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Abacus

Ontario Mathematics Gazette

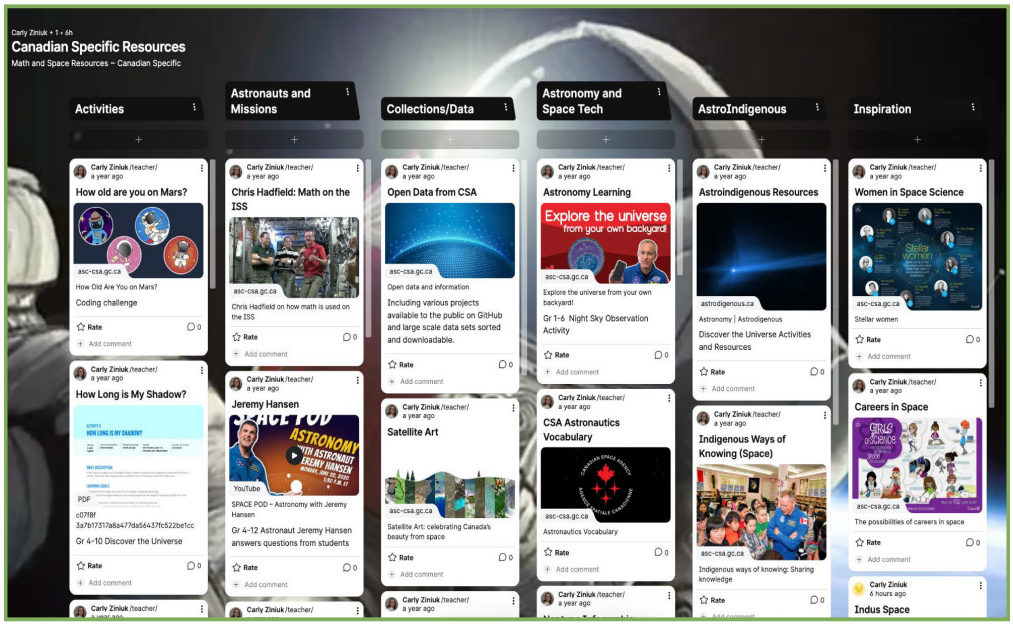
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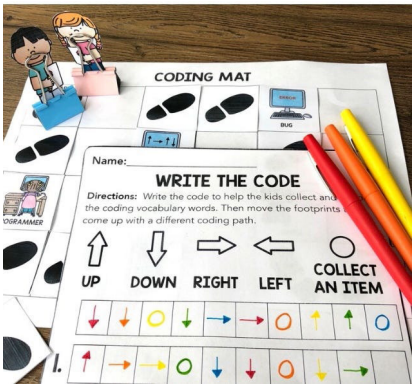
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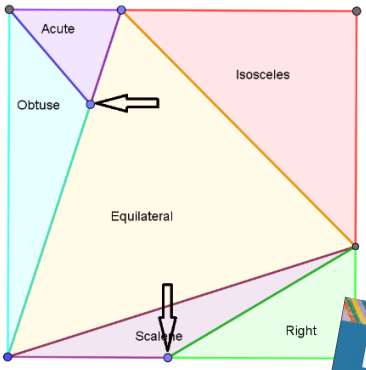
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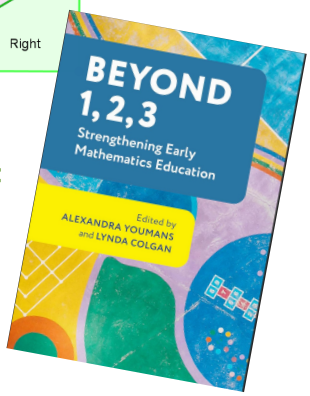


See In the Middle: Made in Canada, When Math Comes Home





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OAME 2026 Let's {Talk} Math

KEYNOTE SPEAKERS



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Pamela Harris

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Octavia Beckles
Dr. Alex Lawson
Chris Luzniak
Cathy Marks Krpan
Marian Small
Ian VanderBurgh

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APRIL 30-MAY 1, 2026

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Submission of Articles

The *Ontario Mathematics Gazette* (*Gazette*) publishes news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *Gazette*:

- The *Gazette* is aligned with the OAME/AOEM Vision, Mission, and Strategic priorities. These can be found on the website www.oame.on.ca under "About OAME" and "Guiding Principles."
- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles are to be word–processed, MS Word is preferred, and prepared according to the *Publication Manual of the American Psychological Association*, Seventh Edition. However, please use single-line (not double) spacing and only one space after each period. Articles should not exceed eight numbered pages of text, though there is some flexibility. Figures, images, and photographs should be placed close to the relevant text and include captions. The photographer's permission is required, and for photos of students under the age of 18, permission of a parent or guardian is required.

Please submit your article in one blind file (i.e., identity of author is not evident), and include author names, contact information including email and mailing addresses, photos—head and shoulders, biographies—less than 100 words, and all content removed for blinding in a second file. Please email these two files to Tim Sibbald at gazette@oame.on.ca.

Upon review, you will be notified whether your article has been accepted for publication (as is, or pending minor or major revisions) or declined. The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME/AOEM.

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The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of OAME/AOEM.

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Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** by email or at the above address. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be print ready, and colour advertisements should have no bleeds.

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Editor's Report



TIM SIBBALD, OCT, PH.D.
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Tim Sibbald is a professor in the Schulich School of Education, Nipissing University, with a focus that includes mathematics education. He is a former President of OAME/AOEM and has held a variety of other OAME/AOEM posts over the years.

What is the purpose of mathematics education? To phrase it another way, what is the overarching intention of our roles as mathematics teachers?

Is it so that students are supported to find employment? Is it so that students become meaningful contributors in our society? Or is it a broader social purpose such as sharing cultures and appreciating we share a rock called Earth? Perhaps the purpose is not based on external rewards or contributions, and instead, it is about learning how to think, that is, thinking in a general sense, in essence freedom by being able to recognize options (i.e., different solutions) and exercise choices. Is that what is meant when the purpose is purported to be self-fulfillment or discovering who one is?

In my view, like so many philosophical questions, there is an error in the question when it implies that there is a singular purpose or overarching intention to education. What is more, it is my contention that the purposes ebb and flow through the years. Is a student in Grade 1 attending with the same purpose as a Grade 12 student? Perhaps, within the limitations of their own expression, they are. Yet, there is surely less clarity for a Grade 1 student, and they are, generally, okay with that. Or perhaps it is weighted, where social purposes may be a Grade 1 emphasis and jobs a Grade 12 emphasis. In my experience, Grades 5–6 highlight enthusiasm for learning, regardless of purpose, before the impact of puberty changes the social–emotional forum of education.

Another issue is whether I have fallen into the trap of casting a general response to the question that side-steps the specific focus of mathematics. Some might argue that mathematics is more strongly aligned with employment than some other subjects—in blunt terms, that math is less about society because of a strong vein of introspection that is commonly observed; or, in another way, that mathematics does not, generally, align with moral or ethical thinking that contributes to one's personal way of being in the world.

Mathematics can be (or at least be close to) value free, where all options are explored and simplistic notions, such as lowest cost, avoid larger contextual challenges of decision making.

At one time, a case could have been made that math class is simply about mathematics. Some of the eldest readers might even argue that when they were in school, the focus was on doing mathematics. However, the historical record does not support these views. Education has always entailed social–emotional considerations and the larger context of content learning. It was done differently, but it was there—teachers have always cared about the larger scope of their students.

Rather than go on at length (knowing most graduate programs have a dedicated course aligned with the purpose of education), let me point out that it is the notion of a purpose that we use to build our pedagogical choices and instruction. Teachers try to address all students and meet them where they are—yet the results vary because, among other complications, students have a limited ability to articulate their perception of a purpose of education (let alone various purposes). What is more, it ignores that their student perceptions are subject to change as they develop (who among us has not felt exhilirated after a notable success, or perhaps more job oriented at one time or another?), yet those may simply be a fleeting sense of purpose.

What I will claim, however, is that thinking about these sorts of philosophical ideas is a technique to keep us out of proverbial ruts. It serves to remind that the spirit of our lessons should not be painted with one brush, nor should it be accepting of one view of any student's needs. Philosophical ideas are challenged in a teacher's daily work through pushes for "improvement plans," standardized testing, learning outcomes, etc., which focus narrowly on specific criteria because the complexity of the whole cannot be captured. Yet, consideration of philosophical questions, such as the purpose of education, reminds that it is the spirit of the teaching endeavour that is important—the so-called "higher calling" perhaps. To put it another way, while we focus on nuts-and-bolts skills, it is the overall development of our students that is going to lead to their doing things that will, years later, "wow!" us. Herein is when the "higher calling" is reinforced and affirmed.

I am confident saying there is no sure way to plan for the myriad applications for which students will ultimately use their education. The best that seems possible is for teachers to periodically mix up their own instruction to avoid having students become blinkered because the instructional diversity has become limited. A rich mix of instruction provokes rich thinking among students, and that may or may not be the purpose of education, but it certainly is beneficial to them, their community, society,…—everyone. Wishing you all the best for the new school year.

News

Ian Vanderburgh's 20-year tenure as Director of CEMC has come to a close. Best wishes on his new role as Associate Vice-President of Strategic Enrolment Management. The new Director is **Jen Nelson**.

In This Issue

While it defies the title of this section, I begin with what is not in this issue. **Iain Brodie** and **Hatice Beyza Sezer** have chosen to discontinue writing "Coding in the Classroom." Their column began in September 2021 and satisfied a need for addressing curricular change. In addition, they bridged their column with interviews that were made available through OAMEtalks. I sincerely appreciate their contributions and thank them for many insightful articles.

Our President, **Kit Luce**, reminds of the various pieces OAME/AOEM has on offer, and encourages members to get involved. The annual OAME/AOEM Awards are featured, and I hope this inspires readers to consider the people around them whom they might nominate this year. **Cristina De Simone** reports on the Canadian Mathematics Education Study Group (CMESG) conference. In the Fields report, **Pamela Brittain** brings news of upcoming events, some of which are in late September.

Jérôme Proulx examines the confusion arising when a percent increase is followed by percent decrease. **Ron Lancaster** focuses on games talking about *Set*, *Rush Hour* and *Abalone*. **Lynda Colgan** clarifies the challenges of directions and misdirections. **Carly Ziniuk** draws a Canadian focus with many resources. **Shawn Godin** tackles a question of dissecting a square as different kinds of triangles. **Jeff Irvine** encourages consideration of stage-environment fit theory. **Sofia Ferreyro-Mazieres** will continue writing the MB4T column, and in this issue, she focuses on instruction pertaining to decimal-place values.

Reports from OMCA and NCTM were not available at the time this issue was going to press. Hopefully we will see them both in the next issue.

Asia Majeed, Lauren Gainsbrook, Tongyu Zhang, and **Raphael Camargo Penteado** explain the implementation of Thinking Classrooms in destreamed Grade 9. **Elizabeth Pearsall** reviews a new book by **Alexandra Youmans** and **Lynda Colgan**. Last, but not least, **Barb Seaton** explains what it takes to be a reviewer for *Gazette* articles.

Thank You

I have recently been updating the *Gazette* reviewer list and wish to thank all the people who have offered their services. Anyone who was missed is welcome to email the Editor to be added to the list (please indicate with which of Pre-K & K, PJ, JI, High School, College, University, Pre-service, and French you would like to be identified). A small lesson that came with this, was when I posted a Google form link on OAMEwrites, and in a short amount of time, had over 30 000 applications of potential reviewers (with 10–20 legitimate)—interesting times we live in.

Anne Yeager, Peter Saarimaki, Gitta Berg, and **Penny Clemens** are the usual suspects, who polish and form every word in the *Gazette* (with the exception of advertising and a few Board of Directors items). In the eyes of this Editor, they are the Fab 4! (The Fab 4 is a reference to the Beatles.) They *Don't Let Me Down*, but instead, *Help!* so that *I Get by with a Little Help from My Friends*. Working with them is a *Day in the Life* of an Editor, and *I Feel Fine*, knowing they *Come Together* and work their magic *Here, There, and Everywhere.* Thank you!

We have also had two additions to the editorial team: **Gurpreet Sambhi** and **Steven Khan**. As the saying goes, many hands make lighter work. While we are not there yet, we certainly appreciate their efforts as they grow into the role. Thank you!

Erratum

On page 3 of the last issue, the MPT sample question asking for the value of b in $b/4 = b - 12$ was mentioned, but erroneously omitted 16 from the multiple-choice answers (the value 18 appeared in its place).



Cartoon by Julianna Kagawa, inspired by working with tangrams

President's Message



KIT LUCE
kit@oame.on.ca

Kit has been an active member of MAC² for many years and joined the Board of Directors in 2022. She is a District Mathematics Facilitator for Grades K–9 with the Simcoe County District School Board. In her work, Kit creates math resources to support educators in their instruction and assessment practice, as well as developing and facilitating professional development. Kit is an Additional Qualification instructor and course writer, and she has written and reviewed work for a variety of publishers. She has been involved in Ministry writing projects and was a Content Lead for the recent OAME/AOEM writing projects. Kit received the 2024 OAME Award for Leadership in Mathematics Education.

Welcome to a new school year, and to this issue of the *Gazette*. As we embark on another academic journey, I hope you've all had a refreshing summer break and are ready to embrace opportunities while navigating the challenges that lie ahead. The past few years have certainly tested our collective resilience and ingenuity, prompting us to adapt and innovate in ways we might never have imagined. Yet, as mathematics educators in Ontario, we have consistently demonstrated our ability to face the unknown and succeed in supporting the growth and development of our students. This new year truly presents a fresh opportunity for growth and learning for us all.

For me, this marks the beginning of my second year as your OAME/AOEM President, and it's a role I embrace with deep gratitude and enthusiasm. My journey with OAME/AOEM began many years ago with my local chapter, MAC² (Muskoka And Cottage Country). Like others, my involvement at the chapter level profoundly shaped my own teaching philosophy and commitment to our mathematics community. I am also immensely grateful for the ongoing support and mentoring I've received from the dedicated members of our Board of Directors; their wisdom and guidance have been instrumental in my growth and development in this role.

At the heart of OAME/AOEM is the belief that we are a community of learners who support one another. We are incredibly fortunate to have a wealth of excellent resources available to us, thanks largely to the tireless contributions of our members. Our commitment to equity and making

mathematics accessible for all students is woven into our resources.

The *Gazette*, along with the *Abacus*, serves as a rich source of professional insight and practical ideas, with an archive stretching back to February 1962. Imagine the wealth of knowledge within those pages! I strongly encourage you to take the time to go back a few years and read a few articles. While the curriculum has changed, the critical and creative thinking about mathematics education has not. Consider sharing your expertise by writing an article for the *Gazette* or the *Abacus*. We all benefit from sharing knowledge and resources with our colleagues.

The Math Curriculum Resource Project, found at ontariomath.support, and a partnership with AFEMO (l'Association francophone pour l'enseignement des mathématiques en Ontario), offers free webinars, lessons, and assessment plans designed by Ontario teachers to support the most recent Grades 1–8 Mathematics Curriculum (2020) and the destreamed Grade 9 curriculum (2021). Crucially, these resources are available to both English and French/French-immersion teachers and students. Similarly, the mathies.ca tools are free, designed by Ontario teachers, and available in English and French, supporting effective technology use in classrooms and encouraging new ways to incorporate them into your math program planning.

The knowledge and expertise within our Board of Directors and among our members are showcased through our premier professional development events. Our Annual Conference in May 2025, and our Leadership Conference in November 2024, provided high-quality professional development. These conferences featured inspiring sessions and speakers, who challenge our thinking, addressing crucial topics like equity, access, and empowerment for students. This coming year will be no different. Be sure to mark the dates for OAME 2026 (April 30–May 1, London, Ontario, www.sites.google.com/oame.on.ca/oame2026?pli=1), and plan now to attend, and to submit a proposal for a workshop. Your voice is needed and valued.

Our OAME Talks podcasts continue to provide valuable insights and conversations with experts. Watch and check for this year's lineup of speakers to be announced. Members also have access to chatOAME, an Artificial Intelligence (AI) tool built for OAME, which is trained on OAME/AOEM content (and other Ontario policies and resources) and aligned with our mission, vision, and strategic priorities. This tool can provide valuable support for educators looking to discuss ideas and receive support based on a foundation of resources from OAME/AOEM, the Ontario Ministry of Education, and Canadian education content grounded in valid and reliable sources.

The value of local chapters for "learning close to home" cannot be overstated. OAME/AOEM has 15 regional chapters across the province, like my own MAC^2. Many chapter volunteers are busy planning events for members, including mini-conferences, workshops, webinars, and book studies. If you're unsure which chapter you belong to, you can easily check your affiliation by logging into the OAME/AOEM website.

I strongly encourage you to get involved and leverage these incredible resources. Whether it's attending chapter mini-conferences, submitting a proposal to present at our provincial conference (a great professional learning opportunity to share the amazing work happening in schools), or nominating an outstanding colleague for one of OAME's annual awards, your participation strengthens our community. Reaching out to your local chapter is a fantastic way to connect and contribute.

Ultimately, our goal is to ensure that you, our members, see profound value in being a part of this organization. OAME/AOEM is constantly evolving to meet the needs of its members and the challenges that confront us. I encourage you to reach out and communicate with us, and share your ideas for future learning opportunities and resources. Keep your eyes on our website for information about upcoming events.

I wish everyone a wonderful school year! May it be filled with rich learning, meaningful connections, and the continued shared pursuit of excellence and equity in mathematics education for all students in Ontario. ▲

Morse Code Graphic Organizer

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Musing on Percentages: Adding and Cancelling %



JÉRÔME PROULX
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Jérôme Proulx is a professor of mathematics education (www.profmath.uqam.ca/~jproulx), and runs the Laboratoire Épistémologie et Activité Mathématique (www.leam.uqam.ca). His research focuses on epistemological issues related to cognition and/in mathematics. His current research program is dedicated to mental mathematics and school mathematics content development. (Photo by Nathalie St-Pierre of Université du Québec à Montréal)

Percent is everywhere—all around us in finance, shops, foods, media, surveys, and so on. Percentages may seem like a rather simple mathematical theme that has little depth. However, when one starts to pay attention to its usage, percentages can be seen as filled with surprising elements that are mathematically rich and conceptually fascinating, hence worth pondering. This is something that became quite salient for our research team, as we have been preparing experiments with students in classrooms about percentages in mental mathematical environments.

These columns endeavour to raise and address some of these peculiarities, which we came to call "curiosities," related to the concept of percentages. These musings can be seen to be as much about our usage of "%" in everyday life, in schools, or in mathematics in general. This fifth column addresses issues related to establishing equivalences between % increases and decreases. It is my hope that these issues, and their underlying mathematical properties, will intrigue you.

Initial Question

Why is it that, when I add 25% to my unit, this addition suddenly becomes 20% of the unit?

This question was posed to me by a colleague one day, and it is still one that fascinates me today. So simple, it leads to work on equivalence links between different percentages that are quite interesting, insisting on the importance of the referent-unit of %. For the above case, the 25% is added to an initial referent-unit, which then gets transformed. As for the 20%, it is to be considered in relation to that new transformed referent-unit, that is, the initial referent-unit augmented by 25%. A sketch of the situation might help to better illustrate the equivalence at stake—see Figure 1.

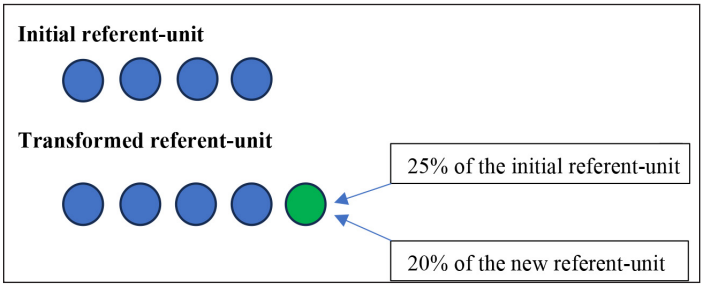


Figure 1: *A picture of the situation*

It is in this sense that 25% becomes equivalent to 20% because the initial referent-unit, augmented by 25%, represents the new referent-unit, for which the added part is now worth 20%. The crux of the enigma resides in the relativity of each %, a core notion about what percentages are. The value of % is never pre-decided; it is always relative to a referent-unit to which this % is associated. That is why it is adequate to say, with a grin, that, in this case, 25% = 20%. A disturbing equality one might say!

Keeping this grin, one can wonder if companies really take advantage of this situation, that is, of the relativity of %, when they advertise their products. For example, using Figure 2, if a box of cookies shows a tag expressing that 20% of its content is free, it is also probably possible to indicate that "25% is free," without necessarily lying. Indeed, if the initial box of cookies is 200 g, and with the bonus, becomes 250 g, then getting 20% more means that 50 g of cookies are free. This 50 g is as much 20% of 250 g as it is 25% of the initial box of 200 g. Although advertising 25% free is more impressive, there are probably strict laws (e.g., from the Canada Consumer Product Safety Act) that prevent companies playing with % in that way. But it is still fun to think them through!

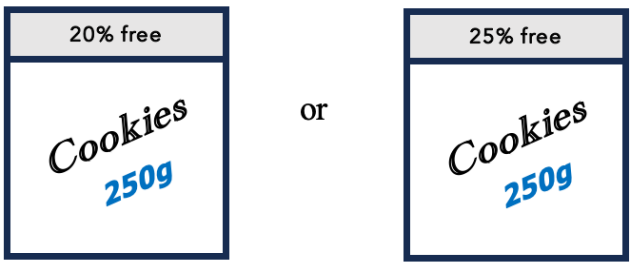


Figure 2: *Comparing packages of cookies*

Symbolizing % Equivalences

In what ways can this equivalence or "equality" be explained at the symbolic level? If my referent-unit is x, adding 25% to it can be written up as $x \cdot (1.25)$. This new

value, that is, $x \cdot (1.25)$, now represents the new referent-unit to consider. How can this new referent-unit be transformed and brought back to the initial referent-unit worth x? In other words, what % would compensate this 25% augmentation? As seen in the last column in Proulx (June, 2025), a 25% reduction does not cancel out a 25% augmentation because $x \cdot (1.25) \cdot (0.75) = x \cdot (0.9375)$. However, applying a 20% reduction leads one back to the initial referent-unit: $x \cdot (1.25) \cdot (0.80) = x \cdot (1.00)$.

What does this mean? This means that an augmentation of 25% can be compensated by a 20% reduction. In fact, this shows that the reciprocal of a multiplication by 1.25 (to obtain an augmentation of 25%) is a multiplication by 0.80, that is a 20% reduction, because $\frac{1}{1.25} = 0.80$. Converting 20% and 25% in decimal form, one obtains the following:

$$\frac{1}{1+0.25} = 1 - 0.20$$
$$\frac{1}{1.25} = 0.80$$

Going one step further, for general cases with A% and B%, it is possible to represent the situation with $\frac{1}{1+A} = 1 - B$ or $\frac{1}{1+B} = 1 - A$. Here, the % to "add" represents the % to "deduct" in order to compensate the transformation and come back to the initial value. This compensating relation is particularly interesting when represented in a Cartesian graph—see Figure 3.

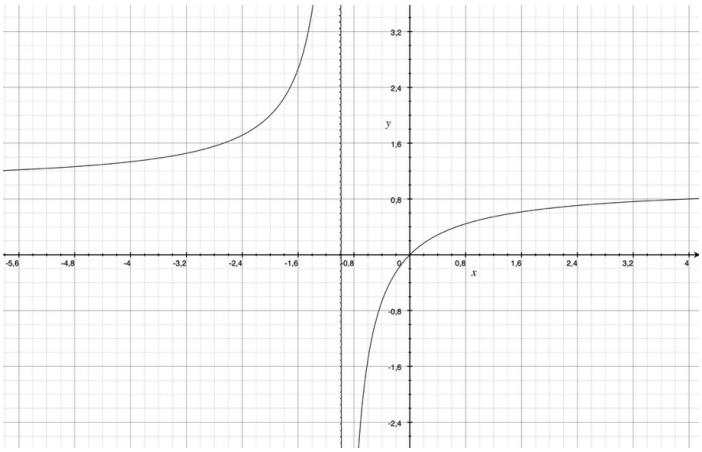


Figure 3: *Graph relating the value after increasing and decreasing*

The part of the graph that interests us most is in the first quadrant. This part from [0, +∞) gives an overview of all the couples of % "compensation". One can see the ordered pair (0.25, 0.20) from the preceding cookies example, but also the (0.50, 0.3), which is a 50% augmentation, followed by a % reduction. And there is also (1.0, 0.5), representing an augmentation by 100%, followed by a reduction of 50%, as well as (1.5, 0.6), which represents a 150% augmentation and a 60% reduction, and the list could go on. (In Parker, 1997, these ideas are even explored in terms of functions.)

All these ordered pairs point to other cases of compensation, as well as other little enigmas, to pose to colleagues. Here are some:

- Why is it that when I add 50% to my unit, this addition suddenly becomes $33.\overline{3}$%?
- Why is it that when I add 100% to my unit, this addition suddenly becomes 50%?
- Why is it that when I add 150% to my unit, this addition suddenly becomes 60%?

I'll let you have fun creating other ones! I'll also leave it to you to create an interpretation of the other parts of the graph. As for me, I'm not yet 100% satisfied with the meaning I make of them. Any ideas?

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Tally Marks with Chinese Characters

BY MICHELLE VUONG

In countries that use Chinese characters, such as China and Korea, tally marks are used to form the character 正(Pinyin: zhèng/zheng⁴—The superscripted 4 and a 3 in the next Pinyin are referring to tones. Chinese is a tonal language, and one of the transcription systems for Chinese to English uses superscripts to denote tone.), meaning "straight" or "correct" in Chinese. The character consists of five successive strokes, shown in Figure 1, with each stroke representing a count of one.

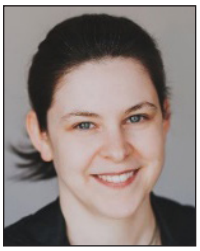


Figure 1: *From https://commons.wikimedia.org/wiki/File:%E6%AD%A3-bw.png*

Coincidentally, it looks very similar to the Chinese character for the number five 五 (Pinyin: wǔ/wu3), although 正 only uses four strokes. Much like the tally mark system used in English-speaking countries (Figure 2), 正 is used for counting, such as keeping score. A set of five is represented by the five-cluster 正, and numbers beyond five consist of 正 plus the remainder.

Continued on page 17

Events at the Fields: Building Capacity in the Scholarship of Mathematics Education, and Getting Ready for a New Academic Year



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Pamela Brittain (Ph.D.) is an educator, author, illustrator, presenter, and math lover, who currently works for the Fields Institute as their K–12 Academic Coordinator. She has been involved in STEM education for over 20 years and strives to make math accessible and enjoyable for everyone.

We begin this edition's column by revisiting the April MathEd Forum and then taking a look at what's coming up at The Fields Institute for Research in Mathematical Sciences this Fall.

MathEd Forum (April)

The April MathEd Forum focused on research relating to the mathematical education of undergraduate students and future teachers. It was sponsored by The Fields Institute and the Mitch and Leslie Frazer Faculty of Education at Ontario Tech.

The days included two panel discussions, lightning talks, working sessions, and three debates. The panel discussions featured a range of speakers from across North America and England. It covered topics such as:

- exploring the discontinuities in math instruction and cultural expectations experienced by teacher candidates, and the impact of these on future teaching practices and methods
- the different ways that undergraduate mathematics students interpret math concepts and communicate mathematically
- how mathematicians think mathematically—how they collaborate and communicate what they do, and how we can use methods of reflection and communication to help others think mathematically
- an introduction to various math journals including the *Canadian Journal for Science Math and Technology Education* (CJSMTE), the *International Journal of Research in Undergraduate Mathematics Education* (IJRUME), and *For the Learning of Mathematics* (FLM), along with information on what type of articles are typically included in each journal and how to submit.

Day 2 of the Forum included three debates and started with a brief introduction by the organizers about how debates provide us with a "rich diversity of perspectives and experiences" and the "importance of encounters with opposing views," yet how there is a trend in educational institutes to shy away from debates, as "individuals don't always feel comfortable exchanging ideas publicly and freely." They then voiced their "hope [that] the debates taking place today will remind us that educational institutions are designed to provide spaces for engaging with contentious issues, exchanging ideas, and expanding our perspectives… although we refer to today's event as debates, the main focus is on fostering discussions on these topics, rather than determining which side is winning." Each debate then started with an introduction of the speakers, and then the moderator asked the group specific questions, before opening it up to audience questions.

The debates featured speakers from universities, associations, and schools from across Canada. It included the following:

- *On the Design of Service Courses in Undergraduate Mathematics:* In our mathematics service courses, such as calculus, we are teaching the wrong things.
- *On Realizing the Vision of the High School Mathematics Curriculum:* In the past decade, the gap between the stated vision of the high school math curriculum and its actual achievements has widened.
- *On the Mathematics Preparation of Future Teachers:* When math content knowledge is secured, pedagogy follows.

The first debate focused on the role and expectations of what are referred to as "service courses," or courses that address a specific niche in education (such as a "Math for Biology Students" course). The discussion looked at exploring what it looks like to "do" math, and how mathematicians and students engage with math. Does their approach, and the approach of a service course, really serve the best interests of the student? Are students developing deep understanding as opposed to memorizing? And, where do each of those have a place, and how does it relate to problem solving and "mathematical habits of mind"? One of the main topics for discussion revolved around asking for whom the course was being created, and what purpose was it trying to solve—is it trying to train students for the jobs of today, or the jobs and opportunities of tomorrow?

The second debate started with the suggestion that "the idea is that we have a positive identity as a mathematics

learner and [students] see themselves as mathematically skilled. I think those two [are] coupled together. And the students have negative views of both of them. They do not see themselves as mathematicians, and therefore it leads into their perception as a mathematical learner." Concerns raised included that students are not going from the concrete to the abstract, are leaving lots of blanks on assignments and tests as they progress to higher grades, and they are not leaving school with a positive view of mathematics or themselves as mathematical thinkers. The debate highlighted deficits in student attitudes toward math and their own capacity to use math. Additionally, concerns were raised that destreaming of Grade 9 has diminished access to advanced topics, and that resources for teaching these students have not been readily available.

The final debate of the day started with discussions about teacher confidence in mathematics content. This confidence affects the ability to implement many of the methods of math instruction taught to teachers, and lacking this confidence often leads teachers to fall back on methods they themselves were taught and are familiar with—whether it is the most widely effective option or not. Discussions took place regarding how math content courses (separate from methodology courses) could increase teacher anxiety about math and drive them away from the field of education entirely. Some presenters argued that you can't teach math pedagogy effectively if you don't fully understand (and are comfortable with) the content, while others argued that math pedagogy courses could strengthen content understanding (and again confidence), while the reverse wasn't always true. Discussions also surrounded procedural versus deep understanding and what teachers should know in order to support diverse learning styles within their classrooms. In the end, while much of the debate surrounded defining pedagogy and content, and determining which order supports the most effective math instruction, all the presenters agreed that confidence in math was key for teachers, even if they disagreed on the specific ways to build it.

Coming Up at Fields



September is a busy month for Fields with the restart of the MathEd Forum after its summer hiatus and a few special events and programs. First, the MathEd Forum will take place on Saturday, September 27, with the Margaret Sinclair Award presentation and talk. This year, we are honouring Dr. Andrijana (Andie) Burazin, Associate Professor, Teaching Stream, in Mathematical and Computational Sciences at the University of Toronto Mississauga campus. You can learn more about Dr. Burazin and the award at www.uoft.me/msa2025.

Also, from September 23–26, the Fields Institute will be hosting the 2025 Fields Medal Symposium, this year focusing on the work of Abacus Medal Winner, Mark Braverman, and the theme of theoretical computer science. The Symposium typically hosts a student night for high school and undergraduate-level students. You can learn more at www.uoft.me/fms2025.

December 10 will once again see the honouring of Ada Lovelace's birthday, with the online conference, *Beyond Chalkboards: Digital Innovations in Teaching Mathematics* (www.uoft.me/ada2025).

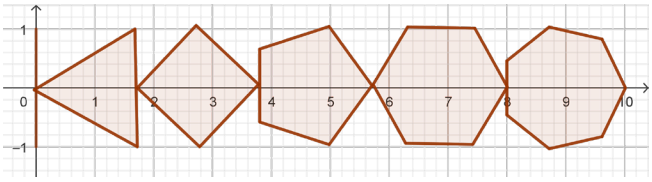
More events, including an exhibit of artwork highlighting dyscalculia, and continued programs for both students and teachers in mathematics, like our *Math Circles* and *Ask a Mathematician* programs, are coming soon, and details can be found at www.fieldsacademy.ca/eep/, or by emailing pbrittai@fieldsinstitute.ca (i.e., the author of this column).

Directory of Math Outreach and Engagement Across Canada

Supported by a generous grant from the Canadian Mathematics Society (CMS), Fields has compiled a list of various Math Outreach and Engagement programs from universities and institutes across Canada. This (growing) list of programs can be found at www.fieldsacademy.ca/directory, and we are always looking for new programs to add to it!

Math in Literacy

We're also very proud to announce the expansion of our *Math in Literacy* series, where we work with authors and illustrators of children's picture books, from communities typically underrepresented in mathematics, and provide math and science activities intended for K–6 teachers. Resources and information can be found at www.fieldsacademy.ca/math-in-books/, and our thanks go to House of Anansi Press and their Groundwood department for supporting this initiative. ▲



Meaning Making in the Classroom: Planning for a Successful Start to the School Year



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David is a principal, who taught and had various board roles in Prince Edward Island. David has also instructed university courses regarding curriculum, differentiation, mathematics, and literacy. He facilitates professional learning in mathematics and school development. Dr. David Costello has published four books, with two of his most recent books being: Messing Around with Math: Ready-to-Use Problems That Engage Students in a Better Understanding of Key Math Concepts, *and* Mathematizing Student Thinking: Connecting Problem Solving to Everyday Life and Building Capable and Confident Math Learners.

The start to a school year can be both exciting and overwhelming. Exciting, in that newness fills the atmosphere and what lies ahead is potential. Overwhelming, in that it can be daunting to craft a plan to ensure that all students are considered, and that their learning needs have been taken into account. It is to address both being excited and overwhelmed that often leads teachers to spend countless hours planning for the school year.

I want to assist teachers in such planning. There are things teachers do, that take considerable amounts of time to plan, but may not have the value added that we seek. As well, there are things teachers do that have considerable value added. It is these things that have considerable value added that I want to share with readers. It is about working smarter, not harder.

In this column, I will share practical strategies that all math teachers can apply to their planning. When they are then applied to instruction, teachers will see how students strengthen their understanding of math concepts, and can work toward becoming the independent math learners that we seek.

Key Concepts

If you have read any of my previous columns, the notion of key concepts will not be new. I do want to emphasize it, as a significant asset, when planning your instructional approach for the school year. Key concepts are curriculum outcomes that not only are important to learn but also play a role in understanding other curriculum outcomes. Key concepts are those curriculum outcomes that meet one or more of the following conditions (Ainsworth, 2003):

- **Leverage:** Curriculum outcome(s) that focus on knowledge and skills used in multiple academic disciplines
- **Endurance:** Outcomes and standards that focus on knowledge and skills that will be relevant throughout a student's lifetime
- **Essentiality:** Outcomes and standards that focus on the knowledge and skills necessary for students to succeed in the next grade level

There tends to be six to eight key concepts per grade level. Selecting these six to eight key concepts is a beneficial professional learning experience, both in terms of reviewing the grade-level curriculum, and in crafting your instructional year plan. First, teachers strengthen their understanding of grade-level expectations, and can see how the key concepts are not only stand-alone concepts, but also have interconnectedness with other concepts. Second, when the key concepts are selected, teachers can prioritize revisiting them throughout the year, so that students will not only strengthen their understanding of the concept, but also their ability to retrieve the concept.

When key concepts have been identified, teachers can then move on to crafting a year plan that ensures students will have opportunities to revisit and review these very concepts throughout the year.

Spiralled Curriculum Year Plan

Far too often, curriculum outcomes are addressed in insolation within year plans. Within such year plans, each outcome is assigned a block of time, and is addressed at one singular point throughout the school year. Such an approach to instruction is referred to as a *blocked practice.* A drawback of this approach to year planning is that mathematical concepts are addressed only once during an entire school year, which does not bode well for retention of learning (Costello, 2021).

Instead of a blocked-practice approach to year planning, let's consider a *spiralled curriculum* year plan. In a spiralled curriculum year plan, students revisit concepts multiple times throughout the year, rather than being concentrated in shorter periods of time. During each revisit of the concept, students explore the concept a little more deeply. Within this approach to year plans, learning moves from a shallow level at the beginning of the year to a deeper level by the end of the year.

When crafting a spiralled curriculum year plan, curriculum outcomes are not to be considered in isolation. Instead, the

teacher organizes instruction under either overarching questions, or big ideas (Costello, 2021). Overarching questions are questions that take into consideration multiple concepts, such as, "How can I use numbers to compare and contrast?" Big ideas are general mathematical ideas that support other ideas, highlight connections among mathematical ideas, or serve to illuminate mathematics as a human process (National Council of Teachers of Mathematics, 2000). An example of a big idea is, "Numbers can be represented in a variety of ways."

When crafting a spiralled curriculum year plan organized within a big ideas approach, teachers can apply the following steps (Costello, 2021):

1. Craft between six and eight big ideas, depending on the grade level.
2. Assign every outcome or standard to one big idea.
3. Divide the teaching time for each big idea into three or four sections.

When the plan is crafted, you then move through the year plan. To begin the year, you teach all the first sections for all your big ideas. Then you teach the second sections. Depending on whether your plan has three or four sections, you then conclude with the third section, or teach the third section and conclude with the fourth section.

Within a spiralled curriculum year plan, students continuously review previously taught material, instead treating mathematical concepts in a one-and-done approach. Revisiting concepts assist students in that they can clarify their understanding of the concept with each revisit. Students tend to engage more confidently and competently with the concept with each visit. Following a spiralled curriculum year plan enhances long-term retention for students. By spacing the learning of concepts throughout the school year, students must exert cognitive effort whenever they revisit a concept, and students are called upon to remember what they have learned about the concept earlier in the school year (Costello, 2021).

Homework

Homework can be a polarizing topic. While some schools choose to assign homework, others adamantly oppose it. For those schools that assign homework, I would like to suggest that the structure given to the homework can play a significant role in its contribution to learning (Costello, 2021). Leinwand (2017) provided a template for homework that involves a range of tasks:

- Two problems requiring a new skill. Two problems are enough for students to approach the concept while avoiding the possibility of reinforcing a misconception.
- Four problems requiring recall of concepts learned in previous lessons. This reinforces students being able to retrieve previous learning from memory.
- Two problems requiring time to think and develop a response, which provides opportunities for communication and problem solving.

A second homework option is a cumulative-review approach (Costello, 2021). Within this homework option, students are assigned six to eight problems that focus on three or four key concepts. Teachers select key concepts that were explored previously in the year. Teachers have the option to focus on the same key concepts throughout the year, or they could cycle through various key concepts during the year, using a bi-weekly or monthly approach.

Final Thoughts

Starting off the school year on the right path can make such a difference for both teachers and students. For teachers, when they have a plan that is thought through with a structure that supports learning, the year becomes less overwhelming, and they can take time to see the gains resulting from their well-developed plan. For students, when they have opportunities to revisit key concepts throughout the year, they clarify their understanding of these concepts, strengthen their ability to retrieve these concepts when solving math problems, and increase their confidence in their mathematical abilities.

The three approaches to a successful September start-up (key concepts, year plans, and homework) do not have to be followed verbatim. You can choose one, two, or three of these planning considerations. However, I do want to emphasize the impact of selecting key concepts for your grade level, and leveraging this work in your planning. By identifying key concepts, you are prioritizing those curriculum outcomes that have a significant impact on student learning. It is about working smarter, not harder.

I look forward to continuing the discussion of mathematical learning and instruction via my website (www.costellomath.com), email (david@costellomath.com), or X (@dr_costello).

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The 2025 Annual Meeting of the Canadian Mathematics Education Study Group (CMESG)



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Cristina De Simone is a middle school mathematics teacher and is a Ph.D. candidate in Mathematics Education at York University. Cristina's current research explores parental experiences of engaging in mathematics with their children, endeavouring to bridge the gap between home and school learning.

I attended my first CMESG in 2022 as a new Ph.D. candidate, fresh off defending my dissertation proposal and just beginning to imagine what it means to conduct my own study. My supervisor, Dr. Tina Rapke, encouraged me to prepare a poster about my research for the "Gallery Walk." She explained that it would be an engaging opportunity to gather insights from others in the field—people whose work I deeply admired. To say I was nervous and overwhelmed is an understatement. I was about to meet the very scholars whose work had made me fall in love with mathematics education.

I had just finished setting up my poster when I felt an arm wrap around my shoulders and someone exclaimed, "Oh! You've cited me!" It was Ann Anderson—known for her influential work on families and mathematics education. In 1997, she described parents as "the more proficient other," an idea that became a cornerstone of my own research. And there she was, discussing my work with me. Since then, I have continued to return to CMESG, and I look forward to this gathering every year.

When I arrived at CMESG 2025 at the University of Lethbridge, Alberta, it felt more like coming to a family reunion than a formal academic conference. I was excited to reconnect with colleagues, hear about their work, and immerse myself in the learning ahead.

CMESG begins with the opening ceremonies and introductions—not just from the executive or keynote speakers, but it includes everyone. Each participant introduces themselves, reflecting the value of shared dialogue and community that underscores the conference. As the program notes, "CMESG is not a typical academic conference, for it is not organized around presentations and audiences. Instead, it is a conference based on *conferring*." That spirit of *conferring* extends into the meals, the excursions, and even conversations over coffee.

The first plenary, similar to a keynote address, was delivered by Hassein Squalli from the University of Sherbrooke. His talk, "Elementary Algebraic Thinking as a Research Subject: Frameworks, Examples, and Perspectives," explored the historical development of algebraic thinking and invited us to consider how it differs from arithmetic thinking. He posed two key questions: "How can algebraic thinking be defined from an operational standpoint?" and "What are the main distinctions between arithmetic thinking and algebraic thinking?" (CMESG, 2025).

At CMESG, questions for plenary speakers are saved for the next day to allow time for reflection. In small groups, we discussed possible questions to bring back to the full community. Our group wondered: *Can students be "folded back" into arithmetic thinking when developing algebraic thinking? Are these ways of thinking hierarchical, or just different?* Dr. Squalli affirmed that arithmetic and algebraic thinking are not hierarchical, but distinct ways of reasoning. Our discussion continued well beyond the session, as his questions stayed with us—provoking deeper thought and conversation throughout the conference.

Working Groups

Five working groups were offered, each inviting participants to engage with a focused topic over three days. Through two-hour daily sessions, filled with rich discussion and collaborative exploration, participants examined complex questions from multiple perspectives. The outcome was a better understanding of the issues and many questions for further thought—a hallmark of CMESG.

Working Group A: Engaging Families in Mathematics: Tensions and Possibilities

Leaders: Kwesi Yaro, Julianne Gerbrandt, Cristina De Simone

This group explored what it means to do research with families, what kinds of methodologies are appropriate for this work, and how we might reimagine community and parental engagement. A key focus was on positioning families and caregivers as rich sources of mathematical knowledge—essential partners in building meaningful school–family relationships.

Working Group B: How can/does/should mathematics education research inform teaching at the post-secondary level?

Leaders: Andie Burazin, Chantal Buteau, Jérôme Proulx

This summary of Working Group B was contributed by Matthew Cheung, who participated in the group.

This group explored how research in mathematics education can inform and enrich post-secondary mathematics instruction. Through selected readings and collaborative discussions, the group examined how educational research sheds light on students' understanding of key mathematical concepts, such as Riemann sums, proof by induction, and eigenvalues and eigenvectors. Participants shared insights on students' experiences with these topics, including common difficulties and conceptual challenges, and reflected on pedagogical strategies that support learning. The group emphasized the importance of grounding instructional decisions in research-informed practices to better address the diverse ways students engage with and make sense of advanced mathematical ideas.

Working Group C: An Introduction to Queering Mathematics Teaching and Research

Leaders: Katryne Dubeau, Jennifer Hall, Shophika Vaithyanathasarma

This group invited participants to critically explore the intersection of queer theory and mathematics education. Together, they challenged dominant narratives and considered ways to foster more inclusive and equitable classroom and research environments.

Working Group D: Three Episodes in the History of Mathematics: Storytelling and Rich Problems as Motivation for Innovative Teaching and Learning

Leaders: Brenda Davison, Viktor Frieman

Participants in this group explored the power of mathematical storytelling—from ancient cryptography to binary systems—to animate history and enrich mathematics learning. Highlights included discussion of unsolved cryptography problems and the evolution of strategies for finding the area of a circle.

Working Group E: Discovering How Amazing and Joyful Mathematics Can Be! Sharing and Analyzing Outreach Activities

Leaders: Frédéric Gourdeau, Annette Rouleau, Zach Wolske

This group focused on the role of outreach in mathematics education, with discussions centred on the design, purpose, and impact of engaging math experiences outside the classroom.

New Ph.D. Sessions

CMESG continues to highlight the work of early-career scholars. Six newly minted Ph.D.'s shared their research across a range of topics in mathematics education. Presentations were followed by contemplative discussion, embodying the spirit of mentorship and reflection that defines the conference.

- **Heather Bourrie** – "Teachers' Perspectives and Pedagogical Documentation in Early Years Mathematics: Tensions and Possibilities"
- **Kelvin Chan** – "Two-Pass Assignments for Guiding Students Through the Transition to Rigorous Logical Reasoning"
- **David Hung Cheong Lan** – "Teachers' Use of Discourse to Promote Student Learning in Grades 7 and 8 Mathematics"
- **Tsubasa Saito** – "Sense Making in Learning Mathematics Across Languages and Countries: Cases of Multilingual Students in a Weekend Japanese School (Hoshuko)"
- **Leslie P. Shayer** – "The Impacts of Contemplative Pedagogy in Mandatory Post-Secondary Mathematics Courses"
- **Pauline Tiong** – "Teacher Noticing and Use of Language as a Resource in the Teaching and Learning of Mathematics"

The final plenary, "Put Your Students in a Pickle!," was delivered by Gord Hamilton. He shared innovative problem sets and mathematical games inspired by deep conjectures, such as the graceful tree conjecture—a puzzle that challenges students to assign labels so that the absolute differences of connected nodes are all unique. His session offered practical strategies for using puzzles and board games to foster rich mathematical thinking in classrooms and homes alike.

Topic Sessions

CMESG also featured shorter topic sessions highlighting emerging research in the field of mathematics education. Notable presenters included:

- **Jo Towers** – "Mathematics Experiences, Images, and Identities"
- **Doug McDougall** – "Supporting Teachers in Early Secondary School Mathematics"
- **Lauren DeDieu** – "Emphasizing Communication in University Mathematics Courses"
- **Fok-Shuen Leung** – "Humility, Pride, and Teaching Math"

Each of these sessions offered pensive provocations and insights into the evolving landscape of math education across Canada.

CMESG 2026 will be held at Queen's University in Kingston, Ontario—the location where the first CMESG was held in 1977. For insights into the conference, I encourage you to visit_www.cmesg.org, where the 2025 proceedings will be published and information about CMESG 2026 will be shared. ▲

▲ MB4T (Mathematics by and for Teachers): When Less Looks Like More – Making Sense of Decimals



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Dr. Sofia Ferreyro-Mazieres is an Assistant Professor in Mathematics Education at Wilfrid Laurier University in Brantford. Her work focuses on destreaming initiatives in mathematics and supporting all learners. Sofia currently teaches Primary/Junior and Junior/Intermediate mathematics education courses to teacher candidates. She has also worked as an elementary and secondary mathematics teacher in the Simcoe Muskoka Catholic District School Board.

As an elementary teacher, I remember a student confidently telling me that "0.64 is bigger than 0.8 because 64 is more than 8." It was a moment that felt both frustrating and entirely understandable. Although we had reviewed place value in depth, the student had not yet internalized that 0.8 represents eight-tenths, not simply "zero point eight." Even some of my other students, who were confident comparing whole numbers such as 432 and 277, felt unsure when they shifted to comparing decimals. Comparing decimals requires an understanding of place value to the right of the decimal point, and the ability to interpret decimals as quantities, not just strings of digits.

Let's pause for a moment and consider what comparing decimals demands of students. At its core, it requires a deep understanding of place value, recognizing that the position of each digit determines its value. It demands that the student look beyond the digits alone and consider the relative size of tenths, hundredths, and thousandths. They need to understand that 0.8 is eight-tenths, while 0.64 is six-tenths and four-hundredths, even though 64 might look larger than 8. Students also need to coordinate multiple representations: symbolic, visual, and verbal. This can be cognitively demanding, especially if they are still developing their understanding of fractions, proportionality, or the base-ten system.

To make sense of values like 0.64 and 0.8, students need a way to visualize decimals. If decimals are just abstract symbols, comparing them can be challenging. However, when students are supported in viewing decimals as measurable quantities, the relationships between them become more meaningful and easier to understand. One of my favourite ways to support students in developing this understanding is through visual models (see Figure 1). A simple hundredths grid can do wonders. When students shade in 0.64 on one grid and 0.80 on another, they can see the difference in size. They begin to realize that 0.8 fills more space. The visual representation, such as the number of squares shaded, often is more persuasive than abstract rules about aligning digits.

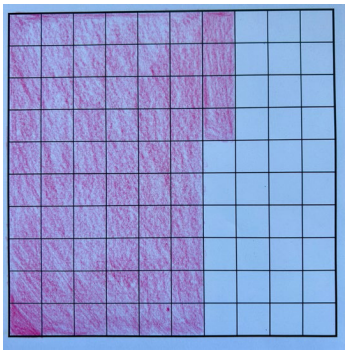


Figure 1: *Visual model of 0.64, six-tenths and four-hundredths*

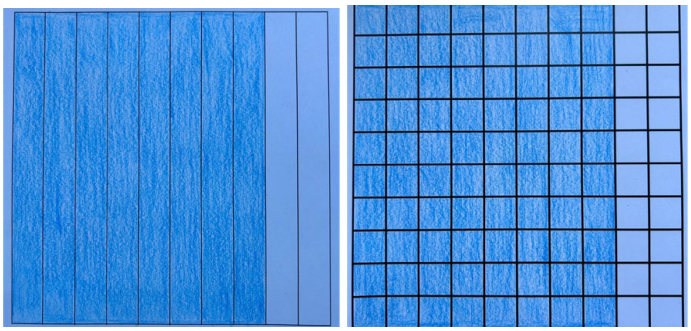


Figure 2: *From tenths to hundredths: Why 0.8 is 80/100, not 8/100*

To make this work with a hundredths grid, students need to understand what these numbers mean in relation to the model. A hundredths grid has 100 equal squares, so each square represents one-hundredth, or 0.01. To represent 0.64, students shade in 64 squares. For 0.8, they shade in 80 squares. However, this is where we as teachers need to be particularly attentive to how students interpret decimals. It is not uncommon for a student to mistakenly think that 0.8 means "8 hundredths" and try to shade in just 8 squares. This misunderstanding usually comes from interpreting the 8 as sitting in the hundredths place. To address this, we go back to place value: in 0.8, the 8 is in the tenths place, meaning it represents 8 tenths, or 8 out of 10 equal parts. Some students may begin by dividing a grid into 10 equal parts and shading in 8 of them, demonstrating their understanding of tenths (see Figure 2). At this point, we

want them to be able to represent that on the hundredths grid, inviting us into rich discussions with our students: what happens if we divide each of those 10 parts into 10 smaller, equal sections? We now have 100 squares, and those 8 tenths become 80 hundredths. The area shaded has not changed. That same space now shows up as 80 out of 100, reinforcing the equivalence between 0.8 and 80/100, not 8/100.

I also find value in exploring decimal comparisons through fraction equivalents. For students who are more comfortable with fractions, they understand that 0.64 means sixty-four hundredths: 64/100. When they look at 0.8, the conversion process itself reinforces place value: 0.8 means eight-tenths, which can be written as the fraction 8/10. To compare it fairly with hundredths, the student finds an equivalent fraction with a denominator of 100 by multiplying both the numerator and denominator by 10, which gives 80/100. Now the comparison can be done, noting that 80/100 is greater than 64/100. This also reinforces that decimals operate within, rather than apart from, our base-ten number system; both decimals and fractions are rational, parts of a whole, and can be expressed in more than one form. Moving between visual models and fractional thinking strengthens students' understanding of number sense and their flexibility with numbers.

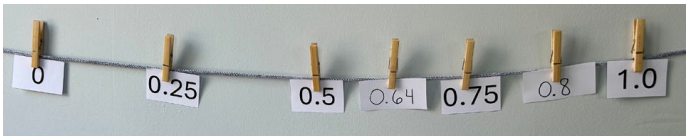


Figure 3: *Hanging number line*

Another helpful model is the hanging number line (see Figure 3). I like to start with benchmarks such as 0, 0.25, 0.5, 0.75, and 1.0. These points act as anchors, giving students a sense of scale, and helping them reason about where other numbers belong. To place 0.64 and 0.8, we begin by identifying the interval within which each number falls. For example, 0.64 is greater than 0.5, but less than 0.75. Similarly, 0.8 is just beyond 0.75 and less than 1.0.

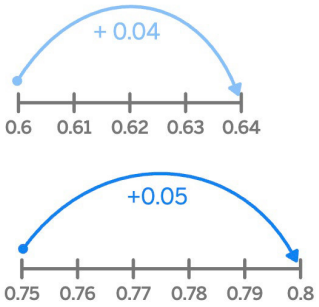


Figure 4: Exploring hundredths

To help students visualize this, I sometimes mark tenths (0.1, 0.2, 0.3, etc.) along the line in addition to quarters. From there, we begin to explore how many hundredths separate each point (see Figure 4). For instance, the distance from 0.6 to 0.64 is 0.04, while the distance from 0.75 to 0.8 is 0.05. We then look at the distance to 1.0 and see that 0.8 is closer to 1.0 than 0.64 is (see Figure 5). This type of reasoning not only supports comparison, but also lays the groundwork for operations with decimals, as students begin to see how values are spaced along a continuum, and how adding or subtracting tenths and hundredths affects position.

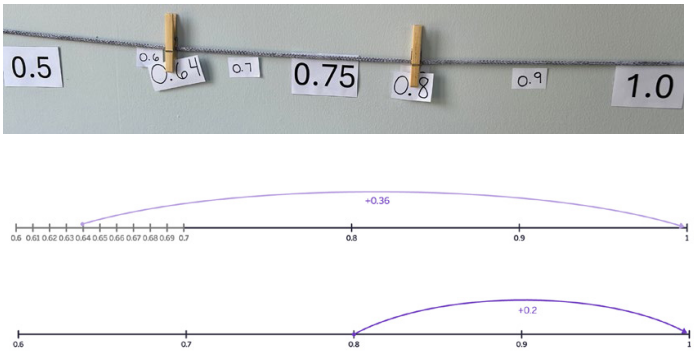


Figure 5: *Relative distance to 1.0*

Another way to address misconceptions is by inviting students into conversation. I often use true/false statements to prompt debate and reflection. An example is: 0.108 is greater than 0.2. Another example is: 0.7 and 0.70 are different numbers. These prompts create low-floor, high-ceiling opportunities for students to articulate their reasoning and challenge assumptions. They give us a window into how students are making sense of decimals. For example, when a student claims that 0.108 is greater than 0.2 because "108 is more than 2," we can help them revisit place value: the digit 2 in 0.2 represents two-tenths, while the 1 in 0.108 represents one-tenth, and the following digits extend into hundredths and thousandths. In discussing whether 0.7 and 0.70 are different, students confront the idea of decimal equivalence. Students begin to recognize that adding a zero in the hundredths place does not change the value, offering an opportunity to deepen their understanding of rational numbers and the idea that multiple representations can express the same quantity. When we engage students in these types of exercises, I make sure to ask them to show or prove their thinking by drawing number lines, building models, writing fraction equivalents, or using place-value charts (see Figure 6). This process externalizes their reasoning and gives them the chance to test whether their explanation holds true.

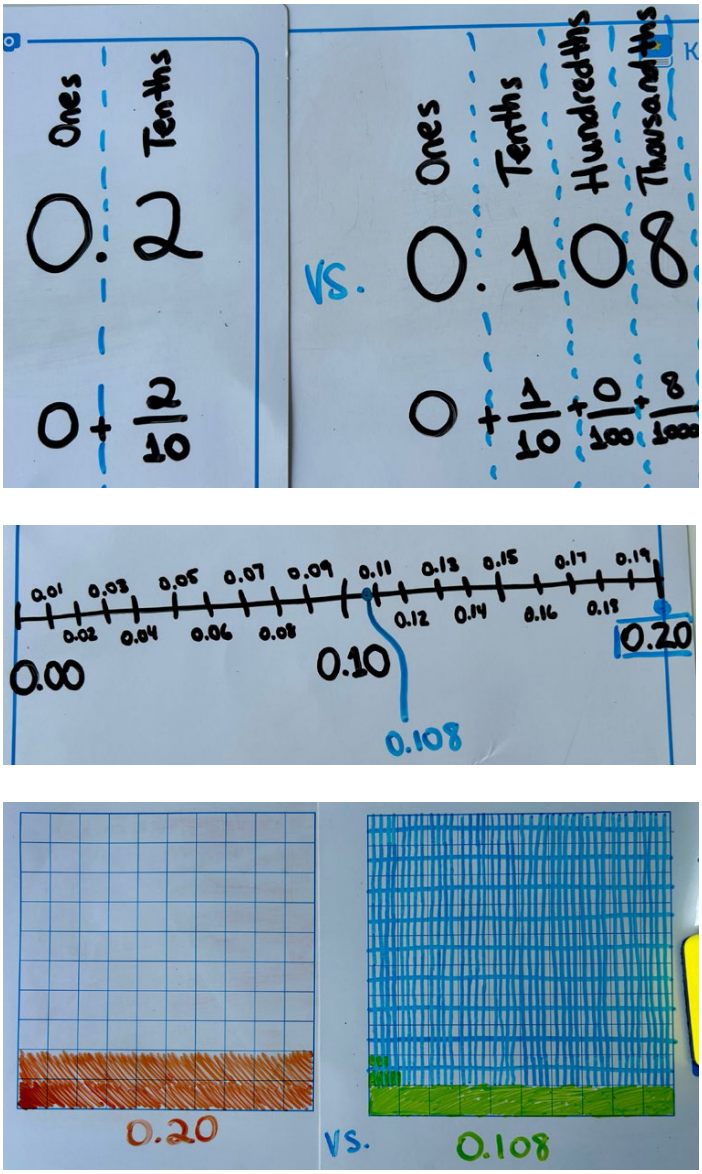


Figure 6: *Sample student responses*

Once students have developed a stronger sense of decimal numbers though models, number lines, place-value charts, and mathematical discourse, they are better equipped to understand them conceptually. This aligns closely with Jo Boaler's message in her recent book, *Math-ish: Finding Creativity, Diversity, and Meaning in Mathematics*. In her book, Boaler invites us to reframe mathematics as a space where flexibility, creativity, and approximation are not signs of weakness, but strengths. She reminds that real-world math often lives in the grey area, the "ish," where estimates, intuition, and sense making matter just as much as algorithms. When students use a number line to estimate 0.64, or explain that 0.8 is "closer to one," they are engaging in exactly that kind of thinking.

Boaler (2024) describes mathematics as a visual experience, especially when working with fractions. She warns that narrowly teaching fractions through algorithms alone can obscure students' ability to make sense of what a fraction is. She clarifies, "I am not arguing that algorithms are not useful; I am arguing that they should not be taught to students until they understand fractions conceptually" (p. 164). This parallels how we might teach decimals, providing space to estimate, model, and compare, before diving into procedures.

Earlier chapters in *Math-ish* also highlight the disconnect between how math is taught in schools and how it lives in the world. Boaler reminds us that most numbers we encounter in life are not exact, they are "ish," and thus encourages students to come up with an "ish" answer before doing a calculation. She notes that many students, when asked to estimate, solve the problem first and then round, a habit that misses the point of estimation entirely. The point of estimation is to help students develop a sense of number size and scale. It allows the student to anticipate whether their final answer is reasonable, whether it is too high or too low, and whether it makes sense in the context of the problem.

In my own classroom, I have seen how students with number fluency can intuitively generate "ish" answers, using guess and check, rounding benchmarks, or visual reasoning. They aren't afraid of being inexact. In fact, that flexibility often opens their minds to be critical thinkers. As Boaler (2024) writes, "When we bring ish into our lives, we are more protected from the dangers of perfectionism—a damaging mindset—and from binary thinking" (p. 113).

We see this play out in everyday conversations too. For example, we often answer questions such as, "How old are you?" or "How much paint do I need?" with "ish" numbers. Let's say you ask a child born on March 10, 2021, how old they are, they probably won't respond with "I am 4.477 years old," but rather, something like, "I am four and a half." That response is more than just rounded—it shows us how people naturally lean into estimation and relational thinking in their daily lives, and why this kind of reasoning should be welcomed in the classroom. This does not mean we abandon precision, but rather, we build it on a foundation of understanding. When we give students room to think, visualize, and talk through their ideas, they come to see decimals not as confusing, but as just another part of the number system, another way of confidently describing how much. What types of "ish" numbers do you use in your day-to-day life or classroom?

Reference

Boaler, J. (2024). *Math-ish: Finding creativity, diversity, and meaning in mathematics.* HarperCollins. ▲

▲ 2025 OAME/AOEM Award Recipients



David Stocker
Toronto District
School Board

The Award for Exceptional and Creative Teaching in Elementary Mathematics 2025

David is an exceptional educator, who integrates social justice into mathematics education, empowering students to use math as a tool for advocacy and change. For nearly 30 years, David has creatively addressed issues like environmental racism, Indigenous rights, and poverty. He's authored two books and shared his expertise through many workshops. David's students benefit from his engaging, differentiated lessons that build deep understanding, confidence, and lasting success in math for all learners. This award is a fitting tribute to his tireless efforts in empowering students through the power of mathematics.



Margaret Quinn
Retired Teacher,
President of TEAMS

The Award for Outstanding Contribution to OAME/AOEM and Mathematics Education in Ontario 2025

Margaret spent many years passionately supporting student-centred math education. She began her career as a classroom teacher, and later took on the role of a K–8 Resource Teacher at the Toronto Catholic District School Board, where she coached teachers and emphasized equity, manipulatives, and engagement. A TEAMS member since 2009 and president since 2015, she has led events like Math Olympics, virtual speakers series, and book clubs, while also co-chairing multiple OAME conferences, including the Equity Counts 2021 Conference. Now retired, she remains active in teacher education and professional learning. Her leadership and dedication make her a deserving recipient of this award.



Shelley Yearley
Curriculum Consultant,
Mathematics Project Lead

The Award for Leadership in Mathematics Education 2025

Shelley has had a profound, far-reaching influence on mathematics education. She has led impactful professional development and research, notably as a Provincial Math Lead with the Ontario Ministry of Education. Her work includes authorship of *Rethinking Fractions*, key roles in action research projects, and developing provincial professional development resources like the EduGAINS series and curriculum writing for Grades 1–9. Shelley's leadership has inspired province-wide improvement in mathematics teaching. With over 25 years of dedication, she remains a passionate, trusted leader, who continues to shape mathematics education. ▲

Continued from page 7



Figure 2: *From https://commons.wikimedia.org/wiki/File:Tally_marks.svg*

Chinese numerals (Figure 3) for one to three appear like the beginnings of a tally system. However, after three, the characters deviate from this pattern. Of course, historically, numerals were written differently, where they eventually evolved into the characters that we have today. This may explain why we have a brief tally-like pattern, it being a result of historical change.

一	二	三	四	五	六	七	八	九	十
1	2	3	4	5	6	7	8	9	10

Figure 3: *Characters for numerals made by Michelle Vuong*

▲ Elementary Math Matters: Just a darn minute—yesterday you said that *X* equals two: Consistency for Conceptual Clarity



LYNDA COLGAN
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Lynda Colgan is Professor Emerita, Faculty of Education, Queen's University; Founder and Faculty Lead for Science Rendezvous Kingston; and Executive Director (Education and Development) for Science Rendezvous National. In May 2025, she was awarded the King Charles III Coronation Medal by the Chancellery of Honours at Rideau Hall for her lifelong contributions to education. Her latest project is as contributing author for an upcoming Springer publication, Teaching and Learning Elementary Mathematics*. Lynda remains deeply committed to her two loves: STEM outreach/public education, and her English Cream Retriever, Annie.*

As adults, we routinely interchange words such as "smarts" and "stings" to mean something "hurts," and that hurt can range from a sore knee to wounded feelings, depending on the context. We do not bat an eye when we read "thanx" in a text message because we know that it is an informal form of "thanks." For the most part, we replace, swap, trade, exchange, substitute, and switch words commonly in conversation without impeding the clarity of the communication. All is well, if the other party in the exchange is another adult. But there are unanticipated consequences when educators haphazardly exchange our words, representations, and symbols in mathematics instruction, especially with young children.

This point arose earlier this year during a Math Night presentation for the Northwestern Ontario Association for Mathematics Education (NWOAME). The main topic for the evening was coding, and my emphasis was on the roots of coding: the solid foundations required before the quantum leap to coding stories or commercial classroom robots. I discussed what coding is and what it looks like in early math curriculum; and argued that neither the computer, nor technological toys, should be the first thing used when teaching young children to code.

The introductory slides made my case for a strong focus on spatial sense through modelling and visualizing, and the development of precise spatial vocabulary. After considering examples about simple and complex positions (e.g., the birthday cake is still *inside* the box, or I put the pickle for my hamburger *above* the patty and *below* the lettuce), I moved on to arguably one the most advanced positional words for young children: "right" and "left," using this quotation from Ginsburg and Oppenzato (2010):

> *Left* and *right* are notoriously difficult for young children to learn, and they need a lot of practice with these ideas. First, children need to remember that one hand is on the right and the other is on the left. This idea is reinforced when children put a sticker on their right hand and then shake hands properly. Then children must apply the idea of right and left to external objects. This is especially difficult because these concepts are always relative to the direction the child is facing. For example, if Mario is looking at a table from one direction, he sees a book on his right and a block on his left. When he moves to the opposite side of the table, he sees the reverse.

I discussed the importance of the teacher and the students facing the same direction when raising the right or left hand, then asked participants to consider the cognitive dissonance that would be felt by students sitting directly across from one another at their desks when working on "right" and "left" tasks. Repeatedly, I reminded the group that we need to step back, look, and relearn through the eyes of the child. One participant raised the point that the hand that makes the "L" shape is always your left hand, regardless of where you are facing—a visual and tactile cue. A question arose about left and right labels for the classroom and led to questions about whose right and whose left. One person suggested that such labels would only work in the classrooms of yore, when students were seated in rows, all facing the chalkboard, and a teacher at the "front" of the room.

Then, to make the point that the development of misconceptions can happen when well-meaning families turn to "educational" resources on television or the Internet, I played a video from Debby Doo Kids TV called the *Right Left Kind of Day* (which had 942 000 views). An observant educator will see the egregious error immediately in Figure 1: Debbie is facing the children, but the children are facing the screen. It is true that the children's *right* and *left* will be correct; therefore, when the song encourages them to raise their right hand and turn to the right, all will be well. However, while Debby is performing the actions, she is in fact raising her left hand and turning left.



Figure 1: *Screenshot from the Debby Doo educational video about right and left*

To create a simple manipulative to aid in consolidating the right/left concept, I suggested using a free "Flat Stanley" (or any other simple cut-out paper doll) template, with the front and back connected to a wooden stick for easy orientation, and a concrete reminder about viewpoint. Students can replace the doll face with their own photo or represent themselves. The doll can be a valuable manipulative later as lessons move on to coding stories, since it can be used to act out movements and turns. I ask you to think about this question: *Is it best to label the right and left hands or not? Why*?

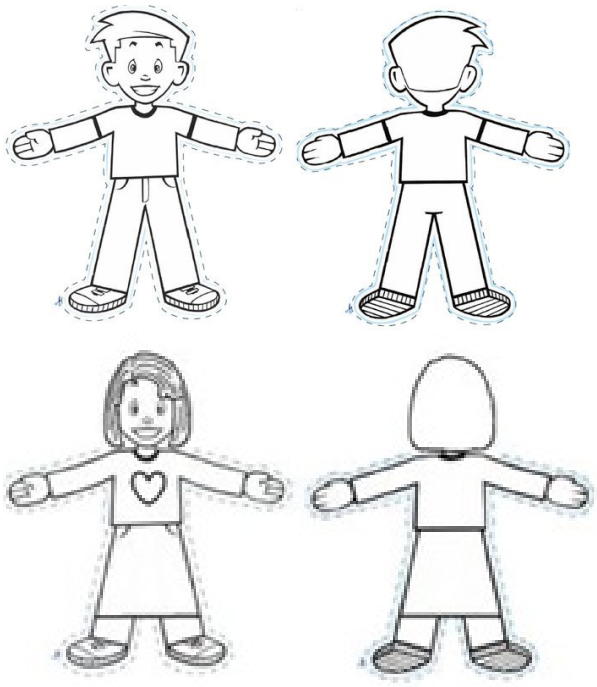


Figure 2: *Free customizable paper doll templates (from www.docformats.com/flat-stanley-templates/) to help young students understand the complex positions of right and left relative to their own position*

The next topic for consideration was the concept of grids—important for introductory coding, but critical for later concepts in mapping, sequencing, and giving directions, and eventually graphing and coordinates. There was consensus that most grids are introduced using squared paper on a chart board, or the equivalent on an interactive whiteboard, before introducing a grid via a masking-tape outline on a classroom carpet or piece of paper.

To segue into the next point that I wished to emphasize, I turned to a simple task from Ginsburg and Oppenzato (2010): an object is placed on a grid, as shown in Figure 3, and the child is asked to describe it so that a partner can locate the object on his own copy of the grid. Initially, the child may respond by saying, "It's over there," or "It's above another square," but those descriptions lack specificity. What the child must do is establish a starting point for locating the object, then a sequence of designated steps that include numerical and directional information.

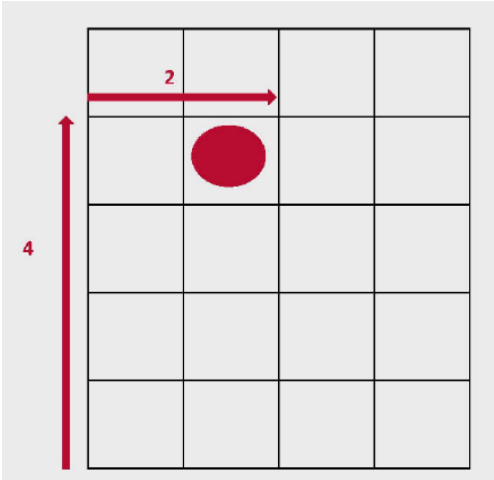


Figure 3: *This simple task involves locating an object in two-dimensions—the 5 x 5 grid has height and width, columns, and rows.*

I mentioned in passing, that typically, children come to Kindergarten with a strong sense of up and down based on their informal experiences at home or in preschool: pointing up to the sky and down to the floor, stretching your fingers up as far as you can reach, or bending down to touch your toes.

I then asked the group to consider what language they would use for a particular task. If the task in Figure 4 were displayed on an interactive whiteboard, the instruction would provide navigational information to a partner who cannot see the grid. The goal of the task being that she can draw the "map" accurately, using those instructions, i.e., identifying the start point as the house, then walking to pick up the bicycle, and finally, riding it to the barn.

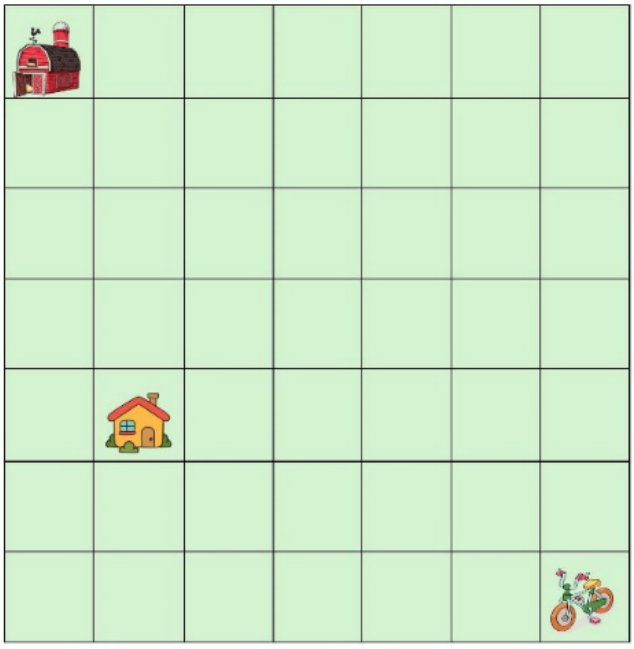


Figure 4: *The challenge presented was to direct a partner to move physically from one place (home) to arrive at the barn by bicycle.*

Immediately, there were responses such as: "Home is three squares up and two squares to the right. Next, walk right five squares, then go down two squares to get the bicycle. To get to the barn, ride up six squares and finally ride left six squares." Without saying whether or not the answers were correct, I then asked, "What happens if I print the pages (one with features and one blank grid) on paper, to place on their desks to use for the task. Would your instructions still hold?"

There was dead air and a collective realization that "up" and "forward" were not interchangeable and that "walk right" or "walk left" did not mean walking sideways (e.g., sidle) to arrive at the destination. While we, as adults, "know what we mean," what we say and omit often translates differently in the child's mind.

Moving from one place to arrive at another, physically and on paper, requires the ability to articulate descriptions of objects relative to other objects. Grid navigational skills can be taught wherever there are floor tiles, e.g., moving around the house or classroom, using simple directions such as: begin at the back door, walk three floor tiles forward, turn right, then walk forward one tile, turn right, and finally, walk forward two tiles to find the hidden treasure.

The next point in the presentation was about the ways in which simple grids themselves can introduce confusion for young children, especially if teachers do not choose examples carefully and are inconsistent in the presentation of tasks involving grids. Figure 5 presents examples of navigational activity pages representative of those found on the Internet. Note the inconsistency in labelling the grid squares, as well as the fact that some grids number the squares themselves, while others label the grid lines. Similarly, some use squares as locations, while others use intersection points on the grid.

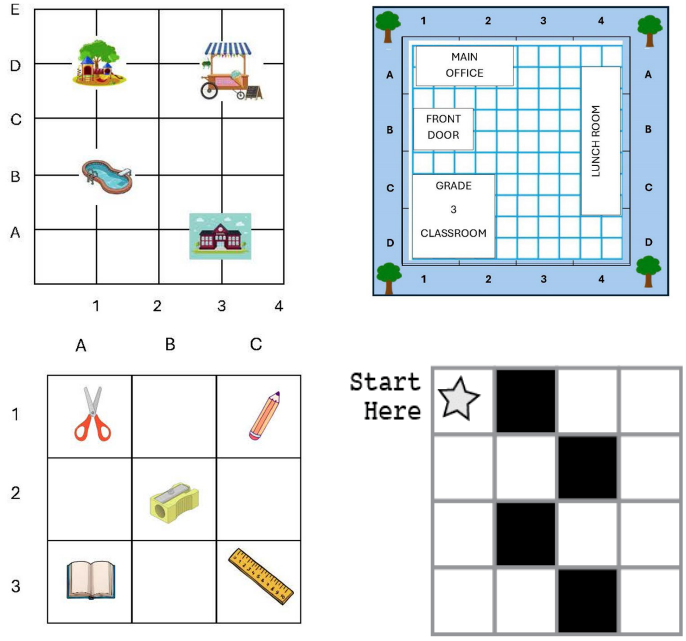
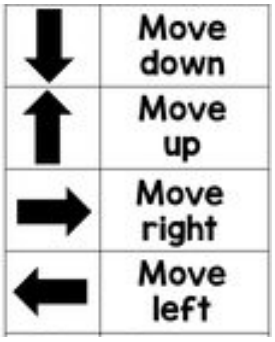
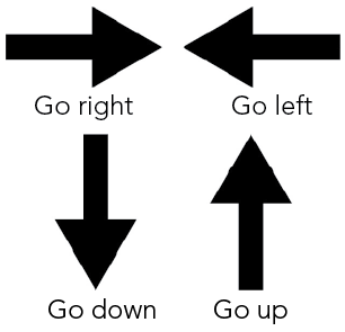


Figure 5: *Sample illustrations that emphasize the disparate systems used to label simple square grids for early mathematics navigational/mapping tasks*

Storytelling and retelling are integral skills for navigational tasks in that they build young children's spatial vocabulary and conceptual knowledge—precursors to reenacting a journey through code, and communicating it using coding conventions.

Such learning activities are broadly described under the catchall phrase "unplugged coding," many of which encourage the use of "coding cards" that use symbols and numbers to record steps in a coding story sequence. In general, the problem with Web-based or commercial sets of coding cards is that there is no consistency in the graphics or language used to communicate the same instruction.





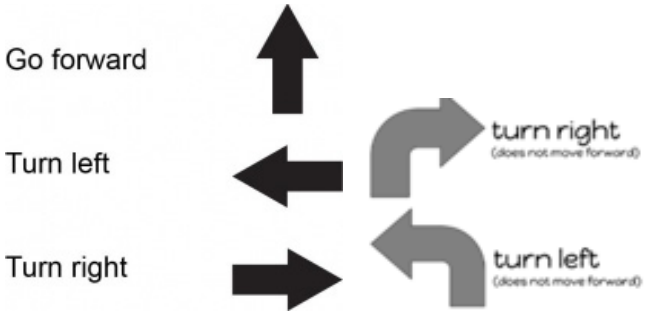


Figure 6: *A sample of directional command "coding cards" used to communicate specific steps used in moving from square to square on a grid to reach a location*

Often classroom activities begin with children challenged to place items on a grid. These can be physical items from the classroom manipulative collection, which are placed on a floor grid, or images drawn by the children placed on a paper grid. After the children create a map, they can "walk" the route or act it out (using their "Flat Stanley" avatar attached to a bulldog clip, as shown in Figure 7) to generate a coding mat. The mat allows them to determine the sequence, using the navigational language, and recording the code.

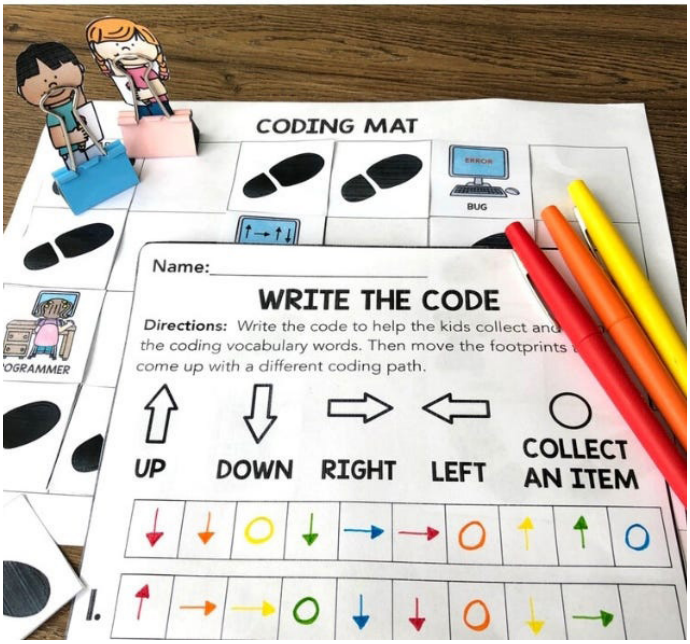


Figure 7: *A representative set of unplugged coding resources from Parthavi Gupta's blog. A glance at available resources suggests that each developer, whether a classroom teacher sharing materials or sets created by commerial companies, use their own symbol cards and vocabulary for paper-and-pencil spatial and navigational tasks.*

Coding in the early mathematics classroom is about multiple opportunities to use spatial, positional, and navigational language and apply that language and, more importantly, the concepts that are represented in other contexts like coding stories. And, like language development, instruction in coding, with its deep roots in spatial sense and geometry, is found in educational processes and experiences that are carefully crafted to anticipate and ameliorate potential childhood misconceptions and confusion. Without some awareness of children's developing abilities, even the most well-intentioned educator may not recognize why a particular task failed.

I close with this example. In April 2025, I received this email from a former B.Ed. candidate, who recently was assigned to a Kindergarten classroom in Kingston after many years in the Junior Division.

> *I'm writing to say a belated "Thank you" for the extremely informative and helpful webinar that you presented in January. Learning from you again came at the perfect time for me. I transferred to [name of public school] this year to teach Kindergarten for the first time. It's been a year of constant learning for me. I'm grateful to be teamed up with a superstar ECE, who has been very patient while I've been learning and making mistakes. I've been overwhelmed at times, but I LOVE teaching Kindergarten! The kids are adorable, unique, and authentic.*
>
> *Thank you for the opportunity to learn about Spatial Sense/Coding for early learners! I had many "aha" moments during your presentation. I didn't know that developmentally, children of Kindergarten age can't see/imagine a different perspective than their own until you spoke about it. This explains why my students weren't able to make this snowman collage as shown. All but one of my students made their snowman facing forward!*



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In the Middle: Made in Canada, When Math Comes Home



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Carly Ziniuk teaches Grade 12 Data Management, Advanced Placement Research, and Advanced Placement Statistics at the Bishop Strachan School in Toronto. She is active in adopting real-life data to engage her students in solving problems. Among 30 global STEM leaders, Carly was part of 2024 NASA Space Apps Collective. As a Science Communicator, she is working to connect astronomy and mathematics for learners of all ages across Canada.

Home is where you learn who came before you.
Home is where you discover who you are
Home is where you imagine who you might become.

From *When the Stars Came Home*, written by Brittany Luby.

For the past few years, I have been using this column to illustrate the connections between astronomy and mathematics by showcasing NASA resources available for middle school learners. In April 2025, Abacus Data reported that 86 percent of those surveyed in Canada say they feel a sense of pride when choosing Canadian-owned brands, while 84 percent see it as a way to regain a sense of control in an unpredictable world. In light of such feelings, I have decided to focus here on resources created here at home. I hope this will amplify the vast array of Canadian content available for middle-level learners and teachers in math classrooms.

Perhaps you have been noticing the "Product of Canada" Maple leaf logo emblazoned on your favourite breakfast cereal, or the "Made in Canada" filter when shopping online. Not surprisingly, these certifications have specific quantitative measures. Under the Consumer Packaging and Labelling Act, "Product of Canada" means that at least 98 percent of the production occurred in Canada. Kevin Maimann of the CBC (2025) summarized this as "Essentially, it was made in Canada by Canadians, with negligible imported ingredients." Although similar in appearance, "Made in Canada," means that more than half, but less than 98% of the total production costs occurred here. The Competition Bureau recommends, but does not require, percentage qualifications for Made in Canada for the labels (such as, "Made in Canada with 60% percent Canadian content and 40% percent imported content").

As an example, French's ketchup might be produced in Canada with 100 percent Canadian-grown tomatoes, but is made with liquid sugar, which is not Canadian—thus, it is "made in Canada." Encouraging a student to look for products with one of these labels might provide some delightful and diverse home-made percentage questions. There are a number of apps and websites, such as madeinca.ca/grocery-store-guide, that provide up-to-date lists of Canadian grocery products. Note, however, that on the website, attention is needed to the table of contents to find French's ketchup, which is made in Canada by a foreign company.



Figure 1: *Official logos from the Government of Canada*

The Fields Institute for Mathematical Sciences recently collaborated with Discover the Universe (and the University of Toronto's Dunlap Institute for Astronomy and Astrophysics) to produce classroom-ready materials supporting their "Math in Literacy" initiative-highlighted Canadian book, *When the Stars Came Home* (fieldsacademy.ca/math-in-books).

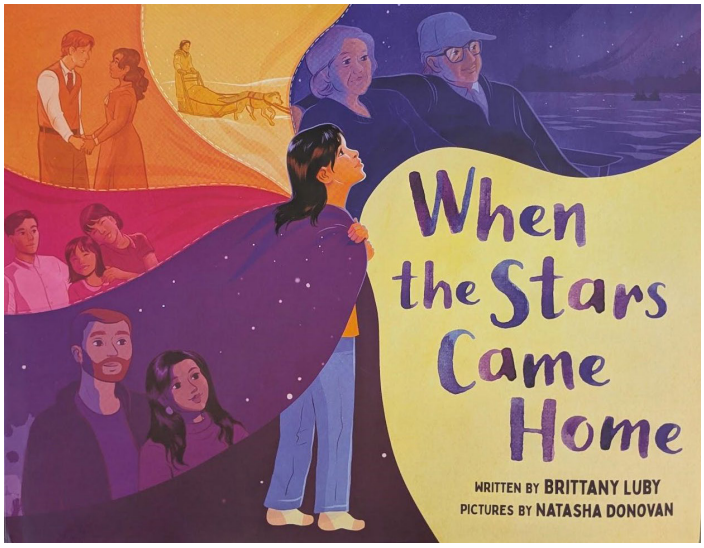


Figure 2: *Cover of the Discover the Universe and Fields Institute collaboration book choice for "Math in Literacy,"* When the Stars Came Home

When the Stars Came Home recounts how Ojiig misses the stars watching over his family when they move to the city. His family gives him a traditional special star-shaped quilt that wraps him in family stories, bringing him the feeling of home. The Fields Institute Grades 6–8 activity, *Travelling to the New City*, asks students to consider trips they have taken, like Ojiig, and how types of transportation differ in terms of time and distance travelled. Connecting this to Ojiig's nostalgia for viewing the stars, you may want to compare these travelled distances to the student's estimated astronomical distances. The true values are included here, which allows you to compare students' estimates to the true values and to distances they have travelled on their journey.

	Estimate of distance above Earth (in km)	Researched distance above Earth (in km)
Von Karman Line (where astronauts enter "space")		100
International Space Station		400
Hubble Space Telescope		525
The Moon		384 400
James Webb Space Telescope		1.5×10^6
The Sun		1.5×10^8

These lessons from Fields also connect the book to quilt patterns of repeated shapes and tessellations, like Ojiig's star blanket. In their lessons from K–8, students investigate the geometry of quilts, and create their own patterns, using tangram tiles. I encourage you to use this as a springboard to explore our *Gazette* archives for other geometric quilt pattern block investigations, including my June 2018 column, "Fun with Fractions" (Vol. 56, #4), and Lynda Colgan's hot-off-the-presses June 2025 (Vol. 63, #4) "Quilts as the Holders of Stories and Geometry."

Last year, the Royal Ontario Museum (ROM) also highlighted Canadian quilts from their permanent collection with an exhibit titled "Quilts: Every quilt tells a story." Although this featured collection is no longer an in-person curated display, many of the exhibit materials are available online at www.rom.on.ca/whats-on/exhibitions/quilts-made-canada.

I visited this exhibit at the ROM last year with my good friend and collaborator, Pauline Martin, who teaches at Beaver Brae Secondary School in Kenora, where we created a series of math questions arising from the exhibit. In addition to being a 2016 Descartes Medalist and extraordinary math teacher, Pauline is an experienced quilter. While looking at many of these quilt stories, she explained the amount of math utilized by the creators of such Canadian quilts, even those over 200 years old. Quilts allow students to surround themselves with these old and new mathematical perspectives.

Zero-waste quilting ensures that no cloth is wasted. One way to do that is to cut angles with such care that they can be rotated and reflected, using each piece mindfully in your design. Many older quilts were made with zero-waste or low-waste pattern cutting, in order to save material and thus cost. The rediscovery of this technique is often ecologically motivated. When Pauline and I explored each quilt in the exhibit, we noted the consistency and repetition of slopes and angles for many of the pieces, likely a testament to zero waste.

Together we observed that the geometric planning required by many patterns matched what we would be teaching in our classes in the coming weeks: the quilters needed to identify and plan with the mathematical concepts of parallel, perpendicular, reflections (opposite slopes, reflections in the *y*-axis and *x*-axis), symmetry, plus horizontal and vertical lines. Additional areas that might need to be considered with quilt blocks include trigonometry (special triangles), similar and congruent triangles, area, perimeter, fractions, and proportional reasoning.



Figure 3: *Lily-pieced 1840's quilt from Lindsay, Ontario, viewed at the ROM Quilts exhibit*

Consider the lily-pieced 1840's quilt, shown in Figure 3, from Lindsay, Ontario, found in the ROM collection.

- How many different slopes does this piece of the quilt illustrate?

- How many different angles would you have to establish for the lily-pieced quilt in order to make the fewest number of cuts?
- What would a quilt block look like with exactly four different slopes?



Figure 4: *From ROM Quilts exhibit, pattern planning examples*

Pauline and I also explored the paper patterns that were included in the ROM exhibit (Figure 4). We noticed a similarity to the tiles we had used in our classroom tessellations, and others that were directly drawn on graph paper. Using a similar approach with my Grade 9 students, I created the grid-pattern block shown in Figure 5, similar to the lily-pieced quilt design, and also evocative of the star blanket in *When the Stars Come Home*.

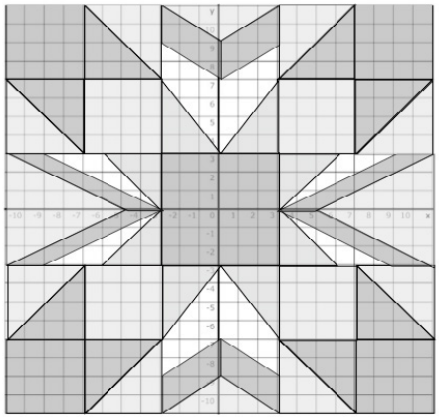


Figure 5: *Sample quilt block on a grid, mathematically adapted from lily-pieced and star blanket designs*

- How many different slopes are there in this one block?
- How do the angles and slopes relate to each other?
- What are the different slopes?
- What are the different angles?

In a different way of considering the stars, the Canadian Space Agency is expanding their Youth and Educators Activities to showcase comprehensive STEM learning. Two such activities that would work well for middle school learners both involve gravity.

In the data collection activity, *Measure the Effect of Gravity on Your Height,* students measure their height at the beginning of the day and the end of the day, noticing the effect of gravity on their spines. Middle school students can consider percentage change overall, and collect more data throughout the day, to notice how the shortening occurs over time. For classes that rotate throughout the day in a tumbling schedule, this might be easier to record by starting each lesson with a height measure. If you do have your students measure their heights, and track their growth throughout the year, an activity I often did with middle school students, you should take care to note the time of day students measure their heights. Without noting the time of day and day of the year, the measurements might lead one to draw very different conclusions, obscuring the effect of adolescent growth over the year, or missing the effect of gravity in a day.

I also suggest combining this with the related activity, *See Why Astronauts Are Taller in Space,* which uses elastic bands, sponges, and textbooks to measure the effect of compression. In the exercise, students show how reducing the number of elastic bands placed around a set of textbooks results in less compression on the sponges. This model represents the spinal cord, where the force of gravity compresses the spinal discs. When we have not had the effect of gravity on our spines downward, such as when we sleep, the effect of that compression is less. Astronauts actually are physically taller in space because the force of gravity is weaker. In math classes, I would expand this activity to explore the effect of altering the variables. Although the activity does not indicate this explicitly, I would use a graph to illustrate these changes and, with higher-level classes, consider mathematical modelling for predictive purposes:

- How does the compression percentage change with additional elastic bands?
- Does the number of books in the stack affect the compression amount?

Once a Canadian astronaut is assigned to a space mission, the Canadian Space Agency works with them to create a patch to wear, reflecting personal preferences and mission objectives. Anishinaabe artist Henry Guimond and Dave Courchene of the Turtle Lodge worked with CSA astronaut Jeremy Hansen to create this equal-sided heptagonal patch (Figure 6) for Hansen's participation in the Artemis II mission to the Moon. This seven-sided shape alludes to the traditional First Nations teaching of the Seven Sacred Laws. In my December 2024 column, I included Jeremy Hansen's Artemis patch, using Polypad within the

Amplify suite. There I noted that Polypad can be used to quickly attempt to tessellate, showing that Hansen's regular heptagonal patch does not! Connecting this to quilt blocks is a beneficial connection. Polypad can also be used for the Fields quilt-block activities with both tessellations.

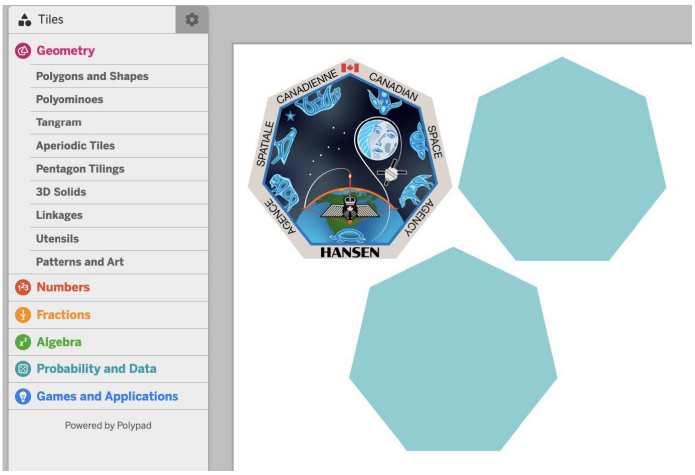


Figure 6: *Polypad using CSA Astronaut Jeremy Hansen's Artemis patch*

In the CSA activity, *Patch Maker, Patch Maker, Make Me a Patch*, students explore the features of human space flight mission patches at www.asc-csa.gc.ca/eng/missions/, then design their own patches. In our classrooms, many students are captivated by the Desmos Art Challenge. Polypad and Desmos are now both part of the Amplify platform and can be integrated into Desmos Activity Builder together. A *Made in Canada* moment for our students would be to express each of the CSA Astronaut's patches, using equations on Desmos, or create them in Polypad. Beyond Jeremy Hansen's heptagonal patch, some other examples I suggest considering are David Saint-Jacques (Expedition 58/59), STS-78 Mission patch including CSA Astronaut Bob Thirsk and featuring Canadian West Coast native cultural elements, and the Artemis Program patch in support of the CSA's involvement in the plan to land humans on the southern lunar pole in 2026.



Figure 7: *David Saint-Jacques CSA patch*



Figure 8: *STS-78 Mission Patch featuring CSA Astronaut Robert Thirsk*

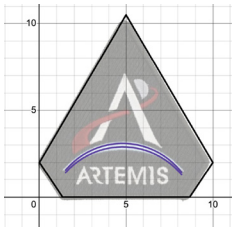


Figure 9: *Artemis Program patch, showing in-progress Desmos graphing*

Co-curricular programming options available right here in Ontario, with strong math connections inside and outside the classroom, include Launch Canada (www.launchcanada.org), Inspire Tech Canada (www.inspiretech.ca), and Stem Racing (www.stemracing.com). Launch Canada and Inspire Tech Canada create workshops for teachers and students, preparing students with interactive, self-guided lessons focusing on technology fundamentals and technologies such as rocketry. Stem Racing (www.stemracing.com) introduces students to STEM through the creation of a miniature Formula 1 (F1) car, with a program designed for teachers, informed by engineers, and endorsed by F1.

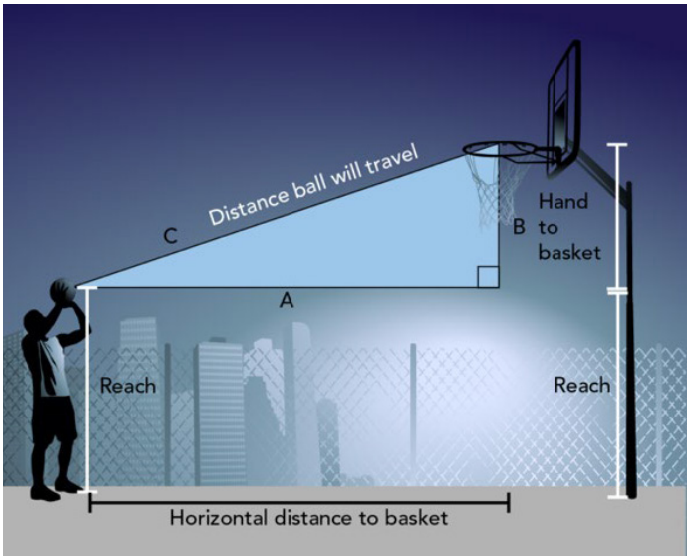


Figure 10: *Illustration from Let's Talk Science* Pythagorean theorem in Sports *activity, which could be altered for different players' heights*

One distance we would need to find out is the horizontal distance from the player to the basket. It is helpful to know that some places on a regulation size basketball court have standard distances from the basket. For example, the **free throw line** is 4.57 metres (15 feet) from the basket and the **three point line** is 6.71 metres (22 feet) from the basket.

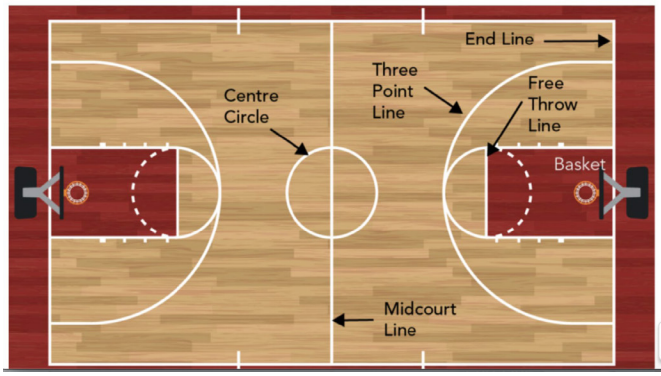


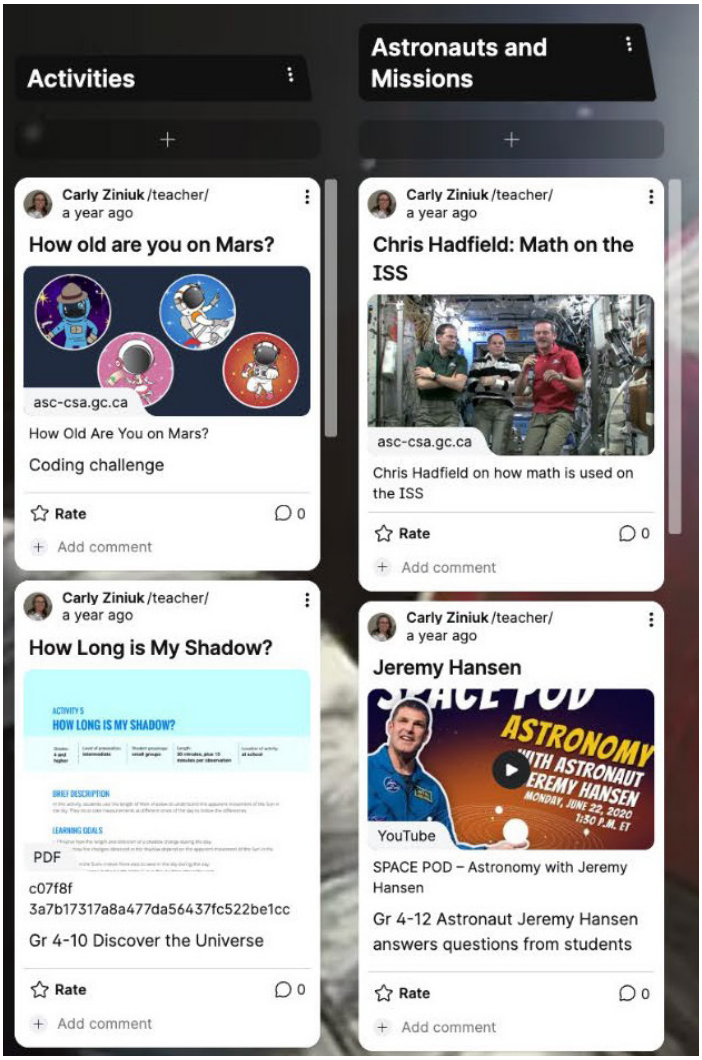
Figure 11: *Sample problem from Let's Talk Science activity,* Running Up the Score: The Math in Sports

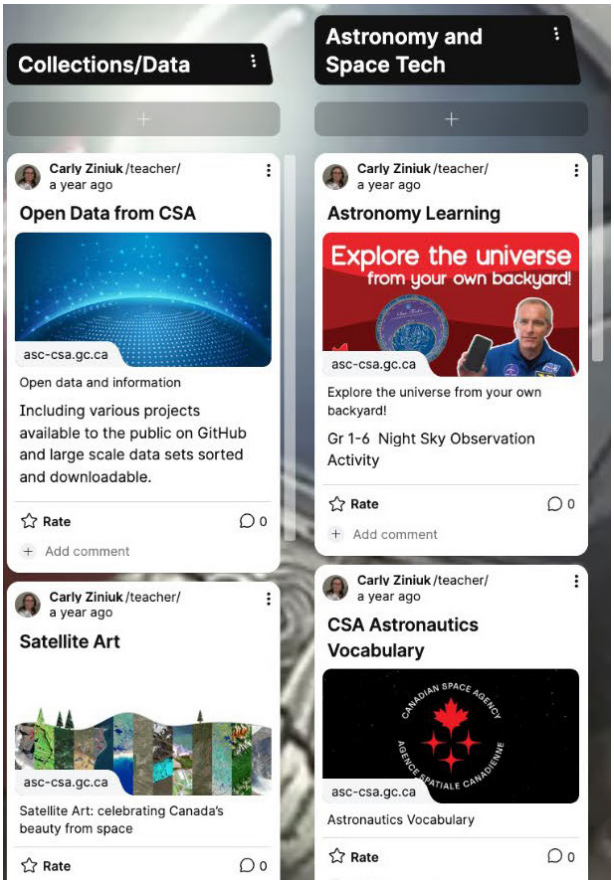
You may also be curious to check out the vast resources at Canadian home-grown Let's Talk Science, https://letstalkscience.ca/taxonomy/term/2976, with the focus on

the math element of STEM learning. The activity, *Running Up the Score: The Math in Sports*, connects Canadian Jim Naismith's invention of basketball with the use of the Pythagorean theorem. This is a wonderful example of a Let's Talk Science classroom-ready lesson, including photocopy-ready worksheets, along with a teacher guide and sample solutions. If you are considering these basketball lessons, it might be interesting to compare the heights of professional NBA players with that of your Canadian middle-schoolers.

Additional activities and lessons of note in Let's Talk Science include:

- *Overcoming Math Anxiety with Confidence*
- *Young Entrepreneurs – Math and Your Dream Business*
- *Cooking Up Math: Learn about how math is used in the kitchen*
- *Buying Power: The Math Behind Money*
- *How Likely Is It? Learn about probability*





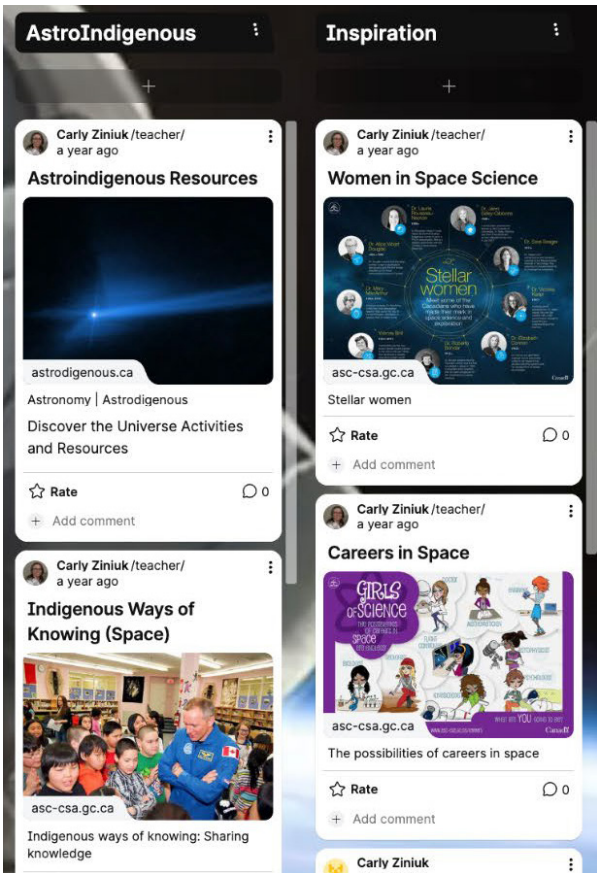


Figure 12: *Padlet showing Canadian space and math resources*

These Let's Talk Science pieces are cross-curricular, with math at the forefront. As I have demonstrated in my previous *Gazette* articles, the connection between space and mathematics works well for integrated middle school STEM classes. Let's Talk Science's collection of Space Resources (https://letstalkscience.ca/topic/space-education) is vast and worthy of exploration.

You can find the links for various resources I have mentioned here on my Padlet of Canadian Space Resources (https://bit.ly/CDNSpaceResourcesZiniuk). I continue to add new activities and lessons to this Padlet easily as math continues to come home in space too. As Brittany Luby writes, *"Home is where you learn who came before you. Home is where you discover who you are. Home is where you imagine who you might become."* I hope, as many of our students start anew with this school year, that these local materials can help our students feel at home in their math classes right here in Canada.

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Ode to Readers

Often when the *Gazette* is read,
Teachers' inspiration is fed,
Students delight, with a laugh,
At the new ideas for teaching math,
And everyone gets ahead.

▲ What's the Problem? Triangulating a Square



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Shawn is a former mathematics teacher, department head, and consultant. He strongly believes in the central role of problem solving in the mathematics classroom. In his retirement, he continues to be involved with mathematical activities as much as possible.

Welcome back, problem solvers. I hope you all had an enjoyable, restful summer. We will start this school year off with a problem suggested to me by *Gazette* Editor, Tim Sibbald:

Your task is to cut a square into triangles. You must have each type of triangle, and each triangle is only allowed to count as one type: scalene, isosceles, equilateral, right, acute, and obtuse. What is the least number of triangles you can find?

It's a cute little problem. By its design, it can be addressed by any student who knows the terminology in the problem. One can just think of terms one at a time and cut the desired triangle out of the remains of the square, although a bit of thought can be used to come up with a solution more quickly. However, what hooked me were the different variations of the problem that can be explored, and the fruits that these explorations yield.

First, the problem presents us with an opportunity to revisit some terminology. We start our triangle classification by first considering angles. Traditionally, we define an acute triangle as one with three acute angles, a right triangle as a triangle with a single right angle, and an obtuse triangle as a triangle with a single obtuse angle. Notice that the acute triangle is defined slightly differently than the other two. What if we slightly changed the wording so that there is a consistency to it? For example, what if we defined the terms in the following ways?

- An acute triangle is a triangle, whose largest angle is acute.
- A right triangle is a triangle, whose largest angle is right.
- An obtuse triangle is a triangle, whose largest angle is obtuse.

Isn't that more satisfying? We can even "justify" these

definitions. For example, if the largest angle is obtuse, which is larger than 90°, then since the three angles add to 180°, the other two angles must sum to less than 90°, which means that they must both be acute. We can do the same for a right triangle, but for an acute triangle, if the largest angle is acute, then the other two angles must also be acute.

This little playing around with semantics might get some students to reflect more deeply and realize a couple of facts:

- All triangles contain *at least* two acute angles.
- All triangles contain an angle that is 60° or larger (why?).

We also have three classifications according to side lengths: scalene, isosceles, and equilateral. We all know what those mean, right? For example, an isosceles triangle has two sides of equal length, an equilateral triangle has three sides of equal length, and a scalene triangle has no sides of equal length. However, if I pick an equilateral triangle and point at any two of its sides, they are equal. Does that mean that the triangle is also isosceles?

If you answered, "No! Absolutely not!," you are correct! However, if you answered yes, you are also correct. How can this be so? Let's look at a property of isosceles triangles to get a little insight.

First, a few more terms. Consider Figure 1 below. We will call the two equal sides the *legs* of the triangle, and the other side will be called the *base*. The angle across from the base is called the *apex*.

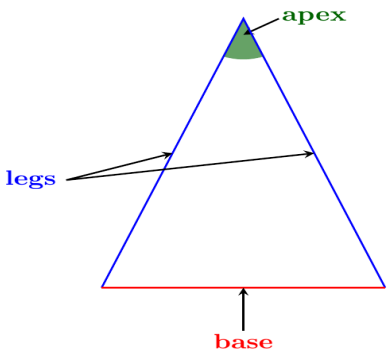


Figure 1: *An isosceles triangle*

We can derive some interesting properties of isosceles triangles by drawing the segment from the apex to the midpoint of the base. As we can see in Figure 2, this divides the isosceles triangle into two smaller triangles. A quick check shows that all three sides of each of the small triangles have a corresponding side: one leg in each triangle, one-half of the base in each triangle, and the median from the apex to the midpoint is a common side. Thus, the two triangles are congruent, by side-side-side congruency. This yields that the two base angles are also equal, another well-known fact about isosceles triangles. Also, we get that the angles at M must be equal, which means they must be right angles. This means that the median from the apex, the angle bisector of the apex, and the perpendicular bisector of the base are all concurrent. This allows us to make another deduction: if we have any line segment AB and draw its perpendicular bisector, and then pick any point P on that perpendicular bisector, triangle PAB will be isosceles with base AB and apex $\angle P$.

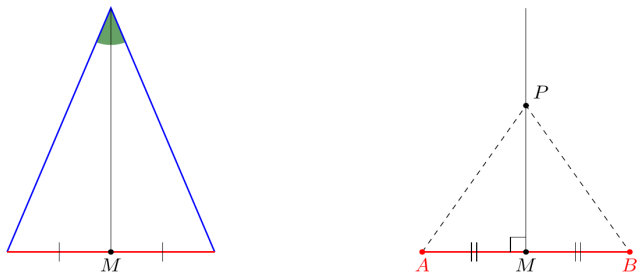


Figure 2: *The perpendicular bisector of the base of an isosceles triangle*

What this means is that, if we have any line segment AB and draw its perpendicular bisector, then picking points on the perpendicular bisector on one side of AB will create every possible isosceles triangle with base AB. Isn't that cool? This can be reproduced easily in any dynamic geometry software package like GeoGebra, and students can manipulate it and see the family of isosceles triangles. Thinking a bit about this family of triangles, if P is close to AB, the apex will be obtuse, and if it is very far away, it will be acute and can be as small as we want it to be (as long as we go far enough away). What that means is we should be able to move P to a location where the apex is exactly 60° and thus our isosceles triangle is, in fact, equilateral!

So, in some cases, it makes sense to think of an equilateral triangle as a "special" isosceles triangle. However, when we are naming geometric figures, we generally give them the *best* name. So, if you were presented with an equilateral triangle and asked to name its type, the best answer would be *equilateral*. However, if I asked if the triangle were isosceles or acute, it would be fine to answer in the affirmative to both because the term *equilateral* implies both of those terms.

All this discussion of semantics is related to my thoughts about the problem. When I first tackled the problem, I was focusing on the "least number of triangles" part and seeing what would happen if I allowed a triangle to fall under multiple categories. As such, a triangle that was equilateral would also qualify as isosceles and acute—three of the six

terms accounted for with one triangle! That would leave us with right, obtuse, and scalene to deal with. Unfortunately, as we saw earlier, a triangle cannot be both right and obtuse, so we will need at least two more triangles to complete the list. Fortunately, it is possible to have a right-scalene triangle as well as an obtuse-scalene triangle. So, if we had three triangles, we could satisfy the list. If we wanted, we could also pair isosceles with obtuse or right and have each of the three triangles account for two terms. As an added challenge to readers, try to come up with a minimal diagram that contains triangles of each of the types (where one can count for more than one term). It doesn't need to be in a square. I will share my example at the end of the column.

Back to the problem at hand, let's think about cutting a square into triangles. How many triangles could a square consist of? You can probably convince yourself, that if we have a convex *n*-gon, it should be able to be cut up into *n* triangles. Just pick some interior point and connect it to all the vertices, and there you have it, *n* triangles. In Figure 3, we can see this depicted in a triangle, quadrilateral, and a pentagon.

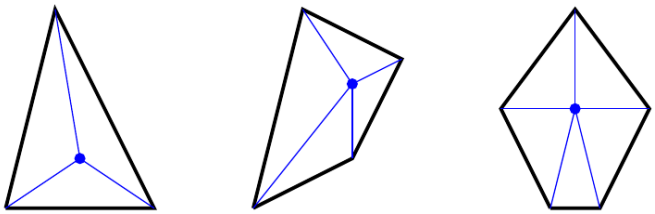


Figure 3: *Cutting* n*-gons into* n *triangles*

However, if we did all our cutting from a fixed vertex (see Figure 4), we could get $n - 2$ triangles from an *n*-gon. This is like the method used when determining the sum of the interior angles in an *n*-gon. Thus, for a hexagon, we would get $6 - 2 = 4$ triangles; for an octagon, we would get $8 - 2 = 6$ triangles; and for a decagon, we would get $10 - 2 = 8$ triangles.

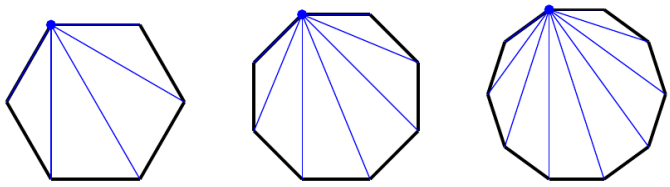


Figure 4: *Cutting* n*-gons into* n *– 2 triangles*

So, starting with a square, the fewest triangles we could cut it into would be $4 - 2 = 2$, by drawing in one of the diagonals. If we picked an interior point, we could cut it into four triangles. How do we get to the six triangles necessary to cover all the terms? This turns out to be straightforward. Anytime you draw a *cevian* in a triangle—a line segment joining a vertex to a point on the opposite side—the triangle is cut into two smaller triangles. Thus, we can always add a new triangle by adding a cevian to an existing triangle. For example, in Figure 5, we start with two triangles by drawing a diagonal, then add cevians to existing triangles to go from two to six.

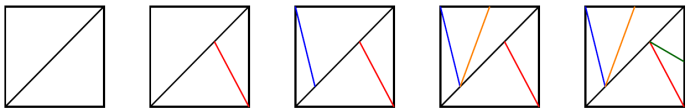


Figure 5: *Using cevians to create more triangles*

Since we are considering the types of triangles and are exploring using the cevian trick from above, it would be interesting to investigate how the new triangles are related to the old triangles in this process. This step alone is an intriguing and worthwhile investigation. Let's see what happens.

If we start with an acute triangle, we can always drop a perpendicular from one vertex to the opposite side (a height, or altitude). This will cut the acute triangle into two right triangles, as shown on the left of Figure 6. If we draw a cevian other than the altitude, then it will meet the opposite side in an obtuse and acute angle pair, which means that the original triangle would be cut into another acute triangle and an obtuse triangle. So, starting with an acute triangle, we can end up with either right/right or acute/obtuse.

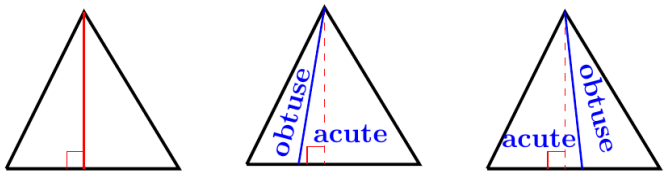


Figure 6: *Cutting an acute triangle with a cevian*

With a right triangle, if we draw a cevian from either of the acute angles, it cannot be perpendicular to the opposite side, or else it would reproduce a side of the triangle, not a cevian. Thus, the cevian must meet at an acute/obtuse pair, separating the original triangle into an acute triangle and an obtuse triangle. However, when a cevian is drawn from the right angle, it can be perpendicular to the hypotenuse, creating a right/right pair, where each of the new triangles is similar to the original, and thus similar to each other (why?). A cevian from the right angle that is not perpendicular to the hypotenuse creates an acute/obtuse pair. So, a right triangle can be cut into either right/right, right/obtuse, or acute/obtuse.

The obtuse triangle case is a little trickier. You can end

up with acute/obtuse, right/right, right/obtuse, and obtuse/obtuse. I will leave it for the interested reader to verify these cases.

We can go through the same exercise with the terms *scalene*, *isosceles*, and *equilateral*. The easiest case to deal with is the equilateral triangle. When a cevian is drawn from one of the vertices, it automatically creates two triangles, each with a 60˚ angle in them, as in Figure 7. Any triangle that has a 60˚ angle means the other two angles add to 120˚, which means they average 60˚. Hence, we have two possibilities: the two other angles are 60˚ and the triangle is equilateral, or one of the angles is less than 60˚ and the other is greater than 60˚. In our case, the cevian cuts one of the original angles into two pieces, each smaller than 60˚. This means that the three angles in each newly formed triangle are unequal, and hence, the sides are unequal, so and equilateral triangle will always be cut into two scalene triangles.

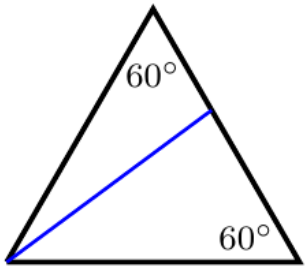


Figure 7: *Cutting up an equilateral triangle*

We saw earlier that the perpendicular bisector of the base of an isosceles triangle always goes through the apex. Flipping this property around means that if we draw the perpendicular bisector of any side of a scalene triangle or of the equal sides of a non-equilateral isosceles triangle, it intersects one of the other sides, not at a vertex. Hence, if one draws a cevian to that point of intersection, an isosceles triangle will be formed. In Figure 8, we can see examples of this in the scalene triangle on the left and the isosceles triangle on the right.

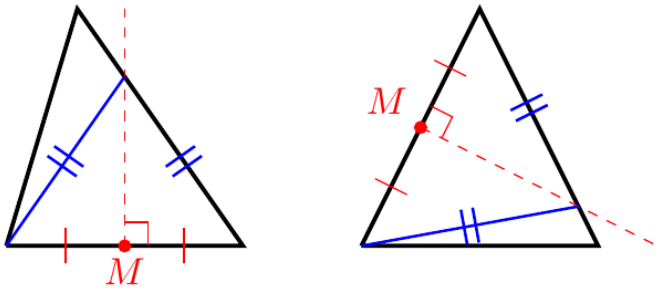


Figure 8: *Cutting an isosceles piece*

Is it possible to cut a triangle into two triangular pieces with a cevian and one of the pieces be equilateral? Since the new triangles will have one of its angles from the original triangle, then we will only get an equilateral piece if one of the angles of the original triangle was . We saw that if we start with an equilateral triangle, both pieces will be scalene, so it won't work in this case. If one angle in an isosceles triangle is 60˚, then all must be, and the triangle must be equilateral. Hence, we cannot get an equilateral piece starting from an isosceles triangle.

However, if we have a Φ,60˚,θ triangle with $\Phi < 60^\circ < \theta$, then we can draw a cevian from the largest angle so that the piece on the 60˚ side of the original triangle is also 60˚, and an equilateral triangle will be formed. What will the other piece be? I will leave that for the interested reader to investigate.

There are still lots of avenues we could pursue with this investigation. We could put all the terms together and classify what can happen to each of the seven types of triangles: equilateral, acute-isosceles, right-isosceles, obtuse-isosceles, acute-scalene, right-scalene, and obtuse-scalene. We could investigate the conditions for an isosceles triangle to be cut into *two* isosceles triangles. Have fun with your investigation.

We are almost ready to address the original problem. However, I am not quite ready to conform to the conditions. Still focusing on the phrase "least number of triangles," I am reminded of puzzles that ask, "How many triangles (or squares, or whatever) are in this diagram?" The key to these types of puzzles is that there are triangles within triangles, so let's use that idea to our advantage.

Drawing in a diagonal, we cut the square into two right-isosceles triangles. Hence, we can use one of them for the term *right*, and one for *isosceles*. Drawing a cevian from the right angle to the hypotenuse of a right triangle creates either two more right triangles (and in this case, they would both be right-isosceles) or an acute triangle and an obtuse one. So, drawing in one more line segment brings us up to four of the six terms taken care of. The one that is going to be interesting is the equilateral. Since we need a 60˚ angle, we cannot use any of the 45˚ angles in our triangle. There are a couple of ways to do this.

One way is to draw a cevian from an unused right angle of the square, at 60˚ to one of the sides. This will meet the diagonal at 75˚, so we can drop a cevian from that point to the base, giving us the desired 60˚ angle and an equilateral triangle. In Figure 9, we can see our six types: right (1 or 2 + 3 + 4), obtuse (2 or 4), equilateral (3), scalene (4 or 2), acute (3 + 4), and isosceles (2 + 3 + 4 or 1).

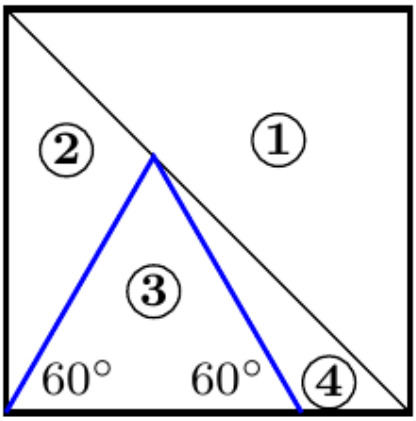


Figure 9: *Six triangle types with four "pieces"*

There is at least one more way to get all six terms with four pieces. I leave it to the interested reader to discover it and to verify that the terms I have claimed above are valid.

However, this doesn't satisfy the conditions of the problem. In the original problem, we are cutting the square into pieces, and the implication is that the pieces themselves must satisfy the terms. So, our solution above, where we combined two or three triangles to make a larger triangle, would not be allowed. With all that fiddling about done, let's finally get to the original problem.

I played around a bit with it and came up with a couple of solutions. As with the last examination, I figured getting the equilateral piece would be troublesome. As such, I started with the equilateral piece by drawing a segment from one of the vertices at 60˚ to one of the sides. In Figure 10, we can cut off a 30˚, 60˚, 90˚ triangle (1), which we could use for the terms *right* or *scalene*.

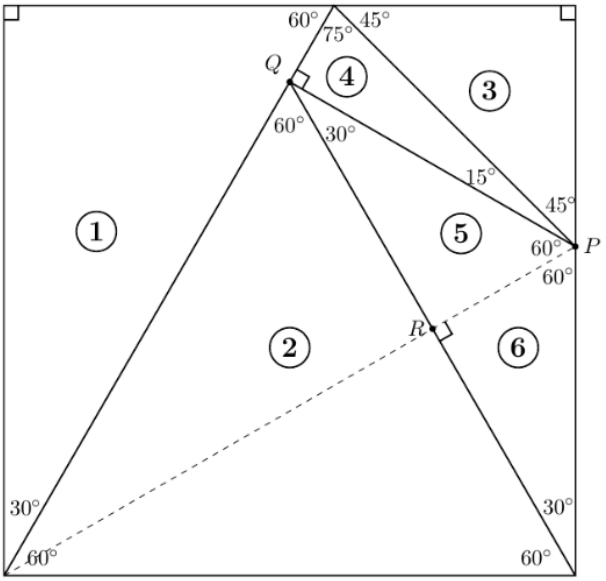


Figure 10: *A solution to the problem*

From the point where the cut meets the side of the square, draw a segment that makes a 45˚ angle, with the side meeting the other side at point *P*. Cutting along here creates a right-isosceles triangle (3), taking care of another term. We can now draw another segment in at 60˚ to the bottom side to create our equilateral triangle (2), marking the top vertex *Q*. Before we cut it off, if the dashed line is drawn as shown in Figure 10, from the starting vertex to the vertex of triangle 3, it can be verified that it is perpendicular to the side of the equilateral triangle, which we mark as point *R*. We can now cut off the equilateral triangle (2), and if we cut along the segment *PQ*, another right-scalene triangle is created (4). With our first four triangles, we have accounted for the terms *right* (1 or 4), *scalene* (4 or 1), *isosceles* (3), and *equilateral* (2). The only terms left are *acute* and *obtuse*. Drawing a cevian from *P* to any point on the opposite side other than *R* creates an acute angle on one side and an obtuse angle on the other. As long as that cevian doesn't make an angle of 30˚ or larger with the dashed line, then both angles at *P* will be acute, and the triangles formed at 5 and 6 will be acute and obtuse, in some order.

When I presented this to Tim, he shared his solution with me, given below in Figure 11. He pointed out that when he created his solution, he started with the triangles and built the square later on. The solution is also designed to be *dynamic*; that is, the two points that have arrows pointed to them can be manipulated (this was created in GeoGebra), and you would get new solutions.

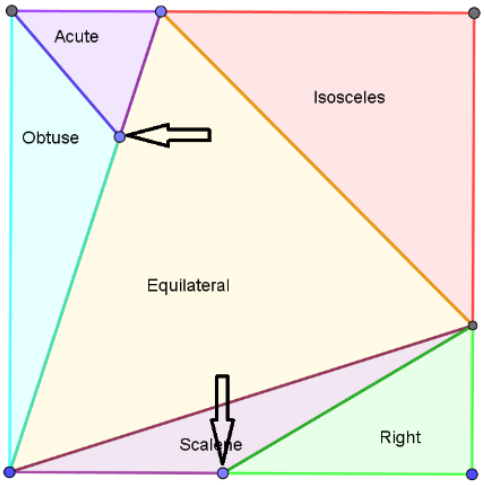


Figure 11: *Tim Sibbald's solution*

Looking at this diagram sent me down another rabbit hole. I got thinking about how an equilateral triangle could be drawn in a square with all of its vertices on the perimeter of the square. I played with it a lot and came up with a nice solution. It is probably a bit much for regular classroom use, but would be a good challenge for an interested student. I

am writing it up for my column, "Problem Solving Vignettes," that appears in the problem-solving journal, *Crux Mathematicorum*. It will be published around the same time as this column, so interested readers can check out the September 2025 issue (and others!) at www.cms.math.ca/publications/crux/.

Before we finish, we have one loose end to tie up. Earlier in the column, I challenged you to come up with a minimal diagram, where all six types of triangles were present, but where a triangle could stand in for more than one term. I present the diagram in Figure 12. This diagram was created by starting with a 30˚, 60˚, 90˚ triangle and drawing the median from the right angle to the hypotenuse. By design, the original triangle is right and scalene, and the median cuts this triangle into two smaller triangles.

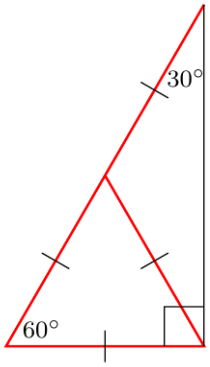


Figure 12: *Six triangle types in one figure*

An interesting property of right-angled triangles is that they can always be inscribed in a semicircle; that is, using the hypotenuse as the diameter of a circle, the circle will also pass through the vertex of the right angle. Since the centre of the circle will be the midpoint of the hypotenuse, this tells us that the median to the hypotenuse is half the length of the hypotenuse, making the top triangle isosceles. Since the top angle is 30˚, this makes the isosceles triangle a 30˚, 30˚, 120˚ triangle, making it also obtuse. This forces the other two angles in the bottom triangle to be 60˚, making it equilateral and acute. We could have also used the fact that the ratio of the side lengths in a 30˚, 60˚, 90˚ triangle is $1:\sqrt{3}:2$, to see that the side lengths of the bottom triangle are all equal. The diagram is minimal, as it only needs four line segments, and any diagram made with only three could draw, at most, one triangle. However, there could very well be other solutions. I would be interested to learn if any readers found any other solutions.

As always, I am interested to receive problems or comments from readers. Until next time, happy problem solving! ▲

▲ OMCA News

GERARD LEWIS
(OMCA PAST PRESIDENT)
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Welcome to a new school year! The Ontario Mathematics Coordinators Association (OMCA) is pleased to begin another exciting year of collaboration and professional growth with mathematics coordinators, consultants, and coaches from across Ontario.



At our final members' meeting in June, outgoing President Gerry Lewis (Toronto Catholic District School Board) passed the ceremonial fan—now transformed into a beautiful plant—to incoming President Melissa Peddie (Near North District School Board). OMCA is delighted to introduce our 2025 Executive Team:

- President: Melissa Peddie (Near North District School Board)
- Vice-President: Alyssa Dixon (Halton District School Board)
- Past President: Gerry Lewis (Toronto Catholic District School Board)
- Secretary: Allison Lawrence (Brant Haldimand Norfolk Catholic District School Board)
- Treasurer: Mariarosa Schniering (York Catholic District School Board)

We are proud to announce the recipients of the 2025 OMCA Award for Excellence and Innovation in Mathematics Teaching: Laurie Clayton and Gail Degagne (Near North District School Board), and Rhonda Hewer (Waterloo Region District School Board). This annual award recognizes OMCA members who demonstrate outstanding contributions to mathematics education in areas such as curriculum implementation, technology integration, equity, assessment, and professional learning. Congratulations to Laurie, Gail, and Rhonda on this well-deserved recognition!

Continued on page 38

▲ Useful Theories to Know About: Stage-Environment Fit Theory



JEFF IRVINE
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Jeff Irvine, Ph.D., has been a secondary mathematics teacher, department head, and vice-principal. He has taught at three faculties of education and at a community college. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. With over 45 years in education, Jeff is particularly interested in the interplay of student motivation and mathematics achievement.

I wish that I had known about this theory when I was in the classroom. This is a well-researched and well-substantiated theory about learning. A Google search for "Stage-Environment Fit Theory" returned 4.2 million results. Since it was first proposed by Jacquelynne Eccles and Alan Wigfield in 1993, Stage-Environment Fit Theory has demonstrated its usefulness to teachers in many ways.

Stage-Environment Fit Theory

Stage-environment fit theory examines how well a student's stage of development and their educational environment match. For instance, as they negotiate being away from their home and familiar surroundings, a student starting Kindergarten needs a warm and empathetic teacher who establishes boundaries and promotes the student's socialization. Stage-environment fit theory is also very important as a student goes through adolescence. As a child grows and develops through adolescence to young adulthood, they progress through a myriad of changes. These changes include puberty and its associated physical and emotional changes, growing cognitive development, increased importance of peers, social activities, and sexual feelings, as well as a growing need for autonomy and decision making (Deci & Ryan, 2008). These changing needs may lead to conflicts with parents, possibly anti-social behaviours, and in extreme cases, delinquency.

Why Should We (Math Teachers) Care About This Theory

Students do not all develop at the same rate. Teachers in Kindergarten to Grade 5 may want to identify alterations to their classroom routines and teaching strategies to address the requirements of students who are developing more quickly (or more slowly) than their peers. For example, some students may need more independence and more opportunities for autonomy earlier than others, while other students may continue to need more structure and direction.

A significant misfit between developmental stages and school environments often occurs in the transition to middle school or junior high school. The student needs more independence and choice, as well as more challenging cognitive tasks and the ability to be included in decision making. Frequently, what the educational environment provides is a greater focus on discipline and classroom management, low-level cognitive tasks, more structure and less self-sufficiency, and little or no role in decision making for the student. The result of this mismatch of environment with developmental stage is reduced student motivation, lack of engagement, apathy, and in extreme cases, total disengagement and dropouts. If teachers are aware of the student's needs, they can take actions to ensure a better fit between the student's requirements and their educational environment.

Secondary school teachers need to recognize and adapt their instructional strategies to address their students' growing thirst for increased autonomy and control over some aspects of their learning, as well as to meet the needs of students' growing cognitive development through more challenging cognitive tasks, with less structure and direction. Clearly, we want to develop our students as interested and independent learners as they progress through high school and into their next phase of learning, whether in university, college, or the workplace.

If You Want to Learn More

Butelo, J. (n.d.). EDUC 331: Review of identity and introduction to stage environment fit [Video]. YouTube.

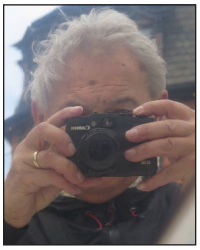
A video introducing the theory in education. (The link for readers of print copies is www.youtube.com/watch?v=rH8MmN3TSSA.)

Deci, E., & Ryan, R. (2008). Self-determination theory: A macrotheory of human motivation, development, and health. *Canadian Psychology, 49*(3), 182–185. doi.org/10.1037/a0012801

Eccles, J., Wigfield, A., & Midgley, C. (1993). Negative effects of traditional middle schools on students' motivation. *Elementary School Journal, 93*, 553–574. doi.org/10.1086/461740. The original formulation of the theory.

Our Animated Box. (n.d.). Who are you? Broadening the theory to person-environment fit theory and occupations outside education. (The link is www.youtube.com/watch?v=GWGbOjlJDkU.) ▲

The Math Teacher's Home Base: Tools, Resources, and Inspiration



RON LANCASTER
ron2718@nas.net

Ron is an Associate Professor Emeritus at the University of Toronto, where he taught mathematics courses for pre-service teachers for 17 years. Ron taught middle and high school mathematics for 21 years in coed public schools and an all-girls school. Ron's professional activities include conference presentations/consultations in North America, Asia, England, the Middle East, Africa, and India. Ron has authored/edited over 200 articles for NCTM and OAME. Ron received the 2015 Margaret Sinclair Memorial Award Recognizing Innovation and Excellence in Mathematics Education, awarded by the Fields Institute, and was the recipient of the 2022 OAME Lifetime Achievement Award.

Welcome to "The Math Teacher's Home Base," where we will explore the power of creating a resource-rich, inspiring space for all things related to teaching mathematics. This column is dedicated to helping you build a personal teaching environment that supports not only lesson planning, but also a lifelong passion for math—whether you're brainstorming lessons, experimenting with activities, or simply enjoying math for fun.

The Toy Office

"The toy office" is how my spouse's two grandsons refer to my home office when they visit us. The room is filled with toys, games, puzzles, and mathematical oddities. Some of the items were acquired recently for teaching, or for the boys, but many have been in my classrooms for decades. In this column, I want to focus on three commercial games that have stood the test of time—*SET*, *Rush Hour*, and *Abalone*. They continue to be sold because they're fun to play, and they offer teachers a powerful way to engage students in mathematical thinking. They also happen to be all-time favourites of my dear friend, Carly Ziniuk, a fellow educator with a deep love for playful learning.

SET: A Mathematical Classic

SET was invented by Marsha Falco and has long been a favourite in classrooms and at math conferences. I first met Marsha and her husband, Bob, at an NCTM conference, where they had a booth that was constantly surrounded by teachers eager to try the game. I've had the pleasure of chatting with them on many occasions, and it's no surprise to me that *SET* has remained popular well beyond the world of education. The game continues to sell widely, and for good reason—it's deeply mathematical, visually engaging, and easy to learn, but hard to master.



Photograph 1: *Box and cards from the game SET*

Each card in *SET* has four attributes:

- Shape (oval, squiggle, or diamond)
- Colour (red, green, or purple)
- Number (one, two, or three symbols)
- Shading (solid, striped, or outlined)

There are three choices for each attribute, and each card is unique. A set consists of three cards, for which each of the four attributes is either all the same or all different. If even one attribute has two the same and one different, it is not a set. (Examples follow in a moment.)

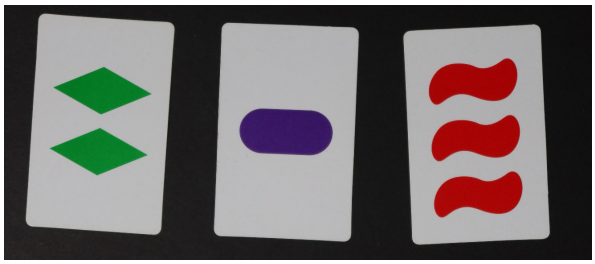
In my classroom, I kept multiple copies of *SET* on hand. It was a go-to activity when students needed a break, finished a test early, or just wanted to play something mathematical and fun. The official rules state that there are no turns—players race to find a set as quickly as possible—but I found that this format didn't always work well in a classroom setting. To ensure everyone had a fair chance to participate and practise their skills, I often modified the rules so that students took turns. This approach slowed the pace slightly, but it encouraged more strategic thinking and made the game more inclusive.

During my years teaching at St. Mildred's-Lightbourn School—an all-girls K–12 school in Oakville—we welcomed visitors from Japan each year. My Grades 7 and 8 students would proudly teach the Japanese students how to play

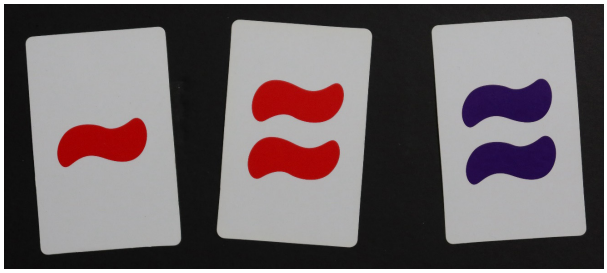
SET. Watching these international exchanges unfold over a shared puzzle was always a highlight of their visit. Mathematics, as *SET* so wonderfully illustrates, truly is a universal language.

To help readers understand what makes a valid set, I've included four images below, each showing a group of three cards. Two of the groups are valid sets, and two are not. Try identifying the valid sets, keeping in mind that each attribute is either the same for all three cards or covers all options for that attribute.

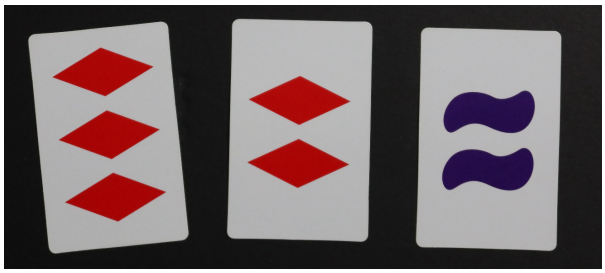
NOTE: For the convenience of readers who are using the black-and-white print version of the *Gazette*, descriptions of all the cards are included in the captions, and all shadings are solid.



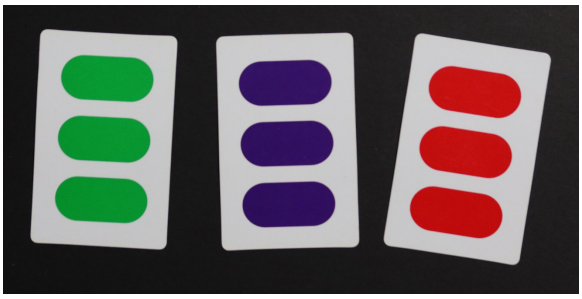
Photograph 2: *Is this a set?*
Left card: Shape diamond, Colour green, Number 2, Shading solid
Middle card: Shape oval, Colour purple, Number 1, Shading solid
Right card: Shape squiggle, Colour red, Number 3, Shading solid



Photograph 3: *Is this a set?*
Left card: Shape squiggle, Colour red, Number 1, Shading solid
Middle card: Shape squiggle, Colour red, Number 2, Shading solid
Right card: Shape squiggle, Colour purple, Number 2, Shading solid



Photograph 4: *Is this a set?*
Left card: Shape diamond, Colour red, Number 3, Shading solid
Middle card: Shape diamond, Colour red, Number 2, Shading solid
Right card: Shape diamond, Colour purple, Number 1, Shading solid



Photograph 5: *Is this a set?*
Left card: Shape oval, Colour green, Number 3, Shading solid
Middle card: Shape oval, Colour purple, Number 3, Shading solid
Right card: Shape oval, Colour red, Number 3, Shading solid

If you'd like more practice, the Daily *SET* Puzzle at www.setgame.com is an excellent interactive tool for playing and building fluency.

After students play *SET*, I like to challenge students with some rich mathematical questions based on the game:

Mathematical Questions Inspired by *SET*

Question 1:

Why are there 81 cards in the deck? If each card had four attributes, with four choices for each attribute, how many cards would be needed for this version of *SET*?

Question 2:

For any two cards, is it always possible to find a third card that can be combined with these two to form a set? How many possibilities are there for this third card?

Question 3:

For any card, is it always possible to find two cards that can be combined with it to form a set? How many possibilities are there for these two cards?

Question 4:

Arrange nine cards in a 3 × 3 grid so that there is a set in every row, column, and main diagonal. Such an arrangement is often referred to as a magic *SET* square.

Question 5:

As you may have suspected while playing, it is possible to have 12 cards that contain no set. Can you find such a collection?

Question 6:

Is it possible to have 13 cards that contain no set? What about 14? 15? 16? And so on.

Question 7:

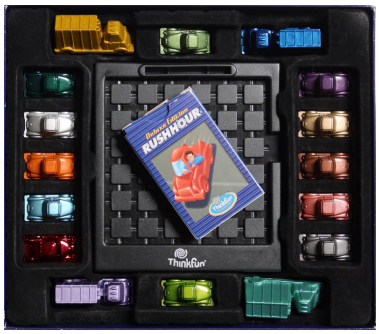
Can only three cards be left at the end of the game?

Rush Hour: A Sliding Puzzle That Teaches Perseverance and Vectors

Rush Hour (Photographs 6 and 7) is a sliding-block puzzle originally made by Binary Arts, later known as ThinkFun. The player's goal is to move cars and trucks out of the way so that the red car can exit the grid (parking lot).



Photograph 6: Rush Hour *game box, ready to be opened*



Photograph 7: *Cars, trucks, tray, and challenge card deck from* Rush Hour

Each puzzle (Photographs 8 and 9) has a set-up card that determines the arrangement of the vehicles, and all movements must follow the constraints of the board—vehicles can only slide forward or backward, not rotate.



Photograph 8: *Challenge Card 1, Beginner level*



Photograph 9: Rush Hour *board set up according to Challenge 1*

I have used *Rush Hour* in my classroom for decades. It encourages strategic thinking, planning ahead (like in chess), and perseverance. It's a perfect off-screen activity that brings genuine satisfaction when a student solves a challenge.

While I don't ask many mathematical questions about *Rush Hour,* I have made the following connections:

The movement of cars and trucks provides an intuitive introduction to vectors, including direction and magnitude.

I challenge students to design an unsolvable puzzle and then explain why the red car cannot escape. This provides a foundation for mathematical proof.

Rush Hour introduces students to the world of sliding-block puzzles, including the famous *15 Puzzle* (Photograph 10). A description of the *15 Puzzle* can be found at https://en.wikipedia.org/wiki/15_puzzle.



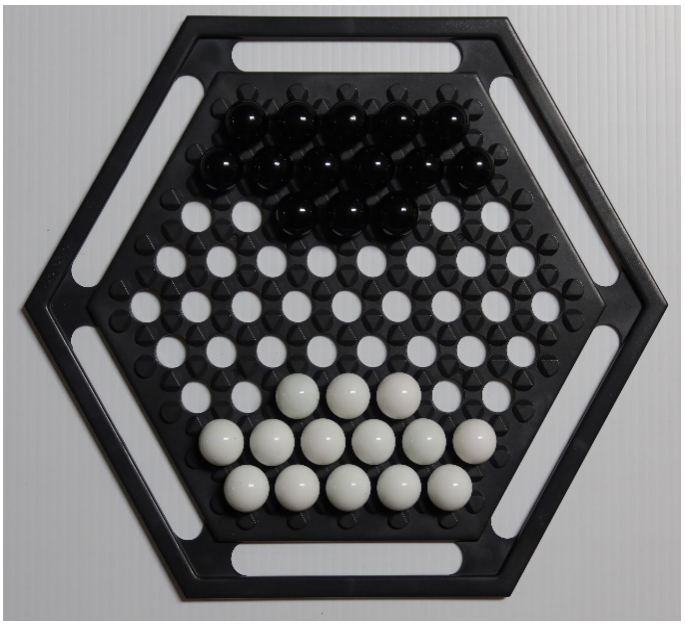
Photograph 10: *The classic* 15 Puzzle *with variations from the author's collection*

Abalone: Strategy, Geometry, and Figurate Numbers

Abalone is a two-player abstract-strategy game, played on a hexagonal board with 14 marbles per player (Photograph 11). The objective is to push six of the opponent's marbles off the board by forming lines of two or three marbles that overpower adjacent enemy marbles. A third player can be added if red marbles are used.

Abalone was developed in 1987 by French inventors Michel Lalet and Laurent Lévi, and it went on sale in 1989. The name "*Abalone*" is a blend of "abalone shell" and "alone," reflecting the goal of isolating your opponent's

pieces. The game has sold millions of copies worldwide and even has travel versions for playing in the car. It has also been incorporated into math programs in French schools to support strategic and geometric reasoning.



Photograph 11: *Initial set-up of Abalone, with 14 black and 14 white marbles*

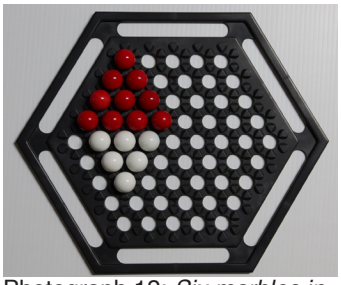
I've used *Abalone* in classrooms for many years, both for its strategic depth and the opportunities it offers to explore geometric ideas. Like *Rush Hour*, it promotes resilience, thinking ahead, and joyful problem solving. Information about the game, along with the rules and various strategies, can be found at https://en.wikipedia.org/wiki/Abalone_(board_game).

Connections to Mathematics

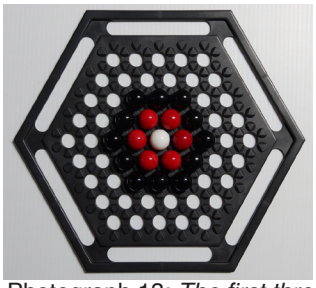
The movement of marbles lends itself to vector discussions, especially when considering how groups of marbles shift along hexagonal paths.

Students can use the marbles to model the sequence of triangular numbers: 1, 3, 6, 10, 15, 21, 28, 36,.... (For a description of triangular numbers, see https://en.wikipedia.org/wiki/Triangular_number.)

For example, Photograph 12 shows six marbles that form a triangle (the third triangular number) and ten marbles that form another triangle (the fourth triangular number). Additionally, when two consecutive triangular numbers are arranged this way, students can see that their sum is a perfect square (6 + 10 = 16). This can be generalized: the sum of two consecutive triangular numbers is always equal to a perfect square (10 + 15 = 25, 28 + 36 = 64, etc.).



Photograph 12: *Six marbles in a triangle below ten: a visual demonstration that the sum of two triangular numbers forms a square value*

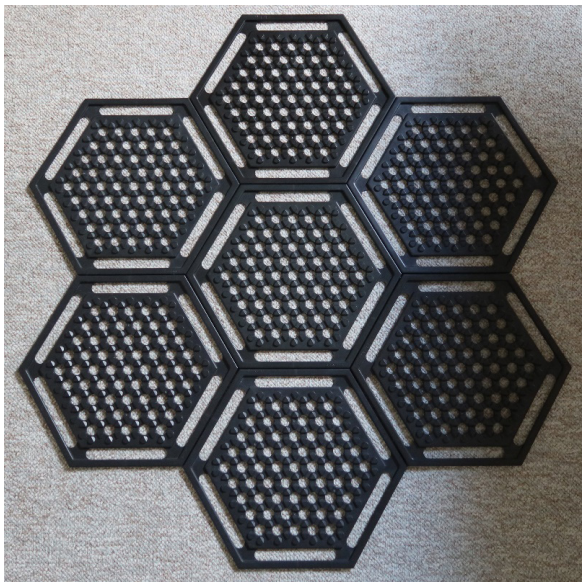


Photograph 13: *The first three centred hexagonal numbers: 1 (white), 7 (white + red), 19 (white + red + black)*

Students can also use the marbles to model the sequence of centred hexagonal numbers: 1, 7, 19, 37, 61, 91, 127, 169,.... (For a description of hexagonal numbers, see https://en.wikipedia.org/wiki/Centered_hexagonal_number.) This is a lovely visual representation of a more advanced figurate number.

Photograph 13 shows marbles being used to form the first three centred hexagonal numbers: 1, 7, 19. The white marble represents the 1; the white and red marbles make the 7; and the white, red, and black marbles make the 19.

Over a long period of time, I purchased about a dozen games of *Abalone* and had students arrange the boards to make 7, the second centred hexagonal number (Photograph 14).



Photograph 14: *Seven* Abalone *boards arranged to form a visual model of the centred hexagonal number 7*

Closing Thoughts

Games like *SET*, *Rush Hour*, and *Abalone* bring mathematics to life in ways that are interactive, challenging,

and fun. They promote habits of mind that are essential for mathematical thinking: pattern recognition, logical reasoning, spatial awareness, and persistence. Just as importantly, they invite play and curiosity into the learning process. I encourage you to try these games in your own classrooms and explore the mathematical richness they have to offer.

These games can also serve as thoughtful gifts or math club staples—extending their value beyond the classroom.▲

Continued from page 32

The 2024–2025 OMCA theme, "Celebrating Math," was embraced through two main initiatives. First, we celebrated our learning communities by offering interactive breakout sessions, where members shared effective practices and opportunities to network. We also made intentional efforts to engage with underrepresented communities, particularly those from Francophone school boards. Second, we celebrated the strong partnerships with our sponsors, whose generous support has enabled us to enhance the quality of our events and initiatives. Thanks to this ongoing support, OMCA is in a strong financial position moving forward.

Each monthly OMCA meeting is hosted by a different school board, and features a reflective land acknowledgement, a highlighted math resource or practice, and an engaging math task. These meetings provide members with valuable opportunities to connect, share successes and challenges, and engage in meaningful professional development. In addition to regular updates from the Ontario Ministry of Education, Education Quality and Accountability Office (EQAO), Ontario Association for Mathematics Education (OAME), and National Council of Supervisors in Mathematics (NCSM), we welcome guest speakers who share current research and insights relevant to mathematics leadership.

If you are a mathematics leader or coach within your board, and are interested in joining OMCA, we warmly welcome new members. Please contact us at OMCAmath@gmail.com for more information about our organization and how to get involved.

We look forward to seeing you at our upcoming Fall meetings, held from 11:45 a.m. to 3:00 p.m. on the following dates:

- Monday, September 15, 2025
- Monday, October 27, 2025
- Monday, November 17, 2025
- Monday, December 15, 2025

We hope to connect with you soon! ▲

▲ *Gazette* Reviewing: What's it all about?



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Barb is an educator and life-long learner, passionate about making mathematics accessible to all. As a Learning Coordinator of Mathematics and a Secondary Math Coach (Thames Valley District School Board, London), Barb built connections with educators around mathematics content knowledge and supporting learning strategies. As a Provincial Mathematics Professional Learning Facilitator, she developed curriculum support materials for Ontario Mathematics educators and students, highlighting the use of appropriate technology in the classrooms. Barb has had the honour of working with district school boards around Ontario and across Canada, delivering professional learning on mathematics, computational thinking, coding, and integrating programmable devices.

I have always appreciated my wonderful and supportive math community, and I felt compelled to give back somehow. An opportunity arose when I noticed on X (@OAMEwrites) that the Editor of the *Ontario Mathematics Gazette* was looking for people who might be interested in being article reviewers. Read a math article, give feedback—how hard could that be?

How does the review process get rolling? The Editor contacts individuals who have agreed to be reviewers with a call-out for reviewers. Each article requires three reviewers to go over it. The Editor explains the gist of the article, a suggested grade level, and a reviewing timeline, most often a couple of weeks. Reviewers can let the Editor know they are interested, or not, as the case may be. Reviewers are in control of the articles they select. Perhaps you just don't have the gift of time to review an article right now—that's fine. At other times, there will be an article that I think sounds exciting and right in my wheelhouse, so I reply quickly, only to find that the article has already been snapped up. Enormous sigh. I guess I'll have to wait for it to appear in the *Gazette* to enjoy!

Some articles pique my interest, but may be geared to grade levels below or beyond my expertise. I may take on these reviews. As we know, math is a continuum. The math I like to play with has grown out of Early Years and Primary-level mathematics, and leads into applications in higher grades. I can often add thoughts on where the math is going

and what came beforehand.

Even starting out, I found that I looked at the articles through a variety of lenses. No matter how hard I tried, I couldn't put away my "teacher red pen" lens and ignore grammatical errors and typos—I'm getting better at that, but maybe I should at least point them out. Other people have that job—once the article gets past reviewers! So, what lenses do I use?

When given an article to read, I first try to decide if it's meant to support math for teaching, or if it's more global and overarching, something research based, or describing a new math philosophy or technology.

When I review an article with my "for teachers" lens, I'm looking for what's in there for a teacher to implement in their classroom. How will it support and hopefully improve math learning for students? Does the theme logically unfold starting with the why, then progress step by step in a sequential or logical order? Are there illustrations to help support understanding of the thrust of the article? Where applicable, are there links for ease of access, or do I need to go off hunting for additional resources myself (note that the PDF version of the *Gazette* has live links, unlike the print version). Teachers are busy, and it is expected that articles are written to make it easy for them! Above all, does the material reflect current pedagogy and enhance understanding of our Ontario 2020 Mathematics Curriculum? At times during article reading, I'll find myself surfing the curriculum to be sure the article is supportive, and that this criterion is being met.

Some articles offer new approaches for teaching content. I may be acquainted with a method, or it may address ideas that are new to me. An example of the latter is taking math outside, which pushes my thinking, and challenges me to be a continuous learner. In either case, as a reviewer, I can offer insight supporting clarity and understanding in the writing.

My second lens tends to be reflective of a bigger picture, which asks: Is the article more global and overarching, rather than classroom or grade specific? We need articles that speak to the larger issues facing mathematics: equity and inclusion, research findings, changing technologies, etc. When reviewing these articles, I keep our curriculum and OAME's Guiding Principles in mind.

No matter my reviewing lens, I take time to look over the references. Who is influencing this article? How relevant or dated are the supporting references? Is there support if a teacher wishes to know more?

Many reviewers take advantage of the reviewing options offered in MS Word (and activated by the Editor). The ability to comment and offer suggestions or probing questions within an article is invaluable. An evaluation form is completed by each reviewer that needs to be reflected upon while reviewing. It is submitted along with any other suggestions. The criteria reviewers consider are as follows (in italics), with reviewers being asked to assess each along a continuum from strongly agree to strongly disagree:

- *The mathematics content is accurate.*
- *The mathematics content is appropriate for our readers.*
    - OAME/AOEM is a respected mathematics association and must be sure to only promote content that is accurate and appropriate for readers. They can't be seen to tacitly endorse anything inappropriate.
- *The ideas/activities are fresh and innovative.*
    - Not all articles are going to provide new, fresh ideas. Sometimes they reinforce good teaching and pedagogy that has been around, but is worthwhile to be revisited.
- *The article is well written.*
- *The ideas are well organized and clearly communicated.*
    - Almost all the articles I have had the pleasure to review fall into the "agree" or "strongly agree" category. However, not to worry if an article is a bit rough, but the content is excellent, there are opportunities for it to be massaged and edited before publication!
- *The article is of interest to our readers.*
    - Teachers' interests vary from teacher to teacher, so it may seem daunting to decide if an article can be of interest. Perhaps I volunteer for just the "good ones," but I haven't been disappointed yet!
- *The article is free of bias: sexual, ethnic, racial, or other stereotypes (equity issues).*
    - This should absolutely go without saying. We live it every day, and this criterion should be part of our lexicon.
- *The article reflects the OAME/AOEM Vision Statement (available on our website).*
- *The article does not endorse, financially or otherwise, any specific commercial products or services?*
    - This criterion is a bit more flexible than the last pair. Sometimes an article may refer to a commercial product like Texas Instruments Graphing Calculators or online math tools like Desmos or Polypad, which are free to use, but require an account to save and share student work. The key is whether the article is about how

to use a product, or essentially an advertisement.

As a reviewer, you offer opinions and advice for articles. The criteria are assessed. This all goes back to the Editor who, along with the other two reviews, handles the decision making. The reviews are anonymized (just as the article you receive is) and are provided back to the author with the Editor's decision. It is also important to recognize that, if the article is accepted, then there is an editorial process, including copy-editing, before the article shows up in the *Gazette*.

Do you have some free time on your hands and are wondering what to do with it? Or do you feel the need to give back to your wonderful mathematics community? Want to have the jump on some of the hot upcoming articles for the OAME *Gazette*? Contact our Editor and give reviewing a try! You won't believe how good it feels to see an article you have touched published in the *Gazette*, and see how it has changed because of your input, a feeling much like a proud parent! ▲

CORRELATION OR CHANCE?

While in a remote area of Canada, Adam Shoalts (2023) noted:

> Oddly enough, at intervals through the day, I'd heard a strange snapping noise coming from somewhere in the woods. When I first heard it, I didn't pay it any attention and just kept plodding along. But when I heard it again, I paused and cast a glance into the dark forest flanking the road, eyeing the spruces and tamaracks suspiciously. The snapping, occurring at occasional intervals, was too loud to be an insect, and I knew of no animal that made any sound like it. I never did figure out what made it, accepting it as one of life's mysteries. (p. 217)

Points for discussion with students:

What do you think is meant by "occasional intervals" and "somewhere in the woods"? Since Adam Shoalts is an experienced outdoors person, do you think these could be random times and/or directions?

Do the noises correlate with the observer's location? Is it possible that such noises occur across the area where he was hiking, but he only observed where he was?

Reference

Shoalts, A. (2023). *Where the falcon flies. A 3,400 kilometre odyssey from my doorstep to the arctic.* Allen Lane (Penguin Random House Canada Ltd.).

▲ Book Review: Early Math: Canadian Content for Canadian Educators

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Youmans, A., & Colgan, L. (Eds.). (2024). *Beyond 1, 2, 3: Strengthening early mathematics education*. Canadian Scholars Press.

One need not imagine what a conference focused on Early Years mathematics with a Canadian focus might be any longer. Editors Youmans and Colgan have gathered authors together to write 16 chapters on early math learning. This book certainly fills that imaginary conference void, where each chapter would make a successful workshop. In book form, the reader can control the order and amount of learning to meet their own needs. The chapters are grouped into four sections: *Creating Mathematically Rich and Supportive Early Years Classrooms* (Chapters 1 to 5), *Enriching Early Mathematics Learning Through Play* (Chapters 6 to 8), *Developing Early Mathematical Thinking with Technology and Coding* (Chapters 9 to 12), and *Engaging Families and Communities in Early Mathematics Learning* (Chapters 13 to 16).

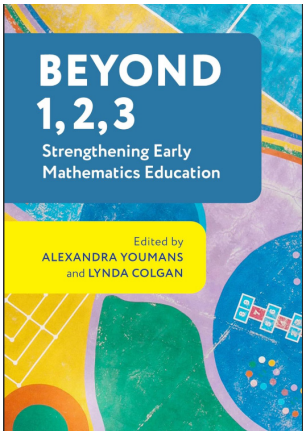
The foci of play, agency, exacting vocabulary, explicit instruction, assessment, accessibility, and engagement are woven through the various chapters, highlighting how they should be in every Early Years classroom.

In today's economy, teachers want to make sure they are spending their money wisely, and that includes purchasing professional books. This review provides a short summary of each of the 16 chapters, to illustrate the width and depth of the topics included, in hopes that Early Years educators will see the value in purchasing this book.

Chapter 1: *Curious Minds Want to Learn: Supporting Curiosity in Early Mathematics Learning* (authored by Alexandra Youmans and Edward Schroeder). This chapter looks at the role of curiosity in an Early Years math class, and its importance in learning and liking math. It acknowledges the realistic problem teachers face—the shear volume of curriculum expectations can create a situation labelled "curriculum overload." I can see this as a chapter that educators come back to periodically, with each rereading being given an opportunity to think further about the changes they could make in their teaching and reflection about their ability to support curiosity in their classrooms.

Chapter 2: *Counting on Early Mathematics: Understanding, Assessing, and Building Children's Early Number Skills* (authored by Heather Douglas, Karen

Coleman, and Jo-Anne LeFevre). This chapter packs a great deal of practical information on informal skills, like verbal counting, and counting small sets. These authors start by explaining what is meant by number knowledge and number relations. A *Number Skills Assessment Tracker* is provided, as well as supporting activities. Explaining flexible math groups and the use of math buddies round out this practical and useful chapter.



Chapter 3: *Teaching Early Mathematics with Manipulatives: Pedagogical Strategies for Early Years Educators* (authored by Helena Osana, Mélanie Barilaro, and Anne Lafay). This chapter begins by discussing the role manipulatives have in developing and supporting children's learning of mathematics concepts. To support the effective use of manipulatives in classrooms, the authors describe and provide examples of two pedagogical strategies: making comparisons explicit between different illustrations, and considering the physical features of the manipulatives.

Chapter 4: *Outdoor Learning and Mathematics in the Early Years* (authored by Janice Novakowski). Having the opportunity to learn from local Indigenous knowledge holders and educators in British Columbia, the author and a small group of her school district colleagues took a Pedagogy of Play course (Harvard Graduate School of Education). They developed and taught lessons focusing on three indicators of playful learning: joy, wonder, and agency. The chapter contains ten routines, each with multiple activities (35 in total) to support mathematics learning outdoors.

Chapter 5: *Too Many Roads Diverge: Insights into Early Mathematics Learning Disabilities* (authored by Derek Berg and Pamela McDonald). The authors of this chapter focus on students with Mathematics Learning Disabilities (MLD). MLD students often find number sense, calculations, and word problem solving more difficult. The authors describe four areas of psychological processing that can impact MLD students' math learning, and discuss three methods of general instruction that are most effective to support MLD students. While there are not as many specific and detailed classroom activities as some other chapters, an extensive list of references (60 in total) provides ample information for a classroom teacher to continue to learn about MLD.

Chapter 6: *Play and Assessment: The Role of Play and Assessment in the Mathematics Development of Early Years* (authored by Hanna Wickstrom, Chris DeLuca, and Angela Pyle). The chapter is organized into three sections: descriptions of play-based learning, the role of assessment, and the integration of assessment into play. Each section contains a relevant literature review and classroom examples. The classroom activities provide clear delineation for Free Play, Guided Play, and Teacher-Directed Play. As the chapter moves to assessment, the authors write the following about the first overarching principle of assessment: "Most assessment tools can be used for either summative purposes or for formative purposes; it is their ***use*** that matters, not the nature of the tool itself" (p. 108). The last section of the chapter, *Integrating Assessment Along a Continuum of Play,* provides examples of observational assessment in free play, withdrawal from free-play assessment, embedded assessment in guided play, and embedded assessment in teacher-directed play, with a concise chart listing the assessment tools for each stage of the continuum.

Chapter 7: *Paying Attention to Spatial Reasoning in Early Years Mathematics: Empowering Children's Learning through a Playful, Spatial Approach* (authored by Tara Flynn, Zachary Hawes, and Catherine Bruce). Throughout this chapter, the authors' focus is to support the concept that "spatial ability is not a fixed ability—something that you either have or do not have. In fact, spatial thinking is a highly malleable skill" (p. 121). They share research to highlight the connection between spatial reasoning and mathematics. The authors discuss three key areas of spatial reasoning connected to mathematical thinking and performance: mental transformations, scaling, and perspective taking. Classroom activities for all three are described, and an exceptional observational guide ends the chapter.

Chapter 8: *Learning to Play. Learning from Play: The Math Minds Initiative* (authored by Martina Metz). The author of this chapter has been part of the Math Minds Research Initiative at the University of Calgary. The research identified key elements of effective mathematics pedagogy, and developed the Math Minds model based on core principles: structure, mindset, memory, change, and overlap. The author ends the chapter, stating that few (if any) early childhood resources have been designed with variation theory specifically in mind. This reviewer would like to have seen more examples of activities described in this chapter, to better understand variation theory.

Chapter 9: *Making Fingers Count: An Introduction to the TouchCounts Program* (authored by Nathalie Sinclair). The author of this chapter was the lead developer of TouchCounts (2015), a free iPad application. The app has two sections: the Numbers World and Operations World. Each is explained

with words, photos, and screenshots. Viewing one of the videos in YouTube before reading the chapter will aid in the understanding of the activities young children can do on the app, in particular, how touching the screen with various numbers of fingers activates different numbers, and the use of colour dots identifies number segmentation (not seen easily in the black-and-white screenshots).

Chapter 10: *The Role of STEM-Based Learning in the Mathematical Development of Early Years Learners* (authored by Scott Compeau, Andrew Michael Bowles, Johanna Whitehead, Desurée DeGagné, and Lindsay Jones). This chapter explores STEM, and in particular, the **E** component, Engineering, in Early Years classrooms. It delineates the engineering design process in a classroom activity involving air-powered cars. The air-powered car activity comes with lesson objectives, assessment opportunities, instructions, and links to the Ontario Math curriculum. There are detailed charts showing the engineering design process by delineating the roles of the educators and the children during the problem, plan, create, and test stages. There is all the information an educator needs to conduct this activity in their classroom.

Chapter 11: *The Role of Coding in Early Elementary Mathematics* (authored by Shaunda Wood). This chapter begins by explaining coding and the differences between a coder and a programmer—the former, common in classrooms, the latter, not so common in classrooms. The author ties the real-life programming processes to learning and the Early Years curriculum. The classroom activity starts with snap cubes to make linear patterns, and then moves to developing multi-directional patterns. After the use of directional symbol cards for pattern and movement, the children use Blue-Bots on grids. Blue-Bots is a device that might be too expensive for some classrooms; however, the chapter is relevant because children acting as robots can follow directions to move around on a grid, and this is a no-cost alternative.

Chapter 12: *Coding: What, Where, Why, When, and How It Matters in Early Mathematics* (authored by Lynda Colgan, Melanie Martin, Edward Schroeter, and Alexandra Youmans). This extended chapter begins by clarifying that JK/SK and Grade 1 educators do not need to feel that coding is an add-on to their curriculum. The concepts of spatial reasoning, positional and directional language, grids, sequencing using symbols, and problem solving are not new to Early Years classrooms. The authors discuss the idea of space (location, position, and direction), and provide a chart showing curriculum expectations from multiple sources. For the remaining portion of the chapter, the authors share lessons they created from The Roots of Coding website, and their experiences and struggles when implementing the lessons. Their insights, coupled with the well-written lessons from this free resource, will be a welcome addition to Early Years classrooms. In addition, interspersed throughout the chapter are Coding Challenges, tasks that help the reader understand and reflect on what it is to code in Early Years classrooms.

Chapter 13: *Parents Matter Too: The Important Role That Parents/Guardians and the Home Environment Play in Children's Mathematical Well-Being* (authored by Erin Maloney, Patrick McLeod, and Elora Wales). This chapter begins by discussing parents'/guardians' roles in math learning, and the role educators have in fostering school–home engagement through communication and interaction. The authors define and discuss mathematical well-being and the effect on math learning. The chapter is peppered with ideas to foster the school–home engagement. The activities are well scripted and can easily be added to a class newsletter or communication platform.

Chapter 14: *Engaging Parents in Their Children's Early Mathematics Learning: Strategies Learned from the Building Parent Engagement in Math Project* (authored by Alexandra Youmans, Lynda Colgan, Lorraine Godden, Michelle Searle, and Barbara Mendes). This chapter is an excellent follow-up to the previous chapter, taking ideas for a single classroom to a school-wide parent engagement project. The Building Parent Engagement in Math Project (BPEiM) involved six schools, each with unique environments. The PIP policy (Parents in Partnership: A Parent Engagement Policy for Ontario Schools), from the Ontario Ministry of Education, has four key strategies: developing a positive school climate, eliminating barriers to parent engagement, providing support so parents can help with their children's learning, and actively reaching out to parents to encourage their involvement. By explaining the different approaches formulated to best suit each school's environment, and how these approaches were adjusted with feedback from the families, the authors truly honoured the role home engagement has on math learning.

Chapter 15: *LittleCounters®: A Community-Based Early Numeracy Program* (authored by Joanne Lee). This chapter describes a program offered in public libraries, and EarlyON Child and Family Centres. The three objectives of the program are: providing information regarding number sense for young children through purposeful play activities; gaining understanding of children's early mathematical ability; and dispelling anxiety, that supporting early mathematical concepts is difficult. The program provides workshops for educators and multiple sessions for families. A visit to the website shows it needs updating. This may prove difficult for educators to contact the author to sign up for the program,

and so this is the weakest chapter in the book with regards to practical information for educators.

Chapter 16: *Exploring Early Numeracy Learning Opportunities with the TOYBOX Community Project* (authored by Sheri-Lynn Skwarchuk, Brenton Button, Alyssa Wright, and 33 additional authors).

This chapter is a positive ending to a book filled with ideas to enhance early math learning. The format developed for "TOYBOX" is one that could be borrowed for other family math bags or other school–home activities. The format contains information for beginners, intermediate, and advanced tasks. The adult prompts, labelled "TIPS," will alleviate many concerns adults might have working with children. An important component of the TOYBOX program is taking user feedback and updating the activities (a visit to the website has a Give Us Your Feedback section, asking, "What activity did you try?," "What did you like best about it?," and "How could we improve the activity?").

Conclusions

There are aspects of the book's format that this reviewer found useful. The no-nonsense titles encapsulate each chapter's content. The sections and layout of each chapter were similar. The format of *tell me what you are going to tell me, tell me, and then tell me what you have told me* is supplemented with subsections titled *Learning Objectives*, *Some Final Thoughts*, and *Discussion Questions*, which makes the readability of the book user friendly. One does not need to read the book in its entirety, and the chapters can be read in any order, based on each reader's needs.

The format suits the book's audience—a class textbook or individual reader. For example, *Discussion Questions* could be an activity for a book-reading group, pre-service teachers attending our Faculties of Education, or for an individual to check in on and reflect on their own understanding of the chapter. Knowing that *Key Terms* are defined in one location means the reader does not need to stray away from the book.

The gray tipping of the edges of the pages marking major sections of the book—Conclusion, Classroom Resources, Discussion Questions, Key Terms, and References—made it very easy to flip back and forth as I read through the chapter, and when I returned to the chapter to locate information.

No doubt this book will be a useful textbook for Faculty of Education students and Additional Qualification courses, but a current classroom teacher should consider purchasing this book on their own. Many of the chapters present information that will build, redirect, and refine experienced educators' practices that it is a very worthwhile read. ▲

▲ Play, Math, and Early Learning: Setting the Stage



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Jennifer Holm is an Associate Professor at Wilfrid Laurier University and works with pre-service teachers, as well as in the field supporting current mathematics teachers. She is interested in developing mathematics knowledge for teaching; focusing on the beliefs and opinions that pre-service teachers hold about mathematics and teaching and the connection they have to past experiences. She uses this research to support future teachers in developing beliefs and knowledge that will encourage and support effective teaching practices in mathematics. She is also the proud mom of two young learners, who are exploring the world of mathematics.

This column marks the first of a new column series focused on mathematics in the early years (birth to Kindergarten, with applicability beyond). The columns will focus on mathematics learning through play and exploration with young children, and look at ways to engage with mathematics in appropriate ways for early learners.

To set the stage for future columns, this issue focuses on language. Many books about raising children and preparing learners for the future focus on the importance of language and literacy development. Some even suggest that when learners are young, to narrate your day to help foster communication and language development. As you are talking about your day, there are many different ways that you could (and likely already do) bring in communication that includes mathematics! You may immediately think about counting items, and this is a simple way to start. Children are naturally egocentric, and their worlds focus on themselves and their own needs, so counting body parts is a simple way to incorporate mathematics. I am sure most early years caregivers (and parents) naturally do this.

Keep in mind as you are counting with young learners, there is a trajectory for counting that begins with the pre-counter, who does not associate the words with quantity (Van de Walle et al., 2018). The next stage in the counting trajectory is the reciter, who simply "recites" the numbers, which may or may not be in the correct order (Van de Walle et al., 2018). Toddlers do not understand how counting works, and eventually understand numbers as a predictable list of words. This means that when you are first counting,

the important part is that children hear the words in the correct order over and over again. It is a much later skill for children to answer the question, "How many?," and understand that when counting, the last word spoken is the number of the quantity. This would indicate that the child has "cardinality" when they are able to do this (Van de Walle et al., 2018).

In this column, we are going to look at other ways, beyond just considering numbers, as important mathematics for preschoolers. I am suggesting there is so much further you can go with language and mathematics, it is a matter of considering what is considered mathematics. As you continue in the column, you likely realize that you are using more mathematics than you realize when you work with young children. If not, try to consider how you might bring in the mathematics more when you are.

Toys and items in a child's environment often have very identifiable shapes. Noting that a ball is a sphere, or that a truck has a rectangular prism on the back, are simple ways to call up the shapes in the real world. As you name the shape in pictures, trace the outside to call attention to the features and characteristics of the shapes. I did not realize how often I did that until I watched my two-year-old reading a story and tracing the shapes that he had watched me trace repeatedly as I read it to him. Try to use the mathematics names for the shapes as you read or come across them in real life (e.g., remember that a "diamond" is not a mathematical shape name, it is likely either a "kite" or a "rhombus" instead, or instead of noting that balls are "circles," use the term "sphere"), and just keep mentioning them. The repetition is helpful for mathematics words as much as it is for language!

Sometimes it is easy to forget that patterns are mathematics! So many children's toys and books are filled with patterns because the repetition is so important for developing brains. Calling attention to the pattern by naming the shapes or colours that are repeated, helps to make children attend to what is happening. Young children often follow predictable patterns when they are doing things because of how much they enjoy routines. Giving words to the pattern is also helpful. When doing a predictable routine, you may also ask, "What's next?," to get the child to tell you what comes next. This can be a good way to lay the groundwork for more advanced patterns because they learn that there is a predictable pattern in the sequence.

When working with children, you likely use a lot of "if, then" statements. Think of something like, "If you clean up the blocks, then you can paint a picture." In coding, you use these types of statements when writing code! Sequences are also crucial in coding. Working on having a toddler put events in a correct sequence, or follow a sequence, sets the stage for looking at coding instructions. Schedules are a natural way in a child's day to do this; however, keep in mind how short a time span a preschooler can consider. For very young learners, consider making sure it is the very near future and a short list (e.g., "First we will have lunch, and then it is nap time".) Following directions to complete a task is another place where you might see examples of how coding is part of day-to-day life. Children may have fun helping and associating lunch with a sequence: first we wash the apple, then cut it into pieces, and then put it onto the plate to eat. A simple task like brushing teeth also has a predictable order that can be helpful to consider for directions.

Another way to consider mathematics conversations is to think about equalities and inequalities. When working with blocks, you can call attention to blocks that are the same size or ones that are different sizes. You do not necessarily have to use the word "equal," but having the conversations where you are laying the groundwork for this skill later in mathematics can be a starting point with young children. I am sure that when you consider the idea of sameness and equivalence, then you can think of many other things in the day of a child that you can call attention to. Anything from the same number of cereal O's on one side of the plate to another. Or looking at how many of the snack fell off the plate, and it is less than or more than what is on the plate. There are many different activities that happen during a young learner's day that could be used to explore the idea of equality.

Young learners absorb language quickly as they are developing, and likely you already communicate with children about mathematics, but maybe you have not thought of it in the same way. In communicating with educators, a lot of individuals immediately think about number sense as mathematics, and they do not readily call up all the other ways that mathematics fit into daily life. As you are speaking with young learners, take moments to bring attention to the ways the world around them holds mathematical power, and help set the stage for future mathematics development. Not only will it help them when they start learning the structured thinking within mathematics, it can hopefully foster a love and amazement for mathematics from the start!

Reference

Van de Walle, J.A., Karp, K.S., Bay-Williams, J.M., & McGarvey, L.M. (2018). Elementary and middle school mathematics: Teaching developmentally (5th Canadian Edition). Pearson. ▲

Implementing Thinking Classrooms in Ontario's Destreamed Grade 9 Mathematics



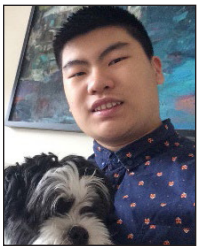
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Mathematics plays a crucial role in developing critical skills that empower individuals so they can make informed decisions in an ever-changing world. However, disparities in mathematics learning continue to reflect broader social inequalities, underscoring the need for innovative and inclusive pedagogical approaches. Research highlights that these disparities can hinder economic opportunities and perpetuate cycles of poverty, emphasizing the necessity of educational frameworks that consider these characteristics (Cavalcante & Savard, 2022).

Recently, Ontario adopted destreaming in Grade 9 mathematics, aiming to mitigate the negative effects of policies that segregate students based on perceived abilities. To support educators in this transition, this article explores how Liljedahl's framework offers transformative practices to engage all learners. Additionally, it highlights the synergistic potential between the Thinking Classroom framework and Paulo Freire's Pedagogy of the Oppressed perspectives, proposing an approach that connects mathematical learning to critical consciousness and social justice (Freire, 2017; Liljedahl, 2020).

Throughout the text, we discuss how practices such as contextualized tasks, visibly random grouping, and the use of vertical non-permanent surfaces (VNPS) foster collaboration and critical thinking (Liljedahl, 2020). We also discuss how practices proposed by Liljedahl can be enriched by Freire's perspectives to enrich independent thinking, reduce students' dependence on the teacher, and promote a collaborative culture, where students support each other's learning. Moreover, we explore how Freire's principles could deepen the impact of these practices, integrating a critical perspective that prepares students not only to solve mathematical problems, but also to foster the formation of participating citizens in the creation of a more equitable and just society (Freire, 2017; Nascimento Silva, 2018).

Reforming Streaming: Equity Challenges in Ontario

Ontario's educational system was unique among Canadian provinces for its practice of streaming Grade 9 students into Academic and Applied pathways. This early division had been criticized for potentially limiting access to post-secondary education opportunities. In December 2019, the Higher Education Quality Council of Ontario (HEQCO) and People for Education convened a meeting with K–12 educators and administrators to address barriers to post-secondary education, focusing on the implications of streaming (Pichette et al., 2020). Concerns were voiced regarding the impact on students, some as young as 13 or 14, who must make consequential decisions about their educational paths.

Participants highlighted the disproportionate steering of racialized and lower-income students toward Applied streams, influenced by both individual biases and systemic inequities.

Following the global anti-racism movement, which underscored the need to confront systemic inequalities in education (UNESCO, 2021), the Ontario government committed to discontinuing the practice of streaming in 2021 (Ontario Ministry of Education [OME], 2021). These changes endeavoured to create a more inclusive educational environment by ensuring all students have equal access to challenging coursework and necessary support.

Students need to feel capable and confident in mathematics. Effective mathematics teaching occurs in welcoming classrooms, where every student feels important, encouraged, and ready to engage in learning and exercising their citizenship. This begins with the teacher knowing each student well, maintaining high expectations for all, providing necessary support, and believing in every student's potential to learn mathematics. It also involves demonstrating the real-life applications of mathematics and employing diverse teaching methods that accommodate all learners and prepare them to actively contribute to their communities. For example, tasks such as assessing water consumption patterns in their neighbourhoods or understanding income tax calculations enable students to connect mathematical concepts with their daily lives and broader societal issues (Freire, 2017; Liljedahl, 2020).

There is an increasing emphasis on engaging students in challenging mathematical problems to deepen their understanding of concepts (NCTM, 2014; Warshauer, 2014). Building on these foundations, initiatives seeking to improve Grade 9 math achievement focus on adapting the curriculum to meet students' needs and enhancing teachers' proficiency in mathematics content and innovative teaching strategies.

Engaging Students in Destreamed Mathematics with the Thinking Classroom

According to Liljedahl (2020), a Thinking Classroom is an environment that not only encourages, but also actively stimulates, critical thinking. It is a space where individuals think both independently and collectively, learning together and constructing knowledge through interactive activities and discussions. This setting fosters an atmosphere of engagement, where students are confident to be active participants in their learning journey, thus becoming engaged learners, who contribute to the collective intellectual growth of the classroom.

Within this framework, Liljedahl (2020) identifies several common behavioural patterns among Grade 9 mathematics students, such as "slacking," "stalling," "faking", "mimicking," and those who attempt problems independently. Slacking students are often distracted by their phones, while stalling and faking students may pretend to be busy, without genuine engagement—stalling students engage in legitimate off-task behaviours, such as getting a drink or going to the bathroom. Mimicking students replicate others' efforts, without genuine understanding, and independent problem solvers are those who actively engage with the material.

The Thinking Classroom framework addresses the needs of all these student types through its inclusive practices. One key strategy is the use of visibly random groups (VRGs), where students participate in the random assignment to groups each day, visibly engaging in the random grouping process (Liljedahl, 2020). This practice breaks up established social structures and encourages students to interact with a variety of peers, promoting inclusivity and collaboration. By working in VRGs, slacking or stalling students are more likely to become engaged due to the group dynamics and peer encouragement. Moreover, the use of vertical non-permanent surfaces (VNPS), such as whiteboards on walls, encourages students to stand and collaborate visibly (Liljedahl, 2020).

This physical engagement could reduce off-task behaviour and keep students focused. It also allows the teacher to monitor progress and promotes immediate feedback, supporting all learners effectively. An example from an Ontario Grade 9 classroom illustrates these benefits: a teacher implemented VRGs and VNPS by designing a task where students analyzed the implications of increasing minimum wage rates on household budgets, leading to greater engagement and collaborative problem solving (see the lesson plan file in the members-only area at www.oame.on.ca).

Another essential engaging practice of the Thinking Classroom is how teachers respond to students' questions during activities. Instead of providing direct answers, Liljedahl (2020) suggests using strategic hints and feedback questions to stimulate independent thinking. For instance, a teacher might ask, "What have you tried so far?" or "What other strategies could you consider?," rather than showing how to solve the problem for the student. This approach promotes a problem-solving mindset, encouraging students to take ownership of their learning and persist through challenges. Additionally, it reduces dependence on the teacher and cultivates a collaborative classroom culture, where students support each other's learning. This practice aligns closely with Freire's principles of dialogic learning, where knowledge is co-constructed through meaningful inquiry and reflection (Freire, 2017).

Challenges and Adaptations in Implementing Thinking Classrooms

Implementing the Thinking Classroom framework in Grade 9 mathematics reveals both the potential and limitations of this innovative pedagogical approach. While the framework offers significant benefits, teachers may encounter resistance from students accustomed to traditional teaching methods.

In our mathematics teaching practice involving three Grade 5 classrooms at an international school in Accra, Ghana, the initial application of the framework (class size 15–16) resulted in students responding positively to collaborative problem-solving activities and the creation of a supportive learning environment that encouraged a growth mindset toward mathematics. These methods fostered engagement and a sense of community among students, demonstrating the framework's potential to enhance mathematical understanding and enjoyment.

However, in subsequent attempts to implement the same strategy the following year in another Grade 5 class with the same teacher (class size 19–20), significant challenges emerged. For a cohort of students with learning needs and a preference for traditional learning methods, adapting to the new routines of a Thinking Classroom was challenging. Many students had difficulty adapting to working on vertical surfaces, in small groups, and working without sitting at a table. Some were unable to stay on task or with their group, causing significant distractions.

In Ontario's Grade 9 destreamed classrooms, similar challenges have been noted. Teachers have found that some students resist the shift from traditional teaching methods to the interactive practices of the Thinking Classroom: students expressed discomfort with the new methods, preferring familiar routines. As Finelli et al. (2018) notes, students often express initial opposition to the introduction of these paradigms as part of their discomfort with accepting personal responsibility for their learning process. To address this, Han et al. (2021) have shown that enhancing students' self-efficacy and providing positive outcome expectancies can significantly influence their engagement. This suggests that educators should start by gradually introducing Thinking Classroom practices as VNPS or VRGs, while providing clear explanations of the benefits and involving students in the process.

Additionally, as highlighted by Warshauer (2014), posing problems of high cognitive demand gives students opportunities to think, reason, and problem-solve. Teachers should carefully select tasks and responses to promote productive struggles, allowing students to focus on understanding both the process and the problem, rather than rushing for quick answers. Educators should foster "buy-in" by demonstrating the relevance of collaborative and self-directed learning to students' mathematical success.

By adopting reflective practices and providing opportunities for student feedback, educators can tailor the approach to meet students' needs. Flexibility is essential in adapting the framework to accommodate students' specific challenges, such as behavioural issues and entrenched learning preferences, to help create an effective and engaging learning environment (Liljedahl, 2016; Finelli et al., 2018; Han et al., 2021).

Using Thinking Classrooms to Enhance Critical Thinking, Collaboration, and Societal Engagement

The Thinking Classroom framework not only engages students academically, but also promotes inclusivity and social consciousness through its practices. In addition to building mathematical knowledge and skills, this framework could be used to promote a citizenship and anti-oppression approach to education, as proposed by Paulo Freire's 1970 pivotal work, *Pedagogy of the Oppressed*. Freire's critical anti-oppressive pedagogy and transformative education proposal highlights the significant role of education in empowering individuals and advancing social justice. This approach centres on dialogic learning, where teachers and students engage in meaningful conversations to co-create knowledge, and problem-posing education, which advances critical thinking and active engagement with real-world issues (Freire, 2017).

In the context of mathematics education, Freire's emphasis on contextualized learning and developing critical consciousness aligns closely with Liljedahl's Thinking Classroom practices. By integrating a focus on societal relevance and empowerment with the Thinking Classroom framework for active engagement and collaboration, educators could cultivate a more inclusive and transformative learning environment. This synergy enhances mathematical understanding and equips students with societal critical-thinking skills, which are necessary to address broader social inequalities and become proactive agents of change (Nascimento Silva, 2018).

Freire's concepts could significantly enrich Liljedahl's methods by infusing a deeper level of societal awareness and relevance into the classroom. For instance, incorporating Freire's idea of contextualized learning into Liljedahl's rich, non-routine tasks could involve designing mathematical problems that reflect students' social and cultural contexts, making the concepts more engaging and meaningful (Freire, 2017; Liljedahl, 2020). Additionally, Freire's dialogic learning

can be used to support Liljedahl's practices of random grouping and collaborative problem solving, fostering an environment where students learn from the tasks and also from each other's diverse perspectives through meaningful dialogue. Moreover, the practice of formative assessments, when informed by Freire's critical-consciousness focus, could serve as a powerful tool to encourage students to reflect on their learning journey and consider the broader implications of their mathematical knowledge. Additionally, self- and peer evaluations could encourage students to reflect on their learning processes and understand the broader implications of learning mathematics, promoting a transformative educational experience (Freire, 2017; Liljedahl, 2020).

By merging these educational framework and perspectives, teachers could create a classroom that is more meaningful for students. Freire's transformative ideas, advocating for education as a tool for social justice, remain profoundly relevant today. By integrating his foundational principles with modern pedagogical innovations like Liljedahl's Thinking Classroom framework, educators can develop practices that improve academic outcomes and could contribute to building a more equitable and just society.

Conclusion

The diverse perspectives presented in this article collectively demonstrate the significant potential of Peter Liljedahl's Thinking Classroom framework to address contemporary challenges in mathematics education. By explicitly aligning the framework with Ontario's Culturally Relevant and Responsive Pedagogy (OME, 2024), we underscore its suitability for promoting inclusivity and supporting diverse learners in destreamed classrooms.

For Grade 9 teachers considering the adoption of the Thinking Classroom framework, professional development and support are essential. Educators must be prepared to address potential resistance by gradually introducing new practices and involving students in the process. Collaboration among teachers can facilitate the sharing of strategies and experiences, contributing to the effective implementation of the framework.

Furthermore, integrating Paulo Freire's educational philosophy enhances the framework's capacity to cultivate critical thinking and social consciousness. This integration could support to advance students' mathematical skills and prepare them to become active contributors to societal change, aligning mathematics education with broader goals of social justice and transformation.

By embracing these innovative approaches, educators could drive considerable progress in mathematics education. The Thinking Classroom framework, enriched by Freire's ideas, can strengthen the promotion of inclusive and engaging learning environments that enhance student societal empowerment. Through collaborative problem solving and contextualized tasks, students develop skills in critical thinking and teamwork that prepare them to address real-world social challenges, such as how to analyze local data to propose solutions for community issues. This approach could contribute to academic success and could foster the formation of engaged citizens, who actively contribute to creating a more equitable and just society, individuals who are proficient in mathematics and are also equipped to use their knowledge to effect positive change in their communities.

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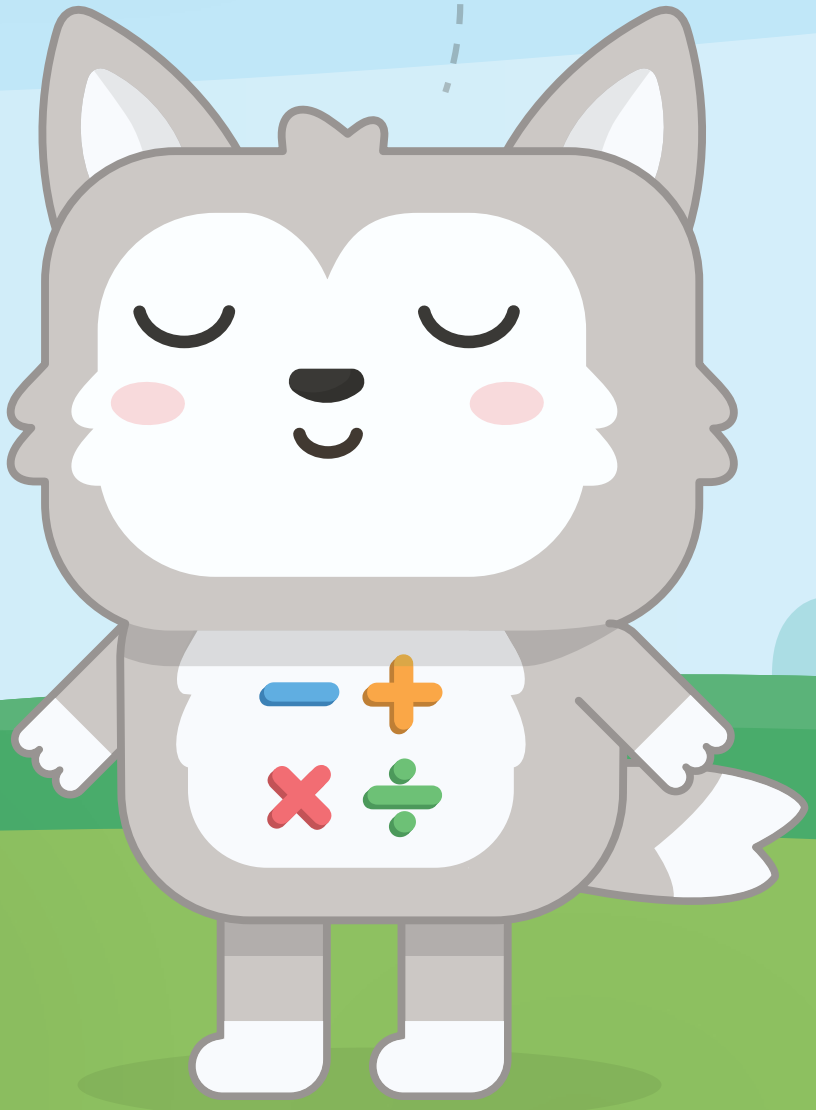
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