* requires $(n - 2)$ conditions on the polynomial coefficients.

This procedure has been applied to polynomials from the fifth to the eighth degrees. The corresponding conditions are included in Table 1.

Summary of the Results

Our main goal in this development was to derive the conditions that a polynomial function needs to meet to qualify for the completion technique like the one used for quadratic functions. The results are summarized in Table 1.

The polynomial functions that verify the derived conditions are related to a type of polynomial equations that can be readily solved. However, the roots may be complex. Since the complex numbers are not part of the curriculum, there is no point in going in that direction, unless we restrict our attention to real roots that exist.

The transformations considered in the mathematics curriculum for university preparation courses include translations, reflections, stretches, and compressions. In the context of the present study, all these transformations conserve the shapes of the basic power functions. Consequently, the polynomial functions that can be considered for a completion process should have one turning point in their graphs when their degree is even, and no turning point if their degree is odd. These limits exclude all polynomial functions having more than one turning point from the process.

The development makes use of the binomial expansion. Consequently, the entries of each line of Pascal's triangle verify the corresponding conditions in the table. This aspect is practical, since it allows one to confirm the results.

Table 1: *Transformations corresponding to completion*

Degree n	Polynomial function	Conditions	Form sought: $y = A(x-h)^n + d$	
			Horizontal translation	Vertical translation
2	$P_2(x) = Ax^2 + Bx + C$	none	$h = -\frac{B}{2A}$	$d = C - Ah^2$
3	$P_3(x) = Ax^3 + Bx^2 + Cx + D$	$B^2 = 3AC$	$h = -\frac{B}{3A}$	$d = D + Ah^3$
4	$P_4(x) = Ax^4 + Bx^3 + Cx^2 + Dx + E$	$3B^2 = 8AC$ $4C^2 = 9BD$	$h = -\frac{B}{4A}$	$d = E - Ah^4$
5	$P_5(x) = Ax^5 + Bx^4 + Cx^3 + Dx^2 + Ex + F$	$2B^2 = 5AC$ $C^2 = 2BD$ $D^2 = 2CE$	$h = -\frac{B}{5A}$	$d = F + Ah^5$
6	$P_6(x) = Ax^6 + Bx^5 + Cx^4 + Dx^3 + Ex^2 + Fx + G$	$5B^2 = 12AC$ $8C^2 = 15BD$ $9D^2 = 16CE$ $8E^2 = 15DF$	$h = -\frac{B}{6A}$	$d = G - Ah^6$
7	$P_7(x) = Ax^7 + Bx^6 + Cx^5 + Dx^4 + Ex^3 + Fx^2 + Gx + H$	$3B^2 = 7AC$ $5C^2 = 9BD$ $3D^2 = 5CE$ $3E^2 = 5DF$ $5F^2 = 9EG$	$h = -\frac{B}{7A}$	$d = H + Ah^7$
8	$P_8(x) = Ax^8 + Bx^7 + Cx^6 + Dx^5 + Ex^4 + Fx^3 + Gx^2 + Hx + I$	$7B^2 = 16AC$ $4C^2 = 7BD$ $5D^2 = 8CE$ $16E^2 = 25DF$ $5F^2 = 8EG$ $4G^2 = 7FH$	$h = -\frac{B}{8A}$	$d = I - Ah^8$
n	$P_n(x) = Ax^n + Bx^{n-1} + \cdots + Z$	$(n-2)$ conditions	$h = -\frac{B}{nA}$	$d = Z - A(-h)^n$

Furthermore, the symmetry of the entries in the Pascal triangle gives rise to similar conditions when the degree of the polynomial is larger than 4. We can see that in the table.

The number of polynomials that satisfy the conditions is far more important than the solutions given by Pascal's triangle. To get a precise idea, consider the case of the cubic function. Pascal's triangle gives the values $A = 1$, $B = 3$, $C = 3$, and $D = 1$. However, in our case, the coefficient D is arbitrary, since any vertical translation would not change the shape of the graph of the function. Furthermore, if we set $A = 1$, the possible choices for B and C are any couple from the parabola $B^2 = 3C$. This corresponds to an infinite number of possibilities. This reasoning is also valid for the coefficients B and C.

The present results can be used to describe accurately how to extend the completion of the square technique to higher-degree polynomials. This is important with regard to the study of the transformations as applied to power functions $y = x^n$. Undoubtedly, the results can be helpful for teachers and useful for students. The same algebraic development can also be used in the context of the binomial expansion and Pascal's triangle.

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Collaborative Learning in a Diverse Classroom and Its Impact on Students' Achievement in Mathematics: A Lesson Learned from the Treisman Model



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Collaboration is one of six learning skills and work habits that are outlined at the forefront of the *Growing Success* document. The policy explains that the evaluation of learning skills and work habits should be separate from the curriculum expectations in a subject or course.

> To the extent possible, the evaluation of learning skills and work habits, *apart from* any that may be included as part of a curriculum expectation in a subject or course, *should not be considered in the determination of a student's grades* (Ontario Ministry of Education [OME], 2010, p. 11).

The document states an exception to this rule when *it is not possible to separate* the evaluation of learning skills and work habits from the evaluation of a student's achievement of a curriculum expectation. This exception is clearly found where the mathematics process intersects with the curriculum expectations. So, what makes these learning skills and work habits impact students' achievement in mathematics?

Picture a mathematics class: students are working together in small groups to develop problem-solving skills. In order for students to achieve this expectation, they need to apply many learning skills and work habits. In fact, students' learning skills and work habits in each of the small groups will have an impact on their learning outcome. Students who work together and share ideas will accomplish group goals, while others who struggle or don't know how to work together will face a different outcome. There is a significant relation between collaborative work and students' achievement. In their review of collaborative learning (CL), Laal and Ghodsi confirm its benefits on students: "CL, compared with competitive and individualistic efforts, has numerous benefits and typically results in higher achievement and greater productivity, more caring, supportive, and committed relationships; and greater psychological health, social competence, and self-esteem" (Laal & Ghodsi, 2012, p. 489). The advantage of collaborative learning goes beyond academic benefits to include social and psychological advantages as well.

As per the *Growing Success* document, collaboration is demonstrated when the student:

- accepts various roles and an equitable share of work in a group
- responds positively to the ideas, opinions, values, and traditions of others
- builds healthy peer-to-peer relationships through personal and media-assisted interactions
- works with others to resolve conflicts and build consensus to achieve group goals
- shares information, resources, and expertise, and promotes critical thinking to solve problems and make decisions

(OME, 2010, p. 11)

The Treisman Model

In the late 1970's and early 1980's, Uri Treisman, a mathematician at the University of California, Berkeley, was working on the problem of high failure rates of minority students in undergraduate calculus courses. Uri noticed that 60 percent of African-American students were failing the class. When examining the data, Uri found out that no Chinese-American students were failing the same class. When Uri posed the question to other faculty members, they thought the reasons could be perhaps that African-American students came into the university with lower math scores or an insufficient mathematical background; perhaps they were from less wealthy homes. In fact, none of these reasons were correct. Uri found that there was one difference: the African-American students worked on math problems *alone*, whereas the Chinese-American students sought peers and worked collaboratively.

Uri observed the Asian-American students, while working collaboratively together, were asking questions, often critiquing each other's work, assisting each other with homework problems, and sharing information related to their common interests. Their collaboration provided them with valuable information that guided their day-to-day study. By contrast, the African-American students worked alone in their dormitory rooms, and when they struggled on problems, they

decided they were just not "math people" and gave up.

Uri and his team set up mathematics workshops for the more vulnerable students. The focus of these workshops was for students to work on math problems collaboratively. Within two years, the failure rate of African-American students had dropped to zero, and the African-American and Latino students who attended the workshops were outperforming their White and Asian counterparts. This was an impressive result, and Uri has continued this approach at Austin, Texas. His approach has now been used in over 200 different institutions of higher education.

I have witnessed how genuine collaboration among students produces surprising and rich learning outcomes. Most importantly, communication while working together allows students to reveal to each other their struggles, embrace different ideas and perspectives, and appreciate other people's contributions. This experience of collaborative work helps students' confidence develop so they are ready to work through various situations and share feelings and information with each other. Phrases like "I actually don't know," "I'm not sure about that," or "I'd love to know how you can solve this" become easier to say when students work together. As a result, students build on their previous knowledge and achieve better learning outcomes.

The images in Figure 1 show students working together on a problem-solving task in a small-group setting. Each student was given a different colour of marker to show their contribution. The first image reveals that students communicated and worked together to solve the problem. This is evident from the intersection of the different-coloured markers. The second image shows that students divided the task among themselves and worked individually on each section.

Collaboration is a crucial learning skill for students to succeed in mathematics or in any other subject. In the twenty-first-century classroom, learning should routinely be happening through collaborative work among students, using a variety of activities such as learning centres, problem solving, inquiry-based tasks, or small-group work. The study results shed light on the reason why some students are successful while others are not. It's important that we teach students "how to collaborate together," and not simply assume that they know how to. It's also important for students to communicate struggle in any classwork with peers, and for teachers to ensure that students don't feel that they are struggling alone. Bossert asserts that "cooperative learning methods seem to promote positive interpersonal relations, motivation to learn, and self-esteem among students" (Bossert, 1988, p. 225). This will help students develop social–emotional learning skills, which will "foster their overall well-being and ability to learn while helping them build resilience and thrive as math learners" (OME, 2020, p. 19). Finally, embracing the diversity of our students will help them to be successful through collaborating and communicating with others, so that they develop more complex thinking, which results in meaningful and more profound learning experiences.

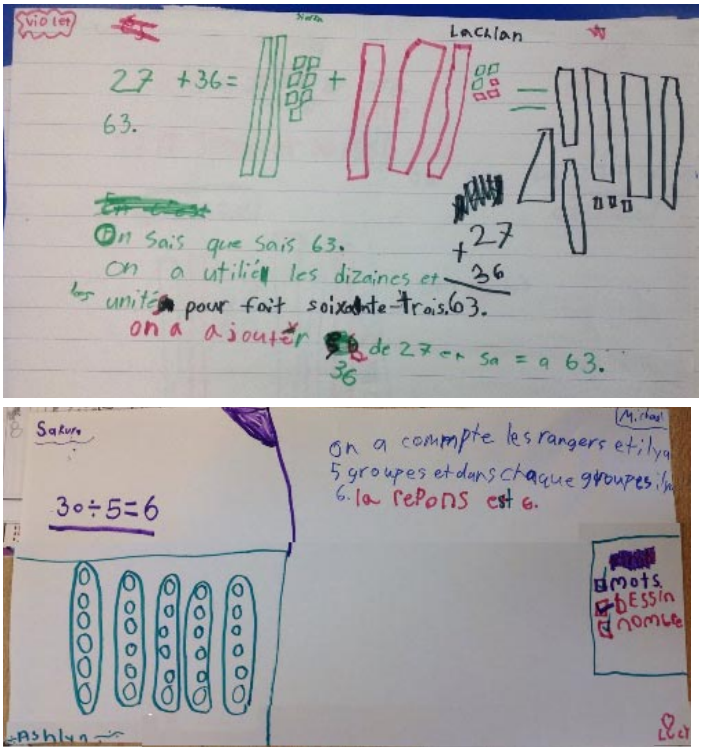


Figure 1: *Images of students' work in Mme. Louis Gr. 2 French Immersion classroom*

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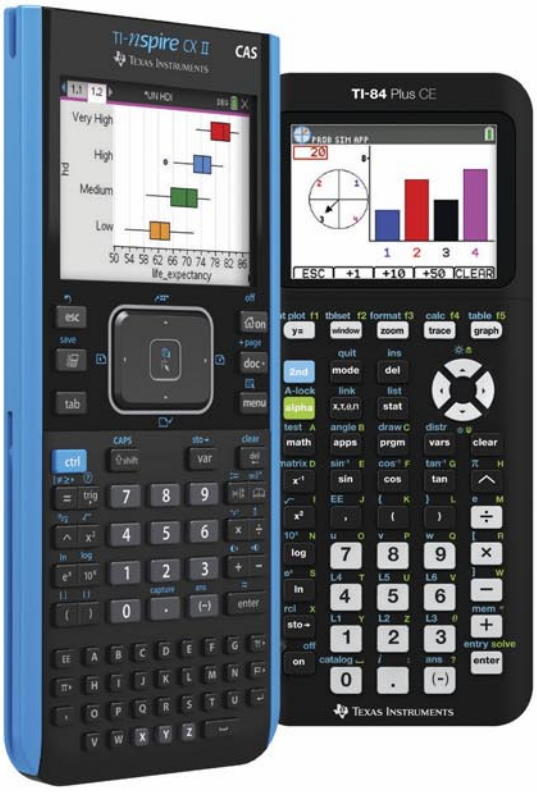
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