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ABACUS

Ontario Mathematics Gazette

OAME - ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

AOEM - ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

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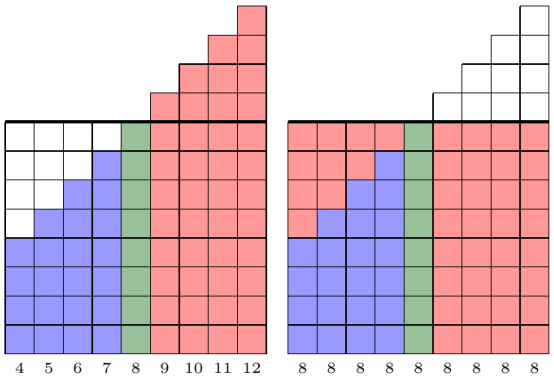




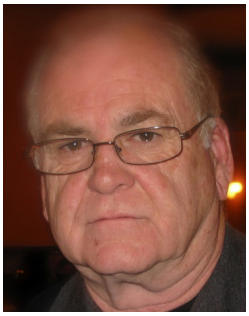


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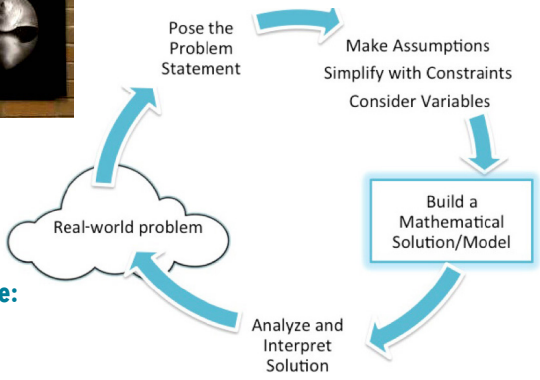
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OAME 2026 Let's {Talk} Math

KEYNOTE SPEAKERS



Dr. Pamela Seda



Pamela Harris

FEATURED SPEAKERS

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Octavia Beckles
Dr. Alex Lawson
Chris Luzniak
Cathy Marks Krpan
Marian Small
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Submission of Articles

The *Ontario Mathematics Gazette* (*Gazette*) publishes news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *Gazette*:

- The *Gazette* is aligned with the OAME/AOEM Vision, Mission, and Strategic priorities. These can be found on the website www.oame.on.ca under "About OAME" and "Guiding Principles."
- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles are to be word–processed, MS Word is preferred, and prepared according to the *Publication Manual of the American Psychological Association*, Seventh Edition. However, please use single-line (not double) spacing and only one space after each period. Articles should not exceed eight numbered pages of text, though there is some flexibility. Figures, images, and photographs should be placed close to the relevant text and include captions. The photographer's permission is required, and for photos of students under the age of 18, permission of a parent or guardian is required.

Please submit your article in one blind file (i.e., identity of author is not evident), and include author names, contact information including email and mailing addresses, photos—head and shoulders, biographies—less than 100 words, and all content removed for blinding in a second file. Please email these two files to Tim Sibbald at gazette@oame.on.ca.

Upon review, you will be notified whether your article has been accepted for publication (as is, or pending minor or major revisions) or declined. The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME/AOEM.

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About the *Ontario Mathematics Gazette*

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Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be print ready, and colour advertisements should have no bleeds.

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Editor's Report



TIM SIBBALD, OCT, PH.D.
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Tim Sibbald is a professor in the Schulich School of Education, Nipissing University, with a focus that includes mathematics education. He is a former President of OAME/AOEM and has held a variety of other OAME/AOEM posts over the years.

Age, it is said, is just a number. Yet anyone who realizes they can no longer win the most prestigious prize in mathematics, the Fields Medal—which is named after a Canadian—perhaps consoles themselves with a senior-discounted coffee at a local restaurant. In case you wonder, the Fields Medal criterion stipulates it is for work done before the age of 40, not that most of us would be contenders (and, to my chagrin, I first got a senior discount at age 41).

In thinking about aging over the last few years, I often gravitate to concerns about education. The traditional instructional idea that students see, hear, and have the tactile experience of writing a note was a fixture for a very long time. It was an early approach to Universal Design for Learning (UDL)—engage with concepts and ideas on enough levels, and connected understanding has the best chance of following. Teachers also orchestrated their instruction to provide varied methods of conveyance: chalkboards, transparency projectors, demonstrations, etc. The technology was different, but in its time, it was effective—we, students of that era, did learn something. There was also encouragement to take pride in notebooks and some degree of metacognition when students were prompted to think about the organization of material that had been written down (more so with binders or "Duo-Tangs" than fixed notebooks).

When I think back, particularly to my elementary school experience, part of the process was the reinforcement of basic skills. Spelling, if I recall correctly, was not being explicitly taught in math class, but was part of the milieu, and similarly, student oration had implicit teachings. In later years, high school textbooks provided an opportunity to look ahead and realize the general nature of what one was going to be learning. Tangentially, the requirement to go textbook shopping in Grade 13 set students up for that experience in university. They were relatively bland textbooks prior to the millennium—text based with some diagrams, maybe a second colour in some, and that served

to keep students focused, albeit it could also be contrived as monotonous.

Around the millennium, considerable visual graphics were infused into textbooks. They became notably more colourful and contained more contextual connections through the use of photos. At the same time, they seemed to have more distractors.…

This is where all teachers need to be careful as they age and think about various experiences they had along the way. It is far too easy to drift into judgmental language and spiral into thinking that what made a generation successful is surely an effective approach. After all, we tend to view ourselves as successes. So, the aging teacher has a knowledge of effective methods, where some are deemed to be out of date, and they naturally try to reconcile the details. Care is needed to avoid simplistic characterizations of the situation and focus on the big picture—students today are not simply opening a digital file and clicking on a blank space to type in a missing word. Much like teacher oration, while a student reads and mechanically reproduces a note, supported learning of skills beyond simply the subject content, so too today, students are learning broad skill sets that support their becoming functional members of society.

We do, however, need to think about what is taking place so that we can avoid difficulties before they happen. As an example, it appears that many teachers have been left to their own devices to provide resources for students. In effect, the preparation of something resembling a textbook has been downloaded to the teacher. While this can have benefits, most older teachers can recall some real gems they found in textbooks—a question or activity that they simply would not have thought of. What then prevents paring down of content and stagnation as we move forward? As a reader of the *Gazette*, you are not a person who will struggle to answer that question, but teachers generally may.

Teaching is multi-dimensional, and the example of raising concern about textbooks ignores too many other dimensions. I could equally well have pointed to the issue of whether hands-on learning includes two-dimensional representations on a device screen, or manipulation of items in a virtual-reality environment. Or, I could have picked on any of a wide range of evolving issues with which teachers are wrestling. The point is that we need to actively engage with these issues and actively pursue reconciliation of changes that take place over our careers, as failing to do so risks having a generation of students missing opportunities—opportunities that could make a difference for them. It is the ability of teachers to make a difference for more students than the previous generation of teachers that is at the heart of UDL.



Let me close this thought with a recollection of a research paper that I have long ago misplaced. It indicated that teaching careers have several common outcomes in retirement. I will not dwell on the circumstances that lead to retirees feeling twisted and used by a system. Rather, the paper described teachers who were innovative and constantly adopting new ideas, firmly oriented to the classroom and their students, as retiring contented, knowing they had made a difference. This always reminds me of a particular teacher who, in their last year before retirement, took on the introduction of graphing calculators. They did not have to—they could have drifted into retirement without doing so—but they did because it made teaching interesting for the teacher, and it helped the students they taught that year.

Favourite Subject?

While Canada was not included in the sampling, the Cambridge Assessment of International Education reported in their 2018 *Global Education Census Report* that:

> … at a global level, the highest proportion of students (38%) say their favourite subject is maths, while an almost-equal proportion say they most enjoy studying English language (37%). Biology is the next favourite, cited by 29% of students globally in our survey. (p. 11)

See www.cambridgeinternational.org/Images/514611-global-education-census-survey-report.pdf for the full report.

In This Issue

Kit Luce considers a gap in student responses to two EQAO (Education Quality and Accountability Office) survey (i.e., non-content) questions, and strategies to address the gap. **Pamela Brittain** brings news from the Fields Institute that includes the Margaret Sinclair awards, an art exhibit, and outreach opportunities. **Gerard Lewis** puts the spotlight on events with the OMCA (Ontario Mathematics Coordinators Association). The NCTM report has returned, and **Gerard Lewis** presents a scavenger hunt, where I know he has a box of prizes for winners! Lastly, Past-President **Paul Lessard** is remembered by **Julian D'Angela**, **Pat Lashmar**, and **Ron Lancaster**.

Shawn Godin brings a cornucopia of ideas revolving around the number 29—no spoilers for why he chose that

specific number. **Sofia Ferreyro-Mazieres** examines various paths that lead to prime factorization. **Jennifer Holm** brings problem solving in the early years to life with the rescue of a lion. **Lynda Colgan** draws the line… well, she brings the number line into focus. **Ron Lancaster** brings out the intersection of poetry and math. **Carly Ziniuk** gives her attention to financial math. **Jérôme Proulx** leverages the commutativity of percent for computational advantages. **Jeff Irvine** explains Glasser's Choice Theory. **David Costello**'s column is missing from the issue, but it is anticipated that he will return in the next issue.

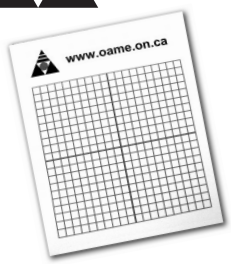
In case that is not enough, there are articles. **Peter Gustainis** explains a project that teaches about error-correcting codes, which he has tested in the classroom and presented at the OAME 2025 Annual Conference. **Evan Throop-Robinson** and **Marc Husband** clarify modelling in an elementary real-world problem solving. A third article, by **Dr. Matthew Man Shing Cheung** addresses the connection between high school and college math needs.

Lastly, you will find an announcement regarding nominations for the Board of Directors of OAME/AOEM. Please consider your strengths, or tap your colleagues on the shoulder and explain to them that they should give due consideration to this important supporting role.

Thank You!

I am indebted to all the columnists and the editorial team for, once again, moving up deadlines to accommodate my travel. It is sincerely appreciated and speaks volumes to the dedication of everyone who contributes to the *Gazette*. Perhaps, however, I should clarify—during the editing of this issue, four members of the editorial team travelled, and the unwritten rule is that no one on the team works while travelling. Yes, one can these days, but no, we don't, if at all avoidable. ▲

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OAME 2026 Let's {Talk} Math

Mark your calendars! From **April 30 to May 1, 2026,** the OAME Annual Conference moves to Fanshawe College in London, Ontario with the inspiring theme **"Let's Talk Math."** This conference is designed for K–12 math educators and students in teacher education programs, offering rich professional learning, networking, and idea exchange. OAME 2026 will showcase keynote addresses by **Dr. Pamela Seda**, co-author of *Choosing to See,* and **Pamela Harris**, author of numerous books designed to help students reason through their math. There will also be presentations by Chris Luzniak (Debate Math), Ian VanderBurgh (University of Waterloo), Kathy Marks Krpan (University of Toronto), Marian Small (author and speaker), Daniel Ansari (Western University), Alex Lawson (Lakehead University), and Octavia Beckles (Ontario Ministry of Education). The conference will dive into how we can more intentionally **talk**—with students, among ourselves, and about mathematics—as a pathway to deeper reasoning, equity, and mathematical community.

At OAME 2026, sessions will centre on five compelling "Talk Math" pillars, giving participants concrete strategies to bring back to their classrooms:

1. **Talk Equity** — Explore how dialogue can surface and disrupt inequities in math learning, and how to create inclusive spaces where all voices are heard.
2. **Talk Assessment** — Examine how to listen to students' mathematical conversations, and then provide timely feedback that nurtures growth.
3. **Talk Technology** — Harness digital tools and platforms to help amplify student voices, through tools that help students show their math reasoning, and foster mathematical discourse.
4. **Talk Content** — Re-imagine how to structure mathematical content so that students talk through ideas, strategies, conjectures, and misconceptions.
5. **Talk Pedagogy** — Engage with other educators in reflection and dialogue about instructional choices, classroom norms, and routines that centre student thinking and discourse.

If you're passionate about cultivating classrooms where conversation is as central as calculation, OAME 2026 promises to equip you with research-informed practices, rich collegial conversations, and renewed inspiration. For more information, please visit www.OAME2026.ca.

WRITTEN BY: MAUREEN SIMS

President's Message



KIT LUCE
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Kit has been an active member of MAC² for many years and joined the Board of Directors in 2022. She is a District Mathematics Facilitator for Grades K–9 with the Simcoe County District School Board. In her work, Kit creates math resources to support educators in their instruction and assessment practice, as well as developing and facilitating professional development. Kit is an Additional Qualification instructor and course writer, and she has written and reviewed work for a variety of publishers. She has been involved in Ministry writing projects and was a Content Lead for the recent OAME/AOEM writing projects. Kit received the 2024 OAME Award for Leadership in Mathematics Education.

Life at work has been busy lately as we prepare impactful professional learning for educators across both elementary and secondary panels. When planning any presentation, I always begin by considering what educators want to know, what students need to learn, and which resources best support those goals.

Recent EQAO (Education Quality and Accountability Office) survey data from May and June 2024 revealed compelling insights into students' attitudes toward mathematics. When asked whether they agreed with the statement, "A person can always get better at math," over 85 percent of students in Grades 3, 6, and 9 responded positively. This suggests a strong belief in the potential for mathematical growth. However, when asked if they agreed with, "I can answer difficult questions," agreement dropped to between 38 percent and 46 percent. It seems students believe in growth for others—but not necessarily for themselves.

What causes this disconnect?

Over the past five years, many resources have emphasized that mathematics should be accessible to all students, and that every learner can engage meaningfully with challenging concepts. Jo Boaler's work on www.youcubed.org offers valuable tools for fostering positive math mindsets. *The Ontario Curriculum, Grades 1–8: Mathematics Curriculum Context 2020* (Ontario Ministry of Education, p. 14) reinforces this, stating: "Having a positive attitude towards mathematics and developing self-efficacy are important elements of students' achievement of the expectations and of all future mathematics learning."

Self-efficacy, as defined by the American Psychological Association, is "an individual's subjective perception of their capability to perform in a given setting or to attain desired results" (https://dictionary.apa.org/self-efficacy). The curriculum's vision (p. 6) is intended to help all students develop a positive identity as mathematics learners—seeing themselves as mathematically skilled, using math to make sense of the world, and making critical decisions based on sound reasoning. This vision comes to life in classrooms filled with enthusiasm, where students receive high-quality instruction and are empowered to reach their full potential.

So how do we help students develop that positive identity?

One area I've been focusing on in professional development is designing lessons that make challenging math concepts accessible, emphasize conceptual understanding, and connect to both real-world contexts and rich mathematical thinking.

A powerful strategy is the use of open questions. These might offer multiple solutions or multiple pathways to a single solution, allowing students to engage based on their prior knowledge and experience. Marian Small has championed this approach at many OAME/AOEM conferences. Her series of books, *Open Questions*, are invaluable resources for educators from Kindergarten through Grade 9 and beyond. In one presentation, she outlined key criteria for a good open question:

- Engenders thinking, not repetition
- Focuses on important math
- Allows entry to all learners
- Leads to rich mathematical conversation
- Extends strong learners
- Provides ample assessment-for-learning opportunities

When students see possibilities in a math question—when they feel invited to explore and reason—it strengthens their identity as capable math learners.

Exploring foundational ideas—like fraction operations, measures of central tendency, and algebraic reasoning—helps students grasp the "Why?" behind procedures. Concrete models and visual representations are essential tools in this process. Consider how you use manipulatives in your classroom. Are they central to instruction, or reserved for "those students who need them"?

I encourage you to reflect on how your lessons help students see themselves as confident mathematics learners. Open questions, manipulatives, and visual models

offer a strong starting point. You might also ask your students directly:

- Do you believe you can solve challenging problems?
- Do you know what steps to take to improve in math?
- Do you believe you can be successful in math?

Gathering this data—and revisiting it in a few months—can offer powerful insights into your students' growth as mathematical thinkers.

As we continue to plan and teach with purpose, let's keep our focus on helping students build confidence and see themselves as capable math thinkers. The OAME/AOEM vision reminds us that every student deserves high-quality instruction and the chance to develop a positive identity as a mathematics learner. Whether it's through open questions, concrete tools, or rich conversations, we have the power to shape classrooms where students feel seen, challenged, and supported. Small shifts in our practice can lead to big changes in how students experience math—and in how they see themselves. That's the kind of impact we strive for, and it's at the heart of our work together.

OAME/AOEM is a community of passionate educators committed to equity and excellence in mathematics. Let's keep leaning on each other, sharing what works, and lifting up every learner—because when we grow together, our students thrive.

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Hypothesize: One Line at a Time

Riddle, riddle, riddle me
Number on a line!
What number can it be?
Like needles on a pine.
Am I large or just wee?
This many could dine.
Do you start to see?
A value less than nine!
A number you can hop to
You would have to move for
How much? Not "too"
Just a little, fore
More than a pair—that's two
But not all the way to four.

▲ OAME/NCTM Report: Scavenger Hunt!



GERARD LEWIS
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Gerry is an Elementary Mathematics Facilitator, who assists teachers of students with IEPs (Individual Education Plans) within the Toronto Catholic District School Board. Gerry has presented over 20 times across Canada and the United States, with a focus on equity, culturally responsive pedagogy, mathematical modelling, and integration of technology in the classroom. In 2024, he completed his Master of Education degree in mathematics education at York University. He is currently the Past President of the Ontario Mathematics Coordinators Association (OMCA), and for his work with mathematics teachers through the pandemic, he was awarded the Prime Minister's Award for Teaching Excellence in 2022.

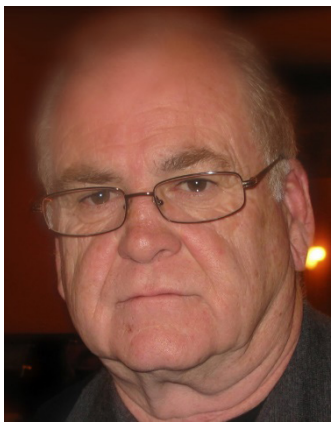
It's an honour to be selected as the NCTM representative for OAME/AOEM for the next few years. I would like to take a moment to celebrate Jacqueline Hill for her years of service and dedication to this position (among others). Her reports have made the information provided by the NCTM accessible to the OAME/AOEM community, so I have mighty shoes to fill with my reports. As a new writer for the report, I would like to celebrate this inaugural report with an NCTM Scavenger Hunt! All the answers can be found on the NCTM website (www.nctm.org). Once you've completed all the questions, send your answers to **gerard.lewis@oame.on.ca** before January 5, 2026. From the correct submissions, we will do live draws on Wednesday, January 7, 2026, where a selection of winners will each receive an NCTM publication as their prize. Details for the event will be emailed to those who enter the draw.

1. What does NCTM stand for?
2. When was NCTM founded?
3. Who is the current NCTM President?
4. As part of the NCTM Mission Statement, what are the four components of the Strategic Framework?
5. The OAME has a charter number as an NCTM affiliate. What is the number?
6. One way to participate with NCTM is to write an article for one of the three journals. What are the names of the three NCTM journals?

Continued on page 14

Remembering Paul Lessard: A Life Dedicated to the Love of Math

JULIAN D'ANGELA, PAT LASHMAR, AND RON LANCASTER



Paul Lessard, B.A., M.Ed., the pre-eminent Canadian mathematics educator, dedicated his 50-year career to a singular philosophy: *"If kids learn to love math, they will love to learn math."* This belief was the foundation of his work as a mathematics consultant, prolific author, and influential leader, touching the lives of millions of students and teachers across the country.

Paul's career spanned 30 years with the Hamilton-Wentworth Roman Catholic District School Board (HWCDSB) and an additional 20 years dedicated to writing and professional development.

- **Consultancy and Partnership:** Starting in 1977 as a Mathematics Consultant, Paul immediately sought collaboration. He teamed up with literacy consultant, Pat Lashmar, an alliance that proved incredibly effective over 27 years. They pioneered self-funded, board-wide mini-conferences and professional development days, bringing keynotes from the Ontario Institute for Studies in Education (OISE), such as Professors David Booth and Don Fraser, to hundreds of teachers and principals.
- **Organizational Leadership:** Paul was deeply involved in provincial math organizations. He was a Past President of the Ontario Association of Mathematics Educators (OAME), Co-Editor of the *Abacus*, and contributor to the *Ontario Mathematics Gazette*. He also held leadership roles in the Ontario Mathematics Coordinators Association (OMCA), which later honoured him with a Life Membership Award, alongside his colleagues Julian D'Angela and George Fawcett.
- **Systemic Influence:** Paul chaired the board's Heads of Mathematics Committee and, for over 25 years, led the math component of the system-wide Teacher-Trustee Education Committee. He also collaborated on preparing Grades 3 and 6 teachers for the introduction of Provincial Education Quality and Accountability Office (EQAO) testing, a massive effort that contributed to the HWCDSB consistently ranking in the Top 10 out of 60 school boards in Ontario. Paul also co-authored one of the EQAO Grade 6 math tests.
- **Hands-On Teaching Aids:** Paul connected with suppliers like Spectrum Educational and Nasco Educational Supplies early in his career, determined to use his budget to put more manipulatives into the hands of teachers.

Paul was a remarkably prolific writer, with his materials forming the backbone of elementary and intermediate math instruction for decades.

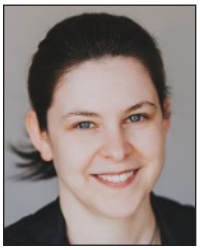
- **Impactful Textbooks:** Paul co-authored a successful series of Intermediate Mathematics textbooks for McGraw-Hill Ryerson (Grades 3–8), with over 200 000 copies sold. This translates to an estimated one million students, who used his materials over ten years.
- **Post-Retirement Writing:** After retiring from the HWCDSB, he continued his work with colleagues Julian D'Angela and George Fawcett, co-authoring 24 math titles over seven years for Tree House Press. These books, focused on Grades 2–8, brought his total book sales to over 300 000 copies, supporting a new generation of students.
- **Real-World Math:** Paul's work was deeply rooted in making math relevant. His publications, like *Finding Math in All the Usual Places* (95 pages of real-world investigations) and *Big Problem Math* (unique math adventures), encouraged students to describe, organize, and explain mathematics through intriguing photos and unique problems. He often photographed tiles, grocery store displays, and even sewer grates to create these investigations.
- **Legacy Donation:** Reflecting his dedication, Paul recently offered to donate his entire collection of self-produced resource materials on elementary math teaching to his alma mater, Wilfrid Laurier University, ensuring his work on the "mathematization of the young" continues to benefit future educators.

Paul had a kind, gentle, and personable nature that allowed him to immediately create a positive, collaborative connection with everyone he met.

- **Creating Math Adventures:** Paul established the HWCDSB's Junior Math Challenge competition, a daylong math adventure involving teams from every elementary school, which continues to this day.
- **Family Man:** For Paul, math was a family affair. His wife, Judy (an elementary teacher), and his children, Janet and Martin, were constantly involved in playing and refining his math investigations. Later, his six grandchildren became his favourite topic of conversation and a new audience for his ideas.
- **Community Service:** In retirement, Paul served as an active committee member for the Ancaster Film Fest (AFF), including serving on its finance committee. The AFF has donated over $300 000 to charitable organizations over the past 20 years.

Paul Lessard's life was a testament to his belief that when math is presented with creativity, passion, and relevance, children will not only learn it, but *learn to love it*.

Events at the Fields: Margaret Sinclair Award, "I don't do math" Exhibit, and Ask a Mathematician



PAMELA BRITTAIN
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Pamela Brittain (Ph.D.) is an educator, author, illustrator, presenter, and math lover, who currently works for the Fields Institute as their K–12 Academic Coordinator. She has been involved in STEM education for over 20 years and strives to make math accessible and enjoyable for everyone.

September MathEd Forum: Margaret Sinclair Award Presentation and Discussion

The September MathEd Forum celebrated the twelfth annual awarding of the Margaret Sinclair Memorial Award recognizing Innovation and Excellence in Mathematics Education. Dr. Sinclair was highly involved in the world of mathematics and math education. She organized junior math Olympiads and was highly involved in both the Canadian Mathematics Education Study Group (CMESG) and OAME through the Scarborough chapter. As attested by one of her advisers, Walter Whitely, she invested her time building communities of people. Her connection to the Fields Institute was solidified posthumously where, in recognition of her significant contributions to the field of mathematics education, she was named a fellow of the Fields Institute. The award in her name seeks to inspire educators to embody her spirit of innovation and dedication to making mathematics a subject that is not just understood, but also appreciated and loved.

This year's winner, Dr. Andie (Andrijana) Burazin (Figure 1), is an Assistant Professor, Teaching Stream at the University of Toronto Mississauga, and jointly appointed to the Institute for the Study of University Pedagogy and the Department of Mathematical and Computational Sciences. Her talk, "Communication at the Forefront of Teaching and Learning Mathematics," focused on building trust and communication. It emphasized the importance of trying to build both with your students, colleagues, and even the mathematics concepts that you teach. She promoted communicating with yourself, using reflections and learning from student feedback. Communicating with students involves connecting with them on a personal level, which might include things like saying hello in a different language each class, using pop-culture references, and "life pro-tips" to build trust and a sense that the instructor actually cares about their students. She also spoke about her love of using mantras and mnemonics to help herself and her students reinforce and remember topics.



Figure 1: *2025 Margaret Sinclair award winner, Andrijana (Andie) Burazin (right) with Deputy Director Patrick Ingram (left). Photo courtesy of The Fields Institute.*

When communicating with colleagues, Dr. Burazin spoke about her involvement with various organizations and organizing committees, including the Canadian Mathematics Society (CMS) Education Committee, and the First-Year Math & Stats in Canada (FYMSiC) group. She spoke of the importance of, and difficulties with, communicating and organizing large groups of people. Finally, when communicating with mathematics itself, she went to the chalkboard to demonstrate the deconstruction of typical textbook presentations of various topics in mathematics (such as limits and integration) and their reliance on "intuitions" that might be misleading or unclear to students, often confusing them and creating misconceptions.

Dr. Andie Burazin's talk was followed by a presentation by Dr. Diana Skrydlo, who is an Associate Professor, Teaching Stream in the Department of Statistics and Actuarial Science, and the current Math Faculty Teaching Fellow at the University of Waterloo. Her talk, "Reflecting Forward: Communication, Reflection, and Growth in Teaching and Learning," followed from Andie's "consider[ing] how communication and reflection work hand-in-hand to create more intentional, responsive, and connected learning environments." Dr. Skrydlo gave some tips and tricks for including reflection in class such as:

- pre-/post-concept tests
- reflective questions about assignments
- exam wrappers or autopsies (deconstructing and classifying incorrect answers)
- allowing multiple attempts with feedback where appropriate—including auto-graded quizzes with unlimited attempts, self-check questions with answers
- early-bird deadlines, where students can submit their assignments early, receive feedback, and then resubmit to be graded

The day ended with a discussion session, where the presenters reiterated many of the topics covered and provided additional advice to attendees. A key discussion point was ways to implement both reflection and communication techniques into their teachings. Recordings of the two talks can be found at www.fields.utoronto.ca/activities/25-26/meforum-Sep.

"i don't do math" Exhibit

From September 25 – October 15, Fields was proud to support the photo-based art exhibit, *i don't do math* (Figure 2), by Ottawa-based artist, Ann Piché. Inspired by Hiroshi Sugimoto's Mathematical Models (www.sugimotohiroshi.com/mathematical-models), Ann sought a way to create something that more clearly reflected her struggles and experiences with math. The *i don't do math* series was the result. The series "highlight[s] how math is everywhere, and although we may struggle to understand the math, we could understand the results."



Figure 2: *The* i don't do math *exhibit at the Ivey Library, by artist Ann Piché (Photos courtesy of Ann Piché)*

The artist described the exhibit:

> *i don't do math* is a photographic exploration of dyscalculia and the unseen struggle with numeracy. Numbers are part of daily life—from passwords and prices to bus timetables and bank balances. For many, navigating this world is effortless. For others, it is an invisible struggle. Dyscalculia, a learning difference that affects the ability to understand and process numbers, is as common as dyslexia, yet far less recognized. Too often mistaken for carelessness or lack of intelligence, it shapes a person's life, limiting opportunities and impacting both mental and physical well-being. Early recognition and awareness are vital, opening the door to support, confidence, and long-term empowerment. This series transforms abstract mathematics into tactile, black-and-white imagery, making the invisible struggle visible. By reimagining equations as photographs, the work invites viewers to consider the human side of numeracy—not as an impassable barrier, but as a deeply personal and often unseen experience.

The exhibit opened with a panel discussion featuring artist Ann Piché, Professors Daniel Ansari (Western University) and Darja Barr (University of Winnipeg), and JUMP Math founder and author of *All Things Being Equal,* John Mighton. During this discussion, the themes of Ann's exhibit were front and centre. Ann also explained the meaning behind the lower-case "i" in her title, saying that initially she had thought of the exhibit as exploring her own struggles with mathematics (an upper-case I), but later realized it was a shared experience of many people, hence the lower-case "i." The other presenters focused on how literacy and dyslexia are treated very differently from numeracy and dyscalculia. They noted how significant pushes at a global level to address gaps in literacy have not been seen for mathematics—something all panelists felt should be changed. You can view a recording of the speaker presentations at www.fields.utoronto.ca/activities/25-26/artsci-salon.

The exhibit was sponsored/supported by the New College NewOne: Learning without Borders program, the Fields Institute for Research in Mathematical Sciences, the University of Toronto, and York University. While the exhibit has passed, those interested in further information can contact Ann directly at apichephoto@hotmail.com, and can check out her website at www.apichephoto.com/.

Ask a Mathematician Returns

For the past four years, the Fields Institute has been bringing mathematicians, math researchers, and those in

industry who use math, virtually into classrooms across Ontario, and we have reached over 4000 students. Last June, we hosted an Ask a Mathematician live event with five experts, who shared their experiences with mathematics and answered questions such as the following:

- "What made you realize you wanted to be a mathematician?"
- "Were you always good at math?"
- "Why should a non-mathematician still study math?"
- "What struggles have you faced with mathematics?"
- "Do you typically work by yourself, or do you collaborate with others?" (spoiler, they all said they work best when collaborating with others)

The "experts" also gave a short talk about a topic in mathematics they find fascinating. The talks included "Catching Ghosts," "Time and Space," "Why does the dark side of the moon have more craters than the Earth side?," "Writing New Numbers," and "Error-Correcting Codes."

The live event highlighted many myths students, and the general population, have about what it means to study math, what it means to be a mathematician, and what types of skills you need to pursue mathematics. The full event, including some bonus questions, can be found at https://fieldsacademy.ca/aam/. If you are interested in requesting a virtual visit for your class, you can fill out the request form on that website as well.

Ongoing Programs at Fields

The *Math Circles* program investigates mathematics challenges and puzzles in a collaborative and guided setting, where students can explore skills such as looking for patterns, making conjectures, finding counter-examples, and constructing proofs. Open to students in Grades 7–12, these sessions are open to anyone who wants to explore math in more detail. Participants don't have to be "traditionally strong" in mathematics to join; they just need to be curious about exploring mathematical puzzles. Math Circles runs on Saturdays from 2–4 p.m., in person, at the Fields Institute (222 College Street, Toronto), and is free to join. You can learn more, and have students registered, at www.fieldsacademy.ca/eep/.

Our newest initiative is the *Math in Literacy* program, where we work with authors and illustrators of children's picture books, who are from communities typically underrepresented in mathematics. The program provides math and science activities for K–6 teachers. Resources and information can be found at www.fieldsacademy.ca/math-in-books/. Thanks to House of Anansi and their Groundwood department for supporting this initiative. ▲

▲ Elementary Math Matters: Spotlight on the Humble Number Line… Warts and All



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Lynda Colgan is Professor Emerita, Faculty of Education, Queen's University; Founder and Faculty Lead for Science Rendezvous Kingston; and Executive Director (Education and Development) for Science Rendezvous National. In May 2025, she was awarded the King Charles III Coronation Medal by the Chancellery of Honours at Rideau Hall for her lifelong contributions to education. Her latest project is as contributing author for an upcoming Springer publication, Teaching and Learning Elementary Mathematics*. Lynda remains deeply committed to her two loves: STEM outreach/public education, and her English Cream Retriever, Annie.*

In August 2025, I had the opportunity to attend the Canadian Early Mathematics Conference, organized by Dr. Sandy Youmans, and sponsored by STEAM+, a research group at the Queen's Faculty of Education.

Sandy kicked off the day with a talk about the work that she has been doing with the Assessment and Instruction in Math (AIM) Collective, a partnership that engages researchers and educators from school districts across Canada to support the development of tools for assessment and instruction in mathematics learning from Kindergarten through Grade 4. Next on the agenda was a session by Dr. John Mighton, founder of JUMP Math, who spoke about the addition of a new set of resources to the suite of JUMP Math curricula, designed to give early learners a strong start to their mathematics education journey: Kindergarten curriculum supports. Closing off the day was Ashley Kozak, Mathematics Curriculum Lead with the Upper Grand District School Board, who presented the results of a large-scale remediation project centred around an instructional program for supporting students identified by their low score on the early math screener that is used across the district.

All three talks emphasized the importance of using interactive and hands-on activities for assessment as well as instructional purposes; recommended concrete tasks requiring body–syntonic reasoning and manipulatives; use of conceptually based games; and spoke of the value of whole-class discussions with carefully chosen teacher

prompts to stimulate students' sharing of individually generated representations, and guiding the development of correct conceptions grounded in students' constructions.

As I looked back on my notes from the day, I was struck by one common element in all three talks—the focus on, and importance attributed to, the number line. Each speaker acknowledged the *spatial–numeric* nature of the number line—a quality that can lend physical and visual support to development of abstract numerical concepts, including quantity, order, and magnitude. The diverse examples of the speakers highlighted the ways in which the humble number line could be a powerful tool in early learners' construction of mental models and strategies for operations with whole numbers. The speakers also touched on the upward scalability of the number line by suggesting ways in which older students could use the number line as a cognitive support for developing proficiency in understanding and using concepts including operations with fractions and decimals.

The presenters' research-based advocacy for the possibilities of the number line's application in instruction and assessment was the impetus for this column—a consideration of the powerful roles and hidden challenges attributed to the number line. Why? The presenters were emphatic that the number line is an *abstract representation* that can be presented in many ways: starting at 0 or 1; having various scales, e.g., 1 to 10 or 1 to 20; or showing milestone numbers only, e.g., 0, 25, 50, 75, 100. Thus, even this harmless-seeming manipulative can have negative impacts on students' understanding, if it is introduced too early in a child's cognitive development in number.

Before the number line should be used as a manipulative for ideas such as counting or skip counting, young math students need to recognize numerals (and their associated quantity) and know some basic counting principles, most of which were articulated with the ground-breaking work of Jean Piaget, who published his foundations of the four stages of cognitive development in the 1936 book, *The Origins of Intelligence in Children.* Of note here is that the assessment, instructional, and remedial programs shared by Sandy, John, and Ashley on conference day, all began with the counting principles, since their research showed that many students in Grades 1–3 have not developed command of these foundational concepts, meaning that students face cognitive barriers that impede growth along the mathematics learning trajectory.

The Counting Principles

The significant period during which children recite number words without understanding their meaning (the equivalent of singing *The ABC Song*) typically occurs before the start of Junior or Senior Kindergarten in the home or childcare settings. Those experiences are important in that they bring familiarity, engagement, and cognitive memory to the act of saying numbers aloud. Although early learning centres are not required to do so, some implement the *ELECT Early Learning for Every Child Today* (ELECT) curriculum. First launched in 2007, the resource is still relevant, but is now complemented by the 2014 policy document, *How Does Learning Happen? Ontario's Pedagogy for the Early Years*. Both provide foundational frameworks and use shared language to describe the content, pedagogy, and classroom culture fundamental to early learning and childcare settings (including the home). Of relevance to this article is this excerpt from the ELECT document that describes *Number Sense* as understanding quantities, counting, and recognizing numerals—all of which are interrelated pieces of a multi-dimensional cognitive construct that moves children beyond simply "saying" numbers and prerequisites to using a number line.

Stable Order

Number stability means that the sequence of number words is always the same and never changes; i.e., *the counting words must be said in the same order every time.* Understanding that the names of numbers have a correct order and cannot be skipped over or left out is important across all situations presented to children as counting tasks.

One-to-One Correspondence

One-to-one correspondence (which Piaget called *numerical equivalence*) is demonstrated by having children say one number each time they point at, touch, or move an object. This is foundational to understanding that each object is assigned one, and only one, number. This concept goes beyond counting blocks one by one, and extends to tasks such as placing a bear in each one of a set of cups or distributing one napkin to each member of the group at snack time, counting aloud in the process.

Cardinality

Knowledge of *cardinality* is demonstrated when a child knows that the *last number* spoken in a counting sequence represents the *total number* of objects in the set. When asked, "How many?," a child at this stage answers by giving the correct quantity in the set *"all together*," without needing to recount.

Conservation

The principle of *conservation* (which Piaget called *stability*) can be assessed by asking students first to count the number of objects in a set when they are close together,

then again after the objects are rearranged so that they are spread out. Students who have mastered the principle of conservation will be able to tell the total for both sets, whether the objects in the set are close together or spread apart, *without recounting*.

Order Irrelevance

This principle refers to the fact that the order in which objects in a group are counted is not important; i.e., the number order does not change just because the physical order in which the objects were counted does. At this stage, the student is able to count a group of objects correctly from left to right, right to left, top to bottom, bottom to top, or arranged in a pattern (e.g., the pips on a die).

Abstraction

Abstraction refers to the ability of the child to count objects, regardless of which objects are being counted, without confusing physical magnitude with quantity. For example, five basketballs and five golf balls both represent "fiveness," despite the difference in their scale; or one bunny, one squirrel, and one rabbit are a total of three animals, despite being different types.

Subitizing

Though not technically a counting principle, the ability to subitize, or rapidly and accurately perceive the number of objects in a small set, without counting, is called *subitizing*, and is important in the early development of number sense. Pictures of pips on die or dot configurations on flash cards are frequently used as subitizing tasks for young children learning to recognize numbers and quantities; however, tasks that involve subitizing are not appropriate until *after* a child has realized the principle of conservation.

Children need to be confident and competent in verbal counting and the basic counting principles discussed, before introducing number *symbols*, i.e., digits that represent numbers. To this point, children's experiences with numbers and counting have been concrete and spoken; therefore, much instructional time must be devoted to supporting the child to link the written digit (numeral) to its quantitative meaning. Though it may not seem that way to us, digits are abstract symbols and require children to construct new cognitive networks before applying them in multiple contexts.

Number Paths Versus Number Lines

After multiple opportunities to link the spatial and numerical representations of digits through activities with counting collections that promote recognition, naming, associating the digit with a quantity, ordering, and describing their meaning, the next stage is to learn about and visualize number relationships with number paths.

A number path, Figure 1, is a simple model to show the natural or counting numbers in order, using a linear representation. Only the counting numbers are included, not zero. Each number is written in a rectangle or other shape, making it a discrete unit, which is an extension of young children's earlier counting experiences. Number paths are used on many board games, providing counting practice as players move their marker in response to rolling a die.



Figure 1: *A number path*

The number path is an important segue to the number line because it puts the numbers in order and illustrates their relative positions. Number paths may be used by children to count forwards or backwards, and provide options to touch the squares or not. For example, a child can use the number path to see that 7 is more than 4 and less that 8, or by counting (physically or mentally) that 7 is two squares less than 9 and four squares more than 3, thereby establishing preliminary understandings of arithmetic operations.

In contrast, from a mathematical perspective, a number line is a *measurement* model that illustrates the *distance between whole numbers*, using equally spaced increments along a continuous line, like a ruler. The number line is a tool that children can use to visualize how far away a number is from the starting point, but are they counting hash marks or spaces? They should be counting spaces.

The "starting point" on a number line raises another important teaching point. For young learners, number lines should begin at "1," to focus on familiar *quantities*. Children need to know that "0" is a digit that represents "none," meaning that it has no quantity (i.e., it cannot be counted), before transitioning to a number line that originates at zero. Zero is a very difficult concept for children to understand because it represents a multiplicity of concepts, i.e., meaning "nothing" in an abstract way; having a preferential position on the number line; and serving as a placeholder in the representation of numbers in the base-10 system.

The goal of the article is to situate the important function of the number line in the tool set used by children, to advance their understanding of early numerical concepts. By applying the number line in active explorations, children can extend their sense of the linear arrangement of numbers in order (e.g., before, after, and between), and develop strong visual and mental representations of how numbers increase or decrease in magnitude according to position on the number line, both of which are important precursors to

the introduction of arithmetic operations and the development of computational fluency.

The Number Line as an Important Diagnostic Tool

Since number lines allow children to physically "move" forward (adding) or backward (subtracting) along the line, they provide a window into visualizing these operations as movement or change, rather than just abstract symbols. As children gain familiarity with number lines and use them for "jumping" in increments (skip counting), the end point on the number scale, the number of increments (labelled or left blank) can be changed to reflect quantities within the range of their developmental understanding, from 20 to 100. This opens the door to the introduction of an important tool in a teacher's arsenal for assessing children's number sense—asking children to estimate the position of numbers on a number line showing only start and end points.

Variations on an early diagnostic assessment that was suggested by all three speakers at the Canadian Early Mathematics Conference included the following activities:

- Provide a blank number line and ask students to estimate and mark the position of a specific number, e.g., 5.
- Ask students to find a specific number, say 8, on a number line, and then show you different ways they could "land" there (e.g., finding the sum by starting at 4 and "jumping" right 4 more spaces, or finding the difference by starting at 10 and "jumping" left 2 spaces).

The student responses to these dynamic assessment tasks provided teachers with robust data about the young child's conceptual gaps—enough to enable the teacher to tailor individualized remedial instructional support. Examples like these were shared by all three speakers at the conference. Why? Because research has shown repeatedly that "of all the early assessments, children's ability to accurately map numbers onto a number line most robustly predicts their achievement in mathematics" (Hawes, quoted in Comay, 2023).

Conclusion

Despite its flexibility and openness, the number line is a non-trivial mathematics manipulative. By this point, the reader will have realized that most of this column has been dedicated to *preparing* students to use the number line. As such, it does not address (perhaps disappointingly to some readers) the full range of applications of the number line to mathematics education across the grades, e.g., as an alternate computational tool for traditional algorithmic or decomposition (block) models in two-digit addition (see Figure 2).

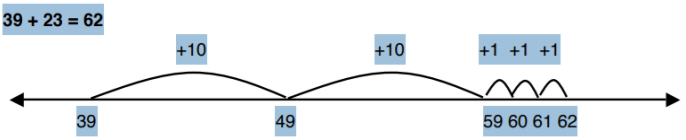


Figure 2: *In this example, the student is representing this addition problem as a series of jumps, starting at 39.*

However, the key take-away from this brief consideration about the number line should be this: though deceptively simple at first glance, the more teachers are aware of both the benefits and constraints of the humble number line, the more likely they are to use it effectively with students as they apply it to instruction about concepts related to numbers other than the whole numbers, i.e., fractions, integers, and decimals. There is no question that the number line has rich potential for teaching and learning, but we have all learned through life experiences that things can prove to be "too good to be true," and that a perfect veneer can mask some catch or downside. That cautionary expression applies even to mathematics education in the case of the number line.

For those readers interested in a more research-based commentary on the number line, I highly recommend the blog article from Canada's foremost early mathematics research centre: the Robertson Program. The post, *The Power of the Number Line: A Model That Scales*, was written by Julie Comay, who was a classroom teacher at the Dr. Eric Jackman Institute of Child Study (JICS) Laboratory School and in Toronto public schools for over 20 years, and is now an Assistant Professor at the Ontario Institute for Studies in Education. For practical ideas based on cognitive and pedagogical research, I recommend the United States-based Erikson Early Math Collaborative, where educators can find invaluable articles such as *What is a Number Path and How Does It Build Number Relationships?* (Brownell, 2023) and hundreds more.

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OMCA News



GERARD LEWIS
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Best wishes and happy holidays from the Ontario Mathematics Coordinators Association (OMCA)! Our members consist of system-level math coordinators, consultants, facilitators, and coaches from across the province. We engage in professional sharing, discussions, and professional development at our monthly meetings.

Our September meeting included an introduction to our executive: Melissa Peddie (President) chaired the meeting, which saw over 110 members in attendance; Gerry Lewis (Past President) provided an overview of OMCA's mission and vision, and he welcomed our Francophone members; Allison Lawrence (Secretary), who invited boards to consider hosting future meetings; Mariarosa Schniering (Treasurer), who reminded members to complete their registrations; and Alyssa Dixon (Vice-President), who shared details for our annual OMCA retreat.

We are excited to host our retreat at Hidden Valley Resort in Muskoka on February 26–27, 2026. We will be welcoming Dan Finkel (Math for Love – www.mathforlove.com/dan-finkel/) to be our featured speaker and moderator for the retreat. Please contact us if you would like more information about this event.

After surveying our membership at the end of last year, our theme of "Math Connections" will help frame our knowledge of content, pedagogy, and coaching as part of our monthly meetings. Our Fall learning began with a mini-series led by Tara Flynn and Shelley Yearley, whose research was guided by the following question: Which instructional strategies support Junior and Intermediate students in the learning of fractions? The September, October, and November sessions engaged our members in the eight core concepts as part of the fraction learning pathway. As the discussion evolved from unit thinking to representations, one particular instructional practice resonated with our members: *Introduce a new concept with a familiar representation; introduce a new representation with a familiar concept.*

In December, January, and March, we will be welcoming Diane Sweeney from Student-Centred Coaching. She will inspire us to think about how coaching makes a measurable impact on the teaching and learning of mathematics. Diane will guide us through the framework for coaching cycles, how we engage in conversations that are student-centred, insights around building a culture for coaching, and methods for supporting and evaluating coaches.

In addition to this professional learning, each OMCA meeting includes updates from recognized organizational delegates (including the EQAO (Education Quality and Accountability Office), the Ontario Ministry of Education, OAME/AOEM, AFEMO (l'Association francophone pour l'enseignement des mathématiques en Ontario), CEMC (Centre for Education in Mathematics and Computing), the Fields Institute, and more); networking opportunities; and resource showcasing. Contact us at OMCAmath@gmail.com to learn more about the organization, find out about upcoming events, and connect with us.

We look forward to seeing you at our upcoming winter meetings on the following dates:

- December 15, 2025—Virtual
- January 12, 2026—Virtual
- February 26–27, 2026—OMCA Retreat at Hidden Valley Resort
- March 9, 2026—Virtual

Continued from page 6

7. In the NCTM Illuminations Interactive called "Bunny Times," which level allows students to practise 11x and 12x multiplication facts?
8. There are two NCTM Spring conferences planned for February and March 2026. Which two cities are hosting the Spring conferences?
9. NCTM position statements define a particular problem, issue, or need, and describe its relevance to mathematics education. In the position statement, "Teaching Mathematics to Students with Disabilities," the NCTM and CEC jointly recognize the important role of educators for students with disabilities. What does the acronym CEC stand for in this case?
10. Research Briefs and Clips provide a foundation for delivering research findings about mathematics teaching and learning to teachers, administrators, school boards, and parents/guardians. Provide the URL for one of the clips, and explain why the clip is interesting to you.

Have a great holiday season and a healthy new year! Happy hunting on the NCTM website! ▲

MB4T (Mathematics by and for Teachers): The Journey to Prime Factorization



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One of the most fascinating parts of teaching mathematics is the way a single problem can draw out so many different ways of thinking. This is evident when we ask Grades 5–8 students to work with factors and multiples. The ideas feel familiar, yet the strategies students bring to them can vary enormously. Give the same task to a class and you see the variety unfold: some lean on divisibility rules they recall, others build and rebuild numbers until they fit, while others look for relationships that allow them to streamline the work. What emerges is not one fixed pathway, but a collection of entry points, each revealing a different way of seeing and interacting with the mathematics.

The problem that begins chapter 10 from *Making Math Meaningful to Canadian Students, K–8* captures this idea (Small, 2020). Small poses the challenge: What is the smallest whole number divisible by 2, 3, 4, 5, 6, 7, 8, 9, and 10? When my students tackled this question, the room filled with curiosity and uncertainty. Some students began with strategies they already trusted, like guessing and checking. Others, unsure of how to begin, turned to partners, questioning and reworking their first attempts. Slowly, patterns of reasoning emerged. A few relied on persistence and divisibility rules; for example, a number is divisible by 5 if its last digit is 0 or 5, and since it is divisible by 2, its last digit is even, so it must end in zero, etc.; others experimented with building and adjusting; some noticed that certain numbers could be set aside because they were already accounted for by larger ones; and a handful of students moved straight to prime factorization.

What struck me in that moment was not how long it took students to reach a conclusion, but the breadth of strategies students brought forward. The variety of approaches opened a window into how students make sense of number structure, and the many ways number sense takes shape. This article explores how students move from forming lists of multiples to reasoning with prime powers, uncovering their journey to prime factorization. By tracing this journey, we can gain insight into how students develop efficiency and abstraction in mathematics, and how each strategy along the way plays a meaningful role in that growth.

For some students, the most natural starting point is a mechanical approach. They take each number (2, 3, 4, 5, 6, 7, 8, 9, 10) and begin listing multiples, one sequence after another. Threes climb: 3, 6, 9, 12, 15…; sevens never quite line up with the others: 7, 14, 21, 28…. The chart fills until, many lines down, one number finally appears across them all: 2520. The strategy works, but the effort is enormous, and perseverance is required. The effort of producing list after list quickly shows students how time-consuming the approach can be. Some pause in frustration and remark: "There has to be a faster way!" That recognition becomes a turning point, as awareness of the length of the process sparks the search for more efficient strategies.

Obviously the number has to be 10 or bigger, so I'm going to use multiples of 10 for trial and error.

10 ÷ 9 = not a whole number

20 ÷ 9 = not a whole number.

Wait. The smallest number divisible by 10 AND 9 is 90.
Because 10 x 9 = 90. So we keep going up multiples of 10 AND 9 from 90. So multiples of 90.

90 ÷ 8 = not a whole number
180 ÷ 8 = not a whole number
270 ÷ 8 = not whole number
360 ÷ 8 = 45, 360 ÷ 7 = not a whole number
450 ÷ 8 = not whole number
540 ÷ 8 = not whole number

Figure 1: *Testing multiples of 90*

Another group of students starts with the realization that

the number must be 10 or larger, so they narrow to testing multiples of 10. They begin with 10, 20, 30…, but then realize: “Wait. The smallest number divisible by 10 and 9 is 90” (see Figure 1). They refine their search and keep going with multiples of 90 to test the next step: is it divisible by 8? They try 180 divided by 8, but that is not a whole number. The same thing appears with 270. Then they find that 360 divided by 8 is equal to 45, which is a whole number. Now they test to see if dividing it by 7 would result in a whole number, which it does not, so they keep going, trying to find a multiple of 90 that is both divisible by 8 and by 7. They purposefully chose to work in decreasing order, recognizing that if a number is divisible by 8, it will also be divisible by 4 and 2, but not the other way around. This strategic decision shows that they were beginning to see the numbers as forming a partially ordered set. Building on this insight, they carried on with multiples of 90 until they reached 2520. At this point in their journey, their conclusion feels earned: “If I have a number that is divisible by 10, 9, 8, 7, and 6, it will be divisible by 2, 3, 4, and 5. So I only have to test my five biggest numbers” (see Figure 2).

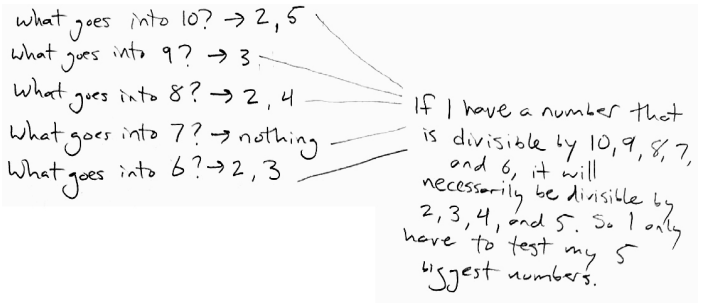


Figure 2: *Reasoning for testing the five largest numbers*

Other students try to build the answer piece by piece (see Figure 3). They approach the problem by incrementally adding each number by which it must be divisible. They start with finding a number that is both divisible by 2 and 3. They begin by listing the multiples of 2: 2, 4, 6, 8, 10, 12; and the multiples of 3: 3, 6, 9, 12. Here they realize that the least whole number that appears in both lists is 6. Now they add on. They make sure that it can also be divisible by 4. Knowing that 6 is divisible by 2 and 3, they list the multiples of 6: 6, 12, 18, 24; and the multiples of 4: 4, 8, 12, 16. Again, they find that 12 is the least whole number that appears in both lists, which implies that 12 can be divisible by 2, 3, and 4. They continue this approach, adding in a factor of 5. Listing the multiples of 12 and 5, they find the lowest common multiple is 60. Since 60 is divisible by 6, they move on to listing the multiples of 60 and 7. They continue this process with 8, 9, and 10, until finally they land on 2520. Students using this approach are reassured that they did account for all the numbers, but start to wonder if there is a way to know in advance which factors will be needed, rather than adjusting step by step.

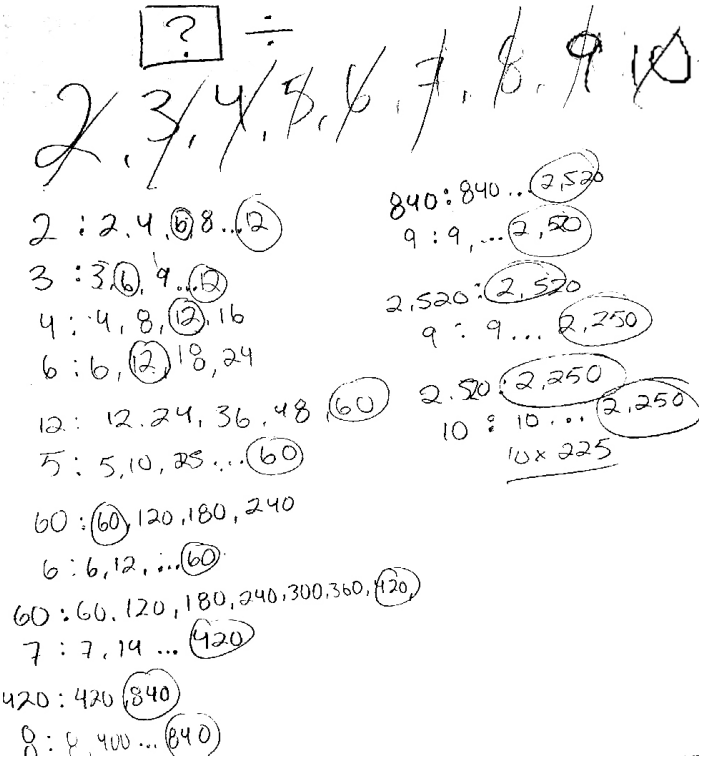


Figure 3: *Integrating divisors one at a time*

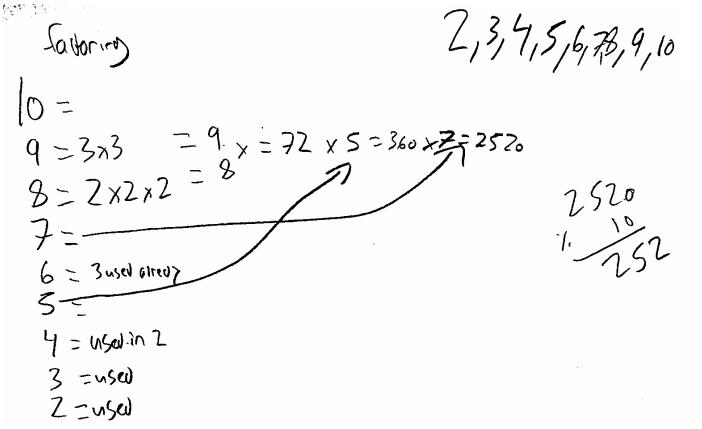


Figure 4: *Narrowing to key divisors*

Another set of students begin by narrowing instead of building. They begin to realize some numbers are redundant and why. For example, if 8 is included, then 2 and 4 are automatically “used” (see Figure 4) because anything that is divisible by 8 is also divisible by 4 and by 2. Similarly, if 9 is included, then 3 is also covered, since anything divisible by 9 will also be divisible by 3. Six is equivalent to 2 multiplied by 3, so the 2 is already present in 8 and the 3 is already present in 9, meaning that anything that is divisible by 9 and 8 will also be divisible by 6. By the end, students have

reduced the list to just 9, 8, 7, and 5. Multiplying them together produces 2520 directly, and then they test to check that it is also divisible by 10. Students in this group often remark how much faster this approach is than listing all the multiples of each number, but they also wonder how they can show that they are not missing any divisors.

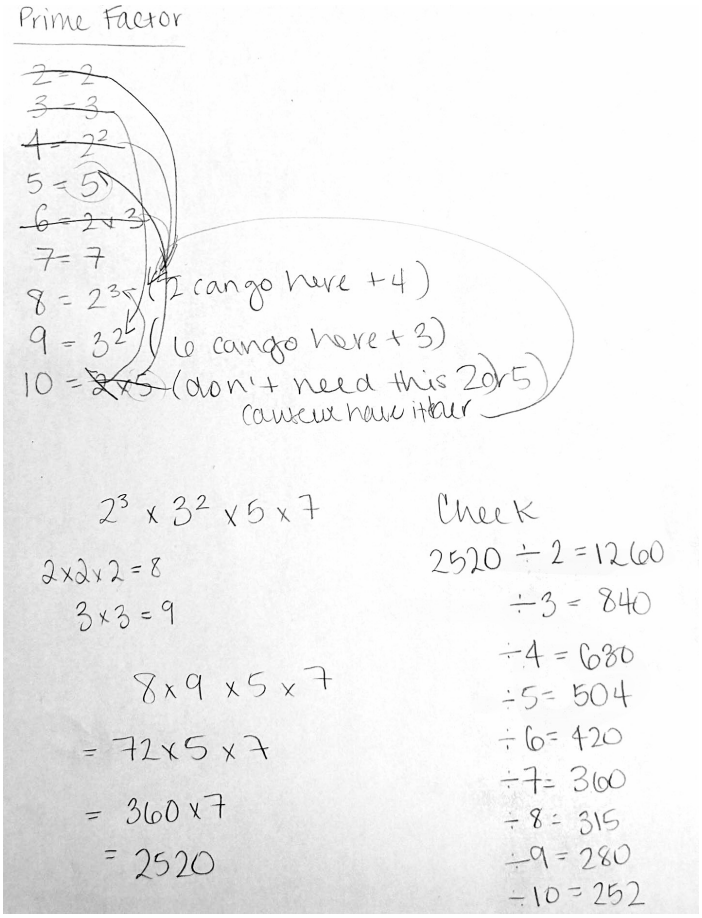


Figure 5: *Reasoning with prime powers*

Finally, some students using prior knowledge leap to prime factorization. They recognize that every number can be written as a product of prime numbers. This gives 2, 3, 4 = 2 x 2, 5, 6 = 2 x 3, 7, 8 = 2 x 2 x 2, 9 = 3 x 3, 10 = 2 x 5. They notice they do not need to keep every prime factor from every number, only the largest powers (see Figure 5). Instead of writing out long overlapping lists, they begin to scan across the set, asking: What is the largest requirement for each prime? From 8, they see that the final product includes three 2's, because 2^3 is the strongest demand placed anywhere in the list. From 9, they take two 3's, since 3^2 is the highest requirement. From 5 and 7, they take a single copy of each because no number in the set contains a higher power of either. By collecting these—2^3, 3^2, 5, and 7—and multiplying them together, the students arrive at the product 2520. When students reach this stage, they comment on how efficient it feels: nothing is left out, but nothing is duplicated either.

What becomes clear across these strategies is a natural progression of learning. Students who skip-count and test multiples are grounding themselves in divisibility rules. Those who build incrementally are beginning to ask deeper questions about what is missing. Those who trim the list are recognizing redundancy and efficiency. And those who use prime factorization are applying abstraction to capture the entire problem in a single general approach. Each pathway has value because each reflects how students make sense of number structure at different stages of development. By attending to these strategies, teachers can see where students are, anticipate where they might go next, and support them in making connections across methods.

For teachers, the value of this problem lies in what it reveals about student thinking. Watching students navigate it offers a diagnostic lens. We see not only what students know, but how they choose to apply it. Are they comfortable with divisibility rules? Do they understand factors as building blocks? Can they reason about redundancy? Have they made the leap to abstraction through prime factorization? The answers to these questions give us insight into their number sense far beyond the correctness of a single solution.

Too often, least common multiples are taught using a single algorithm: factor, take the highest powers, multiply. While efficient, this skips the rich reasoning that should precede it. Allowing students to struggle, test, and adjust, honours the developmental path that leads naturally to prime factorization. Ultimately, the chapter problem from *Making Math Meaningful to Canadian Students, K–8* reminds us that powerful open tasks in mathematics are not about getting students to the answer quickly, but about creating space where multiple entry points exist, strategies emerge, misconceptions are uncovered, and structure comes into view. When the answer, 2520, becomes not just a number, but the result of reasoning, we gain a deeper understanding of both mathematics and how it is learned. Remember, the destination matters, and so does the journey.

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Math Ecologist?

"He has formed new mathematical theory that has allowed scientists model animal movement and territories, identify ecological dynamics in aquatic systems, and determine the spread of invasive species and of disease among wildlife." See student-readable article at https://.news.uvic.ca/2025/uvic-math-ecologist-elected-to-royal-society/.

Play, Math, and Early Learning: Problem Solving



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Jennifer Holm is an Associate Professor at Wilfrid Laurier University and works with pre-service teachers, as well as in the field supporting current mathematics teachers. She is interested in developing mathematics knowledge for teaching; focusing on the beliefs and opinions that pre-service teachers hold about mathematics and teaching and the connection they have to past experiences. She uses this research to support future teachers in developing beliefs and knowledge that will encourage and support effective teaching practices in mathematics. She is also the proud mom of two young learners, who are exploring the world of mathematics.

This column focuses on problem solving in the early years. Young children are avidly exploring their environments and problem-solving on a daily basis. In this column, we explore how to recognize problem solving in mathematics as it is happening during play and unstructured times.

Problem solving in the elementary school classroom is one of the many mathematical processes in the Ontario curriculum (Ontario Ministry of Education [OME], 2020). In Strand A, the curriculum clarifies the expectation in problem solving as "develop, select, and apply problem-solving strategies." In the Algebra strand, the mathematics curriculum also includes the expectation of mathematical modelling. In exploring the Mathematical Modelling process, the curriculum notes that, "Mathematical modelling provides authentic connections to real-life situations. The process starts with ill-defined, often messy real-life problems that may have several different solutions that are all correct" (OME, 2020–2025, para. 1). In this process, the student would include understanding the problem, analyzing the situation, creating a model, and assessing the model. This is not a linear process because students are exploring options as they problem-solve. This model is used for the entire Grades 1 to 8 curriculum. If we consider this is what students learn with problem solving, how can we support and recognize problem solving in our earlier learners?

In *Early Learning for Every Child Today: A Framework for Ontario Early Childhood Settings* (OME, 2007), problem solving is one of the skills identified in the cognition section of the framework for both infants and toddlers. What is expected continues to grow on the continuum as the child ages, but it is a skill that is identified as important to recognize and encourage for early development. In toddlers, they also add "spatial problem solving" as a skill to develop through the toddler years. Problem solving is also emphasized as important for early learners in *How Does Learning Happen? Ontario's Pedagogy for the Early Years* (OME, 2014).

If we start by thinking about the process of problem solving, there are many different problem-solving frameworks that are available. In the previous Ontario curriculum, Pólya's problem-solving framework (Pólya, 1957) was provided as "understand the problem; make a plan; carry out the plan; and look back to check the results" (OME, 2005, p. 12). If we think about problem solving in these terms, seeing how early learners are problem-solving becomes more apparent and easier to recognize.

To illustrate my point, I am going to provide an example from my own early learner. My toddler sleeps with a "lion lovey" that got left in his crib. He wanted to have it with him during the day, but the lovey's head is a little large to get between the bars of his crib. As I watched him trying to figure out the spatial-sense problem presented to him, he worked through the framework for problem solving.

First, he understood his problem was that he wanted his lovey, but lion was currently in his crib after his naptime. His plan was to pull lion through the bars of his crib so that he could have his lovey with him. He attempted this plan, and lion's head quickly became stuck and would not pass between the bars. Further pulling was not helping him remove his favoured toy. At this point, he was checking his results, which were unsuccessful. I watched as he set lion back down on his crib mattress and reconsidered the problem. As I watched, he made a new plan, and he decided to use both hands: one hand lifted lion to the top of his crib bars, and the other retrieved his toy that he had flipped onto the top of his crib. He had successfully solved his problem!

I did not take time to coach or teach my child about problem solving: he intuitively worked through the process on his own. All I needed to do was offer some encouragement when his efforts to pull the lion through the bars were unsuccessful and frustrating. I was impressed with the amount of mathematics processing in this small interaction: he was presented with a problem about working a shape through a space where measurement was an issue, and he had to work out how he could solve the problem when he could not make the space any bigger or the object any smaller.

What if we consider what we see in the classroom with the early learners? Think of all the mathematical potential

there can be. Problem solving might occur when building a tower of blocks and trying to decide how to create a structure that can sustain the tallest building. There can be problem solving in the sandbox when considering how to move sand into a pail when there is either no shovel or the shovel is disproportionately big or small relative to the container. There can be problem solving when trying to get dressed on their own and realizing that they cannot walk when both legs are in the same side of the shorts (but you can walk when your shoes are on the wrong feet). Experience tells us that not every child will approach each of these as a problem: some may know or have been shown a solution or an effective process right away, and they just apply mathematics skills. Keep in mind that true problem solving occurs when a complete solution method is not known in advance (National Council of Teachers of Mathematics, 2000). This means that what one person defines as a problem is not the same as another person (Kajander & Holm, 2025).

To operationalize this in early learning spaces, keeping a problem-solving framework (like Pólya's) in mind is helpful for observing problem solving opposed to mathematical play. The key moment where rescuing the lion became problem solving was the pause after the initial plan did not work, and the consideration of a new plan or strategy before trying again. This iterative process is where I observed the problem solving occurring. He did not have prior knowledge or experience getting his lion from the crib, and his initial plan was unsuccessful, but he considered another method to try again.

In elementary school grades, this process is going to be formalized: students will need to record their thinking and processing to clarify what took place. They will need to communicate how they attempted the problem, when they saw unsuccessful attempts, how they reconsidered when unsuccessful, and how they determined when the problem was "solved." Again, if we consider that problem solving is only occurring when the process is not known in advance, even in elementary school, one needs to be mindful of whether you are seeing problem solving or purposeful practice. To continue my example, every subsequent time the lion needed to be rescued, he used two hands; he was no longer problem-solving, but practising his proven solution method and becoming quicker at it (gaining fluency).

As early learning educators, we need to be observant for a variety of reasons with young learners, but taking the time to consider when you are seeing problem solving can be helpful to setting them up for success. Giving feedback such as, "I liked when you did not give up" or "Since that did not work at first, what could you try next?," or providing helpful tips that do not give away a solution, but may point to another tool, can help to encourage the problem-solving process.

We also need to be mindful of when the problem itself is too advanced for the learner and they are reaching a point where they do not have the social–emotional skills to overcome the frustration and they shut down or stop trying. In my example, there were previous times where my toddler had attempted to remove the lion from his crib, only to become incredibly frustrated and upset to the point that it was clear he was distressed about not having his friend. In those instances, helping him retrieve lion was more appropriate than waiting for him to figure it out. When children are trying to get dressed and happen to put both legs in the same leg hole, they may problem-solve to figure out why they cannot walk properly, or this skill may be beyond their limited motor functions, and they need help. It is the same in elementary classrooms: observing learners to see when a problem is too advanced for them is important so that one can intervene before they become too frustrated and give up. We want learners to explore problem solving, but in a way that does not cause them to become excessively frustrated.

With early learners, we do not need to name the framework in specific terms (although I would do so with elementary students, to give them a process for tackling a problem), but we can make it easier for them to consider that they are problem-solving. To do this, I am going to go through another example while naming the steps that might be suited to use with early learners. Consider watching students building with blocks. We might ask, "What are you trying to build?" This helps them to identify the problem they are going to solve. The child may be trying to build a tower as tall as they possibly can, using all of the blocks. Next, you might ask, "How are you going to do that?" They may be attempting to just pile the blocks one on top of each other until they have used all of them. In this step, I would encourage them to follow the method they suggested and watch the process. Once they have worked on building the tower, the final step is to ask, "What happened?" This calls their attention to what they did to build the tower. If they were just stacking blocks straight up, more than likely the tower is going to fall over, since there is limited structure. At this point, you can ask why they think it happened and encourage them to try something different. Hopefully they will add structure, perhaps starting with a wider base, so that they have a stronger tower in their second attempt.

Not only is it important to make sure that students are not getting too frustrated, it is equally important with young learners to allow experiments to fail and see that not all methods work. There are times when they will not be able to

solve problems, but we want to encourage them to keep trying. Some day, they will be able to build the tallest tower and not have it fall, or they will be able to get dressed on their own, without having help! This is all part of the process of growing up and gaining new knowledge and skills that will help them to tackle the problems that they are faced with each day.

In the end, it is so important to encourage early learners to problem-solve and for us to recognize and encourage them when it is happening. It is important to note that not all problem solving will be mathematical, but the process we use as a framework for mathematical problem solving can extend to other areas of problem solving. Focusing on problem solving in the early years allows for a foundation that will help them to be successful in daily toddler life (rescuing the lion), elementary school, and beyond. Problem solving is a part of every day of their lives, and learning skills to be resilient problem solvers in the beginning will only help them succeed later.

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Impact of Screen Time?

"In this cohort study of 3322 Grade 3 children and 2084 Grade 6 children recruited from primary care settings in Ontario, Canada, between 2008 and 2023, higher parent-reported total screen time and TV and digital media time were associated with lower reading and math achievement on standardized tests in elementary school."

See www.jamanetwork.com/journals/jamanetworkopen/fullarticle/2839927 for details.

▲ In the Middle: Financial Literacy, Beyond Just a Test



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Carly Ziniuk teaches Grade 12 Data Management, Advanced Placement Research, and Advanced Placement Statistics at the Bishop Strachan School in Toronto. She is active in adopting real-life data to engage her students in solving problems. Among 30 global STEM leaders, Carly was part of 2024 NASA Space Apps Collective. As a Science Communicator, she is working to connect astronomy and mathematics for learners of all ages across Canada.

On May 30, 2024, the Government of Ontario announced a series of changes to the Ontario Secondary School Diploma (OSSD) requirements, for the first time in 25 years (Ontario Ministry of Education [OME], 2024a; 2024b). The first change noted in their Press Release was a financial literacy graduation requirement, which prioritized financial skills. The standardized test would be completed as part of their Grade 10 math course and would require students to score 70 percent or higher. The 2020 curriculum revisions included the strand of financial literacy throughout all of the Ontario K–9 Mathematics. The corresponding press release (2024b) justified this additional graduation requirement with the research that, "According to a 2022 Royal Bank of Canada poll, nearly 83 per cent of young Canadians reported needing more information and support on money management, and 68 per cent reported feeling overwhelmed and needed help."

Specifically, this financial literacy test would have a focus on decision making and literacy. The "Backgrounder" accompanying this Press Release (OME, 2024a) noted specifically that:

- students will be able to create and manage a household budget, save for a home or asset, and protect themselves from financial fraud
- the new graduation requirement would be delivered as part of the compulsory Grade 10 mathematics course to build on cumulative financial literacy learning from Grades 1 to 9

To prepare for the upcoming test, during the 2024–2025 school year, I created a checklist of the Ontario Financial Literacy, Grades 1–9 curriculum (bit.ly/4oDc49W). I worked with my Grade 9 students and the K–8 teachers in my

school to review these overall expectations. The section below shows a subset of the expectations from Grades 7–8.

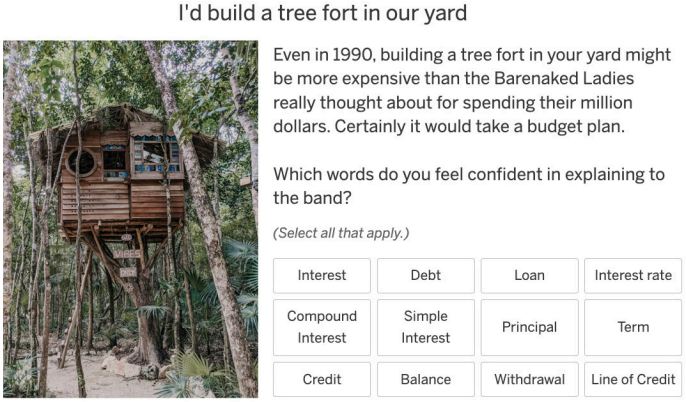
Ontario Financial Literacy 7–8 Curriculum Expectations	
Grade	Theme and Overall Expectation
7	**Money:** Identify and compare exchange rates, and convert foreign currencies to Canadian dollars and vice versa
7	**Financial Management:** Identify and describe various reliable sources of information that can help with planning for and reaching a financial goal
7	**Financial Management:** Create, track, and adjust sample budgets designed to meet longer-term financial goals for various scenarios
7	**Financial Management:** Identify various societal and personal factors that may influence financial decision making, and describe the effects that each might have
7	**Consumer and Civic Awareness:** Explain how interest rates can impact savings, investments, and the cost of borrowing to pay for goods and services over time
7	**Consumer and Civic Awareness:** Compare interest rates and fees for different accounts and loans offered by various financial institutions, and determine the best option for different scenarios
8	**Money:** Describe some advantages and disadvantages of various methods of payment that can be used when dealing with multiple currencies and exchange rates
8	**Financial Management:** Create a financial plan to reach a long-term financial goal, accounting for income, expenses, and tax implications
8	**Financial Management:** Identify different ways to maintain a balanced budget, and use appropriate tools to track all income and spending, for several different scenarios
8	**Financial Management:** Determine the growth of simple and compound interest at various rates using digital tools, and explain the impact interest has on long-term financial planning
8	**Consumer and Civic Awareness:** Compare various ways for consumers to get more value for their money when spending, including taking advantage of sales and customer loyalty and incentive programs, and determine the best choice for different scenarios
8	**Consumer and Civic Awareness:** Compare interest rates, annual fees, and rewards and other incentives offered by various credit card companies and consumer contracts to determine the best value and the best choice for different scenarios

Pauline Martin, a good friend and incredible high school teacher in Kenora, and I noted that our destreamed Grade 9 students were not well versed in the terminology and skills needed, and lack mastery of these Grades 1–8 mathematics expectations. In considering the curricular expectations and what the Ministry had noted about the overall Financial Literacy expectations, Pauline identified the following broad areas of student need:

- Recognize and explain components of a useful Financial Literacy vocabulary including *interest, loan, credit card, budget, debt, term, investment, income, salary,* and *principal.*
- Describe how to apply these financial concepts, using mathematical concepts like percentages, linear and non-linear models, coordinate graphing, use of algebraic formulae.
- Justify how to make different financial choices, to reduce costs, make more money, make more sense, or take less time.

At the end of June, new Minister of Education, Paul Calandra, indicated that the government would be pausing all planned curriculum changes until the 2026–2027 school year, including this Financial Literacy graduation requirement (Fox, 2025). With extra time for more effective preparation, these activities promote spiralling, interleaving practice, and use of financial literacy vocabulary throughout the other curricular components of mathematics instruction. By repeatedly addressing multiple strands with problems like these, teachers can hope to reduce the stress of adding an external, high-stakes assessment into the lives of their students.

Middle school teachers can ensure that their students know the necessary financial vocabulary before applying the concepts. As I have described in the March 2025 *Gazette* issue, I have been using Amplify (previously Desmos) Activity Builder for both student self-assessment and teacher organized assessment. In upcoming issues, I will demonstrate some further uses of Activity Builder for Financial Literacy, but if you are experienced with using this tool, using the check-box option for student self-assessment with vocabulary can be an efficient data-collection tool. The vocabulary shown here is appropriate for middle school level and above, as I modelled for teachers at the CEMC (Centre for Education in Mathematics and Computing) Summer Conference.



(Note that this task is referencing the song "If I had a million dollars" by the band Barenaked Ladies.)

Another way to assess your student's financial vocabulary is through matching vocabulary to definition or application. Card matching, using a platform like Activity

Builder is an option here, but I recommend using physical cards or through a matching paper–pencil activity like the one here.

Match up these financial words with their definitions.				
A	Interest calculated based on the principal amount and interest		Interest	
B	The initial amount, either borrowed or invested		Term	
C	How quickly something changes over time (e.g., price of a stock or the value of an investment)		Borrowing	
D	Money put into an account		Lending	
E	Money owed to a person or a business		Loan	
F	Percentage change of how something decreases over a period of time		Debt	
G	Interest calculated based on the principal amount		Account	
H	The period of time for which money is borrowed or invested		Simple Interest	
I	Charge or fee of borrowing money (borrower's perspective); or the amount earned from lending money (lender's perspective)		Rate of Change	
J	Money lent, usually with an interest charge or fee		Income	
K	A detailed record of all the money that a business or a person receives and spends		Interest Rate	
L	Obtain money for temporary use from a lender		Deposit	
M	When you buy something with the hope of it increasing in value		Compound Interest	
N	The percentage charged on borrowed money or earned on saved money		Principal Amount	
O	Providing money to another person with the expectation that it will be returned		Appreciation Rate	
P	Percentage change of how something increases over a period of time		Depreciation Rate	
Q	Money earned by an individual or business		Investment	

Some of these vocabulary words may seem to be "basic understanding," and yet will need to be explicitly taught to students. Similarly, the curriculum expects Grades 1 and 2 students to explore Canadian coins and make change appropriately for simple cash transactions. This seems to require assessing student understanding before just pushing forward.

The Royal Canadian Mint resources can help introduce your students to the specifics of Canadian currency. Students new to Canada, in particular, may need additional support in this area. Teachers at the CEMC Conference noted that more frequently with card-based transactions (e.g., Interac or credit card), students now have less experience with coins. Perhaps you have some pennies lying around as a curiosity or as a manipulative or both! The discussion of the penny and the necessary rounding when it was withdrawn from circulation is a practical mathematical application for students. In addition, you might be interested in sharing some of the details the Royal Canadian Mint shares about Canadian coins over time, and how they have changed, especially the penny! Take a look at how pennies have changed from 1980 and 2000. (Note that the penny was no longer produced after 2013, but the website says "present.")



1980 – 1981

Composition: 98% copper, 1.75% tin, 0.25% zinc
Weight (g): 2.8
Diameter (mm): 19
Thickness (mm): 1.45

2000 – present*

Composition: 94% steel, 1.5% nickel, 4.5% copper plating
Weight (g): 2.35
Diameter (mm): 19.05
Thickness (mm): 1.45

At the CEMC Conference, some conference participants with experience teaching Chemistry suggested exploring density and electrochemical reactions. Certainly, you could create some appropriate percentage calculations or even determine the volume and surface areas of the coins as thin

prisms. Another challenge I proposed to teachers when I presented at the CEMC Conference was to consider, without looking at the date, the different coins by weight. Consider how balance-scale problems here would work with pennies.

1. If you know that one of the pennies is older (and thus heavy) from a group of 50 (perhaps they are in a roll, remember those!), what is the minimum number of weighings you would need to figure out which penny is heavier?

Moving from one small manipulative to another, I regularly use M&M's to consider unit rates. At the start of the Grade 9 destreamed course, I use M&M's, without a calculator or a ruler, for the following questions. I have students agree or disagree with the following prompts, one at a time, and then spend some time comparing their choice to the class overall.

- A $10 bill has an area of about 40 M&M's.
- 1 000 000 M&M's would cost you over $100.
- Eating one M&M every minute for an entire day (not night) costs over $100.

Listening to students estimate the size of a $10 bill and how many M&M's you can get for a given amount of money is insightful and usually collaborative. As homework, I ask students to answer the following question, with the understanding that this could mean use of calculators, research, and measurements.

2. Is the value of a line of M&M's that stretches for 1 km over a million dollars?

I explain that during the next class, students will share their written solution details with their group. After comparing their solutions, each student gets one caramel M&M, one plain M&M, and a small ruler.

3. How does your answer to the above question change based on the current prices for caramel M&M's from Walmart in August 2025?
   - Individual size, 40 g for $1.99
   - Share and Go size, 109 g for $2.47
   - Family size, 325 g for $8.99
   - Bowl size, 380 g for $9.98

Additionally, I note that purchasing all of these products in Ontario would require 13% HST. Addressing the learning goals of consumer/civic awareness, students will need to calculate the unit rates for goods and services, identify which rates offer best value, and different ways to determine whether something is reasonably priced. To solve this problem, students will need to interpret situations involving adjusting budgets and calculating tax.

The activities and questions presented here are deeply informed by Pauline Martin's suggestions, advice, and contributions to our workshop at CEMC in August. Pauline is retiring from active classroom teaching in February 2026, but will continue to be involved in the mathematics education community. I am truly grateful for her insights, creativity, and practical teaching ideas throughout both of our careers. I am delighted to share our collaborative financial literacy work for this broader audience and will continue to provide more from this longer workshop in the upcoming few issues of the *Gazette* as well. Hint: In the next column, I'll be discussing "tips." A bounty of riches is in store for us all!

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Cursive Probability?

In lower case cursive, 23% of dictionary words and 28% of text need full line height. The rest only need half height.

What's the Problem? Number 29



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Shawn is a former mathematics teacher, department head, and consultant. He strongly believes in the central role of problem solving in the mathematics classroom. He continues to be involved in mathematical activities: presenting workshops, writing articles, working on local projects, and helping create mathematics contests.

Welcome back, problem solvers. We will be doing something a little different in this issue. As many readers may be aware, Canadian hockey icon, Ken Dryden, passed away on September 5, 2025. As a child, Dryden was my idol. As an adult, I had the pleasure to hear Dryden when he was the keynote speaker at OAME 1997—The Name of the Game. Dryden's "fly on the classroom wall" book on education (Dryden, 1995) is worth a read. In this issue, we will talk a little about the number Dryden wore, my favourite number, 29.



Figure 1: *The author with Dryden's equipment at the Hockey Hall of Fame, 2010*

Why 29? On top of being Dryden's number, my birthday is the 29th, fittingly, on National Coffee Day. In cribbage, a game I have played most of my life, a perfect hand scores 29 points. I have never gotten a perfect hand or seen one played by anyone else, but I have seen the next best thing, 28, played *against* me, *twice*, by my wife (who, ironically, has 28 as her favourite number). Another Canadian icon, Gordon Lightfoot, sings "The church bell chimed 'til it rang twenty-nine times for each man on the Edmund Fitzgerald" in a classic from 1976.

From a mathematical point of view, 29 is the tenth prime number. All numbers less than 29 have two or fewer prime factors. Since $2 \times 29 + 1 = 59$, which is also prime, 29 is called a *Sophie Germain prime* after the great French mathematician, physicist, and philosopher. 59 would be called a *safe prime*, i.e., when p and $2p+1$ are both prime, the former is a Sophie Germain prime, and the latter is a safe prime. Writing $29 = 2 \times 3 \times 5 - 1 = p_3\# - 1$, where $p_3\#$ is the third *primorial.* A primorial is like a factorial, the product of the first consecutive positive integers; for example, $3! = 3 \times 2 \times 1$, except it is the product of the first consecutive primes. That makes 29 a primorial prime. The function $f(n) = 2n^2 + 29$ produces prime numbers for the first 29 whole number arguments of 0, 1, …, 28. The number 29 also shows up in some famous sequences like the lesser-known sibling to the Fibonacci sequence, the *Lucas sequence*: 2, 1, 3, 4, 7, 11, 18, **29**,…. Interested readers may enjoy searching other places where 29, or your own favourite number, shows up.

On top of all that, 29, with 31, is one-half of a twin-prime pair. Twin primes are consecutive odd numbers that are both prime. Since 2 divides all even numbers, 2 and 3 are the only possible consecutive primes. Checking a list of prime numbers, one would see that 3, 5, and 7 are a string of three consecutive odd numbers that are prime. Further searches will not reveal any other sets of three, or more, consecutive odd numbers that are prime. However, just because we cannot find such a set, doesn't mean that there isn't one lurking around in the dark somewhere. Let's take a closer look.

Take a minute and write down three consecutive odd numbers. Since we are interested in primes, factor your numbers into primes as you go. Do this for several lists and see if you find any patterns. Don't worry, I will be waiting patiently for you to come back.

So, what did you find? I will go through a few examples:

29	31	$33 = 3 \times 11$
43	$45 = 3^2 \times 5$	47

$91 = 7\times13$	$93 = 3\times31$	$95 = 5\times19$

If you are working with smaller numbers, there is a good chance you might come across some twin primes. However, if you are paying attention, you will notice that you will *always* have one multiple of 3 in your list. Why must this be true? If we are going to list three consecutive odd numbers, we have one of the following three cases:

Case 1: The smallest number is a multiple of 3. For our situation, since we were looking for three consecutive primes, we would probably not start with this. The only way this could work in our favour is if the smallest number were 3, which gives us the list we suspect is the only example: 3, 5, 7. Generally, $3n$, $3n + 2$, and $3n + 4$ only have one multiple of 3.

Case 2: The smallest number is one more than a multiple of 3. Then, we can write the three odd numbers as $3n + 1$, $3n + 3$, $3n + 5$. However, the second number, and only the second number, will then be a multiple of 3. That is, $3n + 3 = 3(n + 3)$. Notice that in my examples, the second (43, 45, 47) and third fell into this category (91, 93, 95).

Case 3: The smallest number is two more (or one less) than a multiple of 3. Similarly, we can write the numbers as $3n + 2$, $3n + 4$, $3n + 6 = 3(n + 2)$, and the third number, and only the third number, is a multiple of 3, like my first example (29, 31, 33).

Hence, any list of three consecutive odd numbers will always contain a multiple of 3. This generalizes to any list of three consecutive terms of any arithmetic sequence, whose common difference is not 0 or a multiple of 3, contains one multiple of 3. Interested readers may wish to explore this idea further.

This categorization of numbers, according to the remainder they leave when divided by a fixed number, is related to a tool in number theory called *modular arithmetic*. Some may have heard of this idea referred to as *clock arithmetic*. Some concepts from modular arithmetic can be visualized easily by putting numbers into columns. In Figure 2, we have the numbers arranged into four, six, and ten columns. In the first table, we see that 10 and 26 are in the same column. We would say that 10 and 26 are *congruent modulo 4*. Congruent tells us that they are in the same column, which really means they leave the same remainder when divided by 4, called the *modulus* (thus the modulo). Mathematicians, being famously lazy, are always looking for some cute notation to write things and would write this as $10 \equiv 26 \pmod 4$.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16
17	18	19	20
21	22	23	24
25	26	27	28
29	30	31	32

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80

Figure 2: *Prime numbers modulo 4, 6, and 10*

You will notice that, since I have chosen even moduli, the columns contain either all even numbers or all odd numbers. Also, if we look at the column of the modulus, it contains multiples of this number. There are other interesting things going on; for example, since 3 is a factor of 6, the 3 column (in the modulo 6 table) contains multiples of 3, but not all the multiples of 3.

An interesting thing pops up in the middle table, modulo 6. Since the 3 column contains multiples of 3, the 6 column contains multiples of 6, and the 2 and 4 columns contain even numbers. Primes (other than 2 and 3) can only appear in columns 1 or 5; that is, all primes larger than 3 are either one more or one less than a multiple of 6. This is another way to see that we cannot get three consecutive odd primes other than 3, 5, and 7.

There is tons of fun to be had with modular arithmetic. You can look for primes and see where they reside in the tables. You can explore operations and see if you can find any patterns. I mentioned modular arithmetic in an earlier column (Godin, 2020) and gave some references for further reading for interested readers. I have created several large tables for you to explore numbers modulo 3 up to modulo 20 that you can find in the members-only section of the OAME/AOME website under “Files.”

Let’s see what else we can find. The first thing I thought of, after knowing that 29 is prime, is that it can be written as the sum of two consecutive integers, $14 + 15 = 29$. That might lead us to explore what other numbers can be written this way. Starting systematically, we get

$$1 + 2 = 3$$
$$2 + 3 = 5$$
$$3 + 4 = 7$$
$$\vdots$$

If we add $0 + 1 = 1$ to our list, we might conjecture that all odd numbers can be written this way. If we think about it, it makes sense that the response is odd, since we are adding two consecutive numbers; then, if the first is even, the second is odd and vice versa. Hence, since the sum of an even number and an odd number is odd, our sum would be odd. If we use a little algebra, then letting the smallest number be n, then the next number would be $n + 1$, so the

sum would be $2n + 1$, which is one more than an even number and hence odd.

That was cute, but not very interesting. However, if we always have our thinking cap on, we can ask "What if…?" questions. In this case, we might be interested in what happens when we add 3 or 4 or any number of consecutive numbers. Let's see what we can get.

Proceeding on to three consecutive numbers, we get

$$0 + 1 + 2 = 3$$
$$1 + 2 + 3 = 6$$
$$2 + 3 + 4 = 9$$
$$\vdots$$

Now that is interesting! It seems that the sum is always a multiple of 3. If we look closer, we can see that $3 = 3 \times 1$, $6 = 3 \times 2$, and $9 = 3 \times 3$. So, it seems that the answer is always three times the middle number, so if we added $29 + 30 + 31$, we should get $3 \times 30 = 90$, which we do. Going back to algebra, letting the numbers be n, $n + 1$, and $n + 2$, we get, $n + (n + 1) + (n + 2) = 3n + 3 = 3(n + 1)$, verifying out conjecture. What if, instead of letting the smallest number be n, we had let the *middle* number be N? Then the other two numbers would be $N - 1$ and $N - 2$ and we would get

$$(N - 1) + N + (N + 1) = 3N.$$

Cool! This is a common trick of using symmetry when we assign our variable names. It might also lead us to conjecture that if we add together any *odd* number of consecutive numbers, we end up with the middle number multiplied by the number of numbers being added together. Looking at the algebra for the first few cases, we get

$$(N - 1) + N + (N + 1) = 3N$$
$$(N - 2) + (N - 1) + N + (N + 1) + (N + 2) = 5N$$
$$(N-3) + (N-2) + (N-1) + N + (N+1) + (N+2) + (N+3) = 7N$$

We could also visualize this with manipulatives. Figure 3 shows the case of the nine consecutive numbers 4, 5, 6, 7, 8, 9, 10, 11, 12, which yields $4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 = 9 \times 8 = 87$.

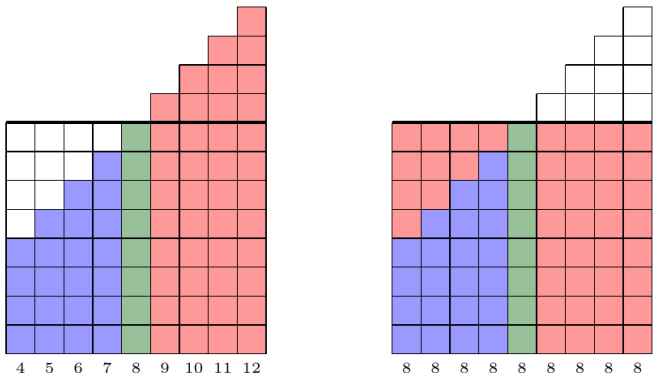


Figure 3: *The sum of nine consecutive numbers*

What happens if we look at the sum of an even number of consecutive numbers? We have already seen that two consecutive numbers yield the odd numbers. Proceeding to four consecutive numbers, we will let them be n, $n + 1$, $n + 2$, and $n + 3$. Adding them, we get, $n + (n + 1) + (n + 2) + (n + 3) = 4n + 6$, which is not a multiple of 4, but if we look closely, we can rewrite this as $4n + 6 = 4n + 4 + 2 = 4n(n + 1) + 2$, or 2 more than a multiple of 4. Looking at the next couple of cases in the same way, we see

$$n + (n + 1) + \cdots + (n + 5) = 6n + 15$$
$$= 6n(n + 2) + 3 = 6(n + 2) + \frac{6}{2}.$$
$$n + (n + 1) + \cdots + (n + 7) = 8n + 28$$
$$= 8n(n + 3) + 4 = 8(n + 3) + \frac{8}{2}.$$
$$n + (n + 1) + \cdots + (n + 9) = 10n + 45$$
$$= 10n(n + 4) + 5 = 10(n + 4) + \frac{10}{2}.$$

There are all kinds of cool patterns going on. Those who know a bit about sequences and series will notice that if we are looking at the sum of k consecutive integers, we get

$$n + (n + 1) + \cdots + (n + (k - 1))$$
$$= (n + n + \cdots + n) + (1 + 2 + \cdots + (k - 1)),$$

which, by Gauss's trick or knowing the sum of an arithmetic sequence, we can rewrite as

$$kn + \frac{(k-1)k}{2} = k\left(n + \frac{k-1}{2}\right),$$

which yields the expressions we have already discovered. The interested reader may want to verify this.

There is no end to your explorations if you keep your mind open to "What if…?" Another thought that springs to mind for 29 is the fact that $29 = 4 + 25 = 2^2 + 5^2$, so 29 can be written as the sum of two squares. You may want to investigate what other numbers have this property (a well-known result in number theory): see, for example, Godin, 2011. If you are keen-eyed, you may recall the smallest Pythagorean triple is $2^3 + 4^2 = 5^2$, which, when combined with our previous result, yields $2^3 + 3^2 + 4^2 = 29$. So, 29 is the sum of three consecutive squares! Interested readers may want to pursue this idea, as we did with sums of consecutive integers above.

Have fun with all your investigations. Thanks for the inspiration, Ken. Until next time, happy problem solving.

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▲ Useful Theories to Know About: Glasser's Choice Theory



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Jeff Irvine, Ph.D., has been a secondary mathematics teacher, department head, and vice-principal. He has taught at three faculties of education and at a community college. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. With over 45 years in education, Jeff is particularly interested in the interplay of student motivation and mathematics achievement.

Do you wonder why a student's behaviour is sometimes unexpected or seems out of character? William Glasser, M.D., has developed a theory of behaviour that not only may explain unexpected behaviours, but also provide teachers with some ways to shape behaviours into more socially acceptable actions. Glasser has applied his theory to education. He has also developed a medical therapy based on his theory to help students with extreme anti-social tendencies modify their positions.

Glasser's Choice Theory

Glasser (1998) states that all people, including children and adolescents, have five basic needs:

- Safety and security (sometimes phrased as survival)
- Belonging
- Freedom
- Power
- Fun

Safety and security as a basic need is somewhat obvious. People need to have a safe environment in which to learn; food and shelter needs must be met; students must be free from mental or physical risk. Belonging is the social need to have friends, feel respected and safe from ridicule, and have a peer group with whom to associate. Freedom relates to student agency. Students need to feel they are in control of some aspects of their learning and their lives. Power refers to psychological power, empowering the student to make decisions about their education and their life. It is the need to demonstrate competence in their decision making and making positive choices. This may occur during class, in an extracurricular activity, in their social lives, or in their home lives. Fun is obvious. Students need to have fun—whether in their learning, their out-of-school lives, or their eventual occupations.

Glasser says that if these five needs are not met, people will act to meet these needs. This may sometimes result in extreme anti-social actions, such as theft, physical altercations, or, in the case of students, behaviours like skipping, acting out in class, or dropping out of school altogether.

Why Should We (Math Teachers) Care About This Theory?

By understanding Glasser's Choice Theory, teachers can act to support students in meeting their five needs in positive ways. We can ensure that students are not subjected to bullying behaviours by others; we may need to refer particular students to a breakfast program, school guidance counsellors, or even to social agencies if their shelter situations are uncertain; teachers can also ensure that we don't contribute, such as through the use sarcasm or belittling students in class.

Employing group activities and co-operative learning strategies help address student needs for belonging, and in the case of co-operative learning strategies, may also help to address the need to demonstrate power and competence. Teachers may also be able to address students' need for belonging by suggesting potential extracurricular activities. By providing choice in some activities, some of the time, students can exercise agency and have some control over some aspects of their learning. This does not mean total freedom, simply that at least some of the time, students get to choose. The choices may involve what they learn, how they learn it, or how they demonstrate their learning. For example, teachers may provide ways for students to demonstrate their academic competence through presentations, free-choice assignments or investigations, taking a lead role in group discussions, as well as encouraging the posing of problems and alternative solutions.

Fun may be the last to be addressed here, but it is not last in importance. There is an abundance of research that confirms the need for students to be motivated and engaged in their own learning. That does not mean that teachers should provide or encourage frivolous fun, simply that coming to your class should be enjoyable and not dreaded by students. When I was in the classroom, I had a simple philosophy. Every day when a student left my class, they should be able to say, "Today I learned something, and I had fun."

Continued on page 29

Musing on Percentages: Families of %



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Percent is everywhere—all around us in finance, shops, foods, media, surveys, and so on. Percentages may seem like a rather simple mathematical theme that has little depth. However, when one starts to pay attention to its usage, percentages can be seen as filled with surprising elements that are mathematically rich and conceptually fascinating, hence worth pondering. This is something that became quite salient for our research team as we have been preparing experiments with students in classrooms about percentages in mental mathematical environments.

These columns endeavour to raise and address some of these peculiarities, which we came to call "curiosities," related to the concept of percentages. These musings can be seen to be as much about our usage of "%" in everyday life, in schools, or in mathematics in general. This sixth column (others available in the *Gazette* archive in the members-only area of the website) addresses issues related to families of %. It is my hope that these issues, and their underlying mathematical properties, will intrigue you.

Initial Steps

One of the properties frequently addressed in school is the commutativity of %:

$$A\% \text{ of } B = B\% \text{ of } A.$$

This property is quite useful to solve a variety of tasks about %. For example, whereas 14% of 50 might be seen as quite a challenge, 50% of 14 seems much more straightforward; both giving 7 as an answer. That said, there are numerous other questions of the *A*% of *B* type that also give 7 as an answer; for example 28% of 25, 35% of 20, 2% of 350, and so on, are all worth 7. Beyond this answer of 7, it is mostly the links existing between these different "tasks" that are of mathematical interest. As such, how are these various "tasks" linked to the point of being equivalent by giving the same answer?

Explaining Equivalencies

As mentioned, 14% of 50 is equivalent to 28% of 25. But why is that? One way to make sense of this is through considering issues of what could be called "compensations." The "28% of" requires taking a % that is the double of 14%, and this doubled % (i.e., 28%) is taken from a number that is twice as small as 50, that is, from 25. It is in that sense that it might be said that there is a compensation: taking twice as much of something that is twice as small. This compensation between the % and the number can help explain the equivalence in the final answer of 7 for both 14% of 50 and 28% of 25. The same thing can be said for 2% of 350: 2% is seven times smaller than 14%, thus it takes seven times less of the number, but the number is seven times bigger than 50, as 350. There is again a compensation between seven times bigger and seven times smaller, with a final answer still being 7.

These compensations are quite demanding to assess, and additional and more direct examples might help in making it easier. Take, for example, 14% of 50 and 7% of 100. The 7% of 100 requires taking one % twice as small as 14%, but for the number 100, which is twice as big as 50. The equivalence of answer is maintained. The same is to be said for comparing it with 28% of 25, where 7% of 100 represents one % that is four times as small as 28%, but for a number that is four times as big as 25, that is, for 100. The fascinating thing about these compensations is that they represent only a handful of examples, merely the tip of the iceberg…

Going Beyond Compensations to Families of %

A few years ago, I wrote an article in the *Gazette* addressing issues arising with families of equivalence in mathematics (see Proulx, 2021). Examples of families that I mentioned were about statistical distributions possessing the same mean, median, mode, standard deviation, and so on, about algebraic equations having the same solution for *x*, and about rectangles possessing the same area. For each of those, I showed that each distribution, algebraic equation, or rectangle was part of a family of (infinite) members that shared the same properties. I now want to build on these ideas to address the issues of compensations of % raised above in terms of families of %.

As such, this type of reasoning by compensation on numbers and % leads to consideration of a panoply of

equivalencies: between 2% of 350 and 10% of 70 (one % being five times bigger for a number five times smaller), between 35% of 20 and 7% of 100 (again five times bigger compensated by five times smaller), between 700% of 1 and 14% of 50 (with here a compensation of 50 times), between 40% of 17.5 and 50% of 14 (with a 4/5 compensation), and so on and so forth. All of these % are equivalent, two by two, in terms of their final answer.

What is fascinating is that these equivalences also stand within each pair! It is indeed possible, with this compensation "game," to move from one pair to the other as one wishes. Doing this gives rise to an entire family of % giving the same final answer of 7: 14% of 50, 28% of 25, 2% of 350, 7% of 100, 50% of 14, and others like 700% of 1, 25% of 28, 40% of 17,5, 35% of 20, etc. These %'s are all members of the same family of %.

Under this view, a family of % has an infinite number of members that have in common the same final answer (here, of 7). Additionally, each member in the family can be used to generate many others (for example, through making compensations from one % to the other, as done above). Therefore, it is possible to start with one %, for example, 14% of 50, and then generate numerous other members of the same family. In addition, not only can an infinite number of members be generated from one single member, but it is also possible to jump from, or link, any member of the family to another through establishing compensations, as was illustrated. These equivalences never end, and it is possible to generate or link members indefinitely. Isn't that fascinating? And, as I'll try to show, this is quite a powerful mathematical idea.

Families of % as a Powerful Mathematical Idea

What makes this idea of families of % even more attractive, mathematically speaking, is that the establishment of these equivalencies under the umbrella of families of % can render some % questions much easier to address. As Antoine, one of the members in my research lab, pointed out, this idea of families of % shows as much that: (1) there exists an infinite number of % questions that give the same answer, but also that (2) any % question can be transformed into another one that might be easier to solve. And it is here that the idea of families of % becomes powerful.

Imagine one is confronted with a difficult % question, for example, 18% of 55. This % question is not easy to calculate mentally. To solve it, however, one could modify it for another member of the same family of % and obtain the same answer. Hence, 18% of 55 can become 9% of 110, or 110% of 9, then compensated to 11% of 90, to 1% of 990, until 100% of 9,9! And, in these series of transformations, depending on what one needs, one can choose one % that appears to be the easiest or most helpful. To some extent, and this is an argument made in the article published on families of equivalences in 2021 mentioned above, this idea of families of equivalences makes it as if all the % questions *were the same*! All of these % questions are simply "clothed" with different numbers, but one is really addressing the same answer "owned" by the family of %.

These equivalencies are then representing a significantly powerful mathematical idea to address % questions, as much through permutations (*A*% of *B* = *B*% of *A*) as through compensations (bigger than, smaller than). And, this idea also insists on the fact that the establishment of these equivalencies shows how these % questions are not isolated, but are part of a tightly knit ensemble. As such, this idea of families of % is not only fascinating, but quite useful, mathematically speaking! I don't know about you, but in our research lab, this idea led a huge % of us to see these questions under a totally different lens…

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Continued from page 27

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Math Kangaroo

Contest for Grades 1–12 takes place March 22, 2026.

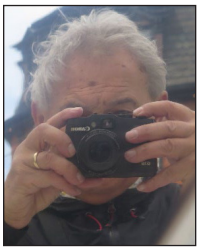
Ran in 111 countries in 2025.

Originated in France in 1991.

In 2025, Canada saw 6832 students engage with it.



The Math Teacher's Home Base: Poetic Patterns—Playing with Words, Numbers, and Structure



RON LANCASTER
ron2718@nas.net

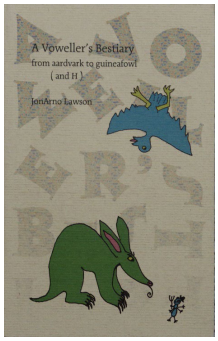
Ron is an Associate Professor Emeritus at the University of Toronto, where he taught mathematics courses for pre-service teachers for 17 years. Ron taught middle and high school mathematics for 21 years in coed public schools and an all-girls school. Ron's professional activities include conference presentations/consultations in North America, Asia, England, the Middle East, Africa, and India. Ron has authored/edited over 200 articles for NCTM and OAME. Ron received the 2015 Margaret Sinclair Memorial Award Recognizing Innovation and Excellence in Mathematics Education, awarded by the Fields Institute, and was the recipient of the 2022 OAME Lifetime Achievement Award.

Welcome back to "The Math Teacher's Home Base," where we continue to explore the intersection of mathematics, creativity, and curiosity. In this column, we're leaving the games shelf behind and turning toward another beloved section of my office: the bookshelves.

This time, I'm focusing on a corner of my collection filled with curious books that celebrate wordplay, structure, and the mathematical beauty of language. These books (Photograph 1) don't just tell stories—they build them through numerical sequences, vowel restrictions, clever poetic rules, and shaped poems that bring out the hidden mathematics of words.



Photograph 1: *Some of the books in my library that celebrate wordplay*



Photograph 2: A Voweller's Bestiary *by JonArno Lawson*

One of my favourite poetic constraints comes from JonArno Lawson's *A Voweller's Bestiary* (Photograph 2), a brilliant collection of 32 poems, each limited to specific vowel combinations. For example, in a poem titled "Raccoons," only the vowels "a" and "o" appear. In "Deer," only "e" is used. That means no "a," "i," "o," or "u"—just "e." Amazingly, Lawson not only makes it work—he makes it beautiful. Even the names of animals in each poem follow the vowel restriction. Each of the 32 poems is built around a different combination (Table 1).

Deer

Deer delve deeper,
peer between endless greens,
gentle breezes tremble the reeds,
tempers seethe,
regrets deepen.

Whenever we freeze,
then flee–
Whenever we're tender,
then severe–

we resemble deer.

In class, students can take on creative challenges based on these ideas. They might brainstorm or photograph a word for each vowel combination shown in Table 1.

Table 1: The 32 letter combinations

case	word	case	word
a		a, e, o	
e		a, e, u	
i		a, i, o	
o		a, i, u	
u		a, o, u	
a, e		e, i, o	
a, i		e, i, u	
a, o		e, o, u	
a, u		i, o, u	
e, i		a, e, i, o	
e, o		a, e, i, u	
e, u		a, e, o, u	
i, o		a, i, o, u	
i, u		e, i, o, u	
o, u		a, e, i, o, u	
a, e, i		none	

The mathematics behind this poetic approach is elegant. With five vowels—A, E, I, O, and U—the general approach allows each either used or not used, there are exactly 2 to the power of 5, or 32, possible combinations or rules for this type of poem. This can be visualized as a binary tree (Figure 1), where each level of the tree represents a yes/no decision for including a vowel. If we consider the letter "Y" to be a vowel as well, that adds another level to the tree, doubling the possibilities to 64.

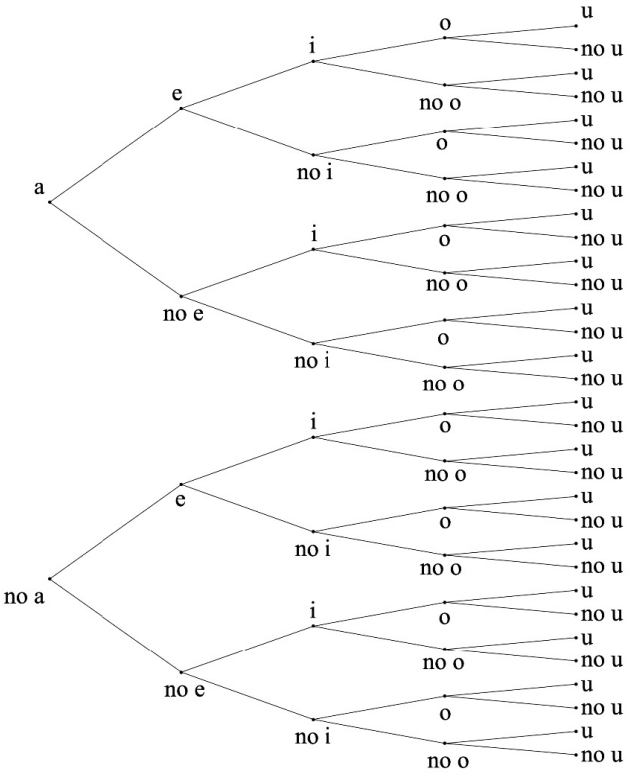


Figure 1: *Binary tree*

Seeing the possibilities laid out like this helps students make the leap from poetic play to combinatorics. This connects beautifully with other binary-based systems like Braille. Each Braille character is made up of six possible dot positions—each either raised or not—resulting in 64 combinations, just like the expanded vowel tree. These binary-based systems are ideal for helping students explore how logical switches, on or off, lead to exponential growth in possibility.

In class, students can take on creative challenges based on these ideas. They might brainstorm or photograph a word for each vowel combination. What's a word that uses only "i," "o," and "u," like "curious"? Can they write a poem with just those vowels? Or even create sentences where the *n*th word has *n* letters? These kinds of investigations blend linguistic play with combinatorics and sequence building—opening doors to rich mathematical questions.

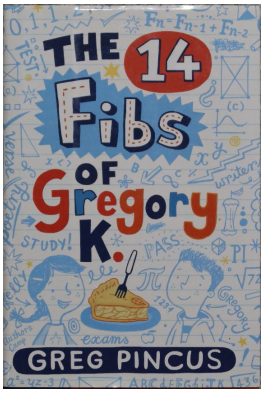
In my collection, you'll also find *The 14 Fibs of Gregory K.* by Greg Pincus (Photograph 3). A Fib is a poetic form, where each line follows the Fibonacci sequence in syllables: 1, 1, 2, 3, 5, 8. Like the famous rabbit population model, these poems build quickly. For example, in a 20-line Fib, the number of syllables in the 20th line would be 6765!

Here's the opening Fib from Pincus's book.

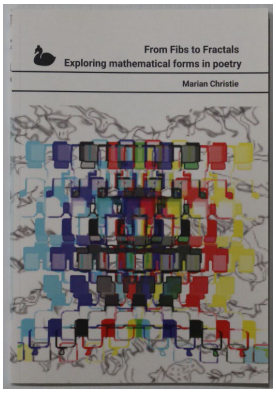
The First Fib

Words
Flow,
Tumble,
Fill the page.
They tell my story.
I promise every word is true.

Two of my favourite books are *From Fibs to Fractals: Exploring Mathematical Forms in Poetry* by Marian Christie (Photograph 4) and *Against Infinity: An Anthology of Contemporary Mathematical Poetry* (Photograph 5), edited by Ernest Robson and Jet Wimp.



Photograph 3: The 14 Fibs of Gregory K. *by Greg Pincus*



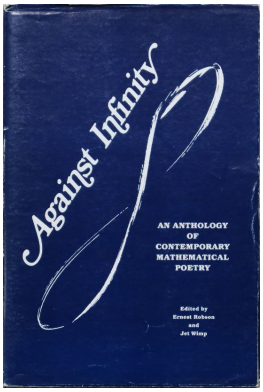
Photograph 4: From Fibs to Fractals *by Marian Christie*

From Fibs to Fractals contains examples of everything mentioned in this column, along with geeky items such as a 10 x 10 square made with 100 words that can be read in many different directions from many starting points. This is worth the price of the book!

Against Infinity contains a poem where the number of syllables per line grows using triangular numbers—1, 3, 6, 10. The first four lines are given below.

A
Tripod is
A perfect plane as well
As the base of a triple point pencil.

Against Infinity also contains a poem that is written in the shape of a spiral, as well as one arranged into the familiar curve of a parabola (Figure 2).



Photograph 5: Against Infinity: An Anthology of Contemporary Mathematical Poetry

Thinking, frail reed,
of you so easily bent
and broken but always knowing it,
I imagine how you shivered
at the thought of far-fetched
vectors, curves and conic sections
slicing planes you occupied
so queasily, caught and propped
between two chairs so as not to slip
and fall unwittingly through
yawning gaps in the parquetry,
to glide between the beams and on
through bedrock, mantle, iron core
and out again through green
antipodes, a rocketing parabola
streaking out toward yet
unsprung infinite maws.

Figure 2: *Pascal and the parabola*

Mathematical Questions to Explore

These challenges invite deeper engagement with the ideas in this column. They are designed to spark discussion, pattern recognition, and exploration of key mathematical concepts across multiple strands of the curriculum.

Question 1

Kangaroo Words contain a synonym tucked inside, like a joey in its pouch. For example, blossom hides bloom, chicken hides hen, and contaminate hides taint.

(a) Can you find a kangaroo word, where the joey has exactly half the number of letters as the larger word?

(b) Explain why the following math-related words are kangaroo words: remainder, proportion, parallel, and triangle.

(c) Find five other math-related words that are kangaroo words.

Question 2

The sentence below contains 20 words. The *n*th word in the sentence has exactly *n* letters.

"I do not know where family doctors acquired illegibly perplexing handwriting nevertheless, extraordinary pharmaceutical intellectuality counterbalancing indecipherability, transcendentalizes intercommunications incomprehensibleness."

(a) Determine the total number of letters in the sentence.

(b) What is the total number of letters in a sentence where the *n*th word is *n* letters long for $n = 1$ to 100?

(c) Can you write a sentence of your own using this same rule for $n = 1$ to 6?

Question 3

(a) Write a Fib poem with six lines. How many total syllables are in your poem?

(b) If you extend the Fib by two more lines, what will be the syllable counts of lines 7 and 8?

(c) Create a poem where the number of words per line (instead of syllables) follows the Fibonacci sequence.

Question 4

The poem *The Great Human Diaspora* by JonArno Lawson has 9 lines, where the number of words follows the pattern 1, 2, 3,…, 9.

The Great Human Diaspora (or, 1 to 9)

I
on my
own see now
what must have been
clear other times - stars, comet
trails - orbits unfold across dreams, people
collect, passive, orderly - observe horizon melting skyward
tranquil. . .peaceful. . .suddenly – enormous nightsky scatters watchers
outwards – wandering separated homegrown multitude, aimlessly
ambitious, traverses limitless distances

(a) How many total words are in the poem?

(b) How many total letters are in the poem?

(c) Write your own poem in which the *n*th line contains *n* words, for $n = 1$ to 6.

Question 5

Choose a curve (circle, parabola, spiral) and write a poem whose words follow that shape.

Summary

The mathematical structure beneath these poems adds rhythm and depth. Students can be challenged to write poems using Fibonacci, triangular, or square numbers—maybe the number of words grows by 1, then 3, then 6. Or perhaps the number of letters per line is 1, 4, 9, 16. Such constraints naturally lead to questions: How many total syllables appear in the first six lines of a Fib? How many words appear in a triangular poem with ten lines? What new rhythms and surprises emerge when we extend these patterns?

Whether we're working with vowel restrictions, number sequences, kangaroo words, patterned sentences, or poems written to form spirals and curves, these language-based activities remind us that mathematics doesn't just belong in the world of numbers—it's woven into words, patterns, and stories. Constraints, rather than limiting us, often bring out creativity. They challenge us to play differently, to think more deeply, and to explore the connections between disciplines.

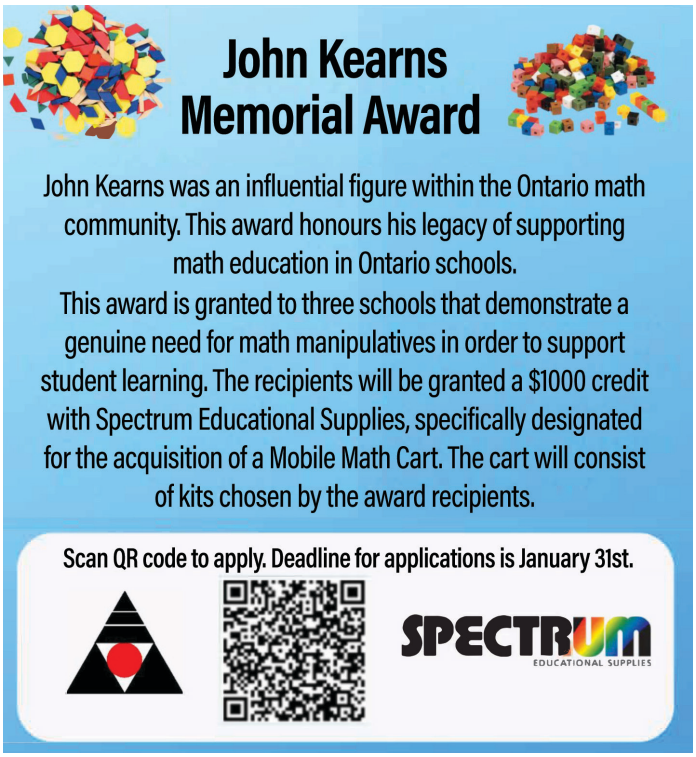
The books in this corner of my home base continue to inspire both poetry and problem solving that form the basis of activities I use in my workshops for teachers and students. They serve as reminders that mathematics and language

are not opposing forces—but rather, dance partners in the choreography of thought.

Bonus Material

The following list of remarkable math-related kangaroo words appeared in Issue 476 of the *Grange Academy Mathematics Department Newsletter*, published almost every week of the year by Chris Smith. Chris is a mathematics teacher at Grange Academy in Scotland. His newsletter contains a gold mine of material for mathematics teachers, including a problem-of-the-week and some mathematical humour. To subscribe to the newsletter, write to Chris at aap03102@gmail.com

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INdeFINITE
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PARALLELogram
TriANgle (as in TANgent)
MULTIPLicitY
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▲ Polynomial Error-Correcting Codes—A Project for MHF4U and MCT4C



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Peter is a secondary math and physics teacher, currently working at the Bishop Strachan School in Toronto. He enjoys exploring connections between mathematics and other fields of study, like science, art, and technology.

Digital information is ubiquitous nowadays, and has been for a while, but one feature of digital information that often gets overlooked is its fidelity: how reliably it can be sent and received or retrieved after storage. It's easy to take for granted that all our digital information stored or sent will remain uncorrupted; that all the zeros and ones will stay just like they are supposed to. Unfortunately, this is not always the case, so it's important to have a method to verify our information is staying the way it's supposed to and fix any mistakes, without having to send or keep multiple copies.

In practice, error-correcting codes are used to detect and fix errors in digital information. Providing details of such codes in a manner accessible to school students is a challenge. Fortunately, it can be done! A good place to start is the educational YouTube channel, Veritasium, which includes a 35-minute video about the history of error-correcting codes that is very accessible to a high school audience (Veritasium, 2024). While the whole video is quite long, luckily the relevant part of the video is short enough to be easily shown during class time. The section from 22 to 25 minutes goes through a toy model of an error-correcting code using polynomials. The toy model uses many polynomial operations that are aligned with the MHF4U and MCT4C curricula. The focus of this article is recontextualizing the polynomial content to be essential parts of error-detection and correction procedures.

This all came together into a project for an MHF4U class I was teaching (all the resources are linked at the end of the article). This project was a lot of fun for both the students and me. It provided many opportunities for students to engage with course content, use a variety of tools to complete calculations, and show their understanding of polynomials in a real-world context.

Getting Started: Illustrating the Problem

Consider starting with paper copies of a large QR code (I printed one QR code per 8.5"x11" page). Then, challenge the students to destroy their QR code as much as possible, while making sure it still works. This is very engaging for all students, and gets them curious about how a QR code can work after being damaged.

Afterwards, play the video clip (minutes 22 to 25; Vertasium, 2024). Follow the first viewing by walking the students through the example below (based on "Student1–…" in the "Student" folder of the project drive).



Figure 1: *Damaged, but functional, QR codes*

The Encoding/Decoding Process

Let's say you want to send the message, "MATH," to someone. You can first convert each letter into a number ("A" = 1, "B" = 2, "C" = 3, and so on). That would turn "MATH" into "13, 1, 20, 8."

You then take these numbers and make them the coefficients of a polynomial with x^2 as your lowest-degree term (you'll see why in a moment): $13x^5 + x^4 + 20x^3 + 8x^2$. Let's call this the *message polynomial*.

Now you need two syndromes (numbers used to encode/decode the message). These could be any numbers, but you and your recipient need to choose the numbers before sending messages. Let's pick easy numbers: 1 and 2. The syndromes will be roots of our final polynomial, and our selection gives factors of $(x-1)$ and $(x-2)$. (Note: Choosing +1 and -1, or +/- any number, for your syndromes causes problems in the error-correction part of the procedure. This will be addressed later on.)

The next step is to divide our message polynomial, $13x^5 + x^4 + 20x^3 + 8x^2$, by $(x-1)$ and $(x-2) = x^2 - 3x + 2$. If you expand $(x-1)(x-2) = x^2 - 3x + 2$, you can use polynomial long division to obtain the following result: $13x^5 + 40x^4 + 114x^3 + 270$, remainder $582x - 540$. (Note: If the syndromes are integers, all coefficients in the quotient and remainder will be integers as well.)

This means that $13x^5 + x^4 + 20x^3 + 8x^2 = (x-1)(x-2)(13x^3 + 40x^2 + 114x + 270) + 582x - 540$.

The remainder is then subtracted from both sides of the equation. Now, the right and left sides are factorable by both $(x-1)$ and $(x-2)$ (which allows us to use the factor theorem to check the message later on):

$$13x^5 + x^4 + 20x^3 + 8x^2 - 582x - 540 = (x-1)(x-2)(13x^3 + 40x^2 + 114x + 270)$$

What you would actually send as the message is the following set of numbers: (Note: You don't actually care about the degree 3 polynomial quotient, since the message is just the coefficients on the left.)

[13, 1, 20, 8, -582, 540]

The receiver of the message can verify that it was sent correctly by using the factor theorem because both the sender and recipient know the syndromes (i.e., two of the roots of this polynomial). The receiver turns the message into a polynomial and substitutes each syndrome in for x to confirm that the results are zero:

$$13(1)^5 + (1)^4 + 20(1)^3 + 8(1)^2 - 582(1) + 540 = 0$$

$$13(2)^5 + (2)^4 + 20(2)^3 + 8(2)^2 - 582(2) + 540 = 0$$

Since both expressions equal zero, the message was received correctly, and the numbers (except the last two) can be turned back into letters to read the message: "MATH"!

Detecting and Fixing Errors

Suppose one of the message numbers gets changed (corrupted, compromised, damaged). You will be able to notice this, and you can fix it! Say, for example, instead of the original message, you receive: [13, **15**, 20, 8, -582, 540] – Which looks like the sender sent: "MOTH"

You can tell right away that the message is incorrect because the value of the polynomial at each syndrome isn't zero anymore!

$$13(1)^5 + 15(1)^4 + 20(1)^3 + 8(1)^2 - 582(1) + 540 = 14$$
$$13(2)^5 + 15(2)^4 + 20(2)^3 + 8(2)^2 - 582(2) + 540 = 0$$

Fortunately, if only one part of the message is incorrect, you can use some math to figure out which part is wrong and correct the number to the intended value.

To do this, start by assuming the first number in the message (the leading coefficient) is incorrect. Replace the

coefficient with a variable, say k, then solve for the value of k, which makes the polynomial zero for each of our syndromes. If you get the same k-value for both syndromes, then that value of k fixes our polynomial!

$$k(1)^5 + 15(1)^4 + 20(1)^3 + 8(1)^2 - 582(1) + 540 = 0 \rightarrow k = -1$$
$$k(2)^5 + 15(2)^4 + 20(2)^3 + 8(2)^2 - 582(2) + 540 = 0 \rightarrow k = 6$$

Since these two values for k don't match, the first coefficient isn't the problem. Let's try the second coefficient:

$$13(1)^5 + k(1)^4 + 20(1)^3 + 8(1)^2 - 582(1) + 540 = 0 \rightarrow k = 1$$
$$13(2)^5 + k(2)^4 + 20(2)^3 + 8(2)^2 - 582(2) + 540 = 0 \rightarrow k = 1$$

Since the values of k now match, you know that the second number was the error. Furthermore, it also tells us that the second value should have been 1 (instead of 15). Using that correction, the message should be [13, 1, 20, 8, -582, 540] = "MATH." (You could check the other coefficients, but there is no need to, since you already found the fix.)

If the message has more than one error, and two of the numbers get changed, you will be able to notice, but you won't be able to fix the message. Say, for example, you receive:

[**16**, **15**, 20, 8, -582, 540]

You can tell quickly it's incorrect, since the values of the polynomial at $x = 1$ and $x = 2$ aren't zero:

$$16(1)^5 + 15(1)^4 + 20(1)^3 + 8(1)^2 - 582(1) + 540 = 17$$
$$16(2)^5 + 15(2)^4 + 20(2)^3 + 8(2)^2 - 582(2) + 540 = 320$$

And if you set each coefficient equal to k and solve for k, assuming the polynomial is zero for both syndromes (x = 1,2), you will get the values shown in Table 1.

Table 1: *Calculations when more than one error is present*

Coefficient set equal to k	k-value when	k-value when $x = 2$
x^5	-1	6
x^4	-2	-5
x^3	3	-20
x^2	-9	-72
x	-599	-742
Constant	523	220

For each line in Table 1, the two k-values do not match. This indicates the message is too damaged to recover, and your sender would need to resend the message.

Releasing Responsibility

Once students have been guided through the process, I suggest having them work collaboratively in groups of three at whiteboards to try the process for themselves. Start with Set 1 of the Check Your Understanding (CYU) questions (see Figure 2; answers in "Student 2 – …"). I suggest splitting up the encoding and decoding explanations so that students go through the encoding explanation; then they try encoding for themselves; then review the decoding explanation; and finally, try it for themselves.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

Set 1	**Syndromes:**	**Message to send:**	**Messages Received:**
	-1, 2	COOL	[14,9,3,5,-213,-210] [14,6,1,20,-202,-212] [5, 1, 9, 3, -77, -66]

Figure 2: *Sample from CYU questions for students*

Most groups in my classes were able to encode the message, "COOL," and decode, then repair (as needed), the first two messages received, but many got stuck on the third message received (see Figure 3). This is because the third message can't be repaired (there is more than one error in the message), and students had trouble recognizing this, or weren't sure what to do to prove that it couldn't be fixed. Many gave up after trying a few of the coefficients when nothing matched—their unfamiliarity with the procedure confounded identifying that the message contained more than one error. This is a good opportunity to offer words of encouragement and suggest groups talk to one another to try to work out what is going on.

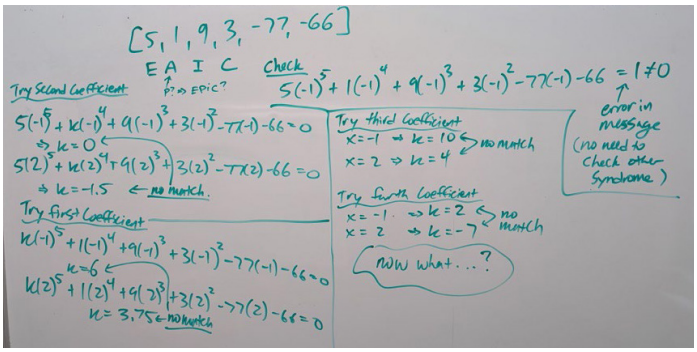


Figure 3: *Getting stuck decoding the third message from Figure 2*

After going through the third message, you can discuss the limitations of the error-correcting procedure, and challenge students to ponder how you could modify the procedure if you wanted to be able to fix messages with two or more errors (this will come up later in the section on project extensions). This is also an opportune time to connect the mathematical procedure to the QR codes activity from the start. You can prompt students to consider how changing the numbers in the message is like destroying the QR code, and that there is a limit to how much you can change the message or QR code before it loses its original meaning.

This whole introduction took my classes about 1.5 hours to complete. The time is somewhat dependent on how long the QR code destruction and testing last, as well as how much of the video is shown.

Automating the Process?

Doing the calculations by hand turns out to be tedious, especially as you get to longer messages. What's more, a single small calculation error can completely derail the process and be difficult to spot. In practice, computers do these types of calculations, so why not leverage technology to streamline the process?

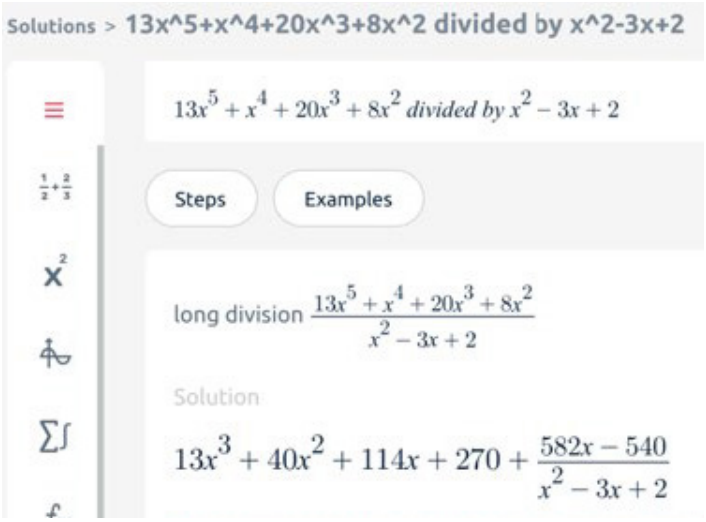
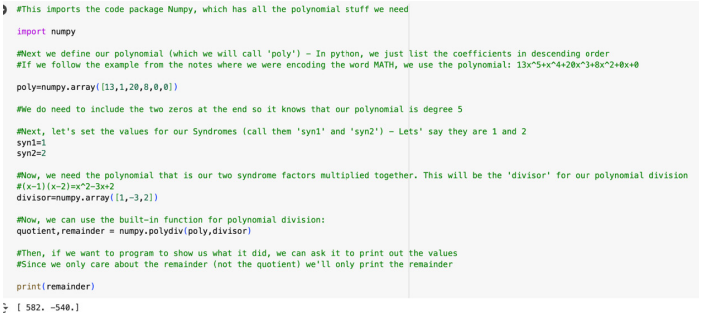


Figure 4: *Encoding calculation from earlier, using Symbolab*

The "Student" folder has a resource explaining how you can use free online tools like Symbolab (www.symbolab.com/), shown in Figure 4, to complete these calculations (see file "Student 3 –…"). Also, in the "Student" folder, you will find some starter code for encoding and decoding messages in Python (see Figure 5). In the "Teacher" folder, I have also included a finished version of the code for encoding and decoding messages. (Don't share with students!) The teacher code is useful for generating questions for students, and checking their answers. I had students try a few calculations in Symbolab (from the CYU), and those who were interested explored how they could make their own code to do the calculations. (This is another project extension, which will be addressed later.)



```
#This imports the code package Numpy, which has all the polynomial stuff we need

import numpy

#Next we define our polynomial (which we will call 'poly') - In python, we just list the coefficients in descending order
#If we follow the example from the notes where we were encoding the word MATH, we use the polynomial: 13x^5+x^4+20x^3+8x^2+0x+0

poly=numpy.array([13,1,20,8,0,0])

#We do need to include the two zeros at the end so it knows that our polynomial is degree 5

#Next, let's set the values for our Syndromes (call them 'syn1' and 'syn2') - Lets' say they are 1 and 2
syn1=1
syn2=2

#Now, we need the polynomial that is our two syndrome factors multiplied together. This will be the 'divisor' for our polynomial division
#(x-1)(x-2)=x^2-3x+2
divisor=numpy.array([1,-3,2])

#Now, we can use the built-in function for polynomial division:
quotient,remainder = numpy.polydiv(poly,divisor)

#Then, if we want to program to show us what it did, we can ask it to print out the values
#Since we only care about the remainder (not the quotient) we'll only print the remainder

print(remainder)
```

```
[ 582. -540.]
```

Figure 5: *Sample of starter code for students*

Once students understood the basics, I moved on to the individual project portion and shared the instructions file with them (see "Student 0 –…"). In the first part, students encoded and decoded some messages of their own. I asked students to choose a message length between four and ten characters, depending on what method they were planning to use for encoding and decoding their messages (four to six, if doing it by hand; six to eight, if doing it with Symbolab; eight to ten, if using code they created).

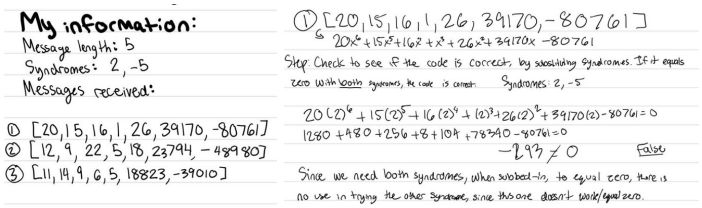


Figure 6: *Student work checking messages received for the project*

Start by generating some random pairs of syndromes (make sure they are not the positive/negative of the same number, e.g., +2, -2). You can also have a discussion with students about the choice of syndromes, asking them, "Do we need to put any restrictions on the values we use for the syndromes?" After assigning syndromes, I use the teacher Python code to create three messages for each student of their chosen length to decode (see "Teacher" folder, "Error-Correcting Project Encoding Examples" file). I made sure each set of three messages sent had one message that was correct, one with a single error students could fix, and one that couldn't be fixed. (The easiest way to do this is to take a correct message and change both error-correcting numbers.) I sent the syndromes and messages to the students, and asked them to choose a message of the same length to encode and send to me. (You can use the teacher code to check what they send you.) I let students complete this part both during and outside of class time.

Syndromes	Message Length	Message 1 decoded	Message 1 encoded	Message 2 decoded	Message 2 encoded
-1,5	6	FUMING (no fix)	[6,21,13,9,14,7,-140611,-140859]	JINXED (correct)	[10,9,14,24,5,4,-163557,-163565]
1,-5	6	REMIND (no fix)	[18,5,13,9,14,4,-227433,225410]	BLOODY (correct)	[2,12,15,15,4,25,-1033,960]
-1,-2	10	DELIGHTFUL (no fix)	[4,5,12,9,7,8,20,6,21,12,-7586,-7412]	SURROUNDED (error in correcting numbers)	[19,21,18,18,15,21,14,4,5,4,-22871,-22994]
-1,-3	8	MASTERLY (correct)	[13,1,19,20,5,18,12,25,-138081,-138096]	BREAKING-> CREAKING	[2,18,5,1,11,9,14,7,23291,23289]
-2,5	7	CHANGED ->CHANCED	[3,8,1,14,7,5,4,-265638,-530660]	MOTHERS (no fix)	[13,15,20,8,5,18,19,-941561,-1885770]
2,-3	4	FALL (no fix)	[6,1,12,12,-379,472]	BELL->BELT	[2,5,12,12,-109,-102]
2,-5	5	PLATE->PLANE	[16,12,1,20,5,29992,-61540]	TABLE (correct)	[20,1,2,12,5,43970,-89400]

Figure 7: *Example messages and syndromes sent to students*

After completing this part, students reflected on how relevant and authentic the project felt to them, using technology to encode and decode their messages just like it's done in the real world. Many said it felt like a puzzle or game, and they got to imagine themselves as secret agents or spies! They also said it was satisfying to use some of the more abstract polynomial calculations (like solving for *k*, or polynomial long division) in a real-world scenario, where they were useful tools, not just practice problems for homework.

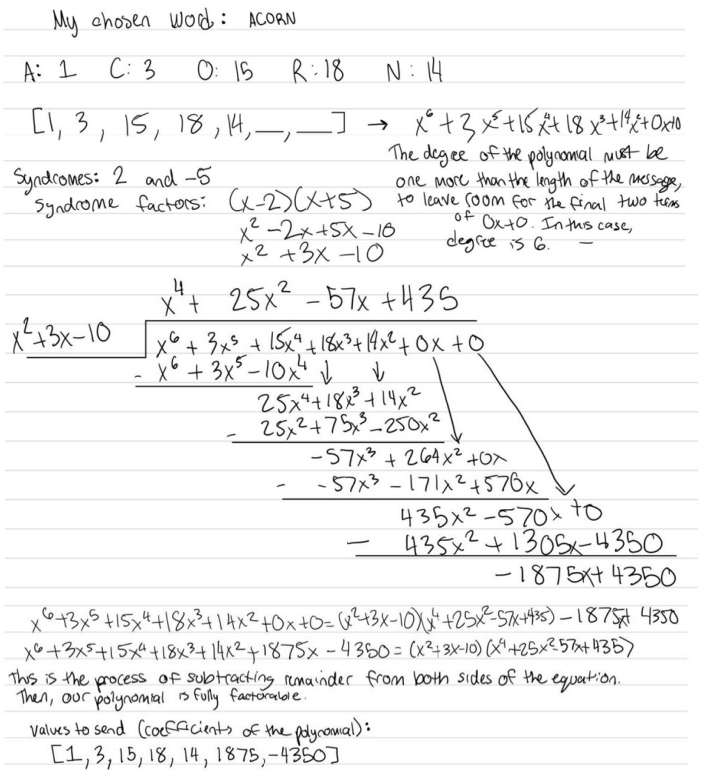
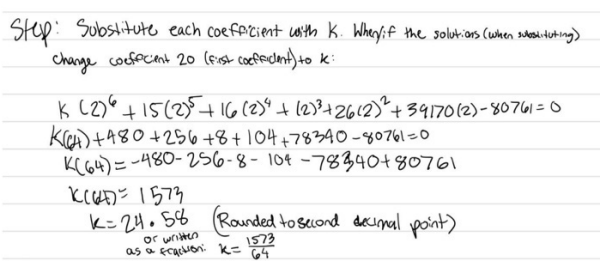


Figure 8: *Student work encoding a message of their choosing for the project*



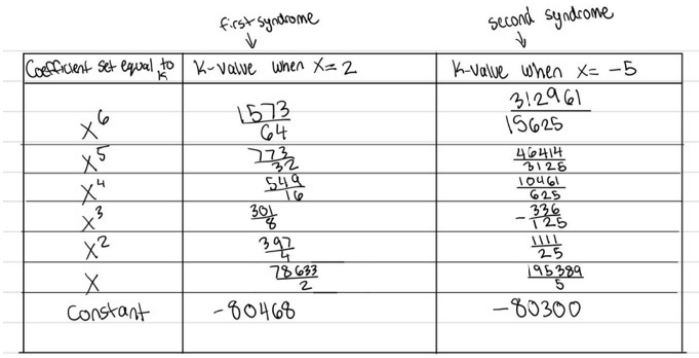


Figure 9: Student work repairing a damaged message for the project

The next part was done in class after students finished the encoding and decoding part of the project (see "Teacher" folder, file "Error-Correcting Project In-Class"). Students printed off the first part of their project to have with them while they completed the in-class portion individually (like a reference sheet). At the end of class, they stapled the first part of their project to the in-class assignment and handed it all in.

The in-class assignment asked students to answer some conceptual questions related to the project, to see how much they understood the "why" behind the procedures they completed in the other part of their project. I would encourage you to think about questions #3 and #4 (see below), as I found them to be suitable indicators for how much students understood the procedure. These are also easy to modify to create multiple versions by changing the key numbers in the following questions from the in-class assignment:

3. Mary receives the following message: [18, 5, 4, -48, 124]. Her syndromes are integers *a*, *b*. She converts this into a polynomial: $18x^4 + 5x^3 + 4x^2 - 48x + 124$, then substitutes her first syndrome into her polynomial and gets zero: $18a^4 + 5a^3 + 4a^2 - 48a + 124 = 0$.
   a. What can Mary conclude from this calculation? Explain.
   b. What should Mary do next? Explain.
4. Betty receives the message [8, 15, 7, -831, 97], but knows there is an error in her message. Her syndromes are integers *a*, *b*. She converts this into a polynomial $18x^4 + 15x^3 + 7x^2 - 831x + 97$, then replaces the first coefficient with a variable, *k:* $kx^4 + 15x^3 + 7x^2 - 831x + 97$. She then substitutes in her first syndrome, sets the equation to zero, and solves for *k*, getting 32: $ka^4 + 15a^3 + 7a^2 - 831a + 97 = 0 \rightarrow k = 32$.
   a. What can Betty conclude from this calculation? Explain.

b. What should Betty do next? Explain.

Questions #3 and #4 from the in-class assignment

For example, in #4, you can change which coefficient is being solved for (whether it is part of the main message or one of the error-correcting terms) and what value of *k* comes out of the calculation. For coefficients in the main message (not the last two numbers), students should recognize that if is not an integer between 1 and 26, it can't be the correct coefficient (since the coefficient represents a letter). If *k* is replacing one of the error-correcting terms, it will also be an integer (but can be any size), as long as the syndromes are integers. Since it's the first letter of the message, *k* being more than 26 means Betty doesn't need to check the other syndrome, since it won't match (or if it does, it doesn't make sense to be her fix). Betty should move on to another coefficient.

5. Why are we unable to repair messages with more than one error?

Question #5 from the in-class assignment

Question #5 hints at some of the extension material posted as a final (optional) part of the project (see "Student 0 – …" instructions file). While optional, I encouraged students to try at least one extension. In my opinion, this is where the real fun happens! I have included six extension opportunities, but you may come up with even more! I am a big fan of extensions a. and f. because they illustrate some of the limitations of the toy model, and how they can be extended. They are a great algebraic challenge for students, but they can be tough to start, so I usually have to offer a hint or two. (If you get stuck on these, send me an email and I'll help you out!)

a. How would we modify the procedure so we can fix two or more errors in a message that we receive?
b. Using Desmos (or another graphing software), create a graphical representation of part of the procedure for encoding, decoding, and/or repairing messages.
c. Code the message, encoding (turning letters into numbers and figuring out the message polynomial) part of this procedure, using the coding language of your choice, and explain your code, using comments in the code.
d. Code the error-correction (checking to see if a message has been received correctly, then if not, solving for *k* to try and fix it) part of this procedure, using the coding language of your choice, and explain your code, using comments in the code.
e. Create a flow chart to illustrate the process of message encoding and error correction.
f. As it turns out, if you choose your syndromes to be the same magnitude, but opposite signs (e.g., +2 and -2), the error-correction part of the procedure no longer works properly. What goes wrong and why doesn't it work anymore?

Extension opportunities for the project

Extensions c. and d. were engaging for those students who were interested in coding, and option c. was accessible even for students with little to no coding experience by using the starter code. Extensions b. and e. were great for those who liked to use visualizations to support their thinking, and extension e. was a nice way for those without coding experience to create something similar to pseudocode, which can support them trying out extensions c. and d. Most students attempted at least one of the extensions, and the varied options gave everyone the opportunity to extend their thinking in ways that drew on their strengths, like algebraic manipulation, creating computer code, or using visualization tools like Desmos or Canva.

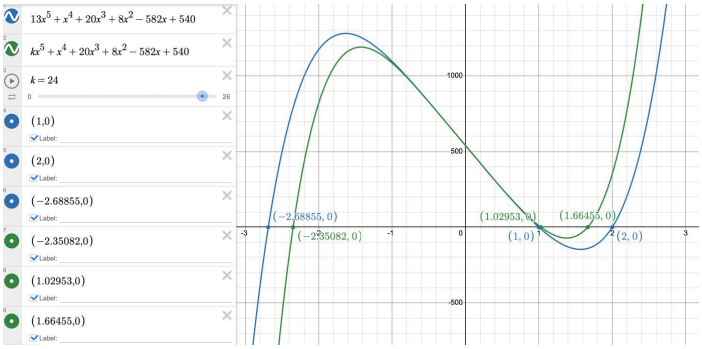


Figure 10: *Extension b. example, using Desmos*

I hope you give this project a try with your students. I am confident that if you do, your students will have an enjoyable and educational time exploring how polynomials naturally connect to the real world outside the classroom. They will have the opportunity to engage meaningfully in the project by leveraging their individual strengths and working collaboratively with peers, all while developing further competence with technical skills related to polynomials.

If you do give the project a try, let me know how it goes, and share what went well (or didn't), so I can continue to improve the project! Here is the link to all the project resources: www.bit.ly/PolyCodeOAME. Feel free to use anything from the linked Google Drive, but please credit the Bishop Strachan School and Peter Gustainis.

Reference

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Mathematical Modelling in Practice: How Elementary Teachers Inquire into Real-World Problems



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With the recent update to Ontario's (2020) elementary mathematics curriculum, mathematical modelling has emerged as a new area of focus—one that many teachers are now expected to incorporate into their classrooms. In Nova Scotia, this focus on modelling provoked us to review our professional learning courses for elementary teachers and prompted the following questions: What exactly is mathematical modelling? And how can teachers be expected to support students in developing modelling competencies, if they themselves have never had the opportunity to engage in it? This article explores a professional inquiry by teachers into mathematical modelling. The inquiry documented their process of identifying a real-life problem and applications of modelling—specifically data collection and analysis—to explore and resolve it. By tracing the learning journeys of six teachers, we endeavour to show how experiencing modelling firsthand can empower teachers to create meaningful, relevant modelling experiences for their students.

What is Mathematical Modelling?

Mathematical modelling is a dynamic and iterative process that empowers learners to explore real-world problems, using mathematics. Rather than focusing on finding a single correct answer, modelling invites students to pose meaningful questions, formulate hypotheses, collect and analyze data, and refine their mathematical thinking to make sense of complex, authentic situations (Lu & Kaiser, 2022). Unlike problem solving, which often begins with a question chosen by the teacher, modelling involves problem posing, ongoing revision, interpretation, and the blending of multiple mathematical ideas. Suh et al. (2021) offer a useful visual framework (see Figure 1) for this process, highlighting how modelling can help students move flexibly between the real world and mathematical representations.

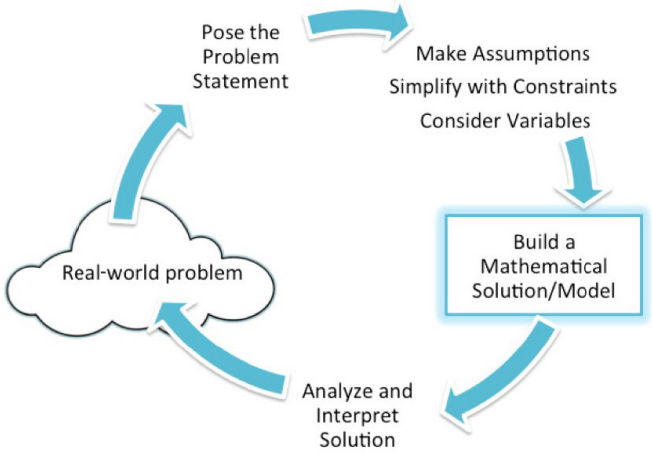


Figure 1: *Mathematical modelling process*

In elementary classrooms, mathematical modelling might look like estimating travel time, using maps, investigating recycling patterns, planning nutritious meals, or even posing questions such as: "How many times can a pencil be sharpened before it is too small to use?" (Kandel, 2023)—projects that draw on students' everyday experiences and interests. Engaging in modelling encourages teachers to think critically, explore different data-collection methods, and embrace the creative process of mathematizing—that is, making sense of their world through mathematics (Björklund et al., 2020). Through modelling, both teachers and students can develop deeper mathematical understanding, by working through problems that matter to them or, at the least, are relatable.

Professional Learning for Elementary Teachers

Mathematics education research highlights the value of

introducing modelling in the early grades (Stankiewicz-Van Der Zanden et al., 2021), yet definitions of modelling vary widely (Lesh & Doerr, 2003), and many available resources rely on pre-made models, rather than inviting students to generate their own (Cirillo et al., 2016). To address this gap, we designed a professional learning experience as part of a course about modelling, in the Certificate in Elementary Mathematics Pedagogy (CEMP) program at St. Francis Xavier University in Antigonish, Nova Scotia. One of ten CEMP courses, the modelling course supports teachers in experiencing modelling firsthand by engaging in inquiry projects rooted in their own professional contexts. Teachers pose real problems, gather and analyze data, and communicate results—mirroring the processes they are expected to facilitate for their students. In doing so, the course helps teachers build confidence, and develop the critical-thinking and problem-solving capacities needed to integrate modelling meaningfully into elementary classrooms.

Teachers' Inquiry Projects

As part of the CEMP professional learning course, teachers were engaged in mathematical modelling by identifying real and relevant problems in their professional lives. Using a problem-posing approach, they collected and analyzed data (often using free online tools), interpreted findings, and raised new questions—mirroring the modelling process they are expected to support in their classrooms. Below are four examples of teacher-led modelling inquiries.

Time for Teachers

Marta (a pseudonym), a Grade 4 teacher, explored how teachers use time in their daily schedules, and how much time they spend working beyond school hours. Using a free online software tool, she surveyed colleagues to gather data on their weekly workloads and analyzed when extra hours were most common (see Figure 2). While limitations of using free online tools can produce skeptical graphics, the data revealed that time demands varied considerably, with 25 percent of teachers reporting seven or more work hours, and 12.5 percent of teachers reporting a range (one to nine) of hours. Anecdotally, teachers reported significant increases in work hours during report card periods. Marta used this data to raise questions about how teachers manage personal and professional responsibilities, and how time pressures affect well-being. She planned to share her findings with school administrators to explore more sustainable scheduling and support practices.

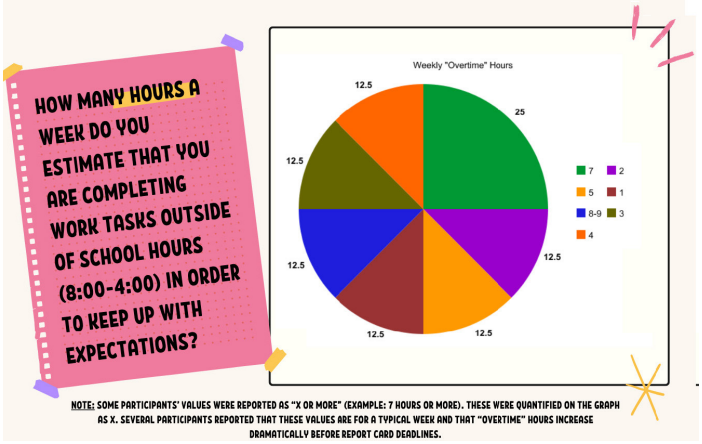


Figure 2: *Hours worked outside of school hours*

Is Free Lunch Being Wasted?

Nina and Saroja investigated food waste in their elementary classrooms following the rollout of a free provincial lunch program. They tracked weekly data on where the lunches ended up, and found that many meals were thrown away, especially when students were absent, or when refrigeration was inadequate, or taken home (see Figure 3). Student conversations revealed preferences for familiar foods (e.g., pizza, French toast) and hesitance around unfamiliar fare (e.g., taco, curry, chili) or recognizable meals that did not taste like they do at home (e.g., macaroni & cheese, pasta, soup). Their findings led to questions about menu planning with student input, managing absences, and helping students understand food insecurity. They intended to share this information with provincial representatives to improve the lunch program.

Nina's Students

Key:
G= Garbage
B= One bite
H= Took home
ab.= absent/food was thrown in the garbage
até
Empty space= they did not order lunch that day

Week 1 and 2

	Macaroni and cheese	taco	Baked potato and chili	French toast	pizza	Pasta with spaghetti sauce	curry	burrito	soup	wrap
Student	Jan 20	Jan 21	Jan 22	Jan 23	Jan 24	Jan 27	Jan 28	Jan 29	Jan 30	Jan 31
M	H	H	BG	ab.	ate	ab.		H	Ab.	Ab.
MC	H	H	BG	ate	ab.	ab.	ab.	H	Ab.	Ab.
S	G	G	BG	ate	ate	G	G	G	G	Ab.
Z					ate					
B										

Saroja's Students

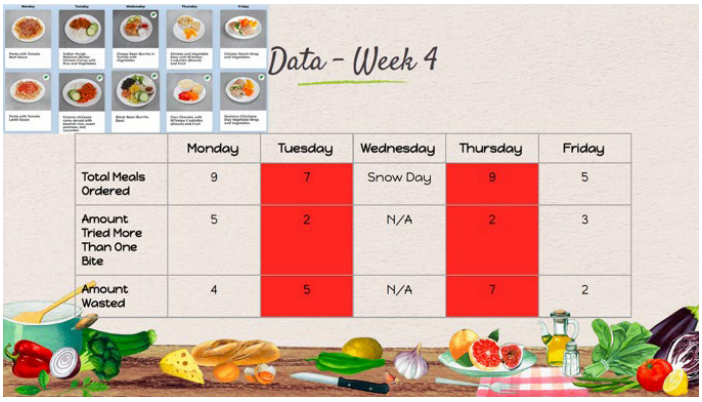
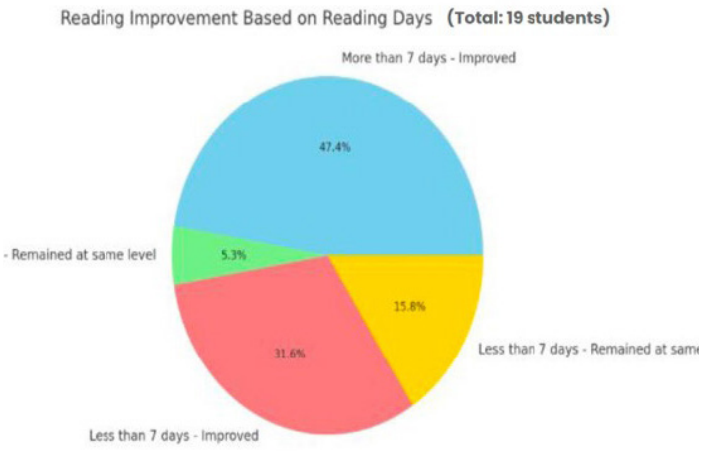


Figure 3: *Where do lunches end up?*

Does Reading at Home Improve Reading Levels?

Kerri and Lise, who team-teach Grades 1 and 2 in French Immersion, studied the relationship between home reading habits and student reading levels. They collected data, limited to the time constraints of the course, on how often students read at home, and compared it to changes in reading performance (see Figure 4). While some patterns emerged—such as improvements among students who read more than seven days for the duration of the study—there were also inconsistencies that led to new questions about variables affecting reading growth (e.g., days, books, students, reading levels). Kerri and Lise planned to continue collecting data by asking: "How might additional time gathering data affect the results?" and "What is the role of parents and guardians in the data, and how might teachers nurture support for students in the home environment?" They intend to use the findings to support communication with parents and guardians about reading at home.

Première année



Deuxième année

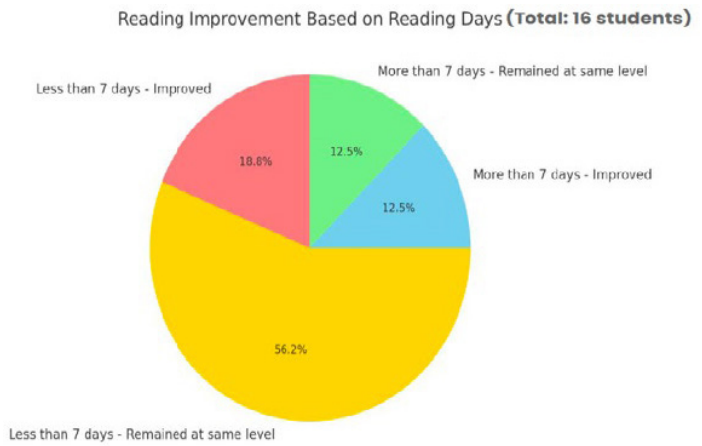
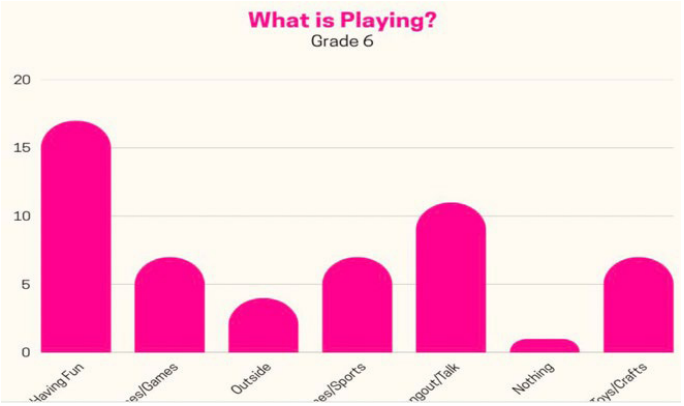


Figure 4: *Reading improvement based on reading days*

Why Aren't Students Playing at Recess?

Jelger, a Grade 6 teacher, wanted to understand why his students weren't engaging in play during recess. He surveyed students to learn how they defined play, and recorded the types of activities in which they participated (see Figure 5). His data revealed a limited range of play behaviours, and prompted further questions about the teacher's role in facilitating play with upper elementary students, the impact of home routines (e.g., screen time), and students' preferences for solitary activity (e.g., drawing) versus social play. Jelger plans to continue gathering data and contribute to school and regional conversations about the role of play in student well-being and mental health.



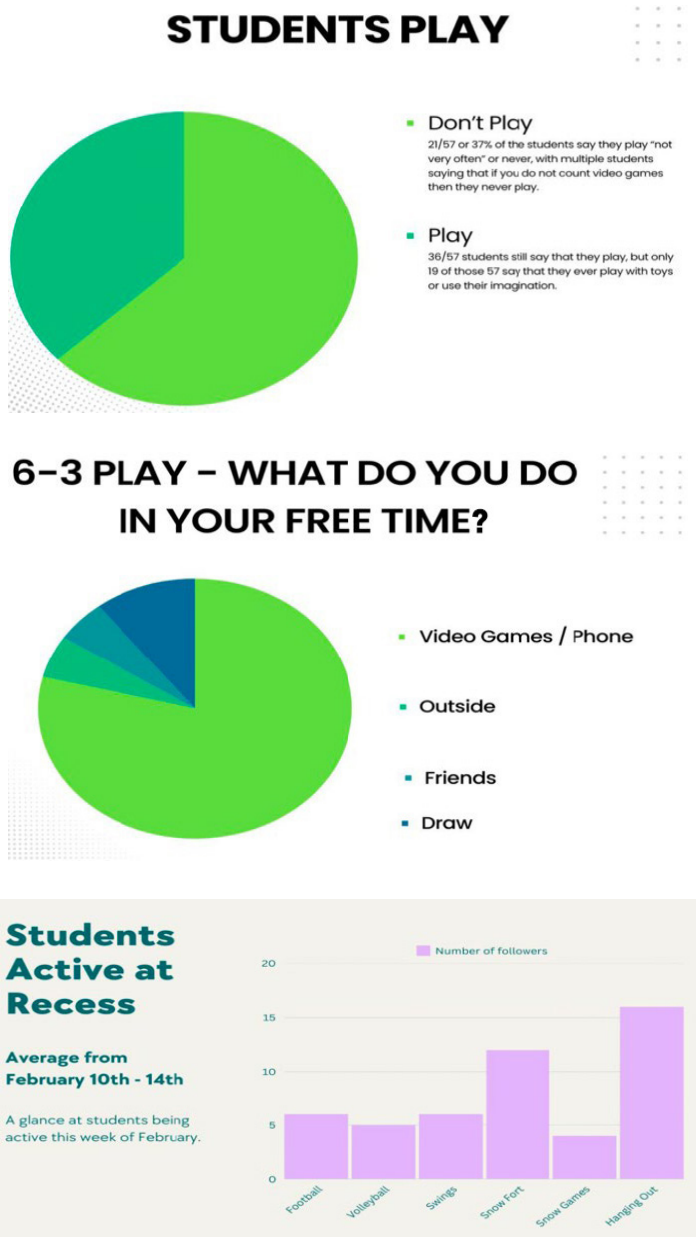


Figure 5: *Students' understanding of play and recess activities*

Teachers' Insights for the Elementary Classroom

The modelling examples from CEMP participants show how experiencing modelling firsthand can empower teachers to turn around and offer similar modelling activities with their students that are as meaningful and relevant for them. Through their participation in modelling activities, teachers also gained valuable insights into the theory and practice of mathematical modelling, using the framework described above. They came to understand that modelling is a non-linear, iterative process—one that rarely results in a neat or definitive answer. Teachers emphasized the importance of encouraging elementary students to pose open-ended questions that are personally meaningful, and to engage in discussion with peers as they interpret their findings. Considering variables, building a model, as well as analyzing and interpreting data, gave teachers necessary hands-on experience to address real-world problems confidently. They also recognized how modelling nurtures curiosity and critical thinking, often leading to new and unexpected lines of inquiry. This became a key take-away for teachers seeking opportunities to spark curiosity and prompt critical thinking with their students. Finally, teachers highlighted the value of identifying a relevant audience for their findings—such as school administrators and community, regional centre staff, or provincial consultants—and communicating results in purposeful and accessible ways. These insights hold the potential to broaden elementary teachers' instructional practices in modelling by emphasizing real-world applications and student-led inquiry in their classrooms.

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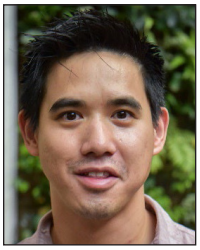
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Time for a Riddle?

When does 11 + 6 make 5?

11am + 6 hours is 5pm

▲ Supporting Productive Struggle to Tackle College Readiness



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Matthew Man Shing Cheung is a Mathematics Learning Strategist at Centennial College and holds a Ph.D. in Mathematics Education from York University. His work bridges research and practice to help diverse learners thrive in college mathematics. Matthew designs success courses, leads drop-in centres, and trains tutors to productively cultivate strategies to address problem-solving struggles, confidence building, and flexibility. His scholarship explores students' and tutors' experiences, noticing, discourse, and identity in math learning environments. He publishes, presents, and collaborates widely to strengthen math-support environments and transition pathways. Passionate about equitable access, Matthew builds resources that support mathematics learning with programs from nursing to engineering.

As a Math Learning Strategist working with college students, I often hear about the many challenges they face beyond just math content. Unlike the more structured and sequential progression common in secondary math classrooms, college mathematics courses are often highly specific to students' fields of study, with topics that do not always follow a linear sequence. Additionally, many students experience more traditional lecture-style delivery in college, which contrasts with the approaches they may have encountered in earlier schooling. Coupled with less classroom instruction time, greater expectations for self-directed learning, heavier workloads, and the need to balance studies with jobs, family, and field-specific or trade courses, students often feel overwhelmed.

For secondary educators, and to some extent all K–12 educators, understanding these realities can help inform teaching practices that prepare students for the transition to college. A focus on productive struggle, the process of engaging thoughtfully and with perseverance in challenging math problems, can develop problem-solving habits and a mathematical mindset essential for success in college and beyond (Hiebert & Grouws, 2007; Warshauer, 2015).

What College Students Face

1. **Reduced Class Time and Increased Self-Regulation**
   College courses typically offer fewer hours of direct instruction compared to K–12. This reduction means students must take greater responsibility for managing their own learning: organizing study time, seeking help when needed, and practising problem solving independently.
2. **Increased Workload and Multiple Responsibilities**
   Beyond mathematics, students often balance demanding field- or trade-specific courses, part-time or full-time work, and family commitments. The resulting time pressures can make staying engaged and persevering through difficult math content more challenging.
3. **Higher-Stakes Assessments**
   College math classes frequently include tests and quizzes that cover multiple topics, requiring not just procedural skills, but conceptual understanding and flexible thinking.

These conditions can exacerbate anxiety and challenge students' confidence, especially if they have limited experience with struggle as a positive part of learning.

How Productive Struggle Supports Transition and Success

Productive struggle encourages students to understand problems, embrace difficulty as part of learning, and develop habits of reflection and perseverance. When K–12 educators emphasize productive struggle, they help students build:

- **Perseverance:** The willingness to persist through challenging tasks by exploring different approaches, and adapting strategies when initial attempts do not succeed
- **Self-Regulated Learning Skills:** Strategies such as planning, monitoring understanding, and revising approaches
- **Flexible Problem Solving:** The ability to apply knowledge to new situations, rather than rote application of memorized procedures
- **Positive Mathematical Identity:** Confidence that struggle is a normal and valuable part of learning, which can help students manage math anxiety

By fostering these skills and mindsets, K–12 teachers can prepare learners for the less structured, more demanding, and multifaceted environment they will encounter in college mathematics courses.

Examples of Productive Struggle in Practice

- A student solving a system of linear equations begins with substitution, but quickly realizes the calculations are cumbersome. They decide to restart using elimination, finding it more efficient. This flexibility in

shifting strategies mirrors the adaptive thinking needed in college, where problems may be structured in unfamiliar ways, and time constraints require a quick evaluation of efficient approaches.

- When confronted with a complex rates-and-mixtures word problem, students first make an educated guess, then notice contradictions in their reasoning. They break the problem into smaller parts and work through each step with peers, revising their approach based on feedback. This iterative process fosters perseverance and collaborative problem solving—critical in college, where multi-step problems often span multiple topics and require both independent reasoning and group discussion in labs, tutorials, or study groups.
- Students exploring fractions use visual models (e.g., area diagrams, fraction strips) to uncover equivalences, rather than relying on memorized rules. By actively noticing patterns and creating their own strategies, they develop conceptual understanding and awareness of their ways of thinking. These skills support success in college courses that demand transfer of knowledge across contexts, such as applying proportional reasoning in dosage calculations in nursing programs.
- During group work, students encounter conflicting solution strategies and must articulate, defend, and refine their reasoning through critical discussion. This process builds mathematical communication skills that help students learn more effectively with peers—listening to different perspectives, clarifying their own thinking, and co-constructing solutions. Such collaborative communication fosters a supportive learning network that extends outside classroom hours.

These examples show how productive struggle builds the flexibility, perseverance, and reflective habits students need to navigate the less structured, high-stakes, and fast-paced environment of college mathematics. By encountering and overcoming obstacles in a supportive K–12 setting, students enter college better prepared to manage the intellectual and emotional demands of mathematical work in specific fields.

Practical Steps for K–12 Educators

Teachers play a crucial role in supporting students' productive struggle by creating environments and opportunities that encourage engagement, reflection, and perseverance. Effective strategies include the following:

1. **Normalize Struggle:** Create classroom cultures, where mistakes are seen as learning opportunities. Use language that values effort and "not yet" understanding.
   - **Example:** During a lesson on factoring quadratics, display two incorrect solutions on the board (with permission or anonymized), and lead the class in identifying where the reasoning broke down. Emphasize the value of learning moments, of not having figured it out "yet," and the opportunity to work through it together.
2. **Encourage Reflection:** Ask students to articulate their thinking, identify where they get stuck, and consider next steps.
   - **Example:** After completing a challenging geometry problem, have students fill out a "thinking log" with prompts: *What approach did you try first? Where did you get stuck? What strategy will you try next time?* This builds awareness of their own ways of thinking that supports self-directed learning in college.
3. **Model Self-Regulation:** Demonstrate how to approach complex problems, check work, and revise solutions.
   - **Example:** Work through a multi-step algebra problem while narrating your thinking: "I'm going to try substitution first… hmm, this is getting messy. Maybe elimination will be cleaner." Pause to check for errors, showing how to catch and correct mistakes before moving on.
4. **Provide Varied Problem Contexts:** Use real-world and multiple representations to deepen understanding and adaptability.
   - **Example:** In a lesson on ratios, present one problem as a recipe adjustment, another as a scale drawing, and another as a speed–distance–time scenario. Ask students to identify what is the same and what is different about each, helping them transfer knowledge across contexts—just as they'll need to in college applications of math.
5. **Scaffold Perseverance:** Support students just enough so they remain challenged, but not overwhelmed, gradually increasing independence.
   - **Example:** Give students a set of progressively harder problems on exponential growth. For the toughest ones, provide a "hint menu" they can access only after they have tried for 5 minutes. Over time, reduce the hints so students build confidence in persisting without immediate help.
6. **Encourage Mathematical Communication:** Create opportunities for students to explain and defend their reasoning.
   - **Example:** During group work, assign one student to present their solution to the class, and have a peer ask a clarifying or challenging question. This practice develops students' ability to explain their thinking and engage in constructive questioning—skills that support peer learning outside the classroom, such

as working with tutors or in study groups.

7. **Highlight Strategy Variation:** Draw attention to multiple ways of solving the same problem.
   - **Example:** After solving a problem involving percentage change, list three different methods on the board that were used by students. Ask the class to discuss the advantages and disadvantages of each, helping them appreciate flexibility in problem solving.

By intentionally adopting these practices, teachers foster productive struggle that equips students with the habits and mindset essential for succeeding in college mathematics and beyond.

Collaboration to Support Productive Struggle in the Transition to College Mathematics

Successfully transitioning students into college mathematics requires strong collaboration between secondary and college educators, centred on fostering productive struggle across educational stages. However, these collaborations are still relatively rare. A number of structural and logistical barriers make them difficult to sustain: the majority of college mathematics instructors are part-time, and often work at multiple institutions, limiting their availability; K–12 educators are deeply committed to their own classrooms, with little flexibility in their schedules; finding mutually convenient times for meetings or classroom visits is often a challenge. In addition, the lack of formal structures for cross-sector engagement means that connections depend heavily on individual initiative.

Despite these challenges, there are practical ways for K–12 educators to initiate and build relationships with college educators:

- **Engage in Shared Professional Learning Focused on Struggle:** Attend conferences, symposiums, and workshops with a lens on transition. For example, collaborate with someone "across the transition" at the Ontario Association for Mathematics Education or Ontario College Mathematics Association conferences, to co-design classroom tasks to support productive struggle. College teaching and learning symposiums and open research groups can also provide spaces to co-develop materials that explicitly prepare students to engage productively with challenge.
- **Facilitate Cross-Sector Reflection and Inquiry:** Encourage educators to share reflective accounts of their own classrooms, such as narratives, anonymized student work, or lesson recordings of their own teaching. These accounts are more readily accessible, require less time commitment, and avoid the need for extensive student consent. Follow-up inquiry sessions can then focus on how students engaged with challenging tasks, and how educators' instructional choices shaped opportunities for productive struggle. Reaching out to college faculty who focus on mathematics support, such as Math Learning Strategists or Math Coordinators, can be a practical entry point for initiating these collaborative reflections.
- **Build Reflective Communities of Practice:** Develop ongoing communities that bring educators together across sectors to interrogate their own practices, share classroom experiences, and collaboratively explore strategies for fostering productive struggle. By engaging in structured reflection and dialogue, participants can uncover new ways to normalize struggle, design tasks that encourage perseverance, and support students in building productive mathematical habits.

By acknowledging the realities that limit collaboration and adopting flexible, creative approaches, K–12 and college educators can overcome many of the barriers, and ensure that productive struggle is not isolated within one educational phase, but is a continuous, supported process that equips students for success in college mathematics and beyond.

Conclusion

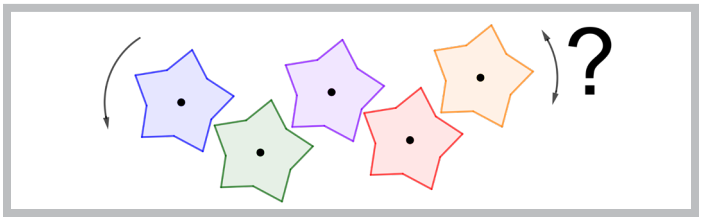
College mathematics presents unique challenges for students—less direct instruction, heavier workloads, and life balance demands—that require students to develop strong perseverance and self-regulated learning. By emphasizing productive struggle in K–12, educators lay a foundation for students to meet these challenges confidently.

Through ongoing collaboration and professional development between K–12 and college educators, we can align supports, share insights, and build a seamless educational pathway. Together, we can foster resilient, capable learners prepared not only for college mathematics, but for lifelong engagement with mathematical thinking.

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Hiebert, J., & Grouws, D.A. (2007). The effects of classroom mathematics teaching on students' learning. In J. Frank & K. Lester (Eds.), *Second handbook of research on mathematics teaching and learning* (pp. 371–404). Information Age.

Warshauer, H.K. (2015). Productive struggle in middle school mathematics classrooms. *Journal of Mathematics Teacher Education, 18*(4), 375–400. doi.org/10.1007/s10857-014-9286-3 ▲



OAME/AOEM Awards for Excellence in Mathematics Education

The purpose of the OAME/AOEM Awards is to formally recognize individuals and educators who demonstrate an outstanding contribution to mathematics education in Ontario. The successful candidate for each award enhances the learning environment for all students, providing opportunities for students to do, see, hear, and touch mathematics in a profound and meaningful way. If you know someone who qualifies for this honour, please nominate her/him/them for one of the following awards.

OAME/AOEM Life Membership Award: This award recognizes an educator who has contributed in a significant way to OAME/AOEM, demonstrates outstanding leadership in mathematics education, and has accumulated 10 or more years of membership in OAME/AOEM.

Award for Exceptional and Creative Teaching in Elementary Mathematics: This award recognizes an exceptional and creative elementary educator who demonstrates excellence in mathematics education and contributes to the overall development of students.

Award for Exceptional and Creative Teaching in Secondary Mathematics: This award recognizes an exceptional and creative secondary school mathematics educator who demonstrates excellence in mathematics education and contributes to the overall development of students.

Award for Exceptional and Creative Teaching in Post-Secondary Mathematics: This award recognizes an exceptional and creative post-secondary educator or team of educators who demonstrates excellence in mathematics education, contributes to the overall development of students, and demonstrates leadership in the mathematics education community.

Elementary School Staff Award for Exceptional and Collaborative Mathematics Teaching: This award recognizes an elementary school (a school accommodating any or all of grades K-8) whose staff fosters collaborative and excellent mathematics teaching, contributes to the overall development of student learning of mathematics, and demonstrates leadership in the mathematics community.

Secondary School Mathematics Staff Award for Exceptional and Collaborative Teaching: This award recognizes a secondary school mathematics staff which fosters collegiality, team work, and excellent classroom teaching, contributes to the overall development of students, and demonstrates leadership in the mathematics education community.

Award for Leadership in Mathematics Education: This award recognizes an educator who has demonstrated leadership by contributing in a significant way to the development of mathematics educators and the enhancement of mathematics education in Ontario through innovation and/or research related activities.

Award for Outstanding Contribution to OAME/AOEM and Mathematics Education in Ontario: This award recognizes an individual who has made a significant contribution to OAME/AOEM and contributed directly or indirectly to the improvement of mathematics education in Ontario but who is not necessarily an educator or a member of OAME/AOEM.

OAME/AOEM AWARDS NOMINATION PROCESS

1. Complete the Google Nomination form appropriate to the specific award, found at www.oame.on.ca (look for the Awards tab on the left of the page). You may include a one page letter of recommendation.
2. Include with the nomination cover page up to three **letters of support**, of no more than two pages each. Please ensure all letters of support:
   - Address the criteria for the award as outlined on the OAME/AOEM website.
   - Include the name and position of the supporter to the application.
   - Include the nominee's name and the award in the letter.
3. Include the **Curriculum Vitae**, of up to five pages, for award nominees except for Elementary and Secondary Staff awards.
4. Submit the Google Nomination form with all required supporting documents by the deadline below.

DEADLINE: Nominations are due by January 31, 2026
The awards will be presented at the OAME Annual Conference in May 2026.

Nominations for the OAME/AOEM Board of Directors 2026–2027

Each year, OAME/AOEM holds elections for open positions. Any OAME/AOEM member may apply for any position. Candidates are needed for the following positions:

- One (1) Director to represent Grades 7–12 for a three-year term from September 1, 2026–August 31, 2029.
- One (1) Director to represent Grades K–6 for a three-year term from September 1, 2026–August 31, 2029.
- One (1) Vice-President to serve on the Executive Committee for a two-year term from September 1, 2026–August 31, 2028.

An online nomination form, as well as a summary of the duties of each position, is available at www.bit.ly/OAMEPositions. The nominee and the nominator must be members of OAME/AOEM. The deadline for nominations is January 23, 2026 at 6:00 pm.

The Nominations/Elections Committee will publish a slate of candidates, including candidate biographies, for consideration by the membership. Voting is open to all OAME/AOEM members and will take place online in March 2026. The election results will be posted on the OAME/AOEM website.

If you require additional information, please contact Paul Alves (paul.alves@oame.on).

Submit completed award nominations with supporting documents to EDs@oame.on.ca

Thank you in advance
for submitting a nomination.

Correlation or Chance?

While in a remote area of Canada, Adam Shoalts (2023) noted:

> Oddly enough, at intervals through the day, I'd heard a strange snapping noise coming from somewhere in the woods. When I first heard it, I didn't pay it any attention and just kept plodding along. But when I heard it again, I paused and cast a glance into the dark forest flanking the road, eyeing the spruces and tamaracks suspiciously. The snapping, occurring at occasional intervals, was too loud to be an insect, and I knew of no animal that made any sound like it. I never did figure out what made it, accepting it as one of life's mysteries. (p. 217)

Points for discussion with students:

What do you think is meant by "occasional intervals" and "somewhere in the woods"? Since Adam Shoalts is an experienced outdoors person, do you think these could be random times and/or directions?

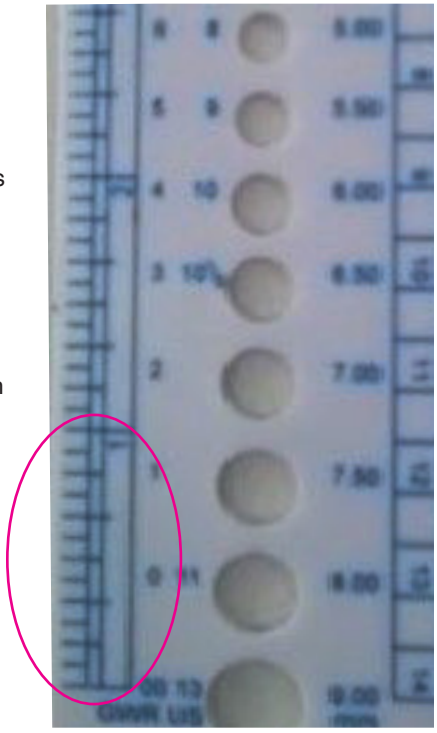
Do the noises correlate with the observer's location? Is it possible that such noises occur across the area where he was hiking, but he only observed where he was?

Reference

Shoalts, A. (2023). *Where the falcon flies. A 3,400 kilometre odyssey from my doorstep to the arctic.* Allen Lane (Penguin Random House Canada Ltd.).

Twelfths of Inch?

A gauge for measuring knitting needles also features a ruler in inches. What is curious about this is that the inches are broken down into twelfths of an inch! Is this for a particular purpose? (What beneficial or realistic properties would twelfths of an inch have over sixteenths?) Or is it an irregular manufacturing design?



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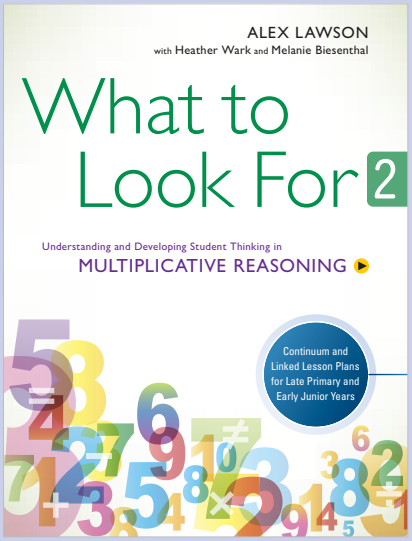
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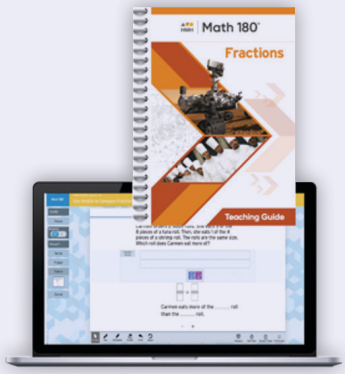
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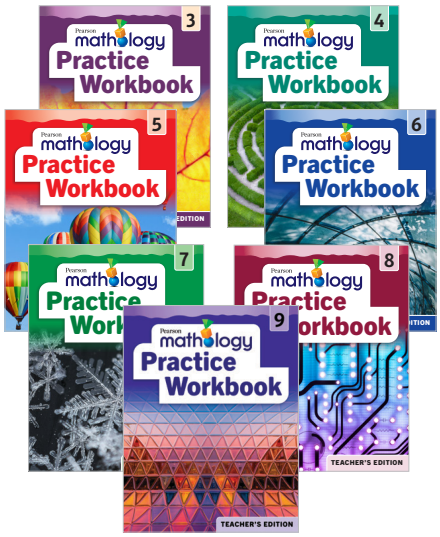
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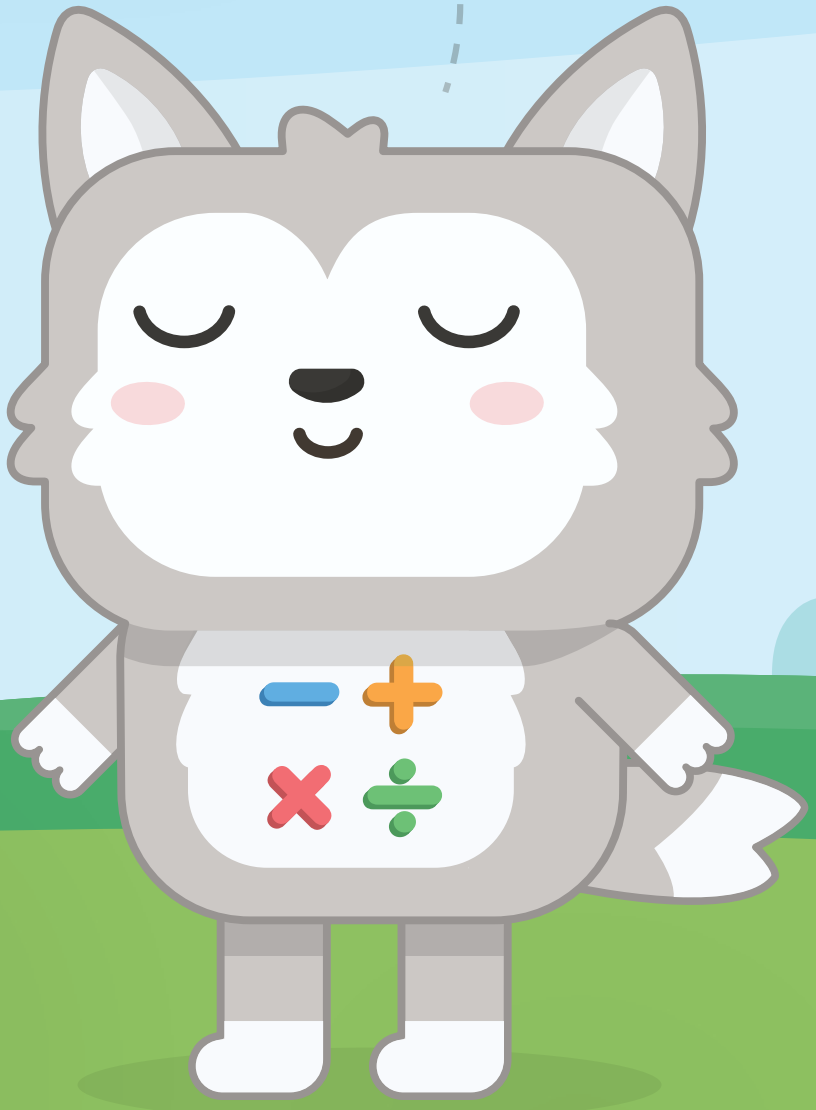
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