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Ontario Mathematics Gazette

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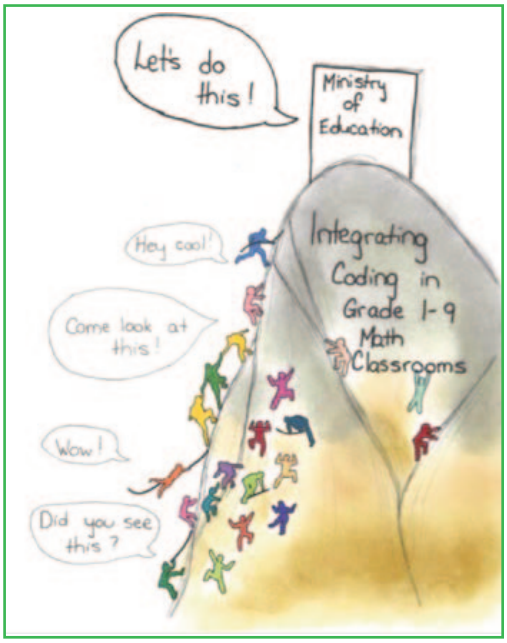
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March 2023

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▲ See In the Middle: The Fun of Playing Games and Learning



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Submission of Articles

The *Ontario Mathematics Gazette* (*OMG*) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *Gazette*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles are to be word–processed, MS Word is preferred, and prepared according to the *Publication Manual of the American Psychological Association*, Seventh Edition. However, please use single-line spacing (not double). Articles should not exceed five numbered pages of text, and figures, images, and photographs should be placed in the text close to where they belong, with captions. The photographer's permission is required, and for photos of students under the age of 18, the written permission of a parent or guardian is required.

Please submit your article in one blind file (i.e., identity of author is not evident), and include author names, contact information including email and mailing addresses, photos—head and shoulders, biographies—less than 100 words, and all content removed for blinding in a second file. Please email these two files to Tim Sibbald at gazette@oame.on.ca.

Upon review, you will be notified whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected. The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME/AOEM.

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Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be print ready, and colour advertisements should have no bleeds.

Advertising Rates
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▲ Editor's Report



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Tim Sibbald is a professor in the Schulich School of Education, Nipissing University, with a focus that includes mathematics education. He is a former President of OAME/AOEM and has held a variety of other OAME/AOEM posts over the years.

The time has come to wade in on an elephant in Ontario's most recent curriculum updates. Specifically,

> When the math curriculum was updated, it included claims that math was racist. "Mathematics has been used to normalize racism and marginalization of non-Eurocentric mathematical knowledges, and a decolonial, anti-racist approach to mathematics education makes visible its historical roots and social constructions," the curriculum stated. This claim was eventually removed. (Zuker, 2022)

The removal of material from the curriculum after it has been made publicly available is common knowledge and the heat of the political issue has cooled, but what do we learn from the events that took place? In my mind, there are two key take-aways.

The first is a trust issue, where having a curriculum produced with an elaborate process of consultations shows a level of attention that builds credibility. It allows naysayers, such as my own personal distaste for the term "real life"—are we denigrating student life as "fake"—to voice their opinions before a consensus stance allows us to adopt a common agreement called the curriculum. The trust issue began with the modification of the curriculum after its release, which was essentially the political issue of the day. But that same trust issue ends with the fact that digital documents, including policy, are subject to change without notice, and there were rumours of other changes. The aftertaste of the trust issue is that many teachers learned of the change in the curriculum from news sources, rather than from the Ontario Ministry of Education. We are left wondering what else has been changed, and whether we will be notified of the next change.

The marginalization of non-Eurocentric mathematics has roots that are not so much a political issue, and the role of this aspect of the curriculum requires careful thought. It is a pendulum issue, where at regular intervals through my career, I have had colleagues bemoan that math history is

missing from classrooms, while at other times, it was pointed to as a history dominated by White, European, males. Often the promotion of teaching math history is because of the many curiosity stories that are available. However, there is no doubt that it also reflects a desire to have a demonstrative human connection to the subject area. It is well intentioned and has many avenues beyond the curiosities.

Math history has patchy regional effects. Consider, for instance, the preponderance of French mathematicians in the seventeenth and eighteenth centuries. The primary language for communication in science and mathematics, at the time, was French. Mathematicians of the day aspired to use the primary communication channel, just as they do today, but at that time, it was French as opposed to English. If you talk to academics, it does not take long to find they have views about prestige in publishing, and it was doubtlessly no different in earlier centuries. The communication channel was not stationary, as we can see from English becoming the primary language of science and mathematics in the early twentieth century.

Perhaps some acknowledgement is needed of publishing in other languages. Russian mathematicians often publish in Russian, Japanese mathematicians in Japanese, etc. This accounts for doctoral programs, until roughly the 1980's, often having a language requirement, where earning a doctorate (i.e., Ph.D.) included demonstrating proficiency in a second language. For most non-researchers, access to scholarship is typically either in the local language or the primary language used by the discipline (i.e., mathematics). This gives the perception that the scholarship in the discipline originates locally or, with prestige, in the primary language of science and math. That is, imagine you were German in the eighteenth century, then you would likely pay attention to publications in German because of familiarity with the language, or French because it was the prestigious publishing language of the day.

Now consider the geography connecting Europe, Asia, and Africa. The Silk Road ran east, joining China and India, among other countries, to Turkey and Greece. Europe had well-developed routes to the Middle East for religious connections, as reflected by the crusades. Africa was connected through Egypt and the Middle East. The region was important as a trading hub and that included the transfer of knowledge. Regardless of which empire was central through centuries of change, the central conveyance allowed for the perception that ideas were coming from that region of prestige.

It is little surprise that Greek mathematics flourished—not that other mathematics didn't. When you tune into one radio station, what you hear is what they have available. There are sure to be details to this process, where one imagines that translators between various languages would be more likely found in the central region. This would lead to a perception that the central hub was producing content in languages the consumers could understand. Imagine being in France and receiving a document from Greece in French or Greek. The perception would be that it originated in Greece, as opposed to it having come from Africa or down the Silk Road and been translated.

This portrayal is a problem for the history of mathematics. It is not a perfect argument because hubs of knowledge tend to be inspirational and facilitate significant changes. Yet, it is a plausible consideration of how communication channels impact the listener's perception, and what it implies is that there were numerous contributors spread across the ancient world and that the documented record does not reflect the genuine reality of the time. In effect, the central region, such as Greece, was a medium for communication, and *the medium is the message*, to quote Canadian Marshall McLuhan.

A common argument against this picture is that the Pythagorean theorem was known in Egypt, China, and India, but (as the argument goes) it is often claimed that these were all particular cases until Pythagoras proved it in full generality. The problem with this version of history is that we are to believe that particular instances, such as the 3–4–5 or 5–12–13 triangles, were known, but not the full set of real solutions. This discredits the importance that the Egyptians

Gazette Articles Needed!

The *Gazette* has a queue of articles that are ready for print. Some of you may be aware that queue was relatively long last year. However, we now have four articles in that queue and will potentially run out of articles after the next issue.

Consider sharing what has been working in your classroom. The professionalism of communicating what you have done will enrich your own understanding and lead you to new ideas! (I speak from experience.)

Your Editor is an email away, if you would like to talk about an idea or the process: gazette@oame.on.ca.

had a direct connection to geometry, rather than simply being a numerical equality. Sure, someone proved the theorem in full generality, but they stood on the backs of many others who sweated studying the relationship.

Egypt, by the way, is in Africa. The historical record is dismissive of mathematics in Africa, generally, yet there are substantial arguments that the situation was much more complex (Anakwue, 2017; Rambane & Mashige, 2007). Sure, there is no evidence of calculus (that I am aware of), and that points to a common attitude that the historical record should focus exclusively on major developments toward the current state of the field. It is this attitude that has led to the history of statistics seemingly taking a back seat to the history of other areas of mathematics. While statistics is diminished in the historical record, if one takes a broader perspective, then it is found that there is evidence of mathematics development in Africa. Consider, for instance, that there is evidence of crop rotation in a systematic fashion since the Bronze Age, and that it was known on the borders of the Sahara Desert. That is indicative of statistical processes being used, perhaps with early theoretical conceptions, in Africa.

A similar dismissive view was adopted by colonial settlers to North America. Yet there is evidence of Indigenous navigation that was significant. Consider, for example, that referring to the shape of North America as "Turtle Island" shows a shape awareness on an enormous scale. How can that happen without mathematics?

All of this is to say that a curriculum that does not conform to the traditional narrative, and challenges teachers to engage with students in learning what other narratives exist, is inclusive. It will take time, and much may have been lost. The rainforest regions of Africa could be hiding all sorts of evidence. Yet we know from the Truth and Reconciliation Commission that the residential school system disrupted and likely caused losses within Indigenous knowledge around us.

So, while I have read many curious stories of mathematicians, the higher ground is clearly to gain a broader understanding. Perhaps we did not need the harsh words that were removed from the curriculum, but doubtlessly many needed a kick in the seat of their complacency regarding sticking to the traditional narrative.

Thank Yous

Anne, Peter, Gitta, and Penny—Thank you for your contributions that ensure every reader, both current edition and future archives readers, can gain a clear understanding. The effort for clarity is both fascinating and an ongoing diligence. At lunchtime, when a teacher announces, "We must eat students!," the editorial team rightfully avoids the cannibalistic with a small, but crucial, fix: "We must eat, students!"

In This Issue…

Our President, **Sandra Jean Price**, draws on perusing her collection of teaching resources and draws out an issue for all of us with stashes of resources. **Angelica Mendaglio** tells of the Margaret Sinclair Memorial Award talk event. **Jacqueline Hill** focuses on growth in her NCTM report. A lot has been happening at the OMCA, and **Heather Theijsmeijer** brings us up to speed.

Among the columns, **Lynda Colgan** examines a report on science teaching that is eye opening. **Iain Brodie** and **Beyza Sezer** give a brief of their interview of **Nicole Myer** and her work in STEM, and, as usual, the full interview is available through OAME Talks. **Shawn Godin** challenges us to consider triangles, where the angles are an arithmetic progression, and he also gets us thinking about 50-digit numbers and the like. **Jennifer Holm** explains how a "bucket of integers" supports conceptual learning for integer addition and subtraction. **Carly Ziniuk** draws on a variety of games and shows many opportunities with educational advantages. **David Costello** considers the benefits of ill-defined problems. **Jeff Irvine** focuses on exams and questions the value of them—perhaps it will provoke a letter or two. We will see. Last, but certainly not least, **Jérôme Proulx** continues with an examination of student responses to a proportion problem that show a variety of solution methods.

Within this issue we have a mix of articles. **Emily Beaulieu** shares a personal story that speaks to the importance of SEL in the curriculum and how teachers can, and do, make a difference through it. **Jessica Sardella**, **Laura Broley**, and **Chantal Buteau** draw on their experiences ascending "Code Mountain." Lastly, **Jeff Irvine** provides insight into crossing the business and mathematics divide with an interdisciplinary article that senior teachers will find useful.

Erratum: The title of Carly Ziniuk's "In the Middle" column in the January issue should have been "Sidewalk Starters." It has been updated in the archives, if you use a digital copy.

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▲ President's Message



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Sandra Jean has been on the Board of Directors of OAME/AOEM for almost 20 years. She is currently teaching in the Durham District School Board and has more than 25 years of teaching experience. She has had many roles, including classroom teacher, Math Coach, and Instructional Leader of online Math Additional Basic Qualifications (ABQs). Sandra Jean has contributed to a variety of Ministry projects and was a coordinator of the most recent OAME/AEMO writing projects. Sandra was the recipient of the 2008 OAME Award for Exceptional and Creative Teaching in Elementary Mathematics for her innovative instructional practices, integration of technology, and belief in success for all.

As this new year began, I found myself going through some bins in my basement in an effort to become more organized and less weighed down by "stuff." Like many of you, I have a lot of education resources stored in a "just in case" room. While going through the wealth of resources, some were timeless, such as bins of manipulatives that can always be used. There were also some resources that at one time were amazing; however, according to current knowledge, they are not so amazing anymore.

Many of the ideas in the resources have some merit. They often encourage the use of manipulatives and technology, math talk, and collaboration. There are resources that are consistent with three-part lessons. Of course, there are also some resources that were likely around when I was a student. The difficulty is that whether the resources are 35 years old or 10 years old, I'd like to think that we have learned during those years and are better able to meet the needs of the students we teach.

We know the importance of connecting with the students in front of us. The connections we make with our students encourage them to take risks and ask questions while learning. But what if the resources that are being used do not reflect the students? The math questions posed in some resources tend to have a Eurocentric view and draw upon cultural norms that may not be as inclusive as we now endeavour to be. What if the content is no longer current or relevant? What if students have lost their parents, or their families look very different from those represented in all of the resources? What if students do not see themselves in the illustrations/photos or represented in other ways? What if students identify as non-binary? What if families do not have cars? What if students have no schema around owning a house, being able to save money, or having a credit card? The "what if" questions could fill pages. The bottom line is that the years of stored resources are not as inclusive as we now strive for, even if they were once found effective in the classroom.

We continue to learn about the importance of acknowledging systemic bias and, as teachers doing our part to mitigate our contributing to it, the unique challenges faced by Indigenous peoples, 2SLGBTQ+ identities, and everyone's right to see themselves reflected in their learning environments. Celebrating diversity makes us stronger as a human race. Inclusivity is key to that strength. It is our responsibility to ensure that our resources and our classrooms are inclusive to all students, regardless of their intersecting identities (i.e., how they overlap through whichever characteristics are considered). Some may argue that the resources actually do reflect their current students, but perhaps that assumption is not always true, as students are still figuring out who they are. Furthermore, we need to model inclusivity and embrace diversity in order to model and teach that everyone deserves to be appreciated for who they are. Does that mean that all of my resources need to be thrown out or recycled? Not necessarily. Some certainly are heading for the blue bin. There are others that have some good ideas…, but will need some updating of the content so that it is relevant, aligns with our current curriculum and understanding of good pedagogy, and is truly inclusive of all students.

I am not an expert, but believe that as educators, we know more now than even ten years ago, and students deserve to benefit from that growth. Developing as an educator sometimes means realizing that misguided judgments based on one's knowledge at a given time may have ignored a reality faced by students. After all, we each have our own biases that are sometimes difficult to recognize. But as we learn more, we have an obligation to be better for our students, for the school community, and for everyone's future. ▲



"Weed? Or remarkable geometry?"

▲ Blast from the Past! Examining Examinations



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Jeff Irvine, Ph.D., has been a secondary mathematics teacher, department head, and vice-principal. He has taught at three faculties of education and at a community college. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. With over 45 years in education, Jeff is particularly interested in the interplay of student motivation and mathematics achievement.

In the beginning (of the *Gazette*, 1962), there were Grade 13 Departmental Examinations, set and marked by the Ontario Department of Education (ODE). Every student in Grade 13 in every school in the province wrote these examinations in June. The exam marks counted as 100 percent of the students' final grades in each subject. Superficially, these appeared to provide unbiased, objective evaluations of the students' knowledge and skills in each subject. The reality, of course, was quite different. I wrote the Grade 13 departmental exam in physics. I estimated that, if I got everything that I answered completely perfect, I could get 43 percent as a final grade. When I received my grade in mid-summer, I was shocked (and pleased) to receive a mark of 87 percent—thanks to the use of the bell curve.

The *Gazette* published sample questions and possible solutions from Grade 13 departmental exams of the previous year (e.g., ODE, 1962; ODE, 1963a; ODE, 1965). Multiple possible solutions were provided, all generated by the ODE. The *Gazette* also published problems and solutions from the Grade 13 Problems Paper; this was an optional contest open to Grade 13 math students (ODE, 1963a) and the forerunner of the Descartes contest. Unsurprisingly, all of this was very academic, and focused heavily on mathematical rigour and good mathematical form (ODE, 1963b), with no attempts to support creative problem solving.

The last set of Grade 13 departmental exams occurred in 1967. After that, schools were free to set their own exams, which in Grade 13, typically were three hours in length and covered the entire year's work. For other grades, most schools had three sets of examinations each year, with exam marks counting for a large portion of students' final grades. There was a heavy focus on knowledge and skills, with little to no problem solving beyond the content taught in class, usually via the transmission method, which dominated teaching during this time period.

After the abolition of Grade 13 departmental exams, articles in the *Gazette* had two foci: sample exams for various grades, and comments expressing the need for a return to provincial exams or for university entrance exams in mathematics. Exams still played a large role in evaluation, and sample exams were a frequent *Gazette* topic (Weiner, 1994, 1995a, 1995b; Schatz, 2000). These samples included a Destreamed Grade 9 exam (Weiner, 1994) for the province's previous ill-fated foray into destreaming.

The articles on instituting university entrance exams, or even on reinstating provincial high school exams, expressed some very real concerns around mark inflation, lack of standardized curriculum, subjective evaluations by teachers, and students being ill-prepared to write university examinations (Gold, 1976; Mullen, 1984; Nicolls, 1993; Taylor, 1996). All these articles viewed examinations as objective ways to provide an equitable method of evaluating students' mathematical knowledge. Nicholls (1993), in advocating for university entrance exams, cited concerns about equitable admissions to university math programs as well as declining international math scores, poor Descartes contest results, and a large increase in Ontario Scholars. He wrote about the need for students to perform under pressure, and that exam writing is a skill that can be taught.

McFarlane (1994) wrote a really interesting rebuttal to Nicholls' article, debunking many of Nicholls' arguments and reflecting that the desire for a return to high-stakes exams was a fallacious argument based on a "rose-coloured glasses" view of the past. Nicholls felt that external exams would result in a more accurate placement of students in university programs. However, McFarlane pointed out that during the departmental exam period, it was common for 70 percent of students enrolling in university math programs to fail to complete their degrees. I had personal experience with this. In my first year of the Bachelor of Mathematics degree at the University of Waterloo, 625 freshmen were brought into a lecture hall and given a "pep" talk: "Look at the person on your left and the person on your right—two of you will make it to second year, one of you will graduate." Really motivating stuff!

There were also a couple of *Gazette* articles on more novel ways to deal with exams. For example, Zichowsi (1992) wrote about using videos for exam review:

Czempinski (1976) made several suggestions under "Some Novel Approaches to Examinations," including giving hints (at a cost of marks) for students who were completely stuck. I actually employed this technique with in-class tests long before I read the *Abacus* article.

All of this emphasis on exams ignored the great strides made in the psychology of learning. It also ignored the growing emphasis on problem solving in mathematics and the need to assess more than knowledge and skills. In 1997, the Ontario Ministry of Education's (OME) new curriculum policy documents fomented a sea change in mathematics education: a standardized curriculum; a constructivist view of mathematics learning; extensive use of technology; assessing the whole child, rather than just knowledge and skills; assessing learning skills; new reporting mechanisms. All of these changes mitigated against a heavy emphasis on examinations as the primary evaluation tool. OME introduced the TACK categories for assessment (Thinking, Application, Communication, Knowledge), with knowledge and skills seen as only one aspect of student learning.

Articles in the *Gazette* began to write about assessment techniques other than paper-and-pencil tests and exams. Suurtamm (1997) described the value of portfolio assessments. She cited the NCTM's (1995) six standards for mathematics education:

- The mathematics standard: Assessment should reflect the mathematics that all students need to know and be able to do.
- The learning standard: Assessment should enhance mathematics learning.
- The openness standard: Assessment should be an open process.
- The inferences standard: Assessment should promote valid inferences about mathematics learning.
- The equity standard: Assessment should promote equity.
- The coherence standard: Assessment should be a coherent process.

Formal written examinations do not fit well when judged by these six criteria.

Articles in the *Gazette* frequently blurred the distinction between assessment and evaluation. This is in keeping with the new emphasis on multiple assessment opportunities and techniques (Irvine, 2019a, 2019b, 2019c, 2020a, 2020b, 2020c; Kajander, 2003; Suurtamm, 1997). We now look at assessment and evaluation as a process, not an event. The last *Gazette* article about examinations was Taylor (1996), and discussed the need for university entrance exams.

So, what is the educational value of an examination? In my personal opinion, based on almost 50 years as an educator, exams are a total waste of time and energy. They unnecessarily stress students; they ignore learning modalities; they ignore ESL factors and special education students' needs; they measure only students' ability to memorize and regurgitate surface learning in a time-constrained environment. Worst of all, they ignore everything that we have learned about the psychology of learning.

When I completed my Bachelor of Mathematics degree, all 24 course grades were based 100 percent on the final exams. I got really good at "spotting" potential exam questions and memorizing the required information. For one course, I memorized 78 pages of mathematical proofs (like "Fred's proof of Joe's theorem"). I did really well on the exam, getting a mark of 100 percent; the next day, I could not remember a single one of the proofs. A total waste of time and energy. Education is a serious and important business. Assessing our students' learning should focus on the big ideas, the important skills and knowledge of our subject; our students' ability to pose and solve problems; our support of all the various dimensions of a student's learning; and our support of the growth of our students into outstanding citizens. Exams don't do any of that and are a relic of a bygone age.

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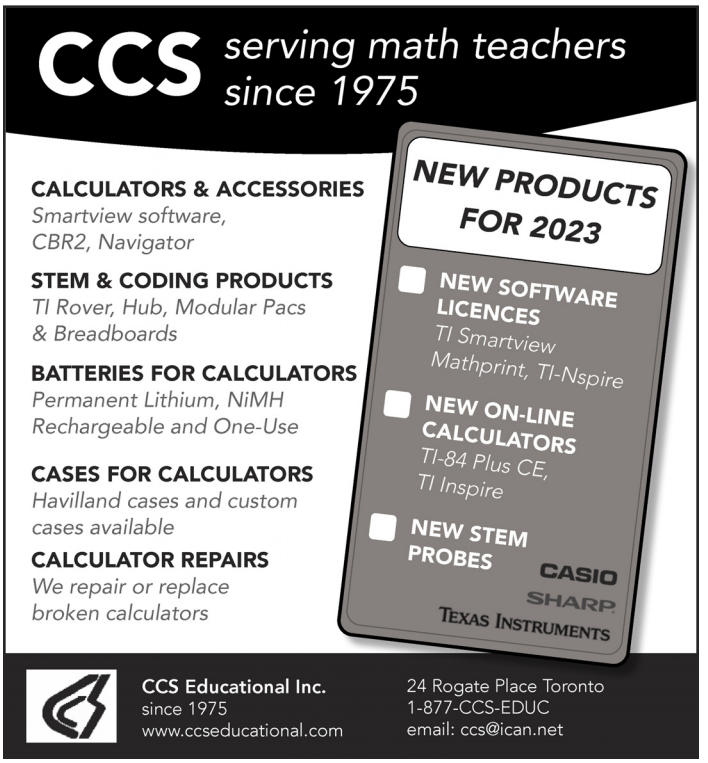
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▲ OMCA News: March 2023

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A belated Happy New Year from the Ontario Mathematics Coordinators' Association (OMCA)! On behalf of our membership of math Coordinators, Consultants, and Coaches from across the province, we hope this finds your year off to a great start.

Our members' meetings continue on a monthly basis throughout the year. Recent meetings have included guest speakers Dr. Pamela Seda on *ICUCARE: A Framework for Equity in the Math Classroom*; a panel presentation by Dr. Juan Carlos Chavez, Dr. Kori Czuy, Dr. Liz McDonald, and Ph.D. candidate Alvin Harvey on facilitating cultural connections in mathematics; David Poras from Amplify (Mathigon) on *Using Virtual Manipulatives to Engage Students in Mathematical Exploration and Discovery*; and Dr. Robert Q. Berry on social justice in mathematics.

In December, OMCA worked with an outreach team from EQAO to offer an armchair session on the new *Power BI* tool for analyzing school and board EQAO data. Members learned how the new platform can provide insights on how to leverage assessment data to inform school and board improvement planning.

OMCA's first in-person retreat since 2018 took place on February 6–7, 2023 at Hockley Valley, and saw over 150 delegates come together to learn more about *Exploring the Power and Potential of Indigenous Knowledge Systems in Mathematics*. It was a full day and a half of presentations and explorations (both indoors and outdoors) by Isaac Murdoch and Jodie Williams. "Chi-miigwetch" to our presenters, and to all who attended, for joining us. It was wonderful to be learning in person again!

Once again this year, we are pleased to offer our members a chance to explore equity-themed resources through Literature Circles, which kicked off in December. OMCA members are able to choose between one of four books: *Choosing to See: A Framework for Equity in the Math Classroom* by Kyndall Brown and Pamela Seda; *High School Mathematics Lessons to Explore, Understand, and Respond to Social Injustice* by Robert Q. Berry III, Basil Conway, Brian Lawler, and John Staley; *Rough Draft Math: Revising to Learn* by Amanda Jansen; *Teaching Math to Multilingual Students, Grades K–8: Positioning English Learners for Success* by Kathryn Chval, Erin Smith, Lina

Continued on page 13

▲ Events at the Fields



ANGELICA MENDAGLIO
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Angelica Mendaglio is an instructional designer at Vretta Inc. in Toronto, Ontario, where she helps to create interactive digital mathematics lessons and activities for middle school students.

Margaret Sinclair Memorial Award

Every year, one of the most engaging talks at the Fields MathEd Forum is given by the previous year's recipient of the Margaret Sinclair Memorial Award. The award is given annually to a Canadian educator, who has demonstrated excellence and innovation in mathematics education. The award winner gives a lecture at the Fields Institute the following year. The winner of the award last year was Dr. John Mighton, founder of JUMP Math, a charity dedicated to helping students reach their mathematical potential. His address to the Forum was titled "All Things Being Equal: How to Use Math to Create a More Equitable World."

Dr. Mighton spoke about the early days of JUMP Math as a tutoring club, and the turning point when he was asked to help in a classroom. As a playwright, Dr. Mighton said that he saw the students as an audience, and that the best way to engage an audience is to take advantage of what is called "collective effervescence." This is a term from sociology that describes the joy that people feel when experiencing something together or working toward a common purpose. In his practice in mathematics classrooms, Dr. Mighton found that tutoring students in groups was much more exciting than individually. From there, JUMP Math grew to include a publications department, professional learning for teachers, and today is used by 200 000 Canadian students as their main classroom program.

When discussing some of the core principles of JUMP Math, Dr. Mighton talked about the danger of survivorship bias when evaluating the success of an educational system. He introduced this idea through an example from Jordan Ellenberg's book, *How Not to Be Wrong: The Power of Mathematical Thinking*. In that book, Ellenberg writes about statisticians supporting a war effort by analyzing the number of bullet holes in different areas of the planes that returned from battle, to determine where additional armour should be added to better protect the planes. It might seem intuitive that armour should be added to the parts of the plane that have the most bullet holes. However, the conclusion was that armour should be added to the parts of the plane with the fewest holes, with the reasoning that the planes that were shot in those areas were so damaged that they were not making it back from battle to be analyzed. This is an example of survivorship bias—how focusing on the examples that "survived" to become part of the data set can exclude critical pieces of the picture. In his lecture, Dr. Mighton compared this to what can happen with education—if we focus on the students who succeed in school and go on to be mathematical thinkers, we will think that the system is more successful than it is. It is paramount that we focus on the students who are not flourishing in our programs and ensure that they don't fall through the cracks.

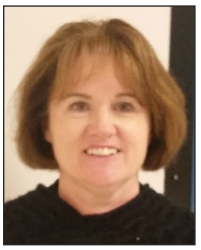
Dr. Mighton also discussed a partnership between JUMP Math and the University of Alberta called Math Minds, an initiative which provides professional learning resources for teachers. They offer an online course for teachers, detailing a structured approach to math education, for which you can sign up for free on their website.

The Fields Institute accepts nominations for the Margaret Sinclair Memorial Award until December 31 each year. The Fields MathEd Forum will convene on the last Saturday in March and April. You can sign up to attend remotely through the Fields website (www.fields.utoronto.ca/activities/workshops/mathed-forum), where you can also see their schedule of other upcoming math education events.

Fields Academy Math Elementary Educator Program

The Fields Institute, through the Fields Academy, is offering new Elementary Educator Courses to help elementary teachers and teacher candidates bolster their mathematics skills. The courses are online and self-paced, with participants gaining access to videos, reading materials, interactive pieces, and forums to connect with other participants. The courses, which were developed by Ontario teachers with Ph.D.'s in mathematics education, are currently being offered under a "pay-what-you-can" model, with the suggested price being $25. There are three courses available (Mathematical Modelling, Financial Literacy, and Operations), with more planned for the future. These first courses are in initial testing and review stages, and the Institute is inviting interested teachers to participate in the courses to provide feedback for future iterations of the courses. You can find out more information on the Fields Academy website. ▲

▲ OAME/NCTM Report: Growth!



JACQUELINE HILL
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Jacqueline Hill is a Grades 9–12 teacher of mathematics. She is a Past President of OAME and OMCA, as well as the recipient of the award for Exceptional Teaching in Secondary Mathematics. She also wrote the "Director's Dialogue" for the Gazette *for a number of years.*

With Spring comes new growth! The snow will leave us, the ground will soak up moisture, the sun will shine, and the world will grow. We as mathematicians also have an opportunity for growth! A new semester in high school, the final push toward June in elementary school. Here are some ideas that may help you with your own personal growth this Spring

G – *Give* – Give to those who are coming up in the ranks of education. Give to those who are in their last few years of education. I recently took many teaching resources to a group of pre-service teachers. I was shocked that they had never even heard (or, perhaps, not heard yet) of the *Growing Success* document. I quoted a few areas to them… such as the advice about ways to prevent a student from earning a zero. If you don't have tangible things to give away at this stage in your career, then consider sharing a daily smile with someone who might need it.

R – *Read* – Read everything you can get your hands on—from works to improve your mind, to works to let your mind wander and escape. The NCTM has just published *Success Stories from Catalyzing Change*, a $35 investment in the journey of a nation to better serve students.

O – *Organize* – Organize and downsize those things that are in the way of your growth. Did you know that the Staples store has a bin to recycle empty pens and markers? I keep an ongoing bag of them in my bottom drawer. When the bag is full, I make a trek to drop them off. Does your school have a place where binders, paper, markers, and such can be shared? Ours was on a table in the mailroom. I once dropped off 12 mugs in my downsizing effort. The very next day, a new teacher came and thanked me, as she had just moved out on her own!

W – *Willingness* – Be willing to learn. Learn new things, or just listen to "old" ideas in a new light. The willingness to push forward has to come from you.

So perhaps the next time your administrative team comes up with just one more seemingly impossible idea… just breathe and listen.

Don't judge. Just listen with an open mind and a willingness to learn.

The NCTM was set up in 1920 to help math educators help students to grow. The NCTM has continued to have face-to-face and online conferences to help educators grow in their practices. OAME and NCTM have both strived to ensure teachers are equipped to grow throughout their careers.

NCTM – Important dates to note:

March 10, 2023 – California Mathematics Council Central Symposium (Mechanics Bank Convention Center, Bakersfield, California)

July 26, 2023 – Pennsylvania Council of Teachers of Mathematics Annual Conference (Ramada Hotel and Conference Center, State College, Pennsylvania)

NCTM – A few things of interest on the website right now:

1) Members-Only Webinars. Your NCTM membership includes exclusive access to 100+ hours of on-demand professional development. NCTM webinars are delivered by experts in mathematics education, including NCTM authors.
2) The 2023 game is free, and a great way to get students thinking. It is on the main page of the NCTM website, but if it has been moved from the main page, just enter "2023 game" into the search bar.

Resource

Graham, K.J., Berry III, R.Q., Bush, S.B., & Huinker, D. (Eds.). (2022). *Success stories from catalyzing change*. NCTM.

The stories in this book represent the efforts along the continuum of change, including work that is just starting, to initiatives in progress, to examples of advanced implementation. Each story shares an approach addressing one or more of the four key recommendations.

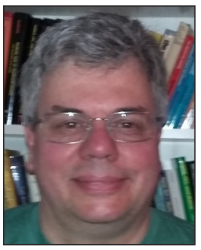
Like OAME, NCTM has a mission statement that helps members keep growing in the right direction.

NCTM Mission Statement – The National Council of Teachers of Mathematics advocates for high-quality mathematics teaching and learning for each and every student.

OAME/AOEM Mission Statement – To promote, support, and advocate for excellence in mathematics education throughout the Province of Ontario

Until next time,
Yours in mathematical fun,
Jacqueline ▲

▲ What's the Problem? *Gazette* Problems



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Shawn is a former mathematics teacher, department head, and consultant. He strongly believes in the central role of problem solving in the mathematics classroom. He continues to be involved in mathematical activities: presenting workshops, writing articles, working on local projects, and helping create mathematics contests.

Welcome back, problem solvers. We are continuing the theme "(mathematical) problems from my life." After high school, I went to the University of Waterloo and did my degree in applied mathematics. Unsure of what I wanted to do with my life, I spent the next couple of years bouncing between things, before yielding to the call of teaching. In September 1991, I entered the profession at St. Joseph-Scollard Hall Catholic Secondary School in North Bay. Very early in my career, I became a member of OAME/AOEM and attended my first conference in Windsor in 1993. It seems appropriate that since OAME/AOEM was such a big part of my career, that we examine a couple of problems from the *Gazette*. Let's look at the first problem from your homework:

The number 24 678 051 is quite interesting because

$$2^8 + 4^8 + 6^8 + 7^8 + 8^8 + 0^8 + 5^8 + 1^8 = 24\,678\,051.$$

There are two other eight-digit numbers that are equal to the sum of the eighth powers of their digits. Find these numbers.

This problem came from Ron Lancaster's "Mathematical Problems" column (Lancaster, 1991) from September 1991. Many *Gazette* readers will recognize the name. Ron has been presenting at the OAME annual conference regularly since 1983 (my last year in high school!). His writing has appeared in the *Gazette* over the years as well, authoring over 60 articles in his columns: "Mathematical Problems" (1986–1992), "Mathematics and the World Around Us" (1992–1994), "Photo Math" (2000–2008), and "Mathematical Snapshots" (2018–2022).

With the emphasis on coding in the curriculum, the first homework problem becomes a nice one to start with. Students can write an algorithm to determine the desired numbers. I wrote a quick program in *Maple* with eight loops, one for each digit, checking if

$$a^8 + b^8 + c^8 + \cdots + h^8 = a \times 10^7 + b \times 10^6 + c \times 10^5 + \cdots + h.$$

After running for a few minutes, the program produced the results 24 678 050, 24 678 051, and 88 593 477. A keen-eyed reader may have observed that when I presented the problem in the last issue, there was a typo (the 8^8 term was missing in the sum on the left). It was interesting to see that in Ron's column, there was a typo in the last result (including an extra 5).

Ron suggested, as I would, that you could investigate other cases. He gives as an example in the 4-digit case below:

$$9^4 + 4^4 + 7^4 + 4^4 = 9\,474.$$

One can easily adjust one's program to search out all kinds of cases of these interesting numbers. Before we get too far, it might be prudent to think about what we are doing. My program for 8-digit numbers took several minutes to run. If we want to investigate 9-digit numbers, using the same model for our algorithm, it will take about *ten times* as long to run. That means that for 9-digit numbers, the program may take close to an hour to run. Ten-digit numbers would require on the order of half a day to run, while 11-digit numbers would take around half a week. Thus, for lucky 13, our program would take about a year to run! For larger numbers, the time needed would continue to grow exponentially.

My Editor and readers would probably be upset if I delayed my column for a year to allow my program to run for 13-digit numbers. This brings up a good point about computers and computer programming. Computers are good at doing many computations very quickly, but even so, there are some problems that they cannot solve in a reasonable amount of time. As such, we need to devise our algorithms thoughtfully. How can we do this? With a little mathematics, of course!

The way I had set up my program, I had, in essence, run through every number from 10 000 000 to 99 999 999 to check to see if the property of the digits held. However, if we look at the given example,

$$2^8 + 4^8 + 6^8 + 7^8 + 8^8 + 0^8 + 5^8 + 1^8 = 24\,678\,051,$$

all numbers that can be formed by permuting the digits of 24 678 051 yield the same result when you take the sum of the eighth powers of its digits. What that means is that as soon as we have found the result above, we know that 15 087 642, 24 180 567, and all other 8-digit numbers containing the same 8 digits are not solutions. Since there are 8! = 40 320 permutations of 8 *unique* digits, then if we redo our program to run through all the possible collections of digits *without respect to order*, we can drastically reduce its running time. I will leave that task to the interested reader.

What else can we learn about the problem, using tools

from the high school curriculum? Let's consider the 8-digit number made up of only one repeated digit. For example, 55 555 555 yields an associated value of $8 \times 5^8 = 3\ 125000 < 10\ 000\ 000$. Hence, no number made up of only digits 0, 1, 2, 3, 4, 5 can have the desired property (why?). Continuing in this line of thought, but for a general number, n, of digits, if $n \times 9^n < 10^{n-1}$, then there would be no numbers made up of n digits or more with this property.

A natural question might be: "How large can a number be, with n digits, that has our desired property?" From the paragraph above, we see that we need to find n such that $n \times 9^n < 10^{n-1}$. Unfortunately, this equation isn't solvable using the tools from the high school curriculum. However, we can get an approximation quite easily. A calculator will show that $100 \times 9^{100} \approx 2.7 \times 10^{97} < 10^{99}$, so 100-digit numbers are too large, meaning there will be no 100-digit numbers with the property. Similarly, $50 \times 9^{50} \approx 2.6 \times 10^{49} > 10^{49}$, so there *may* be some 50-digit candidates. We could bounce around and refine our result, but if we rewrite our inequality as $n < \frac{10^{n-1}}{9^n}$, then we can investigate it by graphing $y = x$ and $y = \frac{10^{x-1}}{9^x}$. You will notice that the graphs cross at about (60.8, 60.8), which means that there will be no numbers with our desired property with 61 digits or more.

What about 60-digit numbers? From our earlier discussion, we may think that the problem is out of reach. It would take more time than the universe has existed before an algorithm like my original would crack the problem. This is true if we just write it and let it run without any thought. Let's see what happens when we do some further analysis.

If we imagine a scenario with no 9's, we can calculate $60 \times 8^{60} \approx 9.2 \times 10^{55}$, which tells us that there is no 60-digit number with the desired property and no 9's. Continuing with our computations, $9^{60} \approx 1.8 \times 10^{57}$ and $10^{59} \div 9^{60} \approx 55.6$. What does this mean? It means that if our number has at least 56 nines, it will be large enough to be a candidate. Since $55 \times 9^{60} + 5 \times 8^{60} \approx 9.9 \times 10^{58}$, we deduce that 55 nines aren't quite enough.

The largest possible sum of sixtieth powers of the digits of our number occurs when all the digits are 9's. Since $60 \times 9^{60} \approx 1.078 \times 10^{59}$, we can deduce a few things. If a 60-digit number exists with our desired property, then:

- the first two digits are 1 and 0,
- the third digit is 7 or less, and
- there must be at least 56 9's.

This means that at least 58 of the 60 digits are known. The information above still leaves the possibility of 57 9's. However, since $57 \times 9^{60} \approx 1.024 \times 10^{59}$ and $57 \times 9^{60} + 8^{60} \approx 1.024 \times 10^{59}$, we can conclude that the fifty-seventh 9 is not possible.

Then, since $56 \times 9^{60} \approx 1.006 \times 10^{59}$, we know that the third digit can be at most 2, and the fourth can be at most 8, so we only have 27 cases to check! I wrote a program and had it print all the sums of the sixtieth powers of the digits. Since a number with our desired property would have 56 9's, it would be easy to spot, and there were none. That is, there are no 60-digit numbers with our desired property.

Even though the mathematics isn't elegant, we still managed to discover some things about n-digit numbers equal to the sum of the nth powers of it digits, without using years of computing time. An interested reader might like to investigate the 59-digit number case.

It is worth noting that when Ron gave this problem to his students, he used his "teacher eyes" to pick the example. Since 24 678 051 worked and $1^8 = 1$, an astute student would notice that since the one is in the units' place, replacing it with a zero would also be a solution. Keep your "teacher eyes" open when looking at problems, as well as trying to use your "student brain."

Let's look at the next problem from your homework:

The sides of a triangle have lengths 3, 7, and 8 cm, respectively. Show that the measures of the interior angles form an arithmetic sequence.

This problem came from John Grant McLoughlin's column, "About Problem Solving" (Grant McLoughlin, 1994), which ran in the *Gazette* from 1993 to 1999. I was a fan of this column when it ran, and I had written John a "fan letter" at the time. I was surprised when, one day, John called me up and we had a wonderful chat. Over the years, I have seen John regularly at the CEMC contest committee meetings. We have also both worked on the problem-solving journal *Crux Mathematicorum*, where we are co-editors of the MathemAttic section, intended for high school students and their teachers. *Crux* and MathemAttic are both available online for free at www.cms.math.ca/publications/crux/. I suggest that you check them out.

We can use trigonometry to solve this problem. Using the law of cosines, the smallest, middle, and largest angles, A, B, C, respectively, are given by:

$$A = \cos^{-1}\left(\frac{7^2 + 8^2 - 3^2}{2 \times 7 \times 8}\right) \approx 21.8°,$$

$$B = \cos^{-1}\left(\frac{8^2 + 3^2 - 7^2}{2 \times 7 \times 8}\right) \approx 60°,$$

$$C = \cos^{-1}\left(\frac{3^2 + 7^2 - 8^2}{2 \times 3 \times 7}\right) \approx 92.8°.$$

Checking the decimal places on A and C seem to confirm that the angles are in arithmetic progression. If the three angles of a triangle are an arithmetic sequence, their

measures can be written as $(\theta - \delta)^\circ$, θ°, $(\theta + \delta)^\circ$ for some values of θ and δ. However, since the angles of a triangle sum to 180°, we get $3\theta^\circ = 180^\circ$ or $\theta^\circ = 60^\circ$. That means that *any* triangle with a 60° angle has its angles in arithmetic progression. A very nice result!

Returning to our triangle with angles A, $B = 60^\circ$, C in arithmetic progression, then the law of cosines (again) gives us $b^2 = a^2 + c^2 - ac$, which means, without loss of generality, if we choose any two numbers a and c with $a \le c$, then setting $b = \sqrt{a^2 + c^2 - ac}$ should make a triangle with sides a, b, and c with a 60° angle.

Some checking shows that with $a \le c$, then $a^2 \le ac \le c^2$ and $a^2 \le a^2 + c^2 - ac \le c^2$. Therefore, $a \le b \le c$, as it should be. Let's do one more check. Every triangle satisfies the *triangle inequality*, that is, the sum of the lengths of two sides of a triangle is always larger than the other side. Thus, since the largest side would have length c, we have

$$a + b = a + \sqrt{a^2 + c^2 - ac}\,.$$

If we let $a = c - x$, for some value $x > 0$, then since a and c are lengths of a side of a triangle, $a > 0$, and hence, $x < c$. Rewriting our expression for $a + b$, given above, in terms of x and c with simplifying, yields:

$$c - x + \sqrt{c^2 - cx + x^2}\,.$$

However, using the same logic that we used earlier, we can deduce that $c^2 - cx + x^2 > x^2$, and hence, $a + b > c$, which means that if we pick any two real numbers with $0 < a \le c$ and set $b = \sqrt{a^2 + c^2 - ac}$, then a triangle with sides a, b, and c exists and its angles are in arithmetic progression (i.e., $B = 60^\circ$).

One might ask, "Is it possible to have all three sides integers, as in the statement of the problem?" As some readers may know, a *Pythagorean triple* is a set of positive integers $(a, b, c,)$ that satisfy $a^2 + b^2 = c^2$. It turns out an *Eisenstein triple* is a set of positive integers $(a, b, c,)$, such that the triangle with sides a, b, and c has either a 60° or a 120° angle. It is beyond the scope of this column to develop the formulas for this case. However, with a little algebra, one can check that

$$\begin{aligned} a &= 2mn - n^2 \\ b &= m^2 - mn + n^2 \\ c &= m^2 - n^2 \end{aligned}$$

generate a triangle with the desired property.

I am very thankful that OAME/AOEM has been a big part of my professional growth. I have attended many wonderful presentations at the annual conference that have helped shape my own teaching style. Early on, I was lucky to work with Dr. Doug Franks at Nipissing University in North Bay, and he encouraged me to present with him at the 1995 OAME Conference in Guelph. This started a long personal tradition of presenting at the annual conference. I was fortunate enough to be on the OAME/AOEM Board of Directors for a few years, where I met and worked with many wonderful people. In 2006, I was comfortable enough with OAME/AOEM that I boldly submitted an article (Godin, 2006) to the *Gazette*. The Editor not only accepted my article, but invited me to write the column that you are reading today.

If you are a practising teacher of mathematics in Ontario, I strongly encourage you to take advantage of all that OAME/AOEM has to offer. Be inspired by the presentations at the annual conference, your local chapter events, or by one of the podcasts. Read the *Gazette* regularly and check out the archives—there are over 60 years' worth of articles for you to discover! Check out the resources on the website www.oame.on.ca.

Finally, take the opportunity to make friends and colleagues beyond your own school. At OAME conferences I have reconnected with people I went to school with, as well as former colleagues. I have met people like myself, with whom I have kept in touch throughout my career. It is well worth making OAME/AOEM part of your career path.

Now, time for your homework:

A sequence is defined by $t_1 = 53$, $t_2 = 79$ *and* $t_n = t_{n-1} - t_{n-2}$ *for* $n > 2$. *Determine* t_{1000}.

Until next time, happy problem solving!

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Continued from page 8

Trigos-Carrillo, and Rachel Pinnow. We will meet three times over the course of the school year for informal conversation around content that resonated with us, as well as how it impacts our practice.

While most of our events remain virtual, our second in-person opportunity to gather this school year will be on Wednesday, April 26, ahead of the OAME2023 conference in Toronto.

From Kenora to Kemptville, new members are always welcome to join OMCA. Contact us directly at OMCAmath@gmail.com, or visit our website at www.omca.website (note the different domain) to learn more about the organization, find out about upcoming events, and connect with us. ▲

Linking Literacy and Math: Making Sense of Ill-Defined Problems and Mathematical Modelling



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David is a principal, who taught in the Primary division before assuming roles of numeracy interventionist, numeracy coach, numeracy leader, and curriculum consultant for Prince Edward Island. David has also instructed university courses regarding curriculum, differentiation, mathematics, and language arts. He facilitates professional learning in the areas of mathematics and school development. Dr. David Costello recently published three books: Mathematizing Student Thinking: Connecting Problem Solving to Everyday Life and Building Capable and Confident Math Learners; Making Math Stick: Classroom Strategies That Support the Long-Term Retention of Math Concepts, *and* Using What Works: Strategies for Developing a Literacy-Rich Environment in Math.

In my previous column, I explored the problems we assign to our students and questioned whether students would find such problems relevant and meaningful. It was during this exploration that I highlighted that often, defined problems are assigned in many classrooms. While there are benefits to defined problems, we must be cognizant to provide students with opportunities to tackle alternative types of problems, problems that are more reflective of the world they live in, and the complexities that are associated with such problems. This is where we started our discussion on ill-defined problems.

Ill-defined problems are missing one or more of the three characteristics of a problem (initial state, goal state, stumbling blocks (i.e., what assumptions and restrictions may be required)). Missing one or more of these characteristics removes much of the structure that is found in traditional defined problems. And, without the structure, ill-defined problems are messy and resemble the problems that we tend to encounter in our experiences outside of school. It is through this messiness that we broaden our perspective of problem solving to include mathematical modelling.

In this column, I want to highlight a few things. First, what exactly are ill-defined problems? Second, why are we not seeing more ill-defined problems in the classroom? Third, why do we need to see more ill-defined problems in the classroom? Fourth, how can we support students in working with and through ill-defined problems?

By methodically examining ill-defined problems and exploring ways to approach and work through such problems, we, as educators, will be in a better position to support our students in recognizing how math can be used to both make sense of our world and solve problems that are relevant and meaningful.

What Are Ill-Defined Problems?

When most people hear of ill-defined problems, they ask to see what one would look like. The following are examples of ill-defined problems—one for each of Primary, Elementary, and Intermediate (Costello, 2022).

Primary

The Kindergarten classroom recognizes that many of their supplies for centres need to be replaced. The teacher shares that they need to buy new supplies, but have a limited budget.

Elementary

The local elementary school has been selected as the site for a cross-country meet. As part of holding this meet, the school has to decide how to accommodate people who want to watch the meet.

Intermediate

In planning end-of-year activities, the school administration decides that the Grade 8 students will go on a field trip. The administration would like to know what the cost per student will be for such a trip.

You may be thinking that these problems do not fit with what you consider traditional math problems. And, you are right. This type of problem is relatively rare, and many teachers will never have seen one when they were a student. These problems lack structure. These problems are messy. Yet, aren't they the type of problems that we encounter in our daily lives outside of school?

Students cannot work with these problems passively or by rote. It is through these problems that students must take an active stance in their learning as they work to make the problem manageable. They make it manageable by using their experiences to frame the problem and to add structure to it.

Why Don't We See More Ill-Defined Problems?

When I was a student in school, I cannot remember working with such problems during math. I wonder how many students today are assigned problems such as the ones above.

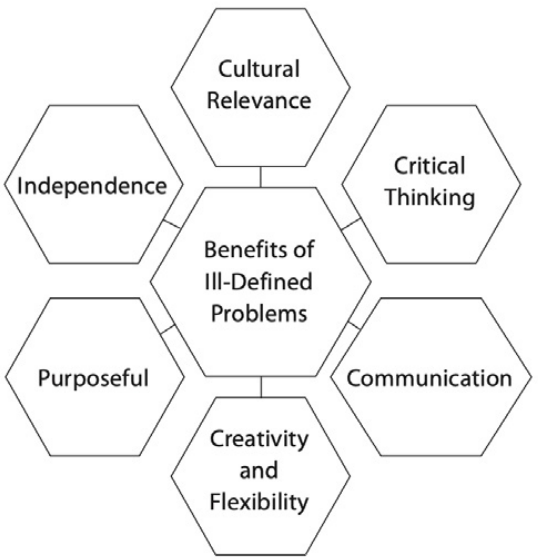
There may be numerous factors contributing to the

infrequency of ill-defined problems in classrooms. In the table below, I highlight five of the most common reasons as to why we may not see many ill-defined problems in the classroom (Costello, 2022).

Reason	Explanation
Lack of Experience	You may find that you have little to no experience with ill-defined problems. If you do have experience with such problems, it may have been later in your education (secondary or post-secondary). Such experiences may result in two realizations: first, that you don't equate ill-defined problems with Primary, Elementary, or Intermediate grade levels; and second, you may not think that ill-defined problems are necessarily an important learning experience for students. Regardless of the realizations, if you have little to no experience with ill-defined problems, your instructional practices may not include much, if any, of this type of problem.
Difficult to Create	As we have read, ill-defined problems are not traditional math problems. As such, there are not nearly as many ill-defined problems available to teachers. Regardless of where teachers find problems, there will undoubtedly be considerably more defined problems than ill-defined problems. And, if you choose to create your own problems, you may find defined problems come more easily because this is the experience you had in mathematics.
Difficult to Assess	Within defined problems, there is a limited number of solutions, frequently only one. This can make assessing seem to be a bit more straightforward for educators. By contrast, the lack of structure in ill-defined problems adds more subjectivity. By this, I mean that there are numerous possible credible solutions. The educator must take a macro perspective to the work to determine how the student framed the problem, how they mathematized (Costello, 2022) their thinking, and if the work makes sense, given the original problem. So, instead of checking the student's final response, there is much more to be considered by the educator.
Standardized Tests	Think of any standardized tests that you may have seen—whether you were a student or a teacher. Many, if not all, of the problems would be defined problems. Knowing the significance that standardized testing assigns to defined problems, there may be less motivation for teachers to provide ill-defined problems to students. Teachers want students to do well on standardized tests, and, as such, may choose to offer problems that students will encounter on such tests. As a result, we see few ill-defined problems in the classroom.

Why Do We Need to See More Ill-Defined Problems?

We have examined the characteristics of ill-defined problems, what they may look like, and the barriers to why more ill-defined problems are not being assigned in class. Now, we move into a discussion of why we need to see more ill-defined problems in the classroom. The diagram below highlights some of the benefits of these problems in the classroom (Costello, 2022).



Ill-defined problems provide educators and students with opportunities to engage in culturally relevant mathematics. When working with ill-defined problems, students must start with the context—a context that is meant to be reflective of their experiences. It brings relevancy to the work they are doing. Instead of context being simply added as an extra, as it is in some defined problems, context is as significant as the math itself. When there is relevancy, there is purpose. Students recognize the purpose of the work and see it as

being more than solving an isolated problem. It is something that they can connect with and recognize in their daily lives.

When working with ill-defined problems, students are required to be active and independent learners, as they cannot simply follow a procedure by rote when working through ill-defined problems. Students must apply their understanding of the situation, most times based on their own experiences, to make the problem workable. During this independent learning, students must think critically about the problem, the parameters included within it, and examine their work throughout the process.

Creativity and flexibility are embedded in opportunities to solve ill-defined problems. Students must be flexible when analyzing the problem and working through various options, and must exhibit creativity in how they approach the problem, how they frame it, and how they choose to solve it. Creativity and flexibility are the skills that not only drive the work of solving an ill-defined problem, but those two skills are also strengthened when working with such problems.

Lastly, I will address the benefit of ill-defined problems with regard to communication. Communication has been identified as a mathematical process that supports mathematical thinking. Communication cannot be removed from thinking. When working with ill-defined problems, it is imperative that students communicate their thinking, both in terms of how they work with the problem contextually and decontextualized. Students must be able to clearly communicate their thinking so that others can recognize how they framed the problem, how they mathematized their thinking, and how they assessed and analyzed their work.

Consider your students. Now consider what your instructional goals are for these students. What would you do if someone asked you if you would like them to experience the six benefits mentioned above (culturally relevant, purposeful, independence, critical thinking, creativity and flexibility, and communication). Ill-defined problems provide students with opportunities to be active, independent learners, who see relevance and purpose in the problems they are assigned.

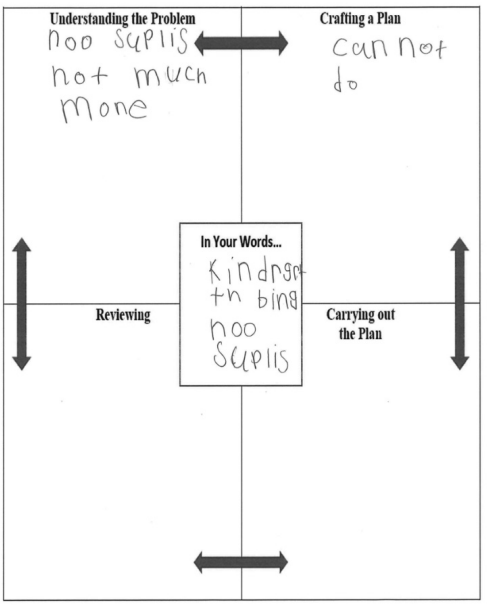
Ill-Defined Problems and Student Work Samples

Let's consider some Primary, Elementary, and Intermediate student work samples related to ill-defined problems (Costello, 2022). However, I want to highlight that these students attempted to solve the problems, using Pólya's four stages of problem solving (understanding the problem, crafting a plan, carrying out the plan, review). Students applied the four-step process in the traditional manner that Pólya's work has been applied in many jurisdictions. Notice how this approach may have contributed to lack of success in solving the problem.

Primary

The Kindergarten classroom recognizes that many of their supplies for centres need to be replaced. The teacher shares that they need to buy new supplies, but have a limited budget.

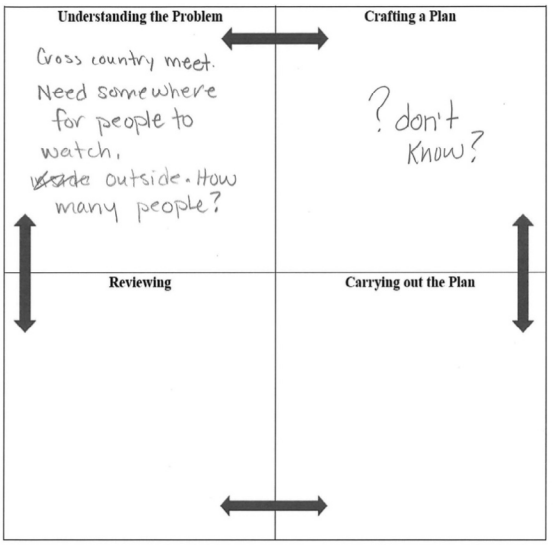
Let's explore the work sample from a Primary student.



Elementary

The local elementary school has been selected as the site for a cross-country meet. As part of holding this meet, the school has to decide how to accommodate people who want to watch the meet.

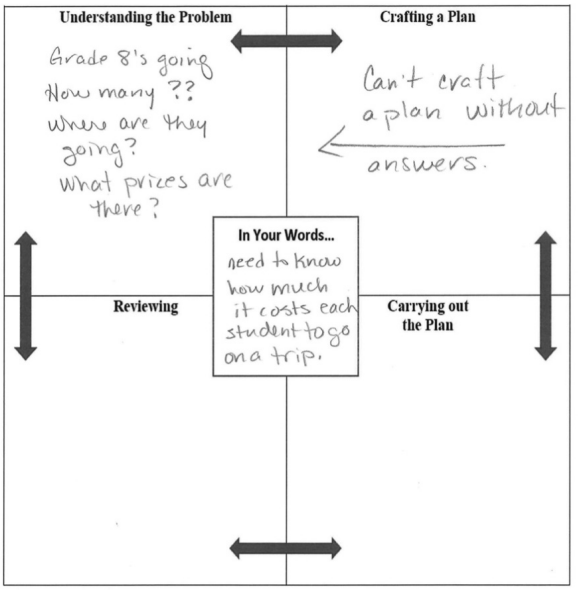
Let's explore the work sample from an Elementary student.



Intermediate

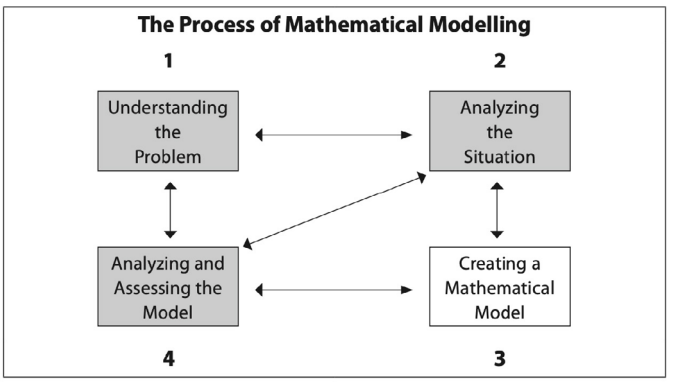
In planning end-of-year activities, the school administration decides that the Grade 8 students will go on a field trip. The administration would like to know what the cost per student will be for such a trip.

Let's explore the work sample from an Intermediate student.



Mathematical Modelling

In the previous section, it is evident that the traditional way that Pólya's four stages of problem solving has been applied to problem solving is not enough to solve ill-defined problems. Instead, students are left with more questions about missing information. It is this missing information that characterizes the messiness of ill-defined problems that makes crafting a plan difficult for students. Instead, we need to expand our perspective of problem solving to account for the lack of structure of ill-defined problems. Students cannot simply retrieve information from a given problem. Students must take an active stance and frame the problem so that it becomes workable. What students need to do is engage in mathematical modelling. Please see below (Costello, 2022).



The four components of mathematical modelling occur within an iterative process, as hinted at with the multiple arrows. Within this process, you will notice that boxes 1, 2, and 4 are shaded, while box 3 is not. The shading is meant to reflect that these particular components are considered within a contextual setting, while non-shading indicates that this particular component is considered within a mathematical setting. Students move from a contextualized setting to decontextualized setting, only to return to a contextualized setting to determine if their work makes sense and if it answers the original problem.

Final Thoughts

The intent of this column is to continue our journey toward providing students with a variety of problems and the necessary tools to solve such problems. It is imperative that students be given opportunities to explore, work through, and solve ill-defined problems. If we want to remove the barrier between school and the lived experiences of students, we must reconsider the problems we assign.

While there may be stumbling blocks to bringing ill-defined problems into the classroom, the benefits far outweigh the negatives. Students are supported in seeing how math is all around them, and that math can be used to make sense of the world. In doing so, we broaden both our own and their perspective of problem solving to included mathematical modelling, whereby both math and context are treated with significance.

In my next column, I will delve into how students make sense and apply mathematical modelling to solve ill-defined problems. Through this exploration, I will share student work examples and highlight the strategic moves of teachers during this process.

I look forward to continuing the discussion of linking literacy and math via my website (www.costellomath.com), email (david@costellomath.com), or Twitter (@dr_costello).

Reference

Costello, D. (2022). *Mathematizing student thinking: Connecting problem solving to everyday life and building capable and confident math learners.* Pembroke Publishers Ltd. ▲

A fence in Perth, Ontario



In the Middle: The Fun of Playing Games and Learning



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In the September 2021 issue, I described how the card game *Timeline* could be adapted for the MTH1W course, and how my students were playing the game co-operatively and competitively, both in person and online. With *Timeline*, my students were excited to play a game that some knew from outside of math class. Playing games can be motivating pedagogically, especially for middle school students, by safely developing social and emotional learning strategies within the structure of the game rules. While teaching online, I discovered Desmos Teacher-mode activities, like *Marbleslides* and *Polygraphs Collection*, that facilitate students working together online in game-based fashion. Colleagues have shown me other game options using Google Slides and Minecraft platforms to create escape rooms and puzzle-building competitions. This reminded me of how effective game play could be for engagement and skill reinforcement.

The basic mechanics of many classic board and card games can be used to explore concepts in graph theory and probability, like *Settlers of Catan*, *Set*, and *Ticket to Ride*. Other traditional games, like *War*, *Dominoes*, and *Pyramid*, use counting techniques as ways to improve on a winning approach. For *War*, in June 2010, our Editor, Tim Sibbald, provided details of how the design is mathematically linked to game characteristics like length (see *Gazette,* 48(4), pp. 30–31, in the archives). The connection between mathematics and games, however, is not always obvious to middle school students.

Inspired by the success of games during online learning, I was keen to revisit game playing in the face-to-face classroom this year. This column provides some suggestions for easy adaptations of board and card games. These may be new to you or your students, or perhaps new in the context of learning math. Although some games directly require mathematical formulas or practice of mechanical skills (e.g., *Math Dice*), my goal here is largely to present games that students might be surprised to play in math class. Most of these do not directly require mathematical expertise and are middle school classroom-friendly adaptations of publicly available games or game mechanics. These adaptations do not require you to purchase the commercially available game, but I do include each game's web link in case you are interested. Many game publishers also include learning resources, starter activities for a class, suggestions for playing with larger groups, and online versions.

Charty Party: All Ages
(www.veryspecialgames.com/products/charty-party-all-ages-edition)

> Note: The original version is for ages 17+, but the All-Ages edition was designed with teacher input so that it could be used in classrooms!

This game started as a crowd-funded Kickstarter project by Josh and Evan Katz. *Charty Party* is a card-based game, where the players use their cards to label the *y*-axis of a graph, a perfect lesson catalyst to present explanatory (or independent) versus response (or dependent) variables. Consider Figure 1, showing the "tendency to be late" card paired with a graph relating it to "hours of sleep." The pairing provides a forum for students to explain their thinking regarding the two variables and the proposed relationship between them. I regularly use this example as a think/pair/share activity, having one student explain what the "graph is saying" to each other, and then having the other choose another potential "orange card" for the vertical-axis label.

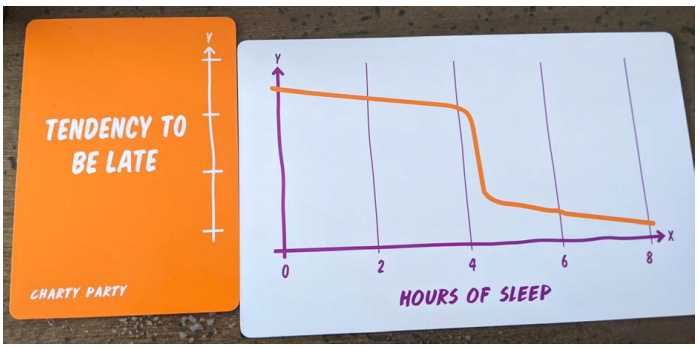


Figure 1: Charty Party *card pair example*

There is a variety of different white graph cards showing different kinds of relationships (see Figure 2). Of course, you can have students quickly create their own graphs on small whiteboards with their partner, creating axis labels, using sticky notes. In pairs, I have also had students create a

graph using Desmos, which has been especially popular with students in Advanced Functions. In MTH1W, linear piecewise functions can work with these as well.

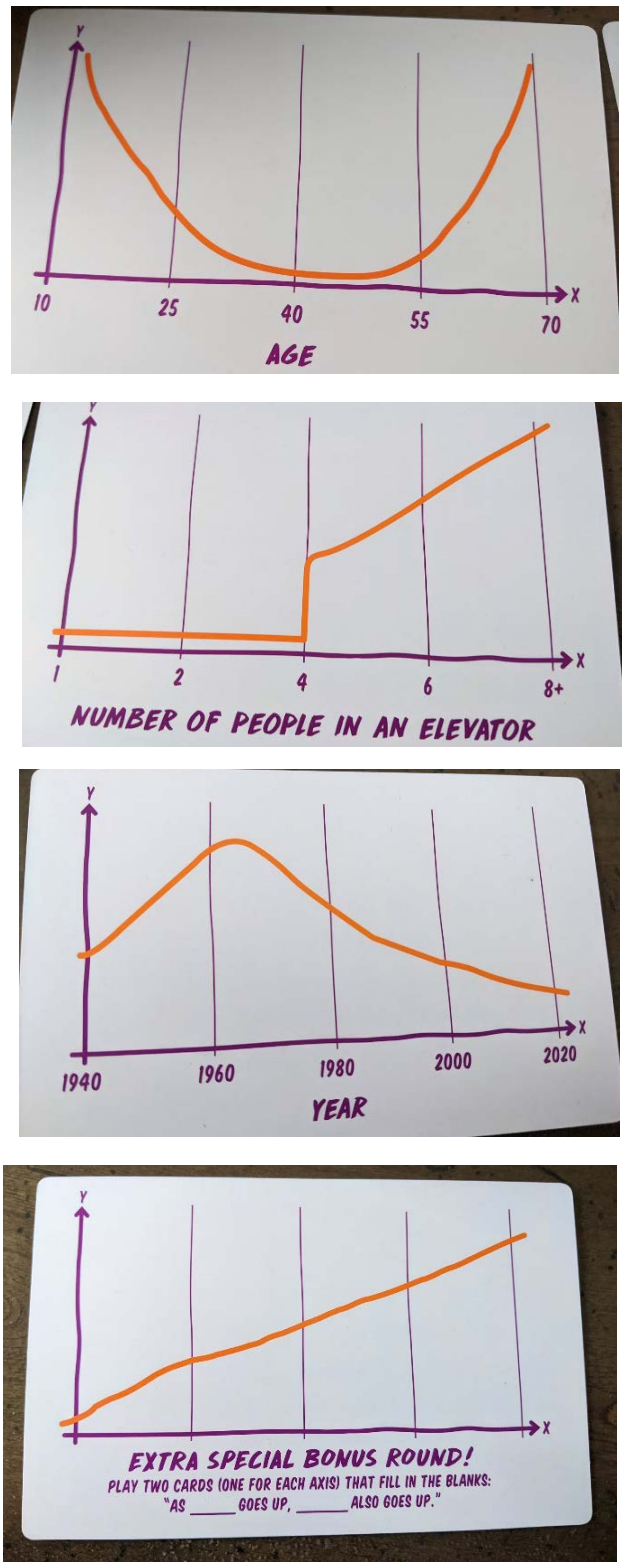


Figure 2: *A variety of graphs from* Charty Party

Charty Party is similar to the active learning on Transum, Human Scatter Graphs (www.transum.org/Software/SW/Scatter_Graphs/), where two walls of the classroom are the axes of a graph, and students "stand in a position in the room that represents your data." One example Transum suggests is: "Stand in a position representing the *number of minutes it takes you to travel to school* on the *x*-axis and the *time you wake up in the morning* on the *y*-axis." I love Transum's suggestion of doing this activity in a larger space and taking photographs from an elevated vantage point.

Scrabble (https://scrabble.hasbro.com/en-us)

Since letters on a *Scrabble* board have numerical score values, the transition to expressing and computing word totals is not new. I first heard about it at the OAME Annual Conference in 1998. At conferences and Math Trails around the world from 2000–2011, my friends, Vince Delisi and Ron Lancaster, and I explored local signs and landmarks to find the highest total scores "*Scrabble*-style." Experimentally, we also explored letter frequencies in travel pamphlets and signs in different languages. This connection to international *Scrabble* versions has an extensive Wikipedia page (https://en.wikipedia.org/wiki/Scrabble_letter_distributions) that includes the Latin distribution of letters determined by the University of Toronto's Centre for Medieval Studies. Looking at how different language letter distributions and their corresponding point-value relationship is one curious correlation to explore. Would this scatterplot look the same for different languages?

I also created and adjusted the original *Scrabble* boards (see Figure 3). They are available in Google Docs for you to download and use (see www.tinyurl.com/OAMEGazette). Feel free to amend it further for your instructional needs. After a class explores the Wikipedia page and notes the English *Scrabble* letter-value table, students can work with the various mimic boards on a larger activity, starting first with words I provide. For my class, this has been our introduction to algebraic notation and proper form, and each year, I include words that speak to the given year or class interest. Considering previous versions, one year, I used *The Simpsons* characters and episode words, while another year, I used words that connected to Spirit Week at our school.

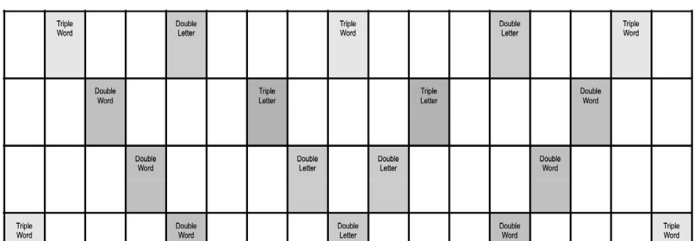


Figure 3: *A section of the "Regular"* Scrabble *board at* www.tinyurl.com/OAMEGazette

Example of instructions and sections of boards for a *Scrabble*-based algebra activity: For each word on the mock-up *Scrabble* board, create a formula and write it in as an equation. For example, Figure 4 shows the word "GRAPH," with the "G" on a triple word and the "P" on a double letter, which might have this equation: GRAPH = 3(G + R + A + 2P + H). Then substitute the letter values, from the value table, into your equation and calculate. Notice that the letter function, double letter or 2P, apply before word function, triple word, which is an interesting variation of order of operations.

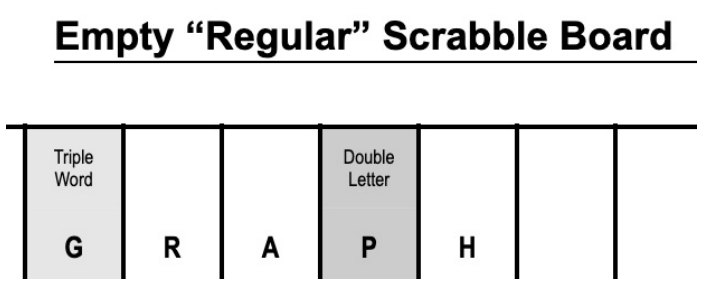


Figure 4: *Example of word on the mock-up board*

The Mathematically Adapted Scrabble Board, Figure 5, was developed by adjusting some of the mathematical operations on the special squares. Students need to use correct mathematical notation, create equations, then substitute letter values and evaluate the expression. Figure 6 shows an example with the word "GAMES," where the "G" is on a square word and the "S" is on the square-root letter, giving the expression: GAMES = $(G + A + M + E + \sqrt{S^2})^2$. Then, with the values for each letter from your value table, substitute the letter values and calculate the score.

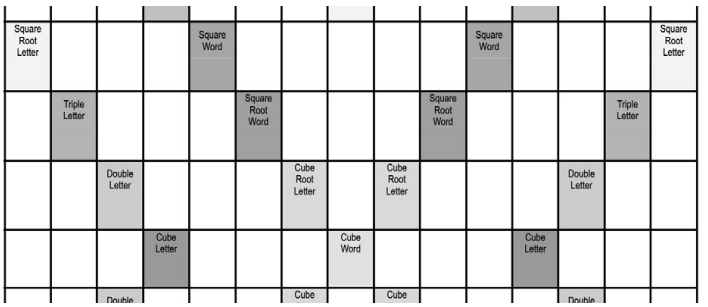


Figure 5: *A section of the Mathematically Adapted Scrabble Board at www.tinyurl.com/OAMEGazette*

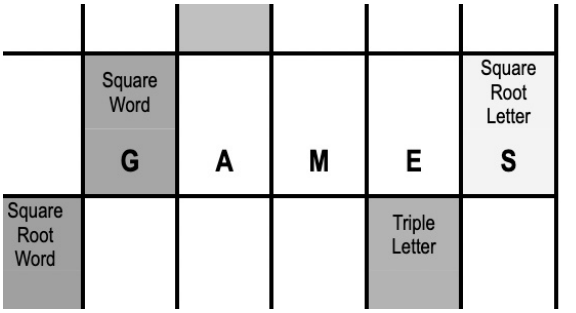


Figure 6: *Example of word on the mock-up mathematical board*

This is often where the fun begins—have students complete the sentence: <*Your name*> LOVES MATH MORE THAN <*Any noun may go in here, making an appropriate, if not truthful, sentence*>. Just as in the game, all of the words in the sentence must link together when placed on the board (see Figure 7). Students also have to show six expressions, subsequent calculations, and a grand total clearly.

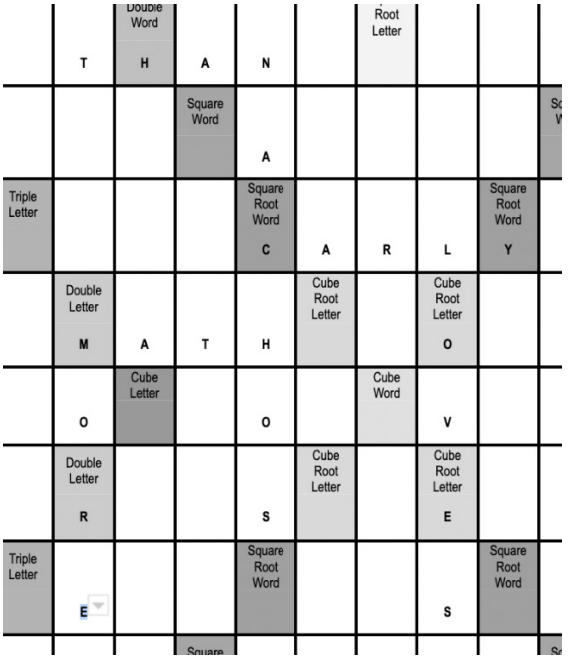


Figure 7: *Example of "CARLY LOVES MATH MORE THAN NACHOS" sentence on the Mathematically Adapted Scrabble Board*

As a game, I have run a "competition" to see who gets the highest score, and have seen overall scores for their "sentences" of over 1020, when students realize that seven letters span two square words, for example. Amusingly, many students "love math much more than 'QUARTZES.'"

Game of Things (www.thegameofthings.com/)

NOTE: The original version is for ages 14+, with expansions that are Adult only. With a little care ahead of time, this game can be used with students as young as 11.

Game of Things can be played with large groups. Present the group with a question like, "Things you wish you had been taught in school," and let the players write whatever answer comes to mind. The group then tries to figure out who provided each answer.

The game itself has some cards that can be used as check-in questions at the beginning of class, such as "Things that don't make sense" or "Things you shouldn't forget," or as checkout questions, such as "in class today" or "in Grade 9 math." I have also adapted this with smaller groups (five or six students) to get to know each other before larger activities or collaborative tasks. When using this with a larger class, I have used it, with lots of student amusement, to demonstrate data organization and management, including Venn diagrams and conditional probabilities.

Some questions I have used with small groups:

- Things you'd buy if you found a $10 bill?
- Things in math class that you really like?
- Things you wish grew on trees?
- Things you want to be doing in five years?
- Things you wish you did in Grade 6?

Spot It! (www.spotitgame.com)

Spot It! is a card game with eight different thumbnail pictures on each card. The mathematical feature of the game is that any two cards selected will always have exactly one matching picture. It doesn't matter which cards you pick! In 2012, my colleagues, Jeff Mahood and Andrew Moffat, and I created these questions considering the captivating design of this game.

1. How many different symbols are there on the deck of cards? What is the minimum number of symbols?
2. How many different cards do there need to be?
3. How would you go about building the cards?
4. Is there a design for how to build this with *n* cards? Does it work for any number of cards?
5. Is there a relationship between the number of cards, the number of symbols, and the number of symbols per card?
6. Does every picture have the same number of matches?
7. Would it work with any number of symbols per card? Could it work with a different number of symbols on each card?
8. What is the relationship between the number of symbols per card and the total number of cards?

Linda Rodriguez McRobbie has explored similar questions and the history of the game, including its connections to 1850's British Mathematics in *Smithsonian Magazine*'s December 2018 issue (www.smithsonianmag.com/science-nature/math-card-game-spot-it-180970873/).

Math Fluxx (www.looneylabs.com/games/math-fluxx)



Figure 8: Math Fluxx

The original card game, *Fluxx*, has a unique game mechanic: players change the goals and rules as part of the game. *Fluxx*, played with two to six players, is fast and unpredictable because everything changes so quickly. There are over 30 versions, from *Anatomy Fluxx* to *Zombies Fluxx*, and of course, *Math Fluxx*. The Keeper cards in *Math Fluxx* are individual digits that the players use to achieve the goal, some of which arise from mathematical operations or language puns. My favourite goal is to find the 33rd Prime Number (137, having the Keepers 1 and 3 and 7). *Math Fluxx*, unlike other versions of *Fluxx*, can be more difficult for younger children, since it includes place value, order of operations, factors, pi, and exponents. Fluency with basic operations for single digits is required. The extended version includes digits from 0 to 9 and then the "Goes to 11" Keeper card 11, which can make the calculations more difficult.

Prime Climb (www.mathforlove.com/games/prime-climb/)

The most obvious choice for this list is the award-winning *Prime Climb* (see Figure 9). The goal of the game is to move your pawn from the starting 0 spot to 101, through the spiral of counting numbers. Players take turns rolling the ten-sided dice, and then use operations to move forward or bump other players back to the beginning.



Figure 9: *The* Prime Climb *game*

My students quickly noted the colours on each of the

numbers in the spiral. The colours assist in finding the factors of each number, as explained in the game's instructions shown in Figure 10. The game includes a multiplication chart to help students identify these factors as well.

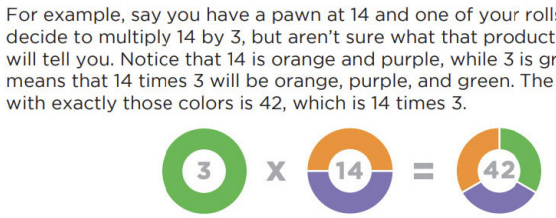


Figure 10: *Instructions regarding colour for factors*

The company Math for Love, which makes *Prime Climb*, provides classroom-ready content to use with the game on their website (www.mathforlove.com/games/prime-climb/teach-with-prime-climb). This includes Instant Warm-Ups and Rich Tasks. One lesson I have used examines the colouring of the first 20 numbers on the *Prime Climb* board, with a conjecture to continue the pattern until 100. It also includes a blank hundred chart to copy and colour.

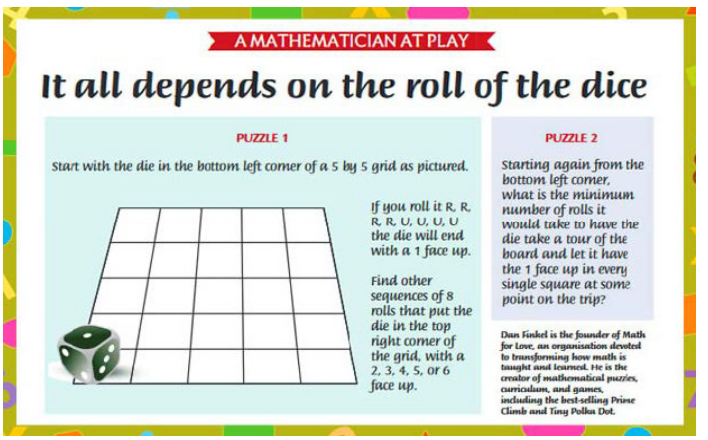


Figure 11: *Sample from A Mathematician at Play*

Dan Finkle, the creator of *Prime Climb*, believes strongly in the power of play. The Math for Love website has printable puzzles and video riddles, which can be filtered by grade level and curricular connections. These include Nim, Broken Calculator, Dots and Boxes, and Pig. Finally, puzzles on the Math for Love website (wwwmathforlove.com/puzzles/) have excellent production value and work well as a strategy to pull students into a collaborative framework. The printable ones from the aptly named "A Mathematician at Play" series are captivating (e.g., *It all depends on the roll of the dice*, see Figure 11) and generalizable to high school mathematics content. Many of these activities, like the dice puzzle above, invite conversation, trial and error, and willingness to make mistakes. All of these puzzles and games show the fun of mathematics and its rich history in play. Mathematicians, let's play! ▲

▲ Dynamical View of Proportionality: Covariations



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For many students, the introduction to proportionality marks a transition from additive to multiplicative thinking when considering the relations between the quantities. In many problems implying proportions, an additive relationship (e.g., 3 to 6 is 3 more, 6 to 2 is 4 less) often misguides students' consideration of a multiplicative relationship (e.g., 3 to 6 is doubling, 6 to 2 is dividing by 3), and can even lead to erroneous answers at times.

There are, however, some student strategies, worthy of pondering, that appear to fit right in the middle of both multiplication and addition. These might be conceived of as elementary, but a mathematical look at them through students' work might tell a different story. In this second column, I continue pondering and exploring some of these additive strategies that students often come up with. In so doing, I raise elements of mathematical significance. These elements, I argue, can in turn lead to an enriched understanding of proportions through a dynamical view of proportionality itself. This column addresses specifically the issue of covariation of quantities within a ratio.

The Problem and the Strategy

Marco, an 11-year-old, was given the following task to solve (taken from Herron & Wheatley, 1978):

> If a strip of metal 8 cm long has a mass of 28 g, what is the mass of a strip of the same type, which is 22 cm long?

Marco first explained that 8 x 3 gives 24, and 28 x 3 gives 84, so a strip of 24 cm had a mass of 84 g. Because 24 cm

of metal was too long, he explained that he needed to take some out. He divided 8 by 4 and got 2, and 28 by 4 to get 7, for both needed to be deduced out of the 24 cm and the 84 g. This new ratio (2 cm for 7 g) led him to a strip of 22 cm by subtracting the 2 cm from 24 cm, which weighted 77 g by subtracting the 7 g out of 84 g. The strip of 22 cm of metal then had a mass of 77 g.

8 cm → 28 g
24 cm → 84 g
2 cm → 7 g
22 cm → 77 g

What is this strategy about?

One main aspect that this strategy reveals is the simultaneous consideration and variation of both proportionally related quantities. As such, a change in the length of the metal strip is *simultaneously* accompanied by a change in the mass of the metal strip; a change in one goes together with a change in the other. Although this is a typical missing-value problem, the strategy does not only focus on that missing value, but on its relationship to other values in the problem. Because the length of the metal strip is intertwined with its mass, and vice versa, transformations in both happen simultaneously.

Marco's strategy shows this simultaneous transformation through the initial multiplication by 3 of both length (8 cm x 3 = 24 cm) and mass (28 g x 3 = 84 g). A change through tripling one quantity goes hand in hand with a change in the other quantity. These quantities are, however, both too big: 24 cm is longer than the required 22 cm; hence, 84 g is also too heavy, and both need to be adjusted as well and *in the same way*. So, when 2 cm is taken from the 24 cm metal bar, it also has 7 g taken out of the mass. These transformations affect both the length and the weight of the bar in the same way *and* at the same time.

This mutual transformation of quantities, here length and mass of a metal strip, is often referred to as the *covariation* of quantities in proportionality problems. The prefix "co-" aims at insisting on the fact that proportionality problems entail the consideration not only of quantities, but mainly of the relationship between them. By focusing on the relation, 8 cm → 28 g or 24 cm → 84 g or 2 cm → 7 g, covariation insists on the fact that in proportional situations, the quantities vary together—they covary. It is more than the fact that every quantity needs to be adjusted in relation to a change in the other quantity; it is that both quantities go together, hand in hand, fluctuating at the same time and in the same way as the other.

Whereas a missing-value problem could lead one to think that the focus is placed only on the value being sought, covariation leads to thinking otherwise. It is thus not merely a question of fixing one value and then noting the resulting effect on the other, but it is focusing on both values simultaneously, on both transformations. In other words, everything is connected and "moves" together.

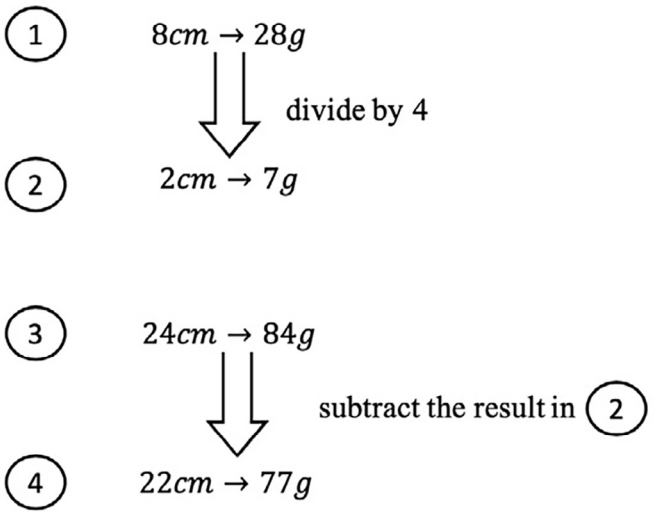
A Dynamical View

Not only is covariation a central element of proportionality, but it also strongly aligns with the dynamical view put forth by Lesh et al. (1988) and discussed in the December column:

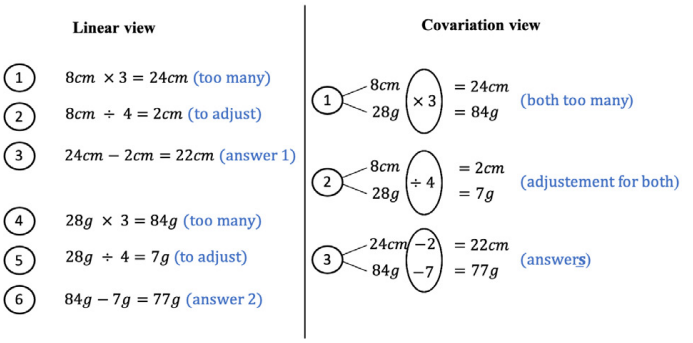
> The equation A/B=C/D can be thought of as a static (=) relationship between two simple mathematical systems which are described independently by the ratio relationships A/B and C/D. However, it can also be thought of as a dynamic transformation which maps one simple mathematical system (described by the ratio relationship A/B) into an "equivalent" system (described by the ratio relationship C/D). It seems to us that the recognition of structural similarity is an essential component of proportional reasoning (p. 100).

What Lesh et al. (1988) insist on is the dynamical interplay linking the ratio A/B and the ratio C/D, one being a transformation *toward* the other. What the notion of covariation does is to make a step even further. It puts forth that this dynamic interplay is also present *within* a single ratio, where A and B are connected and get transformed together. Changing A *simultaneously* changes B, and vice versa, and *in the same way*: tripling will be for both, dividing as well, etc. As for A/B and C/D, the notion of covariation points to the fact that changes made to A and B leads these *toward* C and D, where they *together* become C and D. The ratio C/D embodies in both C and D the same change that happened on, respectively, A and B in A/B.

As such, the subtraction of "2 cm → 7 g" from "24 cm → 84 g" in Marco's strategy points to an additional or even more salient view of this simultaneous transformation. In effect, not only does a transformation from 8 cm to 2 cm require that the 28 g mass be divided by 4 to get to 7 g, but both these new values get simultaneously subtracted from 24 cm and 88 g, respectively. Somehow, taking out 2 cm of length goes with taking out 7 g of mass: it is taking out both, or the same, and each gets taken at the same time, together:



Subtracting one value (2 cm) requires subtracting the other value (7 g). This view somehow eliminates the linearity of the steps that are taken: it is not a series of actions, where each step happens one after the other, but these happen together (a 22 cm bar can never be of 84 g, waiting for the subtraction to be completed, nor would a 24 cm bar be of 77 g). Marco's strategy shows this: when he divides 8 cm to get to 2 cm, he does not directly subtract this length from the 24 cm, but makes sure to first divide also the 28 g by 4 to get to 7 g. And then, and only then, does he subtract both values from both the other two. It simply goes together, at the same time, and in the same way! In that sense, 24 cm and 84 g are linked for the (same) metal strip, and so they get "attacked" at the same time by another set of linked quantities (2 cm → 7 g). In the covariation view, it is almost, beyond doing everything simultaneously, as if the "– 2" and the "– 7" are the one and same thing, taken out at once. The following table contrasts a linear view with this covariational one.



Conclusion

Missing-value problems represent the vast majority of problems given when working with proportions. What Marco shows with his strategy for the metal strip problem is that solving these problems is not only about finding one single value: it can also be about the other quantities to which the missing value is proportionally connected, or about the variation and transformation of all the quantities involved.

This focus on covariation emphasizes the relational aspect of proportionality and makes it impossible to only transform one quantity, as both quantities are connected and a change in one results in a change in the other. Proportionality becomes centred on the study of relations: a relation between both quantities A and B, represented by the ratio A/B, and the relation between both ratios A/B and C/D. Namely, it is about a relation between two relations. The assertion "A/B = C/D" thus can be seen to say much more than the presence of an equality between two objects. Indeed, not only are A/B and C/D related by the equal sign ("="), but A and B are connected to each other in the same way that C and D are connected. A covariational view makes proportions become an explicit case, as Piaget and Inhelder (1975) defined almost 50 years ago, of a second-order relationship: a study of relationships between two relationships. The notion of covariation taps right into these relationships and their dynamic interplay.

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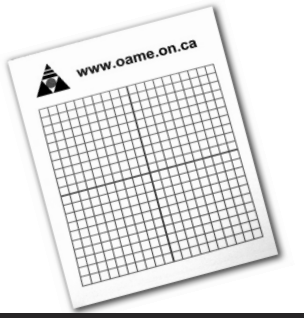
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▲ Elementary Math Matters: The State of STEM Education



LYNDA COLGAN
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Lynda Colgan is Professor Emerita, Faculty of Education, Queen's University. Lynda is currently working on a number of research, resource-creation, and teacher-education projects, funded by the National Sciences and Engineering Research Council of Canada (NSERC), the Social Sciences & Humanities Research Council (SSHRC), and the Mathematics Knowledge Network (MKN). She will carry on her long-established commitment to STEM outreach by continuing to seek out creative avenues to engage students, parents, and educators in mathematics at home and in the classroom.

It would be impossible for an Ontario educator not to know about *Let's Talk Science*—the pan-Canadian STEM outreach behemoth, founded by Dr. Bonnie Schmidt at the University of Western Ontario in 1993. As an organization, Let's Talk Science (https://letstalkscience.ca/) is multifaceted and ever evolving to meet changing classroom and societal needs. A quick glance at the website communicates the depth and breadth of the resources and programs for educators, parents and caregivers, community clubs, and more. From professional development (some with credentialing benefits) to career exploration and kitchen-sink experiments, to research reports, Let's Talk Science does it all.

In its most recent research report, generated in collaboration with Amgen Canada, *Spotlight on Science Learning: Snapshot of a Decade 2012–2022* (https://letstalkscience. ca/spotlight-on-science-learning-snapshot-a-decade), we read a stark and dark picture of the state of STEM education in Canada today.

What Shapes Do You See?
(First Presbyterian Church in Chatham.)



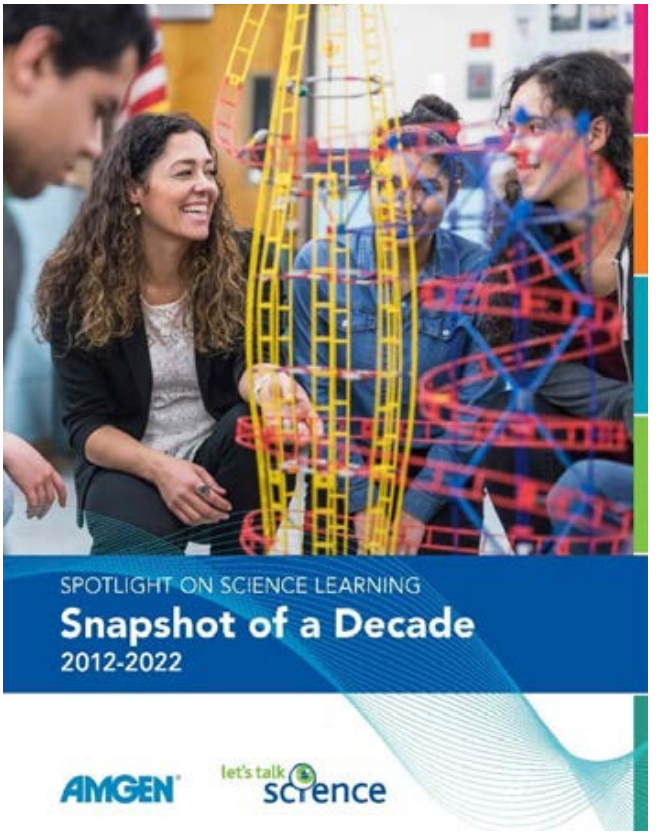


Figure 1: *Ten years after Let's Talk Science and Amgen Canada produced the first* Spotlight on Science Learning *report about STEM education across Canada, the environment has changed considerably. This new report examines what has changed, what hasn't, and what needs to change for youth STEM learning across Canada.*

I urge mathematics educators to download and read the report. Sadly, our siloed approach to curriculum often means that whenever "science" is featured in a title, mathematics educators are likely, and erroneously, to conclude that whatever we have scrolled past does not pertain to us. We would do well to remember that famous mathematician Carl Friedrich Gauss is said to have called mathematics "the queen of sciences."

Perhaps the most startling data from the report used data obtained from Ministries/Departments of Education across the country. Did you know that just 47 percent of Canadian high school graduates leave school with a Grade 12 mathematics credit? While this is the highest national completion rate for a STEM subject (Chemistry has 25 percent; Biology, 22 percent; Physics, 15 percent; and Computer Science, 3 percent), this is no occasion to celebrate. Since admission requirements for post-secondary science and engineering programs typically include a combination of Grade 12 Biology, Physics, Chemistry, and Math credits, it is clear that many doors to a future in STEM are closed for Canadian students.

Jurisdiction	Number of Science Credits Required for Graduation	Includes at least 1 credit at Grade 11 or 12
British Columbia	4	YES
Alberta	2	
Saskatchewan	2	YES
Manitoba	2	
Ontario	2	
Quebec	4	
New Brunswick	1	YES
Nova Scotia	3	
Prince Edward Island	2	
Newfoundland and Labrador	4	
Yukon	4 (follow BC requirements)	YES
Northwest Territories	2	
Nunavut	2	

Figure 2: *Graduation requirements vary by province and territory, but all jurisdictions require at least one science credit for secondary school graduation; most require two science credits. In only four jurisdictions must at least one science credit be at the Senior level.*

Snapshot of a Decade calls upon Statistics Canada data to show that even though the total number of post-secondary students enrolling in STEM subjects has increased over the past decade, domestic students actually represent a declining percentage of total STEM graduates, from 89 percent in 2011 to 72 percent now (Figure 3).

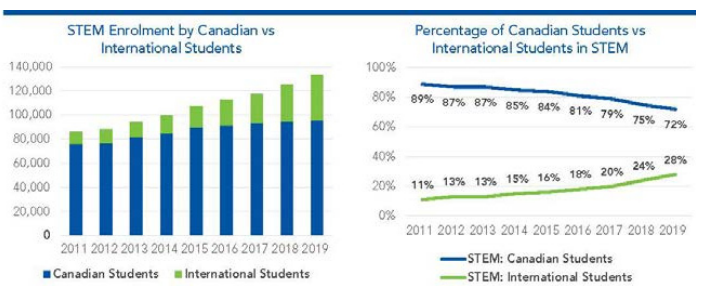


Figure 3: *Statistics Canada Table 37-10-0164-01 (*Snapshot of a Decade*, 2022, p. 9)*

It is clear that "a mass of students are [sic] still abandoning STEM courses at first chance, which is usually after Grade 10" (Let's Talk Science, 2022, p. 7)—a situation that should be of enormous concern to educators and politicians, especially Ministers of Education (Provincial), and Innovation, Science and Industry (National). Yet, after the December 2022 launch of the report at an Ottawa science research summit meeting, there has been little or no uptake in the Canadian popular press or on social media about how the data in the report points to an alarming and growing gap in STEM supply and demand from engineers and data scientists to skilled practitioner tradespeople.

Surprisingly, what did get reported by Canadian news channels early in the New Year was the United Kingdom's new Prime Minister, Rishi Sunak's, first speech of 2023. Sunak stated that in order to combat and better equip young people for the workforce, he planned to force all students in England to study mathematics up to the age of 18. His decision was somewhat influenced by the fact that in the United Kingdom, only about half of students over age 16 study any mathematics at all, and he was significantly motivated by the important and worthwhile goal of better levels of numeracy among the general public that would benefit everyone in our data-driven world.

Reaction to Sunak's speech was furious in both senses of the word, i.e., it came at breakneck speed and bespoke rage. At a time when the United Kingdom was plagued by strikes, saw more people relying on food banks, and faced soaring costs of living, there was outrage that the Prime Minister's answer seemed to be more algebra in school. I suspect that had our own Prime Minister made an announcement in response to the Let's Talk Science longitudinal study, the reactions would echo those from across the Atlantic. There is little public appetite for the obvious fact that there is more to mathematics than studying more mathematics.

Nonetheless, it is sad that those in positions of power have not responded to the damning reality that the *Snapshot of a Decade* report portrays about our low enrolments in STEM subjects, despite massive outreach and public education efforts; huge investments in resource development; concerted efforts to improve equity, diversity, and inclusion; and irrefutable evidence about the central role of STEM in everything from education and health to the economy and climate change.

It is sadder still that they have not heeded the insightful, empirically validated, and actionable recommendations, as well as concrete examples in the report—shining a spotlight on them to catalyze long-overdue educational reforms to teacher preparation and STEM education, making STEM learning:

- more multi-disciplinary
- issues focused
- relevant

- connected to organizations and corporations beyond the walls of the school that are both the producers and end users of STEM research/development/innovation
- authentic in terms of job preparation and responsible citizenship

If you are like Alice in Lewis Carroll's *Through the Looking Glass*, you'll be saying, "There's no use trying. One can't believe impossible things," and, like Alice, you'd be wrong. According to the *Snapshot of a Decade* report, there is a prime example of reimagining the future of STEM education in our own backyard: the *i*-STEM program (where *i* represents innovation).

i-STEM, described as a global skills learning hub to develop innovative minds, is a program for secondary students (Grades 9–12) in Halton (currently at Aldershot High School in Burlington and Elsie MacGill Secondary School in Milton). Students in this new four-year program develop innovation skills related to engineering design and design thinking (Grade 9), entrepreneurial thinking skills (Grade 10), and global competencies (Grades 11–12). In the program, the students work collaboratively, using skills, knowledge, and strategies from multiple disciplines and technology, to develop and implement solutions to real-life social, economic, or environmental global issues, culminating in a public exhibition of their work.



Figure 4: *Alice and the White Queen in* Through the Looking Glass*, artwork by John Tenniel (1865)*

In *Through the Looking Glass*, the White Queen responds to Alice's lament about impossible things by saying, "I daresay you haven't had much practice. When I was younger, I always did it for half an hour a day. Why, sometimes I've believed as many as six impossible things before breakfast." Perhaps more of us should take up the Queen's practice. Clearly, the developers of *i*-STEM did and the impossible is happening.

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▲ Coding in the Classroom: A Holistic Approach and Video Game Design Coding



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Iain has been an elementary school educator for 28 years, and is now into a second career, teaching in-service and pre-service teachers. He teaches face-to-face at Ontario Tech University and virtually at Western University.



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Beyza is a former teacher and long-term researcher. She is currently a research associate in education at Western University, and her research focus is the integration of computational thinking into mathematics education.

This is a column that interviews Ontario teachers who use coding in their practice. Iain and I have a conversation with the teacher to find out how they got started and their favourite pedagogical strategies. In these columns, you will find some of these teachers' best ideas about using coding in their classrooms, but if you want more, you can also access the complete interviews at OAME Talks to get a fuller sense of how these remarkable teachers have struggled and been successful with coding in their classrooms.

(Quotations within this column may have been lightly edited to reflect the difference between oral and written language.)



In this issue, we are delighted to talk to Nicole Myers about her particular slant on coding and her coding experience within the classroom.

Nicole Myers has been a teacher for the last five years after graduating from Ontario Tech University. In addition to teaching Grades 4–6, she holds a Chief Education Officer position at

STEM Minds, where coding and other STEM programs are offered for both students and teachers in K–12 education. Nicole is not only a passionate teacher, but also a content developer, and through these roles, she contributes to increasing access to STEM education, and provides support for STEM topics and technologies.

Holistic Approach

Nicole shared with us how her own understanding of STEM learning and teaching has evolved over the years. Her experiences showed the importance of embracing a holistic approach while integrating coding and robotics into the classroom.

> STEM is about more than just coding and robotics. You were just talking about that idea of a STEM approach, where really, it's not about learning [or] about the technologies per se because those are very quickly going to become obsolete and we're really going to have to move past that.
>
> It's really about how we learn to learn about these things. How do we build our skills in such a way that [we can adapt] when these technologies inevitably change? We are agile enough that we are able to respond to those changes.

Agility of Teaching

From this point, the conversation brought us to a way of understanding integration. Nicole elaborated on it as a connection among all things, such as the relationship between coding and 3D design, video game design, animation, photography, etc. By taking a more holistic approach and including real-world uses, students gain a deeper understanding of what coding is actually all about, while they're actually building the skills they need to be successful in the twenty-first century.

After identifying the benefits of a holistic approach, we moved on to discuss challenges and implications. Two main struggles of teachers were identified by Nicole: fear and lack of understanding of how to integrate STEM coding. In our discussion, these two are connected: while the fear includes not knowing how to bring these new strands into the classroom, it includes bringing STEM coding into the existing curriculum. Nicole's main response to this challenge is taking initial steps and embracing the agility of teaching.

> I've really found in my own experience that a lot of times with teachers, it's not so much that they don't want to, it's that they don't know where to start, and getting started can really be that overwhelming stage.
>
> So oftentimes it's just that initial step that they need to take. And once they take it, then that kind of train keeps going, but it's that initial step that they need support with. And so, I've always found that really sharing meaningful activities with teachers that they can implement, sort of as a recipe, to get started [is important]. That's not absolutely where we want teachers to end up, but it's that good starting place, where they go in feeling confident, they feel like they know what they're doing and they're able to implement it successfully. And then having that success encourages them to kind of keep trying things, keep modifying.

A Meaningful and Impactful Way: Video Game Design

Nicole elaborated on the recipe part and informed us about her area of focus: video game design, which she finds a meaningful way of integrating STEM coding. Video game design, in her experience, is both impactful and easy, while allowing a holistic approach.

> I've found that taking that approach really enables me to deliver impactful experiences to my students. But [it] also helps me show other teachers as well that this is something that you can do, and it can fit into what you're already doing. And with a few small tweaks, I've also really found that's part of why I love video game design so much. I do usually tend to approach coding from the kind of lens of video game design because it's a very practical application of coding. I found that, especially for teachers, but also for some students when they're first getting started with coding, especially in K through 12.

While exemplifying the meaningful and impactful way of integrating coding through video games, it is important to point out that learning here happens through the development of a game, not through gaming.

> The focus is on the actual coding component of developing the game, and being able to play them at the end is a nice experience to see well. Especially, how did other students approach it?
>
> Even thinking back to that idea of a sort of low floor, high ceiling, that's always really the approach that we took at STEM Minds, and I take it in my own teaching as well. The beautiful thing about video game design, when we're talking about coding, is that you can take this kind of basic framework of, you know, we are creating this style of games. A blaster game, for example. There is very much a kind of basic skeleton framework that you can use, where you have a functional game, but it's very basic. And then layered

on top of that are tons of different extensions that you can add on.

We had an insightful talk with Nicole. She highlighted the value of using a holistic approach and the affordances of video game design in integrating coding into the classrooms, while diagnosing the struggles of teachers. In this column, we briefly touched on some points about a holistic approach to integrating coding and STEM into the classroom. There is a lot more in the details of our conversation, so we encourage you to listen to the OAME Talks podcast with Nicole Myers. For our next column, we talked to Nicole about the nitty-gritty details of how to go about integrating coding and STEM into your teaching practice. ▲

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▲ Mb4T (Mathematics by and for Teachers): A Bucket of Integers



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Jennifer Holm is an Assistant Professor at Wilfrid Laurier University and works with Primary/Junior and Junior/Intermediate pre-service teachers, as well as in the field supporting current mathematics teachers. She is interested in developing mathematics knowledge for teaching with both pre-service and in-service teachers. She focuses on the beliefs and opinions that pre-service teachers hold about mathematics and teaching, and the connection they have to past experiences. She uses this research to support future teachers in developing beliefs and knowledge that will encourage and support effective teaching practices in mathematics.

Although integers have been featured in previous columns of Mb4T, there are some lingering questions about how to help students understand the concept of operations with positive and negative numbers. I have read articles about using integer chips, number lines, and even rules and procedures related to teaching integers, but all of them left me with some questions, since they all required some amount of teaching on the use of the model (or had no conceptual connections). When I stumbled across the *Bucket of Zero* model in a presentation, I thought it addressed the majority of my concerns better than other models mentioned previously. In this column, I will focus on just adding and subtracting integers, using the Bucket of Zero model, in order to consider this approach for teaching about integers. I do not take credit for this approach, as this is the work of a collaboration between Dr. Lisa Lunney Borden and Nat Banting. You can read more about the collaboration and approach on Nat's blog (www.natbanting.com/the-bucket-of-zero/). The images in this column are all from using the Bucket of Zero tool on the Mathigon Polypad website (www.mathigon.org/polypad/embed/oeHU1XqNltfew), in case you want to investigate it for yourself.

The initial concept of the Bucket of Zero is very simple: you start with a bucket full of positive and negative integers that equal zero (see Figure 1). This can be a way to start a conversation around zero pairs and then focus on removing positives or negatives from the bucket and consider what

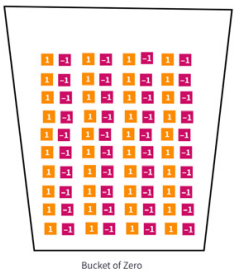


Figure 1: *The "Bucket of Zero" on Mathigon Polypad*

value is left in the bucket. What I find more helpful about this model than the integer chips alone is the help it provides with conceptualizing the operations. Moreover, it does this without having to teach what I might refer to as "procedures with pictures" (Holm, 2018), where a procedure is applied to a model to make the operations "work." The Bucket of Zero allows for students to interact with the concepts of the operations as they learned them in primary school, and then consider the value of what is in the bucket for their answer. To me, this aligns nicely with Dr. Lunney Borden's idea of "verbifying" mathematics (Lunney Borden, 2011), since students are actively doing mathematics with operations instead of just calculating responses.

To consider how the Bucket of Zero may be useful, we start by considering an addition question that has both positive and negative numbers. We can start with 6 + (–4). This question tells us that we need to start with a positive 6 value in the bucket, and then add 4 negative chips to the bucket. Figure 2 shows the progression while considering the operation. This process focuses on addition as adding something new to the bucket, and the answer is what you have left when you create new zero pairs, or in this case, +2. We could ultimately use integer chips on the table to do the same process, but the bucket becomes helpful when we get to more complex questions and operations. This initial look is just to provide a grounding of how the bucket could be useful.

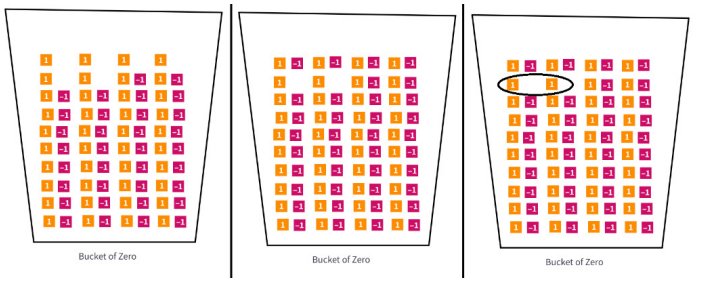


Figure 2: *The far-left image shows 6 in the bucket, the second shows adding –4 into the bucket, and the last shows the result of 6 + –4.*

Using this thought of "adding" to the bucket, how would you use the model to figure out the value of –9 + 16? You may want to pull up the Mathigon site and test it out. Figure 3 looks at the steps and answer to this question. In this case, you would make zero pairs and then add extra positives to the bucket, making a total of +7 as the solution, as shown with the circled amount in the third part of the figure.

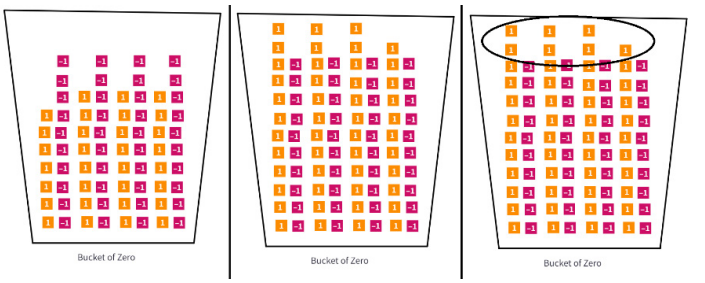


Figure 3: *The far-left image shows –9 in the bucket, the second shows adding 16 into the bucket, and the last shows the result of –9 + 16.*

Addition adds more objects to the bucket. If we were to *add* two negatives or *add* two positives, we would be adding more counters to the bucket, but they would be negative or positive counters. Subtraction removes objects from the bucket. We can see how that might be accomplished using the tool to model the meaning of subtraction. Consider first –7 – (–3) and how you might use the Bucket of Zero to show this question.

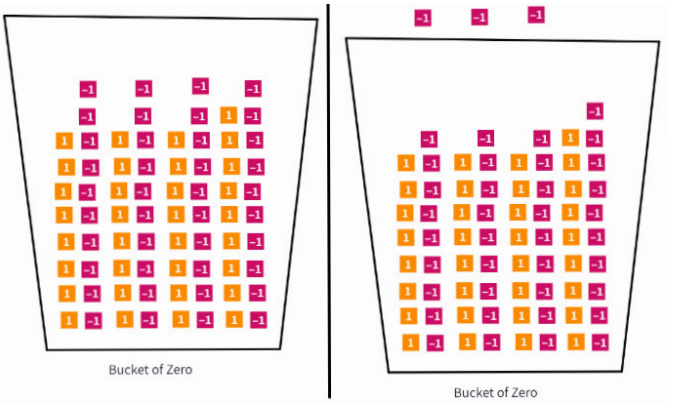


Figure 4: *The image on the left shows –7 in the bucket, and the image on the right shows removing –3, or –7 – (–3).*

Figure 4 shows one way to conceptualize how to consider this question. In the first image, the individual is starting with –7 in the bucket. In the second image, –3 has been removed from the bucket and placed above it. By looking at what is left in the bucket, there is a total of –4 remaining. In this case, the action of "removing" the chips that are being subtracted is shown, but by keeping the chips above the bucket, they are still there so that they can be seen to ensure that only three –3's have been removed.

Now we can consider a more complicated subtraction question to see how that would look. Using the example of –7 – 3, we can compare how it differs from the previous question and discuss why the Bucket of Zero helps to understand the process. Students must understand that the –3 means – (+3)

In this case, you are once again starting with –7 in the initial bucket, but now you are removing 3 positive counters

from the bucket, as shown in the second image in Figure 5. Since positive counters are being removed, fewer zero pairs are remaining in the bucket, and there is a total of –10 remaining in the bucket. This idea of breaking apart more of the zero pairs helps to conceptualize that there would be more negative counters left, so –7 – 3 would leave a negative number that is farther from zero.

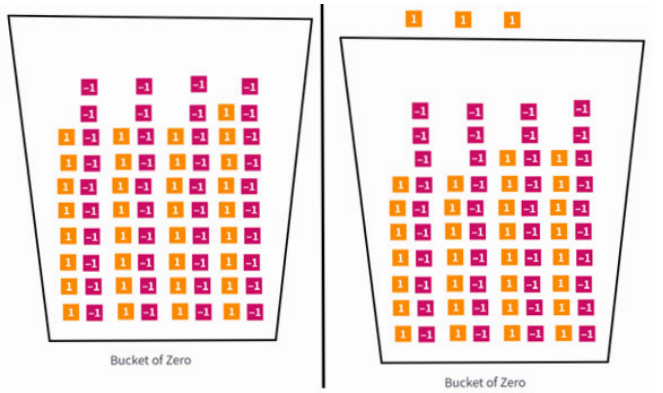


Figure 5: *The image on the left shows –7 in the bucket, and the image on the right shows removing 3, or –7 – 3.*

In the final subtraction example, we are going to look at the subtract-a-negative "rule" that many students memorize. Often students do not actually understand what they are doing or why it works. For this example, we are going to consider 6 – (–10). You likely have heard different variations of the idea that because there are two subtraction signs next to each other, they merge together to make an addition sign. Although this can be a helpful illustration of a rule to help students remember the process, it misses the idea of what subtraction means. It does lead to some interesting misapplications of the rule when students do not understand why this works. If we consider how this would look in our Bucket of Zero, we can begin to consider how to help students develop an understanding of the operation.

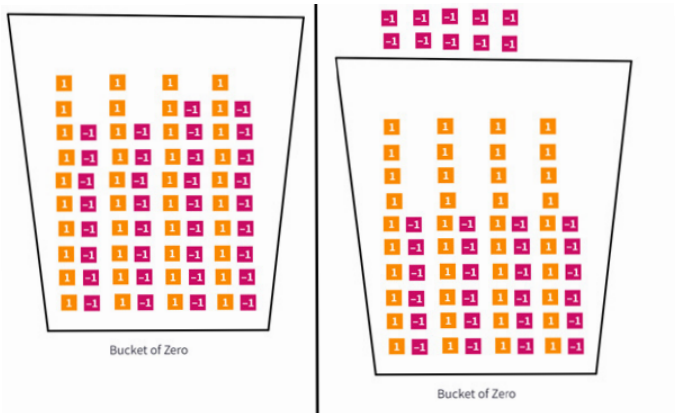


Figure 6: *The image on the left shows 6 in the bucket, and the image on the right shows removing –10, or 6 – (–10).*

Figure 6 considers this question by starting with a bucket with 6 positive counters inside. In the second image, you see the 10 negative counters removed. Again, the meaning of subtraction as removing something can be shown in how negatives were removed from the bucket for subtracting. As we work with students on different examples like this, we can help them to start to understand that since you are splitting more zero pairs apart, subtracting a negative number would give the same answer as adding a positive number to the first value. The Bucket of Zero, I find, also helps to deal with the issue with integer chips, where they would now have to create zero pairs to remove the –10. Since the zero pairs are already in the bucket, students have already focused on the idea that they can have as many zero pairs as they want and still have a sum of zero.

As I noted, there are a multitude of conceptual ways to support students in learning about integers through use of temperature, banking models, number lines, and integer chips. The Bucket of Zero is another way to help students conceptualize integers by basing the understanding on what students already know about the operations using positive numbers. The initial work on practising with the bucket, to see that the zero pairs would give "zero" to start, helps students start to understand what is happening when they are working with positive and negative numbers.

In this column, we highlighted the foundation of how the Bucket of Zero provides a concrete way to discuss addition and subtraction of integers. In the next column, we will focus on multiplication and division.

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Another fence in Perth, Ontario



An Interdisciplinary Opportunity: Mathematics and Accounting



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Jeff Irvine, Ph.D., has been a secondary mathematics teacher, department head, and vice-principal. He has taught at three faculties of education and at a community college. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. With over 45 years in education, Jeff is particularly interested in the interplay of student motivation and mathematics achievement.

It is critically important that mathematics teachers make what we do in mathematics more relevant to our students. One way to do this is to relate our content to the students' real world, where "real world" is considered to be: students could use the mathematics beyond class immediately, for example, in their part-time jobs, budgeting, or sports; students could use the mathematics in another subject, in the near term, such as in science, geography, technical shops, economics, family studies; someone close to the student could or did use the math content, such as a family member, relative, adult acquaintance; there were examples in the real world of people using the mathematics; and the mathematics flowed from an investigation, experiment, or model, in which the students were involved (Irvine, 2015). This article addresses the second of these "make it real to students" examples, applying mathematics to another subject area, Accounting.

We will sometimes have a number of our students who are concurrently taking an Accounting course. Through jointly addressing some of the concepts that occur in both Accounting and Mathematics, we can demonstrate to students that learning is not solely segregated into subject areas, and that significant areas of overlap are possible. This attitude enriches both subjects and allows for collaboration across school departments. A caution, however: while some of the concepts, such as horizontal and vertical analysis, require only knowledge of percent, I do not suggest using these accounting activities with Intermediate students. Accounting is typically a Senior-level course, with Workplace, College, and University levels. Thus, the activities in this article are best suited to Senior students. In addition, you will have students who express no interest in Accounting. Rather than force these students to engage with material in which they have no interest, alternative activities related to other subjects need to be provided for these students. Based on the knowledge of their class, mathematics teachers can select activities from this article that they feel are appropriate for addressing expectations, while maintaining the interest of all their students.

Accounting concepts and structures are very precise, so it can be useful to encourage students to consult their Accounting teachers if they need clarifications. This is also an opportunity for students to explore the ideas of Accounting and Mathematics applied to an area of interest to them. This article uses as an example of financial accounting information for Snap, Inc., the parent company of Snapchat®. Students should be encouraged to choose companies of interest to them, such as other social media platforms, cellphone providers, clothing manufacturers, sports companies, or any other companies in which individual students express interest.

Part 1. Mathematics and Accounting Applied to Ratio Analysis and Mathematical Modelling

Financial Statements

The financial statements of most companies are readily available online. The three standard statements are: Balance Sheet, a snapshot of the company's assets and liabilities at a point in time (similar to a Net Worth Statement for individuals); Income Statement, showing the flow of funds (analogous to a personal budget); and a Statement of Retained Earnings, which indicates what the company is worth to shareholders. Students may be encouraged to choose companies that focus on a single product or service, rather than large diversified companies, whose financial statements may be very complex. Below are simplified Income Statements (Figure 1) and a Balance Sheet (Figure 2) for Snap, Inc. Note: In accounting statements, negative values are indicated by parentheses, e.g., (0.32). In the statements below, these values are represented as they would be in mathematics, i.e., -0.32.

Horizontal and Vertical Analysis of Financial Statements

Horizontal analysis provides a method for comparing financial information by using a base year and expressing all other information as percentages of that base year. In Figure 3, Snap, Inc.'s income statements are compared to the base year of 2015 (columns in **bold** are percents). This makes it easier to identify and quantify trends in the data. For

Snap, Inc.	(US$, millions)				
	2015	2016	2017	2018	2019
Sales/revenue	56.66	404.48	824.95	1180.00	1720.00
Cost of goods sold	182.34	451.66	763.53	864.23	961.81
Gross income	-125.68	-47.18	61.42	315.77	758.19
Sales & admin expenses	164.97	283.78	2360.42	836.59	937.89
Research & development	82.24	183.68	1151.00	739.18	850.3
Net income	-372.89	-514.64	-3450.00	-1260.00	-1030.00
Number of shares	1.16B	1.16B	1.17B	1.30B	1.38B
Earnings per share (EPS)	-0.32	-0.44	-2.95	-0.97	-0.75

Figure 1: *Income Statements for Snap, Inc.*

Snap, Inc. Balance Sheet for the Year 2019	(US$, millions)
ASSETS	
Cash and short-term investments	2110.00
Accounts receivable	492.19
Inventories	0.00
Other current assets	37.81
Total Current Assets	**2640.00**
Property, plant, & equipment	561.67
Other assets	808.33
TOTAL ASSETS	**4010.00**
LIABILITIES	
Accounts payable	46.80
Other current liabilities	452.83
Total Current Liabilities	499.63
Long-term debt	1190.00
Other liabilities	560.00
TOTAL LIABILITIES	**1750.00**
Shareholders' equity	2260.00
TOTAL LIABILITIES & SHAREHOLDERS' EQUITY	**4010.00**

Figure 2: *Snap, Inc. Balance Sheet, 2019*

example, in Figure 3, we can see that revenue exploded over the five-year period, reaching over 3000 percent more by 2019 compared to revenue in 2015. Care must be taken in interpreting negative percentages, and accounting statements frequently omit any computations with negative values.

Vertical analysis uses the same procedure, but compares values for a single year to some base quantity in that year. Figure 4 shows vertical analysis of a Snap, Inc. Balance Sheet, using Total Assets as the base amount. Vertical analysis is often used to examine the efficiency of a company. For example, if Current Assets are a small percentage of Total Assets, a company may have trouble paying its bills when they come due.

Snap, Inc. Balance Sheet for the Year 2019	(US$, millions)	% of Total Assets
ASSETS		
Cash and short-term investments	2110	52.62
Accounts receivable	492.19	12.27
Inventories	0	0.00
Other current assets	37.81	0.94
Total Current Assets	**2640**	**65.84**
Property, plant, & equipment	561.67	14.01
Other assets	808.33	20.16
TOTAL ASSETS	**4010**	**100**
LIABILITIES		
Accounts payable	46.8	1.17
Other current liabilities	452.83	11.29
Total Current Liabilities	499.63	12.46
Long-term debt	1190	29.68
Other liabilities	560	13.97
TOTAL LIABILITIES	**1750**	**43.64**
Shareholders' equity	2260	56.36
TOTAL LIABILITIES & SHAREHOLDERS' EQUITY	**4010**	**100**

Figure 4: *Vertical analysis of Snap, Inc. Balance Sheet*

Snap, Inc.	(US$, millions)									
	2015	2015	2016	2016	2017	2017	2018	2018	2019	2019
Sales/revenue	56.66	**100.00**	404.48	**713.87**	824.95	**1455.97**	1180.00	**2082.60**	1720.00	**3035.65**
Cost of goods sold	182.34	**100.00**	451.66	**247.70**	763.53	**418.74**	864.23	**473.97**	961.81	**527.48**
Gross income	-125.68	**100.00**	-47.18	**137.54**	61.42	**148.87**	315.77	**351.25**	758.19	**703.27**
Sales & admin expenses	164.97	**100.00**	283.78	**172.02**	2360.42	**1430.82**	836.59	**507.12**	937.89	**568.52**
Research & development	82.24	**100.00**	183.68	**223.35**	1151.00	**1399.56**	739.18	**898.81**	850.30	**1033.93**
Net Income	-372.89	**100.00**	-514.64	**238.01**	-3450.00	**-1025.21**	-1260.00	**437.90**	-1030.00	**476.22**

Figure 3: *Horizontal analysis for Snap, Inc. values in **bold** are percents.*

Ratio Analysis

Accountants make extensive use of ratios to evaluate the financial health of companies. They use ratios to measure the company's performance, activity, financing, and liquidity (ability to pay bills).

Some of the more common ratios (illustrated for Snap, Inc.) are:

$$Current\ Ratio = \frac{Current\ assets}{Current\ liabilities} = \frac{2640}{499.63} = 5.3.$$

A number greater than 1 indicates that the company can pay its bills, without having to sell off assets (Snap, Inc. has more than five times enough money to cover its bills). Accounting teachers will tell students that a very high Current Ratio is an indicator of inefficiency, since keeping a large amount of cash prevents the company from deploying the money more productively.

A related ratio is the Quick Ratio:

$$Quick\ Ratio = \frac{Current\ assets\text{-}Inventories}{Current\ liabilities},$$

which is a measure of how quickly a company can pay off its outstanding debts, without having to sell off inventory. For Snap, Inc., the Quick Ratio and the Current Ratio are the same, since Snap is a social-media platform, and the company provides a service, but has no physical inventory. Inventory measures are more important to manufacturing companies that produce tangible products.

Debt to Equity Ratio: This ratio measures whether the company's debt load is greater than the value of the company to its shareholders. A value greater than 1 is dangerous, since if the company went bankrupt, there would be nothing left for the shareholders after the company's debts were paid.

$$Debt\ to\ Equity\ Ration = \frac{Total\ debt}{Shareholder's\ equity} = \frac{1750}{2260} = 0.78$$

Gross Profit Ratio: This measures whether the company has a viable product. When combined with Net Profit Ratio, it can be used to evaluate the efficiency of the company.

$$Gross\ Profit\ Ration = \frac{\text{Gross Profit}}{Total\ Sales} = \frac{758.19}{1720} = 0.44$$

Net Profit Ratio:

$$Net\ Profit\ Ration = \frac{Net\ Profit}{Total\ Sales} = \frac{-1030}{1720} = -0.60 = -60\%$$

Earnings Per Share (EPS):

$$EPS = \frac{Net\ Profit}{Number\ of\ Shares\ Outstanding} = \frac{-1030}{1320} - 0.75$$

Snap, Inc.'s EPS would be reported as (0.75) on accounting statements.

Price-Earnings Ratio (P/E): $\frac{P}{E} = \frac{Current\ Stock\ Price}{EPS}$,

This is used to compute the current stock price as a multiple of current earnings per share. By comparing with other companies in the same industry, P/E is an indicator of whether the stock is overvalued (i.e., costs a lot for what it pays stockholders) or undervalued (i.e., is relatively cheap for the amount shareholders receive) compared to its competitors. This ratio cannot be computed for Snap, Inc., since the EPS is negative.

There are many other ratios used to evaluate a company's financial health. Some additional examples can be found at www.accountingtools.com/articles/ratio-analysis.html and many other Internet sites.

Figure 5 shows an assignment given by a mathematics teacher that uses horizontal, vertical, and ratio analysis. The assignment is done in groups, although each student may choose to analyze a different company.

Company Analysis

You are an accountant who works for a bank. Your job is to analyze company financial statements to help your bank determine whether the company is a reasonable investment for your bank to consider.

1. Choose a company in which you are interested.
2. Download at least five years' worth of Income Statements and the Balance Sheet for the current year. If necessary, you may consult your Accounting teacher to help you identify which lines in these statements are the most important.
3. For the Balance Sheet, perform a vertical analysis. What do these numbers tell you?
4. For the Income Statements, conduct a horizontal analysis. Identify and discuss any trends that you find. What do these numbers tell you?
5. Conduct a ratio analysis on the financial statements. At a minimum, you should compute the current ratio, quick ratio, debt to equity ratio, debt to total assets ratio, and at least four other ratios. For help with ratios, go to www.accountingtools.com/articles/ratio-analysis.html. What do these ratios tell you about the strength of the company?
6. What other information about the company is relevant in making an investment decision? List some of the information that you would want to know.
7. Write a report to your bank manager. Summarize your findings and make a recommendation to your manager as to whether this company is potentially a good investment for the bank.
8. Jigsaw: Make a five-minute presentation to your group about the company that you investigated.
9. Submit your written report electronically by (date).

Figure 5: *Ratio analysis assignment*

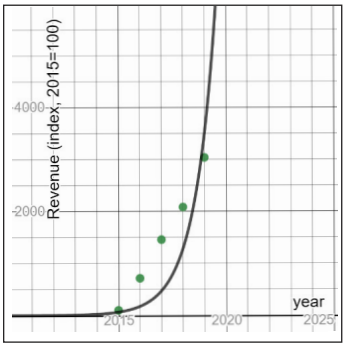
Working with data from a real company provides motivation to consider how mathematical models resemble actual situations. The problem in Figure 6 connects revenue forecasting to concepts learned in mathematics classes, like compound interest. It also provides an opportunity to consider how well mathematical models compare to actual data. How realistic is the model and what are its limitations?

Sales forecasting

- A company forecasts its sales to increase at 5% per year for the foreseeable future. Current sales are $280,000.
- What are the forecast sales for 3 years from now?
- What are the forecast sales for 5 years from now?
- What are the forecast sales for 10 years from now?
- How realistic do you feel this model is?

Figure 6: *Mathematical modelling of sales growth*

Mathematical models based on actual data are a rich source of learning, as well as reinforcement of previously learned concepts, and consideration of relevant range for a model, extrapolation, and justification of conclusions. Figures 7a and 7b show exponential and linear models, respectively, fitted to the revenue growth of Snap, Inc. Surprisingly, the exponential model is not a great fit to the data ($R^2 = 0.6011$), while the linear model is an excellent fit ($R^2 = 0.9937$). This type of modelling activity allows trends to be identified that might otherwise be missed.



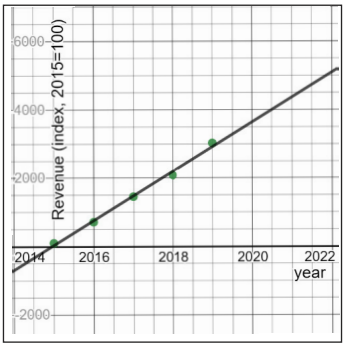


Figure 7a: *Exponential model* *Figure 7b: Linear model*

Figure 8 is a sample modelling assignment given in a mathematics class.

Mathematical Modelling

1. Choose a company in which you are interested.
2. Download at least five years' worth of Income Statements.
3. Construct mathematical models of Total Revenue for this company. Consider a variety of models: linear, quadratic, exponential, cubic, logarithmic, logistic.
4. For each model, identify how well the model fits the given data. Indicate whether you think the model is appropriate.
5. Identify the strengths and weaknesses of each model.
6. Which model would you use to extrapolate revenue for the next few years? Why?
7. In addition to Total Revenue, which other lines in the Income Statements might you want to model mathematically? Why? Give reasons for your choices.
8. You are going to use your models to forecast future revenue for this company. Identify which model you consider the best forecasting model, and justify your choice.
9. Share your findings with your group. Ask your group members to suggest other reasons for the best model choice. Include their suggestions in your report.
10. Submit this assignment electronically by (date).

Figure 8: *Mathematical modelling assignment*

Part 1 of this article provides several mathematical applications to Accounting. If the teacher wishes to go deeper into the links between Mathematics and Accounting, Part 2 illustrates how Accounting and Mathematics can be applied to investment analysis.

Part 2: Accounting Applied to Investment Analysis

One of the most common uses of Accounting is for making investment decisions. Investors access a wealth of information before deciding to invest in a particular asset like a stock. The analysis that we have done so far is called *fundamental analysis*. Fundamental analysis examines the published financial information of a company, to determine the financial health of the company, and to identify possible investment opportunities. Another branch of analysis, called *technical analysis*, is based on the theory that the current stock price already reflects any information contained in the published financial data of a company. This theory is called the *efficient market hypothesis*, which says that any new information is quickly absorbed by the stock market, and that

stock prices change very quickly to reflect that information. Technical analysis makes extensive use of past performance data, especially graphs of that data, and often uses moving averages (Figure 9) to help identify buy or sell opportunities for a particular company's stock.

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9	Day 10
Price	$10.00	$8.00	$8.00	$6.00	$9.00	$9.00	$10.00	$11.00	$12.00	$9.00

5-day Moving Average

Day 1 to Day 5 Average	=	$8.20
Day 2 to Day 6 Average	=	$8.00
Day 3 to Day 7 Average	=	$8.25
Day 4 to Day 8 Average	=	$9.00
Day 5 to Day 9 Average	=	$10.20
Day 6 to Day 10 Average	=	$10.00

Figure 9: *Calculating a five-day moving average*

Figure 10 shows a graph of a moving average. There is a wealth of information about moving averages online. For an additional example of how to calculate a moving average, see https://school.stockcharts.com/doku.php?id=technical_indicators:moving_averages. Care must be taken with the use of online materials, since most videos are produced by companies that want you to purchase their stock-picking systems. These systems often have creative names for what they identify as decision points. In Figure 9, the daily close of the S&P 500 American stock index is shown, together with 50-day (short-term trend) and 200-day (long-term trend) moving averages. When the short-term 50-day moving average drops below the long-term 200-day moving average, that point is called a death cross, and is an indicator that stocks should be sold. When the short-term moving average moves above the long-term moving average, that is called a *golden cross*, and is a buy signal. Analysts attempt to predict future stock price movements based on past performance, without examining company financial data. How successful do you think these systems are?

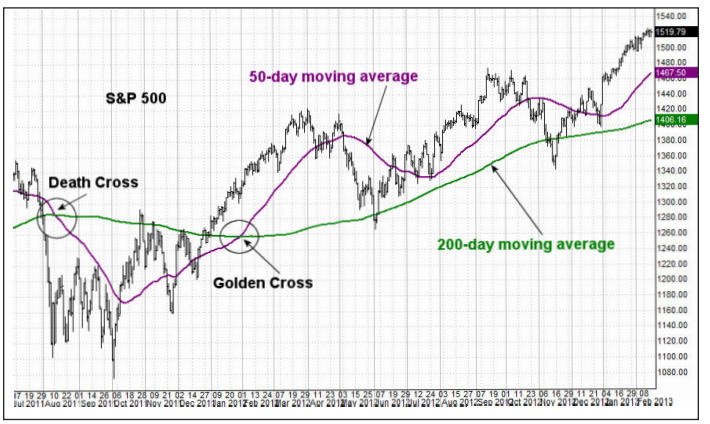


Figure 10: *Moving averages and the S&P 500 stock index*

Student assignment: Do an Internet search for moving averages as a tool for investing decisions. Examine at least three "systems" and write a report on your findings. DO NOT purchase anything from these companies. Submit your report electronically by (date).

Figure 11: *A student assignment on moving averages*

Accounting for Rewards and Risks

Investors are often attempting to decide whether the price of a stock today is worth the risk associated with buying stock, and whether the share price may rise in the future.

Dr. Myron Gordon (1920–2010), former professor at the University of Toronto, invented a simple formula for predicting the price of a company's stock, based on the formula for an infinite geometric series. In the Gordon Growth Model, $P = \frac{D}{g - r}$, where P is the price of the stock, D is the value of next year's dividend, g (%) is the growth rate in dividends, and r (%) is the company's cost of capital, i.e., the cost of borrowing money. The model is only useful for companies that have a long record of dividends and of dividend growth, which is assumed to continue in perpetuity. Here is an example using BCE (Bell Canada): The dividend growth rate $g = 4.7\%$, based on historic records; since BCE is a huge and stable company, its cost of borrowing is very low: $r = 1.1\%$. BCE's dividend this year was $3.10 per share, so based on the historical growth rate of dividends, next year's dividend is forecast to be $D = 3.10(1.047) = \$3.25$ per share. Substituting into the Gordon Growth Model gives $P = \frac{D}{g - r} = \frac{3.25}{(0.047 - 0.011} = 90.16.$ So the Gordon Growth Model predicts a shareprice of $90.16, based on the assumptions of the model. At the time of writing, BCE shares are trading for about $60 per share. If an investor only used the Gordon Growth Model as decision data, then they should buy BCE stock, as the model predicts that the stock price will go up, with a target price of $90 per share by next year. The Gordon Growth Model, with a fairly restrictive set of assumptions, illustrates the impact of assumptions on mathematical models. Our example also demonstrates that a potential investor should obtain multiple sources of data before considering any investment, since the assumptions of the Gordon Growth Model may not be appropriate for the investment being considered. On the other hand, the Gordon Growth Model is relatively easy to understand, and can be an interesting exercise for students (Figure 12).

The Gordon Growth Model: Is Your Stock Investment Overpriced?

1. Choose five dividend-paying stocks. Record their current stock price, their current annual dividend, and previous annual dividends for the last five years.
2. Use the dividend history to compute an average dividend growth rate (g).
3. Use the Gordon Growth Model to compute the price of each stock. Assume that the company's cost of capital is equal to the current prime rate plus 1%.
4. Compare the current stock price to the price predicted by the Gordon Growth Model. Which stocks are undervalued and which stocks are overvalued, according to this model?
5. Undervalued stocks are potential candidates to buy. What additional information would you consider when making this investment decision?
6. Overvalued stocks are potential candidates to sell. If you already owned these stocks, what additional information would you consider when making this investment decision?
7. Write a report of your findings.
8. Submit your report electronically by (date).

Figure 12: *Gordon Growth Model student assignment*

Evaluating Investments, Using Mathematics

We are all familiar with the compound interest formula and the formulas for annuities, a series of regular equal payments. However, some investments produce a series of unequal payments, often at irregular intervals. Two methods are primarily used to evaluate such investments. The first, Net Present Value (NPV), computes the present value of the series of expected payments, using a specified interest rate, called the *discount rate*. If the NPV is greater than the price of the investment, this indicates a potentially profitable investment opportunity. The NVP method is outlined in the video at www.youtube.com/watch?v=vEDEE_h2bMc.

The second method of evaluating such an investment is called Internal Rate of Return (IRR). In this method, no discount rate is assumed. Instead, the price of the investment is counted as a negative, and a compound interest rate is computed based on the investment price and all the potential future cash flows. This method is outlined at www.youtube.com/watch?v=OSDDrZZaV8E. Another video, at www.youtube.com/watch?v=6RztxNwerOA, compares the NPV method and the IRR method. Accountants routinely use both NPV and IRR to evaluate investments. This is a very nice connection to our work on compound interest and annuities done in math class.

A common investment is a mutual fund, which pools the money from a large number of small investors and purchases a basket of stocks or other securities, with each small investor having a proportional share of the basket. The basis of a mutual fund is diversification, which is used to reduce the overall risk of the basket (called a *portfolio*), since if some stock prices fall, they should be cushioned by rising prices in other stocks. Portfolio risk is measured by pooled standard deviation. Historic returns are measured by average annual compound rate of return (AACRR).

The formula for AACRR is $AACRR = [(1 + r_1)(1 + r_2)(1 + r_3)(1 + r_4)(1 + r_5)\ldots(1 + r_n)]^{\frac{1}{n}} - 1$, where r_1 is the percent return in year 1, and so on. This is actually an extension of the geometric mean of two numbers, $G = \sqrt{ab} = (ab)^{\frac{1}{2}}$.

Figure 13 and Table 5 give annual performance data for Templeton Growth Fund, a mutual fund that was started in 1954 and has a long performance record. Investment performance data is usually given with the most recent returns on the left, which is different from most graphs in mathematics.

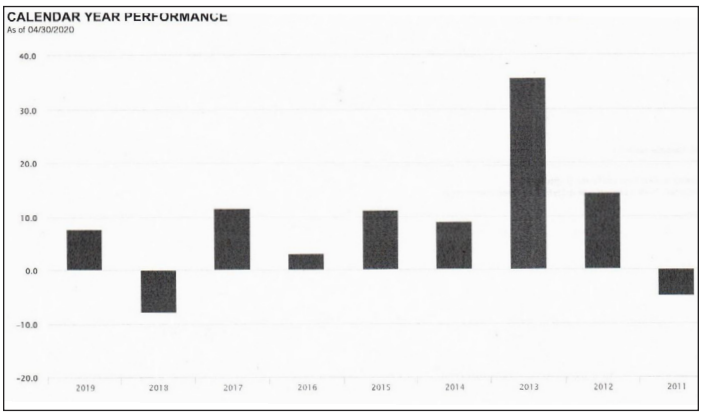


Figure 13: *Annual returns for Templeton Growth Fund, 2019 to 2011*

Year	2019	2018	2017	2016	2015	2014	2013	2012	2011
Performance (%)	7.68	-7.97	11.57	3.06	11.18	8.96	35.72	14.35	-5.04

Table 5: *Annual performance data for Templeton Growth Fund*

The AACRR for Templeton, based on these nine years of annual returns, is

$$AACRR = [(1+.0768)(1-.0797)(1+.1157)(1+.0306)(1+.1118)(1+.0896)(1+.3572)(1+.1435)(1-.0504)]^{\frac{1}{9}} - 1 = 0.082.$$

So, over the nine-year period, the return on investment was equivalent to 8.2% per year, compounded annually. Note that these are historic returns, and there is no guarantee that future returns will perform at this level.

AACRR gives a way of comparing historic returns for different investments, all of which will have had different annual returns each year.

Sharpe Ratio

Nobel Prize winner William Sharpe developed the Sharpe Ratio to compute a risk-adjusted rate of return on an investment. His formula, $S = \frac{R - r_f}{\sigma}$, gives the amount of return per unit of risk, measured by standard deviation. In his formula, is the return on the specific investment; is the risk-free rate of return, such as could be obtained in a savings account; and is the standard deviation of returns of the investment. Sharpe's formula allows us to compare different investments with different returns and levels of risk, in order to make informed decisions.

For example, at the time of writing this article, BCE stock price was $58.86, and its annual dividend payout was $3.10, so its dividend yield was 3.10/58.86 = 5.3%. Returns on a stock are a combination of dividends plus increases in the price of the stock. Since this return is not guaranteed, and historic returns are no guarantees of future performance, standard deviation of returns is a measure of the risk associated with the investment. Annual standard deviation of returns for BCE was 5.86. Risk-free investments, such as Government of Canada Treasury Bills, paid an interest rate of 0.27%. Substituting into the Sharpe's formula gives $S = \frac{5.3 - 0.27}{5.86} = 0.86$. So, for each unit of risk that the investor was willing to assume by buying BCE stock, they were rewarded with 0.86 units of reward. The Sharpe Ratio for one stock is not meaningful; the ratio is only useful for comparing investments, and identifying the investment with the highest Sharpe ratio as a potential investment choice.

Figure 14 shows a student assignment on evaluating investment rewards and risks.

You Have $20 000 to Invest!

For this assignment, you will become a potential investor, with $20 000 to invest. You have a relatively long investment timeframe. You plan to use the money to pay off any student debt that you acquire during your post-secondary studies, or possibly for the down payment on your first home. You should be willing to assume some level of risk with your investments, but you expect to be appropriately rewarded for any risk that you assume.

1. Choose up to ten mutual funds. Record the fund names, annual returns for at least the last five calendar years, and the standard deviation of those returns. If information on the standard deviations is not available, you are to compute the standard deviation, using the formulas learned in class.
2. Compute the average annual compound rate of return (AACRR) for each mutual fund. Remember that these are historical rates of return, and there is no guarantee that future returns will be at the same level.
3. Evaluate the risk–reward ratio for each mutual fund, using the Sharpe Ratio. For the risk-free rate of return, use the one-year Government of Canada Treasury Bill rate.
4. Rate your mutual funds from highest to lowest risk–reward, using the Sharpe Ratio.
5. For the best-rated three mutual funds on your list, investigate further to find the top five companies that each mutual fund holds in their portfolios.
6. Write a report outlining your findings so far. This will become the basis for your investment decisions.
7. Decide how you want to invest your $20 000: do you want to put it all into one mutual fund, or spread the money over several mutual funds. Provide reasons for your decision.
8. IF (a big IF) your investments perform at the same level (AACRR) over the next five years, compute the total value of your investment portfolio at the end of that time.
9. How would you spend this money?
10. Submit your report electronically by (date).

Figure 14: *Student assignment on investment rewards and risks*

The keys to using this material successfully are twofold: allowing students to freely choose which companies they will investigate; obtaining the co-operation of your school's Accounting teachers in assisting students and reinforcing concepts is strongly suggested. In my experience, many Accounting teachers see the benefits of reinforcing what they teach in another venue. The outcomes of a successful collaboration between Mathematics and Accounting include better student understanding of concepts in both areas; greater professional collaborations in schools; and most importantly, greater student involvement, interest, and motivation to study areas of mathematics that clearly relate to real-world concepts beyond the walls of our math classrooms.

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▲ Fostering Healthy Student–Teacher Relationships through Mathematics: A Student's Perspective



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Emily Beaulieu is a Teacher Candidate in the Primary/Junior division at Western University. Emily is thrilled to pursue her career as an elementary teacher. Her goal is to provide an inclusive classroom environment, where students can learn and grow to love mathematics.

I was born with a profound hearing loss. The diagnosis came late (15 months old), as I was born prior to the implementation of the infant hearing test now conducted at birth, and there was no history of hearing loss in my family. I was fitted with hearing aids. Despite the advancements in hearing aid technology, and a lot of effort by my parents and auditory verbal therapists, my speech did not progress. I was accepted into the cochlear implant program at the Hospital for Sick Children and received my first implant just before my fourth birthday. I entered Kindergarten with very limited vocabulary. Despite my belief in inclusive education now, as an individual living with a disability going through the school system, inclusion also presented challenges for me academically and socially. There were many pivotal moments during my educational journey, in which I felt a few teachers were my champions and contributed to my success, and there were a few teachers, who were ineffective and made me feel small. I remember when my Grade 5 classroom teacher sat me down and told me that I would not attend post-secondary school because of my disability, and that I was behind my peers. I successfully managed to prove her wrong.

In Ontario, students spend approximately 1494 hours with their classroom teacher in a typical school year. The number of hours is enough to build a relationship that could ignite a student's lifetime love of learning, but also enough time for the relationship dynamic to go off the rails (Sparks, 2019). Many teachers have a strong ability to engage socially with students. Irvine (2020) made an excellent point: "It is important for teachers to realize that they serve as role models for students. If teachers genuinely display interest, enthusiasm, and joy in the learning of mathematics, students are very likely to mirror these same emotions" (p. 46). Moreover, teachers need to engage with students around their curiosity, interests, and personalities.

Students' personality traits appear to be associated with the degree of closeness, friction, and dependency in the student–teacher relationship. In one European study about the role of student personalities and their effect on student–teacher relationships, the researchers found that the odds of students having a high-quality student–teacher relationship is partially determined by students' disposition and behaviours in the classroom, such as temperaments and personality (Zee et al., 2013). Furthermore, the interpersonal aspects of students' personality traits (i.e., extroversion, agreeableness, and neuroticism) seemed to contribute more than cognitive aspects (i.e., consciousness and autonomy) in explaining the quality of student–teacher relationships (Zee et al., 2013).

Students who have extroverted personalities display behaviour (e.g., assertive) that increases the opportunity for student–teacher closeness; however, by being extroverted, students are more likely to engage in more conflictual relationships (Zee et al., 2013). On the other hand, students who reported high levels of agreeableness are more likely to have closer and less dependent or conflictual relationships with their teachers (Zee et al., 2013), although they are more likely to mistrust others, and feel mistreated and insecure in social relationships. Students with neurotic personality traits (e.g., sensitive and anxious) are less likely to display positive motivational beliefs, are vulnerable to stress, lack confidence, and tend to focus more on their emotional state than on their academic success (Zee et al., 2013). These results suggest that aspects of students' personalities are associated with how well they relate to their teachers and academic performance (Zee et al., 2013). Students' personality traits rarely change; however, teachers' awareness of students' characteristic traits can help them shape a supportive and inclusive learning environment that is more compatible with the student's social and academic needs in school.

Peixoto et al. (2017) report, "Mathematics is one of the most important school subjects. However, it is also considered to be one of the most difficult and therefore is among the most feared by students" (p. 389), myself included. Supporting the development of mathematical skills is critical, not just for teachers to teach skills, but to understand their students' cognitive and emotional experiences (Peixoto et al., 2017). Interestingly, in elementary schools, there is a link between unexcused absences and declining grades, more particularly in mathematics (Gottfried, 2009). Teachers can encourage and

motivate students to work hard and miss fewer lessons. If positive student–teacher relationships can be developed and maintained throughout the school year, this may help close the gap in mathematics education.

The question remains, "How can teachers improve their relationships with students?" This can be accomplished when the teacher exhibits *hope* and *empathy*. According to Chimbganda (2021), "Hope is knowing and understanding: I can do this, I can learn, and I can teach." This really resonates with me. Teachers who demonstrate empathy for and with their students, have the skills to manage students' behaviours and academic engagement better (Sparks, 2019). Students will have the skills to work collaboratively on math problems, express their thinking, listen to the thinking of others, and practise inclusivity in a way that fosters healthy relationships (Ontario Ministry of Education [OME], 2020). Positive student–teacher relationships can help students develop self-regulation skills, particularly autonomy and self-determination (Lee, 2012). With time, students will learn how to evaluate, manage their emotions and behaviours, as well as be able to reach their potential and academic goals (Lee, 2012).

Teachers educate students to not only be good mathematicians, but to be good people, kind friends, creative innovators, and responsible citizens. Problem solving is not only a quantitative challenge, but also a human one. Students solve real-world and mathematical problems by working inclusively with their peers, formulating ideas, and persevering through challenges. While students work through these mathematics practices, they utilize interpersonal and intrapersonal skills. This is supported by social and emotional learning (SEL). An American organization, Collaborative for Academic, Social and Emotional Learning (CASEL), developed a framework that addresses five broad and interrelated areas of competencies: self-awareness, self-management, social awareness, relationship skills, and responsible decision making (CASEL, 2021). This framework emphasizes the importance of establishing inclusive learning environments, and applying these five competencies inside and outside of the classroom (i.e., with families and within communities), to enhance students' social, emotional and academic learning (CASEL, 2021).

Social–emotional learning (SEL) was integrated into the Ontario mathematics curriculum in 2020. The OME (2020) highlighted:

> Strand A focuses on the development and application of social–emotional learning (SEL) skills while using mathematical processes. These skills support students' understanding of mathematical knowledge, concepts, and skills, and foster their overall well-being and ability to learn, while helping them build resilience and thrive as mathematic learners (p. 72).

Effective mathematics instruction can be built upon these

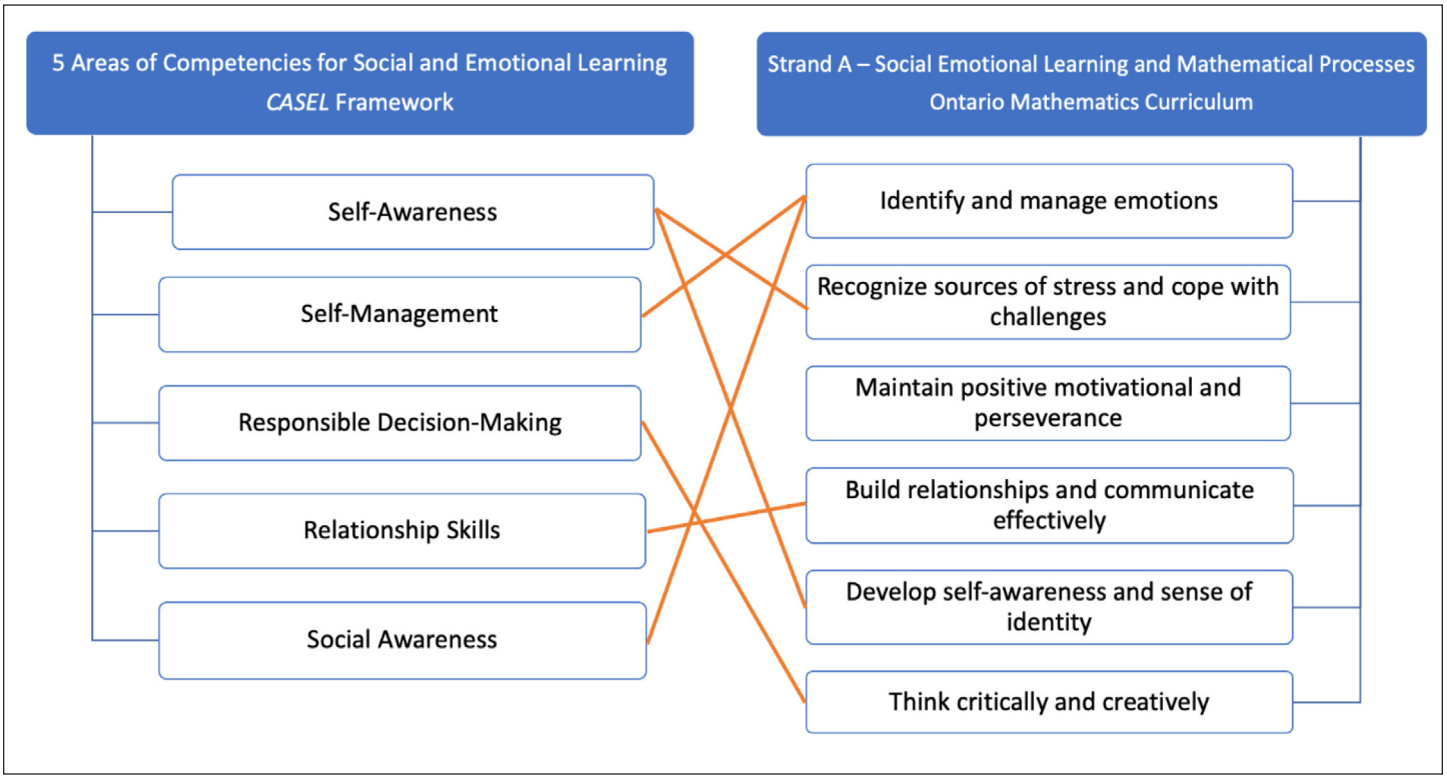


Figure 1: *Comparison chart between CASEL Framework (CASEL, 2021) and Strand A – Social–Emotional Learning and Mathematical Processes (OME, 2020)*

competences to drive student learning and engagement. Billiau (2020) indicated that "the implementation of this new curriculum and instructional practices would benefit students, and teachers certainly welcome the new tools to encourage growth mindset" (p. 54).

When we look at the six key SEL skills shown in the right column in the new 2020 math curriculum and compare it with the five areas of competencies for SEL in the CASEL Framework in the left column in Figure 1, five of the SEL and mathematical processes and expectations are closely linked. As students develop their SEL skills, they will be able to demonstrate a greater understanding and apply what is being taught in the mathematics curriculum, which is critical to supporting learning in mathematics (OME, 2020). Once the students have the skills related to Strand A, they can apply these across the other strands in the mathematics curriculum.

Teaching mathematics is not always about the mathematics itself, but about the experience that we can provide for the students to interact with it through love, hope, wonder, and beauty. Sadly, like many other students, mathematics was not my favourite subject. I often reflect about my time working with my high school mathematics tutor. He was my champion during my struggles and triumphs. He once told me, "It's okay to be scared about math. We are on this journey together." Fortunately, in addition to my high school mathematics tutor, I had many other educational professionals (and family support), who encouraged me throughout my elementary and secondary school years, and these people made a difference. For example, a few teachers did a regular check-in to see how I was feeling on that day. If I was not feeling positive, we worked together to see what needed to be changed for the better. I also learned how to practise mindfulness and breathing exercises in a class setting. By doing this, I was in a better mindset before completing a difficult math task. I still practise this technique in many areas of my life.

While one of my elementary teachers did not see a post-secondary future for me, others did. I graduated from high school. I earned an Honours Bachelor's degree. Now I am pursuing my Bachelor of Education degree. All this, despite one teacher telling me that I would never go on to post-secondary education. One might suggest that one teacher should not have a significant impact. I would surmise that it often only takes one teacher to change the path of a learner. Consequently, SEL is critically important. My story is one example. I am confident there are countless others with similar stories. As a future teacher, it is my goal to provide an inclusive classroom environment and shape the SEL skills for students. It is my goal to be that one teacher who makes a difference.

Acknowledgement

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Yet another, different, fence in Perth, Ontario



▲ Code Mountain: Climbing It Collaboratively



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Jessica Sardella is a fifth-year undergraduate student, majoring in Mathematics as a part of the Concurrent Education program at Brock University. Over the past two years, she has worked as a research assistant for Dr. Buteau and her research team on their work involving computational thinking in university mathematics. Upon completing her education, Jessica plans to become a high school mathematics teacher.



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Dr. Laura Broley studied Mathematics (B.Sc., M.Sc., and Ph.D.), while researching Mathematics Education, eventually as a Vanier Scholar. As a post-doctoral fellow, Laura has recently been examining the teaching and learning of coding in schools and universities, for the purposes of authentic mathematics investigations, and in the context of teacher education. For more information, visit laurabroley.com.



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Dr. Chantal Buteau is Professor of Mathematics at Brock University. Since she joined Brock in 2004, she has been teaching mathematics courses focused on using programming technology for pure and applied mathematics inquiry projects. In 2018, she designed such a course specifically for future math teachers. See www.ctuniversitymath.ca for her recent education research work on this topic. Chantal's interests also include more broadly the use of digital technology for mathematics learning, as well as the areas of mathematics and music.

The new coding expectations in Ontario Mathematics: They're exciting! But also challenging? Many teachers have found coding to engage their students with mathematics in new and wonderful ways (Gadanidis et al., 2017), but there are also teachers for whom adapting to these new curricular expectations may be a particular challenge (Vinnervik, 2022). Indeed, coding is a creative process dependant on concepts, practices, and perspectives that cannot be learned overnight, and many teachers have little prior experience with coding. In addition, teachers are not only expected to teach coding, but also how it can be used as a productive tool for mathematics learning and problem solving.

It's not just happening in Ontario: It's an international phenomenon! A recent survey of 52 countries found that 73 percent have integrated, or are planning to integrate, coding in their curricula starting in elementary school, with teachers reporting feelings of unpreparedness (Dagiene et al., 2019). For some teachers, implementing coding in math class may be like facing a big mountain to climb.

Facing "Code Mountain," in-service teachers in the Niagara Region decided to team up with pre-service teachers to go on a hike! As part of an initiative funded by the Mathematics Knowledge Network (MKN) and facilitated by two numeracy consultants and two university faculties[1], 25 in-service teachers (Grades 5–9) from the Niagara Catholic District School Board collaborated with 36 pre-service Intermediate/Senior mathematics teachers from Brock University. Working in 18 teams (2 pre-service and 1–4 in-service teachers per team), the collaboration involved the preparation and implementation of coding-based mathematics activities in in-service teachers' classrooms. Teams could select or modify an existing activity or create their own. While the pre-service teachers had experience using coding and mathematics together from taking university mathematics courses (namely, Mathematics Integrated with Computers and Applications I, II, and III[2]), many of the in-service teachers had little to no experience with coding.

Figure 1 depicts a timeline of the collaboration.[3] Teams began by getting to know each other and what their journey ahead may look like, before moving on to prepare and implement their activities. At the end of their hike, the teachers took a breather: to look back to admire the view, reflecting on peaks and valleys they experienced; and to look forward on how they might proceed further.

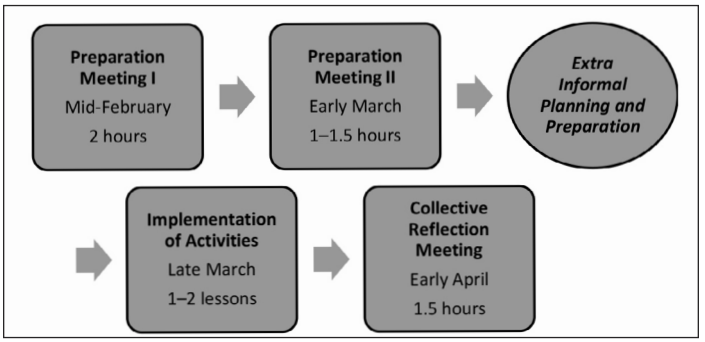


Figure 1: *Timeline of the collaboration*

In this article, we present the experience and reflections of two pre-service teachers—Abbey and Jessica (yes, the first author!)—and two Grades 7 and 8 in-service teachers—Peter and Rebecca[4]—one teacher team that embarked on a journey up "Code Mountain" together. Jessica provides a first-person account of her team's climb and view, based on their reflections from the collective reflection meeting, a report and video detailing the experience, and interviews with Peter and Rebecca. Through this story, our goal is to inform others about the collaboration, how it happened (The Climb), the insights participants may gain (The View on Implementation), and the general benefits it may have (The View on the Collaboration), in hopes that others may consider engaging in similar collaborative climbs. While this article highlights some benefits, challenges, and recommendations from one teacher team, a more comprehensive overview resulting from all teacher teams is available for readers who would like more details (see Broley et al., submitted).

The Climb: Bringing Coding to Math Classrooms

Abbey and I (Jessica) first met Peter and Rebecca virtually at the first preparation meeting, where we began thinking about our journey ahead. The teachers suggested using a Grade 7 activity in *Scratch* from MathUP,[5] a resource with which they and their students were already familiar. A geometry-based activity involving similar shapes and sub-programs was selected. The activity would be implemented in two lessons over two consecutive days, with one lesson per class occurring on each day. At the second preparation meeting, details were finalized. It was decided that Abbey and I would lead the lessons and provide support to students, and Peter and Rebecca would actively help throughout.

Prior to implementation, Abbey and I worked on planning the lessons. We created a Google Slides presentation that would guide students through the activity, which included links to the activity, questions to be asked along the way, and a mini-lesson on sub-programs[6] (what they are, how to use them, and their purpose). We also prepared possible solution codes so that we would be ready to provide guidance to students. Peter and Rebecca prepared their students by going through a series of lessons in *Scratch* involving sequencing; concurrent, repeated, and nested events; and conditional statements (concepts in the Grades 1–6 coding expectations), as well as some other explorations involving shapes. Rebecca described this as a way to get all students "on an even playing field."

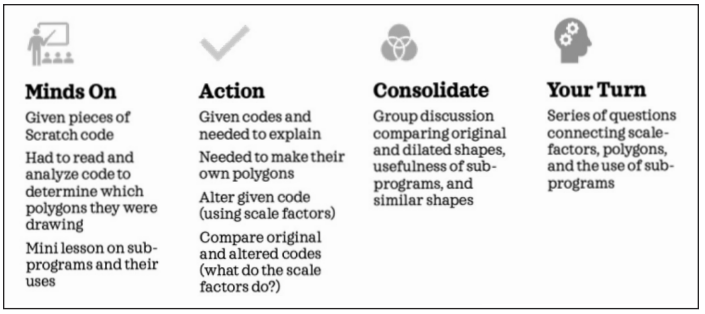


Figure 2: *An overview of the four-part activity selected by the teacher team from the MathUP website*

On the implementation days, Abbey and I introduced ourselves and the activity (outlined in Figure 2) to the class.[7] During the implementation, we realized we would need to adapt our planned route up "Code Mountain." After the first lesson with the Grade 7 students, we added check-ins and explanations to help ensure that students were understanding the math behind the code. For example, we noticed some Grade 7 students had difficulties understanding variables; hence, when we were in the Grade 8 class later that day, we decided to pause the lesson and have a discussion regarding what the scale-factor variables mean and how scale factors need to be set to something for the code to run correctly. Based on a suggestion from Rebecca, we also incorporated a tutorial of how to create a sub-program in the mini-lesson, since this may have been difficult for students who had less experience with *Scratch*.

Following the implementation, Peter and Rebecca continued on the hike, working further on the coding concepts with their classes through a culminating activity in which students used their new skills to create flower patterns in *Scratch*. They also took a breather to debrief and reflect on the implementation with Abbey and me, and the other teacher teams, at the collective reflection meeting. While discussing benefits, challenges, and recommendations related to implementing coding in math class, we also shared how we felt the lessons went. For example, Abbey and I had initially thought that students were less engaged on the second day because they were not asking as many questions; but after reflecting with Peter and Rebecca, we were able to see that the students were actually just working together and collaborating more.

The View: On Implementation

In a report on our experience, Abbey and I highlight how students responded to coding in the math classroom, emphasizing the benefits of *increasing student engagement and motivation* (In this and the next section, we italicize phrases that refer to benefits, challenges, and

recommendations that are further discussed in Broley et al. (submitted).), and *inviting students to be involved in their learning*. We report that, overall, the students were enthusiastic and had a positive response to coding. At the beginning of the activity, many students wanted to get right into executing the given code and writing their own code, and had to be encouraged to first take a moment away from the computer to break down the given code and try to predict what it did. During the activity, students were engaged, trying out new things, working with their table groups, and sharing their work with their peers and the teachers. Many students actively sought out challenges. For example, when one student asked me what shape she should try to make and I suggested a triangle, the student responded, "That's too easy," and wanted a harder shape. Another student called me over to say that she had created a star and started showing her peers. Both students illustrate how coding in mathematics can create a sense of agency and pride, where students take ownership over their work, push themselves to learn, and want to share their work with others. Rebecca also took note of this: "They were proud… you could actually see the pride."

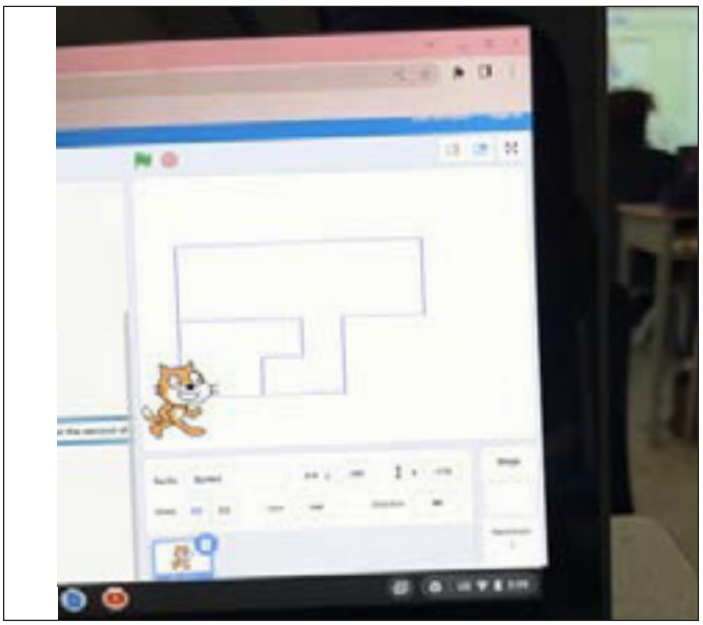


Figure 3: *A student's screen during implementation of coding activity*

Although students were engaged in the activity, they did need help at various points along the way, leading us to experience the challenge of *responding to students' needs*. While just getting started, one student said to Abbey: "I don't know what to do." Abbey responded by giving the student some strategies they could use to persevere and problem-solve, which included reusing and remixing the code that had already been provided, as well as writing it out and breaking down the steps on paper to better visualize and grasp what needed to be coded. During the debugging process, another student was struggling to find the error in their code. I worked through debugging strategies with the student, including reading through each part of the code and breaking it down into individual parts to mimic incremental testing, before eventually finding the error.

When thinking of teachers implementing coding in math classrooms, Abbey and I recommended *fostering an environment that is accepting of errors and questions* and *allowing [or encouraging] students to collaborate*, something we did in the above situations, while providing support and strategies to students, made possible by circulating around the classroom and being available for questions. Moreover, when students did have questions, we often responded by utilizing two general approaches: encouraging students to work together with their table groups and helping to guide discussion, rather than providing a solution right away. This further supported student agency and helped students overcome the fear of making mistakes by encouraging them to feel more comfortable or willing to try things and figure things out. Peter also observed students' increased comfort with taking risks: "The thing about the coding was, they could do something and if they made the mistake… it wasn't a big deal because they could go back and try something different and have a different result… [it was] an environment of high risk and low fear. Like, take some chances." Rebecca agreed and highlighted a new willingness to learn, referencing students that are typically less involved when saying that they were very "into it and trying different things."

Another important challenge Abbey and I experienced was *addressing students' differences* when we found that not all students had the expected knowledge on polygons. Abbey noted one case, in which a student did not know the properties of a rectangle, and so they could not alter the given code to make one during the Action step. This challenge inspired our recommendation to *ensure students have sufficient knowledge*, in the report we wrote about the experience. We suggest ensuring "that the students have the necessary math knowledge prior to implementing an activity like ours," explaining that "even though coding supports many different types of learners, if students do not have the background knowledge, they may not be able to make the connections" and reach the intended learning goals. In our particular activity, for example, the Minds On step could have been adapted to include some classroom discussion on properties of shapes and how these are represented in the given codes to provide a bridge into the Action step.

Interestingly, during their post-collaboration interview, Peter and Rebecca highlighted how meeting the learning goals of our activity was also supported by our recommendation to *allow students to collaborate* and the benefit of *allowing students to learn mathematics in new and different ways*. They mentioned that students naturally took different approaches to the coding, and that the students themselves even recognized this as they were collaborating. Rebecca explained: "You could see the kids actually… reading the code through and seeing what [their peer] had done and how it was different from theirs… some people who had multiple Sprites on different pages and different concurrent codes, whereas someone had just one big block [of code]… they got to see that you could get to the same result using different ways." By seeing how there are different ways to get the same outcome—in this case, with or without the use of sub-procedures—students could also be encouraged to think about the efficiency of their code, one of the learning goals of our activity. Rebecca recalled an interaction with a student, where she asked them the difference between their code and those of their peers, to which the student replied: "Well, mine's a lot longer. I did, I think, a lot more work I didn't have to do."

The View: On the Collaboration

Working collaboratively proved to have many different positive outcomes for Peter and Rebecca, Abbey and me, and the students, as we journeyed up "Code Mountain" together. Peter highlighted how *combining different expertise led to enriched, mutually beneficial experiences* for all: "Watching the [pre-service teachers] interact with the kids, it was… a different dynamic for sure… and [we were] learning from them as much as they were learning from the kids and they were learning from us." Each of us came out of the collaboration with more knowledge and experience than we started with.

For Peter and Rebecca, having additional support in the classroom meant *being free to walk around the room* and work with students they normally do not get a chance to. Peter and Rebecca also noted how they were able to *gain familiarity and confidence in coding* and *learn new pedagogical approaches* through working with both the pre-service teachers and university faculty. Peter explained that by "watching the [pre-service teachers] teach the kids, you actually pick up quite a bit [of the coding]… As a teacher, I was picking up what they were doing as well." Rebecca elaborated further, saying it gave her a "different perspective" to teaching math. More specifically, she spoke about learning to not give the correct answer right away: "We'd have them discover that it was the right answer… and not always jumping in and kind of giving the correction."

Peter and Rebecca were not the only ones to feel positive effects from the collaboration. Abbey and I did too! As pre-service teachers, we had only had one previous visit into classrooms as part of our teacher education program; through the collaboration, we were able to "gain [more] experience in a classroom," while also taking on a leadership role. Working with Peter and Rebecca allowed us "to gain feedback and tips on delivering a lesson and know how [we] should focus on different aspects for the different needs of [our] classes"; it also supported us in "learning real-time how to adapt the lesson to meet [those] needs" of the students. We were able to gain confidence and see the affordances of coding (e.g., agency) and the socio-emotional learning skills associated with coding (e.g., perseverance), which we had learned about through our own courses in a school setting (i.e., *seeing theory in action*). Reflecting on the need to deal with unexpected events that may occur in the classroom, Peter also recognized the importance of pre-service teachers learning in schools: "That's another part of teaching; you're not really going to learn it sitting in a classroom… You're not going to learn it sitting in a university. You're going to learn it hands on in a classroom...."

Peter and Rebecca also suggested that by having Abbey and me lead the activity, students were able to experience *new people and energy in the classroom*, a shift from their normal routine. This encouraged excitement and enthusiasm toward the activity: "Having two new people or two new faces in the room… they were very engaged with the kids… the kids themselves were right into it." Peter and Rebecca indicated that students were excited when we arrived for the second day and that, while usually very quiet, the students were very engaged with us, even calling us over to see their work. As female mathematics majors, we may have also served as *additional role models in the classroom*. Rebecca recalled one female student remarking: "Oh my gosh, it's so cool that we have girl mathematicians!" Hearing this comment made me feel so happy that I could be a source of representation for young girls in mathematics and have the potential to make them feel that they too could be a "girl mathematician" someday.

Conclusions: And What a View It Was!

When Peter, Rebecca, Abbey, and I took a breather (i.e., during our post-implementation discussions in the collective reflection meeting), our view was rich with the experience of coding in math classrooms, and we could all appreciate how working together influenced our hike. We were not alone. The benefits experienced by all teacher teams encourage us to do it again and invite others to partake in similar collaborative climbs!

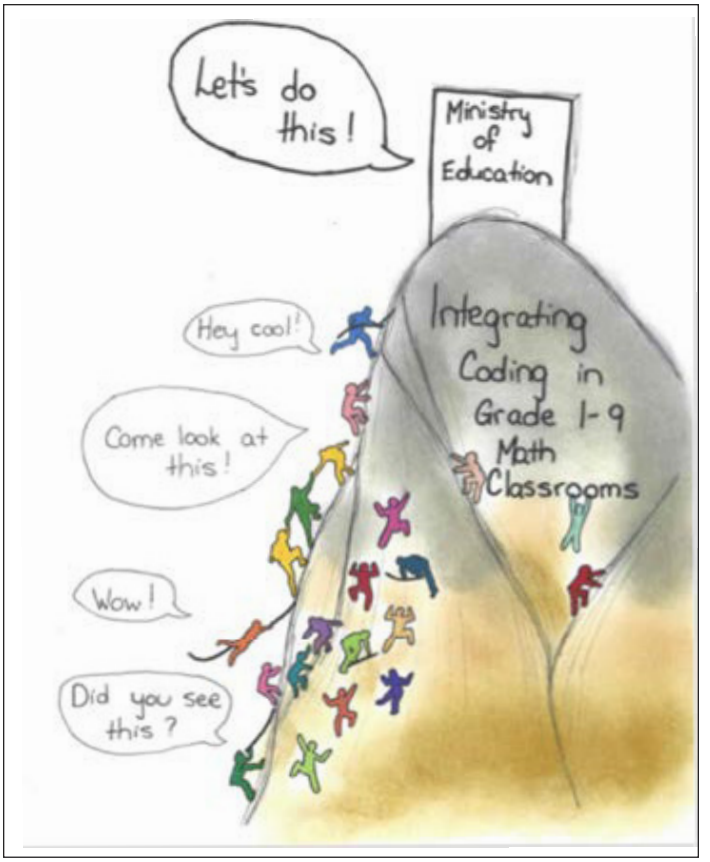


Figure 4: *Teachers and students helping each other on their journey up "Code Mountain" (Drawing by Chantal Lof)*

Although this article endeavours to encourage more collaborations across pre-service and in-service teachers, we also learned, through our initiative, the importance of recognizing a third type of collaborator: the student. When reflecting on her experience in the collaboration, one teacher, Andrea, explained:

> One of the things I think is really important for other teachers, when you're walking through this and you're not comfortable with coding, [is] that you don't have to have all the answers, that it's okay to walk through these problems with the students together. In fact, they'll start teaching you. And so, I think a lot of teachers may want to shy away from this because they don't have the knowledge, but it's something that I learned… you can walk through this with them. Prepare your lesson, but then be prepared for the kids to wow you and let that be okay.

Acknowledgements

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Notes

[1] Laura Cronshaw and Jeffrey Martin from the school board, and Laura Broley and Chantal Buteau from Brock University.

[2] Math majors and future math teachers learn to use programming as a tool for pure and applied mathematical inquiry through this sequence of three project-based math courses. Each course comprises two hours of lecture and two hours of lab time each week for 12 weeks, and ends with a passion project in lieu of a final exam. For those who would like more information, see Buteau, Muller, and Ralph (2015), and course resources posted online at www.ctuniversity math.ca/category/teaching-resources/.

[3] For a 12-minute video providing a synthesis of this initiative, see the video in the members-only area of the OAME/AOEM website under "files and links." For additional details, including meeting guidelines, experience reports from pre- and in-service teacher teams, and videos of teachers reflecting on their experiences, see www.mkn-rcm.ca/niagara-catholic-brock-u-collaborative-coding/.

[4] Peter and Rebecca are pseudonyms for the in-service teachers.

[5] MathUP is an online Canadian resource, designed to support the teaching of mathematics, with direct links to mathematics curricula (including coding requirements) at the provincial (Ontario) and national levels. For more information, visit mathup.ca.

[6] In *Scratch*, a sub-program can be created under "My Blocks."

[7] For more information on the activity and its implementation, see Abbey and my experience report, available at www.mkn-rcm.ca/niagara-catholic-brock-u-collaborative-coding/.

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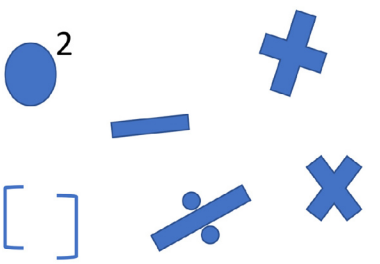
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Did you know… "Code" originally meant a systematic collection of statutes made by Roman emperors.

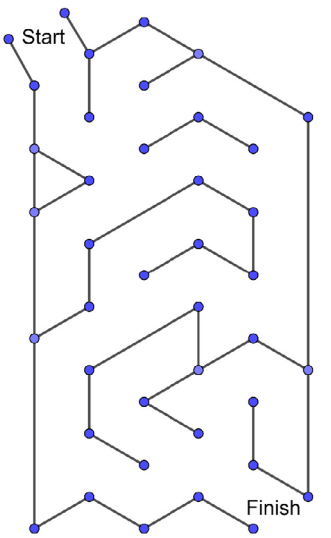
Unscramble each word. Take the marked letters and solve the picture riddle.

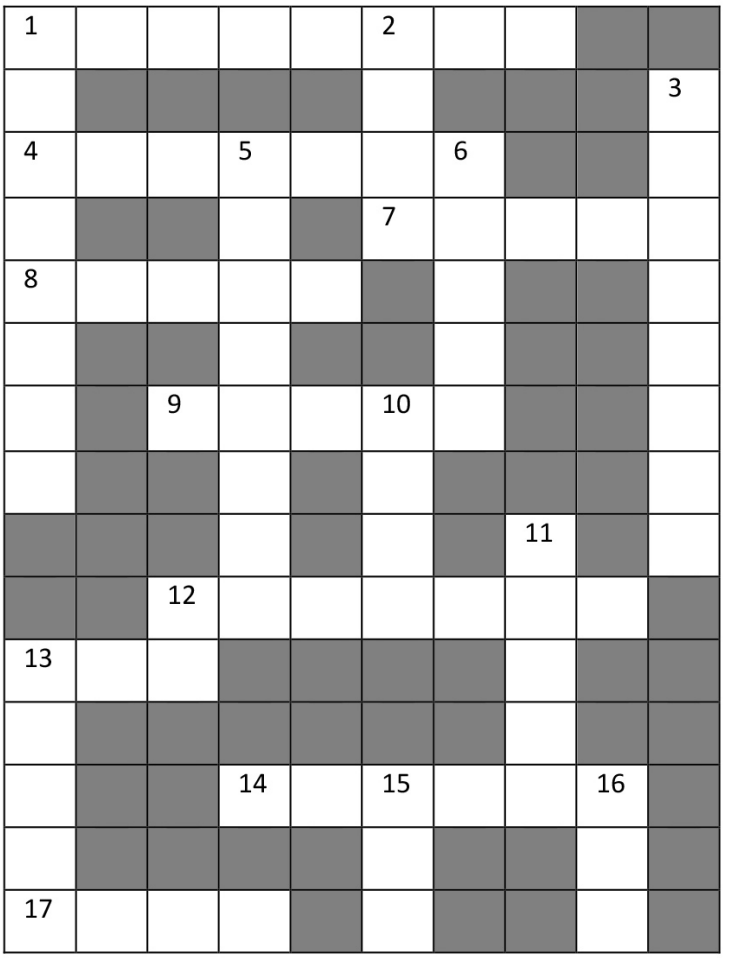
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M	B	S	D	E	A				
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Solution:

	1	6	T	H							Y	!





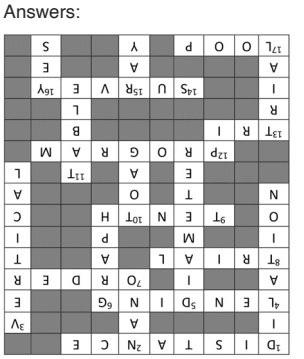
Across

1. Linear space between two points
4. Giving money with the expectation it will be paid back
7. Arrange according to size, amount, or value
8. A single iteration of an experiment
9. The answer comes after ninth
12. After you write code, (e.g., you have a ___?)
13. Prefix for three
14. A method for collecting data in a systematic way
17. A coding control that allows instructions to repeat until a condition is met

Down

1. A transformation that enlarges or reduces by a scale factor
2. A metric prefix for a billionth
3. When something is standing up perpendicular to the horizon, it is said to be? (e.g., be _____?)
5. The distance across a circle
6. A visual representation of data
10. The letters in "goat" in reverse-alphabetical order
11. An arrangement of values in horizontal and vertical rows
12. A number with a value close to 3.14159265358979323
13. Another iteration of an experiment (Same as 8 across)
15. A line that has a starting point, but no end point
16. A common word meaning a person agrees

Answers:



Scramble Solution
1. ORDER
2. OPERATIONS
3. BEDMAS
4. CONVENTION
5. CALCULATE

Answer: "16TH CENTURY!"

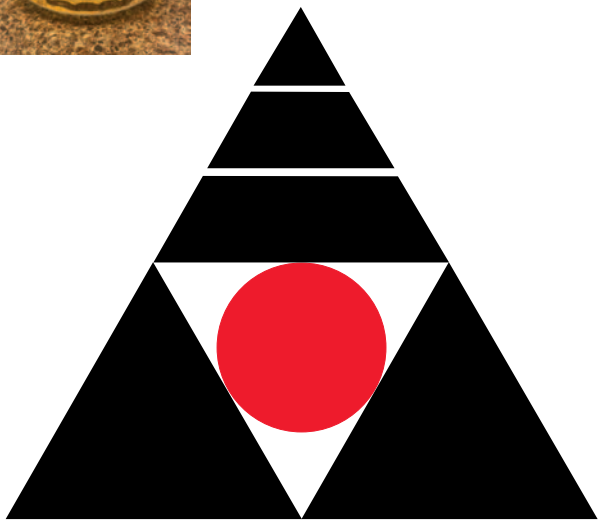














Which Conference was it?

With thanks to Peter Saarimaki.





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