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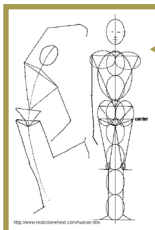
OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

Vol. 63#1
September 2024

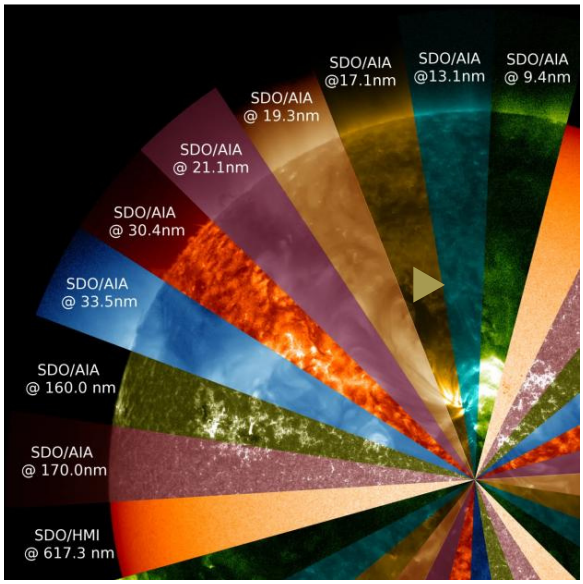
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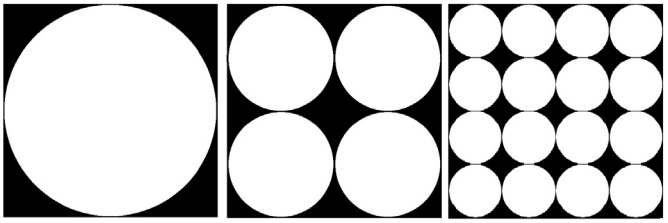
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Submission of Articles

The *Ontario Mathematics Gazette* (*OMG*) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *Gazette*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles are to be word–processed, MS Word is preferred, and prepared according to the *Publication Manual of the American Psychological Association*, Seventh Edition. However, please use single-line spacing (not double). Articles should not exceed five numbered pages of text, and figures, images, and photographs should be placed in the text close to where they belong, with captions. The photographer's permission is required, and for photos of students under the age of 18, the written permission of a parent or guardian is required.

Please submit your article in one blind file (i.e., identity of author is not evident), and include author names, contact information including email and mailing addresses, photos—head and shoulders, biographies—less than 100 words, and all content removed for blinding in a second file. Please email these two files to Tim Sibbald at gazette@oame.on.ca.

Upon review, you will be notified whether your article has been accepted for publication (as is, or pending minor or major revisions) or declined. The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME/AOEM.

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Editor
- ▲ **Tim Sibbald**
  gazette@oame.on.ca

Associate Editors
- ▲ **Anne Yeager**
  anne.yeager@rogers.com
- ▲ **Peter Saarimaki**
  saarimaki@rogers.com

Associate Editor (French articles) /Rédactrice adjointe (articles français)
- ▲ **Kyla Kadlec**
  kyla.kadlec@oame.on.ca

***Abacus* Co-Editors**
- ▲ **Marc Husband**
  marc.husbandl@oame.on.ca
- ▲ **Tina Rapke**
  tina.rapke@oame.on.ca

Copy Editor
- ▲ **Gitta Berg**
  gitta.berg@sympatico.ca

Design and Production
- ▲ **Penny Clemens,** Graphic Designer
  pennyydezign@gmail.com

Printing & Binding
- ▲ **Pole Printing,** Box 69, 1-89 King Street East
  Forest, ON N0N 1J0 (519) 786-5112
  poles@xcelco.on.ca

Advertising Manager
- ▲ **Robert Sherk**
  4366 Snider Rd., Verona, ON K0H 2W0
  Home (613) 374-1515
  robert.sherk@oame.on.ca

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Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be print ready, and colour advertisements should have no bleeds.

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▲ Editor's Report



RALPH CONNELLY
Ralph.Connelly@brocku.ca

Ralph Connelly is a Professor Emeritus in the Faculty of Education at Brock University. He is a former President of OAME (two terms), former Editor of the Gazette *(2+ terms), and has held several other positions with OAME, NCSM, and NCTM over many years! He is pleased and honoured to serve as guest Editor for this edition of the* Gazette *(and equally pleased to turn it back over to Tim for the next issue! ☺).*

"The more things change, the more they stay the same." That saying has often puzzled me, but being "back" as guest Editor for this edition of the *Gazette* has helped bring clarity to its meaning. The modern *Gazette* is obviously a dramatically different journal from the one I last edited close to four decades ago!

So what has changed? Answer: Just about everything! The "old *Gazette*" was essentially all articles. There was the "President's Message" and "Editor's Page," and usually a column relating what OAME Council had been doing. There was a "Problems" column, where it was hoped that readers would share their favourite problems, but which wound up being written by the Editor almost every time. This is in stark contrast to the current *Gazette*, which has an excellent "stable" of columnists! It should be noted that in addition to being superb writers, these columnists are fantastic at meeting deadlines, for which I'm extremely grateful—thank you all for making this guest Editor's job much easier! And speaking of columns and superb writers, beginning with the December *Gazette,* Ron Lancaster will be joining the already impressive list of columnists. In this new column, Ron will take readers on an Open Doors/Drawers tour of his home office to see the books, resources, games, toys, magic tricks, oddities, manipulatives, and found objects that he has used throughout his over 45 years of teaching mathematics. Various items will be selected for each column and will be accompanied by information about how Ron uses them in a mathematics classroom. Ron would like the column to be a place and space where readers can share what resources they have used to teach mathematics. He hopes the column will be useful to teachers looking for new strategies to make their lessons more engaging and to help students better understand mathematics. Readers interested in adding resources to their home or school mathematics office or

closet space will find a gold mine of ideas starting in the December column.

What else has changed? Although the "old *Gazette*" had a fairly long list of folks who were called "Associate Editors," and they were definitely invaluable to the Editor, these were the people who reviewed the articles, made recommendations for changes to them, etc., but didn't really "edit" the material when it was getting ready for publication. That job fell to the Editor. The current *Gazette* has **Anne Yeager** and **Peter Saarimaki**, who as Associate Editors, look over all the columns and articles and suggest changes that will make them clearer, etc. I sincerely appreciate the fine work they did on this issue of the *Gazette*, definitely making it better! The current *Gazette* also has the incomparable **Gitta Berg** as Copy Editor, a position that didn't exist in the "old *Gazette*." Gitta is incredible in more ways than I can begin to describe here! Her "eagle eye" catches every misplaced comma, missing period, incorrectly used capital, etc. (and will probably even "call me out" for using "etc." and emojis, but I claim editorial privilege! ☺). And she does it so quickly that it feels like I haven't finished sending it to her when I'm getting the email back that it's done! **Penny Clemens** then takes over for Design and Production, creating the visually appealing *Gazette* you are now reading—another position that didn't exist when I was Editor. A *Gazette* Editor who came a while after me (*Jack Weiner*) had the foresight to recommend Penny, and the OAME President at the time (some guy named Ralph Connelly), saw this as a terrific idea, and some 30+ years later, is getting to personally reap the benefits! Thank you, Penny, for all the amazing years of service to OAME!

This is where you, the reader, are likely to be thinking, "If the current *Gazette* is so completely different from the *Gazette* this old man is talking about, what could possibly be staying the same?" I'm so glad you asked! Before Tim Sibbald (aka the "actual Editor" ☺) left for his wonderful Australian adventure (including attending the 15th International Congress on Mathematical Education, ICME-15 in Sydney), he told me that there was one article ready for publication, one currently out for review, and no articles "in reserve." Wow, did THAT ever sound familiar! Recall that "my" *Gazette* was almost completely dependent on articles, and there were almost always never enough! It was a constant challenge to get people to submit articles. Indeed, at one point during one of my terms as Editor, I went as far as creating an assignment for a graduate mathematics education course I was teaching–that my students were to prepare an article (e.g., that could be about a teaching approach that worked for them, a great lesson idea that they had, a useful resource they'd found), and the article would be considered for publication in the *Gazette*. It would turn out that after being reviewed, revised, etc., their articles DID make it to the *Gazette*. It would seem that persuading those of you reading this that you indeed DO have something worth sharing with our readers and that you SHOULD consider submitting an article to the *Gazette* is just as much of a challenge today as it's always been! So please—as you read this issue of the *Gazette*—think about your own experiences, lesson ideas, effective resources, etc. (yes—there it is again, Gitta! ☺), and consider submitting to the *Gazette*—it can be short or long, conversational or formal—all will be considered.

In This Issue

Kit Luce delivers her first "President's Message," telling us a bit about herself and her plans for her term as President of OAME. **Jacqueline Hill** fills us in on upcoming NCTM events and offers some suggestions for making the new school year better than ever. **Pamela Brittain** lets us know what's been happening at the Fields Institute and what's coming, while **Heather Theijsmeijer** does the same for the Ontario Mathematics Coordinators Association (OMCA). The OAME Communications Committee provides suggestions for submitting a conference proposal. We also learn about the OAME Award winners.

Our columnists do their usual fine job of providing interesting and insightful information (see how I did that whole alliteration thing there? Not bad for a "math guy," right? ☺). **David Costello** provides some value-added mathematical routines for your classroom. **Jennifer Holm** provides some wonderful explorations involving circles and squares. **Shawn Godin** gives us an exceptionally flexible problem that can be explored in a wide variety of ways across many different grade levels. **Carly Ziniuk** provides an in-depth look at Heliophysics' Big Year. **Lynda Colgan** gives us food for thought regarding anthropomorphic ratios in measurement. **Iain Brodie** and **Beyza Sezer** interview **Derek Tangredi**. **Jérôme Proulx** gives us his musings on percentages. **Jeff Irvine** provides some useful theories about mathematical well-being.

Of course, we also have articles. **Cynthia Shi** talks about the beauty of dual intelligence in the age of AI (Artificial Intelligence), while **Jeff Irvine** discusses key insights into High-Impact Instructional Practices in Mathematics.

It has been a thrill for me to guide this issue of the *Gazette* to print, and to see what a truly impressive journal it has become through the years. Thanks, Tim, for trusting me with the task, and I hope you're as pleased with the result as I am! ▲

President's Message



KIT LUCE
kit@oame.on.ca

Kit has been an active member of MAC² for many years and joined the Board of Directors in 2022. She is currently a District Mathematics Facilitator for Grades K–9 with the Simcoe County District School Board. In her work, Kit creates math resources to support educators in their instruction and assessment practice. She also develops and facilitates professional development for her school board. Kit is an AQ (Additional Qualification) instructor and course writer, and she has written and reviewed work for a variety of publishers. She has been involved in a number of Ministry writing projects, and was a Content Lead for the recent OAME/AEMO writing projects. Kit was the recipient of the 2024 OAME Award for Leadership in Mathematics Education.

September always brings new beginnings, and this year, as I begin my term as President of OAME/AEMO, I am feeling a heightened sense of excitement, anticipation, and some disbelief. I have been involved with OAME/AEMO through my chapter, MAC², since I graduated Teachers College. I had the great good fortune to be part of the organizing committee for the OAME/AEMO 2016 Annual Conference in Barrie. This organization means a great deal to me. From my early days teaching, it has shaped me as an educator, so my disbelief comes from the fact that I'm beginning a role as President for an organization which has been such a positive influence in my life.

This year, it feels as if we have continued to build momentum. In November 2023, we had a Virtual Leadership Conference with Ilana Horn. OAME Talks webinars have continued with great success. Chapters have led book studies, speaker series, and mini-conferences, all of which have been well attended. The 2024 Conference in Kingston was an incredible success, and educators came away feeling inspired and excited (and perhaps a little tired as well). Educators have had a few years to get to know the new curricula for Grades 1–8, and Grade 9 MTH1W. ChatOAME has been launched for members. There is an energy and a desire for ongoing learning.

In the past year, as President-Elect, I have been immersing myself in the history of OAME/AEMO, reading the Constitution and the Position Papers, browsing issues of the *Gazette* and *Abacus*, and reviewing the Mission, Vision, and Strategic Priorities.

Mission Statement

The mission of OAME/AOEM is to promote, support, and advocate for excellence in mathematics education throughout the province of Ontario.

A Vision for Learning Mathematics

OAME/AOEM envisions an equitable mathematics education community in which everyone experiences high-quality, research-informed, and engaging activities that promote critical thinking, while fostering a positive attitude toward mathematics.

Strategic Priorities

- Promoting and advocating for equity in mathematics education
- Supporting all OAME members by providing value for membership, including quality professional learning, resources, and opportunities to network
- Advocating for high-quality, research-informed practices, while building a community to support mathematics education

I also thought back to when I first read the Curriculum Content documents when the new curricula were released. Both the Grades 1–8 and Grade 9 curricula reference the beauty of mathematics several times.

> The beauty in mathematics can be found in the process of deriving elegant and succinct approaches to resolving problems.
>
> At times, messy problems and seeming chaos may culminate in beautiful, sometimes surprising, results that are both simple and generalizable. Elegance and chaos are both integral to the beauty of mathematics itself and to the mathematical experience. In other words, the beauty of mathematics is illustrated and enhanced by students' diverse interpretations, strategies, representations, and identities—not diminished by them. Most importantly, students can experience wonder and beauty when they make exciting breakthroughs in problem solving.
>
> (Ontario Ministry of Education, *The Ontario Curriculum, Grades 9–12: Mathematics*, 2023, p. 7)

The key points of my thinking and reflection are equity, community, and excellence. We see the beauty of mathematics all around us. I know my family laughs at me (and sometimes groans) when I point out, "Hey, that's math!" To engage students, we need to make sure they see the math they are learning as relevant and connected to their lived realities. As an organization, we will continue to work toward increasing access for all educators to contribute to and receive high-quality professional learning. We will strive for

increased access to all levels of mathematics education for all students, providing educators with the resources they need to support their learners. And we will continue to build connections and networks throughout the province. As I begin my term as President, I hope to provide leadership in these areas and ask that you reach out with your ideas, questions, and concerns so that I can listen, learn, and grow in the role.

This organization, from the Board of Directors through to all the chapters and members, is an organization of educators who are dedicated to providing and improving mathematics education for the students of Ontario and beyond. I look forward to working with you through the next few years as we continue to promote, support, and advocate for excellence in mathematics education throughout the province of Ontario. ▲

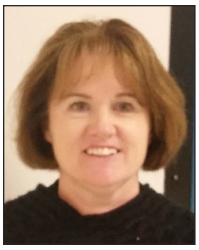
OAME/AOEM Awards for Excellence in Mathematics Education

The purpose of the OAME/AOEM Awards is to formally recognize individuals and educators who demonstrate an outstanding contribution to mathematics education in Ontario. The successful candidate for each award enhances the learning environment for all students, providing opportunities for students to do, see, hear, and touch mathematics in a profound and meaningful way. If you know someone who qualifies for this honour, please nominate her/him/them for one of the following awards.

- **OAME/AOEM Life Membership Award:** This award recognizes an educator who has contributed in a significant way to OAME/AOEM, demonstrates outstanding leadership in mathematics education, and has accumulated ten or more years of membership in OAME/AOEM.
- **Award for Exceptional and Creative Teaching in Elementary Mathematics:** This award recognizes an exceptional and creative elementary educator who demonstrates excellence in mathematics education and contributes to the overall development of students.
- **Award for Exceptional and Creative Teaching in Secondary Mathematics:** This award recognizes an exceptional and creative secondary school mathematics educator who demonstrates excellence in mathematics education and contributes to the overall development of students.
- **Award for Exceptional and Creative Teaching in Post-Secondary Mathematics:** This award recognizes an exceptional and creative post-secondary educator or team of educators who demonstrates excellence in mathematics education, contributes to the overall development of students, and demonstrates leadership in the mathematics education community.

Continued on page 23

▲ OAME/NCTM Report: "Another Day… Another Dollar"



JACQUELINE HILL
jacqueline.hill@bell.net

Jacqueline Hill is a Grades 9–12 teacher of mathematics. She is a Past President of OAME and OMCA, as well as the recipient of the Award for Exceptional Teaching in Secondary Mathematics. She also wrote the "Director's Dialogue" for the Gazette *for a number of years.*

This was a saying my father often used during my upbringing. He frequently emphasized that to achieve success, one must "use your elbow grease" and "stick with the job until it is done." As teachers, we find that receiving a few pieces of sound advice can significantly improve the smoothness of our year.

Here is my "top 5 list for your best year of teaching":

#1 Get to know your students. Build a rapport with them! Use their likes and interests in your lessons.

#2 Begin with the end in mind. Consider the question, "What do your students need to know to be successful this year, in their next math course, and in life?"

#3 Have long-range plans, but be flexible with the students you are teaching each year. Some years, things go off without a hitch, while other years, you have more challenging students, and life skills need to be more of a focus. This is not to say you do not cover the curriculum. It is just to say you need to work with the mix of students you have each year.

#4 Engage in daily exercise and ensure you drink sufficient water. Even modest efforts, such as parking your car at the farthest point from the supermarket entrance or consistently taking the stairs, can be beneficial. You will be more effective and supportive to your students and colleagues if you strive to be the best version of yourself.

#5 Dedicate time for yourself each week. Pursue a hobby or initiate a new one.

NCTM Upcoming Events:

- **September 25–28, 2024** – NCTM Annual Meeting – Chicago, Illinois

Continued on page 10

▲ Useful Theories to Know About: Mathematical Well-Being



JEFF IRVINE
dr.jeffrey.irvine@bell.net

Jeff Irvine, Ph.D., has been a secondary mathematics teacher, department head, and vice-principal. He has taught at three faculties of education and at a community college. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. With over 45 years in education, Jeff is particularly interested in the interplay of student motivation and mathematics achievement.

Clarkson et al. (2010) developed a taxonomy of mathematical well-being to describe the stages of emotions related to a student developing mathematically. The taxonomy is shown in Figure 1. The stages move from simple awareness to independently confident in mathematical activity. The five stages are described in Table 1.

Awareness and acceptance of mathematical activity

Responding positively to mathematical activity

Valuing mathematical activity

Having an integrated and conscious value structure for mathematics

Independently competent and confident in mathematical activity

Figure 1: *Mathematical Well-Being. Based on Clarkson et al. (2010).*

Table 1: *Mathematical Well-Being*

Stage	Description
1) Awareness and acceptance of mathematical activity	• The learner is aware of mathematics as a collection of mathematical activities, rather than a coherent body of knowledge. • The learner is aware that the nature of mathematics is different from that of other subjects. • The learner recognizes that a mathematical activity is different from a language or sports activity, and that it is accepted as a worthwhile pursuit. • The learner feels comfortable in the mathematical learning context, although the learner is passively accepting of the activity and does not seek them out.
2) Responding positively to mathematical activity	• Mathematical activity provokes a positive response. • Beyond acceptance, the learner welcomes the activity and feels pleasure in engaging in the activity and in its achievement. • This pleasure develops feelings of self-confidence and positive self-esteem. • These feelings reinforce the acceptance and worthwhileness of mathematical activity in general.
3) Valuing mathematical activity	• The learner appreciates and enjoys mathematical activity. • There is an active seeking out of these activities and of people with whom the learner can share the activities. • The learner increases awareness of the human development of mathematical knowledge and the learner's place in mathematics. • The learner achieves acceptable (to them) high levels of mathematical competence.
4) Having an integrated and conscious value structure for mathematics	• The learner is aware of their appreciation of mathematics. • The learner is aware of how and why they value mathematics. • The learner is aware of where the valuing of mathematics may lead them in the future. • The learner is confident in their level of skill and competence, and in their ability to judge their own strengths and weaknesses.
5) Independently competent and confident in mathematical activity	• The learner is fully independent in mathematical activity. • The learner can hold their own in mathematical arguments at various levels. • The learner can criticize the arguments of others from well-rehearsed criteria.

From Clarkson et al., 2010, p. 127

Many of our students reach the first stage (awareness) and do not progress beyond that stage. They recognize that math is different from other subjects, but see mathematics as a disconnected series of procedures and facts. This makes our subject far more difficult for them, compared to other subjects, such as history, which seem to be more coherent and connected. A subset of students reaches stage two, liking mathematics, feeling confident in mathematical activities, and building self-confidence and self-esteem as "good" mathematics students. Teachers often refer to these students as "being good at math," "liking math," or "having mathematics ability." Some of these attitudes by teachers are unfortunate, since they tend to separate students into being good at mathematics or being bad at mathematics.

A smaller subset of students reaches stage three, perceiving mathematics as valuable to them and actively choosing to do more, or more challenging, mathematics. For example, students who plan to go into post-secondary studies in engineering recognize the value of not only doing more mathematics, but also becoming competent in mathematics as a useful tool to aid them in their chosen career path. Stage four is the self-regulation stage, where students are aware of their level of competence in mathematics, the utility, and also the beauty of mathematics, and the students possess the confidence to pursue mathematical activities to increase their competence.

A select few of our students reach stage five, independently seeking out mathematical activities and critiquing and formulating mathematical arguments. This may occur in high school or possibly in post-secondary programs.

Why Should We (Math Teachers) Care about This Theory?

It is well documented that there are strong connections with achievement in mathematics among attitudes, engagement, and self-confidence. If we can increase students' self-confidence and motivation to engage in mathematical activities, we will increase our students' achievement in math. In addition, it is incumbent on us to move as many students as we can from stage one (simple awareness and passive acceptance of math) to higher stages. We may have few students who reach stage five, but the more students we can move to higher stages, the more likely it is that our students will achieve their full potential in math. In addition, when we raise the profile of mathematics as a subject, our students' feelings will be more positive toward mathematics and they will perceive mathematics as a valuable and coherent body of knowledge. These are valuable goals for all mathematics teachers to pursue.

Continued on page 10

▲ Meaning Making in the Classroom: Value-Added Mathematical Routines for Your Classroom



DAVID COSTELLO
david@costellomath.com

David s a principal, who taught and had various board roles in Prince Edward Island. David has also instructed university courses regarding curriculum, differentiation, mathematics, and literacy. He facilitates professional learning in mathematics and school development. Dr. David Costello has published four books, with two of his most recent books being: Messing Around with Math: Ready-to-Use Problems That Engage Students in a Better Understanding of Key Math Concepts, *and* Mathematizing Student Thinking: Connecting Problem Solving to Everyday Life and Building Capable and Confident Math Learners.

Teachers place much emphasis on the structure of their math class. An important part of this structure is mathematical routines that will support student learning.

In this column, I will share four strategies that could be used to structure mathematical routines at any grade level. These mathematical routines require little teacher prep work, but offer significant gains in terms of student learning.

Strategies

There is no one-size-fits-all approach to education. These four strategies (Daily Cumulative Review, Exit Tickets, Number of the Day, Interactive Number Line) can be used in isolation or combination. The strategies can take as little as a few minutes each day or every other day. In fact, the strategies can be thought of in terms of weekly, perhaps on a Friday.

Perhaps, most powerful, is the application of one or some of these strategies to provide students opportunities to engage with concepts over a period of time. The strategies do not have to focus on the concept that is currently being explored; instead, they could be used to encourage students to recall concepts from earlier in the year.

Daily Cumulative Review

One strategy that can accomplish this goal is a daily cumulative review. Daily cumulative review provides students daily opportunities to review key concepts. Within daily cumulative review, there are multiple key concepts

addressed, and key concepts are spaced over time. The following is an example of daily cumulative review from an elementary classroom.

ELEMENTARY EXAMPLE: A TEACHER'S EXPERIENCE

Mixing concepts for a week of daily cumulative review

In this example, the teacher identified fractions, place value, decimals, and multiplication as the concepts she wanted to emphasize during the review. Notice how the concepts are arranged in a different order from day to day. (The teacher would continue focusing on these four concepts for several weeks and then revisit them occasionally thereafter.)

Monday	1. Write the following number in words: 583 839.34 2. Arrange the following in ascending order: 0.532, 0.523, 0.352, 0.325 3. Solve: 68×34 4. Place >, <, or = in the circle: $\frac{5}{6} \bigcirc \frac{7}{8}$
Tuesday	1. Arrange the following in descending order: 8.041, 8.140, 8.104, 8.014 2. Write the following number in words: 76 284.481 3. Place >, <, or = in the circle: $\frac{2}{5} \bigcirc \frac{2}{6}$ 4. Solve: 42×26
Wednesday	1. Solve: 98×98 2. Place >, <, or = in the circle: $\frac{4}{7} \bigcirc \frac{5}{8}$ 3. Arrange the following in ascending order: 7.22, 2.72, 2.27, 7.72 4. Write the following number in words: 900 290.075
Thursday	1. Arrange the following in descending order: 23.873, 23.837, 23.738, 23.783 2. Place >, <, or = in the circle: $\frac{3}{4} \bigcirc \frac{2}{3}$ 3. Write the following number in words: 38.098 4. Solve: 56×47
Friday	1. Write the following number in words: 509.006 2. Solve: 38×25 3. Place >, <, or = in the circle: $\frac{3}{5} \bigcirc \frac{4}{9}$ 4. Arrange the following in ascending order: 63.429, 63.294, 63.942, 63.924

(Source: Costello, 2021)

As can be seen in the above visual, students are provided four questions, with each question addressing a key concept. This approach to review will require students to engage with multiple concepts throughout the session so that they are required to read the problem, understand it, select a strategy, apply the strategy, and judge the reasonableness of their response

In addition to multiple concepts being addressed within each review session, concepts are spaced over time so that students will engage with them throughout the year. This revisiting of key concepts will strengthen student understanding and their ability to recall previous learning.

Daily cumulative review is intended to take eight to ten minutes. Students are assigned four questions and are given three to five minutes to solve the questions. During this time, students are to work individually. The teacher subsequently provides the worked-out solutions. This usually takes three to five minutes. Students then check their work against the solutions provided by the teacher.

While some teachers use this daily, others will use it every other day. On the days that they do not use daily cumulative review, teachers will use other mathematical routines (please see options below).

Exit Tickets

Exit tickets usually consist of one or two questions designed to give teachers a quick gauge of student learning. For this reason, many people tend to think of exit tickets as being solely an assessment tool. However, exit tickets can also be a powerful learning activity for students. When the prompt on the exit ticket relates to a concept that was explored earlier in the year, there is a great learning opportunity for students.

Exit tickets typically focus on the concept students engaged with in that particular class. While there is power in such a consolidation task, I want to suggest exit tickets can be used to support students in recalling key concepts from earlier in the school year. By approaching exit tickets in such a manner, students will not only strengthen their understanding of the concept, but also their ability to recall previous learning.

The following prompts can be used to structure the exit tickets (Costello, 2021):

- **Free Write** – In a Free Write, students brainstorm about a concept without judgment. The idea is to generate as much information as possible related to the concept.
- **15 Words or Less** – Similar to a free write, 15 Words or Less is an opportunity to write about a concept, but there is a limit on the total number of words to be used. By introducing the limit, students need to focus on choosing the most crucial aspects from memory and be concise to avoid exceeding the word count limit.
- **Compare and Contrast** – Compare and Contrast requires students to closely examine two concepts and to distinguish their similarities and differences. This process helps the student identify foundational aspects of both concepts. Linkages between the two concepts will be generated as students compare and contrast the concepts.
- **Dual Coding** – Dual Coding is the process of representing understanding, using both text and pictures. Dual coding strengthens learning due to

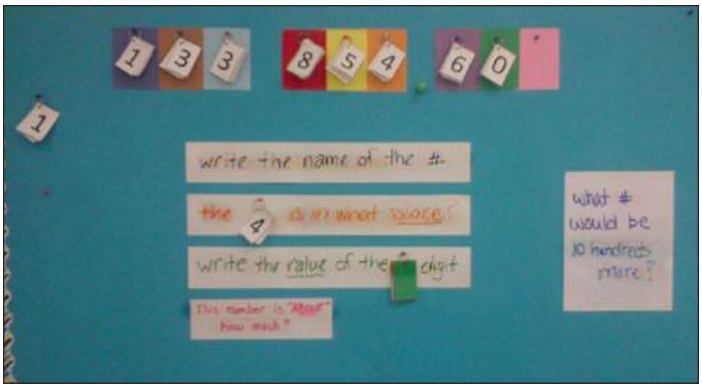
words and visuals being processed through separate channels in the brain.

Exit tickets can take as little as three to five minutes of class time and can be used daily or weekly. In situations where students have writing exceptionalities, exit tickets can take the form of oral sharing, speech to text, scribing, etc.

Number of the Day

A number-of-the-day routine involves students working with a variety of numbers throughout the year. With a number-of-the-day routine, students are provided the opportunity to engage with a particular concept or strategy over days, weeks, or months so that understanding is strengthened.

Number of the day does not have to be a time-intensive task for either the teacher or student(s). The visual below is an example from a combined Grades 5/6 classroom, and illustrates how place value was used as the structure for the number of the day. The teacher would adjust the digit cards daily so that students had a different number to work with each day. Some students in the classroom would only work with the whole-number part, while other students worked with all of the number. The green button between the orange and purple pieces of paper acted as the decimal.



(Source: Costello, 2019)

The number-of-the-day routine could be a warm-up task for students, an exit ticket, and/or a number centre during independent practice. Through observation, the teacher can easily collect assessment data on students as they work through each task.

Interactive Number Line

A daily mathematical routine structured on number lines can be a powerful learning experience for students, not to mention a great assessment opportunity for teachers. The following visual is an example from an elementary classroom.



(Source: Costello, 2019)

The interactive number line provides the opportunity to relate differing representations of number, such as fractions and decimals. As can be seen in the visual above, the white pieces of paper are benchmark fractions. The benchmark fractions were assigned by the teacher. The blue and pink pieces of paper were placed by the students. This particular interactive number line supports the use of benchmark fractions, ordering fractions, ordering decimals, and relating fractions to decimals.

The interactive number line could be a warm-up task for students, an exit ticket, and/or a number centre during independent practice. Through observation, the teacher can easily collect assessment data on students as they work through placing the number representation cards in appropriate places.

All that is needed to implement an interactive number line is a dedicated wall space. Be mindful that when making the interactive number line, you should flexibly alternate between vertical and horizontal markings. Both ways of presenting an interactive number line promote flexibility of thinking so that students do not get used to seeing a number line in a particular way.

Final Thoughts

Teachers are constantly looking for ways to strengthen instruction and support student learning, yet these ways should not be overwhelming and time consuming for teachers. Instead, I have identified strategies that when applied, can have significant value added in terms of instruction and learning. It is about working smarter, not harder.

The mathematical routines shared within this column can be applied to any grade level and can be applied within any instructional model you may be currently using. The routines can take as little as three minutes, and can be used daily, twice daily, or weekly. Along with these aspects, the routines will provide teachers with valuable assessment data that can be used to inform current and future instruction.

Before I end this column, I want to provide one more piece of advice. Take time. Explore the strategies I suggest as mathematical routines, and determine which makes most

sense for you and your students. You may stick with one or you may use multiple.

I look forward to continuing the discussion of mathematical learning and instruction via my website (www.costellomath.com), email (david@costellomath.com), or X (@dr_costello).

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OAME/NCTM Report: "Another Day… Another Dollar".
Continued from page 5

- **September 27, 2024** – Tennessee Mathematics Teachers Association Annual Meeting – Martin, Tennessee
- **October 21, 2024** – Louisiana Association of Teachers of Mathematics – Baton Rouge, Louisiana
- **October 24, 2024** – ATMNE 2024 – Sheraton, Tara Nashua, New Hampshire
- **November 12, 2024** – Reaching All Learners Together – Ann Arbor, Michigan
- **November 14, 2024** – ACTM Fall Forum – Oxford, Alabama
- **February 6, 2025** – SD STEM ED Conference – Huron, South Dakota

Until next time,
Yours in mathematical fun,
Your NCTM rep, Jacqueline ▲

Meaning Making in the Classroom: Value-Added Mathematical Routines for Your Classroom.
Continued from page 7

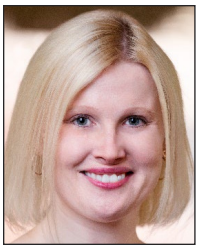
If You Want to Learn More

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▲ MB4T (Mathematics by and for Teachers): Exploring the Circle



JENNIFER HOLM
jholm@wlu.ca

Jennifer Holm is an Associate Professor at Wilfrid Laurier University and works with Primary/Junior and Junior/Intermediate pre-service teachers, as well as in the field supporting current mathematics teachers. She is interested in developing mathematics knowledge for teaching with both pre-service and in-service teachers. She focuses on the beliefs and opinions that pre-service teachers hold about mathematics and teaching and the connection they have to past experiences. She uses this research to support future teachers in developing beliefs and knowledge that will encourage and support effective teaching practices in mathematics.

The exploration of the circle is one that can be challenging for students, with π being part of the measurement. I wrote about π in a previous column (Holm, 2019). Exploring similar shapes, though, can show that there is a similar pattern between circles and other shapes. Take, for instance, the squares in Figure 1. The square on the left has a side length that is double the one on the right. The area of the square on the left (16 cm^2) would be four times the one on the right (4 cm^2), or the one on the right would have one-fourth the area of the square on the left. This is showing similar squares, where the ratio between the squares is kept the same. If you use the same logic with other squares, you will see a similar pattern. Since squares have the same length on each side, you would only need to note one side length difference. With rectangles that are not squares, both the side lengths would need to change in a similar proportion to see the same relationship.

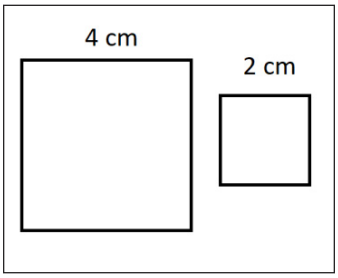


Figure 1: *Similar squares*

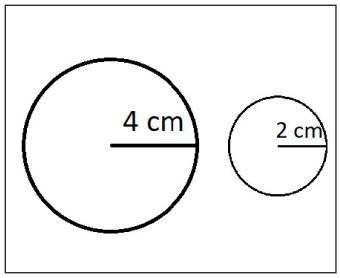


Figure 2: *Similar circles*

Now let's consider circles with the same relationship (Figure 2): the one on the left has a diameter that is double the one on the right. What is the relationship between the two areas? If you calculate the area, you will notice that the same relationship is there: the area of the circle on the left (approximately 50.24 cm^2) is four times the size of the circle on the right (approximately 12.56 cm^2). Again, this is an example of similar shapes and the relationship between them.

One of my favourite problems to use when talking about circles is the circle and square problem (Olson et al., 2014). See Figure 3 for the image, but the question asks, "Which one of the images has the most dark-coloured space? Explain your answer." Each square is the same size, and each circle touches the edges and/or the one next to it. Take a moment for yourself to decide which you would think has the most dark space around the circles.

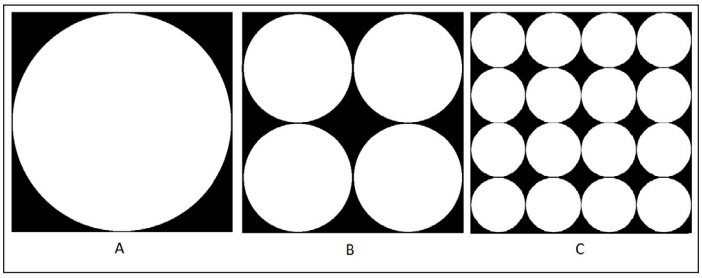


Figure 3: *Circles and squares problem (Olson et al., 2014)*

I have given this problem to students previously, and there are three answers that tend to be the most common: Figure A, Figure C, or they are all the same. In this column, we are going to look at why students may think of those answers, and how a student might be able to reason through the problem to determine the correct solution. Oesterle et al. (2016) discuss giving this problem to adults in teacher education programs.

Students who note that Figure A is the answer usually mention that because the spaces on the outside look bigger, they must have the most black space. They also sometimes note that since the circle is a compact shape, the larger circle must use the space the best, leaving the most black space outside. For the students who note that Figure C has more areas that are black, they note that it must have the most black space.

The correct answer to this problem is that all the figures have the same amount of black space. We are now going to look at different ways that students have figured this out for the answer. The first solution I often see is students who use reasoning to determine that they are the same. Since one circle in Figure B is exactly one-fourth the size of Figure A, the amount of black space around one circle must also be one-fourth of Figure A. Since there are four circles, then adding it together (1/4+1/4+1/4+1/4) equals the same amount as Figure A. Each circle in Figure C is 1/16 of Figure A, so if you apply the same logic with Figure A as the "whole," then you again see that the black space is the same amount.

A similar solution method occurs when the students who divide each shape into quarters to focus on one section of the image at a time (see Figure 4). When looking at the single circle in Figure B, you can see four areas of black space. Comparing it to Figure C, you see there are four times as many circles, making them one-fourth of the circle in Figure B, which also means that the size of the black space has been reduced proportionally and there are now "sixteen" black spaces if you consider each piece of black space around the circle as one piece. Comparing these to Figure A, you see one-fourth of a full circle, which makes it four times as big as one circle in Figure B, and only one piece of black (which would also be four times as large as one space in Figure B), again, showing that all three images have the same area of black space.

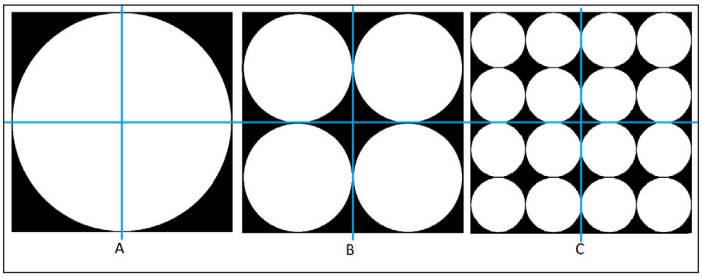


Figure 4: *Dividing the circles and squares problem (Olson et al., 2014) divided into quarters*

The most popular way for students to answer this question is to assign a pretend dimension to each square to calculate the areas. Since the problem notes that each square is the same size, then the side lengths would all be the same. Most students go for something that makes it easy to work with, for example, the side length of 8. See Figure 5 for the example showing the measurements. The area for each of the squares would be equal, so figuring out the area of the circles would show the solution. In Figure A, the circle would have a radius of 4 units, so the area would be $A = \pi 4^2$, or about 50.24 units squared. In Figure B, the circle would have a radius of 2 units, so the area would be $A = \pi 2^2$, or about 12.56 units squared (showing it is one-fourth of the total amount). This would mean that the total circle area would be about 50.24 units squared. In Figure C, the circle would have a radius of 1 unit, so the area would be $A = \pi 1^2$, or about 3.14 units squared. This would mean that the total circle area would be about 50.24 units squared. Since the total circle area is the same for all three figures, then the amount of black space would have to be the same.

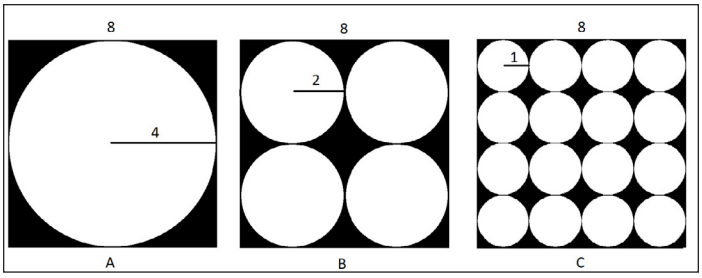


Figure 5: *Assigning measurements to determine a solution for the circles and squares problem (Olson et al., 2014)*

In summary, the circles and squares problem offers different ways for students to reason about the problem, while learning more about similar circles. Students who feel most comfortable in their reasoning can give a solution for the problem, and those who appreciate formulas and "proving" the answer can also do this. Since there are different ways a student may initially "see" the problem, there are lots of opportunities for communication about which solution is correct.

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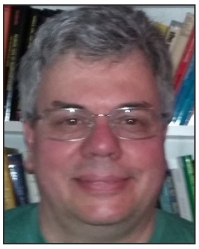
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▲ What's the Problem? Decked Out

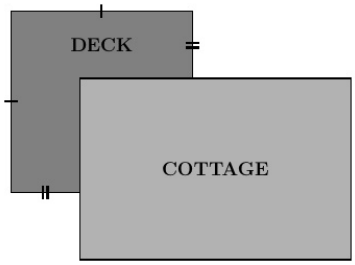


SHAWN GODIN
GodinMathStuff@gmail.com

Shawn is a former mathematics teacher, department head, and consultant. He strongly believes in the central role of problem solving in the mathematics classroom. He continues to be involved in mathematical activities: presenting workshops, writing articles, working on local projects, and helping create mathematics contests.

Welcome back, problem solvers. I hope you all had an enjoyable, restful summer. We are changing things up in the column a little bit. Regular readers may have noticed that there was no homework last issue. I am hoping to solicit problems from the readers and use them in future columns. Our first problem from the field comes from Anne Yeager. Although Anne is retired, many OAME members will remember her from her involvement with OAME and the Ontario Ministry of Education. Anne used the following problem in some of her workshops:

> You have been hired to build a deck attached to the second floor of a cottage, using exactly 48 prefabricated 1 m by 1 m sections. Determine the dimensions of at least two different decks that follow the specifications in the diagram. Will different decks require different amounts of railing? Explain.



I love problems like this because they can be solved by several different techniques, are accessible and challenging to a wide range of students' ages and abilities, and lead themselves to extensions and connections limited only by the imagination of the students (and teacher). When I asked for Anne's thoughts about the problem, her motivation for the problem was virtually identical to my own. She had mentioned the following:

- The problem can be done by a wide range of ages of children.

- The problem lends itself nicely to manipulatives, making it good for younger students and the kinaesthetic learner of any age.
- It is great for the visual learner, especially when looking at the algebraic solutions and seeing the connection to the pictures.
- The problem can be differentiated perfectly seamlessly.
- Many of the mathematical processes are involved. There is lots of opportunity for communication (especially if the students are working in groups), selecting various tools and strategies, using different representations, making connections, and reflecting.

We will investigate this problem first, using a very concrete method, and then moving on to a more abstract method. This is done to reflect learners with increasing experience.

Any student who understands the concept of area can engage with this problem. As the problem involves prefabricated squares (which we will assume are not to be cut), it lends itself nicely to an approach using manipulatives: square tiles, linking cubes, or even graph paper. Starting with 48 square tiles, students should be able to come up with a solution that satisfies the conditions of the problem. Some interesting and complex reasoning and connecting can occur even in this very concrete case, so teachers should be listening to the conversations students are having and asking the right probing questions.

A systematic search works well in situations where trial and error is being used. Students who seem to be trying things at random should be encouraged to use a more organized approached to cut down the chances of overlooking cases. Working systematically, we may even decide to assign names to things like length, for the longer dimension of the deck, and width, for the shorter dimension, to help guide the way we proceed through the trials, as well as aiding the discussion of the results.

Several observations may occur to the students that can help find a solution to the problem, as well as make connections between mathematical concepts. The first observation is that the deck can be broken up into two identical rectangles (shapes 1 and 3) and a square (shape 2), as shown in Figure 1. Students may see that if a perfect square is subtracted from our total area of 48 tiles, the remaining area must be divisible by 2. This observation leads to a couple of conclusions. Firstly, the width must be less than 7 units; otherwise, the square is too big. Secondly, since the two rectangles are identical, their combined area is an even number. Therefore, since the total area is also even, the width must be even. So, by making some observations and making some connections to some concepts from numeration, we started with a problem where we were unsure how many cases we would have to try, determined there were at most six cases, and then reduced that number to three. Students could then be satisfied that when they have checked the three cases, they would have *all* possible solutions.

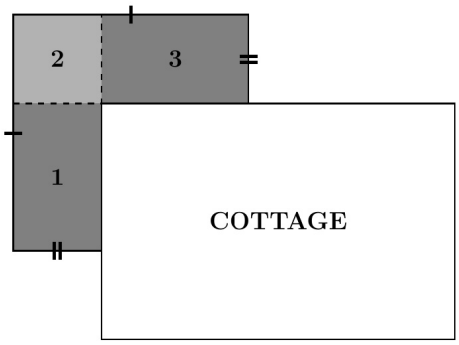


Figure 1: *Deck broken into rectangles and a square*

Students may even be able to generalize their findings, if nudged in the right direction. What can we see in general? Since our deck will break up into the two rectangles and one square, no matter how many prefabricated sections we started with, our width is constrained in the sense that the square of the width must be strictly less than the area. Put another way, the width is strictly less than the square root of the area (number of prefabricated sections). Also, since we have two identical rectangles, the combined area of the rectangles is an even number of sections. Hence, since adding an even number to *any* number doesn't change its parity (odd plus even is odd, and even plus even is even), then the width must be of the *same parity* as the total number of sections used.

Even though 48 is quite large, it is doable with manipulatives. However, if we increased it to something like 175 sections, this might seem too large to even start. Nevertheless, if we have solved the original problem and made the generalizations above, we know that the width is less than $\sqrt{175} \approx 13.2$. Also, since the area is an odd number, the width must also be odd. Therefore, we only have seven cases to explore corresponding to widths of 1, 3, 5, 7, 9, 11, and 13. So even with limited mathematical tools, students can make some deep connections.

If we move up in mathematical sophistication to a student who understands a bit of algebra, we can explore the problem in another way. As with our first solution, at first glance, it might make sense to assign variables for the length and width of the deck, which we will call ℓ and w, as shown in Figure 2.

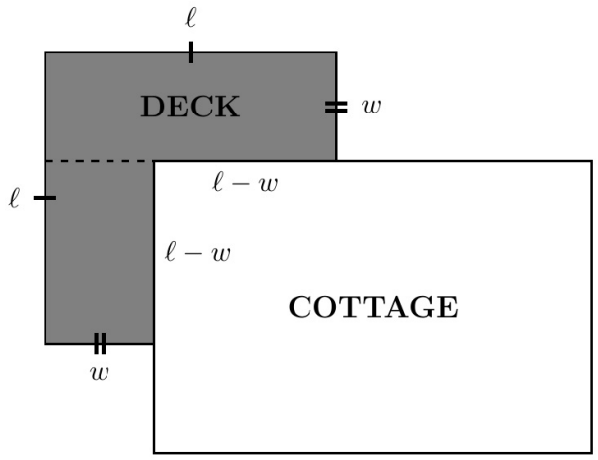


Figure 2: *Assigning variables*

We will dissect the deck into two rectangles, slightly different from Figure 1, from which we can come up with the equation

$$\ell w + (\ell - w)w = 48,$$

which simplifies to

$$2\ell w - w^2 = 48.$$

The interested reader may like to verify that the dissection in Figure 1 leads to the same equation. Once we have the equation, we can start looking for ways that we can use it. Right at the outset, since we have an equation involving two variables, we can explore it, using technology. Figure 3 shows the graph of the equation $2xy - x^2 = 48$ from Desmos, where w and ℓ have been replaced by x and y, respectively. Students could then search for *lattice points* on the graph (points with integer coordinates), which *may* correspond to solutions. Care must be taken, as students may find points like (24,13) on the graph. This point yields values for the variables that satisfy the equation, yet are *not* solutions to our problem (why?).

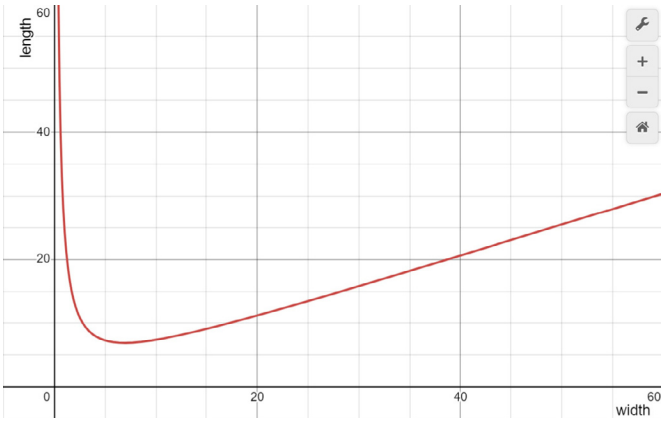


Figure 3: *Desmos graph of our problem*

Similarly, a spreadsheet could be set up with our length and width values to search for solutions, as in Figure 4.

Interested readers can explore this spreadsheet at www.docs.google.com/spreadsheets/d/1XMEoDJL5PeymgtGguY-lbFVH8bj4laae8ji8yu39qPM/edit?usp=sharing. Note in the spreadsheet that I have used *conditional formatting* to colour any solutions green, and "bad" values red (I will leave it to the reader to figure out what I mean by "bad"). Readers interested in exploring conditional formatting can click on one of the red cells, then, under the *Format* menu, select *Conditional formatting*. The rules for the green and red cells open on the right. Interested readers are encouraged to click on either rule to see how it works.

		LENGTH																		
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	1	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39
	2	0	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76
	3	-3	3	9	15	21	27	33	39	45	51	57	63	69	75	81	87	93	99	105	111
	4	-8	0	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144
	5	-15	-5	5	15	25	35	45	55	65	75	85	95	105	115	125	135	145	155	165	175
	6	-24	-12	0	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204
	7	-35	-21	-7	7	21	35	49	63	77	91	105	119	133	147	161	175	189	203	217	231
	8	-48	-32	-16	0	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256
	9	-63	-45	-27	-9	9	27	45	63	81	99	117	135	153	171	189	207	225	243	261	279
WIDTH	10	-80	-60	-40	-20	0	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300
	11	-99	-77	-55	-33	-11	11	33	55	77	99	121	143	165	187	209	231	253	275	297	319
	12	-120	-96	-72	-48	-24	0	24	48	72	96	120	144	168	192	216	240	264	288	312	336
	13	-143	-117	-91	-65	-39	-13	13	39	65	91	117	143	169	195	221	247	273	299	325	351
	14	-168	-140	-112	-84	-56	-28	0	28	56	84	112	140	168	196	224	252	280	308	336	364
	15	195	165	135	105	75	45	15	15	45	75	105	135	165	195	225	255	285	315	345	375
	16	-224	-192	-160	-128	-96	-64	-32	0	32	64	96	128	160	192	224	256	288	320	352	384
	17	-255	-221	-187	-153	-119	-85	-51	-17	17	51	85	119	153	187	221	255	289	323	357	391
	18	-288	-252	-216	-180	-144	-108	-72	-36	0	36	72	108	144	180	216	252	288	324	360	396
	19	-323	-285	-247	-209	-171	-133	-95	-57	-19	19	57	95	133	171	209	247	285	323	361	399
	20	-360	-320	-280	-240	-200	-160	-120	-80	-40	0	40	80	120	160	200	240	280	320	360	400

Figure 4: *Spreadsheet for the problem*

However, if the student has some facility and knowledge of algebra, we can glean some information about our problem. For example, as $w > 0$, we can rewrite the equation as

$$2\ell = \frac{48}{w} + w.$$

Since the length and the width are positive integers, we can deduce that w must be a factor of 48; otherwise, the right-hand side will not be an integer. We could rewrite this equation as

$$2\ell = d + w,$$

where $dw = 48$. Thus, since $d + w$ must be even, w must be part of a pair of factors of 48 that have the same parity. However, since 48 is *even*, if it is written as the product of two numbers, at least one of them must be even. Therefore, w must be even, as we discovered earlier with the numerical generalizations. We could have also discovered this by rearranging the equation a different way and factoring it to get

$$2(\ell w - 24) = w^2.$$

If we revisit our diagram, we can redo Figure 2 as in Figure 5, where we recognize that there are really *three* variables involved in this problem, although they are all linked together.

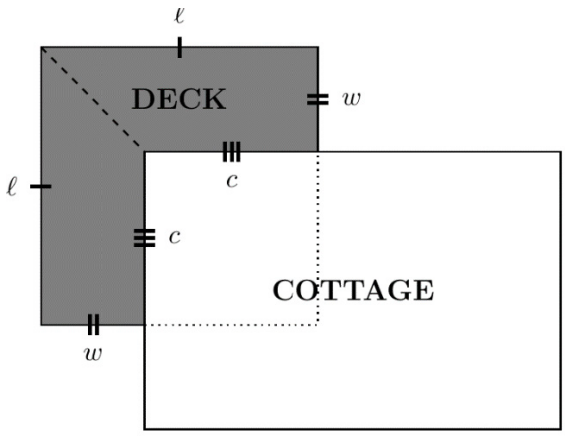


Figure 5: *Three variables*

We have our length and width (ℓ and w) from earlier, but also the length along the cottage, c. Thus, we have

$$c + w = \ell.$$

Hence, we can write out area in several ways. The dissection from Figure 1 can be written as

$$w^2 + 2cw = 48.$$

The one from Figure 2 can be rewritten as

$$\ell w + cw = 48,$$

which is the same as what we would get from Figure 5, if we used the dashed diagonal to dissect the deck into two congruent trapezoids and rearranged them into one long rectangle. If we used the dotted lines from Figure 5, we get something similar to my initial thoughts on the problem. I started with

$$\ell^2 + c^2 = 48,$$

factored and ended up with a system of equations,

$$\ell + c = a$$
$$\ell - c = b,$$

where $ab = 48$. Then elimination yields that a and b must have the same parity, and hence must both be even, which reduced my number of cases to 3, each yielding a solution: $(\ell,w,c) \in \{(13,2,1),(8,4,4),(7,6,1)\}$ or, put another way,

$$\ell = \frac{a+b}{2}$$
$$w = b$$
$$c = \frac{a-b}{2},$$

where $ab = 48$, $a > b$, and both a and b are even.

Again, we can generalize this solution and imagine what would happen if we replaced 48 with some other number. If the new number were odd, each pair of factors would yield one possible deck configuration, except if the new number was a perfect square (why?). If the number was even, all pairs of factors, where both factors are *even*, would yield a possible deck configuration, except if the new number was a perfect square. We can also deduce that we *cannot* construct a deck using an even number of prefabricated sections, if the number of sections is not divisible by 4. That is, we saw that we could build three different decks, using 48 sections, but there are none possible that use 46 or 50 sections. Interesting!

As I am a big fan of number theory, at this point, I could go down a rabbit hole, never to be seen again. The algebraic generalization allows us not only to predict which numbers of prefabricated sections lead to at least one solution, but with further thinking, we can start to classify which numbers will yield a particular number of solutions. Solutions are all tied into the ways we can write the number of sections as a product of two (distinct) numbers. However, in some cases, almost all of these pairs lead to solutions, and in others, very few do. The interested reader might enjoy seeing if they can find the first few values that yield one, two, or three solutions.

Of course, the problem can be explored further in many ways. The original problem asks about the railing (perimeter). We could see what our generalized solutions tell us about the amount of railing needed. We could also loosen some of the constrictions on the problem. What happens if the widths of both parts of the deck do not have to be the same? What if the lengths don't have to be the same? What if neither the length nor width of the two parts must be the same? Does dropping some of the constrictions yield other solutions?

This problem reminds me of a few other problems that have the same "feel" to them. That is, they can be approached and solved by younger students, but are also interesting and full of surprises for older students, using some algebraic modelling. The first one is the *painted cube problem*. I am pretty sure I have mentioned this problem previously in this column, although I have never explored its solution. Anne Yeager had also sent me this problem along with the deck problem. I think the first time I saw this was at a keynote presentation at OAME 1995 in Guelph. I have run into it many times over the years.

A $3 \times 3 \times 3$ cube made up of small cubes is dipped into a bucket of red paint and removed. How many of the small cubes will have three faces painted? Two faces painted? One face painted? Zero faces painted? What happens when we repeat the process with a $10 \times 10 \times 10$ cube? What happens in general for an n cube?

The second problem, the *border problem*, I encountered for the first time at an OAME board of directors meeting about 15 years ago. I have seen it used many times in workshops over the years, and it was a staple of my Grade 9 classes. Here is a version that appears in Jo Boaler and

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▲ OMCA News: September 2024



HEATHER THEIJSMEIJER
(OMCA PAST PRESIDENT)
OMCAmath@gmail.com

Heather is a Past President of OMCA and an active member of OMCA.

Welcome back to a new school year! The Ontario Mathematics Coordinators Association (OMCA) is looking forward to another great year of learning with mathematics coordinators, consultants, and coaches from all across Ontario.

At the final members' meeting of the year on Monday, June 3, outgoing President, Jason To (Toronto District School Board), passed the ceremonial fan to incoming President, Gerry Lewis (Toronto Catholic District School Board). OMCA is excited to welcome our new Executive for the Fall: Jason To (Past President), Gerry Lewis (President), Melissa Peddie (Near North District School Board, Vice-President), Allison Lawrence (Brant Haldimand Norfolk Catholic District School Board, Secretary), and Mariarosa Schniering (York Catholic District School Board, Treasurer).

This year's recipients of the 2024 OMCA Award for Excellence and Innovation in Mathematics Teaching are Brian McBain (Lambton Kent District School Board), and Lisa Tomas (Huron Superior Catholic District School Board). This award is given annually to OMCA members, based on nominations by their peers, in recognition of excellence and innovation related to any areas of mathematics education (integration of technology, equity, implementation of curriculum, motivation of teachers, assessment and evaluation, professional development ideas, etc). Congratulations, Brian and Lisa!

Our membership finished the year with exceptional professional development opportunities. At our May meeting in Kingston, guest speaker Shawn Bredin from the Ontario Ministry of Education spoke about our potential impact as math coordinators, consultants, and coaches within the new Math Achievement Action Plan. At our final virtual meeting of the year in June, Dr. Alex Lawson and Heather Wark shared early findings in their latest research, which will soon be published in *What to Look For 2: Understanding and Developing Student Thinking in Multiplicative Thinking*. Our June meeting also saw members network and engage in open conversations in a digital EdCamp format, planning forward for the new school year. As always, it was great to hear perspectives from across the province.

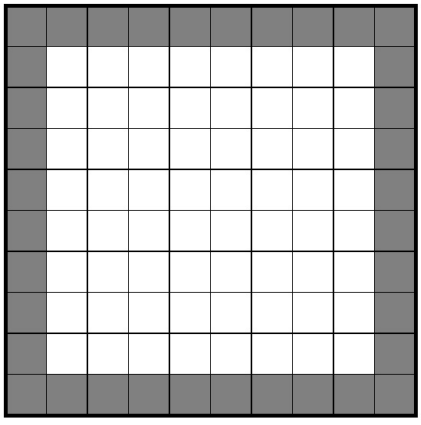
Each OMCA monthly meeting is hosted by a different board, which shares a land acknowledgement, presents a recent in-board practice or resource, and offers a round-table question for discussion by the membership. This gives members a unique opportunity to network with colleagues around shared successes and challenges. In addition to also hearing from representatives from the Ontario Ministry of Education, Education Quality and Accountability Office, OAME, and National Council of Supervisors of Mathematics at our monthly meetings, guest speakers like those mentioned above share their latest findings and research, providing members with timely professional development specific to our roles in math leadership.

Are you a math leader or math coach with your board? Interested in joining OMCA? New members are always welcome. Contact us directly at OMCAmath@gmail.com, or visit our website at www.omca.website (note the different domain), to learn more about the organization and connect with us. Our first virtual meeting of the 2024–2025 school year will be Monday, September 16. We hope to see you there! ▲

Continued from page 15

Cathy Humphreys' book, *Connecting Mathematical Ideas: Middle School Video Cases to Support Teaching and Learning.*

A 10 × 10 grid has the squares on the border shaded. Determine the number of shaded squares in as many ways as possible.



Both problems can be explored in a concrete or abstract way, and are thus accessible for students of a wide range of ages and experiences.

I hope you enjoyed the problems from this issue. Please send me any good problems you have used in your classroom. Until next time, happy problem solving. ▲

▲ In the Middle: Helio's Big Year



CARLY ZINIUK
carlyziniuk@gmail.com

Carly Ziniuk teaches Grade 9 Mathematics, Grade 12 Data Management and Advanced Placement Statistics at the Bishop Strachan School in Toronto. She is very active in adopting real-life data to engage her students in solving problems. Among 30 global STEM leaders, Carly is a member of the NASA Space Apps Collective for 2023–2024. As a Canadian educator, she is contributing toward the pilot program's chosen theme: Building an Inclusive Global Space Community.

Author's Note: I have used the guidelines as presented by NASA's *Technical Documentation Style Guide* throughout this piece, when referring to the Moon, the Sun, and Earth/earth. The NASA Style Guide capitalizes Earth, when it is used as the name of the planet. In this case, it is not preceded by "the", since one would not comparably say "the Mars." If referring to ground or soil, but not the planet, then "earth" is lowercase. Capitalization of "Sun" and "Moon" refers specifically to our Sun and our Moon (but lowercase, for example, with "Jupiter's moons" or the moon Titan). Use of "the" with Sun and Moon is appropriate for emphasis or clarity.



Figure 1: *NASA's Heliophysics' Big Year Logo*

You may have noticed Astronomy is having a moment with young people: April's total solar eclipse had #beautifulmoon trending with North American teens, and just a month later, the breathtaking aurora (Northern Lights) were all over TikTok. Not surprisingly, NASA predicted this celestial excitement, declaring this "Helio's Big Year." The Heliophysics Big Year is a celebration around the planet, recognizing the Sun's influence on Earth and our entire "solar" system!

NASA's Dr. Nicky Fox introduces Heliophysics' Big Year (youtu.be/G3BFbB5FyBc) with the reminder, "The Sun is a living breathing star. It is active, dynamic, and ever-changing. Just like us." Until December 24, 2024, NASA's creating exciting ways to interact with the power of the Sun. I encourage you to consider this Big Year opportunity as a mathematician might: by using exponents and scientific notation with your students.

The Canadian Space Agency's (CSA's) vocabulary list defines Heliocentric as "measured or defined relative to the centre of the Sun" (www.asc-csa.gc.ca/eng/resources/vocabulary). Although we currently believe there's no centre to our universe, the Sun at least is the centre of our solar system: it's right in the name! In our solar system, we relate the bodies in space closest to us, with respect to their relationship with the Sun.

The CSA provides the following fact sheet for our "extraordinary, ordinary star," the Sun.

About the Sun	
Type	Star
Size (diameter)	1.4 million km or about 110 Earths
Mass	2.0×10^{30} kg or about 333 000 Earths
Orbital Period	250 million years
Orbits around	The centre of the Milky Way
Number of planets	8
Average distance from the centre of the Milky Way	2.8×10^{17} km or about 27 200 light years
Temperature	About 5500 degrees Celsius on its surface

Scientific notation is essential to working with space, and especially work on the Sun. Let's start with a "warm-up" by considering these numbers about our heliocentre and re-expressing them, using scientific notation. Adapted from NASA's *Space Math's Scientific Notation I, II, III, Grades 6–7,* pages 12–14, spacemath.gsfc.nasa.gov, I have adjusted the units here to provide the opportunity for metric conversions, making these problems just a little hotter! One interesting task here is to ask students to create questions for each other that require conversion, calculations, and comparisons of the different values.

Space Fact	Standard form	Scientific Notation
Mass of the Sun	1 989 000 000 000 000 000 000 000 000 000 000 g	
Mass of Mercury	330 104 000 000 000 000 000 000 000 kg	
Mass of the Moon		7.3×10^{22} kg
Mass of Earth	5 974 000 000 000 000 000 000 000 kg	
Power output of the Sun	382 700 000 000 000 000 000 000 000 W	
Age of the Sun		4.5×10^{9} years old
Distance from Earth to the Sun		1.5×10^{8} km
Distance from Mercury to the Sun	70 million km	
Diameter of the Sun		1.4×10^{6} km
Hottest Temperature at the core		2.7×10^{6} °C

In looking at public and media explanations of natural phenomena more broadly, I have been discussing science communication with my English and Language Arts colleagues. Enamoured with this year's total solar eclipse, my students and I explored orbital paths to understand how we can predict when and where the next eclipse would be. One task we have worked on considers how to interpret, justify, and express mathematical thinking in context. Adapted from NASA Science's "Our Sun: Facts" (science.nasa.gov/sun/facts), I created the following science journalism questions that could be used for reporting about the Sun.

Our Sun is a medium-sized star with a radius of 700 000 km. Many stars are much larger, but the Sun is far more massive than our home planet: it would take more than 330 000 Earths to match the mass of the Sun, and it would take 1.3 million Earths to fill the Sun's volume.

1. If Earth's radius is 6371 km, and assuming both are approximately spherical, justify the calculations above.

The Sun orbits the centre of the Milky Way, bringing with it planets, asteroids, comets, and other objects in our solar system. Our solar system is moving with an average velocity of 720 000 km/h. But even at this speed, it takes about 230 years for the Sun to make one complete trip around the Milky Way.

2. How long, in distance, is that complete trip?

Temperatures in the Sun top 15 million °C and it's about 138 000 km thick. The density of the Sun's core is about 150 g/cm^3, approximately 8 times the density of gold and 13 times the density of lead.

Number of nuclear particles in the Sun	$\frac{\text{Mass of the Sun in grams}}{\text{Number of grams per particle}}$	OR $\frac{2.0 \times 1033}{1.7 \times 10\text{-}24}$	=
Energy carried by visible light	Energy per cycle × Number of cycles	OR $(6.6 \times 10^{-2}) \times (5.0 \times 10^{14})$	=
Average density of the Sun	$\frac{\text{Mass of the Sun in grams}}{\text{Volume of the Sun in cubic cm}}$	OR $\frac{1.9 \times 10^{33}}{1.4 \times 10^{33}}$	=
Number of sun-like stars within 300 light years	Density of stars per cubic light year × Number of cubic light years	OR $(2.0 \times 10^{-3}) (4.0 \times 10^{6})$	=

3. Using only the information here, what are the density of lead and gold?
4. Research the actual density and see how rounded the calculations are. Do you agree with the use of the word "approximately" here?

Each of the space bodies in our solar system has an atmosphere, the outer layer of gas surrounding the object. Although the atmosphere may surround a planetary core, it can extend thousands of kilometers into space and have a significant mass. As a result, examining these masses requires use of scientific notation. Adapted from NASA's *Space Math's Composition of Planets, Grade 7,* page 7, spacemath.gsfc.nasa.gov, the percentage of these gases in each space object's atmosphere is shown below.

Space Object	Atmospheric Mass (kg)	Nitrogen	Oxygen	Argon	Sodium	Hydrogen	Helium	Other
Sun	3.0×10^{30}					71%	26%	3%
Mercury	1000		42%		22%	22%	6%	8%
Earth	1.4×1021	78%	21%	1%				<1%
Moon	100 000			70%	1%		29%	

Argon is used on Earth in refrigeration, fire extinguishers, and inflating airbags. Comparable in this table, Earth's atmosphere has a larger mass, but a smaller percentage of argon, while the Moon's atmosphere has a smaller mass, but a larger percentage of argon. Alternatively, the atmospheres of the Sun and the Moon have similar percentages of helium.

5. In kilograms, which space object has more argon overall?
6. What fraction of the Sun's atmospheric helium is the total helium in the Moon's atmosphere?

My students often need additional practice with dividing and multiplying exponents. Working with scientific notation,

especially those involving integer exponents, can help justify these rules. In space writing, scientific notation often involves the very small, like the number of grams in a particle, and the very large, like the number of particles in a space object. Space context included with routine scientific notation reinforcement like these below are adapted from *NASA's Space Math's Scientific Notation I, II, III, Grades 6–7*, pages 12–14, spacemath.gsfc.nasa.gov. These examples show a practical reason for the adjustment of the coefficient, after the first computation done with exponent laws.

The graph below shows the planets in our solar system, plotting their diameter versus the distance from the Sun. Although it is a pretty small set of data, consider what you might be able to determine.

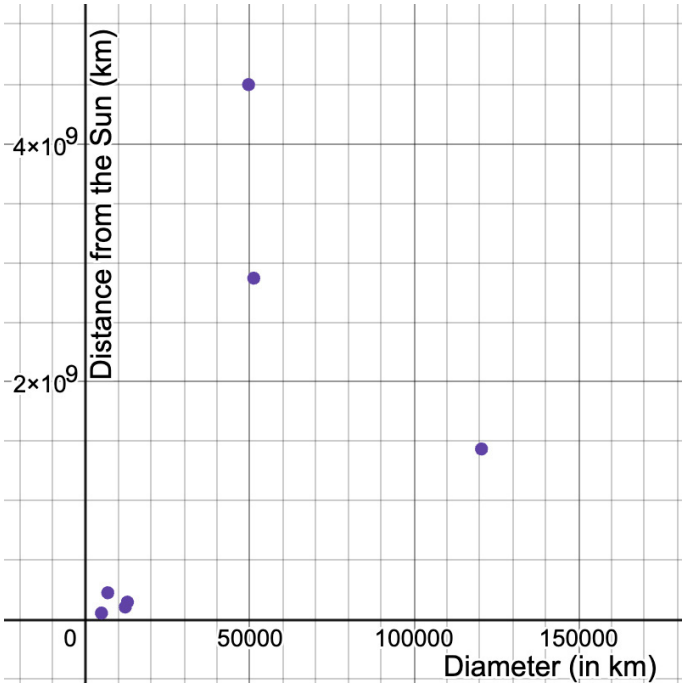


Figure 2: *Desmos-created scatterplot showing Planet Diameter versus Distance from the Sun (both measured in km)*

7. Do you think there is a relationship between the diameter of a planet and its distance from the Sun?

   Can you identify which planet is which?

   Which planets are hardest to determine?

   Why?

As part of the Heliophysics' Big Year, NASA's Goddard Space Flight Center created the scientific visualization below to illustrate NASA's different missions to explore the Sun. You can download a high-definition image or PowerPoint to examine this more carefully.

Heliophysics' Big Year will end on December 24, when the Parker Solar Probe flies closest to the Sun. Parker will be within the orbit of Mercury and will fly into the outermost part of the corona, the Sun's atmosphere. NASA believes it will collect measurements to understand more about the Sun than ever before. Having completed 24 orbits around the Sun in the past seven years, during our winter break, the spacecraft will come within 6.2 million km of the Sun.

8. What fraction of Earth's overall distance from the Sun is covered by each member of the Heliophysics Fleet?
9. How can you use the distance from the Sun to Earth and this image to determine the distances of the other members of the Heliophysics Fleet?

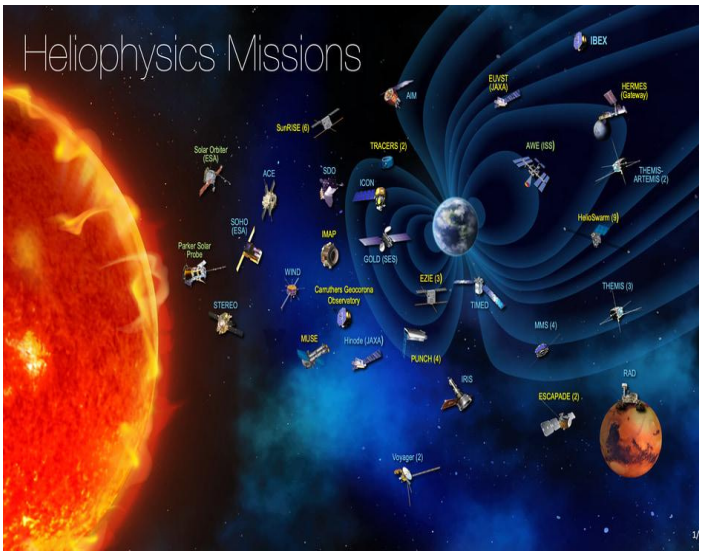
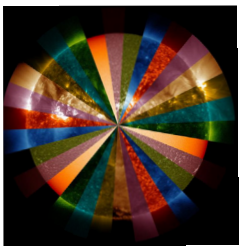


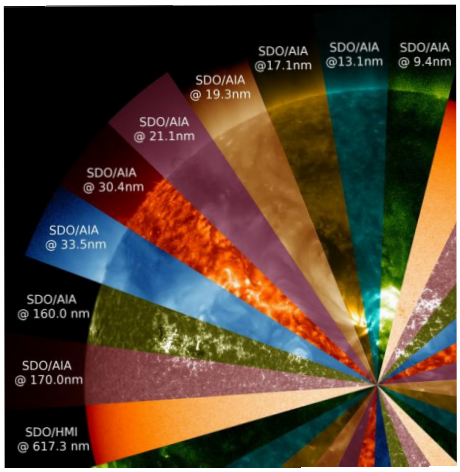
Figure 3: *NASA Heliophysics Fleet Chart, updated January 24, 2024 (science.nasa.gov/heliophysics)*

This probe is named after Eugene N. Parker, who died in 2022. Dr. Parker developed the mathematical model to predict solar winds and was able to view the probe before it left Earth in 2018.

Those beautiful aurorae that captivated many in May resulted from geomagnetic storms. Solar winds' speeds increased the interplanetary magnetic fields, which make the aurora brighter and move further from the poles. CSA presents the many wavelengths of the solar atmosphere (www.asc-csa.gc.ca/eng/astronomy/solar-system/sun.asp). Each colour shows a different wavelength filter moving around an image of the Sun.

The following image maps the dominant colour table to the wavelengths. Understanding solar winds and sunspots could improve space weather predictions, and the Solar Dynamics Observatory (SDO-NASA) uses these images to monitor changes over time at different nanometer filters noted here.





Figures 4 and 5: *Composite image of the solar atmosphere in different wavelengths (NASA/GSFC Scientific Visualization Studio)*

10. What fraction has the Visualization Studio given each wavelength, and how many degrees does the angle measure?

If your students are interested in exploring more about the Sun during the coming year, they might want to join the world's largest hackathon, the NASA Space Apps Challenge. The 2024 NASA Space Apps Challenge takes place on October 5–6, considering Helio's Big Year with the hackathon theme's "The Sun Touches Everything." CSA supports teams across our country in this international competition for coders, storytellers, makers, and innovators, and presents two special CSA challenges in addition to the many offered by NASA. In 2023, in-person events happened in Toronto, Mississauga, Oshawa, Ottawa, Hamilton, Timmins, and Windsor. Some teams are formed well ahead of time and will work together in person, but there are many teams who work from just prior to the start of the event, working in person, virtually, or hybrid.

Each year, thousands of teams submit projects that demonstrate collaboration projects around the world done in a single weekend. Do encourage your students to check out this authentic problem-solving global community! Maybe you would like to mentor or support a team at your school. All ages, skill levels, academic and professional backgrounds are welcome at NASA Space Apps (www.spaceappschallenge.org) through the creative use of science and technology. This year, we'll all have the chance to explore the beauty of the Sun in space and on Earth. ▲

▲ Elementary Math Matters: Anthropomorphic Ratios in Measurement



LYNDA COLGAN
lynda.colgan@queensu.ca

Lynda Colgan is Professor Emerita, Faculty of Education, Queen's University. Lynda is currently working on a number of research, resource-creation, and teacher-education projects, funded by the National Sciences and Engineering Research Council of Canada (NSERC), the Social Sciences & Humanities Research Council (SSHR), and the Mathematics Knowledge Network (MKN). She will carry on her long-established commitment to STEM outreach by continuing to seek out creative avenues to engage students, parents, and educators in mathematics at home and in the classroom.

I recently attended a convocation ceremony at which a former B.Ed. student was being "hooded" after successfully completing a graduate degree. At the reception that followed the formal graduation, we had a chance to talk about her life adventures after becoming a classroom teacher and the impetus for pursuing an M.Ed. She regaled me with story after story about lessons that had gone well (estimating the circumference of a pumpkin), and others that had fallen flat (using the Soroban to reinforce place value in base-10). I heard about attempts to engage parents and guardians through Math@Home packages, and the challenges of addressing the wide spectrum of achievement levels in a typical classroom without appropriate support. There were laments about the shortage of manipulatives, absence of professional development about mathematics, and pressure to include coding in the absence of quality resources. The conversation was chatty and genuine, and I was thoroughly enjoying this "catch-up" with an alumna from a decade-or-so-ago Elementary Mathematics Education course, until she turned to me and asked, her demeanor now somber, if I would still consider teaching "the seven foot" or "perfect square" lessons. The wheels in my mind started turning….

For contextual background, let me explain that "the seven foot" and "perfect square" lessons were about indirect measurement and were intended to be tactile and visual representations of the anthropomorphic ratios first recorded by architect Marcus Vetruvius, then refined by Leonardo da Vinci and illustrated in his iconic, *Vetruvian Man* drawing (depicted below in Figure 1 and described in the paragraphs that follow).



Figure 1: *A young girl stands in front of a depiction of Leonardo's Vetruvian Man to illustrate the concept of a person being a "perfect square," i.e., arm span equal to height.*

If we travel back in time to my mathematics education course and begin at the beginning of the series of lessons, what I recall is introducing indirect measurement through the standard use of paper clips and cubes to measure pencils, desks, and books, and connecting the concept to children's literature, such as Rolf Myllar's *How big is a foot?* and Loreen Leedy's *Measuring Penny*. I also read a short excerpt from *Gulliver's Travels*, written by Jonathan Swift in 1726. In short, the story is about a sea captain, Gulliver, who is the sole survivor of a shipwreck. When he regains consciousness, Gulliver finds himself washed ashore—the prisoner of a race of tiny people less than 15 cm tall, who are inhabitants of the island country of Lilliput. During the shipwreck, his clothes were completely ruined, but the Lilliputian seamstresses assured the gigantic (from their perspective) Gulliver that they would be able to replace his entire outfit (including his hat) by measuring the distance around the base of his right thumb, using a simple length of string.

Gulliver's Travels
by Jonathan Swift

The seamstresses took my measure as I lay on the ground, one standing at my neck, and another at my mid-leg, with a strong cord extended, that each held by the end, while the third measured the length of the cord with a rule of an inch long. Then they measured my right thumb, and desired no more; for by a mathematical computation, that twice round the thumb is once round the wrist, and so on to the neck and the waist; and by help of my old shirt, which I displayed on the ground for a pattern, they fitted me exactly.

Figure 2: *Swift is using da Vinci's ratios in this excerpted text, i.e., that the length of string that fits twice around the base of the thumb is equal to the circumference of the wrist; twice the length of the string equals the circumference of the neck; four times the length equals the circumference of the waist; and three times the length of the string equals the circumference of the head (at the forehead).*

After they experimented with the string ratios on their own bodies, the students began to think about other applications of these ratios. There were many suggestions, but one stands out in my memory: if a pair of jeans folded on a hanger in a store (i.e., the waist folded in half) fits around one's neck, the pants should fit around that person's waist.

I then introduced the generalization that for most adults, our height is the about the same as seven times the length of our own feet. We corroborated da Vinci's ratios by tracing our feet on kraft paper and making seven copies, then lying down on the floor beside the "ruler" made of our feet. We applied the paper feet in other ways: discovering that a person's foot fits almost perfectly on their forearm between the wrist and elbow as well as around the circumference of a clenched fist. That led to suggestions for using arms and fists to decide if a pair of socks or shoes would fit without trying them on.

Next came Michaelangelo's statue of David and a discussion of how da Vinci's ratios were reflected in the classical (and often considered to be ideal) sculpture of a young man. In the case of David, Michaelangelo used da Vinci's observation that the distance from the top of the head to the bottom of the chin is one-eighth of a man's height. In other words, David is eight of his own heads tall. David's head, as shown in the illustration below, was pivotal in determining the width of the shoulders, length of the torso, width of the hips, the distance between the hip and knee, and the length from the knee to foot. This resulted in discussions about how the ratios could be used to help children create more realistic drawings of their family members, proudly leaving stick figures in the dust.

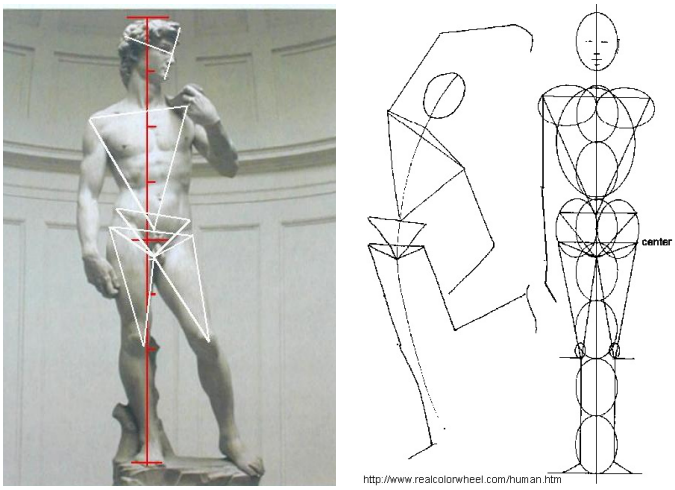


Figure 3: *The statue of David is overlaid with human proportion markings by Professor Brenda Robson in her book,* Lyceum*, which investigates art and the history of ideas. The sketch on the right (www.realcolorwheel.com/human.htm) shows how the head is used as a "unit" in human figure drawing proportions.*

Now back to the post-convocation conversation… When my former student asked me to think about the appropriateness of the lessons in today's classrooms, I felt

that more than a "yes" or "no" reply was needed. So, I began by explaining that at the time, I believed that I was using tasks that sparked curiosity—and I felt strongly that the physical nature of the tasks would make them especially personal and engaging. I also believed that I was integrating mathematics into visual art and capitalizing on rich examples from history.

She listened to my reply and then asked if I thought that the lessons could result in "body shaming" or the development of negative body image by students. "What about the student for whom the ratios do not apply?" she asked.

In response, I agreed that there were dangers to standardizing the human physical shape and form of the body, and admitted that for some individuals, the idealized ratios would indeed be exclusionary; i.e., the bodies of morbidly obese or disabled persons would not conform to the anthropometric measurements established some 500 years ago.

I also went on to describe a fairly recent study that was conducted by American scientists to see how Leonardo's depiction of the Vetruvian Man compared to modern-day humans. Unlike da Vinci, who generated his data manually based on measurements obtained from the analysis of 30 cadavers (including male and female bodies) of all ages, the mathematicians who conducted the modern study conducted body scans of more than 64 000 subjects (63 623 male and 1385 female, aged 17 to 21 years), using high-precision technology. The analysis showed that, "Despite the different samples and methods of calculation, Leonardo da Vinci's ideal human body and the proportions obtained with contemporary measurements were similar" (Thomas, 2020). In fact, the research team found da Vinci's estimates were within 10 percent of the modern human.

I continued by explaining that anthropometric data is used to advantage in many kinds of design, from office chairs to passenger seats in cars; from positioning shelves in stores to calculating the circumference of takeout coffee cups. Some data is used to create clothing patterns and gaming consoles. For instance, in the example of video-game controllers, designers use information about hand size, finger length, and reach to position buttons and other controls for accessibility and usability.

In the end, I realized that my very long answer to a very short question was my way of internally debating the issue through problem-solving and reasoning about what some might consider to be a controversial example of a mathematical concept, i.e., the ratio. I wrapped up my side of the conversation by saying that I would continue to use

Continued on page 25

▲ OAME Award Winners

Exceptional and Creative Teaching in Secondary Mathematics – Andrea Lampo



Andrea Lampo is a veteran teacher, who has not only excelled in her own classroom, but has had an impact on mathematics education in the Thunder Bay area through her role as a mathematics coach and her participation in research projects with Lakehead University. One of her initiatives was to develop and teach a transitions math course at her school to better support students in Grade 9 math. A colleague observed that her resilience toward building a relevant and strong rapport with the students in this course is unmatched in their experience as a math educator. Another stated that Andrea is never satisfied with the status quo and pushes herself to learn more to improve her practice. Her nomination letters made it clear that Andrea truly is an exceptional and creative teacher.

Exceptional and Creative Teaching in Secondary Mathematics – Carly Ziniuk



Carly Ziniuk has had an exceptional impact on the students she has taught. One of her colleagues shared that Carly is one of those teachers who advocates for the underdog, for the student who has difficulty or thinks a little differently, and has an unwavering commitment to student development. She has created many resources that allow students to participate in their learning by doing the math, not just calculating it, and she brings math alive in her lessons. Carly's leadership in mathematics education has been recognized not just by co-workers, who say that working with Carly is like having professional learning opportunities every day, but also by NASA, who chose Carly as one of 30 people worldwide to be part of the Space Apps Collective global engagement platform. She is a regular contributor to the OAME *Gazette*, and has shared her passion and knowledge at numerous conferences and workshops.

Exceptional and Creative Teaching in Post-Secondary Mathematics – Ruth Beatty



Ruth Beatty is a professor at Lakehead University, where she shares her vast pedagogical knowledge with future teachers at the Faculty of Education. Some of her work has included the development of research-backed resources to help teachers take their

students from patterning to algebra, as well as Connecting Indigenous Knowledge and Western Mathematical Ways of Knowing. This research has led to the First Nations and Métis Math Voices Project, which has influenced shifts in research methods and practices, as well as the teaching and learning of Mathematics in countless classrooms across Ontario. As one nominator shared, "She is a leader who leads from beside or behind, by recognizing and nurturing the strength, talents, and knowledge of those around her. Ruth has captivated the hearts and minds of children, families, and entire communities, of pre-service to veteran educators, entire school staffs and boards, as well as University faculty. Ruth has a unique gift for engaging others in the wonder and rigor of mathematics."

OAME Leadership Award – Kit Luce



This award is presented to someone who demonstrates leadership at the school, board, and provincial levels. Kit Luce exemplifies this award. She was instrumental in creating the math Scope and Sequence for her board, and is active at the provincial level as the current President-elect of OAME/AOME and a writer of the University of Waterloo's Problem of the Week. As one nominator stated, "Kit has transformed from a highly knowledgeable and skilled classroom teacher into a district and provincial mathematics leader, who makes a difference for Ontario students and educators every day."

Outstanding Contribution to OAME/AOEM – Kathy Pilon



As the MCIS (OAME Mathematics Conference Information System) administrator, Kathy Pilon has been a key individual for setting up the registration for OAME events since 2012. It is safe to say that if you have attended a conference, you were able to register because of her work and influence on the conference registration system. She demonstrates kindness and patience when helping others and is always quick to answer your questions. Kathy has been an outstanding and valued member of her local chapter, the Board of Directors, and the website team. Many from around the province, including members of OAME, the wider math community, as well as her local colleagues and students, have all benefited from her knowledge, advice, and assistance.

OAME/AOEM Awards for Excellence in Mathematics Education.
Continued from page 5.

- **Elementary School Staff Award for Exceptional and Collaborative Mathematics Teaching:** This award recognizes an elementary school (a school accommodating any or all of Grades K-8) whose staff fosters collaborative and excellent mathematics teaching, contributes to the overall development of student learning of mathematics, and demonstrates leadership in the mathematics community.
- **Secondary School Mathematics Staff Award for Exceptional and Collaborative Teaching:** This award recognizes a secondary school mathematics staff who fosters collegiality, teamwork, and excellent classroom teaching, contributes to the overall development of students, and demonstrates leadership in the mathematics education community.
- **Award for Leadership in Mathematics Education:** This award recognizes an educator who has demonstrated leadership by contributing in a significant way to the development of mathematics educators and the enhancement of mathematics education in Ontario through innovation and/or research-related activities.
- **Award for Outstanding Contribution to OAME/AOEM and Mathematics Education in Ontario:** This award recognizes an individual who has made a significant contribution to OAME/AOEM and contributed directly or indirectly to the improvement of mathematics education in Ontario, but who is not necessarily an educator or a member of OAME/AOEM.

OAME/AOEM Awards Nomination Process

1. Complete the Google **Nomination form** appropriate to the specific award, found at www.oame.on.ca (look for the Awards tab on the left of the page). You may include a one-page letter of recommendation.
2. Include with the nomination cover page up to three **letters of support**, of no more than two pages each. Please ensure all letters of support:
   - address the criteria for the award as outlined on the OAME/AOEM website
   - include the name and position of the supporter to the application
   - include the nominee's name and the award in the letter
3. Include the **Curriculum Vitae**, of up to five pages, for award nominees, except for Elementary and Secondary Staff awards.
4. Submit the Google Nomination form and email the completed package (supporting letters and CV) by the deadline advertised below to EDs@oame.on.ca.

DEADLINE: Nominations are due by January 31, 2025.

The awards will be presented at the OAME Annual Conference in May 2025.

▲ Coding in the Classroom: Why code? For the students.



IAIN BRODIE
iainbro1.625@gmail.com

Iain has been an elementary school educator for 28 years, and is now into a second career, teaching in-service and pre-service teachers. He teaches face-to-face at Ontario Tech University and virtually at Western University.



HATICE BEYZA SEZER
hsezer2@uwo.ca

Beyza is a former teacher and long-term researcher. She is currently a graduate student in education at Western University, and her research focus is the integration of computational thinking into mathematics education.

(Quotations within this column may have been slightly edited to reflect the difference between oral and written language.)

In this column, we are delighted to welcome Derek Tangredi, a multifaceted professional with a wealth of experience in various roles. He teaches at Western University, primarily within the STEM portfolios and engineering capacities, bringing a robust background in these fields. Additionally, he serves as a learning coordinator with a focus on STEM pedagogy and mathematical research. His diverse experience and collective knowledge from these roles contribute significantly to his innovative work. Beyond his multifaceted professional roles, Derek is also a dedicated father of two sons, with whom he shares a passion for coding and all things STEM.

In our conversation, we delved into the specifics of coding and its expansive applications across various areas, such as STEM, engineering, and technological problem solving. One of the main points of our conversation was how coding can be a part of building a person's confidence and pride. Derek emphasized the universal value of making people feel confident and proud of their achievements, regardless of the subject being taught.

When focusing on the educational benefits of coding, Derek highlighted its capacity to provide measurable outcomes and tangible results. He noted that, "It's not always about the tool, but rather the construct," explaining that coding facilitates the creation of simple, yet effective projects, and much like building cardboard models, they are both sustainable and accessible, thereby democratizing STEM education. Furthermore, he explained that integrating elements like microcontrollers and affordable coding tools not only "brings projects to life," but also significantly enhances cognitive skill frameworks.

Derek introduced a compelling metaphor, describing education as a continuum, where learners are constantly progressing and evolving, much like a roller coaster that never stops. This brought us back to John Dewey's timeless wisdom: "Education is not an affair of 'telling' and being told, but an active and constructive process."

In our ongoing discussion on confidence and pride, Derek shared a compelling anecdote from his teaching experience. He described how his students, inspired by their coding achievements beyond the Hour of Code, suggested involving their families in coding sessions during school hours. Emphasizing this initiative's transformative impact on both them and their families, it underscores how it reshaped perceptions of their capabilities and instilled in them a deep sense of pride.

Derek also emphasized the importance of contextualizing learning to give it a deeper meaning, stating, "I think coding needs to be oftentimes contextualized to have meaning... all learning should be more like that to give real purpose and the why around it." Reflecting on this, he highlighted the transformative power of empathy in education, noting, "If there's no empathy, we're just making projects... but if you empathize with something and you're trying to solve a real need or a problem," it elevates the educational experience beyond mere tasks to meaningful learning that resonates.

Following this conversation, Derek brought to light an exemplary program at Thames Valley District School Board, where seventh- and eighth-grade students collaborate with Trudell Medical and local engineers to engage in 3D modelling and design of asthma inhaler caps, using Tinkercad. He emphasized the educational value of this program, noting how it "teaches students the importance of structural integrity and collaboration across diverse teams." He emphasized that the goal is not to push every student into engineering, but to instill the engineering mindset of problem solving, design, and testing, which he believes "would serve all of us, regardless of future career paths."

Building on providing opportunities for agency and empowerment, Derek shared an inspiring coding story involving a Syrian refugee in his class. Despite limited language proficiency, she participated in a "Meet the Teacher"

night, where students created interactive posters, transcending language barriers. Her creativity shone through, leaving a transformative impact on both her and her family, who marveled at her accomplishments. This highlighted the profound effect of enabling students to showcase their capabilities independently, fostering deep empowerment and confidence in their educational journey. Reflecting on this powerful moment, he asserted:

> I think that is what learning to me is really about. It's really creating empowerment and a positive ontological belief in themselves. I actually like education, and I see the real-world practicality of how I can use these skills to teach people how to think.

Please listen to our whole interview with Derek on OAME Talks: Coding in the Classroom 12, to hear his stories and his passion for learning and learners. Our next columns are going to be about the people learning to code in the classroom and what coding brings to their learning: the students. ▲

Elementary Math Matters: Anthropomorphic Ratios in Measurement,. Continued from page 22.

the lessons, but I would do so with sensitivity. I would use the investigations with older elementary school-aged students who showed maturity and demonstrated the ability both to understand approximations and appreciate that differences in measurements/observations were just those, i.e., differences, not judgments. I would use it as an opportunity to talk about the way that statistics can not only describe variation, but disguise it—and that is why we have to be cautious when interpreting words like "average" because they gloss over a whole lot of differences. And going back to my genetics courses in university, I would also explain that this is also why I love the term "wild type," a phrase from population genetics that means "the most common variant of a trait" because it is a LOT less loaded than "normal." In sum, I came down on the side of "let's add some nuance and context" as opposed to the side that avoids the conversation in the first place!

Finally, I thanked my former student for asking the question and turned it back to her, saying, "If the opportunity arises, will you use these explorations of ratio?" She said that she would think about it.

And so, in closing, I ask you, the reader of this column, the same question.

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Liberatore, S. (2020, June 10). *Measurements of Leonardo da Vinci's 'Vitruvian Man'—drawn more than 500 years ago—are very close to modern man's average proportions, study reveals.* DailyMail.com. www.dailymail.co.uk/sciencetech/article-8407479/Leonardo-da-Vincis-Vitruvian-Man-close-modern-mans-average-proportions-study-reveals.html ▲

▲ Creating the Perfect Proposal! AKA What to Consider When Preparing a Proposal for a Conference!

JACQUELINE HILL
OAME COMMUNICATIONS COMMITTEE
jacqueline.hill@oame.on.ca

Some people have found it daunting to consider submitting a proposal to present for an OAME conference. The Communications Committee of the OAME has composed this list to help you in your considerations. The suggestions should not be considered an exhaustive list, but we hope you find the list helpful.

Preparing Your Presentation Proposal

(If you begin with the end in mind, it may help you to craft your proposal.)

- Do your homework. Review some previous session proposals you, as a member of OAME, can find in the members-only section. You may also look to the current OAME sessions.
- Remember that people who will sign up for your session have chosen to attend your presentation, so don't be nervous or afraid. You've got this! Every teacher has worthwhile ideas, suggestions, projects, and classroom-related experiences that can be crafted into a conference session.
- Be aware of the theme of the conference before you craft your proposal.
- Be aware of the following OAME Vision and Mission Statements:

  The mission of OAME/AOEM is to promote, support, and advocate for excellence in mathematics education throughout the province of Ontario.

  A Vision for Learning Mathematics: OAME/AOEM envisions an equitable mathematics education community in which everyone experiences high-quality, research-informed, and engaging activities that promote critical thinking, while fostering a positive attitude toward mathematics.
- Consider co-presenting with someone. If you have presented before, perhaps mentor someone else with similar interests. On the proposal, they will ask you to list your co-presenter (if applicable).
- Think about the size of the audience you would like

(minimum of 15 and maximum of approximately 120).

- Think about how you would like to interact with your participants (lecture style, groups of participants, etc.).
- Make your proposal relevant and interesting to convince the host chapter that your submission is worthy of a conference offering.

Other Things to Consider for Your Presentation

- Have handouts available (both digital and paper).
- Know your audience. You might wish to contact them (as a group) once they have signed up, particularly if there is anything you want them to bring with them to the session.
- Be sure to have a technology presentation, as well as a paper (hard copy) back up presentation, to cover all contingencies.

Giving Your Presentation

- Practise and time your presentation. Have a critical friend listen to your presentation.
- If possible, present your session locally (to your local OAME chapter).
- Make your session interactive and engaging to your audience.
- Know your space! Arrive to the designated room/space early to get it set up to meet your needs.

After the Presentation

- Make notes on what you would like to change for the next time you present this session (don't need "once your session is complete" as heading covers that.)
- Send a follow-up email to your participants after the session (to support them in their learning journeys).

See you at the conference!!

Questions can be directed to jacqueline.hill@oame.on.ca. ▲

▲ Key Insights into High-Impact Instructional Practices in Mathematics



JEFF IRVINE
dr.jeffrey.irvine@bell.net

Jeff Irvine, Ph.D., has been a secondary mathematics teacher, department head, and vice-principal. He has taught at three faculties of education and at a community college. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. With over 45 years in education, Jeff is particularly interested in the interplay of student motivation and mathematics achievement.

In 2020, the Ontario Ministry of Education published a booklet called *High-Impact Instructional Practices in Mathematics*. At that time, teachers and schools were focused on pandemic-related issues, and the publication did not receive widespread notice. That was unfortunate because the booklet has some good, research-affirmed strategies to improve student learning in mathematics. The nine strategies discussed in the document are as follows:

- Learning goals, success criteria, and descriptive feedback
- Direct instruction
- Problem-solving tasks and experiences
- Teaching about problem solving
- Tools and representations
- Math conversations
- Small-group instruction
- Deliberate practice
- Flexible groupings

For each strategy, there is a brief description, followed by a section entitled "In the Classroom," describing what the strategy could look like in action.

You may wonder, however, what makes these strategies "high-impact?" How is effectiveness measured? Are there other, better strategies that we should use as math teachers? Does the booklet give us enough information to actually implement these strategies? Let's take these questions one at a time.

Effectiveness of a strategy is measured by a number called *effect size*. Effect size is a number, *d*, usually between 0 and 1, although some values can be greater, and occasionally, there can be a negative value. For details on how effect size is calculated, see Appendices A and B. The larger the effect size, *d*, the more effective the strategy should be. Here is a Language example of how effect sizes can help us choose the best strategies. In teaching reading, phonics has an effect size of $d = 0.6$, while whole language has an effect size of $d = 0.06$. A teacher using the whole-language strategy would need to be ten times as good as an average teacher using phonics to get the same results with their students. So, the larger the effect size, the more effective a strategy should be. For more on how to calculate effect size, see Appendix A.

As mentioned above, the larger the effect size, the more effective a strategy should be. As math people, we like numbers, so shouldn't we just use the strategy/strategies with the highest effect sizes? Yes, but there are some caveats. First, in teaching, there is no "one size fits all." We teach individuals, and a strategy that works well for one student or group of students may not work as effectively for another group of students. Secondly, effect sizes may differ, depending on how and who is doing the measuring. Let's consider the second problem first.

In 2009, John Hattie published a book called *Visible Learning*, which examined effect sizes for a wide variety of teaching strategies. Some effect sizes from Hattie's book are listed in Appendix C. Hattie's results considered only the effect of strategies on student achievement. He did not address other dimensions such as student engagement, attitudes, and self-efficacy. For example, the direct effect of extra-curricular involvement on achievement is small, but indirect effects, like increasing student engagement, interest in school, and in secondary school, reducing the urge to drop out, all impact achievement, but are not included in Hattie's research summary.

In 2000, Dr. Robert Marzano, along with his colleagues, published a summary of other effect sizes, identifying what they considered to be the most effective classroom strategies. Marzano considered strategy effects on more than just student achievement, but also effects on motivation, engagement, self-confidence, and self-efficacy. The results of the two studies, while similar, are not identical. For more information on inclusion criteria, see Appendix B: Meta-Analyses.

Another factor that influences effect-size calculations is called the *inclusion criteria*. Some meta-analyses may only include studies of a certain size, or only more recent studies, or only studies from certain countries. All of these factors will result in differences in effect-size calculations, although usually, while the numbers may differ, the conclusions about which strategies are most effective will be similar.

A third consideration is what number for effect size is "good." Hattie, for example, felt that just by aging a year, and being in school, would have an effect size of $d = 0.40$. For Hattie, to be effective, the effect size of a strategy had to be above 0.40. For goal setting, he found an effect size of $d = 0.56$ to be effective, since it is above 0.40, while problem-based learning (Hattie's effect size $d = 0.15$) is not effective.

As I mentioned, since all students are different, a strategy that works for one group of students may not be effective for another group. Also, effect sizes are not additive. So, doing goal setting ($d = 0.56$) and also doing problem-based learning ($d = 0.15$) will not result in an effect size of $d = 0.71$. This makes sense, since otherwise doing all nine of the high-impact strategies would potentially move students beyond the 99th percentile, which is impossible.

The Marzano 9

Robert Marzano's list of the nine most effective instructional strategies ("The Marzano 9": Tyson, 2020) consists of the following:

- Setting objectives and providing feedback
- Reinforcing effort and providing recognition
- Co-operative learning
- Cues, questions, and advance organizers
- Non-linguistic representations
- Summarizing and note taking
- Assigning homework and providing practice
- Identifying similarities and differences
- Generating and testing hypotheses

We can see that there is some overlap with the high-impact strategies, but also some differences. Marzano's strategies were neither specific to mathematics, nor to only student achievement. The high-impact strategy of tools and representations is analogous to Marzano's non-linguistic representations. Assigning homework and providing practice is related to the high-impact strategy of deliberate practice. And the strategy of cues, questions, and advance organizers has some elements of direct instruction. The table below provides effect sizes for the high-impact strategies, according to Hattie and Marzano.

High-Impact Strategy	Hattie	Marzano*
Learning goals, success criteria, and descriptive feedback	Goal setting $d = 0.56$ Feedback $d = 0.73$	Goal setting $d = 0.61$ to 0.76
Direct instruction	$d = 0.59$	Cues, questions, and advance organizers $d = 0.59$ to 0.74
Problem-solving tasks and experiences**	$d = 0.15$	
Teaching about problem solving	$d = 0.61$	
Tools and representations Math conversations***		$d = 0.49$ to 0.73
Small-group instruction	$d = 0.49$	
Deliberate practice	Spaced vs. massed practice $d = 0.71$	Assigning homework and providing practice $d = 0.13$ to 0.77
Flexible groupings		

*Variations in effect size due to inclusion criteria.

**Problem-solving can be an excellent motivator (Irvine, 2015).

***For a discussion of math talk learning communities, see Irvine, 2017a.

Omissions

There are some significant strategies not identified in the *High-Impact Instructional Practices in Mathematics* document. For example, Hattie identifies formative assessment as an important strategy impacting student achievement (d = 0.90). From the Marzano 9, both identifying similarities and differences related to concepts (d = 0.66 to 1.61), and students generating and testing hypotheses (d = 0.58 to 0.61) support higher-order thinking skills with students. Specific to mathematics, problem posing (Irvine, 2017b) not only promotes higher-order thinking skills, but also is an excellent strategy to engage and motivate students. On a whole-school basis, Hattie's top strategy to support student achievement is self-reported grades (d = 1.44), which, while not specific to math classes, is a strategy worth considering.

Conclusion

The *High-Impact Instructional Practices in Mathematics* document is a valuable starting point for mathematics instruction. It is a research-affirmed list of several important strategies to improve student learning. The unfortunate timing of its release during the pandemic has resulted in it not being widely implemented. In addition, these strategies need to be supplemented with information on instructional strategies that support the implementation of the high-impact strategies in the classroom. In this way, teachers are better able to meet the needs of their students when utilizing these strategies.

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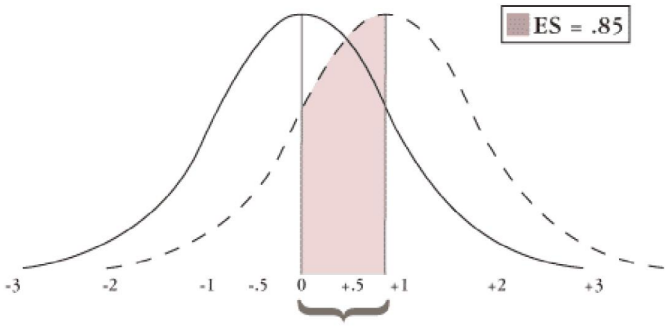
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Appendix A: How is effectiveness measured?

The impact of a strategy is measured by its effect size, *d*. Effect size compares results for a group of students with whom the strategy had been employed (the experimental group) to results from a group of students who had not had the strategy (the control group).



Assuming a normal distribution of results, near the centre of the distribution, 1 standard deviation = 34 percentile points. Since effect size is measured in units of standard deviation, 0.85 standard deviations = 0.85 x 34 = 30 percentile points.

This strategy would move an average student from the 50th percentile to the 80th percentile.

Appendix B: Meta-Analyses

Effect sizes are usually cited based on meta-analyses. To do a meta-analysis, a researcher examines numerous research studies that have already been done, and then publishes an "average" effect size from the studies. The results of different analyses will differ, depending on the goals of the study and the inclusion criteria (what studies qualify for inclusion in the calculations). Hattie looked only at effects on student achievement, while Marzano looked at effects on other student variables such as mood, engagement, self-efficacy, and self-confidence.

Appendix C: Selected Effect Sizes from Hattie (2009)

Strategy	Effect Size	Percentile Gain
Goal setting	0.56	19
Advance organizers	0.41	13
Concept mapping	0.57	19
Worked examples	0.57	19
Feedback	0.73	24
Frequent testing	0.34	11
Teaching test taking	0.22	7
Formative evaluation	0.90	30
Questioning	0.46	15
Teacher immediacy	0.16	5
Spaced practice	0.71	24
Peer tutoring**	0.55	18
Study skills	0.59	20
Matching learning styles	0.41	13
Reciprocal teaching	0.74	25
Teaching problem solving	0.61	20
Problem-based learning	0.15	5
Co-operative learning	0.41	13
Co-operative vs. competition	0.54	18
Co-operative vs. individual	0.59	20
Competitive vs. individual	0.24	8
Team teaching	0.19	6
Homework* (average)	0.29*	9
Homework (elementary students)*	0.15*	5
Homework (secondary students)*	0.64*	21
Phonics	0.60	20
Whole language	0.06	2
Ability grouping	0.12	4
Multi-grade classes	0.04	1
Within class grouping	0.16	5
Retention	-0.16	-6
Acceleration	0.88	29
Enrichment	0.39	13
Classroom management	0.52	17
Classroom cohesion	0.53	18
Quality of teaching	0.44	14
Teacher–student relationships	0.72	24
Professional development	0.62	21
Teacher expectations	0.43	14
Teacher clarity	0.73	24
Reading vocabulary programs	0.67	22
Exposure to reading	0.36	12
Play programs	0.50	17
Creativity programs	0.65	22
Extracurricular programs	0.17	5
Direct instruction	0.59	20

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OAME Award Winners. Continued from page 23

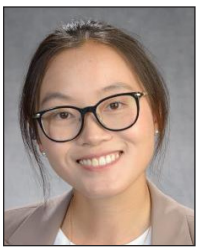
Lifetime Membership Award – Sandra Jean Price



This award recognizes an educator who has contributed in a significant way to OAME/AOEM and demonstrates outstanding leadership in mathematics education. Sandra Jean Price joined the organization in 2003 and since then, she has contributed in immeasurable ways. She has been involved at the local chapter level, planning area events, while also serving on the provincial Board of Directors. She is the current OAME President and was the coordinator of both the OAME/AOEM Grade 9 Curriculum Project and the Elementary Curriculum Project when the new math curriculums were introduced. She has been involved with the CLIPS and TIPS projects, and was a contributor to the *Guides for Effective Instruction in Mathematics* for the Junior grades. She co-chaired the Ontario Math Olympics twice and is currently co-chairing her second annual conference. She also served as a committee chair at three other conferences. Sandra Jean Price has demonstrated an outstanding commitment to OAME and its mission to promote, support, and advocate for excellence in mathematics education throughout the province of Ontario.

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The Beauty of Dual Intelligence in the Age of AI—Engaging with ChatOAME as the "More Knowledgeable Other"



CYNTHIA (XIAOWEN) SHI
cynthia.xiaowen.shi@gmail.com

Cynthia Shi has been a secondary mathematics educator since 2014 and is currently employed at Appleby College. She harbours a profound passion for designing lessons in creative ways and is committed to ongoing reflection of her teaching practice and professional growth. Cynthia is an active participant in the ChatOAME project, serving as a beta tester and demonstrating exceptional skills in prompting.

As I was brainstorming about where to start telling my AI (Artifical Intelligence) adventure story, I remembered immersing myself reading the book *The Genius Makers* over Christmas Break. It brought back vivid memories of how the author outlined pivotal breakthroughs in AI and the individuals behind the advancements. One of the major events that had a lasting impact on me was the DeepMind Challenge Match that took place in March 2016, a five-game Go match between Google DeepMind's AI AlphaGo and the world-renowned Go player, Lee Sedol.

Alpha Go Story

During this match, AlphaGo made its astonishing move 37, unexplainable and baffling to any highly ranked human player watching the game, which led to its triumph over Lee Sedol. And Lee Sedol, in one of the later matches, confounded AlphaGo by his move 78, resulting in the machine's defeat due to its inability to anticipate human behaviour. The evolving development of AlphaGo, honing its skills through matches against elite world Go players, and the reciprocal relationship, where the experience of playing with AlphaGo had "opened eyes to new ways of playing" by human players (Metz, 2021, p. 175).

The story (see textbox above) exemplified the fascinating synergy between man and machine. In a world where creativity is being redefined by this human–machine dual intelligence, it has led to the development of novel strategies and moves that would otherwise be inconceivable without such collaboration. The term "dual intelligence" is used because it implies a co-emergent relationship, where the result is greater than the sum of its parts. I was drawn to explore this machine–person collaboration further, and found an opportunity with ChatOAME—an AI specifically oriented to mathematics education in Ontario. It has been a captivating journey over the past few months, exploring how an educator could effectively collaborate with an AI tool. When harnessed effectively, AI is driving a new era of machine–human collaboration, fostering the creation of impactful educational content, enhancing teaching efficacy and efficiency, and facilitating continuous professional development for educators. I am living proof.

My journey with ChatOAME began in November 2023, when OAME launched it for beta testing. You may recall from the March issue of the *Gazette*, Cal Armstrong's article described the genesis and development of ChatOAME (Armstrong, 2024). Building on that article, I will provide an overview of my experiences using ChatOAME. At first, my interactions with this chatbot involved concise, targeted conversations aimed at completing specific tasks, such as generating assessment questions, rubrics, and success criteria aligned with curriculum expectations. Figure 1 shows the response I received from ChatOAME after I inquired about the tasks it can help accomplish for teachers.

As I became increasingly comfortable and curious about ChatOAME's capabilities, I began testing the chatbot's abilities to accomplish higher-order tasks requiring complex processing, extended dialogue, and advanced thinking. To better navigate my beta-testing journey, I started categorizing my prompts, using Bloom's Taxonomy as a guiding framework. Table 1 shows a selection of sample prompts I've categorized based on Bloom's Taxonomy, reflecting ChatOAME's responses regarding what it can help accomplish for teachers.

As depicted in the Bloom's Taxonomy chart, I approached ChatOAME as a friend of mine, someone capable of varying responses, depending on the complexity of the questions posed. Some inquiries prompted ChatOAME to retrieve information from its memory base, while others demanded higher-order thinking skills. This framework served as a road map, guiding me through the beta-testing process with the chatbot. It was also during this stage where I engaged in more extensive reading, intending to learn more about effective prompting techniques, and how to stimulate greater creativity from ChatOAME. Very naturally, through this human–machine interaction, I found myself being educated on Ministry documents that I was not as familiar with, enlightened by ChatOAME to reconsider aspects of teaching I previously overlooked, and inspired by the machine to conduct self-reflections of my own practice, which might not have been considered otherwise. As you can also see in the chart, at this point, I harboured doubts

Hi my friend, can you please tell me what are the tasks that you think you can help teachers with?

Certainly! As an Assistant to Ontario math teachers, aligned with the Mission and Vision of the OAME (Ontario Association for Mathematics Education), I can help Ontario math teachers in various aspects of mathematics education. Here's how I can assist:

1. Curriculum Planning:

- Provide guidance on aligning daily lesson plans with the Ontario Mathematics Curriculum.
- Offer creative ideas to integrate the Ontario Mathematical Processess into classroom activities.
- Reference specific expectations from the Ontario Curriculum documents for various grades.

2. Resource Development:

- Help in creating and curating educational resources that meet the curriculum expectations.
- Suggest adaptations of teaching methods to include First Nations, Métis, and Inuit perspectives and knowledge.

3. Assessment Design:

- Assist in designing assessment tools in line with the *Growing Success* protocols.
- Develop rubrics, marking schemes, or protocols for various types of assissment tasks including formative and summative assessments.

4. Pedagogical Support:

- Suggest high-impact instructional practices to promote deeper understanding and engagement in mathematics.
- Offer strategies to incorporate the Ontario Learning Skills into mathematics teaching and learning activities.

5. Professional Development:

- Recommend resources from OAME, such as articles from the *Gazette* or the *Abacus* that are relevant to specific teaching goals or challenges.
- Provide summaries and insights from key documents like *Growing Success* and *High-Impact Instructional Practices in Mathematics* for professional learning.

6. Incorporation of Technology:

- Suggest effective use of digital tools to enhance teaching and learning of mathematics.

7. Cultural Integration and Equity:

- Advise on incorporating culturally relevant examples and contexts in mathematics teaching to promote equity and engagement among all students.

Query on Mathematical Content:

- Explain mathematics concepts and provide step-by-step solutions to problems, ensuring precision and correctness.

In everything I do, I aim to uphold the quality of mathematics education, with a focus on excellence, equity, and engagement, following OAME's mission to support and promote superior mathematics education in Ontario.

Figure 1: *Response received from ChatOAME about the tasks it can help accomplish for teachers*

about ChatOAME's ability to handle tasks requiring the evaluation of real-life data and the creation of class-specific materials. However, I soon came to realize that this was a misjudgment on my part. It became an area where I sought continual improvement in my interaction with ChatOAME to date, striving to incorporate more human input to ensure that content generation, lesson planning, and assessment design facilitated by AI were truly dual-intelligent creations tailored to my needs, and those of my classes and students.

I endeavoured to foster this dual-intelligent practice through a recent experience of lesson planning with ChatOAME on an MTH1W lesson. I've annotated the process to highlight both human and machine contributions to the creation process. My lesson planning involved three key stages: content creation centred on high-impact instructional strategies, designing quality questioning, and considerations for class-specific differentiated instruction and formative assessment.

In this lesson, I began by considering instruction choices without using ChatOAME. The objective is for students to revisit angle properties and theorems related to intersecting lines, parallel lines, and a transversal. I'm mindful of the learning goals, which include understanding angle properties and problem solving with deductive reasoning. Recognizing the diverse backgrounds of my students, I understand that while some may find these concepts familiar, others may be encountering them for the first time. I needed a lesson that can address these varying needs, enabling students to apply their knowledge if they have a strong foundation, while bridging any knowledge gaps as needed. Thus, a lesson that supported peer collaboration was required, and preferred that my students' learning takes place through "doing" versus "telling."

I explained my requirements to ChatOAME through a chain of prompting, and ChatOAME suggested to start the lesson with a geometry quest. The word "quest" was suggested by ChatOAME, and it became co-emergent and defined through the planning process. It described the activity, required material, as well as the benefit of this design. Students will be solving some fundamental geometry problems with the help of printed hint cards stating important theorems and angle properties.

I then consulted ChatOAME about how to introduce strategies for solving more complex problems, using deductive reasoning. I sought advice on whether to maintain the entire lesson, using the geometry quest model, or to incorporate some direct instruction. ChatOAME highlighted the benefits of integrating a small amount of direct instruction. Taking this advice into consideration, I chose to review an example with the students before assigning them

Bloom's Taxonomy Levels	AI facilitated teaching example of teacher action sample questions
Remember	Ask Chat OAME to give answers to a specific math course, Ministry document, etc., that require AI to retrieve information from its training data. Ex. Can you provide a list of topics in G10 curriculum? Ex. What do you know about high-impact instructional strategy? Ex. Can you give me an example of a question involving factoring? Ex. Can you show me the solution to this question?
Understand	Ask ChatOAME to provide answers to a specific question about a course, Ministry document, etc., that requires AI to understand its training data and retrieve information based on its understanding of the training material and user input. Ex. Can you break down the solving process of this question and assign marks? Ex. Tell me more about integrating First Nations, Métis, and Inuit (FNMI) perspectives in math education. Ex. What should I consider before planning for assesment according to *Growing Success*?
Apply	Have AI provide suggestions for lesson planning, offer creative ideas, generate numbers, problems, examples, etc., which require AI to apply its knowledge based on training data to provide content for users. Ex. Can you help me generate a sample lesson plan that applies high-impact instructional strategy? Ex. Please help me generate a lesson plan involving project-based learning. Ex. Can you help me generate a variety of questions on ____ that assess students' Knowledge, Thinking, Communication, and Application? Ex. Can you develop a few classroom activities related to decimal operation that involves Thinking and Application? Ex. Can you develop a group activity that facilitate the development of this learning skill?
Analyze	Have AI critique a lesson plan, anesthesiant plan, and two rubrics, and adjust according to AI's advice, which requires AI to first analyze the content given according to its training data, then give suggestions. Ex. I've included a question like the following. Is this appropriate/can you tell me if it's appropriate for a G10 level class according to curriculum document? Ex. I plan to include a lesson like the following (describe the lesson process in detail). Can you tell me if it aligns with high-impact instructional strategy? Ex. Can you please critique the lesson plan described. Ex. Given the assessment plan I've described, what should I consider improving on? Ex. Here's the rationale of how I've marked this response. Could you please comment on my marking.
Evaluate	The teacher feeds in real-life data and feedback from a lesson plan (What went well? What needs improvement? Which group of students seems to have benefited from the lesson, instructional strategy, assessment, etc.) for AI to recognize patterns and provide an evaluation of the situation. (This is not possible with ChatOAME because it won't store memory from new experiences yet.)
Create	The teacher feeds in real-life data and feedback from a lesson plan (What went well? What needs improvement? Which group of students benefited from the lesson, instructional strategy, assessment, etc.) for AI to recognize patterns and re-generate a suitable action plan for better lesson planning. This could be student specific. (This is not possible with ChatOAME because it won't store memory from new experiences yet.)

Table 1: *Sample prompts organized based on Bloom's Taxonomy*

to delve into more complex geometry quests within their groups, followed by some final self-reflection tasks. With the overall structure of the lesson in place, I invited ChatOAME to critique the finalized lesson plan, listing areas where designs were executed well and areas for improvement. Figure 2 shows the response I received from ChatOAME, and how I moved on to further our conversation.

The second stage of lesson planning involved preparing for quality questioning throughout the lesson. I intended to utilize Webb's Depth of Knowledge (DOK) as a framework when crafting quality questions for each stage of the lesson, and I had already developed some questions myself. I was curious to see how ChatOAME could assist me further, enhancing my existing questions. I posed a prompt asking ChatOAME to enrich my list of questions, using Webb's DOK framework, while ensuring alignment to the instructional purposes suggested in the book *Quality Questioning: Research-Based Practice to Engage Every Learner* (Walsh



Critiquing and reflecting on lesson plans is a crucial aspect of instructional planning. Let's evaluate the lesson plan on angle properties for Grade 9 students from a few key perspectives, taking into account differentiation, engagement, cognitive demand, and alignment with instructional best practices, including principles found in documents such as *Growing Success* and *High-Impact Instructional Practices in Mathematics*.

Strengths of the Lesson Plan

1. **Engagement through Variety:** The mix of a warm-up quest, direct instruction, group quests, and self-reflection ensures varied instructional strategies, catering to different learning styles, and keeping student engagement high. This variety supports the Vision of OAME by promoting interactive and engaging mathematics activities.
2. **Use of Prior Knowledge:** Starting with a warm-up that taps into students' prior knowledge serves as a great entry point into the lesson, making learning accessible and building confidence from the outset.
3. **Focus on Higher-Order Thinking:** Incorporating questions at various Depth of Knowledge (DOK) levels encourages students to engage not only in recall, but also in application, analysis, and creation, fostering critical-thinking skills in line with the Ontario Curriculum's Mathematical Processes.
4. **Opportunities for Reflection:** The self-reflection activity toward the end promotes metacognition, allowing students to review their understanding, strategies, and challenges faced during the lesson.

Areas for Enhancement and Differentiation

1. **Scaffolding and Differentiation:** While the plan includes a range of activities, further differentiation could ensure that all students, regardless of their proficiency levels, are adequately supported and challenged. This could involve creating levelled hint cards for the warm-up and group quest activities, providing options that range from basic to advanced complexity. Tailoring questions and tasks to meet diverse learner needs aligns with the principles of *Growing Success* on differentiation.
2. **Formative Assessment:** Consider incorporating more structured opportunities for formative assessment during the lesson, beyond checking for understanding. Quick polls, exit tickets, or even digital tools that allow for real-time feedback could provide more insight into student learning and inform instructional decisions, as emphasized in *High-Impact Instructional Practices in Mathematics*.
3. **Collaboration and Discussion:** While group work is a significant component, adding structured discourse elements can enhance understanding. Strategies like "Think/Pair/Share" or "Numbered Heads Together" during the direct instruction and quest activities could foster deeper discussion and exploration of concepts.
4. **Connection with Real-World Applications:** While the lesson encourages making real-world connections, explicitly integrating these applications within the direct instruction or as part of the quests can help students see tile relevance of geometry in their lives and future careers, promoting a positive attitude toward mathematics.
5. **Differentiated Reflection:** Offering varied formats for the self-reflection activity (written, oral, visual) could accommodate diverse preferences and abilities, ensuring that all students can meaningfully engage in reflection.

In summary, your lesson plan provides a solid foundation for engaging Grade 9 students in a deep exploration of angle properties. Enhancing differentiation, formative assessment, and real-world connections, while capitalizing on collaborative and reflective opportunities, can further promote an equitable, engaging, and high-quality mathematics education, as envisioned by OAME. Considering these enhancements can also support the continual growth of teaching practices in alignment with the principles of both *Growing Success* and *High-Impact Instructional Practices in Mathematics*.

Would you like to delve into specific strategies or tools that could help implement these enhancements and further differentiate instruction within your lesson plan?



Thank you, I've considered #1, #3, #5 already and had them included in my lesson planning. It's fascinating how much we think alike now. However, I am struggling with #2 and #4. Can you recommend what to include in terms of more structured formative assessments? I prefer something that's not a quiz or exit ticket. They've done enough problem solving already. Maybe something along the line of a more structured formative conversation as students work in their groups? As to your point #4, connect with real-world application, I've included as part of their self-reflection at the end to make connections to how deductive reasoning was used in other subject areas as well as outside of school, is there anything else you'd recommend?

Table 2: Su*ggestions made by ChatOAME critiquing finalized lesson plan*

& Sattes, 2016) and indicating when to apply these question designs during the lesson. Table 2 shows the finalized questioning design, which includes contributions from both myself and ChatOAME.

The final stage of lesson planning entailed designing class-specific differentiated instruction and formative

assessment. As seen in the previous image, ChatOAME already prompted me to complete this process by offering

Purpose	DOK1	DOK2	DOK3	DOK4
Set the Stage	*What is the sum of angles in any triangle? Or similar questions asking for facts?*	*Given two parallel lines cut by a transversal, how can we identify pairs of corresponding angles? Or similar?*		
Build Foundational knowledge	What other background knowledge, if any, did you need to solve this question?		-	
Check for understanding	Which theorem was applied in this step? *Can you identify the complementary angles in the given diagram? Or similar?*	Which theorem did you choose to solve this question? Why? *In the given figure, if one angle measures 35 degrees, what is the measure of its adjacent angle or similar?*		
Deepen knowledge	Which theorem or angle property did you apply to make such inference?	What inference can you make with the information provided? Explain your deductive reasoning process. *How do the properties of alternate interior angles help us solve for unknown angles when two parallel lines are cut by a transversal?*	How is this question similar to or different from the warm-up quest? Are you able to identify a question you solved before that may be useful to help you solve this question? Is there anything else that's also solvable with the given information, but wasn't being asked for? *Given a diagram with multiple pairs of parallel lines and transversals, explain how to use angle properties to solve for a particular unknown angle.*	Is there another possible solution? Can you make use of the strategies and theorems you learned today to create a similar question for another group? Critique your peers' solution. What did you like about their reasoning, and what did you not like about their reasoning?
Build metacognitive knowledge		Explain in your own words how you applied deductive reasoning today in class. *Reflect on the strategy you used to find the missing angle. Why did you choose this strategy?* *What concept or theorem did you find most challenging during today's lesson,and how did you overcome this challenge?*	Can you explain how deductive reasoning may be used, or how you have applied deductive reasoning in other subject areas? *When working through the geometry quest, how did you decide which property or theorem to apply in different scenarios?* *Reflecting on today's activities, how do you think the process of using deductive reasoning in geometry can be applied in other areas of mathematics or real-life situations?*	Can you self-reflect and identify when deductive reasoning may be useful for you outside of school? *Design your geometry puzzle, incorporating angle properties and theorems we have learned. Explain how you would solve this puzzle step by step.*

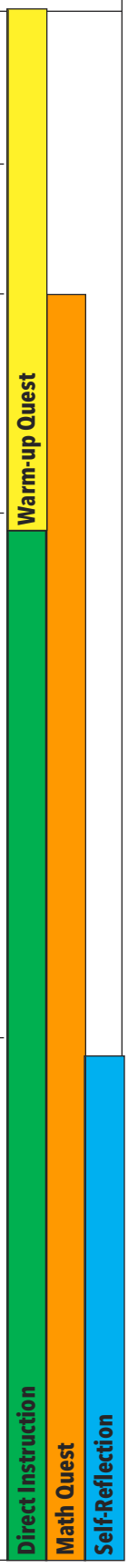


Figure 2: *Suggestions made by ChatOAME critiquing finalized lesson plan*

Thank you for sharing the unique dynamics of your class. It's clear that you have a diverse group of learners, each with their unique strengths, challenges. and ways of engaging with the material and their peers. Your classroom seems to foster a supportive and respectful enviromnent, which is crucial for effective learning and teaching. Creating differentiated instruction strategies and formative assessment plans that cater to these diverse needs, while maintaining this positive classroom culture, will be key to supporting all students' learning. I await your instructions on how I can assist further.

Given the lesson plan, quality questioning design, and the class dynamic described, can you provide suggestions for: 1. grouping, 2. things I should be mindful of when delivering different parts of this lesson, as well as 3. how I can incorporate formative assessment as I circulate the groups? If there's anything else you'd be able to add, please also let me know. Thank you!

Certainly! Given the comprehensive understanding of your lesson plan, question design, and class dynamics, let's explore strategies for grouping, key considerations for lesson delivery, and incorporating formative assessment effectively.

1. Grouping Strategies

- **Mixed-Ability Grouping:** For both the warm-up quest and main quest activity, consider creating mixed-abilitv groups. Place at least one student with strong foundational skills in each member to ensure their engagement. Use proximity and non-verbal cues to redirect when necessary, without disrupting the flow of the lesson.
- **Supporting Frustration:** For the student who gets easily frustrated, tailor challenges to their level of skill, and periodically affirm their efforts. Ensure that stretching and taking breaks are seen as part of their learning process, which can be periodically shared with the class to celebrate diverse ways of learning.

2. Incorporatinng Formative Assessment

- **Observation:** As you circulate, observe not only the solutions, but also the interaction and discussion dynamics within groups. Look for evidence of reasoning, collaboration, and the use of deductive reasoning.
- **Guided Questions:** Use your question design to informally assess understanding as students work in groups. Ask targeted questions from your list to individual students or groups, noting their responses to gauge their understanding and adjust instruction as needed.
- **Exit Tickets for Groups:** At the end of the group activities, provide an exit ticket with a challenging question that requires a collaborative answer. This will give you instant feedback on each group's collective understanding and areas of reinforcement.
- **Reflection Shares:** During the self-reflection activity, invite students to share insights or "aha" moments with the class. This promotes a culture of shared learning and allows you to assess individual metacognitive awareness.

Additional Considersations

- **Peer Teaching:** Encourage students with strong foundational knowledge to lead mini-sessions within their group on a difficult concept, reinforcing their understanding and providing support to peers.
- **Showcase Diversity of Thoughts:** Celebrate the diversity in problem-solving strategies by having groups share their unique approaches. This could foster a rich classroom discussion on the varied ways deductive reasoning can be applied.

The dynamic nature of your classroom offers a vibrant setting for implementing these strategies. By thoughtfully mixing instructional approaches, monitoring progress through formative assessments, and adapting to your students' varied needs, you're well positioned to foster a deeply engaging and effective learning environment in geometry.

How do you envision implementing these strategies in your upcoming lessons to enhance student engagement and learning outcomes?

Figure 3: *Response received from ChatOAME giving class-specific recommendations*

suggestions while critiquing my lesson plan. ChatOAME also suggested potential ways to incorporate real-world applications into this lesson. However, I opted not to include them due to the richness of content and the real-world application-related reflection component already present in the lesson. My priority is to ensure that students maintain a clear focus on their learning goals and achieve a thorough understanding of key theorems and deductive reasoning. Consequently, I kept my conversation with ChatOAME focused on class-specific differentiated instruction and formative assessment.

Figure 3 shows the response ChatOAME gave after I explained the class dynamics of one of my Grade 9 classes. As you can see in the response, ChatOAME provided a very detailed response according to my request of grouping recommendations, special considerations during each stage of lesson delivery, as well as class-specific formative assessment planning. After engaging in three stages of lesson planning with ChatOAME and incorporating its recommendations, I now have a lesson that integrates group work, direct instruction, and self-reflection. It's structured to balance collaborative learning with individual reflection, enhancing both the collective and personal learning experience. Additionally, ChatOAME and I together developed a well-structured list of quality questions, targeting different phases of learning and thinking

processes. This lesson endeavours to help students achieve a comprehensive exploration of geometry concepts and deductive reasoning, supporting students not only in learning, but also in reflecting on their learning process. Together with the class-specific considerations given to differentiated instruction, grouping, and formative assessment, I am very content with the final lesson design. I am confident that the richness of the material and the level of preparedness of this lesson will effectively support the diverse needs of my class.

Timing is a consideration in this exploratory approach. The lesson-planning stage took the longest time, about 1 hour, going back and forth with ChatOAME to finalize the plan. Quality questioning took about half an hour. Differentiation took about half an hour. The above-mentioned time didn't include the time spent creating the actual lesson material, such as handouts and hint cards. ChatOAME has huge potential in helping educators to create quality content and saving time according to an educator's need. One can choose the level of engagement and the amount of time spent with this chatbot when developing the material you need.

The lesson mentioned in this article was a demonstration of how one can engage with this chatbot to a greater extent. It won't be realistic for all educators to do this every single day. However, I think it's realistic for an educator to type up the explanation of a lesson plan and have ChatOAME critique the lesson by providing some feedback; oftentimes I discover a thing or two that's useful from ChatOAME's critique of my lesson. Other things that ChatOAME had done well for me were generating rubrics, creating assessments, and providing creative ideas for projects.

Teaching and learning are deeply human experiences, where only humans can empathize, motivate, and engage in self-reflection for improvement. Reflecting on my own journey engaging with ChatOAME as the "more knowledgeable other" (Vygotsky & Cole,1978), I considered the roles that both machines and humans play in fostering effective teaching practices. Machines excel at processing vast amounts of information and providing consistent evaluations of content, yet humans remain the ones who excel at exercising judgment in the decision-making process because of our intimate understanding of our classes and students. We remain the transformative forces that bring our classes to life. The experience of collaborating and being creative with ChatOAME, while maintaining a firm grasp on my autonomy and voice on behalf of my class and my students, resulted in feeling enlightened and supported. The interaction also improved my skills professionally, enriched

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▲ Musing on Percentages: Exam Scores and Bonus Points



JÉRÔME PROULX
proulx.jerome@uqam.ca

Jérôme Proulx is a professor of mathematics education (www.profmath.uqam.ca/~jproulx), and runs the Laboratoire Épistémologie et Activité Mathématique (www.leam.uqam.ca). His research focuses on epistemological issues related to cognition and/in mathematics. His current research program is dedicated to mental mathematics and school mathematics content development. (Photo by Nathalie St-Pierre of Université du Québec à Montréal)

Being everywhere all around us in newspapers, finances, shops, foods, surveys, and so on, percentages may seem like a mathematically rather simple theme, or one with little depth. At the same time, when one starts to pay attention to them, percentages can be seen as filled with surprising elements, which makes them a rich and fascinating concept with a lot of mathematical potential worth pondering. This is something that became quite salient in our research team, as we have been preparing experimentations with students in classrooms about percentages in mental mathematics environments.

These columns endeavour to raise and address some of these particularities, which we came to call at times "curiosities" related to the concept of percentages. These musings can be seen to be about our usage of % in everyday life, in schools, or even in mathematics in general. The following addresses the usage of percentages in exam scores and their relationship with bonus points. I am 99.9% convinced you'll have lots of fun!

Initial Question

Is it possible to have 100% on an exam? Is it possible to have 102%?

Of course it is possible to have 100%. As for the 102%, most of us have seen it at times. However, the answer might be a simple "no," since theoretically, the ceiling of an exam is fixed at 100%, that is, if you answer all questions correctly. However, what about an exam with bonus points? In that case, one can wonder if the person with 102% succeeded in answering all the questions correctly *and* in obtaining the bonus points, but this is not necessarily so….

Initial Inquiries and Curiosities

First, we can wonder how many bonus points there are in this fictitious exam. If there are 2 bonus points, then the person with 102% has indeed succeeded in answering all questions correctly in addition to the bonus questions. However, if there were 4 bonus points available, then perhaps this person got 98% on the regular part of the exam, and then got all the bonus points. Or there may be other possibilities. In all these cases with bonus points, one might wonder "out of how much" this exam really is with 4 bonus points? Is this exam worth 104 points? If so, what does the 102% score means? Or what would having 100% even mean in this context of 104 possible points?

Here, some additive and multiplicative dimensions related to percentages appear to be playing a role. Bonus points are rarely considered in the official "out of how much" of an exam: there is a score out of 100, and there are bonus points that can be added to it. The bonus points might be transformed in the final score in percent, but in some sense, they are not really part of it (if not, the exam would be out of 104 indeed, which would need to be converted to out of 100, and for which it is impossible to get more than 100% of the posed questions). In that sense, the bonus points are only possible add-ons, and do not represent points that can be subtracted or taken out from the final score. In short, one does not lose points by getting the bonus points questions right. There is thus an exam for 100 points, for which it is possible to obtain bonus points with bonus questions that are not considered in the calibration out of 100. It is in that way that it is possible to get more than 100 points.

This previous formulation raises an important dimension related to scores: the presence of bonus points transforms the score from a relative percentage to a number of points to be considered in itself, in the absolute. From a score of 98, which is initially 98% linked to a fixed whole (e.g., 100 questions of 1 point each), it is now 98 points, to which it is possible to add 4 more points to obtain 102 points in total. Perhaps the 102 points is "talked" in terms of percentages, but it is not theoretically possible to have more than a complete 100%. The 4 points are somehow outside the 100%. They are somehow points themselves, not percentage points. Hence, the operation becomes "98% + 4 points," which is not so simple to compute conceptually, since the nature of "98%" and of the "4 points" differs, and the $\oplus$ operation requires one to operate on entities of the same nature. Obviously, we do this computation without any difficulties, but it is somewhat a curious one, conceptually speaking!

Additional Curiosities

But there is more! Exams are rarely themselves out of 100 points, and the scores in % are more often the result of a conversion. The exam might be out of 20 points and converted out of 100, or it could be out of 32, out of 45, etc., and those, too, converted to 100. What then of the bonus points? If the exam is out of 45, and a student gets all 4 bonus points, for 49/45, for example, these 4 bonus points will be worth around 9% after the conversion (rounded to 109%)! This said, it is a weird 9%. Already, the 4 points were extra to the 45, and thus outside of the final % count or outside of the "out of 45," and they then become near 9% in the final percentage score. As such, the 4 bonus points become some sort of 9 bonus points out of 100 when converted in percent.

Along that line, it is also quite frequent that software programs for entering scores in schools prohibit entering a score higher than 100 (or even 99 for that matter, as it was when I was teaching high school back in the day). But what happens to the bonus points then? One member of our research team highlighted the fact that sometimes students with these bonus points are offered some sort of a raincheck; that is, they are offered to "reinvest them" somewhere else during the year. For example, the student could use these points on another exam. Hence, not only the 4 points got transformed into 9%, or 9 points, but they could also be added to another score, say 81%, to make it 90%. However, this 81% might have been initially an exam out of 32, where the student obtained 26/32. Converted to 81%, and added to 9% to make it a 90%, the student would somehow have obtained 29/32. The 4 bonus points, which became 9 points, then gets retransformed into 3 points! And other possibilities are obviously possible! Even more striking would be if the 9 points could be applied directly to the exam out of 32, for example from 26 to 35, making it a 28% increase in score… but that would be unlikely!

To top this up, and make it even more fun, it is possible that the exam in question, the one the student chooses to apply the 9 additional points to, has a "heavier" weight on the total grade for the semester, which would transform the final value of the 9 points even more so (e.g., this exam could represent 60% of the semester). As well, that semester itself could have a heavier weight in relation to the total school year, where, for example, the first semester could be worth 20% of the final year score, the second semester 30%, and the final one 50% of the year. Here again, the 9 bonus points could be applied strategically to make them more advantageous. Here again, the "final value" of the 9 bonus points could take numerous forms when being bumped up from one exam and one semester to the next!

This completes this first musing on %. I hope you enjoyed it as much as we did. Please feel free to send thoughts and comments on this column and its ideas. I am obviously 100% curious! ▲

▲ Events at the Fields: AI in Mathematics Education, Math Courses for Teachers, and a Peek at Things to Come



PAMELA BRITTAIN
pbrittai@fields.utoronto.ca

Pamela Brittain (Ph.D.) is an educator, author, illustrator, presenter, and math lover, who currently works for the Fields Institute. She has been involved in STEM education for over 20 years and strives to make math accessible and enjoyable for everyone.

The MathEd Forum wrapped up the year at Conestoga College in Kitchener, where the conversation was all about everyone's current favourite topic, AI (Artificial Intelligence). The day included talks on how we use Generative AI (GenAI) in mathematics and education, and even included a sneak peek at the OAME's very own chatbot.

AI in Mathematics Education

The session started with a talk by Nancy Nelson (Conestoga College) titled "Generative AI: From Threat to Thought." Nancy began by sharing how her world changed in 2022, when ChatGPT was released to the public and how it altered how she teaches programming. She had to modify her assessments and her teaching methods to accommodate these changes, and one of the big questions she had to ask was, "How do we make this work (because it's not going away)?"

She then recalled how we've had AI in the past, but that it's been in the background (Siri/Alexa, chatbots, Netflix recommendations), and there have been disruptions in the past (calculator, Internet), but GenAI is the biggest disrupter so far. Her advice, though: "We can use it. Don't think of it as a storm coming in, but as something we can use to help us in an effective, efficient, and ethical way." She then emphasized that we need a foundational knowledge in order to get to that point.

Her talk then went on to introduce various layers of thought, including Macro (policy makers, governments, researchers, educators), Meso (yourself, students, the curriculum), and Micro (you and your students, your thoughts, behaviours, and experiences). From there, she broke down the MACRO acronym into the following:

M: Mindset – The things we know we know, the facts about how GenAI works and what it can (and can't) do well; what GenAI knows about and how can it be used to support learning; what the potential threats are, and what GenAI struggles with.

A: Adapt Authentically – This involves framing our thoughts and understanding how GenAI approaches a problem, and how we can use that to our advantage in terms of cognitive offloading and extending our own capabilities.

C: Consider Competencies – We need to ensure we maintain those critical-thinking skills, sharing, metacognition, analytical thinking, critical evaluation, creativity, precision and accuracy, problem solving, curiosity and inquisitiveness, and the communication of math ideas.

R: Responsible Risk – Protecting privacy and data protections; what inputs and outputs are being used (are they verifiable and up-to-date), and what GenAI is basing its responses on; new cybersecurity risks.

O: Optimize Opportunities – At this point, we are at the same level as our students; we are all learning about GenAI together. How you use it and what you use it for is going to change based on who you are, but we should encourage learning by doing and playing with it. Be creative with it, but be careful what is being uploaded to it (such as student work without permission). Finally, be open and honest with your students about when, where, and how you are using GenAI.

She ended the talk with the following quotation: "AI is at its best when working with human experts, helping them save effort. But, by outsourcing basic tasks to AI, we undermine our ability to create future experts, since expertise is built up from basic knowledge."

Our second talk of the day came from George Gadanidis (Western University) and Jonathan Tan (Western University) on "Mathematics Learning in an Era of Machine Learning" (slides available at bit.ly/AI-fields).

During their talk, George and Jonathan spoke about two directions we can take with AI: as a new literacy, and as a subject of study. Starting with AI as a new literacy, they began their presentation with a historical journal on how we have approached mathematics and math problems over the centuries, starting with using clay tablets in ancient Babylonian society, up to using Scratch to create codes for students to explore concepts and ideas. One of the main concerns they see from teachers is that they love coding,

but they're not seeing the mathematics in it, and they're not seeing the math change, even though the tools are.

On the topic of AI as a subject of study, they spoke on the modern use of neural networks to imitate the human brain. They then showcased some of the amazing resources they have created (including infographics and comics) on how these networks work. They also shared some resources linking AI to Indigenous Knowledge. All these resources can be found on their website: www.ai-ed.ca/.

Their talk concluded with the idea that we need to change school mathematics in order to adapt to, and include, AI in the mathematics curriculum and instruction. They also asked the open question: "What mathematics should we be bringing into this?"

The third talk of the day, "Lessons Learned from Using Generative AI as a Role-playing Tool in an Introductory Data Science Course" by David Espinosa (Conestoga College), gave us some insights into David's experiences with teaching a data science course, using GenAI. He explained how the current state of the course included student storage and retrieval of information, and the gaps he sees (including student misconceptions and echoing information). He then shared his hope for the course, which is to see his students include reconstruction and significance, along with storage and retrieval, and to have the class be a space where the content is significant to his students.

In order to do this, he used GenAI to role-play a job interview and to teach probability (not just the theory, but how it is relevant to the job being applied for). The role playing used ChatGPT/CoPilot to generate prompts, and then students would come up with justifications on how and where probability played a role in their chosen job interview.

David admitted the project didn't go exactly to plan, and that the project needed more scaffolding and an element of significance, but it was a good first start. He also noted that some students were really interested in the AI, felt it saved time and was a promising tool, while others really disliked it and thought it shouldn't be implemented in courses.

Talk number four of the day, "A Bespoke AI for Ontario Math Educators: Design, Use, and Future" by Cal Armstrong (OAME), gave us a sneak peek into a chatbot created for the OAME and how it was made. Cal walked the audience through where content came from (mostly OAME content, Ontario curriculum documents and Ministry support materials such as *Growing Success*, and other materials that were vetted and chosen to be verifiable). First Nations content was pulled from ETFO (Elementary Teachers' Federation of Ontario) and NCTR (National Centre for Truth and Reconciliation) to give Canadian context, as ChatGPT uses American-generated content. The chatbot was deliberately framed in a Canadian context, with a focus on Ontario.

Cal then provided some feedback on how the pilot project was progressing, including the accuracy of math responses (around 95 percent), how the "conversations" between chatOAME and teachers went (most were productive conversations with well thought-out responses), and how the algorithmic work was well done. However, he also found that responses were not creative, visual representations of concepts were missing, and issues arose from how the input information was organized. He also found issues with bias and that much of the input information needs to be rewritten to be useful.

Participants were then treated to a "sneak peek" preview of the chatOAME bot, where they were encouraged to engage with it in many different ways.

The final talk of the day came from Ryan Deschamps (Conestoga College) on "Psycho-motor Approaches to Learning Mathematics."

Ryan started with a magic trick regarding the circumference versus height of a cylinder, and spoke about how we need to actually do the math, even though humans can estimate. He then went on to explain how measurement is a key valuable skill, that math allows us to do very important things, and that the stereotypes of not using algebra outside of school is not really true. Algebra helps us solve really important problems.

Ryan then spoke about the assumptions of AI in that it will continue to improve over time, and common learning tasks, such as solving discrete and even more complex math problems can be done using technology; that industry performance will continue to require judgment, creativity, and critical-thinking skills; and finally, that continuous learning will be in demand.

Ryan then went on to describe how GenAI relates to Bloom's Taxonomy and how it can do the first two areas easily (recognize and understand), the next two most likely (apply, analyze), but it can't quite do the final stages (evaluate, create).

This is where he believes mathematics as a psychomotor activity holds promise, and that mathematics already has traditional psychomotor methods (pencils, chalk, different types of paper, geometric tools, recitation, mnemonics, calculators, and computer-assisted learning (Excel)), but that we have been spending less time on some of these methods.

Mathematics as a psychomotor activity includes things like perception (observing someone solving a problem), set

(establish assumptions and formulas), guided responses (solve with instructor support), mechanism (solve basic problems in real time), complex overt response (increase comfort and aptitude in solving problems), adaptation, and origination. Ryan states that AI can't really do the last three, and he wasn't sure how well it can do the others.

The talk then ended with some open questions such as, "Does GenAI add to understanding or just provide answers?" "Will students do homework when the correct answers are at their fingertips?" "Can we scaffold information gatherers into critical thinkers?" and "Can we trust evaluation is a fair assessment of students' ability to perform math?"

Wrapping Up the 2023–2024 MathEd Forum

And that's it, folks, another year of MathEd Forum complete. Our next sessions start again in September, when you're reading this, and we start with the Margaret Sinclair Award presentation and talk. This year, we honour Dr. Lauren DeDieu, Associate Professor (Teaching) at the University of Calgary. You can read more about Dr. DeDieu and the award at www.fields.utoronto.ca/news/Dr-Lauren-DeDieu-receives-2024-Margaret-Sinclair-Memorial-Award.

You can also watch the April MathEd Forum presentations (along with previous months' talks) at www.fields.utoronto.ca/activities/23-24/meforum-Apr.

Math Content Courses for Teachers

Looking for a way to boost your math content knowledge and skills? Fields is pleased to offer online courses for teachers to help do just that. Based on the current Ontario math curriculum, and math education research, the courses are designed to explain the "why" behind many of the math concepts teachers present to their students every day.

Currently there are three courses available, with more coming soon:

- Operations (Part 1): An in-depth investigation and practice into the four operations (addition, subtraction, multiplication, division) across whole numbers, fractions, decimals, and negative numbers
- Mathematical Modelling: An example-rich exploration into how to use mathematics to model and understand the world around us
- Financial Literacy: A detailed look at financial literacy concepts from Grades 3–8 (and beyond), with practical applications and hands-on activities

All courses are offered online, in a self-paced environment. You can learn more at www.fieldsacademy.ca/eec.

Things to Come at Fields

The MathEd Forum starts again on September 28, 2024, and runs the last Saturday of each month from September to November, and then January to April.

Teachers with classes in Grades 4–10 are invited to request a spot in our highly popular "Ask a Mathematician" program, where we bring mathematicians, math researchers, and those who use math in industry, virtually into the classroom to do an activity with your students and talk about why they find math so fascinating. You can learn more about this FREE program at www.fieldsacademy.ca/aam/.

Want to learn more about Fields and the programming we offer? You can sign up for our weekly newsletter at https://mailchi.mp/fields.utoronto.ca/mailinglist, and for our educator-specific list at www.fieldsacademy.ca/eep/.

Musing on Percentages: Exam Scores and Bonus Points.
Continued from page 36.

my perspectives as a teacher, and enabled me to think more holistically during content creation in general. However, it's also important to recognize that with a machine alone, it will not be able to exhibit motivation or empathy, nor did it recognize the need for self-improvement and reflection. It is both the human behind this AI design and the human utilizing the AI, the combined human–machine dual intelligence, guided by our dedication to our students and love for our profession, that propels this positive change. Traditionally, professional growth occurred mainly through reading or discussions with other colleagues (Armstrong, 2023). The introduction of ChatOAME provides an additional venue for this process to take place, making it more accessible and significantly more efficient for us, and providing a more valuable result for our students.

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OAME website
www.oame.on.ca

DIRECT GENERAL INQUIRIES TO: EXECUTIVE DIRECTORS

Fred and Lynda Ferneyhough
4751 Mack Ave.
Plympton-Wyoming, ON N0N 1J6
Business Phone: 647-466-0674
EDS@oame.on.ca

PRESIDENT

Kit Luce
Barrie, ON
kit.luce@oame.on.ca

PAST PRESIDENT

Sandra Jean Price
Oshawa, ON
sandra.jean.price@oame.on.ca

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Mark Chubb
St. Catharines, ON
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Dave Petro
Harrow, ON
david.petro@oame.on.ca

EMERITUS

Paul Alves
Georgetown, ON
paul.alves@oame.on.ca

CHAPTERS & REPRESENTATIVES

CHAMP: Giovanni Bruccoleri
Mississauga, ON
giovanni.bruccoleri@oame.on.ca

GOLDEN: TBD

GVMA: Glen McMillan
Guelph, ON
glen.mcmillan@oame.on.ca

ISOMA: Cal Armstrong
Burlington, ON
cal.armstrong@oame.on.ca

MAC²: Stacey Collins
Barrie, ON
stacey.collins@oame.on.ca

NOMA: Heather Theijsmeijer
Kagawong, ON
heather.theijsmeijer@oame.on.ca

NWOAME: Chelsea Carlson
Fort Frances, ON
chelsea.carlson@oame.on.ca

$O_3$4ME: Teresa Weselake
McNab Braeside, ON
teresa.weselake@oame.on.ca

PRMA: Jennifer Clark
Bowmanville, ON
jennifer.clark@oame.on.ca

QSLMA: Tanis Lattanzio
Kingston, ON
tanis.lattanzio@oame.on.ca

SAME: Vineeta Krishnan
Toronto, ON
vineeta.krishnan@oame.on.ca

SWOAME: Elizabeth Pearsall
Kingsville, ON
elizabeth.pearsall@oame.on.ca

TEAMS: Alvarine Aldridge
Toronto, ON
alvarine.aldridge@oame.on.ca

WOMA: Sean Cousins
London, ON
sean.cousins@oame.on.ca

Y⁴MA: Gina Micomonaco
Richmond Hill, ON
gina.micomonaco@oame.on.ca

DIRECTORS – TERM EXPIRING 2024

Gerard Lewis
Toronto, ON
gerard.lewis@oame.on.ca

Jason To
Toronto, ON
jason.to@oame.on.ca

Cameron Redsell-Montgomerie
Toronto, ON
cameron.redsell-montgomerie
@oame.on.ca

DIRECTORS – TERM EXPIRING 2026

Elli Weisdorf
Toronto, ON
elli.weisdorf@oame.on.ca

Lisa Rossiter-Thornton
Toronto, ON
lisa.rossitor-thornton@oame.on.ca

DIRECTORS – TERM EXPIRING 2027

Olive Creary-Satchell
Toronto, ON
olive.creary-satchell@oame.on.ca

Michael Frankfort
Richmond Hill, ON
michael.frankfort@oame.on.ca

Jennifer Holm
Kitchener, ON
jennifer.holm@oame.on.ca

NCTM REPRESENTATIVE

Jacqueline Hill
Lindsay, ON
jacqueline.hill@oame.on.ca

WEBSITE COORDINATOR

Greg Clarke
Orillia, ON
web@oame.on.ca

GAZETTE EDITOR

Timothy Sibbald
North Bay, ON
gazette@oame.on.ca

ABACUS CO-EDITORS

Marc Husband
Toronto, ON
marc.husband@oame.on.ca

Tina Rapke
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tina.rapke@oame.on.ca

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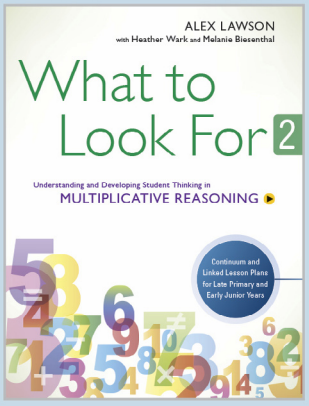
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