8-PAGE INSERT ELEMENTARY MATH
ABACUS

Ontario Mathematics Gazette

OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

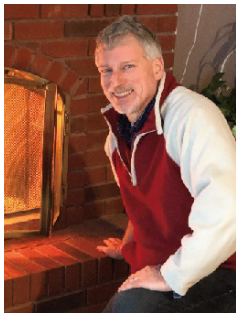
AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

Vol. 61#4
June 2023

IN THIS ISSUE

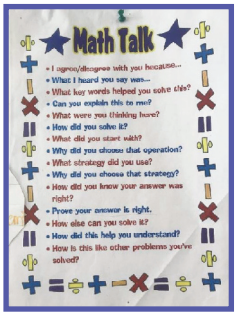
- Editor's Report
- President's Message
- Blast from the Past! The Math Processes
- Events at the Fields
- OAME/NCTM Report: June!
- What's the Problem? Fun with Fibonacci!
- Linking Literacy and Math: Making Sense of Mathematical Modelling
- In the Middle: Rising Ramps and Leaning Ladders
- Dynamical View of Proportionality: Integrity of Structure
- Elementary Math Matters: ChatGPT
- Coding in the Classroom: Curriculum Change
- Mb4T (Mathematics by and for Teachers): A Bucket of Integers, Part II
- Memories of Carol Attridge 1937 – 2023
- Remembering John Kearns
- Annual OAME/AOEM Awards 2023
- Research into Practice: An Application of Self-Determination Theory to the Teaching of Sequences in Grade 11
- Integrating Mathematics and Geography into Intermediate Classroom Instruction
- Making a Difference through Differentiated Instruction: Inclusive Strategies for Differentiated Instruction in the Math Classroom

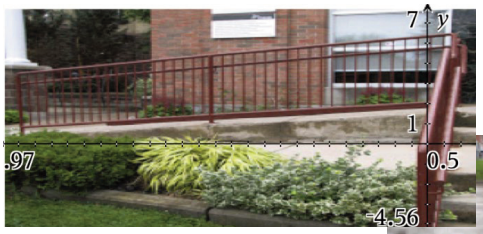


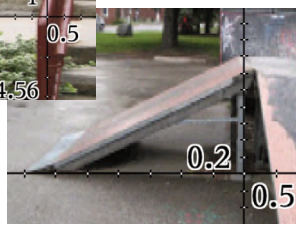


See Annual OAME/AOEM Awards 2023

See Making a Difference through Differentiated Instruction: Inclusive Strategies for Differentiated Instruction in the Math Classroom

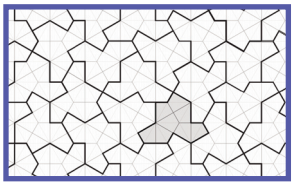




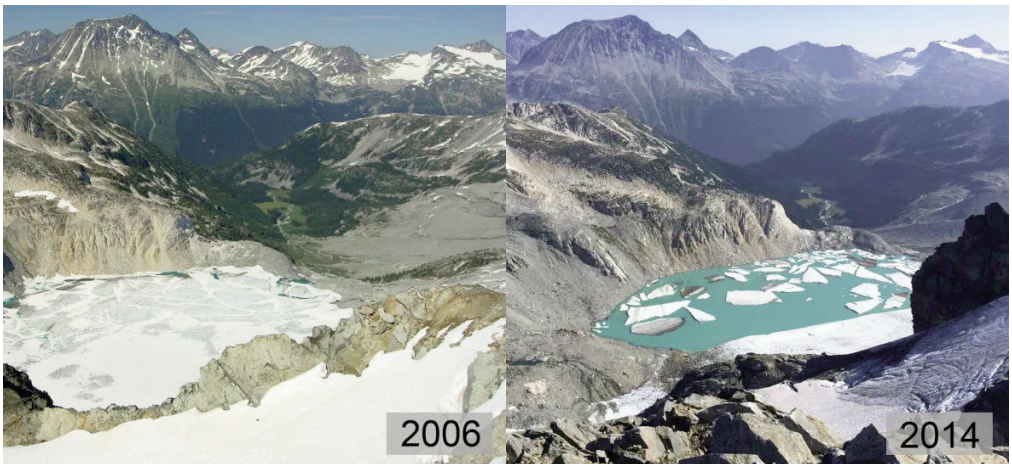


See In the Middle: Rising Ramps and Leaning Ladders

See Editor's Report



See Integrating Mathematics and Geography into Intermediate Classroom Instruction



Submission of Articles

The *Ontario Mathematics Gazette* (*OMG*) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *Gazette*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles are to be word–processed, MS Word is preferred, and prepared according to the *Publication Manual of the American Psychological Association*, Seventh Edition. However, please use single-line spacing (not double). Articles should not exceed five numbered pages of text, and figures, images, and photographs should be placed in the text close to where they belong, with captions. The photographer's permission is required, and for photos of students under the age of 18, the written permission of a parent or guardian is required.

Please submit your article in one blind file (i.e., identity of author is not evident), and include author names, contact information including email and mailing addresses, photos—head and shoulders, biographies—less than 100 words, and all content removed for blinding in a second file. Please email these two files to Tim Sibbald at gazette@oame.on.ca.

Upon review, you will be notified whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected. The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME/AOEM.

PERMISSION TO REPRINT: No part of this publication may be reproduced, distributed, or transmitted in any form or by any means, including photocopying, recording, or other electronic or mechanical methods, without the prior written permission of the publisher, except in the case of brief quotations embodied in critical reviews and certain other non-commercial uses permitted by copyright law. Full credit must be given to the author and to the *Ontario Mathematics Gazette*. For permission requests, contact the Editor of the *Ontario Mathematics Gazette*.

The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of OAME/AOEM.

▲ Table of Contents

Up Front

▲ Editor's Report by Timothy Sibbald 2
▲ President's Message by Sandra Jean Price 5

Regular Features

▲ The *Abacus* . . . . . 8-Page Insert
▲ OMCA News: March 2023 by Heather Theijsmeijer 4
▲ Blast from the Past! The Math Processes by Jeff Irvine 6
▲ Events at the Fields by Angelica Mendaglio 9
▲ OAME/NCTM Report: June! by Jacqueline Hill 10
▲ What's the Problem? Fun with Fibonacci! by Shawn Godin 13
▲ Linking Literacy and Math: Making Sense of Mathematical Modelling by David Costello 16
▲ In the Middle: Rising Ramps and Leaning Ladders by Carly Ziniuk 19
▲ Dynamical View of Proportionality: Integrity of Structure by Jérôme Proulx 22
▲ Elementary Math Matters: ChatGPT by Lynda Colgan . . . 24
▲ Coding in the Classroom: Curriculum Change by Iain Brodie and Hatice Beyza Sezer 28
▲ Mb4T (Mathematics by and for Teachers): A Bucket of Integers, Part II by Jennifer Holm 30

Features

▲ Memories of Carol Attridge 1937 – 2023 8
▲ Remembering John Kearns 8
▲ Annual OAME/AOEM Awards 2023 11

Articles

▲ Research into Practice: An Application of Self-Determination Theory to the Teaching of Sequences in Grade 11 by Jeff Irvine 33
▲ Integrating Mathematics and Geography into Intermediate Classroom Instruction Brett Tomlinson 40
▲ Making a Difference through Differentiated Instruction: Inclusive Strategies for Differentiated Instruction in the Math Classroom by Brenda Stein Dzaldov and Jenna Mandelker 43

Notices

▲ OAME Board Members 2022-2023 . . . . . Inside Back Cover

Advertisements

▲ OAME 2023 Conference . . . . . Back Cover
▲ Texas Instruments . . . . . Inside Front Cover

▲ About the *Ontario Mathematics Gazette*

The *Ontario Mathematics Gazette*—ISSN 0030-3011—is indexed in the Canadian Education Index and is published four times per year. Its Canadian Publication Mail Product Sales Agreement Number is 40051074.

Editor
- ▲ **Tim Sibbald**
  gazette@oame.on.ca

Associate Editor
- ▲ **Anne Yeager**
  anne.yeager@rogers.com
- ▲ **Peter Saarimaki**
  saarimaki@rogers.com

Associate Editor (French articles) /Rédactrice adjointe (articles français)
- ▲ **Kyla Kadlec**
  kyla.kadlec@oame.on.ca

***Abacus* Co-Editors**
- ▲ **Marc Husband**
  marc.husbandl@oame.on.ca
- ▲ **Tina Rapke**
  tina.rapke@oame.on.ca

Copy Editor
- ▲ **Gitta Berg**
  gitta.berg@sympatico.ca

Design and Production
- ▲ **Penny Clemens,** Graphic Designer
  pennyydezign@gmail.com

Printing & Binding
- ▲ **Pole Printing,** Box 69, 1-89 King Street East
  Forest, ON N0N 1J0 (519) 786-5112
  poles@xcelco.on.ca

Advertising Manager
- ▲ **Robert Sherk**
  4366 Snider Rd., Verona, ON K0H 2W0
  Home (613) 374-1515
  robert.sherk@oame.on.ca

Submission of Advertisements
Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** by email or at the above address. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be print ready, and colour advertisements should have no bleeds.

Advertising Rates
Advertising rates are available by telephoning, emailing, or writing to the Advertising Manager.

▲ Editor's Report



TIM SIBBALD, OCT, PH.D.
gazette@oame.on.ca

Tim Sibbald is a professor in the Schulich School of Education, Nipissing University, with a focus that includes mathematics education. He is a former President of OAME/AOEM and has held a variety of other OAME/AOEM posts over the years.

In the winter semester, I had the good fortune to teach a doctoral course about issues in mathematics education. The benefit for the participants was the opportunity to move beyond the usual buzz and explore the research regarding various issues. There are many different perspectives, and often it seems educators vacillate according to the stressors of the day and whether the prevailing view is a glass half full or half empty.

As with all good courses, there was enough on offer to be of benefit to the instructor. While I am well versed in the various issues in education, it was organizing the course and communicating in an orderly manner that provided a deeper sense of each issue. While the conversations were extensive, which is why teachers engage in graduate studies, I will limit my comments to three essential types of issues.

First, there are the perennial issues, where progress is not revealing any specific leaps forward to improve overall student proficiency, for example, the challenge of achieving fraction proficiency for all students. Sure, we can improve the instructional approach, particularly where the breadth of individual teacher instructional strategies is not fully developed, but there is a long-standing problem of achieving high levels of proficiency across the full scope of the topic. There has been considerable research over the years, and virtually all of it highlights that it is a conceptually rich topic, with considerable complexity, that is especially challenging.

Second, there are instances of genuine differences that are not always recognized as such. Many teachers have experienced circumstances, where a colleague does not teach a topic in the same way as themselves, and yet students succeed regardless. This can become an issue when it is not recognized that, despite the instructional differences, both methods may be quite effective—they are simply different. Increasingly I find this divide around

technology, where the *have more's* extoll the virtues and the *have less's* succeed because of other virtues. Another, with deep historical roots, is a distinction between educators who learn about teaching math, and mathematicians who learn about teaching. OAME/AOEM has struggled finding balance over its long history. Parallel to this is the curricular call for cross-cultural respect when the math education differences are authentic. It is in this regime that teacher preparation has found considerable benefit researching the concept of one's identity as a teacher.

A third issue are matters that are resolvable. What comes to mind, at the moment, is ChatGPT, though it is not entirely clear that it will resolve. I suspect it will because math education came to terms with Computer Algebra Systems (CAS) and automated solution generators years ago. While these math education examples are topically specific, they have elements of the same Artificial Intelligence (AI) concepts that are being raised with ChatGPT. It was particularly notable, in the course I was teaching, that students found the *Gazette* article by Robinson and Ferneyhough (*Gazette*, 60(2)) an interesting read to begin to contextualize the role of ChatGPT within education.

It was wonderful to teach the course and observe how structuring the ideas changes perspectives. In research, we use the notion of a theoretical framework, but it is very similar to the structural arrangement in courses—essentially a structure designed with purpose. Moreover, through the course, I found pauses where I would consider the larger role of the *Gazette* as an agent of support and change. Something worth considering in your summer reading. And, something to consider contributing to, perhaps you have tried something and had some success this last year? Why not share it with others?

ChatGPT

I would like to take a moment, as Editor, to express an opinion regarding the use of ChatGPT. In this issue, as you will see, I have accepted some brief quotations with the express purpose of professional discussion about the tool. Overall, however, the international community has made it abundantly clear that ChatGPT has led to active concerns about potential harm to minors, at least one lawsuit for defamation, general concerns about plagiarism, and concerns about bias that is internal to the process it uses. The last is often visible, where the preponderance of material available on the Internet is not an equitable representation of humanity. I will also note a concern with a narrowing of language due to the promulgation of cultures belonging primarily to the people who are most actively engaged with the Internet.

As Editor, until advised otherwise, I will only consider the use of ChatGPT and other AI-bots if the usage serves a benefit for professional learning about the use of such tools. Beyond that, I will take the same stance I do with plagiarism and refer any other usage to the Communications subcommittee of the Board of Directors for guidance.

In This Issue…

Our President, **Sandra Jean Price**, speaks about the power of volunteerism. We also have the OAME/AOEM awards recognizing **Greg Clarke** as an exemplary volunteer and **Heather Lindale** for exceptional and creative teaching. **Jacqueline Hill** advocates for us looking out for the newest teachers to the profession and suggests a variety of summer activities. **Angelica Mendaglio** keeps us up-to-date on recent events at the MathEd Forum. **Heather Theijsmeijer** brings us news of OMCA and may cause readers to look up the location of Nestor Falls.

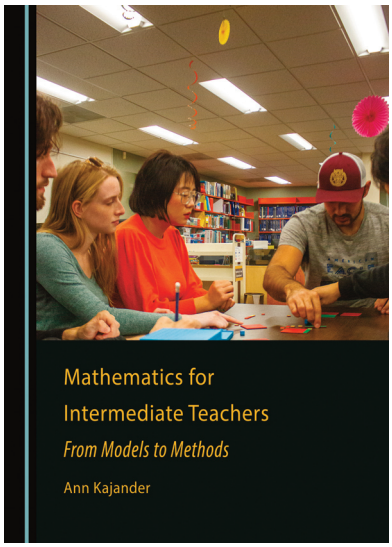
Among our regular columnists, **Lynda Colgan** deserves a special mention. Not only has her writing since retirement continued to be insightful, addressing ChatGPT head-on this time, but she is also the 2023 Margaret Sinclair Memorial Award winner (see www.fields.utoronto.ca/news/Dr-Lynda-Colgan-receives-2023-Margaret-Sinclair-Memorial-Award). **Iain Brodie** and **Hatice Beyza Sezer** revisit their interview of **Nicole Myers** and drop a mention of Hemachandra numbers that will interest anyone considering the history element in the 2020 curriculum. **Jeff Irvine** provides some interesting insights into the *Gazette*—as Editor, I will see what I can do. **Jennifer Holm** explores multiplication and division of integers, using the *Bucket of Zeros*. **David Costello** further develops a model of mathematical modelling. **Carly Ziniuk** tries to get a rise out of us by examining ramps and ladders. **Jérôme Proulx** considers complex strategies students use for handling proportional reasoning. **Shawn Godin** considers series that surprisingly turn out to be periodic.

Thank Yous

Once again, I would like to thank the editorial team for all their efforts: **Peter Saarimaki, Anne Yeager, Ralph Connelly, Gitta Berg,** and **Penny Clemens**. I would also like to thank the columnists for their timeliness. As usual, the editorial team and columnists made it all look so easy, but that is truly because of their professionalism.

Last Words…

The world of math education is vibrant and there is a lot going on all the time. In our own backyard, **Ann Kajander** has recently had another book published—congratulations! (See www.cambridgescholars.com/product/978-1-5275-9090-8 for details.)



The recent discovery of a tile that gives an aperiodic tiling of the plane is remarkable. To put this into context, an aperiodic tiling is a tiling that cannot be periodic—there is no way to rearrange the tiles to make it periodic. What is remarkable is that it is the first time the property has been demonstrated for a single tile. Previous examples have had two or more distinct tiles combined to generate an aperiodic tiling of the plane. The tiling is shown in Figure 1.

In late-breaking news, the government has made a funding announcement. The role of politics is also an issue in math education and not one I addressed in the math-issues course I taught. It is also not one I can reasonably address in this editorial. More importantly, outcomes from funding announcements are very difficult to predict, and it will ultimately be the overall interpretation of political actions that the electorate will consider in the future.

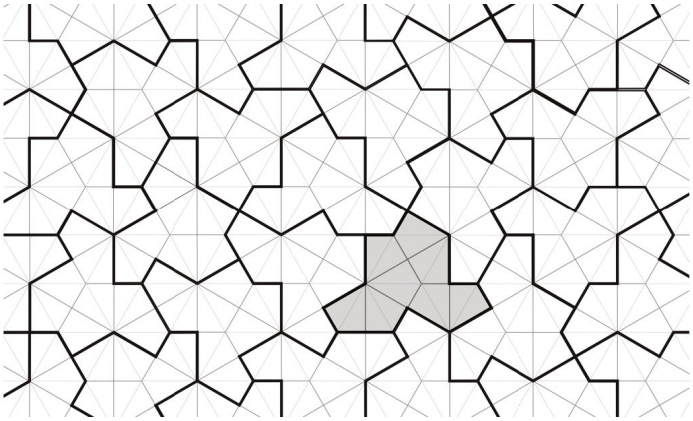


Figure 1: *Small section of the aperiodic tiling showing the tile shape. Reprinted from Smith, Myers, Kaplan, and Goodman-Strauss (preprint), available at www.arxiv.org/pdf/2303.10798.pdf*

▲

▲ OMCA News: March 2023

HEATHER THEIJSMEIJER (OMCA PAST PRESIDENT)
OMCAmath@gmail.com

Greetings from the Ontario Mathematics Coordinators' Association (OMCA)! As the school year winds down, our membership of math coordinators, consultants, and coaches from across the province are reflecting on another great year of learning and networking.

Our members' meetings continued on a monthly basis throughout the remainder of the school year. Recent meetings have included guest speakers Dr. Thomas Guskey on designing and evaluating effective professional learning, and Dr. Molade Osibodu. Our final meeting of the school year is always devoted to conversation and exploration of issues often faced by coordinators, consultants, and coaches in mathematics education.

Hot on the heels of the February Retreat, our April membership meeting was our second in-person gathering of the year. Hosted by the Toronto District School Board, over 60 members met ahead of the OAME2023 conference to engage in networking and capacity-building opportunities. While in-person meetings were standard prior to March 2020, this was the first regular meeting to be face-to-face since then. Special thanks to Jason To for facilitating.

Our equity-themed book explorations continued through the Spring with Literature Circle sessions in March and May. OMCA members were able to choose between one of four books: *Choosing to See: A Framework for Equity in the Math Classroom* by Kyndall Brown and Pamela Seda; *High School Mathematics Lessons to Explore, Understand, and Respond to Social Injustice* by Robert Q. Berry III, Basil Conway, Brian Lawler, and John Staley; *Rough Draft Math* by Amanda Jansen; and *Teaching Math to Multilingual Students (K–8)* by Rachel Pinnow, Kathryn Chval, Erin Smith, and Lina Trigos-Carrillo. In total, we met three times over the course of the school year for informal conversations around content that resonated with us, as well as how it impacts our practice.

As we close out the year, OMCA would like to thank Marilu Deal, who has been our Treasurer for the past few years, and wish her well as she returns to the classroom this Fall. Marilu is a spreadsheet wizard (which, coming from an association of mathematics educators, is high praise!). She

Continued on page 12

▲ President's Message



SANDRA JEAN PRICE
sandra.jean.price@oame.on.ca

Sandra Jean has been on the Board of Directors of OAME/AOEM for almost 20 years. She is currently teaching in the Durham District School Board and has more than 25 years of teaching experience. She has had many roles, including classroom teacher, Math Coach, and Instructional Leader of online Math Additional Basic Qualifications (ABQs). Sandra Jean has contributed to a variety of Ministry projects and was a coordinator of the most recent OAME/AEMO writing projects. Sandra was the recipient of the 2008 OAME Award for Exceptional and Creative Teaching in Elementary Mathematics for her innovative instructional practices, integration of technology, and belief in success for all.

As another school year draws to a close, it is worthwhile spending some time reflecting on the year and the wonderful people who volunteer their time to support others. As educators, many of us willingly give our time to students by offering extra help during non-instructional hours and running or supporting co-curricular activities. It can be a challenge when one has a young child at home, but some still manage to find a little time to give, even then. I was recently reminded of the importance of this donated time when a former student came to visit and expressed that she did not remember much about her elementary school years, but she did remember the field trips, the sports teams, the choir, the band, and a variety of other clubs that she engaged with during her time at the school. She built strong friendships through those extra activities that have lasted through her post-secondary education. Perhaps some of the friendships would have been well-established within the classroom, but she insists participating in the wide variety of activities helped her discover who she really is—and those activities were made available by the staff. The leadership demonstrated by teachers instilled in her a belief in her own leadership ability.

As President, I realize that the message goes well beyond the words of one student or the teachers of one school. Volunteers are incredibly important within our schools, just as they are in all walks of life. They are also integral to our organization. In fact, OAME/AOEM could not exist without the many volunteers who give freely of their time.

Our annual conference relies on chapters stepping up to agree to host the conference. Within the host chapter (or chapters), individual members volunteer to chair or be a member of a sub-committee. Hosting an annual conference is not a simple task and requires three years of planning. Yet members support their chapter along the way so that math educators in the province have this annual opportunity for professional learning. This year is no exception. After three years of only being able to offer virtual professional development, the OAME/AOEM Annual Conference was held face-to-face in Toronto in April, co-hosted by two chapters: TEAMS (Toronto Educators Association for MathematicS) and NOMA (Northern Ontario Mathematics Association). Despite the many challenges throughout their planning, the organizing committee worked diligently to overcome them, and ran an excellent conference. Along with the Board of Directors, I would like to thank every member of the organizing committee for all of their time and energy, without which the conference would not have been such a success.

Within chapters, there are smaller events offered for teachers. These include mini-conferences, seminars, book clubs, and socials. Members of the chapters spend their time organizing and hosting these events in order to support mathematics educators in their region. Also within chapters are the individual members who hold a position on their executive. Positions such as the President, Vice-President, Treasurer, Secretary, and Chapter Representative are filled by volunteers who help their chapter run smoothly. It is these volunteers who help make OAME/AOEM a strong organization. Thank you to everyone who volunteers in their chapter.

Last, but certainly not least, I need to extend a heartfelt thank you to all of the members of OAME/AOEM's Board of Directors. Through times of uncertainty, the Board steered the organization, ensuring that it remained viable, energized, and relevant. This year, we were able to meet in person twice and reconnect in a way that hasn't been possible for the last few years. It was a reminder of the incredible individuals who volunteer their time and energy to support OAME/AOEM, and they surely are representative of all the good things math teachers are doing across the province. Thank you for all you do.

Sincere gratitude needs to be expressed to our Past President, Judy Mendaglio, whose term will be completed at the end of August. As President, she led the organization through the pandemic as well as through joint projects of OAME/AOEM and AFEMO, strengthening the relationship between our two organizations. She has served her term with grace, dedication, and integrity. Thank you for your

mentorship and your commitment to OAME/AOEM.

Our annual elections occurred in March, and it was a bit surprising that some vacant positions did not attract individuals to run for them. As an extracurricular with good professional learning, particularly in leadership, many worthy candidates may have overlooked the opportunity that the Board of Directors represents. It has occurred to me that perhaps there are some who are unsure about stepping into a leadership role, where they know there is a professional learning curve. To be honest, I felt exactly like that, never really thinking of being on the Board of Directors when I was asked to become a Chapter Representative in 2003. I certainly never envisioned myself as one day running for President. I didn't feel like I had the skills necessary to be a leader. But as intimidated as I was at first, meeting incredible math educators, most of whom I had only heard speak at conferences, I grew to realize that we all had something in common—our love of math and our joy in teaching. Their experiences helped me realize that being a leader is not just about always knowing what to do or what to say. It is about listening, caring, and contributing, even if it isn't always immediately clear which path to take. It is about lifting others up when they are down, and about recognizing the abilities of others and encouraging them to share with others. It is about realizing that by engaging in professional learning while working together as a team, leadership skills are developed, practised, and mentored.

Volunteering first with my own chapter, then on the Board of Directors and during conference organizing, helped the organization, but also opened the door to professional learning that helps develop leadership skills. I was able to further develop my own leadership skills, far more than I could in my job. Building those skills made me a better teacher, a better role model at home, and the experiences helped me understand my own potential. Members of the Board of Directors have the opportunity to learn more about what it is like to teach in other areas in the province, and so much more by being able to connect with others. Through my own learning, I have become a leader who wishes to encourage others, empower others, and help them recognize their own potential. It is my sincere hope that you will engage with your chapter and talk to others about stepping up when next January brings the call for nominations leading into the OAME/AOEM election.

"As we look ahead into the next century, leaders will be those who empower others." ~ Bill Gates ▲

▲ Blast from the Past! The Math Processes



JEFF IRVINE
dr.jeffrey.irvine@bell.net

Jeff Irvine, Ph.D., has been a secondary mathematics teacher, department head, and vice-principal. He has taught at three faculties of education and at a community college. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. With over 45 years in education, Jeff is particularly interested in the interplay of student motivation and mathematics achievement.

Beginning in the 1980's, the mathematics community recognized that there was a need for students to see mathematics as more than just a disconnected set of skills and procedures. This need gave rise to the big ideas concept. Big ideas can be viewed as consisting of two branches: big content ideas and big process ideas. There was considerable debate about what constituted big content ideas as well as big process ideas. There tended to be more agreement on the big content ideas (e.g., Small & Lin, 2010), possibly because math teachers were familiar and comfortable with the idea of teaching content. The big process ideas were arguably more important, consisting of concepts such as problem solving, connecting, and communicating mathematics.

Many jurisdictions formulated sets of big process ideas, with labels such as mathematical dispositions, mathematical principles, mathematical practices, and the generic mathematical processes (British Columbia Ministry of Education, 2012; Council of Chief State School Officers, n.d.; Manitoba Department of Education and Early Childhood Learning, 2015; Saskatchewan Ministry of Education, 2010). Ontario settled on mathematical processes, and identified seven of them: problem solving, communication, reasoning and proving, representing, connecting, reflecting, and selecting tools and computational strategies.

These math processes first appeared in the front matter of curriculum documents (Lazarus, 2022), and in subsequent curriculum policy documents, they were integrated into specific expectations (Ontario Ministry of Education [OME], 1998). In the next round of curriculum

revision, the math processes were separated from the specific expectations and returned to the front matter (OME, 2005, 2007). In the current round of curriculum policy documents, the math processes were merged with social–emotional learning and became a separate strand (OME, 2020, 2021). The placement of the math processes as a strand is contentious, since they are deemed to apply across all content strands.

While the provision that the math processes apply across all content strands may account for their current placement with social–emotional learning, this somewhat hybrid strand melds two very different concepts and seems to me to be a strange marriage. According to OME policy, the math processes are to be assessed, but teachers are given no guidance on how to assess them, and they appear to have little to no relationship to social–emotional learning.

Perhaps not surprisingly, there are a fairly limited number of articles in the *Gazette* about the math processes. Suurtamm (2004) identified several jurisdictions that have implemented similar curriculum reforms to address the mathematical processes (Denmark, New Zealand, Quebec, and Singapore). She also addressed the difficulty of assessing the math processes, since they apply broadly across math content areas. Assessment Abby (2016) echoes Suurtamm's contention that assessing the math processes requires a variety of assessment strategies, including rich tasks, observations, conversations, and anecdotal feedback.

An article that I wrote (Irvine, 2019) gives some concrete examples of activities that allow teachers to assess the math processes, and includes a checklist for assessing the math processes informally while students carry out investigations or problem-solving tasks. My article also mentions an excellent resource (OME, 2008), found on the EduGAINS website (www.edugains.ca) that, for each math process, identifies what students are expected to do, how students can demonstrate understanding of the process, sample teacher questions, and sample teacher feedback. While OME no longer supports the EduGAINS website, the information is still available online.

The conflation of the math processes and social–emotional learning has led to more recent articles emphasizing the social–emotional learning portion of the combined strand (Biliau, 2020; Ziniuk, 2020). Ziniuk examines assessment of the math processes during the pandemic and suggests some technology to assist in assessment.

And that's it. Since the introduction of the math processes almost 30 years ago, five *Gazette* articles have been written about them. Anecdotal comments from teachers indicate that the math processes are not frequently top of mind when planning lessons, and are even less frequently assessed. Yet, the big process ideas of mathematics are what we would like students to remember and use long after they graduate from school. So, noting the Editor has said there is a need for articles, I will add the suggestion that the processes and their assessment is a particular topic that would benefit from being included in the *Gazette*.

References

Assessment Abby. (2016). Assessing math processes. *Ontario Mathematics Gazette, 54*(3), 8.

Biliau, R. (2020). Discovering the new (2020) strand: Social–emotional learning skills and the mathematical processes. *Ontario Mathematics Gazette, 59*(2), 52–54.

British Columbia Ministry of Education. (2012). Areas of learning: mathematics—Foundations of mathematics and pre-calculus, grade 10. curriculum.gov.bc.ca/sites/curriculum.gov.bc.ca/files/curriculum/mathematics/en_mathematics_10_foundations-of-mathematics-and-pre-calculus.pdf

Council of Chief State School Officers. (n.d.). *Common core state standards: Mathematical practices.* https://learning.ccsso.org/wp-content/uploads/2022/11/Math_Standards1.pdf

Irvine, J. (2019). Activities for assessing the math processes and learning skills. *Ontario Mathematics Gazette, 57*(4), 33–37.

Lazarus, J. (2022). Secondary curriculum over the last decade. *Ontario Mathematics Gazette, 60*(3), 52–53.

Manitoba Department of Education and Early Childhood Learning. (2015). Conceptual framework for K–9 mathematics. www.edu.gov.mb.ca/k12/cur/math/framework_k-8/concept_framework.pdf

Ontario Ministry of Education. (1998). *The Ontario curriculum, grades 9 and 10: Mathematics.* Queen's Printer for Ontario.

Ontario Ministry of Education. (2005). The Ontario curriculum, grades 9 and 10: Mathematics (revised). Queen's Printer for Ontario .

Ontario Ministry of Education. (2007). *The Ontario curriculum, grades 11 and 12: Mathematics (revised)*. Queen's Printer for Ontario .

Ontario Ministry of Education. (2008). MIN_DICards_MathematicalProcessProblemSolving.pdf downloaded from www.edugains.ca

Ontario Ministry of Education. (2020). *Elementary mathematics (2020).* King's Printer for Ontario. Math_Curriculum%20Context_AODA.pdf

Ontario Ministry of Education. (2021). *Grade 9 mathematics (2021).* King's Printer for Ontario. www.dcp.edu.gov.on.ca/en/curriculum/secondary-mathematics/courses/mth1w

Saskatchewan Ministry of Education. (2010). *Foundations of mathematics and pre-calculus 10.* https://publications.saskatchewan.ca/#/products/33375

Small, M., & Lin, A. (2010). *Big ideas from Dr. Small, grades 9–12 facilitator's guide: Creating a comfort zone for teaching mathematics.* Nelson.

Suurtamm, C. (2004). Assessment corner: Assessing mathematical processes—A complex process. *Ontario Mathematics Gazette, 43*(1), 7–9.

Ziniuk, C. (2020). In the middle: Representing and reinforcing mathematical process amid uncertainty. *Ontario Mathematics Gazette, 59*(2), 27–31. ▲

Memories of Carol Attridge 1937 – 2023

PETER SAARIMAKI, OAME PAST PRESIDENT AND LIFE MEMBER

Carol Eleanor Attridge, RN, was an OAME/AOEM Executive Director (with husband Don) from 1989 until 1995. Don and Carol had been involved in OAME even before OAME existed. Back in the early days, OATM and then OAME conferences were very much seen as family events. While Don was the "math" presence, both Attridges were highly visible in the organization and its many events, social and otherwise, as well as the contest marking committees and socials at Waterloo.



When OAME's inaugural Executive Directors, Morley and Mona McGregor, retired after 15 years, Don and Carol decided to put their hats in the ring. As Don described it, "There was a lot of mail and postage then—there were no emails... Carol was the 'mother hen,' as well as being secretary and summarizing the OAME meetings" (*Gazette*, 60(3), p. 11).

Secretary "stuff" in those days meant using a typewriter and carbon paper—no thought of using a computer. Carol's minutes were clear and concise and helped keep the meetings on track.

As Executive Directors, they became great friends of many of the OAME/AOEM directorate. Don would communicate with the President each week (especially Mickey Sandblom) to determine what the agenda would be for future Council meetings (this was before OAME was incorporated and members became Directors). Carol would work behind the scenes, arranging the meeting space, determining the catering, and organizing the paperwork.

They assisted with organizing the printing and mailing of the *Gazette* (three times per year), as well as administering the financial aspects of the organization. Naturally Carol (and Don) attended all OAME conferences, chatting with delegates, passing out literature, directing the flow of the lectures, and chairing the annual meeting and elections.

Besides her work with OAME, those were busy times for Carol, as she was a nurse at various Toronto and York Region hospitals and was raising a family—three wonderful children: Ian, Karen, and Ross. Over the years, she also worked at the Victorian Order of Nurses, the Ontario Association for Children with Learning Disabilities, Participation House, and Evergreen Hospice.

When they moved on to other ventures in 1995, Carol was presented with the Mona McGregor Award for her significant contributions to OAME/AOEM, and her contributions to the improvement of mathematics education in Ontario.

Then Carol spent her free time increasing her community involvement and adding countless hours as a volunteer, including serving on the Board of Directors for the Living and Learning in Retirement Program at Glendon College. And Carol (and Don) were still to be found at OAME conferences around the province.

Carol was an equal partner to an outstanding team that contributed to the advancement of OAME/AOEM. Furthermore, she was a leader in her own right, brooked no nonsense, enjoyed a good laugh, and her role provided a level of warmth and wisdom.

We are grateful for the countless ways she supported math education in Ontario and enriched the lives of all who knew her.

(See reflections sprinkled through the issue.)

Carol was an amazing and inspiring person.

Ralph Connelly, OAME President, 1987/88 and 1998/99

Remembering John Kearns

We are all saddened by the loss of John Kearns, whom many of us knew as a great supporter of the math community and as a friend. John passed away peacefully on April 28, 2023.

In 2001, John was presented the Award for Outstanding Contribution to OAME and Mathematics Education in Ontario.

We will miss John's energy and the way he made us laugh.



Events at the Fields



ANGELICA MENDAGLIO
angelicamendaglio@gmail.com

Angelica Mendaglio is an instructional designer at Vretta Inc. in Toronto, Ontario, where she helps to create interactive digital mathematics lessons and activities for middle school students.

The March session of the Fields MathEd Forum gathered speakers from Canada, the United States, England, and France to discuss the role of mathematics in math education research. The speakers were invited to share the mathematics that had an impact on their research on how students learn math. The following are examples from two of the exciting speakers.

Jinfa Cai from the University of Delaware delivered a talk titled "Improving Students' Learning of Mathematics: Lessons from Several decades of Research," in which he summarized several research projects with which he has engaged. The first was focused on cross-national studies, with the goal of comparing how students in the United States and China performed on different types of mathematical tasks—computation, simple word problems, complex problems—which could be solved using a standard process or formula, and complex problems which were open and could not be solved using a standard process or formula. The initial study, conducted around the year 2000, found that the Chinese students outperformed the American students in all but the fourth task category (process–open complex problems). Cai revisited the same Chinese school 20 years later and found that in the intervening years, changes had been made to the school's practice and curriculum. As a result, while students' performance had declined slightly in computational and simple word problems, they had seen some gains in process–closed complex problems and significant gains in the open complex problems, which Cai described as a good trade-off.

The second study that Cai described was exploring students' attitudes toward mathematics in different programs of mathematics studies: non-college preparation, traditional college preparation, and a funded college preparation. The last was funded by the National Science Foundation (NSF), an agency in the United States that supports research and education in the sciences. Students in these three programs were asked to answer three questions to reveal how they felt about mathematics: if math were a food, colour, and animal, it would be… because…. The students' responses were then coded as very negative, moderately negative, neutral/ambivalent, moderately positive, or very positive, so that they could be compared among the different programs. The study found that students in the NSF-funded program had, overall, the most positive attitudes toward mathematics. In his talk, Cai observed that all three programs had students whose responses showed a very positive or a very negative attitude toward math, which inevitably led to challenges in the classroom. He also observed that the NSF-funded programs tended to be the most challenging, and this could be related to the positive attitudes the students have toward the subject.

The third study that Cai described looked at 20 middle schools that were adopting the NSF-funded curriculum. In particular, Cai highlighted a comparison between the school's willingness to reform (measured using several different criteria) against the change in the student performance before and after the intervention. The study found that, the more buy-in from the teachers and school administrators, the greater the improvement in the students. By contrast, in the schools that had lower buy-in, the students' performance declined after the intervention.

The final study touched on problem-posing-based learning, in which teachers were trained to teach mathematics through problem posing. In the example given by Cai in his lecture, teachers were given a pattern, and were divided into three groups. The first group was asked to pose three different mathematical problems that could be solved using the pattern. The second group was asked to pose one easy, one moderately difficult, and one difficult mathematical problem that could be solved using the pattern. The third group was asked to make three different mathematical conjectures about the pattern. The idea would be that the groups would come up with these problems and conjectures, and then as a class, they would discuss, solve, prove, or disprove three of the problems or conjectures together. Cai also gave an example of how this approach was deployed in one teacher's classroom. The teacher wrote to him about a lesson that had not gone well in their classroom. The lesson was focused on listing ordered pairs, using a linear function, but students were struggling with the order of the coordinates in the pair and why the order mattered. Cai suggested to the teacher that they take the same linear function and ask students what real-life situation the function could represent. For the students, this act of problem posing allowed them to attach meaning to the abstract variables and helped to clarify the distinction between them.

Anne Watson from the University of Oxford gave a talk

titled "Understanding Opportunities for Learning Mathematics." Watson opened with an anecdote from her time as a student in school, working on exercises in class about multiplying two linear bracketed factors, resulting in quadratic expressions. After the first few exercises, she came across one where her result only had two terms instead of three. She struggled with this for a time, but could not find what she had done wrong, and moved on to the next exercise, which also resulted in two terms. She recalls that she found this very distressing as a student, but that the classroom was not one where she felt comfortable asking for help. Reflecting on this experience later, she realizes that whoever created that exercise sheet intended for the student to have a moment of discovery at that point, but that because of the classroom culture, the moment was one of anxiety, rather than discovery and curiosity. Watson became very interested in how moments of discovery can be facilitated by a combination of carefully selected examples, associated pedagogy, and classroom culture.

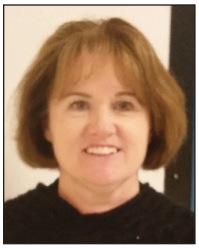
Watson also gave a few examples of books that she found enlightening when she was embarking on her journey as a mathematics educator: *How to Solve It* (Pólya), *Thinking Mathematically* (Mason, Burton, & Stacey), and *Understanding in Mathematics* (Sierpinska). In particular, *Thinking Mathematically* touched on mulling as part of mathematical thinking. Watson became very interested in this idea, and wanted to examine what exactly she was doing when she was "mulling" so that she could teach her students to do the same thing. This led to her writing a book with John Mason, one of the authors of *Thinking Mathematically*, titled *Questions and Prompts for Mathematical Thinking*. Watson shared a few excerpts from it, which included some generic questions to prompt student thinking and model the mulling that Watson had been studying. A few examples of these questions were: Is… an example of…?; What needs to be changed so that…?; Find a counter-example of….; Is it ever false that…?

The Fields MathEd Forum convenes on the last Saturday of each month from January to April, and from September to November. You can sign up to attend remotely through the Fields website, where you can also find a recording of this and other past Forum meetings and see their schedule of other upcoming math education events. There's lots of interesting presentations to be found here. ▲

Carol was a guiding light for me, especially during the year I was President. Her beautiful smile, her sensible solutions were exactly what I needed.

Jeri Lunney, OAME President, 1992/93

▲ OAME/NCTM Report: June!



JACQUELINE HILL
jacqueline.hill@bell.net

Jacqueline Hill is a Grades 9–12 teacher of mathematics. She is a Past President of OAME and OMCA, as well as the recipient of the award for Exceptional Teaching in Secondary Mathematics. She also wrote the "Director's Dialogue" for the Gazette *for a number of years.*

Happy June! Happy June! Happy June!

Whether you are a pre-service teacher, new teacher, or seasoned teacher, June always has a certain ring to it. It is a time to celebrate the end of a school year, rejoice at new beginnings for pre-service teachers, and reclaim balance for first- and second-year teachers! It truly is a time to celebrate and to embrace another chapter in your teaching journey.

This column is dedicated to our "new in the field" pre-service teachers, newly hired teachers, and teachers celebrating the end of their first year of teaching.

Here is a top 10 list of ways to engage new teachers! *If you are a seasoned teacher, please try and take some time to mentor the next generation of teachers.* If you are a pre-service teacher, please consider these as activities to embark on to encourage your journey in teaching.

#10 Take a new teacher out for coffee.

#9 Share a resource with a new teacher.

#8 Ask a new teacher for help in an area you would like to explore.

#7 Introduce new teachers to the OAME/AOEM website with its wealth of resources, and the NCTM website for the North American perspective.

#6 Set up a way for new teachers to contact you in the summer to share ideas and suggestions.

#5 Be an "official" mentor to a new teacher.

#4 Be an "informal" mentor to a new teacher (share your binders and resources).

#3 Go out of your way to meet someone new at the next staff meeting.

#2 Share your classroom management techniques with a new teacher.

#1 Share your personal top 10 list of ways to thrive and grow in education with a new teacher.

Most of all (#0), help new teachers to find and embrace

balance—balance between personal and professional time. Everyone works better with a healthy balance. The summer is a time to recharge and to refill your own personal cup of health up to the brim! September comes around very quickly, and the journey starts all over again! So, rejoice and celebrate summer!

NCTM important dates to note:

July 26, 2023 PCTM (Pennsylvania Council of Teachers of Mathematics) Annual Conference (Ramada Hotel and conference Center, State College, Pennsylvania)

The NCTM was set up in 1920 to help math educators help students to grow. The NCTM has continued to have face-to-face and online conferences to help educators grow in their practices. OAME/AOEM and NCTM have both strived to ensure teachers are equipped to grow throughout their careers.

NCTM has a few things of interest on the website right now:

1) Members-Only Webinars. Your NCTM membership includes exclusive access to 100+ hours of on-demand professional development. NCTM webinars are delivered by experts in mathematics education, including NCTM authors.
2) The 2023 game is free, and a great way to get students thinking. It is on the main page of the NCTM website, but if it has been moved from the main page, just type "2023 game" in the search bar.

Resources

Like OAME/AOEM, the NCTM has a mission statement that helps members keep growing in the right direction.

NCTM Mission Statement—The National Council of Teachers of Mathematics advocates for high-quality mathematics teaching and learning for each and every student.

OAME/AOEM Mission Statement—is to promote, support, and advocate for excellence in mathematics education throughout the Province of Ontario.

Until next time,
Yours in mathematical fun,
Jacqueline ▲

Carol was a classy lady, who always added a spark to the room.

Barry Onslow, OAME President, 1996/97



▲ Annual OAME/AOEM Awards 2023

Award for Outstanding Contribution to OAME/AOEM and Mathematics Education in Ontario: Greg Clarke

The Awards Committee is very pleased to announce this year's recipient of the **Award for Outstanding Contribution to OAME/AOEM and Mathematics Education in Ontario: Greg Clarke**.



Greg's love for mathematics has been long-standing, and his passion shines through in everything he does. He believes that math is fun, that it is relevant, that it is accessible to all, and both children and teachers respond to his messages.

Greg exemplifies everything this award represents. He has demonstrated a high level of dedication to students and educators within individual schools, his district, the province, and beyond. He has also poured countless hours into supporting the goals of OAME/AOEM and his local chapter, MAC^2.

Greg studied math at university and has remained a lifelong math learner. He never presents himself as an expert; instead, he works with students and peers to help them discover their talents within. Upon graduating from the Faculty of Education, Greg became a secondary school Math and Computer Science teacher for the Simcoe Muskoka Catholic District School Board. During his ten years in the classroom, he inspired students in his classes, became a Math and Science Chairperson, and had the time and energy for choir and a weekly school newspaper. His students knew he cared.

Greg then moved his considerable talents to supporting teachers across his board: first as a Computer Consultant, then as Secondary Program Coordinator. His innovative approaches to teaching and learning were shared with many. At the same time, he became an integral part of MAC^2 and the provincial OAME/AOEM, where his enthusiasm, infectious laugh, and supportive demeanour encouraged teachers of math, from neophytes to pros, to join together to share the fun and power of effective math pedagogy.

Throughout this professional career and now as a "retiree," Greg pours skill and time into OAME/AOEM. He saw the importance of a credible organization for math

educators and has contributed in so many ways. At the local level, Greg has been a pillar of the MAC² chapter for years. He continues to play a crucial role in maintaining their website, stepping in to serve as Treasurer as needed, and is always looking for ways to connect educators to chapter events.

Beyond the obvious—the annual conferences, the website, the work on the executive, or local chapter work, he also supported math learning through his stellar work getting Ministry materials onto the OAME/AOEM site so they will be available for future educators. Similarly, he used his math and computer talents to make sure the OntarioMath.Support site would aid teachers in implementing the new math curriculum. He knew this was an important endeavour and, along with the rest of the OAME/AOEM team, he made it happen.

Greg often works behind the scenes of OAME/AOEM and directly contributes to the improvement of mathematics education in Ontario. His influence continues with his contributions at OAME/AOEM Board of Directors meetings, with support of the Chapter Reps, organizing association materials and managing the technology, whether face-to-face or for virtual meetings.

Congratulations to Greg on his outstanding achievements and as recipient of this prestigious award!

OAME/AOEM Exceptional and Creative Teaching in Secondary Mathematics Award Recipient: Heather Lindale

The Awards Committee is very pleased to announce this year's recipient of the OAME/AOEM **Exceptional and Creative Teaching in Secondary Mathematics Award: Heather Lindale**.



Colleagues of Heather Lindale agree that Heather exemplifies a high level of servant–leadership. Her instructional approach is flexible, innovative, and reflective, creating conditions where all students have an opportunity to meet their potential. Through her guidance as the Ottawa Catholic School Board's only Grades 7–12 Math Learning Partner, Heather has helped transform the learning culture in numerous school communities throughout the board. Heather demonstrates a deep understanding of differentiated instruction and works tirelessly to support struggling and reluctant learners, while shifting pedagogical practice by modelling innovative and creative instruction.

Heather's commitment to the growth and well-being of her colleagues and students is evident through her involvement in school, board, and Ministry initiatives. Over the years, Heather's participation and leadership in the Middle Years Collaborative Inquiry, Eastern Ontario School District Network projects, or Building Thinking Classrooms have provided many educators the opportunity to collaborate with her and witness first-hand her excitement and enthusiasm toward learning. Working directly with students within the classroom setting, Heather models innovative instructional practices in a non-intrusive, collaborative manner. Her compassion and creativity are evident in her instructional strategies that are responsive and flexible to meet the needs of all learners.

Heather works diligently to build capacity within school teams, especially with respect to coding, facilitating productive math discussions in the classroom, and effective pedagogical strategies for a destreamed classroom. Currently, Heather is actively collaborating with educators to shift assessment practices in the Gradeless Classroom.

The mark of a successful math learning partner is not in the immediate effect on the classes, whom they support directly, but in the capacity built over the longer term. Heather's professionalism, leadership, commitment to excellence, and collaborative approach serve as an exemplar for other teachers across the province.

Congratulations to Heather on her outstanding achievements and as recipient of the OAME/AOEM Exceptional and Creative Teaching in Secondary Mathematics Award! ▲

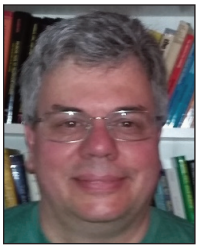
Continued from page 4

kept on top of the association's recent membership expansion and oversaw large events such as the Retreat and Symposium, as well as the minutiae of month-to-month guest fees. You will be missed, Marilu!

This month's meeting (June 2023) will see the membership voting in new members of the OMCA Executive, including the roles of Treasurer and Vice-President. The winners of the OMCA Award for Excellence and Innovation in Mathematics Education will also be announced at the same meeting. Stay tuned for updates in the next issue of the *Gazette*.

From Nestor Falls to Niagara Falls, new members are always welcome to join OMCA. Contact us directly at OMCAmath@gmail.com, or visit our website at www.omca.website (note the different domain) to learn more about the organization, find out about upcoming events, and connect with us. ▲

▲ What's the Problem? Fun with Fibonacci!



SHAWN GODIN
GodinMathStuff@gmail.com

Shawn is a former mathematics teacher, department head, and consultant. He strongly believes in the central role of problem solving in the mathematics classroom. He continues to be involved in mathematical activities: presenting workshops, writing articles, working on local projects, and helping create mathematics contests.

Welcome back, problem solvers. We are wrapping up the theme "(mathematical) problems from my life" that we have been exploring in this volume. During my career, I have done a lot of mathematical reading and exploring on my own. The problem from your homework was one that I often used with my Grade 11 classes:

A sequence is defined by $t_1 = 53$, $t_2 = 79$ *and* $t_n = t_{n-1} - t_{n-2}$ *for* $n > 2$. *Determine* t_{1000}.

This problem came from some details that I had discovered about *linear recurrence sequences*. Let's address the problem as students would before delving more deeply into the mathematics.

I like this problem because students' first instinct is to try to find a formula that they can then substitute 1000 into and be done with it. Unfortunately, coming up with a formula isn't that easy. Since plans to tackle this problem might be few and far between, let's get our hands dirty and start working out some terms:

$$t_3 = t_2 - t_1 = 79 - 53 = 26$$
$$t_4 = 26 - 79 = -53$$
$$t_5 = -53 - 26 = -79$$
$$t_6 = -79 - (-53) = -26$$
$$t_7 = -26 - (-70) = 53 = t_1$$
$$t_8 = 53 - (-26) = 79 = t_2$$

An interesting thing happens. After a certain point, the terms return to the original two terms, which means the whole sequence of six terms just repeats over and over again, giving us the sequence

$$53, 79, 26, -53, -79, -26, 53, 79, 26, -53, -79, -26, 53, 79, \cdots,$$

which is periodic with period 6. Hence, every sixth term will be –26. A little arithmetic gives $1000 \div 6 \approx 166.7$ and $166 \times 6 = 996$. Thus, $t_{996} = -26$ and we can count off the terms until our desired one:

$$t_{997} = 53$$
$$t_{998} = 79$$
$$t_{999} = 26$$
$$t_{1000} = -53$$

When students finish, I ask them if they think there was something special with my starting values that lead to this behaviour. This usually leads them to try some different starting values, and to see that the sequence is periodic with period six each time. I then ask them to "prove" it.

Using algebra, if we let $t_1 = a$ and $t_2 = b$, our sequence becomes:

$$a, b, b - a, -a, -b, a - b, a, b, b - a, \cdots,$$

and we can see that it is periodic no matter what the starting values are. If we look carefully, we can notice that $a - b = -(b - a)$, so the sequence really goes

$$a, b, c, -a, -b, -c, a, b, c, -a, -b, -c, a, b, \cdots,$$

where three numbers and their additive opposites are repeated over and over again. It's very interesting that we get a periodic sequence out of a difference.

Depending on the class, sometimes I provide direct instruction about linear recurrence sequences. Most times, it is after we encounter the Fibonacci sequence, which many students have encountered previously. The Fibonacci sequence is defined by: $F_1 = 1$, $F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n > 2$. The Fibonacci sequence has a rich history that I won't get into here. The interested reader is encouraged to research some of the background of the sequence.

I like to relate the recurrence idea to the sequence $t_1 = a$, $t_n = rt_{n-1}$ for $n > 1$, which yields a geometric sequence $t_n = ar^{n-1}$. I argue that since the Fibonacci sequence is related to "multiples" of previous terms, perhaps it is geometric as well. If we assume that $F_n = ar^n$ (to follow the standard method), then the recurrence relation gives $ar^n = ar^{n-1} + ar^{n-2}$.

Since, for the Fibonacci sequence, it is reasonable to assume $a \neq 0$ and $r \neq 0$, the above equation can be simplified to:

$$r^2 - r - 1 = 0,$$

called the *characteristic equation* for the sequence. Applying the quadratic formula gives us *two* possible values of r:

$$r = \frac{1 \pm \sqrt{5}}{2}$$

How do we deal with two values of r? Perhaps each leads to a solution? This cannot be true of the term with the minus sign, as it is negative, so powers of it will alternate positive and negative, which doesn't match the Fibonacci sequence. Maybe it is an extraneous root, and the positive value is the one we should use. Once again, we can see that this doesn't work, since powers of r will be distinct, and the first two terms of the Fibonacci sequence are the same. Therefore, the Fibonacci sequence isn't geometric, so how do we interpret the two solutions to the characteristic equation?

If we assume that *both* values are part of each term, we will write the terms in the form

$$F_n = A\left(\frac{1+\sqrt{5}}{2}\right)^n + B\left(\frac{1-\sqrt{5}}{2}\right)^n$$

for some constants A and B. Since $F_1 = 1$, we have

$$A\left(\frac{1+\sqrt{5}}{2}\right) + B\left(\frac{1-\sqrt{5}}{2}\right) = 1.$$

Using $F_1 = 1$ we get

$$A\left(\frac{1+\sqrt{5}}{2}\right)^2 + B\left(\frac{1-\sqrt{5}}{2}\right)^2 = 1,$$

which simplifies to

$$A\left(\frac{3+\sqrt{5}}{2}\right) + B\left(\frac{3-\sqrt{5}}{2}\right) = 1.$$

Subtracting the first equation yields $A + B = 0$ or $B = -A$. Adding the equations and solving for A gives $A = \frac{1}{\sqrt{5}}$, which yields *Binet's formula* for the Fibonacci sequence:

$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right],$$

which seems amazing! Here we have a formula riddled with square roots that is *supposed* to yield integer results. Have your students try out this formula for some values of n and see that it does indeed give the terms of the Fibonacci sequence. Adventurous readers may enjoy showing that the above formula satisfies the recursion relation for the Fibonacci sequence.

What else can we learn about the Fibonacci sequence from this formula? Since $\frac{1-\sqrt{5}}{2} \approx -0.618$, as n gets larger, the second term in the bracket contributes less and less to the expression. As such, for $n \geq 2$,

$$F_n \approx \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n$$

is a good approximation. Since the second term is negative, raising it to integer exponents alternately gives us positive and negative results, so the approximation above is alternately a bit too big, then a bit too small, but rounds to the correct Fibonacci number. From the approximation, or from a little calculus, we can see the property of the Fibonacci sequence that

$$\lim_{n\to\infty} \frac{F_n}{F_{n-1}} = \frac{1+\sqrt{5}}{2} = \phi,$$

where ϕ is the *golden ratio*.

The keen-eyed reader will have noticed that the characteristic equation of the Fibonacci sequence *only* required the recurrence relation. The first two terms were used to determine the values of the coefficients A and B. This means that *any* sequence t_n with recurrence relation $t_n = t_{n-1} + t_{n-2}$ has terms of the form

$$t_n = A\left(\frac{1+\sqrt{5}}{2}\right)^n + B\left(\frac{1-\sqrt{5}}{2}\right)^n,$$

where

$$t_n \approx A\left(\frac{1+\sqrt{5}}{2}\right)^n$$

and

$$\lim_{n\to\infty} \frac{t_n}{t_{n-1}} = \frac{1+\sqrt{5}}{2} = \phi,$$

no matter what the initial two values are! This means that if we start with $L_1 = 1$ and $L_2 = 3$ to get the *Lucas sequence*, those terms are closely related to that of the Fibonacci sequence. Interested readers may enjoy working out the closed-form formula for the sequence

$$L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n,$$

which also, surprisingly, produces integer results. The curious may use a little algebraic manipulation on these formulas to verify some known identities such as

$$L_n = F_{n-1} + F_{n+1}$$

or

$$F_{2n} = L_n F_n.$$

But what does all of this have to do with our original problem? Using the same analysis as we did for the Fibonacci sequence, we get the characteristic equation of the original sequence is

$$r^2 - r + 1 = 0,$$

which has no *real* solutions. However, if we explore the world of complex numbers (or just turn a blind eye to the negative value under the square root), we get

$$r = \frac{1 \pm \sqrt{-3}}{2}.$$

Letting $w = \frac{1+\sqrt{-3}}{2}$, its complex conjugate is $\bar{w} = \frac{1-\sqrt{-3}}{2}$, the other solution. The magnitude of each of these complex numbers is

$$|w| = |\bar{w}| = \sqrt{w\bar{w}} = 1,$$

which means the magnitude of any power of w or $\bar{w}$ is 1. What does this mean? Let's take a look at some powers of w to see what is going on:

$$w^1 = \frac{1+\sqrt{-3}}{2}$$
$$w^2 = \frac{-1+\sqrt{-3}}{2}$$
$$w^3 = -1$$
$$w^4 = \frac{-1-\sqrt{-3}}{2}$$
$$w^5 = \frac{1-\sqrt{-3}}{2} = \bar{w}$$
$$w^6 = 1$$

So w and $\bar{w}$ are *primitive sixth roots of unity*. Thus, the pattern then repeats with $w^7 = w$, $w^8 = w^2$, and so on. The powers are periodic with a period of six terms, so it makes sense that the original sequence was periodic with a period of six terms as well. It might also make sense that even though there are complex numbers in the formula, similar to the radicals in Binet's formula for the Fibonacci sequence,

these just yield integer results. You are encouraged to determine the values of A and B and show that they do indeed yield the original sequence.

So why stop here? Let's take what we have learned and see if we can apply it to something new. It seems that if we use complex roots of unity, we can create sequences that will be periodic with *any* period we want. Hence, if we wanted a sequence with period 2, its characteristic equation would be $r^2 - 1 = 0$ (or $r^2 = 1$). Reverse engineering the recursion equation for the sequence would give us $t_n = t_{n-2}$. Choosing $t_1 = 1$ and $t_2 = 2$, we get the sequence 1, 2, 1, 2, 1, 2,…. Not too exciting. Also, using $r^3 - 1 = 0$ yields the recursion equation $t_n = t_{n-3}$, yielding the family of sequences $a, b, c, a, b, c, a, b, c, a,\ldots$ for $t_1 = a$, $t_2 = b$, and $t_3 = c$.

The same line of thought for a period 6 sequence would give the recursive equation $t_n = t_{n-6}$, which isn't what we were originally dealing with. What is going on? The subtle difference is the difference between an nth root of unity and a *primitive n*th root of unity. A *primitive n*th root of unity is an *n*th root of unity that isn't a *k*th root of unity for $k < n$. What does that mean? Well, the only first root of unity is 1. The second (square) roots of unity are 1 and –1, so 1 isn't primitive, but –1 is. The *n*th *cyclotomic polynomial* is the monic polynomial of lowest degree, whose roots are all the primitive *n*th roots of unity.

All the roots of the polynomial $P_n(x) = x^n - 1$ are *n*th roots of unity, both primitive and non-primitive. Hence, the *n*th cyclotomic polynomial is a factor of $P(x)$, while the other factor has all the non-primitive *n*th roots of unity as its roots (and can be factored into cyclotomic polynomials of smaller degree).

For example, for $n = 2$, we started with the characteristic equation $r^2 - 1 = 0$, which we can rewrite as $(r + 1)(r - 1) = 0$. The equation $r - 1 = 0$ has root 1, while $r + 1 = 0$ has root –1, the only primitive square root of unity. Thus, the first two cyclotomic polynomials are $\Phi_1(x) = x - 1$ and $\Phi_2(x) = x + 1$. In general, $x - 1$ is a factor of x^n and, in fact,

$$x^n - 1 = (x-1)(x^{n-1} + x^{n-2} + \cdots + x^2 + x + 1),$$

so the *n*th cyclotomic polynomial is a factor of $x^{n-1} + x^{n-2} + \cdots + x^2 + x + 1$.

Examining our original case, a periodic sequence with period 6, leads us to the characteristic equation $r^6 - 1 = 0$. We can factor this as

$$(r-1)(r+1)\,(r^2 + r + 1)(r^2 - r + 1) = 0,$$

which we can write in terms of the cyclotomic polynomials:

$$\Phi_1(r)\Phi_2(r)\Phi_3(r)\Phi_6(r) = 0$$

Note that the 6th cyclotomic polynomial can be used to reproduce our original recursion equation. The interested reader may enjoy generating periodic sequences of periods 2 and 3, working backwards from $\Phi_2(r)$ and $\Phi_3(r)$, respectively.

The cyclotomic polynomials and their connection to periodic linear recursive sequences are just one example of a topic from outside the curriculum that has interesting ties to topics within the curriculum. Don't be afraid to introduce some idea that is "beyond" the curriculum, as long as it benefits the topics you are studying. "Tangents" of this sort also have the benefit that you may spark the interest of a student who will play with the topic for the sheer intellectual joy of discovery, rather than because it was going to be on "the test."

Back in the dark ages, when I was in school, complex numbers were part of the curriculum. Since I have been teaching, they have been part of the curriculum in a very small way a couple of times (basically just talking about $i = \sqrt{-1}$ and not going much further). There is a lot of mathematics that is within students' grasp that has its ties to the curriculum. These ideas can be used to enrich what you are doing, strengthen students' understanding, show connections between seemingly unrelated parts of mathematics, or provide some enrichment to selected students. The addition and subtraction of complex numbers mirrors what we do when adding and subtracting polynomials, as well as having a connection to vector addition and subtraction for two-dimensional vectors.

Multiplication of complex numbers is similar to multiplication in algebra, with the added wrinkle that powers of i can be simplified, and this also has an unexpected tie to dilations and rotations when looking at the numbers on the complex plane. We can always remove the complex denominator from a complex fraction by multiplication by the conjugate, similar to what is done with fractions involving radicals. Finally, through Euler's formula $e^{i\theta} = \cos\theta + i\sin\theta$, we can see a connection between exponential functions and trigonometric functions. This opens many doors for exploration in calculus, where we can see that this makes sense by taking derivatives. You can also see the connections if you look at the Taylor series expansions of e^x, $\sin x$, or $\cos x$. Have fun exploring mathematical topics and searching for things to intrigue your students.

There are many great "popular" mathematics books out there for your extra-curricular mathematics reading. When not writing for the *Gazette*, I am often moonlighting in the *MathemAttic* section of the problem-solving journal *Crux Mathematicorum*, published by the Canadian Mathematical Society. Although the material in *Crux* is involved, the *MathemAttic* section is intended for high school students and

Continued on page 28

Linking Literacy and Math: Making Sense of Mathematical Modelling



DAVID COSTELLO
david@costellomath.com

David is a principal, who taught in the Primary division before assuming roles of numeracy interventionist, numeracy coach, numeracy leader, and curriculum consultant for Prince Edward Island. David has also instructed university courses regarding curriculum, differentiation, mathematics, and language arts. He facilitates professional learning in the areas of mathematics and school development. Dr. David Costello recently published three books: Mathematizing Student Thinking: Connecting Problem Solving to Everyday Life and Building Capable and Confident Math Learners; Making Math Stick: Classroom Strategies That Support the Long-Term Retention of Math Concepts; *and* Using What Works: Strategies for Developing a Literacy-Rich Environment in Math.

In my previous column, I explored ill-defined problems. Within the exploration, I touched upon what ill-defined problems are, why we are not seeing more of these problems in classrooms, the reasons we should be seeing more of these problems in classrooms, and how we can support students in working through ill-defined problems. The previous column provided an overview of ill-defined problems and introduced mathematical modelling as a method to broaden our perspective of problem solving and to help make sense of ill-defined problems.

In this column, I want to continue to build on my introduction to mathematical modelling. First, I will provide a brief overview distinguishing mathematical modelling from traditional problem-solving approaches used in classrooms. Second, I will dive into the four components of mathematical modelling and share an example of student thinking within each component.

Using concrete examples of student thinking for each of four components provides an opportunity to demonstrate the finer points of mathematical modelling and highlight how students must take an active, independent stance to their learning.

What Is Mathematical Modelling?

Mathematical modelling provides students the opportunity to engage in ill-defined problems—messy problems that are similar to problems they encounter in their experiences outside of school. Consider the student comment below in relation to mathematical modelling and ill-defined problems (Costello, 2022).

INTERMEDIATE EXAMPLE: A STUDENT EXPLAINS

When the teacher assigns mathematical modelling, I know that I will have to do more thinking. It requires me to figure out what is happening in the problem and then I think about what is necessary and what's not. It sounds easy here, but it takes a lot of time. And, how I understand the problem may not be the exact same way as another person understands the problem. What a person brings to the problem will shape how they approach the rest of their work.

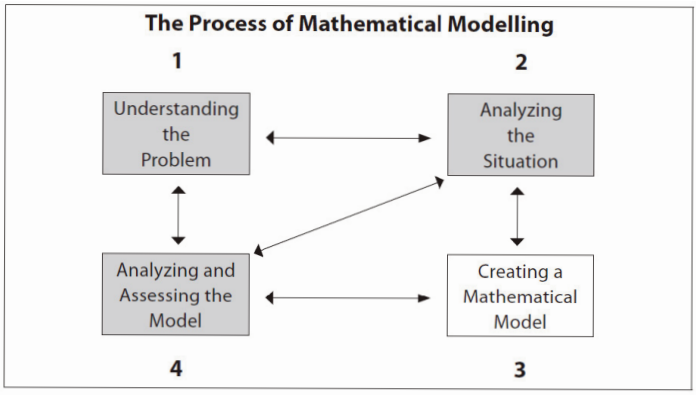
This comment highlights the complexities within mathematical modelling and how students must take an active, independent stance in understanding the problem and working toward a possible solution. Before moving on, it is important to emphasize that the real-world situation of the problem is the starting point in mathematical modelling. It is the point at which students engage with the problem and craft their understanding of it. The context of the real-world situation is not an add-on; it is a crucial aspect of the problem.

Within mathematical modelling, student thinking will shift back and forth from a contextualized perspective of the problem to a decontextualized perspective (Costello, 2022). The following visual highlights this process.



The student will approach the ill-defined problem from a contextualized perspective—the real-world situation. The student shapes the problem, based on their experiences, to make it workable so that they can mathematize the situation and their thinking of this situation. It is here that students construct a mathematical model that can place the real-world problem in mathematical terms. The student uses mathematics to examine the problem so that they can understand it more clearly. In constructing the model and examining it from a mathematical perspective, the student moves into a decontextualized setting. The real-world situation is not considered, as it is the mathematics that is driving this aspect of mathematical modelling. The student then returns to the problem within the context of the real-world situation so that they can determine if their understanding and solution makes sense in relation to what was originally being asked. Mathematical modelling, therefore, is a process that requires students to move between the contextualized real-world problem and a decontextualized mathematical representation of the problem.

There are four components comprising mathematical modelling. Please see the diagram below (Costello, 2022).



You will notice that boxes 1, 2, and 4 are shaded. This is intended to represent the components of mathematical modelling that are contextualized. Box 3, unshaded, is the component that requires students to consider the problem from a decontextualized, mathematical perspective.

Also captured within the diagram are numerous arrows. The arrows are intended to represent that non-linear process within mathematical modelling because mathematical modelling is an iterative process that is built on fluidity. Students may go back and forth between any components based on their understanding, questions, and stumbling blocks.

What the diagram does not capture is the importance of collaboration and communication.

Mathematical modelling is strengthened when it is structured as a collaborative endeavour. Students are encouraged to engage in mathematical modelling within groups—whether it is pairs, small groups, or whole group. This collaboration shapes supports and is supported by communication. When students have opportunities to share their thinking, they can refine and strengthen their understanding.

The Four Components of Mathematical Modelling

The four components of mathematical modelling are as follows (Ontario Ministry of Education, 2020):

1. Understanding the problem
2. Analyzing the situation
3. Creating a mathematical model
4. Analyzing and assessing the model

Although I will address each of the four components of mathematical modelling in isolation, I want to remind the reader that mathematical modelling is an iterative process. In addressing the components, I wanted a common thread and will use the following ill-defined problem for all four components (Costello, 2022).

> *In planning end-of-year activities, the school administration decides that the Grade 8 students can go on a field trip. The administration wants to know what the cost per student will be for such a trip.*

This problem and the student-thinking examples are taken from a Grade 8 classroom.

Understanding the Problem

Within understanding the problem, students make sense of the problem that they are assigned. Students must take the necessary time to understand the ill-defined problem. This understanding is ideally much more than surface level. It is not about identifying relevant and irrelevant information—because some information is not provided to the students. Students must recognize the context of the problem, what is being asked, and what information is required for solving the problem.

The following are suggestions about questions that students can ask themselves or within their grouping to understand the problem:

- What do I know?
- What question(s) need to be answered?
- What information is needed?
- What experience(s) can I bring to this problem?

The following is a work sample of student thinking within this component (Costello, 2022).

STUDENT THINKING

What do I know?

- Grade 8 students are going on a field trip.
- The school needs to figure out the cost per student.

What question(s) need to be answered?

- What is the cost per student?

What information is needed?

- Where is the field trip?
- What are the factors included in the cost (travel, accommodations, food, and chaperones)?
- Are students paying for any of the field trip?

What experience can I bring to the problem?

- When I have been on field trips before, I usually pay for my own food.
- Chaperones usually pay for their own food.
- Most field trips are usually 1-day so a hotel isn't needed.
- Students usually pay a small amount for part of the trip.

Analyzing the Situation

Once students understand the problem, it is imperative that they refine the real-world problem so that it becomes more manageable. Students make assumptions and place

restrictions on the problem and its context in an attempt to refine the parameters of the problem. These assumptions and restrictions provide structure to the messy, ill-defined problem.

The following are suggested questions that students can ask themselves or others when analyzing the situation:

- What assumption(s) can I make regarding this situation?
- How can I make this situation workable? What changes? What stays the same?

The following is a work sample of student thinking within this component (Costello, 2022).

STUDENT THINKING

What assumptions can I make regarding this situation?

- The field trip is one day.
- No need to pay for accommodations.
- Students will pay for a portion of the trip.
- Chaperones will pay for their own food.

How can I make this situation workable? What changes? What stays the same?

- With the field trip being one day, we can remove the consideration of hotels. If the trip goes beyond a day, the price of a hotel needs to be included.
- Chaperones paying for their food will lower the cost per student. This cost per student will increase if students have to cover the cost of chaperone meals.
- If the school decides to not pay for a portion of the field trip, students will have to pay more.

Creating a Mathematical Model

After making assumptions and placing restrictions on the ill-defined problem to make it workable, students will now approach the problem from a mathematical perspective. Students will decide on the mathematical language and concepts to apply, along with a strategy for applying such mathematics. It is in creating a mathematical model that students decontextualize the problem.

The following are suggestions of questions that students can ask themselves or others when creating a mathematical model:

- What mathematical language and concepts can be applied to the now structured problem?
- What representations and support can be applied to build a mathematical model?

The following is a work sample of student thinking within this component (Costello, 2022).

When I was working with Don on curriculum matters, I stayed overnight with them, and they were more than hospitable to me. Carol was a very great lady.

Todd Romiens, OAME President, 1994/95

STUDENT THINKING

What mathematical language and concepts can be applied to the now structured problem?

- Price of items
- Schedule
- Budget
- Survey

What representations and supports can be used to build a mathematical model?

- Numbers
- Spreadsheet

Analyzing and Assessing the Model

In analyzing and assessing the mathematical model, students must examine their work against the real-world problem they encountered at the onset of the experience. To do so, students must move from a purely mathematical lens, as was the case in the previous component, to a contextualized perspective.

In reintroducing contextual components, students examine the appropriateness of the mathematical model as it refers to the real-world problem. And, as part of this analysis and assessment, students review the assumptions and restrictions they identified earlier in the mathematical modelling process. The goal of this fourth component is for students to determine whether their mathematical model provides a reasonable solution to the real-world problem and whether alternative models exist.

The following are suggestions of questions that students can ask themselves or others when analyzing and assessing the model:

- Does my mathematical model lead to a reasonable solution to the problem?
- Do alternative mathematical models exist?

The following is a work sample of student thinking within this component (Costello, 2022).

STUDENT THINKING

Does my mathematical model lead to a reasonable solution to the problem?

- I decided that the trip would be one-day in length, meaning that it would be approximately 8 hours. I thought that the school wouldn't want to spend too much time traveling. I looked at local attractions and chose one that fit the age group.
- I then found the cost to go to the attraction.
- I thought of other costs: meals and travel.

Do other alternative mathematical models exist?

- I could have started with a reasonable cost, based on past field trips, and then determined if any local options were available.

Continued on page 30

In the Middle: Rising Ramps and Leaning Ladders



CARLY ZINIUK
carlyziniuk@gmail.com

Carly Ziniuk teaches Grade 9 Mathematics, Grade 12 Data Management and Advanced Placement Statistics at the Bishop Strachan School in Toronto, Ontario, Canada. She is very active in adopting real-life data to engage her students in solving problems.

Recently, the Ontario Ministry of Education announced that students entering Grade 9 in September 2024 will be required to earn a Grade 9 or 10 Technological Education credit as part of their Ontario Secondary School Diploma. As part of the hands-on activities fundamental to these courses, students will undoubtedly extend their STEM learning in areas such as measurement, programming, geometry, and simple machines. Middle school learners can benefit from similar concrete experiences in their science and math courses to broaden their mathematical thinking. In particular, we do not need to reinvent the wheel, so much as ensure that some good applications are included to effectively prepare our students for the technological education courses they will have to take. Applications like ladders, ramps, roads, and roofs can incorporate learning technical skills with our students in elementary math classes before they begin this revised curriculum.



Figure 1: *Students experiencing sliding ladder problem*

Ladders have been popular in math classes for a long time, with two classic problems often referred to as the *sliding ladder wall problem* and the *ladder box problem*. One visual of the sliding ladder problem using SketchUp is available at www.oame.on.ca/SketchMad/sketches/. My students have enjoyed this version of Ron Lancaster's sliding ladder problem:

> "If you were able to move the bottom of a ladder leaning against a wall to a point that is 10 cm further from the wall, would the top of the ladder slide down the wall a distance of 10 cm?"

My students experience this problem with a meter stick modelling the ladder, using the following instructions:

- Place your meter stick flat against a wall. Use the measuring tape to set the meter stick 10 cm from the wall, then measure the height of the stick above the floor.
- Repeat this procedure by moving the base of the meter stick a further 5 cm at a time. Label a graph with "**Horizontal Distance from the wall (cm)**" along the *x*-axis and "**Vertical Distance from the floor (cm)**" on the *y*-axis.



Figure 2: *Ron Lancaster's version of the sliding ladder problem*

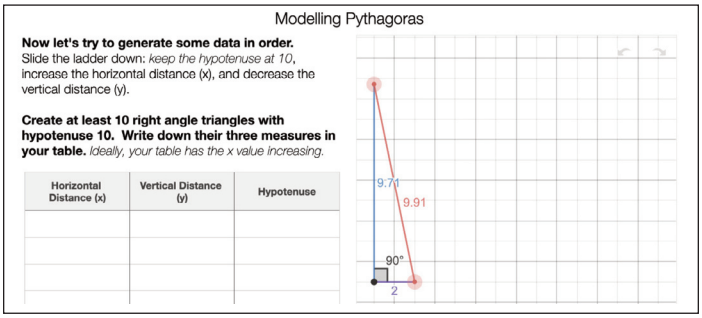


Figure 3: *Desmos version of the sliding ladder problem*

During online learning, I had my students do this using spaghetti noodles and graph paper. Then, my colleague, Dora Bellos, and I adapted a similar approach from Sean Keheller, using the Desmos teacher platform at www.tinyurl.com/DesmosPythaGazette.

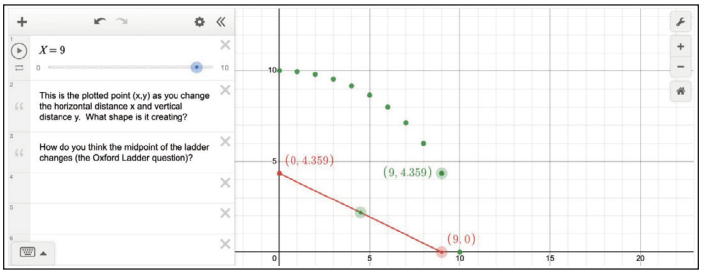


Figure 4: *Graph of the placement of the sliding ladder problem*

Lesson materials from TIPS4Math, Grades 9 and 10 Applied, include practical problems that directly connect to building, design, and job-applicable skills. The measurement unit in the Grade 10 Applied lesson also has several additional ladder questions involving everyday conversions between the imperial and the metric systems of measurement. Ladder safety documents found at the Canadian Centre for Occupational Health and Safety (CCOHS) website, www.ccohs.ca/, can enhance the TIPS4Math questions. CCOHS indicates ladder lengths in meters (see Table 1), yet most commercially available

ladders in Canada are sold in Imperial (6–8 ft., 8–10 ft.).

Table 1: *Ladder lengths from CCOHS*

Grade of ladder	Description of ladder grade	Maximum length
1AA	Construction and industrial use, special duty (170 kg load)	6 m
1A	Construction and industrial use; extra heavy (136 kg)	6 m
1	Construction and industrial use; heavy duty (113 kg)	6 m
2	Trades and farm; medium duty (102 kg)	3.6 m
3	Household, light duty (91 kg)	2 m

The Ladder Association (www.ladderassociation.org.uk/guidance/using-a-ladder/) provides the graphic shown in Figure 5. Correct ladder placement requires a 1:4 ratio and safety angle at 75°. Using trigonometry, can you identify the inconsistency in equating this angle and this ratio?

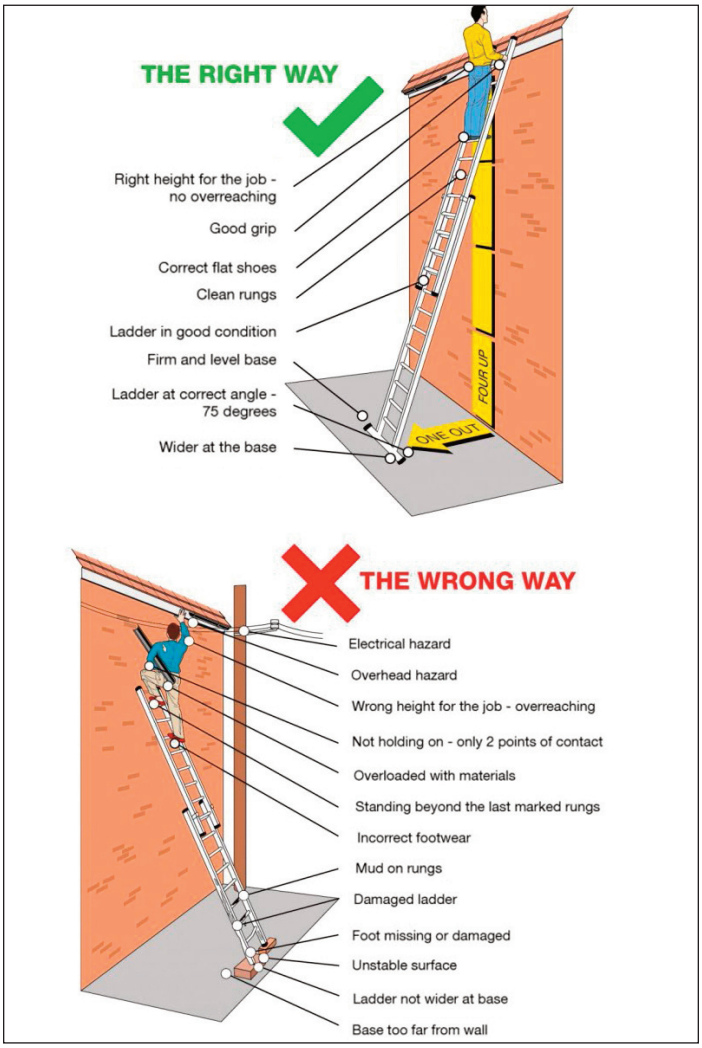


Figure 5: *The Ladder Association graphic to indicate safe ladder placement*

Determining the safe placement of a ladder to reach a second-floor window, for example, is a challenging, multi-step process for students. Middle school students could do this using graph paper and scale, along with conversions between imperial and metric.

The classic ladder box problem asks how high a given ladder reaches up a wall while just touching a specified box that is between the ladder and the wall. Having students experiment with different-size boxes and a meter stick can be an alternate concrete opportunity to explore. We can adapt the sliding ladder problem on SketchUp or Desmos to see how the size of the box relates to how high the ladder can reach. There are many practical constraints for construction, where ladders need to exceed another obstacle or lean against another surface. Seeing whether any possible situations meet the safety criteria is another extension. Presh Talwalker's *Mind Your Decisions* YouTube channel presents this problem more theoretically as the *ladder and box problem* at www.youtube.com/watch?v=CZKD7rffF0M.

Throughout the elementary science curriculum, students explore simple machines including ramps. Math students also can examine ramps throughout their elementary learning. Throughout middle school, investigating and constructing real ramps can be used to measure and draw different angles, determine congruent and similar triangles, calculate Pythagorean theorem, while preparing for slope and trigonometry in high school.

Math trails can reveal many ways of using ramps in practical applications (additional examples of these can also be seen in my December 2017 (i.e., 56(2)) column in the *Gazette* archives). Rolling a ball or toy car to determine how the speed of the item changes with the ramp's angle is one quick math–physics connection (pun intended!).

- How long does it take different items to get down each of the different slopes you find on a math trail?
- If you chose a larger or heavier item, does it go faster? Record your findings in a table. *You may want to take a few different measurements to see if your results are consistent.*
- How far does your item go after leaving the ramp before it comes to a stop? What do you think affects this distance?
- Can you construct a ramp, with other things you can find in your space, that makes your item go faster? Go farther? *Document your findings on what factors make the ramp the best for these features.*

Ramps in the real world must employ the practical applications of construction. For many years, my colleagues and I have been using accessibility ramps to introduce slope

and Pythagorean theorem, while noticing safety considerations and the building code. Initially adapted from Grade 9 Applied TIPS4RM Linear Relations materials, these ramps provide building problems involving technical constraints from the Government of Canada's Accessible Housing document (www.publications.gc.ca/collections/collection_2017/schl-cmhc/NH18-24-63A-2016-eng.pdf).

The Government of Canada's Accessible Housing document states:

> Many people find it difficult to independently use a ramp that has a slope of 1 in 12. Slopes of 1 in 15, 1 in 18, or 1 in 20 require less effort and are recommended if the primary users have limited strength or stamina, as they may be easier to use without assistance. Remember, the less steep it is, the longer the ramp will have to be. (p. 3)

In Canada, where outdoor conditions could be problematic, a ratio of 1:20 is often required for icy conditions, even if a wheelchair is motorized.

My students use graph paper to design ramps to scale (with units) for each client below:

- Andie owns a very powerful motorized wheelchair and often visit parents in Toronto. The parents wish to build a ramp that leads from their sunken living room to their kitchen on the next level. The height of the ramp is 60 cm.
- Bhatia lives in Point Pelee and requires a ramp from the back deck to a patio. Bhatia is of average strength and operates a manual wheelchair. The deck is 25 cm above the patio.
- Charly lives outside of Kenora in Northern Ontario. Charly is healthy, but not particularly strong. The house is a single-level bungalow, but the front door is 0.45 m above ground level. The path that leads directly to the front door from the street is 60 m long, but the ramp does not need to be 60 m long.

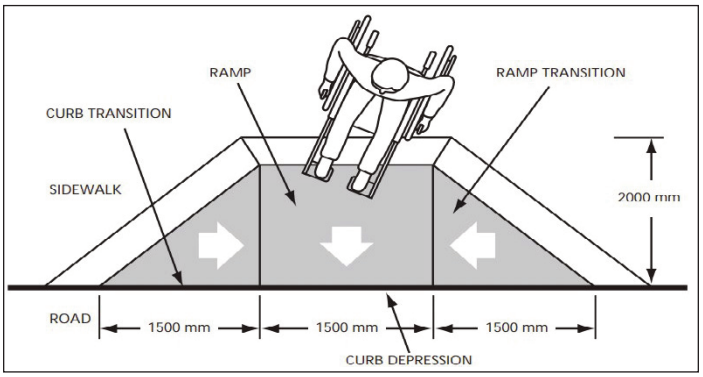
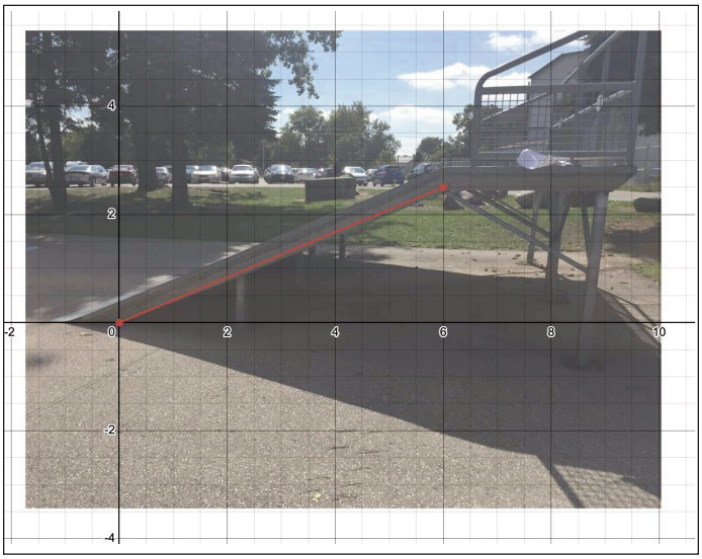


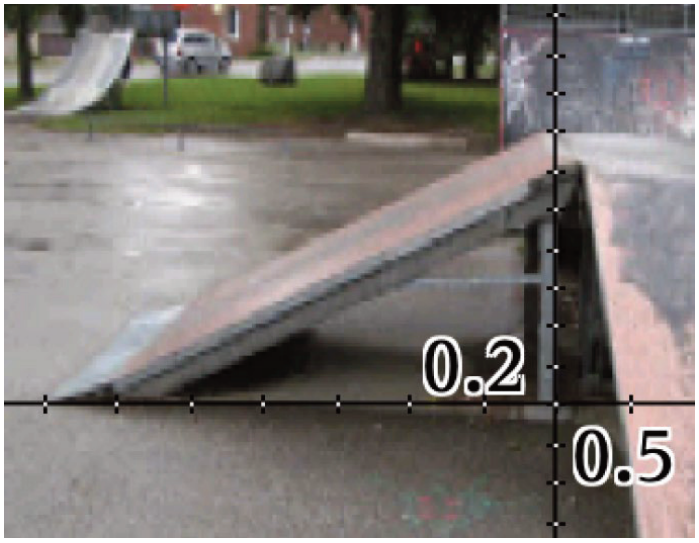
Figure 6: *Curb cuts design from the City of Toronto Accessibility Design Guidelines (www.toronto.ca/wp-content/uploads/2017/08/8fcf-accessibility_design_guidelines.pdf)*

My students analyze various ramps in our community to see if they meet different building codes. The City of Toronto, for example, has Accessibility Design guidelines and resources to investigate. Students can review these real-life requirements and incorporate much of their mathematical learning. We have also examined whether temporary ramps, created by placing wood or metal panels over stairs, are truly safe for wheelchairs. Many communities also document the safe usage of ramps at skateboard parks. Do-it-yourself skate ramp construction projects are easily found online. Overlaying a Cartesian grid on photos with Desmos or a TI-Nspire™ can bring the outdoor environment, and students' real experiences, into the classroom.

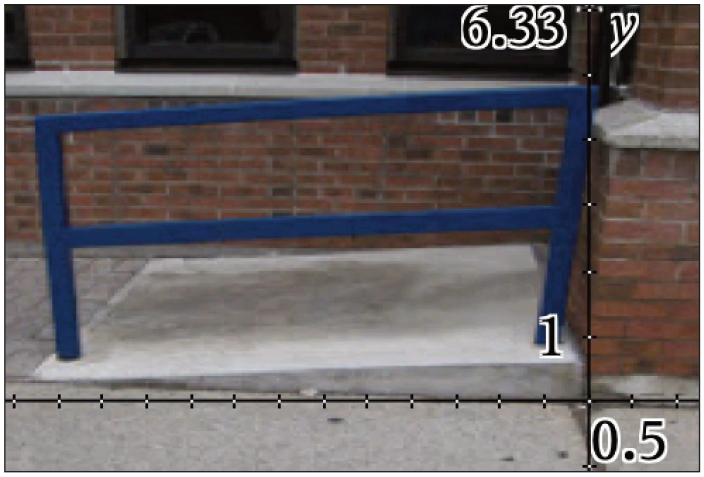
Skateboard Ramps on Desmos and TI-NSpire

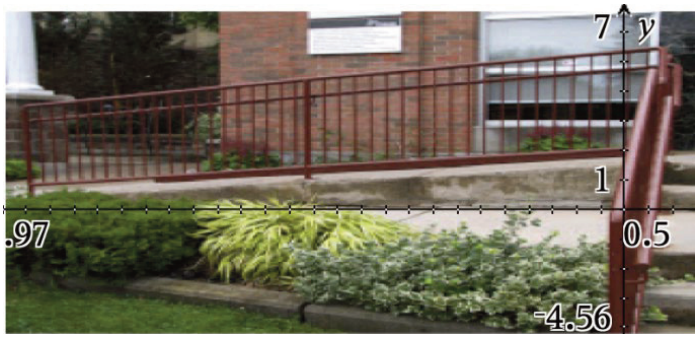
The separation of tick marks for each axis appears below or just to the left of the axis. In some cases, the range for the axis shows, while in other cases it has been cropped. These features are displayed as the TI Nspire (TM) shows them.





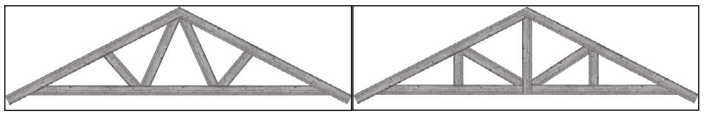
Accessibility Ramps on TI-NSpire





Roofing and road grade explorations involve slope, scale, and proportionality. Interior support beam diagrams reminded students of angle measurement questions they had completed in a previous unit, without any context. We can ask our students to connect their learning to occupational and home repair. How much more would a steeper skateboard ramp cost? Why do steeper roofs require more shingles? Why do "low" road grade percentages, like those above, 16 percent or 1:5, require safety signage for drivers? (See also *Gazette* 53(2), pp. 27–29 for a related article.)

Roof Design with Images from www.mathgiraffe.com/blog/geometry-with-roof-trusses



Roof Calculator Examples (www.roofcalc.org/)



Road Grade Signs from Northern Ireland (www.geograph.ie/, images J3889 and C7234)

Continued on page 28

▲ Dynamical View of Proportionality: Integrity of Structure



JÉRÔME PROULX
proulx.jerome@uqam.ca

Jérôme Proulx is a professor of mathematics education (www.profmath.uqam.ca/~jproulx), and runs the Laboratoire Épistémologie et Activité Mathématique *(www.leam.uqam.ca). His research focuses on epistemological issues related to cognition and/in mathematics. His current research program is dedicated to mental mathematics and school mathematics content development.* (Photo by Nathalie St-Pierre of Université du Québec à Montréal)

For many students, the introduction to proportionality marks a transition from additive to multiplicative thinking when considering the relations between the quantities. In many problems implying proportions, an additive relationship (e.g., 3 to 6 is 3 more, 6 to 2 is 4 less) often misguides students' consideration of a multiplicative relationship (e.g., doubling, dividing in 3), and can even lead to erroneous answers at times.

There are, however, some students' strategies, worthy of pondering, that appear to fit right in the middle of both multiplication and addition. These might be conceived of as elementary, but a mathematical look at them through students' work might tell a different story. In this third column, I continue to contemplate and explore some of these additive strategies that students often devise. In so doing, I raise elements of mathematical significance. These elements, I argue, can in turn lead to an enriched understanding of proportions, through offering a dynamical view of proportionality itself. This third and last column addresses the question of integrity of structure existing between similar ratios.

The Problem and the Strategy

Marco, an 11-year-old, was given the following task to solve (inspired from Herron & Wheatley, 1978):

> If 3 pounds of ground meat cost $10, how much would it cost for 10 pounds?

Marco said that three times 3 pounds gave him 9 pounds, which would cost $30, since $10 times 3 is $30. Then he divided $10 by 3, giving him $3.33 for the pound that is

missing from 9, to get to 10 pounds of ground meat. Hence, $30 plus $3.33 gives $33.33 for 10 pounds of ground meat.

3 pounds	→	$10
9 pounds	→	$30
1 pound	→	$3.33
10 pounds	→	$33.33

What is this strategy all about?

Marco's strategy has strong ties to the previous ones presented in the first two columns (Proulx, 2022, 2023), which combine both additive and multiplicative elements to form the desired answer. What this third strategy particularly puts forth is how the structure of the relation between weight and price of meat is preserved throughout the solving. In effect, the initial ratio of 3 pounds for $10 is reproduced to give 9 pounds for $30, and again as 1 pound for $3.33, making up another ratio of 10 pounds for $33.33 that still preserves the initial relation between weight and price.

This strategy shows how the proportional structure of ratio is kept constant, how its integrity is preserved throughout the generated ratios to get to the one aimed for, that is, the 10 pounds for $33.33. All these ratios follow and reproduce this initial relationship of 3 pounds for $10, and can be seen as part of the same family of ratios that is generated through transformations (as explored in the first column in Proulx, 2022).

Moreover, Marco's strategy highlights how the final ratio of 10 pounds for $33.33 is not a single-obtained ratio, independent of the other ones. It is a ratio that is formed with the other ones, that contains all the other ratios. Indeed, the 10 pounds for $33.33 is composed of, or has been created with, all the other ratios obtained: 10 pounds → $33.33 comprises the 1 pound → $3.33 and the 9 pounds → $30, which itself is composed of three 3 pounds → $10. It is as if the 10 pounds → $33.33 is made of all these other, structure-like ratios, which all together create it.

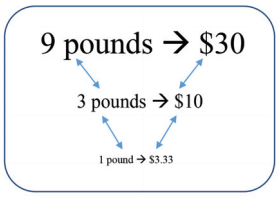


The same thing could be said for another ratio, e.g., 24 pounds for $80. In this case, the 24 pounds → $80 could be seen as comprised, for example, of: 1 pound → $3.33, with 9 pounds → $30, with 3 pounds → $10, and with 10 pounds → $33.33, and then with another 1 pound → $3.33.



Integrity of Ratios

The notion that the other ratios are an integral part of the final ratio, or are seen to compose it, can lead to a different appreciation of these ratios altogether. The preservation of integrity of ratios between 1 pound → $3.33, 3 pounds → $10, 9 pounds → $30, 10 pounds → $33.33 (and now 23 pounds → $70), and their interconnection through being built from one another, enables us to focus on how each one ratio is a transformation of the other. And, more than a transformation, it compels one to see these ratios as *enlargements* or *shrinkages* of one another. Because a ratio contains other members of its family, and is also contained by other members, there is a play on scale. Hence, 1 pound → $3.33 is not only preserving the ratio of 3 pounds → $10, but is a reduced version of it, and vice versa, where 3 pounds → $10 is an enlarged version of 1 pound → $3.33. In the same way, 9 pounds → $30 is to be seen as an enlargement of 3 pounds → $10.



The same can be said of 1 pound → $3.33 and 10 pounds → $33.33, where one could view these as being ten times smaller or bigger than the other. This prompts a "stretching" view of ratios, a zoom-in/zoom-out perspective, when establishing proportions, to get from one ratio to the next. In that sense, ratios are, to a point, connected to one another, that each one is merely a reproduction of the other—cloned along the same relationship, like nesting or matryoshka dolls—preserving the integrity of the relation. Thus, when one enlarges or shrinks it, one cannot escape the integrity of the relationship existing within these ratios.

Dynamic View of Proportions

The zoom-in/zoom-out or nested character of ratios aligns well with Lesh et al. (1988), who gave a dynamic view raised in both previous columns, and again cited here:

> The equation A/B=C/D can be thought of as a static (=) relationship between two simple mathematical systems which are described independently by the ratio relationships A/B and C/D. However, it can also be thought of as a dynamic transformation which maps one simple mathematical system (described by the ratio relationship A/B) into an "equivalent" system (described by the ratio relationship C/D). It seems to us that the recognition of structural similarity is an essential component of proportional reasoning. (p. 100)

In the similarity of structure from A/B to C/D resides this same idea that C/D is an enlarged or shrunken version of A/B, or vice versa from C/D to A/B. Hence, going from one to the other, in opposition to an equality of states, calls for a transformation that preserves the structure of the ratio. Therefore, C/D is simply an A/B transformed, and A/B is also

a C/D transformed; enlarged or shrunken, zoomed in or zoomed out. It then goes beyond a singular relation of equality and becomes more of a fluid movement from one ratio to the next, dynamically.

Conclusion

Conceiving of ratios using a nested view leads to a different conception of what solving proportional problems can be about. That it can be a missing-value problem or a comparison, proportional reasoning entails the same endeavour: looking for a ratio that is the enlarged or shrunken version of another one. It is a fluid search for similarity of structure, for grasping the same ratio, but transformed, as they are all nested together and embody the same relation.

To close this series of three columns on proportionality, a word can be said on how Marco's three strategies all tap into elements of significance for proportional reasoning. These elements of "family of ratios," of "covariation," and of "similarity of structure" all raise explicitly how proportions can favourably be seen dynamically, away from a mere search for answers or for the value of a specific unknown in the problem (the "*x*"!). It thus seems fitting to close with another of Lesh et al.'s (1988) remarks, this one about how proportionality, and mathematics itself, go beyond a search for particular unknown values and are about an intense study of structures and transformations:

> The misconception that the essence of mathematics is "going around looking for lost *x*'s" is an opinion shared by many. Yet, to understand the essence of proportional reasoning, it is important to understand that mathematics is essentially the study of structure and invariance, equivalence and non-equivalence under a variety of different transformations. As students progress to higher levels of mathematics, far fewer problem-solving activities fit the "*x* finding" stereotype. Far more fit into the category of studying structure–transformation–invariance. (p. 100)

References

Herron, J.D., & Wheatley, G.H. (1978). A unit factor method for solving proportion problems. *Mathematics Teacher, 71*(1), 18–21. doi.org/10.5951/MT.71.1.0018

Lesh, R., Post, T., & Behr, M. (1988). Proportional reasoning. In M. Behr & J. Hiebert (Eds.), *Number concepts and operations in the middle grades* (pp. 93–118). National Council of Teachers of Mathematics, Lawrence Erlbaum Associates.

Proulx, J. (2022). Column A – Dynamical view of proportionality: Families of ratios. *Ontario Mathematics Gazette, 61*(2), 17–19.

Proulx, J. (2023). Column B – Dynamical view of proportionality: Covariations. *Ontario Mathematics Gazette, 61*(3), 22–24. ▲

▲ Elementary Math Matters: ChatGPT



LYNDA COLGAN
lynda.colgan@queensu.ca

Lynda Colgan is Professor Emerita, Faculty of Education, Queen's University. Lynda is currently working on a number of research, resource-creation, and teacher-education projects, funded by the National Sciences and Engineering Research Council of Canada (NSERC), the Social Sciences & Humanities Research Council (SSHRC), and the Mathematics Knowledge Network (MKN). She will carry on her long-established commitment to STEM outreach by continuing to seek out creative avenues to engage students, parents, and educators in mathematics at home and in the classroom.

The conversation at many post-secondary institutions of late has been focused on ChatGPT—a natural language-processing tool driven by Artificial Intelligence (AI). One can interact with the ChatGPT bot to have conversations, obtain answers to questions, and compose essays. ChatGPT is relatively new—it was launched on November 30, 2022, and the "Plus" version just became available on February 1, 2023. The benefits and dangers of ChatGPT have been featured in many op-eds, articles, and blogs, using examples of essays that the software has composed using its huge database of information. To gauge for yourself, the current strengths of AI to replicate human written content, I urge you to take a careful look at a wide range of these computer-generated papers at www.ukessays.com/essays/chatgpt/.

I toyed with the idea of experimenting with ChatGPT for this column—asking it to generate a short piece about the current issues in elementary mathematics education—to see if anyone recognized the difference in writing styles between *la machina* and *moi*, and looked carefully enough to question the originality of the text. I would submit to the Editor, Tim Sibbald, and wait for his reaction before responding with "*Punk'd*" in the vein of the once-popular hidden-camera, practical joke, reality television show. In the end, my joke seemed a little too close to home for a university professor like Tim.

Every post-secondary instructor I know is already overburdened with "cut and paste" cheating by students, and most now require that all submissions first be run through a plagiarism checker such as *Turnitin* along with the auto-generated report. *Turnitin* is able to check for originality of text by looking for "similarity," i.e., comparing the text submitted with

a huge body of published works from journals, books, and the Internet. If a student includes a long piece of text without attribution or reference, *Turnitin* "reads" that text as original work and begins the comparison process of "matching" the text to existing work. If the text can be traced to a published source, then it is red flagged and reported.

At the end of an essay, *Turnitin* generates a report that identifies a "Similarity Index" along with a breakdown of where similar/identical material has been detected. A high similarity index tells us that there is concern—work included has not been properly referenced, and has been presented as legitimately quoted material—meaning that there has been a breach of academic integrity.

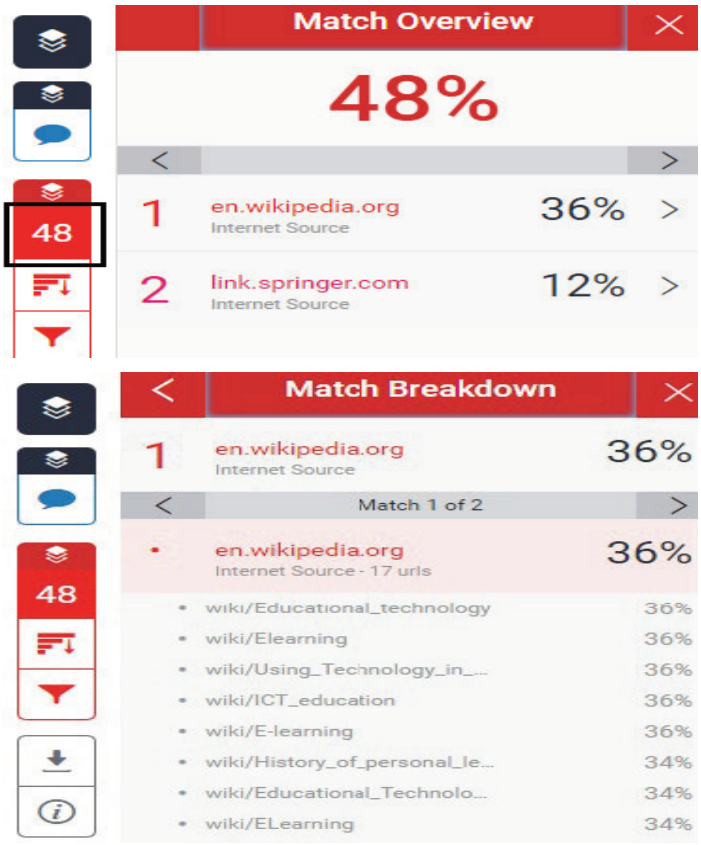


Figure 1: *These images[1] from* Turnitin *show an overall report, as well as a breakdown of how many matches have been found for each source. In this case, 48 percent of the submitted text was identical to authenticated text, with most of the writing taken directly from Wikipedia.*

I have been thinking about why we may need to use tools like *Turnitin* and ChatGPT in other contexts—from curriculum policy documents to Tweets about teaching mathematics and professional learning resources published by school districts.

Recently, I came across a web-based, early math resource, published by an Ontario school district (I won't "out" the board) for educators and families that presented one-page overviews about a variety of topics related to quantity and number—crucial concepts in young children's conceptual development. The resource contained sections on *stable order, one-to-one correspondence, cardinality, conservation of number,* and *order irrelevance*, to name but a few—each colourfully illustrated and explained briefly, some with links to short video clips.

For example, an illustration similar to Figure 2 below was used to illustrate the concept of conservation of number. As you will note, both rows have the same number of candies, yet some children will believe that the bottom row has more candies, when arranged as in the second set, or in the top row, when arranged as in the last set.



Figure 2: *In her study about acquiring conservation of number through play, Watanabe (2017) asked simple questions such as "Which is more?" or "Which number is bigger?"*

As I worked my way through the resource and "clicked" on its many interactive buttons, I was taken to the same resource on a for-fee website that offered lesson plans, podcasts, webinars, and courses. I also found that the resource was freely downloadable, but was attributed to the website, not the school district, which led me to question ownership of the document. I did not need to use *Turnitin* to spot the similarity (identicalness?) of content in the digital board-endorsed resource and on the website. Even if I had scanned the pages with *Turnitin* because the text was so minimal—headers and bullet points or short sentences—the software would not have been able to "run" reliable comparability tests.

After a cursory examination of the rest of the website, I returned to the resource in question and carefully screened the pages for citations and attributions. I found reference to just one scholarly book of note that was published in 1978 by two cognitive scientists: Gelman and Gallistel. The reference list itself consisted of six web-based listings and four research papers. To my dismay, there was no attribution given to the seminal work that was the basis for *all the concepts* in the resource, i.e., the original studies in the field of children's cognitive development in mathematics from which *all* others were adapted, refined, argued, and tested. In other words, Johann Pestalozzi, Jean Piaget, and Constance Kamii were missing, as were Herbert Ginsburg and Stanislas Dehaene, among others.

Next, I turned to ChatGPT and had a very interesting and informative "conversation" about each of the topics listed in the resource's Table of Contents. Each one was explained clearly and included highly accessible examples. For example, ChatGPT told me that, "Conservation of number, also known as

the principle of one-to-one correspondence, is a fundamental concept in mathematics that states that the quantity of objects or entities in a set remains the same even if their arrangement or form is changed." The bot went on to tell me that this cognitive milestone was first studied by Jean Piaget in the 1930's. The only thing missing from the ChatGPT reply to my queries were explanatory images.

After discovering that this freely downloadable counting and quantity teaching resource contained egregious omissions, I turned my attention to many of the recent posts on Twitter that argued that current mathematics curriculum is flawed because of its Eurocentric focus. When asked, "What is Eurocentric math?," ChatGPT replied:

> Eurocentric math refers to a traditional approach to mathematics education that places a heavy emphasis on European and Western mathematical concepts and history[,*] while largely ignoring the contributions of other cultures and civilizations to the field of mathematics. This Eurocentric approach often presents mathematics as a neutral and objective discipline, while failing to acknowledge the ways in which cultural and social contexts shape the development and application of mathematical knowledge. This can result in a narrow understanding of mathematics and limited opportunities for students from diverse backgrounds to see themselves represented in the field. There is a growing movement to decolonize math education and to incorporate a broader range of mathematical perspectives and histories into the curriculum.
>
> * - Our proofreader, Gitta, pointed out the grammatical omission of this comma.

As I next scrolled through many threads on Twitter that presented information on everything from African Quilt Patterns and celestial navigation by early Polynesian peoples, to the discoverers of the concept of zero and the Pythagorean theorem, once again, I was struck by the fact that the tone of the posts suggested that appreciation of culturally relevant mathematics was new and radical (as does the last sentence of ChatGPT's response above)—and today's proponents are pioneers in confronting the myth that math is politically and culturally neutral—something that is patently incorrect.

The concept of ethnomathematics is more than five decades old. The term was first used by Brazilian mathematician Ubiritan D'Ambrosio in 1970, who (according to ChatGPT):

> …used the term to describe the study of mathematical knowledge and practices in different cultural contexts, with a particular focus on the mathematics of non-Western cultures. He argued that mathematics is not a universal and objective discipline, but is shaped by cultural, social, and historical contexts. D'Ambrosio's work has been influential in the development of ethnomathematics as a field of study, which seeks to recognize and value the diverse ways in which people engage with mathematical concepts and practices around the world.

In the decades that followed D'Ambrosio's powerful book, *Ethnomathematics: A Multicultural View of Mathematical Ideas*, numerous scholars have studied and published books and articles about the cultural dimensions of math. Among the most notable were works by Paulus Gerdes, Marcia Ascher, Michele Emmer, Terezinha Nunes and Peter Bryant, and George Gheverghese Joseph. For example, we learn from ChatGPT that Marcia Ascher:

> …was a leading expert in the study of traditional textiles and their relationship to mathematics and culture. Her research focused on the mathematical principles underlying the designs and patterns of textiles from various cultures around the world, including the Andes, Central Asia, and Africa.
>
> In particular, Marica [sic] Ascher was known for her work on the mathematical symmetries and tessellations in textile designs. She also explored the ways in which mathematical concepts such as symmetry, recursion, and iteration are used in textile design and production.
>
> Marica [sic] Ascher's work in ethnomathematics helped to bridge the gap between mathematics and the humanities, and her research has been influential in inspiring new approaches to teaching mathematics and promoting cross-cultural understanding.

I learned about these scholars and their contributions to understanding the cultural contexts in which mathematical knowledge is created and used from my coursework as a student and as an instructor. I had impactful experiences at various conferences and learned about the efforts by valued peers, like Nipissing University's Susan Stuart, who were actively working to be more inclusive and employ more diverse approaches in their efforts to teach mathematics.

It was because of the influence of scholars like Ascher that one of the first purchases I made when coming to Queen's in 1998 was a set of 16 "Math Around the World" posters that hung prominently in my classroom and were referenced often. I like to think that the posters were much more than "eye candy," and, in fact, made a strong statement about my values and deep respect for a field that belongs to no one because it originated from and belongs to everyone, everywhere.

Those colourful and invitational posters were the impetus and backdrop to many lessons and explorations rooted in the history of mathematics. My teacher education candidates were fascinated by the Japanese abacus (also known as soroban)

as a tool for teaching place value; the simultaneous simplicity and sophistication of Egyptian (or its close relative) "Russian Peasant" multiplication; the beautiful symmetry and breathtaking Islamic tessellations of the Alhambra Palace; the vigesimal (base-20) number systems of the Maya and the peoples of Cameroon, Tanzania, and Mozambique; and the regional names given to the ubiquitous (what we call) "Bear Paw" quilt pattern (i.e., Duck's Foot in a Pond, Turkey Tracks, Hands of Friendship). Included in my lessons too were introductions to the first dice (a talus bone); the legends of the tangram and Lo Shu magic square; knots for counting (the Inca and Chinese) or measuring (the Greeks); multiplying using the "bones," developed by the father of logarithms John Napier; and maximizing volume through paper-folding the box, made famous by Italian origami expert, Giuseppi Baggi.

My own small (but I believe) positive steps and those of countless others, to infuse the history of mathematics into existing curriculum, remain silent on ChatGPT and other AI tools because it is the researchers and developers at OpenAI (the company that developed and owns ChatGPT), who select the data that is used to "train" the ChatGPT bot. In turn, they then rely on user feedback to flag errors and report bias to stop the spread of fake information—which should lead us to demand to know "what data," "what data sources," and "what processes" are used to train ChatGPT.

While I wrote several articles about tessellations that were published by the NCTM and are available on the Web (*It's Friezing in Here: Tessellations through Art, Architecture, and Cultural Artifacts*) and paper-folding (*Algebraic Thinking through Origami*), my voluminous course notes, teacher education conference presentations, and workshop packages remain in filing cabinets, on hard drives, and scattered as hard copies to the four winds. I know the same is true for many mathematics education professors and school district leaders, meaning that these examples are far out of the reach of *Turnitin* and ChatGPT.

Thus, a huge body of constructive, applied "proof" that global perspectives using concrete examples have long been used to bring justice to mathematics education is absent from the digital records that can be accessed by *Turnitin* or "learned" by ChatGPT.

While some may argue that the focus on the history of mathematics is less important than addressing the social (mis)justices attributed to mathematics itself, these positive and concrete classroom-based efforts are key to busting the myth that mathematics was largely developed by Greeks and other Europeans.

Now, more than ever, I believe that mathematics educators must take action to confront the distribution of "lite" resources that poach, distill, and oversimplify the complex constructs in children's mathematics cognition in the name of "accessibility." The rich history of who, how, and why particular tasks, specific questions, certain algorithms, and defined strategies have been developed by psychologists and validated for classroom use by mathematics educators is non-existent unless sought by the end user.

It is also time that mathematics educators act to include Baudhayana—the Indian mathematician credited with discovering the "Pythagorean" theorem circa 700 BCE—as well as his early Babylonian and Syrian co-discoverers in the curriculum and our conversations. If we are to have any hope of demarginalizing non-Eurocentric mathematical knowledge, we must talk to our students about debates around controversial topics, including why it is incorrect to attribute the "Pythagorean theorem" to Pythagoras, yet why the name is unlikely to change because it is so deeply engrained in the language of mathematics.

Omissions of cognitive and education researchers and mathematicians alike translate to the flawed understanding of what mathematics education is and has been over time. The time is now (in fact, it was yesterday) to teach the skills to critique and interrogate resources like ChatGPT and professional development documents, which should result in loudly "calling out" the transgressors, be they bot or human.

Perhaps more urgent is the need to make changes to curriculum documents to include explicit, relevant, and inclusive examples from a multiplicity of cultures across thousands of years. If there are changes to curriculum documents to include explicit order irrelevance tasks and topics such as the invention of zero or the efficient, but unconventional computational processes adopted by street children in Brazil, it follows that there must be changes in classroom teaching and learning. And concomitant with the transparent inclusion of such concepts and topics, one would hope to see longer-term positive consequences—historical antecedents informing contemporary understandings.

Postscript

In the hours between asking ChatGPT about Marcia Ascher and my final read before hitting the "Send" button to submit this column, ChatGPT had "learned." When I went to validate its previous response, this is what it said:

> I apologize for my error in my previous response. The correct name of the person you were asking about is Marcia Ascher, not Marica Ascher. As I mentioned earlier, Marcia Ascher was an American mathematician and anthropologist[,*] who specialized in ethnomathematics, the study of the relationship between mathematics and culture.
>
> * - Our proofreader, Gitta, pointed out the grammatical omission of this comma.

Endnote

[1] https://howto.hyms.ac.uk/knowledge-base/turnitin-upload-a-document-and-view-download-a-report/

Reference

Watanabe, N. (2017). Acquiring Piaget's conservation concept of numbers, lengths, and liquids as ordinary play. *Journal of Educational and Developmental Psychology, 7*(1), 210–217. doi.org/10.5539/jedp.v7n1p210▲

Continued from page 15

their teachers. Each issue contains, among other great features, "From the Bookshelf of…", a hearty recommendation of a popular mathematics book. *Crux* is available for free at www.cms.math.ca/publications/crux/. Please check it out and share it with your students and colleagues.

Now for your homework:

Riley has 64 cubes with dimensions 1 × 1 × 1. Each cube has its six faces labelled, with two appearing on two opposite faces and one on each of its other four faces. The 64 cubes are arranged to build a 4 × 4 × 4 cube. Riley determines the total of the numbers on the outside of the 4 × 4 × 4 cube. How many different possibilities are there for this total?

Until next time, happy problem solving! ▲

Continued from page 22



The new technological education course, beginning in September 2024, will cover several different career sectors like construction, transportation, manufacturing, and automation. These examples of ramps, ladders, roads, and roofs revitalize well-established ideas and show how we can already incorporate practical technology with learning experiences that allow students to envision themselves as capable and interested in developing their technical skills. ▲

It was the support from Carol that helped Don and me carry out our duties. They became not only fellow workers, but good friends.

Mickey Sandblom, OAME President, 1990/91

▲ Coding in the Classroom: Curriculum Change



IAIN BRODIE
iainbro1.625@gmail.com

Iain has been an elementary school educator for 28 years, and is now into a second career, teaching in-service and pre-service teachers. He teaches face-to-face at Ontario Tech University and virtually at Western University.



HATICE BEYZA SEZER
hsezer2@uwo.ca

Beyza is a former teacher and long-term researcher. She is currently a research associate in education at Western University, and her research focus is the integration of computational thinking into mathematics education.

(Quotations within this column may have been lightly edited to reflect the difference between oral and written language.)

OAME talks

Any time there has been a curriculum change in our province of Ontario, there has been the problem of enacting it. In the mid-1990's, we shifted to outcome-based curricula. I remember working with my grade-team partners, trying to figure out *how* we were going to implement this change, and planning together *what* we would teach our students in order to achieve this. We were early adopters of the idea of backward planning, and it worked. In fairness, we had decent leadership who helped us understand why the change was necessary. More importantly, we were given the time and some resources to figure out together how and what we were going to do. We were lucky. We were led to understand the *why* of the curriculum and given the time and resources to figure out the *how* and the *what*. As classroom teachers, we were in control of the *how* and *what*, as long as we also delivered the *why*.

Subsequent mathematics curricula in 1999 and 2005 focused on various amounts of *what* in mathematics, exerting control over exactly which skills students must know and demonstrate in each grade. When these specific expectations were introduced to the curriculum as what each student specifically needed to learn in each grade, they numbered in the hundreds. The answer to the *why* of the curriculum became,

"because it's rigorous," and all the *what* was spelled out for us. "How?" we asked. Silence, we heard.

Twenty-one years and one additional curriculum change later, we arrive at the 2020 mathematics curriculum. Why? Read the curriculum context; it is all there. What? Read the curriculum, and get the resources from the OntarioMath.support website. How? Read the curriculum context again. It contains wonderful suggestions, except for one of the biggest changes—Ontario students now need to engage in coding to learn mathematics. For the answers as to why, how, and what, the curriculum gives us a few paragraphs that amount to ¯_(")_/¯ (i.e., a shrug), which is why Beyza and I have been talking to early adopters around the province about what they are doing with coding in their practice, and hearing examples from them about why coding is an important tool for our students' learning.

When we interviewed Nicole Myers for the last issue of the *Gazette*, we knew right then and there that we would have to book more time with her and drill down into her practice to find out how she uses coding in her classrooms through the medium of video-game creation. Video-game playing gets a bad rap from a lot of parents and teachers, but their creation is something else that requires a great deal of critical thinking and problem solving that can lead to learning deeply about coding and other concepts (see our interview with Pekka Reinio, *Gazette* 60(1)). This is just one of the lessons we learned from Nicole about how she has her students begin learning to code. We invite you to listen to the whole interview on OAME Talks, where Nicole shares her experience and knowledge about sustaining that learning and dealing with the inevitable speed bumps that come up.

Lean into Your Strengths

It is safe to say that most teachers do not have an education in computer programming, and this is okay, as we are not really teaching our students how to program a computer (see our interview with Alice Aspinall, *Gazette* 61(2))—we are using code as another tool for students to use in learning mathematics. Nicole is quick to point out that her degree is in English, so she leans into that to teach her students how to code. As she reflected on her own experience, she mentioned that she would not enjoy coding as a profession; however, she finds joy in teaching coding to elementary-age students and teachers who may share a similar lack of expertise in coding. She leans into the idea of story and media literacy by using video-game creation as a way to teach students how to code.

"Video games naturally have a storytelling component to them and a kind of characterization component. There's a really great opportunity to make some really strong language connections there."

Using this and students' common understanding of what a video game is and does allows Nicole to shape lessons in terms of adding features to games. Keeping a score requires the use of variables. Interactions within the game require conditionals and the use of Boolean logic. Any uncertainty within these interactions requires randomly generated values. All of these are inherently mathematical, as well as being integral to modelling mathematical situations that you can use specifically in your math class.

Diversity and Representation

Leaning further into her background, Nicole has her students think critically about diversity and representation in video games. Despite an even split of video-game players between genders, only 10 percent of video games have female lead characters, prompting questions about who makes, buys, and plays video games. She empowers students to create video games that address this issue through diverse characters and storylines, where students write their own characters and game descriptions, while considering how to sell their games to a diverse audience.

"We also look at things like racial representation and all different kinds of representation in video games, with the goal of having students think."

Looking critically at video games as a medium requires the use of data and its analysis for students to be able to make choices about what they are going to create. This is the kind of intensely mathematical thinking and modelling envisioned in the 2020 curriculum. Once the data is gathered and choices made, Nicole has her students "actually do something about this by creating [their] own video games that are going to address this issue of diversity and representation. And so, we have students actually write their own kind of character description. We have them write a game description" to attract potential customers. This idea of creating something for a potential audience is yet another activity that is inherently mathematical, as well as an important literacy concept.

This integration of literacy with mathematics is beneficial for students as they learn how to think critically, use data in decision making, and learn important and useful coding concepts. Perhaps even more important is that they might gain some positive and personal answers to the question "Why math?" as they tackle problems and see how it is integral to something they already know.

For teachers, this allows for rich tasks that include and empower students in ways that individual lessons on these concepts might not. As mathematics teachers, we can look at our schedules for the time we use for literacy and mathematics separately and join them to get more time for both. As well, the concept of video-game creation allows another way for students to demonstrate their mathematical (or other) learnings. For example:

- Math fact practice: Students create a game for others to play, practise, and learn from.

- Number patterns: Students create a game to manipulate, explore, and play with number patterns.
- Flipping coins and probability: Students create a game where a coin can be flipped a huge number of times, allowing them to explore probability.
- Financial literacy: Students create a game to make financial decisions with logical consequences, good and not so good.
- Science: Students create a game that models an ecosystem or keeps it in balance.
- Health: Students create a game that models nutritional choices and consequences.

All of these suggestions require the learning and use of coding skills and concepts that are in the curriculum. Further, all of these skills can be used to model and explore mathematical concepts in ways that we were not able to do previously, such as flipping a coin or rolling a number cube ten thousand times and keeping track of results—imagine flipping or rolling multiple coins or number cubes, too! Imagine exploring what happens to the ratio between consecutive Hemachandra (Fibonacci) numbers when you change the two starting numbers. Imagine… just imagine. ▲

Continued from page 18

Final Thoughts

The intent of this column was to deepen our understanding of mathematical modelling and how it may differ from the traditional approach taken to problem solving. It was important to examine this process from the perspective of student thinking so that, as educators, we can see the active, independent stance students must take with respect to their learning.

In my next column, I will continue our exploration of mathematical modelling. I will examine the process from the perspective of the teacher. The teacher plays a significant role when students are engaged in mathematical modelling, and I will highlight concrete examples of what this looks like, along with the moves teachers can make to support student meaning-making and independence.

I look forward to continuing the discussion of linking literacy and math via my website (www.costellomath.com), email (david@costellomath.com), or Twitter (@dr_costello).

References

Costello, D. (2022). *Mathematizing student thinking: Connecting problem solving to everyday life and building capable and confident math learners.* Pembroke Publishers Ltd.

Ontario Ministry of Education. (2020). *The Ontario curriculum: Mathematics 2020.* Queen's Printer for Ontario. www.dcp.edu.gov.on.ca/en/curriculum/elementary-mathematics/grades-list ▲

▲ MB4T (Mathematics by and for Teachers): A Bucket of Integers, Part II



JENNIFER HOLM
jholm@wlu.ca

Jennifer Holm is an Assistant Professor at Wilfrid Laurier University and works with Primary/Junior and Junior/Intermediate pre-service teachers, as well as in the field supporting current mathematics teachers. She is interested in developing mathematics knowledge for teaching with both pre-service and in-service teachers. She focuses on the beliefs and opinions that pre-service teachers hold about mathematics and teaching and the connection they have to past experiences. She uses this research to support future teachers in developing beliefs and knowledge that will encourage and support effective teaching practices in mathematics.

This column is a continuation of the previous edition's column. The focus here is multiplying and dividing integers, using the Bucket of Zero as a model facilitating an approach for teaching integers. Again, I do not take credit for this approach, as this is the work of a collaboration between Dr. Lisa Lunney Borden and Nat Banting. You can read more about the collaboration and approach on Nat's blog (www.natbanting.com/the-bucket-of-zero/). The images in this column are all from using the Bucket of Zero tool on the Mathigon Polypad website (www.mathigon.org/polypad/embed/oeHU1XqNltfew), in case you want to investigate it for yourself. Also, if you would like to investigate more about the basis for this model, you may want to revisit the previous column, where I set up the context (Holm, 2023).

Just to highlight the idea, the initial concept of the Bucket of Zero starts with a bucket full of positive and negative integers that are paired and equal zero (see Figure 1). The chips are orange "+1" and red "–1," where teachers will need to mention the integer values, particularly for colour-blind students. Let's consider how we can use this model for determining solutions, using conceptual understanding with multiplication and division.

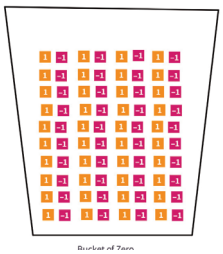


Figure 1: *The Bucket of Zero on Mathigon Polypad*

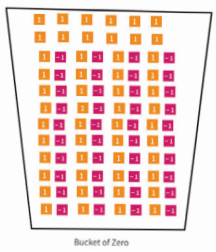


Figure 2: *Adding 6 sets of 2, or 6 × 2*

For this model, we are going to think of multiplication as "sets of" numbers. To start, we are going to use the example of 6 × 2. If we think of the first number as being positive, we can rephrase this as the idea of adding 6 sets of 2 to the bucket. Figure 2 shows how we might do this with the Bucket of Zero. Although this seems like an extra step, this becomes an important process when we think about other numbers. For example, 6 × –2 would be adding 6 sets of –2. This would be similar to the example in Figure 2, but would have red chips put in, representing negatives and showing –12 as the solution.

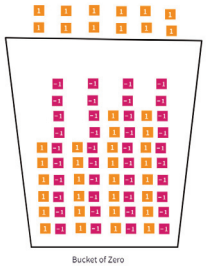


Figure 3: *–6 × 2, as "removing 6 sets of 2"*

Laying this groundwork helps when we think about something new: –6 × 2. Think about how you might help students understand this question. I have noticed the intention is often to tell students just to reverse the numbers to look at 2 sets of –6. If we think about it slightly differently, it will help when we get to a more complicated question. Since it starts with a negative number, we can think about this as "removing" sets instead. In this case, we are removing 6 sets of 2 (see Figure 3). You can see that the 6 sets of 2 have been moved from inside the bucket and are shown on top of it, and there is now a total of –12 inside the bucket, which is our solution. Again, you may be thinking, since it gives the same answer, why not just have students reverse the numbers? The difficulty of reversing the values is the question becomes –6 × –2. In this case, reversing the numbers is not going to help.

The multiplication of a negative by a negative has been the type of question that I have struggled the most with in my teaching career when helping students to understand.

In the past, I have seen individuals talk about using a pattern, such as in Figure 4, to show why –6 × –2 is positive 12. The premise is that as you move down by one group of –2, you would subtract –2, which would show that once you cross the zero, suddenly you have a positive number. It is also common to have students instructed to memorize the rule that a negative multiplied by a negative is a positive, but then, how do you answer the question of why? This is where the Bucket of Zero becomes a useful tool.

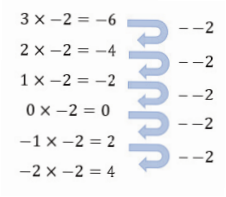


Figure 4: *Showing the pattern of how two negatives multiplied together make a positive.*

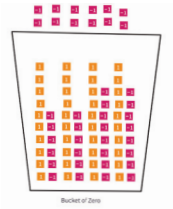


Figure 5: *Showing –6 × –2 with the Bucket of Zero*

If we follow the ideas that we have discussed earlier as the groundwork, then we know that –6 × –2 is removing 6 sets of –2. As you see in Figure 5, the answer is clearly 12, since the 6 sets of –2 that were removed are placed above the bucket. Without the Bucket of Zero, we could use the same language with just integer chips, but then how do you explain to students why this leaves positive integer chips? The Bucket of Zero shows that once those sets are removed, there are positive integers from the zero pairs left representing the solution. Take a moment to consider a couple of additional examples of –3 × 4 and –3 × –4. How would you model them with the Bucket of Zero?

For division, I want to start by talking about two different ways to consider division: a partitive model and a measurement model. I find that many early years teachers focus on the partitive model with children, which we might also call "fair sharing," because it is something that students can grasp. For a question like 10 ÷ 2, we would be taking 10 of something and dividing them into into 2 equal groups. For small children, this is a somewhat familiar concept to grasp, since students are likely familiar with dividing a physical object into groups, and the idea that everyone wants all the groups to be the same size or "fair."

If we only discuss the partitive model of division, students end up thinking the division sign means "making a specified number of groups," which can be helpful, but stops being effective when we think about integers (e.g., how do you divide into 2 groups) and fractions (e.g., how do you divide into ½ groups).

For the Bucket of Zero, thinking about the measurement model is more helpful. If we consider 10 ÷ 2 again, as a measurement model, we think about how many sets of 2 are in 10? If you look at Figure 6, you can see the two different models illustrated.

If we bring in negative integers, we can consider how –10 ÷ –2 can be shown with our model. We are starting with –10

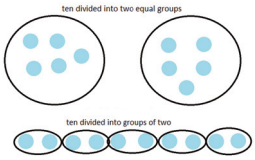


Figure 6: *Partitive model (top) and measurement model (bottom) for 10 ÷ 2*



Figure 7: *Counting groups of –2 in –10*

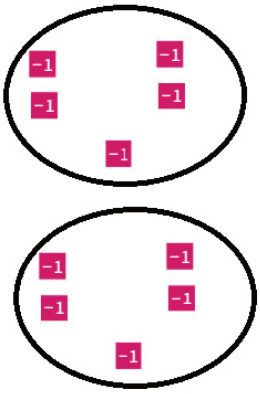


Figure 8: *–10 divided into 2 equal groups (partitive model)*

(represented with ten –1 tiles) and then counting the sets of –2 in the number (Figure 7). This clarifies why the answer is 5 because you can physically count the sets of –2 in the number –10.

If we consider 10 ÷ 2, we could use a partitive model to solve. In this case, think about –10 divided into 2 equal groups with integers (Figure 8), and we can see that each group has –5 as the solution. If we use the Bucket of Zero to think about this one, we are looking for how many sets of 2 are used to make –10 in the bucket. If you look at Figure 9, you can see we start with a Bucket of Zero in the first image. Next, we consider what –10 would look like (shown with the blue circles in the second image). It is visually clear that there are no sets of 2 (i.e., pairs) when we look at the negative number tiles, so our answer has to be negative because there are none! Finally, we have to count the sets of 2 in this number, but it is composed of only negative numbers. We have to "remove" sets of 2 in order to make the –10 with the bucket. If we remove 5 sets of 2, we make –10 in the bucket, showing our answer as –5 (the negative being the "removal"). Consider Figure 10 as another example to work through the process with –6 ÷ 2.

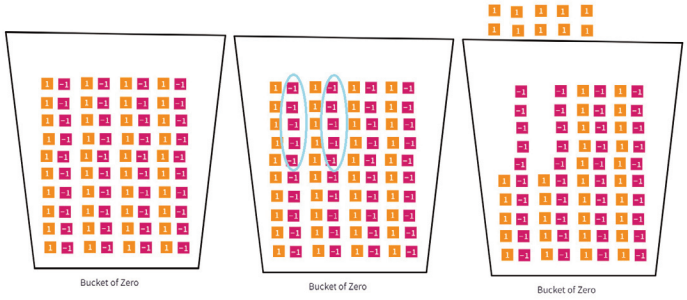


Figure 9: *Using the Bucket of Zero to think about –10 ÷ 2*

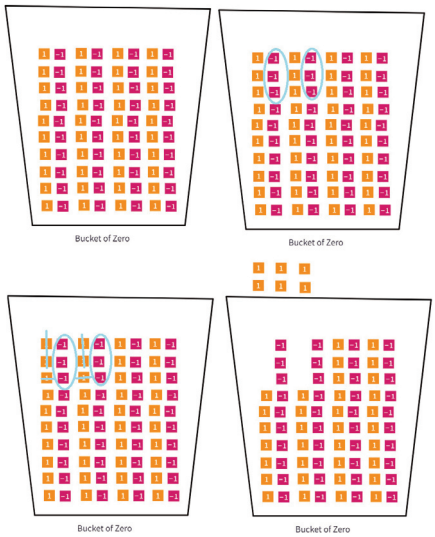


Figure 10: *Using the Bucket of Zero for –6 ÷ 2. The second image shows what 6 would be in the bucket. Next you "remove" sets of 2 shown in the third bucket by crossing off sets of 2. The final image shows that there were 3 sets of 2 (so –3).*

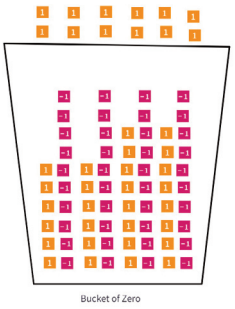


Figure 11: *10 ÷ –2 with the Bucket of Zero*

Where this becomes helpful is that it will work if the first number is positive or negative. Consider 10 ÷ –2. How many sets of –2 are in 10? This would be the same idea as the example in Figure 9, where you are looking at 10, and then you have to remove those sets of –2 in order to make the 10 because they are not there when you have the number 10! See Figure 11, where we look at creating 10 in the bucket, and we have to remove 5 sets of –2, so our answer would be –5 (removed 5).

Integers can be fun if you help students visualize what is happening. When we think about it, a negative number is something that would not physically exist in front of us, and that is why the visualization process is beneficial. Helping students understand how the operations can be conceptualized makes it much easier to work with the operations, and then they can create rules for the operations, like two negatives multiplied together make a positive, or a positive divided by a negative is always negative. By helping them to visualize and "see" the process, they can create the rules, and they are more likely to be remembered as well as understood!

Reference

Holm, J. (2023). Mb4T (Mathematics by and for teachers): A bucket of integers. *Ontario Mathematics Gazette, 61*(3), 29–31.▲

▲ Research into Practice: An Application of Self-Determination Theory to the Teaching of Sequences in Grade 11



JEFF IRVINE
dr.jeffrey.irvine@bell.net

Jeff Irvine, Ph.D., has been a secondary mathematics teacher, department head, and vice-principal. He has taught at three faculties of education and at a community college. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. With over 45 years in education, Jeff is particularly interested in the interplay of student motivation and mathematics achievement.

All students want to have some control over their own lives. This desire for control grows naturally as students mature. However, typically in school, students relinquish most of that control to their teachers: we decide what they will learn, when they will learn it, how they will be assessed, and the consequences for their lives of their learning (or not learning) the specified material. Yet there is abundant research that has found that allowing students some level of control over their own learning, at least some of the time, significantly increases motivation, engagement, and perseverance (Ryan & Deci, 2000). This article highlights the impact of allowing student control when teaching about sequences.

Self-Determination Theory

Self-Determination Theory (SDT) identifies three student needs: autonomy, competence, and relatedness (Deci & Ryan, 2008). Deci et al. (1991) propose that students act intentionally to address needs within these three dimensions, which has implications for student motivation. Autonomy is the need for control, which can often be provided by giving students some level of choice in what they do, and the opportunity to work independently; competence is the need for students to master a particular topic; and relatedness includes the need for students to interact socially with others (Deci, Vallerand, Pelletier, & Ryan, 1991), as well as feeling ownership of the classroom and sense of community within the class or school. There is significant evidence that students who are more engaged in their own learning have higher achievement in mathematics (Connor & Pope, 2013; Deci & Ryan, 2008; Irvine, 2019c).

Hannula (2006) provides an example of the operationalization of self-determination theory in the mathematics classroom. He relates the psychological needs identified in SDT and learning goals arising from the desire to satisfy those needs.

In the context of mathematics education, a student might realize a need for competency as a goal to solve tasks fluently or, alternatively, as a goal to understand the topic taught. A social need might be realized as a goal to contribute significantly to collaborative project work and a need for autonomy as a goal to challenge the teacher's authority (p. 167).

Hannula's example of a goal linked to the need for autonomy is simplistic and quite negative. A better example for such a goal could be to have a choice in activities undertaken or working independently to achieve those goals, thus increasing intrinsic motivation as well as addressing the autonomy need.

SDT has also been found to encourage persistence, effort, student agency, and commitment (Gagné & Deci, 2005). There is a strong link between SDT and intrinsic motivation, the desire to learn for the sake of learning, rather than for some extrinsic (external) reward (Vansteenkiste, Lens, & Deci, 2006). Intrinsic motivation is a major factor in lifelong learning, which is one of the most important goals of education (Kuntze & Dreher, 2015).

SDT-Supportive Learning Environments

The teacher plays a large role in providing environments that encourage SDT. For students to thrive from an SDT perspective, the learning environment needs to be autonomy supportive (Vansteenkiste, Simons, et al., 2004). This means that the learning environment supports and encourages student autonomy, providing choice and the opportunity to make independent decisions about topics and work undertaken. The environment should encourage risk taking, celebrate mistakes as learning opportunities, and recognize student learning growth. Teachers who provide autonomy-supportive environments and who reinforce student competence and growth through a sense of community, as well as providing constructive feedback, have a powerful influence on student engagement and motivation. Teachers need to be clear with students when true opportunities for autonomy are available.

Autonomy-supportive learning environments have been found to promote student self-regulation, work habits, and

persistence (Gagné & Deci, 2005). They have also been found to support deep processing (Vansteenkiste, Simons, et al., 2004), which Campbell and Cabrera (2014) identify as a major goal of pedagogy. On a broader scale, autonomy-supportive environments have been associated with academic competence, school achievement, and greater personal well-being (Vansteenkiste, Lens, et al., 2006).

Feedback

Hattie (2009) found that feedback is one of the five most powerful influences on student achievement. Providing constructive feedback is a strategy that teachers can use to support students' SDT needs. Hughes (2014) identifies three levels of feedback: *feed up*, which allows students to answer questions about their personal learning goals; *feed back*, which lets students respond to their current level of knowledge; and *feed forward*, which provides students with the ability to answer questions about where they need to or wish to go next. The second of these, feed back, is the most common feedback in the classroom, with teachers questioning student decisions and current level of knowledge. However, feed forward is critical to students' learning growth, with student decisions about next steps in their learning. Feedback can be provided by the teacher, by student peers, or by the student's own self-reflection, although providing feedback is a skill that must be taught.

There are four levels of feedback: *task-level feedback*, which assesses how well the task is understood or being performed; *process-level feedback*, which looks at the processes required to perform the task; *self-regulation feedback*, assessing student self-monitoring and regulation of actions; and *self-level feedback*, which involves personal evaluations of the learner (Hattie & Timperley, 2007). Feedback at the task and process levels has been found to be most effective.

There are several dimensions of feedback that enhance its effectiveness: it must be timely, specific, non-judgmental, not personally threatening to the student, and provide opportunities for students to explain their thinking as well as consider next steps (Chubb, 2016; Hattie & Timperley, 2007). General-level praise such as "good work" is not effective. Specific praise of effort, "You worked hard on this problem and succeeded," is effective, if the focus is on effort and not on ability—a growth mindset (Dweck, 2006). According to Dweck (1999), having a fixed mindset predisposes students to see failure as a direct indicator of their low ability, which is distressing and disengaging, whereas students with a growth mindset use failure as a cue to try new strategies and to try harder.

The topic of sequences provides numerous opportunities to meet student SDT needs through judicious choice of instructional strategies and the creation of an autonomy-supportive classroom. By utilizing the instructional strategies discussed in this article, I have found that my students were more engaged, more motivated, persistent, and achieved at a higher level when assessed on the concepts related to sequences. Each activity is followed by Teaching Notes, which link the activity to SDT and provide recommendations for teaching strategies and providing student feedback.

In the Grade 11 University Preparation course, the Discrete Functions strand identifies three overall expectations (Ontario, 2007):

1. demonstrate an understanding of recursive sequences, represent recursive sequences in a variety of ways, and make connections to Pascal's triangle;
2. demonstrate an understanding of the relationships involved in arithmetic and geometric sequences and series, and solve related problems;
3. make connections between sequences, series, and financial applications, and solve problems involving compound interest and ordinary annuities.

This article will focus on the first two expectations, and provide some classroom-tested activities that address the three needs identified in SDT. The third expectation can be related to financial literacy, and teachers may have developed excellent ways to provide students some level of autonomy through choice of activities, assignments, or real-world connections (e.g., Irvine, 2020).

The first activity (Irvine, 2019a) is an introduction to sequences and patterning.

Teaching Notes: The first part of this activity supports students' SDT autonomy need. Teachers need to be clear that any logical sorting rule is acceptable, and that there are multiple valid responses. This activity should be done in groups, but each student should complete the activity independently before sharing with the group in a *table jigsaw*. Table jigsaw is a modification of the usual jigsaw (Irvine, 2019b): each student knows that they will be expected to share their thinking, demonstrate their competence, and respond to peer questions, while still remaining in their group. The grouping strategy may also address the SDT-relatedness need. Group processing usually reduces the number of very simple sorting rules, such as, "These sequences all contain a 3, and the other sequences don't." It is important, however, for the teacher to accept such simple sorting rules as valid, but encourage students to develop more sophisticated sorting criteria. The second part of the activity is focused on developing student competence in handling patterns. Feedback should focus on

asking students to explain their thinking and justify their decisions. The second part of the activity is more focused on competency. Feedback should be centred on student responses to the "Why?" questions, again with an emphasis on explaining their reasoning.

Sample feedback questions:

- *Feed up:* For this activity, do you think that you put in more effort, less effort, or about the same effort as with your other work in math class? Why do you think that?
- *Feed back:* That's a really interesting way of sorting. Can you explain your sorting rule to me?
- *Feed forward:* Can you subdivide each of Navneet's subgroups further? Do some of the sequences in each subgroup differ in the same way from others in the subgroup?

The next activity is a concept-attainment strategy to introduce students to the key differences between arithmetic and geometric sequences.

Similar Sequences

Below are some sequences. Based on the YES examples and the NO examples, conjecture what is similar for all the YES sequences in each group.

YES	NO
3,5,7,9,11,13	2,4,8,16,32,64
1,4,7,10,13,16	1,3,9,27,81,243
5,9,13,17,21,25	12,6,3,1.5,0.75,0.375
9,7,5,3,1,-1	3,15,75,375,1875,9375
6,14,22,30,38,46	8,0.8,0.08,0.008,0.0008,0.00008
-9,-2,5,12,19,26	5,-10,20,-40,80,-160

What is similar about all the YES sequences?

_____

_____

What is similar about all the NO sequences?

_____

_____

The YES sequences are called ARITHMETIC sequences. The NO sequences are called GEOMETRIC sequences. Explain why these names make sense:

_____

_____

Following are some "Tester" sequences. Classify each sequence as Arithmetic, Geometric, or Neither. Give reasons for your decisions.

Tester	Arithmetic	Geometric	Neither	Reason
7,11,15,19,23,27				
8,4,2,1,0.5,0.25				
7,14,28,56,112,224				
6,4,2,0,-2,-4				
3,4,6,9,13,18				
3,4,7,11,18,29				

Toni was planning to do her homework, but the dog ate it. Toni was only able to salvage part of one question, a sequence containing the value "0." All the other terms were blurred beyond recognition. Help Toni decide if this was an arithmetic sequence or a geometric sequence. What should Toni tell her teacher about her decision?

_____

_____

Teaching Notes: The major SDT focus of this activity is student competence. Similar to the previous activity, this should be done in groups, but each student should complete the activity independently before group sharing. Students should be encouraged to justify their reasoning in the "Tester" section. Feedback should focus on students' clear explanations of how they identified the similarities and differences between the two types of sequence. The teacher may wish to confirm with the class that arithmetic sequences have a common difference, and geometric sequences have a common ratio. Identification of similarities and differences is one of the nine most effective instructional strategies identified by Robert Marzano (Tyson, 2020). Feedback should also include introducing correct mathematical terminology, such as common difference and common ratio. Toni's homework question helps reinforce student competence, and should be a focus of feedback questions.

Sample feedback questions:

- *Feed up:* Do you find this activity more interesting or less interesting than other recent activities in math class? Why?
- *Feed back:* How can you relate what you have already learned about finite differences to your work on sequence sorting?
- *Feed forward:* Can you find formation rules for the sequences that are neither arithmetic, nor geometric?

Once students are comfortable with the distinction between arithmetic and geometric sequences, the following think/pair/share activity reinforces their knowledge, while also addressing the SDT need of competence.

Arithmetic or Geometric?

- Make up your own arithmetic or geometric sequence.
- Find the general term for your sequence.
- Trade with a partner.
- Find the general term of your partner's sequence.
- Now do a timed retell to your partner, outlining what you did to answer the question.
- Switch roles and repeat the timed retell.

Teaching Notes: This activity involves the Timed Retell strategy. In Timed Retell, one partner tells the other partner what they did to solve the problem. The other partner then retells what the first partner told them. They then switch roles and repeat the activity. The activity is timed, although I have found that often, time limits are not needed. Using Timed Retell supports the SDT need for relatedness, while reinforcing competence. Feedback can be given by both the think/pair/share partners as well as the teacher. Emphasis should be on explaining reasoning and justifying their decisions.

Sample feedback questions:

- *Feed up:* Would you like to see other strategies like Timed Retell used in our math class? Why?
- *Feed back:* Explain your thinking in developing a general term for your partner's sequence.
- *Feed forward:* Outline the steps that you would take to find the general term of a sequence if you didn't know in advance that it was arithmetic or geometric.

The Inside/Outside Circle activity described below provides students with an opportunity to physically move within the classroom.

Find the Missing Term

- Make up an arithmetic or geometric sequence.
- Find the general term of your sequence.
- Compute two terms in your sequence, such as t_5 and t_8.
- Find the value of another term in your sequence, such as t_{12}.
- On an index card, write the following information: whether your sequence is arithmetic or geometric, and the values of the two terms, in the form t_5=____. Add a question asking the respondent to find the missing term that you have already computed.
- On the back of the index card, write the general term for your sequence and the value of the missing term.
- We are going to use an Inside/Outside Circle activity for you to share your knowledge and demonstrate your proficiency with finding missing terms.

Teaching Notes: This activity supports the SDT need for autonomy and competence through the students' freedom to construct any arithmetic or geometric sequence of their choosing, and then justifying their reasoning to a partner. They also demonstrate competence by solving a problem given two non-consecutive terms of the sequence. Feedback can be provided by peers and the teacher. As usual, the focus of the feedback should be on the problem-solving process, rather than the sequences created. Students can be encouraged to write the problem-solving process in their math journals.

Sample feedback questions:

- *Feed up:* Is it important to you that you have a good grasp of how to solve these problems? Why?
- *Feed back:* Explain your thinking in solving the problem with which you were faced.
- *Feed forward:* How does this type of question relate to your previous work on solving systems of equations?

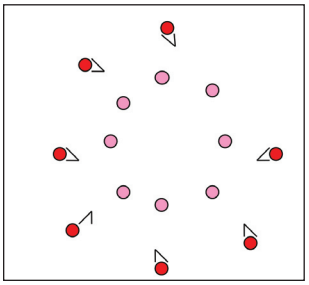


Figure 1: *Inside/Outside Circle*

This activity uses the Inside/Outside Circle strategy for sharing and peer feedback. This strategy, which supports the SDT relatedness need, is described below.

- Students form a circle, facing outward.
- Other students form a circle, facing inwards so that every student is facing a partner.
- The student on the inside circle holds up their index card, showing only the question side. The student on the outside circle then outlines how they would solve the problem, and completes the question. They may want to use a desk and scrap paper to do computations.
- Since the student giving the question can see the back of their own index card, they can troubleshoot the solution being given by the other student and correct misconceptions.
- Once the outside student has completed their solution, they hold up their index card, and the inside student solves the new problem.
- After students from both circles have solved their problem, the inside circle rotates one student to the left. (A variation has both circles rotate in opposite directions to reduce the benefit of listening in on a neighbouring conversation.)
- The new student pairs then repeat the index card

Sequence Sorting

Design a sorting rule and sort the sequences below into two groups.

A: 3,5,7,9,11,13	B: 1,3,9,27,81,243	C: 2,4,8,16,32,64
D: -9,-2,5,12,19,26	E: 5,-10,20,-40,80,-160	F: 9,7,5,3,1,-1
G: 7,8,11,16,23,32	H: 12,6,3,1.5,0.75,0.375	I: 1,4,7,10,13,16
J: 1,4,9,16,25,36	K: 2,7,9,16,25,41	L: 7,11,15,19,23,27

My sorting rule: _____

Group One | Group Two

Another student, Navneet, used a sorting rule based on operations. Sort the sequences above using Navneet's rule:

Formed using + | Formed using ×

In which group should Navneet put the sequence 1,2,2,4,8,32? ____

Why? _____

Now design a sorting rule to help Navneet subdivide each of her two groups into two subgroups.

For Navneet's sequences formed using +, my subsorting rule is _____

Here are Navneet's + sequences sorted using my rule:

For Navneet's sequences formed using ×, my subsorting rule is _____

Here are Navneet's × sequences sorted using my rule:

Where will you put the sequence 3,4,6,9,13,18?

Why? _____

activity.

- Inside circle can be rotated until everyone has shared with everyone, or until the teacher judges that sufficient time has passed.
- For best results, a maximum of eight students should be in each circle.

Students sometimes have difficulty distinguishing between recursion relations and general terms. The activity below helps clarify this difference and build student competence.

Dominos and Parachutes

Conjecture the formation rule for each sequence below:

Sequence	Possible formation rule
A: 3,5,7,9,11,13	
B: 1,3,9,27,81,243	
C: 2,4,8,16,32,64	
D: -9,-2,5,12,19,26	
E: 5,–10,20,–40,80,–160	
F: 9,7,5,3,1,–1	
G: 7,8,11,16,23,32	
H: 12,6,3,1.5,0.75,0.375	
I: 1,4,7,10,13,16	
J: 1,4,9,16,25,36	
K: 2,7,9,16,25,41	
L: 7,11,15,19,23,27	

Domino Lens

Viewing a sequence through a "domino" lens means answering a question like, "To get the value of term 12 if I know the value of term 11, I _____." Think of dominoes standing in a line; to get domino 12 to fall, domino 11 must fall.

We met once in Miami when they were getting off the plane and we were getting on: that was fun. Carol and Don were incredibly welcoming to (my new wife) Marlene. Carol was a delightful lady.

Tom Griffiths, OAME President, 1976/77



Use the domino lens to answer the question for each sequence:

Sequence	"Domino" answer to "To get the value of term 12 if I know the value of term 11, I _____."
A: 3,5,7,9,11,13	
B: 1,3,9,27,81,243	
C: 2,4,8,16,32,64	
D: -9,-2,5,12,19,26	
E: 5,-10,20,-40,80,-160	
F: 9,7,5,3,1,-1	
G: 7,8,11,16,23,32	
H: 12,6,3,1.5,0.75,0.375	
I: 1,4,7,10,13,16	
J: 1,4,9,16,25,36	
K: 2,7,9,16,25,41	
L: 7,11,15,19,23,27	

Parachute Lens

Viewing a sequence through a "parachute" lens means answering a question like, "To get the value of term 100 if I don't know the value of term 99, I do _____ to the number 100." You can "parachute" directly to term 100, without knowing all the terms up to term 99.

Use the parachute lens to answer the question for each sequence:

Sequence	"Parachute" answer to "To get the value of term 100 if I don't know the value of term 99, I _____."
A: 3,5,7,9,11,13	
B: 1,3,9,27,81,243	
C: 2,4,8,16,32,64	
D: -9,-2,5,12,19,26	
E: 5,-10,20,-40,80,-160	
F: 9,7,5,3,1,-1	
G: 7,8,11,16,23,32	
H: 12,6,3,1.5,0.75,0.375	
I: 1,4,7,10,13,16	
J: 1,4,9,16,25,36	
K: 2,7,9,16,25,41	
L: 7,11,15,19,23,27	

For which sequence was it easiest to use the domino lens? _____

For which sequence was it most difficult to use the domino lens? _____

For which sequence was it easiest to use the parachute lens? _____

For which sequence was it most difficult to use the parachute lens? _____

If you wanted the value of term 20 in a sequence, would you use the domino lens or the parachute lens? Justify your choice. _____

Teaching Notes: The SDT need of competence is the focus of this activity. Like many of the activities described in this article, the activity should first be done individually and then shared in groups, using table jigsaw. When confirming student answers, feedback should focus on the decision processes and justification of these decisions. Use of the parachute and domino analogies is also one of Marzano's nine most effective instructional strategies (Tyson, 2020). Deep processing is the focus of the questions following the main activity, with students making decisions on the easiest and most difficult sequences for each of the domino (recursion) and parachute (general term) strategies. Feedback should emphasize answering the "Why?" questions. Correct mathematical terminology can be reaffirmed during the feedback.

Sample feedback questions:

- *Feed up:* Can you think of other "lenses" (other ways of thinking about sequences) that could be applied to your work with sequences? Try to find at least one other lens.
- *Feed back:* You clearly put a lot of effort into your decisions. Which type of lens—domino or parachute—was the more difficult to use?
- *Feed forward:* Is there a way of deciding in advance whether the domino lens or the parachute lens would be most useful? Why do you think that?

Student problem posing is a powerful motivator that addresses all three dimensions of SDT while reinforcing student understanding. This strategy also addresses the mathematical processes of problem solving, communicating, reasoning and proving, and connecting.

What's Your Problem?

Warm-up: Solve these problems. Verify your solutions with your group members.

A jogger runs 300 m in the first minute. The distance covered decreases by 20 m each succeeding minute. What distance will the jogger cover during the 7th minute of her run?

Assume that whenever you wash a pair of jeans, the jeans lose 5% of their colour.

How much of their original colour would remain in the jeans after 10 washes?

A hockey arena has 10 929 seats. The first row of seats around the rink has 220 seats. The number of seats in each subsequent row increases by 16. How many rows of seats

does the arena have?

A lottery promises to pay you $1 000 000, or you can choose to receive $0.01 on the first day of February, $0.02 on the second day, $0.04 on the third day, $0.08 on the fourth, and so on for the entire month of February (in a non-leap year). Which offer should you pick? Justify your answer.

Your mission:

- Make up a problem involving arithmetic or geometric sequences or series.
- Write a complete solution to your problem.
- Give your problem to another group member to solve. Verify their solution.
- As a problem solver, give constructive feedback to the problem poser. Was the problem clear? Was all the necessary information included? Should the wording be modified?
- Give the problem back to the problem poser.
- Each problem poser now writes their problem on large chart paper in preparation for a gallery walk.

Teacher Notes: The activity addresses autonomy, since the participants create their own problems; competence, as each student solves the warm-up problems as well as their group's problems; and relatedness, through the group problem solving and the participation in the Graffiti and Gallery Walk activity. Peer feedback is involved during the group problem-solving activities, as well as during the Gallery Walk. Teacher feedback should reinforce the important features of the problems created, such as necessary and sufficient information in each problem, completeness, interest, and degree of difficulty of the problem, as well as correctness of solutions. Students' thinking should be probed concerning possible alternative solutions to the problem. This activity can be a rich source of not only interesting problems, but also reinforces the need to think outside the box in generating alternative solutions. Feedback should also involve asking students "What If?" questions, to consider the effect of changing parameters in the problems. This probing can be done by both teachers and peers. For those unfamiliar with the Graffiti and Gallery Walk strategy, it is outlined below.

Sample feedback questions:

- *Feed up:* What are the critical components of creating a problem related to this topic? How are they related to problems involving other concepts in this course?
- *Feed back:* Is there more than one way to solve the problem that you created? Can you create at least one alternate solution? How would your solution change if you changed the conditions in the problem?
- *Feed forward:* What was the most difficult part of creating your problem? Why? Can you outline a set of steps that can be used to create problems, on any topic, including sequences?

Graffiti and Gallery Walk

Questions are posted on large chart paper.

- Each group starts in front of their own questions, then moves one question set to the right. The last group moves to the first posted questions.
- After reading the questions, the group writes the first line of their proposed solution to each question.
- The groups then move to the next set of questions.
- Each group writes the second line to the solution for all questions in the set.
- Group rotations continue until complete solutions have been constructed for all the questions.
- Each group then returns to their own questions and analyzes the solutions that have been created.
- Finally, the groups do a Gallery Walk to compare solutions from other groups.

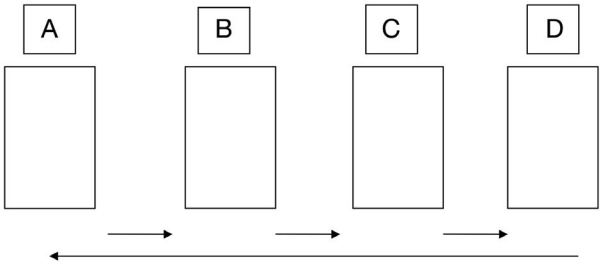


Figure 2: *Graffiti and Gallery Walk*

Self-determination theory provides a research-affirmed strategy for supporting student needs, while encouraging students' engagement, motivation, and persistence (Ryan & Deci, 2000). While not every topic that we teach is conducive to SDT strategies, the more frequently that we can support student needs for autonomy, competence, and relatedness, the more engaged our students will become, and the stronger our students' confidence will grow. Additional benefits include increased student persistence, effort, and agency as they become more responsible for some dimensions of their own learning. The activities in this article are, of course, not the only ways to support SDT. You are encouraged to think about other ways that we can address our students' needs in an active and encouraging way.

References

Campbell, C.M., & Cabrera, A.F. (2014). Making the mark: Are grades and deep learning related? *Research in Higher Education, 55*, 494–507. doi.org/10.1007/s11162-013-9323-6

Chubb, M. (2016). How do you give feedback? *Thinking mathematically*. www.buildingmathematicians.wordpress.com/2016/08/27/how-do-you-give-feedback/

Conner, J.O., & Pope, D.C. (2013). Not just robo-students: Why full engagement matters and how schools can promote it. *Journal of Youth and Adolescence, 42*(9), 1426–1442. doi.org/10.1007/s10964-013-9948-y

Deci, E.L., & Ryan, R.M. (2008). Self-determination theory: A macrotheory of human motivation, development, and health. *Canadian Psychology, 49*(3), 182–185. doi.org/10.1037/a0012801

Deci, E.L., Vallerand, R.J., Pelletier, L.G., & Ryan, R.M. (1991). Motivation and education: The self-determination perspective. *Educational Psychologist, 26*(3–4), 325–346.

Dweck, C.S. (1999). *Self-theories: Their role in motivation, personality, and development.* Psychology Press.

Dweck, C.S. (2006). *Mindset: The new psychology of success.* Ballantine Books.

Gagné, M., & Deci, E.L. (2005). Self-determination theory and work motivation. *Journal of Organizational Behavior, 26*, 331–362. doi.org/10.1002/job.322

Hannula, M.S. (2006). Motivation in mathematics: Goals reflected in emotions. *Educational Studies in Mathematics, 63*, 165–178. doi.org/10.1007/s10649-005-9019-8

Hattie, J. (2003, October). *Teachers make a difference: What is the research evidence?* [Conference presentation]. Australian Council for Educational Research Conference, Melbourne, Australia. http://research.acer.edu.au/research_conference_2003/4/

Hattie, J. (2009). *Visible learning.* Routledge.

Hattie, J., & Timperley, H. (2007). The power of feedback. *Review of Educational Research, 77*(1), 81–112. doi.org/10.3102/003465430298487

Hughes, G. (2014). *Ipsative assessment: Motivation through marking progress.* Palgrave Macmillan.

Irvine, J. (2019a). Using self-determination theory to engage learners in senior mathematics courses and build motivation and higher-order thinking. *Vector, 60*(2), 47–51

Irvine, J. (2019b). Strategies for enhancing mathematics learning for students who are kinesthetic learners. *Ontario Mathematics Gazette, 57*(4), 41–44.

Irvine, J. (2019c). Self-determination theory as a framework for an intermediate/senior mathematics preservice course. *Journal of Instructional Pedagogies, 22*, 1–27.

Irvine, J. (2020). Some alternatives to using tests and quizzes for assessment. *Ontario Mathematics Gazette, 58*(3), 27–33.

Kuntze, S., & Dreher, A. (2015). PCK and the awareness of affective aspects reflected in teachers' views about learning opportunities—A conflict? In B. Pepin & B. Roesken-Winter (Eds.), *From beliefs to dynamic affect systems in mathematics: Exploring a mosaic of relationships and interactions education. Advances in Mathematics Education* (pp. 295–318). Springer, Cham.

Ontario Ministry of Education. (2007). *The Ontario curriculum, grades 11 and 12: Mathematics 2007*. Queen's Printer for Ontario.

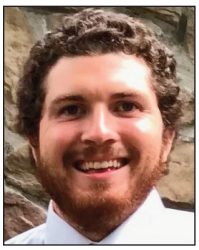
Ryan, R.M., & Deci, E.L. (2000). Self-determination theory and the facilitation of intrinsic motivation, social development, and well-being. *American Psychologist, 55*(1), 68–78. doi.org/10.1037/0003-066X.55.1.68

Tyson, K. (2020). Marzano's 9 effective instructional strategies. www.teachthought.com/learning/marzanos-9-instructional-strategies-graphic/

Vansteenkiste, M., Lens, W., & Deci, E.L. (2006). Intrinsic versus extrinsic goal contents in self-determination theory: Another look at the quality of academic motivation. *Educational Psychologist, 41*(1), 19–31. doi.org/10.1207/s15326985ep4101_4

Vansteenkiste, M., Simons, J., Lens, W., Sheldon, K., & Deci, E.L. (2004). Motivating learning, performance, and persistence: The synergistic effects of intrinsic goals contents and autonomy-supportive contexts. *Journal of Personality and Social Psychology, 87*(2), 246–260. doi.org/10.1037/0022-3514.87.2.246 .▲

▲ Integrating Mathematics and Geography into Intermediate Classroom Instruction



BRETT TOMLINSON
Brettt@nipissingu.ca

Brett is a Bachelor of Education Instructor in the Schulich School of Education, Nipissing University. He has spent time teaching a variety of high school subjects prior to teaching at the university level. Brett appreciates the learning potential and opportunities that come from cross-curricular learning. He encourages teacher candidates and teachers to consider the potential of incorporating themes from mathematics into any other subjects that they teach.

There is a significant focus on mathematics and language arts within the Grades 7–8 classroom. As educators, with regards to math, we are constantly trying to develop our students' skills to help them gain knowledge and understanding. We put such a large focus on ensuring that we teach our students about mathematics and language arts, that we sometimes lose sight of other subjects. Specifically, it is difficult to include all the social studies topics in the school day. This demonstrates the importance of cross-curricular teaching.

Geography is the focus of this article; therefore, I wanted to carefully consider how geography could be incorporated into a unit of mathematics. I want to draw some similarities between the Grade 7 Geography curriculum and the Grade 7 Mathematics curriculum, to highlight the ways in which geography could be taught through a lens of mathematics. I will be focusing on two specific areas in which the subjects connect. The first will be drawing conclusions from data (A2.5 from the Grade 7 Geography curriculum), and using data to gather answers (D1.2 from the Grade 7 Math curriculum). The second area of focus will be about identifying natural and human processes that change landforms (A3.2 from the Grade 7 Geography curriculum), and recognizing a variety of different patterns (C1.1 from the Grade 7 Math curriculum).

Lesson Examples

Graphs and data are frequently used in lessons within geography for the Intermediate grades. For example, when we ask our students in a geography lesson to consider



Figure 1: *Showing glacier changes over time near Whistler, British Columbia (Photo by Jason Krupa, CBC, 2014)*

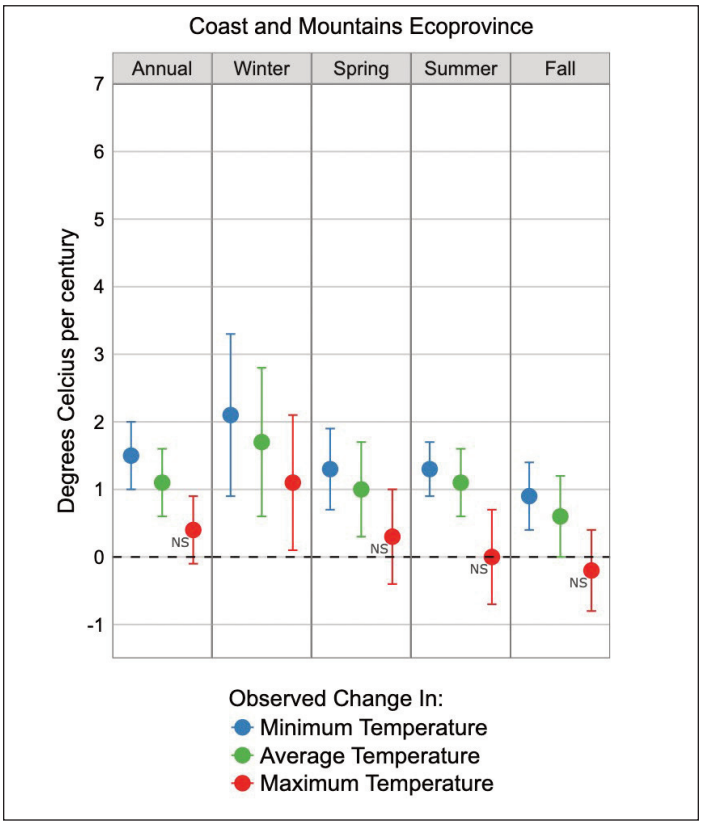


Figure 2: *Changes in temperature statistics (British Columbia Ministry of Environment and Climate Change Strategy, 2015)*

changes in population within parts of Canada, they have to consider changes in numbers and percentages. For example, students are asked to infer the meaning of percentage changes in population. This can be shown through immigration and emigration from province to province or the country as a whole. This connects to mathematics from the perspective of using data to gather answers. On a larger scale, this connects to data management as a whole.

Students can be taught how to recognize trends in population changes and identify what a positive and negative trend could look like. From there, they start using the data that they are given to make a prediction about future outcomes, and thereby see how math is applicable to other subjects. This also gives students an answer to the frequently asked question, “Why are we learning this?” Connecting geography with math is a way to help students understand both subjects at the same time, and further increase learning potential of new concepts in math.

Students are able to learn to incorporate mathematics from the perspective of patterning, while focusing on physical geography. A specific example is to look at the concept of weathering on natural landscapes. This is accomplished while looking at photos of the same area taken over a number of years, to determine how weather has caused erosion and other changes, which can be done for a variety of places. Teachers are then able to layer in aspects of environmental education, for example, by looking at the effect that weather (rising temperatures) has on glaciers in Canada. In an article from the CBC (Krupa, 2014), readers see an example of glaciers shrinking in British Columbia. With graphs and photos, we can measure those effects over time (Weber, 2021). Within this lesson, teachers are given the opportunity to seamlessly blend math and geography by looking at patterns in both graphs and photos. Students are

given an opportunity to apply real-world scenarios to the math theories that they are learning. When data from temperature logs and pictures from the glacier showing changes are included in lessons, students have multiple forms of representation to recognize the pattern and correlation between rising temperatures and glacier melt.

The photo in Figure 1 demonstrates an example of what you could show students about the changes coming with rising temperatures. This picture reflects the pattern that students would see within the data sets and graphs that correspond with the photo. Students connect with these images within their learning, which potentially creates an emotional response to a math lesson. That response helps students stay engaged in the lesson and remember what they have learned. This gives students great potential to recognize the relationship that exists between graphs and images. It can help them in the process of translating data to graphs and representing data in a variety of forms. Students are also given the opportunity to learn more about Canadian geography and issues that are affecting our unique environmental landscapes.

When you are using this information with your students, it is important to highlight the process of interpreting this data. Within a lesson, it would be helpful for teachers to compare and contrast this image to the graph below of rising temperatures in British Columbia. Students could then develop a richer understanding of the interconnectedness between rising temperatures and changes to our planet. This would give them a real-world connection of using statistics to help draw conclusions around their theories and concepts. The graph in Figure 2 indicates temperature rises over the past century within a region of British Columbia, where the Decker Glacier (seen in Figure 1) is located. Students should be encouraged to compare the data within the graph to the picture of the glacier to indicate what they believe to be a factor for the glacier melt.

Extension to Learning

Each of these lessons provides an excellent opportunity to connect to real-world scenarios, as well as provide a chance for students to connect with their local community. When we study populations, it is easy to focus on larger areas instead of the areas where our students live. Rather, we could look at population changes within the towns and cities where our students are from. Inspiring them to study the areas in which they live helps spark more interest in the material they are learning. This could also help students visualize concepts of populations and percentage increase or decrease, depending on the change in the population of their local area. From a physical geography perspective, having students recognize the effects of human or natural impact on a landscape offers an excellent opportunity for them to discover the natural landscapes that surround the communities where they reside. You may need to include some local natural resource sites, but finding topographic photos from the past and present has the potential to really help to demonstrate changes in local geographies. This is also an opportunity to teach students about measurement and patterns involved with changes in their local areas. As natural land features change, there can be a pattern to follow. Specifically with weather and changes in climate, it can be easy to follow a pattern over the past 100 years. Teachers can also find other data sets that relate to different factors within human or physical geography. This can lead to a larger inquiry-based project, where students could continue to study weather patterns and make comparisons between other data sets.

The Importance of Cross-Curricular Learning

Cross-curricular learning is not only a good tool to incorporate more time for topics to be covered, but it also provides richer learning opportunities for students. As outlined in an article from Hickman and Kiss (2010), cross-curricular learning has the ability help students transfer skills more successfully and retain information successfully. Cross-curricular teaching can have a great number of benefits for students, but also seems daunting to teachers to include within their math units. The Ontario curriculum documents demonstrate how achievable it is to make connections between math and geography that go well beyond the examples given here. There is an extensive variety of ways that the two subjects connect, and teaching from a cross-curricular perspective has also been linked to improvements in student engagement (Lester, 2013). This provides a great benefit to students, as they often feel intimidated by math, whereas the cross-curricular perspective helps to eliminate some of that fear. With cross-curricular learning, students are learning about math topics while engaging in material from another course. This will help them stay engaged in the learning process and break some of the routine that is set within elementary school scheduling.

Making Real-World Connections

Educators can create authentic opportunities for students to learn by providing real-world learning opportunities. Creating these real-world connections within learning helps students further develop their knowledge (Ballard & Moore, 2009). By including real-world teaching techniques, students are given some more authentic learning opportunities (Ballard & Moore, 2009). Using cross-curricular teaching techniques, specifically with geography, gives teachers an

opportunity to create real-world connections. This could be connected to local geographies of students and Canada as a whole. Asking students to apply their math knowledge to real-world scenarios gives them a great way to make their learning more relevant.

Conclusion

Teachers have the responsibility of preparing their students for high school when teaching Grade 7 or 8. While teaching a variety of subjects in the school day, teachers need to prioritize their time. Cross-curricular teaching provides teachers the opportunity to cover all of the topics of a school day while utilizing the available time. The math curriculum is assigned a great deal of time within an elementary school day; however, educators recognize that students need more of that time to address new material. By using cross-curricular teaching strategies, the time restraints within a school day can be minimized. This style of teaching also gives students more of a chance to work with the material that they are learning in math and apply that to other scenarios. When students are given a chance to work with math material more frequently, there is a greater chance that they will retain that information. By putting math into practice during geography lessons, students are given more of a chance to develop an understanding of math concepts in a different and more applicable context. Incorporating real-world situations into lessons by using visuals and graphs can give students a different perspective of the material they are learning. This, connected with the authentic learning experience of a real-world lesson, gives students a new opportunity to gain knowledge of math topics.

References

Ballard, J.A., & Moore, K.D. (2009). Connecting to the real world: Authentic learning and telephonic visits. *Proceedings of the Southern Management Association,* January 2009, Asheville, North Carolina. www.researchgate.net/publication/320806608_Connecting_to_the_real_world_Authentic_learning_and_telephonic_visits

British Columbia Ministry of Environment and Climate Change Strategy. (2015). *Indicators of climate change for British Columbia.* Author.

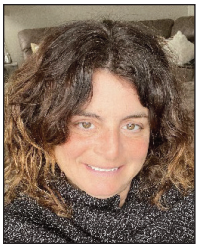
Hickman, R. & Kiss, L. (2010). Cross-curricular gallery learning: A phenomenological case study. *International Journal of Art and Design Education, 29,* 27–36. doi.org/10.1111/j.1476-8070.2010.01635.x

Krupa, J. (2014, August 25). Decker glacier lake at Whistler a sign of melt to come. *CBC News.* www.cbc.ca/news/canada/british-columbia/decker-glacier-lake-at-whistler-a-sign-of-melt-to-come-1.2745216

Lester, D. (2013). A review of the student engagement literature. *Focus on College, Universities, and Schools, 7,* 1.

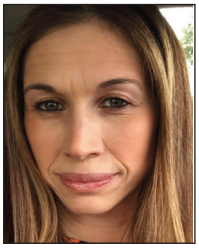
Weber, B. (2021, April 28). The world's glaciers are melting away faster than before, study says. *The Canadian Press.* www.cbc.ca/news/science/glacier-loss-1.6005403

▲ Making a Difference through Differentiated Instruction: Inclusive Strategies for Differentiated Instruction in the Math Classroom



BRENDA STEIN DZALDOV
brenda.steindzaldov@utoronto.ca

Over the course of Brenda Stein Dzaldov's 32-year career in education, she has been a K–8 teacher, consultant, mentor teacher, private school principal, and instructor at OISE/UT. Brenda completed her Ph.D. at OISE/UT in January 2012. Her scholarship demonstrates expertise in curriculum and pedagogy, classroom assessment, and inclusive meaningful learning. In 2019, Brenda won the Excellence in Initial Teacher Education Award at OISE/UT. Brenda's research interests are in the areas of inclusive pedagogies such as culturally responsive pedagogy, differentiated instruction, and universal design for learning.



JENNA MANDELKER
jennamandelker@gmail.com

Jenna Mandelker graduated from the Richard Ivey School of Business at Western University with an Honours in Business Administration (2009) and from the Ontario Institute for Studies in Education with a Bachelors in Education (2012). She holds additional qualifications in both special education and mathematics and has spent the past 11 years teaching middle school-level math and science. Her teaching focuses on differentiating instruction to reach and engage all learners.

Thirteen-year-old Danya and her Grade 7 classmates learn math on a rotary system. Danya has been math avoidant for a couple of years, complaining of stomach aches, and fearful of school whenever confronted with math. Within two months of joining Jenna's class, where differentiated instruction is informed, intentional, collaborative, and invisible, Danya is thriving in math. By December, Danya received an "82%" on a summative assessment and, after reflecting on her own strengths and next steps, Danya told Jenna, "I feel smart in math and it's because of you." An examination of Jenna's daily classroom practices, included:

- her methods of assessment
- her intentional planning
- how she creates differentiated learning experiences invisibly in the inclusive classroom
- her collaboration with her students about their own learning

Through the examination, we seek to rethink and clarify the current well-known framework for differentiated instruction in practice in North American schools.

Differentiated Instruction (DI) is a precise approach in teaching and learning that responds to the strengths and needs of a diverse group of students (Ontario Ministry of Education [OME], 2013). DI uses just-in-time assessment *for* and *as* learning to address specific content, skills, and next steps for all students. In a classroom with little or no DI, only student similarities seem to take centre stage. In a differentiated classroom, commonalities and developmental benchmarks are acknowledged and built upon, but student differences also become important elements in teaching and learning (Tomlinson, 2017). DI is essential when working with academically, culturally, and socially diverse groups of students in any content area, providing appropriate challenges, inspiring meaningful learning, and creating a place to learn, where students feel comfortable and safe (Stein Dzaldov, 2019).

Teachers generally know that in order to be successful with DI, they must create opportunities for ongoing assessment and planning, with an eye to differentiating:

- content (what the students are going to learn)
- process (types of tasks and activities)
- product (how learning is demonstrated)
- environment (the space in which students learn and demonstrate learning) to support all learners (Tomlinson & Eidson, 2003; OME, 2013). Differentiation is based on students' *readiness, interests, and learning preferences.* Too often, this guidance is misinterpreted, and well-meaning teachers end up implementing strategies for differentiating their instruction without a critical and reflective lens as to *why* the strategy is selected, *who* the student is, and *what* the intended purpose is of the differentiation (Subban, 2006).

DI can only occur meaningfully if teachers believe that it is a valuable and inclusive teaching approach that regards all individuals as unique in their interests, abilities, learning strengths, and challenges. Promising practices in DI are flexible and contribute to emotional safety. In order to truly instruct in a differentiated environment, teachers need to start with these beliefs (Tomlinson, 2017). Students approach tasks differently, have different understandings from the same direct instruction, and come to us with different background knowledge, interests, and skills. Knowing each student both academically and social–emotionally is foundational to successful DI (Subban, 2006).

Meet Jenna

Jenna began her career teaching middle school math and science in 2012. Jenna was fresh out of her program of teacher education at OISE/UT. She immediately had a genuine interest in DI to meet the needs of the students in her inclusive classroom, and has since transformed learning through her use of assessment strategies, planning, and flexible instruction. Her work was so unique and effective that Jenna began to instruct a portion of the special education course for OISE/UT teacher candidates by speaking about how she differentiates instruction. Jenna had reframed the conventional understandings of DI, the role of the teacher, and the role of the student to support inclusion. Jenna demonstrated ways to create safe spaces for diverse learners in inclusive classrooms.

Meet Danya

In 2019, when this study began, 13-year-old Danya was one of Jenna's Grade 7 students. Danya came to the class in September 2019 with mathematics knowledge and numeracy skills below grade level, as well as math anxiety. She cried when given math to do and stayed home from school frequently with stomach aches throughout Grades 5 and 6. At that time, there was a clear relationship between anxiety and math performance for Danya. When Danya experienced anxiety, she shut down cognitively and froze, unable to complete math work at home or at school (Sokolowski & Ansari, 2017).

When Danya entered Jenna's classroom, she felt nervous and afraid. Jenna communicated often with Danya's parents, who explained her past experiences. She often lost interest and chatted with others during math class and was reluctant to even try to do her in-class math practice. Upon discussion with Jenna, it became clear that Danya perceived her limitations in math to be fixed and permanent, attributing years of struggle to an innate inability to succeed in math.

However, by early November, Danya started to thrive in Jenna's math class. So, how did Jenna support Danya to accomplish this amazing transformation?

Differentiated Instruction Every Day

This observational study was designed to understand Jenna's approach to differentiated instruction in her daily classes. Weekly in-person observations, videos, and

conversations with Jenna and her students were collected and transcribed over the course of two months. Samples from each lesson (photos, student work, lesson plans, assessment records) were also collected and analyzed as evidence of Jenna's practice.

Results: Taking a Critical Approach to Differentiated Instruction

There are clear similarities and differences between standard approaches to DI and the DI observed in Jenna's class. Within Jenna's class, successful differentiation was framed as:

- **informed** by ongoing assessment by the teacher and the student (observations, conversations, ongoing descriptive feedback on process and products)
- **intentional**, as it is planned into the fabric of each lesson
- **collaborative** between the teacher and the students who have a good degree of choice and voice in their learning
- **invisible** for the purpose of reducing stigma and supporting meaningful learning

Each of these areas was observed during Jenna's instruction and discussed during conversations with Jenna about her practice over the two months of data collection for this study.

DI Is Informed – *Assessment That Guides Instruction*

The foundation of successful DI relies on an informed practice. Jenna shared that she can only effectively differentiate with evidence to inform her about who is in her classroom, what they know, and what they need. Meaningful information about each student is the basis for Jenna's decision-making process.

There are a variety of formal and informal ways in which Jenna gathered information about Danya's learning. These included conversations with Danya both during and outside of instructional time, and reflection on past classroom experiences through conversations with colleagues, parents, and Danya. In conversations with Dayna's parents, Jenna inquired about the routines around homework, as well as Dayna's observed attitude and commentary around school and her perceived level of past success. Observation of her learning behaviours (class participation, collaboration skills/style, environmental preferences) was also key, as Jenna found out Danya had a large, busy family, and parents had little time to work with her. Danya was very social and popular with friends and often very self-conscious about her weaknesses in math. With a strong foundational understanding of who Danya was, and a commitment to allowing herself to be "hopeful and surprised" by changes in Danya's learning at any point, trust was developed, and Danya was able to share more of herself with Jenna. Jenna was careful not to expect Danya to complete math activities that were too hard, gently scaffolding work in Danya's Zone of Proximal Development, as indicated by ongoing assessment (Vygotsky, 1980; Stiggins, 2004).

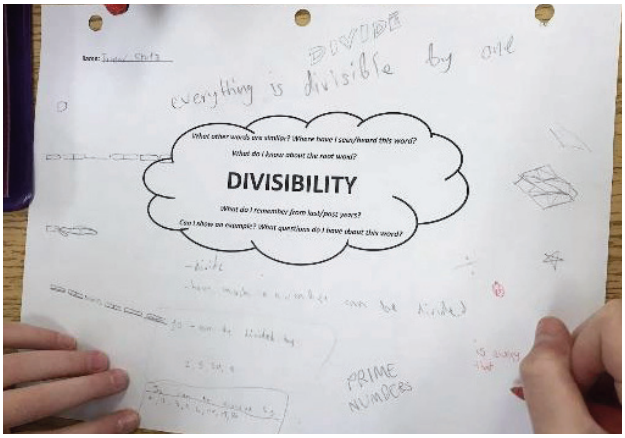
Tools were used in the classroom on an ongoing basis to assess Danya's learning and guide planning and instruction. Danya was familiar with all these assessments, including tickets in/out the door to assess "in the moment" understanding, conferencing using goals and criteria as a guide, homework checks, formative quizzes, anticipation guides, and anecdotal notes on Danya's contributions to class discussions.

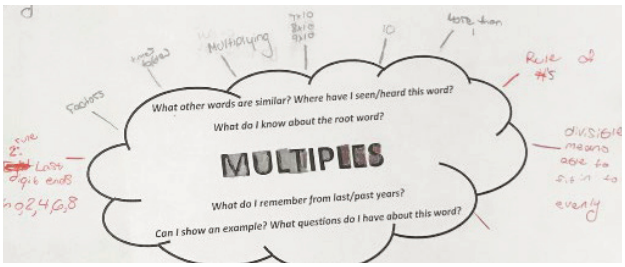
Teachers will know that their practice is **informed** if they are able to accurately describe their learners' strengths, challenges, and next steps at any moment in time. It is important to note that Jenna viewed this process "as ongoing and students *should not* be categorized as 'low, average, or high' as a fixed label." When Jenna took this approach to gather information about her students, students like Danya began to understand themselves as complex and ever-evolving learners as well. It fostered resilience, encouraging reflection and risk taking, and involved Danya safely in the learning process (Willems & Friesen, 2012).

In Jenna's classroom, she used a variety of ongoing assessments to collect information. Some examples include the following:

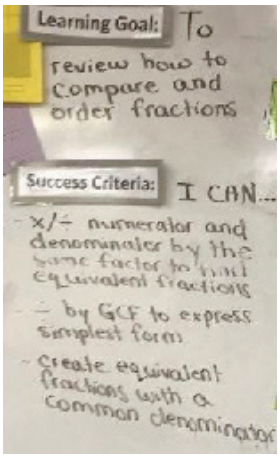
- Anticipation guides were given at the introduction of any new topic, which informed both Jenna and Danya about her readiness relative to the upcoming learning goals.
- Tickets in/out the door were used to immediately assess understanding of a taught concept. These tickets were completed by students with differentiated content, corrected by Jenna, and returned to students in time to support their learning in class that day. This provided both Jenna and Danya with a snapshot of her understanding and the opportunity to provide/receive timely feedback.
- Ongoing individualized brainstorms took place about the mathematical terms being studied, e.g., *Divisibility* or *Multiples*, with prompts such as: "What other words are similar? Where have I seen/heard this word? What do I know about the root word? What do I remember from past years? Can I show an example? What questions do I have about this word? This allowed Jenna to see what prior knowledge Danya was bringing to the topic each day. Further, ongoing learning was

recorded on the page throughout the study of the topic in a different colour so that growth and/or misconceptions could be observed by both the teacher and student.





- Math anchor charts and co-constructed class rules were available in the classroom for all students. All students also had individual copies of rules, goals, and success criteria. These duplicate options for referencing important information were very supportive for Danya.



- Jenna planned her oral questions and prompts that supported learning. For example, "How do you know?" "What do you think?" "Where could you look for that information?" were commonly used when Jenna was facilitating individual and small-group learning. In all cases, students were expected to reference information independently and/or explain their thinking, whether the answer to the math question was "right" or "wrong."

DI Is Intentional – *Planning Based on the Assessment Information*

DI was specific and meant to address identified needs, which is why intentionality is so important. In choosing a grouping/task for a particular student, Jenna was always able to articulate what need was being addressed, what learning goal was being worked toward, and how that choice supported the achievement of both. Intentionality relies on all of the assessment information discussed in the previous section (Wormeli, 2007).

In Jenna's classroom, the assessment information was used to intentionally group students and to vary tasks and teaching methods, with the goal of supporting Danya. She suggested planning ideas such as the following:

Instead of This...	Try This...
Grouping students randomly	Group students based on areas of need. These could be curriculum based and/or learning skills related.
Rotating students through stations for the independent practice of knowledge and skills	Start all students on the content area with which they require the most practice/review/discussion, and allow them to move through stations at their own pace.
Teaching lessons to the whole group	Teach smaller groups. Provide self-guided lesson templates for some students when introducing new concepts, while teaching directly to others.
Using the same practice for consolidation of understanding for all students	Create multiple practice opportunities that address different needs (additional prompts/guided examples embedded, less practice of knowledge and more application questions).

DI Is Collaborative – *Incorporating Student Choice and Voice*

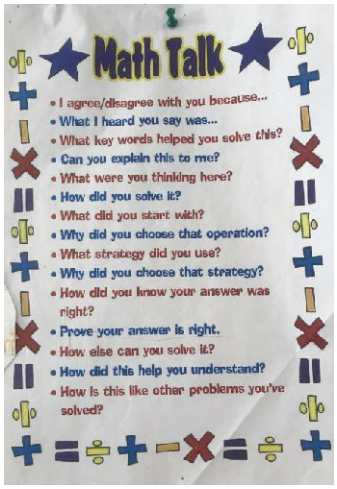
In Jenna's class, differentiation was a collaboration between student and teacher, in which student feedback was routinely solicited and incorporated. The easiest way to get information from Danya was to ask her, "What are you good at?" "What was easy about this?" "What are you struggling with?" "What types of learning activities do you enjoy most?" "What do you think you did well on the assessment?" "Where do you think you need more practice?" These types of prompts were used in formal conferences and during informal conversations.

Providing student choice was another form of collaboration that reinforced self-regulation. When Danya was afforded an opportunity for choice, she was practising the skills of using data to reflect on her learning, and making informed and productive choices for herself.

One of Jenna's central classroom goals was the involvement of students in their learning process. Her collaborative approach to DI ensured students viewed their participation in reflection and self-selected tasks as integral to their progression.

In Jenna's classroom, she used a variety of collaborative approaches:

- Students sat in flexible groupings, usually created somewhat homogeneously, to support differentiation; groupings changed often, based on content and skills (Benjamin, 2006).
- Discussion and small- and whole-group sharing followed individual brainstorms/reflections. Jenna pointed out mathematical understanding (odd/even numbers, division, patterns) and referred often to posted goals and success criteria; Jenna actively listened in and facilitated these discussions through **open-ended questioning and prompting**, such as the following:



  - Each student kept a "reference sheet" in their binders, where they recorded their current understandings of new learning. Each reference sheet was set up the same way, but had different information based on what the student was working on at that time.

Jenna suggested that the partnership between herself and Danya was strengthened by making some of the suggested substitutions below:

Instead of This...	Try This...
"Put your hand up if you need help."	"I am guiding another example on the board. If you would prefer to move on, you may."
Giving visual aids/manipulatives to selected students	Creating a space where visual aids/physical manipulatives are readily available for students to use as needed and any students could access them
Telling students where they need to focus additional practice	Incorporating self-regulation into the assessment process by providing options for next steps from which students need to self-select based on data
Requiring all students to demonstrate their understanding via the same method	Using prompts such as "Using words, numbers, and/or visual representations…," to allow students to demonstrate their understanding in the format that best suits their learning preferences Change your prompts/questions based on the readiness/interests of the child.

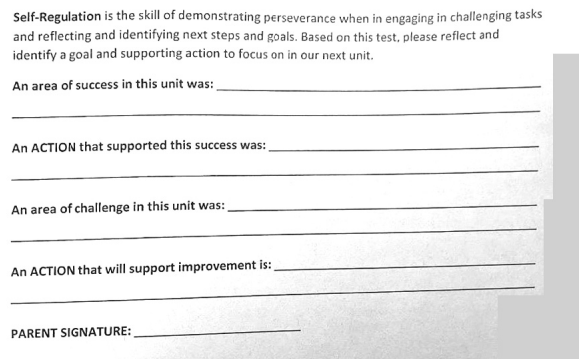
DI Is Invisible – *Removing Labels and Decreasing Stigma*

In implementing DI, Danya's perceptions about herself and her peers as math students had to be considered and planned for. Once Jenna was informed and intentional about their DI, the invisibility approach helped mitigate the stigma and social dynamics at play whenever tasks were assigned, or when groupings were used. This was of particular importance in middle school and in math, where students often believe their ability is fixed. Jenna shared that DI should endeavour to remove labels such as "advanced" or "struggling," which can then allow students to enjoy and engage more openly in the process of their learning. In Jenna's classroom, Danya and the other students were regularly provided different tasks and/or groupings that appeared random.



In reality, Jenna had planned for each student beforehand based on assessment information previously collected. The result of the invisibility of the differentiation was a smooth-running classroom, in which all students were working at their own pace and on the tasks that best supported their learning, without worrying about what others thought of them or what their peers were doing. Jenna used colour as a way to differentiate, but not as a way to show difference.

- Colour was used to make differentiation invisible—therefore, Jenna used the same colour, but gave each individual student their task based on her knowledge of their strengths/needs and ongoing assessment.
- Colour was used to differentiate types of activities (e.g., exit tickets were purple, brainstorming activities were green, differentiated content cards were white).
- Students assessed their learning strengths and learning habits/skills throughout their learning (formative and summative) by responding to prompts such as "areas of success," "actions that supported my success", "areas of challenge for me", "actions that could support my challenges," "areas I'm proud of," and "goals for improvement."


Self-Regulation is the skill of demonstrating perseverance when in engaging in challenging tasks and reflecting and identifying next steps and goals. Based on this test, please reflect and identify a goal and supporting action to focus on in our next unit.

An area of success in this unit was: _____

An ACTION that supported this success was: _____

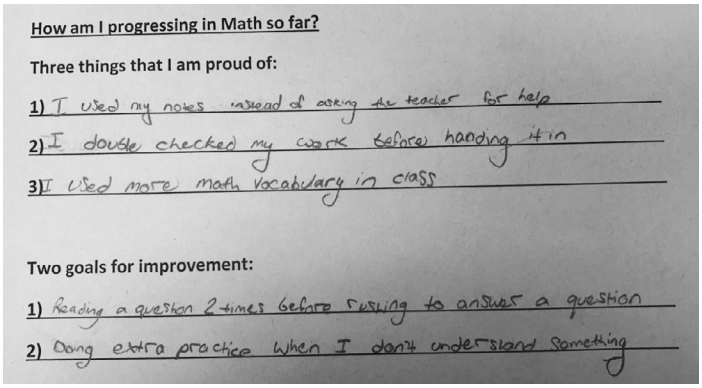
An area of challenge in this unit was: _____

An ACTION that will support improvement is: _____

PARENT SIGNATURE: _____

Danya became much more specific as she completed

these types of tasks regularly:


How am I progressing in Math so far?

Three things that I am proud of:

1) I used my notes instead of asking the teacher for help
2) I double checked my work before handing it in
3) I used more math vocabulary in class

Two goals for improvement:

1) Reading a question 2 times before rushing to answer a question
2) Doing extra practice when I don't understand something

To support invisible differentiation, Jenna regularly substituted common classroom practices with the approaches below:

Instead of This...	Try This...
"Take this sheet if you need more practice, and this sheet if you need a challenge."	Create multiple practice opportunities beforehand that cater to different student needs and are handed out intentionally by the teacher: • One with reduced practice and an extension page included • One with additional practice to consolidate learning • One with guided examples on the page to refer to All pages should appear to look the same (e.g., colour, size), and students should not be informed of the differences.
"Find a partner/group to work with."	Intentionally group students based on data, using an invisible strategy. Groupings appear to be randomly selected to students, despite teacher pre-planning. These may include: • using a deck of cards ordered by the teacher beforehand and handed to predetermined students • using colour-coded handouts to group students
Formatively assessing all students with the same question	Create levelled assessments to check for understanding of a concept and handing them out without noting or announcing any differences. • Tickets can be handed out from the top or bottom of the same pile to minimize any perception of differences.

Conclusion and Discussion

Through the use of differentiated instruction, Jenna was able to move a student's long-held beliefs about her own academic competence and potential from despair to pride. The influence this will have on the trajectory of Danya's educational experience cannot be underestimated. The impact of taking a critical approach to differentiated instruction can often be seen in the dynamic of a classroom and the resulting positive attitudes of students like Danya.

Successful implementation of DI requires forethought and planning, tasks that might feel complicated and time-consuming when a teacher first begins incorporating it regularly. Jenna said: "The magic is all in the planning—the more individualized teaching is, the greater number of students a teacher can reach simultaneously." Over time, the use of informed, **intentional, collaborative, and invisible differentiation** became second nature; it became a teaching philosophy and approach that was heavily grounded ***in the belief that students come to us as unique individuals and thrive when they are educated in a way that honours their differences***. Differentiated instruction is the practical expression of Jenna's commitment to inclusivity, addressing the universally held truth, often not adequately addressed in education, that all students are exquisitely unique and deserve the time, attention, expertise, and support to reach their fullest potential. These principles can and should be applied in any subject or at any grade level to maximize student achievement.

References

Benjamin, A. (2006). Valuing differentiated instruction. *Education Digest, 72*(1), 57–59.

Ontario Ministry of Education. (2013). *Learning for all: A guide to effective assessment instruction for all students.* Queen's Printer for Ontario. www.ontario.ca/page/learning-all-guide-effective-assessment-and-instruction-all-students-kindergarten-grade-12

Sokolowski, H.M., & Ansari, D. (2017). Who is afraid of math? What is math anxiety? And what can you do about it? *Young Minds, 5*(57). doi.org/10.3389/frym/2017.00057

Stein Dzaldov, B. (2019). *Inspiring meaningful learning: Six steps to designing lessons that engage students in deep learning.* Pembroke Publishers.

Stiggins, R.J. (2004). *Assessment FOR learning: Building a culture of confident learners.* Assessment Training Institute.

Subban, P. (2006). Differentiated instruction: A research basis. *International Education Journal, 7*(7), 935–947.

Tomlinson, C.A. (2017). *How to differentiate instruction in academically diverse classrooms* (3rd ed.). Association for Supervision and Curriculum Development.

Tomlinson, C.A., & Eidson, C.C. (2003). *Differentiation in practice: A resource guide for differentiating curriculum, grades 5–9.* Association for Supervision and Curriculum Development.

Vygotsky, L. (1980). *Mind in society: The development of higher psychological processes.* Harvard University Press.

Willems, J.D., & Friesen, S. (2012). Report number 2: The relationship between instructional challenge and student engagement. (What did you do in school today? Research Series). Canadian Education Association.

Wormeli, R. (2007). *Differentiation: From planning to practice, grades 6–12.* Stenhouse Publishers. ▲





The TI-Nspire™ CX II CAS and TI-84 Plus CE graphing calculators help make learning accessible for all

- Support the new Ontario Grade 9 Mathematics course (MTH1W) with tools purposefully designed to bring math learning to life
- Help all students better access and understand important mathematics, including algebra, coding, data and financial literacy

For activities mapped directly to the Ontario Math Curriculum, visit **education.ti.com/canada** and **stemsupports.ca**.

To get started, contact Tom Steinke:
tsteinke@ti.com | 613.601.6535

Texas Instruments

The platform bar is a trademark of Texas Instruments.
©2021 Texas Instruments Printed in U.S.A. AD22878

▲ 2022-2023 OAME/AOEM Board

OAME WEBSITE
WWW.OAME.ON.CA

DIRECT GENERAL INQUIRIES TO: EXECUTIVE DIRECTORS

Fred and Lynda Ferneyhough
4751 Mack Ave.
Plympton-Wyoming, ON N0N 1J6
Business Phone: 647-466-0674
eds@oame.on.ca

PRESIDENT

Sandra Jean Price
Oshawa, ON
sandra.jean.price@oame.on.ca

VICE-PRESIDENTS

Melissa Black
Kanata, ON
melissa.black@oame.on.ca

Bart Vanslack
Oakville, ON
bart.vanslack@oame.on.ca

PAST PRESIDENT

Judy Mendaglio
Mississauga, ON
judy.mendaglio@oame.on.ca

EMERITUS

Bill Otto
Stillwater Lake, NS
bill.otto@oame.on.ca

CHAPTERS & REPRESENTATIVES

CHAMP: Paul Alves
paul.alves@oame.on.ca
Claudio Attanasio
claudio.attanasio@oame.on.ca

GOLDEN: Mark Chubb
St. Catharines
mark.chubb@oame.on.ca

GVMA: Glen McMillan
Guelph, ON
glen.mcmillan@oame.on.ca

ISOMA: Cal Armstrong
Burlington, ON
cal.armstrong@oame.on.ca

MAC2: Stacey Collins
Barrie, ON
stacey.collins@oame.on.ca

NOMA: Heather Theijsmeijer
Kagawong, ON
heather.theijsmeijer@oame.on.ca

NWOAME: Chelsea Carlson
Fort Frances, ON
chelsea.carlson@oame.on.ca

$O_3$4ME: Stephanie Bishop
Carp, ON
stephanie.bishop@oame.on.ca

PRMA: Jennifer Clark
Bowmanville, ON
jennifer.clark@oame.on.ca

QSLMA: Tanis Lattanzio
Kingston, ON
tanis.lattanzio@oame.on.ca

SAME: Franca Genco
Scarborough, ON
franca.genco@oame.on.ca

SWOAME: Elizabeth Pearsall
Kingsville, ON
elizabeth.pearsall@oame.on.ca

TEAMS: Alvarine Aldridge
Toronto, ON
alvarine.aldridge@oame.on.ca

WOMA: TBD

Y^4MA: Gina Micomonaco
Richmond Hill, ON
gina.micomonaco@oame.on.ca

DIRECTORS – TERM EXPIRING 2023

Elli Weisdorf
Toronto, ON
elli.weisdorf@oame.on.ca

Sam Garrison
Toronto, ON
sam.garrison@oame.on.ca

DIRECTORS – TERM EXPIRING 2024

Lori Yee
Pickering, ON
lori.yee@oame.on.ca

Jimmy Pai
Ottawa, ON
jimmy.pai@oame.on.ca

DIRECTORS – TERM EXPIRING 2025

Kit Luce
Barrie, ON
kit.luce@oame.on.ca

Jason To
Toronto, ON
jason.to@oame.on.ca

Kelly McKie
Ottawa, ON
kelly.mckie@oame.on.ca

Cameron Redsell-Montgomerie
Toronto, ON
cameron.redsell-montgomerie
@oame.on.ca

NCTM REPRESENTATIVE

Jacqueline Hill
Lindsay, ON
jacqueline.hill@oame.on.ca

WEBSITE COORDINATOR

Greg Clarke
Orillia, ON
web@oame.on.ca

GAZETTE EDITOR

Timothy Sibbald
North Bay, ON
gazette@oame.on.ca

ABACUS CO-EDITORS

Marc Husband
Toronto, ON
marc.husband@oame.on.ca

Tina Rapke
Toronto, ON
tina.rapke@oame.on.ca

The 50th Annual OAME Conference
OAME2023 EMPOWERING^{ME} Organizing Committees



The **50th Annual OAME Conference, OAME2023 EMPOWERING^{ME},** was held in **Toronto** on **April 27-28, 2023**. The **OAME2023 EMPOWERING^{ME} Organizing Committees** from **NOMA** (Northern Ontario Mathematics Association) and from **TEAMS** (Toronto Educators Association for Mathematics) would to thank everyone involved in making the Mathematics Conference a huge success! We all hope that the opportunity to connect with, network with, and socialize with Mathematics educators from across the province and beyond has inspired and empowered you!

- **Thank you** to the **Keynote Speakers**, the **Featured Speakers**, and the **Breakout Session Presenters** for sharing your knowledge and your expertise with Mathematics Educators in Ontario.
- **Thank you** to our **sponsors** who continue to support professional learning for our attendees financially and also with breakout sessions of their own.
- **Thank you** to the **school boards** that signed up for group codes to support the professional learning of the educators in their school boards.
- **Thank you** to our **dedicated group of volunteers** - **Teacher Candidates, Classroom Teachers, and Retirees from across Ontario** who donated their time to support the presenters before, during, and after their sessions.
- **Thank you** to **everyone at OAME/AOEM, https://oame.on.ca,** for their unwavering support of all mathematical endeavours and for their support of all mathematics educators across the province of Ontario and beyond.
- **Thank you** to our **in-person and virtual attendees** for attending OAME2023 EMPOWERING^{ME}. We hope that you enjoyed the conference!

https://sites.google.com/oame.on.ca/oame2023 NOMA TEAMS https://twitter.com/OAMElearns

- We hope that you are planning to attend the **OAME2024 Mathematics Conference "Embracing Change…Moving Forward…Continuing to Grow"** that will be held at **St. Lawrence College** in **Kingston, Ontario** on **May 2-4, 2024**. The Organizing Committees of the OAME2024 Mathematics Conference are **$O_3$4ME** (The Ottawa Zone for Mathematics Education) and **QSLMA** (Quinte St. Lawrence Mathematics Association).



QSLMA