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Ontario Mathematics Gazette

OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

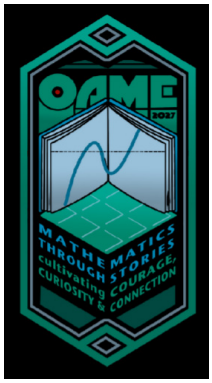
AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

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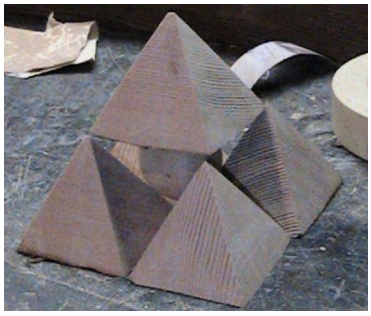
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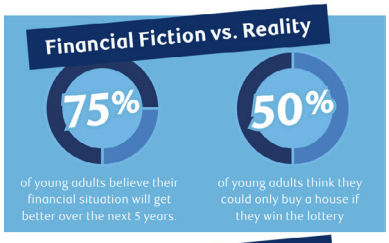
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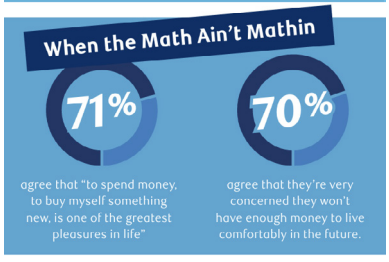


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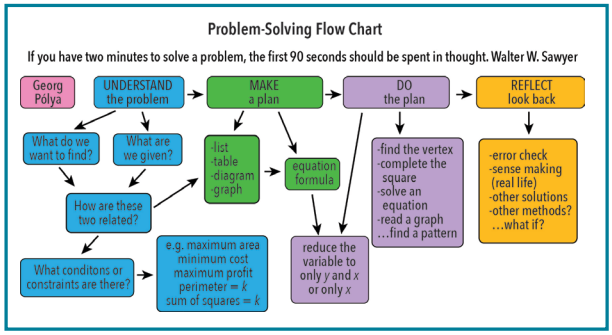


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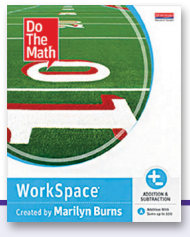
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Submission of Articles

The *Ontario Mathematics Gazette* (*Gazette*) publishes news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *Gazette*:

- The *Gazette* is aligned with the OAME/AOEM Vision, Mission, and Strategic priorities. These can be found on the website www.oame.on.ca under "About OAME" and "Guiding Principles."
- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles are to be word–processed, MS Word is preferred, and prepared according to the *Publication Manual of the American Psychological Association*, Seventh Edition. However, please use single-line (not double) spacing and only one space after each period. Articles should not exceed eight numbered pages of text, though there is some flexibility. Figures, images, and photographs should be placed close to the relevant text and include captions. The photographer's permission is required, and for photos of students under the age of 18, permission of a parent or guardian is required.

Please submit your article in one blind file (i.e., identity of author is not evident), and include author names, contact information including email and mailing addresses, photos—head and shoulders, biographies—less than 100 words, and all content removed for blinding in a second file. Please email these two files to Tim Sibbald at gazette@oame.on.ca.

Upon review, you will be notified whether your article has been accepted for publication (as is, or pending minor or major revisions) or declined. The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME/AOEM.

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The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of OAME/AOEM.

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Editor's Report



TIM SIBBALD, OCT, PH.D.
gazette@oame.on.ca

Tim Sibbald is a professor in the Schulich School of Education, Nipissing University, with a focus that includes mathematics education. He is a former President of OAME/AOEM and has held a variety of other OAME/AOEM posts over the years.

Let me begin by congratulating Shawn Godin on 20 years of columns in the *Ontario Mathematics Gazette*. He brings a particular expertise about problem solving that is particularly accessible and inciteful. His contributions over the years have amassed and The Archives is richer for it. I hope Shawn continues to the end of my tenure as Editor, which, if you look at "Ye Old Archives" on the website under the *Gazette* Editor, you will notice (Greg Clarke's webmastering fun) my term expires in 2040! (The *Abacus* Editors have terms ending in 2030.) It is my hope that I will see more columnists reach Shawn's milestone.





Having noted Shawn's achievement, I have been remiss over the years, and should also acknowledge the elite of *Gazette* contributors, where Shawn is in good company with Ron Lancaster, Lynda Colgan, and Carly Ziniuk. Each of them is an exceptional columnist, who writes with a particular focus within mathematics education. Each of them is worthy of praise for longevity and the creative thoughtfulness they bring to readers. Thank you to all of you—well done!

In recent weeks I have been involved in an overall renovation of an older home. It has involved long days and stretched every muscle in my body along the way—I heartily recommend it for anyone wanting a workout. While I jest, it is the mathematics along the way that has inspired this column—not because it was novel, needed abstract algebra, or somehow squared the circle. It was the contextual applications that stand out.

A pivotal moment arose when mortar needed to be

mixed for laying tiles. The instructions provided the ratio of mortar mix to water by weight. The contextual problem was how to get around this, since we did not have a scale. Some Internet searching came up with varied numbers that seemed to ignore which exact product was being used. An alternative approach was to square up the bag of mortar mix to be as close to a rectangular prism as possible, measure the three dimensions, and then calculate a volume. Since the bag was 50 lbs., the density of the mix could be determined. The density of water is a litre, or a cubic decimeter, and weighs a kilogram; note that in this context, the exactness of distilled water and a particular temperature were the least of my concerns. Thus, the ratio of weights could be converted to a ratio of volumes. But the context held a little more, since my tape measure was only in inches and fractions of inches, so conversions between imperial and metric were necessary. (Related to the mortar challenge is the drywall challenge of filling a hole in such a way that after contraction during drying, the result is as close to flat as possible.)

One of the first challenges was the need for paint. To address this, I sketched the floor plan of the main floor. The sketch followed the blueprint practice of minimizing the number of measurements. This allowed determination of the area, in square feet, of the ceilings. A standard gallon of paint covers 400 square feet (actually, the instructions are less confident, one can say 250–400 square feet), so one could estimate the number of gallons. However, the walls added to this, but all the rooms had either 9.5-foot or 8-foot ceilings, so the perimeter of each room—with a crude subtraction to remove doorways and windows (but also recognizing the closet needed adding)—was multiplied by the applicable height. How big is a pail of paint? Often, I hear they are 5-gallon pails, but sometimes there are 20-L pails, and 20 L is more than 5 American gallons and less than 5 imperial gallons. Frankly, I simply wanted the free pail that came with it.

The porch had been built, but needed railings for the stairs—this brought lots of calculations. Several designs competed for how to mount the support structure of the railings. The issue was whether the supports should be placed on the bottom step, abutting the side of the bottom step, or somewhere in between. A key issue was how to mount the railing down three steps. At issue was that the support posts for the railing were placed in different positions relative to the steps—one was at the back of the bottom step, and the other was on the porch 4¾ inches from the top step. Sure, this is a linear function that needs to be parallel to a line through the top front of the steps. In practice, this was not so obvious, perhaps due to the strains of multi-tasking among physically demanding tasks. We considered measuring angles, but my digital angle-measuring device needs a new battery.

In our uncertainty, we looked at the placement of railings on half a dozen other houses on the street and measured a couple, only to find no consistency. In the end, I realized that I was dealing with 7–10 stairs. For each rise of 7 inches (i.e., the “riser”), there was a 10-inch step (i.e., the “tread”). But it also dawned on me that 3.5 would correspond to 5 inches, and that was important because 3.5 inches is the width of a two-by-four (it is not “four” because milling the lumber removes 0.25 from each face). So, marking a perpendicular line across the two-by-four, and then marking a point 5 inches along one side, allowed drawing the hypotenuse. Cutting along the perpendicular and the hypotenuse gave a triangular prism with the necessary angles. That prism became my template for all other cuts.

There were numerous other challenges, such as painting a ceiling over stairs to the basement (how to set up a make-shift scaffold). The overall effect was a reminder that the applications we use in math class are credible preparation for these real-world scenarios. However, as classroom applications, the realities of particularly challenging details is an important consideration. What is unclear is whether we can reasonably do more to allow the context to reveal itself, and the way in which it alters accessibility of solutions.

Whenever I engage in these sorts of activities, I am reminded of how important all the processes are. Each problem becomes its own sequence of events that chain processes to a resolution. That is something we look for with our students and is a key reason for including contextualized problems. Yet, the reality of hitting a renovation head-on is a gobsmacking reality check of all the math applications most people ever consider. We would be wise to include more applications on the understanding that conceptual understanding may need to be found first through the concrete, before it can develop in more abstract senses for many students.

In This Issue

Our President, **Kit Luce**, brings a message from the final days of her term. She will, of course, remain on the Board of Directors in the Past-President role, which is designed to provide continuity of leadership. Also included in this issue is the latest position paper, that is about the effective use of digital technology. While there is no specific author, I can vouch that it comes from a consensus approach of a dedicated, hardworking team. **Cristina De Simone** who is a Co-Editor of the *Abacus*, is also an award winner. **Gerry Lewis** advises about professional learning opportunities

available through the NCTM. **Gerry Lewis** also brings news of professional learning that has occurred with OMCA, and gives details of their executive for next year.

Jérôme Proulx continues his series of columns about percent and, on this occasion, takes an insightful look at the history of the symbol. **Lynda Colgan**, the ever-dedicated professional, edited her column while in hospital—we wish her a healthy and full recovery—where she considers children's literature regarding mathematics. **Sofia Ferreyro-Mazieres** considers the distributive property as a conceptual continuum that is investigated across grade levels. **Jennifer Holm** delves into the intricacies of early counting experiences. Her columns have become an insightful addition to the *Gazette* because of the developmental elements she is revealing. **Carly Ziniuk** shows us the money with another column related to financial mathematics and, on this occasion, it brings in student perceptions. **Jeff Irvine** draws on perceptual field theory that is interesting in the way it, along with the editorial, points to different features of contexts. **Shawn Godin** addresses a problem that I passed along to him that came from a desk calendar—but Shawn gives it a thorough consideration. **Ron Lancaster** focuses on the legacy of Martin Gardner's work by taking selected examples from three of his books. Lastly, **David Costello** considers metacognition by way of metacognitive regulation.

Articles include one by **Qiye Huang**, who explains her investigation of undergraduate student perceptions of STEM support in Grade 12 math courses. The first two parts of a three-part article by **Jeff Irvine** regarding effective instructional strategies are food for thought over the summer. Lastly, **Tim Sibbald** explains the math required to build a physical three-dimensional OAME/AOEM logo model.

Thank Yous

I would like to begin by thanking the leadership of the OAME/AOEM. Whenever the *Gazette* finds itself in the weeds, there is always leadership available to help get things running smoothly again. Often this is the Communications subcommittee of the Board of Directors, but also consultations with the President (thank you, Kit) and the Executive Directors. It is always good to know that the community is there to help move past the speed bumps of moving forward.

As always, I owe gratitude to **Penny Clemens, Gitta Berg, Peter Saarimaki, Anne Yeager,** and **Steven Khan**. They are a dream team of expertise and a world of experience. Thank you all for all you do. I would also like to thank reviewers who have been providing their time deciding on the suitability of articles for the *Gazette*.▲

▲ President's Message



KIT LUCE
kit@oame.on.ca

Kit has been an active member of MAC² for many years and joined the Board of Directors in 2022. She is a District Mathematics Facilitator for Grades K–9 with the Simcoe County District School Board. In her work, Kit creates math resources to support educators in their instruction and assessment practice, as well as developing and facilitating professional development. Kit is an Additional Qualification instructor and course writer, and she has written and reviewed work for a variety of publishers. She has been involved in Ministry writing projects and was a Content Lead for the recent OAME/AOEM writing projects. Kit received the 2024 OAME Award for Leadership in Mathematics Education.

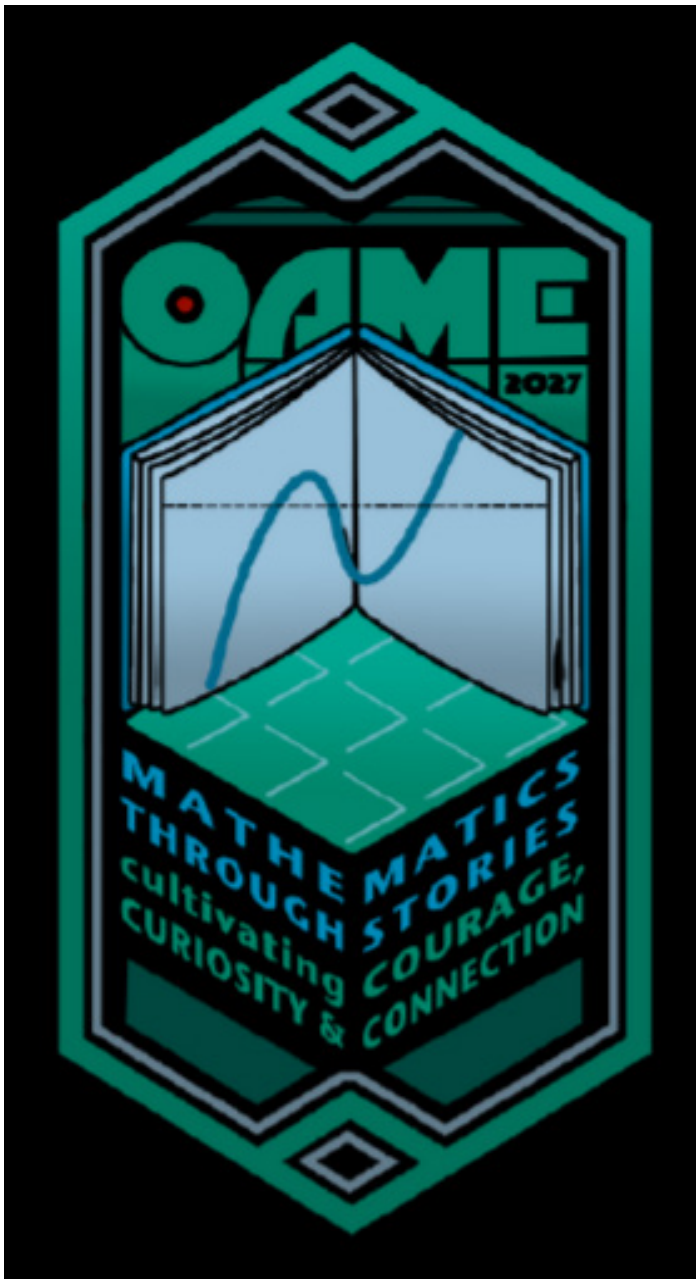
As June arrives and another school year draws to a close, I find myself reflecting on the familiar rhythm of teaching, with the cycle of beginnings, growth, challenge, and renewal that shapes our work. This year, that rhythm feels especially poignant. At the end of August 2026, I will complete my term as President of OAME/AOEM, a role that has been both an honour and a profound learning experience. These past two years have been filled with anticipation, collaboration, reflection, and the pleasure that comes from participating in shared vision and purpose.

When I stepped into this role, I carried a deep appreciation for the organization that shaped me as an educator from my earliest days. What I could not have anticipated was how much I would continue to learn from the collective wisdom, generosity, and passion of mathematics educators across Ontario. Our mission to promote, support, and advocate for excellence in mathematics education has always been more than a statement. It is a living commitment, one that comes to life through the actions of our chapters, the dedication of volunteers, and the ongoing conversations that connect research to classroom practice.

Over the past two years, I have witnessed this mission in action in ways that have continually inspired me. Chapters across the province have created spaces for educators to learn together, challenge assumptions, and deepen their practice. Whether through miniconferences, book studies, math nights, or speaker series, each chapter has contributed to a vibrant professional community grounded in equity, curiosity, and collaboration. These local initiatives embody

our vision of an equitable mathematics education community, one where every educator has access to high-quality, research-informed learning, and every student is supported in developing a positive mathematical identity.

Our collective learning has also been strengthened by the conversations sparked through OAME Talks and our Leadership Series. These sessions invite us to rethink assessment, investigate neurodiversity, braid Indigenous mathematical ways of knowing, and consider how storytelling can foster belonging in mathematics classrooms. Each speaker reminds us that equity and excellence are not competing priorities; they are interdependent. When we design learning environments that honour students' identities, experiences, and ways of thinking, we create the conditions for deep mathematical understanding.



Thank you to the hard-working members of the organizing committee for the **OAME2026 Annual Conference: *Let's Talk Math***, which was held in London, Ontario. The theme invited us to consider how mathematical conversations—between students, between educators, and between communities—shape understanding and foster belonging. Our keynote and featured speakers deepened this exploration, offering research-informed practices and perspectives that continue to resonate in classrooms across the province.

As we look ahead, the momentum continues. Planning is already underway for the **OAME2027 Annual Conference: *Mathematics Through Stories: Cultivating Courage, Curiosity, and Connection***, which will be offered in a fully virtual format. This shift reflects our commitment to accessibility and our desire to reach educators across all regions of Ontario. The theme invites us to consider how narrative—our own stories, our students' stories, and the stories embedded in mathematics—can deepen understanding and strengthen relationships. I am excited to see how this conference will continue to expand our collective thinking.

Attending a conference, whether in person or virtually, allows participants to choose their own learning journey by selecting workshops and sessions that speak to their interests, questions, and professional goals. These experiences remind us that professional learning, like teaching itself, is dynamic, personal, and continually evolving.

Throughout my term, I have often returned to the Mission and Vision Statements, our Strategic Priorities, and our Position Papers to guide the direction and decisions made by the OAME/AOEM Board of Directors. These texts articulate who we are and who we aspire to be as a community. They remind us that our work is grounded in equity, informed by research, and sustained by collaboration. They call us to advocate for high-quality mathematics education for all students, and to support educators in creating learning environments, where every student can experience the beauty, wonder, and power of mathematics.

That beauty is something I have always cherished. I often find myself pointing out mathematical patterns in the world to my friends and family. I believe deeply that mathematics is not just a subject; it is a way of seeing and interacting with the world. When students experience mathematics as relevant, connected to their lived realities, and reflective of their identities, they begin to see themselves as capable mathematicians. That is the heart of our work.

As I conclude my presidency, I am filled with gratitude. To the Board of Directors, thank you for your leadership,

your thoughtful deliberation, and your unwavering commitment to our mission. To the chapters, thank you for the countless hours you devote to creating meaningful professional learning opportunities. To the volunteers who support conferences, publications, committees, and events, thank you for your generosity and your belief in the power of community. And to every educator across Ontario who engages with OAME/AOEM, whether by attending a webinar, participating in a chapter event, reading the *Gazette* and *Abacus*, or sharing ideas with colleagues, thank you for the work you do every day to support students in their mathematical journeys.

Our new Executive Director, Sandra Jean Price, has taken over the role. Sandra Jean brings a wealth of experience to the position, beginning as a Chapter Representative in 2003, being part of the conference organizing committees for OAME 2003 and 2025, as well as being instrumental for the quick pivot to a virtual format in 2020, and ensuring that educators benefited from valuable learning during a time of upheaval and uncertainty. She served as President-Elect in 2021 to 2022, President for September 2022 to August 2024, and Past President in 2024/2025. Sandra Jean won the Exceptional and Creative Teaching in Elementary Mathematics Award in 2008, and the Life Membership Award in 2024. We look forward to her ongoing leadership and guidance as we continue the work of OAME/AOEM.

Although my term as President is ending, my commitment to this community remains steadfast. I look forward to my ongoing work as Past President, supporting our incoming President, Mark Chubb, and continuing to learn with you, to contribute where I can, and to celebrate the ongoing work of an organization that has meant so much to me. The future of OAME/AOEM is bright, sustained by the passion, expertise, and dedication of educators across the province.

As we move into summer, I hope you find time for rest, reflection, and renewal. May you carry forward the "aha" moments from this year, the questions that continue to spark your curiosity, and the connections that sustain your practice. Thank you for the privilege of serving as your President. Together, we will continue to promote, support, and advocate for excellence in mathematics education throughout Ontario.

Order of Operations Choose Your Own Adventure (OOCYOA)

Pick a number and then pick whether to go to "OOCYOA" on page 21 or page 31.

Effective Use of Digital Technology in Mathematics Education OAME/AOEM Position Statement June 2025

The goal of each OAME/AOEM Position Statement is to outline OAME/AOEM's beliefs on issues surrounding math education by providing transparency and guidance to those involved with mathematics education in Ontario. Any clarification regarding a position, or how to proceed in accordance with it, can be brought to the attention of the OAME/AOEM Executive Committee.

Focal Question

How does OAME/AOEM support, promote, and enhance the use of technology in classrooms in ways that are pedagogically sound, equitable, responsive, and reflect the ever-changing nature of technology?

Summary

There is a meaningful place for technology within the mathematics classroom. It can be used by students to develop their mathematical proficiency, as well as by educators for planning meaningful activities.

With respect to learning mathematics, technology can be used to:

- help students visualize and make sense of mathematical concepts,
- help students develop a productive disposition toward mathematics, and
- enhance students' ability to process and communicate mathematical thinking.

With respect to teaching mathematics, technology can be used to:

- differentiate instruction,
- engage and excite students in learning math, and
- support academic honesty.

OAME/AOEM supports the effective use of any technology in mathematics education, provided the technology augments–not replaces–student thinking. This organization commits to modelling ethical and responsible use of technology in our publications and teacher support materials.

Use of Technology in Mathematics Learning

Mathematical proficiency goes beyond computational skills and the application of algorithms. It includes conceptual understanding (visualizing and making sense of the math), productive disposition (persevering in learning), and adaptive reasoning (including providing explanations and justifications) (National Research Council, 2001). As an instructional tool, technology can be used to help students develop mathematical proficiency in several ways:

1) **Technology can be used by students to help visualize and make sense of mathematical concepts.** Conceptual understanding, or an integrated and functional grasp of mathematical ideas, includes visualizing and making connections with prior knowledge (National Research Council, 2001). In some instances, such as using number lines and arrays, the visual representations are the math (Lambert, 2024). Understanding of mathematical concepts often begins with students using concrete materials, which may include virtual, dynamic manipulatives (NCTM, 2002). Digital tools can be used by students to interact with multiple representations. Virtual manipulatives, graphing software, and geometric tools can all be used to explore the dynamic nature of mathematical relationships, and make mathematics accessible to all students. Through thoughtful discussion, questioning, and consolidation, teachers can highlight connections between various digital, symbolic, and concrete representations.

   However, technology is not a substitute for conceptual understanding of basic mathematical skills. For example, technology can perform calculations, but students still need to learn the meaning and application of operations such as addition, subtraction, multiplication, and division. When students engage with math in various contexts and make connections between operations through problem solving, reasoning, and proving, they are better able to understand how, when, and why each operation is used. These connections to the Mathematical Process Skills listed in the Ontario Mathematics curricula (elementary, secondary) are foundational to learning math and should be developed alongside the use of technology.

2) **Technology can help students develop a productive disposition toward mathematics.** Productive disposition includes the belief that a steady effort in mathematics is rewarding, as well as the ability for students to see themselves as learners and doers of mathematics (National Research Council, 2001). Where computation is a barrier to completing a complex sequence of steps, technology such as a computer algebra system can provide a scaffold for students. In addition, students can use technology to compare a computer-generated solution with their thinking about a problem after they have engaged in the problem-solving process and found a solution. This then becomes a means to make sense of different solutions. Digital tools can also be used to facilitate opportunities to engage in complex tasks by gathering primary and secondary data. This can include relevant and engaging tasks that address issues in local communities and around the world and support the belief that students can make a societal impact using math.

   While technology might be used for repeated practice of skills to improve computational fluency (and hence, improve students' confidence as doers of mathematics), teachers should be judicious in the frequency with which they assign such tasks. Targeted practice goals can be revisited and adjusted frequently, using technology to maintain an appropriately high level of expectation for individual students. If the primary focus of a technological tool is advancing levels or collecting rewards at the expense of deep thinking, the use of the tool should be reconsidered.

3) **Technology can enhance students' ability to process and communicate mathematical thinking.** Adaptive reasoning, which includes students' ability to explain and justify their thinking, can be facilitated through the use of technology in the classroom (National Research Council, 2001). Digital tools, such as a collaborative virtual whiteboard space, can offer students the opportunity to work with their peers to identify multiple strategies to approach a problem and discuss questions to clarify the task. This allows students to make connections between diverse perspectives when problem-solving, as well as receive feedback from other learners. Students can also use technology as an aid when providing both informal and formal justifications of their process and solutions. Teachers can use digital tools to help create a classroom culture of collaboration, with a focus on making connections among concepts and situations.

   While access to technology allows students to share technologically generated solutions with their classmates and teachers, it also makes it easier to misleadingly present this work as their own. When emphasis is placed on the initial thinking and the process of collaboration during problem-solving, it becomes more difficult for students to replace these processes with solutions they did not create themselves.

Use of Technology in Mathematics Teaching

Teaching is a complex process which involves designing instructional experiences that connect to, and apply,

students' background knowledge. To do this effectively, educators need both deep mathematical content knowledge and pedagogical strategies (Loewenberg Ball et al., 2008). Technology has several uses when it comes to teaching, including the ability to make instruction easier to plan, learning easier to assess, and lessons more engaging.

1) **Technology can be used to differentiate instruction.** Students have different learning needs due to their diverse backgrounds, as well as varying prior knowledge and experiences with mathematics. Technological tools can help educators with determining and meeting such needs. Responding to students' math learning needs also involves a deep understanding of curricular concepts and how they develop. Technology can support teachers in deepening their math content knowledge for teaching through the vast array of interactive teacher guides, online simulations, and developmental continua. Technology can support teachers to be more precise in their planning and respond in a timely manner to student areas of need.

Technology can provide targeted instruction and practice opportunities to help close gaps in student knowledge and reinforce learning. However, technology is not a replacement for educators' thinking or being responsive to students' needs, nor should it be applied indiscriminately and simply because it is available. With digital practice tools, there is the potential to collect large amounts of assessment data. It is nonetheless important to triangulate balanced assessment data between conversations, observations, and products from a wide range of sources when making instructional decisions and evaluating learning. In addition, when technology is used as a one-size-fits-all solution for all learners, opportunities for differentiation may be overlooked.

2) **Technology can be used to engage and excite students in learning math.** Gamification of mathematics learning has been shown to increase engagement, improve problem-solving skills, and reduce math anxiety (Jutin & Maat, 2024). While technology can be used by teachers to gamify the learning process, the primary purpose of any technology or digital tool in a math class is to promote and facilitate mathematical thinking and reasoning. As such, the strategy of gamifying with technology should not be misused as a classroom management technique or as a tool that keeps students quiet, busy, and working alone. Furthermore, students may feel isolated when technology is used primarily to engage students in independent practice.

3) **Technology can be used to support academic honesty.** Teachers play a leading role in guiding students to understand the difference between technology augmenting versus replacing student thinking. For example, teachers can model how to assess the reasonableness of an answer and then use online tools to check solutions and receive immediate feedback. Similarly, online graphing tools can be used to confirm a student's initial sketch of a function. However, teachers are understandably concerned about the potential use of technology by students to misrepresent work and thinking as their own. Placing a greater emphasis on the learning process, rather than final products, shifts the nature of students' utilization of technology from deception to development.

Regardless of how an educator chooses to use technology in their classroom, they have a responsibility to ensure that technology used in mathematics teaching and learning is accessible to all students, regardless of socio-economic status or other demographic factors. Unequal access to technology among students can exacerbate existing inequities and harm student learning outcomes.

The Role of OAME/AOEM in Supporting Math Educators' Use of Technology

OAME/AOEM supports the effective use of any technology in mathematics education, provided the technology augments—not replaces—student thinking. Teachers considering the use of any form of technology, from calculators to generative Artificial Intelligence (AI), must be intentional in its purpose to support specific learning goals. OAME/AOEM commits to supporting teachers with ongoing learning to ensure technology use in classrooms enhances students' abilities to apply mathematical processes.

OAME/AOEM is positioned to play a lead role in technology use within mathematics education in Ontario. The organization commits to modelling ethical and responsible use of technology in its publications and teacher support materials. In addition, through the development of chatOAME, an AI chatbot designed to support Ontario mathematics teachers, OAME/AOEM is leading the way in leveraging AI ethically and responsibly to enhance teacher practice. Finally, OAME/AOEM calls on school boards to invest ample time and resources toward teacher training so that all mathematics educators are supported to meaningfully integrate technology into student learning experiences.

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▲ Bridging Research and the Classroom: Cristina De Simone Honoured with OAME Award



The Ontario Association for Mathematics Education (OAME/AOEM) is proud to recognize Cristina De Simone with the Award for Exceptional and Creative Teaching in Elementary Mathematics. A dedicated Grades 7 and 8 educator in the Peel District School Board, De Simone is celebrated for her rare ability to bridge high-level research with transformative classroom practice.

Nominated by Dr. Marc Husband of St. Francis Xavier University, De Simone is described as an "exceptional educator, whose work is deeply grounded in classroom practice, while simultaneously shaping the broader professional landscape." Her classroom is a hub of mathematical agency, utilizing "low-floor, high-ceiling" tasks and purposeful discourse to ensure conceptual understanding for all learners.

Beyond the classroom, De Simone's influence extends through her role as Co-Editor of the *Abacus*, where she translates complex research into accessible insights for practitioners across the province. Her doctoral research into family relationships with mathematics further highlights her commitment to student confidence and identity. By centring students' lived experiences and fostering equity and inclusion, De Simone exemplifies the spirit of innovation in mathematics education. Her leadership continues to enrich the learning experiences of students, families, and colleagues alike.

▲ OMCA News



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As another school year comes to a close, the start of the summer offers members of the Ontario Mathematics Coordinators' Association (OMCA) the perfect opportunity to reflect on challenges and areas of growth. On behalf of our membership of math coordinators, consultants, and coaches from across the province, we hope this month offers you the opportunity to celebrate the successes from the past school year!

Monthly OMCA Meetings and Professional Learning

For the Spring season, we completed our mini-series on student-centred coaching with Diane Sweeney. We discussed finding common entry points for coaching, implementing coaching cycles, and creating authentic engagement in coaching. Following our professional learning on supporting teachers, we immersed ourselves in Indigenous ways of learning mathematics with Ruth Beatty and Christina Ruddy. Their work has influenced shifts in research methods by developing ethical relations, by centring instruction, and by prioritizing Indigenous knowledge systems. We closed out our year with a presentation by Graham Fletcher, whose work with fact fluency, problem-based lessons, and progression videos help to support teachers' breadth of knowledge, and students' curiosity.

Ad-Hoc Committees

Our membership articulated two specific needs in the Spring: approaches to teaching mathematics to students with Individual Education Plans (IEPs), and addressing numeracy programming for the new Kindergarten curriculum. The two ad-hoc committees met and established goals surrounding curriculum planning, resource curation, and community building within the special education and early learning communities. Our goal is to collaborate and share these conversations with our school board members to help with system planning for both of these learning communities.

Counted Out Screening

On May 21, we invited our members to watch a screening of the documentary *Counted Out*. Individuals had a choice to watch the movie on their own, or teams of members had

the opportunity to screen the film together as a department. Our coordinators and consultants found connections between their classroom practice and the impact of mathematics on real-world issues presented in the film. We hosted an online group discussion in the evening, to share our reactions; reflect upon the theme of mathematics as a path to power and agency; and develop actionable steps within our coaching roles to reveal the ways in which mathematical literacy is a tool for discovery, communication, empowerment, and democratic participation.

New Executive

Our final meeting of the school year is an opportunity to reflect upon the professional growth we experienced from the year, and to consolidate the issues often faced by coordinators, consultants, and coaches in mathematics education. We also look ahead with fresh ideas, as the membership will also vote to select at least two new members of the OMCA Executive, filling the roles of Treasurer and Vice-President. We will also take time to celebrate the winners of the OMCA Award for Excellence and Innovation in Mathematics Education. All of our nominees have shown exemplary leadership and creativity in their roles, and we are thankful that our community continues to support each other as we break for the summer. ▲

What plane shapes contribute to this tricycle?

Explain how any shape you find contributes to the fit for humans.



▲ OAME/NCTM Report: Summer 2026 NCTM Highlights for Ontario Math Educators



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Gerry is an Elementary Mathematics Facilitator, who assists teachers of students with IEPs (Individual Education Plans) within the Toronto Catholic District School Board. Gerry has presented over 20 times across Canada and the United States, with a focus on equity, culturally responsive pedagogy, mathematical modelling, and integration of technology in the classroom. In 2024, he completed his Master of Education degree in mathematics education at York University. He is currently the Past President of the Ontario Mathematics Coordinators Association (OMCA), and for his work with mathematics teachers through the pandemic, he was awarded the Prime Minister's Award for Teaching Excellence in 2022.

Scavenger Hunt

Congratulations to Adriana Silvestre, Jillian Kee, and Nisha Patmanathan for completing the NCTM Scavenger Hunt. They will receive their choice of available NCTM publications. We thank everyone who participated in this challenge. Our respondents were particularly drawn to this research clip about classroom practices that support equity-based mathematics teaching (www.nctm.org/Research-and-Advocacy/Research-Brief-and-Clips/Classroom-Practices-That-Support-Equity-Based-Mathematics-Teaching), which articulates three lenses that help frame these practices: reflecting, noticing, and engaging in community. One of our teachers appreciated thinking about how students experience mathematics in the classroom, where strong math teaching is not only about curriculum, but also about student inclusion, identity, positions, and beliefs as mathematical thinkers.

Conferences

Registration is open for the NCTE–NCTM Joint Conference for Elementary Literacy and Mathematics (www.ncte.org/2026-ncte-nctm-joint-conference/), which will take place from August 3–5, 2026 in Charlotte, North Carolina. Elementary teachers and leaders will strengthen their practice in mathematics and language arts, while exploring instructional strategies that support connected

learning and student engagement across both disciplines.

The NCTM Annual Meeting and Exposition (www.nctm.org/denver2026/) will take place from October 28–31, 2026 in Denver, Colorado. The theme of the conference is "Mathematics for a Changing World: Belonging, Learning, and Leading," and the event will bring together mathematics educators from all over the continent. Educators, administrators, and changemakers will exchange ideas that amplify student voice, thoughtfully integrate technology, and enhance instruction for all learners.

Membership Highlight: Webinars

(www.nctm.org/online-learning/)

NCTM members have access to a wide array of resources to support their growth as mathematics educators. This month, I want to highlight the live and recorded webinars that cover a variety of topics relevant to your teaching practice. Members can search by a specific topic and grade band, and webinars are archived for future viewing.

One recent webinar presented by Jelena Byers was "Using Generative AI to Strengthen Math Questions" (www.nctm.org/online-learning/Webinars/Details/795), where the presenter explored a mathematical task appropriate for students, and uses generative Artificial Intelligence (AI) to refine, revise, and enrich questions that support meaningful classroom discussion centred on the task. With the use of ChatOAME, OAME/AOEM members can use the strategies from this webinar to create custom tasks that satisfy Ontario curriculum expectations.

The webinar presented by James O'Neal Jr, "Motivating and Supporting Hard-to-Reach Students" (www.nctm.org/online-learning/Webinars/Details/793), shares a research-informed framework rooted in four powerful classroom cultures: praise, belonging, competence, and autonomy. These cultures provide conditions where students re-engage, participation rises, and teachers feel their joy and confidence return.

If you have any questions about being an NCTM member, or would like to connect regarding NCTM membership events or benefits, please reach out to me any time. ▲

OOCYOA (Instructions page 6)

This piece demonstrates an ordering of operations that entails choice.

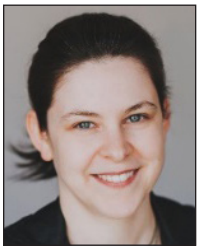
N led to *N* + 2 or 3*N* (p. 21) but could lead to *N*+3 or 2*N*+1 (p. 31).

In all cases, the number is bigger than you started with.

The possible remainders are 0, 1, 2, 3.

This could be used for self-forming groups of four by requiring groups have a person with each remainder. It could also be used to compare experimental to theoretical probability—do you think the probabilities would be close, or does the human condition interfere?

▲ Events at the Fields: Research Day, the Math Transfer Nexus, and Assessments



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Pamela Brittain (Ph.D.) is an educator, author, illustrator, presenter, and math lover, who currently works for the Fields Institute as their K–12 Academic Coordinator. She has been involved in STEM education for over 20 years and strives to make math accessible and enjoyable for everyone.

Research Day

The January MathEd Forum was our annual Research Day. The day involved a keynote talk, short presentations from multiple speakers, and a poster presentation. Getting us started was Marina Milner-Bolotin (University of British Columbia), whose keynote speech was on **When Research Meets Reality: Lessons from STEM Education for Addressing the Canadian K–12 Teaching Crisis**. The talk introduced the purpose of the day and her belief in the need for a research-informed view of education, and that those who do academic research and those who practise in classrooms should not be separate. One of her main beliefs is that the core of STEM needs to focus on these four areas (and that even the "T" might be unnecessary, as it's already considered inherent in the other fields). She also believes that the inclusion of other areas, like the "A" in STEAM, can dilute the focus on core content. Her main tool for integrating the STEM fields is the use of modelling, which can help make abstract ideas more concrete and focused. The main thesis of her discussion was the need for research evidence in education, and she warns against practices that can imitate research-based intervention, but lack the hard evidence required. She advocates that researchers must be constantly critical of their studies, and be transparent when publishing results, even for failed interventions. Her talk went on to discuss the role of Artificial Intelligence (AI) in education and how it can provide opportunities for one-on-one tutoring for all students, something not feasible with human tutors. She stated that technology use in the classroom needs to be deliberate, and caveats this with the note that technology tools are only effective if teachers implement them deliberately and carefully, with the goal of

empowering students through their use versus making them less able or helpless. The rest of the day consisted of a number of short talks on current and recent research.

Mathematics That Matters for Teaching: Connecting Mathematical and Teaching Practices in a Professional Development Workshop. Cody Wilson (University of Toronto, OISE) presented on how generalization, or noticing common properties and then extending them to larger sets, is a central practice in mathematics, and if teachers can develop a stronger knowledge of this area, often neglected in teaching mathematics, they could provoke students to develop stronger generalization skills themselves. This concept was demonstrated in a three-day workshop with teachers to show them how to make connections between recognizing patterns and generalization.

Enacting Analogical Reasoning: The Critical Role of Instructional Quality in Structured Strategy Comparisons. Haoyi Wang (University of Toronto, OISE) and Jon R. Star (Harvard Graduate School of Education, Cambridge, Massachusetts) focused on various problem-solving strategies to support student learning and comparing different solutions to problems to investigate various math concepts. These compared strategies were evaluated on areas such as efficiency, accuracy, and "beauty," and comparing different solutions in a structured way improved students' abilities to be flexible in how they implement various procedures.

The Examination of the Cross-Curricular Content Within the Ontario Grade 12 University-Level Mathematics Curriculum. Qiye Huang (Ontario Tech University, Oshawa, Ontario) surveyed 95 Ontario STEM undergraduate students to determine how well the Grade 12 math curriculum prepared them for university. The results found that calculus and vectors classes were considered to be effective, but many students found them irrelevant to their specific fields (such as nursing or biology) and recommended more realistic examples and scenarios.

Para-Imaging: What Theoretical Framing Would You Take? Marc Husband (St. Francis Xavier University, Nova Scotia), Tina Rapke (York University), Evan Throop-Robinson (St. Francis Xavier University), and Allison Tucker (St. Francis Xavier University) presented. This group looked at "Para-Imaging," as applied to Number Talks, where they ask students to take their mental images of math problems and make them visible to the class so they can be discussed and revoiced, to understand the processes and approaches students take. They did note that these representations have both the ability to help and distract from a student's original mental image of the problem, and this needs to be taken into account.

Mathematics and Coding in Teacher Education was presented by Lisa Anne Floyd (Wilfred Laurier University, Waterloo). Lisa spoke on understanding pre-service teacher perspectives on teaching through the use of coding. Her research sought to understand how teachers' perspectives on learning mathematics through coding could change and evolve. Her research was grounded in constructionism and was conducted over six years and six cohorts of teacher candidates, focusing on 53 pre-service teachers. Her research noted four main areas: that candidates went from being nervous to excited about learning coding and implementing it with their students; that they began to recognize the benefits of coding as they engaged with literature; that they experienced a "culture of enjoyment" and that it fostered an inclusive environment in the classroom with students becoming experts, while teacher candidates became facilitators; and that by the end of the study, the teacher candidates felt more confident and competent.

Overall, the January session highlighted how research and practice in mathematics education are closely intertwined, and how evidence-informed research can be invaluable for educators and students. In addition, providing educators with knowledge in both subject matter and teaching practices are vital to helping students understand and succeed in the subject.

The Mathematical Transfer Nexus

The focus of the February MathEd Forum was on **The Mathematical Transfer Nexus**. The day went through four areas of mathematics education (elementary, secondary, post-secondary, and teacher training), and looked at how they connect with one another. Some themes that arose from the day included:

- looking at how we build foundational knowledge, and that math shouldn't be taught in individual silos, but be integrated across concepts and grade levels, and the importance of spatial reasoning
- how we need to create classroom cultures that encourage students to be brave and curious when exploring math, and that mistakes should be modelled and encouraged
- that post-secondary education needs to provide support and build trust with students to overcome the "math PTSD" and low confidence that many students face upon entering their programs, and to show them where and how to seek help for both these issues and the "culture shock" they may face, moving from small classrooms to large lecture halls
- finally, that teacher candidates need to be prepared to teach math and to be able to explain mathematical concepts to their students; furthermore, that these

programs need to address negative feelings teacher candidates may have toward math themselves and how this needs to be addressed so it is not passed on to their students. The need for mentorship and cross-collaboration between math and education departments was also raised. The presenters for this day came from a wide variety of backgrounds.

Kit Luce (Simcoe County District School Board). Her topic, **Building the Foundation for Long-Term Transfer**, focused on the "continuous journey" of mathematics and the lack of connection that is often seen in elementary schools between concepts across years. How these "grade-level silos" often create foundational gaps in students and a "rush to procedures" without conceptual understanding leads to lack of deeper understanding that progresses from year to year. She also highlighted the importance of spatial reasoning as an essential area of mathematics, along with low-floor, high-ceiling activities and think-alouds to help students build confidence, efficacy, and make visible any gaps in learning.

Rachel Bearse (Kingston Secondary School). She spoke on **Encouraging Joyful and Courageous Math Classes**, and focused on encouraging bravery and collaboration in her classes, specifically early high school. She often randomizes her students, and whom they are working with on any given day, to normalize working with others and collaborating on ideas in math. Like Kit, she utilizes the low-floor, high-ceiling activities to engage all students, and encourages risk taking and models the benefits of making mistakes by intentionally modelling this in the class. She does this to show that speed is not the goal in math—rather, it is building understanding and learning how to think.

Cameron Redsell-Montgomerie (Mohawk College, Hamilton). This talk on **Fostering Mathematical Resilience in Post-Secondary Pathways** looked at how many college students come to his class with "math PTSD," and they have a lack of confidence in their own skills and abilities. To address this, Cameron focuses on early, low-stakes testing to show students they can succeed, and to build trust and confidence before any high-stakes exams or assignments are brought before them. One other method he offers is "Late Passes" that students can use to give themselves a certain number of extensions throughout the year. By giving each student the same number of passes, and leaving it up to them as to when and how to use them, he builds confidence, but also transparency and a sense of ownership in his students. He also makes use of his school's AI-powered Learning Management System (LMS) system to send messages to students to encourage them, make them aware if they are falling behind, and to congratulate them for achievements.

Francis Duah (Toronto Metropolitan University) spoke on the **Transition from School to University Mathematics Study: Observations and a Call to Action**, and focused on the "culture shock" that many students experience when entering university from high school. He spoke about how many students are lacking skills in knowing whom and where to turn to for help, and that they often fail to attend office hours or utilize math supports offered to them. To help lessen the pressure on his students, he often uses surveys instead of more high-stakes diagnostic tests to help him and his students identify where their mathematical gaps may lie upon entry into the program.

The day ended with a summary presentation called **Expanding the Horizon: Where do we go from here?**, by Jennifer Holm from Wilfrid Laurier University, and a group panel discussion. Jennifer's summary acted as a recap of the day's presentations and concepts, and sought to bring together the connected ideas from the day. Her main take-aways from the day included the inseparability of "knowing math" and "teaching math," and that teachers need a flexible understanding of mathematics to help their students effectively. She also noted how teachers can perpetuate negative attitudes toward mathematics with their students, and how they need their own preconceptions to be challenged to help prevent this. She ended her talk with a call for changes to teacher education training to incorporate both strong math skills and strong pedagogical knowledge.

This was followed by a panel discussion, where the day's presenters responded to audience questions and provided further insights into their presentations.

Assessment in Math

The March MathEd Forum was on **Reframing the Impact of Assessment in MathEd: From Measurement to Meaning** and had presenters from university programs both here and abroad, along with members of the Education Quality and Accountability Office (EQAO) presenting on various assessment measures, their impacts, and how to make meaning from them.

The day started with a discussion on **PISA, Math Literacy, and Student Outcomes Across the Industrialized World: An Ordinary Least Squares Analysis with Policy Implications**, by Louis Volante (Brock University, St. Catharines). In his talk, Louis spoke from the perspective of an educator, not a mathematician, though he does have a background in statistics. He focused on comparing Canada (a "highly educated country") to other countries, using PISA (Programme for International Student Assessment). Some of the metrics he was looking at related

not just to completion of students in high school and them moving on to post-secondary education (what was originally considered student success), but also specific outcomes in mathematics and "resilience," or the ability of students from lower socio-economic brackets to achieve at the same level as their more affluent peers. In 2022, it was noted that scores dropped globally and, while Canada still ranks about thirteenth in math, that rank is slipping, and the drop was sharp between 2018 and 2022. Some of the trends he mentioned included declining math scores, which started before the COVID pandemic—although the pandemic certainly accentuated these differences, especially when comparing socio-economic status and gender. The findings of the PISA scores showed that countries that had the most school shutdowns had the biggest impacts on achievement and equity. He also hypothesized that girls' scores dropped more substantially due to a shift in online schooling, which came with increased online bullying (which affects girls more frequently), and that students who are frequently bullied perform about one and a half grade levels below their peers. The final area he raised included issues related to grade inflation and how students entering university often struggle in first-year calculus courses due to this.

Next was **Assessment for All – Addressing Assessment Issues in Mathematics Education**, by Guri A. Nortvedt (University of Oslo), who joined from the mountains of Norway. Her talk focused on issues with individual assessments being seen as "telling the whole truth," and her belief is that instead, assessments need to be seen as a long process. One of her key issues was that the word "mathematics" often means different things to different individuals and groups. Some see it as methods and procedures, while others tend toward problem solving and critical thinking. The idea of math being "culture free" also came up as an issue, as math is often taught differently across different cultures, including what is, and isn't, valued. Her talk then turned toward issues of equity and access, and how many students often don't have access to the full curriculum or the curriculum of their peers. These differences in access show up in large-scale assessments, where some groups have had access to only a small portion of the curriculum—such as minority or low-performing students, who are often only taught arithmetic and miss out on problem solving and modelling. She ended her talk by noting that assessment has the power to improve education, but it can also distort it, if the purpose and design aren't aligned with the curriculum.

The third talk was **Tracking Pathways in Mathematics Achievement: EQAO Cohort Data for Classroom Reflection**, by Maggie Dunlop and Elli Weisdorf (EQAO). Their talk focused on research from a group of students, whom they followed from 2022 to 2025. The talk started with background information on the EQAO assessments, where students take them in Grades 3, 6, and 9, and how the Grade 9 test now accounts for 10 percent to 30 percent of a student's final grade. What they found was that 81 percent of students, who did not meet the provincial standard in Grade 3, also did not meet it in Grade 6. A lack of strong skills early tends to result in significant struggles in later grades. From this, they called for early interventions to help students build these skills as early as possible. Since the pandemic, new trends in gender gaps were also seen, with girls underperforming boys in Grades 3 and 6, though they do gain some back by Grade 9 (see the talk by Voltane, above, for similar findings and hypothesized reasons). The rest of the talk focused on the Ontario Ministry of Education's "Math Achievement Action Plan," where the focus is on knowing the learner. Their recommendation from this plan was for educators to focus on Tier 1 interventions that bring universal design into the classroom. They've also seen that school boards who focus on deep dives within the data, to discover exactly where their students are struggling, have seen good gains within their results. Finally, they advocated for using high-impact instructional practices, like small-group instruction and using tools and representations, to make abstract concepts visual.

The final talk of the day was on **Mining Insights from an Assessment Task**, by Robin Sharma (McGill University, Learning Design Collab, Montreal). In his talk, he focused on how he uses Large Language Models (LLMs) as a tool for creating and evaluating assessments with the APOS (Action, Process, Object, Schema) framework, that allow him and his team to create knowledge graphs of math topics. Students solving questions in a digital environment have the data related to how students approach a problem fed into the LLM, not just the final answer. This allows the system to look at how they approached the problem and where they potentially went wrong. For example, a student solving a geometry question might use the Pythagorean theorem correctly to find radius, but not double it for diameter, which is what is needed to solve the problem correctly. Instead of marking the final answer incorrect or wrong, the LLM can identify where the error occurred and why. Doing this, and providing teachers with a visual representation of this, allows teachers to do more targeted interventions. The talk did end with a caveat about LLMs and their limitations, and how they can often create incorrect assumptions and conclusions. However, Robin and his team found that basing the LLM in a theoretical framework has made them more reliable.

The afternoon session was then spent in small groups discussing a variety of assessment tasks from multiple sources, and asking various questions regarding what they target, what strategies might be used, whether they effectively measure what they are seeking to measure, whether they support or hinder student reasoning, and whether they could be improved. If you'd like to try the assessment tasks yourself, the questions and the tasks can be found here at www.fields.utoronto.ca/talks/Breakout-Workshop-Tasks-Responses-and-Cognitive-Models.

Storytelling for Mathematics

The March MathEd Forum was followed by a workshop led by Dr. Erica Walker, Dean of OISE, and Kitty Yan (Ontario Institute for Studies in Education (OISE)), entitled **Nurturing Mathematical Minds Through Multimedia Storytelling**. In this workshop, participants were shown videos of mathematicians from cultures and ethnicities typically underrepresented in mathematics, speaking about their history with math and various math concepts they find fascinating. The videos come from the **Storytelling for Mathematics** project, where "[i]n 2021, we interviewed 31 mathematicians—their rich mathematical stories form a library of 90+ short videos for people of all ages." The session featured videos and discussions on how to integrate the resources into classrooms, and the implications of telling such stories for different students and classrooms. You can learn more and view the collection at www.fields.utoronto.ca/activities/25-26/math-storytelling. This program was offered in collaboration with the Fields Math in Literacy program (www.fieldsacademy.ca/math-in-books/). ▲

What shapes (2d and 3d) do you see?



▲ Elementary Math Matters: Good Book, Bad Book? Children's Literature Under the Lens



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Lynda Colgan is Professor Emerita, Faculty of Education, Queen's University, and Founder and Faculty Lead for Science Rendezvous Kingston. In May 2025, she was awarded the King Charles III Coronation Medal by the Chancellery of Honours at Rideau Hall for her lifelong contributions to education. Her latest project is contributing author for an upcoming Springer publication, Teaching and Learning Elementary Mathematics. *Lynda remains deeply committed to her two loves: STEM outreach/ public education and her English Cream Retriever, Annie.*

Appealing as it may be to integrate children's books into mathematics teaching and learning, mathematics teachers must be vigilant; not all children's books with a mathematics theme are created equal.

On what basis should a picture or storybook be used to complement and/or supplement mathematics lessons? Does it have:

1. visual appeal, i.e., colourful, attractive, and well-designed illustrations?
2. an engaging storyline, i.e., a compelling narrative, relatable or memorable characters, a fun and imaginative context?
3. age-appropriate content and language, i.e., alignment with reading and comprehension levels?
4. culturally sensitive and inclusive themes and contexts?
5. promotion of a positive, interactive relationship with mathematics?
6. curriculum alignment?

While all these criteria could be included in the evaluation of a children's book to determine its suitability as a tool in mathematics instruction, first and foremost, the mathematics must be correct, i.e., mathematical accuracy is a crucial feature. Unfortunately, in many instances, critical elements get subordinated to other categories of satisfaction, and this is certainly true in the case of some books that contain a mathematical theme.

Consider the popular "Sir Cumference" book series by

Cindy Neuschwander. The longevity (over 20+ years) of this series speaks to its continued popularity, although the mathematical errors contained in multiple books from the collection have been well documented over time. For example, the original book, *Sir Cumference and the Dragon of Pi*, contains information that suggests that π is equal to exactly 3.14, no more and no less. *Sir Cumference and the First Round Table* may lead to misconceptions because it is suggested that complex geometric shapes (like a long, rectangular table) can be cut easily and rearranged into other shapes (a square or circle, without losing material or creating structural issues). It is also suggested that perimeter is the main factor in table design; i.e., the story focuses on changing the perimeter to fit more people, but overlooks how changing a table's shape affects the actual area and utility: a mathematical nuance often skipped to focus on the story.

Sir Cumference and the Fracton Faire is a medieval story featuring the Earl of Frackton. In the story, characters measure cloth and cheese, using numerators, denominators, along with equivalent fractions, leading to the use of "fracton" numbers to solve a mystery. The terminology "fracton numbers" is an unnecessary introduction of confounding vocabulary, yet plays a major role in this story, while being easily confused with the correct word, "fraction." All of the books in the series rely on wordplay, riddles, and Arthurian-themed character names (e.g., Lady Di of Ameter)—features that may be more accessible and appealing to adult readers.

Pre-Implementation Examination and Analysis

So, how can mathematics educators avoid the selection of books that do not include mathematical accuracy as a key element? One suggestion is to apply the continuum suggested by Hellwig et al. (2000) that is shown below in Figure 1.

Less Effective	Criterion	More Effective
Represents factual information inaccurately; uses incorrect terminology or principles, or both.	**Accuracy**	Represents mathematics and other information accurately; depicts relationships correctly.
Format and presentation are pedestrian; book is overpowered by mathematics—a modified "textbook."	**Visual and Verbal Appeal**	Both format and presentation are visually and verbally appealing.
Depicts mathematics as isolated or neutral; connections are hard to find.	**Connections**	Provides a context for learners to make meaningful connections between mathematics and their own experiences.
Targets a specific audience with no range for age or ability.	**Audience**	Presents concepts in a way that appeals to a range of audiences and abilities.
Contains little or nothing beyond the expected; has no surprises.	**"Wow" Factor**	Offers layers of richness beyond the predictable or expected; presents exciting new views or ideas.

Figure 1: *A continuum for evaluating literature to enhance mathematics instruction (Hellwig et al., 2000). The gradient (from 0 on the left [weak] to 6 on the right [strong]) is used to evaluate each of the criteria.*

In their article, Hellwig et al. (2000) also include a worksheet (Figure 2) that teachers can use to assist in the evaluation process. The worksheet could be used in a professional development program, during which teachers evaluate books from their school or classroom collections and then share their findings. The peer-completed evaluation pages could then be organized into a resource binder, to be used to support decision making in lesson or unit planning.

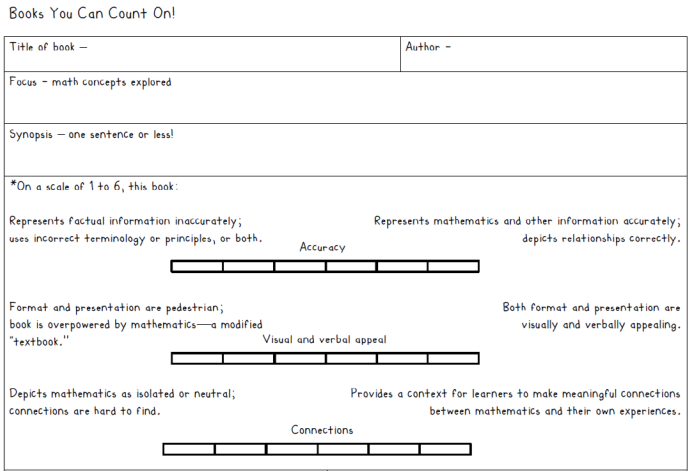


Figure 2: *An application of the Hellwig et al. (2000) continuum. The guide focuses the teachers' attention on the key criteria from the continuum and asks them to apply a six-point rating scale to the evaluation of each. From* Numeracy for All: Making Literature—Math Connections *(Alberta Education, 2014).*

The *Whitin and Whitin Framework*, published in 2004, builds upon the Hellwig et al. continuum, and has been widely acknowledged as a trustworthy instrument for selecting children's books. Developed by Dolores and David Whitin, they are key figures in mathematics education and authors of multiple books on the topic of using children's books to complement and supplement mathematics instruction such as *Read Any Good Math Lately?: Children's Books for Mathematical Learning, K–6* and *New Visions for Linking Literature and Mathematics* (Whitin & Wilde, 1992). The Whitin and Whitin framework serves as a guide for teachers to evaluate children's math books critically by focusing on accurate, relevant math content; an appropriate (and explicit) developmental level; an engaging presentation; and deep mathematical thinking.

The framework focuses on selecting books with high mathematical integrity, aesthetic appeal, and potential for open-ended, varied responses. It emphasizes integrating accurate, engaging math in natural contexts, rather than purely didactic, "formulaic" counting books.

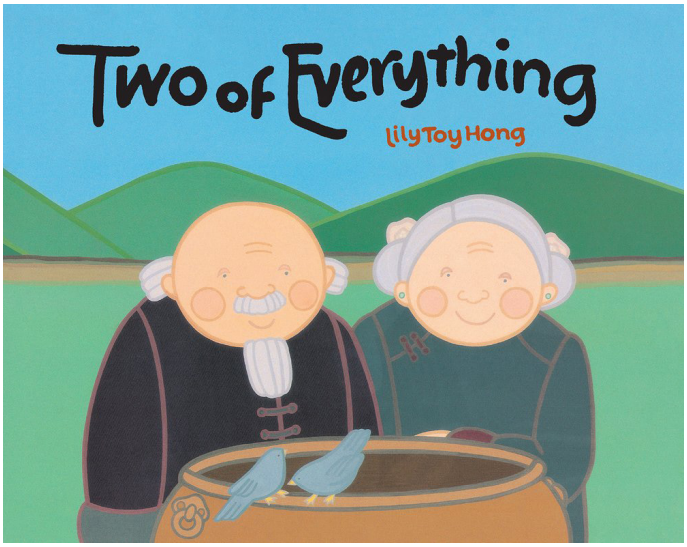


Figure 3: Two of Everything *is a traditional Chinese folk tale about a couple who find a magical pot that duplicates anything put inside it. The story involves counting and doubling, introducing the concept of multiplication in a narrative form.*

As an example, consider the book *Two of Everything* by Lily Toy Hong (Figure 3). Applying the Whitin and Whitin framework as an evaluation standard for *Two of Everything*, the following is noted:

1. *Mathematical Accuracy*
   - *The doubling (multiplication by 2) is accurately depicted through the magical pot's effect.*
   - *The mathematics concept (doubling) is straightforward and correctly represented.*
2. *Mathematical Content and Themes*
   - *The central math theme is doubling, related to multiplication and number sense.*
   - *The story is built around repeated doubling, making mathematics essential to the plot.*
3. *Developmental Appropriateness*
   - *The book is suitable for early elementary children who are beginning to learn about addition, multiplication, and number patterns.*
   - *The story's language and mathematics complexity match young learners' cognitive levels.*
4. *Engagement and Interest*
   - *The magical/mystical story format captures children's imagination.*
   - *The plot is fun, which encourages children to predict what will happen next and think about the mathematics involved.*
5. *Ability to Encourage Mathematical Thinking*
   - *Children can be encouraged to count objects before and after doubling.*
   - *The book sparks curiosity about multiplication and can lead to hands-on activities (e.g., doubling objects physically).*
6. *Language and Illustration Quality*
   - *The language is simple and clear, making mathematics concepts understandable.*
   - *Colourful illustrations enhance comprehension by depicting scenes and objects being doubled.*
7. *Cultural and Social Relevance*
   - *The book introduces children to a folk tale from Chinese culture, broadening cultural understanding.*
   - *Inclusive storytelling respects diversity.*

Mathical Books You Can Count On

There is no precise, up-to-date tally of how many children's books have a mathematics theme. New titles are published regularly, and books can vary widely in how prominently mathematics features in their content, but it is reasonable to estimate that there are thousands of children's books in print that incorporate math concepts. Available books range from basic counting and shapes to more advanced topics like patterns, problem solving, geometry, and even early algebra. Major publishers and independent authors have produced picture books, storybooks, activity books, and educational series focused on math.

The practical challenge of evaluating new and existing books prior to classroom implementation, even with the support of the Hellwig et al. continuum and the Whitin and Whitin framework, would require uncountable hours of teacher preparation time. While I still recommend that teachers take some time to apply the scales in advance of implementation to familiarize themselves with the content and its presentation, there is a trustworthy alternative.

Since 2016, the *Mathical* Book Prize has been awarded by the Mathematical Sciences Research Institute (MSRI), now known as the Simons Laufer Mathematical Sciences Institute (SLMath). The award is presented in partnership with the National Council of Teachers of English (NCTE) and the National Council of Teachers of Mathematics (NCTM), in coordination with the Children's Book Council (CBC). To be awarded a *Mathical* designation, the book must inspire children to see math in the world around them.

In February 2026, the latest *Mathical* awards were announced. They are shown in Figure 4.



Figure 4: *From left to right, the covers of the 2026* Mathical *award winners for preschool students and early readers (Pre-K); young readers (grades K–2); elementary readers (grades 3–5); middle school readers (grades 6–8); and young adults (grades 9–12). Details about each book can be found at www.mathicalbooks.org/2026/02/2026-mathical-winners-announced/.*

Over the years, the selection committee has chosen hundreds of books. They can be used in the classroom with confidence because of their multiple high qualities—the most important of which is the accuracy of the mathematics. According to its website, *Mathical* books:

> inspire a love of math by connecting with children's interests, from magic, sports, art, music, and poetry, to biography, adventure, graphic novels, and of course, picture books for the very young. These titles are as varied as the intersection between literature and mathematics—that is to say, they encompass picture books, novels, poetry collections, puzzle books, biographies, and more!

In Conclusion

As educators, we must be judicious when using books in our mathematics lessons to avoid superficial learning; ensure stories do not oversimplify concepts; or provide inaccurate, forced mathematical connections. We can use tools such as the ones developed by Hellwig et al. or Whitin and Whitin to ensure that the books we select: (1) enhance, not replace, active problem-solving, and (2) ensure the mathematics is developmentally appropriate, accurate, and truly integrated… or we can go directly to the expansive *mathical* collection, confident that any choice from the juried set will enhance classroom instruction and support student learning.

Continued on page 24

▲ MB4T (MATHEMATICS BY AND FOR TEACHERS): SEEING THE DISTRIBUTIVE PROPERTY ACROSS A CONTINUUM



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I often notice that the distributive property is introduced as though it were something entirely new. A teacher writes brackets on the board, draws arrows, and begins a sequence of symbolic steps. Before long, you hear, "…use FOIL: first, outside, inside, last." For many students, that becomes the whole lesson. They learn a sequence to follow, but not always the conceptual mathematics understandings underneath it. It is easy to see why this can make algebra feel sudden and unfamiliar. One day, students are multiplying whole numbers, and the next, they are expected to expand algebraic expressions with variables. It can seem as though algebra is a different kind of mathematics altogether. But is it actually new? In most cases, students have been building the foundation for years. So rather than presenting the distributive property as a brand-new rule or collapsing it into a shortcut like FOIL, we can help students see that they are working with familiar mathematics concepts all along. To make that progression visible, I will guide you through a six-example distributive-property continuum. Each example is shown in three forms: concrete, representational, and abstract. The examples begin with whole-number multiplication and gradually move into algebraic expressions.

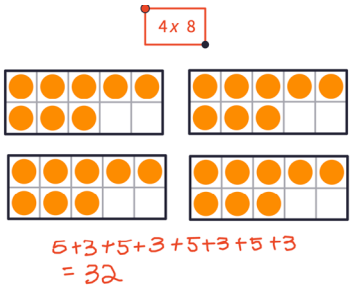


Figure 1: *4 × 8 concrete model*

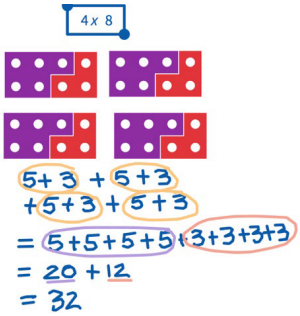


Figure 2: *4 × 8 representational model*

Let us begin with the first example: 4 × 8 (see Figure 1). In the concrete model, using ten-frames and counters, students see four groups of eight, arranged in a way that invites them to decompose each group into 5 + 3. Rather than viewing 8 as one fixed quantity, they begin to see that it can be broken apart into friendlier parts. A student may first describe what they notice additively, saying 5 + 3 + 5 + 3 + 5 + 3 + 5 + 3, before realizing that this is also four groups of five and four groups of three. That moment matters. It shows that the student is beginning to see multiplication as something that can be reorganized and reasoned about, not simply recalled. In the representational form (see Figure 2), this same idea is redrawn as a rectangle partitioned into four regions with five pieces and four regions with three pieces. Students can now see that the earlier model can be represented as four groups of five and four groups of three. In the abstract form (see Figure 3), this becomes 4 × 8 = (4 × 5) + (4 × 3) = 20 + 12 = 32. This example connects directly to Grade 3, expectation B2.1, where students are expected to use the properties of operations, and the relationships between multiplication and division, to solve problems and check calculations (Ontario Ministry of Education [OME], 2020). In Grade 3, students are already being asked to think flexibly about multiplication. The distributive property is present there, even if it is not always named in formal algebraic language.

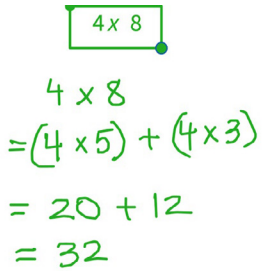


Figure 3: *4 × 8 abstract model*

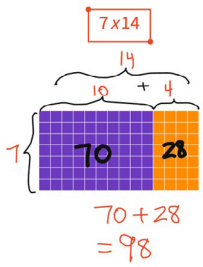


Figure 4: *7 × 14 concrete model*

The next example, 7 × 14 (see Figure 4), builds on this idea, but now place value becomes more central. In the concrete form, using base-ten blocks, students see seven groups of fourteen, partitioning 14 into 10 and 4. Students begin to recognize that 14 is not simply one number to be multiplied all at once. It is composed of one ten and four ones. In the concrete model, using tens and ones, students identify seven groups of ten and seven groups of four. This leads naturally to the partial products 70 and 28. In the representational form (see Figure 5), the area model makes this even clearer. One part of the rectangle has the area 7 × 10, and the other has the area 7 × 4. In the abstract form (see Figure 6), students can show this through the written form 7 × 14 = (7 × 10) + (7 × 4) = 70 + 28 = 98. This example connects explicitly to Grade 4, expectation B2.1, where students are expected to use the properties of operations, and the relationships between addition, subtraction, multiplication, and division, to solve problems involving whole numbers, including those requiring more than one operation (MOE, 2020). This is where students begin to deepen their understanding that numbers can be decomposed when using multiplication.

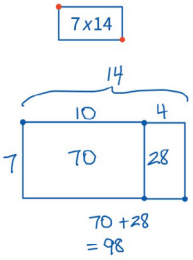


Figure 5: *7 × 14 representational model*

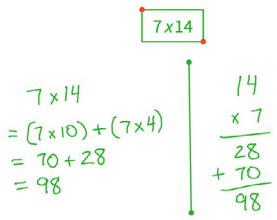


Figure 6: *7 × 14 abstract model*

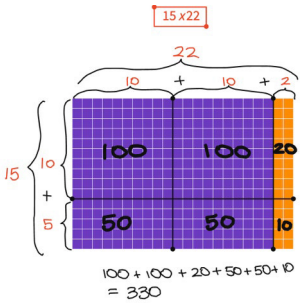


Figure 7: *15 × 22 concrete model*

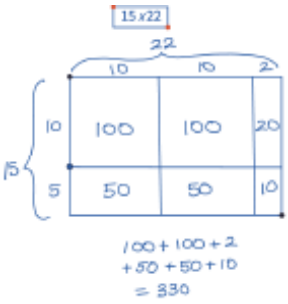


Figure 8: *15 × 22 representational model*

The third example, 15 × 22 (see Figure 7), extends this reasoning even further because students are no longer decomposing just one factor. In this continuum, 15 is composed of one ten and five ones, while 22 is represented with two tens and two ones. In the concrete form, students can use their base-ten blocks to create areas of 100, 100,

20, 50, 50, and 10. In the representational form (see Figure 8), these partial regions become easier to visually see. Students can begin to appreciate that every part of one factor must be multiplied by every part of the other. Once these partial areas are combined, the total product is 330. In the abstract form (see Figure 9), some students may record this as the full sum of the partial products visible in the model, while others may begin to compress it into larger chunks, such as (15 × 20) + (15 × 2) = 300 + 30 = 330. This example aligns strongly with Grade 5, expectation B2.1, where students are expected to use the properties of operations, and the relationships between operations, to solve problems involving whole numbers and decimal numbers, including those requiring more than one operation, and also with Grade 6, expectation B2.1, where students are expected to use the properties of operations, and the relationships between operations, to solve problems involving whole numbers, decimal numbers, fractions, ratios, rates, and whole-number percents, including those requiring multiple steps or multiple operations (MOE, 2020). By this point in the curriculum, the distributive property is becoming more explicit.

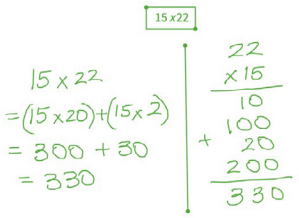


Figure 9: *15 x 22 abstract model*

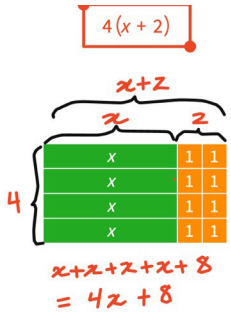


Figure 10: *4(x + 2) concrete model*

Once students have had repeated opportunities to reason this way with whole numbers, we can move into algebra with 4(*x* + 2) (see Figure 10). This is a critical transition because many students interpret the appearance of a variable as though the mathematics has suddenly changed. Yet structurally, nothing has changed. Algebra tiles require a careful clear introduction, with students working with their own tiles. In the concrete form, using algebra tiles, students see four groups of *x* + 2. Each group contains one *x* tile and two unit tiles. As they build the model, they can count four *x* tiles and eight unit tiles. This is important because it allows students to reason multiplicatively with an algebraic expression before writing any symbolic steps. The algebra is grounded in something visible.

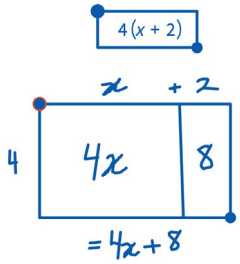


Figure 11: *4(x + 2) representational model*

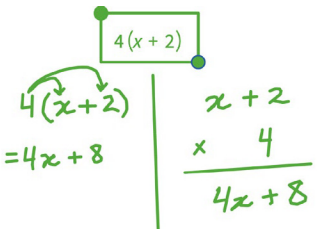


Figure 12: *4(x + 2) abstract model*

In the representational form (see Figure 11), the rectangle has side lengths 4 and *x* + 2, and the side labelled *x* + 2 is partitioned into *x* and 2. This creates two regions with areas 4*x* and 8. In the abstract form (see Figure 12), students can then write 4(*x* + 2) = 4*x* + 8. This example aligns directly with Grade 9, expectation C1.4, where students are expected to simplify algebraic expressions by applying properties of operations of numbers, using various representations and tools, in different contexts (MOE, 2021). This is a powerful moment in the continuum because it helps students see that algebra is not a new kind of mathematics. The same distributive structure they used with whole numbers still applies. The only difference is that one of the quantities is now unknown.

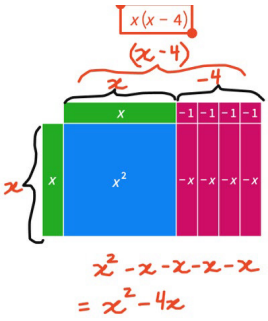


Figure 13: x*(x – 4) concrete model*

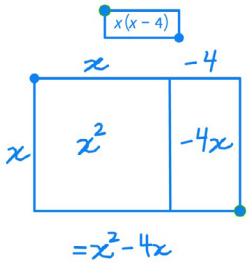


Figure 14: x*(x – 4) representational model*

The fifth example, *x*(*x* – 4) (see Figure 13), deepens this Grade 9 work by introducing subtraction into the binomial. This often creates uncertainty for students, especially if they have previously experienced area only in positive terms. The concrete model helps make this less mysterious. Students begin with one dimension of length *x* and another dimension of length *x* – 4. In the area model, they can identify one large x^2 region and four negative *x* regions. This is important because students begin to conceptually see what the subtraction in *x* – 4 signifies. In the representational form (see Figure 14), the students can now see what the area in each rectangle represents, with *x* times *x* creating an x^2 region and *x* times –4 creating a –4*x* region.

In the abstract form (see Figure 15), students can write $x(x - 4) = x^2 - 4x$. This example continues to align with the Grade 9 expectation C1.4, where students are expected to simplify algebraic expressions by applying properties of operations of numbers (MOE, 2021). More specifically, it supports students in multiplying a monomial by a binomial, while reasoning carefully about why the negative term appears. Instead of accepting the result procedurally, students can trace the $-4x$ back to the structure of the area model and the subtraction embedded in one dimension of the rectangle.

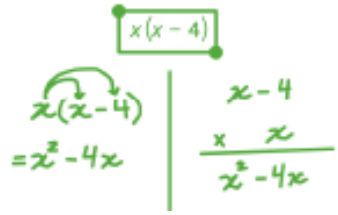


Figure 15: x*(x – 4) abstract model*

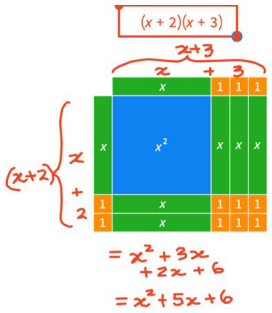


Figure 16: *(x + 2)(x + 3) concrete model*

Finally, the example $(x + 2)(x + 3)$ (see Figure 16) invites students to work with second-degree polynomial expressions. This is often the point at which a shortcut such as FOIL is introduced, and students are asked to follow a sequence without necessarily understanding the structure of the multiplication. Yet the area model allows them to see all the components of the mathematics. In the concrete form, using algebra tiles, students build a rectangle, whose side lengths are $x + 2$ and $x + 3$. As they fill the interior, they can see one x^2 region, three x regions, two x regions, and six-unit regions. In the representational form (see Figure 17), those four regions are laid out clearly in the partitioned rectangle. In the abstract form (see Figure 18), students can then determine each partial product in turn: x multiplied by x gives x^2, x multiplied by 3 gives $3x$, 2 multiplied by x gives $2x$, and 2 multiplied by 3 gives 6. At this stage, it is important not to rush to the simplified form. Students benefit from first recording the full expression as $x^2 + 3x + 2x + 6$. Only then do they combine like terms to conclude that $(x + 2)(x + 3) = x^2 + 5x + 6$. This example aligns explicitly with the Grade 10 expectation that students expand and simplify second- degree polynomial expressions, using a variety of tools, such as algebra tiles and diagrams (MOE, 2005). What is powerful here is that the model accounts for every term in the product. The $5x$ does not appear mysteriously. It comes from the $3x$ region and the $2x$ region, which are already visible in the rectangle. This protects the distributive property from becoming a list of procedural instructions to imitate.

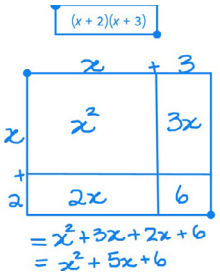


Figure 17: *(x + 2)(x + 3) representational model*

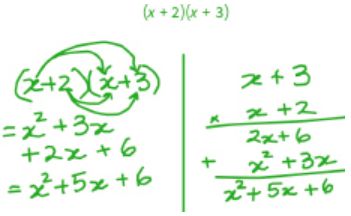


Figure 18: *(x + 2)(x + 3) abstract model*

Seen together, these examples make something very clear: the distributive property is not a standalone algebra topic that suddenly appears in secondary mathematics. It is a curricular throughline. It begins with whole-number multiplication in the Junior grades, becomes more explicit as students work with larger numbers and more complex decompositions, and then extends naturally into algebraic and quadratic expressions. When students experience this progression through concrete, representational, and abstract forms, they are far more likely to see mathematics as connected, rather than fragmented.

In my own teaching, I have found that this kind of continuum changes the tone of the lesson. Students are less likely to respond to algebra with hesitation and more likely to recognize something familiar. They begin to say, “Oh, this is like what we did before,” and that shift matters. It positions mathematics as coherent. It invites students to carry forward prior knowledge, rather than abandon it at the start of a new unit. When the distributive property becomes visible across models, diagrams, and symbolic notation, students can begin to see algebra not as a collection of disconnected procedures, but as mathematics that has been there all along.

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OOCYOA (Instructions page 6)

Either add 2 or multiply by 3.
Go to “OOCYOA” on page 42.

Play, Math, and Early Learning: What Counts with Early Number



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Jennifer Holm is an Associate Professor at Wilfrid Laurier University and works with pre-service teachers, as well as in the field supporting current mathematics teachers. She is interested in developing mathematics knowledge for teaching, focusing on the beliefs and opinions that pre-service teachers hold about mathematics and teaching, and the connection they have to past experiences. She uses this research to support future teachers in developing beliefs and knowledge that will encourage and support effective teaching practices in mathematics. She is also the proud mom of two young learners who are exploring the world of mathematics.

This column takes an in-depth look at counting with early learners. Although I have heard many teachers talk about how counting is incredibly easy, teaching counting well requires a depth of knowledge that may go overlooked. Incorporating counting activities into play is simple, but understanding the complexities of determining where young learners need supports is nuanced and anything but simple. Early learning frameworks include counting as one of the skills for young learners (e.g., Best Start Expert Panel on Early Learning, 2007), which extends into Kindergarten and the elementary curriculum, with larger numbers being required at each grade level (Ontario Ministry of Education, 2020, 2026). Table 1 shows the continuum of counting in Ontario from pre-Kindergarten to Grade 2. As you can see from this continuum, the quantity of numbers that are required at each grade level increases, as do the complexities of what students need to know.

It is important to consider the trajectory of counting. Van de Walle et al. (2018, p. 118) has a wonderful description of how learners move through learning to count. Figure 1 shows the trajectory.

Precounter	Does not associate a word with quantity
Reciter	Says number words, but not always in correct order
Corresponder	Uses one-to-one correspondence
Counter	Accurately counts in organized manner and has cardinality
Producer	Can count out objects
Counter Backwards	Count down
Counter and Producer	Has previous two (i.e., is a Counter and Producer), but can also respond to random arrangements and see which have not been counted
Counter from Any Number	Does not have to start at one to count
Skip Counter	Skip-counts with understanding

Figure 1: *Counting trajectory from Van de Walle et al. (2018, p. 118)*

The counting trajectory provides a basis for helping to understand where learners are in a framework, but counting principles also need to be considered when observing learners. Make Math Moments (n.d.) provides a detailed explanation of these counting principles, and examples of

Pre-Kindergarten (Best Start Expert Panel on Early Learning, 2007)	**Kindergarten** (Ontario Ministry of Education, 2026)	**Grade 1** (Ontario Ministry of Education, 2020)	**Grade 2** (Ontario Ministry of Education, 2020)
4.12 Counting: counting in meaningful ways in play and daily living (p. 55)	A6.2 Understand when counting forward, the quantity gets bigger, and when counting backward, the quantity gets smaller	B1.4 Estimate the number of objects in collections of up to 50, and verify their estimates by counting	B1.3 Estimate the number of objects in collections of up to 200, and verify their estimates by counting
4.13 Determining quantity: counting to determine quantity (p. 55)	A6.7 Estimate the number of objects in small collections and check by counting	B1.5 Count to 50 by 1's, 2's, 5's, and 10's, using a variety of tools and strategies	B1.4 Count to 200, including by 20's, 25's, and 50's, using a variety of tools and strategies
4.14 Comparing quantities: counting to compare quantities; counting using one-to-one correspond-dence (p. 55)	A6.8 Count to 20 by 1's, starting from any number		

Table 1: *The continuum of counting from pre-Kindergarten to Grade 2*

what they look like in practice, as well as activities for supporting learning development. For the purposes of this column, we are going to focus on three of the principles, as defined by Make Math Moments: stable order, cardinality, and one-to-one correspondence. Stable order refers to "using a list of words to count in a repeatable order. This ordered or 'stable' list of counting words must be at least as long as the number of items to be counted" (Make Math Moments, n.d., #1). One-to-one correspondence occurs when a learner assigns one number to each object counted, and only counts each object one time. This may be observed by watching children tag or touch items as they are counting. Cardinality refers to "understanding that the last number used to count a group of objects represents how many are in the group" (Make Math Moments, n.d., #3). A child who does not yet have cardinality will often recount the items when asked "How many?"

How children move through their learning about counting is a complex process. To provide an example, my two-and-a-half-year-old loves numbers and counting. He recites numbers and has figured out that he can count things that he sees in his environment. When given up to three items, he is able to use one-to-one correspondence (uses one number for each item in the set), can recite the numbers in a predictable and correct order (stable order), and shows cardinality (when asked, he can tell you there are three items). He is also beginning to show an understanding of subitizing because, if the items are grouped in a simple way, he does not always need to count them to be able to tell you how many there are. If you consider all of this, he would be a Counter on the counting trajectory—which seems incredible, considering his age.

Every morning, as we climb the stairs into his school, we have counted the stairs as he steps on each one going up the staircase. Recently, he has decided he wants to count them himself as we climb the steps. He shows one-to-one correspondence as he works his way up the stairs, in that he names one number as he steps on each stair. Quite often, what I hear is the following: "one, two, three, four, seven, eight, nine, ten." What is clear, is he has not internalized a correct order for the numbers, although he does say the same number of words as there are steps, but this puts him between Reciter and Corresponder, depending on which skill you are considering in the counting trajectory. What is evident is that, for numbers up to three, he is at one level, and for numbers over four, he is now at a different level.

To make it even more complicated, if you give him over four items to count, he shows he is unable to give a stable order. and he does not have one-to-one correspondence. Once he hits four, it is not uncommon for him to just move his hand around over the objects and recite numbers in whatever order he thinks of, up to ten. Although sometimes he can produce the numbers to ten in the correct order, he is unable to do this consistently, and often uses a similar number pattern as he does when counting the stairs. If you consider this, then he is definitely at Reciter stage in the trajectory.

These observations show an important point to consider with young learners: context and the quantity matter, for considering next steps for students. If students are always given objects to count on the table in a certain pattern, they may be at one level, but when tested with a different environment, they may be somewhere else. It is the same with quantity: young children may show advanced counting skills with smaller numbers, yet struggle when they move to a larger number of items. Starting with amounts of five and less provides a good starting point, but varying the amount will allow educators to determine where students may have already developed more advanced counting skills.

Returning to the example of my son, I can consider what my next steps may be for his development in counting. For numbers up to three, my goal is to work on him being able to produce the quantity (e.g., Please give me two cars). Or I could also work on having him count different arrangements, to see if there is a point where he is unable to count to three (see Figure 2 for some examples). For numbers over three, I need to work on the stable order of counting words. Practising places where he can recite the numbers to five as a starting place would be helpful. He could also use some practice with counting objects up to five, until he is able to show an understanding of one-to-one correspondence with objects. His goals for number need to be varied, but based on where he is, but also taking into consideration the complexity of what he does already know about number.



Figure 2: *Four different ways to present three cars for counting*

In the end, it is important for educators to understand the counting trajectory and the counting principles, along with

how to recognize when young learners have achieved them. In addition, we also need to be mindful that different objects and different quantities can produce different results. It is not enough to just consider students as being at a single level: they may show different levels of skill, depending on the difficulty of the task for them. As educators, we need to consider different types of activities, such as counting stairs versus counting cars, as well as what developmentally appropriate numbers work for every student.

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Geometry and Context?

What is the same? What is different?

Left – Anishnaabe, Nipissing First Nation, Ontario
Right – Spanish, Montevideo, Uruguay





▲ Meaning Making in the Classroom: Redefining Metacognition



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David is a principal, who taught and had various board roles in Prince Edward Island. David has also instructed university courses regarding curriculum, differentiation, mathematics, and literacy. He facilitates professional learning in mathematics and school development. Dr. David Costello has published five books, with two of his most recent books being: What's Missing in Math Instruction: Test Preparation and Classroom Strategies That Show Students How to Apply What They Know in Meaningful Ways, *and* Messing Around with Math: Ready-to-Use Problems That Engage Students in a Better Understanding of Key Math Concepts.

Metacognition is the ability to think about one's thinking, and it is essential to the learning process. Metacognition is a prerequisite for independent learning. When students understand their thinking, when they know they are making progress, and when they recognize they must adjust their plan, students will persevere through struggles. It is this understanding of their thinking that serves as a compass. It informs students of next steps. And, when they have this understanding, they will persevere through the problem-solving process because they know that they are making progress toward the solution.

Marzano (1998) referenced metacognition as the "engine" that drives thinking. Garofalo and Lester (1985) stated that mathematics instruction without metacognition is inadequate. While both references are more than 25 years old, teachers still share that they struggle with students being dependent learners.

Metacognition has been used so often in education that its meaning can become so broad that it loses significance. When thinking about instruction and learning, teachers must be cognizant that much goes into the ability to think about one's thinking. In this column, I will share an approach to metacognition that will assist teachers in having a clearer depiction of what metacognition means in terms of the learner and thinking process. This approach will include a refined approach to metacognition and will provide teachers with a strategy for students to strengthen their metacognition as they become independent learners.

Metacognitive Regulation

Teachers must be intentional in fostering the metacognitive skills of students. And, to do this, teachers must be aware of what metacognition entails. Under the umbrella of metacognition is a concept referred to as *metacognitive regulation* (Stanton et al., 2021). Metacognitive regulation refers to how one controls one's thinking throughout the learning process. There are three key components within metacognitive regulation:

- **Planning:** Deciding on suitable strategies to use for a task and when they will be used
- **Monitoring:** Tracking one's understanding, progress, and the quality of performance during the task
- **Evaluating:** Assessing the success of the task and the chosen strategies, and then making any necessary adjustments to strengthen future performances

Metacognitive regulation involves actions that people take to learn (Stanton et al., 2021). It begins at the onset of the task and carries on through to completion. It is crucial that students strengthen these three components (planning, monitoring, and evaluating), if they are to work through a problem without having to rely on others to determine if they are making progress. If students apply metacognitive regulation, they will be able to work through the problem and persevere when they encounter stumbling blocks. They will be able to make informed plans and make adjustments throughout the task as they see fit.

Applying Metacognitive Regulation

To approach instruction and learning from a metacognitive-regulation perspective, the teacher must determine a strategy (or set of strategies) that will enable the student to control their thinking throughout the learning process. One specific metacognitive strategy is *self-questioning*. Self-questioning closely aligns with the three components of metacognitive regulation. First, students can self-question throughout the planning stage of their work. They can question the strategies that they will employ as well as the task that they are assigned. As students monitor their work, self-questioning techniques will allow them to examine if they are making progress toward their goal and whether their plan is effective. Evaluating the task is a reflective endeavour that is best done when students question their plan, their delivery of the plan, and whether they must make any adjustments when working on future tasks.

For students to strengthen their metacognition through metacognitive regulation, they must self-question throughout the task, as opposed to only at the conclusion. Far too often, students are asked if their work makes sense only after they have finalized their response. While this is a reflective exercise, it does little to encourage students to question their thinking throughout the task. Teachers must engage students in self-questioning from the onset of the task through to completion. By doing this, teachers will support students in becoming independent learners, who can monitor their thinking from start to finish.

There are numerous opportunities for students to self-question throughout the task. When assigned a problem to solve, students move through four commonly accepted stages of problem solving (Pólya, 1945):

- Understanding the Problem
- Crafting a Plan
- Carrying out the Plan
- Reviewing Your Work

Teachers must incorporate opportunities for students to engage in self-questioning throughout the problem-solving process, if the goal is to strengthen student metacognition. When students work independently, they can rely on self-questioning as an approach to metacognitive regulation. By controlling their thinking process, students can ensure they understand the problem, they select and carry out a plan that makes sense for the problem, and they can judge the reasonableness of their response. Students will not be reliant on the teacher for next steps.

Role of the Teacher

Teachers play an important role regarding students self-questioning as a way to build metacognitive regulation. Janssen (2002) emphasized that self-questioning should be explicitly taught in the classroom, as it is a higher-order skill that needs ongoing support. Teachers need to foster self-questioning in students by creating a learning environment that encourages and values self-questioning. Teachers must model self-questioning for students and provide opportunities for students to receive feedback on their self-questioning. Finally, teachers will need to provide ongoing support so that students can strengthen and refine their self-questioning skills as a way to build metacognitive regulation (Stanton et al., 2021).

Final Thoughts

Self-questioning, as a metacognitive-regulation strategy, can have a significant impact on student learning and their independence when problem-solving. Teachers must explicitly model self-questioning and must be aware that they will need to communicate the importance of self-questioning, model how and when to apply it, and then support students in sharing their experiences with this

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In the Middle: Calculated Confidence



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In my last few columns here, I have provided financial literacy resources in anticipation of the graduation requirement test in the coming school year. After collecting feedback from my colleagues and CEMC (The Centre for Education in Mathematics and Computing) workshop participants last summer, I have been adjusting some activities and filling in gaps. Furthermore, recent discussions with my young adult children and my teenage nieces made it clear that the need for learning about how to manage their finances extends beyond just taxes, tips, and interest. This column attempts to consider some financial literacy topics like budgeting, debt, and borrowing, once again utilizing some elements of the Amplify Activity Builder platform, and including some of the new available resources.

RBC's *Talking Money with Our Kids*, released in November 2025, shows that it is not just my family seeking help to explain financial literacy elements: think credit card fees or household expenditures. The report notes that for parents of teens and young adults (ages 13–24), 43 percent of parents surveyed "reported they're not using any resources—or don't know what's available—to assist them in helping their children prepare for financial independence." A related RBC/Environics poll, *The Cost of Keeping Up*, surveyed people belonging to Gen Z in September 2025, concerning how they felt about their real-life and social media money mindset.

From *The Cost of Keeping Up*, Figure 1 shows that young people are concerned about their financial independence.

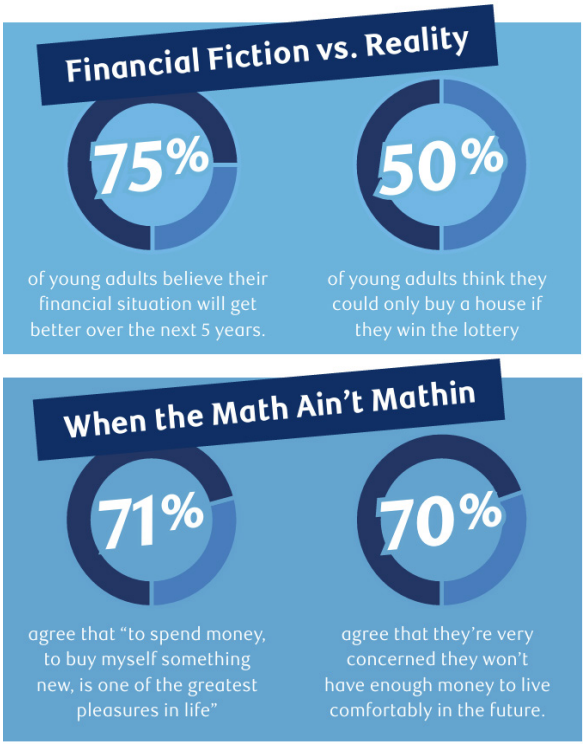


Figure 1: *Data from RBC/Environics report,* The Cost of Keeping Up*, responses from a sample of Gen Z people*

In my December 2025 column, I shared the checklist I created of the Ontario Financial Literacy, Grades 1–9 curriculum expectations (bit.ly/4oDc49W). All expectations in this column are math curriculum expectations, though the titles are categorizations from the checklist. One such expectation from Grade 7 is:

> **Financial Management:** Identify various societal and personal factors that may influence financial decision making, and describe the effects that each might have.

Expectations such as this one can be less familiar for math teachers to address. One strategy can be to incorporate Social–Emotional Learning strategies and visible thinking tools, such as Project Zero's *Thinking Routines*, especially those in the *Perspective Taking Category* (Harvard Graduate School of Education, 2022). Using the statistics in Figure 1 with the *Step Inside Thinking Routine*, students can "step inside" the role of the person who would agree with each of these statements. Students then respond to the prompts:

What might this person believe?

What might this person care deeply about?

What might this person wonder about?

Using these thinking routines can help students to identify personal and societal factors that influence financial decisions through different lenses. Perspective taking with this set of statistics, for example, can prompt students to

think about financial decision making with respect to their own futures.

With half of the respondents in the RBC/Environics poll worrying about buying a house, I reviewed the perspective about inflation that I presented in my previous column (March 2026). There I shared some Amplify Activity Builder slides, adapted from the Barenaked Ladies song and the Bank of Canada activity exploring inflation since 1990, "If I had a million dollars… I'd be reasonably well off." You can find the student version at bit.ly/MathFinanceStudent, and the teacher version at bit.ly/MathFinanceTeacher. The scatter plot shown in Figure 2 illustrates how much Canadian house prices have increased since the song was written. How might students interpret this graph with a perspective-taking routine?

When I presented a different version of this scatter plot in the last column, I proposed using it to find the average rate of growth per year, having students connect slope and average rate of change financially. In conversations with teachers this year, they have reinforced my opinion that financial literacy learning has to happen like this, while covering other content strands.

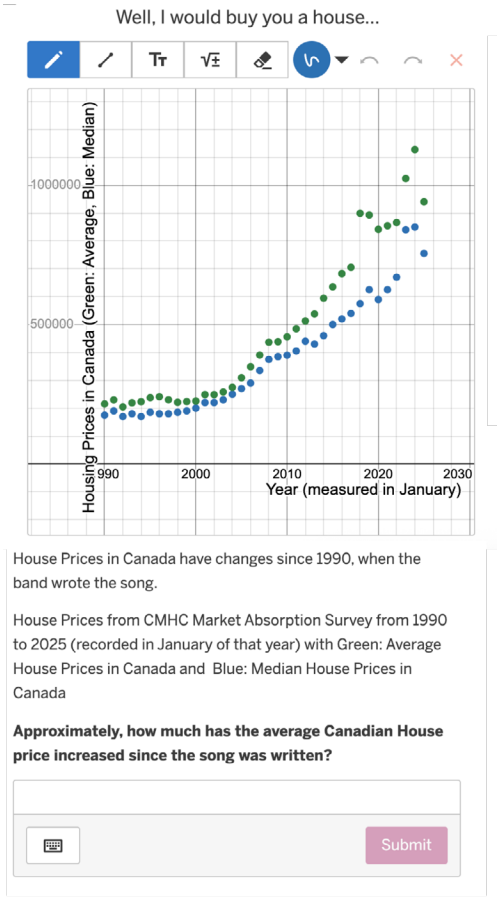


Figure 2: *House prices from Canadian Mortgage and Housing Corporation (CMHC) Market Absorption Survey in Amplify Activity Builder*

Asking middle school students to interpret financial graphs from popular media provides a similar opportunity. Last year at the CEMC Conference, I presented an article from the CBC with dynamic data presentation, allowing the reader to compare household spending and hourly wages, such as shown in Figure 3 (Lam, 2025). As teachers at the conference explored different views of the graph, they discussed the effect of different variables on median hourly wages. A classroom approach to use here, for financial perspective taking with these different variables, can be Harvard Project Zero's *See Think Wonder*:

- What do you **see**?
- What do you **think** is going on (or what do you **think** about that)?
- What does it make you **wonder**?

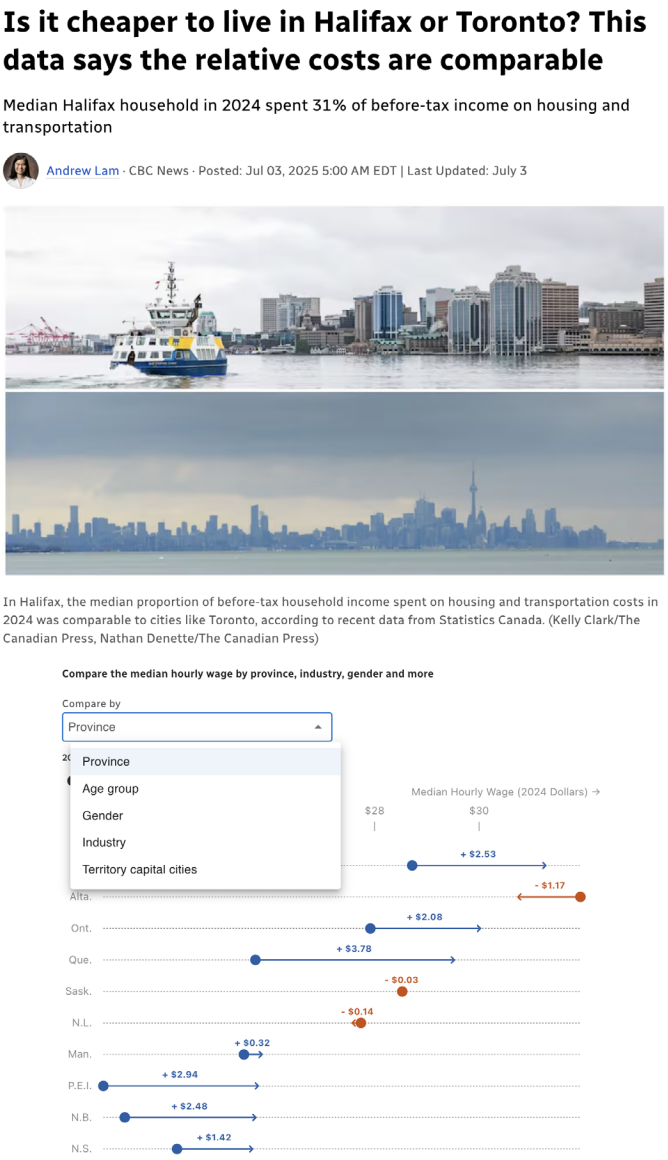


Figure 3: *CBC July 2025 article connecting household spending and hourly wages and gender*

Jenna Laib's excellent work with Slow Reveal Graphs provides a further opportunity to understand graphical representations in context; she has several examples that address financial literacy. Laib (2023) notes that "(the) Slow Reveal Graph instructional routine promotes cross-curricular sense making about data. As more and more of the graph is revealed, students refine their interpretation and construct meaning, often in surprising ways."

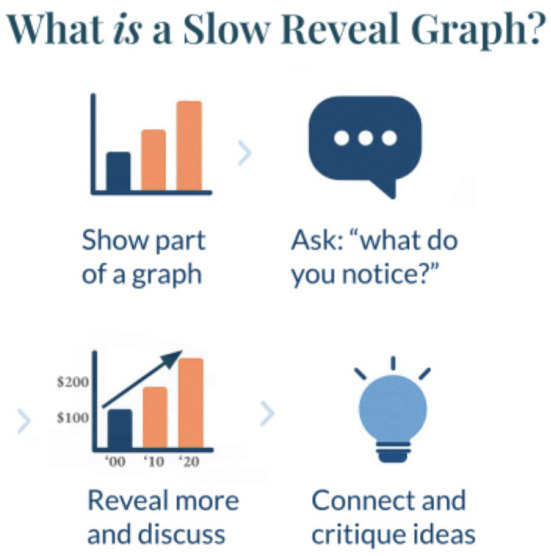


Figure 4: *Slow Reveal Graph instructional routine steps*

Laib's website, slowrevealgraphs.com, shows the steps (Figure 4) used in a classroom, which a teacher can adapt by making a copy of Laib's Google Slides deck. Laib has powerful three-minute videos modelling her use of Google Slides. Slow Reveal Graphs could also be adapted into Amplify Activity Builder, but this requires the *pace, pause,* and *sync to me* features, which may take some practice and finesse for classroom facilitation.

Figure 5 shows the first and last slides from the slow reveal slide deck "The income you need to purchase a home in Canada," which includes support materials for teachers. Sample supports include the following questions as you move through the steps (Laib, 2023):

- Estimate the *x*-value for several points.
- What scale do you think is being used along the *y*-axis? Why?
- How much would you need to buy a typical home in Vancouver?
- How much would that typical home be worth?

The Bank of Canada Museum (2024) uses a similar approach in their activity illustrating types of debt in the activity *Decoding Canadian Economic Data*. As part of the activity's pre-work, Figure 6 asks students individually to consider the following questions:

- What age group would you assign to each stack of coins (bar)?
- What would you use as a title for this chart?

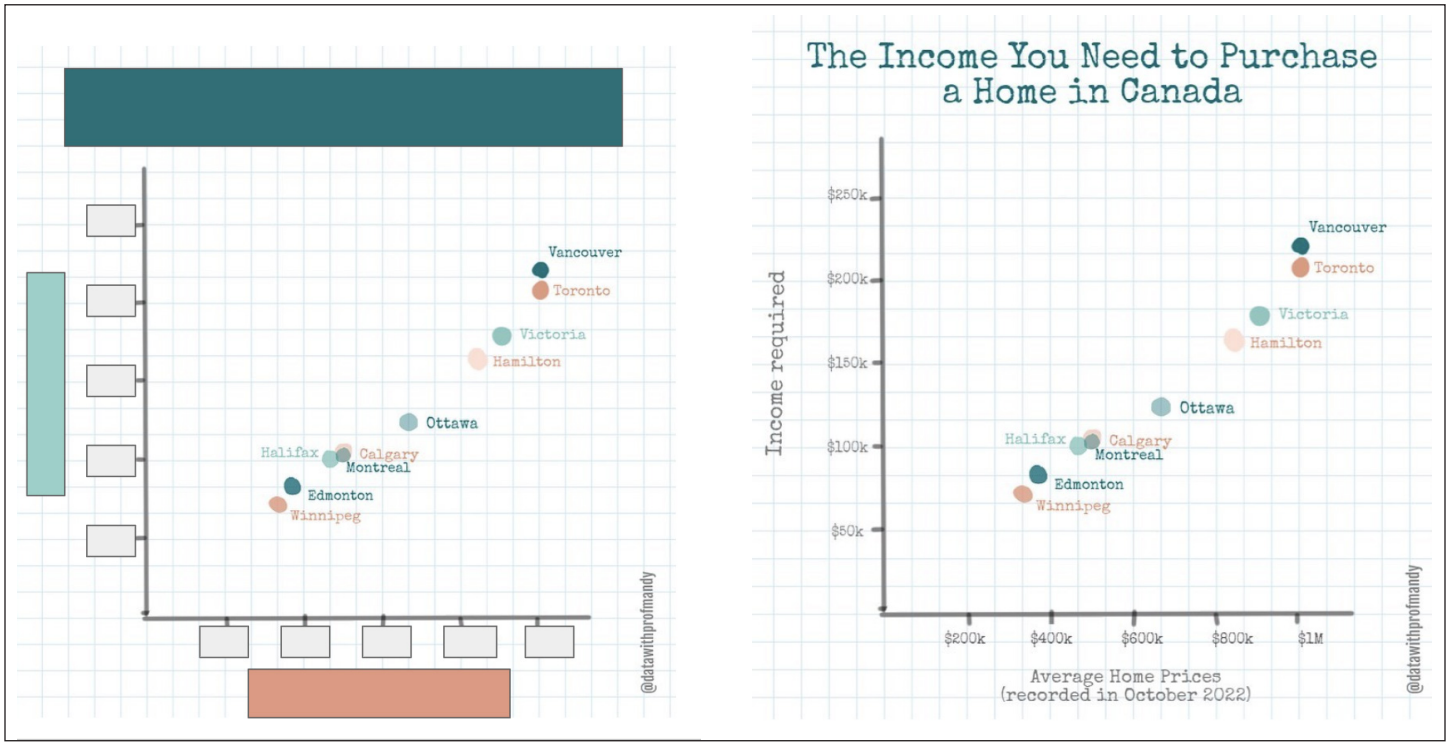


Figure 5: *Starting and Ending slides of Slow Reveal Graph about Canadian home prices*



Figure 6: *Pre-work Bank of Canada activity asking students to create the age categories and graph title*

Many teachers have explained that they felt more confident dealing with interest in the context of percentage calculation problems, and that this addresses this expectation:

> **Consumer and Civic Awareness:** Explain how interest rates can impact savings, investments, and the cost of borrowing to pay for goods and services over time.

Extending this Bank of Canada activity can incorporate thinking and perspective-taking questions beyond just calculating with questions, for example:

- Mathematically, how would higher interest rates affect these values and this graph?
- What type of debt contributes the most to the overall Canadian debt?
- Which type do you think has the highest interest rate? Why?

The Explainer (Bank of Canada Museum, 2024) component of the activity notes states:

> The data in this chart do not include mortgage debt, which would increase the amounts even more. In 2023, around two-thirds of Canadians owned their homes, with many in their 30s and 40s owing around half a million dollars for them on top of other debt. Canadians have a significant amount of household debt, more than most comparable countries.

Once they have completed and corrected their pre-work answers, students are asked to take on perspectives by engaging in a larger class discussion or a written analysis.

- What are some possible risks for the Canadian economy if older Canadians hold more debt?

The Answer Guide (Bank of Canada, 2024) presents the following response to this question. (When you read this, consider what possible strategies you might give your students to encourage perspective taking?)

> Owning a home doesn't mean just having shelter at a reliable cost—it's also an investment. Older Canadians have fewer working years—and fewer years of life—than younger Canadians for making significant payments. Traditionally, home ownership has been higher in older age groups, and they can use their homes as collateral for other debts. However, if upcoming generations of aging Canadians are less likely to own homes, or have enough other forms of debt collateral, it may be harder for them to borrow at a low interest rate. Worse, they may default on their loans.

TVO has prepared a series of financial literacy practice materials for both teachers and students. It includes a specific section, "Graduation requirement," as a curated collection of their activities. These include *Choosing the Right Credit Card*, which includes the table presented in Figure 7.

1. What do you notice about each?
2. Who might be the target audience?
3. What are the incentives and fees attached to each type of card?

Financial Institution	Royal Bank of Canada	TD Canada Trust	Scotiabank
Card type	World Elite Mastercard	Platinum Visa	Gold American Express
Interest rate	19.99%	19.99%	19.99%
Annual fee	$0	$89	$129
Minimum credit limit	N/A	$2,000	$5,000
Features	Get an annual World Elite round-trip companion voucher starting at $119 CAD (plus taxes, fees, and other ATC charges). Get up to 450 WestJet dollars – 250 welcome WestJet dollars plus earn an additional 200 WestJet dollars when you spend $5,000 in the first 3 months.	Earn 3 TD Rewards Points for every $1 in grocery purchases and regularly recurring bill payments. Earn 5 TD Rewards Points for every $1 when you book travel online through Expedia. Earn 2 TD Rewards Points for every $1 you spend on all other purchases.	Earn 5 times Scotia Rewards points for every $1 CAD spent on eligible grocery stores, restaurants, fast food, and drinking establishments - includes popular food delivery and food subscriptions.
Minimum income	$80,000	none	$12,000

Figure 7: *TVO Learn details about different kinds of credit cards with incentives*

This example can inspire mathematical comparisons, including systems of equations. However, financial literacy often requires a more complicated set of variables. Further

perspective-taking questions could be:

- Usually, the credit card company asks for a minimum payment each month, which can be a flat rate of $10 or a percentage of the amount that you owe. Is it advantageous to you, or the credit card company, if you don't pay the full amount owed? Explain your answer.
- Which of the following student credit cards (Figure 8) would you recommend as a first credit card, and why?
- What would you have to consider before making a decision?

In preparation for a test-like situation, TVO Learn provides a student self-check, sorting credit card features into cost and incentive, which can help to address the expectation:

> **Consumer and Civic Awareness:** Compare various ways for consumers to get more value for their money when spending, including taking advantage of sales and customer loyalty and incentive programs, and determine the best choice for different scenarios.

RateHub.ca (Owainati, 2026) provides a comparison of student credit cards (Figure 9) and highlights that the best credit card choice depends on the different scenarios, as the curriculum suggests. As an in-class activity, each student can review a different credit card and then explain why RateHub.ca made their different recommendations. Do they agree with the rating?

Best credit cards for students — by category.

Student credit card	Best for	Annual fee
BMO CashBack Mastercard – student	Cash back	$0
PC Financial Mastercard	Groceries	$0
KOHO Essential	Credit building	$48
TD Rewards Visa	Travel	$0
Scotia Scene+ Visa for students	Entertainment	$0
Rogers Red Mastercard	Phone and Internet bills	$0

Figure 8: *RateHub.ca's rating for best student credit cards*

Additional resources are now available for broader financial literacy for both teens, and adults of all ages. The Canadian Foundation for Economic Education (CFEE) recently prepared FinLit101 (www.finlit101.ca/en) to provide a free program with real-life scenarios, including high-quality videos. Free enrolment for students and teachers provides a point system to mark progress. One activity series worthy of note is "The ups and downs of Marvel collectables," which could be combined with some classic percentage questions on appreciation and depreciation.

The federal government also includes further information for all Canadians to learn more about money and finances, with up-to-date information on how to manage debt and interest rates, consumer fraud protection, and United States tariffs. Student-friendly resources to be found there include independent budget calculators, mortgage and vehicle loan calculators, bank and credit card comparison tools, and a financial literacy self-assessment quiz.

The provincial government also has some financial literacy modules for student learning (Ontario Ministry of Education, 2025). These self-guided modules are intended for high school students and are divided into three distinct areas:

- Earning, spending, and budgeting
- Bank accounts and credit cards
- Borrowing and investing

This provincial content directly addresses the vocabulary and qualitative learning in the financial literacy strand. The provincial government has also funded third-party organizations, including Le Centre Franco and United for Literacy, to develop supportive resources, largely directed toward educators and parents. United for Literacy (Chan et al., 2023) provides downloadable resources for financial literacy resources for Grades 4–12, especially for students living in Northern and remote Indigenous communities. This Culturally Responsive and Relevant Pedagogy (CRRP) provides an additional perspective, by taking teacher-talk moves and journal prompts to address expectations for each grade. Lessons and activities include a downloadable PowerPoint presentation, a printable assignment with suggested answers for teachers, and academic language definitions throughout. The MTH1W "Adjusting budgets" activity contains four different assignments to reflect varied financial contexts, where students can adjust individual, household, school club, and business budget templates, in response to unexpected financial changes.

Finally, as with all of my columns oriented to financial literacy, I have been thinking about the importance of inequality, privileges, and social forces in financial decision making. I encourage you to review the financial literacy expectations, and how you teach them, in connection with your students' backgrounds and culture. I hope the resources encourage you and your students to reflect on different perspectives and grow more confident in developing financial independence.

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OOCYOA (Instructions page 6)

Either add 3 or multiply by 2 and add 1.
Go to "OOCYOA" on page 42.

▲ What's the Problem? Tricky Tilings



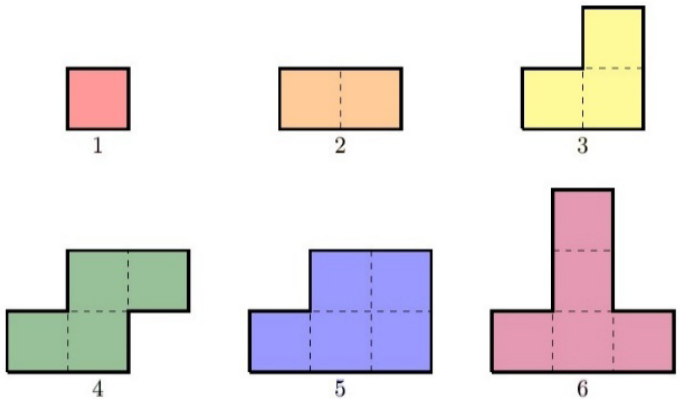
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Shawn is a former mathematics teacher, department head, and consultant. He strongly believes in the central role of problem solving in the mathematics classroom. He continues to be involved in mathematical activities: presenting workshops, writing articles, working on local projects, and helping create mathematics contests.

Welcome back, problem solvers. In this issue, we will revisit something we saw, in passing, in an earlier column (Godin, 2024).

Mental Blocks

Put together three of these pieces to make a square. You may rotate pieces, but not flip them to make their mirror images.



This was the *other* problem from Editor Tim Sibbald's puzzle calendar, besides the one featured in Godin (2024).

This is a cute little problem. Students could cut shapes out and try to piece them together, or try to draw a square made of these shapes on graph paper. However, the areas of the pieces are 1, 2, 3, 4, 5, and 5. Thus, the largest area we can make with three shapes is 4 + 5 + 5 = 14, while the smallest is 6. The *only* perfect square in that range is 9.

So, we could play with our shapes, trying to make a 3 × 3 square, or go one step further and look at the possible combinations of three areas that will yield 9, which are 5, 3, 1 and 4, 3, 2. Looking at shape 6, the only shapes that will fit inside a 3 × 3 square with it are shapes 1 and 2, so it

cannot be used. However, if we were allowed to use multiple copies of a shape, a 6 with two 2's would work. This perhaps gives us a hint of what we can do with this problem after we solve it.

Returning to the other possibilities, we see that pieces 5, 3, and 1 can be used, as shown in Figure 1 on the left. However, to use piece 4, which is three units wide, both "outer" edges (with length 1) must touch parallel sides of the 3 × 3 square. As well, one of the sides with length 2 must coincide with the outside of the 3 × 3 square. This leaves a 1 × 1 hole that cannot be filled with either piece 2 or piece 3, as seen on the right in Figure 1. Thus, there is no solution with pieces 4, 3, and 2.

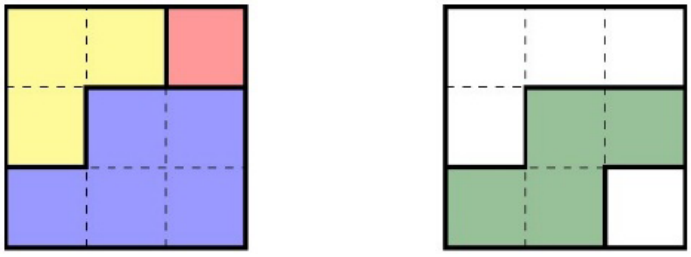


Figure 1: *A solution with pieces 5, 3, and 1, and proof of no solution with pieces 4, 3, and 2*

So where can we go with this problem? One place is to re-examine the part of the problem statement, which states: "*You may rotate pieces, but not flip them to make their mirror images.*" What restrictions does this place on our problem? If we remove that condition, does it add solutions?

Since all pieces are made by gluing together 1 × 1 squares along their edges, we can assume, without loss of generality, that all sides will be either horizontal or vertical. Hence, any rotations will be of multiples of a right angle. Piece 1 is then trivial, in that rotations (or reflections) leave the shape "unchanged," since the shape has order 4 rotational symmetry and order 4 reflectional symmetry. Piece 2 will be either horizontal or vertical. Each of these states has order 2 rotational symmetry and order 2 reflectional symmetry.

Piece 3 might not be as obvious. However, if we look at the four rotational states, we notice that reflecting any one of them in a horizontal or vertical line then yields another of them. That is because piece 3 has one axis of symmetry, so again, reflection does not add anything.

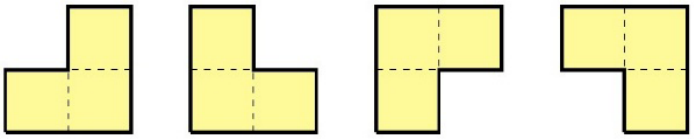


Figure 2: *Piece 3 under rotation*

Piece 6 behaves similarly to piece 3, in that it has one axis of symmetry, so reflections do not add anything new. However, for both of pieces 4 and 5, there are orientations that we can only reach by reflection. In Figure 3, we can see the two rotational states of piece 4 (it has order 2 rotational symmetry), as well as the two orientations that can only be reached by reflection (unshaded in the diagram).

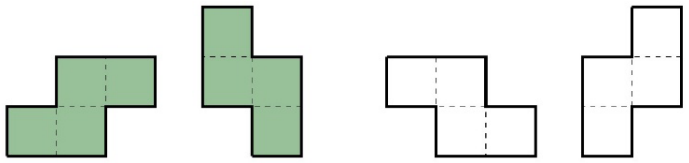


Figure 3: *Piece 4 under rotations and reflections*

The same thing happens with piece 5. However, because of its shape, we can handle the analysis in a different way, more easily. Looking at piece 5, it consists of a 2 × 2 square with a square attached to the outside. Looking at a 2 × 2 square, we see that there are eight places where we can attach the smaller square. However, using just rotations of piece 5, we can only generate four of them. The other four could only be produced if reflections were allowed.

Does any of this help us with the original problem? Not really, since adding in reflections doesn't add any new solutions to the problem. However, moving forward, if we want to extend the problem, we might want to allow the reflections (or not), to allow us to make the most shapes possible.

So how do we extend the problem? We hinted at an idea earlier: we can allow pieces to be used multiple times. If we also drop the necessity of three pieces, we might want to investigate how we can tile squares of different sizes with the given shapes.

Trivially, n^2 copies of piece 1 can be used to make any $n \times n$ square. Similarly, since two copies of piece 2 can be used to make a 2 × 2 square, then n^2 copies of that 2 × 2 square can be used to make any $(2n) \times (2n)$ square. Some of these trivial squares are shown in Figure 4.

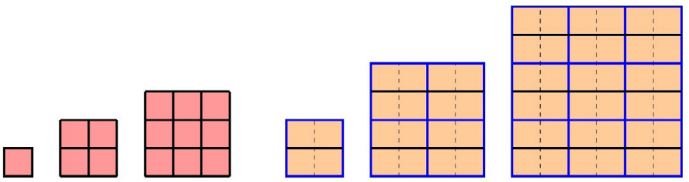


Figure 4: *Squares that can be made from just piece 1 or piece 2*

Piece 1 is very versatile; it can be used to "fill holes" left by other pieces. Piece 2, being made up of two squares, cannot be used to solely tile an $n \times n$ square when n is odd. However, it can *always* be tiled using one copy of piece 1

and the rest filled with piece 2. It may seem obvious, but take a moment to see if you can justify this claim.

Suppose we have an $n \times n$ square when $n > 1$ is odd. Then we can write $n = 2k + 1$ for some whole number k. Now, cut a $(2k) \times (2k)$ square from the corner of the original square. Since its dimensions are even, it can be tiled with piece 2. What are we left with? We are left with a big skinny shape that looks like an "L," similar to piece 3, as shown on the left in Figure 5. We can slice the "L" into two $(2k) \times 1$ rectangles, which can each be tiled with k copies of piece 2, and a lone 1×1 square, that can be filled with a single piece 1, as seen on the right in Figure 5.

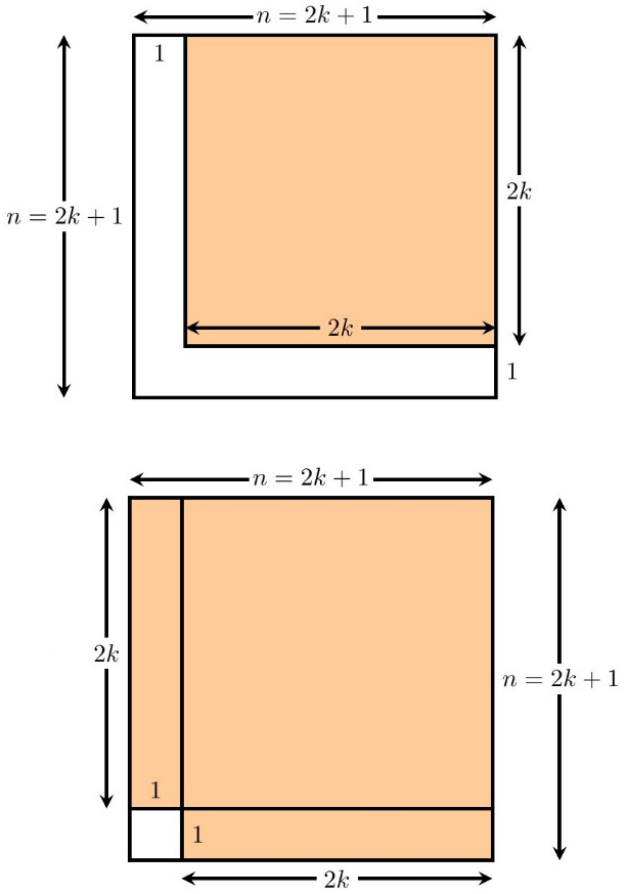


Figure 5: *Tiling an* n × n *square, where* n *is odd, with one piece 1 and the rest piece 2*

If we are careful, we can place a single piece 1 in a little more than half of the allowed positions. To illustrate, we will examine a 9 × 9 board, shown in Figure 6. Notice that we have shaded the squares in a pattern like a checkerboard. We claim that the shaded squares are the possible positions for piece 1. Using the added coordinate system, note that if we look at the bottom left corner of any of the red-shaded squares, the x and y coordinates have the same parity. This makes sense, as the bottom left corner, (0,0), has even coordinates. If we move just horizontally or vertically, since we have the checkerboard pattern, we must move an even number of spaces, which preserves the parity of the coordinates. However, if we want to move to the next row or column, we must move 1 unit in each direction to get to another shaded square. Thus, the parities of both coordinates are changed, keeping them identical to each other. This also means that the distances from the sides of piece 1 to the closest edge of the big square all have the same parity.

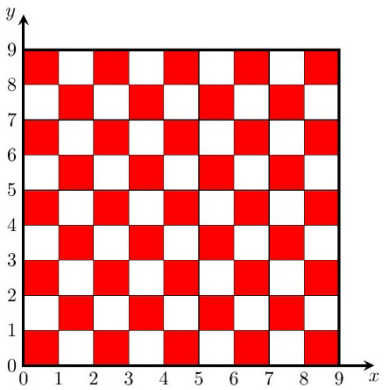


Figure 6: *Possible positions of piece 1*

Suppose we want to place piece 1 so that its bottom left corner has even coordinates, as shown in Figure 7. If we extend the sides of piece 1, it breaks the checkerboard into rectangles with dimensions of even length. Since piece 2 is a 1×2 rectangle, we can use it to cover any rectangle with at least one even dimension by stacking the pieces so that the sides of length 2 are parallel to the side with even length. Hence, all the rectangular pieces of the checkerboard can be covered by copies of piece 2, so the whole checkerboard can be covered by a single piece 1 and a number of piece 2's.

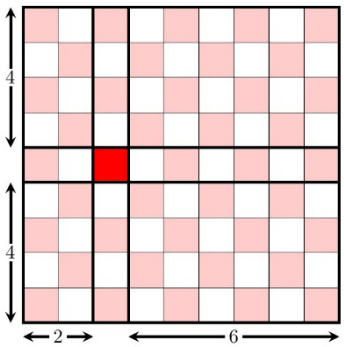


Figure 7: *When piece 1 is an even distance from all sides*

If the bottom left corner of piece 1 has odd coordinates, as in Figure 8, we need to make an adjustment. If we extend the sides, as we did with the even case, we are left with rectangles with odd dimensions, shown on the left in Figure 8, which cannot be covered with piece 2. However, if

we alternately remove four of these borders, combining the four "strips" to adjacent rectangles, we end up with four rectangles with one even dimension, and one odd dimension, shown on the right in Figure 8, which can be covered by piece 2 tiles. Therefore, any square with odd dimensions can be covered by one piece 1 (on one of the shaded squares) and a number of piece 2 tiles.

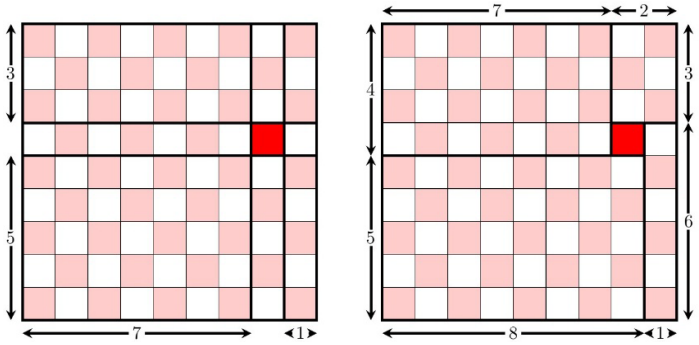


Figure 8: *When piece 1 is an odd distance from all sides*

It is interesting to note that if piece 1 is placed on one of the non-shaded squares, the rest of the figure cannot be covered by copies of piece 2. The easiest way to see this is by noting that piece 2, no matter where it is placed, will cover one shaded and one unshaded square. If we are dealing with a square with an odd side length, there are an odd number of squares, which means there is not the same number of shaded and unshaded squares. It turns out that there will always be one more shaded square, so if piece 1 is placed on a shaded square, there will be equal number of shaded and unshaded squares left, which can then be covered by a collection of piece 2's. However, if piece 1 is on a non-shaded square, there will now be two more shaded squares than unshaded squares. This means, at best, that we will be able to cover all but two squares with a collection of piece 2's. This is related to the well-known puzzle that asks you to cover an 8 × 8 checkerboard, where two opposite corners are removed with 31 dominoes, each of which cover 2 squares. This task turns out to be impossible. I will leave it to the reader to convince themselves that this is true.

Moving on to the other tiles to see what they can cover becomes even more complicated. Those who are up to the challenge can check out question 9 from the 2018 Euclid contest, which deals with rectangles that can be tiled using only piece 3. You can find this contest, and more, on the CEMC website at www.cemc.uwaterloo.ca/.

What else can we do with this problem? All the pieces in the original problem are examples of *polyominoes,* figures that can be created by affixing unit squares along their edges. The term was coined by mathematician Solomon W. Golomb, who wrote a great treatise on them (Golomb, 1994). Pieces 1 and 2 are the only monominal and domino, respectively. There are two triominoes: piece 3 and a line of three squares. Piece 4 is one of five tetrominoes: can you find the other four? Finally, pieces 5 and 6 are pentominoes, a term you may have heard before. Many schools probably have sets of pentominoes for students to work with.

There are many directions you could go with students. For any given number of squares, how many "different" polyominoes of the given size can be made. We can treat different orientations as different polynomials, while getting students to group similar ones together (leading to discussions of symmetry and transformations). Or we could treat the different orientations as the same object, but ask how many different orientations each piece has. We could have students create their own puzzles, like the original, that is, come up with a set of pieces, polyominoes, so that a square (or multiple squares) can be made. There are many resources for polyominoes, so there are plenty of challenges to be had. For example, one such challenge is to make "copies" of a polyomino, using only that type of tile. Figure 9 shows the right triomino, piece 3, used to make some larger copies of itself.

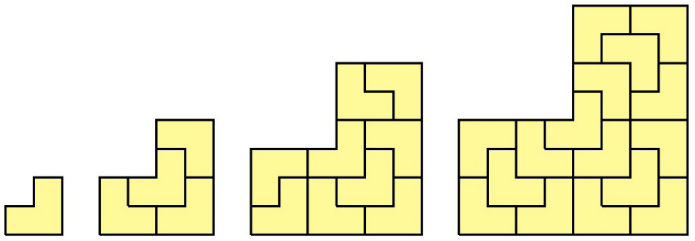


Figure 9: *Using piece 3 to make larger copies of itself*

Have fun exploring polyominoes!

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Calculus inspired furniture?



Musing on Percentages: Some Historical Notes About the % Symbol



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Percent is everywhere—all around us in finance, shops, foods, media, surveys, and so on. Percentages may seem like a rather simple mathematical theme that has little depth. However, when one starts to pay attention to its usage, percentages reveal surprising elements that are mathematically rich and conceptually fascinating, hence worth pondering. This is something that became quite salient for our research team, as we have been preparing experiments with students in classrooms about percentages in mental mathematical environments.

These columns endeavour to raise and address some of these peculiarities, which we came to call "curiosities," related to the concept of percentages. These musings are about our usage of "%" in everyday life, in schools, or in mathematics in general. This column continues the previous columns, and addresses the not-so-obvious history of ways in which the % symbol came about.

Preliminary Set-up

A first thing to say about the % symbol is that, although quite useful for remembering it, there is no historical links between the form of the % symbol—with its bar and two circles—and the bar and the two 0's in the number 100. What follows endeavours to better understand how this % symbol came about, though the history is sufficiently complex to require more than this column to be comprehensive. As such, although inspired from and combining numerous sources and documents, this column should be interpreted as an informative invitation to immerse oneself in one "story" about the % symbol. This said, I certainly believe that it is worth the adventure! (See the reference list for sources.)

Initial Origins

The notion of % appears to have its source in merchants' practices of taxation. These taxation practices did not, however, initially refer to 100, but mainly to various ratios. A taxation practice from authorities could be "for each haystack that you produce, one haystack needs to be given" or "two haystacks per five produced," and so on, depending on the produce (fruits, metals, etc.). All these taxes were grounded in an idea of establishing ratios and calculating proportions. Slowly, using 100 in these ratios became established.

This transition to 100 is not well detailed in historical accounts, meaning that it is not clear why 100 was chosen as a basis of comparison in ratios. Numerous hypotheses exist. Some of these hypotheses are linked to our base-10 numerical system, but it is not clear why 100 was the precise number targeted. Some associate this decision to the emergence of systems of money, which often uses cents, dollars, etc. However, many of these systems have emerged after the notion of %, making this justification a little unclear. Other hypotheses relate to ancient practices, for example, the Roman taxations of 1/100 on commercial trade. Finally, some, like Baruk (1995), assert simply that: taking for granted our base-10 decimal system, it is the "reasonable" character of 100 that made it a privileged choice, where 100 is seen as not too small, like 10 would be, and not too big, like 1000. Hence, to this day, the question about the reason for having opted for 100 seems debatable, since other numbers such as 12, 50, or 60 could easily have fulfilled similar criteria, while at the same time, having some advantages because of their connections with hours, divisibility patterns, social habits, and so on.

Whatever the historical reason that explains the place of 100 in %, its usage in calculation of taxes and incomes was mainly practical, a way of doing. In other words, percent as a mathematical concept did not exist at this time, despite there being references to 100 in calculations. In that sense, the idea of % was not something people could refer to and was simply a calculational habit. As such, without a clear concept, no specific symbolizing practices existed for it. This is one reason why its historical genesis is difficult to pin down. However, it is through these calculational habits of using 100 in ratios, and the willingness to disseminate this practice and its techniques more widely (e.g., in merchant books), that things began to evolve.

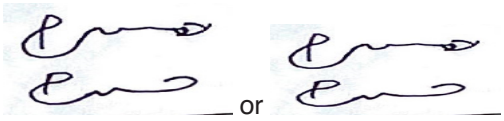
Italian Manuscripts

Numerous Italian manuscripts linked to commerce, dating back to 1400, are considered to be the first to contain traces of some sort of symbolization that might have led to our modern % sign. Consider, however, that these manuscripts were often written or annotated by hand. This detail appears to have played a significant role at the symbolic level. Indeed, the constant repetition of the same words can lead one to some sort of hand fatigue or a willingness to go "calligraphically" faster, as shorthand did. In return, this carelessness can provoke reduction of quality on the writing, or simply to the development of abbreviations.

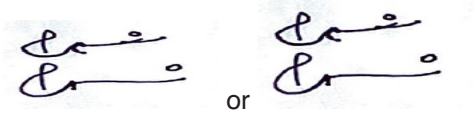
The expression "for 100" in Italian is read *per 100* or *per cento*. The *per* was often handwritten in an abbreviated way with a suffix of capital P, separating the number from the 100. This gave, for example, "*7 P 100*" or "*7 P cento*." This *cento* was also at times reduced to some sort of "c," with an elongated stem ending in a curve to represent the final "o" of *cento*. This is reminiscent of our frequent ways of ending a word written in cursive with a tail to go faster; a tail that somehow adapts to abbreviate the word. For example, in a grocery list, as a francophone, if I often have to write the word "légumes," it might end up like this:

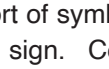


Here, the context of groceries helps remind one of what is written and its meaning, whereas taken alone, these scribbles of the word "légumes" might be quite difficult to decipher. Hence, in a similar way, the *cento*, when rapidly written and joined with the capital "P," can give these sorts of results (author-developed examples):



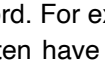
Taken in a merchant book and repeated many times, both on the same page and paragraph, this cursive expression does not cause any problem. Through repetition and sometimes slight modifications, as shown above, these writing habits became more familiar. It is possible to see, in some Italian books, that the curve at the end of the word *cento* slowly becomes, as pages go by, a little detached from the tail and is elevated a little above it, looking like this:



This small dot on the right also started to be linked to the usual Italian way of ordering numbers, from the first (1^{o} or 1^{o}), to the second (2^{o} or 2^{o}), to the third (3^{o} or 3^{o}), etc. In that way, *cento* could become c^{o}. Writing *per cento* became of the form  or . And, when it was to be written at the end of a line, in the margins, it could also be condensed and presented as .

The writing practices were obviously varied and have evolved, and the capital "P" in *per* appears to also have taken different forms from one manuscript to the next, to the point of having been left aside at times. This ended up, occasionally, that merely a reference to *cento* was maintained to indicate the procedure to follow. And, of course, through the handwriting, the form of  was diverse and different in height and in length from one transcription to another, where the curve of the initial "c" in *cento* could be in close proximity with the small circle (which, as explained, could be seen as much as representing the ordering in 1st, 2nd, 3rd, etc., as the "o" in *cento*).

Numerous Transformations

The growing presence of a symbol like  from one manuscript to another also implicitly enabled people to make reference to the associated procedure of "per 100." Slowly, but surely, the concept became established and different manuscripts referred to it explicitly: it was not sudden or surprising to see it appear, since this symbol became more widely used by merchants.

The symbols , c^{o}, or c^{o}, taken in themselves, do not permit to tracing their origin with *cento* and *per*, nor to the rationale explaining the addition of the small circle to determine order or the end of *cento*. This rupture from its origin opened up, again because of handwriting, to what could be called productive modifications, adjustments, and contractions.

Around 1650, the symbols , c^{o}, and c^{o} were slowly transformed, preserving mostly a lateral bar and two "curves," which at times, even became transformed themselves into two simple dots. From  and , the use of symbols like  and  started to implicitly take place. As mentioned, the "P" of *per* is sometimes present, sometimes hard to read, or is even simply absent, where some sort of fusion between the "P" and the curve of the "c" appears to take place, leading to symbols like , , , or . There is thus an oscillation between the small "o" or circle and the dots to represent each of the two circles above and below the bar. By setting aside the "P" of *per*, the symbol started to be used more frequently for representing the percentage calculations, for example, with . And, this bar was somehow made oblique, leading

to our known % symbol, with writing 7%.

Standardization and Solidification of the Concept

This new way of writing, that is, the $\frac{\circ}{\circ}$, but mostly the actual %, might also have been reinforced by its links with the fraction bar in *a*/*b*, which was by then also conceived as representing a division. This link with the writing of fractional values might have played an important role in the standardization of % in writings and communications, where many elements contributed to its solidification. For example, writing fractions in the vertical form, as in $\frac{a}{b}$, was causing technical problems in transcriptions because it took more than one single line and was overflowing onto the line below it. This form of writing fractions was rapidly substituted by the *a*/*b* form, which could be written on a single line, without modifying the rest of the lines below. This became an incentive to go from $\frac{\circ}{\circ}$ to %. As well, the advent of the % symbol coincides with the years of Leibniz's quest to establish more standardized mathematical symbolizations in mathematical texts, particularly around fractions, divisions, ratios, and proportions (see Cajori, 2013, pp. 180–195). The symbol % was well aligned (no pun intended), in its form at least, with the much sought-after way of writing mathematical symbols.

As the symbol % was getting established in different texts, the usual association of its bar and two zeros with the number 100 did not go unnoticed. This link had the effect of strengthening its relevance for representing percentages, but also to guide the creation of other symbolics. Thus, by way of analogy with %, the symbol ‰ was created to represent the ratio "per mil," with the addition of a "third" zero, as found in 1000. As much as the symbol ‰ took its source from the percentage symbol %, it also reinforced the relevance of % to represent percentage with its visual association with 100. To this, other ratios such as "parts per million," "parts per billion," "parts per trillion," etc., have had their source in the idea of percent, rather than in its symbolism: ppm, ppb, ppt, etc. Here again, there is an influence of percentage that participates in giving rise to additional concepts, while in turn, these concepts also solidified the relevance of percentage as a concept. It is therefore in that sense that the slow generation and evolution of the symbol % for representing percentages contributed to the solidification of percentage as a mathematical concept, be it by itself or through the generation of other related concepts.

Final Remarks

So, this is a brief story about the symbol %, quite compressed and synthesized through a few words; a little like how the % symbol came about! A story like this shows how the emergence and development of specific mathematical symbolisms are far from being linear, and also how it plays a fundamental role in the emergence and development of mathematical concepts themselves (the very ones that it aims to represent). Form and content are quite aligned!

Again, I am always interested in your comments, so feel free to send them to proulx.jerome@uqam.ca or jerome.proulx@kcl.ac.uk, whether they are word-processed or hand scribbles!

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Continued from page 25

strategy. Teachers need to be aware of how students are doing in terms of self-questioning so that they can provide them with targeted feedback to further develop this strategy.

In the next issue of the *Gazette*, we will take a deeper dive into applying self-questioning as a strategy to strengthen metacognitive regulation. All four stages of problem solving will be explored, and there will be numerous self-questioning prompts provided to immediately use in the classroom.

I look forward to continuing the discussion of mathematical learning and instruction via my website (www.costellomath.com), email (david@costellomath.com), or X (@dr_costello).

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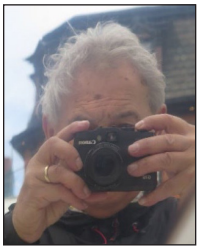
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The Math Teacher's Home Base: The Legacy of Martin Gardner



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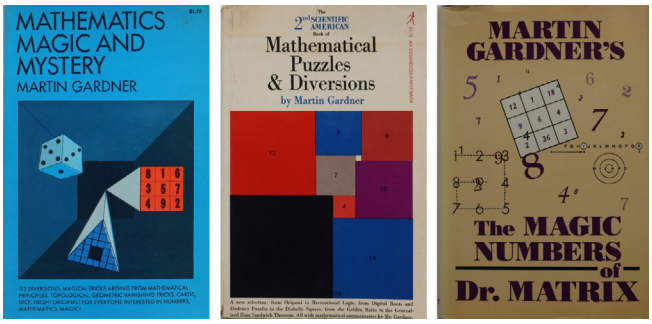
A Summer of Discovery

My home office—the "Toy Office"—is more than just a collection of objects; it is a library of influences. At the very heart of this space is a deep row of books by Martin Gardner. While my fascination with recreational mathematics began in university, it was Gardner's famous "Mathematical Games" columns in *Scientific American* that truly shaped my pedagogical DNA.

I often think back to a summer when I spent time with my colleague, Cliff Lelievre, at Oakville Trafalgar High School. We were given 25 years' worth of *Scientific American* issues by our school library. We spent hours going through every single column by Gardner, making notes and organizing the material by mathematical topics. It was one of the best learning experiences of my career. It turned Gardner's body of work into a living curriculum for my classroom.

I have been fortunate to know other teachers who shared this passion. Educators like Julian D'Angela and Paul Lessard had long careers with the Hamilton Catholic District School Board, and Ken Collins taught at Charlotte Latin School (Charlotte, North Carolina) for 50 years. Sadly, Ken passed away several years ago, and Paul died last year. I continue to see Julian, who remains passionate about puzzles. These colleagues understood that math is a living thing, meant to be shared.

Today, I want to share how I have moved ideas from three specific Gardner books into my classroom.



Photograph 1: *Featured books by Martin Gardner (Photographed by Ron Lancaster)*

Mathematics, Magic and Mystery: Heath's Tappit

Throughout my high school teaching career, and now in the workshops I lead for teachers and students around the world, I have used a version of this trick to spark mathematical curiosity. The original mechanics of this effect are detailed in Gardner's *Mathematics, Magic and Mystery* (see Figure 1). While I often perform it using a set of wooden tiles I purchased years ago, I also demonstrate it using six large sheets of coloured paper spread out on a desk or the floor (see Photograph 2).

At the start of the performance, the tiles or sheets are all face down so that no one can see what is written on the other side. I turn my back while a participant turns over just one item, looks at the number written there, and returns it face down. At this stage, they do not know what is on any of the other items.

I turn around and begin tapping the face-down papers. For every tap I make, the student silently spells one letter of their chosen number (for example, if they chose "13," they would spell T-H-I-R-T-E-E-N). When they reach the last letter, they yell "Stop!," and I am always touching the paper that corresponds to their chosen number.

The Investigation

After the performance, the room usually buzzes with theories. Only then does everyone get a chance to see what is on all the cards—the numbers 88, 77, 85, 13, 16, and 49. I invite the group to look closely at these specific values to see if they notice anything unusual. I perform the trick several more times, encouraging the group to track my hand movements.

If the group is stuck, I provide a prompt: "Does it matter which papers I tap for the first few letters? When exactly does my tapping become *deliberate*?" Eventually, someone realizes that the first few taps are random, but once I reach

a certain point in the spelling, I must follow a specific sequence to ensure a "collision" with their choice on the final letter.

Generalizing the Magic

Once the secret is out, I challenge the participants to move beyond my specific set-up:

- Variable Sets: I first have the students stay with the six tiles or sheets, but challenge them to use the names of countries or students' names instead of the original numbers. How does the spelling of these new words affect the tapping sequence?
- The Universal Rule: The final task is to generalize to a set of more or less than six items. I ask the group to find a general formula or strategy: How does the magician ensure the "downward" count of the taps always matches the "upward" count of the letters in any chosen word, regardless of how many items are on the floor?

Heath's "Tappit"

An application of the tap principle to the *spelling* of numbers was made by Heath in 1925 when he marketed "Tappit." The trick employs six small tiles, each bearing a number, and each a different color. They are pictured in Fig. 53.

The tiles are placed on the table, numbered sides down. While the magician's back is turned a spectator looks at the number on one of the tiles, then shuffles them about. The performer turns around and begins tapping the tiles with a pencil. As he taps, the spectator spells the number silently to himself, calling out "Stop" when the spelling terminates.

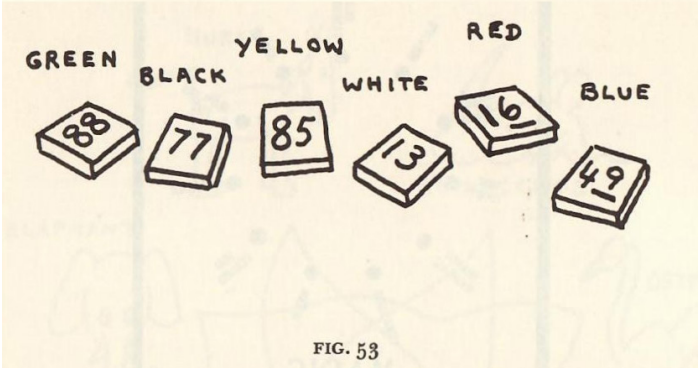


FIG. 53

The tile on which the pencil is now resting is turned over. It proves to be the tile bearing the chosen number.

Method: The first six taps are made at random. The next six touch the tiles in the following order: 16, 13, 49, 85, 88, 77. The performer is able to tap them in this order because he has memorized the corresponding color sequence. The working depends, of course, on the fact that "sixteen" spells with seven letters, and each succeeding number has one additional letter in its spelling. A set of plastic tiles, with complete instructions, can be obtained from any magic dealer.

Figure 1: *An excerpt from Martin Gardner's* Mathematics, Magic and Mystery*, detailing the "Heath's Tappit" effect*



Photograph 2: *A classroom-ready version of the Tappit trick, using six sheets of coloured paper (Photographed by Ron Lancaster)*

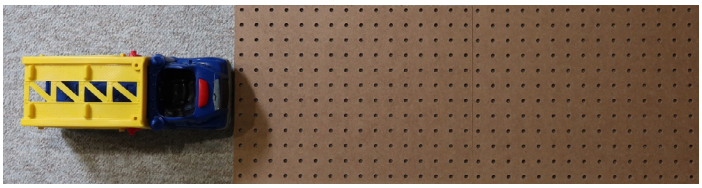
The Second Scientific American Book of Mathematical Puzzles: The Crossing the Desert Problem

One of the things I like most about many of Gardner's books is that they often contain sections filled with truly great problems. The Crossing the Desert problem (sometimes called the Jeep problem) is one of my favourites.

The Challenge

An unlimited supply of gasoline is available at the edge of a desert that is 800 miles wide. There is no source of fuel in the desert itself. Your task is to get a vehicle across, but the vehicle you will be using can only go 500 miles on a full tank of gas (see Photograph 3).

You are allowed to set up refueling stations (caches) anywhere in the desert, and these caches may be of any size. You will, of course, need to figure out a way of delivering the gas to these caches. You should find that it is, in fact, quite easy to drive across the desert. The more difficult question to resolve is this: What is the minimum amount of gasoline required to drive across the desert?



Photograph 3: *A visual representation of the Crossing the Desert problem, using a toy truck and a desert grid (Photographed by Ron Lancaster)*

The Investigation

In my classes and workshops, I always let students know that I am not initially looking for the correct or optimal solution. I want everyone to give it their best shot within the time set aside for the task. I simply ask: How much fuel did you use and what was your thinking?

This approach lets everyone know there is no pressure in trying to solve what feels like a very difficult optimization problem. It allows the students to relax and dive in. Everyone can come up with a strategy—perhaps not the most efficient one at first, but a strategy that works. This low-floor, high-ceiling entry point is vital for getting students to engage with the logistics of the problem.

Gardner's Discussion and the Connection to the Harmonic Series

The key to solving this problem effectively is to shift focus away from the total distance and instead ask: How far can I get with one full tank (or load) of gas? This is where the solution truly begins.

As detailed in Gardner's book, the strategy is built on maximizing the distance covered as more loads of fuel are added. Since one full tank of gas (one load) allows the vehicle to go 500 km, we can think of 500 km as one unit of distance.

Getting Farther with Two Loads:

Two loads can carry the truck a maximum distance of 1 and 1/3 units (666.6 km). This is done in four trips by first setting up a cache at a spot 1/3 unit from the start. The truck begins with a full load, goes to the cache, leaves 1/3 load, and returns. It then picks up another full load, arrives at the cache, and picks up the 1/3 load waiting there. The truck now has a full load, which is sufficient to take it the remaining distance of one full unit.

Getting Farther with Three Loads:

Three loads will carry the truck 1 and 1/3 plus 1/5 units (766.6 km) in a total of nine trips. The first cache is established at 1/5 unit from the start. Three trips from the start to this point allow the truck to place 6/5 loads in the cache. The truck then returns to the start, picks up the final full load, and arrives at the first cache with 4/5 load remaining in its tank. This 4/5 load, combined with the 6/5 loads already in the cache, provides the two full loads needed to carry the truck the remaining 1 and 1/3 units, as described above.

This reveals a fascinating pattern. The general formula for the distance *d* is given by the series: *d* = 500 times (1 + 1/3 + 1/5 + 1/7 + ...).

To cross an 800 km desert (1.6 units), the truck requires a little more than 3.46 loads of gasoline.

A Challenge: The Infinite Sum

As a nice extension to this problem, I often pose a challenge involving the sum:

1 + 1/2 + 1/3 + 1/4 + 1/5 + 1/6 + 1/7 + 1/8 + ...

I ask students to look at the terms in groups, as shown in Figure 2.

$$\frac{1}{3}+\frac{1}{4} > \frac{1}{4}+\frac{1}{4}=\frac{2}{4}=\frac{1}{2}$$

$$\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8} > \frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{4}{8}=\frac{1}{2}$$

$$\frac{1}{9}+\frac{1}{10}+\frac{1}{11}+\frac{1}{12}+\frac{1}{13}+\frac{1}{14}+\frac{1}{15}+\frac{1}{16} > \frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{8}{16}=\frac{1}{2}$$

Figure 2: *Grouping terms to demonstrate that the sum increases without limit by consistently exceeding 1/2*

As the diagram illustrates, 1/3 + 1/4 is greater than 1/4 + 1/4 (which is 1/2). Similarly, 1/5 + 1/6 + 1/7 + 1/8 is greater than 1/8 + 1/8 + 1/8 + 1/8 (which is also 1/2). By showing that we can keep finding groups of terms that sum to more than 1/2, students can see that the total sum must get larger and larger, without ever stopping.

This means that there is actually no limit to the width of a desert the truck can cross!

Gardner also poses a second challenge for those who want to push further: What if you need to return to the starting point? For a round trip, the distance formula shifts to using even denominators: 1/2, 1/4, 1/6, 1/8, and so on. This highlights how a small change in logistics can shift the underlying mathematical structure, while still relying on a series that grows without end.

The Magic Numbers of Dr. Matrix: The Four Fours Problem

My introduction to the Four Fours problem came from the pages of Martin Gardner's book, *The Magic Numbers of Dr. Matrix*. It has been a staple in my work for decades, and it remains a key part of the sessions I lead with students and teachers around the world today. It is also a favourite of Carly Ziniuk, who writes the "In the Middle" column for this same journal.

The Challenge

Using only four fours (4, 4, 4, and 4), create a mathematical expression that equals every number from 1 to 100.

Participants are encouraged to use a wide array of mathematical tools to reach their goals: addition, subtraction, multiplication, division, exponents, square roots, factorials, decimal points, and juxtaposition (for example, placing 4 and 4 side by side to make 44).

The Investigation

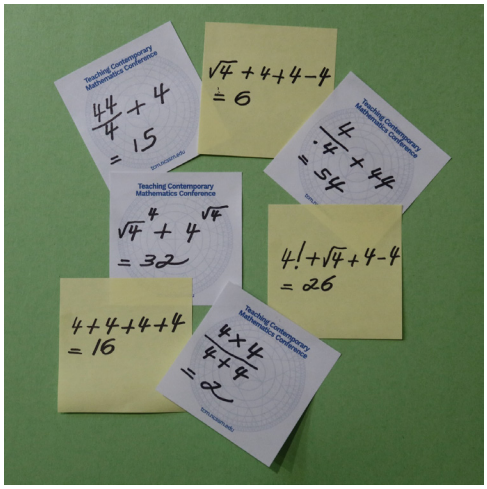
In the workshops and sessions I lead, I like to make this a highly visible, collaborative project. I have students and teachers put their successful answers on sticky notes and post them on a wall (see Photograph 4). This allows the whole group to see which numbers are being solved and

which remain elusive, turning a difficult set of puzzles into a collective achievement.

To help guide these explorations, I often provide a resource like the one shown in Figure 3. This specific guide was created by a teacher to help organize expressions and keep track of progress while working toward the goal of 100.

Mathematical Art: The Four Fours Clock

The problem also offers a wonderful opportunity to blend mathematics with art. Figure 4 shows a clock where every hour is represented by a unique "Four Fours" expression. This is a project that can easily be replicated, with students creating the artwork for their own functional or decorative clocks to show how math defines the passing of time.



Photograph 4: *A solution wall, where sticky notes are used to share mathematical expressions for the Four Fours problem (Photographed by Ron Lancaster)*



Figure 3: *A teacher-created resource used to help organize and track progress through the Four Fours challenge (Source: www.mathequalslove.net/fours-challenge-puzzle/)*

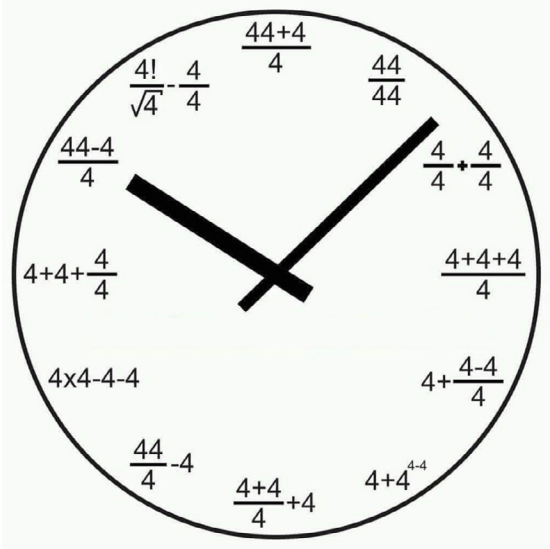


Figure 4: *A creative clock face, where each hour is represented by a mathematical expression using exactly four fours (Source: www.reddit.com/r/JhinMains/comments/cdqknl /the_perfect_clock/)*

Variations of the Challenge

Over the years, I have found that the Four Fours problem is easily adaptable, allowing it to remain a staple in my work across different contexts.

The Year Challenge

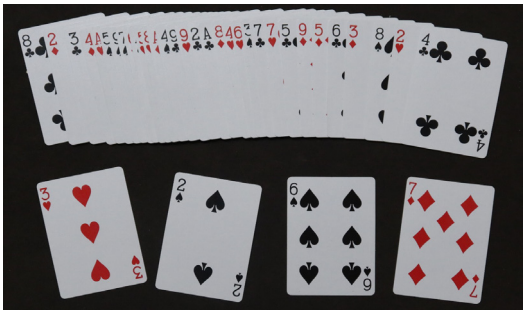
One popular variation is to replace the four fours with the digits of the current year. For example, using the digits from the year 1998 (see Photograph 5), students would use 1, 9, 9, and 8 to create their expressions. While this version became quite difficult in the early 2000's due to the prevalence of zeros, it is becoming a more accessible and varied challenge as the years progress.

The Playing Card Challenge

Alternatively, I often have students pick four cards from a standard deck, excluding the 10's and face cards (see Photograph 6). This adds an element of chance to the task, as students must work with whatever set of four digits they happen to draw. It's a wonderful way to generate a near-infinite supply of new puzzles while using the same mathematical rules.



Photograph 5: *House numbers representing the year 1998, which can be used as the four source digits for a numerical challenge (Photographed by Ron Lancaster)*



Photograph 6: *A set of four playing cards (3, 2, 6, and 7), drawn from a deck to create a randomized version of the Four Fours puzzle (Photographed by Ron Lancaster)*

The Global Legacy of Martin Gardner

As I look across the shelves of my "Toy Office" at the rows of books that have travelled with me across continents, I am reminded that the true magic of Martin Gardner was not just in the puzzles themselves, but in the community he built. Gardner never considered himself a mathematician; he was a journalist who fell in love with the elegance of logic. It was that very perspective—that of a curious explorer, rather than a distant expert—that allowed him to democratize mathematics.

Through his "Mathematical Games" columns, Gardner turned a subject that many found intimidating into a playground for the imagination. He didn't just teach people how to solve problems; he taught them how to play with ideas. This spirit of play is a universal language, transcending borders and classroom walls. Whether in North America, Asia, Africa, India, or the Middle East, I have seen firsthand how Gardner's influence continues to spark the same "Aha!" moments in students and teachers today, as it did decades ago.

Martin Gardner didn't just write about mathematics; he invited the world to fall in love with it. He showed us that behind every number and shape lies a story, a mystery, or a piece of art waiting to be discovered. As we carry these challenges into the next generation of classrooms, we aren't just teaching math. We are keeping alive a tradition of curiosity, wonder, and joy that began with a man who simply wanted to share the beauty of a good puzzle with the world.

If your own shelves do not yet contain any books by Martin Gardner, it is not too late. Check out new and used independent bookstores, or online retailers such as Alibris, Biblio, Powells, AbeBooks, Better World Books, and Amazon, and start your own collection. ▲

OOCYOA (Instructions page 6)

Determine the remainder when you divide your current value by the number you started with.
Go to "OOCYOA" page 11.

▲ The Mathematics of a Logo: Suggestion for a Workshop or Project



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Figure 1: *OAME/AOEM logo*

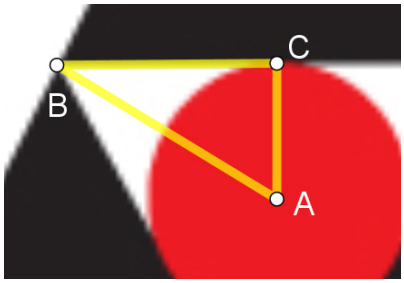


Figure 2: *Radius and semi-edge length relationship*

The OAME/AOEM logo, Figure 1, has a red circle surrounded by three black equilateral triangles. A three-dimensional version of the logo generalizes the triangles to equilateral square-based pyramids and the circle to a sphere. The two versions of the logo are mathematics hiding in plain sight!

Before delving into the mathematics, historically: the red circle corresponds to the letter "O"; the outer sides of the overall shape, along with the horizontal line above the red circle, correspond to the letter "A"; the two non-horizontal sides of the two lower black triangles correspond to the letter "M"; and the two white lines in the upper small black triangle (along with white at the left edge and white along the edge above the red triangle) correspond to the letter "E." Et voilà, "OAME" is in the logo.

The two-dimensional logo inspires the question of how the radius of the circle is related to the edge length of the sides of the equilateral triangles. Readers may recognize the circle as being inscribed in a white equilateral triangle. Using Figure 2, angle *BCA* is a right angle, while *ABC* is 30° because it bisects the angle. This facilitates using the tangent ratio to determine the relationship between the radius R (i.e., AC) and the edge length L (i.e., $2BC$) is $L = 2\sqrt{3}R$.

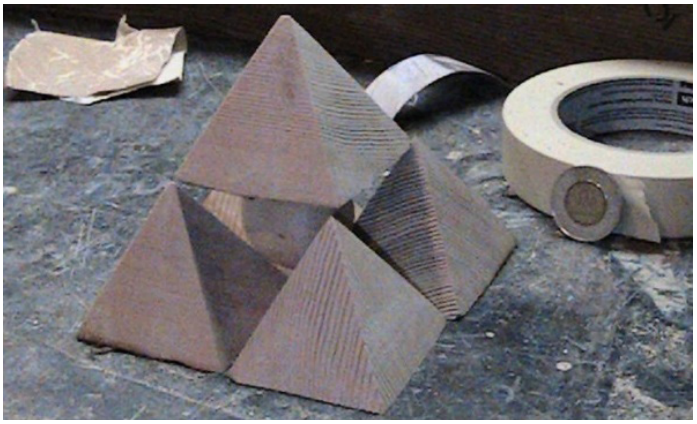


Figure 3: *Workshop OAME/AOEM logo*

At some point in OAME's history, the three-dimensional version was created (by Greg Clarke) using four square-based pyramids, with a fifth placed on top (see Figure 3). In amongst the pyramids is a sphere that touches the four-square pyramids that form the base level and the middle of the bottom of the pyramid on top. The mathematical challenge is a trades question of how to render this in a workshop. What are the measurements making it practical to get correctly-sized material, and to make appropriate cuts? Perhaps you have seen the wooden logo displayed at the annual conference? But did you consider how it was made?

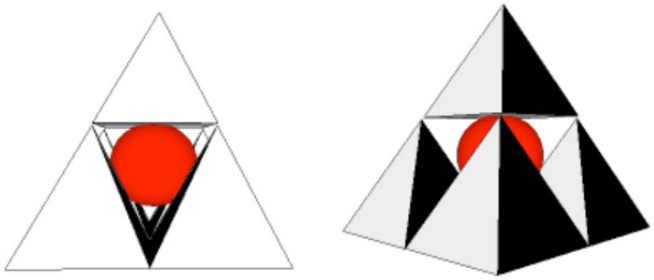


Figure 4: *Three-dimensional logo from two different angles*

The construction of the logo requires two key measurements from which all other measures can be derived. The choice of measurements is made to fit, with workshop equipment used to construct the shape. One needs to define the edge length for the base of a square pyramid (L) and the angle (α) that the triangular side face makes with the square base. The angle is taken as the minimum angle by using the centre line of the base and centre line of a side face. The measures that can be derived from this that are workshop relevant are: the radius of the ball (R) required to fit in the centre, the angle of the pyramid edges (θ) relative to the diagonal of the square base, and the height of a pyramid.

A key for design is the development of the relationships between the different measurements, especially since wooden spheres are difficult to make—but available in stores in 2", 2½", and 3" sizes. Generally, however, the prices for 3" are prohibitive, and larger sizes are not available in wood, suggesting that custom construction for sizes of choice is the way to proceed. (Larger spheres are available in Styrofoam, which is left for the reader's imagination.)

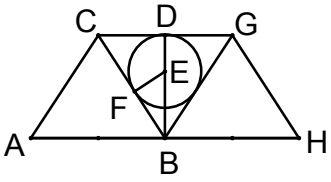


Figure 5: *Cross-section of the bottom tier*

Figure 5 shows a diagonal cross-section of the bottom tier of the logo. Length AH is the diagonal of the base of the full shape. Angle $\angle ABC$ is θ, and both EF and ED, in the middle, are radii of the sphere. The sphere is tangent to BC, BG, and CG, making the angles $\angle CFE$ and $\angle CDE$ (along with a third angle not marked) right angles.

Because the diagonal is shown, $AB = 2L$ and $CG = 2L$. AH is parallel to CG, so $\angle ABC = \angle BCD$. Since $\angle CBD = 90^\circ - \angle BCD$ and $\angle BEF = 90^\circ - \angle CBD$, we can say that $\angle BEF = \angle BCD$, which means that right triangle $\angle BCD$ is similar to right triangle $\angle BEF$.

Consideration of angle $\angle FBE = 90^\circ - \theta$ gives

$$BE = \frac{R}{\mathrm{Sin}(90^\circ - \theta)} = \frac{R}{\mathrm{Cos}(\theta)}.$$

Consideration of angle $\angle BCD$ gives,

$$\mathrm{Tan}(\theta) = \frac{R + BE}{CD} = \frac{R\left(1 + \frac{1}{\mathrm{Cos}(\theta)}\right)}{\left(\frac{L}{\sqrt{2}}\right)},$$

which leads to $R = \dfrac{L\,\mathrm{Sin}\theta}{\sqrt{2}\left(1 + \mathrm{Cos}(\theta)\right)}$.

Rearranged, it gives $L = \dfrac{\sqrt{2}R\left(1 + \mathrm{Cos}(\theta)\right)}{\mathrm{Sin}\theta}$, which is more convenient for deriving the edge length from a particular wooden ball size.

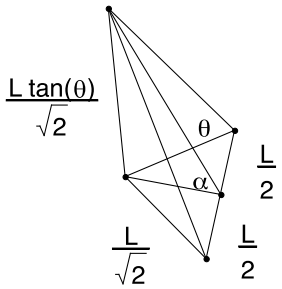


Figure 6: *Quarter section of a square-based pyramid*

Figure 6 shows a one-quarter section of a single square-based pyramid. A bit of tinkering provides the height, and that allows a relationship between the angles to be determined as $\tan(\alpha) = \sqrt{2}\tan(\theta)$.

This equation, along with the earlier equations for the radius and edge length, allow calculation for different angles and diameters. This has been done in a spreadsheet for standardized radii, with the results shown in Table 1.

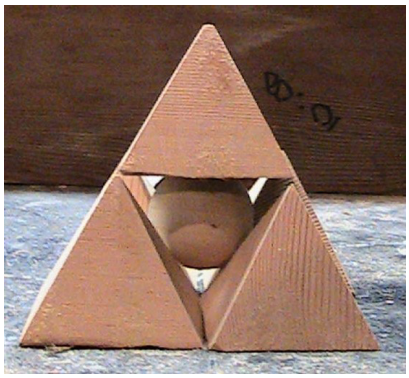
Table 1: *Side lengths for different angles and radii*

Angle alpha	Angle theta	Diameter Radius	1.00 0.50	1.50 0.75	2.00 1.00	2.50 1.25	3.00 1.50	3.50 1.75	4.00 2.00	4.50 2.25	5.00 2.50
80	76.00		0.91	1.36	1.81	2.26	2.72	3.17	3.62	4.07	4.53
75	69.25		1.02	1.54	2.05	2.56	3.07	3.58	4.10	4.61	5.12
70	62.76		1.16	1.74	2.32	2.90	3.48	4.06	4.64	5.22	5.80
65	56.60		1.31	1.97	2.63	3.28	3.94	4.60	5.25	5.91	6.57
60	50.77		1.49	2.24	2.98	3.73	4.47	5.22	5.96	6.71	7.45
55	45.28		1.70	2.54	3.39	4.24	5.09	5.93	6.78	7.63	8.48
50	40.12		1.94	2.90	3.87	4.84	5.81	6.78	7.75	8.71	9.68
45	35.26		2.22	3.34	4.45	5.56	6.67	7.79	8.90	10.01	11.12

The final detail for constructing the logo is the height of a pyramid. This is useful because it indicates the thickness of wood needed for a solid pyramid—or how many layers of Styrofoam sheet are needed to be glued together. Table 2 shows the heights for the same parameters as used above.

Table 2: *Pyramid heights for different angles and radii*

Angle alpha	Angle theta	Diameter Radius	1.00 0.50	1.50 0.75	2.00 1.00	2.50 1.25	3.00 1.50	3.50 1.75	4.00 2.00	4.50 2.25	5.00 2.50
80	76.00		2.57	3.85	5.13	6.42	7.70	8.98	10.27	11.55	12.83
75	69.25		1.91	2.87	3.82	4.78	5.73	6.69	7.64	8.60	9.56
70	62.76		1.59	2.39	3.19	3.98	4.78	5.57	6.37	7.17	7.96
65	56.60		1.41	2.11	2.82	3.52	4.22	4.93	5.63	6.34	7.04
60	50.77		1.29	1.94	2.58	3.23	3.87	4.52	5.16	5.81	6.45
55	45.28		1.21	1.82	2.42	3.03	3.63	4.24	4.84	5.45	6.05
50	40.12		1.15	1.73	2.31	2.88	3.46	4.04	4.62	5.19	5.77
45	35.26		1.11	1.67	2.22	2.78	3.34	3.89	4.45	5.01	5.56



Continued on page 45

Useful Theories to Know About: Perceptual Field Theory



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In 1998, Arthur Combs proposed perceptual field theory. This theory gives us a lens through which we can examine the motivation behind our decisions and actions, as well as the decisions and actions of our students. While not an astonishing revelation, perceptual field theory gives us a unified way to look at and consolidate our stances about education and about life.

Perceptual field theory (also referred to as perceptual theory) posits that one's perceptions are coloured by one's previous experiences, prior knowledge, emotions, values, and beliefs. This set of personal characteristics comprise one's perceptual field. The perceptual field influences one's perceptions of reality, perceptions of other people, and perceptions of oneself. Each person's perceptual field is unique. Thus, when one encounters a new experience, it is compared to one's existing perceptual field; the new experience will then be added to the perceptual field, modify one's perceptual field, or be rejected as conflicting with one's existing perceptual field. Perceptual field theory thus explains why each person perceives the same experiences differently, based on their own personal existing perceptual field.

Why Should We (Math Teachers) Care About This Theory?

Every teacher's perception of curriculum is viewed through the lens of their perceptual field. Their perceptual field is based on their beliefs about teaching and learning, their past experiences with teaching and learning, their

personal philosophy of teaching, their personal repertoire of instructional strategies and the personal valuation of each strategy, their understanding of students and students' individual needs, and the degree to which the curriculum impacts the teacher's personal perceptual field. So, if curriculum is not congruent with the teacher's own perceptual field, the teacher may reject the curriculum entirely, or perhaps only gloss over the curriculum with their students. If, for example, a teacher believes that Socratic teaching is superior, they are unlikely to embrace co-operative learning strategies, unless they are shown numerous examples of successful co-operative learning strategies in actual classrooms. Only then might the teacher modify their personal perceptual field to include co-operative learning in their repertoire of instructional strategies.

In the same way, each student's own perceptual field influences their perceptions of every lesson they encounter. They may accept the new information, partially accept the new information, or reject the new information, depending on how it compares to their personal perceptual field. Students' perceptual fields will be modified when they perceive value in the activities and have an expectation that they can complete these successfully. Therefore, the teacher's knowledge of their students is critical, in that they need to connect subject content to the students' various perceptual fields. Only then can the material being presented engage the students in their learning and ensure that students absorb, retain, and understand the subject matter being taught.

If You Want to Learn More

Combs, A. (1999). *Being and becoming: A field approach to psychology.* Springer. ▲

Continued from page 44

It is interesting to reflect on how the mathematics calculations needed to be transformed to provide a utilitarian interpretation regarding material thickness. Additionally, the size of the sphere could be made to any size, using a lathe, but the reality that specific sizes can be purchased is a contextual consideration that impacts the design choices.

In all, the intention of this article was to highlight some interesting math emerging from the OAME/AOEM logo. The practical aspects of manufacturing a version in wood highlights what it means to consider the real world. It is an insightful, albeit simple, example of that contextual shift. While the wooden model has been around for quite a while, perhaps this article will encourage the production of larger models (there are 20-inch beach balls available!). ▲

▲ Implementing Effective Strategies for Mathematics Classrooms – Part 1



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Jeff Irvine, Ph.D., has been a secondary mathematics teacher, department head, and vice-principal. He has taught at three faculties of education and at a community college. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. With over 45 years in education, Jeff is particularly interested in the interplay of student motivation and mathematics achievement.

I have always been impressed that so many math teachers intuitively do things that benefit their students. Not surprisingly, many of these strategies have research evidence to confirm their effectiveness. There has been a lot of research in which instructional methods are most effective. In 2000, based on an extensive literature review, Dr. Robert Marzano compiled a list of the nine most effective instructional strategies (the "Marzano 9") (Marzano et al., 2000). In 2009, Dr. John Hattie, who received his doctorate from the Ontario Institute for Studies in Education (OISE), synthesized the results of over 800 meta-analyses involving over 3 million students, and published his results in a well-known book called *Visible Learning*. Hattie ranked the effectiveness of strategies that impact student achievement, but did not consider the impact on other dimensions of student learning, such as motivation, engagement, self-confidence, and self-esteem. Dean et al. (2011) echoed Marzano's results, with minor measurement differences. Killian (2024) found that Marzano and Hattie agreed on eight of the nine strategies. None of this research was specific to mathematics. In 2020, the Ontario Ministry of Education (OME) published a booklet entitled *High-Impact Instructional Practices in Mathematics* that identified eight effective strategies for mathematics teaching.

This three-part article describes how nine very effective strategies (the "Marzano 9") can be applied in secondary school mathematics classrooms. It is important to note that the strategies apply equally well to elementary mathematics classrooms. I have used every strategy in this paper in the

classroom, and I can attest to their efficacy. The majority of the references in this article are classroom-tested examples taken from provincial mathematics journals including *Ontario Mathematics Gazette*, *Vector* (British Columbia mathematics teachers), *Delta K* (Alberta mathematics teachers), and *The Variable* (Saskatchewan math teachers—no longer published, but available in archives). These journals are a rich source of teaching and learning ideas. The "Marzano 9" are:

- learning goals and providing feedback
- reinforcing effort and providing recognition
- co-operative learning
- cues, questions, and advance organizers
- non-linguistic representations
- summarizing and note taking
- assigning homework and providing practice
- identifying similarities and differences
- generating and testing hypotheses

How Is Effectiveness Measured?

The effectiveness of a strategy or intervention is measured by *effect size*. This is a numerical value that indicates how effective a strategy is, and allows for comparison of strategies to identify the most effective. The most common measure of effect size is Cohen's *d*, usually a decimal value from 0 to 1. The larger the value, the more effective the strategy. For example, highly effective strategies include formative assessment (d = 0.90), descriptive feedback (d = 0.73), worked examples (d = 0.57), and spaced versus massed practice (d = 0.71). Less effective strategies include teaching test taking (d=0.22), team-teaching (d = 0.19), and surprisingly, problem-based learning (d = 0.15). All effect sizes are taken from Marzano et al. (2000), Dean et al. (2011), and Hattie (2009). For details on the computation of effect sizes and on meta-analyses, see Irvine (2024).

Setting Learning Goals, Success Criteria, and Providing Feedback (∂ = 0.61 to 0.76)

Learning goals should be shared with students at the outset of learning, in student-friendly language (OME, 2010). This is in line with the old adage, "If you don't know where you're going, how do you know when you get there?" Learning goals should specify what students will come to know and be able to do by the end of the learning activities. Part of the teacher's role is to relate the curriculum content to the learning goals. The students' role is to monitor their progress toward the learning goal, and take appropriate actions to move toward the learning goals.

To assist students in moving toward the learning goals, success criteria need to be clearly articulated. "Success" in mathematical thought often includes solutions or arguments that could be considered elegant, innovative, thoughtful, or that connect previously learned mathematics in thoughtful ways. Thus, "brute force" methods, while computationally correct, would not rank high on the success criteria scale, unless the focus was coding. For clarity, success criteria can be co-constructed with students. One strategy for developing success criteria is the Post-It® Pileup (Irvine, 2020a). Working in groups, students write success criteria on sticky notes, one criterion per page. Students then group the success criteria based on similarities. The resultant piles of sticky notes represent the consensus success criteria for the group, and these can be shared with the class, to result in a class-generated set of criteria. This strategy makes it clear that the success criteria are being generated by students, with teacher input. Post-It Pileup can be done face-to-face or virtually, using technology. To assist in constructing success criteria, exemplars can be provided. Student-generated exemplars have the most impact, since students learn to identify aspects of what constitutes a quality solution or a valid argument.

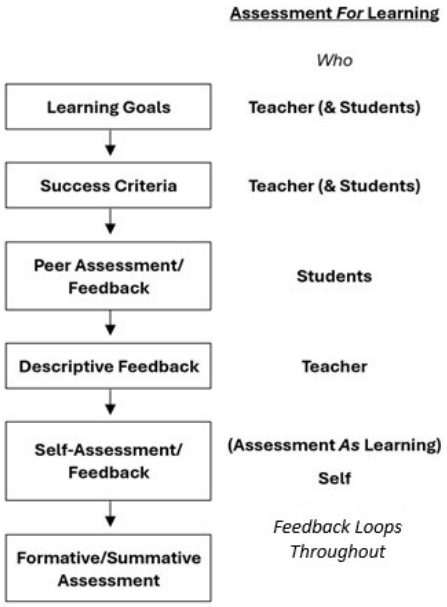


Figure 1: *Learning goals, success criteria, and feedback processes, based on work done by the Peterborough Victoria Northumberland Clarington Catholic District School Board*

Descriptive feedback is critical to improve student learning, as it "helps students learn by providing them with precise information about what they are doing well, what needs improvement, and what specific steps they can take to improve" (OME, 2010, p. 34). Hattie (2009) found that feedback is one of the five most powerful influences on student achievement, with an effect size of 0.73. Feedback should be timely, specific, non-judgmental, and supportive for the student, and provide opportunities for students to explain

their thinking as well as consider next steps (Hattie & Timperley, 2007; Irvine, 2023). Feedback can be provided by the teacher, peers, or the student themselves. Figure 1 illustrates the connections among learning goals, success criteria, and feedback, with links to assessment *for* and *as* learning. Feedback may also involve recognition and praise, discussed below.

Reinforcing Effort and Providing Recognition (d = 0.80)

Student recognition needs to recognize effort, which can contribute to developing a *growth mindset* (Dweck, 2006). The praise of effort should be specific; for example, "You worked diligently on this problem and you created a really unique solution." Contrast this with general praise such as "good work," which has been found to have no impact on student achievement. Dweck (1999) found that having a fixed mindset predisposes students to see failure as a direct indicator of their low ability, which is distressing and disengaging, whereas students with a growth mindset use failure as a cue to try new strategies and persist. A growth mindset stresses that through effort, students can improve, while a fixed mindset sees achievement limited by innate ability, such as "having the math gene," so no amount of effort will lead to improvement.

Praise of effort should promote intrinsic motivation. Intrinsic motivation is the desire to learn for the sake of learning, or because the activity is inherently interesting to the student. This differs from using extrinsic motivation exclusively, where the student is encouraged to learn only because of external rewards such as marks, or parental pressure to succeed. Studies have shown that intrinsic motivation is the best predictor of student achievement (Irvine, 2018a, 2018b). Motivation, as a whole in mathematics, is a major concern. Shernoff et al. (2003) found very low levels of engagement in mathematics classes where teachers reported that the classes had very high cognitive intensity, but very low motivation. Students reported being more negative about mathematics and less engaged than in any other subject. Therefore, it is important for teachers to provide student recognition when effort produces the expected learning, while giving descriptive feedback to assist students in reaching their learning goals.

Co-operative Learning (d = 0.44 to 0.73)

Co-operative learning is a large collection of pedagogical techniques and not just having students work in groups. Johnson et al. (1994) list five criteria for co-operative learning: mutual positive interdependence, individual accountability, face-to-face interactions, social skills, and group processing. The most commonly recognized co-operative learning strategies are jigsaw, four corners, placemat, and think/pair/share (Irvine, 2019, 2020b, 2021a).

Here is an example of how I have used jigsaw in my math classroom.

When teaching word problems in Grade 10, students first move to home groups of four (they could choose their own group). Members then move to expert subgroups to learn about how to solve specific types of word problems (distance, mixture, interest, etc.). Each expert subgroup is provided with sample worked examples and a worksheet of eight unsolved problems. The expert subgroup members work collaboratively to solve the odd-numbered problems, and check to ensure that all subgroup members understand the solution method. Students then return to home groups and take turns teaching the solution method for their word problems to their home groups. Home groups then collaboratively solve the even-numbered problems.

Mathematics is often treated as a sequential curriculum, so there are some content considerations that must be attended to if a strategy like jigsaw is to be used (Irvine, 2019).

- Content needs to be minimally consecutive (e.g., applications of functions: quadratic, exponential, logarithmic, trigonometric; power laws; solution methods for systems of equations; intersection of planes) so that expert subgroups do not require content knowledge from other groups.
- All members of expert subgroups need to contribute to the home group's understanding (mutual interdependence).
- All members of expert subgroups need to thoroughly understand their content, since they will be teaching their home group (individual accountability).
- Strategies can be used inductively as well. (Each expert subgroup investigates a specific case. Then students return to home groups to pool their results and draw conclusions.) See, for example, Irvine (2021b).

An example of multiple co-operative learning strategies used to teach one concept appears in Irvine (2021a).

The student-talk dimension of co-operative learning has been shown to stimulate deep learning and higher-order thinking skills (Dean et al., 2011). Math-talk learning communities may also be examples of co-operative learning, if they are structured to meet the five criteria specified above. For an example of math-talk learning communities in a Kindergarten to Grade 5 school, see Irvine (2017). Strategies such as jigsaw allow students to demonstrate their understanding and competence in a constructive way, and encourage engagement and self-confidence in our students.

Part 2 of this article looks at the next three strategies of the "Marzano 9."

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▲ Implementing Effective Strategies for Mathematics Classrooms – Part 2



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Part 1 of this article looked at ways to apply the first three strategies of the "Marzano 9" in secondary mathematics classrooms. Part 2 continues with the next three strategies, again applied in secondary mathematics programs.

Cues, Questions, and Advance Organizers (∂ = 0.59 to 0.74)

Strategies such as cues, questions, and advanced organizers support intrinsic motivation and are useful when introducing new material, activating students' prior knowledge, or encouraging interest and curiosity. Activating prior knowledge is important, since it has been shown that there is a significant relationship between prior achievement and future achievement, with an effect size of 0.67 (Hattie, 2009; Marzano, 2004). To use these strategies most effectively, teachers need to focus on what is important in upcoming material, use explicit cues, and ask inferential and analytic questions. For example, students might be presented with questions about a relationship, such as, "Is there a relationship between height and arm span?," and asked to brainstorm possible relationships and how to investigate them. The extensive use of technology during the COVID-19 pandemic demonstrated that many of these strategies can be dealt with using online resources (Irvine et al., 2023). Other ways to activate prior knowledge include: narrative stories such as, "Think of a time when you wanted to solve a problem that required mathematics. What were the circumstances and what resources did you need to solve the problem?"; using a short video about earthquakes

and the Richter scale to introduce logarithmic scales (Irvine, 2022); and brief role-play exercises, in which students are asked to imagine themselves in a role such as investment adviser or home buyer (Irvine, 2019a, 2020), accountant (Irvine, 2023b), or model rocket builder.

Anticipation guides are a very effective way to introduce new material. In this advanced organizer, prior to studying the material, students are asked to respond to some true/false or short-answer questions. Students make reasoned conjectures about the answers, and after studying the material, they return to the anticipation guide and compare their answers (this also supports recognition of SEL [Social–Emotional Learning] skills). Figure 1 illustrates anticipation guides for quadratic functions. An anticipation guide for trigonometry can be found in Irvine (2019b).

	Before the unit starts		After the unit is over	
The graph of a quadratic relation will be a straight line.	Agree	Disagree	Agree	Disagree
The graph of a quadratic relation will be a curve.	Agree	Disagree	Agree	Disagree
The graph of a quadratic relation will always have one y-intercept.	Agree	Disagree	Agree	Disagree
The graph of a quadratic relation will always have two x-intercepts.	Agree	Disagree	Agree	Disagree
The graph of a quadratic relation will always be symmetric.	Agree	Disagree	Agree	Disagree
The graph of a quadratic relation will always have a minimum value.	Agree	Disagree	Agree	Disagree
The graph of a quadratic relation will always have a maximum value.	Agree	Disagree	Agree	Disagree

Figure 1: *Anticipation guide for graphs of quadratic functions (e.g.,* $y = 2x^2 - 12x + 18$*)*

Generating student interest and activating students' prior knowledge is important for student achievement. Motivating students to think about new situations stimulates students to become engaged in their own learning and take an active part in their mathematics classes.

Non-Linguistic Representations (∂ = 0.49 to 0.75)

We routinely represent mathematical information using symbols, equations, graphs, diagrams, and manipulatives. Non-linguistic representations also include physical models, graphic organizers, mental pictures, diagrams and illustrations, pictographs, videos, and kinesthetic representations. Psychology tells us that the brain represents information non-linguistically using *schema*, which are integrated relationships similar to mind maps (Irvine, 2016). Fosnot (2005) states that learning occurs when students modify or build onto their existing schema for a particular topic.

The *Abacus* column, "Links to Manipulatives," in the *Ontario Mathematics Gazette* (*OMG*) is a rich source of ideas for using manipulatives in math classrooms, as are Dr. Jennifer Holm's (2018, 2020) and Irvine's (2018b) articles in *OMG*. A particularly useful strategy is time-lapse diagrams, illustrated in Figure 2. Time-lapse diagrams ask students to illustrate situations at different points in time, to help clarify relationships. Figure 2 illustrates a 6 m ladder leaning against a wall, with the top of the ladder touching the wall at a 4 m height. The ladder slides down the wall at a rate of 0.5 m/min. The time-lapse diagrams show students that the foot of the ladder moves at a different rate than the top of the ladder, a concept that is often confusing for students. This problem can be an application of the Pythagorean theorem. When the problem is initially posed, many students will claim that the foot of the ladder moves at the same rate as the top of the ladder. Time-lapse diagrams, combined with the Pythagorean theorem, will result in cognitive dissonance and more thoughtful student analysis.

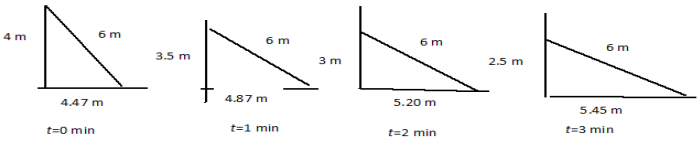


Figure 2: *Time-lapse diagrams for a sliding ladder*

Time-lapse diagrams can also clarify problems about two objects that are moving relative to each other. For example, "A ship is sailing due North at 25 km/h and passes another ship that is travelling South 30 degrees East, at 18 km/h. How far apart are the two ships two hours after they pass each other?" Time-lapse diagrams also allow students to answer questions from the past, using negative values for time. For example, they can draw a diagram to determine: "How far apart were the two ships three hours before they met?"

One sometimes neglected representation is kinesthetic models, such as quadratic aerobics and live demonstrations of normal distributions (Irvine, 2019a). I have also found student understanding is improved by using a living Cartesian grid, in which a Cartesian grid is laid out with tape on the floor of a gymnasium, and students physically move to locations specified by ordered pairs. The living grid can also be used to kinesthetically represent linear growing patterns.

Graphic organizers such as mind maps and webs can help students relate concepts to each other. Decision trees help students prioritize decisions. Decision trees can be straightforward, such as ordering the steps in solving an

equation (Irvine, 2023a), or more complex, such as (Figure 3), considering the steps and relevant information in problem solving and Georg Pólya's problem-solving heuristic (Irvine, 2018a).

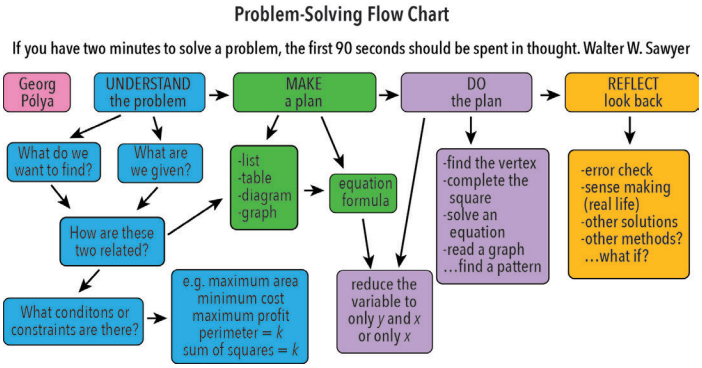


Figure 3: *Problem-solving decision tree*

Venn diagrams can help students identify attributes in common concepts. Other graphic organizers, such as cause–effect diagrams, fishbones, and similarities–differences charts can help students organize and understand concepts and relationships. Templates for these and other graphic organizers can be found free of charge on the Internet, and may be interactive, allowing students to construct the graphic organizer online, a good fit for today's Internet-knowledgeable students. Our students ably demonstrated their technological expertise during the COVID-19 pandemic (Irvine et al., 2023). Non-linguistic representations are a staple of mathematics teaching. Marzano's research supports multiple representations of information and relationships, which addresses different learning styles.

Part 3 of this article looks at applying the final three strategies of the "Marzano 9" in mathematics classrooms, summarizing how these effective strategies can enhance student achievement in mathematics.

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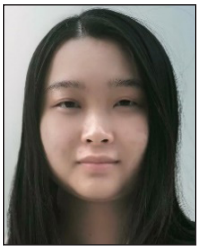
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What Do Current University STEM Students Think About the Cross-Curricular Content in the Ontario Grade 12 Mathematics Curriculum?



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This article shares the results of thesis research that investigated the current Ontario Grade 12 University-level Mathematics curriculum. The project gathered students' thoughts on the cross-curricular content in the Ontario Grade 12 University-level Mathematics curriculum to resolve various questions: How do current university STEM students perceive the cross-curricular content in the Ontario Grade 12 Mathematics curriculum? Do they find it sufficient? If so, how have they applied this knowledge in their university STEM studies? If not, what changes would they recommend?

The Survey: Who, Where, and How?

The target population for the survey was STEM students at Ontario universities, who took the current Ontario Grade 12 Mathematics curriculum. The survey was sent to STEM communities like FYMSiC (First-Year Math & Stats in Canada) and to 354 STEM professors across various Ontario universities. It was also distributed through university departments, such as first-year math offices, and posted on university STEM program Facebook groups.

What Did the Survey Ask?

To make sure participants met the sampling criteria while still protecting their privacy, the survey began with a participant information session. First, the survey asked about their high school background to confirm they took the current Ontario Grade 12 Ontario Mathematics curriculum by asking which of the three math courses (Calculus and Vectors, Advanced Functions, and Data Management) they took, their high school graduation year, and so on. Next, the survey checked their university status by asking about their STEM program, university start year, and current year of study.

The second part of the survey assessed participants' perceptions of cross-curricular content in the Grade 12 Mathematics curriculum; the survey asked them to rate: (1) their learning experiences, and (2) how well these prepared them for university STEM studies. For convenience, they were labelled as experience ratings and effectiveness ratings. These five-point Likert-scale ratings ranged from 1 (very limited cross-curricular content) to 5 (abundant cross-curricular content) for experience, and from 1 (not enough preparation) to 5 (substantial preparation) for effectiveness. For clarity, participants were first given an example question (e.g., a logarithmic function problem related to pH in chemistry for cross-curricular contents in Advanced Functions) to illustrate the cross-curricular topic. These questions were either from Grade 12 Ontario Mathematics curriculum or textbooks addressing that curriculum (Crippin et al., 2009; Kirkpatrick et al., 2008; Ontario Ministry of Education, 2007; Zimmer, 2003). After viewing the sample problems, participants provided experience and effectiveness ratings for that topic. This process was repeated for all major topics in the three courses—Advanced Functions, Calculus and Vectors, and Data Management.

In addition to the quantitative scales, students' responses to open-ended questions were collected. These questions asked them to: (1) provide specific examples of how they applied the cross-curricular content in a Grade 12 course to their university STEM studies, and (2) suggest potential improvements to enhance the cross-curricular elements in light of their university experience.

What were the results?

There were 95 valid responses used in data analysis. The average experience rating for Advanced Functions, Calculus and Vectors, Data Management were 3.41, 3.55, and 3.49, respectively. This rating indicates students' satisfaction with their learning experiences in these courses. Next, the effectiveness ratings were 3.36, 3.71, and 3.60, respectively. This rating is indicative of how much students consider that they applied these materials in their university STEM studies. It was observed that the modes were generally higher than the averages (or the mean) for these ratings. The mode ratings were all four across the three courses in experience and effectiveness ratings. This observation suggested that some of the participants gave much lower ratings than averages on these topics and lowered the averages. Notably, Calculus and Vector topics consistently received high effectiveness ratings, particularly for concepts like rate of change, demonstrating that many students felt these cross-curricular elements effectively prepared them for university STEM studies.

Analysis of variance (i.e., ANOVA) tests were conducted to examine whether significant differences existed in experience and effectiveness ratings among the three courses. When comparing the resulting p-values to the standard significance level ($\alpha = 0.05$), the analysis revealed that there was no statistically significant difference between the mean experience ratings across courses. However, there was a statistically significant difference between the mean effectiveness ratings. To investigate the source of this effectiveness rating misalignment, three paired t-tests were performed between courses. The comparison between Advanced Functions and Calculus and Vectors produced a t-statistic exceeding the adjusted alpha (it was adjusted to compensate for multiple tests). The results have shown that the statistically significant differences in the mean effectiveness ratings across the three courses were primarily driven by the ratings for Advanced Functions. In other words, the effectiveness ratings for Advanced Functions were statistically significantly lower than the effectiveness ratings for the other two courses. The ANOVA test performed between topics within each course has shown that there was no significant difference between the experience and effectiveness ratings of each topic within the same course.

The analysis of experience and effectiveness ratings by university programs encountered limitations due to participants frequently selecting multiple options when indicating their primary university program. For example, a biophysics student can select physics and biology, or they could type in their name of program if the option was not available. However, many participants understood this question as to select all courses they were taking, instead of their primary program. Participants in their early years of university might be unsure of their primary programs as well, which led to insufficient data for analysis in this section. Among programs with at least three analyzable responses (programs with at least three participants), distinct patterns emerged in the ratings. Health Science participants rated Data Management highest for both experience and effectiveness, while giving Advanced Functions the lowest ratings. Biology students valued the cross-curricular content in Advanced Functions more highly than other courses. Chemistry participants reported that Calculus and Vectors provided both the best learning experience and most effective preparation for university chemistry studies. Mathematics and Statistics students, in contrast, provided remarkably consistent ratings across all three courses for both learning experience and effectiveness.

The ratings were further analyzed by participants' high school graduation years to account for potential factors affecting their perceptions. This categorization addressed two possibilities: (1) that recall accuracy of cross-curricular content in the Grade 12 Ontario Mathematics curriculum might vary with time since graduation, and (2) that different university

years might influence how students perceive the value of their high school mathematics preparation. Results indicated that participants who graduated from high school between 2021–2023 (typically university years 1–3, if they entered directly after high school) provided mid-range ratings for both experience and effectiveness. However, a notable decline appeared in the 2020 graduate cohort. Interestingly, 2019 graduates (likely in their fifth year of university studies or beyond) gave the highest ratings for cross-curricular content effectiveness and experience. This pattern may reflect either the students' advanced university standing or the disruptive impact of the COVID-19 pandemic, which began in Canada during early 2020 and may have affected students' educational experiences.

The Open-Ended Responses

The open-ended responses were a clarifying part of the results, delivering students' most direct thoughts. Most participants who provided comments about Advanced Functions perceived the course primarily as a mathematical foundation that helped develop their problem-solving skills and logical-thinking abilities, rather than seeing direct connections between the course content and their university STEM studies. For instance, one Health Science student noted how working with exponential and logarithmic functions improved their ability to work with a calculator, while a Biomedical student highlighted that the course encouraged them to "think outside the box." However, many participants, including those in Computer Science and Chemistry, expressed that they found limited explicit cross-curricular content in Advanced Functions that directly supported their STEM studies at university.

Participant feedback about Calculus and Vectors primarily addressed two key aspects: the course's challenging nature and its predominant connections to Physics and Engineering. Many respondents, particularly those who took the course online during the COVID-19 pandemic, reported struggling with concepts like rate of change. Several students expressed a desire to go beyond memorization of derivative rules, with more cross-curricular applications or real-world contexts. STEM students outside of Engineering, Physics, and Mathematics often questioned how these mathematical concepts related to their specific fields of study. Additionally, some participants suggested that introducing integrals during high school could better prepare students for University-level STEM coursework.

Data Management received significantly more positive feedback regarding its cross-curricular content compared to the other two courses. Many participants reported finding the material particularly beneficial, especially appreciating the integration of technology in the curriculum. Several students suggested expanding the range of technological platforms used in instruction. However, some participants offered constructive criticism about the course content. They noted that certain cross-curricular problems relied too heavily on hypothetical scenarios (e.g., "assuming there is..."), rather than practical applications. Others observed that probability and counting concepts focused excessively on card games and poker examples, and some suggested Data Management could be more impactful if tied to real-world situations like understanding taxes, mortgages, or other financially relevant scenarios.

A primary limitation of this study was the low response rate to the online survey, especially on the open-ended responses, which also impacted on the reliability of the data collected. Despite this, the survey collected 95 students' thoughts on the Grade 12 University-level Mathematics curriculum, which was a significant voice for reflecting on Grade 12 Math course experiences.

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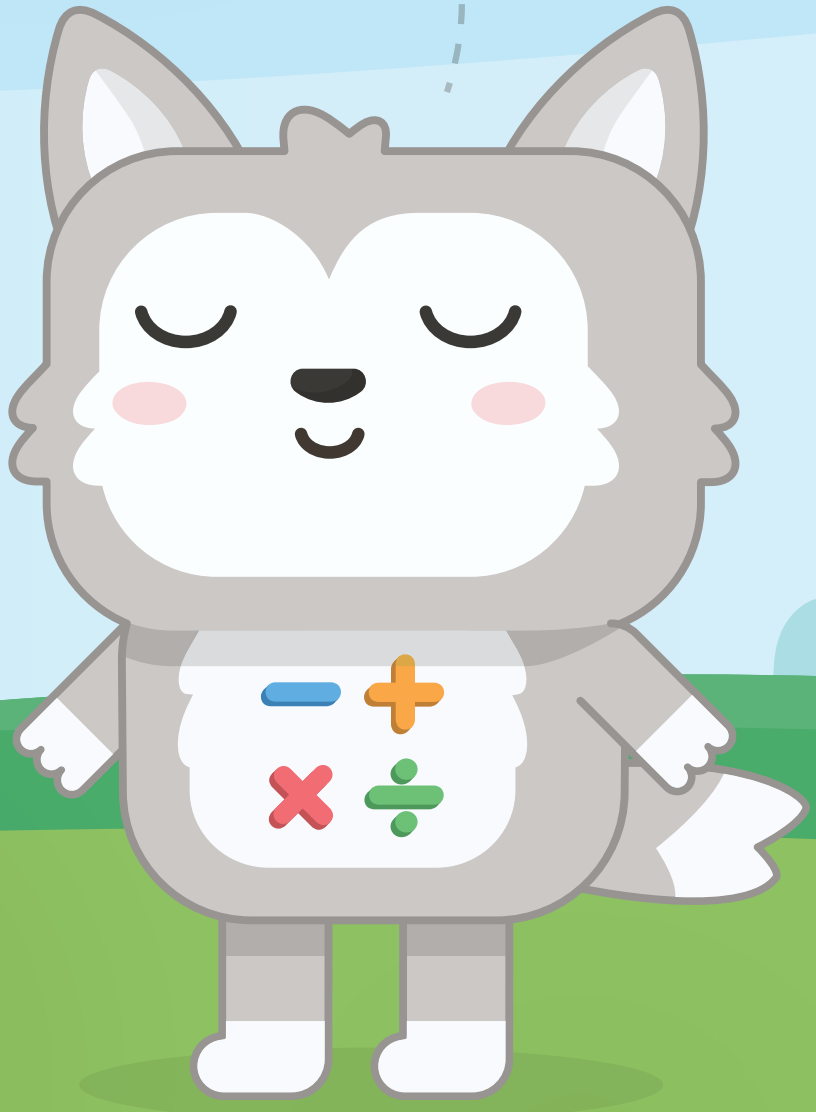
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