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ABACUS

Ontario Mathematics Gazette

OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

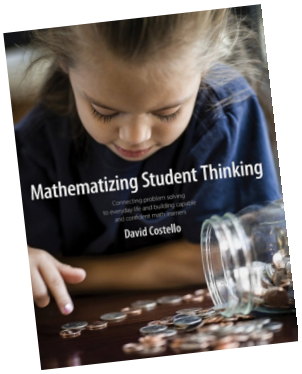
AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

Vol. 62#1
September 2023

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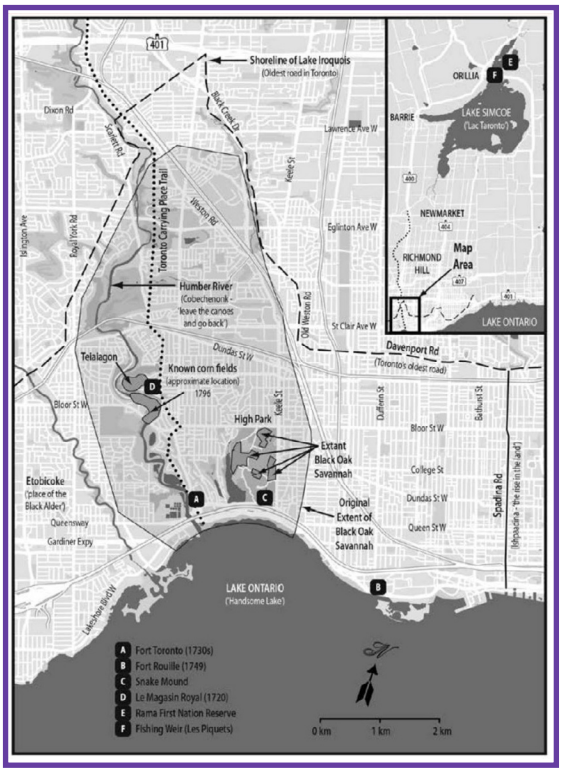
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- Uncovering High Park's Pasts and Presents: Storying the Land through Mathematics and Storywork

▶ See Uncovering High Park's Pasts and Presents: Storying the Land through Mathematics and Storywork



▲ See Book Review: *Mathematizing Student Thinking*

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OAME/AOEM Presidents
Then and Now



Submission of Articles

The *Ontario Mathematics Gazette* (*OMG*) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *Gazette*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles are to be word–processed, MS Word is preferred, and prepared according to the *Publication Manual of the American Psychological Association*, Seventh Edition. However, please use single-line spacing (not double). Articles should not exceed five numbered pages of text, and figures, images, and photographs should be placed in the text close to where they belong, with captions. The photographer's permission is required, and for photos of students under the age of 18, the written permission of a parent or guardian is required.

Please submit your article in one blind file (i.e., identity of author is not evident), and include author names, contact information including email and mailing addresses, photos—head and shoulders, biographies—less than 100 words, and all content removed for blinding in a second file. Please email these two files to Tim Sibbald at gazette@oame.on.ca.

Upon review, you will be notified whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected. The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME/AOEM.

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The *Ontario Mathematics Gazette*—ISSN 0030-3011—is indexed in the Canadian Education Index and is published four times per year. Its Canadian Publication Mail Product Sales Agreement Number is 40051074.

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Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** by email or at the above address. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be print ready, and colour advertisements should have no bleeds.

Advertising Rates
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▲ Editor's Report



TIM SIBBALD, OCT, PH.D.
gazette@oame.on.ca

Tim Sibbald is a professor in the Schulich School of Education, Nipissing University, with a focus that includes mathematics education. He is a former President of OAME/AOEM and has held a variety of other OAME/AOEM posts over the years.

Can we, collectively, put the pandemic behind us? Or, perhaps I should ask, when will each person put the pandemic behind them? The latter question reflects the need for some individuals to protect their own health above and beyond societal norms.

The question arises because the evidence, particularly as evident on the World Health Organization dashboard, shows that the pandemic is essentially over. There continue to be infections, but not in the sorts of numbers, nor with the mortality rates, seen not so long ago. There remains a mortality rate, and those who are at high risk rightly have a need to remain cautious. That said, much of society has returned to many normative practices—plastic barriers in stores have come down, the wearing of masks is far less, and attitudes toward social gatherings have been regaining the pre-pandemic norms. All this points to the pandemic being behind us, and yet, there are signs that something is not right.

Two things seem to be lingering on in the post-pandemic world. One is the use of online resources that serve a direct need or have a direct benefit. For example, I have graduate students in different parts of the province, and meeting online is clearly effective and, while it could be on the telephone, the ability to share screens or show diagrams can aid conversation. Similarly, events around the world have discovered that providing online access has many benefits. I attended conferences in Shanghai and Tokyo during the pandemic—not in person; rather, my presence was remote and virtual.

The other lingering effect in the post-pandemic world for teachers is a malaise. Professional engagement does not seem to have bounced back from the pandemic for many teachers. Conference attendance is down, engagement in professional activities seems to be slow in returning to normal, and many teachers at the end of the last school year seemed to be beat. Never did a summer seem so important as an opportunity for teachers to recuperate and recharge.

Can we blame the pandemic for the malaise? Perhaps,

but the political climate in education seems a more significant influence. Sure, the pandemic led teachers into a very tough circumstance that was collectively handled the way teachers do—with care and respect for the needs of their students. It was a huge shift and exhausting. However, teachers feeling beat at the end of the last school year seems to have more to do with circumstances in the administration of education than the pandemic itself. Was it due to changing the curriculum in June 2020, to stall teacher recuperation early in the pandemic? Was it perhaps due to the failure to address the teacher shortage that has been a consistent trend since 2015, and there are now unreasonable demands because of poor management of the teacher supply? There are also challenging social issues, where society itself seems torn over some of the policies with which teachers are being required to work.

This editorial is not a political soapbox from which to spout rhetoric and stilted opinions. The non-political point is that, quite apart from the pandemic, education has seen enormous shifts taking place. These are shifts beyond the influence of teachers as a collective—the profession—and that seems to be the reason so many teachers were exhausted at the end of the last school year. As a profession, teachers are self-regulating and have a relatively high degree of autonomy to do what they need to do to support student learning. Teachers have thrived as professionals and are well known to innovate and enact changes of their own accord.

The current milieu has seen the professional body modified, and many imposed changes that have hindered teachers exercising their professional muscles to the extent they would like. It is somewhat akin to a teenager who wishes to choose their activities, but cannot escape overwhelming parental demands under the auspices of chores. Teachers have been struggling to adapt to one circumstance, and before that is done, another change requiring more adaptation occurs. No wonder many teachers feel like they are on a treadmill of chores—doing their professional best, but struggling to keep up.

The professional status of teachers is the solution to the issue. We need to wipe the pandemic dust off our arms and return to engaging in collective professional efforts, as opposed to responding to domineering external obligations for change. Responding to change through professional engagement was recognized as an efficient way to address the needs of our students. The key here is that collective professional efforts refers to collections of professional teachers, such as subject associations, chapter events, and as you surely know, the *Gazette* (there is a reason it has been around for 60 years). There is a need to share practices so that we efficiently share beneficial approaches among teachers. That is our value as an association, as a professional collective.

News?

Long-time columnist, Dr. Lynda Colgan, has been recognized twice recently. She is the 2023 winner of The Fields Institute for Research in the Mathematical Sciences Margaret Sinclair Memorial Award. As if that isn't enough, she has also been awarded the 2023 Distinguished Citizen Award by the City of Kingston for her work in public education and STEM outreach.



Do you recall in 1991 when the eighth Dedekind number was discovered? Admittedly, I may have overlooked that too. However, breaking news that last Spring, the ninth Dedekind number was found.

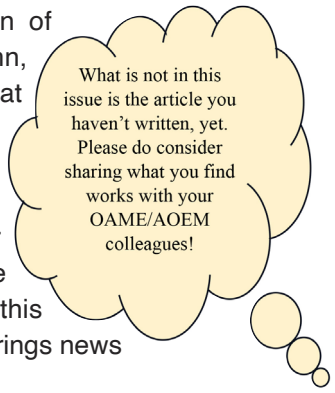
Congratulations to Jeff Irvine on reaching 50 years in education. It is quite an achievement, and I know there are quite a few other OAME/AOEM members who are in the 45+ years club. Kudos to all of them. It is that level of experience that enriches our organization to the benefit of all members and the students we teach.

Erratum

In Jeff Irvine's article, "Research into Practice: An Application of Self-Determination Theory to the Teaching of Sequences in Grade 11," there is a difficulty with the layout. On page 34, where it says, "The first activity (Irvine, 2019a) is an introduction to sequences and patterning," this should be followed by examining the activity before continuing to the teacher notes. The activity has inadvertently ended up in the left column on page 37.

In This Issue…

Our President, **Sandra Jean Price**, begins her second year as President with a message regarding equity and inclusion. **Jacqueline Hill** brings an NCTM perspective to the promotion of self-care. In her final column, **Angelica Mendaglio** looks at the overall role of the Fields Institute for Research in Mathematical Sciences. She will be missed, and the Editor would like to hear from anyone who can step up to assume this role. **Heather Theijsmeijer** brings news from the OMCA.



By way of columns, **Carly Ziniuk** brings puzzles, games, and books to our attention. **Lynda Colgan** considers the complexity of relative positions. **Iain Brodie** and **Hatice Beyza Sezer** look at the history of coding and interview **Jeff Irvine**. In his own column, **Jeff Irvine** has changed focus, leaving the "Blast from the Past" and considering "Useful Theories to Know About." **Jennifer Holm**, who needs congratulating for achieving tenure and promotion to Associate Professor—Congratulations!—considers various options for using the area model. **David Costello** looks at teacher facilitation of problem solving involving ill-defined problems. Lastly, but certainly not least, is **Shawn Godin**'s column that is explicit about the math processes through the process of solving a problem. Missing from this issue is **Jérôme Proulx**, who will be back in the next issue.

The issue contains an article and a book review. **Dahlia Benedikt** writes about storying the land and storywork in an article that received very positive comments as it went through the review and editorial process. **Tim Sibbald** reviews *Mathematizing Student Thinking* by **David Costello**.

Thank Yous

I would like to take a moment to recognize the ongoing commitment of our columnists. They routinely generate ideas and share them with the readership, while meeting deadlines and responding as needed. They are dedicated and, frankly, are an interesting bunch. As Editor, I have the pleasure of reading every column from beginning to end, and there is always food for thought and reminders of good teaching practices.

In the "Blast from the Past" column in the last issue, we have the suggestion, "So, noting the Editor has said there is a need for articles, I will add the suggestion that the Mathematical Processes and their assessment is a particular topic that would benefit from being included in the *Gazette*" (p. 7). I passed along a mention of this to the columnists, and several have responded with their intent to do something over the coming year. It is notable that "What's the Problem?" specifically addresses the Mathematical Processes in an excellent and insightful way (that I think suits all teachers, my pre-service teacher candidates as well). So, a special thank you for the added attention by our columnists.

Behind the scenes, Anne Yeager, Peter Saarimaki, Gitta Berg, and Penny Clemens have been the usual—~~great~~, stupendous! They are a powerhouse of making the *Gazette* the professional tour de force that it is. Their efforts are appreciated by all readers and writers, but because they complete their tasks so well, readers and writers rarely know to show appreciation! Please consider letting them know with a letter to the Editor. ▲

▲ President's Message



SANDRA JEAN PRICE
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Sandra Jean has been on the Board of Directors of OAME/AOEM for almost 20 years. She is currently teaching in the Durham District School Board and has more than 25 years of teaching experience. She has had many roles, including classroom teacher, Math Coach, and Instructional Leader of online Math Additional Basic Qualifications (ABQs). Sandra Jean has contributed to a variety of Ministry projects and was a coordinator of the recent OAME/AEMO writing projects. Sandra was the recipient of the 2008 OAME Award for Exceptional and Creative Teaching in Elementary Mathematics for her innovative instructional practices, integration of technology, and belief in success for all.

As we welcome the beginning of another school year, it is important to re-examine the importance of mathematics within our world, the ways in which it is taught and learned, and how we, as educators, can empower all students through mathematics. The world has changed. In what ways have/should changes in the world impact your thinking about being an educator, who is responsible for a student's mathematical experiences?

In June, NCTM president, Kevin Dykema, included in his president's message a discussion regarding what mathematics is. He cited the growing debate on social media that is, at best, an open and honest discourse, but at times is divisive, abusive, and marginalizing. But as he pointed out, it all stems from how one defines mathematics. He stated, "Because mathematics must be about reasoning and sense making, it should be taught and learned differently than if mathematics were only about getting answers" (Dykema, 2023). This distinction is very important. Procedural fluency, the ability to get the correct answer quickly, using an algorithm, is not necessarily mathematical proficiency. Proficiency truly means that a person can understand a concept and how it fits within other mathematical concepts. It is not just memorizing a formula and being able to apply it.

Thinking back to my days as a student, I had some incredible math teachers who helped me understand the concepts. But at the end of the day, I had a really good memory and could just memorize what I needed to do. I was very adept at applying procedures. But, for example, why did a negative integer multiplied by a negative integer result

in a positive integer? I couldn't have told you or done anything to prove that thinking. I just knew that it was a "rule." It wasn't until I was a teacher that I started asking and being able to answer the "why" questions that I had always had, but was afraid to ask. Consequently, as a teacher, I have always strived to teach beyond the rules and procedures and allow students to understand the concepts, which eventually leads to their ability to apply the same mathematical procedures, but with a conceptual understanding of how and why the procedures work within the world of mathematics. This is the main difference between knowledge and understanding. By having a strong conceptual understanding of an algorithm or procedure that goes beyond memorization, a student is able to apply that understanding in a variety of different contexts.

Mathematics can be a game changer for many students. In the past, if a student struggled to be successful in math, often they were told that they just aren't good at math. In my second President's Message, I related my own experience with this and how it affected my feelings of self-worth and my self-efficacy in other areas. Mathematics can also be a positive influence on individuals, allowing students to make sense of the world around them through, for example, understanding data graphics, to be better able to think critically about the data and differentiate that from how the data is being presented. Feeling capable and being successful often breeds more success. To that end, educators have an obligation to use their expertise in pedagogy to help and give all students the opportunity to be successful in mathematics.

To do this, however, it is necessary to understand how math and its applications have perhaps helped to marginalize specific populations in the past. The most obvious way is through the IQ test. While the original IQ test was created by Alfred Binet in 1904 to help identify students who would be most likely to require extra help to be successful in school, the use of the modified Stanford-Binet IQ test in the immigration process in the United States in the 1940's led to generalizations about entire populations (Cherry, 2023). In fact, low test scores, or low IQ's, were used by eugenicists to try to convince the U.S. Congress to restrict immigration of those who they felt were genetically inferior into the U.S. What was originally intended to be a helpful tool to address the needs of students was altered and then applied in an entirely different context to support a very narrow and biased agenda. The misinformation that grew out of this movement continues today.

Another example is our very Eurocentric mathematics system. The Indigenous peoples in Canada have long had their own mathematics systems in place, very much rooted in culture, in their relationships with each other and the land, and in the world. Over the last 20 years, mathematics education has begun to recognize that many of the research-proven pedagogical strategies align with how Indigenous groups in Canada view and learn math. Outside of Canada, it is interesting that we have learned that the Pythagorean relationship was actually discovered by many others. In fact, there is concrete evidence that Babylonians had some knowledge of this. There are four tablets from around 1900–1600 BCE that show calculations of the square root of 2, which is the length of the hypotenuse of a right triangle, with the length of the legs being 1 unit. It also lists what we know as Pythagorean triples that satisfy this relationship. The theorem is mentioned in other cultures around the world including, but not limited to, India in the Baudhayana Sulba-sutra, which was written between 800 and 400 BCE. It appears that this relationship was discovered by many different cultures over a span of time. This is something that I was never aware of as a student. It was never discussed.

I identify as a White cisgender female who, although born in Canada, is a settler. I am not a person of colour, nor do I identify as being Indigenous, despite having some Indigenous ancestry. And I do not consider myself an expert in how systemic oppression has impacted our youth from underserved and marginalized communities. But I am a teacher and believe every student has a right to learn, experience, and be successful in mathematics. If this means that time needs to be invested in exploring the context and history of bias within math, then it is time well spent. To clarify, the math itself isn't racist. What we choose to define as math, what we choose to teach as math, who has been included and identified as mathematicians, and how we do the teaching is where we see racism and Eurocentric bias. And if there are some people out there who argue that math has nothing to do with racism and that these discussions should not take place within a math classroom, then it is important to remember that for years, there were many other things that were not discussed in any classes. Colonization and its impact on our Indigenous communities was not discussed until relatively recently. Just because it wasn't discussed does not mean that the atrocities that we have learned about did not happen. They did. We just weren't ready to listen. It would be naïve to believe that mathematics has only been used to promote justice and equity. However, believing teachers can teach mathematics while promoting justice and equity is not naïve; it is what educators should be aspiring to.

"Education is the most powerful weapon we can use to change the world." ~ Nelson Mandela

"Those who pursue justice in mathematics cultivate the

virtues of *empathy for the marginalized* and *concern for the oppressed*. We sometimes don't recognize the constant burden of oppression that is experienced by the powerless until we start opening our eyes to see what they see. Those of us with power need to assist those who have no power." ~ Francis Su (Su, 2020)

References

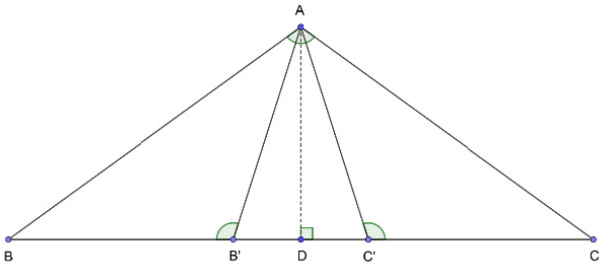
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Su, F. (2020). *Mathematics for human flourishing.* Yale University Press.▲

Thabit ibn Qurra's Generalization of the Pythagorean Theorem

Using similar triangles, a classroom task is to prove:



Then, when the common angles *BAC*, *AB'B*, and *AC'C* are 90º, this becomes the Pythagorean theorem.

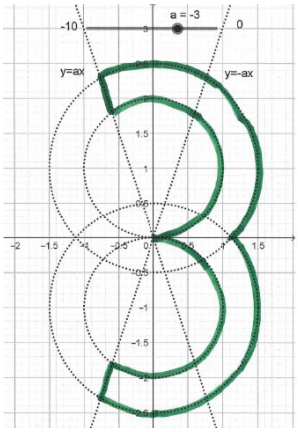
(From: *The Crest of the Peacock: Non-European Roots of Mathematics*, by George Joseph, pp. 492–493)

Make Your Own Font?

What are the equations used to create the value of "3" in this font?

How does the "*a*" slider affect the appearance? What range of "*a*" values is reasonable?

What is the domain and range for each equation?



▲ Useful Theories to Know About: The Four-Phase Interest Model



JEFF IRVINE
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Jeff Irvine, Ph.D., has been a secondary mathematics teacher, department head, and vice-principal. He has taught at three faculties of education and at a community college. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. With over 50 years in education, Jeff is particularly interested in the interplay of student motivation and mathematics achievement.

Welcome to my new column. This column will describe theories that relate to teaching mathematics, and more importantly, why practising math teachers might want to know about them.

We all have personal beliefs about how mathematics should best be taught. Some feel that drill and practice are keys to succeeding in mathematics. Others believe that problem solving or investigations are most important. Some math teachers believe students should work in small groups to develop their mathematical understanding. Others believe that learning mathematics is mainly an individual pursuit.

The big difference between a belief and a theory is research evidence. Theories start out as beliefs; however, theories are supported by research. The more research evidence that supports a theory, the better the theory is likely to represent a useful way to think about teaching mathematics.

Some of the theories described in this column are directly about mathematics teaching. Other theories apply more broadly to teaching many subjects, but are important for math teachers to be familiar with.

The column will take this form: a description of the theory; why math teachers can use the theory in their professional practice; additional resources in case you want to learn more about the theory.

This first column describes the Four-Phase Interest Model, proposed by Suzanne Hidi (Ontario Institute for Studies in Education) and Ann Renninger (Swarthmore

College).

The Four-Phase Interest Model

The four-phase model of interest development (Hidi & Reninger, 2006) is shown in Figure 1. The theory states that initial interest is triggered by a situation or topic (Triggered Situational Interest). This interest may be fleeting, and may be positive or negative. If interest in the situation becomes more sustained (Maintained Situational Interest), there will be positive student focus and persistence with the material. If the student develops Emerging Individual Interest, they are likely to independently re-engage with the material or classes and ask curiosity questions, building their knowledge about the material. Finally, at the Well-Developed Individual Interest stage, the student willingly re-engages with the content, self-regulates to reframe questions, and seeks answers. At this level, the student has positive feelings toward the material, persists through frustration and challenges, and actively seeks feedback on their learning. The four-phase model has abundant research evidence supporting it.

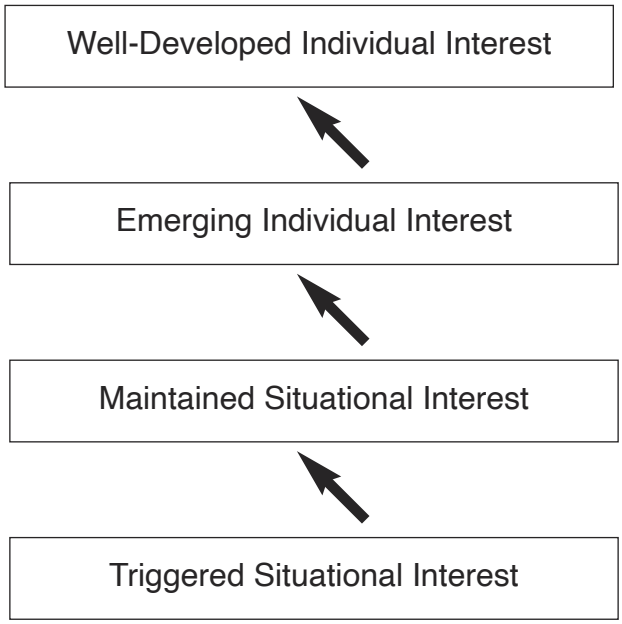


Figure 1: *The Four-Phase Interest Model*

Why Should We (Math Teachers) Think about This Theory?

There is much evidence that when students are interested in what we are teaching, they are more engaged. More engaged students tend to persist when work becomes difficult, and more persistent students tend to achieve at higher levels.

When first introducing a topic in mathematics, we want to trigger situational interest in as many students as possible. Then we need to find ways to get our students to the sustained situational interest level. The longer students remain interested and engaged, the more likely they are to be successful with the material.

Interest is a tricky concept, since it is generally very personal. Some of our students are interested in sports, others in fashion, cars, social issues, magic tricks, art, music, and a host of other topics. We can also find interesting topics within mathematics, such as number patterns like Fibonacci sequences and Pascal's triangle, or the golden ratio. We are unlikely to be able to find topics that interest all of our students all of the time, but finding a variety of topics, such as real-world applications of mathematics, helps.

A few students may reach the emerging individual interest stage with some topics. These students will independently want to learn more and become more deeply engaged with the topic. As teachers, we need to support these students on their learning journey. That doesn't mean that we need to know everything there is to know about a topic, but offering pathways for these students to grow their own knowledge may be sufficient. Finally, a select few students will reach the well-developed individual interest stage. These students are a gold mine for teachers, since the students will typically know far more about a topic than their teachers. They can be a resource to encourage other students to grow their own interest levels.

Knowing about the four-phase interest model lets math teachers recognize what their students care about, and lets us support and encourage our students as they engage and learn about mathematics and how it relates to our students' worlds.

Want to Know More?

The original research article:

- Hidi, S, & Renninger, K.A. (2006). The four-phase model of interest development. *Educational Psychologist, 41*(2), 111–127. www.dx.doi:10.1207/s15326985ep4102_4K

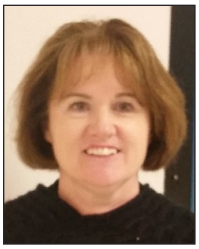
The following has over 14 studies looking at how to generate interest in math and society:

- Renninger, K.A., Nieswandt, M., & Hidi, S. (Eds.). (2015). *Interest in mathematics and science learning.* American Educational Research Association. www.dx.doi:10.3102/978-0-935302-42-4

Lastly, a video that addresses how to move from interest to passion:

- Anonymous. (2023, April). *Short story: How is passion created? (The four-phase model of interest development)* [Video]. YouTube. www.youtube.com/watch?v=HEfb4u3U1uk ▲

▲ OAME/NCTM Report: Filling Your Cup! Self-Care for Teachers by Teachers!



JACQUELINE HILL
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Jacqueline Hill is a Grades 9–12 teacher of mathematics. She is a Past President of OAME and OMCA, as well as the recipient of the Award for Exceptional Teaching in Secondary Mathematics. She also wrote the "Director's Dialogue" for the Gazette *for a number of years.*

Last year, my columns revolved around embracing change and growing. The June issue was about encouraging those young and seasoned in our profession to find the good in everyone and share it!

This year, we embark on the year of YOU! We will start by looking into how NCTM, OAME/AOEM, and the individual strengths of each other can generate some ideas to help you be the best teacher that you can be. I believe it is an ancient saying that "you can't give from an empty cup"; that is, in modern terms, you have to make sure to take care of yourself before reaching out to aid others. Take, for instance, the idea of oxygen masks in an airplane. If the oxygen masks are needed, then you are instructed to put yours on first, then come to the aid of others. The same is true for your well-being, and the well-being of others in your building.

Top 5 List: Suggestions for Self-Care

#5 – Identify the communities that make you feel good. Is there a walking club at school, or a yoga class, or a mindful meditation group you can join? Is there a local OAME/AOEM chapter that holds events or meetings in which you would like to participate? Gravitate to the communities that bring you joy. Stay clear of those that don't; fill your cup with joy!

#4 – If you do a search on the nctm.org website, there are 24 relevant articles on self-care. One such article talks about filling your gratitude cup. Each day, start and end your day with something for which you are grateful.

#3 – Set (and say) your intention for the day. In the morning, I would look in the mirror and state "Today I am going to walk during a portion of my lunch hour," or "Even though I will be very tired at the end of the day full of students, I will go to yoga with my colleagues."

#2 – Do some 3–4–5 breathing through your nose when you can. In for 3, hold for 4, and out for 5. It may even work best when some sweet child throws a zinger your way, and you need time for them to reflect on what they have said before you reply. In through the nose, out through the nose is the way normal breathing should occur. Your nose is for breathing, your mouth is for eating.

#1 – Know your limits and play within them! That means saying "no" to commitments that make your plate too full to balance. A phrase I make all new teachers practise is, "Thank you for thinking of me! But my plate is very full at the moment." This phrase honours the person who is asking, but also lets them know that you can't take on more right now.

Consider ways you can look after yourself as the new school year gets underway. That will make you better able to help those around you, including your students, colleagues, and your family.

NCTM 2024 Regional Conference & Exposition

The NCTM 2024 Regional Conference & Exposition, which will take place February 7–9 in Seattle, Washington, is open for submissions. NCTM regional conferences and expositions give thousands of your peers in math education the chance to meet face-to-face and focus on developing skills to help their students.

There are two types of individual memberships into NCTM.

A $99 per year essential membership allows you entrance into an online member community, provides you with a journal, access to members-only classroom resources, and a 20 percent off discount at the NCTM store/event. A premium membership provides access to more journals and 30 percent off.

Until next time!
Yours in mathematical fun,
Jacqueline ▲

Congratulations

Dr. Jeff Irvine
On reaching 50 years as an educator!

▲ OMCA News: September 2023



HEATHER THEIJSMEIJER
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Heather is a Past President and an active member of OMCA.

Happy New Year! The Ontario Mathematics Coordinators Association (OMCA) welcomes math educators back for another school year. We are looking forward to another great year of learning with mathematics coordinators, consultants, and coaches from all across Ontario.

This year's recipients of the 2023 OMCA Award for Excellence and Innovation in Mathematics Teaching are Melissa Peddie (Near North District School Board), and Rosanna Mortillaro (Toronto Catholic District School Board). This award is given annually to OMCA members, based on nominations by their peers, in recognition of excellence and innovation related to any areas of mathematics education (e.g., integration of technology, equity, implementation of curriculum, motivation of teachers, assessment and evaluation, professional development ideas). Congratulations, Rosanna and Melissa!

Recent OMCA meetings have included guest speakers Dr. Thomas Guskey on "Elements of Strong Professional Development"; Dr. Molade Osibodu on "Authoring New Stories: Embracing Fantasy and Fiction in Mathematics Education"; and Liv Rondeau on "The Legacy We Leave: Teaching for Future Generations." Our last meeting of the school year in June gave members a chance to network and engage in open conversation through a digital EdCamp format. Some of the topics up for discussion included supporting equitable assessment and evaluation practices, improving instruction using AI, diagnostics and screeners in math, and digital math tools. As always, it was great to hear perspectives from across the province.

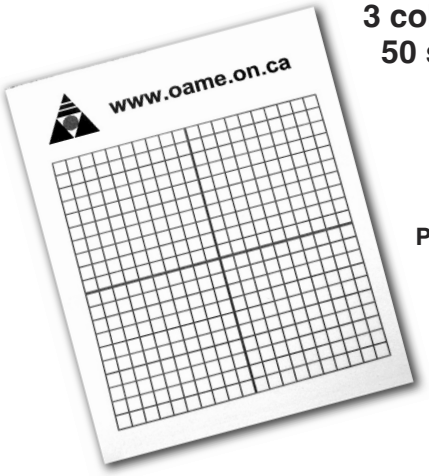
At the final members' meeting of the year on Monday, June 12, outgoing President, Jason To (Toronto District School Board) surprised the membership with a poetic tribute to the year, and passed the gavel to incoming President, Jackie Decker (Limestone District School Board). OMCA is excited to welcome our new Executive for the Fall: Jason To (Past President), Jackie Decker (President), Gerry Lewis (Toronto Catholic District School Board, Vice-President), Allison Lawrence (Brand Haldimond Norfolk Catholic District School Board, Secretary), and Mariarosa Schniering (York Catholic District School Board, Treasurer).

Sadly, OMCA is bidding adieu to two leaders within the organization. Marilu Deal (Ottawa-Carleton District School Board) joined the Executive as Treasurer in 2021, and has been an integral part of the organization as the association rapidly grew in membership through the pandemic. She will be returning to the classroom in September. Greg Clarke, one of our technical wizards, is also stepping away to focus on family. Greg's online presence was indispensable to remote members when meetings were hybrid, and he helped make the association's transition to virtual meetings seamless. Best of luck to both Greg and Marilu in their new adventures!

Are you a math leader or math coach in your board? Interested in joining OMCA? New members are always welcome. Contact us directly at OMCAmath@gmail.com, or visit our website at www.omca.website (note the different domain), to learn more about the organization and connect with us. Our first meeting of the 2023–2024 school year will be Monday, September 18. We hope to see you there! ▲

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Meaning Making in the Classroom: Making Sense of Mathematical Modelling (Part Two)



DAVID COSTELLO
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David is a principal, who taught in the Primary division before assuming roles of numeracy interventionist, numeracy coach, numeracy leader, and curriculum consultant for Prince Edward Island. David has also instructed university courses regarding curriculum, differentiation, mathematics, and language arts. He facilitates professional learning in the areas of mathematics and school development. Dr. David Costello recently published three books: Mathematizing Student Thinking: Connecting Problem Solving to Everyday Life, Building Capable and Confident Math Learners, Making Math Stick: Classroom Strategies That Support the Long-Term Retention of Math Concepts, *and* Using What Works: Strategies for Developing a Literacy-Rich Environment in Math.

In part one of this two-part column, I explored mathematical modelling from the perspective of student thinking. I was cognizant to highlight each of the four components of the mathematical modelling process and share examples of student thinking for each component.

The previous column was meant to situate students as independent learners who must take an active stance in their learning. Students engage in mathematical modelling by making sense of an assigned ill-defined problem, using their lived experiences as points of reference, and making the problem workable by applying assumptions and restrictions. It is from this work that students then apply a mathematical lens to work through the problem to a solution that is meaningful. Within this process, students are confident and capable math learners.

I have been asked by educators what role the teacher plays in mathematical modelling in the classroom. It was my intent to focus on the role of students first so that educators can see the end goal—students engaged in a process that makes sense of problems they encounter in their lives, and applying a process that can lead to a solution that is both meaningful to the problem and to the students' lived experiences. Once teachers have the perspective of students as confident and capable math learners, it becomes important to take a step back and examine the role of teacher as facilitator.

Similar to the previous column, I will delve into the four components of mathematical modelling and share examples of how a teacher facilitated student learning during that particular component. While the student takes the lead in their learning during mathematical modelling, the teacher does play a significant role as a facilitator. And, to be an effective facilitator, the teacher must have specific skills in supporting students as they engage in mathematical modelling (Costello, 2022).

Think about your classroom and your facilitation skills as you read through this column. There will have been times when you were leading the conversation, times when you were walking alongside your students, and times when you were following their lead. It is not the teacher's role to dispense knowledge during mathematical modelling, but instead, to support the independence of students as they make meaning of the ill-defined problem throughout the process.

Teachers and Mathematical Modelling?

Mathematical modelling provides students the opportunity to engage in ill-defined problems—messy problems that are similar to problems they encounter in their experiences outside of school. Consider the comments in Figure 1, made by teachers when asked to define their role as teacher during mathematical modelling (Costello, 2022).

Primary Example: A Teacher Explains

Mathematical modelling can be tricky in Grade 1 through Grade 3. I don't know how to support my students in starting the work without being prescriptive. If I am prescriptive, doesn't that negate the whole purpose of mathematical modelling?

Elementary Example: A Teacher Explains

Finding the right question is an important part of math modelling. The problem is finding such questions. My math program doesn't really do much with mathematical modelling, and I don't feel that I can craft such questions. So, without having the questions, I really haven't been doing much mathematical modelling in the classroom.

Intermediate Example: A Teacher Explains

I appreciate that mathematical modelling will provide my students with opportunities to see the relevance in math, and how math can be used in their lives outside of school. However, I struggle with knowing how to best bring mathematical modelling into the classroom. Students are supposed to be independent and creative when working through the problem, but how do I support this when I'm not sure where they are coming from or whether their approach to solving the problem is a great way to do it.

Figure 1: *Comments made by teachers about mathematical modelling*

As can be taken from the three comments in Figure 1, there is a level of uncertainty from teachers when considering their role during mathematical modelling.

Therefore, it is important to pause and consider subtle moves that teachers can make in the classroom to facilitate the mathematical modelling process. Whether you are a novice or an expert teacher, I am hopeful that you can identify some "next steps" in this column when working with students in the classroom.

The Four Components of Mathematical Modelling

The four components of mathematical modelling are as follows (Ontario Ministry of Education [OME], 2020):

1. Understanding the problem
2. Analyzing the situation
3. Creating a mathematical model
4. Analyzing and assessing the model

Although I address the four components of mathematical modelling in isolation, it is important to note that mathematical modelling is an iterative process. In addressing the components, I want to develop a common thread. To meet this objective, I will use the following ill-defined problem for all four components (Costello, 2022). This is the same problem as I highlighted in my last column. I decided to use this same problem so that the reader could see the relationship between the teacher moves and student thinking.

> *In planning end-of-year activities, the school administration decides that the Grade 8 students can go on a field trip. The administration wants to know what the cost per student will be for such a trip.*

This problem and the examples of facilitation that teachers can apply are from a Grade 8 classroom.

Before moving on, I want to pause and emphasize the importance of teachers taking time to anticipate the possible responses of students to their questions. If teachers can anticipate possible responses, they will be better prepared to provide a higher-quality reply and avoid possibly giving hints or providing next steps to the student.

Understanding the Problem

While understanding the problem, students make sense of the problem that they are assigned. It is necessary that students take time to understand the ill-defined problem. When students ask the teacher to help them in making sense of the problem, it is important that teachers facilitate the exchange so that it is students making sense of the problem, as opposed to the teacher telling them.

Consider some of the following questions that the teacher could ask students during this component of mathematical modelling (Butler Wolf, 2015):

- Have you read the whole situation?
- What is the problem about?
- What are you supposed to find out?
- Can you tell me what the problem is about in your own words?
- Can you draw a picture of the problem?
- Can you represent the problem, using concrete objects?
- Have you seen another problem like this one?
- Does this problem remind you of anything you have experienced?

Figure 2 highlights questions that can be asked by the teacher to support students in understanding the problem (Costello, 2022).

Teacher Facilitation Questions

- What is the problem about?
- What are you supposed to find out?
- Can you rephrase the problem in your own words?
- What experiences can you bring to understanding the problem?

Figure 2: *Questions to facilitate the understanding component*

Analyzing the Situation

Within this component, students make assumptions and place restrictions on the problem as a way to provide structure to the messy, ill-defined problem. As many students have spent much of their time in math being assigned defined problems, students may struggle with this particular component of mathematical modelling. They may have few or limited experiences in crafting assumptions and placing restrictions on problems. As such, teachers can support students in this work by asking questions (Costello, 2022). The key is that the questions are of a facilitative nature, as opposed to being directive. We still want students to be independent learners, who are not reliant on the teacher to direct them in next steps.

Figure 3 highlights the questions asked by the teacher when supporting students in analyzing the situation (Costello, 2022).

Teacher Facilitation Questions

- Have you ever been on a field trip?
- What type of field trip would you be interested in taking?
- What happens if students have to pay for all the expenses on the trip?
- When you were working through understanding the real-world problem, you identified information you need to know.
- Have you made any decisions regarding that missing information?

Figure 3: *Questions to facilitate analyzing the situation*

Creating a Mathematical Model

It is during this component of mathematical modelling that students approach the problem from a mathematical perspective. Students will decide on the mathematical language and concepts to apply, and the method of applying such mathematics. If students struggle in selecting a mathematical model, the teacher can assist the student by asking questions that will move students toward independent thinking. Consider the following questions that teachers may ask students in such a case (Costello, 2022):

- What pictures, diagrams, or graphs might help people understand your information, model, and results?
- How can you organize your data to share with an individual unfamiliar with your project?

These two examples of questions asked by the teacher serve the purpose of facilitation. The questions are not directive, but are meant to place the productive struggle on students so that they are working through the problem. In addition to this, these two questions can also assist students in thinking of alternative mathematical concepts to use as the framework of the mathematical model.

Figure 4 highlights the questions asked by the teacher when supporting students in creating a mathematical model (Costello, 2022).

Teacher Facilitation Questions

- What is involved in planning a trip?
- What math concepts would help you finalize plans for the trip?
- How would you organize your data?

Figure 4: *Questions to facilitate the modelling component*

Analyzing and Assessing the Model

In this component of mathematical modelling, students examine their work against the assigned ill-defined problem. Within this examination, students consider their work from the contextualized perspective of the problem. While we may think that students are well versed in analyzing and assessing their work, often much of their experience has been with defined problems. As such, students may view this particular aspect of mathematical modelling as a time to double-check the math in their work. This is a simplistic way to approach this component.

The teacher needs to provide an opportunity for students to talk with classmates, question their thinking, and share their thoughts related to their work and the problem. This is not a superficial task. Instead, there is much thinking that goes into this particular component. It is a reflective experience, whereby students review the many steps and choices that they have taken throughout the entire mathematical modelling process and communicate the thinking behind this process.

Consider the following questions that teachers could use to support students in analyzing and assessing their model (Costello, 2022):

- Will your created model generate approximate or precise solutions? Does it matter?
- In what types of contexts would your model provide important information?
- If you change the assumptions you made to the real-world problem, how much would your mathematical model and solution change?

Figure 5 highlights the questions asked by the teacher when supporting students in analyzing and assessing their model (Costello, 2022).

Teacher Facilitation Questions

- Do the results make sense to the real-world problem?
- Would it improve the model to change something decided at an earlier step and resolve the problem?

Figure 5: *Questions to facilitate appraisal of the model*

Final Thoughts

The intent of this column was to deepen our understanding of mathematical modelling, and strengthen our understanding of the role teachers play in facilitating mathematical modelling in the classroom. It is important to examine this facilitation within each of the four components of mathematical modelling so that we, as educators, can see the influence teachers have when facilitating the process. Understanding our own facilitation helps us to support students to become active, independent learners.

In my next column, I will continue our exploration of mathematical modelling. This time, however, we will examine the process from the perspective of social–emotional learning. As many educators are aware, social–emotional learning is included in a strand within the revised math curriculum in Ontario (OME, 2020). It is important to take time to recognize and understand how social–emotional learning is not an add-on, but is embedded in the learning process.

I look forward to continuing the discussion of meaning making in the classroom via my website (www.costellomath.com), email (david@costellomath.com), or Twitter (@dr_costello).

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▲ In the Middle: What Fun Can Be Found in a Book?



CARLY ZINIUK
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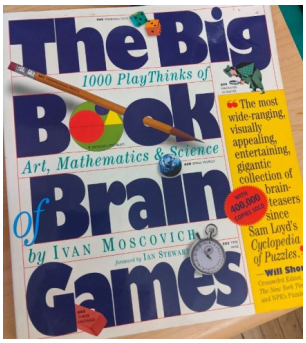
Carly Ziniuk teaches Grade 9 Mathematics, Grade 12 Data Management, and Advanced Placement Statistics at the Bishop Strachan School in Toronto. She is very active in adopting real-life data to engage her students in solving problems. Among 30 global STEM leaders, Carly is excited to have been selected for the NASA Space Apps Collective for 2023–2024. As a Canadian educator, she will be contributing toward the pilot program's chosen theme: Building an Inclusive Global Space Community.

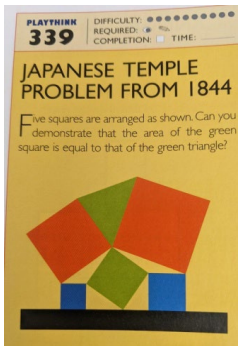
A friend recently asked me to suggest some tech-free "mathy" fun for him and his children to do over the summer. I quickly showed him some classic pencil-and-paper games like *dots and boxes, chomp,* and *nim* (simple instructions for these can be found at UCLA Professor Tom S. Ferguson's site, www.math.ucla.edu/~tom/Games/). I later remembered playing John Conway's *sprouts* with my now-adult children on a train vacation for what seemed like hours, a game that I had learned from Martin Gardner. Gardner's mom was a teacher, and his interest in mathematics started as a boy reading books about puzzles and magic. Martin Gardner's *Scientific American* column (running from 1957–1981) often is considered the greatest popularizer of recreational mathematics, with games like *cram*, creations like *hexaflexagons*, and puzzles like *fork in the road* (truth tellers and liars), mathematically inspiring students and teachers of all ages.

Many of my favourite recreational mathematics books directly expand upon Gardner's work, and this column highlights those that are appropriate for middle school learners. For each book, I have summarized a few activities that I have used with my classes or in my one-week math camps. Liljedahl's *Building Thinking Classrooms* recommends starting classes with oral non-curricular problems like these as a way to get students moving quickly to discuss what is being asked and increasing collaborative engagement. The activities chosen from these books can be quickly explained orally or demonstrated with brief instructions from the teacher and minimal (non-commercial) materials like pencils and graph paper.

The Big Book of Brain Games: 1,000 PlayThinks of Art, Mathematics & Science by Ivan Moscovich, Workman Publishing Company, 2006

Moscovich's heavy tome has 1000 engaging "playthinks" with difficulty level and required materials noted, all brightly coloured. Many of them, like this *Japanese temple* problem from 1844, are curricular and can be used directly in class with solutions that show direct algebraic solutions, but then also show playful, creative opportunities. These two activities are also curricular examples which I have used with younger students to discuss counting the entire sample space (with #48) and symmetry (with #119). Both are highly engaging tasks that promote discussion, as Liljedahl suggests, and problem solving within the first five minutes.





Sample Activity: #48 Murphy's Law of Socks (p. 19)

Imagine that after washing five pairs of socks, two socks are missing. Which is the more likely situation?

- The two missing socks make a complete pair and you are left with four complete pairs.
- You are now left with three pairs of socks and two orphan socks.

SOLUTION: The first situation happens only 5 ways. The two missing socks being unmatched can happen in 40 different ways (consider socks A_1, A_2, B_1, B_2, C_1, C_2, D_1, D_2, E_1, E_2… and write out all the unmatched pairs, the "sample space"). Unmatched occurs 8 times more than matched.

Sample Activity: #119 Symmetry Alphabet (p. 49)

Which capital letters of the alphabet have no symmetry?

(Hint: Check vertically? Horizontally? Rotationally?)

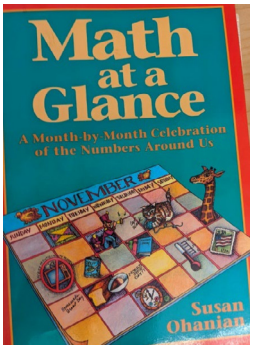
SOLUTIONS: Letters that are only symmetrical vertically are A, M, T, U, V, W, Y.

Letters that are only symmetrical horizontally are B, C, D, E, K.

Letters that are symmetrical both horizontally and vertically are H, I, O, X.

Letters that are only symmetrical rotationally are N, S, Z.

This leaves only F, G, J, L, P, Q, and R as having no symmetry.



Math at a Glance, A Month-by-Month Celebration of the Numbers Around Us by Susan Ohanian, Heinemann, 1995

Ohanian describes her book as "a series of provocative (occasionally zany) math facts and anecdotes, arranged chronologically by date, that will inspire exciting mathematics investigations throughout the year." During online learning, I found it helpful to have something weird to be waiting on the screen while the students logged in, and this has persisted to my in-person class while I wait for them to arrive. Sometimes, I connect the seemingly random facts to the lessons at hand. These activities help students see the mathematics they have learned as directly relevant to their daily lives, or as the two sample questions will demonstrate—tasty and amusing.

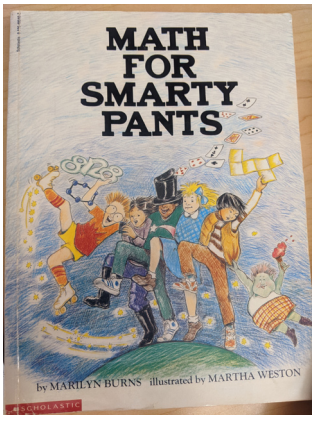
Sample Activity: July 11 World Record Chocolate Chip Cookie (p. 64)

On this date in 1992, a record-size cookie from Ripon, Wisconsin had an area of 907.9 square feet. If its diameter was 34 feet, how many chocolate chips do you think they needed for this tasty treat?

Sample Activity: January 2 Melting Chocolate (p. 3)

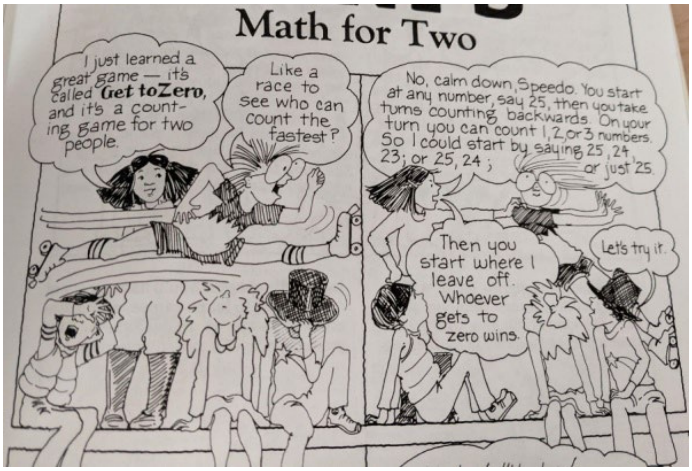
On this date in 1980, the Blommer Chocolate Company in Chicago caught fire. The fire caused many 100-pound bags of chocolate to melt, creating lakes of chocolate. One of these lakes was reported to be 30 feet across and 3 feet deep. How many pieces of chocolate do you think created that lake?

Using these facts together: How big would the lake created by the melting chocolate from Ripon World Record Cookie be?



Math for Smarty Pants by Marilyn Burns, Scholastic, 1982

As a book I used to pore over in my small elementary school library as a child, this Marilyn Burns classic has a special place in my heart. I loved the cartoon illustrations, the sense of whimsy about mathematics, and thinking about myself as someone who could actually do math. The two sample activities I have chosen aptly come from the chapter titled "Math for Two." The second sample, Burns' *dollar words* activity, has many versions, including online calculators, and is often chosen by teachers as a *100 Days of School* activity.



Sample Activity: The 31 Game (p. 64)

From a deck of cards, collect the cards from Ace through 6 for each of the four suits, and lay the 24 cards face up. The first player turns any card face down and says that number out loud. The second player turns over another card and adds their new number to the old number. Continue taking turns, keeping a running total. Whoever reaches the sum of 31 first wins!

Sample Activity: Dollar Words (p. 15)

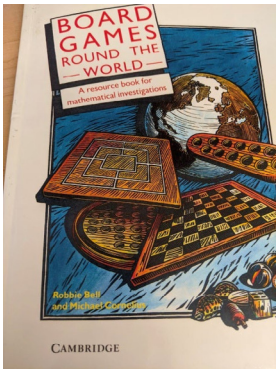
Assign a cent value to each letter in the alphabet (A =1, B = 2, etc.). The object of the "game" is to find words whose value is exactly $1.00.

One variation that enriches the activity for middle school students is showing them these dictionary definitions of mathematically themed dollar words and having them figure out what word has been defined. Once you have collected a series of these dollar words, examine them statistically to consider what they have in common (length? number of vowels? etc.).

- Someone who uses a ruler or a thermometer ~ *Measurer*
- The act of finding the biggest appropriate value ~ *Maximize*
- The ability to use and work with numbers ~ *Numeracy*
- Limited in scope or nature ~ *Finitely*
- A four-sided plane figure, especially a square or rectangle ~ *Quadrangle*
- A solid having four plane triangular faces; a triangular pyramid ~ *Tetrahedra*
- The state or fact of something's being likely; probability ~ *Likelihood*
- A structure of hexagonal cells of wax, made by bees to

store honey and eggs ~ *Honeycomb*

- In a curve corresponding in direction to the typical forward movement of the hands of a clock ~ *Clockwise*
- The middle point of a line segment ~ Midpoint
- The quantity obtained by dividing the number one by a given quantity ~ *Reciprocal*
- The inquiries starting from given conditions to investigate or demonstrate a fact ~ *Problems*



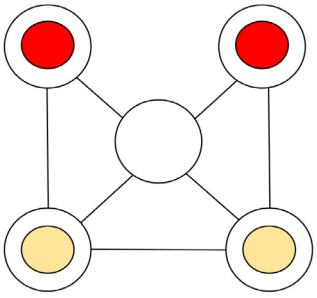
Board Games Around the World: A Resource Book for Mathematical Investigations by Robbie Bell and Michael Cornelius, Cambridge University Press, 1995

I have used this book in the Data Management course with the probability of Mancala, and in the Grade 8 course, practising fractions and percentages with dice. Bell and Cornelius include images of Chinese filigree dicing cups from the eighteenth century, and astragali "dice," sheep bones from the third century CE Egyptian tombs. The Pong Hau K'i game below can be done with stones or marbles outside, but also can be drawn.

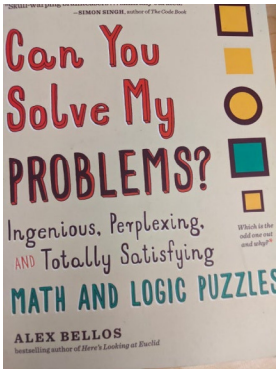
Sample Activity: Pong Hau K'i (p. 98)

This traditional Chinese two-player game has variations in Korea (Umul Gonu), Thailand (Sua tong tok), and India (Do-guti).

Each player has two stones of different colours, placed as in the diagram below. Alternately players move one stone to an adjacent empty point, with the intention of blocking the other player's stone.



Does matter who goes first or second? Does it matter which colour you choose? Can you force your partner to lose (and thus you win)?



Can You Solve My Problems?: Ingenious, Perplexing, and Totally Satisfying Math and Logic Puzzles by Alex Bellos, The Experiment Publishing, 2017

I love to start my mathematical week with Alex Bellos' Monday Puzzle, "*Can You Solve It?*," in the *Guardian*. Bellos has published three book collections of these puzzles, with this book being my personal favourite. I have argued about these problems with students of all ages. They can be proved with formal mathematics (as below), but also reasoned fully.

Sample Activity: #72 A Sticky Problem (p. 122)

You have a stick and you saw it in two. If the cut is made randomly anywhere along the stick, what is the length, on average, of the smaller part?

SOLUTION: Randomness means that any point on the stick has an equal chance of being chosen. If the stick is held horizontally, half the time, the point will fall on the left half of the stick, and half the time, on the right half. If it falls directly in the middle, there is no smaller part. All of the points on the smaller side have an equal chance of being chosen as the cutting point. The length of the smaller piece on the stick, on average, will be half the length of the half stick, a quarter of the whole stick.

Sample Activity: #86 Coins in the Dark (p. 144)

A blindfolded magician asks the audience to put ten coins on the table and tell you how many of them are showing heads. If the magician truly cannot see the coins, nor tell which way up they are by touching them, how can they separate the coins into two groups, such that each group has the same number of heads?

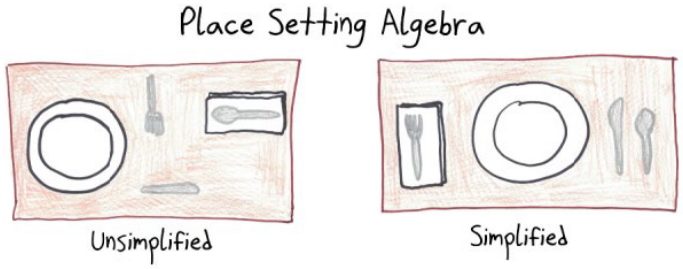
Hint: First try to figure out this problem without a blindfold. Get ten coins and lay them on a table. For any different number of heads, try to separate the coins into two groups, such that each group has the same number of heads. Use this to see if you can generalize your decision making that would work while blindfolded.

SOLUTION: If a member of the audience tells the magician that there are x *heads, then the magician should select any* x *coins in one group and flip them over (the other group will have 10 –* x *coins). You can show this works by demonstrating it for all integer values of* x*, such that* $0 \leq x \leq 10$ *(by exhaustion), or you can prove it by algebra. Interestingly this "trick" happens for any number of coins, not*

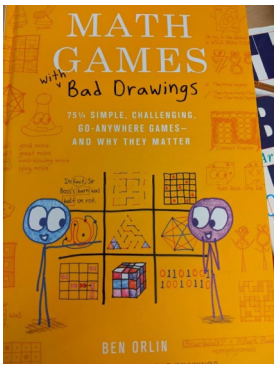
just ten. If you know the total number of heads, you can separate the coins into two groups that contain an equal number of heads by selecting a number of coins that equals the number of heads and flipping them.

Math Games with Bad Drawings: 75 ¼ Simple, Challenging, Go-Anywhere Games—And Why They Matter by Ben Orlin, Black Dog & Leventhal Publishers, 2022

Orlin's book has been an amusing favourite of many of my middle school students. We regularly review this visual reminder of the importance of algebraic simplification.



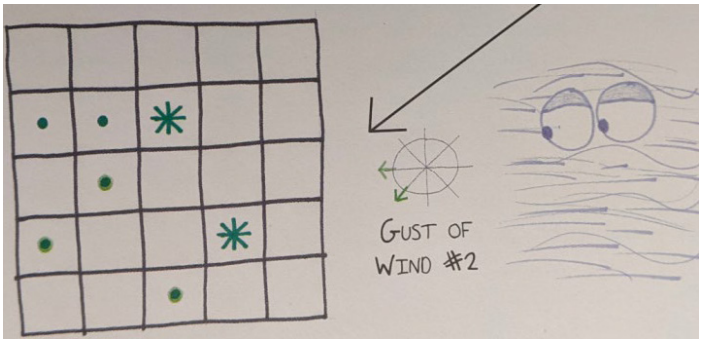
This new book includes classics like the aforementioned *sprouts* and *dots and boxes*, with charming variations and connections to mathematical history and simple rules. Two new games that I have been playing with anyone who will try to beat me, that are explained below, are *dandelions*, a paper-and-pencil spatial turn-taking game, and *undercut*, a physical strategy game akin to *pig*.



Sample Activity: *Dandelions* (p. 55)

This is a pencil-and-paper two-player game, akin to the *sprouts* and *dots and boxes* games. The "Dandelion" (Player 1) endeavours to cover the whole meadow (a 5x5 graph-paper square) with flowers (asterisks). The "Wind" (Player 2) endeavours to leave at least one square of the meadow uncovered from the Dandelions. The Wind draws a compass rose indicating eight directions North (N), North-East (NE), East (E), South-East (SE), South (S), South-West (SW), West (W), and North-West (NW).

The Dandelion first places a flower (asterisk) anywhere on the grid. The Wind then chooses a direction in which to blow the seeds. Any empty square downwind of the Dandelion is now occupied by a seed (a dot). During the game, the Wind blows *exactly once* in each of the eight directions on the compass rose. After each blow, the Wind should mark it off on the compass rose as used.



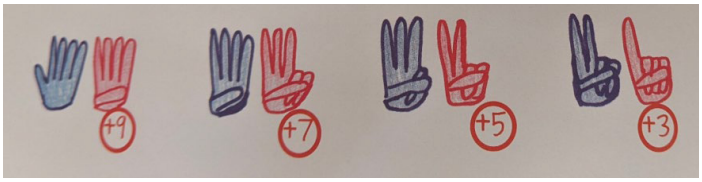
Continue taking turns. A Dandelion is planted on either an empty square or on top of an existing seed, and then the Wind blows in a new direction, carrying seeds of all Dandelions on the board and planting them on downwind squares. Seeds emerge from all Dandelions. The game ends after seven turns, leaving one direction unused. Dandelions win if every square is covered, while the Wind wins if any blank square remains.

Sample Activity: Undercut (p. 213)

Each five-fingered player thinks of a number from 1 to 5, and then on a count of three, reveals it, earning one point per finger, *except* if the two players' numbers differ by exactly one.



If that happens, then the larger numbered player has been undercut, and the smaller numbered player wins all of the points.



The game continues until one player is ahead by 11 or more points.

Although Martin Gardner was not a mathematician or teacher by training, these books are wonderful opportunities for our classrooms. I hope these activities have inspired you to read a book, and like my friend, play a game or two with some students! ▲

MATH: ***M**istakes **A**llow **T**hinking to **H**appen*
Dr. Patricia Dickenson

▲ MB4T (MATHEMATICS BY AND FOR TEACHERS): THE WONDERS OF THE AREA MODEL



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Jennifer Holm is an Associate Professor at Wilfrid Laurier University and works with Primary/Junior and Junior/Intermediate pre-service teachers, as well as in the field supporting current mathematics teachers. She is interested in developing mathematics knowledge for teaching with both pre-service and in-service teachers. She focuses on the beliefs and opinions that pre-service teachers hold about mathematics and teaching and the connection they have to past experiences. She uses this research to support future teachers in developing beliefs and knowledge that will encourage and support effective teaching practices in mathematics.

This month, the focus is on a single model that can be used right from the start of the concept of multiplication, with single-digit numbers, right to a discussion of algebra. I am going to develop the model and move it through stages that support students in a conceptual understanding of multiplication, which can also lead to an understanding of the area formula for rectangular shapes! Let me also note that all images used Mathigon polypad manipulatives to create the models (www.mathigon.org/polypad).

As an elementary school teacher, I tended to focus on multiplication interpreted as "groups of," since it seemed to be the easiest way for students to grasp the concept (e.g., 6 × 3 is six groups of three and can be shown in this way). This definition breaks down when you start working on more complex concepts. Think for a minute about $(x+2)(x+3)$ or $\frac{1}{4} \times \frac{3}{4}$. How would you make $x + 2$ groups of $x + 3$ or $\frac{1}{4}$ groups of $\frac{3}{4}$? Using this definition of multiplication exclusively, requires teaching something new for multiplication as number (and algebra) concepts get more elaborate, so students start to think that multiplication changes as the concepts do.



Figure 1: Left shows 6 × 3 modelled as an array. Right shows 6 × 3 in an area model or rectangle shape.

This does not mean that "groups of" should not be taught; however, it should not be the *only* definition of multiplication discussed with elementary children. Let's think about how we might bring in a different way to visualize multiplication that is transferable to other situations/concepts as students mature mathematically. My favourite way to support children developing math concepts is through the use of contexts. Equal groupings is a way to set up word problems or contexts, or we could think about multiplication as "rows of" to help push another model. In this case, we may give word problems about creating gardens with six flowers in each of three rows.

Although students may use the model for the left image in Figure 1 to represent the problem in an array format, as a teacher, we could work toward showing the right image in Figure 1 to represent it in a rectangle. Using square tiles makes this transition easier than circular counters because square tiles naturally align to make a rectangular array. I have tried two methods to shift this thinking: either drawing the rectangle as they explain their model, or using the manipulatives in the array to push them together to create the rectangle. This helps students start to make observations about how multiplication relates to finding the area of a rectangle. The bonus is that with repeated practice, they do not need to worry about a formula, since they will understand that knowing the two side lengths is all that is needed to find the area!

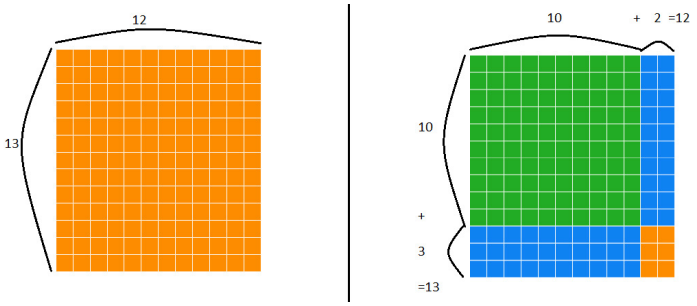


Figure 2: The left shows an area model that is 13 × 12. The area model on the right shows decomposing 12 and 13 to make an area model.

This model is also helpful as we move into different forms and bigger numbers. Consider 13 × 12. Yes, we could set it up as 13 groups of 12, but that is tedious to do! If we use the idea of considering it as 13 rows of 12, now we can create an area model! If we consider the area model on the left of Figure 2, it shows the two side lengths and the individual squares make the answer. What we want students to work toward is the area model in Figure 2 on the right, where there are different sections of the number (separating 10 + 3 and 10 + 2). We can work on building this understanding with students by looking at the initial rectangle built in Figure 1 and focusing in on ways to make it easier to

count the tiles instead of having to do individual rows. The idea would be to work on splitting or decomposing the numbers and then focusing on how we can use those to create the area model. For example, in the right image in Figure 2, you see a section that is 10 × 10, so a *hundreds block* can be used. If we continue through the other parts of the model, we can easily build it using the different *base-ten blocks* instead of just ones cubes.

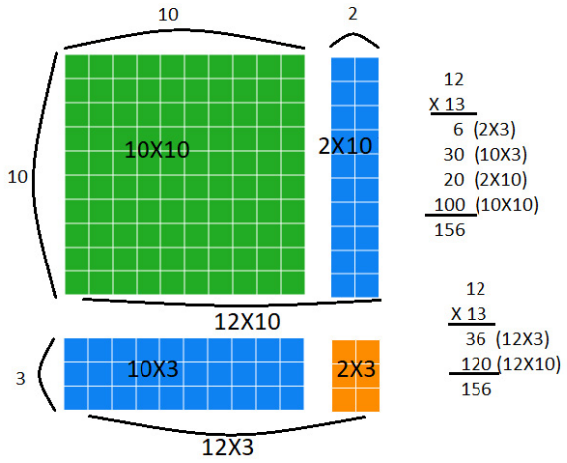


Figure 3: *Showing the relationship of the area model to the procedure for multiplication*

When we look at Figure 3, this also fits nicely with thinking about the parts of the standard algorithm, so now we have a model that helps to justify the method that students learn to make things routine! The procedure makes a nice intermediate step for a procedure for multiplication that relates directly to the model created. Underneath is the "standard algorithm" that can be extrapolated from there. What is nice with the first procedure is that students can see that it does not matter the order they multiply the sections, as long as they multiply all the parts of the numbers.

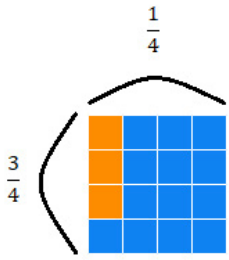


Figure 4: *Area model for* $\frac{1}{4} \times \frac{3}{4}$

As we proceed through the curriculum, students move into fractions and thinking about $\frac{1}{4} \times \frac{3}{4}$. If we consider that the idea here is thinking about side lengths of $\frac{1}{4}$ and $\frac{3}{4}$, we can model an image like Figure 4. In this image, we see that the top is coloured in to show $\frac{1}{4}$ of the side coloured in orange (one of the four tiles on the top). The right side is coloured in to show $\frac{3}{4}$ of the side length is orange. We have created an entire rectangle that has the side lengths of the two factors, and if we keep it rectangular, then we can determine and justify $\frac{3}{16}$ of the square as the solution. See, for example, a second area model using more complicated numbers of $\frac{2}{7} \times \frac{3}{5}$ in the left of Figure 5. As we see on the right of Figure 5, this helps to show the basis for the standard formula for multiplying fractions and why you need to multiply the numerators together and the denominators together to solve!

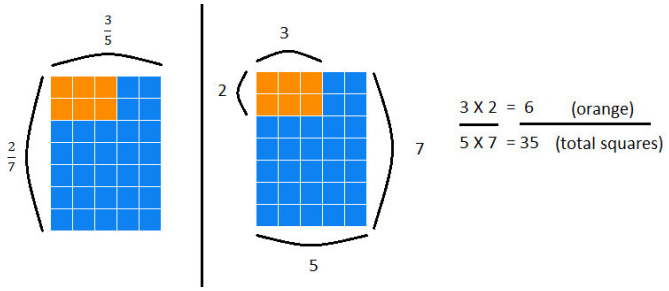


Figure 5: *The left shows the area model for* $\frac{1}{4} \times \frac{3}{4}$. *The right shows the connection between the model and the standard procedure for fraction multiplication.*

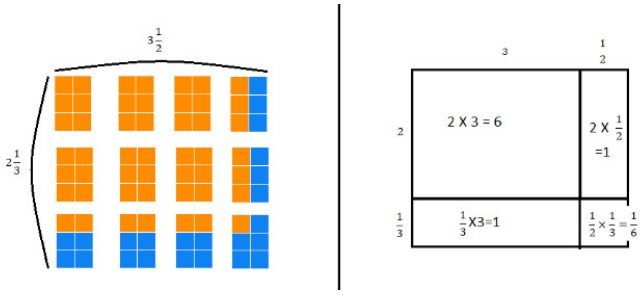


Figure 6: *The left shows the concrete area model for* $3\frac{1}{2} \times 2\frac{1}{3}$. *The right shows a model to represent the process using decomposed numbers.*

If we use a larger rectangle, such as the 13 × 12 rectangle in Figure 2, we can even think about how to use this concept for mixed fractions, like $3\frac{1}{2} \times 2\frac{1}{3}$ and suddenly, multiplying does not seem as daunting. Again, the focus is on creating a rectangle with the correct side lengths, and we can start with something concrete, like the right in Figure 6, but then move to the more abstract pictorial version on the right in Figure 6 as students gain confidence before removing the model all together! In the right of Figure 6, we have once again decomposed the numbers so that students can see that in order to multiply, you focus on the parts.

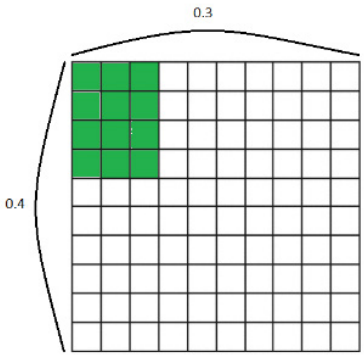


Figure 7: *Area model for 0.3 × 0.4*

Using the same model, we can consider decimals. If we start simply with something like 0.3 × 0.4, we see the model in Figure 7 showing the two side lengths of 0.3 and 0.4, and justification of why the answer is 0.12! If we look at more complicated amounts, such as 1.2 × 2.6, we can see in Figure 8 the manipulative model and its pictorial version that we can use as a basis to create the model.

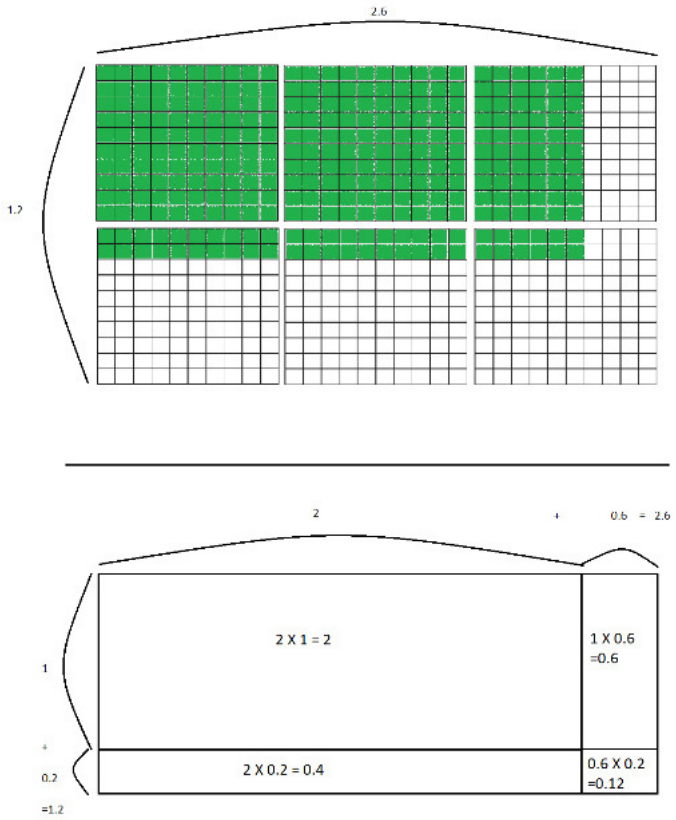


Figure 8: *The left shows a concrete representation for 1.2 × 2.6. The right shows how to multiply using the concept of the area model with decomposed numbers.*

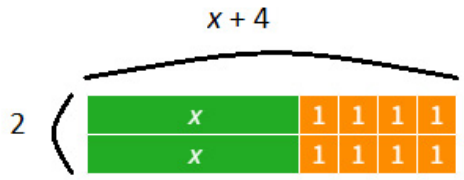


Figure 9: *Area model for 2(x + 4)*

Having spent so much of their mathematics career learning about the area model in different concepts, we can now consider how students can apply this idea to algebra. If we start with a question like 2(x + 4), students can use algebra tiles to make the image in Figure 9. The underlying concept is the same: one side needs to be 2 long and the other side is x + 4 long, making the area and answer 2x + 8 when you multiply. This can also allow students to build to the idea behind (y + 2)(y + 3). If we start with the concrete representation in the left of Figure 10, students can use what they have learned to create the pictorial representation in the right of Figure 10. This represents the distributive property and allows students to connect it with the "FOIL" acronym, without needing to be shown it to start! It becomes intuitive that you would need to multiply the *First* parts of the expression, the *Outside*, the *Inside*, and the *Last*. What helps with this is that students can realize that the acronym order does not matter (they could use "FILO"), as long as they make sure each of the parts has been multiplied (much like they did with mixed numbers and two-digit numbers earlier).

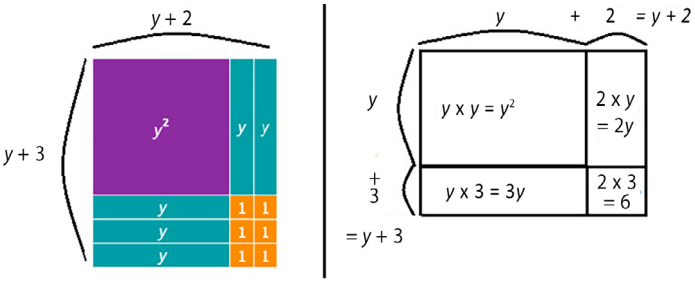


Figure 10: *The left shows the concrete area model for (y+2)(y+3). The right shows an area model with decomposed parts of the multiplication question.*

Spending the time in early elementary school to build the understanding of the area model allows for students to consider that the definition of multiplication does not "change" as the numbers get more complicated; just the representation of the numbers changes. The theory of the side lengths being the parts of the question, and the inside giving us the total, allows for a model that will let students explore any form of multiplication as they move through their schooling! ▲

▲ Elementary Math Matters: When "up" isn't…



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Lynda Colgan, Professor Emerita, Faculty of Education, Queen's University, is also the Executive Director (Education and Development) for Science Rendezvous, a pan-Canadian organization that celebrates and showcases Canadian science and scientists. She is currently leading two NSERC PromoScience projects and two PromoScience Supplement initiatives that make it possible for her to carry on her long-established commitment to bring equity and diversity to STEM outreach and public education. Lynda's columns reflect her current work on several research, resource-creation, and teacher-education projects, and are rooted in her years of experience as a classroom teacher, district-level coordinator, and pre-service and in-service instructor.

For the past few years, I have been involved in a fascinating project with Dr. Sandy Youmans and two expert Kindergarten educators, Edward Schroeter (now retired) and Melanie Martin. We have been collaborating on a curriculum-development venture called *The Roots of Coding*—a free, downloadable seven-module resource (earlymath.ca) that begins with lessons about simple positional language and moves along a developmental trajectory to include complex positional language, perspective, grids and mapping to pathfinding and transformations.

Our goal for the collection of lessons has been to illustrate that coding in the early years is math; in particular, geometry and spatial-sense concepts that are familiar components of early years curriculum—and not an "add-on." Curriculum expectations, as they are written in policy documents, e.g., "**Describe the relative locations** of objects or people, using positional language (over, under, above, below, in front of, behind, inside, outside, beside, between, along)" are misleadingly simplistic. On the surface, expectations like these seem to emphasize language or vocabulary—the way they are presented captures neither the depth nor complexity of the multiple math concepts that fall under one simple word like *position*.

Undoubtedly, the most rewarding facet of the work has been the lessons learned from "observing" the students in Melanie's Millbrook, Ontario, Kindergarten classroom. While I have not been there in person to be part of the field tests (nor the re-tests or re-re-tests) of each lesson, I have had the privilege of being a "fly on the wall" through the magic of video capture while Melanie "does her thing" in large- and small-group settings or while working with individual students.

Talk about surprises… watching the video recordings was to experience more unanticipated twists and turns than a ride at *Canada's Wonderland*. One thing is certain: the machinations of Kindergartners' brains are not for the faint of heart! American poet David Kresh was über-correct when he so wisely stated that "the understanding of atomic physics is child's play, compared with the understanding of child's play" (Kresh, 1961, p. 288).

As a small team, we used the videos to reflect on and refine our lessons so that they included intentional instruction supports for those implementing our lessons. For example, we developed purposeful prompts to expand upon the implicit math connections made by the author and illustrator of a children's book to unpack the colloquial vocabulary that does not communicate the layers embedded in words that have everyday as well as mathematical meanings. We also used the students' responses to identify success criteria and make suggestions for scaffolding—all part of our focus on dynamic assessment from an asset stance.

There were many "aha" moments, despite having more than 100 years of collective experience. We worked to structure unplugged coding activities that were simultaneously engaging and fun. Yet the activities need to be conceptually accurate and challenging enough for students to construct new understanding in the moment and set the stage for their future computational thinking experiences.

Which brings us to "up." After researching many existing resources (a quick "Google" search for "coding in Kindergarten" returns 15 600 000 "hits"), including print and digital, we were startled to see the misuse of "up" (and by extension, "down") by educators and curriculum developers. Over and over, we observed teachers using (or being directed to use) grids on chart-paper stands (or projected onto interactive whiteboards) as children were asked to describe positional movements, using directional words and numbers to arrive successfully at a destination, e.g., getting a duck across the street to the park and, ultimately, to the safety of the pond.

As we reviewed videotaped lessons or checked the teacher notes, we noted that the vertical orientation of the chart paper typically resulted in student (or suggested) responses such as "move two squares up, turn right, move three squares right, turn left and move down five squares." While this makes intuitive sense to the Kindergartner: the

top of the chart paper is facing the ceiling, and like the sky, ceilings are "up" and the bottom of the chart paper is oriented toward the floor, so one must look "down" to see it; the use of "up" and "down" in this context, using the grid as a tool and this vocabulary, is conceptually wrong.

Equally egregious were the typical, suggested progressions to follow this introductory lesson in the video or print resource—immediately moving to a paper grid on the children's desks, where they were asked to act out a storyline, using the same erroneous coding commands with the addition of arrow cards, introduced, and reinforced through repeated exposure. But moving toward the top of the grid is not "up" when moving a tractor on a 2D grid representing a farm—the correct coding command is to move "forward."

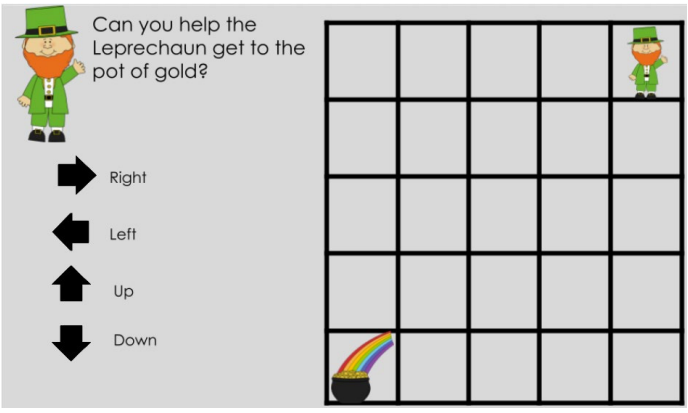


Figure 1: *A free, downloadable resource from the website Teachers Pay Teachers, advertised as a* Coding Grid for Digital Learning in Kindergarten *that misuses the words and symbols for "up" and "down"*

It is important to lay strong and accurate foundations in spatial sense—concepts that are, by their nature, the roots of coding (and hence the title of our resource)—by correctly formalizing and extending children's intuitive and budding mathematical knowledge. Since young children are, by nature, egocentric and see things only from their own perspective, we need to support the growth of their developing spatial sense from other perspectives, gradually and purposefully.

And so, to begin, our small team used explicit examples of "silly play" in our introductory lessons. Playful (but intentional) learning is an essential component of particular note in one of our lessons: *2-D Simon Says "Snake."* The lesson opens by inviting children to move their own bodies to personalize positional and directional language. This is a whole-body activity on the floor or ground, in which children crawl on their bellies like snakes and follow Simon's commands. It is designed to teach children that there is no "up" or "down" on a two-dimensional plane (or in coding), that 90-degree (one-quarter) turns are used to change direction, that a one-quarter turn on the spot counts as one move to change their orientation, but not their position.

When an educator participates in "silly play" (as you see my "snaky" colleague, Edward Schroeter, do in the video included in the lesson plan), he is modelling a way of experiencing learning as safe and fun, setting a positive emotional culture in the classroom. Given that research suggests that the learning process is 50 percent social–emotional and 50 percent cognitive (Schonert-Reichl & O'Brien, 2012), as educators, we designed our lessons not only to focus on content instruction, but to develop positive dispositions to math as well as risk taking.

By the time we transitioned to a large floor grid on which the students (and teacher) could walk, and finally to a desk-sized paper grid, we found that most children were successful when acting out the steps of a positional/directional storyline. We found that success was also increased when the children were moving a "Flat Stanley-like" paper doll or an animal figurine because of their definite "front" and "back" that support the notion of moving forward or turning left. We knew that the lesson sequence from serpentine slithering to directing imaginary zoo animals into their respective pens (on paper) was a resounding success when we observed one child saying, "We need to go up five squares," and his peer light-heartedly correcting him by saying, "If you went up [emphasis added] that high, you'd be flying near the ceiling!"

While interactive whiteboards, document cameras, and chart-paper grids may be powerful tools to support learning for young children, it is critical to remember that the cognitive jump for the child is huge when changing perspective from the vertical representation of a page projected on a screen to a sheet of paper in a different orientation on a flat, horizontal surface such as a desk. These teaching aids should come a very distant last after multiple kinesthetic experiences that build children's capacity to imagine in their own mind's eye, the positions of objects, their orientation, their spatial relations to one another, and the directional movements they must make to form new spatial relations as well as the precise vocabulary, coding symbols, and quantities that are required for their communication.

The moral of this story is that describing position, identifying location, giving instructions using distance and direction, finding objects using landmarks, perspective taking relative to other objects, and pathfinding—foundational concepts and skills in geometry and coding all seem deceptively simple to us as adults; however, they are highly complex, multi-layered, and nuanced when seen through the eyes of a child.

It is not enough to provide random, non-research-based activities about these interrelated yet incremental concepts that develop over time. Classroom spatial sense (also known as pre-coding and early coding activities) require well-designed tasks that grow in complexity by small steps. They need to guide the educator using them about how to observe carefully to recognize how the children are representing and communicating their mathematics knowledge. Resources need to support the teacher to help the children to build on and extend their understanding through prompting and questioning, as well as multiple opportunities to use the correct language and concepts in various contexts before moving on.

Our small and experienced team quickly learned that we could not assume that our students had the prerequisite skills required for some of the most preliminary tasks, such as having the most basic positional vocabulary. At the same time, we learned that specifying positional instructions was no simple matter.

For example, in the *Build a Robot* activity from our resource *The Roots of Coding*, each student was given a small, clear bag containing ten connecting cubes sorted by colour and quantity: one each of white, red, blue, and green, two each of yellow, black, and orange. In this small-group activity, students were challenged to listen to very specific block-placement instructions, which included colour and position of an interlocking cube, with the goal of creating identical block robots. Students needed to listen carefully to the instructions, visualize each instruction, and then place the blocks in the correct position and orientation.

We found that young students often tried to build the robot while holding it in the air, while we had imagined that the build would take place on the flat surface of the desk; therefore, position descriptions such as "in front of" become meaningless because the airborne robot could be turned to face in any direction.

We also learned that seating arrangements contribute to learning in important ways. When young students sit facing different directions or in a circle, they have different perspectives on what direction is "left" and what is "right," and tend to confuse each other because they are watching their classmates work in mirror image to themselves; therefore, there are times when it is important for students to sit side by side, facing the same direction to be successful.

Since curricular language includes colloquial vocabulary, it does not communicate the layers embedded in words that have "everyday" as well as "math" meanings. We found that it was important to provide focused exposure to this language as it is used in various contexts, and revisit concepts regularly to give children multiple opportunities to use the math term with precision.

We must never forget that our youngest learners are reliant on their teacher to help them move forward using mathematical language and representations to describe and understand the spatial ideas that lead to proficiency in coding and simple programming languages. This can be done by explicitly "unpacking" the multiple meanings and uses for a word as seemingly trivial as "up."

We hope that you will log on and download our resource found at earlymath.ca. Our classroom-tested pre-coding and coding challenges involve no special equipment beyond that found in most early years classrooms: interlinking blocks, counters, carpet spots (for taping grids), and puzzles. Through wide-ranging contexts, from construction sites to the kitchen table, from a farmyard to a parade, it is our sincere hope that students, like Melanie's and Ed's, will see ordinary words transform into math words as they engage in dramatic play, build 3D shapes, move themselves and 2D objects in space, and choose appropriate arrow and turn cards from a set. As they learned about their school and community as well as how to make sandwiches, our children learned math and language content as well as conflict resolution and social problem-solving skills (collaboration, co-operation, and helping); and communication using multiple representations of ideas (oral, diagrammatic, symbolic). Together, these academic achievements form a coherent and comprehensive foundation for the various technology-based coding experiences to follow in higher grades. Beyond the combination of subjects and the wealth of physical, cognitive, emotional, and social development opportunities, you will find that our lessons are built upon the principle of curriculum integration for the fact that it allows flexible grouping, brings novelty and authenticity to learning, promotes creativity, stimulates self-directed learning, and plants the seeds of lifelong coding and learning.

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"On December 10, 1949, the Department of Education issued a circular which stated Grades 1 to 13 were divided into four divisions: Primary, Junior, Intermediate and Senior." (Hoey, 1989, p. 72)

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▲ Coding in the Classroom: What Is Coding? An Interview with Jeff Irvine



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(Quotations within this column may have been lightly edited to reflect the difference between oral and written language.)

What is coding?

This is a question that we are going to explore over the course of the next few issues of "Coding in the Classroom." It is a question that arose in our working group at the recent "Coding, Computational Modelling, and Equity in Mathematics Education" symposium, held at Brock University in Spring 2023 (www.cpmath.ca/ccmeme2023/).

There are many hypotheses about what coding might be for learners, and what type of thinking they are engaged in while coding. Each of these hypotheses provides fuel for interesting discussions about what our learners are doing while coding, and each has important implications for what and how we will teach our learners in the future.

We have many questions we need answered before we can get to what coding is or might be. There are people to talk to and learn from, and we hope to sit down with as many of the thinkers and doers of coding as we can.

Coding is:	
What?	**Author(s) you should read**
A mathematical immersion	Seymour Papert
Computational thinking	Jeannette Wing
Computational problem solving	David Weintrop et al.
Computational modelling	George Gadanidis, Immaculate Namukasa
Computational participation	Mitch Resnick, Yasmine Kafai
Computational fluency	Conrad Wolfram
Computational action	Mike Tissenbaum
Computational literacy	Andy diSessa, Clifford H. Lee

We have read all of these thinkers, and each one of their conceptions of coding is fascinating and provocative. They are provided here as an illustration of the diversity of thinking around coding and as a reference of the authors we think people might read first.

Let Us "Just" Start Here

What if coding is just a tool to be used in mathematics classrooms? Granted, it is a powerful tool with multiple uses that we have talked about in previous columns, and will be exploring going forward—think circular saw versus a handsaw, or a Swiss Army knife versus a regular pocket knife. There have been many tools introduced into mathematics classrooms over the years, and their adoption for regular use has been spotty at best. Some have been physical and representational (manipulatives) and some have been technological. It is these technological tools that we are interested in, as coding definitely fits into this category, even though coding can be compellingly and surprisingly concrete, according to George Gadanidis.

To find out more about how new tools are incorporated into classroom teaching, we had to reach out to Jeff Irvine. Jeff has been teaching since the 1970's and has been a teacher, an administrator, a researcher, and a general education officer for the Ontario Ministry of Education.

Revitalising Traditional Tools

Jeff began with one of the earliest technological tools employed in mathematics classrooms: the scientific calculator. In the mid-seventies, he highlighted a remarkable technological advancement by making effective use of available technology, particularly the scientific calculator, which proved to be a valuable asset in teaching. Technology in the mathematics classroom provided immense freedom, enabling teachers to engage students with real-world problems and foster problem-based learning. This approach, as Jeff implied, eliminated questions of relevance and motivated students to actively participate, resulting in increased engagement and genuine motivation.

The introduction of calculators in the seventies and eighties marked a significant milestone. Then, in the nineties, the arrival of graphing calculators brought about more transformative changes in the curriculum. This remarkable development had a profound impact on education.

Jeff exemplified the transformative impact of graphing calculators on mathematics teaching and learning. With graphing calculators, tasks like function transformations and data analysis became more accessible. Students could understand the effects of transformations using function notation, eliminating the need for manual calculations and graphing. Engaging activities like the "Stormy Weather" investigation (Ontario Ministry of Education, 2000, unit 4, p. 15) ignited creativity, challenging students to find functions resembling storm clouds. Jeff, as a teacher, not only focused on functions, but also requested screenshots of calculator settings, enabling visual manipulation without altering functions. Beyond data investigations, graphing calculators facilitated experiments and hypothesis testing, opening a world of exploration and discovery in the classroom.

Jeff also pointed out that there are high school mathematics departments that have shelves full of these calculators just sitting there unused. In trying to understand why teachers would not want to use calculators, we came upon the unfortunate fact that not all teachers view mathematics the same way. Some view mathematics as a series of calculations to be carried out to get an answer. With this view, calculators are doing the calculations the learners should be doing—using a calculator is cheating somehow.

If coding is indeed a tool, is it one that teachers will see as replacing some vital mathematical doing? Or will it be viewed as something that allows learners to dive into and understand more complex and interesting mathematics through its use?

Unleashing the Potential of Coding: What is a tool for mathematical learning?

Jeff believes that tools in mathematics enhance clarity and understanding by making concepts more explicit and transparent. These tools allow mathematicians to tackle real-world problems without getting too caught up in complex details. He emphasizes the importance of not letting intricate mathematics hinder the learning of fundamental principles.

> To me, a tool in mathematics is something that makes the mathematics more explicit or more transparent or maybe is a better word. [...] It allows us to tackle real-world problems to some extent, without getting bogged down in the details [...] in the sense of don't let the mathematics get in the way of learning important mathematics.

So, what is coding?

Jeff sees coding differently from just another technological tool, and believes many teachers share this view. Coding is considered by some to be an add-on related to mathematics: he feels it may not be seen as particularly useful compared to alternatives. He raises concerns that coding is often perceived as an extra element added to the curriculum, without much being removed. Modelling and socio-emotional learning were also introduced, and many mathematics teachers, particularly at the elementary level, have limited experience with these concepts. Jeff observes that coding is often seen as an unfamiliar subject that teachers are expected to teach while grappling with the challenges of covering the core curriculum.

If coding is a tool, it is not just another technological tool. It is one that can give learners access to real mathematics just like the calculator freed us from long, tedious calculations in order to get at more.

We would like to hear from you as we continue our journey. How do we get to teachers who are skeptical, and how do we show them what these tools allow learners to do? How might we get over the hurdle of teachers seeing coding as just another add-on—one that you can cover with a quick unit or a visit to do the Hour of Code (www.hourofcode.com/)?

If you would like to listen to our entire interview with Jeff, where we cover more than just coding and get historical insights into technological mathematics teaching tools, it is available at OAME Talks (www.talks.oame.on.ca/) or some of your favourite podcast providers.

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▲ Uncovering High Park's Pasts and Presents: Storying the Land through Mathematics and Storywork



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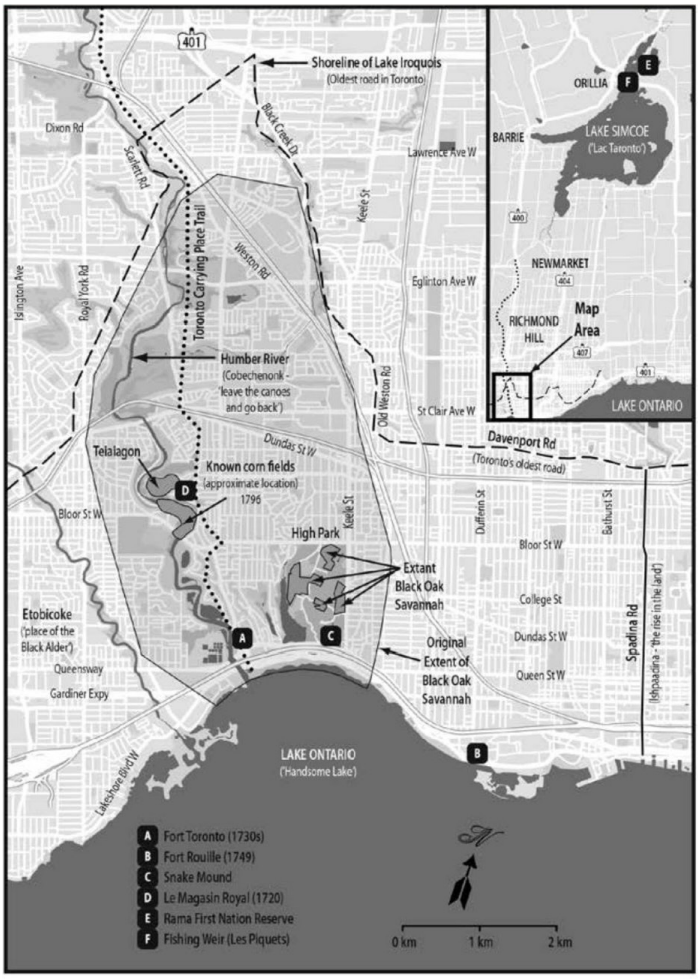
Dahlia is a community-based educator in Toronto. Having been involved in social-movement education since her youth, Dahlia now teaches at Seneca College and supports active teaching and learning at Toronto Metropolitan University. Previously, Dahlia worked as an educational researcher at the Ontario College of Teachers and at the University of Toronto's Centre for Critical Qualitative Health Research. She has an M.A. from the Ontario Institute of Studies in Education (OISE) and is currently completing an M.Ed. in Mathematics Education in Community at the University of British Columbia. Dahlia is committed to changing the way we do education and living up to our land responsibilities.

Introduction

> *"I was just thinking how if a plant has a different angle, then maybe something happened to it. It could be a way to see if something affected its growing, like an animal, or a human."*
> — Rosa, 2023, 14-year-old participant

The sun had already set over High Park in Toronto on a Sunday evening in late February, when a group of six 14-year-olds and two facilitators gathered for their monthly program. A fresh coat of snow blanketed the remains of southern Ontario's Black Oak Savannah as the dispersal of the day's human activity gave way to a bustling network of winter life. The group of youth are affiliated with a Jewish outdoor education organization that runs after-school programs and camps around Jewish nature connection. Last year, these six teens took part in a year-long *b'nei mitzvah* (coming of age) program, where they formed lasting relationships and sought to continue their learning together. Now, they meet for monthly programming in parks and ravines around the city.

My sister, Iris, is the program lead for the group. Iris and I are both Jewish community educators in continual conversation about how our own cultural frameworks may be engaged as collective sources of guidance for how to act



The Toronto Carrying Place Trail (Johnson, 2013)

on our responsibilities as non-Indigenous settlers on Anishinaabe and Haudenosaunee lands. We had recently been discussing mathematics and story, how stories shape how we come to understand our environments, and considered how mathematics might help us reveal elements of story not initially visible. Aligned in our educational philosophies, but with very different relationships with mathematics, Iris invited me as a guest co-facilitator, and together we developed a program that would apply mathematics to further inform the group's ongoing land-based explorations. The central problem that shaped our activity was to investigate how mathematics could be utilized to deepen our understandings of some of the past and present activities of High Park, including those that often get missed or disregarded. Our inquiry questions were:

> *What can we trace about High Park's activities? What stories can we tell?*
>
> *How can mathematics help us to reveal some of the hidden histories of High Park?*
>
> *How does our perspective inform what we find (and what we overlook)?*

As educators devoted to community in the age of global capitalism and climate change, we respond to a reality in which certain stories are privileged over others, and thus, central to our educational work, including this activity, involves unsettling the colonial master-narrative to reveal multitudes of living histories enduring prior, within, against, and beyond it. The activity, which involved traversing from the park's north entrance down the forest trails to Grenadier Pond, was punctuated by text excerpts narrating the history of the park that we sporadically gave participants to read aloud. We started with the official placard of John Howard's benevolent gift of the "wooded resort" to the citizens of Toronto, and was confounded in turn by pre-existing activity of Indigenous peoples along an important Indigenous trade route dating back 11 000 years. These excerpts told just some of the myriad stories that make up the park's past and present.



Forest trail leading down to Grenadier Pond in High Park

Stories are important ways of understanding and orienting us to our sense of place. Sto:lo educator Jo-Ann Archibald Q'um Q'um Xiiem's (2008; 2019) storywork pedagogy outlines seven principles—*respect, reciprocity, reverence, responsibility, holism, interrelatedness,* and *synergy*—to help educators and learners ethically engage in the Indigenous stories and life experiences that make up our environments and guide us in becoming "storywork ready." This essay tells the story of our activity through the lens of mathematical inquiry and the seven storywork principles, focusing on three "math moments" that took place during the activity, to examine how mathematics can be used to reveal hidden stories about the land and help us to become storywork ready.

This paper is written for fellow community-based educators who are interested in exploring how mathematics could be incorporated into community-oriented programming as a way of formulating the world more justly and truthfully, and as another way of looking at social and practical issues. With participants' permission, I audio-recorded segments of our activity. All participant names are pseudonyms.

Math Moment #1: Storying the Land – Animal Tracking and Spatial Imagination

Guiding question: What stories of the land's activity can we trace?

Math concepts: spatial recognition, logical and spatial reasoning, patterning, probability



Rabbit tracks in snow

As we descended off the paved road and into the forest trail, participants were instructed to put on their math tracking glasses and see what they could uncover about the land's activity. The group had been practising animal tracking and identification skills over the last year, but had not considered the range of mathematical strategies they had already been employing in the process. As soon as they looked, the group began spotting evidence of life activity throughout the contours of the snow, counting the segments and examining their arrangements to identify the animal prints with the help of identification guides. Before we could name it as such, I was observing participants employing their mathematical thinking to decipher the prints based on their numbers and shapes, their level of pressure and impact in the snow, and their spatial progression; indicating the size of an animal, how and where it may have been moving, and how its motion might be traced in the surrounding fauna, such as bent or broken branches or trampled debris. The group hovered over sets of four prints along a sequence, two longer parallel prints behind two smaller ones, applying spatial reasoning to gather that the animal was travelling in

a hopping motion, and verifying from its prints to be a kind of rabbit. Another set of prints was said to resemble a canine track, and Iris asked how they might tell the difference between a domesticated dog and wild dog, such as a fox or coyote. She explained that compared to pet dogs, which often walk in a relaxed fashion with their paws spread out, coyotes and foxes walk carefully with their paws more narrowly spaced, and tend to place their back paw in the same spot as their front paw to conserve energy. Participants looked for evidence of these behaviours in the landscape, visualizing the print patterns these strategies would produce. "It's not just about the prints, it's also about the context," Iris reminded, pressing them to consider the probability of their conjectures. The small likelihood of a wolf, for instance, passing through High Park, simultaneously narrowed its plausibility, while indicating that it was still possible.

Using these points of reasoning, we then had participants break off into groups to explore the area and develop stories of animal activity based on what they could trace on the land. Given the low visibility (we were using headlamps and flashlights), we told participants to remain where they were still in earshot of the group, and I thought about how this in itself was a calculation in spatial awareness. By storying the land with possibilities they could trace, we were having participants proliferate the number of stories and interwoven activities conceivably at play across High Park's present and recent past. Placed together, we were constructing a story of stories centred on the wildlife and interplay of ecological forces, to which human narratives merely at times play a part. When we came to listen to each group's stories and examine their evidence, the rest of the group was given the task of responding to the group by suggesting some counter-narratives to the one they presented.

One group hypothesized that the same rabbit they had identified earlier was rushing to have tea with his grandma, breaking a few small branches as he brushed past and up the hill. One participant, Shlomo, challenged the story by questioning its spatial soundness:

Shlomo: Okay so, I think a lot of your story makes sense, except for two key things. First of all, how would an animal go in a straight line here and break this piece of wood in a different direction?

Leah: That's what I was wondering.

All: Hmmm.

Eli: Maybe it fell over.

Leah: If it fell over, then there would be other marks on the ground.

Adina: He wanted a walking stick, so he reaches out, grabs this—

Leah: With what arm?

Adina: His other arm.

Shlomo: Anyways, also, how could it be the animal we saw, because the other animal we saw before had two tracks at a time, 1-2, 1-2, 1-2. And it also wasn't really this shape.

In their decoding of the story being told in the land, the participants employed mathematical reasoning to challenge each other's accounts and refine their understanding of what could conceivably have taken place there. They used spatial vectors to visualize the scope of the animal's dimensional shape and motions, scaffolded with imaginative thinking to help rationalize their findings. Shlomo helped to reveal how the team may have been overlaying their own prior assumptions onto the stories they found, by conjuring a familiar animal and applying it to what they had seen previously. This teaching connected back to an opening text we read by Rabbi Benay Lappe (2014), who quoted Rabbi Lisa Edwards saying, "If donkeys read the Torah, all the donkey stories would jump out at them." It demonstrated how, as we read the world around us, we are often programmed to see our own images in its textual landscapes; that it takes an added layer of awareness to be able to see those stories that do not pertain to us, or that stories may emerge in unfamiliar forms and ways, or that donkeys might also tell their own stories, or that we too may be donkeys. How would these stories differ from the ones humans tell themselves?

Math Moment #2: Age of a Tree – Circles, Rings, and Growth Factor

Guiding question: Can we find out the age of a tree without cutting the tree down?

Math concepts: geometry, spatial reasoning, problem solving, measurement, pattern recognition

Our mathematics continued when we arrived at a landing at the bottom of the trail. We talked about the Black Oak Savannah that once densely covered a large portion of southern Ontario, a rare ecosystem that was maintained by Indigenous peoples through controlled burns that encouraged plant growth and nourished wildlife (Johnson, 2013). This raised a question from Rosa of how old the forest is. Our research told us 4000 years, and that today, as a result of excessive human interference and invasive plants, High Park's Black Oak Savannah represents less than 0.5 percent of what was here previously (Johnson, 2013). Participants began to imagine the swaths of wildlife activity that characterized the past eras of High Park, and

the legacy of the trees as habitats for different forms of wildlife. Following our question about the age of the forest and our reverence for its longevity, we wondered if we could measure the age of a tree.



High Park's Black Oak Savannah in winter

"How do we typically measure the age of a tree?" we asked the group. Participants had heard that the rings of a tree reflect its age in years. But can we do it without cutting the tree down and exposing its rings? "Well, we can see the tree's measurements, sort of," remarked Adina. Participants began to brainstorm ideas, suggesting that they could measure one ring of the tree and apply it to how many times it would fit inside it. They observed that one ring they could access was the outermost ring encompassing the tree's width. However, some participants raised a few problems: first, each ring would be a bit smaller than the last, and second, would the extra layer of the bark create an added distance from the rings inside?

Benny recalled what he had learned in school about circles. He reasoned that if they know the diameter of the tree, they can estimate the distance between each ring and discern the number of rings spanning the diameter. He suggested that they can use the circumference to find the diameter. Participants identified two equations for circumference: $C = 2\pi r$, and $C = \pi d$. We noticed that these were the same because the diameter, d, is equal to twice the radius, r. They worked together to measure the circumference of a large tree nearby, using a string and measuring tape. The group found the circumference to be 99 inches. While Eli used a calculator to divide by π, other group members began to estimate what the diameter would be. Leah used the mathematical habit of generalization (Su, 2020) to guess that it would be around 33 inches because π was a little bit more than 3, and a third of 99 was 33. "So will it be a little bit less or a little bit more?" I asked. Responses varied, but the answer, 31.5 inches, confirmed that it would be a little less. "Oh yeah, because dividing by a higher number will make a smaller number."

From there, the group considered how the diameter would reveal the number of rings in their tree. They recalled their idea to estimate the distance between the rings and use division to determine the number. By this point, a shared mental picture of the tree's circular cross-section was informing the group's thinking. Benny then pointed out that the diameter would cross each ring not once, but twice, indicating that what they needed was in fact the radius, not the diameter. They used mental math to determine that the radius would be 15.75 inches. But how would they estimate the distance between the rings? Leah correctly surmised that the distance between rings may differ based on the kind of tree, and Shlomo pointed out that they may vary even within a single tree as a result of changes in the weather and climate.

Tree Species	Growth Factor
Red Maple	4.5
Linden	3.0
Austrian Pine	4.5
White Birch	5.0

Table of growth factors (excerpt)

That's when I introduced the concept of *growth factor*: an index of tree growth rates based on species. I handed them a table of growth factors for about 50 tree species, and the next task of the group was to identify what kind of tree theirs was. The group observed the tree's winter needles, branching patterns, and distinctly high canopy; they noticed the texture and colour of the bark, and what looked like sap running down the trunk. Based on the smell of the sap, the group guessed it to be a pine, but not a white pine, because they had recently made tea with a white pine and noted them to be shorter. They guessed their tree to be an Austrian pine based on the darker needles

Diameter (in.) x Growth Factor = Approximate Tree Age (years)

and how high they were growing. They found the Austrian pine's growth factor on the table to be 4.5. From there, they could calculate the estimated age of the tree by multiplying the growth factor with the diameter. Benny paused. Didn't we say it was the radius? We thought about it. Adina suggested, "Is the growth factor already doubled… or half?" We rationalized that the equation of the age of a tree, using the growth factor, must already account for the extra step of dividing for the radius.

Based on their calculation, the age of the tree was estimated to be around 142 years old. Did that seem believable? The group found it to be probable. Using more mental math, they calculated that the tree would have been born in the 1880's. We wondered together: what was happening here in the 1880's? What surroundings must have propagated the birth of this tree? What lifetimes of activity has this tree borne home and witness to? Was this a native tree? If not, how did it get here? The name, "Austrian pine," is suggestive of the tree's origins; at the same time, as Eli noted, "Lots of people get credit for something that isn't theirs." We talked about the tree's longevity as a reminder of the significance of old-growth trees and their role in the surrounding ecosystem, that a planted sapling was not the ecological equivalent of a felled tree. I wondered how further dwelling on the perspective of the tree was helping us to turn our focus to the more-than-human inhabitants as the subjects and overseers of the forest, who have maintained and sustained the land over thousands of years, and who have increasingly come under threat by anthropogenic primacy.

Math Moment #3: Claims to Land and Truth – Value, Ratio, and Sacred Sites

Guiding questions: How does power shape the stories we focus on? What does it feel like to not be believed?

Math concepts: value, ratio, relativity, number sense, economic reasoning

The group proceeded further down the forest trail and settled in a circle at the north point of Grenadier Pond. We explained that in 1805, the City of Toronto purchased the land from the Mississaugas of the Credit for 10 shillings and a small handful of provisions. This is known as Treaty 13. We examined a map of the Toronto Purchase, the scale of which made High Park too small to locate. Some participants had learned about the Toronto Purchase beforehand, recalling that officials had taped the signatures of Indigenous peoples onto the agreement, and that the document was written in English, meaning Indigenous peoples were likely excluded from reading it. Treaty 13 is surveyed to be 250 830 acres—now probably worth billions of dollars, Rosa observed.

We continued: thirty years later, in 1836, John Howard purchased High Park's 160 acres from the City of Toronto for $1000. The math task was clear: participants immediately began to think through the land and monetary value transfers of the two purchases, and consider the relative worth placed on each transaction. The first question they asked was, "How much was a shilling worth?" I told them that from what we could find in our research, one shilling was about a day's work for a labourer. Shlomo suggested that this would be equivalent to $40. Rosa responded, "It might not even be that, because a lot of labourers at the time were really underpaid." Leah added, "I researched this one time. It's not that much." They postulated that a day's work might be closer to $10 at that time. Following this math, they concluded that the City would have paid the Mississaugas around $100, their estimated equivalent of 10 shillings. By comparison, the group noted that the City of Toronto received ten times as much from John Howard, totalling $1000, for the purchase of High Park. Using a calculator, they calculated 160 acres out of 250 830 acres to determine that High Park encompassed about 0.06 percent of the land of the Toronto Purchase. Participants digested the reality of these numbers: that the City of Toronto profited from *less than one-thousandth* of the land by tenfold what they had purchased it for. We asked them to imagine how much would have been paid to the Mississaugas had the land been valued at this price point. "How much is Toronto worth?" Rosa rephrased. Unfortunately, we did not have time to verify the calculation. Benny indicated probably over a million dollars; Leah suggested two million. We considered the relative worth of that amount of money in today's terms. We asked, "Why do you think the City of Toronto placed such little value on the treaty with the Mississaugas of the Credit, compared to the purchase from John Howard?" "Racism," Adina suggested. "Profit," declared Benny.



A camp set up among BMX bicycle jumps to protect ancient native burial grounds in May 2011 (Red Power Media, 2016)

At this point, we turned our attention to the place where we had been sitting. The next recited quotation read that in

1921, during roadside construction, the remains of ten Algonquin people were discovered in a burial site at the north ridge of Grenadier Pond (Donaldson & Wortner, 1995). In the years since, there have been estimates of 57 ancient Haudenosaunee burial mounds said to remain throughout the park, dating back 3000 years (Johnson, 2013; Red Power Media, 2016). Despite the 1921 evidence, whose current whereabouts is unknown, city officials and archeologists have not formally recognized the presence of ancestral remains. In recent years, the area began to be used as a BMX bike course, and an Indigenous-led group began lobbying for the preservation of sacred sites in High Park, whose ancestral remains and ecologically sensitive nature were potentially being disturbed (Red Power, 2016; Jackson, 2011). Following community pressure, the City of Toronto eventually conducted an archeological assessment, concluding that no archeological remains were found, and clearing the site of further archeological concern (Robertson et al., 2009).

As they processed these events, participants recognized what was required for the City of Toronto to believe the accounts of Indigenous peoples: hard evidence of human remains via excavation that they noted was likely disrespectful and potentially damaging. We prompted participants to reflect on cultural practices in our own Jewish tradition around death and the burial sites of our loved ones. "Do not unearth dead people," Eli asserted. Others noted customs of putting stones on graves, stepping around rather than over graves, saying prayers, showing respect, wishing them well. The absence of "evidence" to corroborate the sacredness of High Park has been used to justify the area's mistreatment, and to dismiss Indigenous claims as stories unfounded by history. I reflected aloud: how do our beliefs inform what is deemed worth protecting?



A City of Toronto sign at the edge of the site with a handwritten addition (Photo by Nick Kozak)

As we headed quietly back up the hill, we asked group members to think about times when they weren't believed, as well as times when they may have contributed to the disbelief of others. In the closing circle, participants drew on their tracking knowledge as they critically analyzed the situation: "It's not all about proof. It's also about context. What does the City of Toronto have to gain by not believing it? And what would the Indigenous peoples have to gain by lying?" Shlomo remarked. I added, "The stories we find will often be the ones we look for. Either we will find the ones that benefit us, or we can have the humility to notice that there is more going on; the ecosystems that are here, the many histories that have come through here, the trees that have been witnesses here." Iris closed us off with a reminder of Rabbi Lappe and the donkeys who read the Torah. "Tracking is a bit of a judgment. And the City of Toronto made a judgment based on their capacity to track. They didn't look from a different perspective. They didn't look from the donkey's perspective, nor from Indigenous peoples' perspectives. They didn't look from the land's perspective."

Discussion

Guided by Archibald's (2019) storywork principles, this activity sought to explore how uncovering and utilizing mathematics as a way of storying the land can help to disrupt the colonial master-narrative and expose hidden histories that guide us in becoming storywork ready. Participants' passions effortlessly fuelled the inquiry and their desire to do the math to uncover more. As we explored the storied landscapes of the park, we (as educators and participants) had opportunities to deepen our understandings of the storywork principles and how they orient us both explicitly and implicitly. For instance, learning about the Indigenous histories that predated High Park's founding garnered our **respect** for them as the cultural context upon which colonial narratives were oppositionally imposed. Indigenous stewardship practices of controlled burning deepened understandings about **reciprocity** and mutually beneficial relationships that maintain the health and integrity of the ecosystems that sustain them. Uncovering a range of land activity through mathematics ignited the group's **responsibility** as shomrei adama (guardians of the Earth) to pay attention to the stories shared, and opened opportunities for continuing conversations around their reciprocal responsibilities to protect, to steward, to advocate, to respond to colonial imposition, to show solidarity with land protectors, and to keep alive the stories of the land.

The activity embodied holism by engaging the land as a **holistic** text for multiplicities of experience; connecting the physical (spatial tracking and mathematics), the intellectual

(storying and extrapolating), the emotional (response to disbelief and hidden truths), and the spiritual (relations to the dead). Learning about High Park's burial sites brought the group to pay reverence to the dead and to sacred rituals, connecting their own cultural practices to honour and respect Indigenous protocols, and recognizing the more-than-human elements contained in the land. Participants reflected on the **interrelatedness** between their own connections to the dead and those buried in High Park, and sacred cultural practices that tend to the spirit world. Overall, the activity's progressive deepening of the stories of High Park sought to spark moments of **synergy** and new understanding about the power of story and of mathematics, and the implications of colonial authority on life forces. Mathematics offered another story into the extent of these different histories, and their connectedness when it comes to the ways that the land is treated. For Iris and myself, being guided by these principles as educators taught us the value and meaning of orienting our pedagogies around storywork as a continuous process.

Conclusion

When the group had first gathered at the park that night, we had asked them to check in by sharing their current relationships with mathematics. Their responses were to be expected; a mixed bag, largely centred on their math performance and experiences with specific teachers. In the closing circle, a new appreciation was expressed for mathematics as participants highlighted mathematical patterns they had not seen before in the land, and Iris and I reflected on math's ability to bring communities together around matters of importance. American Mathematician Francis Su (2020) talks about the love and community that is built among people through mathematics. He quotes mathematician Simone Weil, writing, "The soul empties itself of all its own contents in order to receive into itself the being it is looking at, just as he is, in all his truth [sic]" (p. 212). We ended off our activity in this spirit with a closing song by queer Jewish songwriter Batya Levine, singing,

> *May I be empty, and open to receive the light. May I be empty, and open to receive.*
>
> *May I be full, and open to receive the light. May I be full, and open to receive.*

As the youth dispersed to their guardians waiting at the park gates, the opening words we read of Rabbi Lappe now resounded with new meaning, as they saw mathematics too in a new light:

> *All the donkey stories, they'd see. All the donkey stories that we completely missed. When I began to learn Talmud, all the donkey stories, my donkey stories, started jumping out at me. And when I connected the dots, I saw a tradition that was smarter, more sophisticated, more courageous, compassionate, bolder, and more radical than anything I could've imagined. And I fell in love.*

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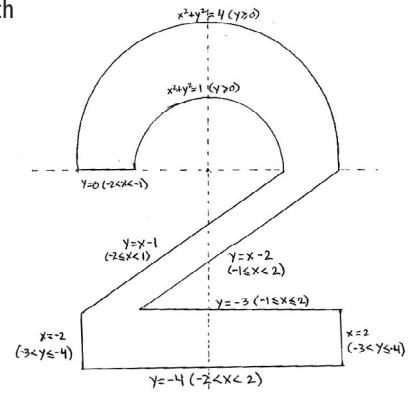
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Fun With Fonts

This "2" looks a little wonky. Suppose you could change one equation and its connections on each end, what would you alter?

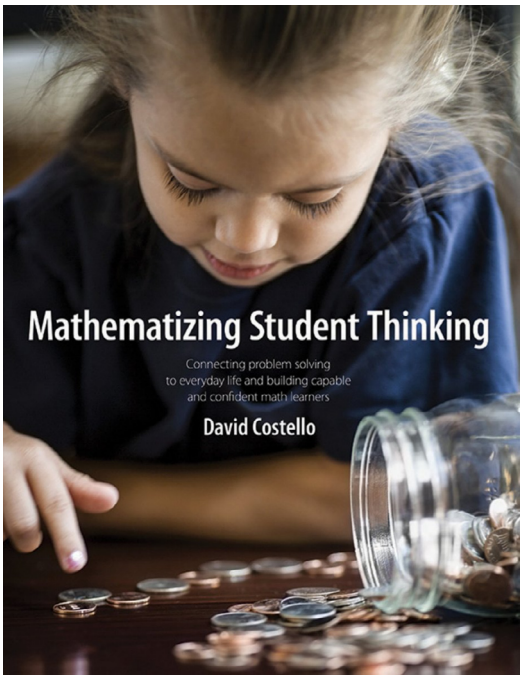
Use colour to show which segments include endpoints and which don't. Is the boundary continuous or are there points missing?



▲ Book Review: *Mathematizing Student Thinking*

TIM SIBBALD
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Costello, D. (2022). *Mathematizing student thinking.* Pembroke Publishers Limited.



As Editor of the *Gazette*, I am well versed in David Costello's column in the *Gazette*. I knew this book was referenced in his columns. There is overlap, but the book contains more detail, and if you find his columns of interest, then this book provides extensions of his thinking.

The instructional views are reasoned and with purpose. They focus on Grades 1 to 8 and do not attempt to expand to the senior courses, which is a reasonable scope. The writing uses some dichotomous classifications, such as defined and ill-defined problems, where the reader may need to consider the implications further for what resides in the middle ground. That said, readers may appreciate that David makes his case relatively succinctly at 153 pages, which many teachers are likely to appreciate.

I do feel conflicted by the book. When I put on my academic hat, I find the argument that Pólya's problem-solving heuristic needs updating to be challenging—but not necessarily incorrect. The argument is essentially that in practical contexts, particularly with ill-defined problems, the heuristic does not fit the level of guidance that students need. In particular, the proposal is to be more explicit about the extraction of details from the context, the use of modelling, and interpretation that suits the original context. On one hand, Pólya's heuristic is very powerful; on the other hand, it is clear that Pólya's examples in the various books I have on hand tend to focus on mathematical problems, with minimal contextual details.

While Costello is not explicit around curriculum change in Ontario, I found his book interesting reading for understanding how bringing modelling into the 2020 curriculum might oblige changes to a problem-solving heuristic. We now have modelling and coding explicit in the curriculum, and it is not clear that one heuristic fits all aspects of the curriculum. So, the personal conflict is between my awareness of the effort Pólya gave to his heuristic and the need for careful attention to the development of any modified or new heuristic. Costello has a point, and, academically, I would like to see the argument developed further.

Despite the conflict I feel between a point well made and an academic desire for considerable attention before changing a long-standing heuristic, I think many teachers will find that this book is the pragmatic bridge they need to support increased attention to modelling. It effectively opens a conversation about what students need by way of suggested heuristics in math. What is more, you may find yourself wondering, as I have, whether there might be a larger instructional conversation to have with students about metacognition and what heuristic might work in what situation.

It is an interesting book for thinking about ill-defined problems. Teachers will find it can help them, but you may find yourself wondering if Dr. Costello is holding out on considering how metacognition and student choice regarding heuristic design might inform instruction. For now, the current book clearly causes thought and that is its real benefit—as to where David goes with his future scholarship, that goes beyond a review of this book.▲

Consider nominating someone for an award!

(These folks seem to enjoy their awards.)







Events at the Fields



ANGELICA MENDAGLIO
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Angelica Mendaglio is an instructional designer at Vretta Inc. in Toronto, Ontario, where she helps to create interactive digital mathematics lessons and activities for middle school students.

The Fields Institute is a centre for research in mathematics and mathematical sciences. Its yearly calendar of activities includes lectures from research mathematicians, and events for mathematics enthusiasts, educators, students, and the general public.

Mathematical Art at the Fields Institute

The Fields Institute building itself is a lovely structure in downtown Toronto, with several rooms with books and chalkboards for collaborative work, and many mathematical sculptures and other artifacts.

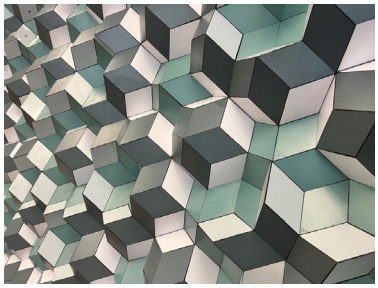
In 2017, the Fields MathEd Forum was visited by mathematical sculptor George Hart, who led attendees in the creation of a geometric sculpture made from cardboard (pictured right). You can find the patterns for his sculptures and build your own at home or with your class by visiting his website: www.georgehart.com/sculpture/sculpture.html.



Outside the building is a sculpture of three reflective triangular Borromean rings, titled "Intuition," that was unveiled at a 90th birthday celebration for Donald Coxeter at the Institute. Coxeter was a British-Canadian mathematician, who worked for 60 years at the University of Toronto, and made immeasurable contributions to the study of geometry. (I highly recommend *King of Infinite Space: Donald Coxeter, the Man Who Saved Geometry*, a lively biography by Siobhan Roberts, written for a general audience.)

The latest sculpture to be installed is the beautiful Wieringa Wall by University of Toronto graduate Ross J. Cocks, inspired by Penrose tiling. A Penrose tiling, named for British mathematician Roger Penrose, is a covering of the plane with polygons (think of square tiles covering a floor), but in a way that does not have any repeating patches. Here is a link to an interactive demonstration of Wieringa roofs on Wolfram (www.demonstrations.wolfram.com/PenroseTilingsAndWieringaRoofs/).





Left: *A photo of Frederick Helson, John Chaddam, Ronnie Brown, and Donald Coxeter unveiling Intuition at the Fields Institute.* Right*: A photo of the Wieringa Wall at the Fields Institute. Source: Fields Institute (www.fields.utoronto.ca/programs/ scientific/96-97/coxeter/coxeter.html)*

John Charles Fields and the Fields Medal



John Charles Fields Source: www.fields.utoronto.ca

John Charles Fields was a Canadian mathematician, born in 1863. Fields believed very strongly in the value of research in mathematics, and, as there is no Nobel Prize for mathematics, he created an international medal to recognize outstanding research in mathematics. It is awarded every four years at the International Congress of Mathematicians to (typically four) research mathematicians under the age of 40. The medal itself features a profile of Archimedes, and the original prototype of the medal is on display at the Fields Institute.



The Fields Medal Source:www.fields.utoronto.ca

Fields Medal Symposium

Every Fall, the Fields Institute hosts a presentation from a Fields Medallist, designed for a general audience. The presentation offers great insight into the work of that mathematician. Past symposium public opening lectures are available on the Fields Institute's YouTube channel. I highly recommend the lecture from Fields Medallist Manjul Bhargava titled "Patterns in Numbers and Nature."

The Symposium also includes a student night, which is intended to foster engagement between high school and undergraduate students and a Fields medalist, which is a terrific opportunity for students.

Math Circles

On Saturday afternoons, the Fields Institute is open to

high school students from the Toronto area to prepare for mathematics competitions or work on challenging problems.

Fields MathEd Forum

The Fields MathEd Forum convenes on the last Saturday each month from January to April, and September to November. Each month, the organizers invite speakers to give talks or lead discussions about topics on a theme in mathematics education, from Kindergarten to undergraduate studies (two themes from last year were "Coding, Computational Modelling, & Equity in Mathematics Education" and "First-Year College Mathematics: The Reality, the Expectations, and What's Ahead"). You can sign up to attend remotely through the Fields website, and can watch recordings of past Forums on the Fields Institute's YouTube channel.

The Margaret Sinclair Memorial Award Recognizing Innovation and Excellence in Mathematics Education

The Margaret Sinclair Award is awarded by the Fields Institute each year to an outstanding mathematics educator in Canada. Margaret Sinclair was a passionate educator, who had an unusual path to math education. Margaret attended Teacher's College at age 19, but began her studies in mathematics later in life, completing her bachelor's degree at 38, her master's degree at 45, and her doctorate at 51. In 2012, the Award was created by her husband Larry and their friends and family to recognize excellence in mathematics education. The list of past winners includes many names familiar to OAME members and conference attendees, such as Kathryn M. Kubota-Zarivnij, Ann Kajander, Ron Lancaster, Peter Liljedahl, and Chris Suurtamm. The winner each year gives a talk at the Fields Institute (typically as part of a MathEd Forum meeting), and it is always one of the most engaging Forum meetings of the year. Nominations are accepted until December 31—you can find out more on the Fields website.

Public Lectures

While the Institute hosts lectures intended for research mathematicians, it also hosts other lectures intended for a broader audience, on topics ranging from climate change to cryptography. You can join the Fields' mailing list on their website to stay up-to-date on the events at the Fields.

A Note from the Author

This will be my final report for the *Gazette*. It has been an honour to be a part of the gathering of mathematics educators, both here at the *Gazette* and at the Fields Institute, for the past five years. I hope this article inspires you to take a look at some of what the Institute has to offer. I wish everyone a happy and healthy school year full of rich mathematical experiences!

Editor: Many thanks to Angelica for reporting on an important mathematical resource in Ontario. Clearly, we need someone to continue the column, and if you attend the MathEd Forum regularly, please let the Editor know (gazette@oame.on.ca). ▲



Nominations for the OAME/AOEM Board of Directors 2024–2025

Each year, OAME/AOEM holds elections for open positions. Any OAME/AOEM member may stand for any position. Candidates are needed for the following positions:

- One (1) Vice-President for a two-year term from September 1, 2024 to August 31, 2026
- One (1) Director to represent Grades 7–12 for a three-year term from September 1, 2024 to August 31, 2027
- One (1) Director to represent Grades K–6 for a three-year term from September 1, 2024 to August 31, 2027

No nominations are needed for President-Elect at this time. Since we have gone to two-year terms for the Presidency, we only need a candidate for elections held in odd-numbered years.

A nominee and the nominator must be members of OAME/AOEM. The Nominations/Elections Committee will publish a slate of candidates, including candidate biographies, for consideration by the membership. Voting will take place online in March 2024, and is open to all OAME/AOEM members. The election results will be posted on www.oame.on.ca.

An online nomination form, as well as a summary of the duties of each position, is available at www.bit.ly/OAMEPositions. If you require additional information, please contact Paul Alves (paul.alves@oame.on.ca). The deadline for nominations is January 12th at 6:00 p.m.

Thank you in advance for submitting a nomination.

▲ What's the Problem? Playing with Processes: Part 1



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Shawn is a former mathematics teacher, department head, and consultant. He strongly believes in the central role of problem solving in the mathematics classroom. He continues to be involved in mathematical activities: presenting workshops, writing articles, working on local projects, and helping create mathematics contests.

Welcome back, problem solvers. I was intrigued by Jeff Irvine's column in the June *Gazette* (Irvine, 2023), where he mentioned the lack of articles in the *Gazette* on the mathematical processes. I have always thought that, by the nature of this column, I am implicitly talking about the mathematical processes throughout. Upon reflection (one of the processes, which even I do!), I see that I am not being explicit in my discussion of them, so they may not always be apparent to the readers. Similarly, when students employ the mathematical processes in our classrooms, it is not always obvious to the teacher. As a result, in this volume, I am going to shine a light on the processes through the problems that I choose, and try to highlight how you can collect evidence of student engagement in mathematical processes.

Before we get to the problem from the last issue, let's review the mathematical processes. The processes as they appear in the current curriculum are: problem solving, reasoning and proving, reflecting, connecting, communicating, representing, and selecting tools and strategies. As we encounter examples of these throughout this volume, I will highlight them in parentheses.

As Jeff pointed out in his column, the mathematical processes have been around for a long time. They were integrated into specific expectations (Ontario Ministry of Education [OME], 1998), appeared in the front matter and were given the status of "expectations" for the first time (OME, 2005, 2007). Currently they reside in the social–emotional learning strand in the elementary curriculum (OME, 2020) and the mathematical thinking and making connections strand in the Grade 9 curriculum (OME, 2021). At one point, I had all the Ministry guidelines dating back into the 1970's. The only one that I currently have, prior to 1998, is one from 1985. Even in this one, the processes are there—although all are not stated explicitly—in the section "Process Components in the Mathematics Program" (OME, 1985, pp. 17–21).

I have always felt that the mathematical processes are the things that we, as teachers, want our students to take away from our classroom. Much as I *love* the content of *mathematics*, few of our students will solve quadratic equations or use trigonometric identities in their lives beyond school. However, they will be presented with situations where they must solve problems and communicate their thinking clearly. The mathematical processes are just higher-order thinking skills that are useful beyond the mathematics classroom and beyond all classrooms in general. As such, helping students engage in these processes should be a central part of our role as a classroom mathematics teacher.

I have always liked the graphic for the Mathematical Processes from a Ministry of Education poster (OME, 2008a), shown in Figure 1. This website also has a link to the "Mathematical Process Package" that remains current.



Figure 1: *Mathematical processes graphic from an Ontario Ministry of Education resource*

The graphic depicts how problem solving is the driving force any time we use the other processes. As well, it shows how communication becomes vital for us to uncover the processes used by the student. It also depicts how we can use the mathematical processes to move from merely facts and procedures to deeper conceptual understanding.

It is also interesting to note that the categories from the achievement chart—Knowledge and Understanding, Application, Thinking, Communication—are shown in pink around the outside of the graphic. It seems to imply that certain processes fall under certain categories. This is also reflected in the rubric for the processes from the Ministry of Education (OME, 2008b). In the rubric, Selecting Tools and Computational Strategies and Connecting are placed under Application; Representing and Communicating are placed

under Communication; and Problem Solving, Reasoning and Proving, and Reflecting are placed under Thinking. These are convenient and the most natural place for the processes, but don't think of them as hard and fast rules.

The lines between the processes, and their placement in the categories, are blurry, not well defined. As such, for example, Representing shows up in the students' communication of the solution; hence, it is under Communication. However, the representations that the student chooses are tied in with the students' thinking about the problem, understanding of the concepts, and the connections they have made between the problem and various mathematical concepts. Hence, an instance of Representing may also be showing some Reasoning and Connecting, while falling under the categories Thinking, Communication, and Knowledge and Understanding.

Thus, there are going to be things that we observe that fall under multiple processes and multiple categories. In some circumstances, we may not agree how someone else would classify things, and this is okay. You may not agree with some of the labels that I have used. Use it as an opportunity to reflect on your own thinking. Can you understand why I used the classification I used? In some cases, you may change your mind about something. In others, you may understand why I classified as I did, yet still not agree with me. This too is fine. Just as different people may solve problems in different ways and may deem their own method as "the best," there is something to be gained by seeing the other method and thinking about it. The same is true for us thinking about students' thinking. It is good to reflect on our own thinking, and it is always valuable to discuss things with others. Some of the best professional development I have ever had involved talking with other teachers about student work in a very specific way. In an earlier part of my career, we had board-wide exams for certain math courses, and we would do conference marking. We would gather from across the board, bringing a couple of examples of student work, with copies. We would pair up, mark each other's papers, and then discuss our evaluations. This is very powerful, and I would suggest you try this with teachers in your own school, who are teaching the same course.

Let's take a closer look at how the processes are interconnected. In his well-known book, *How to Solve It* (Pólya, 1957), George Pólya breaks the problem-solving (**Problem Solving**) process into four steps. He also uses some guiding questions for each, some of which are reproduced below in italics.

- UNDERSTAND THE PROBLEM. It is impossible to solve a problem if you don't understand what is being asked, the notation or terms in the problem. This involves, in some cases, accessing prior knowledge. (**Reflecting**)
- DEVISE A PLAN. (**Selecting Tools and Computational Strategies, Representing, Reasoning and Proving**) *Have you seen it before? Do you know a related problem?* (**Reflecting, Connecting**)
- CARRY OUT THE PLAN. As you work through each step of your plan: *Can you see that the step is correct?* (**Reflecting**) *Can you prove that it is correct?* (**Reasoning and Proving**)
- LOOKING BACK. *Can you check the result?* (**Reflecting**) *Can you derive the result differently?* (**Reasoning and Proving**) *Can you use the result, or method, for some other problem?* (**Connecting**)

Throughout the four columns in this volume, I will focus

Reflecting				
Criteria	Level 1	Level 2	Level 3	Level 4
Uses metacognitive skills to determine which mathematical processes to revisit in order to reach the goal	Applies metacognitive skills with significant prompting in determining which mathematical process to revisit in order to reach the goal	Applies metacognitive skills with minimal prompting in determining which mathematical process to revisit in order to reach the goal	Applies metacognitive skills independently in determining which mathematical process to revisit in order to reach the goal	Applies metacognitive skills independently in determining which mathematical process to revisit in order to reach the goal, with a broader view of the goal
Reflects on the reasonableness of answers	Makes minimal connections between a prior estimate and the solution	Makes some connections between a prior estimate and the solution	Makes appropriate connections between a prior estimate and the solution	Makes appropriate connections between a prior estimate and the solution, and provides insightful comments

Figure 2: *Rubric for Reflecting*

on one of four of the mathematical processes: Reasoning and Proving, Reflecting, Connecting, and Representing. I will go in-depth for the focal process, yet I will still point out the other processes as we go along.

I will not focus on, or highlight, Problem Solving, as the column focuses on using problem solving in the classroom, so it should be obvious. Selecting Tools and Computational Strategies will also not be a focus. I find it to be the most obvious one, and I will just indicate it during the problem-solving process.

You may have noticed that Communicating hasn't been indicated above. This is because communication isn't necessary during problem-solving. However, it is a vital component for the teacher and the student, and for group problem-solving activities. For the teacher, it is our way of observing the other processes in action. If we see only the answer, we have no idea how the student arrived at that answer. There may be some cases, where an incorrect answer had ticked all our boxes to a high degree, but the only problem was a slight miscalculation, a mis-transcribed number, or some other tiny error. On the other hand, a student with the correct answer may have arrived there by accident, using misguided reasoning and sloppy work. For the student, encouraging them to communicate helps them strengthen their conceptual understandings, as well as fine tuning the other mathematical processes along the way.

Another thing to keep in mind is that having the students provide a well-written solution is *not enough*! Lots of the thinking processes and problem solving goes on behind the scenes and isn't apparent in the written solution. Therefore, the teacher needs other tools at their disposal. Interviewing students is insightful, but takes a lot of time. As such, you can interview students on an ongoing basis, with a few every week, so that you can go through the one-on-one process several times during the span of the course. Another revealing way to see the processes in action is to have the students working in groups at the board. Stopping by and listening to the students' conversations as they solve a problem offers you a window into their thinking process, their use of the mathematical processes, and their communication skills. Sticking your head into a group and asking, "Why are you doing…?" or "What if…?" can give you a lot of insight into several students' thought process.

For the reasons indicated above, I will not focus on Communicating; however, I will give some examples of how to use it within the indicated problem.

In this issue, I want to focus on **Reflecting**. This is a very powerful process that sometimes gets short-changed. Many people think of Reflecting as just "checking your answer," and even then, many students don't do that. Reflecting should be used in an ongoing way during the problem-solving process, to see if the results of our plan seem reasonable and that the investigation is going in the right direction. This is reflected in our discussion of Pólya's four steps earlier in this column. This can also be seen from the Ministry's Rubric for Reflecting (OME, 2008b), shown in Figure 2.

At the end of the process, Pólya's guiding question, "Can you derive the result differently?," is rarely, if ever, considered. A student may wonder, "Why would I want to re-solve a problem I have already solved?" The reason is there may be a "better" solution out there, or one that allows you to make connections to different parts of mathematics (**Connecting**). It is also useful for students to discuss their solution with someone who solved the problem differently, as they may each learn something, even if they were both able to correctly solve the problem. This is also a reason why, when students are working in groups, it is good for the teacher to purposefully select a couple of groups to present their findings, so that multiple perspectives, techniques, and tools can be brought to light. As such, it is important for the teacher to explore the problem, anticipate difficulties that students may encounter, as well as possible methods that they may use to help them along the way.

Also important, the final "step" is a place where we can also ask our own questions. I have pointed out in many of my columns that the "answer" to the problem that you were given may just be the starting point to some very rich mathematical exploration. It is imperative that we condition our students to ask "What if…?" questions when they are done, and encourage them to carry out their own investigations. This may suggest another criterion to add to the rubric in Figure 2: *Reconsiders the problem, once solved, for alternate solutions, extensions, generalizations, and new investigations.*

Now we are ready for the problem from the last issue (finally!):

> *Riley has 64 cubes with dimensions 1 × 1 × 1. Each cube has its six faces labelled with a 2 on two opposite faces and a 1 on each of its other four faces. The 64 cubes are arranged to build a 4 × 4 × 4 cube. Riley determines the total of the numbers on the outside of the 4 × 4 × 4 cube. How many different possibilities are there for this total?*

This problem was #23 from the Team Problems from the 2022 Canadian Team Mathematics Contest, hosted by the Centre for Education in Mathematics and Computing (CEMC) at the University of Waterloo. A group of six students work as a team to solve problems presented in the

contest. A limited number of teams sign up to compete at the University of Waterloo; other teams can compete at their own school. The competition is made up of three parts: an individual event (students work on problems individually), a team event (students work in their teams to solve problems), and a relay event. For more information on the Canadian Team Mathematics Contest and other contests and resources from the CEMC, check out their website at www.cemc.uwaterloo.ca.

Let's follow Pólya's steps:

1. UNDERSTAND THE PROBLEM. We are dealing with 64 cubes, labelled with 2 on two opposite faces and 1 on each of the other four faces. What does this look like? We may decide at this point to introduce some manipulatives to help us out (**Representing, Selecting Tools and Strategies**). Linking cubes with little pieces of tape on which we could write 1 or 2 might be helpful. Also helpful for visualizing the problem would be a net like the one in Figure 3 below.

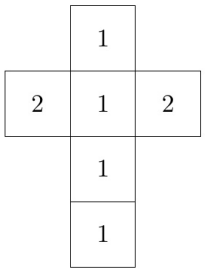


Figure 3: *Net of the cubes used in the problem*

Notice that the four faces that touch a face with a 2 on it all have 1's on them. Similarly, a face with a 1 is touched by two faces with 2's and two faces with 1's. This will be useful for solving the problem.

Since the cube is 4 × 4 × 4, then each face has the faces of 4 × 4 = 16 of the unit cubes visible, as shown in Figure 4. Hence, for the whole cube, 6 × 16 = 96 unit cube faces are visible. Thus, the largest the sum could possibly be is 96 × 2 = 192 and the smallest it could possibly be is 96 × 1 = 96, although we may not be able to arrange the cubes to give either of these results (**Reasoning and Proving**).

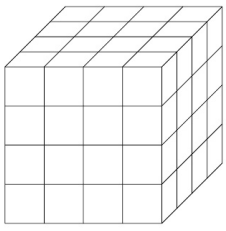


Figure 4: *A* 4 × 4 × 4 *cube*

Another important thing to understand is that the problem asks for the number of possible *totals*, not configurations. For example, Figure 5 shows two possible faces, which are different configurations, yet have the same total. We must be careful to enumerate the different totals for the cube.

2	1	2	2
2	2	1	1
1	2	2	2
1	1	2	1

2	2	2	1
2	2	2	1
2	2	2	1
1	1	1	1

Figure 5: *Two possible faces on a 4 × 4 × 4 cube with same total*

2. DEVISE A PLAN. Now it is time to decide how to proceed. The important idea will be to see how the numbers can appear on the faces. This comes down to visualizing the problem. The easiest way to do that, is to use a physical model like the one involving linking cubes, discussed in step 1. Students likely will not use the model, but may be visualizing it in their mind or drawing a picture (**Representing, Selecting Tools and Strategies**).

If a plan doesn't come immediately to mind, playing around with the problem may help. You may look at some different configurations of a 4 × 4 × 4, looking for patterns. We will use a tried-and-true problem-solving method: *solve an easier problem*. We will look at easier configurations to see if we can see any patterns (**Selecting Tools and Strategies**). By examining the 1 × 1 × 1 cube, the 2 × 2 × 2 cube, and the 3 × 3 × 3 cube first, we will hope to discover some properties that all cubes of this form have that will allow us to solve the problem for the 4 × 4 × 4 by doing our experimentation on easier cases.

3. CARRY OUT THE PLAN. At this point, we have half of a plan. We are going to look at some cases to see if we see a pattern. This reflects the nature of the problem-solving process. We don't necessarily go through the process 1–2–3–4, then dust off our hands because we are done. Sometimes we have to cycle back to earlier steps to get a better grasp of the problem, refine our plan, or start solving a new problem that we devised from the last step. Let's proceed with our plan.

Case 1: 1 × 1 × 1 cube

This case it trivial because it is the cube we are given. All sides are visible, so the sum is 8; thus, in this case, there is only one possible sum for the exposed faces. This is not really that helpful, yet it is good to consider the easiest case, as it may sometimes be useful (and it is the easiest one to analyze).

Case 2: 2 × 2 × 2 cube

This case is a bit more interesting, since some of the faces are hidden where the cubes come together. However, no matter how the cube is assembled, the sum is 32. Yet there is still something to be discovered. All eight cubes are "corner" cubes, with three faces showing. No matter how a cube is placed in the corner of the larger cube, a 2 and two 1's are visible. Why does this happen? If we look at a corner cube highlighted in Figure 6, we see three faces: one red (top), one blue (front), and one green (side). Each of the hidden faces will be directly opposite of a visible face. Hence, for each pair of opposite faces, exactly one of them is visible. Since each cube has two 2's on them, and they are on opposite faces, exactly one 2 will be visible for each corner piece. Therefore, for each corner piece, a 2 and two 1's will be visible (**Reasoning and Proving**). Many students will observe that each corner piece has a 2 and two 1's visible, but not think to justify it. Others may have justified it for themselves, but not given any indication that they have. This is an excellent place for a well-asked question from the teacher to either uncover the thought process or to highlight the importance of the step (**Communicating**).

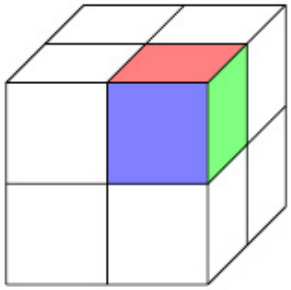


Figure 6: *A corner cube*

In our understanding of the original problem, we reasoned that the minimum and maximum possible sums for the 4 × 4 × 4 cube were 96 and 192, respectively (**Reflecting**). Since each corner has one 2 and two 1's exposed, it is impossible for the visible faces to be all 1's or all 2's, so our minimum is larger than 96; in fact, it must be at least 104 (why?). Similarly, our maximum must be lower that 192 and must be, at most, 176 (**Reasoning and Proving**).

We may now wonder what other types of configurations might occur in the 4 × 4 × 4 cube (**Reflecting**). Let's continue with easier cases first.

Case 3: 3 × 3 × 3 cube

This is where things get interesting. We discover that there are four different possible configurations for the cubes within the larger cube, which we can see will extend beyond the cube to all larger cubes (**Reasoning and Proving**). There are corner cubes (in red), edge cubes (in blue), face cubes (in green), and interior cubes (unseen). We saw the corner cubes in case 2, and we know their configuration.

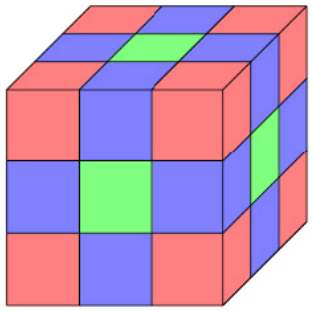


Figure 7: *The "types" of cubes on a 3 × 3 × 3 cube*

At this point, we should wonder what could be showing on the faces of the different "types" of cubes within the large cube. In case 2, we saw that the sum of the faces visible on a corner cube was always 4. Is this the same for the others? We need to find out (**Reflecting**). As such, we can refine our plan to see what we can figure out about the other types of cubes.

Examining the edge cubes (blue) first, we see that two adjacent faces are showing. We know the two 2's are opposite to each other, so an edge cannot show two 2's. It **is**, however, possible to have two 1's. Thus, an edge cube can show two 1's, for a sum of 2, or a 1 and a 2, for a sum of 3. This would reduce our maximum further (**Reasoning and Proving**).

The next two are the easiest. The hidden cubes obviously do not contribute anything to the total. The face cubes show just one face, which may be either a 1 or a 2. Thus, we can refine our plan again. For the 4 × 4 × 4 case, we must determine how many cubes of each type are present and how they contribute to the sum.

A cube has eight corners, twelve edges, and six faces. Using Figure 4 (i.e., the 4 × 4 × 4 cube), we see that there are two edge cubes per edge and four face cubes per face (although, not important, this may remind us of Euler's formula for polyhedra, $V - E + F = 2$, which we can use to check our results for the cube 8 – 12 + 6 = 2 (**Reflecting, Connecting**)). Thus, the 64 1 × 1 × 1 cubes end up in the following configurations:

- 8 corner cubes
- 12 × 2 = 24 edge cubes
- 6 × 4 × 24 face cubes
- 8 hidden cubes

We can now refine our minimum and maximum values from what we know about the types of cubes. Hence, the minimum value of the sum is

$$8 \times 4 + 24 \times 2 + 24 \times 1 + 8 \times 0 = 104,$$

and similarly, the maximum value of the sum is

$$8 \times 4 + 24 \times 3 + 24 \times 2 + 8 \times 0 = 152$$

(**Reasoning and Proving**). We know that these sums are attainable, but what other values are allowed? Again, we must now refine our plan (**Reflecting**) to see how the total will depend on our various cube configurations. This may remind students of the painted cube problem, if they have ever done it (www.math4thenines.ca/resources.html#paintedcube) (**Connecting**).

Thinking about it, or playing with a physical model, we discover that we could easily change any face cube to go from showing a 1 to showing a 2 or vice versa. This would change the overall total by 1 (increasing when a 1 is replaced by a 2 and decreasing otherwise). Similarly, an edge piece could be changed from showing two 1's to a 1 and a 2.

If we imagine starting with the minimal sum of 104, each face must be showing a 1, and each edge must be showing two 1's. We could change the faces one at a time from 1 to 2, and then change every edge one at a time from showing two 1's to a 1 and a 2. This means that we can have every possible value of the sum from 104 to 152, inclusively, which means that there are 152 – 104 + 1 = 49 possible sums (**Reasoning and Proving**).

4. LOOKING BACK. We have come to our result. As we have been checking and refining as we go, the result seems valid. No other method may come to mind. However, if you have your mathematician's hat on, you may wonder what happens for a 5 × 5 × 5 cube. We could use our method and solve this and that would be an *extension* of the problem. However, let's reach for the brass ring and *generalize* the problem and try to solve it for an $n \times n \times n$ cube (**Reflecting, Reasoning and Proving**).

Using what we discovered solving the original problem, we know that we need to determine how many cubes of each type are on the larger cube. Since we know how many edges, faces, and vertices a cube has, no matter the size, we just need to know how many cubes are on each edge, face, and vertex in terms of n (**Connecting**).

The vertices are the easiest, as there are eight vertices and so, eight corner cubes for all $n \geq 2$ (why?). To determine the number of cubes per edge and face, let's look at a single face of the $n \times n \times n$ cube (Figure 8).

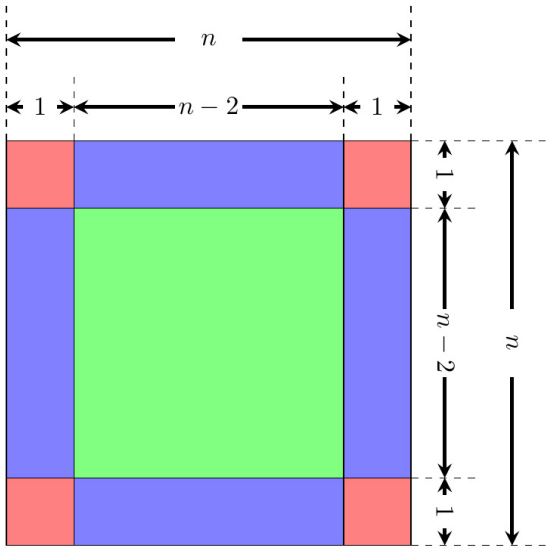


Figure 8: *Cube types on a face of a n × n × n cube*

Each edge will contain $n - 2$ cubes, and each face will contain $(n-2)^2 = n^2 - 4n + 4$ cubes (**Representing**). From our earlier solution, we see that since there are twelve edges, where each edge cube shows a total of 2 or 3, and there are six faces, with each face cube showing a sum of 1 or 2, we can determine the minimum value of the sum of all cubes to be

$$8 \times 4 + 12 \times (n-2) \times 2 + 6 \times (n^2 - 4n + 4) \times 1 = 6n^2 + 8.$$

We can check that for the original problem, which corresponds to $n = 4$, the above expression gives 104, exactly what we determined in the problem (**Reflecting**). Similarly, we can determine the maximum value of the sum of all cubes to be

$$8 \times 4 + 12 \times (n-2) \times 3 + 6 \times (n^2 - 4n + 4) \times 2 = 12n^2 - 12n + 8.$$

We can again check that for the original problem, the above expression gives a sum of 152, the same as when we solved the problem. This means the total number of values for an $n \times n \times n$ cube will be

$$(12n^2 - 12n + 8) - (6n^2 + 8) + 1 = 6n^2 - 12n + 1.$$

Hence, we have generalized our solution to any $n \times n \times n$ cube for $n > 2$.

Observant readers may have noticed an algebraic identity hidden in our generalization. If we consider the four types of cube–hidden, face, edge, and corner–we can come up with the following (**Connecting, Representing**):

$$(n-2)^3 + 6(n-2)^2 + 12(n-2) + 8 = n^3.$$

There are many other things you could do with the problem. You could look at what happens if you label the cubes differently. You could look at the problem with tetrahedral dice. There is room to play, so it would be worthwhile to play.

As always, it is important to consider the students you are teaching. If you are dealing with younger students, an algebraic representation may be beyond their current skill set (although some students may be able to explain the patterns with words). It is important for teachers to work through the problem themselves, to see how many different directions that the exploration could go. This is valuable for setting your own expectation for what the students should be able to do with the problem.

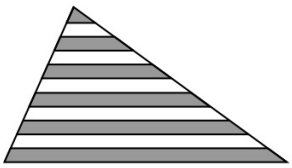
In many cases, when you go "beyond" the problem, you are getting into deeper and more abstract mathematics. Students may not think to do this unless it is modelled for them and shown that this step is valuable. It helps to create creative critical thinkers, which should be one of the goals of all teachers.

Looking back at this problem, if we imagine what a student may write up as a solution, it is easy to see that much of the evidence of the student's use of the mathematical processes would not be evident. Even if they were to discuss their solution, some details would be left out. The teacher should be ready to ask, "Why did you do that?" "Did you try/think of any other methods?" "What would happen if…?" A good balance of conversations and observations along the way, as well as products, will give us the best picture of our students.

As I mentioned earlier, the mathematical processes are always on my mind when I write this column, even if I may not explicitly refer to them. Interested readers may enjoy looking over past columns for examples of Reflecting. Below are a few problems that I have looked at in past columns, where there was some looking back, extending, or generalizing.

1. The right triangle, having sides 3, $\sqrt{7}$, and 4, has the strange property that the two integer lengths sum to the value under the square root sign for the length of the third side. Find another such triangle. (Godin, 2008)
2. Calculate: $6^2 - 5^2$, $56^2 - 45^2$, and $556^2 - 445^2$. Find and describe the pattern in the questions and answers, and prove that the pattern continues. (Godin, 2013)
3. A band of eight pirates is going to disband. They have divided up all of their gold, but there remains one GIANT diamond that cannot be divided. To decide who gets it, the captain puts all of the pirates (including himself) in a circle. Then he points at one person to begin. This person steps out of the circle, takes his gold, and leaves. The person on his left stays in the circle, but the next person steps out. This continues with every second pirate leaving, until there is only one left. Whom should the captain point at if he wants to make sure he gets to keep the diamond for himself? (Godin, 2017)

Now, time for your homework. Keep the mathematical processes in mind when you think about this problem, and see if you can figure out which process we will focus on in the next issue.



A triangle of area 770 cm^2 is divided into 11 regions of equal height by 10 lines that are all parallel to the base of the triangle. Starting from the top of the triangle, every other region is shaded, as shown. What is the total area of the shaded regions?

Until next time, happy problem solving!

The author would like to thank fellow $O_3$4ME chapter members, Marian Small and Christine Suurtamm, for their insightful feedback and comments. Their suggestions helped to greatly improve this column. I look forward to working with them on the remaining columns in this volume.

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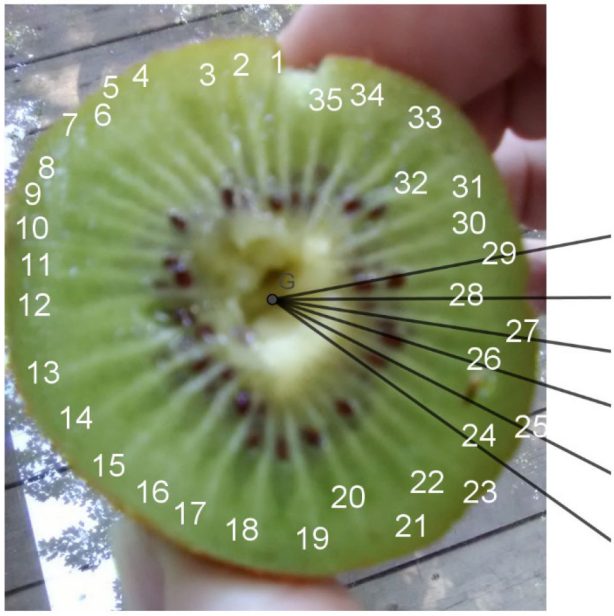
1. Complete the Google **Nomination form** appropriate to the specific award, found at www.oame.on.ca (look for the **Awards** tab on the left of the page). You may include a one-page letter of recommendation.
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   - include the nominee's name and the award in the letter
3. Include the **Curriculum Vitae**, of up to five pages, for award nominees, except for Elementary and Secondary Staff awards.
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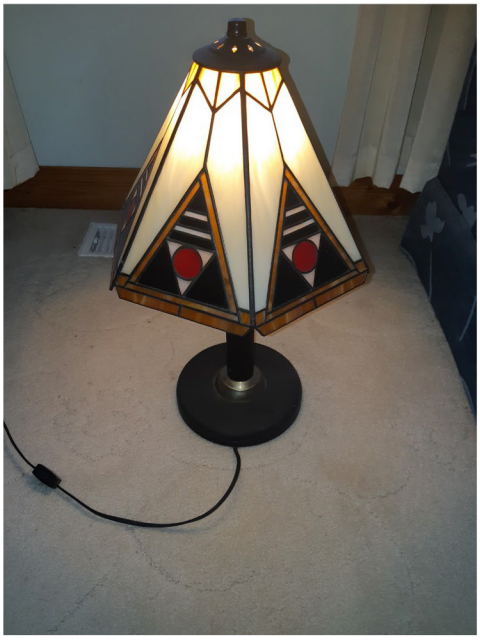
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The awards will be presented at the OAME Annual Conference in May 2024.

Fun with Fruit!

Students can use a slice of kiwi for counting or measuring angles. In the picture, there is a notch at the top that was used as a starting and ending point for counting. Notice that 35 rays are close to evenly spaced, making the angles approximately 10 degrees. Perhaps kiwi orienteering in the schoolyard can become a new sport?

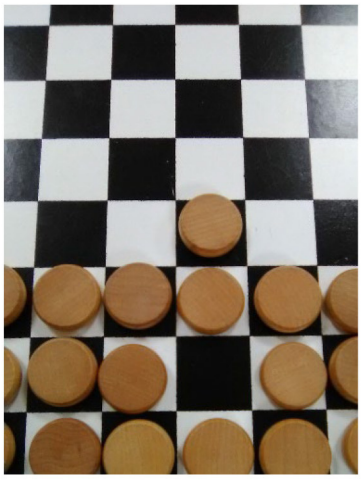
The other beneficial use in this is for students who are learning how to work with photos in GeoGebra (that was used here) or Geometer's Sketchpad®. What aspects of this photo do you think assist communication? What might be clearer?





Conway's Soldiers

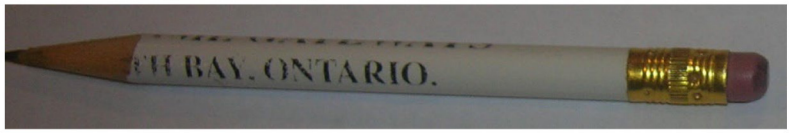
An infinite checker board is completely filled below a line, and empty above. Pieces can only hop as in checkers (one piece moved in picture, and the hopped piece needs to be removed). How far can the infinite army advance?



OAME Lamp?
Photo: Jacqueline Hill



Which conference is each pencil from?
Can you guess the conference name, year, and location?
Photos: Ralph Connelly





Which conference?
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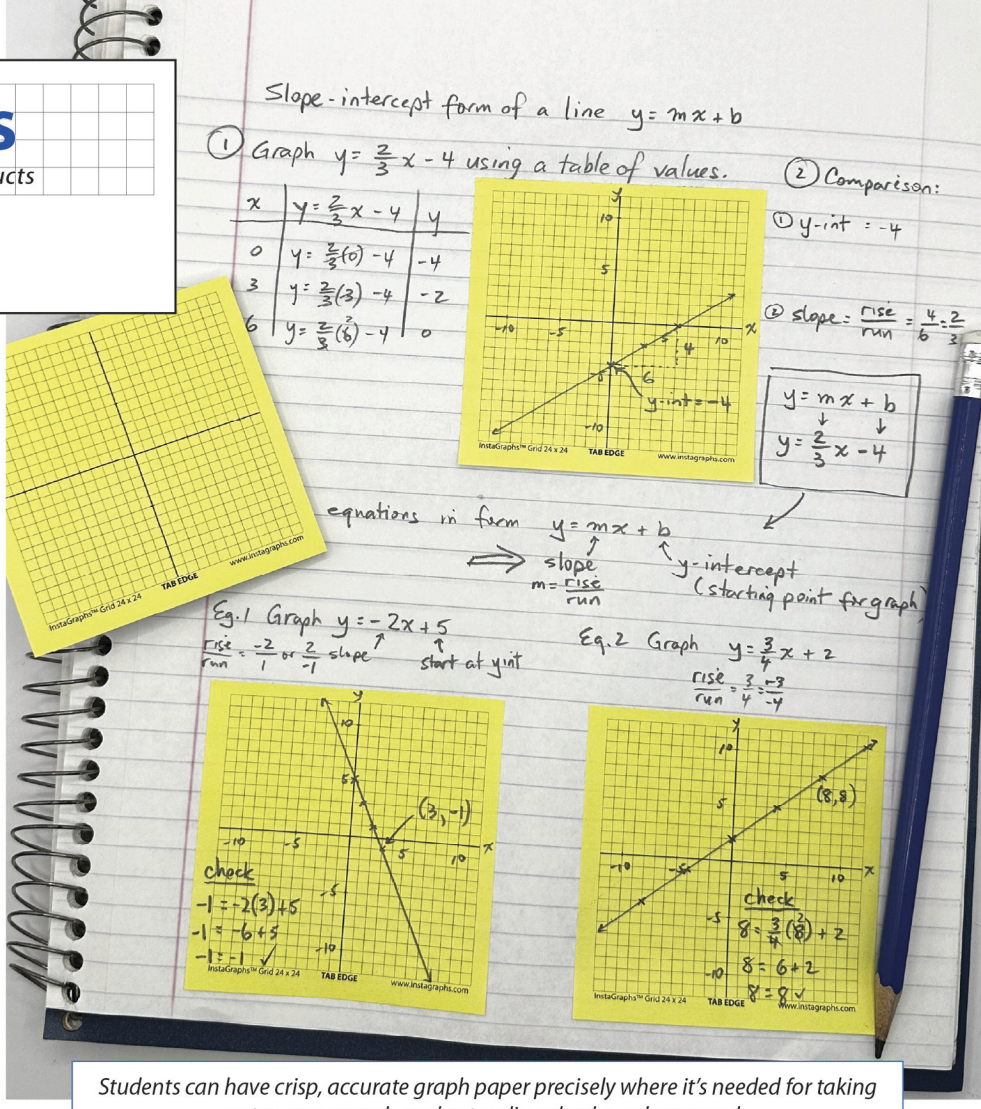
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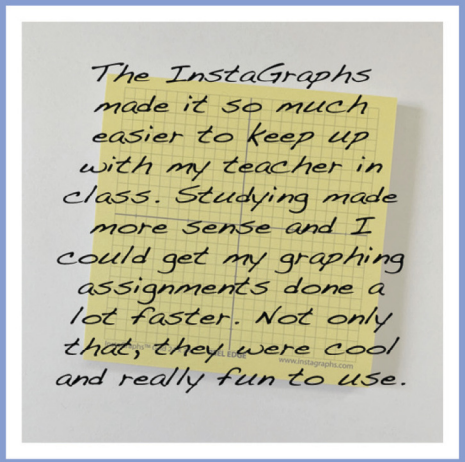
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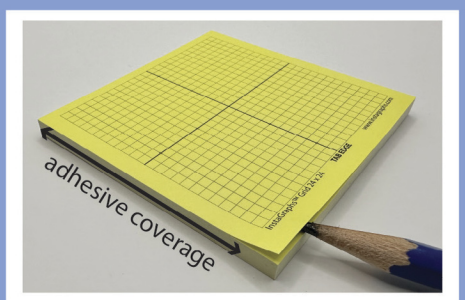
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