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ABACUS

Ontario Mathematics Gazette

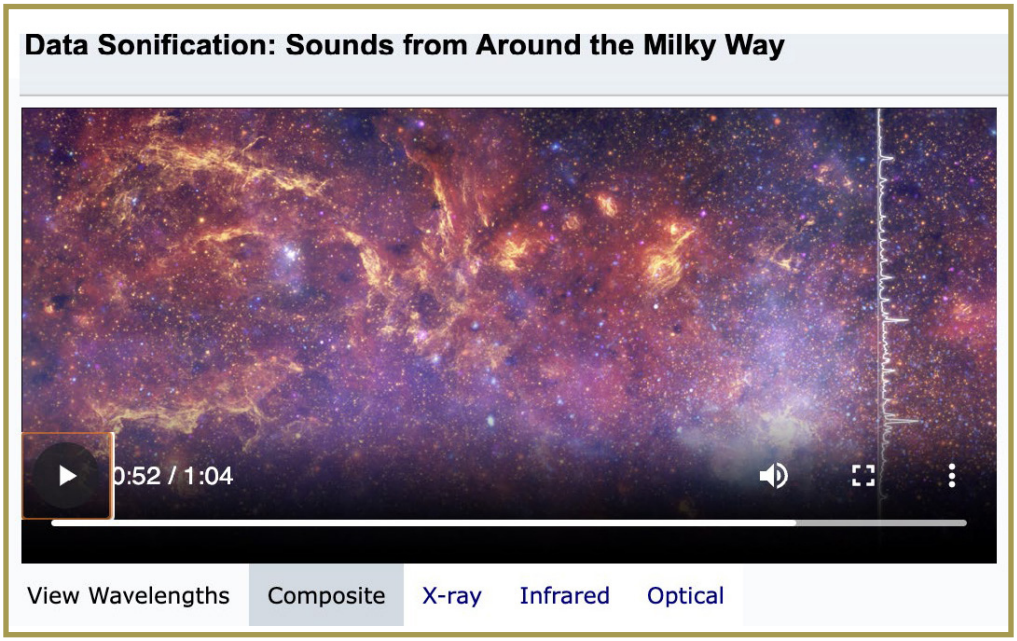
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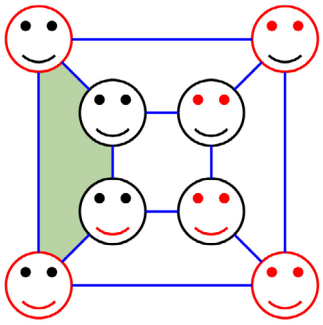
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December 2024

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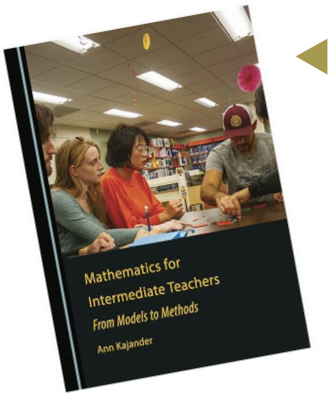
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See What's the Problem? K.I.S.S.



See Book Review: Relearn Mathematics, Using Representations

See Elementary Math Matters: Scissors, Symmetries, and Dancing Dolls... on a (Air)Plane



Submission of Articles

The *Ontario Mathematics Gazette* (*Gazette*) publishes news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *Gazette*:

- The *Gazette* is aligned with the OAME/AOEM Vision, Mission, and Strategic priorities. These can be found on the website www.oame.on.ca under "About OAME" and "Guiding Principles."
- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.
  Articles are to be word–processed, MS Word is preferred, and prepared according to the *Publication Manual of the American Psychological Association, Seventh Edition*. However, please use single-line (not double) spacing and only one space after each period. Articles should not exceed eight numbered pages of text, though there is some flexibility. Figures, images, and photographs should be placed close to the relevant text and include captions. The photographer's permission is required, and for photos of students under the age of 18, permission of a parent or guardian is required. .

Please submit your article in one blind file (i.e., identity of author is not evident). In a separate file, provide contact information for all authors. Please email these two files to Tim Sibbald at gazette@oame.on.ca.

When received, the Editor will provide you with the "Author Agreement" that clarifies copyright requirements. The *Gazette* uses double-blind peer review. Upon review, you will be notified whether your article has been accepted for publication (as is, or pending minor or major revisions) or not. The Editor reserves the right to edit manuscripts prior to publication.

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The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of OAME/AOEM.

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Editor
- **Tim Sibbald**
  gazette@oame.on.ca

Associate Editors
- **Anne Yeager**
  anne.yeager@rogers.com
- **Peter Saarimaki**
  saarimaki@rogers.com

Associate Editor (French articles) /Rédactrice adjointe (articles français)
- **Kyla Kadlec**
  kyla.kadlec@oame.on.ca

***Abacus* Co-Editors**
- **Marc Husband**
  marc.husbandl@oame.on.ca
- **Tina Rapke**
  tina.rapke@oame.on.ca

Copy Editor
- **Gitta Berg**
  gitta.berg@sympatico.ca

Design and Production
- **Penny Clemens,** Graphic Designer
  pennyydezign@gmail.com

Printing & Binding
- **Pole Printing,** Box 69, 1-89 King Street East
  Forest, ON N0N 1J0 (519) 786-5112
  poles@xcelco.on.ca

Advertising Manager
- **Robert Sherk**
  4366 Snider Rd., Verona, ON K0H 2W0
  Home (613) 374-1515
  robert.sherk@oame.on.ca

Submission of Advertisements
Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** by email or at the above address. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be print ready, and colour advertisements should have no bleeds.

Advertising Rates
Advertising rates are available by telephoning, emailing, or writing to the Advertising Manager.

▲ Editor's Report



TIM SIBBALD, OCT, PH.D.
gazette@oame.on.ca

Tim Sibbald is a professor in the Schulich School of Education, Nipissing University, with a focus that includes mathematics education. He is a former President of OAME/AOEM and has held a variety of other OAME/AOEM posts over the years.

Thank you, Ralph, for taking on the September issue of the *Gazette*! Thank you, also, to the editorial team, who again demonstrated their capacity to adapt. Having been Editor since September 2017, it was worthwhile taking a break for an issue, and I sincerely appreciate Ralph and the editorial team for facilitating that.

The conference I mentioned in the June editorial was quite interesting, but with 2311 participants, it is near impossible to capture the experience. I did, however, catch up with the group of roughly 40 Canadian math education-focused attendees and include a couple of photos (see Figures 1 and 2).



Figure 1: *Can you name the columnist? And the* Abacus *Editor?*

I will share one item from my travels that I found at the Australian Museum in Sydney. It appears in Figure 3, and I am including it because it highlights Indigenous knowledge. The particular item is a "mattang," or navigational chart, from the Marshall Islands. In all likelihood, I will share more photos going forward, but that is enough for now.



Figure 2: *Can you name the $O_3$4ME chapter member in this picture? (Hint: It's not the NOMA member.)*

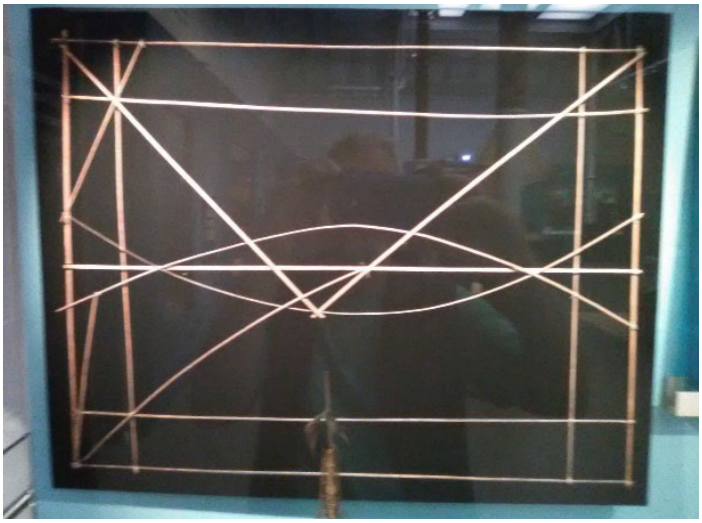


Figure 3: *Indigenous navigation chart*

Destreaming

I am a reluctant critic because I generally trust that decisions are made because they are well informed or a consensus view. While I may dissent, I generally exercise tongue holding where I am not aware of details. However, I increasingly worry about where education in Ontario is being led, and I feel a professional obligation to speak up.

The concern I see is a confluence of two issues. First, in recent history, EQAO discrepancies between Grade 9 Applied and Academic students showed a long-standing performance difference, and now we have destreamed scores. Second is the interpretation that discrepancies in performance are equity issues. I have no objection to the claim, but I find the outcome of the attempted solution alarming.

Following are some data to set the stage for this. Table 1 shows the provincial Level results reported by EQAO for English Language Learners as the percentage of students who achieved at or above the provincial standard. That standard is a Level 3, corresponding to achievement of 70 percent or above. Table 1 provides a summary for the last few years, where the year reflects when the school year started—so, "2023" means September 2023 to June 2024.

	2018 (%)	2019 (%)	2020	2021 (%)	2022 (%)	2023 (%)
Grade 3	61	58		59	60	61
Grade 6	49	48		47	50	50
Grade 9 Applied	45	44				
Grade 9 Academic	84	84				
Grade 9 Destreamed				52	54	54

Table 1: *Percentage of students at or above the provincial standard reported by EQAO*

Table 1 shows that Grades 3 and 6 have been relatively consistent. It is important to note that a change of 1 percent is substantial because of the number of students, but it is a 4 percent range. Public media publish articles about the drops, and seem to always focus on Grades 3 and 6, and less often do they include the Grade 9 results. In Grade 9, the same consistency was observed before destreaming. Grade 9 Academic was 84 to 85 percent from 2012 to 2019. Grade 9 Applied was 44 to 47 percent from 2012 to 2019.

The Grade 9 destreamed 2023 results are based on 147 212 students. In 2019, the data were derived from an estimated 147 945 students (I say estimated because EQAO results prior to 2022 are not available from the Province or EQAO as best I can tell, so I have used provincially reported enrolment figures). According to provincial data in 2019, there were 38 855 Applied math students, and 111 800 Academic students.

Consider a statistical estimate of what the achievement in destreamed should be. It is not a simple average because of the disparity in the number of students. Instead, a weighted average is required. Specifically: $\frac{44 \times 38855 + 84 \times 111800}{38855 + 111800} \doteq$ 74 percent.

The expected result is that 74 percent of destreamed students should achieve the provincial standard. The actual results show that destreaming has effectively reduced the opportunities for achievement for over 100 000 students.

So, with that concern, how do I reconcile the possible inequity that was the explanation for destreaming in the first place? The challenge, as I see it, is that Applied and Academic results showed a huge achievement gap of roughly 40 percent, which was not formed in one school

year. Instead, it points to the need for streaming earlier so that the students who are underachieving receive the support they need—this is equitable in the sense that supports meet the needs for support. I suggest that it would be better to use the Grade 6 EQAO results to stream students and provide intensive support during Grades 7 and 8 to students who clearly need the support. Following intensive Grades 7 and 8 support, the Grade 9 EQAO could be used to re-examine the stream for each student. Clearly, thought and research would need to be used to develop an equitable policy for interpreting the re-examination of stream assignment.

The worry I have is, quite simply, that high-achieving students, over 100 000 of them, have seen a 20 percent drop below the provincial standard, so that lower-achieving students, slightly under 40 000, demonstrate that 10 percent of them can achieve the provincial standard. In simple terms, approximately 22 000 students have achieved less, so that 4000 could ascend.

Equity is a tricky concept, which does not require all students to achieve the same. It does require that they are all afforded opportunities to demonstrate their ability to achieve. That, in my opinion, is why the underperformers need much more support. It is also why the vast number of students who have now sunk below the provincial standard require that equity considerations are applied regarding what destreaming has done to them. (I will bite my lip about raising the place of gifted students.)

A further challenge is that we, as a community of teachers, are being led to debate along equity lines when a leadership vacuum seems to be the fundamental challenge. Unfortunately, destreaming is unlikely to see the rapid change I think it needs, given the way the manufactured teacher shortage has been ignored. On that note, I will move on to avoid my editorial turning into a proverbial soap box.

In Other News…

Over the past few years, Jeff Irvine and I have been working on indexing the historic record of the *Gazette*. It covers 1(1) from February 1962 to 58(4) in June 2020. This was for research purposes, but it did not escape us that it would also be useful to others. It was a lot of work and a long and tangled effort that ultimately has been refined as a spreadsheet that is available in the file resources area of the members-only section of the OAME website.

The index was constructed based on professional opinions, but following a qualitative research methodology. Notably, for those who wish to search the *Gazette* from the last decade or so, it is feasible to use the "Find" feature in your Adobe reader. The issues were prepared digitally, unlike the older issues.

A goal, moving forward, is to have indexing as an ongoing feature of every issue—because indexing and searching for words are distinctly different—which brings this into an interesting realm. The columnists and editorial team have lots to do already, and so the idea has been to automate the preparation of an index for each issue. Even a semi-automated system can make it easier. That process has yielded insights into how statistics of language usage—computational linguistics—is a key link in the chain that has led to Aritificial Intelligence (AI). More about this as the effort continues.

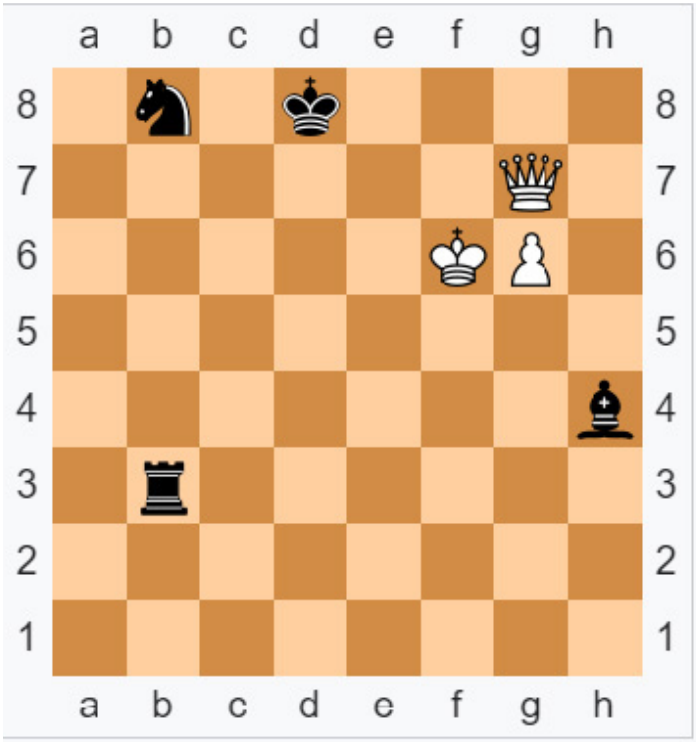


Figure 3: *Computer-discovered chess problem*

While the inspiration may lead to better understanding of AI in the modern age, let me provide an entry point. Consider challenging your students to write an algorithm for playing snowman (which apparently is the politically correct name for what was formerly known as hangman). If a word of 6–12 letters is chosen at random, what order do statistics suggest you should pick letters? Suppose, however, you find that the word does not contain the letter "e," then how do the statistics change, and what should you pick for the next letter? Students can, of course, test their algorithms and justify their method.

To my mind, this is much more tangible than challenging students with computer-discovered chess problems, such as shown in Figure 3 (source: https://en.wikipedia.org/wiki/Endgame_tablebase).

The theme for the International Day of Mathematics (aka "pi day") is "Mathematics, Art, and Creativity."



In This Issue…

Our President, **Kit Luce**, brings an upbeat approach to some aspects of the curriculum. **Jacqueline Hill** gives an overview of what NCTM provides to members. **Jason To** brings details from OMCA that include a mention of **Cal Armstrong** (ISOMA). **Pamela Brittain** brings news of the Margaret Sinclair Memorial Award recipient, Dr. Lauren DeDieu, along with details of a discussion of math outreach.

Carly Ziniuk highlights an impressive array of math resources related to space. **Lynda Colgan** reminisces about flying with scissors, but then draws out Frieze patterns. **Iain Brodie** and **Hatice Beyza Sezer** provide resources for a coding task, and challenge readers to generalize some aspects. **David Costello** highlights the *Problem Toolbox, Problem of the Week*, and *Resources* sections of his website. **Jérôme Proulx** examines the commutative property applied to percent—a column that has subtle challenges. **Jeff Irvine** explains Expectancy Value Theory. **Shawn Godin** is making faces after I sent him a page from a calendar. Last, but not least, **Jennifer Holm** considers three-dimensional extensions of a problem involving circles that appeared in her September column.

We have a feature invited article that will be of particular interest to high school teachers. A group focused on **First-Year Mathematics and Statistics in Canada** (FYMSiC) has been growing for seven years. It focuses on instructional issues in the early years of university. For all of us, it is a chance to see that professional conversations are taking place to enact change. Teachers of senior grades might find that engaging in the group is a way to help bridge the gap between high school and university.

By way of articles, **Stacey Jacobs** addresses the challenging topic of math anxiety and strategies to mitigate it. **Jeff Irvine** writes about the important topic of student agency and strategies to encourage it.

Second to last, I do not generally make a point of mentioning notices; however, the call for nominations for the Board of Directors is important to our community. It is an excellent opportunity for professional growth and worth taking time to discuss with others in your chapter. Please give it due consideration.

Lastly, I owe many thanks to the wonderful editorial team. As this issue was being handled, two of us had significant travel. Yet, as a team, everyone adapted and came through. It truly is marvelous to see the inner workings and marvel at it like the workings of a well-made watch. As I write this acknowledgement in Spain and had some email issues that were resolved seamlessly by the team, it impresses me to see how readily technology is handled. I sincerely appreciate all that **Anne Yeager, Peter Saarimaki, Gitta Berg,** and **Penny Clemens** do to make the *Gazette* come together for our readers.

▲ Letter to the Editor

Happy Monday!

I have had several emails with regards to our timely writings about "How to Write an OAME Conference Proposal." I think we should do the next step and have Tim:

a) either separate out the part that talks about "before, during, and after presenting" and republish it
or
b) have Tim write a small "refresher from a speaker/presenter's point of view

Again, let me know if you want me to get the ball rolling with Tim.

Yours in mathematical fun, Jacqueline

Editor Response:

It is wonderful to hear that being explicit about how to write a conference proposal has been useful to members. This is the sort of sharing we need as a community. It is reminiscent of the video *The Right Write Stuff* (at the bottom of www.oame.on.ca/main/index.php?lang=en&code=pub second), which offers support for writing *Gazette* articles. I am, however, not inclined to republish because readers can always revisit the article for the advice it offers about presenting. So, I will offer a few words from my own experience. Before I do that, however, let me note that when you submit a proposal, there was a question for experienced presenters, asking if they would be willing to mentor new(er) presenters—I don't know the details, but it is an opportunity to investigate if you are a new(er) presenter.

It is exciting to receive word that one's proposal has been accepted—it is an affirmation of sorts. Personally, my next steps are often to start a list of ideas of what the presentation looks like. This is often a crude list, but like initial lesson planning, the ideas then can be ordered, modified, or built on as one mulls the presentation. Often in the excitement, I imagine what people might wonder or ask, and often I find aspects where I need to broaden an idea to suit a larger audience. For example, can an idea for Intermediate teachers be crafted to include simplifications that make it accessible to Junior classrooms, or intensified to work in Senior courses.

The time between acceptance of a proposal and the presentation facilitates considerations that often lead to a more fulsome plan for the presentation. Yet, it is all just sideas. I then ask teachers questions: What is the action? Are there hands-on elements and, if so, what is their orchestration? Whom might the audience include, and how do I engage as many of them as possible? Often these thoughts come as I convert the list of ideas into a presentation plan—whether speaking with slides or running a hands-on workshop.

Typically, I endeavour to have presentations drafted a month ahead. This allows time to set it down for a couple of days and revisit it with relatively fresh eyes. Then I go over it with a view to addressing details. A key one is, what if I run short? With presentations being 70 minutes at OAME 2025, I will plan for 90 minutes and anticipate having leftovers to seed ideas for future conferences. I also look for details I might have glossed over during the preparation process. I recall, on one occasion, realizing that I had not considered how I would get heavy resources to the actual workshop site. These are solvable problems, but very much in the final week or two ahead.

The night before a presentation requires a quiet review of the plan—and sleep. Ahead of presentations, whether hours before or the day before, I will visit where the presentation takes place to familiarize myself with the layout. It is rare, but surprises can arise. Then I attempt to arrive at the presentation space in a timely manner to get set up and to shift into presentation mode. It's not dissimilar to teaching, but one also wants to respect the audience with one's professionalism.

I suspect, like lesson planning, that there are many different approaches to preparing conference presentations. Please consider writing your thoughts and sharing how you prepare in a Letter to the Editor. Perhaps a bevy or multitude of supportive letters will inspire others!▲

▲ President's Message



KIT LUCE
kit.luce@oame.on.ca

Kit has been an active member of MAC² for many years and joined the Board of Directors in 2022. She is a District Mathematics Facilitator for Grades K–9 with the Simcoe County District School Board. In her work, Kit creates math resources to support educators in their instruction and assessment practice, as well as developing and facilitating professional development. Kit is an Additional Qualification instructor and course writer, and she has written and reviewed work for a variety of publishers. She has been involved in Ministry writing projects and was a Content Lead for the recent OAME/AOEM writing projects. Kit received the 2024 OAME Award for Leadership in Mathematics Education.

As I am writing this message, the sun is shining outside my window and the Fall colours are glowing red and orange. By the time this appears in the *Gazette*, we will be well into the winter season and all that brings with it. Winter can be a time of rest and rejuvenation, a time to consider and reflect on our teaching.

The past few weeks, I have been in schools with educators who have voluntarily come to professional learning on their own time. Their dedication to learning and reflecting on their practice to support their students has been clearly evident. The energy and enthusiastic talk while working with concrete manipulatives, models like concept circles and number lines, has been both inspiring and contagious. I've also been involved in developing and facilitating professional learning focused on Universal Design for Learning. Reading the feedback from these learning sessions makes one thing very clear—educators have a strong desire to meet the needs of their learners and help them develop as capable and confident mathematicians.

There is so much we need to juggle and prioritize as educators, whether we are in elementary or secondary schools or alternative environments. We all have specific concerns about how to address the entire curriculum in a short period of time. We worry about finding and using the best resources for planning lessons. We wonder how we are to get it all taught and still have time for our families, friends, and interests. At the heart of all of these concerns is a clear focus on the students in front of us each day. Teachers want

their students to feel confident in their abilities to engage in challenging mathematics. They want to be able to open up access so that all of their students can enter into math tasks and move their learning forward.

We are also aware that our students face incredible challenges in their lives. They come to us with a variety of school and life experiences and don't always bring a positive mindset toward math. As Dan Meyer said in his TED Talk, "Math class needs a makeover" (www.ted.com/talks/dan_meyer_math_class_needs_a_makeover):

> I sell a product to a market that doesn't want it, but is forced by law to buy it. Larry Martinek stated, "Children don't hate math. What they hate is being confused, intimidated, and embarrassed by math. With understanding comes passion, and with passion comes growth—a treasure is unlocked."

The question for many educators is how to undo experiences of confusion and embarrassment and help our students get to that place of passion and growth.

I love this section of the Mathematics Curriculum Context document.

> The vision of the mathematics curriculum is to help all students develop a positive identity as a mathematics learner and see themselves as mathematically skilled, to support them as they use mathematics to make sense of the world, and to enable them to make critical decisions based on mathematically sound principles. This vision is attained in a mathematics classroom filled with enthusiasm and excitement—a classroom where all students receive the highest-quality mathematics instruction and learning opportunities, interact as confident mathematics learners, and are thereby enabled to reach their full potential (Ontario Ministry of Education, p. 62).

When we bring our enthusiasm and excitement to the classroom, we can help to create an environment where students see the possibilities of using math to explore their world, to make connections between subject matter and solving real problems they see in their lives and communities. We can create opportunities for students to play with math, to see the beauty in the patterns, and experience the joy in discovering mathematical truths. I clearly remember driving to work one day and listening to a news story on the radio about an accident involving a Brinks truck and a tractor trailer, resulting in over three million dollars in loonies and toonies, along with candy, being spilled over the highway and surrounding area. When I entered my classroom, I shared the story, and we spent the rest of the day creating math problems and working on them together.

"How much would three million dollars in coins weigh?"

"What would that look like if the coins were stacked up in towers?"

"Would 3 million loonies fit into our classroom?"

"How many toonies tall am I?"

"If we laid that many toonies and loonies in a line, how long would it be?"

"Are coins magnetic? How many could a giant magnet pick up?"

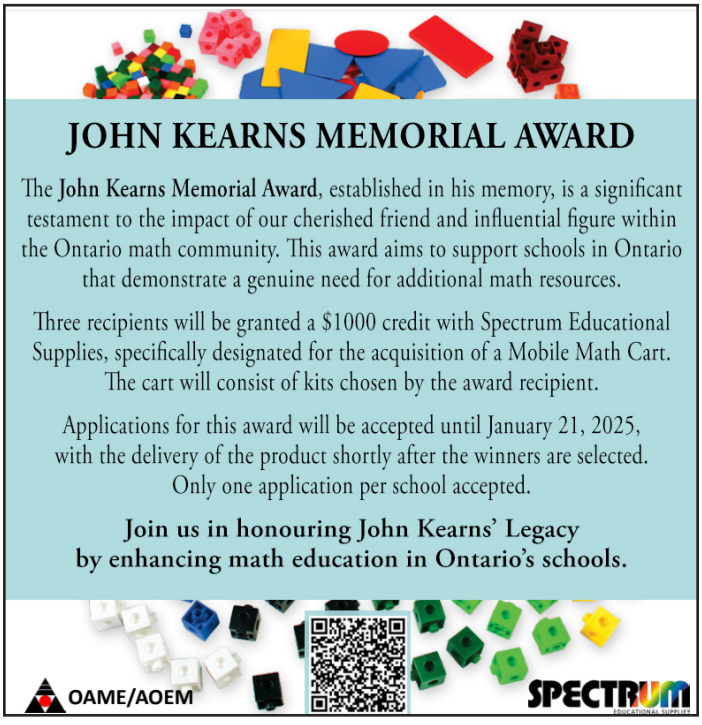
The students' creativity was endless. We made problems and put them up in the hallway for other classes to read and solve. They wrote persuasive letters to our administrator, suggesting we go on a field trip the following Spring, since they were sure there would still be money to be found in the area. (Sadly, the field trip was not approved.) They noticed and wondered and persevered to solve problems about which they cared.

While we can't rely on interesting news stories to jump-start our lessons each day, we can bring our passions to the classroom. We can help our students discover their own enthusiasm for learning, and open their minds and hearts to the math in their world. We can unlock that potential for growth.

"It is impossible to be a mathematician without being a poet in soul." ~ Sofia Kovalevskaya

Reference

Ontario Ministry of Education (OME). (2020). *The Ontario curriculum, grades 1–8: Mathematics 2020.* ▲



OAME/AOEM Awards for Excellence in Mathematics Education

The purpose of the OAME/AOEM Awards is to formally recognize individuals and educators who demonstrate an outstanding contribution to mathematics education in Ontario. The successful candidate for each award enhances the learning environment for all students, providing opportunities for students to do, see, hear, and touch mathematics in a profound and meaningful way. If you know someone who qualifies for this honour, please nominate them for one of the following awards.

- **OAME/AOEM Life Membership Award:** This award recognizes an educator who has contributed in a significant way to OAME/AOEM, demonstrates outstanding leadership in mathematics education, and has accumulated ten or more years of membership in OAME/AOEM.
- **Award for Exceptional and Creative Teaching in Elementary Mathematics:** This award recognizes an exceptional and creative elementary educator, who demonstrates excellence in mathematics education and contributes to the overall development of students.
- **Award for Exceptional and Creative Teaching in Secondary Mathematics:** This award recognizes an exceptional and creative secondary school mathematics educator, who demonstrates excellence in mathematics education and contributes to the overall development of students.
- **Award for Exceptional and Creative Teaching in Post-Secondary Mathematics:** This award recognizes an exceptional and creative post-secondary educator or team of educators, who demonstrates excellence in mathematics education, contributes to the overall development of students, and demonstrates leadership in the mathematics education community.
- **Elementary School Staff Award for Exceptional and Collaborative Mathematics Teaching:** This award recognizes an elementary school (a school accommodating any or all of Grades K–8) whose staff fosters collaborative and excellent mathematics teaching, contributes to the overall development of student learning of mathematics, and demonstrates leadership in the mathematics community.
- **Secondary School Mathematics Staff Award for Exceptional and Collaborative Teaching:** This award recognizes a secondary school mathematics staff, who fosters collegiality, teamwork, and excellent classroom teaching, contributes to the overall development of students, and demonstrates leadership in the mathematics education community.
- **Award for Leadership in Mathematics Education:** This award recognizes an educator who has demonstrated leadership by contributing in a significant way to the development of mathematics educators and the enhancement of mathematics education in Ontario through innovation and/or research-related activities.
- **Award for Outstanding Contribution to OAME/AOEM and Mathematics Education in Ontario:** This award recognizes an individual who has made a significant contribution to OAME/AOEM and contributed directly or indirectly to the improvement of mathematics education in Ontario, but who is not necessarily an educator or a member of OAME/AOEM.

OAME/AOEM Awards Nomination Process

1. Complete the Google **Nomination form** appropriate to the specific award, found at www.oame.on.ca (look for the Awards tab on the left of the page). You may include a one-page letter of recommendation.
2. Include with the nomination cover page up to three **letters of support**, of no more than two pages each. Please ensure all letters of support:
   - address the criteria for the award as outlined on the OAME/AOEM website
   - include the name and position of the supporter to the application
   - include the nominee's name and the award in the letter
3. Include the **Curriculum Vitae**, of up to five pages, for award nominees, except for Elementary and Secondary Staff awards.
4. Submit the Google Nomination form and email the completed package (supporting letters and CV) by the deadline advertised below to EDs@oame.on.ca.

DEADLINE: Nominations are due by January 31, 2025.

The awards will be presented at the OAME Annual Conference in May 2025.

▲ What is FYMSiC?



ANDIE BURAZIN
a.burazin@utoronto.ca

Andie Burazin is a passionate mathematics educator at the University of Toronto, Mississauga, who enjoys supporting students in any capacity. You might catch her watching hockey or football.



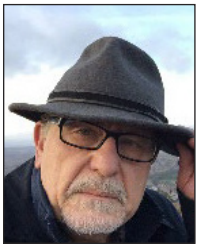
REBECCA CARTER
rebecca.carter@queensu.ca

Rebecca Carter is a Ph.D. student in the Department of Mathematics and Statistics at Queen's University, with interests in teaching and curriculum development. She enjoys the outdoors and reading when not in school.



LAUREN DEDIEU
lauren.dedieu@ucalgary.ca

Lauren is an Associate Professor (Teaching) in the Department of Mathematics and Statistics at the University of Calgary. She enjoys travelling and taking pictures of birds and squirrels.



VESELIN JUNGIĆ
vjungic@sfu.ca

Dr. Veselin Jungić is a Teaching Professor in the Department of Mathematics at Simon Fraser University, and the coordinator of the Math Catcher Outreach Program.



MIROSLAV LOVRIC
lovric@mcmaster.ca

Miroslav Lovric is a professor at McMaster University, with a passion for mathematics education and teaching. When not in school, he loves to travel, read, and drink coffee.

FYMSiC (First-Year Math and Stats in Canada) Beginnings

The landscape of teaching university mathematics courses in Canada has seen significant changes in the past decade or so, not just because of COVID. These changes are attributed to the creation of more teaching-stream faculty positions, which are not standardized across institutions in terms of:

- workload, pay, and acknowledgement
- the increase in enrolment of a very diverse student population
- the impact of modern technology affecting the teaching and learning of mathematics
- most importantly, providing the support needed for students to succeed by addressing and accommodating their personal, academic, and career goals

In the early 2010's, having all the above in mind, university mathematics instructors had limited opportunities to connect with one another to discuss the successes and challenges of the teaching and learning of mathematics. No Canadian community at the time provided a comprehensive picture of the state of undergraduate mathematics. It almost seemed as if each university mathematics department, and even its faculty, was its own island, rarely communicating to other departments across Canada, solitarily navigating the rough changing waters of university education. Given that many incoming students faced significant obstacles when completing a "gatekeeping" first-year mathematics course(s), and most university instructors were not formally trained to deal with the complexities of teaching and learning, something had to be done.

In 2017, the *Canadian Mathematical Society Notes* published an education article, calling for a national dialogue to critically analyze the state of teaching mathematics at a first-year undergraduate level in Canada. The goal of the dialogue was to:

> unite Canadian teaching and research faculty in their efforts to provide the best post-secondary math education possible, [and to] provide a forum for all involved to learn about the complexity of issues related to teaching Level 1 mathematics at universities across Canada (Jungić & Lovric, 2017).

It could be said that this call was the beginning of a grassroots movement called First-Year Math and Stats in Canada, known by the acronym FYMSiC. FYMSiC became a community of practice for the teaching and learning of mathematics, whose goal is to share experiences, gather data, and look at research-based decisions and strategies.

With this in mind, the group would discuss possible ideas and suggestions for university faculty and instructors to move forward to address concerns.

Initially, FYMSiC's main purpose was to focus on first-year mathematics courses in Canada. The discourse was shaped by questions surrounding teaching and learning practices:

- What is common, what is different, and what are the driving forces behind the current situation?
- From a horizontal perspective, what are the goals and practices at different universities?
- From a vertical perspective, what are the goals and practices at the high school and university levels?
- What roles do first-year mathematics courses play in other academic programs?
- What, if anything, beyond mathematics content should be done in first-year mathematics courses?

At the inaugural in-person FYMSiC conference in April 2018 at the Fields Institute in Toronto, these questions were discussed. As well, attendees were asked to voluntarily fill in the FYMSiC Course Repository, a searchable online database, with information and details of first-year mathematics courses at their institution. It was shocking to learn how little knowledge a university mathematics department had about other departments in terms of organizing curriculum, courses, and instruction. Moreover, a disconnect and gap between high school and university mathematics instruction has not been narrowed nor closed. The conversations within FYMSiC also moved to related issues occurring in upper-year university mathematics courses, since the challenges students experience in earlier mathematics courses propagate into advanced mathematics courses.

Presently, FYMSiC activities are coordinated by five FYMSiC team members, who have authored this article, and two of whom are the co-founders of this community of practice.

FYMSiC Members

There is no fee to become an FYMSiC member. At most, an individual is asked to pay a small registration fee for an in-person conference, where the fee is used solely toward the conference expenses. There is no fee for online programming.

Any individual interested in the teaching and learning of mathematics at any level can join FYMSiC, independent of their affiliation and position: students, teaching assistants, post-doctoral fellows, high school teachers, college and university instructors and administrators, researchers, publishers—all are welcome. Members are added to an email list, which presently contains over 700 names from both Canada and internationally.

FYMSiC strives to better connect its members, while trying not to take their time and resources away from other commitments, and other communities of practice. This community of practice is happy to hear the voices and ideas of its members suggesting how to improve and benefit the community, as well as benefit from partnerships with other teaching and learning communities such as the Canadian Mathematical Society, the Canadian Mathematics Education Study Group, the Pacific Institute for the Mathematical Sciences, and the MathEd Forum at the Fields Institute.

FYMSiC Advocacy

Notably more than half of the FYMSiC members are female university mathematics instructors, which reflects the fact that a large group of female mathematicians and statisticians are involved in teaching early years of undergraduate courses. What is still missing is the full and explicit acknowledgement of the tremendous contributions of talented, knowledgeable, and hard-working female mathematics and statistics educators, who are the true role models and inspiration for students. Through its activities, FYMSiC endeavours to advocate for this important cause.

Since the beginning, FYMSiC has also advocated for a major shift in the way university mathematics and statistics departments across Canada view and treat their teaching faculty, in particular, the large number of instructors, who hold precarious contract, part-time, or limited-term appointments. It is these instructors, who are routinely assigned to teach important first-year courses. FYMSiC believes that mathematics departments should create a respectful and supportive environment for teaching faculty, as well as offer more full-time, permanent employment opportunities, with a smaller number of (necessary) short-term contractual positions.

Lastly, for the OAME/AOEM audience, it might come as a surprise that opportunities for professional development for university teaching faculty in mathematics is relatively limited and not mandatory. Yes, it is common that Canadian post-secondary institutions have general-purpose teaching and learning centres with support for instructors and offering an array of resources as well as activities in which to participate. However, the expertise of the centre might not adequately help a mathematics instructor with teaching a group of students, who possibly lack the necessary skills and knowledge required to succeed in a mathematics course. This is precisely one of the reasons why an important objective of FYMSiC's existence is to create opportunities

for its members to learn from each other and to share their knowledge and experiences about the teaching and learning of mathematics.

FYMSiC Activities

At this time, FYMSiC members have several ways to contribute to fostering the community spirit throughout the academic year. Engagement is purely voluntary.

FYMSiC Website and Email List

The FYMSiC website (www.firstyearmath.ca) and the messages to the email list are major sources of information for the community. This is where its members can find announcements of conferences and various education events, job postings, members' milestones, links to teaching resources, and much more.

FYMSiC Annual Conference

Since 2018, FYMSiC has hosted annual conferences, with the exception of 2021, when it was cancelled due to the COVID fatigue. The annual conferences in April 2018 and May 2019 were in person, and the remaining conferences have been online. Although hosting an online event is more accessible for many individuals, it does present challenges for engagement and generating fruitful discussions. FYMSiC hopes to bring back their in-person annual conference in May 2026, alternating between online and in person.

Previous conferences have addressed numerous issues of interest and importance to the FYMSiC community:

- 2024 FYMSiC One-Day Online Conference: *Why are we teaching mathematics today?*
- 2023 FYMSiC One-Day Online Conference: *Teaching: The Backstage of Our Courses*
- 2022 FYMSiC One-Day Online Conference: *Everyday Life of a Post-Secondary Math & Stats Practitioner – In Memory of Alfonso Gracia-Saz*
- 2020 FYMSiC One-Day Online Conference: *Teaching First-Year Math and Stats in Interesting Times (to say the least)*
- 2019 FYMSiC Conference: *Time to Rethink Our Curriculum?* University of Alberta
- 2018 FYMSiC Conference: *Facts, Community, and Vision.* Fields Institute

A typical conference schedule consists of several keynote speakers, who have done work within the scope of the conference theme. FYMSiC members are invited to present either in 25-minute presentation parallel sessions or the staple 5-minute lightning talks. When in person, over the course of a few days, socializing and networking opportunities are purposely embedded within the schedule, as some of the best ideas happen during informal conversations. When the conferences were in person, there were about 75 attendees, and when the modality was online, the number of active participants was around 100 throughout the day.

FYMSiC Online Teaching Meet-ups

The participants of the 2020 FYMSiC Conference expressed a need to stay better connected during the pandemic. The solution for FYMSiC was to host frequent online teaching meet-ups. To this day, the meet-ups continue to be offered three or four times per semester. The format is informal, where individuals volunteer to lead a meet-up, or are invited to present, or have discussions guided by the focus of the meet-up. Some of the meet-up titles in the past academic year were:

- August 2023: *An Open Discussion on Managing Our Academic Workload*
- September 2023: *A Summer Theatre for MAT102 (Intro to Math Proofs)*
- October 2023: *Applying Cognitive Science Principles for Effective Math Instruction*
- Winter 2024: *Let's Focus on Content and Good Examples in First-Year Calculus*

A rich variety of topics is explored in each meet-up. FYMSiC asks its members to suggest topics, and welcomes its members to moderate a meet-up. On average, there are about 30 participants from across Canada and beyond, joining the conversations at each meet-up.

(Details from: www.firstyearmath.ca/fymsic-meet-ups)

FYMSiC Newsletter

After the 2019 FYMSiC Conference at the University of Alberta, FYMSiC created its own newsletter, which is available at www.firstyearmath.ca/newsletter/. Since then, 15 issues have been published, typically two or three issues per year. FYMSiC members are invited to submit articles about classroom resources, book reviews, or short communications based on their own teaching and learning experiences; as well, career milestones, publications, and mathematics education events are promoted. With this, there is a very healthy spectrum of articles in every newsletter issue.

FYMSiC is delighted that Issue 15 (released in July 2024) had close to 300 individuals opening the newsletter. (Details from: www.firstyearmath.ca/newsletter)

FYMSiC Profiles

As of Summer 2024, supported by the Pacific Institute for the Mathematical Sciences (PIMS) and the Canadian Mathematical Society (CMS), FYMSiC Profiles are available

on the website. It is an accessible online repository, whose purpose is to better connect the teaching mathematics community in Canada and beyond. This centralized, password-protected, user-verified repository contains self-entered data about FYMSiC members, including professional contact information, courses taught, research interests, research projects, publications, and resources. It is easily searchable so that individuals can use it to find other FYMSiC members with similar interests.

Since the launch of FYMSiC Profiles, over 100 members have created a profile, with the number continuing to grow. FYMSiC will remind the community annually to update their Profiles so that their information remains relevant and fresh.

Joining FYMSiC Profiles has been advertised to the FYMSiC community, the Canadian Mathematics Education Study Group, and the Canadian Mathematical Society.

See the landing page to learn about FYMSiC Profiles: www.firstyearmath.ca.

Future for FYMSiC

FYMSiC has been fortunate to have survived seven years, and is proud of successfully running the annual conference, creating newsletters, and providing online teaching meet-ups. The hope is to continue supporting and bringing the university mathematics teaching community in Canada together. The challenge is keeping the momentum and interest going, as the landscape of mathematics teaching is rapidly changing. FYMSiC continues to evolve and is listening to the needs of its members.

Recently, a mathematics graduate student was introduced to the FYMSiC team, to gain experience and insight in the world of academia. The hope is to have fresh ideas from early career individuals, who may be more attuned and relatable to the students, so that the community can have a better perspective on how to support the mathematics teaching and learning environments. As well, younger talent joining the FYMSiC team will help in smoothly passing the torch to maintain the spirit of the community as it transitions.

With all this, FYMSiC cordially invites interested OAME/AOEM members to contribute to the *FYMSiC Newsletter* by writing about their teaching-related experiences and practice, as well as to join the FYMSiC Profiles.

Reference

Jungić, V., & Lovric, M. (2017). Call for national dialogue: The present and future of teaching first-year mathematics at Canadian universities. *Canadian Mathematical Society Notes, 49*(5), 10–*12.* www.notes.math.ca/archives/Notesv49n5.pdf ▲

▲ OMCA News: December 2024



JASON TO (OMCA PAST PRESIDENT)
OMCAmath@gmail.com

The first meeting this school year of the Ontario Mathematics Coordinators Association (OMCA) took place virtually on Monday, September 16. It was the first meeting chaired by OMCA's new president, Gerry Lewis.

"Celebrate Math" is going to be a central theme for OMCA this year. From opening meetings with math problems to celebrating our members' successes, we're looking to make joy central, while also meeting the commitments of the Province's Math Achievement Action Plan.

In line with the theme of "Celebrate Math," this year's annual OMCA retreat will be welcoming Libo Valencia as the keynote presenter. Libo is a mathematics educator from New York, and the author of *Math Play*. He will be joining OMCA members and guests at Blue Mountain for this year's retreat.

OMCA monthly meetings typically feature a guest speaker to lead learning with our members. At the September meeting, Cal Armstrong provided an insightful presentation on ChatOAME, the AI (Artificial Intelligence) tool that he has designed to support OAME/AOEM members with all things mathematics for teachers. Cal shared how ChatOAME is designed, and some of the functionality of the tool, including lesson planning and task creation.

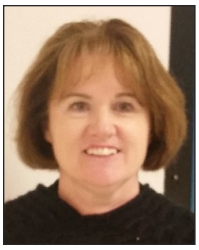
At the next OMCA meeting on October 21, Dr. Kara Dymond from OISE will be speaking to members about designing mathematics instruction so that no mind is left behind. Kara is an educator, researcher, and advocate for students with ADHD.

If you are a math leader or math coach with your school board, we'd love for you to join OMCA! New members are always welcome. Please contact us at OMCAmath@gmail.com, or visit our website at www.omca.website (note the different domain), to learn more about the organization and connect with us. ▲

NCTM Reporter Needed!

Mentoring available to June 2025.
Contact the Editor today!

▲ OAME/NCTM Report: Happy Holidays, OAME/AOEM Members



JACQUELINE HILL
jacqueline.hill@bell.net

Jacqueline Hill is a recently retired Grade 9–12 teacher of mathematics. She is a Past President of OAME and OMCA, as well as the recipient of the Award for Exceptional Teaching in Secondary Mathematics. She also wrote the "Director's Dialogue" for the Gazette *for a number of years.*

I hope this column finds you happy and healthy!

NCTM has been keeping busy updating their website with recent and relevant articles and activities. Here is a summary of what NCTM has to offer.

About NCTM

The National Council of Teachers of Mathematics is the public voice of mathematics education in the United States. They support teachers to ensure equitable mathematics learning of the highest quality for all students, through vision, leadership, professional development, and research. They were founded in 1920. NCTM is the world's largest mathematics education organization, with 90 000 members and more than 230 Affiliates throughout the United States and Canada.

Principles and Standards

Principles and Standards for School Mathematics (the "Standards," published by NCTM in 1998 and revised in 2005) outlines the essential components of a high-quality school mathematics program and presents a common foundation of mathematics to be learned by all students. *Curriculum Focal Points*, published in 2006, identifies three important topics at each grade level pre-K–8, and is the next step in implementing the Standards. *Focus in High School Mathematics: Reasoning and Sense Making* (2010) and related topic books set forth a vision of high school mathematics, putting reasoning and sense making at the core of learning. Lastly, *Principles to Actions: Ensuring Mathematical Success for All* (2014) describes the policies and actions required for a high-quality mathematics education for all students.

Publications and Resources

NCTM publications include: *Mathematics Teacher: Learning and Teaching PK–12* (2020), Mathematics Teacher Educator, Legacy Journals and Blogs, *Journal for Research in Mathematics Education*, and a catalog of more than 200 educational books and other materials.

NCTM's website, www.nctm.org, features resources such as *Student Explorations in Mathematics*. As well as information about jobs and new products, it includes information about NCTM and its programs.

Professional Development

The Council holds an annual meeting and exposition along with three regional meetings each year, with a combined attendance of 25 000. Teachers of mathematics and others attend lectures, panel discussions, and workshops, and view exhibits of the latest mathematics education materials and innovations.

NCTM offers a diverse array of professional development events, services, and resources. The Professional Development website (www.nctm.org/profdev) has information about future offerings, including institutes for teachers focused on teaching mathematics, with core ideas focused on reasoning and sense making.

Research

NCTM works with Council leaders, staff, researchers, and practitioners to synthesize and summarize existing research that supports the organization's mission and objectives, products, and services. NCTM's *Research Clips and Briefs* focus on research that link mathematics to education practice.

Legislative and Policy Leadership

To represent the interests of its members, NCTM engages in government relations activities dedicated to constructive policy discussions among all interested parties to determine what is best for students. *The NCTM Advocacy Toolkit*, available to NCTM members, is a collection of materials to give individuals the tools and guidance they need to advocate for mathematics and education.

Grants and Awards

NCTM's *Mathematics Education Trust* (MET) provides funds directly to teachers, Affiliates (which includes OAME/AOEM), and institutions, to enhance mathematics education. MET offers grants, scholarship and award programs, and a lifetime achievement award.

Dates to put on your calendars:

- February 6, 7, & 8, 2025 – SD STEM ED Conference, Crossroads Event Center in Huron, South Dakota

Until next time,

Yours in mathematical fun,
Your NCTM representative, Jacqueline ▲

Events at the Fields: Margaret Sinclair Memorial Award Presentation and Making Math Outreach Meaningful



PAMELA BRITTAIN
pbrittai@fields.utoronto.ca

Pamela Brittain (Ph.D.) is an educator, author, illustrator, presenter, and math lover, who currently works for the Fields Institute. She has been involved in STEM education for over 20 years and strives to make math accessible and enjoyable for everyone.



Figure 1: *Dr. Kasra Rafi (left), Interim Deputy Director, presents Dr. Lauren DeDieu (right) with this year's Margaret Sinclair Memorial Award (Photo Credit: Fields Institute)*

September saw the return of the MathEd Forum and the presentation of this year's Margaret Sinclair Memorial Award. This year's winner was Dr. Lauren DeDieu from the University of Calgary (see Figure 1), whose inspiring talk, "How do we want students to experience and remember our courses?," provided thought-provoking insights on how we need to focus on not only what we teach (the content), but also what we want our students to take away from our courses, in a broader context.

Lauren teaches various courses at the University of Calgary, from 1300 first-year calculus students to smaller second- and third-year math courses, along with some math education courses. Across all her courses, Lauren focuses on asking the following question: "We have content we need to be teaching, and we have 'eager, young, bright students sitting in our classrooms,' so how do we want to 'shape their experiences'? What do we want them to take away and use to make meaningful experiences?" Her talk focused on answering these questions.

For her courses, Lauren wants her students to both learn the content at a deep level, and be prepared for where they are heading. But in terms of remembering things from the course, it's less about the content and more about "habits of mind and skills that will stick with them and serve them well in their careers and degrees."

So how do we do this? Through active engagement. What does this look like? In class, Lauren uses both overview and big-picture questions, along with group work. Outside of class, there are frequent low-stakes formative online assessments for the students to practise their skills. What Lauren made sure to emphasize, though, is that there is a need for a balance of high expectations of deep learning, along with enjoyment of content. Her big-picture questions often highlight common misconceptions, and the students discuss with each other these misconceptions and "sell" their idea of why their answer is correct to a classmate. This "helps combat illusions of mastery by helping students identify areas they need to improve," helps them with retrieval and strengthening learning, and "primes their brains for learning." She also spoke about how reducing barriers under her control, such as cognitive load, has been helpful for her students as well; including things such as transition slides, using colours and titles to break things up, and showing where students are during the lesson, to help in developing metacognition skills.

Lauren then highlighted some of the outreach work she does, which includes a SNAP Math Fair with a twist. In this fair, it is the undergraduate students she works with who make the exhibits, and elementary students are invited to come, view, and participate—rather than the elementary students making the exhibits. Each year, they see around 200 Grades 4–6 students explore various logic-based puzzles that have very little arithmetic, and lots of hints and extensions to make the puzzles engaging and accessible to all students. Through this program, education students get to interact with the students they will eventually be teaching.

Another outreach initiative near and dear to Lauren is Girls Excel in Math (GEM) Calgary. This event focuses on introducing girls to lots of unique topics in math, and provides access to accessible topics and activities. Teachers are provided with undergraduate volunteers and a guide containing solutions and tips for the various activities. In addition, Lauren is currently working on making short

modules about coding and how things work, such as, "If you go to a website, how does it know what picture to show you?" The motivation for these programs was the realization that girls were under-represented in enrichment programs at the University of Calgary, and she wanted to create safe spaces for girls, where they could have fun doing math and hang out with one another. She also spoke on gender bias and how it affects girls in STEM. Other biases (including implicit ones) also reduce the number of girls who see themselves in STEM, and this creates a "leaky pipeline." GEM helps expose students and teachers to female role models (the undergraduate students), and provides opportunities to see themselves in math and STEM fields.

Finally, Lauren spoke about the Math Attack Summer Camp for Girls, an eight-day overnight camp that is similar to GEM, but with more undergraduates and colleagues, who run more advanced sessions. This program has 21 spots, but is open to anyone who wishes to apply. Both programs, GEM and the Camp, are free or accessibly priced, and those unable to gain a spot in the overnight camp are offered an option for a three-day day-camp option. Both programs utilize junior chaperones (those just out of high school), senior chaperones (undergraduate students), and other University of Calgary members.

In her courses, Lauren provides lots of opportunities to show growth, and reflect on that growth. This includes mulligan tests and rewrites, dropping scores, and low-stakes assessments. She wants her students to build critical-thinking and problem-solving skills, along with how to build logical arguments. A key take-away for her students is that they "…may never use content again, but will use the skills." Finally, learning how to communicate mathematically, and to communicate to non-math experts, is an important skill in terms of future employability. To do this, Lauren has her students engage in argumentative writing, which has been shown to enhance learning.

Lauren's talk wrapped up with the question, "What do I want students to remember?" Her answer: skills and habits of mind. She spoke about wanting her students to be brave, try something new, and build confidence to do hard things. The thinking here is that if a student can say to themself, "If I can do a hard abstract algebra course, then I can do anything." Lauren's belief is that "mathematics is an excellent vehicle to build perseverance," and it helps to build positive feelings about math. She sees her students having more confidence than at the start of the course, and this is especially true for prospective teachers.

Her talk ended with the question, "How do YOU want your students to remember your course?"

Congratulations to Lauren on being the well-deserved recipient of this year's Margaret Sinclair Memorial Award!

Short Talks: Varieties of Mathematical Outreach: Chris Wu, Paul King, Julia Liu

Our next session was a panel discussion with three individuals involved in various math outreach initiatives.

Chris is connected to CMS (Canadian Mathematical Society) math camps, where elite students who score highly on math competitions are invited to attend. He spoke about how the approximately 50 participants are chosen for the camp, and about how the focus of these camps is more on making connections and learning from others, rather than competing with each other. He also spoke about bringing back former campers to act as role models and "trailblazers" for camp attendees.

The microphone was then passed to Paul, who oversees a math club run by students, who use it to prepare for contests. He described how the program runs and how the students interact with one another. He also emphasized the importance of interacting with the students at a different level.

Finally, Julia introduced herself as a high school teacher in the Toronto District School Board in a specialty program, where she oversees a math clinic. The math clinic itself is run by students, who help each other out. Julia and her colleagues coordinate and gather students to come to the program, but the students themselves provide math assistance. The moderator of the panel prompted Julia by asking, "What's the draw? How do you convince them to come? What do you do?" Julia's response was that, for students who want to succeed and can't afford private tutors, they are motivated to come for extra help; for the senior students who provide assistance, they get volunteer hours, and they also want to help students. The senior students also learn valuable skills in interpersonal communications.

Julia was then asked about the STEM-focused courses and how they differ from the general classes, along with how students are chosen to attend. Julia responded that for the specialized programs, there is an application that is done by lottery. The lottery is not fully random: there is a 50 percent ratio of female students, along with representation from groups typically under-represented in STEM. While it is open to anyone who wants to apply, most students who apply are engaged and want to be there. When asked about the program's goals, Julia replied that many of the students enrolled are interested more in the science side than the math side, but a main goal is building connections and transferable skills.

The moderator then asked all the speakers, "What kind of institutional credit do you get for doing this work?" Julia

replied that she volunteers, and it's just something she chooses to do. Paul spoke of being one of those "freaky people," who were "just tolerated" by their colleagues for running a math club, and that he has faced a lot of pushback for starting a club. He stated, "No one said, 'Don't do this,' but no one said, 'We're glad you're doing this.'" Chris stated he receives appreciation from the community and the students, but there isn't anything formal from the institution. All of the speakers reiterated that they do this because they enjoy the work, making a difference, and finding a different way to connect with students.

Panel: Building Relationships through Mathematics Outreach: Lauren DeDieu, Chris Wu, Paul King, Julia Liu, Moderated by Zack Wolske

The session ended with a panel discussion that covered various topics. Members of the audience were invited to vote on and submit questions to be posed to the panelists.

Question: *Not all students enjoy competition questions, or they associate math with those types of questions. What strategies and activities can we focus on to change that?*

We can use these questions to build perseverance, and to motivate and develop their thinking and logic. They can build fundamental math thinking, using topics like cryptography, prisoners' dilemma, and logic. Competition questions can help students learn techniques. You can also use questions that don't need prior knowledge to do. Some strategies can include forming small groups with mixed abilities, where they teach each other, using low-floor, high-ceiling questions, to challenge them to develop their mathematical independence. A very important point raised by a panelist was also the idea that when students know you believe in them, they will reach up.

Question: *What kind of activities might appeal to students for whom basic math is difficult or frustrating? (i.e., students who may have some kind of learning disability)*

Monopoly, card games, number operations, Battleship, SNAP Math Fairs—basically anything that is accessible and fun, including logic puzzles that even a Kindergarten student could solve based on the knowledge they have.

Question: *How does your outreach get students to see mathematics as something they can do, no matter what they want to do in higher studies or life?*

The student going through a course or program has learned they can do something hard, and has learned about the idea of being brave. They are more willing to try when it's not being evaluated. Also, it's about normalizing the experience of getting things wrong and focusing more on metacognition—the process versus the end result.

Question: *What are you doing to reach everyone (apart from the very top performers)?*

Changing the structure of the program will change who is attracted to participate; and there are lots of options for the layout and presentation of questions.

Question: *How are you potentially exploring outreach opportunities with elementary schools to connect in terms of transitions between the two panels?*

Having older students help younger ones, and situations where the teacher takes a less hands-on role.

Question: *You need people to run these programs, and they rely on volunteers. How are you getting these volunteers and why are they doing this?*

High school students need volunteer hours, but snacks are also a great motivator. A lot of the volunteers come from a love of math already, and they want to be involved, they love to do it, and it makes them feel good about themselves. Many competitive International Baccalaureate students just want to be involved, and former students come back for math camp. Yes, they get all their volunteer hours, but they really enjoy sharing their love and enjoyment of math with other math people. Undergraduate volunteers often like giving back, and it's a way to connect with professors, and get reference letters. In general, they want to give back and share their love.

The panelists then shared some stories and anecdotes of their experiences, some resources that they've found useful, and discussions about how money and time are needed to run many of these projects.

Closing Remarks

There are many options for outreach with a lot of different groups. A major theme was to model what you want your students to emulate, and students will take on the personality of what they see. Show them "you may have had a bad experience with math, but that's okay; the more you learn and interact with these types of conversations, the more tools you will have to help out your students."

In Other News

Do you know a high school student who is interested in developing their math puzzle and logic skills, and is willing to travel to downtown Toronto? If so, we'd like to ask for your help in promoting Fields Math Circles, held every Saturday, in person, at the Fields Institute (222 College Street) from October 2024 to June 2025. These sessions focus on building math logic skills through engaging and fun puzzles with a group of like-minded students. More details can be found at www.fieldsacademy.ca/eep/, and students can join anytime! ▲

▲ Elementary Math Matters: Scissors, Symmetries, and Dancing Dolls… on a (Air)Plane



LYNDA COLGAN
lynda.colgan@queensu.ca

Lynda Colgan is Professor Emerita, Faculty of Education, Queen's University, and Executive Director of Science Rendezvous. Lynda collaborates with the Assessment and Instruction for Mathematics (AIM) Collective, which engages researchers and educators across Canada to support the development of tools for assessment and instruction in mathematics learning from Kindergarten through Grade 3. She continues to work on a number of funded research, resource-creation, and teacher-education projects. Lynda remains deeply committed to STEM outreach and can be found regularly at events that engage students, parents, and educators in science, technology, engineering, and mathematics at home and in the classroom.

I keep dating myself. Far too often, I find myself saying, "I remember when…," and then diving into a story about a book, event, student, classroom, or colleague, whose image from the distant past pops up out of nowhere with remarkable clarity.

The latest disclosure came at a recent professional development workshop, during which one participant was knitting while listening to the speaker. What amazed me was her ability to knit long stretches of stitches without once looking down at the wooden needles that danced in her hands. We got talking over lunch about how she was a life-long knitter—something she first learned for a "badge" as a Girl Guide many years ago. She talked about taking her knitting everywhere, and how it was a running joke that she was more attached to her skeins of wool than she was to her own family members. She spoke of the days when she would knit while travelling by car, rail, or… airplane.

The word "airplane" was a trigger for me, and I instantly recalled, to her amazement and mine, how once I travelled to Germany for the Mathematics and Creativity Conference in Muenster to present a paper with Dr. William (Bill) Higginson called *(Re)Learning and Teaching Through the Eyes of a Child: Reflections on a Pre-Service Elementary Mathematics Education Course*. I regaled her with a story about how I spent the entire nine-hour flight on Swiss Air, cutting out paper-doll chains in preparation for the session. "You took scissors on the plane? That must have been a long time ago." As it turned out, it was 1999.

As part of an extracurricular series for pre-service teacher candidates at Queen's Faculty of Education, Bill, his then graduate student, Nathalie Sinclair (now Distinguished Simon Fraser University Professor), and I ran an extended enrichment program called *The Joy of X: Helping Prospective Teachers See the Creative Potential of Mathematics*. One of the sessions that the teachers found most engaging, *Matisse Mathematics*, was the inspiration for the workshop in Germany and an article, Colgan and Sinclair (2000). It was also a session that challenged my own spatial sense; hence, the need for extra practice on the plane.

The year 2000 had been designated the World Mathematical Year by the International Mathematical Union (IMU) under the sponsorship of UNESCO. The promotional poster was the image of a circle of friends—paper dolls—decorated in the costumes of cultures from around the globe. Building on that powerful image, and that of Henri Matisse's *La Dance*, our paper doll mathematics exploration was a tribute to the upcoming special celebratory year. For us, the paper dolls represented the intersection of paper folding and mathematics, and invited kinesthetic exploration of symmetry, transformation groups, and coding systems.



Figure 1: La Dance*, Henri Matisse – State Hermitage Museum, Saint Petersburg, Russia, PD-US, https://en.wikipedia.org/w /index.php?curid=3523871*

To do justice to the investigation, our small team would have required more than the two hours allotted for the optional session—held at the end of a long day of mandatory classes. To give the 60+ attendees the gist of the mathematics abounding in paper-doll chains and paper folding, we needed to curtail some explorations by assigning specific tasks and relying on a few teacher candidates who had volunteered to participate in an extra preparatory period

so that they could become leaders for small breakout groups.

Before introducing the paper cut-outs, we asked the students (in pairs) to create, code, and identify the geometric transformations of seven specific patterns, using a "stamp" of a bare foot with an ink pad.

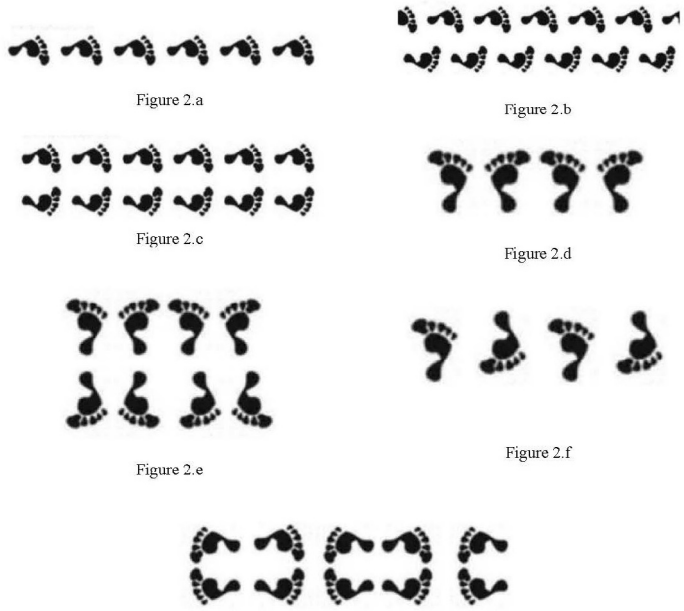


Figure 2a-g: *The seven "footprint" patterns that the students created as visual cues*

Next, using their stamped reference pages, we asked the students to stand up and act out the patterns by imagining they were leaving their footprints in the sand or snow. The physical experience of using their own bodies, we believed, would help them to concretize how geometric transformations are actualized in space. How would they move to replicate the pattern? What could they name the moves? What geometric transformations and symmetries did the footprint patterns represent? How could the moves be coded using this system of letters: *b*, *d*, *p*, and *q*? Why were these specific letters chosen?

There was much laughter as concentrated efforts were made to slide, hop, walk, and turn, first on one foot and then on two. As I watched, John Cleese in Monty Python's *Ministry of Silly Walks* came to my mind (and once again, I am dating myself). After some minutes of experimentation, the following labels and codes were established and geometric transformations identified for the footprint patterns shown in Figure 2:

a) The hop on one-foot pattern *bbbb* representing horizontal-translation symmetry
b) The simple walking-steps pattern *bpbp* representing translation and glide-reflection symmetry
c) The two-foot jump pattern *bbbb* representing translation and horizontal-reflection symmetry *pppp*
d) The cross-over pattern *bdbd* representing translation and vertical-reflection symmetry
e) The spinning cross-over pattern *bdbd* representing translation, vertical-reflection and 180°-rotation symmetry *pqpq*
f) The one-foot spinning-hop pattern *bqbq* representing translation and 180°-rotation symmetry
g) The two-foot spinning jump pattern *bdqpbdqp* representing translation, horizontal- and vertical-reflection, and 180°-rotation symmetry

We called the dancing and pirouetting to an end so that we could debrief and discuss; then explain why we had selected these specific sets of repeated-design footprints, known as *Frieze Patterns* (see also Sibbald, 2024). We answered questions, the most common of which was to ask why there were only seven possible combinations of transformations of 2D shapes involving translations, glide reflections, half-turn rotations, vertical reflections, and horizontal reflections.

Next, we demonstrated our own efforts at creating the seven possible paper-doll chains by using oversized construction paper to improve visibility. Our simple doll design (which appears further on in this article) was intended to reduce student anxiety: nothing fancy was required beyond a non-symmetric generator (in our case, a doll) that had specific characteristics such as a prominent nose, flowing hair, a ball cap, or distinctive right/left boot shape, so that it was possible to distinguish its left-right and up-down orientation. When the students began planning their own generators in small table groups, we noticed that many used outlines such as elephants (connected by their trunks and tails) and machine-like objects with tubes and pipes.

In advance, we had prepared long strips of construction paper for the table groups. As a class, we asked the students to make the classic paper-doll chain by making successive book folds (folding edge to edge along the vertical axis) until they had a region that was approximately rectangular. The next step was to draw their generator on top, making sure that there was a region on each side for the generators to stay connected, then cut out the design and unfold the paper. The students quickly identified this chain as the cross-over pattern: their two feet have vertical reflection, and identifying "crossing over" as something found in many folk dances, i.e., right leg over left and left leg behind right.



Figure 3: *The classic paper-doll chain,* bdbd *(as in Figure 2.d), in which the dolls are reflected about the fold, to produce a chain showing translation and vertical-reflection symmetry*

The students readily recognized that if they added a horizontal fold first, before doing the book folds, they could create a chain with both horizontal and vertical symmetry.

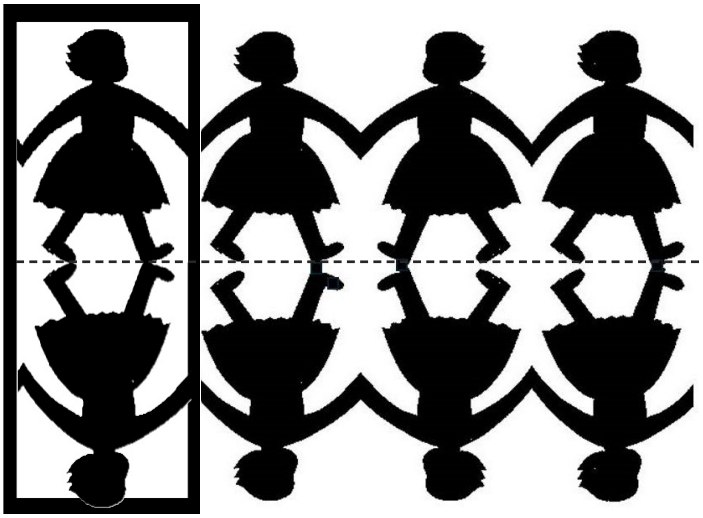


Figure 4: *A paper-doll chain with both horizontal and vertical reflections, coded* bdb *(where the underline represents the centreline reflection)*

At this point, the students were arranged into small groups, each with a facilitator. How could a chain with simple horizontal translations be achieved? What about a chain with horizontal reflections and translations? The facilitator demonstrated how to trace a generator on a cylinder so that the "arms" or other features are connected, and then added a horizontal fold to a wider strip, before folding it into a cylinder, resulting in the following chains:

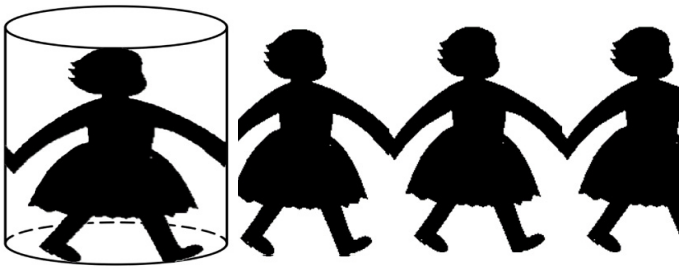


Figure 5: *Paper-doll string, cut from a cylinder, to generate translation chains (*bbbb*)*



Figure 6: *Paper-doll string, cut from a cylinder, to generate translation chains having a horizontal reflection, as in the two-foot jump pattern,* … bbbbbbbb … / … ppppppppp …

Given time constraints, we did not fold the other three chains during our session, but we did demonstrate that each chain could be achieved through paper-folding hexagons and specialized strips, folded and rolled into a cylinder. These are shown in Figures 6–8. More detail can be found in the references identified at the end of this column.

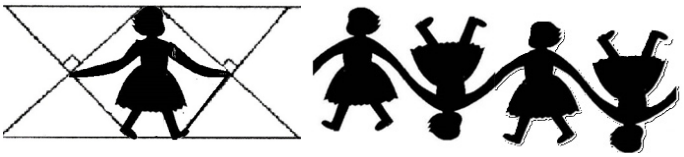


Figure 7: *By folding a hexagon as shown (top left), we can generate the one-foot jump-turn pattern chain shown below it (*bqp*) by 180° rotations.*

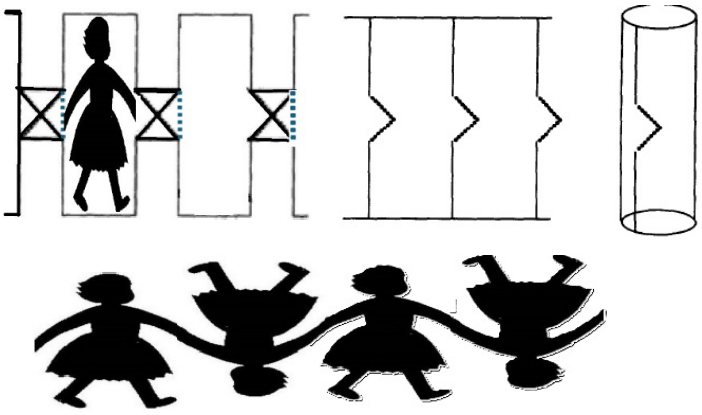


Figure 8: *By folding the small connecting squares as shown, and then rolling the strip into a cylinder as shown (top right), we can generate the simple walk pattern (*bpbp*) chain below it.*

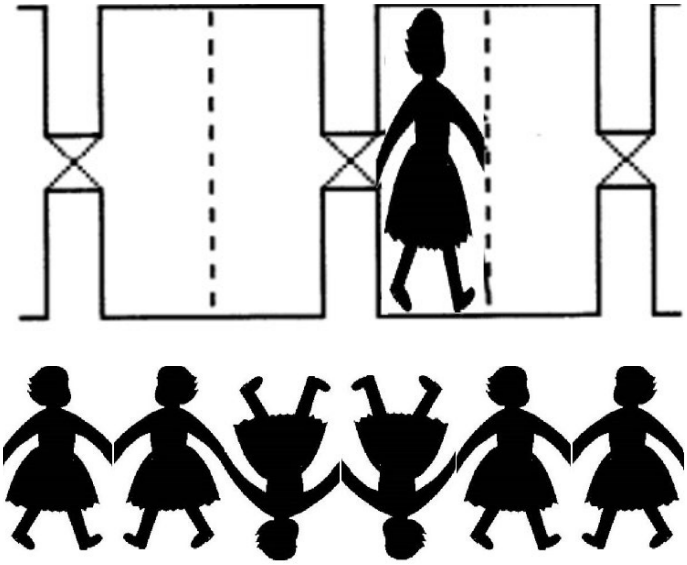


Figure 9: *This paper-doll chain is formed by a series of reflections and rotations* ... bdbd ... ... pqpq ...

On reflection (pun intended) about this *Joy of X* session, I became quite maudlin. The workshops ran throughout my second year as an elementary mathematics instructor in the B.Ed. program at Queen's. Over 60 teacher candidates attended regularly, to be challenged and to challenge Bill, Natalie, and me, to keep the enrichment and inspiration coming. There was time in the teacher candidates' schedule for instructors to offer special programs like *The Joy of X*, and for students to choose from a menu of opportunities, none of which was for credit beyond a certificate of participation. I loved the passion and hunger that the students had for learning about all and any mathematics, and were willing to sing, dance, and laugh as members of a community in the pursuit of understanding. Our broad enrichment topics were often tasks and topics such as Fractal Dragon Curves, that might never filter down into an elementary classroom; however, others, like making a giant Sierpinski kite, constructing a cubic metre from rolled newspaper, or creating a DIY Plinko board, had enormous potential for modelling tetrahedra, volume, and probability in fun and engaging ways.

The current "back to basics" climate in Ontario is pervasive—impacting K–12 classrooms and faculties of education by removing any possibility of enrichment, like the *Joy of X* afforded novice teachers, and, by extension, young learners in school. How will early mathematicians be inspired to fold paper in the process of learning mathematics, gaining the spatial skills necessary to become an origami professional in the field of space engineering? When will students be given interesting challenges, such as disproving the statement that a piece of paper can only be folded eight times? In 2002, Brittany Gallivan, a Grade 9 student from California, saw my challenge on Connect-ME (the online community that continued the *Joy of X* experience beyond graduation) and then folded a single piece of paper in half 12 times to set a new world record… just for fun! Will today's students know that Alexander Graham Bell lost interest in the telephone after securing its patent, and turned to building giant kites that could lift people off the ground, becoming an aeronautical pioneer through his knowledge of 3D geometry? Can they visualize a cubic metre of soil when they order one from the garden centre, or will their driveway become a mountain of dirt? Could they explain to someone at an art gallery how Escher created his famous tessellated horse from a simple square?

There are some who will say that educator enrichment like the *Joy of X*, with its focus on creativity, beauty, play, and fun, is illustrative of all that is antithetical to "back to basics," and an impediment to serious mathematics teaching and learning. But can we afford, as a community of mathematics educators, to reduce mathematics in school to a "life skill" for calculating interest on a loan or computing area to purchase tiles for a floor. If we use a mathematical lens, the paper-doll/footprint example in this column, from one-quarter century ago, shows that there was a focus on facts/definitions and procedures, as well as an introduction to coding—like those specified in "back-to-basics" curricula—but addressed using decidedly "back-to-the-future" methods, whose time has come.

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Which Frieze is this? (Punta Delgada, Azores)



Meaning Making in the Classroom: Costello Math Website as a One-Stop Shop



DAVID COSTELLO
david@costellomath.com

David is a principal, who taught and had various board roles in Prince Edward Island. David has also instructed university courses regarding curriculum, differentiation, mathematics, and literacy. He facilitates professional learning in mathematics and school development. Dr. David Costello has published four books, with two of his most recent books being: Messing Around with Math: Ready-to-Use Problems That Engage Students in a Better Understanding of Key Math Concepts *and* Mathematizing Student Thinking: Connecting Problem Solving to Everyday Life and Building Capable and Confident Math Learners.

Teachers spend considerable time looking for resources to support instruction and student learning. In fact, I have heard from many Primary, Junior, and Intermediate teachers that it seems like a never-ending scavenger hunt. As a result of this frustration, I have created a website (www.costellomath.com) that offers support to teachers, to meet the vast learning needs of the classroom, in a quick and manageable way.

In this column, I will highlight key components of the website and how it can be used as an instructional resource. My goal is to support teachers by providing tasks which can be easily incorporated into daily lessons that enhance teaching and learning in a meaningful way.

Key Concepts

All curriculum outcomes are important; however, not all of them have the same impact on student learning. Some curriculum outcomes have greater potential to add value to understanding. Such curriculum outcomes can be referred to as *key concepts.*

For a curriculum outcome to be referenced as a key concept, one or more of the following conditions must be met (Ainsworth, 2003):

- **Leverage:** Curriculum outcome(s) that focus on knowledge and skills used in multiple academic disciplines
- **Endurance:** Outcomes and standards that focus on knowledge and skills that will be relevant throughout a student's lifetime
- **Essentiality:** Outcomes and standards that focus on knowledge and skills necessary for students to succeed in the next grade level

Applying a key-concept lens to instruction and learning does not, and should not, preclude the instruction and assessment of other curriculum outcomes. All curriculum outcomes are important.

In short, key concepts are curriculum outcomes that allow other outcomes to make sense and be learned more effectively (Ainsworth, 2003). Key concepts form the basis for further learning, whether the learning takes place in the current school year or in later years.

To provide structure to many supports provided by my website, I have used key concepts as the content focus. The instructional supports that I share here address key concepts. I know that by doing such, teachers will see significant value added in terms of student achievement. It is about working smarter, not harder.

Instructional Supports

The focus of the website is to provide educators with support in developing and facilitating meaningful mathematics instruction that moves student learning forward. Three such instructional supports are *Problem Toolbox, Problem of the Week,* and *Resources.*

Problem Toolbox

It is important that students experience a variety of problems, and that these problems are offered to students throughout the school year. When students are offered a variety of problems, the teacher can accommodate multiple problem-solving scenarios and can address the vast learning needs of their students. The visual in Figure 1 provides the range of problems offered with the Problem Toolbox found on my website.

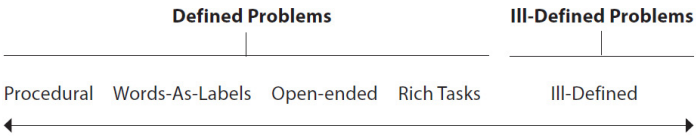


Figure 1: *The range of problems (Source: Costello, 2022)*

Defined problems and ill-defined problems are two categories of problems that are referenced within the Ontario Math Curriculum (Ontario Ministry of Education, 2020). I have written about them in past columns, but clarify them here.

Defined problems are problems that have a clear structure, present readers with obstacles, and have a single or limited range of possible solutions. These are the

problems that we have tended to see in many math classrooms in the past. Within defined problems, there are four types. *Procedural problems* are problems where the strategy has already been identified. Students are required to apply a series of steps accurately to find the solution. *Words-as-Labels* problems, often referred to as word problems, provide a context whereby students must apply a mathematical concept. *Open-ended* problems are problems with multiple points of entry and multiple options that can be considered in order to arrive at a solution. *Rich tasks* are similar to open-ended problems, with the only difference being that rich tasks are presented within a realistic context.

Ill-defined problems have little to no structure. Ill-defined problems can take on many forms and can look quite different from one to the other. And, perhaps most importantly, context is not an add-on. Instead, the context of the problem is as important as the mathematics used to solve it. In addition to the importance of context, ill-defined problems also have little to no structure.

A continuum of problems is provided within the visual above. By offering students both categories of problems, instruction and learning are based on the full continuum of problems available. And, in doing so, students are provided with a variety of experiences that strengthen their mathematical understanding.

Kindergarten	Primary	Elementary
• Compare Numbers • Compare Objects • Count a Collection • Number Sequence • Partition Numbers • Represent Numbers • Repeating Patterns • Subitize	• Addition and Subtraction • Collect, Organize, and Analyze Data • Count Collections • Estimate Quantities • Fractions • Identify and Describe Patterns • Measurement • Money Concepts • Multiplication and Divisions • Whole Numbers	• Addition and Subtraction (Decimals) • Addition and Subtraction (Whole Numbers) • Collect, Organize, and Analyze Data • Cost of Transactions • Decimals • Fractions • Integers • Measurement • Measure Angles • Multiplication and Division (Decimals) • Multiplication and Division (Whole Numbers) • Solve Equations • Whole Numbers

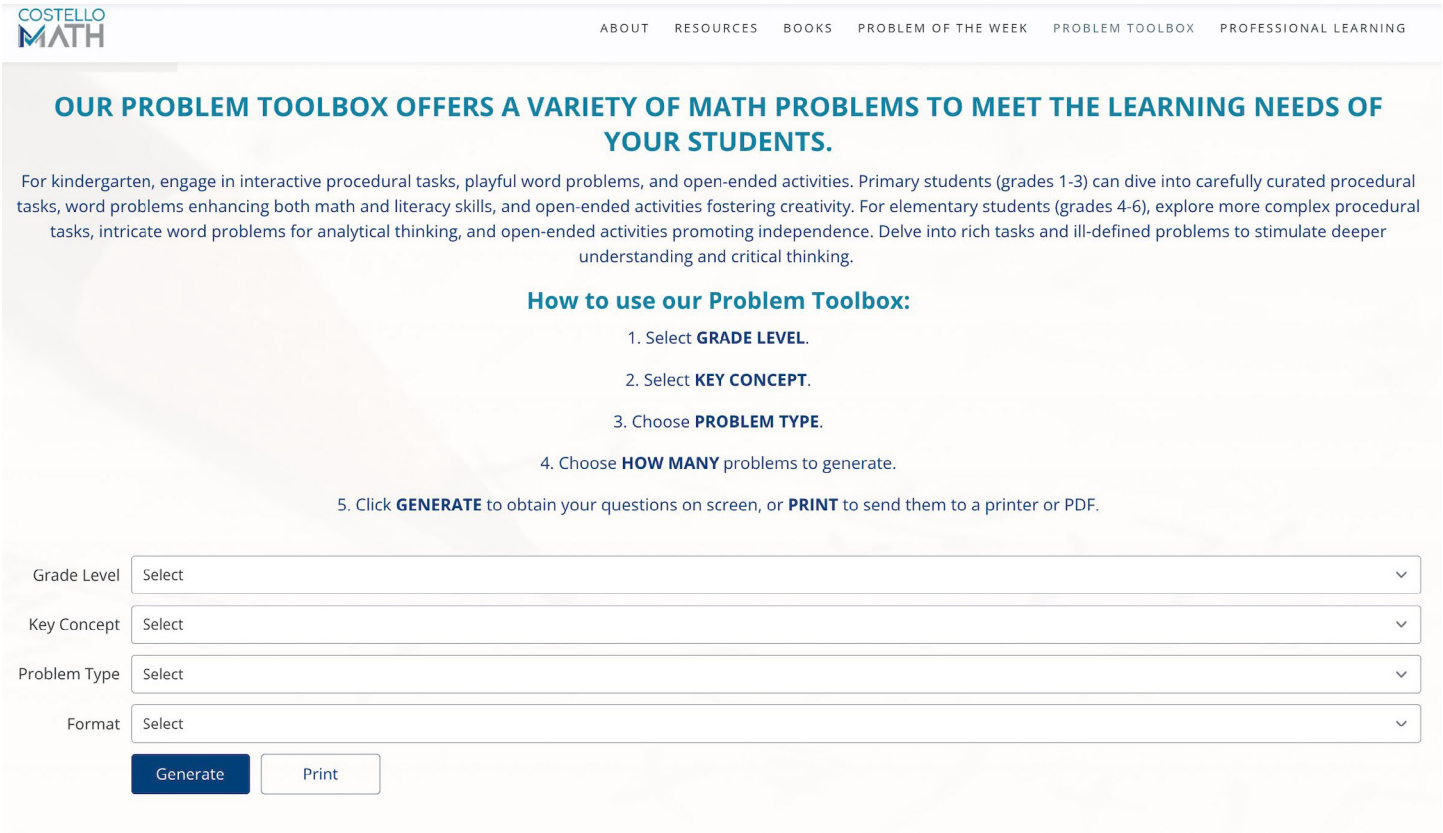


Figure 2: *The Problem Toolbox*

The Problem Toolbox appears as shown in Figure 2. To use the Problem Toolbox, the educator must select the grade levels that they are considering. Currently, there are three options: Kindergarten, Primary (1–3), and Junior (called Elementary) (4–6). Please note that Intermediate (7–8) will be added in the near future.

From there, the educator must select the key concept that they are exploring in the classroom. These key concepts are reflected within the math curriculum.

Next, the educator selects the type of problem.

Problem Type
• Procedural Problem
• Word Problem
• Open-Ended Problem
• Rich Task
• Ill-Defined Problem

Finally, the educator needs to select the format. The format allows them to choose either Teaching Prompt (one question) or Independent Practice (five questions).

Overall, there are approximately 2900 problems within the Problem Toolbox. This number will increase as I continue to add more problems and as Intermediate tasks (7–8) are included in the Toolbox.

Problem of the Week

It is important to provide students with problem-solving scenarios. As such, I have developed a Problem of the Week section on the website. These problems are open-ended so that students can engage from multiple points of entry. This openness is intended to promote opportunities for mathematical discourse within the classroom.

The Problem of the Week problems are connected to the curriculum strands. In particular, problems align with the Number, Algebra, Data, Spatial Sense, and Financial Literacy strands. Within the problems, students have opportunities to apply social–emotional learning skills and mathematical processes.

Each problem addresses a key concept. Then, I craft a prompt that is grade-level appropriate for the concept. There are three prompts within each Problem of the Week: Primary (K–3), Elementary (4–6), and Intermediate (7–8).

The following is a sample Problem of the Week.

Be mindful that during these prompts, it is important the teacher listens to student conversations and asks questions that illicit student thinking.

Primary	There is an expression on both sides of the equal sign. Each number is made with different digits from 0–9. One side of the equal sign has both an addition and subtraction sign, while the other side only has one sign. What could the equation be?
Extension	The digits 0–9 are all used only one time. Different number of signs on either side of the equal sign.

Elementary	There is an expression on both sides of the equal sign. There are both whole numbers and decimal numbers. One side of the equal sign has both multiplication and subtraction, while the other side only has one operation. What could the equation be?
Extension	Each of the digits can only be used up to two times. Different number of signs on either side of the equal sign.

Intermediate	There is an expression on both sides of the equal sign. There are both proper and improper fractions. One side of the equal sign has both multiplication and subtraction, while the other side only has one operation. What could the equation be?
Extension	Each digit can only be used one time. Different number of signs on either side of the equal sign.

Resources

There are many instructional resources within the Resources section of the website. These resources can be used across different grade levels and as supplements to many different instructional models and math programs. These are resources that I have implemented in classrooms across Canada and the United States, where I have seen them be effective across grade levels. Resources range from interactive slides to exit tickets to daily cumulative review.

One of the most visited and downloaded sections of the Resource section is Daily Cumulative Review post. Daily Cumulative Review targets *key concepts*. Daily Cumulative Review is a daily mathematical routine (often used as a warm-up activity) that provides students with opportunities for recalling key concepts throughout the school year. The more that students recall previously learned information from long-term memory, the easier it will be for them to recall this information.

Daily Cumulative Review consists of two parts:

- Students work *individually* to solve three or four questions over a four- to five-minute period.
- The teacher takes four to five minutes to model the solution for each question. The teacher will think aloud during this modelling.

Within the Daily Cumulative Review resource, I provide questions for Grades 3–8 (for both Ontario math curriculum

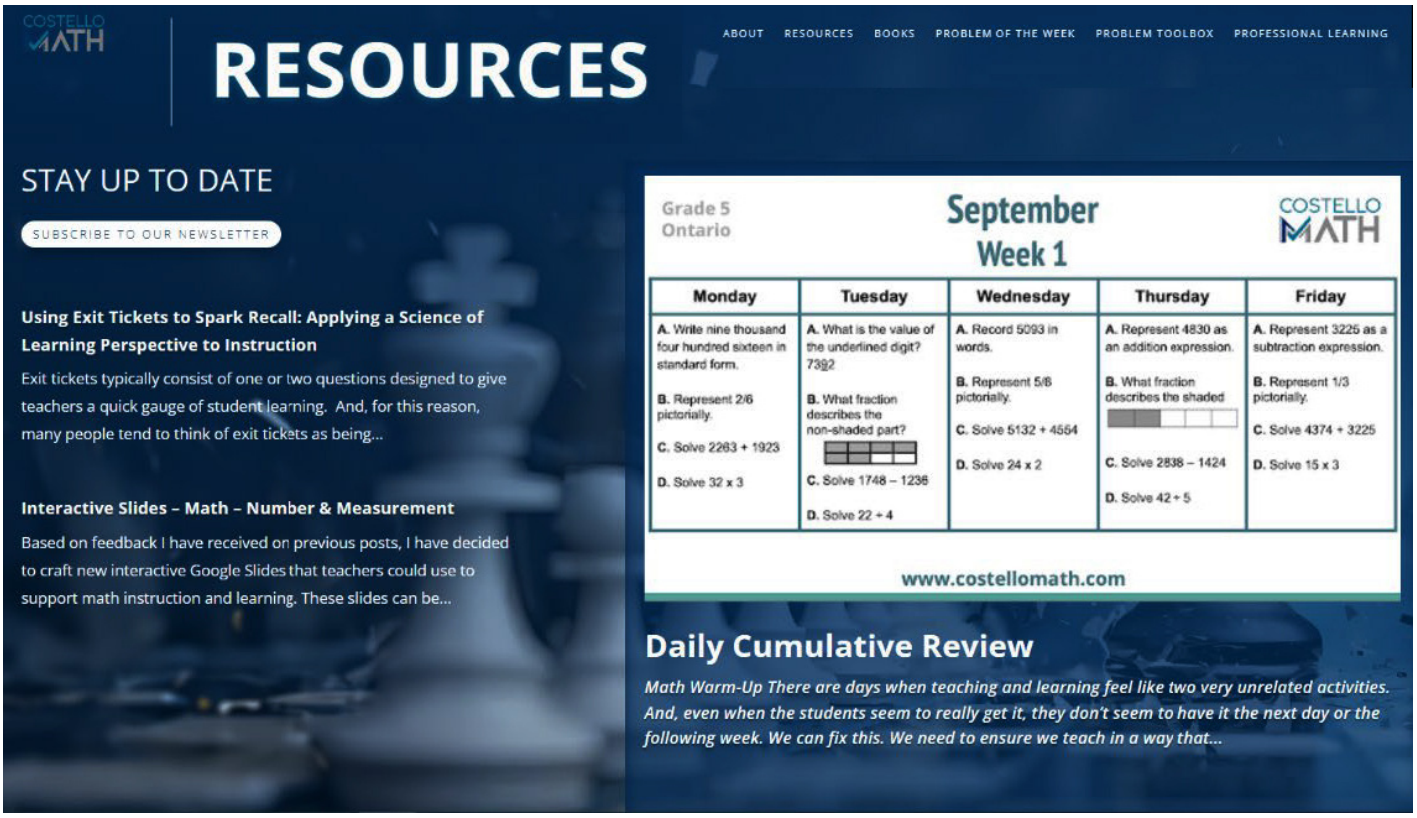


Figure 2: *The Problem Toolbox*

and Canadian math curricula generally). The intent is to update this throughout the year so that educators have access to questions for each day of the school year.

Figure 3 shows a weekly example from the Daily Cumulative Review Resource for Grade 5.

Grade 5
Ontario

October
Week 2

COSTELLO MATH

Monday	Tuesday	Wednesday	Thursday	Friday
A. Arrange the following numbers in descending order: 1/5 2/3 4/8	**A.** Use <, > or = to compare the following fractions: 7/8 and 1/2	**A.** Arrange the following numbers in ascending order: 4/5 0/1 2/4	**A.** Use <, > or = to compare the following fractions: 6/8 and 6/8	**A.** Arrange the following numbers in descending order: 5/6 2/5 1/10
B. Solve 51.4 – 5.38	**B.** Solve 43.6 + 31.55	**B.** Solve 93 – 2.42	**B.** Solve 677.9 + 12.4	**B.** Solve 8.00 – 1.02
C. Solve 85 ÷ 4	**C.** Solve 60 x 7	**C.** Solve 95 ÷ 5	**C.** Solve 46 x 3	**C.** Solve 52 ÷ 2
D. Record 80.01 in words.	**D.** Represent 62.4 as a subtraction expression.	**D.** Write 7.030 in expanded form.	**D.** Write three hundred ninety-eight and two tenths in standard form.	**D.** What is the value of the underlined digit? 23.470

www.costellomath.com

Figure 3: *Example of the Daily Cumulative Review*

Final Thoughts

Educators spend countless hours searching for math resources—to support whole-group instruction, as well as independent student practice. My website is a one-stop resource for math educators.

Over the years, I have had the pleasure of digging into mathematics, from both content and pedagogy perspectives. It was during this experience that I identified key concepts and instructional resources that make a difference for both teachers and students. I want to share these, and I am using www.costellomath.com to bring them to the classroom.

What I have posted, and what I will continue to post, on the website can be used at any point throughout the math lesson. It can be a warm-up, a mini-lesson, the during portion, or a consolidation task. You can use it to introduce a mathematical concept, to strengthen understanding of the concept, and to recall previous learning.

The examples I have shared in this column are a sample of the many items posted on the website. My goal is that you, as a math teacher, can use this website to support your instructional practices. I know that we cannot work any harder. I want to support us in working smarter.

I look forward to continuing the discussion of mathematical learning and instruction via my website (www.costellomath.com), email (david@costellomath.com), or X (@dr_costello).

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In the Middle: Math Needs Some Space



CARLY ZINIUK
carlyziniuk@gmail.com

Carly Ziniuk teaches Grade 9 Mathematics, Grade 12 Data Management and Advanced Placement Statistics at the Bishop Strachan School in Toronto. She is very active in adopting real-life data to engage her students in solving problems. Among 30 global STEM leaders, Carly is a member of the NASA Space Apps Collective for 2023–2024. As a Canadian educator, she is contributing toward the pilot program's chosen theme: Building an Inclusive Global Space Community.

Author's Note: I have used the guidelines as presented by NASA's *Technical Documentation Style Guide* throughout this piece, when referring to the Moon, the Sun, and Earth/earth. The NASA Style Guide capitalizes Earth, when it is used as the name of the planet. In this case, it is not preceded by "the", since one would not comparably say "the Mars." If referring to ground or soil, but not the planet, then "earth" is lowercase. Capitalization of "Sun" and "Moon" refers specifically to our Sun and our Moon (but lowercase, for example, with "Jupiter's moons" or the moon Titan). Use of "the" with Sun and Moon is appropriate for emphasis or clarity.

Eighteen-year-old Morgan believes, "Space proves there's a reason to do all this learning." Astronomy is having a moment with young people like my student Morgan: the total solar eclipse (see Figure 1) had #beautifulmoon trending with North American teens, and then just a month later, the breathtaking Northern Lights (Aurora Borealis) were all over TikTok.



Figure 1: *Young people enjoying the darkness during April 2024's total solar eclipse*

Over the last calendar year, I have spent a lot of time thinking about the connection between space and mathematics. Space exploration fuels learners' curiosity and imagination in a way that many math teachers want to reignite in their students. The connection between mathematics and astronomy has a legacy that continues with Earth and Space Science applications appropriate for K–12 learners. The Canadian Space Agency (CSA) believes, "Role models like astronauts inspire students to dream, and consequently, achieve their goals."

The major project I created as part of my year in the NASA Space Apps Collective provides an astronomical abundance of free, easily accessible space and climate resources for classroom teachers (bit.ly/MathNeedsSpaceZiniuk). Working as the only Canadian educator in this NASA-supported 30-member cohort of STEM professionals inspired me to create this curation project: a series of curriculum-organized Padlets with the NASA/CSA education materials appropriate for math learning.

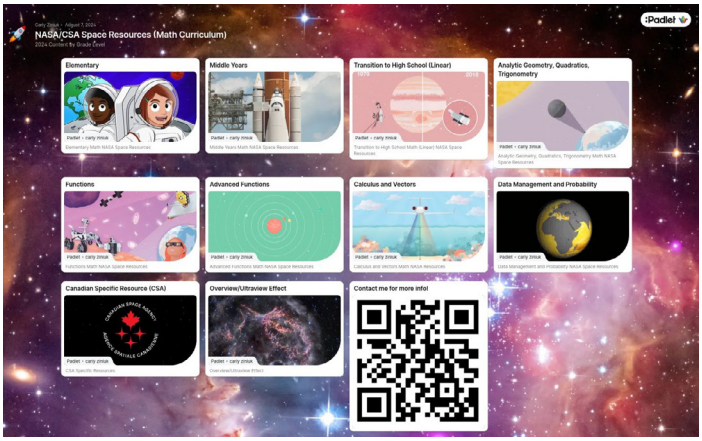


Figure 2: *NASA/CSA Space Resources (Math Curriculum) Padlet Curation Project*

Each tile connects to NASA activities, projects, and problems at different levels, with engaging materials for all areas of K–12 Math: lessons and activities appropriate for classroom learning tomorrow. Also included are problems and questions to practise math learning in a different context.

Beyond social media's astronomy excitement, my students, including Morgan, were inspired by incorporating satellite data to draw mathematical conclusions. Predicting whale movement changed how Morgan experienced math: "It's a way to take it beyond numbers, into my own life. Applications of space make me understand why I would even want to do (math)."

Leka Hattori, who worked with me as part of the NASA Space Apps Collective, has been simulating space missions on Earth. She is hoping that the teachers she works with in Brazil can also delight in space's beauty to motivate student

learning. She is captivated by a post that Mission Little Prince, set to launch in 2027, plans to grow roses on the moon. “I just saw that the Little Prince [mission] will be bringing flowers to the moon.” The simulated space missions have been focused on self-sustainable capsules, where “analog astronauts” wear spacewalk suits, recycle their own waste, and produce their own food in greenhouse modules. Similar capsules could host roses on the moon.



Figure 3: *Mission Little Prince’s model of a rose-growing capsule on the Moon*

I have expanded this curation project so that parents (including guardians) and students can use this to launch their mathematical thinking outside of their math classrooms. Working with the Collective to first filter and then choose the most appropriate resources during this past year, I can now share these more broadly. Any teacher, parent, or young person can find classroom-appropriate, gap-closing practice materials, lesson plans, and more, to illustrate math concepts and explain astronomy terms for all students.

Each tile on the previous Padlet opens a new Padlet, arranged by the curricula strands. Many activities include answer keys and assessment materials, which can help to close gaps in skills with engaging content.

The NASA Space Apps Collective was launched to explain space’s beauty in the daily lives of young people. Hattori agrees with a focus on providing resources for teachers: “How do we connect space into the daily lives of everyday people? If teachers don’t know about any of these things, how can the kids know?” The benefits have been evident in my classes, where one of my students, Addie, noted in her end-of-year reflection, “We worked with real climate change data from satellites to learn about normal distributions. I wish all students could learn math like this.”

“Students don’t know that math really goes beyond high school,” related Addie who, like Morgan, used the NASA curation project materials.

Teachers talk about what the names of the planets are and stuff, but not how it impacts us. I think children are really

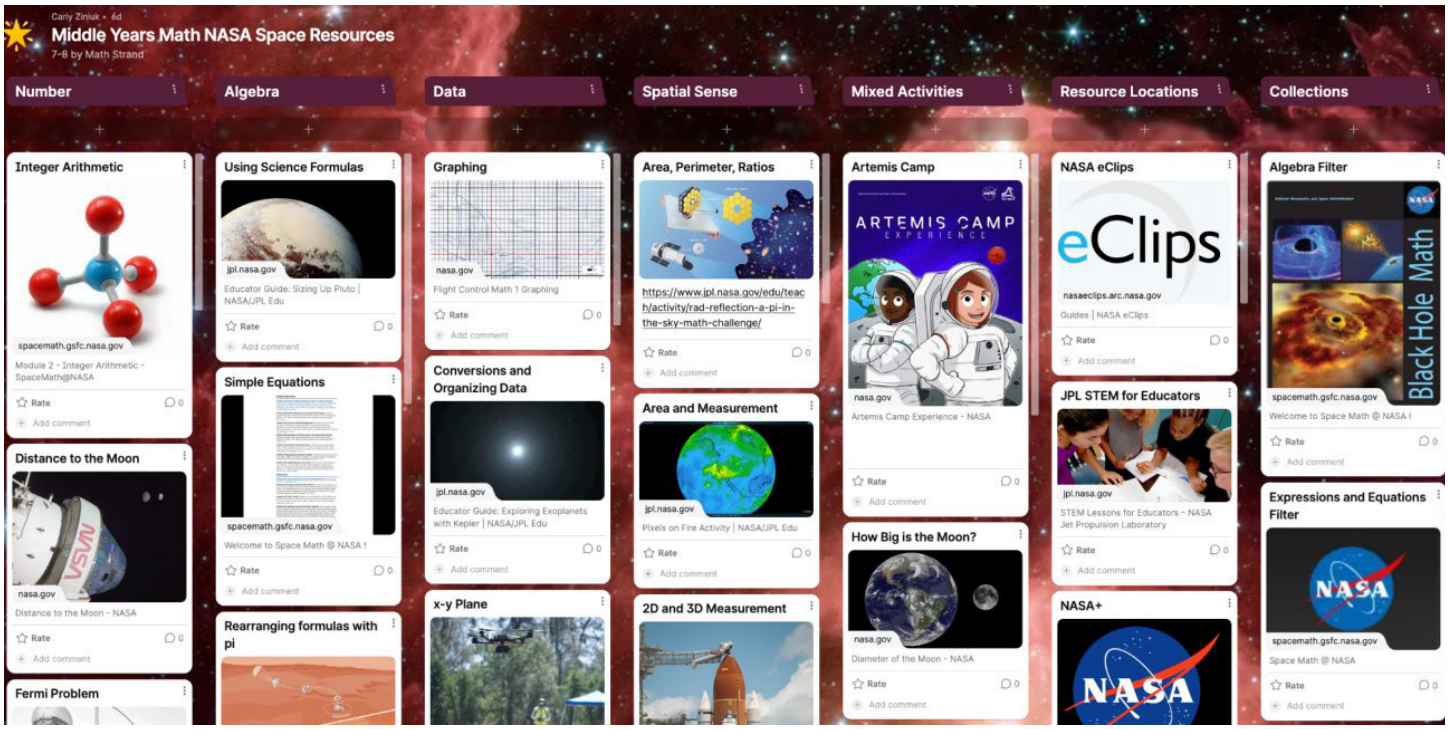


Figure 4: *Padlet with the middle years content organized by strand*

motivated about space, but there's no resources to connect it to their learning, especially math. Until this class, I didn't know there was much math in this.

You think about planets and stars and stuff, but you don't even think about how space is helping us in this world, such as how satellites are tracking the whales. I really like math, but I didn't know how I really could use it until this.

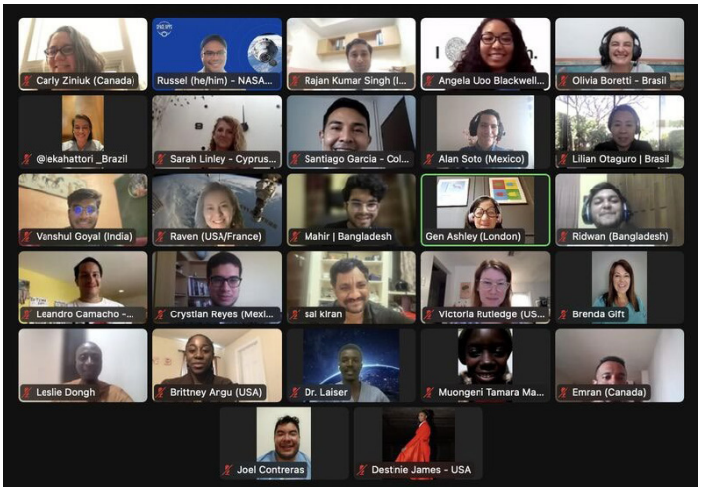


Figure 5: *NASA Space Apps Collective Cohort first meeting in August 2023*

Hattori has explained to Brazilian teachers how NASA monitors ocean currents to help big ships along the coast unload faster.

> Together we talk to the teachers to get them to see how to introduce space into their own subjects, into their own classrooms, and connect to the lives of their students. How would Uber exist if there's no space technologies?

Hattori laughs. Uber uses geospatial data to match each user with the closest driver who is prepared to take a new rider, which harkens to shortest distance problems (such as the travelling salesman problem) often shown in math classes.

Many Collective members, including Hattori, led local events in the world's largest *Hackathon*. They mentored over 8000 teams of young people to use NASA open data to solve authentic problems in a single October weekend. One of eleven 2023 global winners, "Team Arcobaleno" from Brazil, created a method to convert NASA images into sounds, allowing people with sight impairments to explore the cosmos. This project can reveal to your students how scientific data collected from space as digital signals are commonly turned into visual imagery. The conversion of imagery to sound, or sonification, provides a purpose for working with the data. Similar activities with Scratch coding translate visual data into astronomical music! In Figure 6, sounds represent the position and brightness of the sources, while objects' light at the top of the image are heard as higher pitches, and the intensity of the light controls the volume.

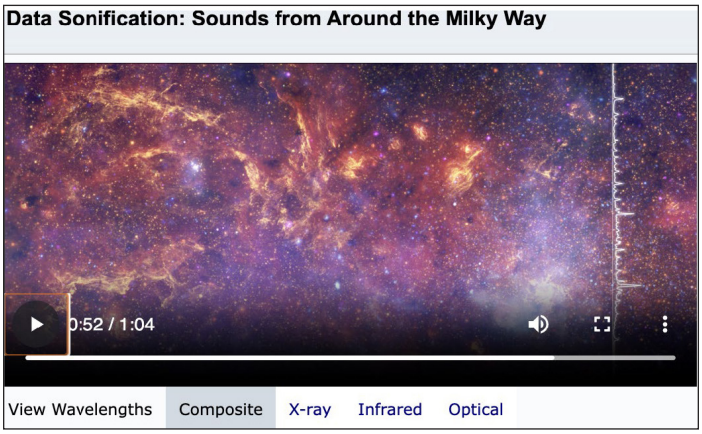


Figure 6: *NASA's sonification project, with* Sounds from the Milky Way *showing all wavelengths in composite*

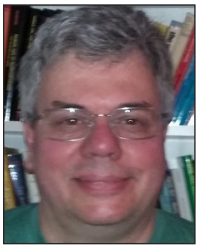
Leka Hattori also maintains that these potential collaborations expand space's inspiration. "When we work together, we build some very good stories, [which is] motivating for more learning. Learning about space answers the questions of why, the story of our heart." The International Astronomical Union asserts that astronomy, "is a tool for improving people's mental health and wellbeing." While Morgan was barely a teenager when pandemic disruptions began, Morgan found confidence with the "space math." "Other applications are scary: the pandemic, climate change, there's fear in math. Space is a topic where it is so much less stressful. It's hopeful."

Morgan notes that the Collective's work sparked them to learn more beyond the once-in-a-lifetime eclipse. Addie grins when she's asked about one thing she's learned about space from the work of the Collective: "Small introduction to the big possibilities that space has will encourage kids to learn more." By uniting STEM leaders worldwide with diverse perspectives, the Collective seeks to foster connections and encourage the exchange of lived experiences with a curiosity to explore Earth and space. Using the materials in this curation project (bit.ly/MathNeedsSpaceZiniuk) is one way math teachers can expand the universe for students, since math truly needs some space. ▲

Which Frieze is this? (Punta Delgada, Azores)



▲ What's the Problem? K.I.S.S.



SHAWN GODIN
GodinMathStuff@gmail.com

Shawn is a former mathematics teacher, department head, and consultant. He strongly believes in the central role of problem solving in the mathematics classroom. He continues to be involved in mathematical activities: presenting workshops, writing articles, working on local projects, and helping create mathematics contests.

Welcome back, problem solvers. I hope your school year is off to a good start. We continue this volume of "problems from the readers" with one from none other than *Gazette* Editor, Tim Sibbald. Tim sent me several problems to consider, one of which is MENTAL BLOCKS, from the right of Figure 1. This problem came from Tim's desk calendar from roughly 20 years ago.

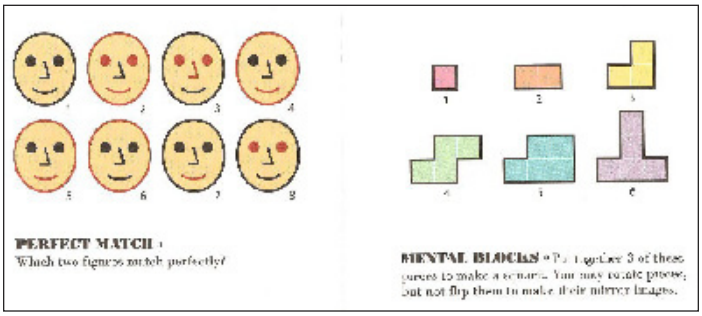


Figure 1: *Two puzzles from a desk calendar*

Finding problems that engage your students can be difficult. You want something with a low floor so that all students can *engage* with the problem. However, you would also like something with a high ceiling so that there are also *challenges* for everyone. This can be tough. In many cases, when you find something with a low floor, the ceiling is on the floor and the problem is trivial. Similarly, finding a problem with a high ceiling often raises the floor and, as such, excludes a portion of your class. Finding that balance can seem impossible. There is an adage that comes in handy here – keep it simple…. When you find a cute problem, play with it. Be aware of any observations or internal questions you ask yourself. In many cases, there will be some natural things that can make the problem easier or harder. There may also be some questions that one may ask, that can lead the problem in different directions. If you play with something and find these multiple branches, you probably have a problem worthy of exploration. If not, it might not be worth your time. Spending time trying to add to a problem that doesn't want to be added to can lead to something that feels contrived and lacking the richness you were trying to add.

As I mentioned earlier, the problems in Figure 1 come from Editor Tim Sibbald. I have known Tim for many years and greatly respect him and his opinions. For that reason, I will ignore his suggestion (for the moment) and focus on *PERFECT MATCH* from Figure 1.

At first glance, it might not seem like there is much going on in the problem. Which two are the same? Easy-peasy, 4 and 5, and we are done! This may seem like the very definition of a closed task. There is one answer. You just need to check things to find it. It may seem like there is only one way to do the problem, but there are a few subtle variations in methods you could use. One could:

- glance at them all, looking for the two that look the same to "jump out" at you
- systematically check 1 and 2, 1 and 3, 1 and 4, etc., until a match is made
- focus on features one at a time and eliminate things

Let's look at the third suggestion first. Each "feature" of the face comes in one of two colours: red or black. If we focus on the outer circle, the face, we get two groups: 1, 3, 7, 8 have a black face, and 2, 4, 5, 6 have a red face. So, any match must come from within these groups. Next, we look at the eyes. Image 1 has a black face and eyes, and so does image 7. However, image 1 has a black mouth, and image 7 has a red mouth. As such, both images can be eliminated. If we systematically check the pairs, as in the second approach in the list, we would have to make seven comparisons before we could eliminate image 1. The nice thing here is that after a couple of checks, we have eliminated two images! On top of that, the remaining two images, 3 and 8, can be quickly compared and eliminated as well. Continuing, we will quickly find our culprits, 4 and 5.

There is always value in discussing methods of solving problems. Many students will look at each other and say, "I got 4 and 5," and the other will say, "Me too!," and be done with it. However, they may have each approached the problem in different ways and would find the other's solution enlightening. Always try to engage that dialogue between students in small-group settings, or based on some eavesdropping, with the whole class.

Personally, I made a table of features like the one in Figure 2 and found the solution that way.

Image	Face	Eyes	Nose	Mouth
1	black	black	black	black
2	red	red	black	red
3	black	red	red	black
4	red	black	black	red
5	red	black	black	red
6	red	black	black	black
7	black	black	black	red
8	black	red	black	black

Figure 2: *Table of image features*

In retrospect, the solution I discussed earlier would have probably solved the problem more quickly for me. However, focusing on the features allowed my mind to go in several different directions. First, since there are two choices for each of the four features, what happens if we "translate" the problem to binary? That is, using 1 for black and 0 for red, image 1 would be $1111_2 = 1 + 2 + 4 + 8 = 15_{10}$. Similarly, if we code every image in binary, and then convert to decimal, we would get 15, 2, 9, 6, 6, 7, 14, 11, which makes the answer obvious.

In what other directions could we go with this? Using the same set of images, we could instead ask: Which images are the *least alike*? Binary comes in handy here as well. The least alike pair of images could occur when each feature is coloured differently. In terms of binary, that means where one figure has a 1, the other has a 0, and vice versa. What that means is that if we add the two numbers together, in binary, each pair of digits will be a 0 for one image and a 1 for the other, which add to 1. Thus, these two numbers would add to $1111_2 = 15_{10}$. The only two numbers from our list that add to 15 are 9 (image 3) and 6 (images 4 and 5). Therefore, image 3 is exactly opposite of images 4 and 5.

What else is possible? An obvious question comes to mind: How many faces are possible? Since we have four features—face, eyes, nose, mouth—with two choices for colour each—red, black—then the number of possible faces is $2 \times 2 \times 2 \times 2 = 2^4 = 16$. Generalizing the situation so our face has f features, and we are using c different colours, we would have c^f possible faces.

That doesn't give us much to work with, but if we wanted to create a similar puzzle, we have a boundary; that is, if I use more than 16 faces, using the constraints of the problem, there must be at least two that match. This is an example of the *pigeonhole principle* (or the Dirichlet principle). Hence, we can create a problem with at most 17 faces and have at least one pair match.

Let's dig a little deeper into some of the things that are swimming around under the surface. To make life a bit easier, we will eliminate the nose, so we are only dealing with three features: face, eyes, mouth. Staying with two colours, red and black, we have $2^3 = 8$ possible faces. All possibilities are shown in Figure 3. Take a minute before reading on, and see if you notice anything about the way the faces have been arranged in the figure.

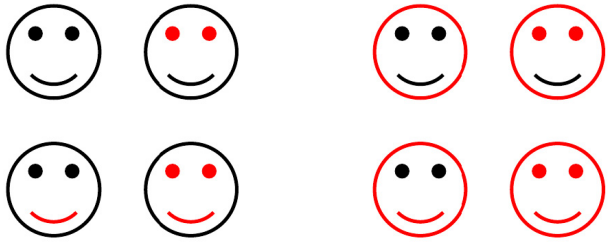


Figure 3: *Faces with three features and two colours*

You may have noticed that the faces are arranged in two groups of four. The group on the left has black faces, while the ones on the right are red. Looking closer within each group of four, you see the two faces on the left have black eyes, while the two on the right have red eyes. Similarly, the mouths of the faces on the top row are black, and the ones on the bottom are red.

Going one step further, if we connect either of the groups of four into a square, the edges of that square will lead us from one face to another so that the two faces only differ by one feature. Pretty cool! Unfortunately, the ones from different squares that are adjacent differ in two ways. However, with a little insight, we can come up with the arrangement in Figure 4.

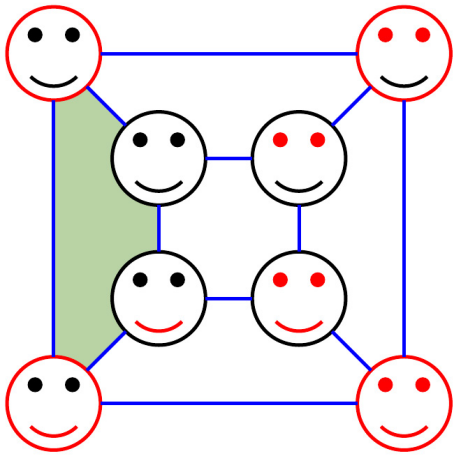


Figure 4: *Graph of faces*

Travelling from one face to another, following a blue edge, leads us to a face that differs by just one feature. Moreover, if we move left/right, the eyes change colour; moving up/down changes the colour of the mouth; and moving inner/outer changes the colour of the face. The

arrangement of the faces is something that is called a *graph*, with the faces being at the *vertices* and the blue line segments marking the edges. Furthermore, this is also isomorphic (i.e., corresponding or similar in form or relationships) with the graph for the vertices of a cube. Examining Figure 4, you should see a common representation of a cube (specifically a cubical box, where we are "looking in"). The shading in the figure represents one of the faces (of the cube). For our representation of the faces (with three features), the shaded face (of the cube) corresponds to the faces with black eyes. Further observations will reveal that each of the faces (of the cube) corresponds to four faces (with three features) that have one feature in common.

We can define the *distance* between two faces as the number of features that differ between them. In Figure 5, starting with the face with all black features and changing things one at a time, we can see that each face is at most a distance of 3 from any other face (the maximum distance occurring by changing all of the features). This can be verified by going back to the graph in Figure 4, or by labelling the vertices of a cube with faces and travelling along its edges. It is interesting to note that if we use Figure 4 to label the vertices of a cube without eight faces, then:

- pairs of faces that are a distance of 3 apart are on "opposite" corners, that is, separated by a space diagonal
- pairs of faces that are a distance of 2 apart are separated by a diagonal of one of the cube's faces
- pairs of faces that are a distance of 1 apart are on the same edge

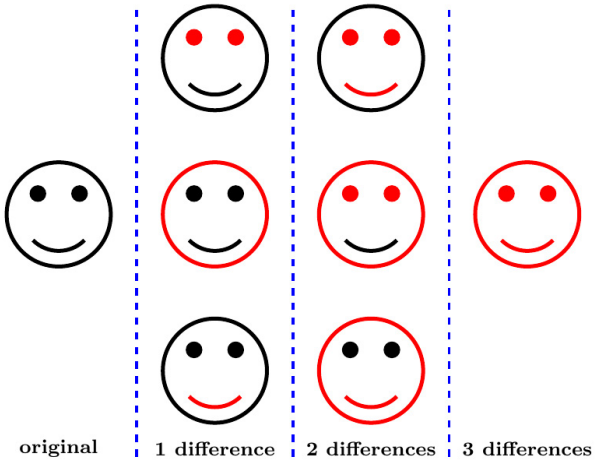


Figure 5: *Distance between faces*

Another interesting thing is happening in Figure 5. If we consider that the original is a distance of 0 from itself, then the number of faces a distance of 0, 1, 2, 3 from the original would be 1, 3, 3, 1. Do these numbers look familiar? If we went through the process for the simpler case with only two features, we would find the number of faces a distance of 0, 1, 2 from any particular face would be 1, 2, 1. These are the entries in Pascal's triangle, shown in Figure 6.

$$
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 1 & & 1 & & \\
 & 1 & & 2 & & 1 & \\
1 & & 3 & & 3 & & 1 \\
1 & & 4 & & 6 & & 4 & & 1
\end{array}
$$

Figure 6: *First few rows of Pascal's triangle*

One of the things known about Pascal's triangle is that its entries represent the number of combinations of choosing a specific number of items from a group. Thus, the entries 1, 3, 3, 1 are the number of ways to choose 0, 1, 2, 3 items from a set of 3 items. Why does this make sense for our investigation? The second 3 in the list 1, 3, **3**, 1 corresponds to the number of ways we can we pick two of three features to change: symbolically, $\binom{3}{2}$ (read 3 choose 2), that is: eyes and mouth, eyes and face, or face and mouth (see Figure 5).

There is one more related tidbit waiting for us. If you look at each row of Pascal's triangle, you see that the entries add up to a power of 2, that is:

$$
\begin{aligned}
1 &= 1 = 2^0 \\
1+1 &= 2 = 2^1 \\
1+2+1 &= 4 = 2^2 \\
1+3+3+1 &= 8 = 2^3 \\
1+4+6+4+1 &= 16 = 2^4
\end{aligned}
$$

This tells us that the number of subsets of a set with n elements is 2^n. How does this relate to our investigation? We can think of it as a set of three features: face, eyes, mouth. Our subsets will correspond to the features that are black on our face; that is, the subset {face, mouth} corresponds to a face with black face and mouth (and thus red eyes). The Venn diagram in Figure 7 shows all our faces for three features and how they lie in the set's black face, black eyes, black mouth.

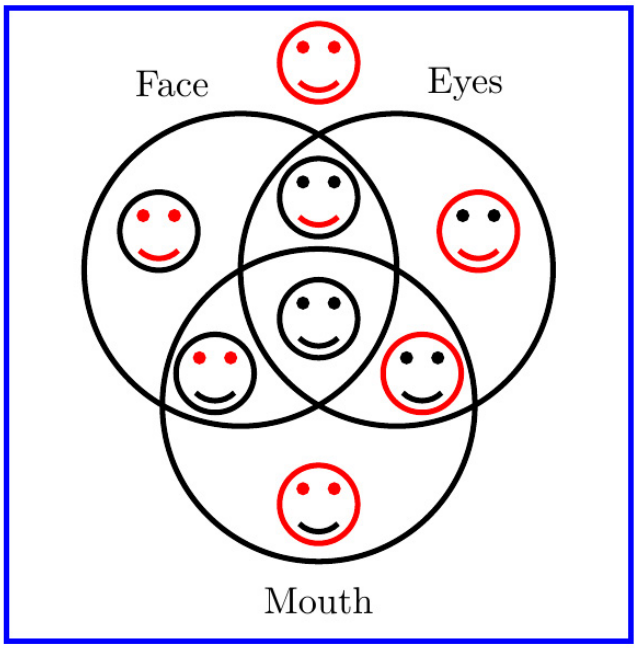


Figure 7: *Venn diagram of facial features' colouring*

We started with a seemingly simple closed problem and have found patterns, used many different representations, and found connections to different areas of mathematics. There are more that came to my mind, but I am sure the readers, including the Editor, are not interested in a whole issue of the *Gazette* dedicated to this problem (although I would have fun). Interested readers may enjoy seeing how the patterns we found for the three featured faces hold for the situation of the original problem. Some other ideas you may like to explore:

- Represent the different faces as branches on a tree diagram.
- Represent the different faces as ordered triples for the simplified case, to see the connection to the vertices of a cube.
- Create a full set of faces on individual cards and make up various games that could be played using these cards. (If three or more colours were used, you could play a variation of the game "set.")

You are only limited by your imagination.

Allow yourself to think beyond any problem you encounter. Play with different methods of solving, as well as keeping yourself open to your own internal questions. The more you do this, the easier it will become. The downside is that you may become more like me, who is very easily distracted by any little problem.

Until next time, happy problem solving. ▲

▲ Useful Theories to Know About: Expectancy Value Theory



JEFF IRVINE
dr.jeffrey.irvine@bell.net

Jeff Irvine, Ph.D., has been a secondary mathematics teacher, department head, and vice-principal. He has taught at three faculties of education and at a community college. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. With over 45 years in education, Jeff is particularly interested in the interplay of student motivation and mathematics achievement.

Why do some students consistently attempt every homework question, while others rarely, if ever, even attempt any? How does a student decide whether to engage with an assigned task? Can we, as teachers, take actions that will increase the likelihood that our students actively participate in their own education? Expectancy Value Theory can provide some answers to these questions.

Expectancy Value Theory

Jacquelynne Eccles and Allan Wigfield proposed expectancy value theory before the turn of the century. This theory posits that students' decisions to engage are based on two factors. The first factor is the student's subjective belief that they can be successful at the task. This belief may or may not reflect their real probability of success (which is based on their prior knowledge and experience), their level of confidence in their mathematical ability, their attitude toward mathematics, their risk tolerance level, and other factors.

The second decision criterion is the value that the student places on the task. If the student feels that the task has value for them, they are more likely to engage in the task, and to persist when they encounter obstacles. On the other hand, if the student places a low value on the task, they are less likely to engage at all.

Value can be subdivided into four components. The *intrinsic value* of a task is the amount of interest or enjoyment the student will achieve from doing the task. The *attainment value* is how much the task will impact the

student's identity by completing the task, or how the student will feel if the task is done successfully. Being able to complete the task successfully is valuable to the student's self-esteem. The *utility value* is how useful the student perceives the task to be for them personally. This may be for future plans, for immediate gratification, or for self-esteem. And finally, the *cost* component of a task is the *opportunity cost* (i.e., value) from choosing this task over another task. This cost may be in time commitment, effort, or emotional cost associated with the task. Since everyone has limited resources, committing to this task means foregoing other tasks that may also be useful or enjoyable. All four components combined represent the value of the task to the student.

Thus, the expectancy value theory boils down a myriad of details that go into a student's engagement decision into two factors: the student's expectancy of success, and the value that the student places on the task.

There is a massive amount of research supporting expectancy value theory, which has been researched in many countries, across various education levels, and across many subjects. The theory is particularly important in mathematics, since, for our subject, engagement is critical to successful achievement and understanding.

Why Should We (Math Teachers) Care about This Theory?

With respect to the student's expectancy of success, math teachers can influence expectancy in several ways. We can build students' self-confidence; we can ensure that their prior knowledge is sufficient to tackle the task; we can create tasks that have low floors (every student has an access point to begin the task), high ceilings (students can grow their mathematical understanding beyond the immediate demands of the task), and wide walls (the task is sufficiently broad to allow the student to go down multiple paths and investigate areas of interest to them); we can work on creating a positive attitude in our classrooms that influences student attitudes toward mathematics.

We can affect the value that a student places on a task by creating tasks that are clearly connected to the real world; link to students' interests; and allow students to investigate, solve problems, and pose questions that they can then investigate. If our students see that the work that they are asked to do has value to others and to society, they are more likely to see value in the tasks for themselves. In the same vein, we need to avoid make-work and boring, repetitive tasks that do not appear to be useful. That doesn't mean that we need to avoid assigning tasks that provide support and skill building, just that those tasks need to have sufficient variety and clear applications to real-world situations: by carefully choosing or creating tasks that add value to the students' academic growth and understanding, without resorting to a textbook style list of routine questions.

Increasing both students' expectancy and their perceptions of value derived from tasks requires effort on the part of teachers. However, the impact on student engagement is well worth the effort and will pay off through students' increased engagement in their learning of mathematics, as well as their positive attitudes toward our subject.

Opportunities to Learn More

Jones, B.D. (2021, March 11). *Expectancy-value theory – Motivating others ep. 3* [Video]. YouTube. https://youtu.be/De5U-GE4YkE

Focuses on implications for teachers of expectancy value theory.

Vroom, V.H. (2019, March 21). *Expectancy theory of motivation* [Video]. YouTube. Expert Program Management. https://youtu.be/GIXUg6N-eOs

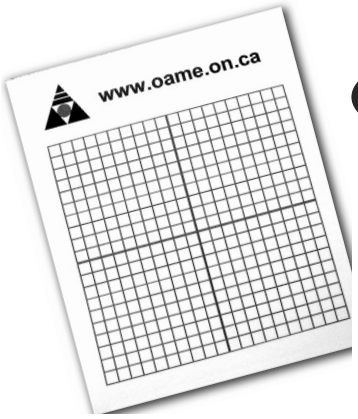
Expectancy value theory in the workplace.

Vroom, V.H. (2023, September 27). *The expectancy-value theory* [Video]. YouTube. Rise Above Life. www.youtube.com/watch?v=Wk4hA5j3dxM

Provides a history of expectancy-value theory and applications in the digital age. ▲

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JENNIFER HOLM
jholm@wlu.ca

Jennifer Holm is an Associate Professor at Wilfrid Laurier University and works with Primary/Junior and Junior/Intermediate pre-service teachers, as well as in the field supporting current mathematics teachers. She is interested in developing mathematics knowledge for teaching with both pre-service and in-service teachers. She focuses on the beliefs and opinions that pre-service teachers hold about mathematics and teaching and the connection they have to past experiences. She uses this research to support future teachers in developing beliefs and knowledge that will encourage and support effective teaching practices in mathematics.

This column builds on the previous column (Holm, 2024), where the focus was on circles. I presented a problem that I have used when teaching about circles (Olson et al., 2014). The problem is, which one of the images shown in Figure 1 has the most dark-coloured space? Explain your answer. As a reminder, each square is the same size, and each circle touches the edges and/or the one next to it. In the previous column, we discussed how the black space would be the same in each square. You may want to review the column to see different ways of reasoning through that problem, if you need more information.

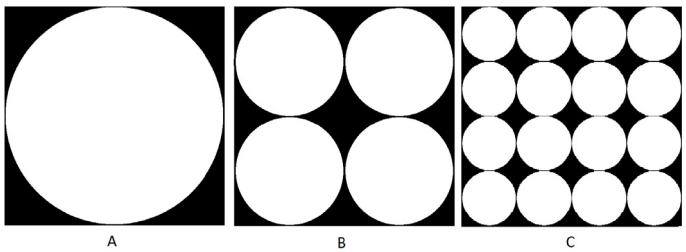


Figure 1: *Circles and squares problem (Olson et al., 2014)*

In this column, we are going to expand this idea to consider what happens in three-dimensional space. Consider the image in Figure 1, and now imagine these circles represent a three-dimensional image inside a cube-shaped box. What figures might your students suggest? What probably comes to mind the quickest are the base of right circular cylinders, the base of cones, and spheres. We will now consider each shape and see if the same pattern from two-dimensional images holds true when we move into three-dimensional space. For consistency in this article, we will consider that the cube-shaped box has a side length of four units for each of the scenarios. This means that the boxes each have a volume of $V = (4 \times 4) \times 4 = 64$ *units*^{3} (since the volume formula for the box is $V = A_{base} \times h$). Please keep in mind that throughout this article, we are using π in the formulas and approximating the value of π when calculating the volumes. The calculated volumes of the right circular cylinders, cones, and spheres will all be approximate values.

Right Circular Cylinders

If we consider that each of the boxes is filled with vases that are right circular cylinders, where the height of the vase reaches from the bottom to the top of the box, we could pose the question, "Which box would need the most sand to fill the boxes between the vases?" If we look at Figure 1, the bases of the vases are represented by the circles, and the black space would represent where the "packing sand" would be. Before you continue, take a moment to consider how you might reason through this problem, and hypothesize which would use the most sand.

To find the approximate volume of a right circular cylinder, we could use the following volume formula: $V = A_{base} \times h$. Since students would have done the previous problem with two-dimensional circles, they would hopefully be able to reason that the amount of sand would be the same in each of the boxes, since the heights would not be changing, only the area of the base, which was the same problem as with the circles. If not, students could always calculate the volume and determine their hypotheses. In scenario A (i.e., left in Figure 1), the diameter of the vase is equal to the side length of the box, which was defined as four units. So, the volume of the vase is $V = A_{base} \times h = \pi 2^2 \times 4 = 16\pi = 50.26544$ *units*^{3}. In scenario B, the diameter of one vase is half the width of the box, or two units, and the volume of the vase would be $V = A_{base} \times h = \pi 1^2 \times 4 = 4\pi = 12.56636$ *units*^{3}. In scenario C, the diameter of one vase would be one unit, and the volume of the vase would be $V = A_{base} \times h = \pi \left(\frac{1}{2}\right)^2 \times 4 = 1\pi = 3.14159$ *units*^{3}.

The relative volumes of the right circular cylinder show that the single vase in B is one-quarter of the volume of the vase in A. Similarly, the vase in C is one-quarter of the volume of B, and one-sixteenth of the volume of A. This shows that when you calculate the total volume taken up by the vases in each box—one vase in A, four vases in B, and sixteen vases in C—they are identical, which means the amount of sand needed to pack the boxes would be the same as well. To make sure we clarify this, we can also

calculate the volumes for each scenario: $V_A \approx 50.26544$ *units*^{3}, $V_B \approx 12.56636$ *units*$^3 \times 4 = 50.26544$ *units*^{3}, and $V_C \approx 3.14159$ *units*$^3 \times 16 = 50.26544$ *units*^{3}. This shows clearly that the same relationship holds for three-dimensional right circular cylinders in a box as it does for the two-dimensional circles in a square.

Cones

An alternative is to consider the image in Figure 1 as showing the bases of bronze cones that are going to be used for decorative fences. Which box would need the most amount of sand to fill the boxes around the cones? Take a moment to reason through the problem and see how you might figure out the answer to the question.

To find the volume of a cone, we use the volume formula: $V = A_{base} \times \frac{h}{3} = \pi r^2 \frac{h}{3}$. We can start by determining the volume of one cone in each of the boxes to see if the same relationship occurs as we have seen with both circles and right circular cylinders. In scenario A, the diameter of the cone is equal to the side length of the box (defined as four units), and the volume of the cone is $V = \pi r^2 \frac{h}{3} \approx 3.14159 \times 2^2 \frac{4}{3} = 16.75515$ *units*^{3}.

In scenario B, the diameter of one cone is two units, and the volume of the cone is $V = \pi r^2 \frac{h}{3} \approx 3.14159 \times 1^2 \frac{4}{3} = 4.18879$ *units*^{3}. In scenario C, the diameter of one cone is one unit, and the volume of the vase would be $V = \pi r^2 \frac{h}{3} \approx 3.14159 \times \frac{1}{2}^2 \frac{4}{3} = 1.04720$ *units*^{3}.

By calculating the volume of a single cone in each box, you can see that the same relationship does hold here as well: the cone in B is one-quarter of the cone in A, and the cone in C is one-quarter of B or one-sixteenth of A. This means that the total volume of all the cones in each box is exactly the same.

Editor note: It may be beneficial, for enrichment and differentiating instruction, to challenge students to consider that scenario B allows one upside-down cone in the middle, and scenario C allows a three-by-three (i.e., nine) arrangement of upside-down cones as well. Does that change the outcome? But realize this detracts from the main theme of this column.

Spheres

Another alternative is if each circle is actually a sphere inside a cube. In scenario A, there is one sphere that fully fits in the box. In scenario B, there are eight spheres (four on the top, but then you would have to consider the depth as well, and a second layer is feasible). In scenario C, there would be 64 spheres (the 16 on the top, but then to consider the depth, it would be four layers deep as well). If we reconsider the problem for this new context, we can now determine the volume of the spheres and consider which of the cubes has the most empty space (represented by the black in the images).

Kajander (2007) presented the same problem in "Gumball Boxes" and asked the question, "Do more, but smaller, things always fill a space better?" (p. 77). In this activity, Kajander posits using gumballs in a box, to help students visualize the activity (or better yet, bring in the actual items to illustrate the scenarios for themselves). Take a moment now to consider this: Do you expect the same results with areas of circles will happen when we consider the total volume of spheres, as well as with total volumes of right circular cylinders and cones? Before reading further, consider the questions for yourself, how you might think about it, and how your students might consider the problem.

Let's consider the three boxes that were presented in Figure 1, to see what happens to the empty space. To find the volume of a sphere, the formula is $V = \frac{4}{3} \times \pi \times r^3$. Now we determine the volume of one sphere in each of the boxes. In scenario A, the sphere fills the entire box, so its diameter would be the same as the side length of our box (defined as four units), making the radius two units: $V = \frac{4}{3} \times \pi \times r^3 \approx \frac{4}{3} \times 3.14159 \times 2^3 = 33.51029$ *units*^{3}. In scenario B, the diameter of two spheres together makes the entire side length, so each sphere has a diameter of two units and a radius of one unit: $V = \frac{4}{3} \times \pi \times r^3 \approx \frac{4}{3} \times 3.14159 \times 1^3 = 4.18879$ *units*^{3}. In scenario C, the diameter of the four spheres would be equal to the side length, so each sphere would have a diameter of one unit and a radius of ½ unit: $V = \frac{4}{3} \times \pi \times r^3 \approx \frac{4}{3} \times 3.14159 \times \frac{1}{2}^3 = 0.52360$ *units*^{3}. Once again, if you consider the volume of individual spheres in each box, all of the volumes are approximately proportionate to the previous one! The volume of B is one-eighth the size of A, and the volume of C is one-eighth the size of B!

For this next step, we calculate the total volume of all the spheres in each box. In A, the total volume is $V \approx 33.51029$ *units*^{3}, since there is only one gumball. In B, the volume is $V \approx 4.18879$ *units*$^3 \times 8 = 33.51032$ *units*^{3}. In C, the volume is $V \approx 0.52360$ *units*$^3 \times 64 = 33.51040$ *units*^{3}. (Note that five decimal-place accuracy requires the zero in the fifth decimal place.) If we had not rounded the amounts or used an approximate for π, then all the calculated volumes would be exactly the same! Even with spheres, the amount of empty space between the balls would not change, even as we used smaller spheres to fill the boxes!

Summary

In this column, we explored three-dimensional shapes to consider relationships with volume and whether having more smaller shapes would fill the space better. What we

discovered, as we worked through each scenario, was that the smaller shapes did not fill the space better!

This is a beneficial activity to work through with your students as an extension from the work in two-dimensions using area, and it gives them opportunity to posit what will happen before performing the calculations. It is worthwhile to have students consider other shapes that may or may not have a similar relationship as was found in this article. Many students (and some pre-service teachers) are surprised by the results, since they may interpret smaller pieces filling more space. Having a conversation about why this does not happen can bring a surprise to mathematics class, but also encourages reasoning and problem solving.

References

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Pattern? Knot!

Look carefully at the pattern in this floor design:



If you were to try to make the curved centre knot, how many pieces of string would you need?

Can your students, in groups, make similar patterns that require different numbers of strings?

Consider the striped pattern on the cathedral shown below:



Students can be challenged to discuss whether the stripes on the cathedral illustrate parallel lines. The argument "against" is that they are not lines. The argument "for" is that regions are parallel line segments and that the lines lay in planes like a stack of cards. Photo: Duomo di Orvieto, Orvieto, Italy

▲ Coding in the Classroom: Why code? We Need Your Voice



IAIN BRODIE
iainbro1.625@gmail.com

Iain has been an elementary school educator for 28 years, and is now into a second career, teaching in-service and pre-service teachers. He teaches face-to-face at Ontario Tech University and virtually at Western University.



HATICE BEYZA SEZER
hsezer2@uwo.ca

Beyza is a former teacher and long-term researcher. She is currently a research associate in education at Western University, with a focus on the integration of computational thinking into mathematics education.

For this issue and the next two, we will not be interviewing coders in the classroom. We have talked to people at all different levels of the education system, but the person we really want to talk to is you, the one who might not have coded in your classroom as much as you would like to. We would really like to talk to some newcomers, who have taken on the challenge to integrate coding into all the other strands of the mathematics curriculum.

To that end, we are going to be offering a guided tour of how we started coding in our classes, so that they could dive into some rich mathematical (and other) topics. In this issue, we are going to start with our favourite set of beginning activities, and how we spin them into some beautiful geometric art, as well as a deep look at three of the most fundamental shapes: the square, the triangle, and the circle. In *Coding for Young Mathematicians* (Gadanidis & Floyd, 2019), the first thing we learn to code is a square. How we get to that is a tried-and-true method for any coding lesson—start unplugged with no computers. Students are asked to come up with a set of instructions that will tell their teacher how to walk in a square. Listening to and following these instructions allows the teacher to model the idea that computers are actually not very smart at all, and that they will only do exactly what you have told them to do. Now, plugging in and introducing the students to Scratch, and

seeing how the blocks work to program the animated cat to walk in a square, is a really fun, and sometimes challenging, task.

When we talked to one of the co-authors, Lisa Floyd, we shared that George Gadanidis had challenged all of us to come up with a new starting activity, and that *walking a square* was getting old. Neither Lisa nor we could come up with anything that had as much breadth and depth. So, let us set out what you can do with this start.

Coding a Square

We learn how to move the cat ahead and turn it 90°, using a series of basic move and turn commands. A natural fit here is to let our students know about the repeat block, and how it functions, because it lets us reduce the overall number of blocks (commands). Also, for good measure, it never hurts to show how to get pen blocks in Scratch, so that the cat can draw our squares.

For those interested, a sequence of instructions is a Grade 1 expectation, and using the repeat function is Grade 3. Technically, it is also more efficient to use a repeat, so there is a case for repeats addressing the efficiency aspect of the Grade 6 expectations.

Congratulations, you and your class have coded your first program. We'd love to hear about your experience—please contact us through the *Gazette*! In our guide, where links are given at the end of this article, we have three options for follow-up coding activities.

Both of our activities start with a challenge: How can you modify your program so that instead of drawing a square, it draws a triangle? What will you need to change in your code for the square in order to accomplish this? This challenge comes with some surprises, and the opportunity for learning more about some properties of 2D shapes, which is all outlined in our guide.

One path from here involves challenging the students to draw other regular polygons, and to code the computer to calculate for us using operator blocks, notably the division block, because who really likes doing long division? In our activities in this option, you can learn various other functions, and improve your coding skills.

The second path is perhaps a bit more complex, with a strong undercurrent of inquiry: Code the different types of triangles. This activity can be done at all levels of mathematics, from early elementary grades to late secondary. It can also be done by mixing methods to generate coding instructions, while students are simultaneously learning what exactly the different triangles are. This exploration we use less often, as we are almost always pressed for time with our students, but it is a wonderful activity that pushes our students to think deeply about triangle geometry, at whatever their grade level.

The third path is from *Coding for Young Mathematicians*. Using the program that drew a square, we demonstrate how to repeat our drawn square with a turn after each square is completed, creating beautiful mandala-like art made with a computer. We now let the students loose with hints about trying out the different pen blocks to see what they might add to the art they have created. They are also using a nested event (creating a square that is nested in the larger construction) to create their art—a Grade 4 math expectation for those keeping track.

During these activities, we also set the norms for our students when they are coding.

1. Knowledge doesn't belong to anyone. We share our knowledge freely and gratefully by helping each other out. You are not alone. Besides, someone using your coding idea is complimenting your genius in coming up with it in the first place.
2. Coding something incorrectly can be fun, and it is definitely a good learning experience. Finding where our code has gone wrong is one of the most effective ways to learn to read and understand code. It is also quick to see the results when we try testing our fixes. This process is called "debugging," and it is one of the most important coding skills.

We're eager to hear from you! Again, if you have tried any of the follow-up activities with your students, we would like to talk with you and learn from your experiences. If you would like to share examples of your students' code, with all due permissions provided, feel free to contact us through the *Gazette*, and we might feature it in an upcoming column. Imagine showcasing real student coding in the classroom—that would be truly exciting!

References

Gadanidis, G., & Floyd, L.A. (2019). *Coding for young mathematicians, using Scratch and Python* (3rd Ed.). WORLDiscoveries.

Our Guide: An outline of these activities is available to download for free in the file resource area in the members-only section of the OAME website. Or at the following link: www.drive.google.com/file/d/1PN6FM1ND-IGfyI_xHNsF-L0-WcqmYxKB/view?usp=sharing

TinyURL: www.tinyurl.com/2249v7jn

QR code: ▲

▲ Musing on Percentages: Commutative %



JÉRÔME PROULX
proulx.jerome@uqam.ca

Jérôme Proulx is a professor of mathematics education (www.profmath.uqam.ca/~jproulx), and runs the Laboratoire Épistémologie et Activité Mathématique (www.leam.uqam.ca). His research focuses on epistemological issues related to cognition and/in mathematics. His current research program is dedicated to mental mathematics and school mathematics content development. (Photo by Nathalie St-Pierre of Université du Québec à Montréal)

Percent is everywhere—all around us in finances, shops, foods, newspapers, surveys, and so on. Percentages may seem like a mathematically rather simple theme, or one with little depth. However, when one starts to pay attention to its usage, percentages can be seen as filled with surprising elements that are mathematically rich and conceptually fascinating, hence worth pondering. This is something that became quite salient in our research team, as we have been preparing experiments with students in classrooms about percentages in mental mathematical environments.

These columns endeavour to raise and address some of these peculiarities, which we came to call at times "curiosities," related to the concept of percentages. These musings can be seen to be as much about our usage of "%" in everyday life, in schools, or in mathematics in general. This second column addresses issues related to the multiplicative structure underlying the notion of percentages. Again, I am 99.9% convinced you'll have lots of fun!

Initial Question

One well-known task about percentages is the *Warehouse* problem, formulated by John Mason in his book *Thinking Mathematically* (Mason et al., 1982, p. 1). It can be outlined this way:

> In a warehouse, you obtain [a] 20% discount, but you must pay a 15% sales tax.
>
> Which would you prefer to have calculated first: discount or tax?

A common spontaneous reaction is to say that there probably exists, for the client or the merchant, a favourable order to complete the calculation with percentages. One's first attempts at calculating, however, can lead to some surprises: the % is grounded in a multiplicative structure, and the order in which the tax and the discount are realized does not matter. For example, if a pair of jeans is worth \$100, taking 20% off first gives $\$100 \times 0.80 = \$80$. This \$80 is then taxed at 15% to give $\$80 \times 1.15 = \$92$. If the order is reversed, it is then $\$100 \times 1.15 = \$115$ for the tax first and $\$115 \times 0.80 = \$92$ for the discount second. In the first, tax is only charged on the discounted amount, where in the second, the discount is applied to the price and to the tax. In both cases, it leads to the same final amount: the order has not changed the final price to pay. In other words, there is commutativity in the notion of percentage, where in our example, $100 \times 0.80 \times 1.15 = 100 \times 1.15 \times 0.80$. The multiplicative factors can be interchanged without impacting the final answer.

This said, even if one is aware of the multiplicative structure of percentages, leading in turn to accept the commutativity aspect and the fact that the order of operation does not matter, it is possible to conceive that there are some differences in meaning for each situation. Indeed, a frequent reply to the *Warehouse* problem is that "legally speaking," both are not equivalent, since there is an amount that needs to be given to the government with the taxes. Even if the consumer is paying the same final amount, of \$92, doing the tax before the discount leads to more dollars in taxes than one has to (taxing \$100 leads to a bigger amount than taxing \$80). Hence, the "logical" order for the retailer would be to calculate the taxes *after* the final discounted price. One could also add that obtaining a discount after having calculated the taxes leads to receiving a bigger discount in terms of the amount itself, since calculated on an augmented, already taxed, price, the jeans at \$100 becomes \$115 after the sales tax, and it is then on this new \$115 that the 20% discount is applied, worth \$23. This could also mean that there is a discount made on the tax, which could again appear illegal or simply awkward. Is a vendor allowed to discount the tax amount? In sum, although there is no difference in the final price to be paid by the client, if one determines the tax or the discount first, for the seller, the government, and the legal taxation policies, this does not end up being the same.

These diverse reactions to the *Warehouse* problem reveal that, beyond the mathematical commutative property for the %, the context of the problem can play a role. Since % is very much a notion present in everyday life situations, thus highly contextual, this seems important to take into consideration. Therefore, all this raises questions that can generate better understandings of the meaning to give to commutativity in % situations.

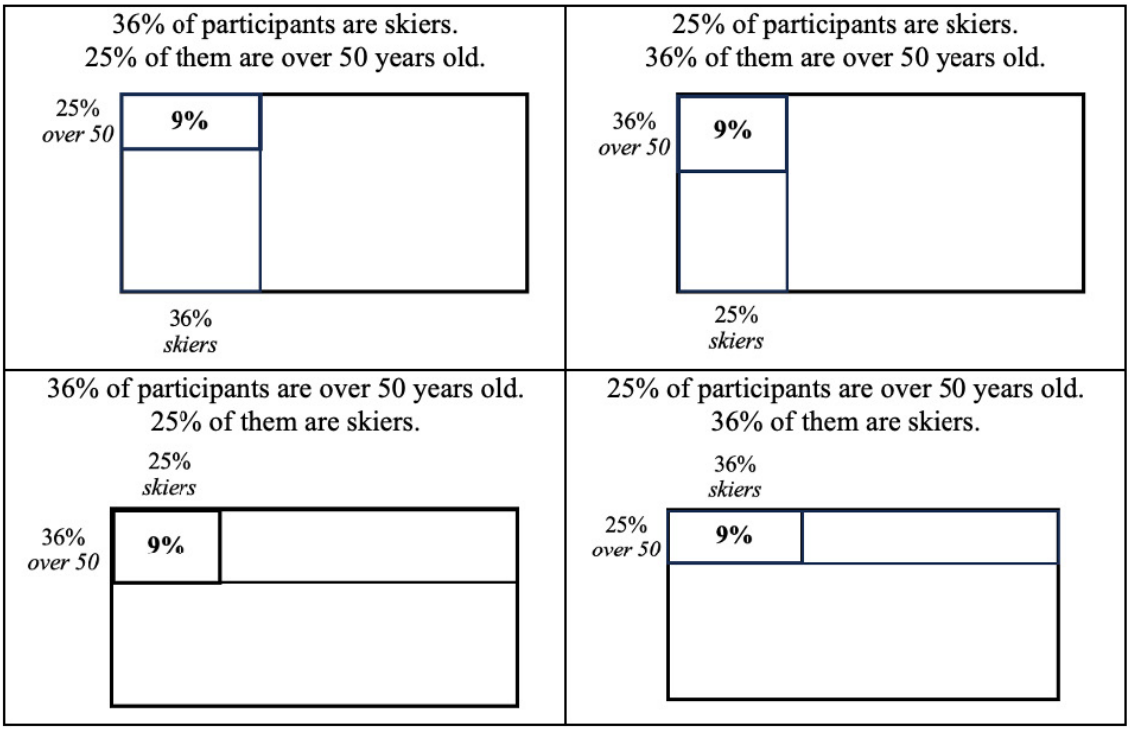


Figure 1: *Illustrations of the different scenarios for combining %*

Consider a second situation, inspired by an example found on a francophone Wikipedia page about percentage (see https://fr.wikipedia.org/wiki/Pourcentage):

> In an organized outdoor trip, 36% of the group are skiers, and 25% of these skiers are more than 50 years old.

Here as well, independent of the number of persons participating in the activity, calculating 36% and 25% of that number can be done in any order: (Number of participants) × 0.36 × 0.25 = (Number of participants) × 0.25 × 0.36. It is commutative. But, here again, what about the meaning to be given to this situation? Asserting that 36% are skiers, and that 25% of these are over 50 years old, is not the same as asserting that 25% of participants are skiers and that 36% of them are over 50 years old. And this is what that Wikipedia page hinted to: even if in both cases, the final number of skiers said to be over 50 years old represent 9% of the whole group, the makeup of the participants in the activity is not the same. Asserting that 25% of the participants are over 50 years old, and that of these, 36% are skiers, does not mean the same, and neither does saying that 36% of participants are over 50 years old and that 25% of them are skiers; or any other imaginable combination leading to 9% of the whole group being skiers aged over 50. In each of these examples, the composition of the group itself would simply not end up being the same. Figure 1 shows (approximative) drawings to illustrate these situations.

Each of these situations end up at 9% of skiers of over 50 years old; however, as mentioned, the representations implied by intermediate stages in the solving process are not the same.

What does all this mean? When the notion of "%" as a multiplicative structure is considered at a calculational level, it is commutative: the final result—the final price to pay and corresponding final % of the price tag, and the number of skiers over 50 years old were the same. In other words, the computational order does not matter. However, percentages rarely appear in questions outside of any context. Thus, when considering the context of the problem and by giving concrete meaning to the numbers as well as the calculations they require, this commutativity of % does not maintain the same meaning, even to the point of being seen as inadequate. It is as if there were a "contextual commutativity" in these situations. Could it be in this contextual commutativity, that is, in the interaction between "calculations" and "significance of these calculations," that the notion of % takes most of its meaning?

So, that's some nice food (0%, fat free) for thought! I hope you enjoyed these second musings as much as we did. Having reached 50% of my columns for this volume, and always being 100% curious, feel free to send me your thoughts and comments.

Reference

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Leveraging Student Agency to Enhance Motivation and Engagement in Mathematics Classrooms



JEFF IRVINE
dr.jeffrey.irvine@bell.net

Jeff Irvine, Ph.D., has been a secondary mathematics teacher, department head, and vice-principal. He has taught at three faculties of education and at a community college. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. With over 45 years in education, Jeff is particularly interested in the interplay of student motivation and mathematics achievement.

There is abundant research on student motivation that identifies students' freedom to choose as a major motivating factor. Glasser (1998) found five factors that students need to be addressed in their learning: safety and security, freedom, fun, belonging, and power. In self-determination theory, Deci and Ryan (2008) list three needs that require satisfying: autonomy (freedom), competence (power), and relatedness (belonging). Plentiful research has identified the most powerfully motivating factor as autonomy (freedom). In the classroom, this need can be addressed through providing students with some level of choice in what they do. The choice doesn't need to happen all the time, and of course, the choice can't be without limits. But giving students some choice some of the time is a powerful motivating factor.

There is also brain research that has found that when students have freedom to choose, the brain secretes the enzyme serotonin, which enhances cognitive function; when students face no choice, the brain secretes cortisol, which interferes with cognitive functions such as problem-solving. This can be seen occurring in timed tests and exams, when some students "blank" due to cortisol production (for example, Graham et al., 2022).

Reeve and Lee (2013) identified a component of student engagement: agentic engagement. Essentially, students are more engaged in their learning when they have some control over their learning environment. Agentic engagement can be described as "students' constructive contribution into the flow of the instruction they receive" (Reeve, 2013, p. 579). Agentic engagement is proactive, involving student self-advocacy, such as asking questions, offering opinions, identifying areas of student interest, and stating preferences (Reeve & Lee, 2013). So, agentic engagement involves the student in making choices for their own learning.

This article describes some ways that mathematics teachers can increase student agentic engagement and motivation through providing choice. I have used all of these strategies in my classes and attest that they work. An example is shown in Figure 1.

Fraction Follies

1. Write down your favourite fraction.
2. Add 1 to the numerator and 1 to the denominator.
3. Is this new fraction larger or smaller than the previous fraction? How do you know?
4. Repeat steps 2 and 3 several times. What happens?
5. Explain your results.
6. What if you started with a smaller fraction?
7. What if you started with a larger fraction?
8. What if you started with a mixed number?

Figure 1: *Fraction Follies*

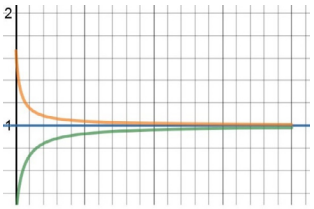


Figure 2: *Graphs of Fraction Follies*

In this activity, which I have used in Grades 6 to 12, the students have two places involving choice. In the first step, they choose their own fraction. More importantly, in the third step, students choose how they justify their decisions. This could involve logic, conversion to decimals, graphing, building a model, or some other method. Figure 2 illustrates the results for a proper fraction (bottom line) and for an improper fraction (top line). Starting with a proper fraction yields a monotone increasing sequence bounded above by 1. Starting with an improper fraction gives a monotone decreasing sequence bounded below by 1. Students in calculus can evaluate the limit of the fraction $(a + n)/(b + n)$ as n approaches infinity to prove this result.

Open Questions and Parallel Tasks

With open questions, students exercise agency through control over the question(s) and/or the parameters of the question. This can be extremely motivating for students. Questions may be fully open, with no single correct answer, or they may be open-routed, with one correct answer, but with several ways to get there (like Fraction Follies).

Marian Small has written two excellent books about open questions and parallel tasks (Small, 2009; Small & Lin, 2010). Dr. Small gives a wide range of examples, and also describes

how to make a closed question, such as a typical textbook question, into an open question. Here is an example: Change "Find the volume of a cylinder with a diameter of 10 cm and a height of 8 cm" to "A cylinder has a diameter of 10 cm. Choose a possible height of the cylinder and find its volume." This simple strategy of removing one of the parameters makes the question open, and supports student agency by giving them some control over the question.

Another activity that employs student agency in an open way, and allows for student agency in their learning, is Sequence Sorting, shown in Figure 3 (reproduced with permission from *The Variable*, Irvine, 2020).

Design a sorting rule and sort the sequences below into two groups.

A: 3, 5, 7, 9, 11, 13	B: 1, 3, 9, 27, 81, 243
C: 2, 4, 8, 16, 32, 64	D: -9, -2, 5, 12, 19, 26
E: 5, -10, 20, -40, 80, -160	F: 9, 7, 5, 3, 1, -1
G: 7, 8, 11, 16, 23, 32	H: 12, 6, 3, 1. 5, 0.75, 0.375
I: 1, 4, 7, 10, 13, 16	J: 1, 4, 9, 16, 25, 36
K: 2, 7, 9, 16, 25, 41	L: 7, 11, 15, 19, 23, 27

My sorting rule: _____

Group One Group Two

Navneet used a sorting rule based on operations. Sort the sequences above, using Navneet's rule:

Formed using + Formed using ×

In which group should Navneet put the sequence 1, 2, 2, 4, 8, 32? ____

Why? _____

Now design a sorting rule to help Navneet subdivide each of her two groups into two subgroups.

For Navneet's sequences formed using +, my subgroup sorting rule is _____.

Here are Navneet's + sequences sorted using my rule:

For Navneet's sequences formed using +, my subgroup sorting rule is _____

Here are Navneet's × sequences sorted, using my rule:

Where will you put the sequence 3, 4, 6, 9, 13, 18?

Why? _____

Figure 3: *Sequence sorting*

When using this activity, you need to be able to accept very simple sorting rules, such as, "These sequences contain a 4" and "These sequences do not contain a 4." However, the growth in student understanding is valuable. By allowing students to make their own decision rules and justify them, students find this a powerful motivational activity. I have frequently been impressed by the sophistication of some students' sorting rules, demonstrating high levels of cognitive thought.

Parallel tasks can be described as two or more tasks involving the same concept, but with varied levels of complexity or abstraction. This provides multiple entry points for students. A student is free to choose whichever task with which they are most comfortable. Parallel tasks are designed so that all students can participate in consolidation at the end of the class. Here is an example (based on Small and Lin, 2010):

Draw a pattern that would help someone figure out the value of the 100th term in the sequence.

Option 1: 1, 3, 5, 7, 9,…

Option 2: 4, 7, 10, 13,…

Option 3: 5, 11, 17, 23,…

Problem Posing

Problem posing is often thought of as being the domain of strong, motivated mathematics students. However, I have found a lot of success using this strategy with all students, including those who are sometimes unmotivated or disinterested in mathematics. This is a great opportunity to tie mathematics to real-world concepts that are of interest to students. The initiating task can be an article from the Internet, from a student newspaper, or from the local newspaper. Students may work individually, in pairs, or in groups. The students identify an article of interest to them and pose several problems based on the article. They then formulate solutions to their problems, and trade their questions with another student group, who then solve the problems and have their solutions verified by the original group. For more details on this strategy, see Irvine (2017). Problem posing can be a strong motivator, and by giving students control over the initiating article as well as the problems that are posed, students have a high degree of control over their learning environment.

Jigsaw

Jigsaw is a co-operative learning strategy. Co-operative learning tasks have five facets:

- Mutual positive interdependence
- Individual accountability
- Face-to-face interactions

- Social skills
- Group processing

(Johnson & Johnson, 1994)

See Irvine (2020) for details on how to set up jigsaw, a terrific co-operative learning strategy with mutual interdependence, but individual accountability. Students are allowed to choose their home groups, as well as which subgroup they attend.

Menus

This differentiated-instruction strategy, patterned on a restaurant menu:

- addresses student agency, since students have control of their own choices
- has all tasks based on a common topic/theme/big idea
- has side dishes that can vary based on learning style, interest, or readiness
- offers desserts as consolidation, "fun," or enrichment
- may be a multi-day structure

Figure 4 illustrates a typical menu.

Today's Menu Choices

- Side Dishes (Choose 2)
  Task SD1, Task SD2, Task SD3, Task SD4, Task SD5
- Main Course (Must be chosen)
  Task MC
- Desserts (Optional – may choose 1, if you wish)
  Task D1, Task D2, Task D3

Figure 4: *A typical menu*

Depending on the teacher's decisions, menus can be straightforward choice tasks, or they can be used to insert some fun into math class, while still focusing on productive mathematical activity. I used menus a couple of times each semester with all my classes for a "restaurant day." The menus were printed on large cardstock and folded to resemble an actual restaurant menu. The tables were covered with plastic checkered tablecloths. Dressed in a tuxedo, I met my students at the door and gave them the menus for that class. We had a lot of fun and accomplished productive work, while satisfying the students' desire for autonomy and choice.

Tiered Assignments

Tiered assignments provide another strategy that allows student agency in their learning. Students choose one tier and attempt all questions in the tier. The tiers can be different levels of complexity or abstraction.

Typically, three tiers are sufficient. The tiers can be based on level of difficulty (straightforward, medium difficulty, challenging); on student interests (questions on three different applications); on learning modalities (auditory, visual, kinesthetic-tactile); on three of the assessment categories (Communication, Application, Thinking); or on any basis that lends itself to tiering.

RAFT (Role, Audience, Format, Topic)

RAFT allows students to select how they will demonstrate their knowledge of a topic by choosing one of several options. For example, to demonstrate their understanding of exponential functions, students could choose one row from Table 1.

Role	Audience	Format	Topic
Investment adviser	Client	Presentation	Compound interest
Scientist	Other scientists	Scholarly paper	Radioactive decay
Waste manager	General public	Newspaper article	Disposal of radioactive waste
Salesperson	Manager	Job interview	Predicted sales growth
Environmentalist	Factory owners	Model	Growth of carbon dioxide emissions

Table 1: *RAFT choices*

Cubing

This is a fun homework or direct-practice choice strategy, where each student rolls a die and must complete at least the number of questions on the upward face of the die, with the caveat that they must be able to explain the concept to a peer or teacher the following day. Students have total choice on which questions to complete, and frequently decide to complete more than the required number of questions to confirm their mastery of the concept. See Irvine (2019a) for other homework strategies involving student agency.

Four Corners

The student chooses one corner of the classroom based on the task described in that corner. For example, if the topic is solving systems of equations, the corners might be Elimination, Substitution, Graphing, and Technology (e.g., CAS). After completing the task, corner groups discuss advantages/disadvantages with other corner groups.

Choice Boards

Students choose a specified number of tasks from a selection such as shown in Figure 5. All choices relate to the same topic, but may differ in difficulty, applications, interest, or real-world context. In Figure 5, the tasks are labelled, K=Knowledge, A=Application, C=Communication, and T=Thinking, to indicate the focal category of knowledge.

A	T	C
T	K	A
C	A	T

Figure 5: *TACK choice board*

Task #1	Task #2	Task #3
Task #4	Task #5	Task #6
Task #7	Task #8	Task #9

Figure 6: *Think-Tac-Toe*

Each square on the choice board should display the actual student task. Students choose from a specified number of tasks to complete—for example, students may have to choose four of the tasks from the options in Figure 5. A variation on choice boards is Think-Tac-Toe (Figure 6), where students choose three tasks that mimic a tic-tac-toe win. Choice boards carry the risk of ego depletion; when there are many choices, students may focus on choosing the tasks, rather than on the tasks themselves. Bounded choice can reduce ego depletion; for example, rather than choose any four tasks, students can be instructed to choose one task from each row.

Learning Contracts

Learning contracts are another example of student agency in action. The student commits in writing to accomplishing a specified task or set of activities, usually within a specified time period. I have had great success using contracts in classes that lack motivation, since the student controls their own learning environment and makes a written commitment, which they almost always honour.

Providing students with some level of choice enhances agentic engagement, as well as puts some fun into math class. When promoting student agency through use of student choice, there are some considerations to keep in mind:

This is an excellent way to provide motivation and stimulate engagement.

- It promotes student autonomy and student agency.
- It can be used in instruction or assessment.
- It can be used as differentiated instruction.
- Use bounded choice to avoid ego depletion.
- A maximum five choices (e.g. choose three out of five) are usually sufficient.
- If a mix of compulsory and choice, lead with choice.

Student agency, giving students some control over their learning environment and academic work, is a powerful method of increasing student engagement and motivation. Many of these strategies do not require extensive preparation, and the rewards for both teachers and students can be dramatic. In my experience, many of my non-academic students have benefited most from these strategies. These students often feel a lack of control over their own learning, and the strategies outlined in this article help students to take ownership of what they do in mathematics classrooms.

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Pattern

What patterns do you see?

How might you describe them?

Why might two people disagree about some pattern ideas?



▲ Break Free from Math Anxiety: Reflective Practices to Transform Math Fears and Fuel Future Success



STACEY JACOBS
s.jacobs@utoronto.ca

Stacey Jacobs is an Education Consultant and Course Instructor in the Master of Teaching Program at OISE/UT. Stacey has served as a K–8 classroom teacher, curriculum coach, has her Principal Qualifications, and works with the International Baccalaureate Organization (IBO), providing Teacher Training and School Authorizations. She pioneered a STEM and Robotics Program at an Independent School in Toronto, Ontario. Her practice focuses on child-centred learning. Stacey is interested in supporting teachers in mathematical literacy and competency to support deep and meaningful learning in Mathematics.

Have you considered how we develop our math learner and math teacher identities? How personal narratives impact our roles as teachers and learners? Or consequently, how personal narratives shape our math teacher identity? In my work with pre- and in-service teachers, I have witnessed how math anxiety is compounded with teacher identity, and how it impacts a teacher's confidence, planning, and instruction. Math anxiety has been defined as "…a persistent fear, tension, and apprehension related to situations that require math" (Ramirez et al., 2018, p. 1). Many of the strategies and teaching approaches that I have used with pre- and in-service teachers have helped educators who are silently suffering from math anxiety. Uncovering and processing our roles as both learners and teachers takes time, and requires deep, honest reflection, and a willingness to change our mindset. Understanding the impact past experiences have on our teacher identity formation is profound.

The approach presented here has been organized into six reflective practices to help identify and alleviate math anxiety: The Learner Experience, Looking Inward, Identifying Myths, Leaning In, Shifting to a Positive Mindset, and Taking Action. Included are some reflections from teachers who have used these approaches, and prompts to work through their math anxiety.

The Learner Experience

The acknowledgement of math anxiety is needed to begin to change one's narrative. The Learner Experience Reflective Prompts (Figure 1) explores some possible origins of math anxiety in relation to the learner's past experiences. One must seek to understand their narrative if they want to change it. We must delve into our past and remember what it felt like to learn math growing up. Was it easy? Hard? Were there any specific experiences that can be recalled that have shaped self-confidence and learning with respect to math?

Reflective Practice: Ask or Journal:
What were/are your experiences as a "Math Learner" as far back as you can remember?
Teacher Reflection: *"Growing up in my family, it was part of my culture to do well in math. It came so easily to my older brother and sister, that I was ashamed that it was so hard for me. The harder I tried, the more anxious I became. It got to the point where I would have panic episodes when having to study for math tests."*

Figure 1: *The Learner Experience Reflective Prompts* (Please note these reflective practices can be directed to oneself or reworded to support teachers or students.)

Varied experiences and narratives contribute to a teachers' math identity and anxiety. Whether it be feelings and experiences they have negatively internalized, or the progression of abstract and difficult concepts, narratives may have negatively contributed to our math identity and can fuel math anxiety within us. There is no one simple or single cause as to why some teachers struggle with math anxiety. Its causes and roots are unique to every learner. Understanding our narratives is an important first step in beginning to reconcile with it.

Looking Inward

The Looking Inward Reflective Prompts (Figure 2) endeavour to set the stage for developing a *positive mathematical mindset* in which the learner understands that math "is a subject of growth, and their role is to learn and think about new ideas" (Boaler, 2022, p. 40). This reflective practice is used to begin to shift mindsets from negative associations and math anxiety to form new, positive associations. Without remembering and creating positive associations to math learning and our math identity, the negative narrative and anxiety will prevail.

Reflective Practice: Ask or Journal:
Who/what inspired you as a learner? How is this different from the identity you have now as a learner and teacher of mathematics?
How was your "math identity" formed? Is there a connection between your own learning and experiences and your current math identity? Is there something that needs to change?
Teacher Reflections: *"I had a tutor who never gave up on me. They made a game of my mistakes, which helped me see them as learning opportunities, not mistakes."* *"My friend and I would do our homework together. She said by helping me, I was helping her. This made me feel less guilty of needing help."*

Figure 2: *Looking Inward Reflective Prompts*

Many teachers and students share stories of favourite teachers, mentors, tutors, or friends who have inspired them. What is often at the heart of these sources of inspiration is the relationship, and that someone believed in them. If one can start to recall these positive associations when the negative thoughts and feelings creep in, it gives way for a change in narrative, leading the way for a revised and renewed math identity formation. It takes deliberate practice and intentional self-reflection to change our negative feelings.

Identifying Myths

Understanding some of the many myths around mathematics learning, coupled with personal narratives, is another way for many to understand the root of their math anxiety, and to begin reconciliation. I conducted a meta-analysis of data related to myths surrounding mathematics learning, and created the following chart (Figure 3: Math Myths) debunking many of the long-held myths that surround mathematics learning. Teachers can use this chart to identify their stance on these myths (or others they share), specifically in relation to their lived experiences, and then examine the research that debunks them.

The following prompts (Figure 4) are used to further uncover how/if the myths are framing current narratives.

Reflective Practice: Ask or Journal:
Is there a myth that resonates with you? (One that is shared here or another one that exists)
What do you need to do to shift your learning and understanding? (This may take time to curate.)
Write down the incident or experience that has led to your math anxiety (if you can recall) and what your reaction to it was. (This helps with visualizing.)
Write down what your feelings are about math now, at this point in your life, then add the reasons you want to succeed at math. (This makes our unconscious visible, helps us to take control of our anxiety, and start to reconcile with it.)
Teacher Reflection: *"I don't want to panic when I teach math; I don't want my childhood experiences to continue to form my identify as a math teacher. I want other children, who might look and feel like me, to know that math is not easy for everyone. I need to let go of and grow from my past experiences."*

Figure 4: *Identifying Myths Reflective Prompts*

By doing a deep dive into what has shaped and is shaping our narratives, we can begin to reframe and take possession of math anxiety, befriending it, unravelling the fear of what has challenged us for so long.

Leaning In

"Leaning in" refers to the willingness to acknowledge and understand that math anxiety may be part of our narrative. In leaning into math anxiety, understanding where it lies and its triggers, we can uncover something very powerful within our stories. The following chart (Figure 5: Keys to Developing a Positive Math Mindset) provides some strategies to support teachers when trying to grow their math identities into a positive mathematical mindset (Boaler, 2022).

Myth	Truth
Boys are better at math than girls.	The gender myth is largely perpetuated by various social and cultural factors, and has more to do with differences in instruction, environment, and cultural beliefs than the ability and capability of either gender (Kane & Mertz, 2012).
There is a "right way" to solve math problems.	Students can better understand and conceptualize mathematics tasks when working in small groups, with the ability to communicate and collaborate. When they have access to how others approach and solve problems, they are better equipped to make sense of what they are learning, rather than simply memorizing a set of rules and formulas (Frank, 1990).
There are certain cultures that are better at math.	The Myth of the Model Minority, which is based in the stereotype that Asian-American and Asian-Canadian children are math geniuses, does a disservice to an entire sect of our population, and not only perpetuates inequality, but also fosters math anxiety in many individuals (Chang & Au, 2009).
You can be born with a "math gene."	A math gene has not yet been discovered. It has been found that with effective teaching and the proper mindset, success and achievement in mathematics is attainable for every student (Chestnut et al. 2018).

Figure 3: Math Myths

What Not to Do	What You Can Do
Allow the negative self-talk to take over. Quiet that voice.	Turn up the volume on positive self-talk.
Avoid tasks/topics/concepts. Doing so will negatively impact your students' learning and understanding.	Celebrate mistakes!
Skim over topics quickly. Students need time to create the new pathways, strengthen them, and make connections.	Engage in professional learning. When we learn, our students also reap the benefits.
Assign concepts that you are not comfortable/confident with as self-directed learning for students. There is a much greater chance for students to develop misconceptions.	Relate mathematics to real life and engage your students in critical-thinking tasks. In a world full of instant gratification, allow them to develop the muscle of productive struggle and perseverance.
Assign difficult concepts for homework. Students will need guided and varied practice in the classroom .	Create engaging, collaborative activities in class, and students will begin to see the joy in the journey/process of solving mathematical tasks, rather than fixating on the destination/correct answer.
Teach a concept once and in one way. Strengthening the connections includes a variety of learning experiences and real-world connections.	Remember that mathematics class should be seen as a performance subject. Provide opportunities for students to work together and take shared responsibility for completing tasks, learning from one another, and valuing diversity of thought and strategy.
Use only one modality.	Make mathematics visual and hands on. Math is a universal language in many ways, yet we take away the visual and physical component as students get older, weakening connections for them, and forcing them into the abstract. Visuals and manipulatives allow for connections to prior learning and different strategies.

Figure 5: *Keys to Developing a Positive Math Mindset*

Once teachers have identified the root of their math anxiety and the contributing factors, they can begin the journey of reframing their narrative. The next series of reflective practices (Figure 6: Leaning In Reflective Prompts) endeavours to cultivate a positive mathematical mindset (Boaler, 2022).

Reflective Practice: Ask or Journal:
Where is the shift for me?
What do I need less of?
What do I need more of?
Teacher Reflection: *"I need to make sure I am familiar with core concepts prior to teaching them to my students. I need to acknowledge any mistakes I make myself when learning and teaching, to normalize mistakes. The shift will be to incorporate different ways/strategies in math problem solving. I need to take the fear out of math by creating a safe space to talk about any doubts/confusion."*

Figure 6: *Leaning In Reflective Prompts*

Reframing thinking to what is needed, instead of what we cannot do, will allow teachers to break free from their math anxiety and past math experiences, and begin to form a new math teacher identity, in which confidence can grow and thrive.

Shifting to a Positive Mindset

It is at this point where teachers often say to me: "Now what? I have figured out what has caused my math anxiety, I have identified the 'what' and 'how I want to feel,' but how do I get there?" I invite you to view the menu of strategies that is provided here as a resource (Figure 7: Menu of Strategies to Foster Positive Mathematical Mindsets). This list is by no means exhaustive, and I suggest you add to it or create one that is meaningful to you, as you begin to shift and create a positive mathematical mindset.

Menu of Strategies to Foster Positive Mathematical Mindsets	
Practise.	Use manipulatives to make the concepts concrete.
Find examples and non-examples of concepts.	Have a beginner's mindset, trying to unlearn what might be holding you back.
Lean on colleagues for suggestions on how they teach a particular concept.	Work to dispel harmful myths and misconceptions.
Attend professional development.	Vary assessment practices.
Use different teaching and learning strategies.	Work through examples with solutions as practice.

Figure 7: *Menu of Strategies to Foster Positive Mathematical Mindsets*

Taking Action

The last reflective practice (Figure 8: Taking Action Reflective Prompts) invites teachers to put their learnings into practice. This step is what will truly allow for a change in the narrative.

Reflective Practice: Ask or Journal:
How will I be kind to myself and support the growth of a mathematical mindset?
What actions will I take to support a mathematical mindset? (Set realistic and tangible goals with dates and resources, if applicable.)
Teacher Reflections: *"I will replace negative self-talk with positive affirmation. I will talk about math holistically with my students. I will seek to understand my students' past experiences with math. I will discuss what math anxiety is, so they are aware of it."* *"I will look at many ways/contexts to apply the concept I'm going to teach (increase flexible thinking). I will endeavour to discover different ways that manipulatives can be used to illustrate/solidify concepts so I can show students who get stuck. I will practise the language that I will use to explain concepts. I can look at different approaches to solving problems within a concept (i.e., multiplication or division), so that if one doesn't resonate with a student, I have other tools that can help."*

Figure 8: *Taking Action Reflective Prompts*

What is evident from my work with educators, new and seasoned alike, is the importance of continued support for teachers who struggle with math anxiety. I invite you to use the strategies suggested in this article in ways that are meaningful to you. For many teachers to continue their learning and professional development in mathematics, we must recognize that math anxiety is a gatekeeper. Providing teachers with tools and strategies to work through their math anxiety, and to form positive mathematical mindsets and teacher identities, is a necessity, not only for themselves, but for the lasting impact it will have on their students' learning.

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Nominations for the OAME/AOEM Board of Directors 2024–2025

Each year, OAME/AOEM holds elections for open positions. Any OAME/AOEM member may apply for any position. Candidates are needed for the following positions:

- One (1) President-Elect for a four-year term from September 1, 2025 – August 31, 2029 (one year as President-Elect, two years as President, and one year as Past President)
- One (1) Vice-President for a two-year term from September 1, 2025 – August 31, 2027
- One (1) Director to represent Grades 7–12 for a three-year term from September 1, 2025 – August 31, 2028
- One (1) Director to represent Grades K–6 for a three-year term from September 1, 2025 – August 31, 2028
- One (1) Director to represent Colleges for a three-year term from September 1, 2025 – August 31, 2028

The nominee and the nominator must be members of OAME/AOEM. The Nominations/Elections Committee will publish a slate of candidates, including candidate biographies, for consideration by the membership. Voting will take place online in March 2025, and is open to all OAME/AOEM members. The election results will be posted on the OAME/AOEM website.

An online nomination form, as well as a summary of the duties of each position, is available at www.bit.ly/OAMEPositions. If you require additional information, please contact Paul Alves (paul.alves@oame.on.ca).
The deadline for nominations is January 16, 2025 at 6:00 p.m.

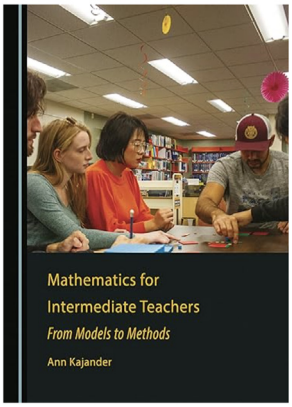
Thank you in advance for submitting a nomination.

▲ Book Review: Relearn Mathematics, Using Representations

MICHELE MELLOW
michele.mellow@tcdsb.org

Kajander, A. (2023). *Mathematics for intermediate teachers: From models to methods*. Cambridge Scholars Publishing.

"Ours is not to reason why. Just invert and multiply." Many of us learned how to divide fractions following this saying. It was fast and gave us the correct answer. But how is this algorithm derived? What tool could a teacher or student use to represent what happens when you divide a fraction by another fraction? *Mathematics for Intermediate Teachers: From Models to Methods* answers these questions and more as it shows educators how to teach Intermediate math with manipulatives.



Mathematics for Intermediate Teachers: From Models to Methods encourages you to think deeply about mathematical concepts, increase your understanding, and make your teaching more effective. Kajander, a mathematics professor at Lakehead University, addresses various topics including area and perimeter, linear patterns, integers, and quadratics. The book focuses on the specialized content knowledge teachers require for teaching mathematics, providing step-by-step instructions on how to use representations to understand the "why" of mathematics. Depending on your mathematical background, some visuals may give you insight into how math works. For example, I now appreciate how to use the area model, such as algebra tiles, to represent $3x - 2$.

Throughout the book, Kajander encourages teachers to recognize that students will be at different stages in their learning and to view knowledge as a developmental continuum. She argues that students need time to use appropriate representations to fully comprehend a mathematical concept. "Mathematics is about reasoning, and procedures are constructed based on, and to support that reasoning, and speed up computations. And yet, once available, procedures are often elevated to surpass reasoning…. Teachers have to work hard to put aside our own past learning trajectories and mathematical biases, in order to think developmentally alongside our students" (p. 39). The book encourages teachers to let students find the procedure through a guided activity with models. Rushing students to the algorithm leads to students' incomplete comprehension of a concept and may result in math anxiety.

Kajander's enthusiasm for teaching Intermediate mathematics shines through in each chapter. Her passion is infectious and will convince you to use models to solve the many practice questions at the end of each chapter. Learning how to find the formula for the area of a circle, and the exploration of limits using Kajander's model, gave this reviewer fresh insight and the necessary confidence, if I had to teach it to students. Similar exercises with fractions, quadratics, and angles also increased my understanding.

Educators in their first few years of teaching at the Intermediate level will appreciate the book's descriptions of procedures and stages of learning. They will find explanations of why students have misconceptions when they enter the Intermediate classroom, as well as suggestions for how to remediate those misunderstandings with models.

To further enhance the content, links to YouTube videos demonstrating how to use the suggested manipulatives, like algebra tiles and fraction bars, or how to derive the formula for the area of a circle, would be helpful. Some readers would benefit from seeing the models in action, rather than just reading descriptions to understand how to incorporate representations in their lessons. Teachers may also wish to seek school funding before purchasing *Mathematics for Intermediate Teachers: From Models to Methods.* The book must be ordered from a UK-based publisher, costing over $50 (CAD) for paperback or over $100 (CAD) for hardcover.

Mathematics for Intermediate Teachers: From Models to Methods is a worthwhile choice for Intermediate teachers or department heads, who want to start a book-club discussion group or wish to expand their content knowledge of the mathematical processes from the Ontario Mathematics (2020) curriculum: representing, reasoning, and selecting tools and strategies. Testing the suggested models and answering the practice questions with colleagues would generate productive discussions, increase content knowledge, and improve classroom teaching.

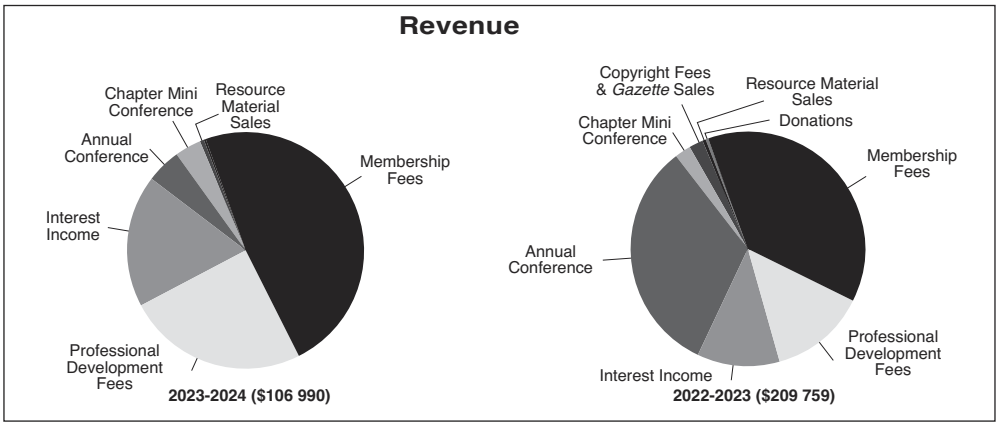
If you learned math without using manipulatives, take the time to read *Mathematics for Intermediate Teachers: From Models to Methods.* You will master how to model $4y - 1$ and appreciate why you need common denominators to add and subtract fractions. Best of all, you will no longer rely on abstract rules like "invert and multiply" to teach mathematical concepts. ▲

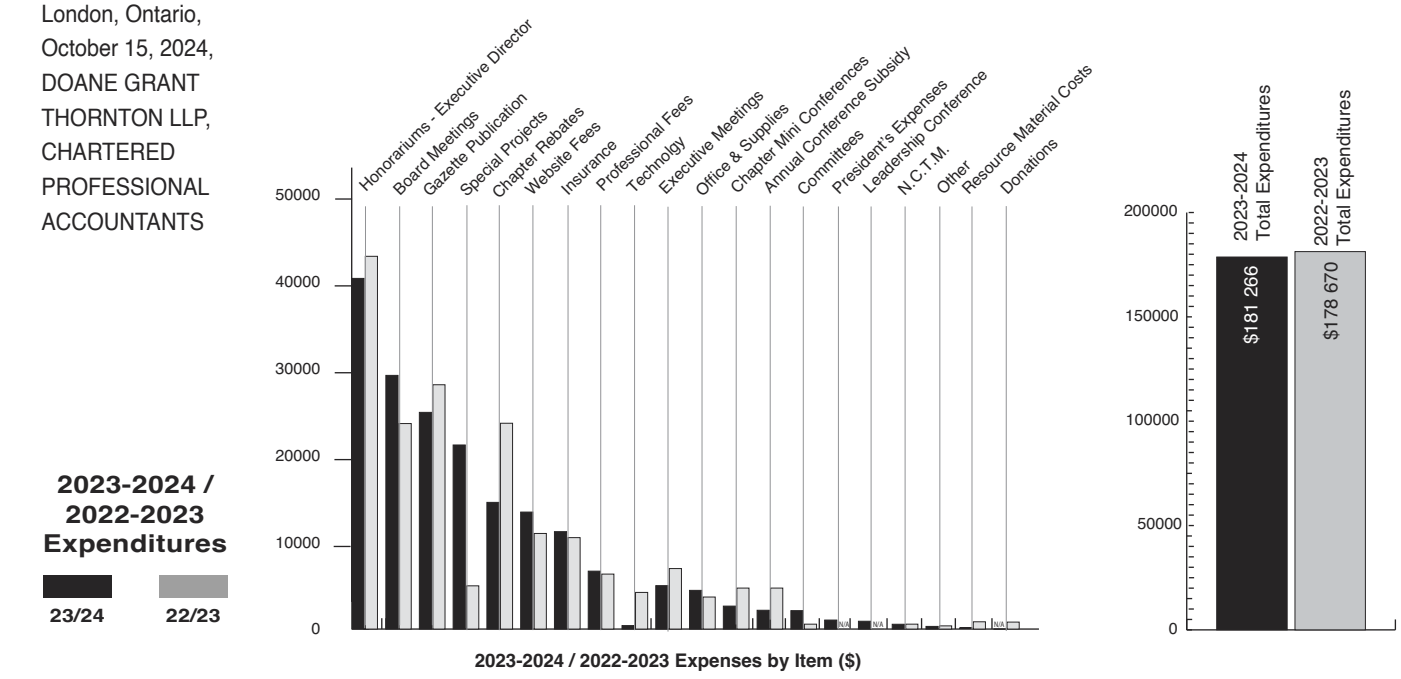
▲ OAME/AOEM Financial Statement 2023 – 2024

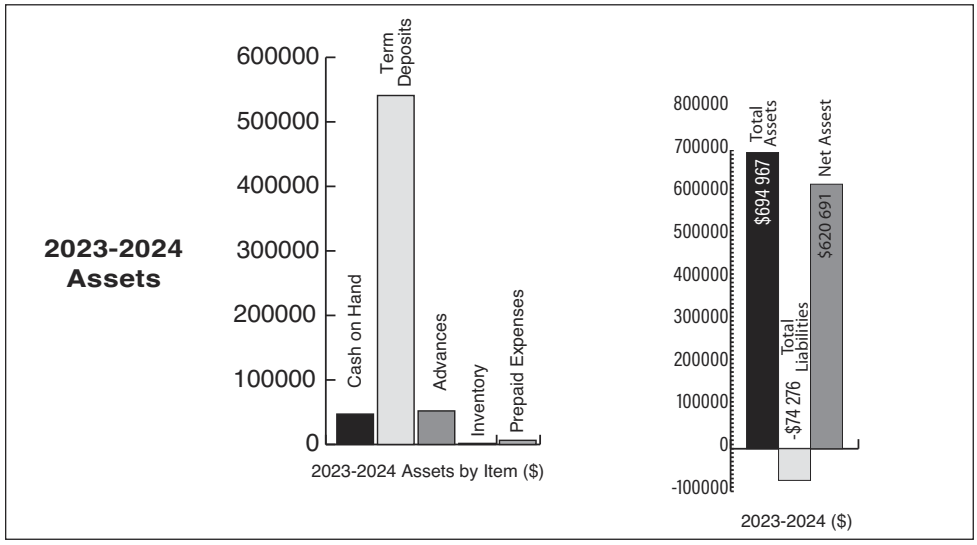
Notice to Readers

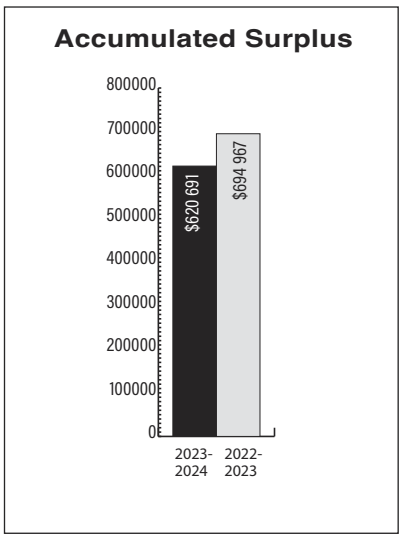
On the basis of information provided by Management, we have compiled the statement of financial position of Ontario Association for Mathematics Education, Association Ontarienne pour L'Enseignement des Mathematiques as at August 31, 2024, and Note 1, which describes the basis of accounting applied in the preparation of the compiled financial information ("financial information").

London, Ontario,
October 15, 2024,
DOANE GRANT
THORNTON LLP,
CHARTERED
PROFESSIONAL
ACCOUNTANTS









▲ 2024-2025 OAME/AOEM Board

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DIRECT GENERAL INQUIRIES TO: EXECUTIVE DIRECTORS

Fred and Lynda Ferneyhough
4751 Mack Ave.
Plympton-Wyoming, ON N0N 1J6
Business Phone: 647-466-0674
EDS@oame.on.ca

PRESIDENT

Kit Luce
Barrie, ON
kit.luce@oame.on.ca

PAST PRESIDENT

Sandra Jean Price
Oshawa, ON
sandra.jean.price@oame.on.ca

VICE-PRESIDENTS

Mark Chubb
St. Catharines, ON
mark.chubb@oame.on.ca

Dave Petro
Harrow, ON
david.petro@oame.on.ca

EMERITUS

Paul Alves
Georgetown, ON
paul.alves@oame.on.ca

CHAPTERS & REPRESENTATIVES

CHAMP: Giovanni Bruccoleri
Mississauga, ON
giovanni.bruccoleri@oame.on.ca

GOLDEN: Jessica Drake
St. Ann's, ON
jessica.drake@oame.on.ca

GVMA: Glen McMillan
Guelph, ON
glen.mcmillan@oame.on.ca

ISOMA: Cal Armstrong
Burlington, ON
cal.armstrong@oame.on.ca

MAC²: Stacey Collins
Barrie, ON
stacey.collins@oame.on.ca

NOMA: Heather Theijsmeijer
Kagawong, ON
heather.theijsmeijer@oame.on.ca

NWOAME: Janet Fuchek
Thunder Bay, ON
janet.drake@oame.on.ca

$O_3$4ME: Teresa Weselake
McNab Braeside, ON
teresa.weselake@oame.on.ca

PRMA: Jennifer Clark
Bowmanville, ON
jennifer.clark@oame.on.ca

QSLMA: Tanis Lattanzio
Kingston, ON
tanis.lattanzio@oame.on.ca

SAME: Nkiru Azza
Scarborough, ON
nkiru.azza@oame.on.ca

SWOAME: Elizabeth Pearsall
Kingsville, ON
elizabeth.pearsall@oame.on.ca

TEAMS: Alvarine Aldridge
Toronto, ON
alvarine.aldridge@oame.on.ca

WOMA: TBD

Y⁴MA: Gina Micomonaco
Richmond Hill, ON
gina.micomonaco@oame.on.ca

DIRECTORS – TERM EXPIRING 2024

Gerard Lewis
Toronto, ON
gerard.lewis@oame.on.ca

Jason To
Toronto, ON
jason.to@oame.on.ca

Cameron Redsell-Montgomerie
Toronto, ON
cameron.redsell-montgomerie
@oame.on.ca

DIRECTORS – TERM EXPIRING 2026

Elli Weisdorf
Toronto, ON
elli.weisdorf@oame.on.ca

Lisa Rossiter-Thornton
Toronto, ON
lisa.rossitor-thornton@oame.on.ca

DIRECTORS – TERM EXPIRING 2027

Olive Creary-Satchell
Toronto, ON
olive.creary-satchell@oame.on.ca

Michael Frankfort
Richmond Hill, ON
michael.frankfort@oame.on.ca

Jennifer Holm
Kitchener, ON
jennifer.holm@oame.on.ca

NCTM REPRESENTATIVE

Jacqueline Hill
Lindsay, ON
jacqueline.hill@oame.on.ca

WEBSITE COORDINATOR

Greg Clarke
Orillia, ON
web@oame.on.ca

GAZETTE EDITOR

Timothy Sibbald
North Bay, ON
gazette@oame.on.ca

ABACUS CO-EDITORS

Marc Husband
Toronto, ON
marc.husband@oame.on.ca

Tina Rapke
Toronto, ON
tina.rapke@oame.on.ca

Cristina De Simone
Mississauga, ON
cristina.desimone@oame.on.ca

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