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Abacus

Ontario Mathematics Gazette

OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

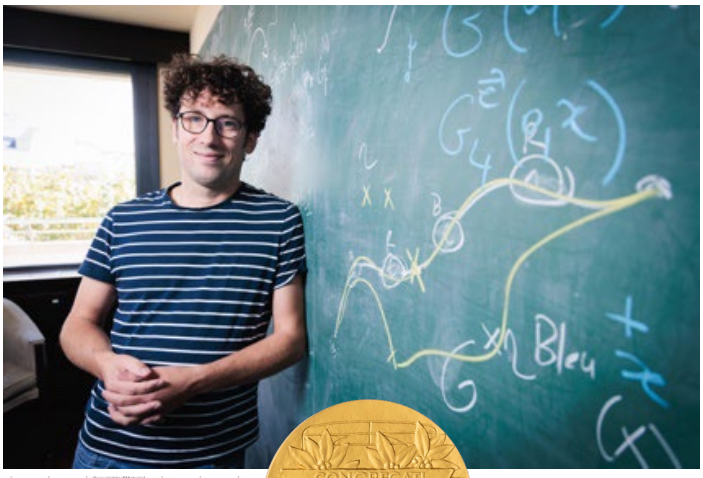
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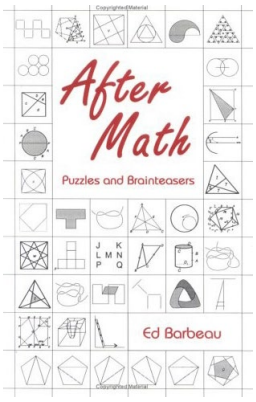
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See Elementary Math Matters: Where did all the fun stuff go?



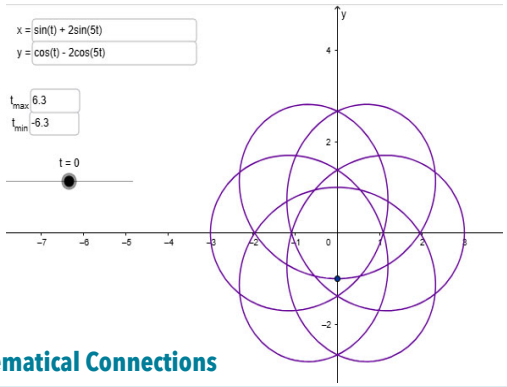




See Events at the Fields: Two Debates and an Invitation to the Fields Medal Symposium

See Mb4T (Mathematics by and for Teachers): Making Connections to Solve Equations





See Turk's-Heads and Rope Rugs: Mathematical Connections

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Submission of Articles

The *Ontario Mathematics Gazette* (*Gazette*) publishes news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *Gazette*:

- The *Gazette* is aligned with the OAME/AOEM Vision, Mission, and Strategic Priorities. These can be found on the website www.oame.on.ca under "About OAME" and "Guiding Principles."
- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles are to be word–processed, MS Word is preferred, and prepared according to the *Publication Manual of the American Psychological Association*, Seventh Edition. However, please use single-line (not double) spacing and only one space after each period. Articles should not exceed eight numbered pages of text, though there is some flexibility. Figures, images, and photographs should be placed close to the relevant text and include captions. The photographer's permission is required, and for photos of students under the age of 18, permission of a parent or guardian is required.

Please submit your article in one blind file (i.e., identity of author is not evident), and include author names, contact information including email and mailing addresses, photos—head and shoulders, biographies—less than 100 words, and all content removed for blinding in a second file. Please email these two files to Tim Sibbald at gazette@oame.on.ca.

Upon review, you will be notified whether your article has been accepted for publication (as is, or pending minor or major revisions) or declined. The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME/AOEM.

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The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of OAME/AOEM.

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About the *Ontario Mathematics Gazette*

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Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** by email or at the above address. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single-sided. Each advertisement should be print-ready, and colour advertisements should have no bleeds.

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Editor's Report



TIM SIBBALD, OCT, PH.D.
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Tim Sibbald is a professor in the Schulich School of Education, Nipissing University, with a focus that includes mathematics education. He is a former President of OAME/AOEM and has held a variety of other OAME/AOEM posts over the years.

Last Spring, a book found me. This often occurs when I am travelling, with many serendipitous finds through the years—these are not necessarily easy reads: often they are more challenging in personal ways. The most recent, Haidt (2024), argues that what we are seeing in education is essentially a foundational change in the lifestyles of youth. The implication is that issues around statistics, such as declining math scores, are symptomatic of a significant non-math issue.

Haidt focuses on issues to do with social well-being. He notes that anxiety and mental illness are up substantially in the United States since 2010. For example, self-harm by girls is up 188 percent. He reports that suicide in the 10–14 age range, in the United States, is up 91 percent for boys and 167 percent for girls. Psychological distress in Nordic countries is up 76 percent for girls and 51 percent for boys. Haidt indicates that the number of eight-, tenth-, and twelfth-graders combined (in the United States), who are getting less than seven hours of sleep has increased 10 percent since 2010, and a similar 10 percent decline has been observed with satisfaction with oneself. Between 2010 and 2018, high school seniors in the United States who indicate "Life is meaningless" has increased 10 percent. In pre-COVID years, time spent with friends had declined from 150 min/day in 2003 to less than 50 min/day in 2019. These statistics are shocking, but again symptomatic.

Clinical depression in the United Kingdom has been found to increase with social media use. Yet, this is just a correlation and does not necessarily correspond to causation. So, researchers looked at the increase in high-speed Internet in Spain and found a corresponding increase in negative effects for girls. Rates of having close friends, among high school seniors, has been declining since 1998, and loneliness has increased. Risk taking has also been declining since 2000.

A premise of the line of thinking derived from these statistics is that the Internet, along with ease of access to

handheld devices, has fundamentally changed the fabric of society. It is not uncommon to see couples sitting in restaurants, each gazing at their device and not conversing. Adults model behaviours to their children.

If we pause and consider how cellphones were used in the half-decade before the pandemic, it contrasts in many ways with usage today. There seemed to be a wow factor in years gone by, but now the novelty seems to have worn off and they are perceived more as a routine tool. Yet, socialization seems slow in returning, with many people appearing to be more narrowly focused, with a choice of friends who are seamless because of online access. But the choice, while not problematic, seems to detract from the building of new circumstantially based relationships.

The situation seems to align with the notion that the Internet allowed the monetization of addictive social tools—that there is a degree of healthy usage—but efforts were made to extend engagement beyond that in the interests of profits. Policies have been slow to respond because these are complex issues making it difficult to establish what is taking place and how to craft suitable policies. Certainly, the Internet is not the Wild West it used to be—thank goodness—much has been done to reign in bad actors, but more needs to be done.

How this intersects with math is concerning. The use of myopic tools seems to focus the issue on a narrow set of problems. EQAO scores suggest an issue, and some suggest that requires more focus on math. We do not know, however, whether geography has the same issue because there is no standardized testing in geography. Evidence of declines in geography are arising from anecdotal observations of people using navigation systems—it now appears to be okay to say one does not know which way they are going, and declare dependence on a device. The point is that there is a narrative that is portraying declines in math, and it is applying some degree of persecution on teachers (as evidenced by the Math Proficiency Test), yet the evidence suggests that this is the tip of an iceberg—an iceberg where social change is being delivered by large corporations who are parasitic on the human condition.

Sure, EQAO scores are declining, but where is the evidence that this isn't due to years of challenges getting policies into place? As an example, it was evident that cellphones in classrooms were an issue 15 years ago and the handling of that issue is sure to have contributed, as recent policies have indicated. Let's stop blaming math educators—the fact that they are on the tip of the iceberg should not make them a target, as that would mean the rest of the iceberg is being ignored.

Hyperlinks

The *Gazette* provides active hyperlinks in the PDF version, available in the members-only area. Readers may notice that they are not underlined or in a different colour—who needs the distraction?—but they are hyperlinked regardless.

In the News

Gurnoor Kaur, a Grade 11 student in Kitchener, realized that a mathematical instability in cardiac modelling can be addressed by adding a missing term. This discovery will lead the way to less racial bias in the accuracy of oxygen sensor measurements, particularly more accuracy for people of darker skin colours. See www.cbc.ca/news/canada/kitchener-waterloo/gurnoor-kaur-canada-wide-science-fair-project-innovation-award-wow-9.7217194.

Growing Success has been updated to include requirements for final evaluations, and details for the mandatory financial literacy component. It seems odd that financial literacy is being tied to the Career Studies course. However, looking over the resources that are available, it is pragmatic content, as opposed to analytical interpretations, that mathematics classes would focus on. See www.ontario.ca/page/growing-success-assessment-evaluation-and-reporting-ontario-schools-kindergarten-grade-12#section-1.



Figure 1: Thank you, Peter Saarimaki, for organizing another Past President's gathering!

Note about AI (Artificial Intelligence)

The *Gazette* continues to navigate the waters that are characterized as "Artificial Intelligence" derived from Large Language Models (LLMs)—and I will use the LLM term because machine-generated responses are not intelligence; rather, they are regurgitating what they have copied or plagiarized from other people. While there are issues with such models, it is also foolhardy to simply ignore them. Personally, I began using a spell-checker when it initially

became available (around 2002, I think). Grammar checkers were a slower adoption because the quality seemed to be slower coming, and I remain vigilant about what they suggest. Neither of these tools is as good as an effective copy editor (Thank you, Gitta!). Often overlooked is the statistical basis that is much larger for modern LLMs, and obscured by the use of artificial neural networks.

> Neural networks are adaptive statistical models based on an analogy with the structure of the brain. They are adaptive in that they can learn to estimate the parameters of some population using a small number of exemplars (one or a few) at a time. They do not differ essentially from standard statistical models (Abdi et al., 1999, p. 1).

While neural networks are challenging to model mathematically (hence the lack of articles), they are highly responsive to statistical details, although none provide the creativity and presence of mind required to draft and generate a meaningful column for readers.

In talking to columnists about usage, I heard LLMs were being used for polishing writing that had already been crafted, drafting translation between languages, and as a writing tool for low-level bumps (much like a thesaurus of old). Care has been taken to preserve personal integrity, where one columnist described using a personalized system trained on their own writing. Suffice it to say that the *Gazette* is evolving just like everyone else, and that our checks and balances indicated a measured and reasoned usage that is in keeping with the OAME/AOEM position paper, "Effective Use of Digital Technology in Mathematics Education."

In This Issue

Mark Chubb, who succeeded **Kit Luce** as President, considers the importance of listening at all scales of the education. A Code of Conduct for OAME/AOEM is presented in this issue, and it is introduced by **Elizabeth Pearsell**, who was involved in the committee that led to its development. **Pamela Brittain** brings news of debates and other opportunities at the Fields Institute for Research in Mathematical Sciences. **Gerard Lewis** brings news from the NCTM, which includes the recent release of another position paper. **Gerard Lewis** also updates us about the OMCA.

Sofia Ferreyro-Mazieres addresses the teaching of equation solving and how to effectively migrate from concrete models to representational and then to an abstract model. **Carly Ziniuk** is building a collection of postcards that explore mathematical questions across a geographical area—there is quite a mix of opportunities in her column! **David Costello** discusses how the teaching of metacognitive regulation is important among metacognitive skills. **Jérôme Proulx** considers some surprising details of how percentage keys on calculators operate. **Lynda Colgan** infuses the issue with mathematical fun that can serve to engage students through mystery and surprise. **Ron Lancaster** finds a collection of books in his office by Brian Bolt that sound quite intriguing. **Shawn Godin** finds ways to manipulate digits to achieve desired results. **Jennifer Holm** considers what is the same and different through the eyes of children. The regular column, "Useful Theories," is missing from this issue.

The articles include the third part of **Jeff Irvine**'s trilogy about effective strategies. The other two parts appeared in the June issue, and if you missed it, you can find it in the archive. We also have an article by **Robert Chisnall** who explains some mathematics related to knots and rugs.

Thank Yous

In many respects, the Editor is the hub of a wheel. Pieces of writing come in and then go out for others to take a look before returning. It is a distinguishing character of a professional publication and illustrates how peer review regulates clarity and quality. The editorial team does this, though it is often invisible, but I do see the goings-on. Much as it sounds like clockwork, it has always surprised me how indispensable the various members of the team are. In my early days of the Editor role, I was under an illusion that seeing, for example, the copy editor's (i.e., **Gitta Berg**'s) feedback would be training and, over time, I expected the number of items to decrease—this simply has not been the case.

Just as students focus on mathematical ideas, while communication has to be considered separately, so it is with the *Gazette*. This is where having every piece gone over by myself and one of **Peter Saarimaki**, **Steven Khan**, or **Anne Yeager** is invaluable. The focus is ensuring clarity of what is articulated so that readers can focus on the ideas without being distracted by disruptions in the diction.



Figure 2: *Penny Clemens, our beloved graphic designer, holds the record for the most years involved in the production of the* Gazette*! She first appeared in the December 1993 issue—an impressive 32 years!*

Even when clarity is achieved, **Gitta Berg** goes over every piece with an eye to ensuring consistency. Then the layout requires a special talent to pull it all together into an issue. The page constraints, needs of the printer, development of covers, handling of different types of content, all receive the attention

of **Penny Clemens** (Figure 2).

Finally, a special mention: **Pole Printing** first appeared in the *Gazette* in March 1977. Thank you for 48 years of dedication to making the *Gazette* happen!



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Abdi, H., Valentin, D., & Edelman, B. (1999). *Neural networks.* Sage Publications Inc.

Haidt, J. (2024). *The anxious generation: How the great rewiring of childhood is causing an epidemic of mental illness.* Penguin USA Inc.

Ontario Ministry of Education. (2022). *Growing success: Assessment, evaluation, and report (kindergarten to grade 12).* King's Printer for Ontario.

Letter to the Editor

Dear Editor and the entire *Ontario Mathematics Gazette* Team:

I am writing to send a huge shout-out and a deeply deserved thank you to everyone involved in bringing the *Ontario Mathematics Gazette* to life—from the editorial board and authors to the production and publishing teams.

Over the years, I have watched the magazine evolve and grow, and it has undeniably become one of the absolute best journals available for mathematics educators today. Every single issue contains outstanding articles that are practical, well written, engaging, and deeply thought-provoking. To put it simply: the moment the *Gazette* arrives in my mailbox, I drop everything else and devour it. It is a true treasure.

To provide some context for my praise, I have had the privilege of working with teachers and students across 41 states, and I have seen a vast array of journals published by various NCTM affiliates. While those publications certainly have something to offer, none of them come close to the *Gazette* in terms of article quality and day-to-day classroom practicality.

Thank you all for your vision, your dedication, and your tireless work to support the mathematics education community. You are making a profound difference, and your efforts do not go unnoticed.

With highest regards and gratitude,
Ron Lancaster
Associate Professor
Emeritus, OISE/UT ▲

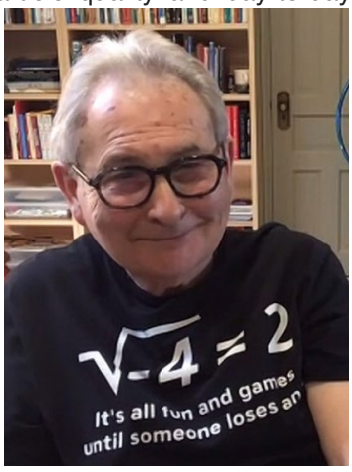


Figure 3: Ron Lancaster with a humorous "fashion statement!"

▲ President's Message



MARK CHUBB
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Mark Chubb has been part of OAME/AOEM for 15 years, including as Chapter Representative, member of the Board of Directors, Chair of the Curriculum Committee, Vice-President, and now President. He has been a teacher and instructional coach for the District School Board of Niagara for 26 years.

I've always been fascinated by fractals. In mathematics, we see these self-similar patterns everywhere, from the way a coastline jaggedly repeats itself as we zoom in or out, to the branching of a single fern, and even the OAME2009 logo, which contains a Sierpinski triangle. The beauty of a fractal is that the smallest parts mirror the larger whole.

Lately, I have been thinking a lot about how our work in education actually behaves like a fractal. As a provincial subject association, how we organize ourselves at the provincial level should mirror how we support our chapters, which in turn should mirror the interactions we have with students in our classrooms. If we hope to support our students as they learn to think mathematically, then the practice of listening needs to be the pattern that repeats at every single level of our system.

The Power of Listening

When we look at the smallest scale of this fractal, we are looking at the interaction between a teacher and a student. Last year, I joined a classroom teacher as their students were being introduced to the magnitude of decimal numbers. This teacher recognized that for her students to learn a complicated concept deeply, she needed to offer them time to think, to discuss, to be heard, and to hear others. To do this, she had them place numbers on student-created number lines on the floor of her classroom. Students were challenged to find numbers between other numbers, and estimate the numbers other groups had placed on their lines. In this lesson, the teacher recognized that listening to her students was a powerful way to learn about their current thinking, and the best way to guide the next instructional decision. She used listening as a mini-diagnostic, focusing heavily on the mathematical strengths her students brought to the table at this early stage in their learning.

Over the past three years at our leadership conferences, we have heard from some amazing math leaders, including Dr. Rachel Lambert, Dr. Ilana Horn, and Dr. Elham Kazemi, on this

informative and critical topic. Each, in their own way, pointed toward the idea that listening is a powerful and rigorous professional competency, rather than a soft skill.

Rachel Lambert reminded us that to hear the brilliance in our students, we have to provide tasks with multiple entry points. She challenged us to rethink how we view "disability" in mathematics, and showed us concrete ways to help our students succeed. However, this requires both tasks that allow students room to learn, and a teacher who is listening and learning from their students.

Ilana Horn suggested to us that educational leaders need to move away from telling teachers the "right" approach to teaching (moving away from certainty) and instead, engage in real dialogue, where teachers can learn from each other. This too is about listening. Teachers who feel valued are more likely to mirror this practice in their classroom. When we do this, we move beyond just checking for right answers. We start practising what Brent Davis calls hermeneutic listening, where we allow our students' thinking to actively shape the direction of instruction.

This is where math becomes human. Elham Kazemi's sharing at our most recent leadership conference gave us a glimpse into how to make this possible in our busy classrooms. Her work on intentional talk showed us that whether we are leading a professional development session for teachers or facilitating a lesson for children, our job is to listen for the links between ideas, and to use those ideas to shape the learning. When we model responsive listening at the leadership level, we set the stage for classrooms to mirror the same pattern, moving us from listening evaluatively, and into spaces where our students' mathematical identities can flourish.

All of these examples from our previous leadership conferences emphasized one thing: we all benefit when we treat listening as an active process of learning, used to better know how to serve the students in front of us. We have been fortunate to have such thoughtful leaders help shape the teaching and learning of mathematics here in Ontario.

Listening as a Community

The three leaders mentioned all did something powerful. They genuinely demonstrated the power of listening, both with students and with adults. This pattern has to repeat within our 15 regional chapters. The strength of OAME/AOEM has always been its grassroots foundation. When I look at the work being done in chapters, such as the Rethinking Fractions journey taken by Golden Section Math Association (GSMA) or chapters leading book clubs, I see spaces where we stop being tellers of best practices and start listening to each other's lived classroom realities.

These local gatherings are the mid-sized branches of our fractal. When our chapters listen to members, it creates a community of practice that is responsive and connected. When we gather together to read through a professional resource, we come together to develop a shared vision. We are at our best when we are a community of learners who support one another.

Listening as Leadership

At the largest scale, OAME/AOEM has to operate as a listening organization too. Our role on the Board of Directors is to witness the potential of our members and to lift others up.

Our commitment to equity and excellence is not a top-down statement. It is a pattern that has to be present in everything we do, from how we curate our *Gazette* archives to how we develop new tools like chatOAME. We are practising a form of institutional reciprocity. We aren't just listening to see if people got the message. We are listening to be changed by the feedback and the needs of Ontario's educators. Listening as leadership means actively seeking out a variety of viewpoints, intentionally providing opportunities for members to share their thinking, and genuinely learning and growing because of what we have learned. Our goal is to intentionally select provincial speakers who directly address the evolving equity and pedagogical needs of our classrooms. We are actively building new pathways to engage math educators across every corner of Ontario, ensuring our systemic work directly improves daily student experiences.

A Chance to Reflect

- As the school year is starting, I invite you to look for the fractals in your own teaching.
- Are we providing tasks that are loud enough so that our students can be listened to?
- How can we move from evaluating an answer to witnessing the process?
- How can we continue to engage in more professional discussions and learning opportunities?
- Whose voices are being heard (both at the student and adult levels)? How can we expand who is being listened to?

Mathematics is not a solo performance; it is a collective movement that begins with the simple, profound act of hearing one another. We know our students are more successful when they are empowered to share their thinking, but that empowerment must be mirrored at every level of our system. For our "math fractal" to thrive, we must lead by example: listening to our students, listening to our colleagues, and listening to the diverse perspectives that make our community strong. I look forward to learning with you, listening to you, and seeing the patterns we build together throughout the next two years as OAME/AOEM President. ▲

▲ Introducing the OAME/AOEM Code of Conduct

ELIZABETH PEARSELL
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Elizabeth was a member of the Code of Conduct Ad Hoc Committee and asked to introduce the code.

The OAME/AOEM Code of Conduct was approved at the Board of Directors meeting on May 29, 2026.

As one of the members of the Ad Hoc Committee tasked with developing the Code of Conduct, I was asked to provide some background for this document. The reflections provided here are mine alone. For our members, codes of conduct are commonplace for professional organizations, places of work, institutions of learning, and volunteer organizations. Indeed, in today's world, phone messages, sporting events, and signs posted in shops and restaurants, outlining behaviour guidelines, are becoming more and more commonplace. The overall goal of the committee was to create a document that would set clear expectations for behaviour and to promote a safe, respectful, and inclusive environment for all OAME/AOEM members and other individuals, groups, organizations, and businesses associated with OAME/AOEM. During the drafting and writing processes, we kept *concerns for* and *support of* a person, who was a victim of violations of the Code of Conduct, centremost in our deliberations, along with the continued viability and reputation of OAME/AOEM.

The development of the Code of Conduct followed the steps OAME/AOEM takes to create Position Statements. Those involved conducting individual research and, where possible, reaching out to people involved in the area under discussion. For the Code of Conduct, we looked at the Codes of Conduct of other professional associations and institutions. We then met and shared information, which led to development of key questions to guide the development of the document. We decided on several sections that, upon reflection, seem to answer the questions of Who?, What?, Where?, Why?, and How? Over time, our discussions went from big ideas into drilling down, to describe details and provide examples, and to the use of individual words. The document was shared several times with the OAME/AOEM Executive for their feedback, before the Board of Directors voted for its acceptance as our organization's Code of Conduct.

At all times, the committee understood the importance of this task, as well as the changing nature of education and, so, I would like to start off this article by referencing the last paragraph of the code.

> ***The Board of Directors recognizes that the Code of Conduct needs to reflect the evolving nature of the profession. This Code of Conduct will be reviewed every two years by the Board of Directors. This Code of Conduct is subject to change at the discretion of the OAME/AOEM Executive Committee.***

Have we thought of all possibilities? Have we covered all the concerns? Have we provided a system for our members and those associated with OAME/AOEM to follow, if an environment is not respectful or professional? If the answer to these questions is *maybe not*, we can adjust the Code of Conduct to meet those needs. This, to my mind, is an important part of the code.

The document has six sections: 1) Purpose and Scope; 2) Professional Conduct When Engaging with OAME/AOEM Members and at OAME/AOEM Events, Conferences, and Workshops; 3) Responsible Sharing and Engagement in OAME/AOEM Spaces; 4) Ethical Use of Content; 5) Online Conduct & Virtual Events; and 6) Reporting Violations & Enforcement.

Purpose and Scope outlines one of the problems we tackled—Who is included in this Code of Conduct? At first glance, one might think that OAME/AOEM is an organization of only professional Ontario teachers. While our membership does indeed include current and past educators, our outreach with meetings and conferences, in-person and virtual, extend to other populations. So, while we state, "*OAME/AOEM is committed to fostering a professional, ethical, and inclusive environment that promotes the advancement of mathematics education and aligns with the Ontario College of Teachers' Ethical Standards of care, respect, trust, and integrity*," others such as volunteers, guest speakers, sponsors, and vendors do not fall under the Ontario College of Teachers (OCT) standards and are therefore included in this code.

Professional Conduct When Engaging with OAME/AOEM Members and at OAME/AOEM Events, Conferences, and Workshops lists some examples to clarify "Professional Conduct" in five areas: Respect, Inclusion, and Equity; Academic Integrity & Ethical Teaching Practices; Professional Collaboration & Open-Minded Inquiry; Respectful Engagement & Collaboration; Zero Tolerance for Harassment or Discrimination; and Exhibitor & Sponsorship Guidelines.

In the document's statements, we incorporated words to describe the actions expected, and the boundary with what is tolerated and/or not tolerated. For example, *engage, treat, ensure, support, uphold, acknowledge, embrace,* and

foster are a few of the actions that support "fostering a professional, ethical, and inclusive environment" and the standards of "*care, respect, trust, and integrity*." Also, please note the last subsection directly speaks to exhibitors and sponsors, who, while associated with OAME/AOEM events, are not necessarily members of OAME/AOEM or familiar with the OCT.

Responsible Sharing and Engagement in OAME/AOEM Spaces and **Ethical Use of Content** focus on guidelines for the presenting, sharing, and exchanging of ideas. The statements are directed to presenters and audience members, to ensure respectful discourse.

Once upon a time, OAME/AOEM and its chapters offered only in-person events. The **Online Conduct and Virtual Events** section provides guidelines for the multiple platforms used today. The use of social media during and after an event is addressed as well.

The final section, **Reporting Violations and Enforcement**, outlines several key points. Both those who experience misconduct and those who witness code violations are encouraged to report their concerns. Recognizing the need to provide a safe and trustworthy path that covers a wide variety of provincial and chapter activities, participants can contact OAME/AOEM leadership or event organizers. From there, the OAME/AOEM Executive will respond fairly, sensitively, and promptly, ensuring that victims are treated with compassion, respect, and appropriate support.

From the point of view of an organization, by outlining shared values, responsibilities, and standards of behaviour, the Code of Conduct supports OAME/AOEM in achieving its mission, protects its reputation, and fosters a culture of accountability and trust.

From the point of view of a member or an associate of OAME/AOEM, by adhering to the standards of behaviour in the Code of Conduct, you will contribute to a culture of trust, accountability, and excellence that supports learning, collaboration, and the well-being of all participants. In fact, the Code of Conduct is actively promoting the guiding principles (www.oame.on.ca/main/index.php?lang=en&code=principles) of OAME/AOEM.

OAME/AOEM Code of Conduct

1. Purpose and Scope

This Code of Conduct applies to online and public communications made by members, volunteers, presenters, and other participants whenever they represent or could reasonably be perceived as representing OAME/AOEM. This includes, but is not limited to:

- OAME/AOEM members, volunteers, and leadership
- OAME/AOEM speakers, presenters, and panelists
- attendees at in-person and virtual events and meetings, including individuals engaged on social media connected to OAME/AOEM social media accounts
- OAME/AOEM exhibitors, sponsors, and vendors.
- posting or sharing content that displays the OAME/AOEM name, logo, event materials, or other identifiers (e.g., badges, merchandise, conference slides)
- referring to OAME/AOEM membership, positions, or roles within the organization
- using "OAME" or "AOEM" in usernames, bios, hashtags, or captions

In such cases, members are expected to uphold the values of professionalism, inclusion, and respect outlined in this Code of Conduct. Public statements or actions that undermine these values may be addressed under this policy, even if they occur outside OAME/AOEM-managed platforms.

OAME/AOEM is committed to fostering a professional, ethical, and inclusive environment that promotes the advancement of mathematics education and aligns with the Ontario College of Teachers' Ethical Standards of care, respect, trust, and integrity.

2. Professional Conduct When Engaging with OAME/AOEM Members and at OAME/AOEM Events, Conferences, and Workshops

- Respect, Inclusion, and Equity
- Engage in professional dialogue that honours different perspectives in mathematics education.
- Treat all individuals with fairness, dignity, and respect, regardless of gender, race, ethnicity, ability, socio-economic status, or mathematical background.
- Avoid exclusionary language or behaviour.

Academic Integrity & Ethical Teaching Practices

- Ensure all research, resources, and teaching practices being shared are accurate, evidence based, and transparent.
- Acknowledge sources of ideas and resources, respecting intellectual property.

Professional Collaboration & Open-Minded Inquiry

- Embrace multiple ways of knowing and learning mathematics, including Indigenous, culturally responsive, and community-based approaches.
- Foster a professional environment in all OAME/AOEM activities (in-person and online), where critical thinking, creativity and diverse problem solving are valued.
- Remain open to constructive feedback and engage

in professional learning.

Respectful Engagement & Collaboration

- Support a culture of mathematical curiosity and professional discourse at all OAME/AOEM events.

Zero Tolerance for Harassment or Discrimination

- No verbal abuse, personal attacks, intimidation, or harassment will be tolerated.
- Avoid language or conduct that belittles participants' mathematical abilities or professional contributions.
- Uphold the ethical principles of trust, integrity, care, and respect in all professional interactions (as per OCT Ethical Standards).

Exhibitor & Sponsorship Guidelines

- Exhibitors must align with OAME/AOEM's values in their marketing and engagement strategies.
- "Suitcasing" (unauthorized solicitation and advertising during OAME/AOEM events) is strictly prohibited.

3. Responsible Sharing and Engagement in OAME/AOEM Spaces

OAME/AOEM encourages the exchange of ideas to support professional learning and enhance mathematics education. Whether you are sharing original content or engaging with the work of others:

- Ensure that visual aids, slides, and digital resources are accessible and inclusive.
- Respect privacy laws when using student work, learning analytics, or classroom data in presentations.
- Share and respond in context – Accurately represent the intent of the content when sharing or commenting on it.
- Give credit – Acknowledge original creators when sharing slides, articles, videos, or ideas by tagging or naming them clearly.
- Respect boundaries – If a presenter, author, or participant asks that something not be shared (in part or in full), honour that request.
- Engage with integrity – Whether you're posting, sharing, or responding, ensure your engagement reflects OAME/AOEM's values of inclusion, professionalism, and constructive dialogue.
- Protect our reputation – Public posts referencing OAME/AOEM should uphold our commitment to respectful discourse and improving mathematics education for all.

4. Ethical Use of Content

- Presentations, resources, or examples—especially those referencing geopolitical, cultural, or social content—must be used with clear pedagogical intent and awareness of context.
- Avoid endorsing or opposing political movements or causes unless the content is directly relevant to mathematics education.
- Ensure all shared examples and graphs are accompanied by transparent context, source credibility, and sensitivity to the audience.

5. Online Conduct & Virtual Events

- Maintain professionalism and ethical engagement in OAME/AOEM's online platforms (forums, webinars, social media, etc.).
- Encourage respectful discourse about different mathematical teaching methodologies.
- Do not spam participants during and after virtual events.
- Avoid sharing confidential student or research data and images without proper permissions.

6. Reporting Violations & Enforcement

The enforcement of this Code exists to uphold the safety, integrity, and inclusivity of OAME/AOEM spaces. The goal is not to punish, but to ensure that all members can participate in a respectful and professional environment. We believe in education, dialogue, and fair process. Where conduct falls short of the standards outlined here, OAME/AOEM Executive will address concerns in a manner that is proportionate, transparent, and focused on restoring trust within the community.

- Participants who experience or witness violations of this Code of Conduct should report concerns to OAME/AOEM leadership or event organizers.
- Reports will be handled confidentially, and individuals found violating this Code may face some or all of the listed consequences :
  - Warnings or removal from events/publications
  - Suspension or revocation of OAME/AOEM membership
  - Legal action
  - Reporting to outside agencies, if necessary.

The Board of Directors recognizes that the Code of Conduct needs to reflect the evolving nature of the profession. This Code of Conduct will be reviewed every two years by the Board of Directors. This Code of Conduct is subject to change at the discretion of the OAME/AOEM executive committee. ▲

What is the pattern?

A, E, F, H, I, K, __, __, __, __, ...

How many terms does the sequence have?

OMCA News



GERARD LEWIS
(OMCA PAST PRESIDENT)
OMCAmath@gmail.com

Welcome to a new school year! The Ontario Mathematics Coordinators Association (OMCA) is pleased to begin another exciting year of collaboration and professional growth with mathematics coordinators, consultants, and coaches from across Ontario.

At our final members' meeting in June, outgoing President, Melissa Peddie, passed the ceremonial plant to incoming President, Alyssa Dixon. This September, OMCA will be led by our 2026 Executive Team:

- President: Alyssa Dixon (Halton District School Board)
- Vice-President: Laura Cronshaw (Northeastern Catholic District School Board)
- Past President: Melissa Peddie (Near North District School Board)
- Secretary: Allison Lawrence (Brant Haldimand Norfolk Catholic District School Board)
- Treasurer**:** Nisha Patmanathan (Upper Grand District School Board)

We are proud to announce the recipient of the 2026 OMCA Award for Excellence and Innovation in Mathematics Teaching: Lisa Pilgrim (Halton District School Board). This annual award recognizes an OMCA member who demonstrates outstanding contributions to mathematics education in areas such as curriculum implementation, technology integration, equity, assessment, and professional learning. Congratulations to Lisa on this well-deserved recognition!

Each monthly OMCA meeting is hosted by a different school board and features a reflective land acknowledgment, a highlighted math resource or practice, and an engaging math task. These meetings provide members with valuable opportunities to connect, share successes and challenges, and engage in meaningful professional development. In addition to regular updates from the Ontario Ministry of Education, the Education Quality and Assessment Office (EQAO), the Ontario Association for Mathematics Education (OAME), the National Council of Supervisors of Mathematics (NCSM), and other educational organizations, we welcome guest speakers who share current research and insights relevant to mathematics leadership.

Continued on page 14

OAME/NCTM Report: Fall 2026 NCTM Highlights for Ontario Math Educators



GERARD LEWIS
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Gerry is an Elementary Mathematics Facilitator, who assists teachers of students with IEPs (Individual Education Plans) within the Toronto Catholic District School Board. Gerry has presented over 20 times across Canada and the United States, with a focus on equity, culturally responsive pedagogy, mathematical modelling, and integration of technology in the classroom. In 2024, he completed his Master of Education degree in mathematics education at York University. He is currently the Past President of the Ontario Mathematics Coordinators Association (OMCA), and for his work with mathematics teachers through the pandemic, he was awarded the Prime Minister's Award for Teaching Excellence in 2022.

As teachers prepare for a new school year, the National Council of Teachers of Mathematics (NCTM) Classroom Resources page (www.nctm.org/Classroom-Resources) is an excellent place to find lessons, tasks, and professional learning that can help teachers engage with students in ways that promote rich mathematical thinking, rather than rote procedures. Whether you teach Kindergarten or Grade 12, the resources can help spark meaningful classroom discussions and reduce planning time. On the right-hand side of the web page, you can find free monthly activities that are specifically curated for different grade-level bands.

This past August, NCTM co-hosted the third Joint Conference for Elementary Literacy and Mathematics with their colleagues from the National Council of Teachers of English (NCTE). The concept of literacies transcends subject boundaries, and this conference demonstrated that mathematical literacy goes beyond numbers and includes reading, interpreting, communicating, and applying mathematical ideas in meaningful ways. In the NCTM "President's Message" from July 2026, "Literacy and Mathematics: Using Literature to Support Math Instruction" (www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Latrenda-Knighten/Literacy-and-Mathematics_-Using-Literature-to-Support-Math-Instruction/), Latrenda Knighten shares that integrating literacy strategies into mathematics helps students build conceptual understanding, strengthen reasoning skills, and

connect mathematics to real-world situations. By using authentic children's literature in mathematics, teachers can create learning experiences that foster curiosity, joy, and deeper mathematical thinking. Diverse books also support equitable teaching by helping students develop positive mathematical identities and see themselves—and others—as capable mathematicians from a variety of backgrounds. In her message, Knighten also provides several links for resources that teachers may use to support the integration of literacy strategies in the mathematics classroom. Especially with the launch of the new Kindergarten curriculum in Ontario, these resources may help to support increased access to numeracy integration throughout the school day.

NCTM published its latest position paper, "Advancing Excellence and Access in Mathematics Centering Joy, Powerful Teaching, and Equitable Assessment" in June 2026, and it articulates that excellence, access, and equity require intentionally created learning environments in mathematics, where every student feels a sense of belonging, agency, and joy. Students from all backgrounds, identities, and abilities, including those from historically marginalized communities, should have equitable access to high-quality mathematics learning and opportunities to reach their full potential. As a Math Facilitator whose focus is on supporting teachers of students with IEPs, I can reference this paper to shape conversations I have with administrators, classroom teachers, and special education teachers about options to best support students who are working at different grade levels or who belong to a variety of cultural backgrounds. The paper can be found at www.nctm.org/Standards-and-Positions/Position-Statements/Advancing-Excellence-and-Access-in-Mathematics-Centering-Joy,-Powerful-Teaching,-and-Equitable-Assessment.

For educators who regularly use NCTM resources, becoming a paid member offers significant value. Membership provides full access to NCTM's peer-reviewed journals, such as *Mathematics Teacher: Learning and Teaching PK–12*; members-only classroom lessons; webinars; conference discounts; and a professional network of mathematics educators from across North America. As the school year begins, these benefits can provide fresh instructional ideas and ongoing professional learning that support effective, engaging mathematics instruction throughout the year. Please let me know if you need assistance with NCTM membership or access to resources.

Some upcoming NCTM events include:

- October 22 – ATMNE 2026 Conference Bridging Ideas, Building Futures in Warwick, Rhode Island
- October 28 – NCTM 2026 Annual Meeting & Exposition in Denver, Colorado ▲

▲ Events at the Fields: Two Debates and an Invitation to the Fields Medal Symposium



PAMELA BRITTAIN
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Pamela Brittain (Ph.D.) is an educator, author, illustrator, presenter, and math lover, who currently works for the Fields Institute as their K–12 Academic Coordinator. She has been involved in STEM education for over 20 years and strives to make math accessible and enjoyable for everyone.

Two Lively "Debates"

The April MathEd Forum, entitled "Tensions and Partnerships: Two Debates on Mathematicians in Teacher Preparation and Research Collaboration," was all about prompting discussions about the involvement of mathematics departments and mathematics researchers in math education, and about creating and nurturing collaborations among various mathematical and educational communities. While titled "debates," the sessions were set up to promote discussion between colleagues, as opposed to formally debating each other. This, however, didn't dissuade some opposing, and sometimes heated, dialogues from taking place, both in person and online. By the end of the day, everyone agreed on the importance of such discussions.

The first debate was "On Involvement of Mathematicians in Teacher Education" and asked the overarching question, "Should departments of mathematics be seriously involved in preparing future teachers, for example, by offering service courses that focus more on the process of doing mathematics?" The speakers (Figure 1) were Wesley Burr (Trent University), Cameron Morland (University of Waterloo), Sarah Mayes-Tang (University of Toronto), and Sa'diyya Hendrickson (Ontario Institute for Studies in Education [OISE]). Ami Mamolo (Ontario Tech University) was also scheduled to participate, but was unable to attend.



Figure 1: *Speakers for the first debate*

Presenters were first asked to give their individual answers to the debate question, and then the panel was asked questions by a moderator and members of the audience. For the opening remarks, the presenters all agreed that mathematics departments, and mathematicians should be involved with math education. It was suggested that there needs to be a cultural shift in how departments of mathematics and education view each other and the things they can learn from each other, such as how to form mathematical mindsets, and how to build foundational skills like logical strategies and mathematical definitions. However, they warned that many university mathematics departments don't recognize instructor involvement with mathematics education (such as pedagogical innovation) in tenure and grant applications, and that this causes barriers to involvement. Furthermore, many mathematics instructors are not trained in mathematics pedagogy. This leads many of them struggling to understand effective ways to address gaps in the foundational mathematics knowledge of educators, and that many educators have deeply engrained math anxieties and fears related to the subject. The presenters also discussed how service-learning courses in some institutions have and could be effectively implemented to bridge the divides between various departments.



Figure 2: *The first debate*

In the end, the presenters agreed that math departments should be involved with math education; however, they did so with three main caveats: that both sides must actively collaborate; that pedagogy and teaching need to be recognized just as much as traditional research; and that there needs to be a greater focus on the learner's emotional needs, along with building their mathematical knowledge.

The second debate was "On Research Collaboration Across Communities," which asked "Should a culture of research collaboration between mathematicians, math education faculty, and teachers be established and nurtured, for example, with a SSHRC (Social Sciences and Humanities Research Council)-funded grant?" The debate featured speakers Amanda Cooper (Ontario Tech University), George Gadanidis (Western University), Asia Matthews (Quest University), the *Gazette*'s own Tim Sibbald (Nipissing University), and Cody Wilson (OISE).



Figure 3: *Speakers for the second debate*

This debate was a bit more mixed, with presenters encouraging collaborations, but with varying degrees of support. Some were cautious, warning us not to jump in with initiatives such as the mathematician-led "New Math" of the 1960's, while others focused on the need for professional development and training of mathematicians, with a focus on pedagogical mathematics practices. There were also calls for defining core community values and ways to bridge research and practice in pragmatic ways. Presenters cautioned that we need to recognize things such as gender equity differences between university mathematics departments and mathematics education (including education faculties), and also how major academic sources of funding view mathematics and education differently.



Figure 4: *The second debate*

By the end of the discussion, all the presenters agreed that, while there are a lot of institutional constraints and roadblocks to overcome to make collaboration work, it is vital to improving and addressing issues in mathematics instruction and education. There was also an acknowledgement that the new, compressed teacher education program, which limits mathematics instruction time even further, is an additional foreseeable roadblock.

The day ended with an "open mic" discussion, where attendees were asked to voice their thoughts and opinions on the topics of the day, and the comments were fascinating. Many speakers brought up issues of addressing math traumas and anxieties, with a view of recognizing them as an important part of the mathematical learning process. There were also comments about how many interventions need to start earlier than post-secondary education, as early as elementary schooling, to set a foundation for students to build on. One speaker also brought up the slightly controversial question of whether the goal of education is competence or genuine enjoyment of the subject.

Adding to the conversation was a lively side debate in the online chat, where participants spoke about the need to recognize math anxieties and traumas, and teaching instructors at all levels how to effectively and empathetically address these attributes. A significant discussion also arose regarding the question: "What is mathematics?" This led to questions including: How do we define it?, Do we share a common definition of it? (the consensus was that we don't), and What do we mean when we use the term "mathematics"? This was followed up by the equally interesting question regarding the different values people place on mathematics and how a shared space can move past value differences.

Overall, participants and panelists alike discussed the pitfall and benefits of building more collaborative environments between those who are considered mathematics educators and those considered mathematics researchers. In this author's humble opinion, this dichotomy is a false one. We are facing a situation that clearly requires "all hands-on deck," and while approaches may be different by various constituents, we are all working toward the same goal, and the best way to do that is through empathy, understanding, and true collaboration. It's not about a distinction of us and them. To effectively address this issue, all sides recognize what the others have to offer and build on that together.

You can view the debates at www.fields.utoronto.ca/activities/25-26/meforum-Apr.

And now for something completely different…

An Invitation to the Fields Medal Symposium

Each year, the Fields Institute hosts a Fields Medal winner, who presents a series of talks about their area of research. For those unfamiliar with it, the Fields Medal, named after Canadian mathematician, John Charles Fields, the namesake of the Institute, is often (somewhat misleadingly) referred to as the "Nobel Prize in Mathematics," as there is no Nobel prize in mathematics. There are many wild stories out there for why there is no Nobel prize in math—including one that calls it Nobel's revenge after his wife had a torrid affair with a mathematician (even though Nobel was never married). But most believe the reason is simply that Alfred Nobel felt that mathematics was too detached from the "real world" and did not meet the spirit of the award. As well, there were already other awards for mathematicians available at the time in Sweden. In any case, it was after World War I that John Charles Fields suggested creating a medal to specifically honour mathematics, and this led to the creation of the Fields Medal.

The Fields Medal is awarded every four years by the International Congress of Mathematicians (ICM) to between two and four mathematicians who are under the age of 40. The purpose is "to recognize outstanding mathematical achievement for existing work and for the promise of future achievement." The printing plates for the medal itself are housed at the University of Toronto. You can learn more about the medal (including what the inscriptions mean) and a list of past winners, at www.mathunion.org/imu-awards/fields-medal.



Figure 5: *The Fields Medal (front and back)*

This year, the Fields Institute is honoured to host Hugo Duminil-Copin (Institut des Hautes Études Scientifiques, University of Geneva), who was awarded the Fields Medal in 2022 "for solving longstanding problems in the probabilistic theory of phase transitions in statistical physics, especially in dimensions three and four." "Educated in France and based in Europe, Duminil-Copin is known not only for the depth of his work, but also for his collaborative spirit and exceptional clarity as an expositor. His lectures and writings reflect a belief that deep mathematics should be shared, discussed and, most importantly, enjoyed."

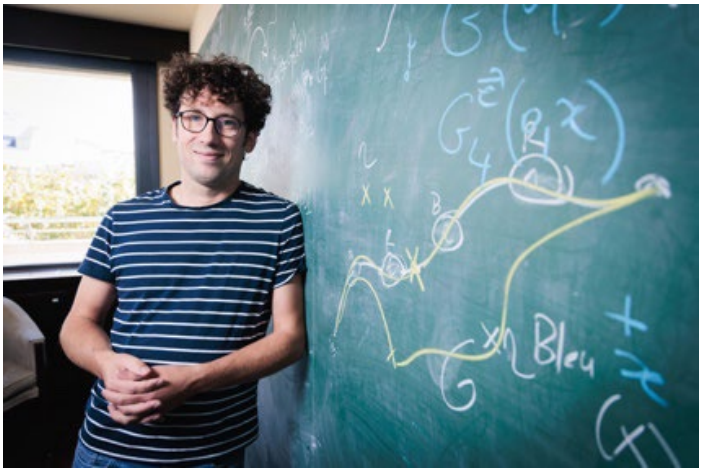


Figure 6: *Fields medalist (2022), Duminil-Copin*

Public Opening and Student Night – Your Invitation to Join Us

The Fields Medal Symposium opens on Monday, October 13 with a public talk, open to all who are interested. Details and registration can be found at www.fields.utoronto.ca/activities/26-27/fieldsmedalsym.

Then, on Tuesday, October 14, the Fields Institute invites high school and undergraduate students to attend an informal evening to meet with Professor Duminil-Copin over pizza and refreshments. Attendees will have the opportunity to ask him questions about a career in mathematics and what it's like to win a Fields Medal. Registration for this event is free, but space is limited. You can sign up at www.fields.utoronto.ca/activities/26-27/FMS-student.

Ongoing Programs at Fields

As we start a new school year, please be reminded that the Fields Institute has a number of programs to help educators promote mathematics in fun and interesting ways. These include:

- the *Ask a Mathematician* program, where we bring math researchers and those in industry virtually into the classroom
- the *Math in Literacy* program, which provides educators with math lessons and resources on diverse topics linked to children's books by authors and illustrators typically underrepresented in math
- numerous other resources, activities, fun videos, and more

Learn more about these free programs at www.fieldsacademy.ca/eep/.▲

Continued from page 10

The 2025–2026 OMCA theme was "Math Connections," and we explored this theme throughout the year. We started the season connecting research to practice through the lens of fractions, then we engaged in strengthening the connections between coaches and teachers through student-centred coaching practices. Our annual retreat focused on reframing math play to build connections among students, teachers, coaches, and communities. The panel discussion at the OAME Conference brought together math leaders to share stories and reflections about our leadership journeys.

If you are a mathematics leader or coach within your board and are interested in joining OMCA, we warmly welcome new members. Please contact us at OMCAmath@gmail.com, or check out our website (www.sites.google.com/view/omcamath/home) for more information about our organization and how to get involved.

We look forward to seeing you at our upcoming Fall meetings, held from 11:45 a.m. to 3:00 p.m. on the following dates:

- Monday, September 28, 2026
- Monday, October 19, 2026
- Monday, November 16, 2026
- Monday, December 14, 2026 – Open House

We hope to connect with you soon!▲

▲ Elementary Math Matters: Where did all the fun stuff go?



LYNDA COLGAN
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Lynda Colgan is Professor Emerita, Faculty of Education, Queen's University, and Founder and Faculty Lead for Science Rendezvous Kingston. In May 2025, she was awarded the King Charles III Coronation Medal by the Chancellery of Honours at Rideau Hall for her lifelong contributions to education. Her latest project is contributing author for an upcoming Springer publication, Teaching and Learning Elementary Mathematics*. Lynda remains deeply committed to her two loves: STEM outreach/public education and her English Cream Retriever, Annie.*

I am a huge fan of Britbox and Acorn TV. On those streaming services, I indulge in mystery series such as *Death in Paradise, A Taste for Murder, The Brokenwood Mysteries*, and *My Life Is Murder.* Most of the programs are "cozy" mysteries that substitute graphic content with small, close-knit communities, charming settings, and endearing sleuths.

One of the latest police procedural series to capture my imagination is *Patience*. The main character, Patience, is a brilliant, self-taught, autistic archivist, who becomes directly involved in criminal investigations, thanks to her exceptional pattern recognition and puzzle-solving abilities. Patience applies her instinctive skills to new and old files, making original connections among cases that help her detective colleagues find solutions to curious cases.

The first episode of the second season of *Patience* ended when she finally realized that a numerical clue left for her by her mother was a *vampire number.* As Patience rattled off the characteristics of such a number, her companion just stared at her in disbelief. My own curiosity piqued—I turned to the Internet for more detail.

It turns out that a *vampire number* is a type of interesting number in recreational mathematics with a specific property involving its digits, defined as follows:

A vampire number is a composite natural number with an even number of digits, which can be factored into two whole numbers called *fangs*.

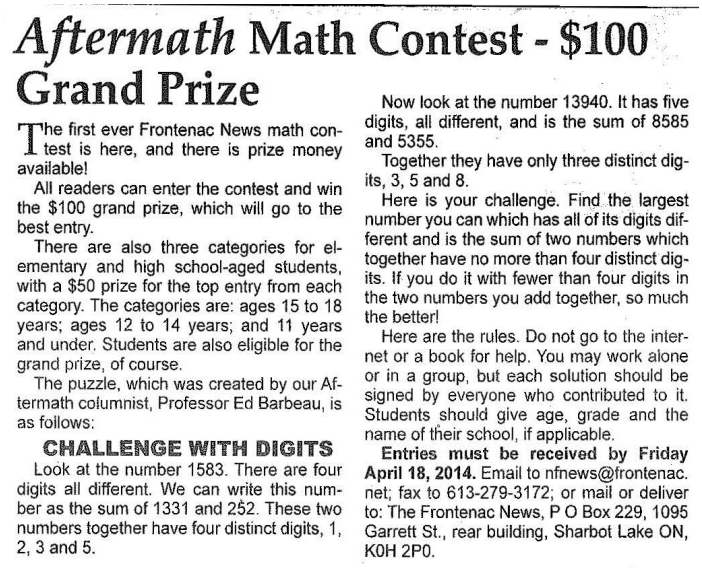
- Together, these fangs contain all the digits of the original number, in any order (they are a rearrangement of the original number's digits).
- Each fang is half the length of the original number.
- Neither fang ends with zero simultaneously (to avoid trivial cases, like both ending with zero).
- The product of the fangs equals the original number.

If we apply the rules above, we can conclude that **1395** is a vampire number because:

- it has four digits
- it factors into **15** and **93**, so that 15 × 93 = 1395
- each fang has two digits
- digits of 15 and 93 are 1, 5, 9, 3, which match the digits of 1395

Armed with the definition and an example (obtained from *LibreChat,* Queen's University's own secure Artificial Intelligence [AI] interface), I then spent some time "playing" to find other two-fanged numbers.

Yes, *playing*—scrap paper, pencils, and calculator at hand as I dusted off my trial-and-error skills. "Playing" was once a standard activity in mathematics class, used well beyond Kindergarten. "Playing" came in the form of recreational mathematics—mathematics just for fun. Recreational mathematics had enormous appeal, with many problems and puzzles appearing in many publications, including small community newspapers such as the *Frontenac News* (as shown in Figure 1).



Aftermath Math Contest - $100 Grand Prize

The first ever Frontenac News math contest is here, and there is prize money available!

All readers can enter the contest and win the $100 grand prize, which will go to the best entry.

There are also three categories for elementary and high school-aged students, with a $50 prize for the top entry from each category. The categories are: ages 15 to 18 years; ages 12 to 14 years; and 11 years and under. Students are also eligible for the grand prize, of course.

The puzzle, which was created by our Aftermath columnist, Professor Ed Barbeau, is as follows:

CHALLENGE WITH DIGITS

Look at the number 1583. There are four digits all different. We can write this number as the sum of 1331 and 252. These two numbers together have four distinct digits, 1, 2, 3 and 5.

Now look at the number 13940. It has five digits, all different, and is the sum of 8585 and 5355.

Together they have only three distinct digits, 3, 5 and 8.

Here is your challenge. Find the largest number you can which has all of its digits different and is the sum of two numbers which together have no more than four distinct digits. If you do it with fewer than four digits in the two numbers you add together, so much the better!

Here are the rules. Do not go to the internet or a book for help. You may work alone or in a group, but each solution should be signed by everyone who contributed to it. Students should give age, grade and the name of their school, if applicable.

Entries must be received by Friday April 18, 2014. Email to nfnews@frontenac.net; fax to 613-279-3172; or mail or deliver to: The Frontenac News, P O Box 229, 1095 Garrett St., rear building, Sharbot Lake ON, K0H 2P0.

Figure 1: *An example of a recreational mathematics problem that appeared in the* Frontenac News *in 2014. The contest puzzle was taken from a collection by Dr. Ed Barbeau, published in 1995.*

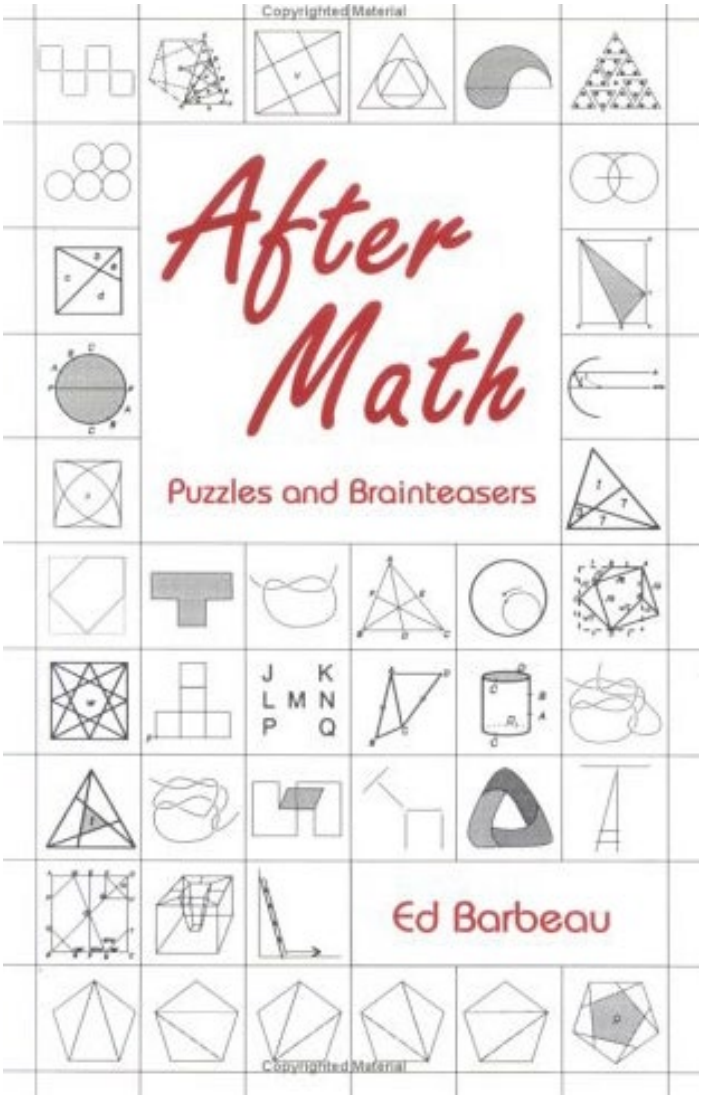


Figure 2: *The cover of Dr. Ed Barbeau's 1995 collection of problems: just one example of his award-winning contributions to the enrichment of mathematics learning through the stimulation of mathematical curiosity*

As I mused about the puzzle in the *Frontenac News* (to which a dear friend had submitted a solution all those years ago), and the *Aftermath* collection from which it was taken (Figure 2), I was reminded of perhaps *the* most highly influential American mathematician and popularizer of mathematics and recreational math puzzles, Martin Gardner. Gardner is probably best known for his "Mathematical Games" column in *Scientific American* (1956 to 1981), through which he introduced a wide range of mathematical concepts, puzzles, and paradoxes to millions of readers. I have several of Gardner's books in my personal library, including *The Colossal Book of Mathematics* and *The Annotated Alice.* In classrooms and professional-development workshops, I often used one of my favourite Gardner puzzles as an "icebreaker" because it never failed to spark lively discussion among learners of all ages:

> *Bill sold his motor scooter to Tom for $100. After driving it around for a few days, Tom discovered that it was in such poor condition that he sold it back to Bill for $80. The next day Bill sold it to Herman for $90. What was Bill's total profit?*

Many of you will remember the variation of Gardner's question, posed by Marilyn Burns, called *Dealing in Horses* (www.youtube.com/watch?v=Wq9gjzZ5Zuw), in which a farmer talks about buying his horse for $50, then selling the horse to a friend for $60. The farmer bought the horse back for $70, but then resold the horse to his friend for $80. The farmer then wonders whether through all these transactions, he made money, lost money, or broke even.

Remembering the spirited debates about the solutions to these questions prompted many other happy reflections about other activities I used in the classroom for "fun." The students quite happily practised their computational or algebraic skills as part of the "games" that I introduced.

Palindrome Addition Practice

I recalled that students shared my enthusiasm about palindromes, especially when they realized that not all palindromes were simple words (e.g., *mom*, *radar*), but could be phrases (e.g., *never odd or even*) and numbers (e.g., *3663*). While not all numbers are palindromes, with a little addition, most numbers can be transformed into palindromes. For example, when you add a number (35) to its reverse (53) you get a palindrome (88). In some cases, more than one addition is required, as in this example: 57 + 75 = 132 and 132 + 231 = 363, which is a palindrome created after 2 steps.

Equipped with a hundreds chart, the students were tasked with using different-coloured pencils to shade all the palindromes, as well as all the numbers that become palindromes in 2, 3, 4, or 6 steps, with kudos going to those students who persevered to find the pair of numbers that is a 24-step palindrome.

Here's a fun fact: *Lychrel numbers* are natural numbers that do not form a palindrome through the reverse-and-add process, at least within a reasonable number of iterations. The smallest and most famous suspected Lychrel number is 196 because despite extensive computational effort, no palindrome has been found by reversing and adding 196 repeatedly.

Divisibility Rules

Not only do people of all ages enjoy magic tricks; they want to know why they work. To introduce the basic divisibility rules, I whet the students' curiosity by starting with this "trick":

1. Pick your favourite number between 1 and 8.
2. Multiply by 3.
3. Add 3.
4. Multiply by 3.
5. You'll get a two-digit number....
6. Add the digits together.
7. On the count of three… shout it out!!!

There are surprised looks when the room resounds with "9" and the students turn to ask their neighbour what their favourite number was—how is it possible that everyone had a different starting number, but everyone ended up with 9? The reason is that everyone ended up with a two-digit number that is a multiple of 9 because two of the steps required multiplication by 3, and $3 \times 3 = 9$. In fact, the sum of all the digits of the multiples of 9 add up to a multiple of 9. For example, 18 is a multiple of 9, and $1 + 8 = 9$. Similarly, 198 is a multiple of 9 because $1 + 9 + 8 = 18 = 2 \times 9$ and $1 + 8 = 9$.

That discussion led to a series of investigations. Is 729 divisible by 9? What about 12 345 or 45 000? How can knowing this rule be helpful? Are there rules for other numbers that can we apply to quickly and mentally test for divisibility, improving our computational speed and accuracy without a calculator?

Number	Rule Description	Example
2	The number is divisible by 2 if its last digit is even (0, 2, 4, 6, 8).	134 (last digit 4, yes)
3	The sum of the digits is divisible by 3.	123 → $1 + 2 + 3 = 6$, 6 divisible by 3, so yes.
4	The last two digits form a number divisible by 4 or are 00.	1,204 → last two digits 04 divisible by 4, yes.
5	The last digit is 0 or 5.	325 (last digit 5, yes)
6	The number is divisible by both 2 and 3.	54 → even and sum digits 9 divisible by 3, yes.
7	Double the last digit, subtract it from the remaining leading truncated number. If the result is divisible by 7 (including 0), then the original is divisible by 7. (You can repeat the process as needed.)	203 → 20 - (2 * 3) = 20 - 6 = 14 divisible by 7, so yes.
8	The last three digits form a number divisible by 8 or are 000.	12,000 → last three digits 000, yes divisible.
9	Sum of all digits is divisible by 9.	729 → 7 + 2 + 9 = 18, 18 divisible by 9, yes.
10	Last digit is 0.	150 (last digit 0, yes)
11	The difference between the sum of the digits in odd positions and the sum in even positions is 0 or divisible by 11.	2728 → (2+2) - (7+8) = 4 - 15 = -11 divisible by 11, yes.
12	Divisible by both 3 and 4.	144 → sum digits 9 divisible by 3, last two digits 44 divisible by 4, yes.

Figure 3: *A summary of the divisibility rules for the numbers 2 to 12*

Secret Numbers

Introducing the notion of variables as a first step in algebra is often fraught with challenges for students. Building on the "9" puzzle proved to be a positive and concrete strategy for teaching the notion that *x* or *n* (or *any* letter) can represent *any* number, as we saw in the earlier puzzle, when everyone started with a personal favourite number, but collectively ended up with *9* as a final result.

Another example is as follows. I ask students to perform this sequence of steps:

- Think of a secret number.
- Triple it.
- Add 12.
- Add your secret number.
- Divide by 4.
- Add 5.
- Subtract your secret number.

Then I would say, "I predict that everyone has '8' on their paper," and smile at the astonished "How did she know?" responses.

Next, to prove that there was no magic, just mathematics behind the "trick," I would "act out" the puzzle, using tiles to represent the secret number (or *x*) and counters to represent the numbers, as shown in Figure 4 below:

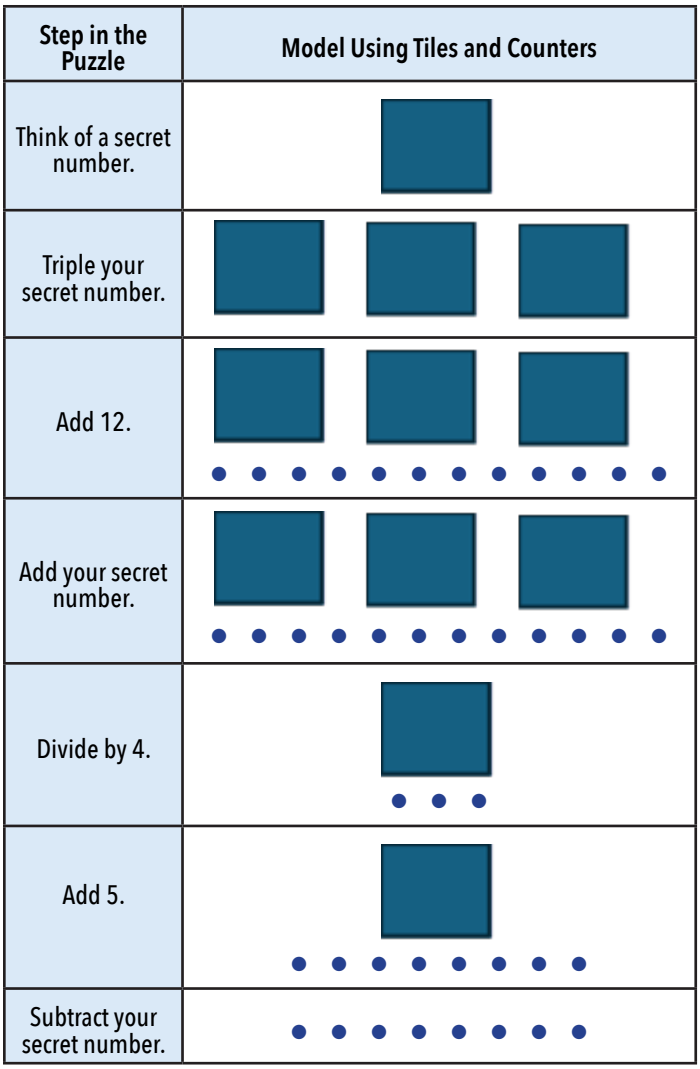


Figure 4: *Using manipulative materials to "act out" a secret-number problem, in which everyone ends up with the same numerical answer*

There was much excitement as students generated their own puzzles; took on the role of mathemagician by asking

their peers to follow the steps in their sequence of instructions; and announced the answer that everyone should have obtained.

Next came a different kind of "secret number" problem:

- Think of a secret number.
- Double it.
- Add 9.
- Add your secret number.
- Divide by 3.

Tell me what you have written on your paper, and I'll tell you your secret number.

This class of problem generated more excited questions about the mathemagic behind the "trick," which was, of course, the impetus for the exercise. So, if the final result is "7," the secret number is 4. Once again, tiles (to represent secret numbers or variables) and counters (to represent numbers) were used to act out the puzzle so that students could then use the sequence to design their own secret number problems.

Step in the Puzzle	Model Using Tiles and Counters
Think of a secret number.	
Double it.	
Add 9.	
Add your secret number.	
Divide by 3. This is the value students say.	
Subtract 3 from the number stated by the student, to get the secret number.	

Figure 4: *Using manipulative materials to "act out" a secret-number problem, in which it is possible to determine* **x** *using subtraction*

Conclusion

The current elementary curriculum is crowded—and while the Ontario Ministry of Education curriculum documents do not provide a single total count explicitly, if one refers to the published documents, one finds roughly 800 specific expectations for Grades K through 8 in total. A crowded curriculum like the one in Ontario, which contains so many important mathematics concepts (in addition to problem-solving and critical-thinking strategies) means that it is challenging (if not impossible) for teachers to "cover" them all thoroughly within the instructional time available. The corollary is that there are fewer opportunities for "just for fun" mathematics activities that have enormous educative value from a cognitive and attitudinal perspective.

Many examples that I used with students (and parents, guardians, and educators) come to mind—making and learning to use a Soroban (please see Figure 5), to consolidate understanding of place value; introducing Egyptian, Gelosia, and "Peasant" multiplication as alternatives to the traditional algorithm for multi-digit multiplication; reading the legend of the Lo-Shu turtle as a starting point for activities with Magic Squares; using string and the story of Gulliver in Lilliput, to open the conversation about ratio and proportion (with the help of da Vinci's Vetruvian Man); learning about the history of hexaflexagons (discovered by a bored student because the size of standard paper in England and North America differed by about one inch); how to fold a basic hexaflexagon that can be used as a fidget spinner; and making metamorphosis tessellations in the style of Escher.



Figure 5: *A mother and son exploring anchor numbers and base-10 place value, using a large-scale and small-scale Soroban (Japanese Abacus) at a Family Math event*

So, thank you, Patience, for igniting my curiosity with your excitement about Vampire Numbers. The reflective journey that ensued is attributable to your genuine excitement about *doing* mathematics puzzles. My hope is that the few examples that I shared will have the same outcome with *Gazette* readers. ▲

▲ Play, Math, and Early Learning: What's the Same?



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The focus of this column is consideration of how young learners conceptualize the ideas of "same" and "different." At first glance, these may not seem like important mathematics concepts, but understanding them is foundational in being able to classify shapes, sort objects, and determine patterns. Classifying shapes refers to organizing shapes based on their properties and attributes. Sorting shapes is an important aspect of the Spatial Sense curriculum in the early years in Ontario (Ontario Ministry of Education [OME], 2020, 2026). In patterning, students determine which aspects of a pattern are the same and which are different so that they can determine the pattern rule or structure.

As in previous columns in this series, the focus is on how the young learner views the concepts of "same" and "different," and how these initial understandings may be different from what an adult would expect or see as they are presenting mathematical situations. To illustrate some of the intricacies of considering what children see, we are going to start with a couple of vignettes about children, and then explore how this relates to mathematics understandings and learning.

As toddlers begin to foster their own independence, choosing their own outfits and dressing themselves become a focus for many young learners. This can also be a place of mathematics learning as you observe how they choose their outfits and ask questions. This is where I discovered how different my son's idea of "same" is from my own. To give an example, my son loves to choose shorts and a shirt that are the "same" to him. He defines this as ones that have the "same" character, no matter the colour or pattern. If the shirt and shorts both have a dinosaur on them, they are the same. Similarly, if they both have Spidey on them, they are the "same." Although he has a shirt and shorts that have an identical pattern of Spidey on them, he did not consider them to be any more the "same" as the patterned shirt with a pair of shorts that just had Spidey's face on them. My colleague, Ann, shared a story about her son wearing shirts and shorts that were dramatically unmatching, and he was convinced they "matched" because "they both have stripes." This led me to consider how this might impact mathematics understandings, and how educators can meet students where they are.

Early concepts in sorting note that the idea is for children "to sort by discriminating same from different" (OME, 2007, p. 29). Although this document does not set this out as "mathematics," it is definitely the early start to ideas in spatial sense and geometry. The idea of "sameness" in mathematics is important when we think about how all the triangles in the group have the same attributes, and this means that they are all classified as triangles. We can further specify the attributes (e.g., acute triangles versus right-angle triangles), but the underlying concept is being able to identify what relates between them. If we consider the vignettes of the two young learners choosing their outfits, what does that tell us about their mathematics learning?

In the vignettes, both learners were identifying an "attribute" of their clothing, and then determining that all clothes with that particular attribute could be worn together. To a parent, the combinations that are created may be horrifying, but the mathematics understanding is clearly developing as they are noticing the "sameness." In the same vein, it is important to help learners refine their understandings of attributes to consider more precise identifying features; in the example of the dinosaur clothes, helping to identify how they can tell the difference between different dinosaurs. In a previous column, we reviewed how children begin to sort objects by determining how they relate to each other and how those schema help to determine the way to group items (Holm, 2026).

These conversations help to lay the groundwork for patterning. In determining the pattern rule in a repeating pattern, the student would need to identify that some of the objects in the pattern are the same and some are different. Especially when you move into patterns with multiple attributes (see Figure 1), these concepts become even more confusing, if students are struggling with identifying how the objects are the same. They may be able to identify

the different shapes as being a pattern, but if they cannot notice that the shapes also have different colours (as may be the case for a child with colour blindness), they may miss the actual pattern rule.



Figure 1: *Two-attribute repeating pattern: The shapes show an ABC pattern, but the colour is an AB pattern. Putting the two attributes together shows this is an ABCDEF pattern.*

When working with young learners, it is important to ask questions to help you see their thinking, since it may not be the same as yours, and to determine the level of mathematical understanding. For example, asking a child with two wildly conflicting sets of stripes how they thought they "matched" would allow some insight into how the child conceptualizes "sameness" as they explore the world around them. It is helpful to set up activities and games, where children can help begin to explore how items are the "same" or "different." Matching games can be helpful for beginning to identify attributes. An initial play of the game may have children matching two cards because they are both "fish," even when the fish look nothing alike. Asking questions and probing thinking can help start to draw their attention to how the fish are different as well. This also helps to begin conversations about attributes, and how one object is actually different, even when they are the same. For example, a Tyrannosaurus Rex is a very different dinosaur from a Brontosaurus, yet they are both dinosaurs. As children advance in creating their own schemas for how items are connected, they can start to make more sophisticated matches.

Where matching games may miss some of the nuances that educators need to see in determining attributes, they are often created in such a way that the objects are vastly different from each other. The card game *Uno* provides a more complex way of helping children see how same and different can be played. In this card game, there are pictures or numbers that are the same, but the colours are different. We have one of the novelty games of *Uno*, where there are characters on each card, but they are different colours from one another. Currently we are at the stage where my son can identify the pictures that match, but he cannot be convinced that the colour matters. The monkey on the card is the same, no matter what the colour happens to be. This early understanding of how they relate will continue to develop as his understanding of attributes develops.

Continued on page 24

▲ MB4T (MATHEMATICS BY AND FOR TEACHERS): MAKING CONNECTIONS TO SOLVE EQUATIONS



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This article examines how students develop an understanding of solving equations through their early experiences with equality of and relationships between quantities. Although equation solving builds on ideas students have been developing for years, it is often presented in the Intermediate grades as something entirely new, with an emphasis on procedural fluency over conceptual understanding. For many students, equation solving becomes a sequence of steps to memorize, such as moving a number to the other side, changing the sign, and isolating the variable, rather than an opportunity to connect new learning with the mathematical ideas they developed in earlier grades. As students compare equal quantities, solve for unknown values, and reason about equivalent expressions, they develop an understanding of preserving equality. By making these connections explicit, teachers can help students use their prior knowledge and see equation solving as a continuation of familiar mathematical thinking, rather than the introduction of a completely new concept.

To make the progression visible, we will explore an equation-solving continuum across concrete, representational, and abstract forms. The examples move from recognizing equivalent quantities with balance models to solving increasingly complex equations involving variables, decimals, and multiple terms on both sides (covering content from the Ontario Mathematics curriculum, Strand C, from Grades 4 to 9).

Let us begin with the first example: $8 + 6 = 4 + x$. In the



Figure 1: *Concrete example of 8 + 6 = 4 + x*

concrete model, students use a balance scale to compare two quantities (see Figure 1). One side contains blocks representing 8 and 6 in different colours, while the other contains a block representing 4 with an unknown quantity, *x*. Here students are asked to determine the value of the unknown by reasoning about what will preserve the balance. As students recognize that one side has a total value of 14, they can remove 4 from both sides, leaving 10 on one side and *x* on the other. They then can verify that inside the *x* envelope, there were 10 blocks. Rather than memorizing a rule about "moving" numbers across an equation, students experience equation solving as a process of preserving equality.

Using the same equation, 8 + 6 = 4 + *x*, the representational form (see left in Figure 2), the concrete materials have been removed, and students can still visualize why subtracting 4 from both sides maintains equality. In the abstract form (see right in Figure 2), students record the equation as 8 + 6 = 4 + *x*. Then 14 = 4 + *x*, followed by 14 – 4 = 4 – 4 + *x*, before concluding that 10 = *x*. Each step corresponds directly to an action already performed on the balance model. Because a balance can be viewed from either side, or physically turned around without changing the relationship between the masses, students can also recognize that 10 = *x* and *x* = 10 communicate the same equality. Once this conceptual understanding has been established, teachers may choose

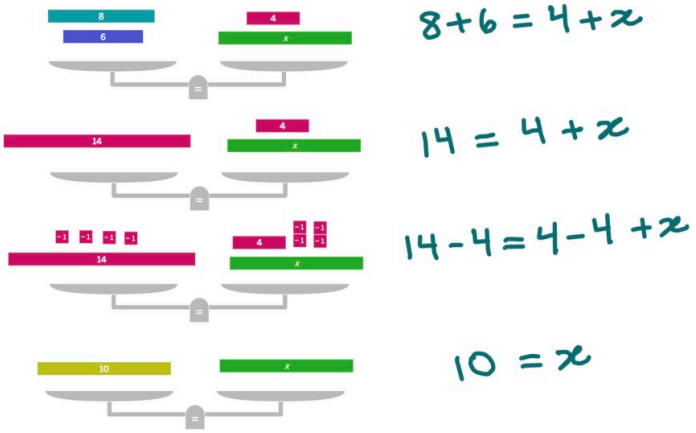


Figure 2: *Representational and abstract example of 8 + 6 = 4 + x*

to adopt the conventional communication practice of writing the variable on the left (*x* = 10), as this is the notation students will encounter more frequently in later grades. This example aligns with Grade 4 expectation C2.2, where students solve equations involving whole numbers up to 50 in various contexts, and verify their solutions (Ontario Ministry of Education [OME], 2020). At this stage, students are beginning to see that solving an equation is not about applying a procedure to isolate a variable, but instead, it is about reasoning carefully to preserve equality.

A second example, 2*x* + 2 = 5 + 6 + 7, extends this reasoning by introducing an equation that requires more than one operation to determine the value of the unknown. In the concrete form (see Figure 3), students use the balance to model two groups of the unknown along with 2-unit blocks on one side (left in the figure), and 5-unit blocks with 6-unit blocks, and an additional 7-unit block on the other (right in the figure). Instead of solving the equation in a single step, students can first determine the total value on one side (right) of the balance and recognize that the other (left) side must represent the same quantity. They then remove 2-unit blocks from both sides, before partitioning the remaining value equally to determine the unknown.

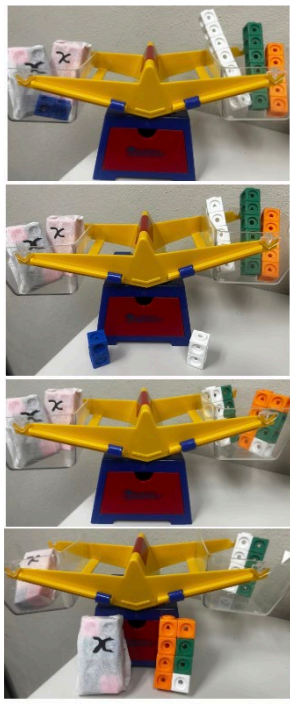


Figure 3: *Concrete example of 2x + 2 = 5 + 6 + 7*

In the representational form (see left of Figure 4), the balance is replaced with a diagram that captures the same mathematical relationships. Students can trace the same reasoning by removing equal quantities from both sides and partitioning the remaining value into two equal groups. The representation reinforces that solving equations may involve multiple operations, but each step is guided by the same idea of preserving equality. In the abstract form (see right of Figure 4), students record the equation as 2*x* + 2 = 5 + 6 + 7. They first simplify the right side to obtain 2*x* + 2 = 18, before subtracting 2 from both sides to find 2*x* = 16, and then divide both sides by 2 to conclude that *x* = 8. Each step mirrors the reasoning developed through the concrete and representational models, with the notation providing a more efficient way to record the conceptual understanding that

has been developed. This example aligns with Grade 5 expectation C2.3, where students solve equations involving whole numbers up to 100 in various contexts and verify their solutions, and naturally extends into Grade 6 expectation C2.3, where students solve equations involving multiple terms and whole numbers in various contexts (OME, 2020).

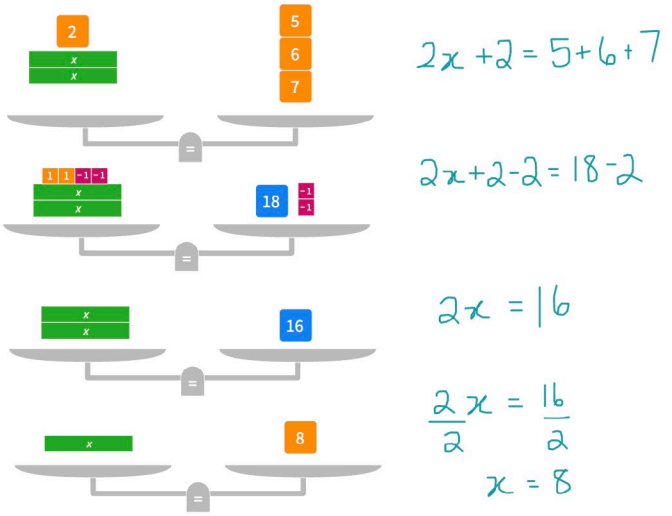


Figure 4: *Representational and abstract example of 2x + 2 = 5 + 6 + 7*

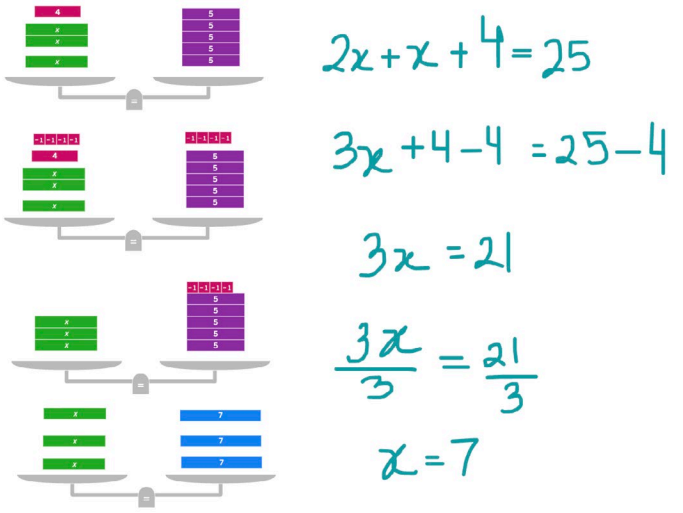


Figure 5: *Representational and abstract example of 2x + x + 4 = 25*

A third example, $2x + x + 4 = 25$, extends this reasoning by introducing multiple unknowns on the same side of the equation (see Figure 5). In the representational form, students recognize that the two variable blocks, together with the single variable block, represent three equal groups of the same unknown quantity. At this stage, the emphasis is on seeing the three equal groups, rather than immediately rewriting the expression as $3x$. Once students recognize the left side as three equal groups, together with 4-unit blocks, the subsequent actions follow naturally from maintaining balance. Removing 4 units from each side isolates the three equal groups, without changing the relationship between the two sides. The remaining total of 21 can then be interpreted as the combined value of the three equal groups, making the final step one of equal sharing, rather than the application of a relatively abstract division. Students record the equation as $2x + x + 4 = 25$. Having already identified the three equal groups in the representational model, they now express $2x + x$ as $3x$, giving $3x + 4 = 25$. They then subtract 4 from both sides to obtain $3x = 21$, before dividing both sides by 3 to conclude that $x = 7$. Students continue to justify each step by preserving equality. This example aligns with Grade 6 expectation C2.3, where students solve equations involving multiple terms and whole numbers in various contexts, and verify their solutions (OME, 2020).

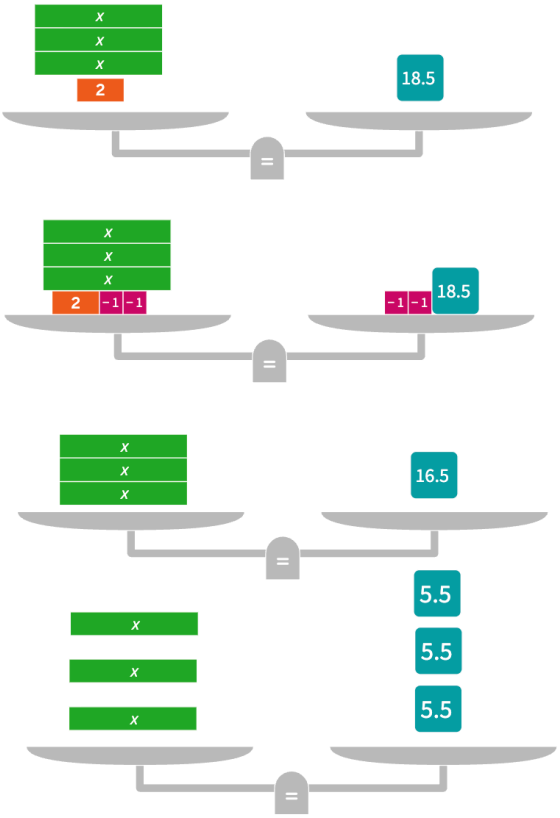


Figure 6: *Representational example of 3x + 2 = 18.5*

A fourth example, $3x + 2 = 18.5$, extends this work by introducing decimal numbers (Figure 6). Although equations involving decimals may appear more challenging, students continue to interpret the equation as three equal groups of an unknown quantity, together with 2-unit blocks balancing a total value of 18.5. Once the 2-unit blocks are removed from both sides of the balance, the remaining value of 16.5 is understood as the combined value of the three equal

groups. Determining the value of the unknown therefore becomes an act of equal sharing, with each group representing 5.5. In the abstract form, students mirror this reasoning by writing $3x + 2 = 18.5$, subtracting 2 from both sides to obtain $3x = 16.5$, and then dividing both sides by 3 to conclude that $x = 5.5$. Here students are encouraged to generalize that the principles of preserving equality and interpreting division as equal partitioning apply equally to whole numbers and decimals. This example aligns with Grade 7 expectation C2.3, where students solve equations involving multiple terms, whole numbers, and decimal numbers in various contexts, and verify their solutions (OME, 2020).

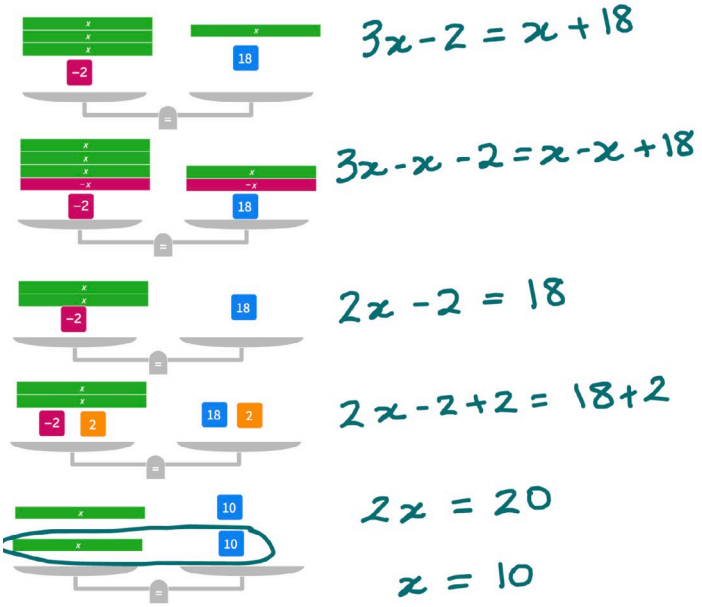


Figure 7: *Representational and abstract example of 3x – 2 = x + 18*

A fifth example, $3x – 2 = x + 18$, extends this reasoning by introducing variables on both sides of the equation (see Figure 7). This is often when students are introduced to procedural rules, such as "move the variable to one side," and even if these strategies help students arrive at a solution, they can hide the conceptual understanding that makes each step valid. Getting Junior students to use the balance model (in concrete and/or representational form) emphasizes that the same principles developed in earlier grades continue to apply. Students recognize that both sides contain equal groups of the unknown, and that removing one variable block from each side preserves the balance while simplifying the relationship. Then the remaining equation can be interpreted as two equal groups, together with two negative-unit blocks, balancing a total of 18. Removing the negative blocks isolates the two equal groups, and the remaining value of 20 is then partitioned equally to determine that each unknown has a value of 10. The abstract form mirrors this reasoning, where students solve $3x – 2 = x + 18$ by subtracting x from both sides to obtain $2x – 2 = 18$, then adding 2 from both sides to obtain $2x = 20$, before dividing both sides by 2 to conclude that $x = 10$. This example aligns with Grade 8 expectation C2.3, where students solve equations involving multiple terms, integers, and decimal numbers in various contexts, and verify their solutions (OME, 2020).

The final example, $\frac{x}{2} = \frac{25}{5}$, extends the continuum into Grade 9 by introducing an equation involving division (see Figure 8). The representation shows that half of an unknown quantity is equal to 5. Rather than viewing division as an operation to be reversed, students reason from the relationship itself. If one-half of the unknown is equal to 5, then two equal halves must represent the whole, giving a value of 10. The model makes visible why multiplying both sides by 2 preserves equality. This example aligns with Grade 9 expectation C1.5, where students create and solve equations in a variety of contexts and verify their solutions (OME, 2020).

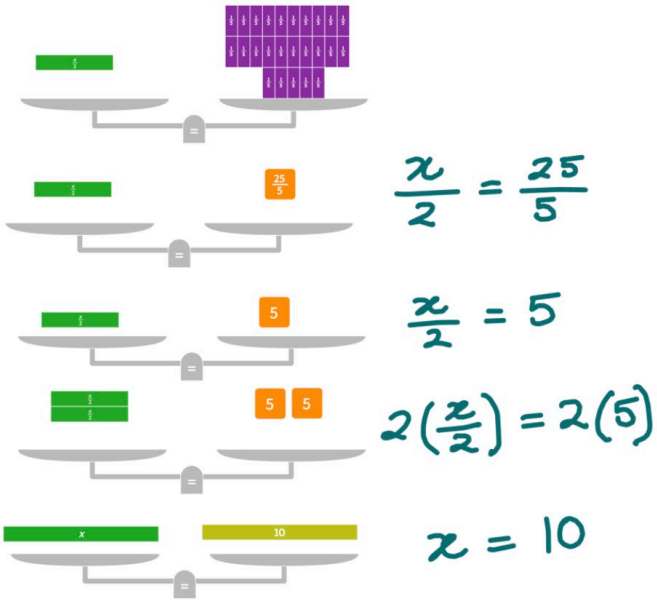


Figure 8: *Representational and abstract example of x/2 = 25/5*

When students experience this progression through concrete, representational, and abstract forms, they are far more likely to view equation solving as connected, rather than procedural. The balance model provides a powerful visual representation that supports this development. It allows students to justify each step they take, seeing that every operation performed on one side of an equation must also be performed on the other to maintain equality. Instead of memorizing rules such as "move the number to the other side and flip the sign," students develop a conceptual understanding, and over time, the visual models gradually

become internalized.

In my own teaching, students have become less concerned with remembering which operation comes next, and more interested in explaining why a particular step is mathematically justified. Instead of asking, "What rule do I use?," they begin asking, "What can I do to both sides to keep the equation balanced?" This shift in thinking is an important one. Students begin to recognize that the equations they encounter in Grade 9 are built upon ideas they have been developing since the Junior grades. For additional ideas for your classroom, I recommend trying Pyper's E-Kwatee-Ons games (Pyper, 2013). I am interested to hear how you introduce equation solving, and whether you use learning progressions similar to those described here. Sharing our experiences is one of the best ways to continue growing as mathematics educators, so feel free to reach out!

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Continued from page 20

As with other columns, play and conversation with early learners leads to interesting investigations of how children begin to understand mathematics. These conversations are so important in seeing how a child works through mathematics because their developing understandings may miss some of the sophistications that an adult may see. For children to continue to feel confident in their knowledge and investigation of mathematical concepts, being able to meet them where they are is key. Asking questions and observing how a child interacts with their world can lead to observations about how they are working with early mathematics concepts. They do not need to know it is "mathematical" to work with it, but as educators, we need to understand the mathematical power in their play.

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▲ Meaning Making in the Classroom: Metacognitive Regulation and Problem Solving



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In my previous column, I took time to highlight the importance of metacognition and how it is the engine that drives thinking (Marzano, 1998). This column adds to the topic by considering metacognitive regulation. Metacognitive regulation refers to how one controls one's own thinking throughout the learning process (Stanton et al., 2021). There are three key components within metacognitive regulation:

- *Planning:* deciding on suitable strategies to use for a task and when they will be used
- *Monitoring:* tracking one's understanding, progress, and the quality of performance during the task
- *Evaluating:* assessing the success of the task and the chosen strategies, and then making any necessary adjustments to strengthen future performances

Self-questioning can be encouraged as a strategy that promotes metacognitive regulation. When students self-question, they refine their thinking and develop a better understanding of the concept being explored and themselves as learners. This column will continue the exploration of metacognitive regulation by suggesting a framework as to how it can be applied throughout the problem-solving process.

Considerations for the Teacher

Self-questioning, as a metacognitive strategy, can have

a significant impact on student understanding. When used effectively, self-questioning can contribute to a student's metacognitive regulation. It is important to recognize the importance of the teacher in supporting students with such self-questioning within the problem-solving process, from start to finish. Teachers, however, are encouraged to be mindful in applying this instructional shift:

1. *Explicit Modelling*
   The teacher must emphasize the importance of self-questioning, model how and when to apply it, and provide support to students in sharing their experiences with this strategy.
2. *Communication*
   The teacher must be cognizant of how students not only self-question during their work, but also about how students communicate their self-questioning after the work is completed.

Teachers must provide opportunities for students to engage in self-questioning throughout the problem-solving process. When teachers make space for this, they encourage students to take ownership of their learning and become independent learners.

Metacognitive Regulation Throughout the Problem-Solving Process

Metacognitive regulation is not intended to happen at only one part of the problem-solving process. It can and should happen throughout the problem-solving process—from start to finish. When students engage in metacognitive regulation throughout the four stages of problem solving, students strengthen their understanding of the problem and themselves as learners.

The following table provides self-questioning prompts that students can ask themselves as they move through the four stages of problem solving (Costello, 2026).

Stage of Problem Solving	Self-Questioning Prompts
Understanding the Problem	• How does the problem relate to other problems I have solved? • What connections can I make to the problem? • Can I draw a picture to represent the problem? • Can I restate the problem in my own words? • Can I break the problem into smaller parts?
Crafting the Plan	• Do I need an exact answer or an estimate? • Have I attempted a problem like this before? If yes, what strategy did I use? • Does my plan make sense based on my understanding of the problem? • What representation will help me solve the problem: visual, concrete, numbers, graph, table, etc.? • How will I know if my plan is working?
Carrying Out the Plan	• Have I correctly applied the strategy? • How can I verify the result? • Am I following my plan? • Did I carry out the plan to completion? • Am I missing any steps in the plan? • Do I need to reconsider my plan?
Reviewing Your Work	• Does my response address the original question? • Could I have used another strategy to solve the problem? • Were there opportunities to be more efficient in solving the problem? • What can I take from this experience to help me with future problems? • How do I know that my response is correct? • How would I communicate my process to someone else?

The self-question prompts included in this table are not intended to be a checklist or a list that students must go through when solving problems. The prompts are intended to be suggestions for how students can strengthen their metacognitive regulation through self-questioning as they move through the four stages of problem solving. Metacognitive regulation can be emphasized from the onset of the problem-solving process through to the completion.

Final Thoughts

Metacognition cannot be thought of as an add-on. It is too important to think of it as being that simplistic. Instead, metacognition must be thought of in terms of metacognitive regulation—the actions that people take to control their thinking. When considered in this way, we can all agree on the crucial role it plays in supporting students as problem solvers.

As I stated in my previous column, self-questioning, as a metacognitive-regulation strategy, can have a significant impact on student learning and their independence when problem-solving. When students can use self-questioning to control their thinking during problem solving, they strengthen their ability to be self-reliant problem solvers.

I look forward to continuing the discussion of mathematical learning and instruction via my website (www.costellomath.com), email (david@costellomath.com), or X (@dr_costello).

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▲ In the Middle: Made in Canada: STEAM Comes Home



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Carly Ziniuk teaches Grade 12 Data Management and Advanced Placement Statistics at the Bishop Strachan School in Toronto. She is active in adopting real-life data to engage her students in solving problems. Among 30 global STEM leaders, Carly was part of the 2024 NASA Space Apps Collective. As a Science Communicator, she is working to connect astronomy and mathematics for learners of all ages across Canada.

At the wonderful OAME 2026 Conference in May, "Let's Talk Math," in London, I was delighted to work with Paul Alves, Ron Lancaster, and Kevin Spry to create a Math Walk. We found math everywhere around Fanshawe College. I was inspired to explore further—with places across Canada viewed through a math lens and supported by STEAM (Science, Technology, Engineering, Art, Math)-connected questions that this column reveals. No doubt these questions have been influenced by Ron's "Mathematical Snapshots" column (see the *Gazette* archives), and I am tremendously grateful for the mathematical creativity I have experienced with Ron, both in person and in these pages, for over 30 years.

This column will start with some special Canadian summer travel experiences! Perhaps this exploration will encourage you to have some mini-adventures here at home, taking your own photo-based "Mathematical Snapshots," or adapting mine with your students.

I have been developing these photos as a series of postcards, which you can adapt for your students from www.canva.com, or print for your classes. My website, www.linktr.ee/CarlyZiniuk, contains Canva designs, largely based on my own travels, and photographs, along with math questions on the back. There are separate versions of the postcards with teacher answers as well. I hope to continue with this series for the next few columns, sharing even more Made in Canada STEAM adventures.

Made in Canada: Little Canada

As an example of the postcards, "Little Canada" is shown in Figure 1 and explained below.

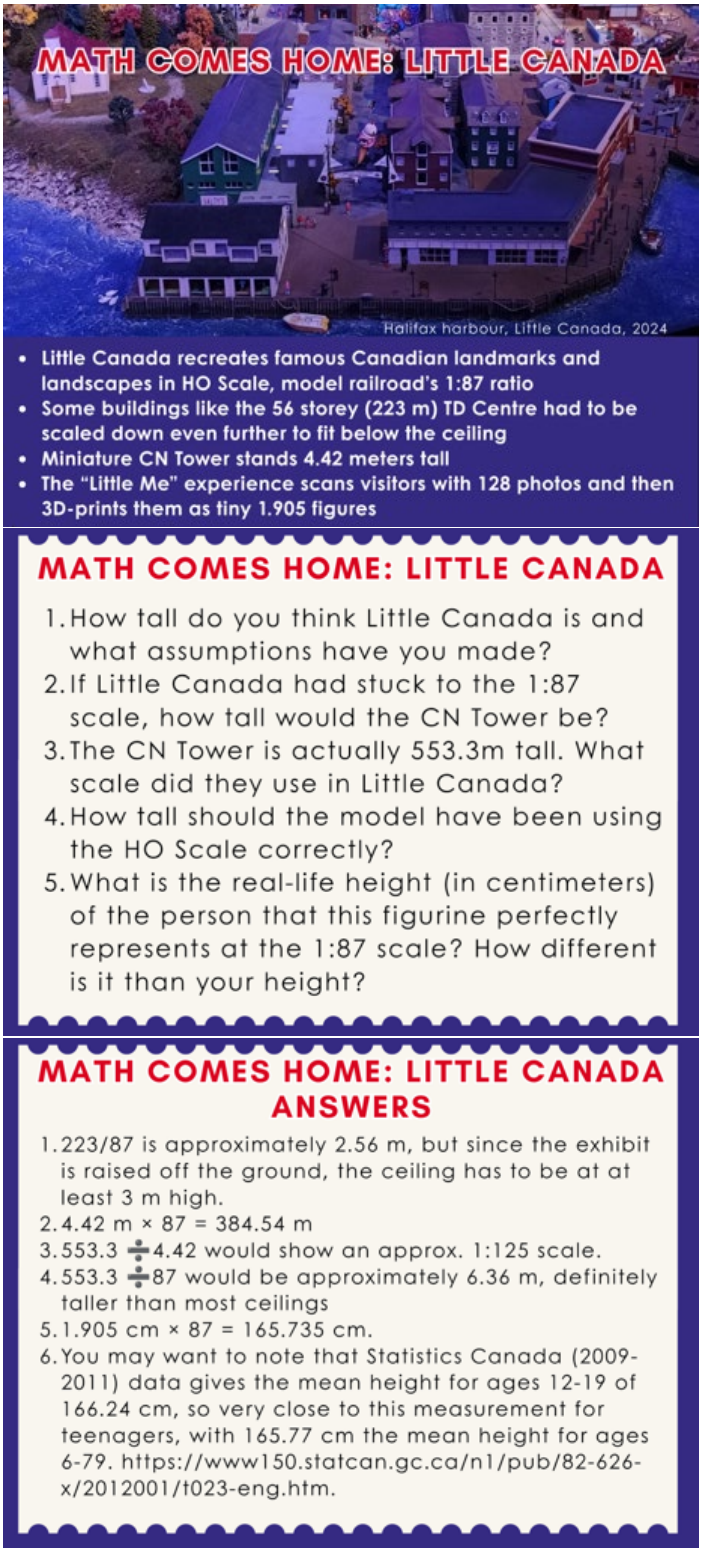


Figure 1: *Little Canada sample of the Made in Canada series, postcard front images with: information (left), postcard back with student questions (middle), postcard back with teacher solutions (right)*

Celebrating its 5th birthday this summer, Little Canada represents Canadian landscapes and landmarks (see Figure 2), using HO Scale. This most popular model train

gauge represents the real world with a 1:87 ratio, where 1 unit on the model equals 87 units in the real world. What began as Dutch-Canadian Jean-Louis Brenninkmeijer's model railroad hobby, Little Canada has become a tourist attraction in downtown Toronto, featuring high-tech mechatronics and soundscapes. A collaboration with nearby Toronto Metropolitan University provides students with the opportunity to design 3D-printed structures such as the Japanese-Canadian War Memorial in Vancouver's Stanley Park.



Figure 2: *Little Canada's recreation of Halifax's waterfront*

Little Canada notes that some exceptionally tall buildings had to be scaled down from the 1:87 scale so that they can fit below the ceiling, including the miniature CN Tower standing at 4.42 meters tall. Visitors to Little Canada can purchase the "Little Me" experience, where they are scanned with 128 photos used to produce a 3D-printed tiny figure, which you can have sent to your home or included within the displays. The 3D prints are then made at the "Littlization Station," with a height of 1.905 cm.

Questions about Little Canada

1. How tall do you think Little Canada is and what assumptions have you made?
2. The CN Tower is actually 553.3 m tall. What scale did they use in Little Canada?
3. If Little Canada had stuck to the 1:87 scale, how tall would the CN Tower be?
4. How tall should the model have been, using the HO Scale correctly?
5. What is the real-life height (in centimeters) of a 3D print from the Littlization Station, perfectly represented at the 1:87 scale? How different is it from your height?

Little Canada Answers

1. 223/87 is approximately 2.56 m, but since the exhibit is raised off the ground, the ceiling has to be at least 3 m high.
2. 553.3 ÷ 4.42 would show an approximately 1:125 scale.
3. 4.42 m × 87 = 384.54 m
4. 553.3 ÷ 87 would be approximately 6.36 m, definitely taller than most ceilings.
5. 1.905 cm × 87 = 165.735 cm.

You may want to note that Statistics Canada data (2009–2011) present the mean height for ages 12–19 of 166.24 cm, so it is very close to this measurement for teenagers, with 165.77 cm the mean height for ages 6–79 (www150.statcan.gc.ca/n1/pub/82-626-x/2012001/t023-eng.htm).

If you choose to go to Little Canada, be on the lookout for many whimsical touches throughout your visit, like the many pop-culture references in the hotel rooms of Chateau Laurier. One of my favourite mathematical jokes is nearby: check out the teeny signage of the Connaught Building, home to the Canadian Revenue Agency (Figure 3).

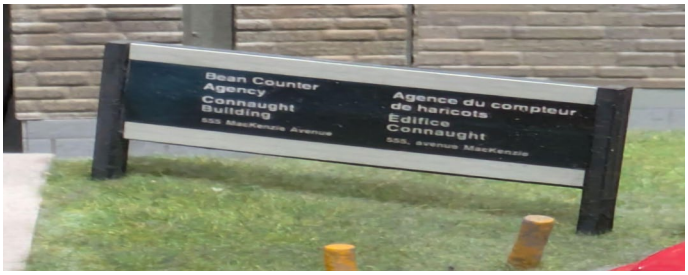


Figure 3: *The sign outside the Connaught Building representation from Little Canada's Ottawa*

Made in Canada: Lego® Mosaic



Figure 4: *Lego mosaic in Union Station, near UP Express platforms*

Built during the 2024 International Auto Show in Toronto, then moved to Union Station, this miniature representation of Canadian landscapes is made fully of Lego (Figure 4). The mosaic, highlighting a cross-country journey across Canada, took ten days to make and contains approximately 222 000 bricks. The details surrounding the mosaic (Figure 5)

claim a cross-country distance calculation that would be 627 million bricks end to end, creating a Lego pathway from Vancouver to St. John's.



Figure 5: *Details surrounding the Lego mosaic*

Research by University of Montreal's Denis Cousineau (1996, 2000, 2002) indicates:

> The most accurate technique to measure small things is to measure a lot of them, and then divide by the number of measured objects. To measure the length of a brick, I noted that in 1 meter (100 cm), exactly 125 bumps could be assembled side to side. This led to the simple calculation that 1 bump is four-fifths of a cm, or 0.8 cm. Similarly, in a meter, I can put 104 bricks on top of each other, so that their height is 100/104 cm.

Cousineau (2002) later indicates that 440 bricks together weighed 1.1 kg on a grocery scale (www.mapageweb.umontreal.ca/cousined/lego/3-Physics/Measurement/index.htm). This facilitates determining the weight of a single brick, which would require a more sensitive scale.

Questions about the Lego Mosaic

1. How far is a 627-million brick path across Canada in km?
2. The CN Tower's height is 553.3 m, while Figure 5 indicates the mosaic details showing the CN Tower would be 57 626 bricks. If this is true, how tall would a single brick be?
3. Does your brick calculation match the experimental result of 104 bricks stacked to one meter?
4. Using a decimal "equivalent" of 0.96 cm for brick height, how far off would the 57 626 bricks be when compared to the CN Tower?
5. How much would the 57 626 Lego bricks weigh?

Lego Mosaic Answers

1. Multiplying the 627 million bricks by 0.80 cm gives 501 600 000 cm. When converted, this is exactly 5016 kilometers, which is the straight-line or flight distance between the two cities (as verified by Google Maps).
2. The CN Tower is 553.3 m, or 55 330 cm. Dividing 55 330 by 57 626 gives each brick approximately 0.9601568736 cm.
3. Multiplying the 57 626 bricks by 100 and dividing by 104 cm gives approximately 55 409.6 cm, or about 553.4 meters, slightly higher than the CN Tower!
4. Multiplying the 57 626 bricks by 0.96 cm gives 55 320.96 cm, or about 553.2 meters. 57 626 bricks × (100 ÷ 104 cm) = 55 409.6 cm. This shows that two slightly different scales, with a tiny difference of 0.0015 cm per brick, correspond to almost a 1-meter difference when multiplied by 57 626 bricks!
5. 1.1 kg is equivalent to 1100 g. Thus, if 440 blocks weigh 1100 g, then a single standard Lego brick weighs 2.5 g. The weight of the 57 626 bricks needed to stack up to the height of the CN Tower would be 144 065 g, or 144.065 kg. For your reference, a Giant Panda or an upright piano is about 150 kg, so approximately the same weight.

Made in Canada: CN Tower

The CN Tower is celebrating its 50th anniversary in 2026, having first opened in 1976. Until 2007, its 553.3 m height held the Guinness World Record for the world's tallest freestanding structure (www.cntower.ca/50). To celebrate this half-century milestone, the Royal Canadian Mint has released a $2 commemorative coin. The CN Tower hosts an annual World Wildlife Fund (WWF) fundraiser, "Climb for Nature." Since 1991, more than 140 000 participants have climbed the 1776 steps to the top of the CN Tower. Each climber must raise a minimum of $125 in donations for WWF.

Questions about the CN Tower

1. What is the average height of one step? What assumptions have you made?
2. How many Lego bricks tall is one average CN Tower step?
3. The minimum donation of $125 dollars corresponds to how many cents per step?
4. If you placed exactly one new $2 commemorative coin on every stair, what would be the value of these coins?
5. What is the average number of WWF climbers who have conquered the steps each year?

CN Tower Answers

1. 553.3 m is 55 330 cm. Divide the total height by the total number of steps, 1776. 55 339 ÷ 1776 steps are approximately 31.2 cm per step.
2. Divide the total number of bricks by the total number of steps. 57 626 bricks ÷ 1776 steps = 32.4468... bricks, so each step is approximately 32 bricks tall. Another

way to consider this is one brick is 0.96 cm, so the previous answer, 31.2 cm per step, would be 31.2 cm per step ÷ 0.96 cm per brick, would be 32.5, which rounds to 33 bricks!

3. Convert dollars to cents, optional, but helpful for the final answer: $125.00 = 12 500 ¢. Divide by the number of steps: 12 500 ¢ ÷ 1776 steps = 7.038... cents per step. The participant raised approximately 7 cents for every single step they climbed!
4. Multiply the number of steps by the value of the coin, 1776 steps × 2.00, so the total value of the coins on the staircase would be $3552.
5. Calculate the total number of years the event has happened: 2026 – 1991 + 1 = 36 years. Calculate the average participants per year: 140 000 people ÷ 36 years = 3888.88... people. The event has had on average approximately 3889 climbers per year.



Figure 6: Union Station historical numbers

An informational display in Union Station (Figure 6) states that there are 22 columns lining Union Station's Front Street entrance, and each column is 12 m tall. The display also notes that if you stacked all 22 columns, one on top of the other, their combined height would be exactly half the height of the CN Tower.

What questions might your students come up with to connect Union Station's columns and Lego blocks by way of the CN Tower?



Figure 7: *Postcard designed around the 50th-season logo for the Toronto Blue Jays*

Made in Canada: Toronto Blue Jays

The Toronto Blue Jays are also celebrating their 50th season, with their first official game held on April 7, 1977. Did you know that for this first game, the Jays needed snow to be removed from the CNE's Exhibition Stadium by a Zamboni on loan from the Toronto Maple Leafs? This year's celebration includes a special logo and some historical trivia www.mlb.com/bluejays/fans/50-seasons! You may recall that the Toronto Blue Jays won the World Series in 1992 and 1993. Sadly, they came in second in 2025, losing to a nail-biter in Game 7.

NCTM each year runs the Year Game, with the rules to use the digits in the year and the operations +, –, ×, ÷, ^ (exponentiation), sqrt (square root), and ! (factorial), along with brackets, absolute value, and concatenation, to write expressions for each of the counting numbers 1 through 100 (www.nctm.org/Classroom-Resources/Year-Game/The-Year-Game/). Many people challenge themselves to also keep the digits in the year's order.

Questions about the Toronto Blue Jays' 50th Anniversary

Using the NCTM Year Game rules:

1. How many ways can you create the number 50, using the digits 1, 9, 7, and 7?
2. How many ways can you create the number 1, using the digits 1, 9, 9, and 2?
3. How many ways can you create the number 1, using the digits 1, 9, 9, and 3?
4. How many ways can you create the number 2, using the digits 2, 0, 2, and 5?

Toronto Blue Jays' 50th Anniversary and Year Game Sample Solutions

Some fun example solutions with the year digits in order:

- $1^9 + 7 \times 7 = 50$
- $-1x\,(9 \div 9) + 2 = 1$
- $-1 + -(9 \div 9) + 3 = 1$
- $(2 + 0!)! - (2 \div 5) = 2$

You can also find many online versions of the Blue Jays classic logo recreated by students on Desmos. Perhaps we could challenge our students to adapt the 50th-season special logo. Where can you see some symmetry in this 50th-anniversary logo? What functions would you use?

Made in Canada: Our Canadian Flag

You may recall that last year, the Canadian flag celebrated its 60th birthday, raised for the first time on Parliament Hill on February 15, 1965. Perhaps you noted vertical line symmetry in the maple leaf on the Toronto Blue Jays logo and some rotational symmetry in the 11 points on the leaf. Canadian flag protocol carefully states where the 11 points on the maple leaf on the Flag must be (www.canada.ca/en/canadian-heritage/services/flag-canada-description.html). The original 1964 design (by historian George Stanley and artist Alan Beddoe) had 13 points on the leaf, but was simplified for easier recognition as a maple leaf at a distance.

Officially the flag is twice as long as it is wide, with a white square in the middle. The flag is shown as 64 units in length and 32 units wide, with a 32-unit square (Figure 8, taken from the website link above). Take a note of the algebraic representation at the bottom of the figure: $A = 32$. You could also consider an orientation with the origin (0,0) placed in the flag's bottom left corner.

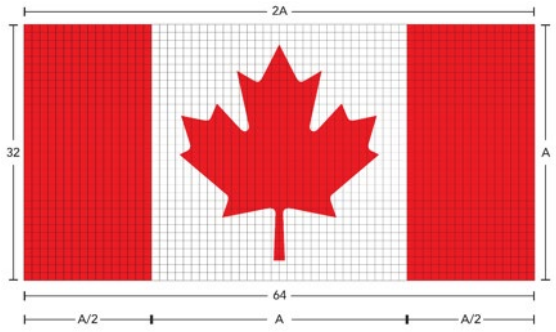


Figure 8: *Technical specifications on the Government of Canada flag description web page*

Questions about Our Canadian Flag

1. What percentage of the flag's total area is the white square (ignoring the leaf)?
2. What are the dimensions of the two red bands on the flag's left and right sides?
3. With the origin (0,0) in the bottom left corner, what are the coordinates of the 11 leaf points? *Hint: Using symmetry here could help checking for accuracy!*
4. What would be the equation of the vertical line of symmetry for the entire flag?
5. What are the equations of the boundaries where the white square meets the rectangular red sections?

Canadian Flag Answers

1. The flag is 64 units long and 32 units wide, and its centre features a white square that is exactly the width of the flag (32 × 32 units). This means that the square is exactly half of the flag.
2. Subtracting the white square's 32-unit width from the total 64-unit length leaves 32 units to be split equally between the two red sides. The red rectangles must be distributed on either side, one-quarter each. Since the flag's base is 64 units, each red rectangular band is 16 units wide by 32 units high.
3. Use the Desmos file here: www.desmos.com/calculator/lretjr1umq (Figure 9).

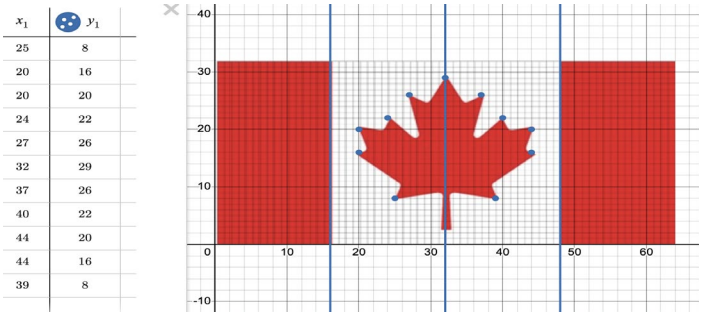


Figure 9: *Desmos version adapted from the official Canadian flag*

4. $x = 32$
5. $x = 16$, $x = 48$

Artists often use grids to create accurate representations of images. They use scaling, and closely examine only one section of the grid at a time, instead of a much larger picture. For example, if you want to paint a giant Canadian flag on a gym wall that is 4.8 meters long, how many centimeters does a single "unit" on this grid in Figure 9 represent for a 4.8-meter-long flag?

Another 50-year anniversary that many sad fans will honour this year is the Toronto Maple Leafs' last Stanley Cup victory on May 2, 1967. Interestingly, the current Toronto Maple Leafs logo, among the logos in Figure 10, includes many more points on the maple leaf, and you may be curious to create your own math questions, using these meaningful numbers. It was a significant change upon the team's 100th birthday in 2017 (https://ssl.mlse.digital/leafs/sweater/index.html).

- The Maple Leaf logo has 31 points representing Maple Leaf Gardens' opening in 1931.
- Thirteen veins at the top consider their winning 13 Stanley Cups.
- Seventeen veins total to consider the 1917 founding of the team.

Looking at Figure 10, what mathematical questions

might you ask in comparing the team logos? How does the team logo compare to the maple leaf on the Canadian flag?



Figure 10: *Toronto Maple Leafs logo evolution since 1963*

Made in Canada: Pride!

Many things have changed around Canada since the building of the CN Tower, the establishment of the Canadian flag, the opening of the Toronto Blue Jays, and the Toronto Maple Leafs' last Stanley Cup victory. The foundation for 2SLGBTQI+ rights in Canada was laid in 1971, with the first demonstrations in Ottawa and Vancouver. By 1973, the movement had grown, with Pride events taking place in Montreal, Saskatoon, Toronto, and Winnipeg. To represent this expanding and diverse community, the first rainbow Pride flag was designed a few years later in 1978. Since then, the flag has evolved alongside the movement. In Canada, it has been a decade since Prime Minister Justin Trudeau first raised a Pride flag on Parliament Hill on June 1, 2016. Since 2024, the Inclusive-Intersex Pride Flag (often called the Inclusive-Progress Pride Flag) has been the official Pride flag of the federal government (Figure 11). Let's consider the mathematical proportions behind the evolution of these newer symbols across Canada (www.canada.ca/en/women-gender-equality/pride-season.html).

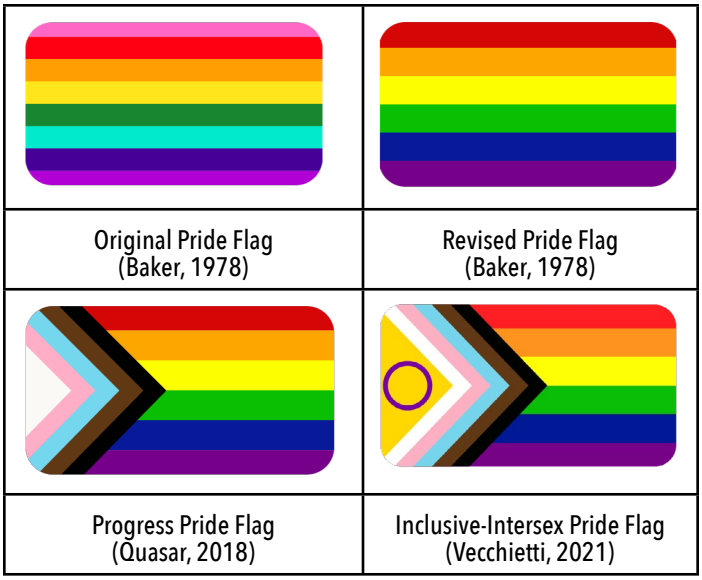


Figure 11: *Evolving Pride Flag designs since 1978, as represented on the Government of Canada website*

Applying the strict 2:1 proportions and the 64 × 32-unit grid of the Canadian flag to the evolution of the Pride flag provides excellent opportunities for geometry and fraction problems. The total area of the 64 × 32-unit flag is 2048 square units. A similar activity that was inspirational for me is Nathan Day's activity in Interwoven Math: www.interwovenmaths.com/pride-flag-maths.

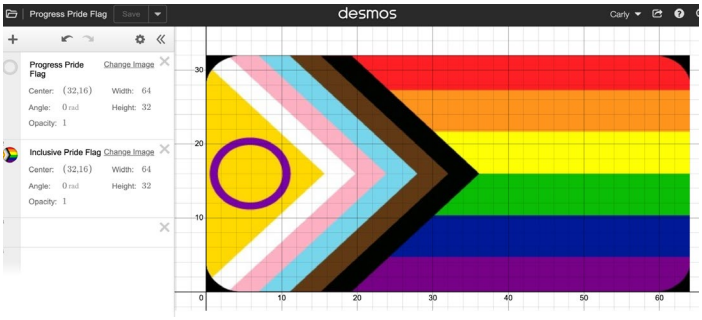


Figure 12: *Desmos file showing the Inclusive Pride Flag on a 64 × 32 grid*

I have placed the image of the Government of Canada: Pride Season's Flag (Figure 12) into Desmos (www.desmos.com/calculator/jtmtrob99g), and it appears that the angle of the Chevron is 45 degrees, with the apex of the black chevron slightly more than halfway, sitting at 36 units.

Questions about the Pride Flags

1. What is the exact height of each horizontal stripe on the original eight-stripe flag?
2. What is the exact height of each stripe for the revised six-stripe flag?
3. What percentage of the flag's total area is covered by the chevron? If the purple circle is drawn with a ten-unit

diameter, what is its circumference?

4. To ensure the chevron and its circle maintain symmetry across the middle of the flag, what must the *y*-coordinate be for the exact centre of the purple circle?
5. What are the equations of the six lines that provide boundaries to the light blue region?

Pride Flags Answers

1. Dividing the 32-unit height by 8 means each stripe has an exact height of 4 units.
2. Dividing 32 by 6 gives an exact height of 16/3 units as a fraction. As a repeating decimal, this is 5.33... units.
3. The full rainbow rectangle formed on the right (to the point of the chevron) begins at 36 and goes to 64 units. Thus, the full area will be 28 multiplied by 32, or 1008 square units. There are two additional triangles of rainbow above and below the chevron, with the base going from 20 to 36, which is 16 units and height of 16. So, both triangles together would be 16^2, or 256 square units. The total without the chevron would be 1008 + 256, so 1264 square units. As a fraction of the total, 1264 divided by 2048 would be approximately 62%, leaving 38% for the chevron.
4. The circumference is 10π units, or approximately 31.4 units.
5. The horizontal line of symmetry must be at $y = 16$ units.
6. The lines $y = x - 7$, $y = x - 11$, $y = -x + 39$, and $y = -x + 43$ are good middle-level approximations, given the scale presented here.

Public transit across Canada, including Waterloo, Ottawa, Vancouver, and Fredericton, uses the rainbow motif to wrap buses and trains in their fleet. Every June since 2008 in Toronto, the Toronto Transit Commission (TTC) has featured a custom-wrapped bus, which operates on various high-visibility routes and appears in major community marches throughout Pride Month. The 2026 Pride Bus celebrates key historical milestones for both the TTC and the city of Toronto (Figure 13). What math questions could we create about the dates on the bus?



Figure 13: *TTC 2026 Pride Bus*

For many years, I have found public transit to be a source of many mathematical curiosities. In 2008, in fact, I designed a math trail on the TTC subway system, specifically highlighting public art underground, the speeds of the trains, and the distances between the stops. My next column will spend more time looking at the math of public transit and transportation across our incredible country. Happy mathematical travels!

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What is the pattern?

9, 8, 11, 6, 13, __, __, __, __, ...

Can you think of two distinct ways to define the pattern?

▲ What's the Problem? Mysterious Multiplication



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Shawn is a former mathematics teacher, department head, and consultant. He strongly believes in the central role of problem solving in the mathematics classroom. He continues to be involved in mathematical activities in his retirement. The resource page for this column can be found at www.bit.ly/4g1JjC5.

Welcome back, problem solvers. I hope you had a wonderful, restful summer that left you recharged and ready for a new school year. I was at home, relaxing, in my pyjamas, coffee in hand, thinking of you all on that first day. Let's start the school year off right, with a problem.

> *The number 2013 is multiplied by a positive integer* n*. The last four digits of the result are 2025. What is the sum of the digits of the smallest possible value of* n*?*
>
> *(A) 17 (B) 13 (C) 15 (D) 14 (E) 16*

This was problem 23 from the 2025 Gauss Contest for Grade 8 students. The Gauss Contest is one of many contests run by the Centre for Education in Mathematics and Computing (CEMC) at the University of Waterloo. Check out the CEMC website, www.cemc.uwaterloo.ca, for past contests and other resources for teachers and students.

Before attempting the problem, we need to clarify a few things. First, the original problem was in a multiple-choice contest, hence the choices and the desire for the sum of the digits, which avoids giving away the answer. Also, since we are considering this for classroom use, we can drop some of the constraints contest writers would face; that is, we will allow the students to work together, as well as utilize any technological aids they wish. As such, let's reword the problem for use in the classroom.

> *Find a positive integer* n*, such that when the number 2013 is multiplied by* n*, the last four digits of the result are 2025.*

If students can find *any* such number, we will be happy. Hopefully this will lead us to want to investigate *all* such numbers, and hence we might also be interested in the smallest one. Let's start by playing around with the problem. In cases where a strategy does not readily come to mind, trying some values in the problem can lead us to see some patterns, which may suggest a strategy. We could try some systematic trials with a calculator, but that might take a while. Let's use a spreadsheet like the one shown in Figure 1.

n	2013n	n	2013n	n	2013n	n	2013n
1	2013	5	10065	25	50325	25	50325
2	4026	10	20130	50	100650	125	251625
3	6039	15	30195	75	150975	225	452925
4	8052	20	40260	100	201300	325	654225
5	10065	25	50325	125	251625	425	855525
6	12078	30	60390	150	301950	525	1056825
7	14091	35	70455	175	352275	625	1258125
8	16104	40	80520	200	402600	725	1459425
9	18117	45	90585	225	452925	825	1660725
10	20130	50	100650	250	503250	925	1862025
11	22143	55	110715	275	553575		
12	24156	60	120780	300	603900		
13	26169	65	130845	325	654225		
14	28182	70	140910	350	704550		
15	30195	75	150975	375	754875		

Figure 1: *Using a spreadsheet to solve the problem*

In the two columns on the left, we have *n* and its product with 2013. We could search down the list (a long way) until we find a solution. However, if we notice that 2025 ends with a five, and that when *n* is a multiple of five, the product ends with a five or a zero. This means we can adjust our strategy; rather than looking at all values of *n*, we can look at multiples of five, focusing on the last digit of the product.

In the next pair of columns (columns 4 and 5), we consider when *n* is a multiple of five. Since 2025 ends with 25, we look for products 2013*n* that end with 25. As with the multiples of five, it becomes apparent that when *n* is a multiple of 25, the last two digits of the product are multiples of 25 as well: 25, 50, 75, or 00. So, we can adjust our strategy again and look at when *n* is a multiple of 25, which we do in the next pair of columns (columns 7 and 8). From there, focusing on the last two digits being *exactly* 25, we see that it happens every hundred numbers: 25, 125, 225, … . The last two columns (columns 10 and 11) focus on these values of *n*, and we find a solution 925 × 2013 = 1 86**2 025**.

There are some adjustments we could have made to streamline our strategy. In the first step, rather than looking at all the multiples of five, we could have concentrated on 5, 15, 25, 35, …, that is, the ones that end in five. Similarly, instead of looking at the multiples of 25, we could have jumped immediately to the last step. This is the power of having students discuss their solutions—rather than just "checking answers"—they can see the subtle differences in methods and can learn from one another.

There are some things we could have done to make our search a bit easier. Since we are only interested in the last

four digits of our product, we could have set up our spreadsheet to give us this, rather than the whole product. This can be done using the MOD function, which returns the remainder of a division. That is, MOD(1 862 025, 10 000) would return the remainder when 1 862 025 was divided by 10 000, which would return the last four digits (why?). I discuss how to set up the spreadsheet in Figure 1, as well as a few modifications we could do in a YouTube video on my resource page. You can access the video through the What's the Problem? resource page at www.bit.ly/4g1JjC5.

This is also a nice problem to solve with coding. The following pseudocode program will yield the solution we found with the spreadsheet. Try it out by writing a program in your favourite programming language.

```
n = 1
prod = 2013
while prod <>2025 do
      n = n + 1
      prod = prod + 2013
      if prod > 10000 then prod = prod – 10000
end while
print n
```

Notice at each step we add 2013 to the product and add one to *n*. The "if" statement allows us to just look at the last four digits. That is, if *prod* surpasses 10 000, and becomes a five-digit number, we just subtract 10 000, returning it to a four-digit number while preserving the last four digits. The loop keeps repeating this process until we reach our goal.

When programming, one must be careful to avoid infinite loops—sets of instructions that the program runs repeatedly, never stopping. The problem implies that there is a solution, so we should be alright. If you were unsure, you could pick a limit for *n* and build it into the loop so that if a solution isn't found by a certain point, you abandon the effort.

Let's use some clues discovered so far, to move from a technological solution to one based on mathematical properties. Since we want the last four digits to be 2025, a number with units-digit 5, we know that the product is a multiple of five. How can we use this to our advantage? How can we represent any number that ends in 2025?

If we look at the solution we found with the spreadsheet, it can be written as

$$1\ 862\ 025 = 1\ 860\ 000 + 2025 = 186 \times 10\ 000 + 2025.$$

In general, for any positive integer m, the number will be a number ending in 2025, whose leading digits are the digits of m. Thus, we can write our problem mathematically as

$$2013n = 10\ 000m + 2025.$$

In this form, it becomes clear that n must be a multiple of five, since 10 000 and 2025 are multiples of five—and hence, so is $10\ 000m + 2025$—while 2013 is not. We can go even further in this form, since 10 000 and 2025 are both multiples of 25, n must also be a multiple of 25. Letting $n = 25N$, our equation becomes

$$2013(25N) = 10\ 000m + 2025,$$

which, after multiplication, yields

$$50\ 325N = 10\ 000m + 2025.$$

Writing $50\ 325N = (5 \times 10\ 000 + 325)N = 325N + 5N \times 10\ 000$, we can then subtract the multiple of 10 000 from each side of the equation, yielding

$$325N = 10\ 000(m – 5\mathrm{N}) + 2025$$

or, for simplicity,

$$325N = 10\ 000M + 2025,$$

where $M = m – 5N$. Since each term in the equation is divisible by 25, we can divide through by 25, giving us

$$13N = 400M + 81.$$

We have reduced the problem to one with smaller numbers. However, the reduction does not seem to have helped. Nevertheless, let's continue pursuing divisibility, keeping in mind that we are after the smallest N. The right-hand side of the equation above must be divisible by 13. Looking at multiples of 13, our equation could be rewritten as

$$13N = 390M + 10M + 78 + 3$$

or

$$13N = 13(30M + 6) + 10M + 3.$$

Since we are after the *smallest* value of N, we need $10M + 3$ to be a multiple of 13, and be as small as possible. Trivially, if $M = 1$, $10M + 3 = 10(1) + 3 = 13$, a multiple of 13. This works and gives us

$$13N = 13(30 + 6) + 13 = 13 \times 37,$$

which yields $N = 37$. Working back, we get $n = 25N = 25 \times 37 = 925$ and, ultimately, $925 \times 2013 = 1\ 86\mathbf{2\ 025}$.

So, where do we go from here? There are several lines of investigation we could follow. We have found one solution. Are there others? If so, how are they related? Let's see what we can get from this line of reasoning.

Assume that a and b are positive integers, with $a > b$, such that when either is multiplied by 2013, the last four digits are 2025. That is, we can write

$$2013a = 10\ 000A + 2025$$
$$2013b = 10\ 000B + 2025$$

for some positive integers A and B, with $A > B$ (why?). Subtracting the equations yields

$$2013(a – b) = 10\ 000(A – B).$$

Continuing with divisibility, we know that 2013 and

10 000 share no prime factors. Hence, $A - B$ must be a multiple of 2013, and $a - b$ must be a multiple of 10 000. From this, we can reaffirm that 925 was the smallest value that works, since any two numbers that work must be a multiple of 10 000 apart, so there are no positive values less than 925 that work. The family of solutions is

$$925 \times 2013 = 1\,86\mathbf{2\,025}$$
$$10\,925 \times 2013 = 21\,99\mathbf{2\,025}$$
$$20\,925 \times 2013 = 42\,12\mathbf{2\,025}$$

Hopefully, this gets you thinking. There was nothing special about the last four digits being 2025. When we looked at the difference between two values that worked, the 2025 disappeared from our analysis, leaving only the 2013 and 10 000. Thus, the argument above would work for *any* four digits. That is, if we wanted to find numbers n such that $2013n$ ends with 4321, we know that the solution set of numbers would be an arithmetic sequence with common difference of 10 000. Since there are 10 000 different possibilities for the last four digits of the product $2013n$, this seems to suggest that if we look at the last four digits of $2013n$ as n moves through the positive integers, then they will go through every possible four-digit number and then repeat with a period of 10 000. Amazing! This turns out to be true and hints at a result from group theory, although, in this case, it would be difficult to observe. A student, or their teacher, might need a bit of evidence to convince themselves.

Just as 2025 was a "random" choice for the last four digits, who says we must stick to four digits? Using the same logic as earlier, suppose we wanted multiples of 2013 to end with 5, rather than 2025. If we adjusted our argument above to look at only the last digit, we would expect it to repeat every 10, which it does, as shown in the spreadsheet in Figure 2.

n	2013n	last digit	n	2013n	last digit	n	2013n	last digit	n	2013n	last digit
1	2013	3	11	22143	3	21	42273	3	31	62403	3
2	4026	6	12	24156	6	22	44286	6	32	64416	6
3	6039	9	13	26169	9	23	46299	9	33	66429	9
4	8052	2	14	28182	2	24	48312	2	34	68442	2
5	10065	5	15	30195	5	25	50325	5	35	70455	5
6	12078	8	16	32208	8	26	52338	8	36	72468	8
7	14091	1	17	34221	1	27	54351	1	37	74481	1
8	16104	4	18	36234	4	28	56364	4	38	76494	4
9	18117	7	19	38247	7	29	58377	7	39	78507	7

Figure 2: *Examining the last digit of multiples of 2013 with a spreadsheet*

Similarly, if we look at the last two digits of multiples of 2013, they cycle through every possible two-digit number (including 00), and then repeat with a period of 100. We seem to have discovered a property of the last digits of multiples of a number.

However, we must be careful before making sweeping statements. Going back to the original problem, if we reversed the roles of the numbers, we would be looking for a multiple of 2025 that ends in 2013. It shouldn't take much to convince yourself that there are no such numbers. So, what happened in this case? The key is, once again, divisibility.

When we started working mathematically, we deduced that since 2025 and 10 000 are multiples of 25, while 2013 is not, we needed to multiply 2013 by a multiple of 25. If the roles of 2013 and 2025 were reversed, our starting equation would be

$$2025n = 10\,000m + 2013,$$

which can be rearranged to yield

$$2025n - 10\,000m = 2013.$$

Since each term on the left is divisible by 25, the term on the right must also be divisible by 25, but it is not, so there are no solutions. Hence, if we look at the last four digits of multiples of 2025, they can *only* be four-digit numbers that are multiples of 25. There are only 400 such four-digit numbers, so the last four digits of multiples of 2025 cycle through the 400 four-digit multiples of 25 (i.e., 0000, 0025, 0050, 0075, …) in some order and repeat with period 400.

Let's see if we can solidify this idea by exploring the easiest possible case, that is, single-digit numbers. In Figure 3, we look at the last digit of multiples of two, three, four, and five. We see in each case that the pattern repeats. If we look at the entry before the first repetition, in each case, it was a zero. Once we hit zero, our next multiple will end with the same digit that we started with, and therefore, the pattern of digits will repeat. I'd like to thank the editor (Steven Khan), who pointed out the use of string-art patterns (modulo arithmetic on a circle or clock arithmetic), which is used at times in the elementary grades, may be a useful connection for some readers. Here is a nice web sketchpad link: www.sineofthetimes.org/stars-polygons-and-multiples/. Interested readers should also check the archives for this column in the June 2020 issue.

So how do we get a number ending with zero? It must be a multiple of ten. When do we arrive at a multiple of ten? Obviously, if n is a multiple of ten, then multiplying it by any single-digit number will give a product that ends in zero. We see, from the table, that for some digits, this also happens at other places. Where else can it happen?

Suppose we are dealing with the single-digit number d. If we want a multiple of d to end with a zero, then we need $dn = 10k$, for some integer k. If d and ten are relatively prime (share no common factors other than one), then n must be a multiple of ten. However, if d and ten share a factor greater than one, what happens then?

n	2n	last digit	3n	last digit	4n	last digit	5n	last digit
1	2	2	3	3	4	4	5	5
2	4	4	6	6	8	8	10	0
3	6	6	9	9	12	2	15	5
4	8	8	12	2	16	6	20	0
5	10	0	15	5	20	0	25	5
6	12	2	18	8	24	4	30	0
7	14	4	21	1	28	8	35	5
8	16	6	24	4	32	2	40	0
9	18	8	27	7	36	6	45	5
10	20	0	30	0	40	0	50	0
11	22	2	33	3	44	4	55	5
12	24	4	36	6	48	8	60	0

Figure 3: *Looking for patterns in the last digit of multiples of two, three, four, and five*

Since $10 = 2 \times 5$, there are only two cases we need to look at: d is even (two, four, six, or eight), or d is five. If d is even, then we can write $d = 2D$ for some positive integer D, and then we must have $2Dn = 10k$ or $Dn = 5k$. Since $D \in \{1,2,3,4\}$, n must be a multiple of five. Checking Figure 3, we see that for $d = 2$ and $d = 4$, this is indeed the case. That is, the last digit of the product runs through the five multiples of two in some order (2, 4, 6, 8 for two; 4, 8, 2, 6 for four), before becoming zero and repeating the process. The reader can verify that similar things happen when $d = 6$ and $d = 8$. When $d = 5$, we can go through the same analysis to see that the final digit alternates between five and zero.

The fact that the final digit of multiples of five is either a five or a zero is well known. Similarly, most people will know that the final digits of multiples of two cycle through the even single-digit numbers in order. However, many may not have recognized the similar feature for the last digit of the multiples of four, six, and eight. Furthermore, since the final digit of the multiples of two, four, six, and eight cycle though a periodic sequence of period five, we have discovered a shortcut for remembering our times tables for these values. That is, we only really need to know the products of these numbers with 1, 2, 3, and 4. Multiplying by five gets us halfway to the product with ten, so we can make a reduction. For example, when I was young, for whatever reason, I always had trouble remembering the value of 8×7, so I would always calculate it another way. Here we can take advantage of our pattern:

- 8×5 is halfway to $8 \times 10 = 80$, or 40
- 7 is 2 more than 5
- $8 \times 2 = 16$, so $8 \times 7 = 16 + 40 = 56$

This algorithm works for the product of any even digit with a number greater than five. After some practice, it would come quicker so that you could calculate, for example,

$$6 \times 9 = 30 + 24 = 54.$$

Where can we go from here? We can put some of our discoveries together with the fact that $100 = 2^2 \times 5^2 = 4 \times 25$ to come up with a modification of our problem. If we pick any multiple of four, for example 76, and look at the last two digits of its multiples, they will cycle through the multiples of four from 00 to 96 in some order. I will leave it to the reader to convince themselves that this is true. Hence, we can pick another multiple of four, for example, 44, and reword our original problem as:

> *Find a positive integer* n *such that when the number 76 is multiplied by* n*, the last two digits of the result are 44.*

This problem retains all the elements present in the original. However, since we are dealing with a multiple of four and looking at the last two digits, there are only 25 numbers to check (actually, 24, since multiplying *any* multiple of four by 25 yields a number ending with 00). Thus, a brute-force search could find a solution and get the investigation started.

The original problem is tricky, and many of the elements might not naturally come out in the students' investigations because the numbers are large. However, by playing around with it, we discovered several interesting offshoots, as well as discovering the underlying structure of the problem. This allows us to rework the problem into one where students are more likely to engage successfully. When they solve the easier problem, they could be pointed down any of the avenues we have explored, or they could be pointed to the original problem. There is a lot of bang we can get for our buck with this problem.

It is worth checking out the official solutions to the problem from the CEMC website. Their second solution uses some elements of divisibility, like some of the investigations above. However, their first solution uses the multiplication algorithm in an interesting way. I suspect many students might use a similar method to address the problem, so have a look.

Over the years, I have posted online supplementary material for this column. A few times, I have set up websites, where the resources could be stored. However, the methods of sharing have become outdated, and many of the shared resources have been lost. To fix this problem, I am setting

up a section of my Google Drive, dedicated to What's the Problem? and its resources. You can access an index to the resources at www.bit.ly/4g1JjC5 (which, for the record, is short for www.docs.google.com/document/d/1ht2W3v_ROEXCAU1INQNU1LKt8oU-0LJ56ejNnKdOLAg/edit?usp=sharing). I have also been going back through my old columns and building resources, where previously there were no resources, adding some videos to explain some of the technological components. Check out the page from time to time to see what has been added.

A few years ago, I suspended the homework section of this column. My intention for doing this was to try to make the column more interactive and to solicit some problems, and perhaps some classroom insights from some of the readers. However, my original idea for using homework was also to make the column interactive. This gave readers a few months to play with the problem on their own, before they got to see what sort of fun I had with the problem. I regretted getting rid of the homework, so I have reinstated it. I am always delighted to hear from readers about their own investigations with the problem, or with problem suggestions of their own. Please feel free to reach out to me by email about anything to do with problem-solving.

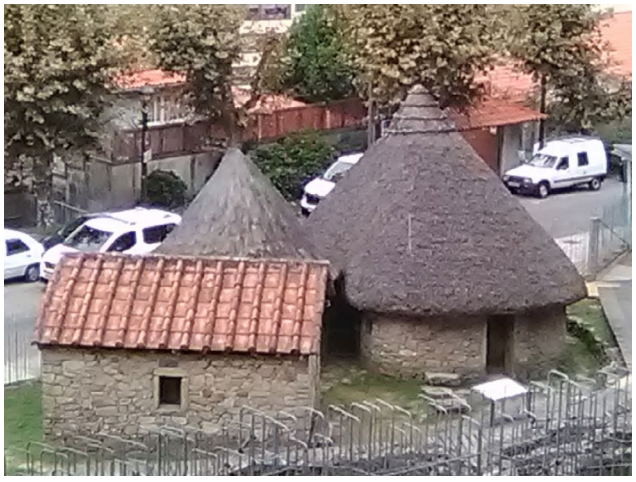
And now—after a long hiatus—it's time for your homework.

Triangle ABC *is right-angled with right angle at* C*, and legs* AC *and* BC *have lengths 6 and 8, respectively. Determine the length of the segment from* C *to the midpoint of the hypotenuse* AB*.*

Until next time, happy problem solving! ▲

Conehead Spotted!

An archaeology site in Vigo, Spain, features thatched rooves that are cone shaped.



▲ Musing on Percentages: Calculators and %



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Percent is everywhere—all around us in finance, shops, foods, media, surveys, and so on. Percentages may seem like a rather simple mathematical theme that has little depth. However, when one starts to pay attention to its usage, percentages reveal surprising elements that are mathematically rich and conceptually fascinating, hence worth pondering. This is something that became quite salient for our research team as we have been preparing experiments with students in classrooms about percentages in mental mathematical environments.

These columns endeavour to raise and address some of these peculiarities, which we came to call "curiosities," related to the concept of percentages. These musings are about our usage of "%" in everyday life, in schools, or in mathematics in general. This column continues the previous ones (available in the archives on the website) and addresses the functioning of calculators with %.

Introduction

The functioning of a calculator is actually quite surprising. Because percentages are underpinned by a multiplicative structure (as discussed in Proulx, 2025 about the order of calculating discounts and taxes), it is through multiplying the reference number, the % *of what*, that the desired percentage is obtained. For example, calculating 6% *of* 50 amounts to doing 50 × 0.06, which gives 3. Somewhat counter-intuitively, however, the calculator relies more on additions than on multiplications in its workings. But how exactly does it do this? That is what I propose to explore in this column, using a number of cases and examples that help understand how it works. I would suggest that you take a calculator yourself, the one on your phone, for example, and that you perform the operations

proposed in each case—you will better grasp what is happening in these calculations. Using other calculators might offer other results, which are also quite fascinating.

Case 1: Determining the Value of 6%

Entering [6], followed by the [%] key (and possibly the [=] key), the calculator automatically displays the number 0.06. This represents a normal and usual conversion between a percentage and a decimal number, where a numerical value is directly assigned to the percentage. In other words, 6% here equals 0.06.

Case 2: Calculating 6% of 50

Entering [50], followed by the [+] key, the [6] key, the [%] key, and finally the [=] key, the calculator displays 53 as the answer. However, the intermediate steps are also of interest. Although nothing happens when the [50], [+], and [6] keys are pressed, everything changes when the [%] key is pressed. Indeed, before pressing the [=] key, the screen displays 3. This value of 3 represents 6% of 50, that is, a percentage relative to its reference number. At that moment, 6% is equal to 3 because it is converted relative to a reference whole, which is 50 (as opposed to Case 1, where 6% equalled 0.06). Afterwards, pressing the [=] key, the calculator adds 50 and this new 3, to give 53. This "calculation" gives 106% of 50, that is, the value of 50 increased by 6%.

Case 3: Calculating 6% of 50 (*bis*)

Entering [50], followed by the multiplication key, then [6], then [%], and finally the [=] key, the calculator displays 3 as the final answer. Here again, the intermediate steps deserve particular attention. When entered, [50], the multiplication key, and [6] produce nothing on the screen. However, when [%] is pressed, the calculator automatically displays the value 0.06, representing, as in Case 1, a direct conversion of 6% into 0.06. This value of 0.06 is not relative to 50 as in Case 2, but rather, is the number by which 50 is to be multiplied. Afterwards, entering [=] gives the value of the product of 50 with 0.06, which is 3.

Some comments seem necessary at this stage, before continuing with other cases. The examples considered above show that the [%] key, as soon as it is pressed, causes a value to be displayed on the calculator screen (which is not the case for all the other keys). This obtained value is nonetheless different, depending on the keys pressed *before* [%]. If [%] follows the entry of a single number, the calculator then performs a direct conversion between the percentage and a decimal number, as seen in Case 1. It does the same when the number is preceded by a multiplication, as in Case 3. Here, the conversion to 0.06 provides the number with which to multiply the starting number (in Case 3, the 50). Thus, in Cases 1 and 3, the [%] key offers a direct conversion between the percentage and the associated decimal number (6% becomes 0.06). Case 2 is different, where the value displayed after pressing [%] gives the value of this percentage relative to the initial number entered: a 3 is obtained and not 0.06, because this 3 is relative to the 50 and 3 is 6% of 50. Pressing the [+] key afterwards significantly changes the situation or the sequence of operations, shifting the 6% from 0.06 to 3, that is, from a direct value to a value relative to the number initially entered in the sequence of operations. However, the sequence is simply interpreted differently by the calculator: the [+] key turns the sequence into a request to *add* a percentage to a number, this percentage then being relative to that number (50 + 6% of 50 = 50 + 3 = 53). In Case 2, this results in obtaining 50 increased by its 6%, or 106% of that same 50. Taking these differences into account, the following cases allow us to explore even further how the calculator works with percentages.

Case 4: In the sequence "60 + 50 + 6%," what is the value of 6%?

Entering [60], [+], [50], [+], [6], [%], and [=] displays 116.6 on the screen. An analysis of the intermediate steps helps understand the value assigned to 6% by the calculator. In fact, after entering [60], pressing [+], and then entering [50], nothing happens. However, the second [+] evidently adds 60 and 50, and displays 110 on the screen. This explains why, in the following steps, pressing [6] and [%] displays 6.6 on the screen. The value 6.6 is in fact 6% of 110 (and not of 50, nor of 60). It is this 6.6 that is added to 110 to give 116.6 after pressing [=]. The value 116.6 therefore represents 106% of 110. The 6% here is 6% relative to 110 and is equal to 6.6 (not 3, nor 0.06).

Another noteworthy comment is that, although the percentage is grounded in a multiplicative structure, this is not taken into account in the sequence of Case 4. The percentage, as a multiplicative operation, is not given priority in the sequence of operations: 6% is not automatically assigned to 0.06, nor is it directly multiplied by the 50 that precedes it. In other words, the percentage does not have priority over addition here! The 6%, with value 6.6, is indeed added to the reference number, here 110. It is only at this moment that the calculation of 6% is considered multiplicatively between 6 and % because the calculator does not display 116 after pressing [6] to perform "60 + 50 + 6 = 116," but instead, computes 6% of 110. It is the [+] pressed before 6% that transforms the interpretation of the percentage into an additive mode, but this 6% is neither 0.06 as in Case 1, nor 3 as in Case 2, but rather 6.6, since it is 6% of 110. It is *also* possible, somewhat strangely, to see that the [+] transforms the entire sequence into a multiplicative mode, namely a multiplication of 110 by 1.06. However, there would then be a necessary and prioritized link with the [50] in the sequence of operations that would give 106% of 50 and not 6%

of 110. The comparison of this situation with the next Case 5 shows the difference at the level of operation priorities.

Case 5: In the sequence "60 + 50 × 6%," what is the value of 6%?

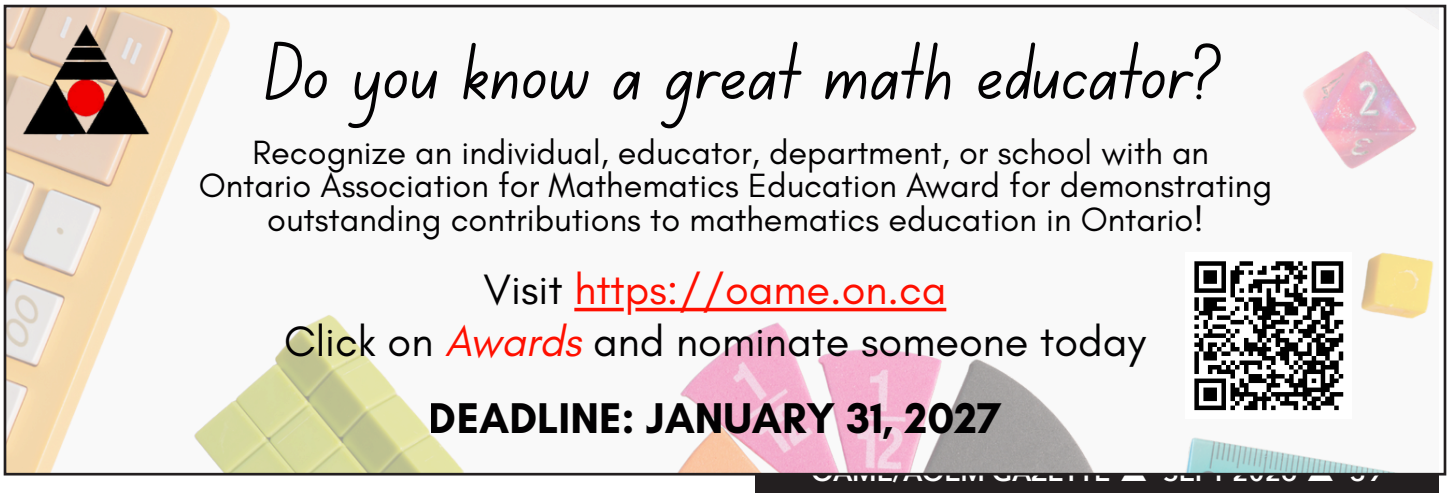
In this sequence of operations, everything changes, thanks to the multiplication applied after entering [50]. At that moment, priority is given to multiplication and brings us back to Case 3, where the sequence [50], [×], [6], [%] gives 3 as the answer. The addition of 60 with the sequence [50, ×, 6, %] gives 63 as the final value. In a certain sense, it is not possible to combine the 60 with Case 2, that is, to add 60 with 106% of 50. Adding 60 with 106% of 50 must be done afterwards, once the answer for 106% of 50 has been obtained (with Case 2). It is possible to perform the sequence [60, +, 50, ×, 1.06, =], which gives 113, but this does not involve the [%] key—unless one enters [106] and [%] instead of [6] and [%]! Here, as in Case 3, 106% is converted directly into the decimal 1.06.

From all these cases, several observations can be made. In a certain sense, the calculator is very efficient from a "commercial" perspective, and its functioning with percentages was probably designed that way from the start. Case 2, through the input of the addition of a percentage with the sequence [50, +, 6, %, =], gives the value of the initial number increased by the desired percentage. In other words, this sequence reproduces the calculation of a tax; here, a tax of 6% that would be added to an amount of 50, giving 106% of 50. The calculation is direct and efficient!

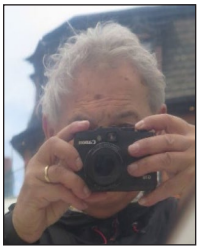
However, if someone wants to know the value of a percentage of a number, that is, the precise value of the tax to be paid, it is Case 3 that allows one to obtain it. Indeed, to obtain the precise value of 15% of 25, one must enter [25, ×, 15, %, =], which gives 3.75. The total amount with tax, namely 115% of 25, is obtained by [25, +, 15, %, =], which gives 28.75. Finally, to know the price of an item after a discount, it is the subtraction key that is to be used. An item priced at $25 with a 40% discount is obtained on the calculator by [25, –, 40, %, =] and gives 12. In a certain way, the [+] and [–] keys serve to calibrate the percentage relative to 1, or rather, to one time the number at stake, to its 100%. Pressing [+] ensures that the entire initial number is kept and that the percentage is added to it: one full unit of the number plus its percentage. With 6% of 50, calculated by [50, +, 6, %, =], this amounts to 106% of the number, that is, of multiplying it by 1.06: 100% and 6%, or 1 and 0.06. For subtraction, the same thing happens relative to the initial number: one full unit of the number minus the desired percentage. This subtraction involves the complement to 100 of the desired percentage. Thus, [50, –, 6, %, =] gives 94% of 50, that is, 1 – 0.06 = 0.94 of the latter. It is therefore possible, when considered from the angle of the impact on "1," on "one time" the initial number, on its 100%, to see in the background the multiplicative aspect taken into account by the operations carried out with the calculator. There is thus a calculation of what must be added (a tax, for example) or removed (a discount, for example), and then the multiplication by what this addition or subtraction represents.

The calculator thus resides at the heart of the interplay between additive and multiplicative reasoning associated with percentages. However, it does so in a different way that mixes up the roles in this additive and multiplicative interplay. Why? Because the relative values of percentages are usually considered from a multiplicative perspective with "so many percent of …": in multiplicative mode, the value of a given percentage is not fixed—it varies, depending on its reference whole. Conversely, the additive dimension is more often viewed from the "absolute" perspective of percentage values: percentage values are fixed in additive mode, where 6% + 8% is always 0.06 + 0.08, independently of any reference number. Thus, contrary to the calculator, in Case 2 through addition that offers relative percentage values (6% of…), and it is Case 3 through multiplication that offers absolute percentage values (6% *is* 0.06). In a way, it is a bit of a topsy-turvy world with the calculator and percentages! But it is nonetheless fascinating, isn't it?

Again, I am always interested in your comments, so feel free to send them to proulx.jerome@uqam.ca! ▲



The Math Teacher's Home Base: Bolt Out Now and Get These Books!



RON LANCASTER
ron2718@nas.net

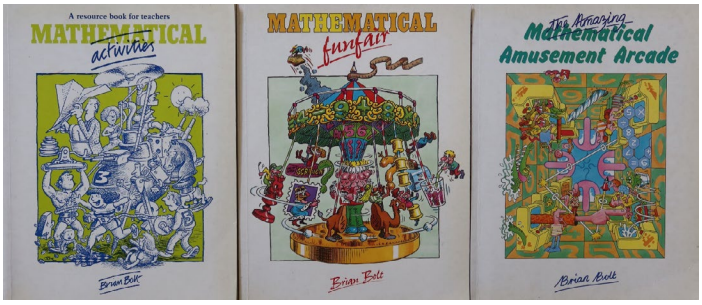
Ron is an Associate Professor Emeritus at the University of Toronto, where he has taught mathematics courses for pre-service teachers for 17 years. Ron taught middle and high school mathematics for 21 years in coed public schools and an all-girls school. Ron's professional activities include conference presentations/ consultations in North America, Asia, England, the Middle East, Africa, and India. Ron has authored/edited over 200 articles for NCTM and OAME. Ron received the 2015 Margaret Sinclair Memorial Award Recognizing Innovation and Excellence in Mathematics Education, awarded by the Fields Institute, and was the recipient of the 2022 OAME Lifetime Achievement Award.

As mathematics educators, we are constantly on the hunt for resources that can transform our classrooms from spaces of passive listening into dynamic hubs of active thinking. Sometimes, the best inspiration doesn't come from a new website or a trendy app, but from the well-loved pages on our own office bookshelves.

For decades, the prolific British mathematics educator, Brian Bolt, has been a quiet hero for teachers seeking to spark genuine mathematical joy. Author of numerous collections of puzzles, games, and essays, Bolt had a rare gift for uncovering the playful side of mathematics without sacrificing its depth. His books are packed with quirky items, rich activities, and accessible challenges that are perfect for modern classrooms.

In Ontario, we often talk about building a "Thinking Classroom"—a concept popularized by Peter Liljedahl that emphasizes collaboration, perseverance, and problem solving. Brian Bolt's activities fit this philosophy perfectly. They are classic "low-floor, high-ceiling" tasks—easy for a group of students to start working on, yet deep enough to reward sustained curiosity and collaborative debate.

Here is a curated look at some classic gems from three of his books resting on my home office shelves, complete with solutions and ideas on how to use them to get your students talking, collaborating, and sticking with it.



Photograph 1: *Three books by Brian Bolt*

Part 1: Items from *Mathematical Activities*

Activity 45: How many triangles can be made on a 3 x 3 pinboard (Figure 1)?

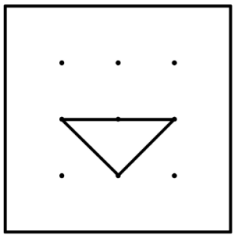


Figure 1: *Triangles on a 3 x 3 pinboard (geoboard, dot paper)*

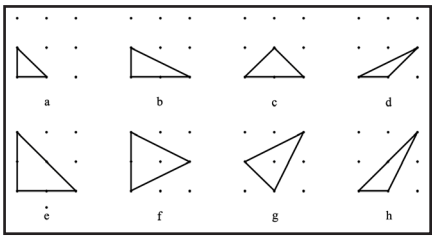


Figure 2: *Eight kinds of triangles*

Solution:

The diagrams in Figure 2 show the eight kinds of triangles that can be formed on a 3 x 3 pinboard. Table 1 shows the number of each type that can be made. The sum of these numbers is 76, which equals the number of triangles that can be made.

Table 1

type	number
a	16
b	16
c	8
d	16
e	4
f	4
g	4
h	8
total	76

Students with a background in counting (combinatorics) can also solve this problem by noting that there are 9 pins in total, and to form a triangle, we need to choose 3 pins. However, as the calculation below shows, we need to

subtract 8 cases where the pins are collinear (3 horizontal lines, 3 vertical lines, and 2 main diagonal lines).

$$\binom{9}{3} = \frac{9 \times 8 \times 7}{3 \times 2 \times 1} - 3 - 3 - 2 = 84 - 8 = 76$$

Thinking Classroom Connection:

This task is very suitable for collaboration. If you hand a geoboard (or dot paper) to a group of students, they will initially count randomly and lose track. To succeed, they must co-operate to find a systematic strategy—perhaps grouping triangles by size, type (right-angled, isosceles), or orientation. It builds "stick-to-it-iveness" when their early counts conflict, forcing them to audit each other's work and reason out the definitive answer.

Activity 62: The letter dice

A word game uses dice with a letter on each face. Three views (Figure 3) of one of the dice are shown below. Which letter is on the face opposite "H"?



Figure 3: *Three views of a die*

Solution:

The letter opposite the "H" is an "S." There are in fact 2 letters "S," as shown in the net below (Figure 4).

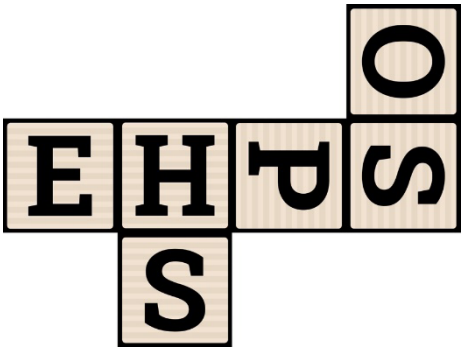


Figure 4: *Net of die*

Thinking Classroom Connection:

This puzzle is great for getting students to build a physical model. By drawing a flat net on paper and folding it into a cube, they can directly test their ideas. The twist of having two "S" faces forces them to look more closely, as it breaks the common assumption that every side must have a different letter. It naturally gets groups talking and checking each other's logic to find the match.

Activity 100: Know your vowels!

This table (Figure 5) contains each of the five vowels—A, E, I, O, U—five times. Show how to cut the 5 x 5 square into five different-shaped pieces, each of which contains all the vowels only once.

When you have solved this one, try making a similar one for yourself.

E	A	I	O	I
U	E	U	E	O
O	I	A	O	A
I	U	E	A	I
A	O	U	E	U

Figure 5: *Know your vowels*

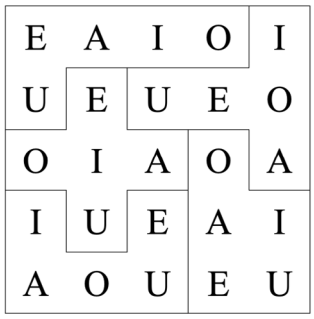
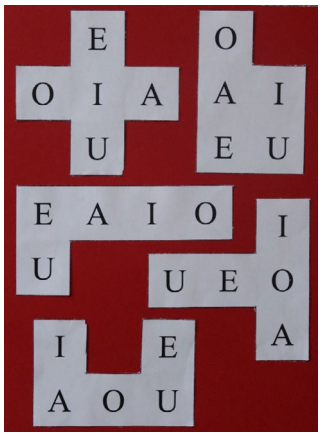


Figure 6: *How to cut the square into five pieces*

Solution:

One way of cutting the 5 x 5 square is shown in Figure 6 and Photograph 2. The natural appearance of pentominoes provides a gateway to study them.



Photograph 2: *Five pentominoes solve the puzzle*

Thinking Classroom Connection:

This puzzle is a strong exercise in perseverance. Because it requires dividing a grid into five unique shapes that each contain all five vowels, early trial-and-error attempts will often fail. It keeps students talking and working together because a single change to one piece immediately impacts the rest of the board, forcing the group to constantly adjust their strategy.

Part 2: Items from *Mathematical Funfair*

Activity 49: As easy as abc!

Let *abca* be a four-digit number. Find numbers *a*, *b*, and *c* such that

$$a^b \times c^a = abca$$

Solution:

$$2^5 \times 9^2 = 2592$$

Thinking Classroom Connection:

This problem stops students from aimlessly guessing by forcing them to look closely at how multiplication and exponents interact. Because the four-digit answer begins and ends with the letter *a*, students can narrow down their options by looking at the last digit of their calculations.

When groups work on this at the boards, encourage them to test values for *a*. For example, if $a = 2$, the final four-digit number must end in 2. This means the calculation $2^b \times c^2$ must result in a number ending in 2. Since c^2 is a perfect square, its last digit can only be 1, 4, 5, 6, or 9. By multiplying these possibilities by different powers of 2, students can rule out combinations that do not end in 2. This structural clue turns a blind search into a game of elimination based on basic multiplication facts.

Activity 52: Fascinating fractions

The digits 1, 2, 3, ..., 9 can be arranged to form two numbers, whose ratio is 1/2, like this:

$$\frac{7329}{14658} = \frac{1}{2}$$

Your challenge is to find a set of numbers for every other unit fraction up to 1/9. Working with your group, use your calculators to find at least one valid arrangement of the nine digits for each of the following ratios:

$$\frac{1}{3} \quad \frac{1}{4} \quad \frac{1}{5} \quad \frac{1}{6} \quad \frac{1}{7} \quad \frac{1}{8} \quad \frac{1}{9}$$

Solution:

Multiple ways of expressing each unit fraction, except for 1/5, are given below. Are these the only ways? How do you know?

$$\frac{1}{2} = \frac{7293}{14586} = \frac{6729}{13458} = \frac{7269}{14538}$$
$$\frac{1}{3} = \frac{5823}{17469} = \frac{5832}{17496}$$
$$\frac{1}{4} = \frac{7956}{31824} = \frac{5796}{23184} = \frac{3942}{15768} = \frac{4392}{17568}$$
$$\frac{1}{5} = \frac{2697}{13485}$$
$$\frac{1}{6} = \frac{2943}{17658} = \frac{4653}{27918} = \frac{5697}{34182}$$
$$\frac{1}{7} = \frac{2394}{16758} = \frac{2637}{18459} = \frac{4527}{31689}$$
$$\frac{1}{8} = \frac{3187}{25496} = \frac{4589}{36712} = \frac{4591}{36728} = \frac{6789}{54312}$$
$$\frac{1}{9} = \frac{6381}{57429} = \frac{6471}{58239}$$

Thinking Classroom Connection:

Using calculators at the vertical boards makes this heavy task manageable, but the real power of this problem lies in the extension questions: “Is this the only way? How do you know?” This shifts the task from a repetitive calculation exercise into a deep mathematical argument. To prove why 1/5 only has one solution, students must shift from guessing to using number properties. For instance, a group might realize that the five-digit number on the bottom must end in 5 (since it cannot end in 0). This immediately fixes the position of the 5 and restricts where the remaining digits can go, helping the group systematically rule out other combinations.

Activity 101: Think again!

What is the next line in Figure 7? When the pattern is extended, in which line does 4 occur first.

1							
1	1						
2	1						
1	2	1	1				
1	1	1	2	2	1		
3	1	2	2	1	1		
1	3	1	1	2	2	2	1

Figure 7: *A sequence that says it all*

Solution:

This is the “Look and Say” sequence, where each line describes the digits of the line above it. Reading the last line given (“one 1, one 3, two 1’s, three 2’s, one 1”), the next line is 1113213211. A 4 will never occur in this sequence because generating a 4 requires the previous line to contain four identical digits in a row, which the rules of the sequence prevent.

Teacher Note/Fun Fact:

John Conway proved that this simple sequence leads to some unbelievably deep mathematics. The excerpt shown below, from *The Book of Numbers*, by John H. Conway and Richard K. Guy, is astonishing, stunning, and downright bizarre. How is it even possible that this sequence is related

to a polynomial equation (Figure 8) of degree 71? For information about this result, read the article given below.

Ekhad, S.B., & Zeilberger, D. (1997). Proof of Conway's lost cosmological theorem. *American Mathematical Society, 3*, 78–82. https://sites.math.rutgers.edu/~zeilberg/mamarimY/Zeilberger_y1997_p78.pdf

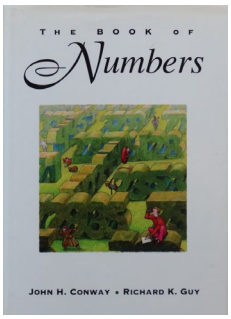
It can be shown that the number of digits in the nth term is roughly proportional to

$(1.30357726903429639125709911215255189073070250465 94\ldots)^n$,

where the simplest algebraic equation defining the number in parentheses is

$$x^{71} - x^{69} - 2x^{68} - x^{67} + 2x^{66} + 2x^{65} + x^{64} - x^{63} - x^{62} - x^{61} - x^{60} - x^{59}$$
$$+ 2x^{58} + 5x^{57} + 3x^{56} - 2x^{55} - 10x^{54} - 3x^{53} - 2x^{52} + 6x^{51}$$
$$+ 6x^{50} + x^{49} + 9x^{48} - 3x^{47} - 7x^{46} - 8x^{45} - 8x^{44} + 10x^{43}$$
$$+ 6x^{42} + 8x^{41} - 5x^{40} - 12x^{39} + 7x^{38} - 7x^{37} + 7x^{36} + x^{35}$$
$$- 3x^{34} + 10x^{33} + x^{32} - 6x^{31} - 2x^{30} - 10x^{29} - 3x^{28} + 2x^{27}$$
$$+ 9x^{26} - 3x^{25} + 14x^{24} - 8x^{23} - 7x^{21} + 9x^{20} + 3x^{19} - 4x^{18}$$
$$- 10x^{17} - 7x^{16} + 12x^{15} + 7x^{14} + 2x^{13} - 12x^{12} - 4x^{11} - 2x^{10}$$
$$+ 5x^9 + x^7 - 7x^6 + 7x^5 - 4x^4 + 12x^3 - 6x^2 + 3x - 6 = 0.$$

Figure 8: *A wild excerpt from* The Book of Numbers



Photograph 3: The Book of Numbers*, a beloved book in my home office*

Thinking Classroom Connection:

Sharing a mind-blowing fact, like Conway's constant at the end of the lesson, completely transforms how students view mathematics. It shows them that a game with no traditional formulas or fractions can secretly hide incredibly deep, advanced algebra. When groups are working at the boards, this story changes the vibe from "doing a simple riddle" to "exploring the edge of a massive mathematical mystery." It is a highly effective way to spark genuine curiosity and prove to students that math is full of unexpected surprises.

Items from *Mathematical Amusement Arcade*

Activity 69: Two at a time

Put ten coins in a line, as shown in Photograph 4. A move consists of picking up a coin, leaping over two coins, and landing on another coin. Show that in five moves, it is possible to arrange the coins into five pairs equally spaced. Not as easy as it seems!



Photograph 4: *Ten coins in a line*

Solution:

Put 7 on 10, 5 on 2, 3 on 8, 1 on 4, 9 on 6.

Thinking Classroom Connection:

This puzzle beautifully highlights how a simple condition—"leaping over two coins"—can completely derail a student's intuition. When groups try to tackle this at the vertical non-permanent surfaces (VNPs), they often make the common mistake of assuming a "pair" counts as just one item to leap over, or they lose track of the empty spaces left behind.

To keep the thinking moving when groups get stuck, encourage them to use a classic mathematical strategy: working backward. Have them start with the final state—five completed pairs—and try to unpack a move in reverse. This subtle shift in perspective forces students to analyze the spatial logic of the constraints from the inside out, turning a frustrating game of trial-and-error into a structured lesson on system thinking.

Activity 70: Heads and tails

Take eight pennies and place them alternately head and tail touching each other in a line, as shown in Photograph 5. A move consists of moving two touching coins to one end or to a suitable space of the row without changing their order.

Show that in four moves, it is possible to arrange them in the order TTTTHHHH, with the coins touching and in line.



Photograph 5: *Heads and tails*

Solution:

Table 2 shows how the coins can be moved in four steps.

Table 2

Initially			H	T	H	T	H	T	H	T
First move	T	H	H	T	H	T	H			T
Second move	T	H	H	T			H	H	T	T
Third move	T			T	H	H	H	H	T	T
Fourth move	T	T	T	T	H	H	H	H		

Thinking Classroom Connection:

The trickiest part of this puzzle, for students at the boards, is managing the "empty spaces" created by moving a pair of coins. Human intuition usually wants to slide coins over to close a gap immediately, but this puzzle requires students to leave those gaps open and purposefully use them as landing zones for future moves.

When facilitating this, watch for groups who get stuck trying to solve it entirely in their heads or trying to move one coin at a time by mistake. Encourage students to actively move the physical pieces (or draw clear slots on the board) to see how moving a block of two completely alters the available real estate. It's a wonderful task for building spatial foresight and teaching students how to plan multiple moves ahead.

Activity 78: Square relations

The number 24 has the property that it is one short of a square number, and its double is also one short of a square number. Figure 9 shows a geometric view of this property.

$$24 = 5^2 - 1 \qquad 48 = 7^2 - 1$$

What is the next number with the same property?

Generally, if x and $2x$ are two positive integers that are both one less than square numbers, how many values of x are there?

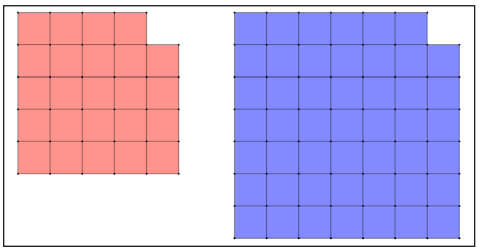


Figure 9: *Geometric view of two numbers that are one less than a square*

Solution:

The next number with this property is 840.

$$840 = 29^2 - 1 \qquad 1680 = 41^2 - 1$$

There are infinitely many values of x. While advanced mathematicians analyze this, using a tool called Pell's Equation, students can discover the solutions by using technology to build a table of values.

Thinking Classroom Connection:

This activity offers an excellent opportunity to pivot students from basic guessing into using a targeted strategy. When groups work on this at the boards, they will easily verify 24 and 48, but trying to find the next solution by testing numbers one by one on a calculator will quickly lead to a dead end and stall their momentum.

If a group gets stuck, challenge them to write out the basic algebraic relationships:

$$x = a^2 - 1 \qquad 2x = b^2 - 1$$

Since solving these equations by hand is far too complex for an average student, this creates the perfect opportunity to introduce technology. Encourage students to move to a computer, open Excel or a Google Sheet, and create a table, using their formulas. By letting the spreadsheet do the heavy lifting to generate the numbers and search for matching values, students get to experience how basic formulas, combined with a simple table, can unlock solutions that are far too large to find by hand.

Additional books by Brian Bolt (all highly recommended and worth the price):

- *101 Mathematical Projects*
- *A Mathematical Dictionary for Schools*
- *A Mathematical Jamboree*
- *A Mathematical Pandora's Box*
- *Even More Mathematical Activities*
- *Mathematical Activities: A Resource Book for Teachers*
- *Mathematical Cavalcade*
- *Mathematics Meets Technology*
- *More Mathematical Activities: A Resource Book for Teachers*
- *The Amazing Mathematical Amusement Arcade*
- *The Mathematical Funfair*



Photograph 6: *Eight more books by Brian Bolt (using pictures from Amazon.com)* ▲

"Pope Leo XIV, who taught mathematics and physics, sends a message on the occasion of the International Day of Mathematics, reiterating that having a great deal of knowledge is not enough if we do not know who we are or what the meaning of life is."

Lubov, D.C., (2026, March 13). Pope Leo on Mathematics Day: Moral dimension must be upheld. *Vatican News.* www.vaticannews.va/en/pope/news/2026-03/pope-leo-sends-mathematics-day-message.html

Implementing Effective Strategies for Mathematics Classrooms – Part 3



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Jeff Irvine, Ph.D., has been a secondary mathematics teacher, department head, and vice-principal. He has taught at three faculties of education and at a community college. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. With over 45 years in education, Jeff is particularly interested in the interplay of student motivation and mathematics achievement.

Summarizing and Note Taking (d = 0.90 to 1.00)

Summarizing and note taking are skills that must be taught, but have a large impact on student achievement. Summarizing involves identifying key concepts, clarifying misconceptions, and verifying and deepening understanding. Math journals are often used as a location for summarizing. Mathematics lends itself to summarizing, using equations, symbols, graphs, and diagrams to summarize large volumes of information (e.g., see Figure 1).

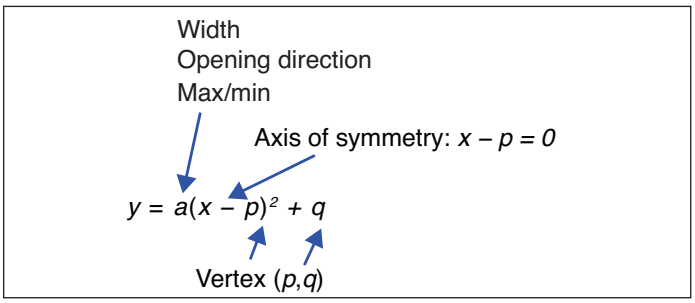


Figure 1: *Summarizing quadratic transformations with an equation*

In a school where I worked, a portion of every Friday's math class was devoted to students writing summaries of the week's work (Irvine, 2015, 2020a). Then, at the end of every month, students summarized their weekly summaries for the month. All summaries were restricted to one side of one page. Completed summaries could be used during tests and other evaluations. Summarizing and note-taking strategies were explicitly taught in our math classes, helping our students to recognize key concepts and big ideas. For exceptional students, consider alternative ways of summarizing, such as verbal reports or recordings.

A strategy that I found very effective is a placemat activity involving a cartoon with blank speech bubbles (Figure 2). Working in groups, students are asked to summarize a skill or procedure, such as factoring a specific polynomial type, completing the square, or solving a specific type of word problem. Students write their explanations in their portion of the placemat. The group then comes to a consensus on the clearest explanation, which is placed in the blank speech bubbles. This process helps ensure clarity of thought and concise expression. As an extension, each group can be given a different procedure to summarize, and the results shared using a gallery walk.

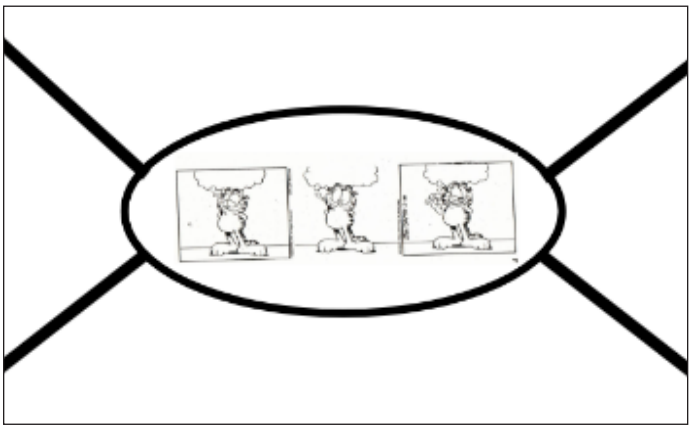


Figure 2: *Placemat*

Summarizing also involves deepening learning and understanding. Some strategies for summarizing can be explicitly linked to higher-order thinking skills (Dean et al., 2011). An example of this linkage in mathematics is the use of Frayer models (Irvine, 2020a; Ontario Ministry of Education [OME], 2004). Frayer models (Figure 3) allow students to summarize information about a concept (placed in the centre rectangle), as well as use higher-order thinking skills to add ideas to associated boxes, such as essential and non-essential characteristics or examples/non-examples. After completing a Frayer model individually, students share in groups to clarify and correct misconceptions.

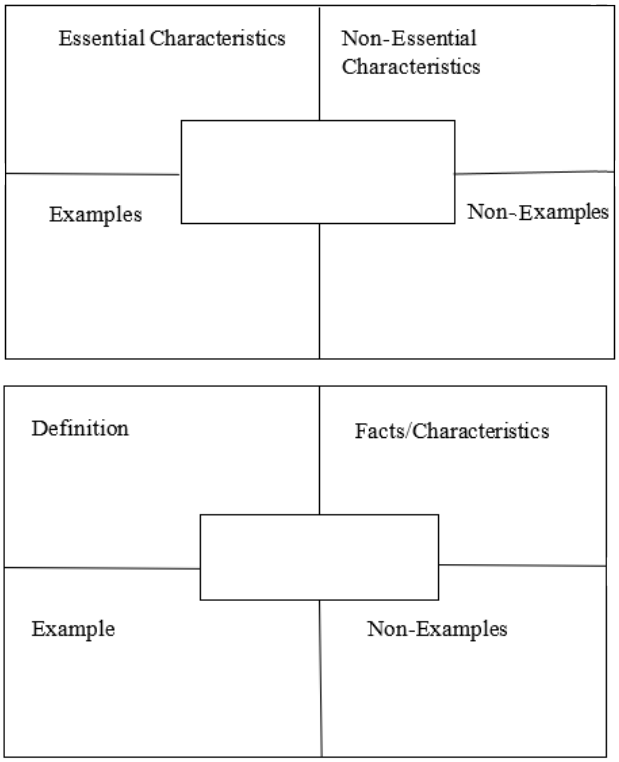


Figure 3: *Two versions of Frayer models*

Summarizing can also be enhanced in interactive ways, using strategies such as Timed Retell. In Timed Retell (Irvine, 2023), students work in pairs. One student explains a concept to the other student. Then the second student reiterates and clarifies what the first student said. The students then switch roles and repeat the exercise. The activity can be timed to encourage brevity and clarity.

Two strategies that can be used for summarizing and requiring higher-order thinking skills are Quescussion and Duggleby Connecting Words.

Quescussion

In this strategy, students work in groups to discuss a topic, but all statements must be in the form of a question. The teacher starts the process by asking a question or making a statement. Each student has an opportunity to contribute, but the discussion uses only questions. No answers are given. Groups must create norms to allow everyone to speak. Questions must not be restatements of declarative statements and should be constructive. Quescussion promotes higher-order thinking skills and deep learning. When first using Quescussion, starter stems may be needed, such as:

- What about…?
- Are you saying that…?
- Have you considered…?
- How might you justify saying…?

Duggleby Connecting Words

This is an activity that I got from my friend, Sandra Duggleby, an instructor at the University of Calgary. While details of the strategy were reported in a previous *Ontario Mathematics Gazette* Issue (Irvine, 2019g), the strategy is so powerful, it is described here as well. Each connecting word card (see example in Figure 4) contains the word, its definition, and some examples of its use. Each student group decides on a discussion topic (e.g., Pythagorean theorem). The deck of connecting words is shuffled and placed face down. A student takes the top card and uses the word on the card in their statement. The next student takes the next card and begins their statement with that word. This activity requires students to think critically and formulate coherent statements.

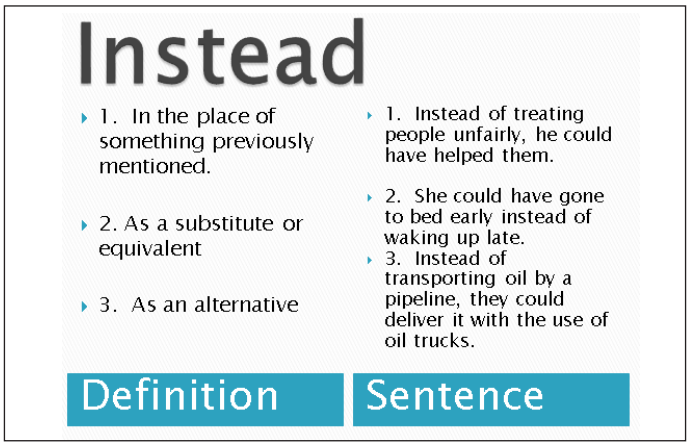


Figure 4: *Sample Duggleby card*

Sandra's research has shown that using this activity results in students exhibiting higher-order thinking skills. Sandra developed 76 different cards, broken into five categories (Table 1), which I usually print on assorted colours. Some sample words are shown below. The deck can be edited by removing certain cards, if the teacher finds some of the words too difficult for students, or inappropriate to the topics under discussion. The activity can be used in any subject, but I have used it successfully in Grades 9 through 12 math classes and at all the levels (with some significant editing if you want to use it in Workplace classes).[1]

Table 1: *Categories for Duggleby connecting words*

Connective Category	Word Examples
Additive	also, for example
Adverb	typically, certainly
Adversative	although, instead
Temporal	when, before
Causal	because, if… then….

Note taking has been shown to improve retention as well as stimulate higher-order thinking skills (Lalchandani & Healy, 2022). Similar to summarizing, note taking needs to be taught, helping students identify what is important and stimulating connections to prior learning.

There have been considerable advances in technology since Marzano's research was published. Technology may be used in note taking and summarizing, especially given the wide range of currently available platforms. Eason (2017) pointed out that electronic note-taking apps, such as those available on tablets and smartphones, allow voice to text, handwriting to text, and provide a kinesthetic dimension to note taking that is congruent with today's technology-savvy students. Summarizing tools such as mind maps and concept maps are freely available and allow students to efficiently construct graphical organizers and summaries for large volumes of information. These tools may be interactive, such as CMAP®, a free concept-mapping tool that allows online construction of concept maps relating ideas and concepts to current and prior learning. However, in Marzano's research, note taking was considered to be handwritten, a personalized representation through a kinesthetic method to derive something with student ownership. Additional research may be needed to consider whether teacher-generated notes sent using technology or photos of whiteboard summaries have the same impacts on student learning and retention.

Assigning Homework and Providing Deliberate Practice (d = 0.13 to 0.77)

"Homework is a powerful tool that can contribute to the advancement of children's education, or it can do more damage than good to their education and development" (Hong et al., 2004, p. 203). Homework consists of anything that is done outside of class: practice, review, preparing or studying for assessments and evaluations, finishing assigned work. Homework should also include investigations, problem solving, and research. Effect sizes for the impact of homework and practice on student achievement vary widely, from 0.13 to 0.77, perhaps indicating that homework requires a level of individualization and there is no "one size fits all" for homework. Since homework is a staple of most math classes, teachers need to understand the very personal nature of homework, especially the impact on student motivation.

Hong et al. (2004) have proposed a homework model that considers student motivation and learning preferences. In their model, motivation considers the source of student motivation (self-motivated, parent-motivated, teacher-motivated) and motivation strength (promptness and persistence); learning preferences, including organization (structure, order, place, time), surroundings (sound, light, temperature, furniture design), mobility, and intake—food/drink, as well as interpersonal preferences (alone, with peers, with authority figures). They found that motivation is key, especially self-motivation (i.e., intrinsic motivation).

Some educators propose the "10-minutes per grade rule" for homework, an arithmetic sequence beginning with 10 minutes per night in Grade 1, 20 minutes in Grade 2, and so on. However, Hattie (2009) found that assigning homework in elementary school had a negligible effect on student achievement, while homework in secondary school had a greater impact. Homework also has influencers in parents and the students themselves. Parents believe that homework is important and encourages time-management skills. However, in my opinion, there is no conclusive research evidence to support this claim. Homework may also interfere with family time, extra-curricular activities, recreation, or part-time jobs.

Given the highly personal nature of homework, teachers should consider ways of addressing the varied student needs, for example, varying the product or processes involved in homework by using strategies such as RAFT (Role, Audience, Format, Topic). Students choose one of several options to demonstrate their knowledge. For example, to demonstrate their understanding of exponential functions, students could choose from Table 2.

Table 2: *Sample RAFT choices*

Role	Audience	Format	Topic
Investment adviser	Client	Presentation	Compound interest
Scientist	Other scientists	Scholarly paper	Radioactive decay
Waste manager	General public	Newspaper article	Disposal of radioactive waste
Salesperson	Manager	Job interview	Predicted sales growth
Environmentalist	Factory owners	Model	Growth of carbon dioxide emissions

Other homework strategies include alternating among practice, research, investigations, and problem solving; varying the length of homework assignments; involving videos or other multimedia resources; or allowing student input into homework decisions.

There is abundant research that student learning is enhanced when students have some level of autonomy and choice in their learning (Deci & Ryan, 2008). Since

homework plays a large role in mathematics education, it is important to provide students with opportunities for some autonomy and control in their homework assignments. Three strategies that I have used: Cubing (Irvine, 2018), which lets each student roll a die and then complete at least the number of homework questions on the up-face of the die, while still being expected to demonstrate overall competence; "1-3-1," where the student completes a minimum of one easy question, three medium-difficulty questions, and one hard question (students decide on degree of difficulty of each question); and "You decide," where the student is told, "Read each question. If you are sure that you can successfully do the question, don't do it," resulting in the student only attempting questions for which they are not positive they can be successful and view the questions as a challenge. By allowing student choice at least some of the time, students take more ownership in their own learning.

It is also important that students receive feedback on their homework. Consider varying the ways feedback is provided, such as sometimes utilizing a flipped classroom model.[2] I have frequently used homework groups (Irvine, 1997) to provide feedback on homework assignments. Homework groups is a strategy that I got from Dr. Geoff Roulet at Queen's University. In pre-assigned groups, students discuss the previous day's homework, emphasizing understanding, alternate solutions, clarifying misconceptions, and identifying issues. The teacher circulates and addresses issues of concern to all or most of the group. If multiple groups identify the same issues, the teacher may choose to stop the group process and conduct a whole-class discussion. Homework groups avoid spending valuable class time on questions where everyone was successful, and the strategy has the added benefit of peer pressure to push students to complete homework assignments so that they can be meaningful contributors to group discussions.

With respect to skills practice, there is substantial research evidence of the efficacy of spacing practice versus assigning many questions at once (Seabrook et al., 2004). So, if you have 20 questions that you want students to practise on a particular skill, it is much more effective to have them do sets of five questions each time over a period of several days or even weeks, rather than doing all 20 questions as one assignment. In this way, students retain the concepts longer and gain deeper understanding. Spaced versus massed practice has been shown to be effective across multiple academic subjects, as well as in other fields such as sales training (Kauffeld & Lehmann-Willenbrock, 2010) and medical education (Cleland & Durning, 2022).

Homework and practice will remain contentious in education. Some evidence (Hattie, 2009) has shown that homework can be demotivating to students and thus is counterproductive. However, by varying the parameters of homework and practice, and by knowing your students' prior knowledge, readiness, and learning preferences, homework can be an effective part of mathematics pedagogy.

Identifying Similarities and Differences (d = 0.66 to 1.61)

Within the category of identifying similarities and differences are comparing, classifying, creating metaphors, and creating analogies (Dean et al., 2011). These are strategies that focus on promoting higher-order thinking skills (HOTS) and deep learning for students (Irvine, 2021), so student achievement gains can be substantial when these strategies are employed. In mathematics, we routinely ask students to classify objects, such as geometric shapes, based on attributes. Comparisons can be assisted using graphic organizers such as Venn diagrams and Frayer models. Teachers also invoke metacognitive similarities and differences when we encourage students to recognize types of questions, for example, recognizing various word problems associated with solving systems of equations (mixture, distance, percent, interest, age, and so on).

Comparing similarities and differences can be explicitly included in content acquisition. For example, we might ask a student, "How are $y = 3x + 5$ and $y = 3x^2 + 5$ similar? How are they different?" This can assist students in making connections between new content and prior learning. This can be particularly important in problem solving, where students relate new problems to previously solved problems.

Identifying similarities and differences can also be related to presenting students with situations involving *cognitive dissonance*. When something occurs that doesn't fit students' preconceived notions, students encounter psychological discomfort, which results in them questioning their beliefs. Examples of cognitive dissonance in mathematics include solving systems of equations involving parallel or coincident lines, resulting in no solution or an infinite number of solutions; similarly, determining the equation of a line that happens to be vertical; and the formula for slope of a line results in an undefined value (Irvine, 2019b). Students then identify similarities and differences compared to their previous work and draw conclusions.

Metaphors can also be used to stimulate students' higher-order thinking skills, such as comparing the solving

of an equation to unwrapping a birthday present (Irvine, 2023), or discussing slope of a line with reference to black diamond ski slopes. Creative mathematics teachers may find ways to include analogies in their mathematics teaching. Identifying similarities and differences plays a central role in mathematics teaching and supports higher-order thinking skills and deep learning for our students.

Generating and Testing Hypotheses (d = 0.58 to 0.61)

This category includes decision making, problem solving, experimenting, investigating, and generating and testing of hypotheses from *what if?* questions. All of these subcategories are components of HOTS (Irvine, 2015, 2019a, 2019b, 2019e, 2019f, 2020b). Hypothesis testing can involve both deductive and inductive thinking (see, e.g., Irvine, 2019c). Many *what if?* questions arise naturally from students' work in mathematics, when students ask questions such as, "What if the leaking cylinder had more than one hole at the bottom?" or "What if the shape were an ellipse instead of a circle?" or "What if we had more than four kinds of candy in the box?" (Irvine, 2019c). Students need to be encouraged to think about the parameters and constraints in a particular situation, and possible consequences if those conditions were changed. After students ask *what if?*, they then need to formulate hypotheses and design plans to test those hypotheses (Irvine, 2019d; Irvine et al., 2016).

Providing a learning environment in which *what if?* questions are encouraged and valued has a motivating impact on students. Further, it allows students to pursue mathematical investigations that are grounded in student-generated questions. This is closer to the work of real mathematicians than much of what we do in our classes, and supports students' creativity, as well as contributing positively to students' self-confidence as learners of mathematics.

Putting It All Together

Dean et al. (2011) divide "Marzano's 9" into three categories. The first category is *creating the environment for learning*, and comprises strategies for setting objectives and providing feedback, reinforcing effort and providing recognition, and co-operative learning. Decisions about these strategies generally occur before or early in the learning process. The next four strategies—cues, questions, and advance organizers; non-linguistic representations; summarizing and note taking; and assigning homework and providing practice—are grouped under *helping students develop understanding*. The final category, *helping students extend and apply knowledge*, comprises identifying similarities and differences, as well as generating and testing hypotheses. Viewing the strategies in this way assists teachers in identifying which strategies might be employed and when. Marzano (2003) categorizes all nine strategies as teacher-level factors, since decisions about what classroom strategies to implement occur at the teacher level. Marzano identifies teacher-level factors as the most important factors in supporting student achievement, in a framework that includes school-level and student-level factors, which typically are mainly beyond the control of the classroom teacher. Hattie (2003) identifies the teacher as the second-most important factor in student learning, behind only student-specific factors such as prior knowledge or student attitudes. Thus, teachers have the greatest controllable in-school impact on student achievement.

Many of the strategies discussed in this article are familiar to math teachers. It is comforting, however, to know that what we do in class is affirmed by research evidence. "Marzano's 9" provides teachers with confirmation of exactly which strategies are likely to be most effective in impacting student achievement, as well as increasing student motivation and self-confidence in their study of mathematics.

Notes

1 *Sandra has generously agreed to provide the entire word deck to anyone who wants to use this activity. Email her at sduggleb@ucalgary.ca and put CONNECTIVES in the subject line.*

2 *For example, see www.youtube.com/watch?v=wdcVXJSLDcE for a math classroom example, or www.edweek.org/ew/articles/2012/08/29/02el-flipped.h32.html for an evaluation of the flipped classroom model.*

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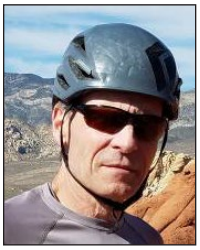
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▲ Turk's-Heads and Rope Rugs: Mathematical Connections



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Robert Chisnall taught at Queen's University in the Faculty or Education during the 1990's and 2000's. His research on the forensic analysis of knots and ligatures and climbing safety has been published in journals such as The Canadian Society of Forensic Science Journal, Forensic Science International, *the* Journal of Forensic Sciences*, and the* International Journal of Mountaineering and Climbing*. He is a retired educator, who is still active in research, safety education, accident investigation, and international forensic case work. He enjoys watching* Veritasium *on YouTube, doing math puzzles, and rock climbs with his wife to relax.*

Practical knots and rope activities offer connections to a variety of mathematical concepts and curriculum subjects at all levels. The spatial–visual nature of hands-on knotting endeavours can be appealing to students with aesthetic interests, as well, as those keen to delve into more esoteric subjects. The idea of teaching mathematics using knots and rope was briefly explored in a previous *Ontario Mathematics Gazette* article (Chisnall, 1989), wherein topics such as plane geometry and tying pentagons, sequences and the Möbius strip, catenaries and parabolas, transformations and group theory, and several other areas, were highlighted.

Activity Overview

The exercises provided in this article are an opportunity for students to explore various interconnected mathematical concepts by creating a knotted craft—either a cylindrical Turk's-Head bracelet (Figure 1), or a flat, circular coaster or rope rug (Figure 2).[1] It satisfies Seymour Pappert's notion of objects to learn with (Strohecker, 1991), and the mathematical approach is the same for both projects. To produce these crafts, a precise plan is required, and there are at least four strategies for creating patterns or maps. In mathematical terms, these maps are called *roulettes*. The simplest approach is a freehand drawing following a few geometric rules. The second is more complex because it requires exact geometric construction, using ruler and compass, to generate a map following the same rules. The third method utilizes the popular drawing toy, Spirograph®,

thereby revealing the mathematical concepts the toy embodies. The fourth method enlists the assistance of online graphing applications to explore the algebra and geometry involved.



Figure 1: *Turk's-Heads*



Figure 2: *Rope rugs*

1. Free-Hand Method (Grade 9: Geometry and Measurement)

The procedure for constructing roulette maps for Turk's-Head bracelets, coasters, and rope rugs follows and is illustrated in Figure 3.

1. Draw a regular polygon freehand. Three to six sides provide sufficient roulette complexity without getting too cumbersome. With practice, more complex roulettes are feasible.
2. Connect adjacent vertices with an exterior curve. This creates a one-level map.

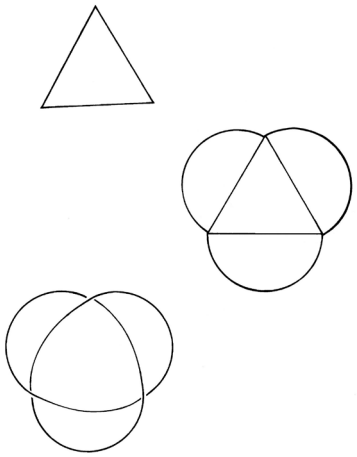


Figure 3: *Construction of roulette maps*

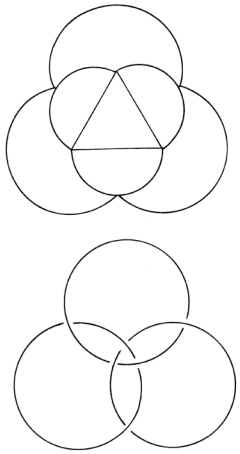


Figure 4: *The addition of a second layer and Borromean rings*

3. An equilateral triangle is the simplest regular polygon. With the addition of three curves, the map will have three crossings and form a generic Trefoil Knot, as illustrated. It consists of a single path and requires only one length of cord. To determine the continuous path of a roulette, start anywhere and trace along the curve, moving clockwise or counter-clockwise. When an intersection is encountered, take the centre curve or line. Following this basic rule, every line and curve will be traced once, and you will arrive back at the starting point. Smooth out the path.
4. Establish over-under crossing points. In order for a flat or cylindrical roulette craft to hold together and have structural integrity, the rope or cord crossings must consistently alternate. Consecutive crossings cannot be the same.
5. To add more levels, repeat step 2. Then finish with steps 3 and 4. (See Figure 4, which illustrates how to add a second layer to the Trefoil Knot.) More complex patterns with added layers may require numerous separate paths, rather than one continuous path, which is the case with the second Trefoil Knot layer. The bottom drawing in Figure 4 illustrates what are known as Borromean Rings. They consist of three paths and require three separate lengths of cord. (In this particular instance, the roulette map can be drawn more quickly using three intersecting circles.)

2. Construction Method (Grade 9: Geometry and Measurement, Number, and Algebra)

The same procedure described above can be followed. Instead of freehand drawing, however, use a ruler and straight edge to construct the initial polygon. Precise curves can be added with a compass. Care and strategic application of the construction rules must be applied to generate the desired regular polygons and curves. A regular polygon with *n* sides can be constructed with a straightedge and compass if, and only if, *n* is the product of a power of 2 and any number of distinct Fermat primes, of which five are known: 3, 5, 17, 257, and 6,5537. (Fermat primes are determined by the Gauss-Wantzel formula $F(n) = 2^{(2^n)} + 1$.) Therefore, constructed polygons have the following number of sides: 3, 4, 5, 6, 8, 10, 12, 15, 16, 17, and so on. For a regular polygon that cannot be constructed, a protractor is required to establish its vertices on a circle. They are separated by the angle 360/*n*, where *n* is the number of sides. For example, a heptagon, also known as a septagon, has seven apexes located 360/7 or about 51.43 degrees apart. A nonagon or enneagon has nine apexes 360/9 or 40 degrees apart.

3. Spirograph Method (Grade 11 Functions; Grade 12 Advanced Functions)

There are many types of roulettes, but we are concentrating on four basic types: epitrochoids and epicycloids (also known as hypertrochoids and hypercycloids), along with hypotrochoids and hypocycloids. Picture a smaller circle rotating within another larger stationary circle. Pick a point anywhere on that smaller circle. The pattern it traces as the inner circle moves



Figure 5: *The upper coin rotates around the lower coin twice.*

produces a hypotrochoid. Hypocycloids are special cases. The point selected is on the outer rim, or circumference, of the smaller inner circle. In contrast, epitrochoids are created by rotating one circle around the outside of a stationary circle. Similarly, epicycloids are created by using a point on the perimeter of the moving circle.

Think of Spirograph, the classic drawing toy children have been using for decades. Spirograph wheels come in various sizes with different numbers of gear teeth. Each rotating wheel has a series of holes at increasing distances from the centre of the wheel. Insert a pen or pencil into one of these holes to draw a roulette pattern. In order to determine how many times one wheel (rotor) must rotate within a stationary ring or around a stationary wheel (stators), so as to have your pencil or pen arrive at its starting point, there are several variables that have to be taken into account: the diameters or circumferences (numbers of gear teeth) of the two wheels, and the location of the pencil or pen hole. Use the least common multiple (*LCM*) of the gear teeth to determine the number of rotations.

$N = LCM(R, W)/W$, where:

N = number petals or lobes created

R = the number of teeth on the rotator wheel (or the diameter of the rotating circle)

W = the number of teeth on the stator wheel (or the diameter of the stationary circle)

$LCM\ (R, W)$ = least common multiple of R and W

How Spirograph Wheels Relate to Rotating Coins

The notion of rotating a circle within or around another circle is reminiscent of a question that appeared in a 1982 Scholastic Aptitude Test (SAT), for which the given answer was incorrect. The question was: How many times does a smaller circle rotate when circumnavigating the rim of a circle that is three times its radius? The answer was assumed to be three times, but the correct answer is four. Versions of that problem have appeared frequently in puzzle books and online. For example, visualize two quarters lying on a table. Both heads are facing up and oriented as illustrated in Figure 5. Similar to the action required to create an epitrochoid, the task is to keep the lower coin stationary while moving the upper coin around its perimeter, allowing the moving coin to rotate, but not slip. This must be imagined without using actual coins. How many times does the moving coin rotate, and what is the position of the coin's head when it returns to the twelve o'clock position? The moving coin will perform one full rotation by the time it arrives at the six o'clock position, with the head orientation as it was at the start, and it will complete another rotation to return to the top in its original position.

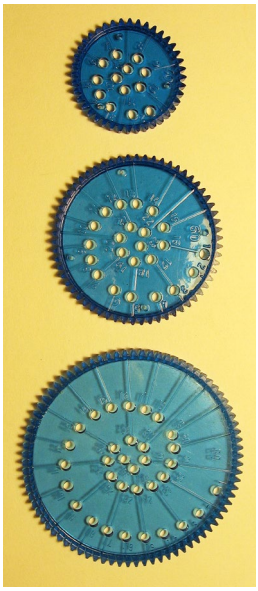


Figure 6: *Spirograph® disks, from top to bottom, containing 40, 60, and 80 gear teeth, which correlate with their circumferences.*

Depending on the circumferences, diameters, or radii of the coins or circles involved, the number or rotations (N) can be generalized using the following formula:

$N = R/r + 1$, where:

R = circumference, diameter, or radius of fixed circle

r = circumference, diameter, or radius of moving circle

R/r accounts for the circumferential travel distance, and the addition of 1 accounts for the rotation of the moving coin. The relationship between rotations and diameters is an interesting pattern to explore. Spirograph disks can be used, once the number of gear teeth for each disk is known, thereby providing a relative measure of disk circumference (Table 1 and Figure 6).

$R \rightarrow$ $\downarrow r$	40	60	80
40	2.0	2.5	3.0
60	1.67	2.0	2.33
80	1.5	1.75	2.0

Table 1: *Number of rotations a moving wheel makes to travel around a stationary wheel once, using combinations of different sizes of Spirograph disks*

4. Plotting Applications (Grade 12: Trigonometric Functions)

Roulettes can be created using general parametric equations, as show below:

Hypotrochoids: $x(\theta) = (R - r)\cos\theta + d\cos[\theta\ (R - r)/r]$, $y(\theta) = (R - r)\sin\theta - d\sin[\theta\ (R - r)/r]$

Epitrochoids: $x(\theta) = (R + r)\cos\theta - d\cos[\theta\ (R + r)/r]$, $y(\theta)$

$= (R + r) \sin \theta - d \sin [\theta (R + r)/r]$

r = radius of the moving circle

R = radius of the fixed circle

d = distance from the centre of the moving circle to the point tracing the curve

θ = the angle formed by a line running from the origin to the centre of the moving circle relative to the x axis, like a polar angle

There are ways of plotting roulettes, using these parametric equations with online plotting applications. (See the reference section for a few sample links.) Figures 7 through 10 illustrate a few examples of roulette plots. Note that the number of annular layers of crossing points, moving from the origin outward, varies in these examples: Trefoil, one layer; Quatrefoil, two layers; Cinquefoil, three layers; Sectfoil, four layers. The number of layers can be varied to make knotted configurations simpler, or more complex. (Bights (*B*) and Leads (*L*) will be explained below.)

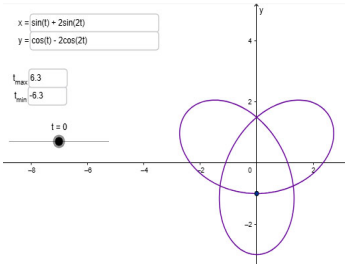


Figure 7: *Trefoil Knot.* B = *3,* L = *2.*

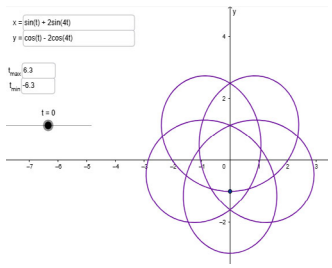


Figure 8: Quatrefoil Knot. B = *4,* L = *3.*

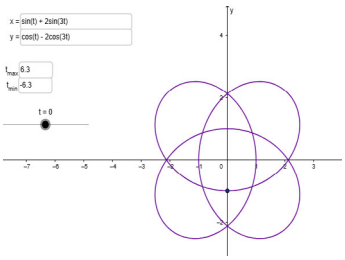


Figure 9: *Cinquefoil Knot.* B = *5,* L = *4.*

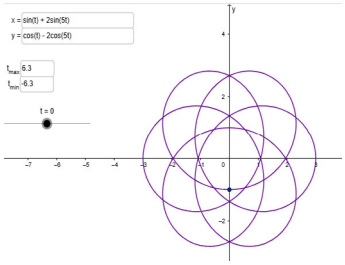


Figure 10: *Sectfoil Knot.* B = *6,* L = *5.*

More Mathematical Connections

The Law of Common Divisor
(Grade 9: Linear Relationships; Grade 11: Functions)

The Trefoil Knot roulette can be regarded at the simplest regular Turk's-Head. Turk's-Heads comprise bights and leads. Bights can be identified by the obvious outer lobes. In the case of the Trefoil, like its crossing points, there are three bights, as shown Figure 11. Leads refers to how many times the map traces around the knot's central point. Starting at the dotted line, trace along the solid line, either clockwise or counter-clockwise, and count how many times the central axis of the configuration is wrapped. The Trefoil contains two leads. This same exercise can be performed for larger complex formations, although it can be challenging. In all cases with regular Turk's-Heads in two dimensions, the lead and bight never share a common divisor. Discovering this commonality makes for an interesting exploratory exercise. A more difficult question is, why does that rule apply?

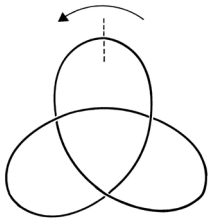


Figure 11: *The Trefoil Knot has three obvious lobes or bights, and the knot wraps its axis two times.*

The Greatest Common Factor Rule
(Grade 9: Linear Relationships; Grade 11: Factoring)

Simple roulettes, like the Trefoil, can be tied using a single cord. More complex structures require two or more separate cords. The rule of the greatest common factor states that the number of separate cords required to create a Turk's Head with *B* bights and *L* leads is the greatest common divisor of those two variables (Coleman, 1997). By drawing or tying different Turk's-Heads, and referring to Ashley's Turk's-Head table (1944, p. 234) or by creating their own table, students may be able work out this relationship. A truncated version of Ashley's chart appears in Table 2. As will be seen shortly, planar projections of topological knots reveal several variables other than bights and leads. Edges or spans between crossing points, and bounded areas, deserve attention as well.

Leads → ↓ Bights	2	3	4	5	6
2	2	1	2	1	2
3	1 (a)	3	1	1	3
4	2	1 (b)	4	1	2
5	1	1	1 (c)	5	1
6	2	3	2	1 (d)	6

Table 2: *This is a truncated version of Ashley's Turk's-Head chart, tabulating structures up to 6 bights and 6 leads, featuring Coleman's data (1997). The number of separate cords required to tie Turk's-Heads is shown for each intersection of bight and lead values based on the greatest common factor rule. The number of crossing points are not indicated. The cells with letters refer to knots suitable for parametric plots: Trefoil (a), Quatrefoil (b), Cinquefoil or Pentafoil (c), Sectfoil (d). See Figures 7 to 10.*

Euler's Polyhedron Formula Applied to Knots
(Grade 9: Linear Functions; Grade 10 Academic: Analytic Geometry)

As some students may know, Euler's formula works for any convex polyhedron, where:

$V - E + F = 2$ when: V = vertices E = edges F = faces

This formula can also be applied to planar projections of topological knots, with the following conditions. Crossing points represent vertices, cord segments between crossing points represent edges, bounded areas represent faces, and the area surrounding a knot image counts as an additional face. This relationship affords another interesting discovery exercise. By drawing or tying different topological knots and tabulating V, E, and F in each case, students may be able to work out how those three variables are connected by way of Euler's formula. With regard to the Trefoil Knot, $V = 3$, $E = 6$, and $F = 5$.

Torus Knots (Enrichment Topic)

The Trefoil Knot is a torus knot, a three-dimensional entity that can be traced around the surface of a torus or donut-shaped object (Figure 12). The subject of torus knots ventures into a specific area of interest in topology. It offers a different perspective regarding the question of why the law of common divisor works. Torus knots are characterized according to two parameters, denoted as (p, q), and the Trefoil Knot is a (2, 3) torus knot. The number of times the knot travels around the circumference or meridian of a torus determines the value of p, while q is determined by the number of times the knot wraps around the torus cylinder longitudinally. These are just different ways of expressing leads (p or L) and bights (q or B), offering an alternative way of looking at the law of common divisor.

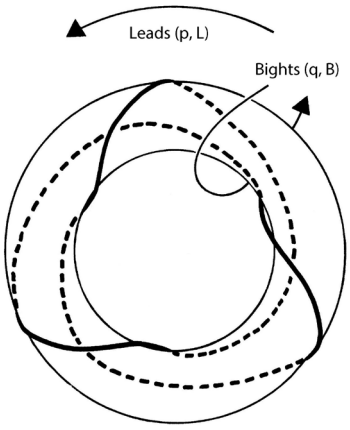


*Figure 12: The Trefoil Knot is a torus knot. It wraps the meridian twice (*p, L*) and the torus cylinder three times (*q, B*).*

Craft Materials and More Intricate Patterns

For large classes with limited space, drawing pleasing two-dimensional roulette patterns may be the goal. However, this exercise can be adapted to a hands-on exercise by supplying sufficient cord or rope of various colours so that students can make their own real-world roulettes. The length required for a project depends on the diameter of the material available and the complexity of the pattern chosen. Thinner cord—which can be acquired from hardware outlets, craft shops, and outdoor equipment suppliers—is ideal for bracelets and coasters. Marinas and climbing gyms may have retired ropes waiting to be discarded that might be suitable for rugs. Scissors, duct tape, and glue are needed to finish off loose ends. Turk's-Head Bracelets and flat coasters comprising thin cord can be worked on at a table or desk. You will need 3 or 4 metres of cord measuring 3 or 4 millimetres in diameter for simple patterns. Rugs require more space, and at least 20 metres of rope measuring around 10 millimetres in diameter will be needed. Overestimate to ensure there is sufficient length to complete the desired craft.

The result can be rendered more aesthetically pleasing if the pattern is retraced several more times in parallel—usually three repetitions—and snugged up. (A repetition of the pattern is called a ply.) The latter task may take some patience, especially if the pattern is quite intricate and considerable cord is involved. The ends can be attached with glue or tape and concealed behind overlying plies. The finished project can be deformed and manipulated to form a hollow cylinder or Turk's-Head, which may work as a bracelet, depending on its size. Alternatively, the pattern can be flattened out to create coasters for mugs and tea cups. If tied in long lengths of rope, rugs can be fashioned.

Conclusion

This is just the tip of the iceberg. Multiple roulettes can be combined using alternating crossing points and different-coloured ropes or cords for attractive results. More intricate rosette designs can be created as well (Rossing, 2012c). Another avenue of exploration is the set of complex harmonic motions called Lissajou or Bowditch patterns. (Think of the wave pattern seen on oscilloscope screens.) These patterns can be the basis for rectangular rope rugs, and they are defined by generalized parametric equations.

This was very a quick overview of the many possibilities roulettes such as Turk's-Heads, rope rugs, and other knotted structures offer in the way of unique, hands-on learning opportunities, with links to many fields in mathematics—both basic and advanced. Additional topics can include symmetry, Fourier transforms, art and mathematics, and Celtic knots, to mention a few. Further, there is an abundance of printed material and online resources to assist in designing lessons. See the following references.

Note

[1] Knot names are capitalized in forensic science and certain major knot references, like Ashley (1944), and Graumont and Hensel (1943). However, this has not been consistent in all knot books published over the years.

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Plotting Sites

Epitrochoids, Hypotrochoids, Peritrochoids – www.walter-fendt.de/ html5/men/epihypotrochoids-en.htm

Geogebra – www.geogebra.org/

KnotPlot – www.knotplot.com/

WinPlot – Plotting utility software. www.cs.hofstra.edu/winplot/ ▲

Conehead? Pyramid-head?



Architecture at the University of Winnipeg combines a cylinder with a cone and an octagonal prism with a pyramid.

Inuit Geometry

An explorer in the early 1900's, speaking about some Inuit hunters, "noted that although they had never before seen a printed map, they could grasp its meaning at once and with a pencil, fill in all sorts of details regarding distances and features" (Shoalts, 2025, p. 51).

Reference:

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