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Ontario Mathematics Gazette

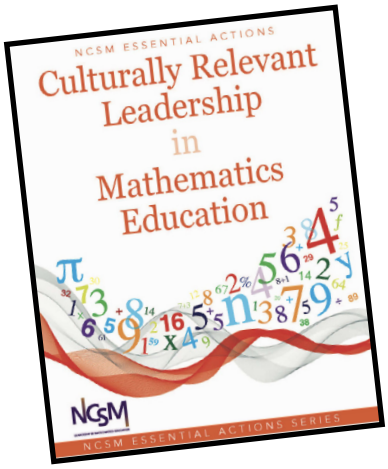
OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

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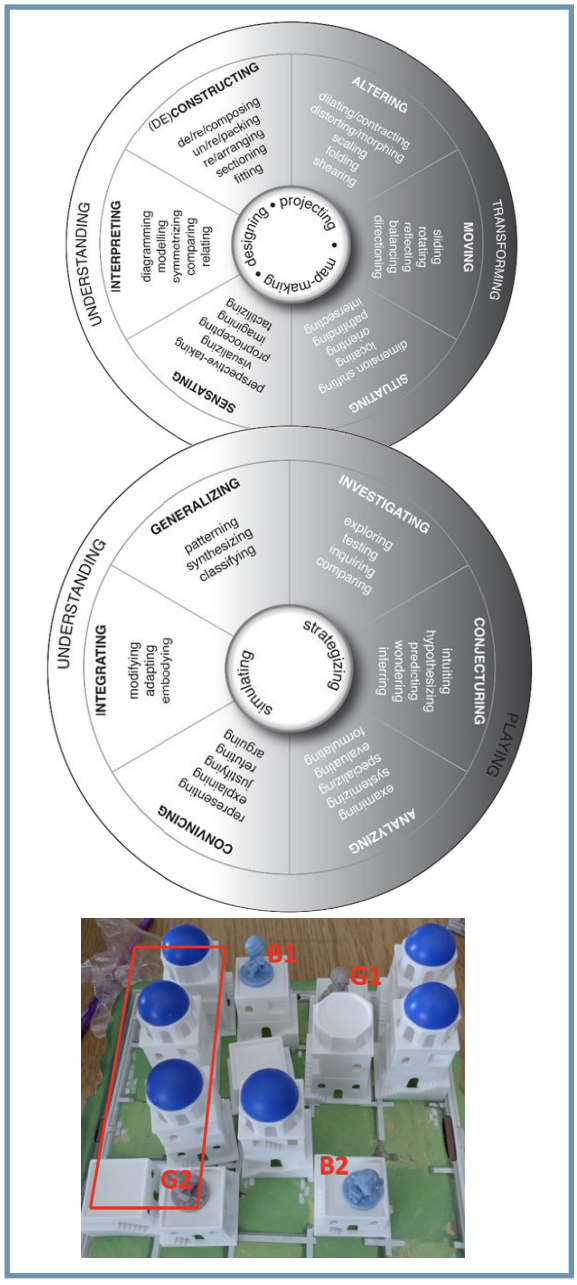


See Book Review: Take Action with Culturally Relevant Pedagogy

See President's Message



See Spatial and Logical Reasoning in Game Play: The Case of Santorini



Submission of Articles

The *Ontario Mathematics Gazette* (*OMG*) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *Gazette*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles are to be word–processed, MS Word is preferred, and prepared according to the *Publication Manual of the American Psychological Association*, Seventh Edition. However, please use single-line spacing (not double). Articles should not exceed five numbered pages of text, and figures, images, and photographs should be placed in the text close to where they belong, with captions. The photographer's permission is required, and for photos of students under the age of 18, the written permission of a parent or guardian is required.

Please submit your article in one blind file (i.e., identity of author is not evident), and include author names, contact information including email and mailing addresses, photos—head and shoulders, biographies—less than 100 words, and all content removed for blinding in a second file. Please email these two files to Tim Sibbald at gazette@oame.on.ca.

Upon review, you will be notified whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected. The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME/AOEM.

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Submission of Advertisements

Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** by email or at the above address. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be print ready, and colour advertisements should have no bleeds.

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▲ Editor's Report



TIM SIBBALD, OCT, PH.D.
gazette@oame.on.ca

Tim Sibbald is a professor in the Schulich School of Education, Nipissing University, with a focus that includes mathematics education. He is a former President of OAME/AOEM and has held a variety of other OAME/AOEM posts over the years.

Inspiration is a key element of engaging teaching. I recall in my classroom days, pursuing ideas that crossed my mind, but wanted to know the details before I would use the ideas with students. However, as I read more about project teaching, discovery methods, and self-determination, it dawned on me that students find it motivating, if there is a good idea, when the teacher does not know all the details. It was with significant trepidation that I dabbled with this approach in the classroom. I only used it now and then, but every time that I did, it went quite well.

I was explicit with my classes about not knowing all the details. That stance facilitated student motivation because they understood they might develop an understanding before the teacher. At the same time, I did have to highlight, for the least confident students, that I knew enough that there was an important instructional point to pursuing the idea.

The approach helped develop instructional serendipity—in the sense that one could find inspiration that connects to classroom expectations and pursue it with an authenticity of recognizing a mathematical opportunity, without necessarily knowing all the details.

An example came to my attention this last week when the OAME 2024 Annual Conference logo caught my eye. Consistent with my experiences of serendipity, my mind went in multiple directions at once! I let it wander, initially, before reigning it in. "Embracing change" speaks volumes for our current days in teaching; it is necessary to respond to changing local circumstances, addressing modifications underlying policy shifts, or our own professional growth that often entails altering or expanding our own perspective of instruction. Yet the element that caught my attention for instruction is the wonderful pattern of circles.

The mathematics of the patterns of circles within the logo was clearly a consideration of the graphic designer, and it is gold for talking about patterning in math class. An open-ended approach to questioning can elicit a variety of ideas about the patterns that are arising. Do you see a large circle

filled with shrinking circles? Or a very small circle surrounded by increasingly larger circles? If you pick circles of one colour, do you notice that they form a spiral—is that mathematically necessary or something the graphic designer imposed? And what is the shape of those spirals?



At this point, I should address that these are general brainstorming questions, and some tweaking to make them grade-level appropriate is likely to be necessary. Consider that describing the spirals with and without analytic geometry may be two different things—can high school students actually develop a closed-form algebraic description of the spirals? It does not matter! Why not go after characterizing the pattern, perhaps identify points or coordinates that would be on the curve, and simply see what can be done. Do we need a definitive answer, or is there merit in challenging students to see what they can do with it?

Some teachers might enjoy the ideas until they dig in and realize that there are ill-defined aspects lurking in the inspiration. Suppose you take the big outer circle and begin filling it with smaller circles. The first ring of circles in the logo includes 12 smaller circles, but why 12? Other values could have been chosen, and that might change the outcome. Yet, this opens the door to using parallelism instructionally in the classroom, where maybe we take an educated guess as to how much importance we wish to place on the detail. It might be beneficial for it to be a multiple of four, corresponding to quadrants, so perhaps split students into groups investigating 8, 12, and 16? First, however, consider how the pattern changes—it would be counter-productive to do this if the 12 small circles lead to 8 circles in the next iteration—in fact, the initial number of smaller circles is fixed for each iteration, and 12 circles generates 12 circles for the next ring of the pattern. Therefore, splitting a class into groups makes sense…, but consider that 16 could result in a lot of busy work that might reduce development of mathematical thinking; perhaps 10 would be more effective?

As I write this, it becomes apparent how the ill-defined aspect is giving rise to broad instructional consideration of how to open the task for students, but reign in the ill-defined aspects enough to provide a genuine accessible opportunity to engage in the task. Meanwhile, it has pedagogical opportunities emerging, where the different groups can explain to the class what they found. Comparison of patterns based on different starting numbers will likely become hypothesized or apparent.

I have a personal love of letting inspirations like this percolate for a day or two while mulling the general parameters of the inspiration. In an early version of this editorial, I focused on angular details and then subsequently the distances from the centre. I will leave that to the reader's curiosity and suggest coding using a GeoGebra script to generate the pattern could challenge students. There is also the cross-curricular opportunity to explore the design, using aesthetic constructivism. Perhaps dripping candle wax onto paper from different heights could create splatter circles suited to the creation of such designs? Or tracing different-sized Canadian coins?

I will cut this off at this point, as I could go on and on and…. You get the message. Inspiration is like that and, as a teacher, my own motivation and passion for playing with a problem that is ripe with patterns is coming through. That has a clear benefit for your students. I have, over the years, had success with this approach, as an infrequent, but very powerful technique in the classroom. It can be surprising who exercises perseverance and comes out with insightful contributions. Perhaps the most inspiring aspect, for me personally, has been that it is not uncommon that a few weeks later, a student will approach me about what they have found, and it becomes evident that they continued playing with it.

I have done this with pre-service teacher candidates. It was a pleasant surprise last year, when two students caught up to me a semester after I presented a particular problem. They had made progress! Over a chat of half an hour, and with meaningful contributions by each of us, we came to fully understand the details related to the question. Curiously, a decade earlier, one of my Grade 9 students had hypothesized exactly the result. Serendipity to see a question, and success, span such a range of students.

In This Issue…

Sandra Jean Price brings perspective to the challenges facing teachers and some of the professional learning occurring with the Board of Directors. There is a special recognition of **Bill Otto** for his many years supporting OAME/AOEM. **Jacqueline Hill** touches on the history of

NCTM. In her first column, **Pamela Brittain** highlights many things that are going on with the Fields Institute for Research in Mathematical Sciences. **Heather Theijsmeijer** brings news from the OMCA.

Carly Ziniuk blasts off into the resources NASA has for teachers. **Jeff Irvine** draws attention to cognitive load theory. **Lynda Colgan** addresses details of coding that were given considerable attention in the early days of computer use in the classroom, but seem to need refreshing in the context of coding. **Iain Brodie** and **Hatice Beyza Sezer** interview **Lisa Anne Floyd**, who highlights different approaches to computational thinking. **Shawn Godin** continues to consider the math processes through an examination of a triangle problem. **David Costello** considers social–emotional learning during modelling.

What is not in this issue is MB4T. It will return, but **Jennifer Holm** needs time to adjust to her baby boy! Also missing from this issue is **Jérôme Proulx**, who intends to be back in the next issue.

The issue also includes a book review by **Michele Mellow** and an article about the game Santorini by **Janelle McFeetors**, **Qingna Jin**, and **Nimrah Ahmed**.

Thank you!

I owe a round of thanks to many people in this issue.

The Communications subcommittee of the Board of Directors gives oversight to the *Gazette*, and their ongoing support is appreciated. Thank you! I also need to thank Greg Clarke and Heather Theijsmeijer, NOMA Chapter Rep, for facilitating an online meet-up of *Gazette* columnists and editorial team members.

The columnists got their columns in early to facilitate travel by members of the editorial team. Their diligence and timeliness are impressive and speak to the professionalism driving the authorship. Thank you!

The editorial team is the other side of the professionalism. Where would I be without them? It is wonderful to have met with Anne, Peter, Gitta, and Penny face-to-face last summer because so often we address issues back and forth through email, that the personal side slides a little. There was also support from John Rodger and Ralph Connelly, who continue to help out whenever needed—so good to simply know I can call on them when the need arises. Through and through, the editorial team is the heart of the *Gazette*! The attention to detail, every last word and punctuation mark, is exactly the reason most readers do not notice the details—proper polishing shines! ▲

▲ President's Message



SANDRA JEAN PRICE
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Sandra Jean has been on the Board of Directors of OAME/AOEM for almost 20 years. She is currently teaching in the Durham District School Board and has more than 25 years of teaching experience. She has had many roles, including classroom teacher, Math Coach, and Instructional Leader of online Math Additional Basic Qualifications (ABQs). Sandra Jean has contributed to a variety of Ministry projects and was a coordinator of the most recent OAME/AEMO writing projects. Sandra was the recipient of the 2008 OAME Award for Exceptional and Creative Teaching in Elementary Mathematics for her innovative instructional practices, integration of technology, and belief in success for all.

You are likely reading the *Gazette* and this message in December, having completed several months of school, and are looking forward to a well-deserved winter break. As I am writing this message, however, it is still September, and we are in the midst of a heatwave… summer's last hurrah. It seems very strange to be writing a "President's Message" so far in advance, but there is much to do behind the scenes to ensure the publication is ready on time, while maintaining the high quality we expect.

Even so, the first part of the school year seems to have vanished. We are all busy delving into the curriculum and giving our very best efforts to ensure success for all of our students. We need to remember, however, that sometimes our very best may differ from day to day, and that we need to look after ourselves as well.

In many ways, this year feels like the school years we had prior to the global pandemic. It should feel closer to normal when one considers the challenges of the last few years, but it doesn't really feel "normal." Teachers are already tired, often being pulled in multiple directions, both at school and at home. Students are still working to close gaps that were exacerbated by the pandemic. For some, these are purely academic gaps, while for others, these are more social and emotional. We all experienced strains and even trauma during the pandemic, and the education system is still coping with the aftermath.

Additionally, this year, many educator unions are, yet again, dealing with the uncertainty of beginning the school year without a contract. The last time job action was taking place was just prior to the world shutting down. This is a

further layer of stress added to the lives of teachers, students, and parents.

Ultimately, it is up to us to choose how to manage the stresses of everyday life. Many things cannot be controlled, but one thing we can control is our attitude. It is my hope that you remain positive and take some time for yourself to refresh and enjoy your family. This is likely to have benefits after the fact with your classes and students. Recently, I have begun scrapbooking again, and have started doing some yoga for the first time in a few years. These activities help me remain mentally strong and resilient. In essence, they nourish my soul. I have also invested time in my professional learning. During the last few years, I have been learning much about Indigenous Ways of Knowing and have been working on creating my own personal land acknowledgement that holds deep meaning for me. Recently I have been reading *21 Things You May Not Know About the Indian Act* by Bob Joseph, a book that has made me realize that it is only when we learn about the *Indian Ac*t and the resulting atrocities, that true reconciliation becomes possible.

Regardless of how you remain strong and healthy, OAME/AOEM can play a role. One way that I hope OAME/AOEM helps you is through professional learning and the building and strengthening of connections. Last April, we had the first face-to-face post-pandemic conference, "Empowering Me in 2023," which was incredibly successful. Educators were able to attend high-quality sessions, be inspired by exemplary mathematics leaders, and connect with each other in a way that hadn't been possible for a while. Unfortunately, some were not able to attend due to a shortage of supply teachers. Hopefully, this will be resolved in time, as the organizing committee of OAME 2024: Embracing Change is planning the next annual conference, which will take place May 2–4, 2024 at St. Lawrence College in Kingston.

As I am writing this, season 6 of OAME Talks has recently begun with its first podcast, in which David Petro talks about empowering you and your students' learning with math podcasts.

At the local level, there are several chapters that have held professional learning events, including mini-conferences and book clubs. Please visit the OAME/AOEM website and MCIS (Mathematics, Computer, and Information Sciences) to stay up-to-date with these local offerings.

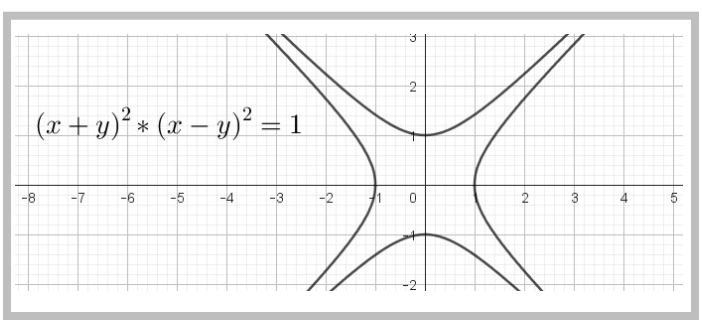
The *Gazette* and *Abacus* continue to offer high-quality content to our membership. We are always looking for new contributions to our publications. It is a wonderful way to share ideas, research, and to connect with other educators.

The OAME/AOEM Board of Directors has also been investing some time in ourselves as an organization. At the June Board meeting, we had a professional learning session, led by Fred Ferneyhough, in which we explored coding through the use of TI-Innovator™ Rovers. There were many moments of frustration and celebration as groups worked together to guide the Rovers through a maze.

At the October Board meeting, there will have been a professional learning session, led by Dr. Ruth Beatty from Lakehead University, and Christina Ruddy, who is the Indigenous Trustee for the Renfrew County District School Board and a member of The Algonquins of Pikwàkanagàn First Nation. They will present examples from their multi-year, multi-site Indigenous mathematics research study, *The First Nations and Métis Mathematics Voices Project*. Board members have a chance to do some designing, do some math, view videos of students thinking, explore some connections to LYNX coding, and generally get a sense of what is involved when centring mathematics education in Indigenous cultural practices. The Board members will connect with each other while learning how to "integrate Indigenous knowledge and teaching methods into classrooms" (*Calls to Action of the Truth and Reconciliation Commission (TRC) of Canada,* 2015, p. 7).

Although our days have a limited number of hours, if possible, invest in yourself as an educator. I have found a renewed sense of purpose and fountain of energy by investing some time in me, and I hope that you find some time to do the same. In doing so, you allow yourself to grow, learn, and connect with other educators. And, as an added benefit, we model a passion for life-long learning for our students. After all, we want them to also grow, learn, and connect with others.

I wish all of you who are celebrating at this time of year a very happy holiday and a wonderful winter break. ▲



Recognizing Bill Otto

BY FRED FERNEYHOUGH, EXECUTIVE DIRECTOR

About 35 years ago, OAME/AOEM accepted a new chapter to represent the mathematics teachers of North-Western Ontario Association for Mathematics Education, abbreviated NWOAME.

I'm always amazed when I see chapters that are thriving due to the enthusiasm and efforts of a few dedicated teachers. From what I can gather from old documents, the formation of the NWOAME chapter was spearheaded by Marilyn Hurrell and Bill Otto. Despite the incredible distances, they engaged in organizing events for their very large geographic region; NWOAME has been a vibrant chapter throughout its history. One of the very interesting aspects of this chapter is the close connections that the chapter enjoys with Confederation College, where many early events were held, and Lakehead University. Even now, you can see the names of several Lakehead University people listed on their chapter executive.



Bill Otto appears as the chapter representative for NWOAME in September 1991. Since that time, Bill has served OAME/AOEM not only as a chapter representative, but later on the Board of Directors, a Vice-President, and our first Emeritus Executive member. Our organization has only had five couples who have served as Executive Directors, and Bill has worked with all of them.

By the time that this is published, Bill will have stepped down from the OAME/AOEM leadership, with a total of 32 years of volunteer service to the Board. Think of that—32 years is a full teaching career for many people; and he had already taught for over a decade when he first started as a chapter representative.

Obviously, Bill has stayed on the Board as a valuable member since retiring from teaching. Through his leadership, the Constitution, By-Laws, and Terms of Reference of OAME/AOEM have been fine-tuned and, as a result, we have revamped the size and direction of the Board of Directors, cutting costs and putting the organization in a much better financial position.

Many times, I am required to look back over minutes and motions from OAME/AOEM's past, and I'm always amazed at how a few thoughts from Bill have changed the direction of a conversation for the better. It is not an exaggeration to say that our organization is all the better for his presence over the past 32 years.

Several years ago, the Executive of OAME/AOEM developed the concept of an Emeritus Executive member of the Board of Directors, someone who would not have voting privileges, but would remind the Executive at all times of our Mission, Vision, and Strategic Priorities, as well as considerations of equity in our work. The position was created with Bill in mind, as he not only "fit the bill" for what we were thinking, but also was able to explain how and why many of our policies and practices were put into place. He has fulfilled that role better than we could have hoped.

Bill and I have worked closely for almost 20 years on the official documents of OAME/AOEM and, despite the distance separating us, have become great friends. And I know that friendship will continue. I've also asked others to comment on Bill's impact on the Board. Some of those thoughts follow:

From Paul Alves, a Past President of OAME/AOEM and Emeritus Executive Member

I think the overriding impression I have in reflecting on my interactions with Bill Otto over the course of my service with OAME/AOEM is that there was rarely a conversation where Bill was sharing with the group that I didn't feel I received a lot more from the exchange than I offered.

When Bill would quietly raise a hand (virtual or real) to signal that he had something to relate, I always knew that his thoughts had been slowly churning for a period and he was ready to offer his perspective. And that perspective usually cut through the confusion to offer a clarity that we all needed. It was "Otto"-matic: his clarity derived, in part, from years of service to OAME/AOEM at the chapter and board level. Every organization needs an institutional memory—an understanding of where we have come from and why certain operational procedures and mechanisms are in place. It is not an understatement to say that Bill is OAME's institutional memory. But what I admire most about Bill is his recognition that simply because things are the way they are doesn't mean that they should be or should remain that way. His measured and considerate thoughtfulness balanced the current governance with what is best for the organization. I certainly benefited, like many other Past Presidents, from having his voice at the table when we were mulling and debating how to proceed on a host of issues—I will miss that. We serve and volunteer in this organization in these

roles, but over time, you grow close, and I am happy and humbled to call Bill Otto a friend. Thank you, Bill, for what you have done for the mathematics education community in this province. It is not an exaggeration to say that your impact in quiet and understated ways will continue to be felt for some time to come.

From Greg Clarke, the Webmaster and Recipient of the OAME Life Membership Award

When I first joined the Board of Directors as a chapter representative in the Fall of 2002, Bill was already a ten-year veteran, and was one of many who warmly welcomed me to the group. Throughout the years, as we continued to serve the Board in various positions, I got to know him better, and truly appreciate him for the wonderful, kind, thoughtful, and humourous human being that he is. His contribution to OAME/AOEM and the Board of Directors cannot be understated—he has so often provided the sound and reasoned advice that we have needed in meetings, especially in difficult times, when we were changing directions and needed to plan wisely for the road ahead. In all things, I have always found Bill to be a man of sincere honesty, and good will. I have the deepest respect for him and am honoured to call him friend.

Jacqueline Hill, a Past President of OAME/AOEM and Recipient of the OAME Life Membership Award

Bill was a very reserved and reflective member of OAME/AOEM; he had a true balance between fun, good times, and the work of OAME/AOEM. Consider a quotation (by some Anonymous person) that really holds true for Bill: "Leadership is not about being in the spotlight; it is about illuminating the path for others."

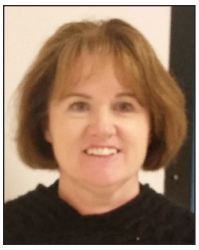
Bill is one of those people who is loyal to the *n*th degree. He has been loyal both to OAME/AOEM and the Leafs forever and a day! He can often be heard to say, "Keep your stick on the ice!"

And even though it would never be enough, thanks, Bill! ▲

Head Scratcher?

Can you find a three digit number *ABC* so that when it is multiplied by *CBA* (where its digits are reversed) the product is divisible by three but not by nine?

▲ OAME/NCTM Report: NCTM… Let's dig a bit deeper!!



JACQUELINE HILL
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Jacqueline Hill is a Grades 9–12 teacher of mathematics. She is a Past President of OAME and OMCA, as well as the recipient of the Award for Exceptional Teaching in Secondary Mathematics. She also wrote the "Director's Dialogue" for the Gazette *for a number of years.*

Newly founded in 1920, NCTM sought to bring mathematics educators together. It is now the world's largest math education organization. In 1950, MPAO (Mathematical and Physical Association of Ontario) became an affiliate of NCTM. (Note: OAME was formed in 1973 by the joining of math organizations: OATM [Ontario Association of Teachers of Mathematics] and OMC [Ontario Mathematics Commission]). So, Ontario has been an affiliate of NCTM for 73 years and counting.

Covering the United States (and its 17 territories) and Canada, NCTM seeks to provide a meaningful avenue for the entire continent to actively engage in mathematics education.



Team… NCTM is a team of like-minded individuals. John Egsgard (see photo) was the first Canadian elected to the NCTM Board of Directors, and John was the NCTM President from 1976 to 1978. At least one other Canadian has been on the NCTM Board since.

Mission (NCTM's mission) is to provide the vision and leadership necessary to ensure a mathematics education of the highest quality for all students. NCTM advocates for high-quality mathematics teaching and learning for each and every student. It supports teachers to ensure equitable mathematics learning of the highest quality for all students through vision, leadership, professional development, and research.

If you decide to join NCTM, mention that you belong to the OAME/AOEM affiliate. This has a positive benefit for OAME/AOEM behind the scenes.

NCTM offers virtual and in-person conferences.

Virtual: April 10–13, 2024

In-person: Washington, DC – September 25–28, 2024

Until next time!

Yours in mathematical fun, Jacqueline ▲

▲ Useful Theories to Know About: Cognitive Load Theory



JEFF IRVINE
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Jeff Irvine, Ph.D., has been a secondary mathematics teacher, department head, and vice-principal. He has taught at three faculties of education and at a community college. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. With over 45 years in education, Jeff is particularly interested in the interplay of student motivation and mathematics achievement.

We ask our students to think about mathematical ideas and utilize their cognitive functions. Cognition involves two kinds of memory: long-term memory and working memory. Human long-term memory is analogous to computer memory on hard drives or cloud storage. It is virtually unlimited, and features a huge interconnected storage facility. Working memory, analogous to a computer's Central Processing Unit (CPU), is where learning occurs. Human working memory is quite limited in capacity and duration, with items remaining in working memory for only a few seconds. Research has shown that working memory is limited in capacity to about seven "things." These things may be facts, procedures, algorithms, or schema. Schema are connected networks of information, and are often referred to as mental mind maps. Learning occurs when students integrate new knowledge with existing knowledge stored in long-term memory.

Cognitive Load Theory

John Sweller's cognitive load theory (1988) focuses on the limited capacity of working memory. Sweller separated cognitive load into two categories[1]. The first, intrinsic cognitive load, is the thinking demanded for completing the task or solving a problem. Extraneous cognitive load is everything else that might take up or impact precious working memory capacity. Extraneous cognitive load might be environmental, like the temperature of the room or the noise level; it might be physiological, like a student's lack of sleep or hunger; it might be emotional—break-up of a relationship or an argument with parents the previous night. Also included in extraneous cognitive load are the instructions for doing the task and their clarity.

Why Should We (Math Teachers) Care About Cognitive Load Theory?

For our students to optimize their learning, as much working memory capacity as possible should be focused on intrinsic cognitive load (the demands of the task). Since working memory has a limited capacity, we want to minimize extraneous cognitive load, leaving as much capacity as possible for working memory to deal with the intrinsic cognitive load of the task.

One way to do this is to make our instructions exceptionally clear and unambiguous. We often use a lot of instructional words in mathematics: solve, expand, factor, explain, justify, and simplify. Students need to know exactly what each of these instructions means and what students are expected to do. We can't have precious working memory taken up by students needing to think about what the instructions indicate they are supposed to do.

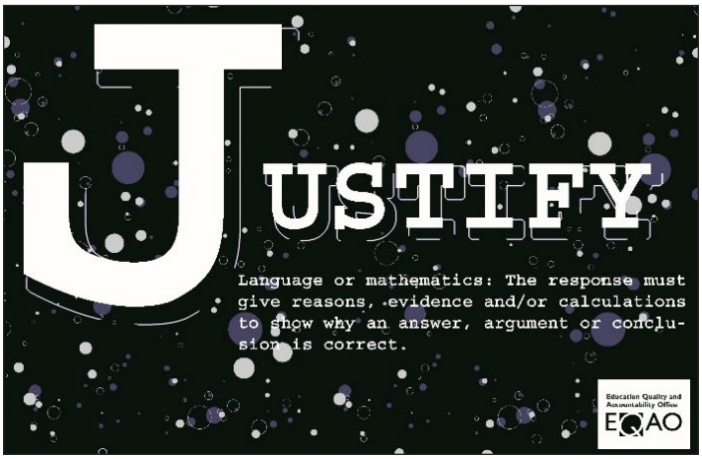


Figure 1: *Sample instructional poster*

At a school where I was vice-principal, I asked the art department to design posters explaining the instructional words used in EQAO assessments (e.g., Figure 1). These posters were on the walls of every classroom in the school, and every classroom in each of our feeder schools. As a result, all the teachers used the word in the same way, and students could look at the poster on the wall, if they needed clarity. So if a student was asked to justify their reasoning, they didn't have to think about what the instruction meant, and working memory was freed up to focus on the intrinsic cognitive load of the task.

Teachers can also address some of the other factors in extrinsic cognitive load, like noise levels, temperature, etc., and we can allow, or provide, water so that students do not focus part of their working memory on being thirsty. Other examples include opening/closing a window (for temperature)

or adjusting a thermostat; allowing a "5-minute" break, where students can have a healthy snack; providing instructions verbally and written on the board. There are also things like "brain gym" for 5 minutes about 15 minutes into the period, where students stand, stretch, and do a series of specific movements along with the teacher. The reference, *Fueling the Mind*, provides many other simple suggestions.

Cognitive load theory is valuable to us as math teachers, since it provides us with a strategy for optimizing our students' learning. It serves to give us a way to consider cognition and the demand of particular tasks and problems we use in our classes. Thinking about cognitive load is a means to address and improve clarity in what we teach.

Note [1] Some researchers identify a third cognitive load: germane load. Germane load is a subset of intrinsic cognitive load and is focused on integrating new learning with existing knowledge stored in long-term memory.

Acknowledgement

The author expressed appreciation for details Anne Yeager constructively provided during the editorial process.

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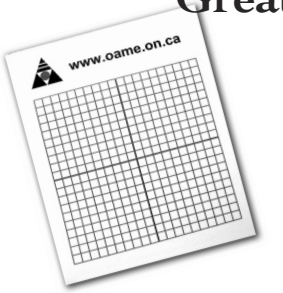
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The original article explaining the theory. ▲

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NOMINATIONS FOR THE OAME/AOEM BOARD OF DIRECTORS 2024–2025

Each year, OAME/AOEM holds elections for open positions. Any OAME/AOEM member may stand for any position. Candidates are needed for the following positions:

- One (1) Vice-President for a two-year term from September 1, 2024 to August 31, 2026
- One (1) Director to represent Grades 7–12 for a three-year term from September 1, 2024 to August 31, 2027
- One (1) Director to represent Grades K–6 for a three-year term from September 1, 2024 to August 31, 2027

No nominations are needed for President-Elect at this time. Since we have gone to two-year terms for the Presidency, we only need a candidate for elections held in odd-numbered years.

A nominee and the nominator must be members of OAME/AOEM. The Nominations/Elections Committee will publish a slate of candidates, including candidate biographies, for consideration by the membership. Voting will take place online in March 2024, and is open to all OAME/AOEM members. The election results will be posted on www.oame.on.ca.

An online nomination form, as well as a summary of the duties of each position, is available at www.bit.ly/OAMEPositions. If you require additional information, please contact Paul Alves (paul.alves@oame.on.ca). The deadline for nominations is January 12 at 6:00 p.m.

Thank you in advance for submitting a nomination.

The 2023 OCMA Conference



MALLORY ANDREWS
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The 43rd annual Ontario Colleges Mathematics Association (OCMA) Conference saw math educators from colleges across the province gather at Fern Resort in Orillia from May 24 to 26. Upon arrival at the resort, conference attendees received a pair of aviator sunglasses and scarf to set the theme of this year's event: *Top Sum—Elite Mathematics Academy*. Call signs were chosen by many attendees and proudly displayed on name badges, furthering the fighter pilot theme.

Two days of presentations included a wide range of topics. Presenters explored pedagogical techniques such as gamification, investigated the connection between cognitive neuroscience and math education, demonstrated how to incorporate specific technologies like Padlet into the classroom, participated in discussion about current research in college mathematics, and even took a calculated approach to using humour while teaching. Many presentations addressed the underlying theme of navigating the new frontier of education by including discussion around artificial intelligence (AI), such as ChatGPT. Some presenters shared strategies for circumventing AI, while others looked to leverage its power and incorporate it into the classroom.

Representatives from several publishing companies attended the conference to share resources during presentations and at the annual PUBlisher night, which was casino themed this year. The popular casino night saw resort employees staffing several games tables, where guests could put their probability knowledge into practice. (Disclaimer: No real money was won or lost.) The hospitality of Fern Resort was excellent as always, with guests enjoying delicious meals and entertainment. Thursday afternoon saw the educators participating in many of the leisure activities that the resort offers, including golf, archery, table tennis, and fishing. A lively interactive mystery theatre dinner performance by Mysteriously Yours of Toronto rounded out the entertainment Thursday evening.

The annual OCMA Conference is an excellent opportunity to connect with and learn from college math educators from all over Ontario. The committee also hosts professional development workshops during the winter season. ▲

Events at the Fields: Some Highlights of Fields' Offerings for Educators



PAMELA BRITTAIN
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Pamela Brittain (Ph.D.) is an educator, author, illustrator, presenter, and math lover, who currently works for the Fields Institute. She has been involved in STEM education for over 20 years and strives to make math accessible and enjoyable for everyone.

Greetings, *Gazette* readers! My name is Pamela Brittain. I am honoured to be taking over the "Events at the Fields" column from Angelica, and hope to provide as much great information as she did. In one of my other roles, I am the K–12 Academic Coordinator for the Fields Institute for Research in Mathematical Sciences, and as such, am very excited to report on the MathEd Forum and bring details of new programs we are working on, as well as other talks and events of interest.

The Fields Institute has long been involved in mathematics education and offers/supports many programs, including the MathEd Forum—featured in this column—which is housed under the Centre for Math Education. Fields also provides funding that supports math education programs at its partner organizations, and it is now offering programming directly to teachers and their students through the Fields Academy. For my first column, I will give a brief introduction to some of the activities going on at Fields. For all of the events and opportunities described below, specific details can be found on the website www.fieldsacademy.ca/eep/.

MathEd Forum

The MathEd Forum is back for its 26th year, bringing math researchers and math educators together to talk about topics of current interest. The talks run on the last Saturday of each month from September to November, and January to April, and are offered both in person and online. Talks are also recorded for those who can't attend or want to revisit the content. This September's Forum honoured this year's Margaret Sinclair winner, Lynda Colgan—a columnist well known to *Gazette* readers (her talk will be featured in the March issue of the *Gazette*).

Elementary Teacher Courses

The Fields Institute offers low-cost ($25 per course) online, self-paced mathematics courses to help elementary teachers build not just their skills in mathematics, but also their

understanding of why the "tips and tricks" we use in math actually work, for example, why multiplying two negatives has a positive result, or how we know how many decimal places are in the product when multiplying two decimal numbers together. Currently there are four courses available (Operations: Part 1; Financial Literacy; Mathematical Modelling; and Logic, Games, and Coding).

Ask a Mathematician (AAM)

What better way to show students what a "real-life mathematician" does than to give them the opportunity to speak to one directly? The AAM program does just that. It brings mathematicians into Grades 4–10 classrooms virtually, and has them interact with students to talk about the math they love, their journey with mathematics, and how they got to where they are. The program is free and only requires a connection to Zoom. The program also provides short videos to help students learn concepts and post-presentation activities.

Fields Medal Symposium

Every Fall, the Fields Institute hosts a Fields Medalist to give a public lecture on their field of research. This year, the symposium honours the work of 2018 Fields Medalist, Caucher Birkar, and his work in algebraic geometry. The symposium also offers a student night, giving high school students a chance to get up close and personal with a Fields Medalist and ask them about their journey.

Fields Notes

Our bi-annual newsletter features articles and information on the many programs taking place at the Institute. It also addresses issues of interest to readers, including many pertaining to mathematics education and opportunities for getting involved.

YouTube Fun

Looking for a bit of amusement for you and your students? The Fields Institute has put together a number of fun videos on topics like "Valentine's Day Math" and "Easter Math" (which involves a lot of multiplying bunnies). There are also recordings of YouTube live events with our Ask a Mathematician presenters, where they talk about Math Modelling and their journey to becoming a mathematician. There are also recordings of Fields-hosted talks called "What the Numbers Say" and "Behind the Numbers," which are publicly accessible YouTube live talks on a variety of current issues (including inflation, climate change, and the COVID-19 pandemic), presented by experts in the field.

As you can see, there is a lot going on at the Fields Institute, and I am excited to bring it to you here in the *Gazette* as Angelica and others have done previously.

Further information on all the programs listed above can be found at www.fieldsacademy.ca/eep/. ▲

▲ What's the Problem? Playing with Processes: Part 2

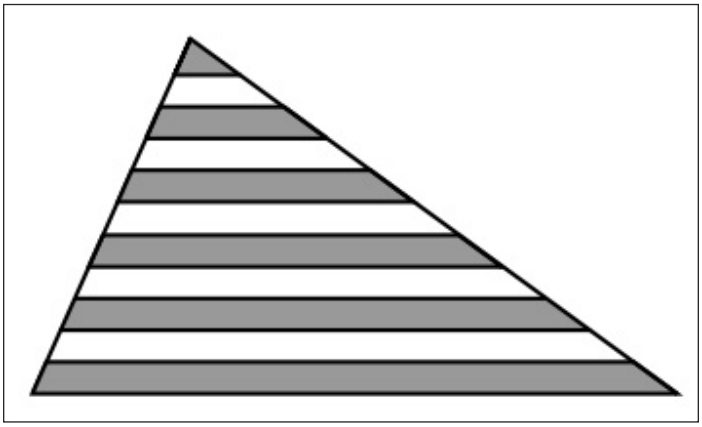


SHAWN GODIN
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Shawn is a former mathematics teacher, department head, and consultant. He strongly believes in the central role of problem solving in the mathematics classroom. He continues to be involved in mathematical activities: presenting workshops, writing articles, working on local projects, and helping create mathematics contests.

Welcome back, problem solvers. In the last issue (Godin, 2023), we started a four-part series focusing on the mathematical processes. We continue in this issue with the following problem from your homework:

> *A triangle of area 770 cm^2 is divided into 11 regions of equal height by 10 lines that are all parallel to the base of the triangle. Starting from the top of the triangle, every other region is shaded, as shown. What is the total area of the shaded regions?*



This problem was question 6a from the 2023 Euclid Contest. The Euclid Contest is hosted every year by the Centre for Education in Mathematics and Computing (CEMC) at the University of Waterloo. The contest is a combination of questions requiring only an answer, and others requiring full written solutions. This issue's problem required a full written solution, not just the answer. If you are interested in this contest or any other resources from the CEMC, you can check out their website at www.cemc.uwaterloo.ca.

As in the last issue, we will keep in mind George Pólya's

four steps in problem solving from his book *How to Solve It* (Pólya, 1957). As a reminder, they follow this paragraph with some of his guiding questions in italics. As in the last issue, relevant mathematical processes are noted in bold within parentheses throughout the column.

1. UNDERSTAND THE PROBLEM. It is impossible to solve a problem if you don't understand what is being asked, the notation, or terms in the problem. This involves, in some cases, accessing prior knowledge. (**Reflecting**)
2. DEVISE A PLAN. (**Selecting Tools and Computational Strategies, Representing, Reasoning and Proving**) *Have you seen it before? Do you know a related problem?* (**Reflecting, Connecting**)
3. CARRY OUT THE PLAN. As you work through each step of your plan: *Can you see that the step is correct?* (**Reflecting**) *Can you prove that it is correct?* (**Reasoning and Proving**)
4. LOOKING BACK. *Can you check the result?* (**Reflecting**) *Can you derive the result differently?* (**Reasoning and Proving**) *Can you use the result, or method, for some other problem?* (**Connecting**)

In this issue, I want to focus on **Representing**. Although this process is transparent, a bit of thinking about representing reveals its importance. We can think of this process from two points of view: how the student thinks about the problem, as well as how they will try to communicate their thinking to others (**Communicating**). Some representations, like physical models, allow us to "see" results more easily, yet lack rigor. As such, we may occasionally "see" something that isn't there. Other representations are more abstract. In some cases, two representations may allow us to see different things and make different connections (**Connecting**). Representing is closely tied to mathematical modelling, selecting tools and computational strategies, as well as connecting. This can be seen in the Ontario Ministry of Education rubric for Representing (Ontario Ministry of Education, 2008), shown in Figure 1.

We are ready to tackle our problem, using Pólya's four steps.

1. UNDERSTAND THE PROBLEM. This is straightforward. We are given the area of the triangle, and we want the shaded area. The formula for the area of a triangle and other figures may be important in our solution (**Reflecting, Selecting Tools and Computational Strategies, Connecting**). From the diagram, there are six shaded regions and five unshaded regions. Looking from the bottom up, we can pair each shaded region with the unshaded region above it. In each case, the shaded region is larger. Additionally, there is an extra shaded region that is unpaired. We can deduce that the shaded region must be more than half the area of the triangle (**Reasoning and Proving**).
2. DEVISE A PLAN. Let's start simplistically. We will use the diagram from the problem to see what we can find out (**Representing, Selecting Tools and Computational Strategies**). We will cut the diagram into strips and manipulate them, looking for patterns.

Representing				
Criteria	Level 1	Level 2	Level 3	Level 4
Creates a model to represent the problem (e.g., numerical, algebraic, graphical, physical, or scale model, by hand or using technology)	Creates a model that represents the problem with limited effectiveness; representing little of the range of the data	Creates a model that represents the problem with some effectiveness; representing some of the range of the data	Creates a model that represents the problem with considerable effectiveness; representing most of the range of the data	Creates a model that represents the problem with a high degree of effectiveness; representing the full range of the data
Makes connections between numeric, graphical, and algebraic representations	Makes limited connections between numeric, graphical and algebraic representations	Makes some connections between numeric, graphical, and algebraic representations	Makes appropriate connections between numeric, graphical and algebraic representations	Makes strong and insightful connections between numeric, graphical, and algebraic representations
Translates from one representation to another as appropriate to the problem	Translates representation with major errors when solving a problem	Translates representations with some errors when solving a problem	Translates representations appropriately when solving a problem	Translates representations appropriately and with insight when solving a problem

Figure 1: *Rubric for Representing*

3. CARRY OUT THE PLAN. Cutting the diagram into shaded and unshaded shapes, it seems we get ten trapezoids and one triangle, as shown in Figure 2 on the left.

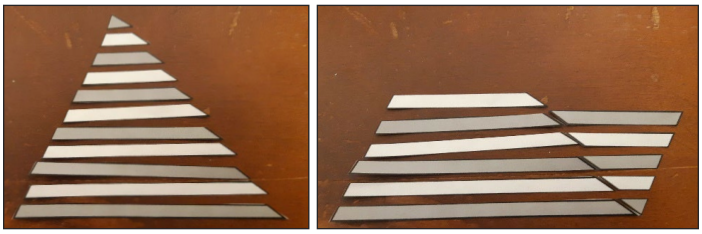


Figure 2: *Cutting the diagram in the problem*

The figures all have the same height, as was given in the problem. The lengths of the parallel sides seem to decrease at a constant rate as we move from the bottom to the top. So, if we move smaller pieces from the top to pair up with larger pieces from the bottom, we end up with something like the picture on the right in Figure 2. In this diagram, it seems that we have five equal parallelograms and a trapezoid that looks to be half as big as the parallelograms. If we were to cut this trapezoid through the middle, parallel to the two parallel sides, the two pieces could be put together to make a parallelogram with the same base as the others, but half the height (**Reasoning and Proving**).

Thus, it seems the shaded part makes up 3 of the 5.5 parallelograms or $\frac{6}{11}$ of the total area. Hence, the shaded area would be $770\,\text{cm}^2 \times \frac{6}{11} = 420\,\text{cm}^2$.

4. LOOKING BACK. From our original observation, we knew that the shaded area would be larger than half the area of the triangle, which agrees with our result. However, "more than half" isn't a very precise estimate. If we came up with something less than 385 cm^2, it would have told us that there was something wrong, but that's about all it is good for (**Reflecting**).

Our computation is based on a visual image, where it looks like we have several equal parallelograms, but sometimes images can be misleading. Take, for example, the two triangles in Figure 3. In this variation of a classic puzzle, six pieces are arranged into a right-angled triangle with base 34 units and height 13 units shown on the top. However, when the pieces are rearranged, you end up with the same triangle, but one square unit is missing. Where did the area go? If you are interested in this and similar puzzles, check out Tyler Somer's excellent series of articles on the topic (Somer, 2022a, 2022b, 2023a, 2023b, 2023c).

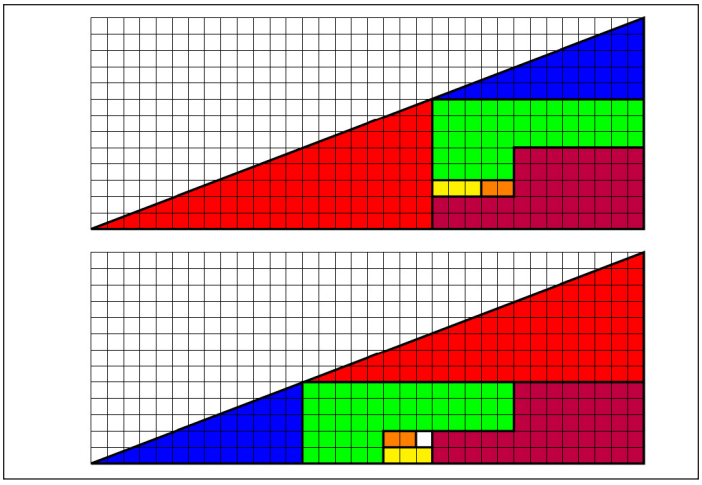


Figure 3: *Expanding triangles*

Something subtle must be going on in the puzzle, and perhaps something subtle is happening in our problem. If that were the case, our answer could be close, but incorrect. We will have to be more precise if want to be sure of our answer (**Reasoning and Proving**). One of Pólya's questions at step 4 is, "*Can you derive the result differently?*" Let's take another look at the problem and see what we can come up with (**Reflecting**).

In our first solution, we cut the triangle up into pieces to make parallelograms. What if we made one big parallelogram? If we take two copies of the original triangle, rotate one and piece them together, it seems we have a parallelogram broken into 11 *equal* strips, as in Figure 4 (**Representing, Selecting Tools and Computational Strategies**).

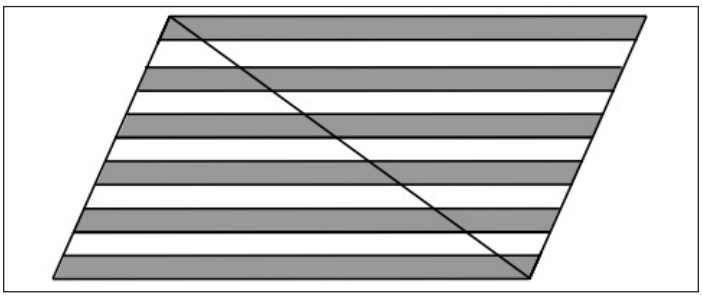


Figure 4: *Building a parallelogram*

Let's be sure that what we are seeing is really happening. We will ignore the strips and concentrate on one "line" through our "parallelogram." In Figure 5, we see that if we label the angles of our original triangle α, β, and γ, then since the lines that form the strips are parallel to the base, we can find a bunch of equal angles and see that the small triangle at the top (blue), formed by the new line, is similar to the original triangle. Similarly, in the rotated triangle, the smaller triangle at the bottom (red) is similar to the original triangle. We can quickly deduce that all the strips in the

original triangle are parallel to the strips in the rotated triangle and that the figure is indeed a parallelogram. Hence, since the strips are of equal height, the strips do indeed join in straight lines (**Reasoning and Proving, Connecting**).

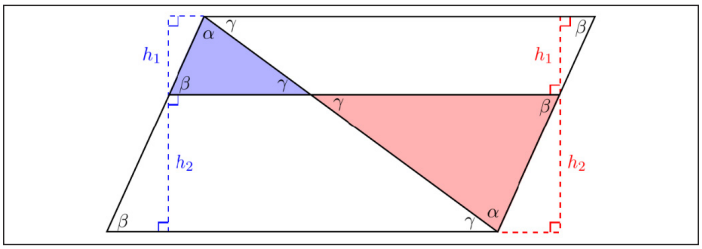


Figure 5: *Justifying our result*

Therefore, the big parallelogram is made up of 11 equal, smaller parallelograms and the area of the shaded parallelogram is $\frac{6}{11} \times (2 \times 770\,\text{cm}^2) = 840\,\text{cm}^2$.

Half of the shaded area belongs to each triangle, which agrees with our initial result.

So why stop there? Let's re-examine the problem using another representation—manipulatives (**Selecting Tools and Computational Strategies, Representing**). Let's generalize (**Reflecting**) what is going on in the problem by building a triangle from smaller versions of itself, as in Figure 6, which could be done with pattern blocks. Here the blue and red triangles represent the shaded and unshaded portions, respectively. In this version, we are dealing with a triangle with 3 shaded (blue) strips.

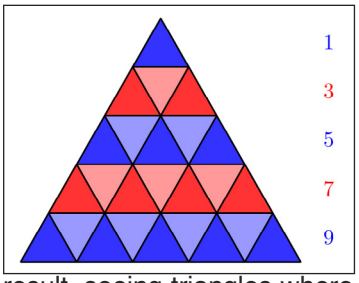


Figure 6: *Building a model with triangle tiles*

Several patterns become apparent. We can imagine building this from the top down and, as a result, seeing triangles where the base is made up of 1, 2, 3, 4, or 5 smaller triangles. For each of these, we see that the total number of little triangles needed is a perfect square: 1, 4, 9, 16, or 25. This makes sense, as the larger triangle will be similar to the smaller triangle, no matter the shape of the smaller triangles used in the tiling. Hence, from what we know about similar triangles, if a triangle is similar to another triangle with a scale factor of s, then its area will be s^2 times the area of the other triangle (**Connecting**). We can see that there are an odd number of triangles on each strip, which can be used to "see" that the sum of n consecutive odd numbers is n^2 (**Reasoning and Proving**), that is:

$$1 + 3 = 4 = 2^2 \qquad 1 + 3 + 5 = 9 = 3^2$$
$$1 + 3 + 5 + 7 = 16 = 4^2 \qquad 1 + 3 + 5 + 7 + 9 = 25 = 5^2$$

As we move along, it might be beneficial to move from the concrete to the abstract. Thus, we will use our concrete model (manipulatives) and describe our results numerically and finally algebraically (**Representing**). If we look at the strips, the nth row, starting from the top, contains n darkly coloured triangles and n lightly coloured triangles. This confirms that the nth odd number is $2n - 1$ (**Representing, Connecting, Reasoning and Proving**). We can verify this because the lightly coloured triangles on one row correspond to the darkly coloured triangles on the row above (thus, the $n - 1$). If we think of adding the triangles in the strips, we see that they contain sums of alternate odd numbers; that is, for the odd strips we would have 1, 5, 9,… triangles, and for the even strips, we would have 3, 7, 11,… triangles. If we rewrite in terms of the light and dark triangles, we can rewrite these numbers as (0 + 1), (2 + 3), (4 + 5),… and (1 + 2), (3 + 4), (5 + 6),… (**Representing**). Hence, the number of triangles in the first k odd rows would be

$$(0 + 1) + (2 + 3) + (4 + 5) + \cdots + ([2k - 2] + [2k - 1]) = \frac{(2k - 1)\,2k}{2} = k(2k - 1),$$

and similarly, the number of triangles in the first k even rows would be

$$(1 + 2) + (3 + 4) + (5 + 6) + \cdots + ([2k - 1] + 2k) = \frac{2k(2k + 1)}{2} = k(2k + 1).$$

(**Reasoning and Proving, Connecting, Reflecting**). If these hold in our problem, then the number of shaded triangles would be the number of triangles in the first six odd rows, or $6(2(6) - 1) = 66$, and the number of unshaded triangles would be the number of triangles in the first five even rows, or $5(2(5) + 1) = 55$. Thus, the shaded area would be $\frac{66}{66 + 55} = \frac{6}{11}$ of the area of the whole triangle, which corresponds with our earlier result (**Reasoning and Proving, Reflecting**). We can further verify this method by drawing in some lines parallel to the other sides of the triangle, as shown in Figure 7, where the 66 shaded and 55 unshaded triangles come to light (**Representing, Reasoning and Proving**).

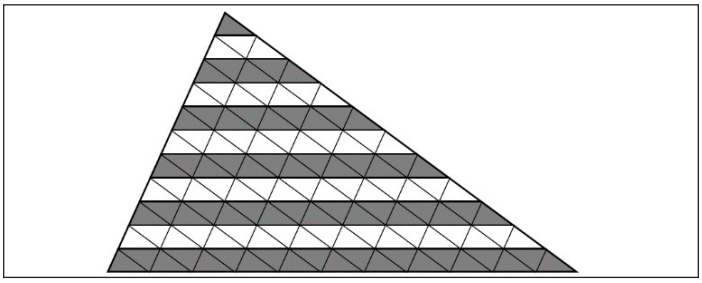


Figure 7: *Breaking the original triangle into smaller similar triangles*

We can look at some extensions of the problem (**Reflecting**). For example, if we cut the triangle into n strips of equal width, shading the top strip and every second strip after that, as in our problem, what happens to the shaded

area? If we have an odd number of strips, $2k-1$, then we will have k shaded and $k-1$ unshaded. From our equations above, we see that we will have $k(2k-1)$ shaded triangles and $(k-1)(2[k-1]+1) = (k-1)(2k-1)$ unshaded triangles. Thus, the portion of the triangle that is shaded is

$$\frac{k(2k-1)}{k(2k-1)+(k-1)(2k-1)} = \frac{k}{2k-1},$$

which generalizes our "strange" result (**Connecting**). That is, if we have an odd number of strips, the fraction shaded is

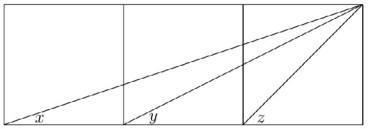
$$\frac{\textit{number of shaded strips}}{\textit{total number of strips}},$$

even though the strips are all different sizes. In our case, we had 6 shaded strips, 5 unshaded, and the shaded area was $\frac{6}{11}$ of the total area. However, if we have an even number of strips, then this idea doesn't work. If it did, half of the area would be shaded, but we know this cannot happen, as we discussed earlier in the column. In this case, we would have $2k$ strips, k each shaded and unshaded. Thus, our equations now give us the shaded area as $\frac{2k-1}{4k}$ of the total area (I will leave the details to the interested reader). We can rewrite this as $\frac{1}{2}-\frac{1}{4k}$, which is less than half (**Reasoning and Proving**). It is interesting to note that if we let n represent the total number of strips, then the shaded part is given by $\frac{n+1}{2n}$ when n is odd, and by $\frac{n-1}{2n}$ when n is even. If we want to get slick, we can combine the two equations into one formula, $\frac{n-(-1)^n}{2n}$, since $(-1)^n$ is -1 when n is odd and $+1$ when n is even (**Representing**). This means that the more the triangle is divided into strips, the closer the shaded area will be to $\frac{1}{2}$. See how this different algebraic representation lets us see our result in a slightly different light.

You may want to check out the official solution to the problem from the CEMC website. In it they also explore some direct calculation of the shapes involved, as well as use some similarity and insightful reasoning to make connections between this sum and the sequence of similar triangles. Although all solutions lead to the same result, they pass through different mathematical territory. It is always worthwhile to explore these paths untaken.

As in the last column, here are some problems from previous columns, where multiple representations were used. Feel free to explore them for yourself and revisit the columns in the OAME archives (available in the members area of the website).

1. The diagram shows three squares and angles x, y, and z. Find the sum of the angles x, y, and z. (Godin, 2012)



2. At the beginning of the winter, there were at least 66 students registered in a ski class. After the class started, 11 boys transferred into this class and 13 girls transferred out. The ratio of boys to girls in the class was then 1 : 1. Which of the following is not a possible ratio of boys to girls before the transfers? (Godin, 2014)

   (A) 4 : 7 (B) 1 : 2 (C) 9 : 13 D) 5 : 11 (E) 3 : 5

3. Pick any whole number between 20 and 100, inclusive. Add the digits together and subtract the sum from the original number. Finally, add the two digits from your last result and concentrate really hard. (Godin, 2016)

Now time for your homework:

You play a game where you have 15 piles containing 1, 2, 3,… 15 counters, respectively. On each move, you may choose any subset of piles and take the same number of counters from each pile. For example, on your first move, you could remove 5 counters from each of the piles with 6, 9, 11, and 14 counters (leaving 1, 2, 3, 4, 5, 1, 7, 8, 4, 10, 6, 12, 13, 9, 15 counters). What is the fewest number of moves required to remove all the counters?

Until next time, happy problem solving!

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▲ OMCA News: December 2023



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Heather is a Past President and an active member of OMCA.

Happy holidays from the Ontario Mathematics Coordinators Association (OMCA)! The school year is off to a fantastic start for our members, consisting of Math Coordinators, Consultants, and Coaches from across the province.

Our first meeting of the year in September welcomed new Executive members, Gerry Lewis as Vice-President and Mariarosa Schniering as Treasurer. Newly appointed President Jackie Decker chaired the meeting, which saw over 80 members in attendance, and followed an EdCamp format in lieu of a guest speaker. Jason To moved into the role of Past President, and handled the hoodie draw that occurs at every meeting. Rounding out the Executive, Allison Lawrence continued in the role of Secretary.

We are excited to be planning a return to Hockley Valley for our annual retreat in February 2024. This will include a day and a half of networking, collaborating, and learning from guest speakers on the topic of "Math Discussions." Details are still being finalized; keep an eye on our website for more information.

We continue to hold our monthly members' meetings virtually. With the new provincial math funding model, our membership now includes many in-school instructional coaches. To reflect this new focus, our guest speaker at the October, November, and December virtual meetings has been Jennifer Hlavka, from The Instructional Coaching Group. These Fall sessions explored aspects described in the workshop series title, "Focus on Instructional Coaching: Partnership Principles and Goal Setting." Sessions on this topic will continue as part of our January and March meetings. Non-member guests are always welcome to attend for the guest speaker component of the meeting for a fee of $30.

Curious about our meetings? Each meeting includes updates from stakeholder groups sharing interests similar to those of OMCA (such as the Education Quality and Accountability Office, the Math Knowledge Network, the Ontario Ministry of Education, and more), networking opportunities, resource showcasing, and a guest speaker. Contact us directly at OMCAmath@gmail.com, or visit our website at www.omca.website (note the different domain), to learn more about the organization, find out about upcoming events, and connect with us. ▲

▲ Meaning Making in the Classroom: Recognizing the Importance of Social–Emotional Learning Skills in Mathematical Modelling



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David is a principal, who taught in the Primary division before assuming roles of numeracy interventionist, numeracy coach, numeracy leader, and curriculum consultant for Prince Edward Island. David has also instructed university courses regarding curriculum, differentiation, mathematics, and language arts. He facilitates professional learning in the areas of mathematics and school development. Dr. David Costello recently published three books: Mathematizing Student Thinking: Connecting Problem Solving to Everyday Life, Building Capable and Confident Math Learners, Making Math Stick: Classroom Strategies That Support the Long-Term Retention of Math Concepts, *and* Using What Works: Strategies for Developing a Literacy-Rich Environment in Math.

In my previous two columns, I have taken time considering mathematical modelling and what it means for students and teachers. Within this exploration, I spent considerable time looking at the four components and identifying notable aspects that occur during each. For this column, I want to consider mathematical modelling and the role that social–emotional learning plays within the student experience.

I have been asked by many educators about the role social–emotional learning plays in mathematics. Within these same discussions, the notion came up that addressing social–emotional learning in the classroom is one more thing to do. However, I want to counter this notion because social–emotional learning is not an extra, but is embedded in the learning experience of students.

When engaged in mathematical modelling, students must take an active stance in their learning. Within this engagement, students must make the ill-defined problem manageable with suitable framing, work through stumbling blocks, and make sense of their work when communicating with others. It is during this engagement, an active stance in learning, that students have opportunities to develop their social–emotional learning skills (Costello, 2022). In this

column, I will examine mathematical modelling from the perspective of social–emotional learning.

Social–Emotional Learning Skills and Mathematical Modelling

Social–emotional learning skills are highlighted in the Social–Emotional Learning (SEL) Skills in Mathematics and Learning Processes strand of the revised Ontario Mathematics Curriculum, Grades 1–8 (Ontario Ministry of Education [OME], 2020). This particular strand focuses on students developing and applying social–emotional learning skills as they engage with mathematical concepts and processes. For the purpose of this column, let's consider six specific social–emotional learning skills: identification and management of emotions, stress management and coping, positive motivation and perseverance, healthy relationships, self-awareness and a sense of identity, and critical and creative thinking (OME, 2020).

The following table (see page 18) attempts to outline the six social–emotional learning skills in relation to mathematical modelling (Costello, 2022, pp. 83–84).

Putting It Altogether

Social–emotional learning skills are embedded within student learning experiences. When engaged in mathematical modelling, students must take an active stance, and it is within this active stance that students will need to manage emotions, work through stumbling blocks, persevere, communicate effectively, develop self-awareness, and think critically. Within this engagement, it is clear that we cannot isolate social–emotional learning from mathematical modelling. It is the social–emotional learning skills that students apply within their experience that helps them move through the mathematical modelling process.

Consider the visual in Figure 1. It is my attempt to highlight the relationship among the task, struggle, mathematical processes, and social–emotional learning skills. Students are assigned a task—let's consider it an ill-defined problem. Students can engage in productive struggle or unproductive struggle. If productive struggle, students will continue to apply their social–emotional learning skills and mathematical processes to arrive at a meaningful solution. However, if students are engaged in unproductive struggle, they need to alter course by first applying social–emotional learning skills. By doing this, they will move to productive struggle. Then, and only then, will they continue to apply social–emotional learning skills and mathematical processes to arrive at a meaningful solution. Without social–emotional learnings being applied, students will not be able to effectively work through the four components of mathematical modelling.

Final Thoughts

The intent of this column was to highlight social–emotional learning skills in relation to mathematical modelling. While there is much more emphasis being placed on social–emotional learning skills with the new curriculum than in the past, it is important to note that these skills are not an extra or an add-on. Instead, as I attempted to highlight in this column, social–emotional learning skills are inherent in mathematical modelling. It assists the students in moving from unproductive struggle to productive struggle, in being able to apply the mathematical processes, and in arriving at meaningful solutions.

I look forward to continuing the discussion of mathematical learning and instruction via my website (www.costellomath.com), email (david@costellomath.com), or X, formerly known as Twitter, (@dr_costello).

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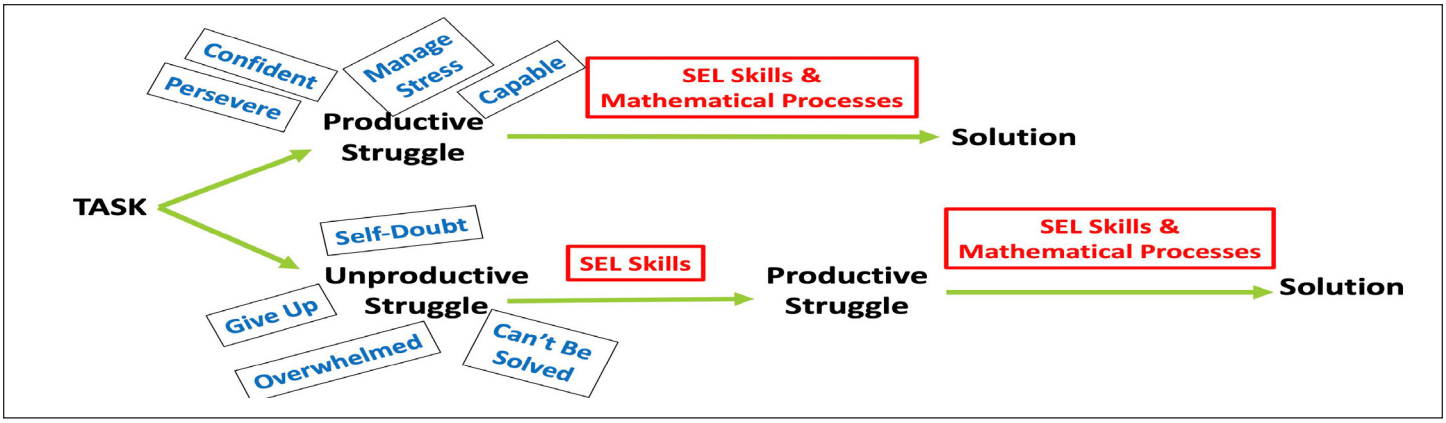


Figure 1: *The relationship among task, struggle, processes, and SEL*

Social-Emotional Learning Skill	Brief Description	Embedded Strategies in Relation to Mathematics	Mathematical Modelling
Identification and management of emotions	The ability to identify and manage emotions and emotional responses when engaged in learning.	• Recognize the range of emotions that they may encounter. • Understand the connections among thoughts, feelings, and actions. • Manage emotions when problem-solving, and have the ability to apply self-regulation.	• Students will need to identify their initial responses to the ill-defined problem. These problems may be unlike the problems they typically encounter. • Students will need to self-regulate their emotions as they work through the four components of mathematical modelling.
Stress management and coping	The recognition that stress will arise when engaged in problem solving, and that they can apply coping strategies to build resilience.	• Work through stumbling blocks. • Understand coping strategies builds resilience.	• Within the mathematical modelling process, students will undoubtedly encounter stress. • Stress may be encountered when trying to understand the problem, connecting the problem to their lived experiences, applying mathematics, or communicating their thinking. • Stress may also be encountered when students collaborate with their peers during mathematical modelling.
Positive motivation and perseverance	Having a positive disposition is a support as students work through a problem. Recognizing that mistakes are opportunities for learning will support students in persevering.	• Practise perseverance. • Embrace mistakes as a necessary part of learning. • Try different methods when problem-solving.	• Struggle is inherent in mathematical modelling. Students must recognize this struggle and remain motivated through their effort to solve the problem. • Students may have to try multiple approaches to solve the ill-defined problem. They cannot be deterred by this occurrence. Perseverance is a must when engaged in the mathematical modelling process.
Healthy relationships	The ability to work co-operatively and collaboratively with others will strengthen the work and lead toward determining a solution.	• Be co-operative and collaborative with others. • Consider the perspectives of others.	• Mathematical modelling is a collaborative process. Students will work with peers to make sense of the ill-defined problem, make the problem manageable, apply mathematics, and arrive at a meaningful solution. • In working collaboratively, students must accept the feedback and perspectives of others, whether it is working through the problem or sharing their solution with the class.
Self-awareness and sense of identify	Knowing oneself as learner will assist students as they work through a problem. Having an accurate awareness of their understanding will support students as they frame and support their abilities to problem-solve effectively.	• Know oneself. • Communicate their thinking about mathematics. • Build identity as a math learner. • Monitor progress in skill development.	• There is a lack of structure within the problem students are assigned. As such, students must have a self-awareness so that they recognize their contribution to the collaborative nature of mathematical modelling. • Students must also monitor their progress as they encounter and work through ill-defined problems. This will assist them in being confident and capable math learners.
Critical and creative thinking	The ability to critically examine a problem and the decisions they have made while working through the problem enables students to have an overall picture of where they are at in the problem-solving process. The ability to creatively determine a problem-solving approach leads to students being able to think outside the box and increase their chances of determining the solution.	• Make connections between mathematical concepts. • Evaluate the thinking within the problem-solving process. • Assess possible strategies. • Think outside the box when a traditional approach isn't effective.	• Mathematical modelling requires students to be critical thinkers. Students cannot be passive or rely on the assistance of the teacher. They must scrutinize the ill-defined problem so that they understand it and can make meaning with it. • Students cannot work throughout the mathematical modelling process without creativity. Creative thinking is a must, as there is no prescribed path to take. Instead, students must take risks and approach the problem from a flexible stance.

In the Middle: Out-of-This-World NASA Resources for Math Classrooms



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Carly Ziniuk teaches Grade 9 Mathematics, Grade 12 Data Management and Advanced Placement Statistics at the Bishop Strachan School in Toronto. She is very active in adopting real-life data to engage her students in solving problems. Among 30 global STEM leaders, Carly is excited to have been selected for the NASA Space Apps Collective for 2023–2024. As a Canadian educator, she will be contributing toward the pilot program's chosen theme: Building an Inclusive Global Space Community.

Last July, as you may have noticed in my biography, I started working with a group of international STEM professionals known as the NASA Space Apps Collective. As part of my work, I am exploring NASA's free and open data to spark innovations and knowledge within a theme of "Building an Inclusive Global Space Community." This column will explore some teacher-accessible middle school resources available from NASA. If you are interested in learning more about NASA resources available for classroom teachers, please don't hesitate to contact me. This column is just a glimpse at the intergalactic materials for use with students.

To start with, if you are looking for video content, *NASA+* is a new ad-free, no-cost, family-friendly (thus classroom-accessible) streaming service that includes original NASA video series, live video feed of future launches, and documentaries. In particular, *NASA+* looks to "better tell the stories of how NASA explores the unknown," and especially the stories that have, until now, been untold. (*Editor:* The post-testing website URL for *NASA+* is not known at the time of writing.)

STEM resources at www.nasa.gov/stem include many contemporary untold stories. Some of these resources include varied media types such as the *AstroOlympics* project (with connections to space and Olympics through science concepts), the *Modern Figures K–12 Toolkit* (with connections to the 2016 *Hidden Figures* film), and *Women in STEM Interviews and Activities*, which are presented for both students and educators.

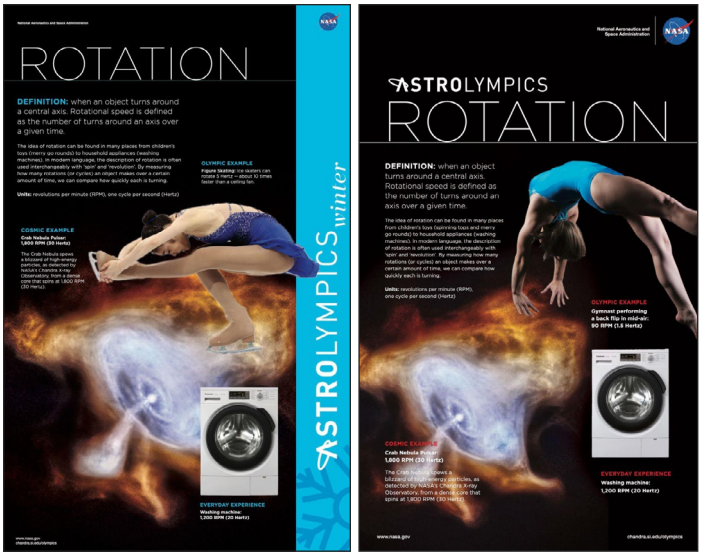


Figure 1: *Connecting Olympic athleticism and cosmic phenomena*

The *AstrOlympics Project,* www.nasa.gov/stem-ed-resources/astrolympics.html, connects the scientific and mathematical concepts common to both Olympic athleticism and cosmic phenomena. Examples are shown for both Winter and Summer Olympians. For example, rotation is explained (see Figure 1) with Winter Olympians as:

> The neutron star at the center of the Crab Nebula is moving at 30 Hertz, in other words, making 30 rotations in just one second. That's almost as fast as Olympic ice skaters, which is amazing, especially when you consider that the neutron star is over 10 miles, or 16 kilometers, across!

The *Let's Go to Mars* activity, part of the *Modern Figures Toolkit*, asks students to use planetary position data and algebra to determine when the next opportunity to launch to Mars will be (see www.jpl.nasa.gov/edu/teach/activity/lets-go-to-mars-calculating-launch-windows). A linked article, *Teachable Moments: When Computers Were Human*, discusses how early (human) computers first noted that determining when the two planets were the closest would minimize the amount of rocket fuel necessary. This motivation for finding an optimal launch window to Mars is the modern version. The activity also includes a video story featuring Sue Finley. Finley worked as a human computer in 1958, and still works in NASA's Jet Propulsion Laboratory (JPL) as a test engineer. Similar videos, along with printable posters and written biographies of the many JPL researchers, can be found at women.jpl.nasa.gov. These resources describe how they directly use mathematics in their daily work. Similarly, *Women in STEM* (see www.nasa.gov/stem/womenstem.html) has examples of STEM careers at NASA, including applications of each of

the four (i.e., S, T, E, and M) disciplines. Michun North, a program analyst for the Commercial Crew Program, for example, discusses how managing resources and financial planning mathematically ensures that the crew has what they need.

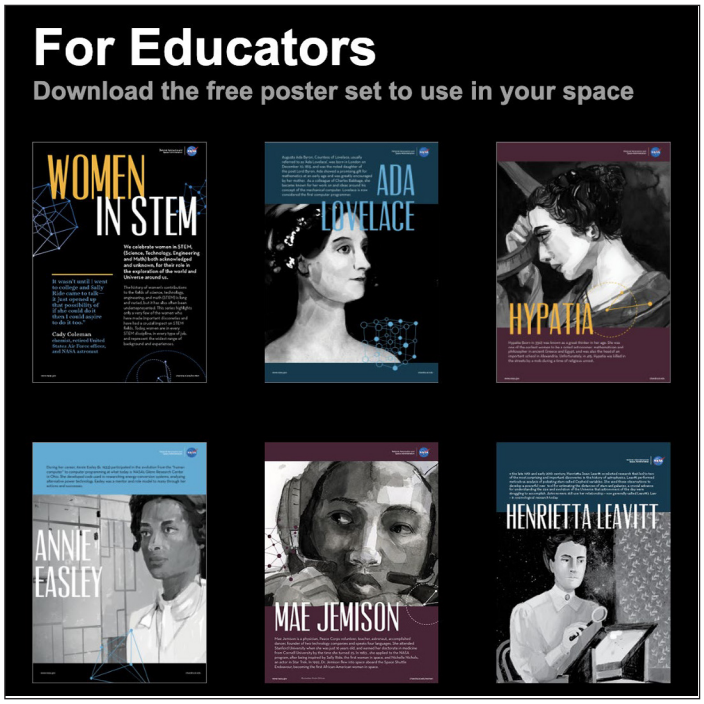


Figure 2: *A sample of available posters*

NASA's flagship mission for X-Ray Astronomy, the *Chandra X-Ray Observatory* and its website, chandra.si.edu/women/index.html, has downloadable posters, as shown in Figure 2. There are also free Augmented Reality (AR) experiences, which can be used on phones or tablets for students to "Meet your female space science heroes (virtually)." Another free web-based AR experience and graphic novel designed for younger students (Grades 5–8), *First Woman*, tells the fictional story of Callie Rodriguez, www.nasa.gov/stem-ed-resources/first-woman-graphic-novel.html, the first woman to explore the Moon. As part of the *Camp Experience* materials accompanying *First Woman*, there is a design challenge: "Design a spacecraft drag system like NASA." Students explore drag used to slow down a weighted spacecraft by considering varied 3D shapes. By exploring various shapes and calculating their volumes and surface areas, students will learn how geometry changes the amount of drag, and thus reduces the speed of a spacecraft so that it can enter the atmosphere safely.

If you are looking for quick examples to use in your classroom to illustrate a specific mathematical concept, NASA's *Space Math* archive, spacemath.gsfc.nasa.gov, has an abundance of over 700 problems. Many middle school problems can be used without much set-up or preparation. The following examples have a brief context, ready-to-use questions, and photocopiable handouts with full solutions:



- **Percentages** (Launch into Math, Exercise 6): How much water is needed to rehydrate food for astronauts on the International Space Station?
- **Rates/Ratios/Units** (Launch into Math, Exercise 1): How long will the Orion spacecraft of the Artemis Mission take to get to the moon?
- **Perimeter and Area of Irregular Shapes** (Exploring Space Mathematics, Problem 7): How much electricity can be collected from solar cells with different shapes on the satellites?
- **Applications of Scientific Notation** (Space Math II, Activities 8 and 9): How much energy and luminosity are produced by different astronomical phenomena?
- **Conversion and Computation with Numbers in Scientific Notation** (Space Math IV, Activities 9, 10, and 11): How do different astronomical quantities compare?
- **Equations with One Variable** (Space Math V, Activities 8, 78, and 79): How can Hubble's law and other formulae be rearranged to solve for different quantities, convert between different temperature scales, and calculate pulsar spin rates?
- **Fractions, Rates, and Ratios** (Space Math, Activities 24 and 25): How do the different planet sizes compare, and what are the relative sizes of the sun and stars?

NASA astronomers must scale satellite and telescope images to determine actual space sizes. *Image Scale Math* (see www.nasa.gov/sites/default/files/atoms/files/image_scale_math.pdf) has student groups perform real-life scaling similar to those done by scientists.

- Each activity includes a solar system image and a measure of the actual field of view.
- Students then use a ruler to measure the size of the image.
- They will calculate the image size and scale.
- They will then investigate the features of the image and determine the real-life size.

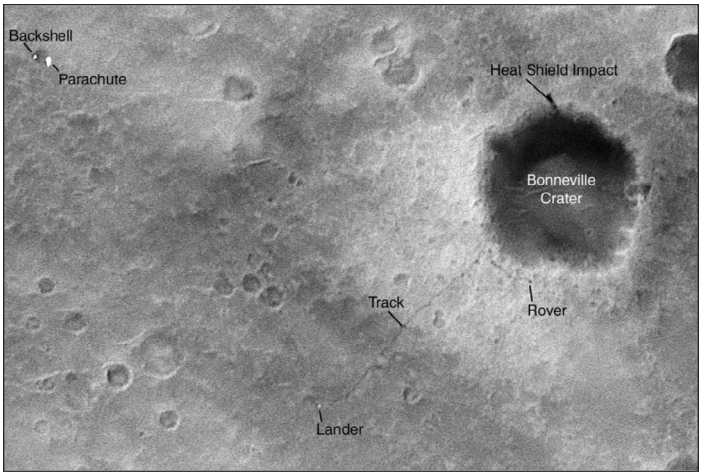


Figure 3: *Image of a Mars Rover* (Credit: NASA/JPL/MSSS, Image Scale Math, p. 12, Activity 4)

Figure 3 shows an image of the Mars Rover, Spirit, landing area near Bonneville Crater. It includes debris from the landing and the track of the rover. The actual width of this image is 895 meters. Students then can discover the length of the track, the size of the parachute, and more. It also includes images taken by NASA's Mars Reconnaissance Orbiter of an avalanche on Mars, solar surface pictures from the Swedish Vacuum Telescope by the Royal Swedish Academy of Sciences, and an image of Jupiter with its satellite Io, taken by the Cassini spacecraft.

My NASA Data, mynasadata.larc.nasa.gov, offers petabytes (2^{50} bytes; 1024 terabytes, or over a million gigabytes!) of free and open global Earth science data collected from satellites. The *My NASA Data* platform helps students and teachers import, analyze, and interpret this data. There are instructions on how to import the data into Google Sheets, and advice on how to produce appropriate graphs for different audiences. The *My NASA Data Quick Start Guide* allows you to find data/imagery appropriate for student investigations. This Guide shows the overall scientific area addressed, the ease of use in data analysis, and the source of the data.

Students who have not experienced larger-scope, real-life problems, problem-based or project-based learning, can be overwhelmed as they struggle to know where to begin. A five-minute introductory video, mynasadata.larc.nasa.gov/basic-page/big-questions-video, with real NASA investigators, can set the tone. Starting with a big question ("Why are the trees where they are in a local environment?") provokes students to create smaller, more solvable questions. The *My NASA Data* platform also contains prepared mini-lessons categorized by grade level, including the following two middle school-appropriate lessons addressing the climate crisis. Note that for these, you must register as a teacher (with a teacher email address) to gain access to the answer key.

- **Global Air Temperatures:** Students analyze a graph that illustrates changes in global air temperatures relative to 1951–1980 average temperatures (mynasadata.larc.nasa.gov/mini-lessonactivity/global-air-temperatures-graph).
- **Extreme Temperatures in July 2022:** Students explore daily mean and monthly surface temperatures, and investigate a model displaying these extreme summer temperatures, then consider the impact of excessive heat (mynasadata.larc.nasa.gov/mini-lessonactivity/extreme-july-2022-temperatures).

As *NASA Global Climate Change* plan (climate.nasa.gov) asserts, "From space, sky, sea, and land, NASA provides detailed climate data and research to the world." At climate.nasa.gov, you can find the *Vital Signs of the Planet, Earth Minute* videos, updated news and scientific findings, quizzes, colouring pages, and interactives. The *For Educators* section, climate.nasa.gov/for-educators, provides larger modular collections of linked resources on a similar theme, sorted by grade level and topic (including specific mathematical areas like number, algebra, data analysis, geometry, and measurement). The *Grade 6 Module I: Dividing Multi-Digit Whole Numbers*, for example (see spacemath.gsfc.nasa.gov/Modules/6Module1.html), addresses the objective, "Students will learn about satellites that gather data about Earth systems through reading a NASA press release and viewing a NASA eClips video segment. Then students will practise dividing multi-digit numbers, using satellite data related to the Earth's ozone layer."

The Climate Modules connect some of the smaller questions in the Space Math series, culminating in a larger, multi-step challenge problem. One example, *Grade 7 Module II: Integer Arithmetic*, investigates methane gas, considers how it contributes to the Earth's changing climate, and discusses why it is vital in the search for life on other planets (spacemath.gsfc.nasa.gov/Modules/7Module2.html). The challenge problem in this module is as follows:

> A maltose sugar molecule has 12 carbon atoms, 22 hydrogen atoms, and 11 oxygen atoms. A single molecule of tannic acid has 64 more carbon atoms, 30 more hydrogen atoms, and 35 more oxygen atoms than a molecule of maltose. How many methane molecules could you form from one tannic acid molecule if a single molecule of methane has one carbon atom and four hydrogen atoms?

Finally, on April 8, 2024, some areas in Ontario will

experience a total solar eclipse during the afternoon. If you are looking to prepare for this significant celestial event with your students, "How Do Different Locations on Earth Experience a Solar Eclipse?" (see mynasadata.larc.nasa.gov/mini-lessonactivity/how-will-different-locations-us-experience-2024-solar-eclipse) addresses the mathematics of eclipses, eclipse safety, and the path of totality. Local information closer to Kingston and other cities in the totality path can be found through the physics department at Queen's University (see www.queensu.ca/physics/2024eclipse). As I write this, the *Queen's Eclipse Task Force* is being established to provide regional schools with educational activities, resources, viewing events, and eclipse viewers for the actual day. A parallel *Ontario Eclipse Task Force* is also being developed by the Dunlap Observatory at University of Toronto, but unlike Kingston, most of the Greater Toronto Area is not in the path of totality. For many Ontario students, this once-in-a-lifetime experience could mean further inspiration to explore the cosmos. I hope these resources can help students see the important role mathematics has in exploring our planet and beyond! ▲

OAME/AOEM is starting the process of looking for a new Executive Director(s). The position would begin Sept. 1, 2025. The position includes, but is not limited to:

- managing all correspondence related to OAME/AOEM
- maintaining the membership records: reminding when expiry dates are looming, adding new members or information, sending receipts
- managing finances and budget of the organization from year to year and working with the accounting firm to prepare annual statements
- arranging for the printing and mailing of the *Ontario Mathematics Gazette* quarterly
- organizing and helping to plan all meetings through the course of the year
- being a liaison with OAME/AOEM's local chapters and board committees
- assisting with visioning and forward planning of the organization
- attending annual conferences and staffing a table in the sponsors area

The successful candidate(s) will be provided with training during a transition period.

Anyone interested in being considered for the position should contact Eds@oame.on.ca.

▲ Elementary Math Matters: Turning a Coding Craft into a Purposeful Mathematics Lesson



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Lynda Colgan is Professor Emerita, Faculty of Education, Queen's University. Lynda is currently working on a number of research, resource-creation, and teacher-education projects, funded by the National Sciences and Engineering Research Council of Canada (NSERC), the Social Sciences & Humanities Research Council (SSHRC), and the Mathematics Knowledge Network (MKN). She will carry on her long-established commitment to STEM outreach by continuing to seek out creative avenues to engage students, parents, and educators in mathematics at home and in the classroom.

It is always a pleasure to be in touch with former B.Ed. teacher candidates after graduation, when they get back in touch to tell me about their upcoming classrooms and curriculum planning. It is a time of sharing creative ideas; reflecting back on lessons and readings, assignments and workshops; and evaluating some of the freely available resources on the Internet.

One of my exceptional alumnae is a gifted young educator, who is deeply committed to the principles of STEAM…, in other words, integrating the Arts into Science, Technology, Engineering, and Mathematics. As a pre-service teacher candidate, her contributions to the class were many—thought-provoking questions, inspired demonstration lessons, and "knock-it-out-of-the-park" assignments.

What set her apart from her peers was the depth of her curriculum-integration efforts. It is difficult to integrate mathematics into other subject areas and maintain the integrity of both disciplines. Often, it is the mathematics that falls short, relegated to mere measurement as one example.

Recently, en route from one city to another, she stopped in for a visit and soon after, we were deeply immersed in her ideas about teaching coding with a creative spin. She told me about the current "rage" in STEAM coding activities: binary bracelets. A Google search of "binary bracelet" returned 1 410 000 results (0.40 seconds). She showed me images (see Figure 1) and explained that one colour of bead represented "1," and the other colour, "0," and a spacer bead separated the letters to make "reading" the code possible.

She went on to tell me that a "Binary Decoder Key" or "Binary Code Alphabet" was used to convert the letters of the alphabet into zeros and ones.

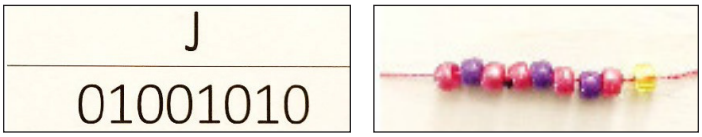


Figure 1: *The letter "J," extracted from a table showing its binary equivalent, and the result of beading the letter, using pink for "0" and purple for "1"*

When she was finished, she asked me what I thought of the activity. I scrolled through the myriad websites that had popped up, and finally said, "This is a nice craft idea, and that's where it ends. Where's the mathematics beyond matching the colour of bead with its appropriate digit?" I continued, "I see that this idea is called by different names, usually labelled as an unplugged coding activity, but I would disagree."

She asked me to explain my thinking. Here is the gist of my evaluation:

- The concept of binary place value is conceptually well beyond children in the Primary division (the target age for most of the web-based resources on the topic).
- The binary alphabet charts are provided without context.
- The binary alphabet charts are not supported with conceptual or foundational explanations.
- The lesson plans do not help us to understand why the "0's" and "1's" are important operationally to a computer.

To my delight, she said, "Whew! I was thinking the same thing. I was asked to do a workshop on binary bracelets at my school to show the possibility of arts integration into mathematics, and how to address some of the new coding expectations. Should I say, "No"?

"Absolutely not. Let's turn this into a worthwhile exercise with a fun twist. Now hand me those napkins." For the next hour or so, we shared in an animated sequence of lessons along with question-and-answer breaks, beginning with binary notation, moving on to bytes, ASCII code, and the role of ON/OFF switches in computers.

I began my lesson by reviewing the base-10 number system. We talked about how base-10 likely emerged from the fact that we have ten fingers, and I lightheartedly spoke of some cultures in Africa and the Maya, who used base-20 because they also used their toes to count—likely attributable to weather and the absence of shoes. When I mentioned that the Babylonians used base-60, and the Egyptians base-12, my former student giggled and said, "Wonder what they were counting?"

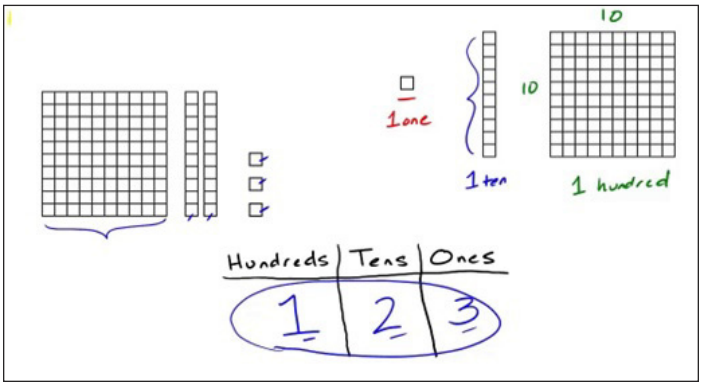


Figure 2: *My refresher sketch about place value in the base-10 system*

What would place value look like if, instead of base-10, we were using base-2. *What would the columns be labelled? Would you be able to write "2" in the 1's column?* As my student sipped her latte, I quietly gave thanks to Mrs. Harvey from Bruce Public School, who, many years ago, was charged with teaching my class how to compute in multiple bases other than 10—a consequence of the "New Math" after Sputnik 1's success for the Soviet Union.

"Let's think about the prefix "bi-" and what that means." Her response was, "Well, two, of course. I understand that binary means two, but what I don't understand is why there are no '2's' in binary, if that's what it means."

"What's the largest single digit in base-10?," I asked.

She replied, "9, of course." (Note that with zero, this gives ten digits: 0, 1, 2,…, 9.)

"Why?"

"Because if you count any higher, you need to go to the next place-value column." And after a moment, she continued, "So you can only use '0' and '1' because if you count up to 2, you need to go to the next place-value column." (Note that with zero, this gives two digits: 0, 1.)

On a new napkin, I sketched a chart showing the place-value headers in a format reminiscent of the headers in base-10: 1's, 10's, 100's, 1000's, 10 000's, and so on, but in base-2, as shown below.

128	64	32	16	8	4	2	1
One hundred and twenty-eight's place	Sixty-four's place	Thirty-two's place	Sixteen's place	Eight's place	Four's place	Two's place	One's place

Figure 3: *Binary place-value chart to eight places*

On studying the chart, my student said, “Now I see. In base-10, we multiply by 10 between columns: 1, 1 × 10 = 10, (1 × 10) × 10 = 100, and so on. In base-2, we multiply by 2: 1, 1 × 2 = 2, (1 × 2) × 2 = 4. This is starting to make sense.”

“Good,” I said. “Grab a napkin and make me a chart of the alphabet—with ‘A’ representing the first letter in the alphabet or ‘1’ and ‘Z’ representing the last letter in the alphabet, or ‘26’.”

Base-10 Counting Number	Base-2 Binary Number
1	1
2	10
3	11
4	100
5	101
6	110
7	111
8	1000
9	1001
10	1010

Figure 4: *The list of binary numbers from 1 to 10*

Within a few minutes, she had a completed chart and felt very accomplished.

“Ready for the next step?” I asked.

“Yes,” came the reply.

“We’ve established that binary means using only ‘1’s’ and ‘0’s’ to represent information. Each ‘1’ or ‘0’ in binary code is known as a bit. When you put together eight *bits*, you get a line of binary code called a *byte*.”

“I’ve heard of bits and bytes, and not just the snacks, but I never knew what they were about.”

“Back in the 1960’s, when computers were being developed, it was decided that there was a need for a standardized language so that computers could communicate. That language is called ASCII—American Standard Code for Information Interchange. Every letter, number, character, and symbol is represented by its own special byte. So, when you press ‘A’ on your keyboard, the computer reads this as ‘0100 0001’.”

A quick Google search turned up 24 000 000 ASCII tables showing upper- and lower-case letters, numbers, and special characters.

Character	Binary Code	Character	Binary Code
A	01000001	o	01101111
B	01000010	p	01110000
C	01000011	q	01110001
D	01000100	r	01110010
E	01000101	s	01110011
F	01000110	t	01110100
G	01000111	u	01110101
H	01001000	v	01110110
I	01001001	w	01110111
J	01001010	x	01111000
K	01001011	y	01111001
L	01001100	z	01111010
M	01001101	!	00100001
N	01001110	"	00100010
O	01001111	#	00100011
P	01010000	$	00100100
Q	01010001	%	00100101
R	01010010	&	00100110
S	01010011	'	00100111
T	01010100	(	00101000
U	01010101	)	00101001
V	01010110	*	00101010
W	01010111	+	00101011
X	01011000	,	00101100
Y	01011001	-	00101101
Z	01011010	.	00101110
a	01100001	/	00101111
b	01100010	0	00110000
c	01100011	1	00110001
d	01100100	2	00110010
e	01100101	3	00110011
f	01100110	4	00110100
g	01100111	5	00110101
h	01101000	6	00110110
i	01101001	7	00110111
j	01101010	8	00111000
k	01101011	9	00111001
l	01101100	?	00111111
m	01101101	@	01000000
n	01101110	_	01011111

Figure 5: *A typical ASCII-Binary chart*

My former student next asked, “So what does all of this have to do with how a computer operates?”

I said that my knowledge of that was a bit rusty (*Editor: That’s better than a byte rusty!*), and while I remembered a little about electrical switches and bytes, I would prefer to get some help from a Google search myself. Together we looked at the screen on my new IOS 64-bit iPhone (which, we learned, stores and accesses information in groups of 64 binary digits—which means that it’s capable of storing 2^{64} values, or more than 18 000 000 000 000 000 000 unique 64-bit combinations of binary integers1), and worked our way through a short tutorial from a free IT website.

	128	**64**	*32*	*16*	**8**	*4*	**2**	*1*	
BINARY OCTET	0	**1**	0	0	**1**	0	**1**	0	74 Decimal

In the table above, think of the 1's in the BINARY OCTET row as if they were individual light switches on a wall, and up is "on" and down is "off." Each light switch is labelled with a number. When you add up the numbers on the labels of the light switches that are turned on, you get a total. In this case, the light switches labelled 64, 8, and 2 are all turned on. Doing the addition (64 + 8 + 2), you get the total value of all the light switches in the byte that are turned on (set to 1), which is 74. Within a computer, a transistor stores the "on" and "off" state. While no physical switches are used, the transistors are essentially electrical switches.

Figure 6: *This screenshot from InetDaemon.Com explains how the number 74 is written and what it means in terms of ON/OFF switches that send operational instructions to the computer. A byte, or a string of 8 bits, may also be called a binary octet.*

"Whew," was what we said in unison by the end of our lessons and investigations. I then asked my former student the grade level for which she thought binary bracelet making would be most appropriate. Her reply was quick, "Grades 6 to 8."

After the table was cleared and the napkins neatly folded and tucked away for future reference, our discussion turned back to coding in the elementary mathematics curriculum and the rush to design activities, rather than lessons, that superficially addressed the curriculum expectations. We talked about how the binary bracelet activity was indeed "unplugged" in that it did not require any technology, but to have integrity, the focus needed to move far beyond coloured beads and pre-made tables. We agreed that our students needed to be respected by holding ourselves as educators accountable—teaching ourselves about the concepts underlying simple vocabulary like "binary," "bit," and "byte," along with demonstrating how and what we learned on their behalf to make activities mathematically meaningful.

Before she headed back to her car, my young mentee said, "I've got some work to do, but I'm going to rock that workshop for the teachers in my school."

I assured her that she would be an excellent workshop leader, and before she left, I said, "I need to tell you one last story. Effective learning stays with you. As I mentioned, I learned about computing in multiple bases in Grade 5—six decades ago—with my amazing teacher, Mrs. Patricia Harvey, and I was introduced to bits, bytes, and ASCII. Four years later, in Grade 9 as part of an enrichment computer programming course with Mrs. Patricia Hall, I was still able to call upon (most of) my learning to share with you, all these years later. And I didn't even have a bracelet."

Note: [1] www.sciencefriday.com/educational-resources/write-your-name-in-binary-code/▲

▲ Coding in the Classroom: Computational Participation



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Iain has been an elementary school educator for 28 years, and is now into a second career, teaching in-service and pre-service teachers. He teaches face-to-face at Ontario Tech University and virtually at Western University.



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Beyza is a former teacher and long-term researcher. She is currently a graduate student in education at Western University, and her research focus is the integration of computational thinking into mathematics education.

(Quotations within this column may have been lightly edited to reflect the difference between oral and written language.)

Lisa Anne Floyd has been teaching people to code, and how to teach with code, for most of a decade. Currently a doctoral candidate at Western University, she is a teacher,



coordinator, developer, author, researcher, and has worked at TVOntario, Inksmith, and other places with an interest in math, science, and/or coding. After interviewing Jeff Irvine (September issue), we had even more questions about what coding actually is, even as he helped us understand that it is more than just a tool to do mathematics. We hoped to get Lisa's views on the subject, based on her diverse experience in the field as an educator and researcher.

What we learned is an exciting view of computational thinking that is a direct descendant of the ideas and values of Seymour Papert and all the others who followed him and made contributions to this new idea in mathematics classrooms.

Computational Thinking 3.0

Computational thinking was first conceived by Papert as something students did while learning mathematics through interaction with a computer. He stated that learning math with a computer was like learning to speak French in France. Coding the computer as a way to learn math allowed students to engage with mathematical ideas on a level that they would not be able to reach alone. This is the idea of a low-floor, high-ceiling approach. In this Computational Thinking 1.0, Jeannette Wing later contributed that the thinking that goes on when we program a computer to make a mathematical model is a process of decomposition, pattern recognition, pattern abstraction, and algorithm design used recursively to solve a problem.

Moving onwards after Wing's contribution to the field that helped to spark the present movement of coding in the classroom, we get to Computational Thinking 2.0 with the addition of wide walls to the low floor, high ceiling of Computational Thinking 1.0. Mitch Resnick, Yasmin Kafai, George Gadanidis, and many others recognized that coding allowed for learning in many more areas than just mathematics. These wide walls allow for a much wider context for mathematical learning through the act of coding. There are now projects being conducted that have students creating games, for their peers to play, that incorporate storytelling, the arts, and potentially all the other subjects as well. What is the one thing that is common to all of these games? The underpinnings of all of them are fundamentally mathematical in nature.

Exciting as Computational Thinking 2.0 is, Lisa told us about the idea of a Computational Thinking 3.0 that she has been learning and thinking about from the writings and speeches of Yasmin Kafai. Kafai is one of the people who think about the idea of computational participation, or Computational Thinking 3.0. In Computational Thinking 2.0, the wide walls are those of the classroom or school. What happens if we allow ourselves to widen our scope beyond these wide walls? Think of it this way: a student who understands simple, basic coding will be able to think about important topics, like the use of artificial intelligence, in a much different way than someone who doesn't. They will have also been taught ~~and absorbed~~ a set of problem-solving skills that lend themselves well to solving problems other than computational ones. Our students would be able to look at problems much more broadly than before, and put to rest the old adage that "if all you have is a hammer, every problem looks like a nail."

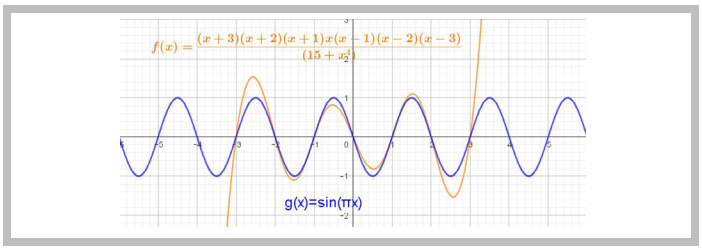
So what?

What does this mean for the mathematics classroom and mathematics learning? We are not sure. But… we can imagine. We know in classrooms, having seen it, how the use of coding allows our students to automate boring, repetitive math in order to get at bigger answers and seek bigger patterns by producing data. Imagine if we were able to teach our students how to use coding to interpret that data, and get the computer to help us codify that data, even if it is set at a very low floor. The things outside our classroom or school walls that are being built right now that use various forms of machine learning might not be so opaque, and we would not have to just accept them at face value. Students would be able to reason mathematically and logically about things presented to them. As a pie in the sky—moon shot—we hope it might help the coming younger generations to answer the age-old question of "Yes we can, but should we?" in a better way than we have until now.

"Seymour Papert talks about [how] low-floor, high-ceiling coding can be accessible for anyone, and you can push it. There's no end to what you can do with it… there's a lot of different aspects other than just the kind of narrow view of computational thinking. We're seeing it come to light with all of this, like all of this momentum and movement with coding."

We would really like your thoughts about the journey we are taking this year. You may write to us care of the *Gazette*.

If you would like to listen to our entire interview with Lisa Anne Floyd, it is available at OAME Talks (www.talks.oame.on.ca) or some of your favourite podcast providers. ▲

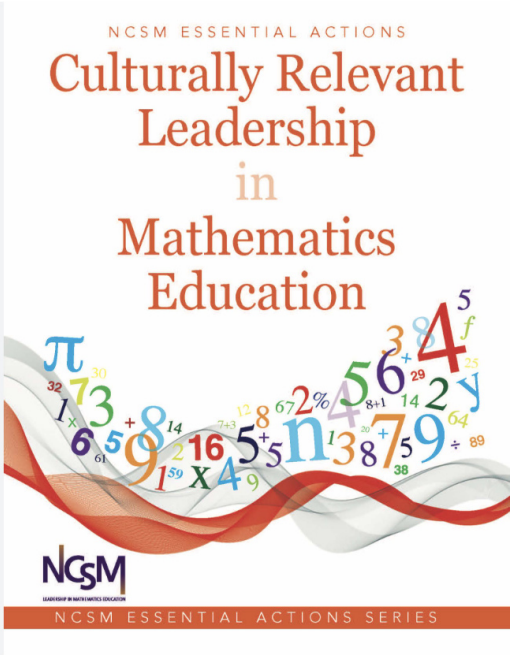


Book Review: Take Action with Culturally Relevant Pedagogy

MICHELE MELLOW
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Gray, P. et al. (2022). *NCSM essential actions: Culturally relevant leadership in mathematics education.* National Council of Supervisors of Mathematics (NCSM).

For educators who want to incorporate culturally relevant pedagogy into their math classroom, and have been wondering how to get started, begin by reading *NCSM Essential Actions: Culturally Relevant Leadership in Mathematics Education*. At only 132 pages, it is a succinct, actionable work that gives the reader clear examples from elementary classrooms showing how to examine a math task and recreate it as culturally relevant. If you and other staff at your school have already begun your equity journey, the reflection activities suggested in every chapter will help discern the effectiveness of your lessons and actions taken in the classroom and the school. The authors consistently challenge the reader, urging you to ask thought-provoking questions: "How will you empower those you serve?" and "How do we ensure that students who have traditionally been marginalized see themselves as mathematicians?" You will find this publication a valuable addition to your professional library.



This book is part of the NCSM Essential Action Series, which includes *Coaching in Mathematics Education, Instructional Leadership in Mathematics Education,* and *Framework for Leadership in Mathematics Education.* NCSM (National Council of Supervisors of Mathematics) is a North American mathematics education leadership organization that wrote this book in response to their vision to "guarantee all students engage in equitable, high-quality mathematical experiences," using the guiding question, "Do your students see themselves in your instructional tasks and classroom activities?" This work supports the first principle of the Ontario Mathematics (2020) curriculum, that a mathematics curriculum is most effective when it values and celebrates the diversity that exists among students. So even though the authors are from the United States, their suggested framework is applicable in Ontario classrooms. As well, the authors' suggestion of beginning with rich mathematical tasks fits with the current pedagogy espoused by Peter Liljedahl and Marian Small, Canadian scholars whose research many Ontario school boards promote among teachers.

NCSM Essential Actions: Culturally Relevant Leadership in Mathematics Education lives up to its promise to show educators, at all levels, the actions necessary to becoming culturally relevant math leaders. Using a friendly tone and relatable classroom anecdotes, it effectively reveals the progression of a lesson, from emerging to exemplary, and provides reflection questions for teachers, curriculum leaders, and principals to consider. Each chapter explains one of the four guiding principles: advocate, design, empower, and monitor. As well, it includes related leadership actions to enact these elements.

This book is suited to a wide variety of elementary educators. The detailed examples of mathematics task design focused on cultural relevance is necessary reading for K–8 teachers and curriculum leaders learning how to ensure students see themselves in their mathematics class. Principals would benefit from the many suggested leadership actions, such as partnering with families and community stakeholders, developing a shared understanding of culturally relevant pedagogy, and supporting the implementation of equitable mathematics teaching practices. Educational leaders such as coaches, resource teachers, and principals will appreciate the book's website, where they can download "reproducibles" to use for staff professional development.

There are some areas where the authors could have improved the content. Further explanation or examples of how to enact some of their recommendations would be helpful. For example, the book asserts that teachers need to believe all students are capable of doing mathematics. For those teachers who are not convinced of this statement, how can you persuade them that there is a moral imperative to apply culturally relevant pedagogy? What could that conversation sound like? I also wondered about the practicality of other suggestions. I agree that leaders should give time and space for teachers to reflect on their mathematical identity, and how

their identity and own math experiences impact their instructional practices. But when would this happen, especially when supply teachers are difficult to find and release time is not within school budgets? To effectively incorporate all the authors' suggestions, educators require system-wide support.

From the Introduction to the Epilogue, *NCSM Essential Actions: Culturally Relevant Leadership in Mathematics Education* offers many opportunities to contemplate our teaching practice. Give yourself time to reflect and ponder while reading this book, preferably with colleagues, because the material may challenge deep-seated assumptions about your teaching practice. Take time to complete the beliefs survey and reflect on the biases it uncovers for you. I found many statements throughout the book that resonated with me, such as pointing out the power we hold as mathematics teachers and that we should use this influence to offer students opportunities to see themselves in mathematics. So much of classroom teaching is beyond our control. It was an important reminder of what we can affect.

NCSM Essential Actions: Culturally Relevant Leadership in Mathematics Education is a worthwhile investment for your personal professional library, whether you are a new or experienced teacher. As all educators learn how to ensure students see themselves in math class, the many detailed recommendations will show a way forward for those who want to improve, but are not sure how or where to begin. Heed the call to action and get started with culturally responsive mathematics. ▲

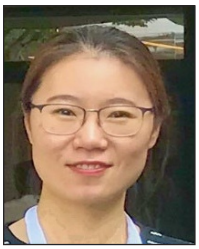


▲ Spatial and Logical Reasoning in Game Play: The Case of Santorini



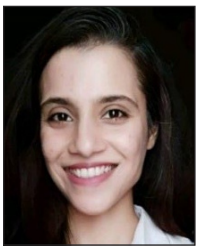
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Nimrah Ahmed is a doctoral student in Elementary STEM Education. She has a master's degree in clinical psychology. She is interested in using playfulness to understand the difficulties students face in learning.

Every move solves a problem, but also poses a problem for the opponent. When our elementary school students played Santorini, the board game served as their playground. With a 5 × 5 grid, the three-dimensional board of Santorini is a vibrant and visually appealing setting (see Figure 3). Following a handful of rules to govern the movement of the workers and placement of the blocks, students build and traverse multi-tiered towers. However, in deciding how to move a worker and place a block each turn, they encounter a problem-solving context. Two factors must be considered: first is to advance their position, and the second is to hinder the opponent's progress. Each decision

made then becomes a complex interplay between strategic tackling of moves and devising a plan against the opponent, gaining control and blocking the opponent, and *problem solving* and *problem posing*.



Figure 1: *Cora and Fayola playing*

During their play, Grade 6 students, Cora and Fayola, began their game at one corner of the board. As the turns progressed, it was Cora's turn to make a move and build. After a moment of studying the board, Cora successfully prevented Fayola from achieving a win by strategically moving a worker beside a three-level tower and placing a dome on top of it. Fayola identified and resolved the first challenge posed by Cora through her strategic response. While Cora explored a different section of the board, Fayola chose to remain at the initial corner and strategically constructed two adjacent towers—one at the one-level and the other at the two-level. Fayola's implementation of the two-tower strategy, involving the movement of workers between these towers to build a three-level tower for a winning move, posed a challenge for Cora. Realizing the implications of Fayola's move, Cora swiftly returned to the corner and strategically built to hinder Fayola's progress. Utilizing the two-level tower Fayola had constructed, Cora established two adjacent two-level towers. This successful resolution of Fayola's challenge also introduced a new problem for Fayola to tackle. Faced with this new problem and unable to find a solution, Fayola conceded the victory to Cora, acknowledging her opponent's strategic game play.

Cora and Fayola's game play was driven by the dynamic interplay between problem solving and problem posing. Their proficiency in employing logical reasoning (LR) and spatial reasoning (SR) played a critical role in navigating the complexities of the game. Relying on their reasoning, they transitioned between problem solving and problem posing, overcoming challenges and introducing new obstacles for the opponents.

Spatial Reasoning and Logical Reasoning in Mathematics Class

The Ontario curriculum emphasizes reasoning for students to "begin to justify or prove their solutions by providing evidence" (Ontario Ministry of Education [OME], 2020, p. 75). Under the Spatial Sense strand, *spatial reasoning* (SR) has been integral in developing geometry and measurement. In addition, "spatial reasoning is not a separate content area," rather, a process that should be integrated to support mathematical thinking (OME, 2014, p. 19). We define SR as the ability to see with the mind's eyes, imagine, visualize and mentally manipulate shapes, and navigate and relate to the physical world around us (Cohen & Hegarty, 2012). SR components are organized into six categories: altering, moving, situating, sensating (using senses like sight and touch to process spatial information), interpreting, (de)constructing, as shown in Figure 2a (Davis et al., 2015).

Logical reasoning (LR) has been identified in the Ontario curriculum under Problem-Solving Strategies as "methods that can be used to solve problems of various types" (OME, 2020, p. 75). We view logical reasoning as a systematic pattern of behaviour (Reid, 2002) to "develop lines of thinking or argument" (Brodie, 2010, p. 7). LR consists of conjecturing, analyzing, convincing, integrating, generalizing, investigating, etc. (Lannin et al., 2011; Mason et al., 2010; NCTM, 2000), as shown in Figure 2b. Although SR and LR are identified separately in the Ontario curriculum, we noticed their simultaneous enactment—co-enactment—by students during Santorini game play.

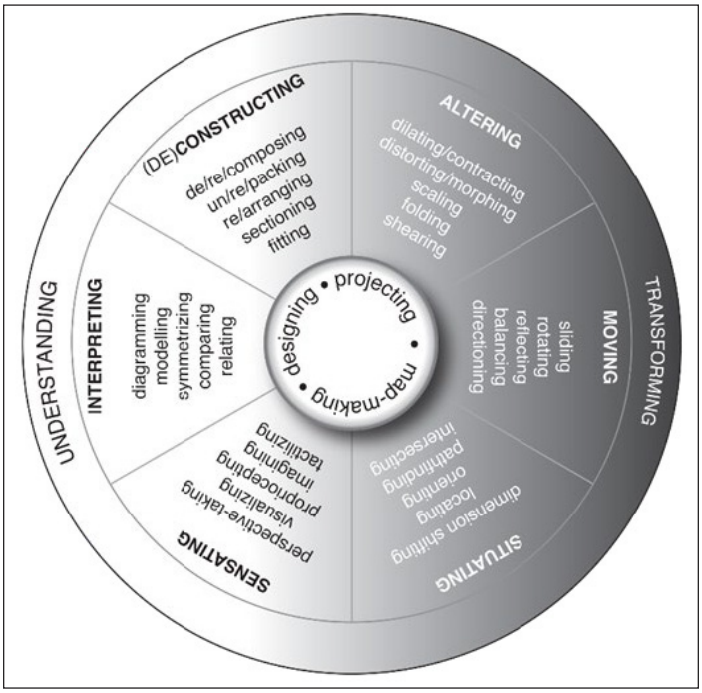


Figure 2a: *Spatial reasoning wheel*

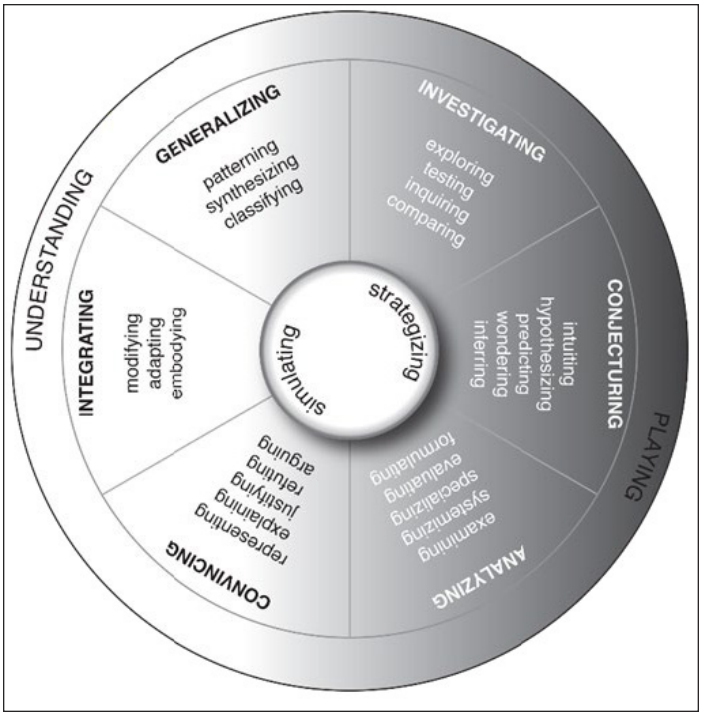


Figure 2b: *Logical reasoning wheel*

Incorporating Santorini in Elementary Mathematics Classes

Since we were interested in commercial games as a potential site for students' problem solving, in which they could use spatial reasoning and logical reasoning (SLR), we evaluated abstract reasoning games. One game we selected was Santorini. We found Santorini to be a great fit in our classes because it incorporated three-dimensional play, had no element of chance, took about 15 minutes to play, had a simple rule set, elicited analysis of several possible moves each turn, and allowed students to develop "an increasingly sophisticated understanding of size, shape, location, movement, and change, in both two and three dimensions" (OME, 2020, p. 88). In addition, the grid "helps to show where one object is in relation to another and can be a guide to determining distances and pathways" (p. 170). We appreciated the curricular relevance.

Santorini is played on a 5 × 5 grid with 3D game pieces (Figure 3). On every turn, a player moves one of their two workers into an adjacent space, so long as that space would not require the pawn to leap upward more than one level (although they can jump down any number of levels), nor move into another pawn's space. Then they construct a tower level on a space adjacent to the pawn just moved. A dome is placed on the fourth level, removing that space from play. A player wins by moving a worker onto the third level or by trapping their opponent's workers. Rule-bending god cards can change a player's abilities to move or build, as well as alternate winning conditions, which make every play of the game unique. We invite readers to visit an online board-game site, like Board Game Arena or Tabletopia, to try a game or two! For limited classroom budgets, students can play Santorini online for free or create a homemade version (e.g., use cardstock 5 × 5 grid, square colour tiles for tower blocks, centicubes for workers).



Figure 3: *Santorini*

As mathematics education researchers, we were fortunate to incorporate Santorini into three Grades 4–6 classes, though Santorini could be played well into high school. Each class played Santorini in a weekly 40- to 60-minute session over five weeks. Their teachers led the sessions, beginning with a 10-minute class discussion that showcased interesting reasoning from the students' play in the previous week to highlight components of spatial and logical reasoning (SLR). Next, the students were paired up in teams and played several rounds of the game, usually fitting two to four rounds into a class period. The teacher and researchers observed student game play and posed prompts such as—"How are you arranging your towers to move up a level?" or "What did you consider when you moved your worker?"—to encourage students' reasoning. At the end of the class, students completed reflection sheets that invited them to share how specific moves impacted their game, analyze board situations, and justify their emerging strategies. Students wrote explanations and drew accompanying pictures. Lesson resources are available at www.learnmathwithgames.com.

Student Examples of Co-enactment of Spatial and Logical Reasoning

We found that while our students engaged in Santorini

game play, they enacted SLR simultaneously for developing effective strategies. In fact, these two processes were intertwined and occurred rapidly in students' play, to the degree that it became meaningless to separate these cognitive actions. We became interested in how SLR was co-enacted and so, as we watched more closely and across our three classes of students, we noticed that pairs of SLR were enacted often by students. Therefore, we highlight three pairings—*locating and exploring*, *pathfinding and analyzing*, as well as *sectioning and modifying*—to demonstrate the co-enactment, illuminating each with student examples. Pseudonyms are used for student names.

Locating and Exploring. During the first rounds of Santorini, students were *exploring* the rules, pieces, and interactions around the board. They were figuring out how to make moves and build, including where to *locate* their workers to gain an advantage. At one table in the Grade 5 class, Eden's (Grey) and Amara's (Blue) initial placement of workers shows they were *exploring* two aspects of *locating* on the board (Figure 4a). First, Eden *located* almost opposite sides of the board for her workers to *explore* the effect of using a strategy of symmetrical piece placement from Tic Stac Toe, a game they played in the previous game cycle. Second, Amara chose to *explore* a game approach by putting her workers adjacent to Eden's workers, *locating* a close proximity as worth playing in her first round of Santorini.

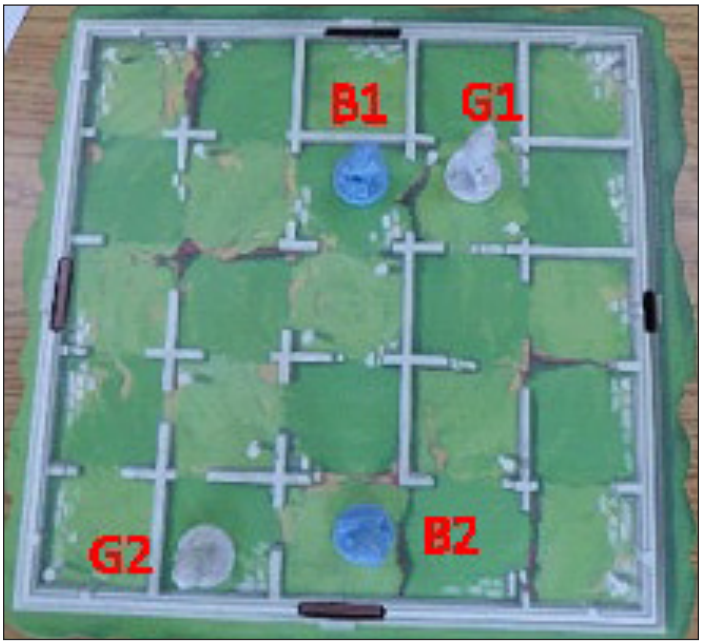


Figure 4a

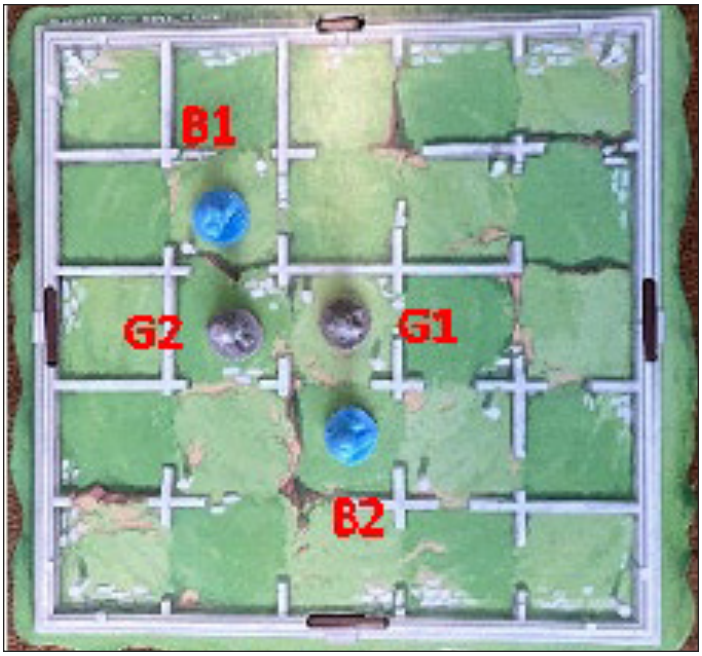


Figure 4b

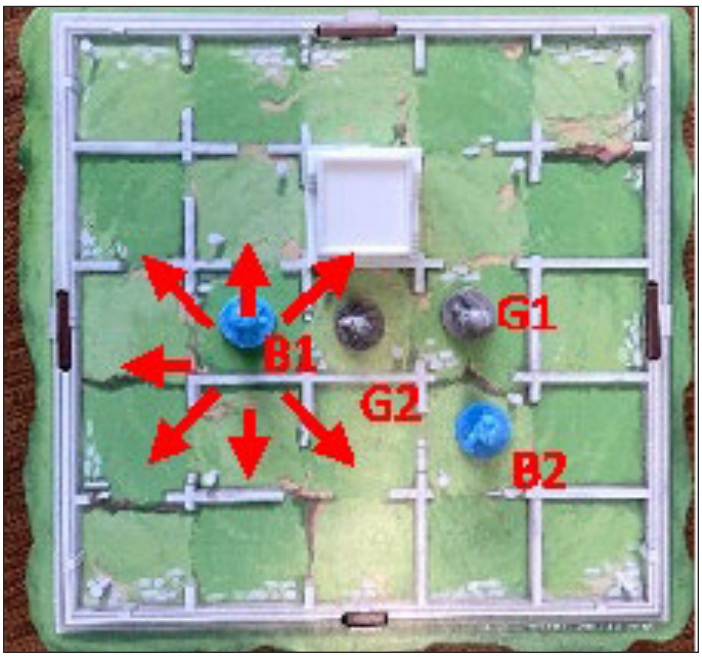


Figure 4c

At another table, Ana and Aisha (Blue) placed their workers against Dylan and Edison (Grey) intentionally (Figure 4b). Ana explained in her reflection sheet: "I thought it would be easier when they would be near each other." Her tentativeness in suggesting "I thought" shows she was *exploring* the game mechanic of worker placement to set up a good game by realizing how to *locate* her workers in relation to each other. After B1 and B2 were initially placed, Dylan (G1) and Edison (G2) chose to mirror the strategy by locating their pair of workers in even closer proximity to each other and between the Blue workers, *exploring* how they

might control the centre of the board through this placement. After initial worker placement, Ana and Aisha took their first turn by moving B1 diagonally and staying adjacent to G2 (Figure 4c). Their next decision was to determine where to place the tower piece. While *exploring* spaces adjacent to B1 (orange lines), Ana was able to *locate* possibilities for placement of her first tower level. As co-enactment of SLR, while students were initially *exploring* Santorini, *locating* workers and towers to discuss what to do on a turn was critical to learning the game rules to become more analytical.

Pathfinding and Analyzing. *Pathfinding* involves understanding spatial relationships and navigating through space to reach a specific destination. During Santorini play, students employed *pathfinding* as they mentally and gesturally manipulated the spatial information, such as levels, directions, and (un)available spots for moving workers. *Pathfinding* allowed them to devise paths for their workers and towers and obstruct routes for their opponents. We observed that *pathfinding* was frequently implemented in tandem with *analyzing*, where students considered the game-board state and possible future moves/builds for both teams (available evidence) to find the value of a turn and the turn's effects. We illustrate this pairing with a Grade 6 student below.

By *analyzing* the game board and potential legal moves, Travis (Blue) identified the possible paths that Kirsten's worker (G1) could take to escape, and used *pathfinding* to pinpoint the exact route (as seen in Figure 5a). Since G1 was unable to move up more than two levels, there was only one viable path available (shown with red arrow in Figure 5a), which Travis blocked by moving his worker (B1) down one spot and constructing a dome in the adjacent space (as shown in Figures 5b and 5c). Through the strategic move and build, Travis was able to successfully trap G1 within the top right corner (Figure 5d). As co-enactment of SLR, Travis engaged in *analyzing* when tracing out possible *pathways* for both players. And, *pathfinding* yielded more possibilities for *analysis* for a strategic turn.

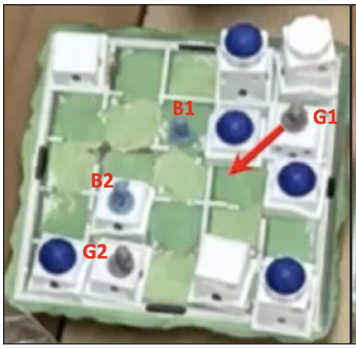


Figure 5a



Figure 5b

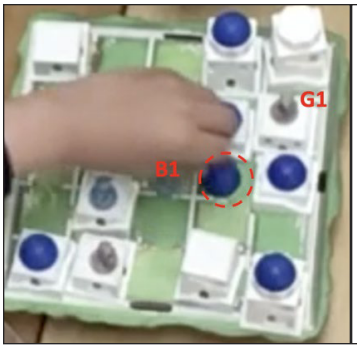


Figure 5c



Figure 5d

Sectioning and Modifying. In playing Santorini, students were *sectioning* as they divided the game board into different areas and controlled specific zones by strategically placing their workers and/or constructing towers. The Grade 4 students, especially, identified strategic sectioning in their play. Students wrote in their reflection sheets that, "Have your own little section" for winning, and "The corners [are] more useful, so you can create a mini-zone." "Corners" was a *modified* strategy from games played before Santorini. Here, instead of a corner as part of a winning line (e.g., Tic Stac Toe), the *sectioning* of a corner as a *modified* strategy could be used defensively to section off a part of the board to trap their opponents' workers.

Students *modified* imminent plans in response to opponents' turns, too. For example, when a pawn was trapped by *sectioning*, players reacted by *modifying* their strategy to creatively find ways to escape and perhaps even win the game eventually. Below, when Jack (Blue) noticed a line of three domes on the top left section of the board, and that G2 was close to that section (Figure 6a), Jack said, "There is a way I can win!" Jack sealed off that section with additional domes and trapped G2 (Figure 6b). Bilal (Grey) *modified* his plan to win, explaining: "He trapped me, but a turn later I figured out" how to *modify* the two-tower strategy to win the game (Figure 6c). It is worth noting that the closure of the *section* not only prevented G2 from leaving, but it also prevented Jack from entering the area to prevent a win. Thus, Jack scrambled to *modify* his imminent plan to win by trapping to try to get to the third level in another *section* of the board. Bilal used the two-tower strategy to move G2 to the top of the third level in his *section* and claim victory (Figure 6d). Jack and Bilal co-enacted *sectioning* and *modifying* during their game play for both offensive and defensive moves.

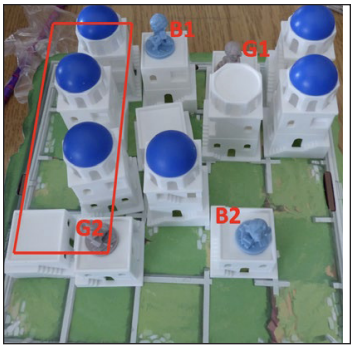


Figure 6a

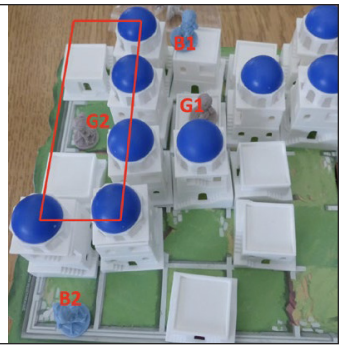


Figure 6b

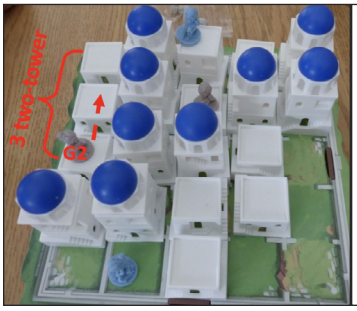


Figure 6c



Figure 6d

Reflections

We investigated spatial reasoning (SR) and logical reasoning (LR) as fundamental mathematical processes that play essential roles in problem solving, critical thinking, and development of mathematical literacy, which are explicitly highlighted in the Ontario Mathematics curriculum (OME, 2020). Santorini game play provided a context which invited students into challenging and joyful moments of mathematical thinking by engaging in SLR processes. As we observed and reflected on our students' Santorini play, we were thrilled to see many moments of both SR and LR. In fact, it was often challenging to make note of each instance of SR and LR because the students were rapidly and continually enacting SLR in each of their plays. However, key plays or moments where we could ask students to express their reasoning gave us an opportunity to slow down and identify instances of SR and LR. We realized that it was not only difficult to tease apart SR and LR, but that it was not meaningful to separate these moments to understand students' actions and mathematical thinking. Through Santorini game play, the students were co-enacting these two types of reasoning to either find solutions or pose challenges for their opponents. This co-enactment—SLR—is essential in problem-solving scenarios, where students need to leverage both types of reasoning simultaneously. The integration of SR and LR allows students to approach problems with flexibility and efficiency, fostering the development of a holistic and flexible mathematical mindset, as emphasized in the Ontario curriculum (OME, 2020). Additionally, our students across the grades were learning to "describe and represent… location, and movement by applying… spatial relationships" (OME, 2020, p. 257) when locating and exploring, to "describe and perform translations" (p. 314) when pathfinding and analyzing, and to "decomposing… into shapes with known areas" (p. 377) when sectioning and modifying.

Santorini provided a valuable context for actively involving our students in SLR; thus, we encourage teachers to consider incorporating this game into their instruction. We invite teachers to embark on an exploration of other commercial board games, too, that offer similar opportunities for students to immerse themselves in SLR. Drawing from our research, we suggest a selection of pedagogical approaches that can help effectively nurture and elicit students' SLR abilities. These include: (1) posing open-ended questions that empower students to articulate their reasoning through language, drawing, and gesture; (2) providing reflection sheets to encourage students to pause and reflect on their own thinking and reasoning; and (3) cultivating peer discussions that encourage collaboration among students. We are inspired by Lina, one of our Grade 4 students, who expressed during her interview, "I was using examining (LR) and locating (SR) together to make a good move." Why don't you ask your students how they are using SLR in their mathematical thinking and game play?

Acknowledgements

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- Remember that people who will sign up for your session have chosen to attend your presentation, so don't be nervous or afraid. You've got this!
- Be aware of the OAME/AOEM Vision and Mission Statements.

The mission of OAME/AOEM is to promote, support, and advocate for excellence in mathematics education throughout the Province of Ontario.

A Vision for Learning Mathematics: OAME/AOEM envisions an equitable mathematics education community in which everyone experiences high-quality, research-informed, and engaging activities that promote critical thinking, while fostering a positive attitude toward mathematics.

- To cover all contingencies, be sure to have a technology presentation, and a paper (hard-copy) back-up presentation as well.
- Consider co-presenting with someone. If you have presented previously, perhaps mentor someone else with similar interests.
- Have handouts available (both electronic and paper).
- Know your audience (contact them (as a group) once they have signed up), perhaps two weeks before the session. You can send a list of needs (e.g., computers).

Giving Your Presentation:

- Practise and time your presentation. Have a critical friend listen to your presentation.
- If possible, present your session locally (to your local OAME chapter).
- Make your session interactive and engaging to your audience.
- Know your space! Arrive to the designated room/space early to get it set up to your liking.
- If you have student examples and samples, be certain to have no faces in the photos (or be certain to have media waivers for students faces you have in the presentation).

After the Presentation:

- Once your session is complete, make notes on what you would like to change for the next time you present this session.
- Send a follow-up email to your participants after the session (to support them in their learning journeys).

See You at the Conference!!

Submitted by the OAME/AOEM Communications Committee. Written up by Jacqueline Hill.

Questions can be directed to Jacqueline.hill@oame.on.ca.

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The purpose of the OAME/AOEM Awards is to formally recognize individuals and educators who demonstrate an outstanding contribution to mathematics education in Ontario. The successful candidate for each award enhances the learning environment for all students, providing opportunities for students to do, see, hear, and touch mathematics in a profound and meaningful way. If you know someone who qualifies for this honour, please nominate them for one of the following awards.

OAME/AOEM Life Membership Award: This award recognizes an educator who has contributed in a significant way to OAME/AOEM, demonstrates outstanding leadership in mathematics education, and has accumulated ten or more years of membership in OAME/AOEM.

Award for Exceptional and Creative Teaching in Elementary Mathematics: This award recognizes an exceptional and creative elementary educator, who demonstrates excellence in mathematics education and contributes to the overall development of students.

Award for Exceptional and Creative Teaching in Secondary Mathematics: This award recognizes an exceptional and creative secondary school mathematics educator, who demonstrates excellence in mathematics education and contributes to the overall development of students.

Award for Exceptional and Creative Teaching in Post-Secondary Mathematics: This award recognizes an exceptional and creative post-secondary educator or team of educators, who demonstrates excellence in mathematics education, contributes to the overall development of students, and demonstrates leadership in the mathematics education community.

Elementary School Staff Award for Exceptional and Collaborative Mathematics Teaching: This award recognizes an elementary school (a school accommodating any or all of Grades K–8), whose staff fosters collaborative and excellent mathematics teaching, contributes to the overall development of student learning of mathematics, and demonstrates leadership in the mathematics community.

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1. Complete the Google **Nomination form** appropriate to the specific award, found at www.oame.on.ca (look for the **Awards** tab on the left of the page). You may include a one-page letter of recommendation.
2. Include with the nomination cover page up to **three letters of support**, of no more than two pages each. Please ensure all letters of support:
   - address the criteria for the award as outlined on the OAME/AOEM website
   - include the name and position of the supporter to the application
   - include the nominee's name and the award in the letter
3. Include the **Curriculum Vitae**, of up to five pages, for award nominees, except for Elementary and Secondary Staff awards.
4. Submit the Google Nomination form, and email the completed package (supporting letters and Curriculum Vitae) by the deadline advertised below to EDs@oame.on.ca.

DEADLINE: Nominations are due by January 31, 2024.
The awards will be presented at the OAME Annual Conference in May 2024.

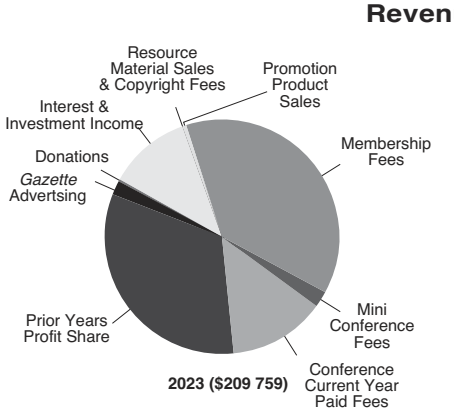
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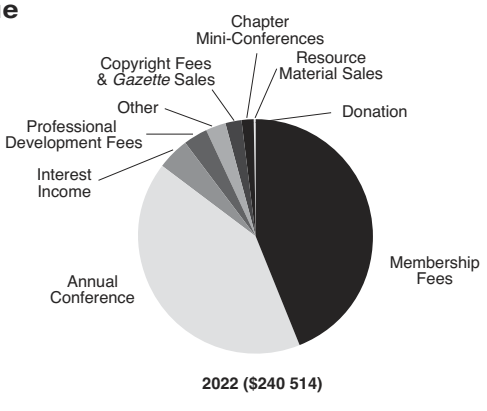
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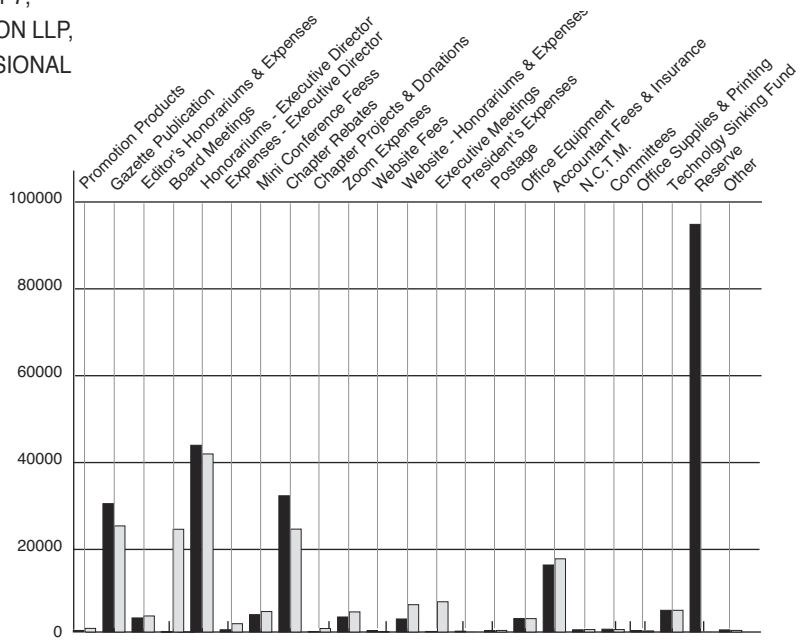
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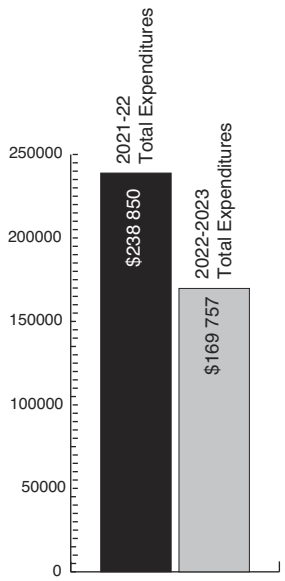




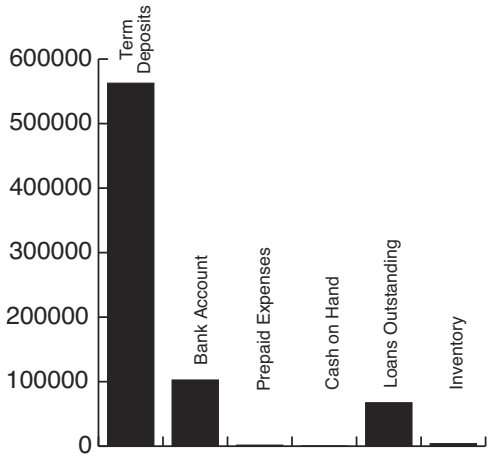
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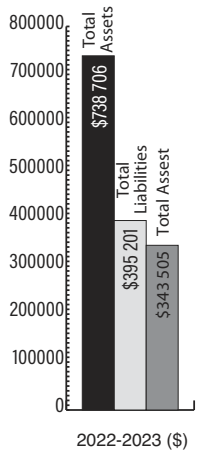
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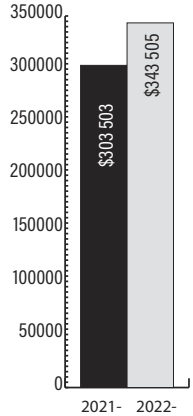


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