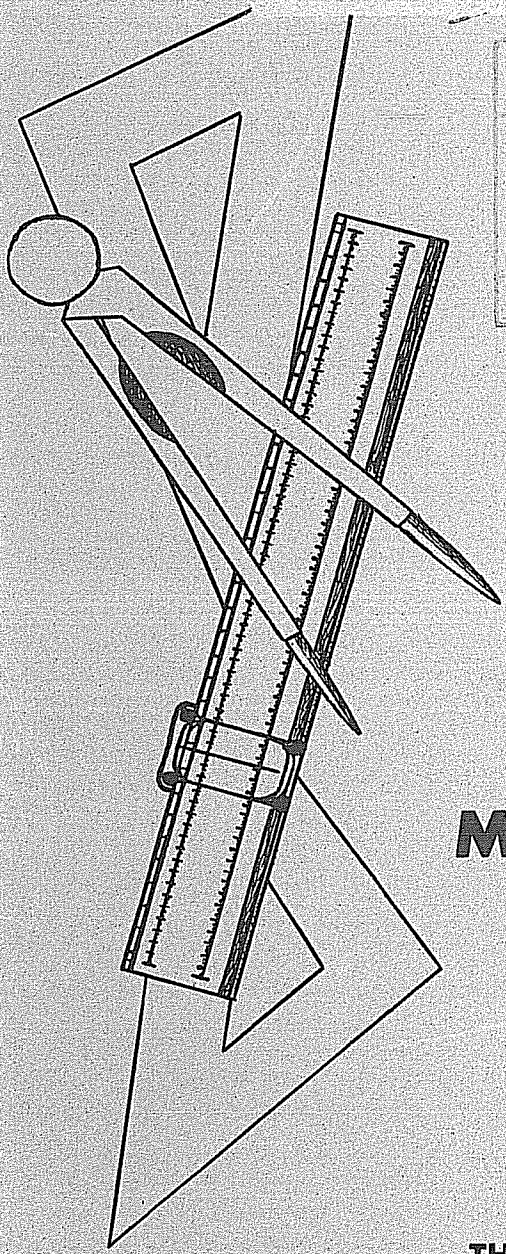


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E D I T O R I A L

It is both a pleasure and a privilege to introduce an issue of the Gazette devoted entirely to Mathematics in the new Colleges of Applied Arts and Technology. This new system of Colleges is only in its second year of operation, yet already it shows tremendous vitality in attempting to meet the needs of the great number of High School graduates who require further education in our complex society.

The issue consists of two parts.

The aim of the first part is to give the reader a prospective of the philosphy, and problems of teaching mathematics in these new colleges. First, we are enabled to see why and how the colleges came into existence. Next, we are given insight into the deeper meaning of engineering mathematics as an analytical tool in extending human ability to deal with our environment. A very interesting commentary and analysis of student difficulties follows, which pinpoints the need for better teaching at earlier levels and for intelligent co-operation between the schools and the CAATs. Lastly, the emphasis swings to the needs and goals of the students, and it is cogently pointed out that rote learning will not suffice for the widespread and varied planning and problem solving needed in the world of business and government today.

The second part consists of two reports of projects undertaken by the recently formed Advisory Committee on Colleges of Applied Arts and Technology, of the Ontario Mathematics Commission.

The first report deals with the achievement and ability of High School Grade 12 graduates (both 4 and 5 year programs) as they enter the C.A.A.T.'s. The three studies undertaken are valuable in indicating reasonable levels of achievement, and areas of curriculum in which CAAT beginning students are weak. The basic recommendation is for better communication

and cooperation between High Schools and CAAT's, particularly at the local levels.

The second report outlines in considerable detail a common first year course for the 3 year technology program. It emphasizes understanding of language, development of problem-solving ability via exposure to many problems, and applying mathematics, and is a well-balanced program involving arithmetic, algebra, trigonometry, and calculus. It is a most valuable document which should be studied by all concerned with post-elementary education in mathematics.

I close with a suggestion - why not use this issue as the theme of your next professional development day? In any case, don't file it away, keep it handy for frequent reference. I am sure it will be worth the effort.

D.H.C.

P.S. Professor D.R. Campbell and his helpers are due our sincere thanks for their work in producing this issue.

THE COLLEGES OF APPLIED ARTS AND TECHNOLOGY

by

M.K. Straka, Ontario Department of Education.

It is a well recognized fact that the human resources of a country are by far the most important ones a modern nation can possess. To develop those human resources to a maximum poses a gigantic educational problem. One may postulate that every citizen must be educated to the limit of his capacity. Due to the rapid change in technology this educational process can never be finished but must be continued over the life span of the individual.

The complexity of occupations and the high level of skill and knowledge demanded in them requires education and training beyond the high school graduation level. But the only institutions that offered such a continuation in the past were the universities. Therefore two distinct orders developed among the employees; university graduates and high school graduates, with a large gap between them. While technological progress removed many unskilled jobs (and the possibility of finding employment without high school graduation) it created new jobs in the aforementioned gap. As a result, many professionals had to perform operations far below their capacity and capable individuals were frantically groomed for more complicated jobs.

In Ontario the Department of Education quickly attacked this problem by creating the Ryerson Institute and four other institutes of technology to produce graduates to fill the gap in business and industry. Three year programs were devised, to relieve the shortage in certain areas. Institutes of trades and the Ontario Vocational Centres produced graduates from 2-year courses and apprentices filling the demand

in other areas of the gap. In the meantime the professional levels were drastically raised, thereby widening the existing gap still more. Successful as the two new types of institutions were, they were unable to produce the number and variety of graduates still urgently required. This need led to the legislation[1] creating the Colleges of Applied Arts and Technology. It may be well to remember that the purpose of these colleges is to fill a gap between high school and university graduation level and not to take over undergraduate training for the universities.

In individual cases, however, the universities will accept graduates from the Colleges of Applied Arts and Technology who have demonstrated their academic ability during their college years. The door is open to the capable student for a university education.

The major responsibilities of the Colleges of Applied Arts and Technology may be described as follows:

(1) to provide courses of types and levels beyond, or not suited to, the secondary school setting;

(2) to meet the needs of graduates from any secondary school program, apart from those wishing to attend university; and

(3) to meet the educational needs of adults and out-of-school youth, whether or not they are secondary school graduates.

The province of Ontario was divided into twenty areas, in each of which one college was established, with the possibility of further colleges as the need arose. The location of these areas is shown diagrammatically in figure 1, while figure 2 gives the name and address of each individual college.

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1 "C.A.A.T. Basic Documents", available on request from the Department of Education, Information Branch.

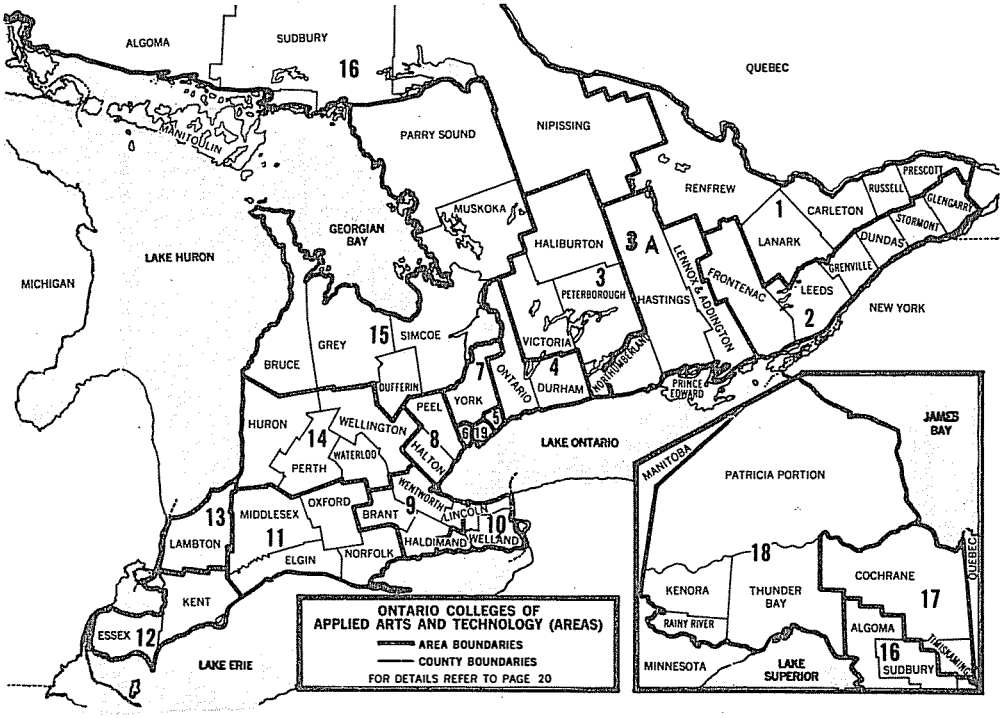


FIGURE 1 - DISTRIBUTION OF COLLEGES ACROSS ONTARIO

LIST OF COLLEGES OF APPLIED ARTS AND TECHNOLOGY

Further information regarding any of the courses shown may be obtained by writing to the college concerned.

AREA 1:
Algonquin College
President: Dr. F.T. Rosser
Woodroffe Campus

Ottawa 5
Business Campus

Elmar Campus

AREA 2:
St. Lawrence College
President: R.C. Short
Kingston Campus

Cornwall Campus

1500 Cumberland St., Cornwall

AREA 3:
Sir Sandford Fleming

Sir Sandford Fleming Campus
President: D.B. Sutherland

AREA 3A:
Loyalist Campus
President: J.R. Bradford

AREA 4:
Durham College

President: Dr. G.E. Willey

AREA 5:
Centennial College

President: J.L. Haar
T.I.P.T. Campus

AREA 6:
Humber College

Toronto 14
President: G. Wragg

AREA 7:
Seneca College

Willowdale
President: W.T. Newnham

AREA 8:
Sheridan College
President: J.M. Porter
Main Campus
Toronto
Brampton

Lorne Park Campus
School of Design
Toronto

Port Credit

AREA 9:
Mohawk College

Hamilton
President: J.W. Hazelton

AREA 10:
Niagara College
P.O. Box 248

President: Dr. W.G. Bowen

AREA 11:
Fanshawe College

President: Dr. J.A. Colvin

AREA 12:
St. Clair College

President: Dr. R.C. Quittenton

AREA 13:
Lambton College

President: W.E. Franke

AREA 14:
Conestoga College

President: J.W. Church

AREA 15:
Georgian College

President: R.P. Crawford

AREA 16:
Cambrian College
President: J.T. Koski
Sudbury Campus

North Bay Campus

Sault Ste. Marie Campus

Ryerson Polytechnical Institute

President: F.C. Jorgenson

AREA 17:
Northern College
President: O.E. Walli
Main Office

Timmins
Porcupine Campus

Kirkland Lake
Haileybury Campus, School of Mines

AREA 18:
Confederation College

President: D.A.R. Bradshaw
Administrative Offices–

–Campus–
c/o Sir Winston Churchill Collegiate and Vocational Institute

AREA 19:
George Brown College

Nassau Campus

Dartnell Campus

Many colleges have satellite campuses in addition to the various locations given above.

FIGURE 2 – DETAILS OF INDIVIDUAL COLLEGES

The Council of Regents, by law the Minister's assisting body for planning, establishing and co-ordinating programs of instruction, set up the Boards of Governors for every college. The organization chart of a typical college is shown in figure 3. To illustrate the variety of courses offered in a college, a schematic of a typical technology division is shown within the framework of other education levels in figure 4:

Examples of the different available programs are the following:

Technology: Chemical, Civil, Mechanical, Electronics
Senior Engineering Technician: Automotive, Surveying, Geology
Engineering Technician: Computer Operator, Drafting, Heating
Apprenticeship: Baking, Millwright, Plumbing, Watch Repair

Similar schematics could be drawn for the business and applied arts divisions. In the year of study 1968/69 the colleges will offer approximately 230 different courses on all levels. Please note that this information applies only to the day school operation. In addition the extension department of each college duplicates as many opportunities as practically possible, (most extension classes are offered at night) often in satellite locations.

None of the courses are terminal in the sense that no further training after completion is possible. On the contrary, everybody is given a chance to advance as far as his individual abilities will carry him. Many possibilities of transfer are built into the system. (They are shown in figure 4 by dotted arrows). It is noted that two years of technician training is not equated to two years of technology and so on. The reason for this is the basic philosophy of technical training, best explained in a diagram: (See figure 5, page 11). The emphasis on skill and academic knowledge

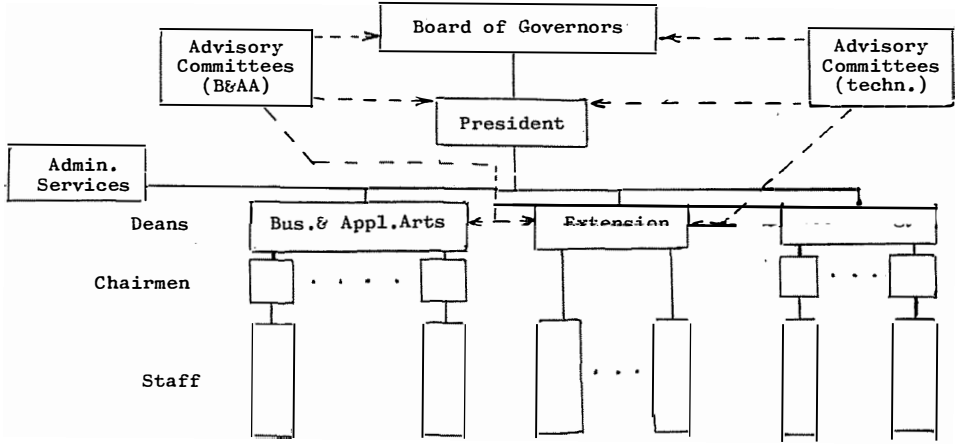


FIGURE 3 - ORGANIZATION CHART OF A TYPICAL COLLEGE

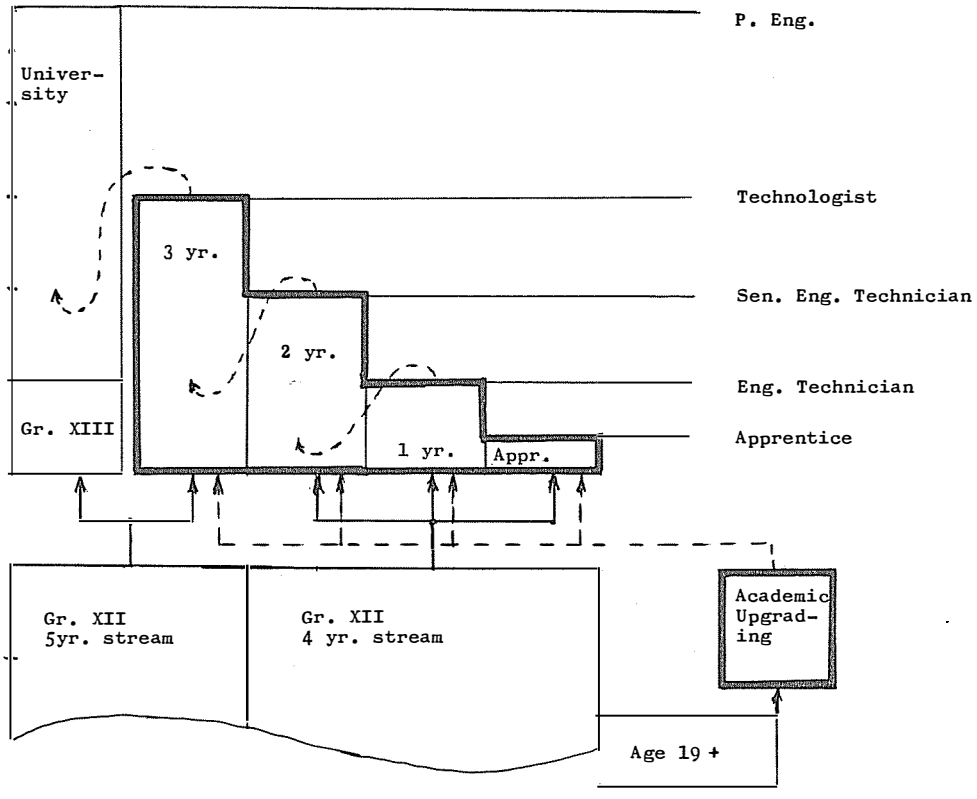


FIGURE 4 - ORGANIZATION CHART OF A TYPICAL TECHNOLOGY DIVISION

is different for every type of training. In order to fit into a new stream, the academic content must be consolidated and upgraded to a certain extent (as shown by the dotted arrows).

There is, of course, transfer in the opposite direction also. A student who finds it difficult to cope with the abstractions of a three-year course may do quite well in a two-year program. A well trained counselling staff is available not only to advise students on choices and changes in his academic career, but also to be of assistance in any matter the student may wish to bring up.

The colleges are well equipped to deal with any future changes that may come upon us suddenly. Through the close liaison with business and industry via the advisory committees and professional organizations the content and/or types of programs are kept up to date continuously.

There are still many problems to be solved before the system will function at maximum efficiency. To name one: many programs in the high schools that used to be terminal have now corresponding continuations in the colleges. An adjustment of the curriculum is necessary at both ends. It may be interesting to note that the Ontario Mathematics Commission has responded immediately to the need. Right now several committees are engaged, in cooperation with the educational authorities, in advising on the changes in the mathematics curriculum in the high schools as well as in the colleges.

Although the system of colleges is only one year old, it has already demonstrated a tremendous potential. A few statistical data may illustrate the rapid growth projected over the next years:

Year of Study	Enrolment*
1966/67	12,500
1967/68	20,000
1968/69	30,000 (expected)

* Including Ryerson Polytechnical Institute

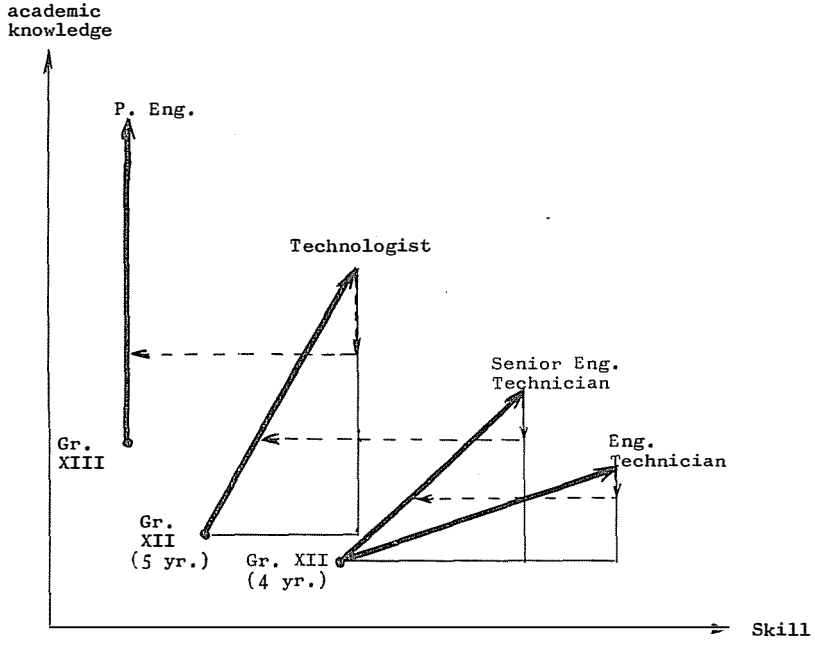


FIGURE 5 - BASIC PHILOSPHY OF TECHNICAL TRAINING

In spite of the official (and rather lengthy) title of "The Colleges of Applied Arts and Technology", the term "community colleges" is frequently used. Though descriptive, it brings about confusion with the equally named institutions in the States, which are quite different from most of our colleges.

In Canada the aim is to establish a true "community college" that will look after all phases of education other than that of the university--technical, business, arts and recreational--for all the members of a given community according to the local (and provincial or even national) needs. But whatever name is used, it is of the utmost importance that everybody understand the basic ideas and principles of the new institutions of higher learning. It is hoped that this short article has contributed a little to this basic understanding.

WHAT IS ENGINEERING MATHEMATICS?

by

Andrew I. Vertesi,
Mohawk College of Applied Arts and Technology

Mathematics is a man-made logical structure and in its purest form it is more of a reflection of the workings of the human brain than a true image of the physical world surrounding us. It must be remembered however, that much human thinking is focused on the basic problems of our existence in a physical world. Logic in this area of human thinking is based on observations made possible by our ability to see, to hear, to smell, to taste, to touch and to move. Our ability to make logical predictions as to the outcome of certain physical events and to make value judgements relating to them is predicated upon outside stimuli reaching our brain through our sensory systems with sufficient frequency and in a stable pattern. It is perhaps for this reason that Mathematics as a structure built by human thinking incorporates in its foundations the experiences we gained in our encounters with the physical world, and provides in some degree abstract models of human abilities which made possible these experiences.

Thus though the structure rises high above the clouds - by following the pattern of the foundations - it emerges as a powerful tool in our efforts to extend human abilities and to achieve control of our environment, which are the basic objectives of all engineering activities.

It would be impossible to discuss in a short article the many faces of engineering mathematics. However, perhaps I can illustrate its character by showing how the transition

from a pure mathematical concept to physical reality could provide an analytical tool for the extension of a human abilitiy.

Let us take the mathematical concept of division by zero as it relates to the extension of the human ability to judge distances. In a purely mathematical investigation we may propose a problem such as "Divide a by 0 " where "a" is any number other than 0 itself. Assuming that the operation leads to an answer we may write $\frac{a}{0} = x$ where the value of "x" is the answer to the division. However, if $\frac{a}{0} = x$, then $a = 0.x$ must be true, and remembering that any number multiplied by zero equals zero the statement $a = 0$ is also true. It is easy to see that assuming the existence of an answer to the proposed division leads to the conclusion that all numbers are equal to zero. Finding this an intolerable situation we conclude that proposing a division by zero is not a decent thing to do and declare the operation to be undefined. This would be the end of the problem unless we ask a disturbing question: "Is the conclusion that division by zero is undefined only a necessity to safeguard the structure of Mathematics, or is it also a fact in line with our physical experiences?"

To answer this question we may think of a physical analogy of mathematical division such as a process which could be used to divide, or separate if you wish, a given steel rod into shorter rods of <u>equal length</u>. For example if we have a rod 6 ft. in length and wish to "divide" it into 3 shorter rods of equal length the process could be described using mathematical symbolism as: $\frac{6}{3} = 2$. Interpreting the elements of this equation we can recognize that the numerator on the left side represents the original length of the steel rod, the denominator on the left side indicates the number of shorter rods we must form by using our physical "division" process, and the quantity on the right side is the length of the newly formed rods.

The statement $\frac{6}{1} = 6$ will then call for no division at all. Indeed, the original rod cannot be separated into less than two smaller parts indicating that we cannot divide by a number less than 2 in case of this physical division process. But how about $\frac{6}{1.5} = 4$? Well, in this case the number 4 at the right side is indicating that after the division is completed the newly formed rods should all have a length of 4 ft. In order to make this possible we could stretch the original length of the rod to a length of 12 ft. and then divide it into 3 equal parts. Using symbols this could be indicated as $\frac{6 \times 2}{3} = 4$ which of course is an equivalent of the original form $\frac{6}{1.5} = 4$. Investigating a division by 0.5 in the same way would show that really only stretching takes place without any division. We are ready now to investigate the proposition of $\frac{6}{0} = ?$. This is really an order to take a rod of 6 ft. length and to separate it into zero number of shorter parts by using our physical division process. Simply speaking $\frac{6}{0}$ is calling for the complete destruction of the matter contained in the original rod. However, our experience expressed in the laws of Physics indicates that really there can be no such physical division process. Division by zero then is excluded from the structure of Mathematics not only as a matter of convenience, but also as a reflection of physical reality. We have evidence now that when the mathematical investigation of a physical problem is leading to a division by zero this may be an indication of limitations imposed by the nature of the problem or by the selected values of the parameters.

Our next step is the building of a mathematical model of the mechanism humans use in judging distances. We know by experience that while most people are in general very good at judging short distances, the reliability of their judgement rapidly deteriorates as the distance in question increases. The objectives of our investigation will be to find out why this is so, and to discover methods which could

be used to extend this human ability. Fig. 1 shows that a distant object and the two eye-balls are located at the vertices of a triangle. A mathematical formula relating

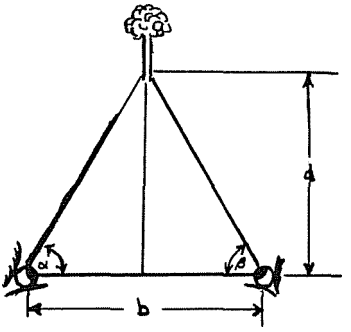


Fig. 1

the quantities involved could be written in the form:

$d = \frac{b}{\cot\alpha + \cot\beta}$. Clearly our ability to judge distances depends on the distance "b" between the two eye-balls, and on the fact that whenever we look at an object we form a left and a right picture with the two eye-balls registering the angles α and β . The sensing of these angles may involve a certain amount of errors affecting the reliability of our judgement regarding the magnitude of distance "d" . It is important to note that the error in judging distance "d" , caused by the inaccurate sensing of angles α and β , increases as the angles α and β get larger. If we let distance "d" become very large now relative to distance "b" we can se (Fig. 2) that the angles α and β may be sensed as having a magnitude of 90 degrees. However, $\cot 90^{o}$ equals zero and the distance formula will appear as $d = \frac{b}{0}$. We have encountered a division by zero indicating that in this case distance "d" becomes undefined. Note that in this case the fact that we have encountered a divison



Fig. 2.

by zero is not an indication of error. It is really the "result" of our investigation though it is not a numerical answer.

We have now a clear indication that our ability to judge distances is failing when the distance to be judged is very large relative to the distance between our eye-balls. This finding immediately suggests a method of extending our ability to judge distances: "We have to increase distance "b", or in other words we have to set our eye-balls more apart." This may not appear to be a practical idea; however, this is the point where engineering picks up the results of mathematical analysis. It is quite possible for instance to develop prism systems which would measure the angles α and β forming left and right pictures many feet apart and then transmit these pictures to our eye-balls. Or in case of very large distances we may set up observation stations which may be miles apart. With this we have completed our investigation.

Of course the continuation of this study would be the actual design of an optical system which would "set our eye-balls further apart". Such a design would certainly involve numerical considerations and would therefore be recognized relatively readily as an application of engineering mathematics.

It is my hope however, that in a small way I have succeeded in showing the reader that numerical considerations represent only one aspect of engineering mathematics and that engineering mathematics is really a science relating pure mathematics to physical reality in our efforts to extend human abilities and to achieve control over our environment.

TECHNOLOGY STUDENTS' DIFFICULTIES IN MATHEMATICS

by Curt Moser,

Algonquin College, Ottawa

I have been invited to make comments on the difficulties in Mathematics encountered by students who enter the first year of a School of Technology of a College of Applied Arts and Technology. The comments relate to the following areas of concern--language retention of basic mathematics, and the common first year course.

The Language Problem

At the level we are concerned with, Mathematics can certainly be considered as a language rather than an object of research. The short-comings of the students in mastering the mathematical language are numerous and I will mention two which are most apparent.

Translation of worded problems into mathematical expressions is a major weakness. A part of the difficulty in the translation seems to arise from a limited vocabulary. The translation of words and expressions such as proportional to, inverse of, product, factor, magnitude, displacement, initially, decrease, at rest, odd-numbered, per unit length, etc. into mathematical terms seems to create problems.

Another part of the same difficulty lies in setting up equations with the unknown quantity designated by some symbol, say U. The student fails to treat U in the same way as a known quantity and write down the mathematical relation that exists between U and the other known quantities.

Still another part of the difficulty arises from careless reading habits.

A second major weakness is the failure of students to apply the algebraic axioms in concrete cases even when they are familiar with them. The same holds for the manipulation of trigonometric identities. There is a continuing need for skill in manipulation in all mathematical subjects of the technology program. Some of the problems which have proved to be difficult are:

factor $B^{2m} - A^{4m}$

rationalize $\dfrac{1}{x - \sqrt{5}}$

simplify $\dfrac{\sin \beta}{1 + \cos \beta}$

simplify $\lg \lg (10^{a^2})$

solve the equation $\dfrac{P}{Po} = \left[1 - \dfrac{k-1}{2} \left(\dfrac{v}{vo}\right)^2\right]^{\frac{k}{k-1}}$ for v

if $\lg i = - \dfrac{RT}{L} + \lg A$ (find i)

find the x--intercept of the curve $y = \dfrac{1}{x^2} - 2$

if C is constant sketch the curve $I = \dfrac{C}{\lambda^4}$

A considerable number of our first year students are what may be called "xy" students. These students can handle mathematical problems expressed in terms of x, y and a,b but are confused if the symbols are changed, say into p and σ . Since all letters of the Latin and Greek alphabets are constantly used, in science and engineering, it may be advisable to expose the students to these symbols more often.

Retaining Basic Mathematics

There is nothing more frustrating than teaching integration by partial fractions only to discover that half of the class has forgotten how to bring two or more fractions to the smallest common denominator.

I do not think that the lack of retention of basic information is due to bad memory on the part of the students. I do believe, however, that the failure to retain facts is based on at least the following deficiencies:

(a) Lack of practical understanding. A typical case is that of logarithms. A few examples will demonstrate this.

(i) The student has been taught that if $b^x = A, A > o$, $b > 1$ then we call $x = \lg_b A$. If later in the course we ask a student to solve $u = \lg v$ for variable v he is most likely not able to do it.

(ii) All students will find that $\lg 2 = 0.30103$. Again, most of them do not see the implication that $2 = 10^{0.30103}$.

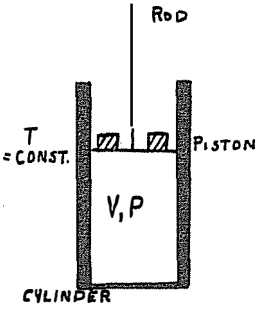
(iii) If we write $\lg 0.2 = 0.30103 - 1 = -0.69897$ students will object that the log of a number cannot be negative. Hence, the identity $0.2 = 10^{-0.69897}$ is unfamiliar to many students.

(iv) Frequently students can read the laws of logarithms only from the left side to the right as illustrated below:

$\lg \sqrt[3]{\frac{a^2}{b}} = \frac{1}{3}\left[2\lg a - \lg b\right]$ but cannot find the antilog of, say $\frac{1}{3}\lg P - (\lg v + 2\lg M)$

(v) Statements such as $\lg u - \lg v = \frac{u}{v}$ are met quite frequently. In all these examples the practical understanding of the basic definitions of logarithms is missing.

(b) <u>The attitude of students to Mathematics</u> is very often detrimental to their progress in it. Work in Mathematics is done to obtain marks but not to increase knowledge and to gain deeper insight into other areas. "Who wants to know that stuff, all I want is marks". Such an attitude is nourished by a lack of proper motivation and also when subjects such as Mathematics, Physics, Chenistry, etc. are placed in compartments and students cannot see inter-relations. The following example which I have personally observed will illustrate this point. Teacher Z explained Boyle's law as follows.



Referring to the figure shown drawn on his blackboard he stated that if a weight is placed on the piston the gas in the cylinder is under a pressure P and occupies the volume V. Now if some of the weight is taken off the piston the pressure is reduced, the piston moves up and the volume is increased. In this process, the product PV = constant, which is called Boyle's law. A student then remarked that if one writes said law in the form of $V = \frac{\text{constant}}{P}$ it states, exactly as before, that if the denominator (P) is decreased the quotient (V) increased, and if the denominator (P) is increased the quotient (V) decreased. Here Mr. Z interrupted the student by saying: "Your explanation is Mathematics and has nothing to do with the physical facts". Thus teachers themselves can contribute to poor student attitudes.

(c) Study Habits. Some students seem to think that Mathematics can be learned by cramming. If a student wants marks only he may succeed in the first test. We have made an extensive study of the value of "crash programs" and found that such a cram course may be effective momentarily, but the degree of understanding is low and the retaining value over a period of 3 months is practically nil.

It is better to study mathematics one hour every other evening three times per week than three hours in one night. Repeated action is less tiring, and much more effective than one single attempt. Further, practical understanding of Mathematics comes with doing exercises and problems rather than by watching the teacher at the blackboard. Nobody learns to play the piano by watching a piano player.

(d) Student Maturity. It is a sign of maturity in a subject when one knows what to study. The question "Sir, are we responsible for this?" is revealing to the teacher. We try to reach a certain maturity among the students by teaching basic principles and how to think. I strongly believe in the economy of theory in the classroom: that is, with a minimum of theory we cover a maximum of ground. For this reason our tests and examinations are conducted with open books and notes. We do not want the students to memorize theories and formulas. In addition, we want the students to be able to read textbooks and the like because this is what they will be doing on the job when nobody is around to explain.

Proper study habits should be cultivated very early in a student's life. At the last College of Applied Arts and Technology Curriculum Conference of Mathematics it became quite apparent that a student who knows how and why to study is the one who will be successful, but not necessarily the one with the highest entrance qualifications.

The Common First Year Course

The first year course in the Colleges of Applied Arts and Technology is common for several technologies. For obvious reasons this arrangement is very desirable and useful. For the mathematics teacher, however, it has presented problems of considerable magnitude.

Up to quite recently a first year class contained grade 12 students from the Four-Year Program, from the Five-Year Program with "New Maths", without it, some with grade 13 subjects, and also mature students of various backgrounds. The composition of first year classes is not uniform for all. This of course, has created considerable hardship for the students as well as for the instructor.

The following is an excerpt of the Admission Requirements established for the Schools of Technology:

> "The minimum educational requirements for admission to the first year of any course in the School of Technology is the Ontario Secondary School Graduation Diploma obtained after four years of successful study in a Five-Year Program. Applicants should have passed the English, Mathematics, Physics and Chemistry subjects of grade 11 and 12 with an overall average of at least 60 per cent."
>
> "Note: All applications are received by the Board of Admission, which is empowered to reject or defer the admission of any applicant whose qualifications, in the opinion of the Board, are not adequate to the demands of the course in which the applicant wishes to register.

> Applicants who have obtained the Secondary School Graduation Diploma with high standing in the Four-Year Program may, at the discretion of the Board of Admissions, be admitted to the course in the School of Technology."

It is common practice to interpret "high standing" as "70% and better". The admission of graduates from the Four-Year Program has created a major problem, as these students, no matter how good the standing, are often lacking subject matter in various topics of trigonometry, analytical geometry and algebra.

It is thus apparent that a remedial program in Mathematics is of importance to any College. The necessity for remedial training is accentuated by the fact that some of our best students are in fact graduates of the Four-Year Program who have overcome their initial deficiency in Mathematics. In consideration of this fact some Secondary Schools provide remedial training for those students who lack such subject matter but who are otherwise qualified to enter the first year technology course. These schools provide a real service to the students and it is suggested that they maintain contact with the Mathematics Department of the College of their region to obtain detailed information.

For students of the Four-Year Program who cannot meet the requirements, the door to the College is by no means closed as they can enroll in one of the numerous Technician or Technical Programs.

The Effect of the "New Mathematics"

It may be of interest to the reader to learn some comments on the effect of the "New Mathematics" on our

first year students

The fact that in general the students of the Four-Year Program are not exposed to the "New Maths" emphasizes again the heterogeneous character of the first year classes in the School of Technology. For the reasons indicated below, however, we are not overly concerned about this deficiency.

With regard to the use of the concepts, terminology and notation of sets the following general remarks may be of interest:

(i) So long as these means serve to clarify basic concepts and promote understanding they have proved to be very useful. Once, however, the students have become familiar with the subject, the terminology and the notation in particular should be replaced by the conventional ones. A scientist or engineer does not talk about "open sentences", "sets of ordered pairs" etc., and nobody writes Ohm's law in the form $\{(e, i) | e = Ri, e, i, R \in Re\}$

(ii) At various places too much emphasis is placed on "New Maths" at the expense of training manipulative skill. For a Technologist the latter is more important than the former. This is particularly so in learning numerical analysis for the purpose of computer programming.

It is felt that the pendululum of reform of Mathematics has reached the maximum amplitude and is bound to swing back to more realistic and stable position.

In teaching mathematics from an applied point of view, one cannot escape the impression that the High School curriculum is still heavily University oriented. This seems

to be so despite the fact that a minority of students of the academic programs enters a University, despite the fact that for each scientist and engineer a supporting staff of 3 to 4 technologists or technicians is desperately needed and despite the fact that at the present time the ratio is just reversed.

As a result of the general overemphasis of University education many of our first year technology students have entered the College AS A SECOND CHOICE TO UNIVERSITY. So have the University drop-outs who have no other place to go. This attitude, of course, is all wrong as it does not lead to satisfaction, happiness and success in a profession. In "Seminar Number Two" of the Association of Professional Engineers in March 1968 Mr. J.H. Craighead, Administrator, C.A.A.T. stated

> "We should not downgrade the Colleges, by suggesting that the programs are designed for students not good enough to go to University, but rather for those students interested in applying knowledge to the solution of specific problems in industry, in business and in society".

We feel that the parents of the High School students should be better informed about the role a technologist will play in the future. For a start it may be a good idea if the guidance counsellors would approach the C.A.A.T. of his area to obtain first hand information on what we do; if not to satisfy his own curiosity than to help the students.

Conclusion:

It can be seen that the birth of the new Colleges have indeed brought problems. However, already their problems have been recognized,and solutions are being sought. The goodwill, understanding and cooperation of Secondary School mathematics teachers can be a significant help in realizing these solutions.

OBSERVATIONS OF A FORMER BUSINESSMAN ON TEACHING AT A C.A.A.T.

by R.C. Hamilton

Fanshawe College, London

Students Today

The other day, someome asked me if I was ready to go back to business. Business is the main reason for our strong economy and affluent society, and I had many happy days there. Back to business? Not me! Sure, there are lots of problems--I prefer to call them challenges--in teaching, but there are many, many rewards, too. I believe that the most important assest in a community is our young people. To work with them is a satisfying and stimulating experience.

Oh yes, we have a few students who won't work or apply themselves, who are passing the time, or who may be glad to have a place away from the cold of winter. However, it is my feeling that a large majority of the students at the College of Applied Arts and Technology appreciate the opportunities for further education and training. Many of them did well in a four or five year program in secondary school. However, in some cases, students did not achieve well in secondary school, and many of these students are grateful for a second chance to apply themselves, and they take the opportunity to broaden their practical training and cultural background.

As far as I am concerned, we have a tremendous number of very fine teenagers and young people. We hear far too much about the few who create problems, rowdyism and destruction. These form a very small percentage of the young people and the publicity they receive from news-hungry media of communication--newspaper, radio and T.V.--

is far out of realistic proportions. We must remember the contribution and good citizenship that the vast majority of teenagers and other young people are giving to our society.

Mathematical Background of Students

At Fanshawe College, I have taught mathematics ranging from mathematics for apprentices (fractions and other arithmetic and simple algebraic concepts) to mathematics for technicians (such as differential and integral calculus). My main field of endeavour has been mathematics for students in Business Administration and other business courses, including Mathematics of Finance, algebra and statistics. In each case, we attempt to adapt the mathematics to the course; for example, with data processing students, we emphasize the algebraic concepts which have greater applicability to the computer, such as number systems and matrices.

One of our chief problems is that the mathematical backgroud of the students has such a wide range--little or no algebra (or other mathematics beyond arithmetic) in some cases, while in other instances, students may have Grade 13 Math A and even Math B. Those who are light in mathematical background do need extra help. The upgrading courses which our own College will offer, will help to alleviate this situation. It is encouraging to see that many secondary schools are giving broader mathematical courses to students in their four year programs. It is worth noting, also, that liason is improving between the colleges and the secondary school system to co-ordinate their respective mathematics programs. This is a must and the Ontario Mathematics Commission is to be commended for the part which it is playing in this regard.

There are many challenges for all of us who are involved in mathematics at these various levels. During the past year, I was interested to see that several of my students with a poor mathematical backgroudd visited their former

secondary schools to voice the opinion that they should have had broader training in mathematics. Perhaps when such a student was in the high school, he did not take the opportunity to learn more. Certainly, we want to encourage young people to be responsible for themselves, but perhaps here lies a real challenge to us as teachers--to create a greater enthusiasm for mathematics at various levels of schooling.

Development of Thinking Processes

In business it seemed that there were never enough people on staff to undertake assignments or projcets and see them all the way through. That is to say, there were not enough individuals who could do the planning as well as the routine detail involved in investigating a business problem, e.g., to streamline an office procedure, design a cost improvement program, develop a promotion for a new line of merchandise, or draw up a sales plan. There was usually no problem in securing people to do the routine work, but it was more difficult to find someone to do detailed planning which required careful thought.

Fifty years ago, (much less in many businesses), a very few people--often only one person in a good-sized organization--could do the planning and problem-solving for the Company. This involves considerable thinking, the ability to organize thoughts and the foresight to visualize what will happen in a given set of circumstances. In today's world, our society is much more complicated, the problems in business and government are much more complex, and far more planning and problem-solving are necessary. Many people are needed who will assume the responsibility, have the desire and ability to think through the detailed planning, and solve the wide range of problems with which business organizations are confronted today.

As we progress more and more into the computer age, this situation will be compounded. It will be necessary for management to learn to use computers to the best advantage and to interpret the results effectively. It will also be necessary to have efficient programs to make the best use of the computers.

Hubert A. Simon, Professor of Computer Science and Psychology, and Associate Dean of the Graduate School of Industrial Administration at Carnegie Institute of Technology, has had extensive consulting experiences with business firms, government agencies, and foundations, and he has been concerned with decision-making processes in organizations. He states: "We can still accept the classical list of factors of production--land, labor and capital. But the content of several items, notably capital and labor has greatly changed. As 'capital' became 'power driven machinery', labor became 'thinking'. What a modern worker-----rents out when he accepts employment is his brain-----."[1]

Despite the large numbers of graduates from universities, business is not able to get sufficient of them to fill their needs in various levels of management. In some cases, the university graduate does not fit into the business pattern. Similar comments may be made about the industrial situation. For this reason, there is a very great need for the training of personnel to handle the "in-between" levels of management, and this is where the Colleges of Applied Arts and Technology can be very helpful.

How does all this relate to teaching mathematics in a Community College (C.A.A.T.)? I have a firm conviction that if our educational processes could encourage and help students to develop their thinking, we would not have to worry about students learing mathematics, English or any

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[1] SIMON, Herbert A., Canadian Imperial Bank of Commerce Commercial Letter, October, 1966.

other subject. I would hope that a good start could be made in developing thinking processes in the secondary school system. If it has not, I feel that we should really encourage students in this direction in the Colleges of Applied Arts and Technology.

Mathematics should be helpful in developing thinking processes. Some students tend to learn by rote, even in mathematics, and in many cases, pass examination on this basis. It can be helpful to the student if we encourage him to see the logic in the various topics--how we proceed from one step to another, giving full consideration to what we know at the beginning, what we want to find out, and the logical steps needed to arrive at the solution. In so doing, our minds learn to take concepts and principles and organize them in problem-solving.

Challenges for Four Year Program Students

Many of the students who come to the Colleges of Applied Arts and Technology have come throuh the Four Year Program in secondary school. My college teaching experience has been brief, and no doubt I will change my mind on many things as time goes on, but I have wondered if at times we have been afraid to give the four year student a greater challenge. Specifically, in mathematics, for example, do we try to encourage the student to see why he is taking certain steps rather than let him learn a particular problem by rule of thumb. Many people, unconsciously, I think, rebel against learning something new, but there is a great deal of gratification for them when they accept the challenge to do so.

In the process of solving for an unknown in an equation or formula, I feel it is more helpful to a student to learn to manipulate the equation and end up with the unknown on the left hand side by itself, than to learn a different rule of thumb for the solution of an unknown

in each different situation. I feel that if the student learns why he is doing certain operations, his thinking processes will benefit. In this regard, I interviewed and hired many people into business with this type of background. In working with these people day by day and in interviewing many others, I found that a very high percentage wished that they had stayed in school longer, that they had learned more. Many indicated regret that they had not learned the "why" of many things they had studied.

I am sure that each of us often recalls comments made by individuals and circumstances surrounding them. These are often excuses and, in many cases are used to cover up for bad performances. But in some instances perhaps we should look beneath the surface to find the real meaning.

I recall one student who was having considerable difficulty with mathematics and needed extra help, came by my desk several times at the end of the period and said "This is great". Two or three times he commented "You know, I really like this but I got bogged down and then I became frustrated". In working with him, invariably I would find that his problem was not so much the point of the immediate lesson, as the handling of a fraction, the manipulation of an equation, or some other elementary algebraic problem. The teacher must first look at himself and try to improve his own presentation, and I found that this could often be done. This particular student had come to Fanshawe College with no training in algebra, chiefly perhaps because of his own lack of effort. It would have been helpful if he had taken some algebra courses.

I have referred earlier in this article to the broader mathematical curriculum and co-ordination of plans between secondary schools and the C.A.A.T.'s. In addition to this if teachers encourage the students at various levels to strive for more than basic minima, more doors will be left open for the student in future. In this vein of thought,

many people involved in counselling and guidance feel that the average student graduating from school today will change his vocation four times during his lifetime. The more that is learned at each level, the easier it will be for him to make a change.

The Relation of Other Experiences to Teaching

In many ways I like teaching mathematics for its own sake. However, I find that I am able to refer to many applications or incidents in my own business, army and other experiences which help to clarify a point or create interest. I suppose that training staff is another form of teaching. These experiences, together with an excellent teacher training program at Althouse College of Education, have made it easy to transfer from business to teaching mathematics. To work with the most important asset in our community--our young people--is a privilege.

Acknowledgement

Much of this material has been discussed with my good friend, Lt. Col. G.T. Kirk, who took early retirement from the permanent army to take secondary school teacher training and is presently on staff at Fanshawe College. I am deeply indebted to him for his comments and criticisms.

R.C. Hamilton
Fanshawe College
London

ONTARIO MATHEMATICS COMMISSION

Report on Project 1 of the Committee on Colleges of Applied Arts and Technology

Summary and Recommendations

Aims:

1. An examination of the present and proposed courses in mathematics in secondary schools and CAAT's with the purpose of facilitating the transition to the colleges of students who have completed Grade 12 of the 4 or 5 year programme.
2. To identify gaps, problem areas and differences in emphasis between secondary schools and CAAT's and to provide recommendations for a smoother transition from secondary schools to the colleges of applied arts and technology.
3. To assist the committee working on Project 2 to draw up a realistic programme for the sudents in first year Technology at the CAAT's.

Personnel:

Mr. A. Vertesi, Head of Mathematics Department, Mohawk CAAT, Hamilton, Ontario (Chairman of the CAAT committee of OMC)

Mr. K.A.C. Jones, Head of Mathematics Department, Anderson High School, Whitby, Ontario

Mr. E.S. Michael, Head of Mathematics Department, Danforth Technical School, Toronto, Ontario

Mr. F.G.B. Maskell, Mathematics Department, Algonquin CAAT, Ottawa, Ontario

Mr. D.R. Campbell, Mathematics Department, Althouse College of Education, UWO, London, Ontario (Chairman for Project 1 of the CAAT Committee)

Work Plan for Project 1:

1. A study of the High School Courses, examinations, texts, etc. to determine what skills and knowledge in Mathematics could reasonably be expected of a Grad 12 graduate in both the 4 and 5 year programme. The study should show some sample questions to indicate the degree of difficulty students could handle in the various topics.
2. A study of entrance tests and first year programmes ot the 3-year Engineering Technology courses at the CAAT's to determine prerequisites for the first year of the 3-year Engineering Technology course.
3. A study of the new Department Curricula for the 4 year courses in Mathematics in the Secondary Schools.
4. A report on the Mathematics Commission material in use with Grade 10 and Grade 11 4-year and being prepared for Grade 12 four-year.

Meetings:

Meetings on Project 1 were held on May 27, August 30, September 17 and December 9, 1967 and January 27, 1968.

Reports:

1. Jones Report: Mr. Jones, working with colleagues in the Secondary Schools, completed a study of Secondary School Grade 12 5-year course to determine a reasonable level of skill and knowledge for the 5-year Grade 12 graduates. It is hoped that this document will be helpful to the CAAT's in setting up their Mathematics programme. The complete report is attached.
2. Michael Report: Mr. Michael prepared and distributed a questionnaire on the mathematical ability of Grade 12 graduates of the 4-year ST$T course. The questionnaire and results are attached. Considerable interest in the questionnaire was expressed by those replying. The attached report should also be helpful to the CAAT's.
3. Questionnaire to CAAT Instructors: (a) A questionnaire

was sent to all mathematics instructors at the CAAT's to determine their opinion of the mathematics competence of secondary school graduates. There was an excellent response and 28 replies were received, along with many comments. The results are tabulated on the attached questionnaire, indicating the number of responses in each category. Remarks on opinion of student ability are also attached.

(b) Comments on Questionnaire: Math instructors at CAAT's do not have a high opinion of the mathematical competence of the high school graduates. The following areas were considered particularly weak: estimating answers, formula manipulation, exponents, surds, Theory of Logs, problem solving, slide rule. Also considered weak were factoring relations and functions, solving quadratics.

Recommendations of Project 1 Committee:

1. More and better communications between instructors at CAAT's and secondary schools so that each will understand the aims, limitations and requirements of the other.
2. More participation by CAAT instructors in secondary school programmes.
3. Upgrading the 4-year course so that more graduates from Grade 12-4-year course may have a reasonable chance of success at the CAAT's.
4. More emphasis by secondary schools in weak areas noted above.
5. To implement recommendation #1 and #2, the following actions are suggested:
   (a) Intervisitation between instructors of mathematics in secondary schools and CAAT's.
   (b) Formation of a faculty committee of secondary school instructors and instructors at CAAT'S in each local area, to discuss problems and priorities in teaching mathematics.

Conclusions and Intentions

Project 1 Committee feels that there is a large amount of work to be done in co-ordinating the mathematics of CAAT's and secondary schools, and that this work whould continue over several years. The material of the Jones and Michael reports should be helpful to instructors at CAAT's who are planning courses or preparing orientation (entrance) tests. However, it is recognized that constant revision is necessary in this area to keep up with changing curriculum and emphasis.

More study should be given to the new 4-year programmes which are just being developed to see if the graduates can cope with the 3-year Technology courses at the CAAT's. Also, some time should be spent looking at the material prepared by the O.M.C. writing team for the 4-year course as soon as the material is available.

I. JONES REPORT

A STUDY OF THE HIGH SCHOOL GRADE 12 MATHEMATICS COURSE FOR THE FIVE YEAR STREAM TO DETERMINE A REASONABLE LEVEL OF SKILL AND KNOWLEDGE ACQUIRED BY A GRADUATE OF GRADE 12.

This report consists of a series of topics and related questions. The topics summarize the Mathematics Curriculum S.12B issued by the Ontario Department of Education for the Grade 12 five-year programme and represents the material that a grade 12 graduate in the five-year stream could be reasonably expected to cover.

The degree of skill acquired by the student has been illustrated by related questions arranged in groups of three levels of difficulty. Most graduates should be able to answer the first question in each group; about half of the students, the second question; and only the above average (university bound) students would handle the third question successfully. Therefore for the purpose of this report, students wishing to enter CAAT's should be able to handle the first two questions in each group.

1) Algebraic solutions of various types of relations in one variable.

Inequalities: Solve for x (i) $4(x+2) \leq 3-3(2x-5)$

(ii) $(3x+1)(2x-7) \geq (x-3)^2+7$

(iii) $\{x \mid -2 < 2x+5 < 7, x \in I\}$

Fractions (restrictions):

Solve for x (i) $\frac{7x+2}{5} = \frac{4x-1}{2}$

(ii) $7 + \frac{6}{x-3} = \frac{5x-9}{x-3}$

(iii) $\frac{x+2}{x-1} - \frac{7}{3} = \frac{4-x}{2x}$

Factoring trinomials:

Solve for x (i) $x^2 - x - 2 = 0$

(ii) $6x^2 + 5x = -1$

(iii) $(x-1)^2 - 3(x-1) + 2 = 0$

Factor theorem: Solve for x (i) $x^3-3x-2 = 0$

(ii) $x^3-x^2-x-2 = 0$

(iii) $6x^3-17x^2 = 3-14x$

Surds (extraneous roots):

Solve for x (i) $\sqrt{2x}-6 = 0$

(ii) $\sqrt{4-3x} - x = 12$

(iii) $\sqrt{x-3} + \sqrt{x+14} = 7$

General formula:

Solve for x (i) $6x^2 - 7x + 2 = 0$

(ii) $2x^2 - 3 = 13(x-1)$

(iii) $x^2-6ax + 3a^2 = 0$, $(a \in R)$

Literal coefficients:

Solve for x (i) $3mx + 2mx = p$

(ii) $ax + bx + cx = 6$

(iii) $a^2x - a = b^2x - b$

2) **Graphing linear relations.**

Draw the graph for the following relations

(i) $A = \{(x,y) | y \geq 2x+3,\ x,y \in R\}$

(ii) $B = \{(x,y) | y > x,\ -3 < x \leq 3,\ x,y \in R\}$

(iii) $C = \{(x,y) | y > x, |x| \leq 3,\ x,y \in R\} \cap$
$\cap\{(x,y) | x + y < 5,\ x,y \in R\}$

3) **Description of graphs under headings x and y intercepts, domain and range.**

Sketch a graph to determine the intercepts, domain and range for:

(i) $A = \{(x,y) | 2x + y = 5,\ x,y \in R\}$

(ii) $B = \{(x,y) | 2x + y = 5,\ |x| < 3,\ x,y \in R\}$

(iii) $C = \{(x,y) | 2x + y \leq 5,\ |x| < 2,\ x,y \in R\}$

4) **Algebraic solution of the intersection of two linear equations.**

Determine algebraically $A \cap B$ for:

(i) $A = \{(x,y) | x + 2y = 7,\ x,y \in R\}$
$B = \{(x,y) | x - y = -2,\ x,y \in R\}$

(ii) $A = \{(x,y) | 2x + 3y + 4 = 0,\ x,y \in R\}$
$B = \{(x,y) | x - 2y = 3,\ x,y \in R\}$

(iii) $A = \{(x,y) | 2x + 3y + 4 = 0,\ x,y \in R\}$
$B = \{(x,y) | x = -\frac{3y}{2} + 2,\ x,y \in R\}$

5) **Extend (4) to include fractional and literal coefficients.**

Solve the following systems of equations for x and y :

(i) $ax + y = r$
$ax - y = s$

(ii) $hx = ky$
$3hx - 2ky = hk$

(iii) $\frac{x}{a} + \frac{y}{b} = 2$
$ax - by = a^2 - b^2$

6) **The use of linear functions in the form $y = mx+b$ to determine a line that satisfies certain conditions.**

Determine the linear equation representing a line having:

(i) y-intercept 6, and slope -2 .

(ii) passing through point (2,6) with slope 1/2.

(iii) perpendicular to $5x + 2y = 3$, with x-intercept -1 .

7) The role of paramters.

Determine the family of lines with:

(i) slope 3.

(ii) parallel to $2x + y = 3$.

(iii) perpendicular to $Ax + By + C = 0$.

8) Functional notation.

Given $f = \{(x,y) | y = \frac{x-1}{x^2+1} , x,y \in R\}$,

$$g = \{(x,y) | 2x + y = 1, x,y \in R\}$$

compute: (i) $f(3)$

(ii) $f(3 + a)$

(iii) $f[g(-2)]$

9) Description of the graphs of quadratic functions under headings, intercepts, domain, range, vertex and axis of symmetry.

For each of the following quadratic functions determine by algebraic methods (a) the intercepts, (b) the range, (c) the domain, (d) the vertex, (e) the axis of symmetry, (f) from the features determined above sketch the graph.

(i) $A = \{(x,y) | y = x^2 - 6x + 8, x \in R\}$

(ii) $B = \{(x,y) | y = 4x^2 - 15, x \in R\}$

(iii) $C = \{(x,y) | y = -x^2 + 2\sqrt{2x} - 3, x \in R\}$

10) Maximum, minimum problems.

(i) Find two positive real numbers whose sum is 13 and whose product is a maximum.

(ii) A rectangular parking lot is to be enclosed by 1000 yards of wire fence. What is the maximum area which may be enclosed? What are the dimensions of this lot?

(iii) If a farmer harvests his crop he will have 1200 bushels worth $2.00 per bushel. Every week he waits, the crop increases by 100 bushels, but the price drops 10 cents per bushel. When should be harvest the crop?

11) Determination of the roots of a quadratic equation using the general formula.

Determination of the character of the roots of a quadratic using the discriminant.

Checking the roots of a quadratic by means of the sum and product of the roots.

(i) Without solving the equation determine the character of the roots of $3x^2 - 4x + 10 = 0$.

(ii) Determine the real values of k for which $3x^2 - 4x - 3+k = 0$ has real and equal roots.

(iii) Determine the real values of k for which $x^2 - 15 - k(2x-8) = 0$ has real and equal roots.

12) (i) Without solving the equation determine the sum and product of the roots of $8x(x+1) = 8x+4$.

(ii) If the roots of $ax^2 - 11x + 2 = 0$ are reciprocals, determine a .

(iii) Show that 1.25 and 4.16 are the roots of $100x^2 - 541x + 520 = 0$.

Form the quadratic equation which has the given roots:

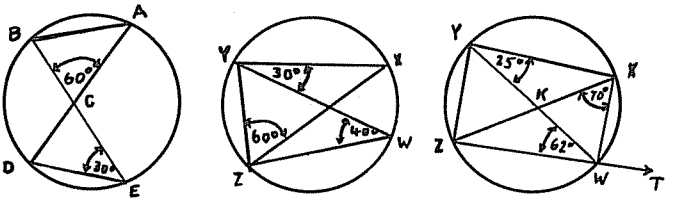
(i) 1 and -3 .

(ii) $-1 + \sqrt{5}$ and $-1 - \sqrt{5}$

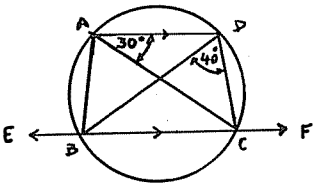
(iii) $\sqrt{5} - 3$ and $\sqrt{3} + 2$

13) Chord and angle properties of a circle. (Inscribed angle theorem and 4 corollaries).

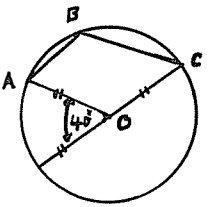
(i) For each of the following state the measurement of each of the angles whose measurement is not indicated.



(ii) From the diagram determine the measure of ABE, ADB, ACD, DCF .

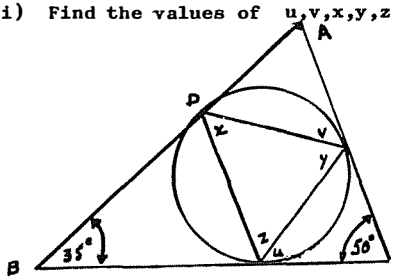


(iii) From the diagram determine the measure of ABC .

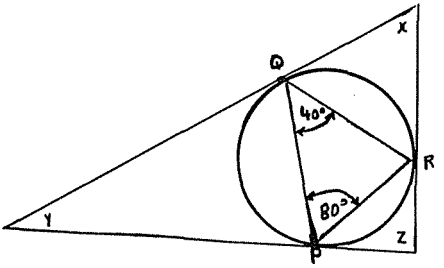


14) Tangent properties of a circle. (Tangent segment properties, incircle and escribed circle properties, tangent-chord property).

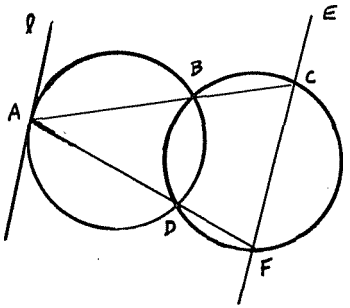
(i) Find the values of u,v,x,y,z in the diagram.



(ii) Find the values of x,y,z in the diagram.

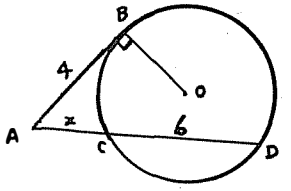


(iii) Given the conditions as shown in the diagram, prove l is parallel to EF .

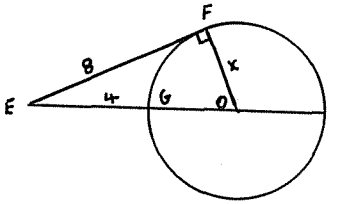


15) Tangent secant properties of a circle.

(i) Calculate x in the diagram.



(ii) Calculate x in the diagram.



(iii) If two circles intersect, the tangent segments to each circle from any point on the common chord extended are equal.

16) Graphing a circle with centre at the origin.

Draw the graphs of the following relations:

(i) $A = \{(x,y) | x^2 + y^2 > 16, x,y \in R\}$

(ii) $B = \{(x,y) | x^2 + y^2 < 49, x > 0, y > 0, x,y \in R\}$

(iii) $C = \{(x,y) | x^2 + t^2 \leq 4, x + y \geq , x,y \in R\}$

17) Solution of systems of one linear and one quadratic equation in two variables.

(i) Determine graphically

$S = \{(x,y) | x-y = 1,\ x^2+y^2 = 25,\ x,y \in R\}$

(ii) Determine the length of the tangent segment from the point $A(5,4)$ to the circle defined by $x^2 + y^2 = 9$.

(iii) Prove that the centre of the circle defined by $x^2+y^2 = 25$ lies on the right bisector of the chord determined by $x+y = 5$.

18) Trigonometric functions.

Evaluate: (i) $\cos\frac{\pi}{3} \sin\frac{\pi}{6} + \cos\frac{\pi}{6} \sin\frac{\pi}{3}$

(ii) $\cos(-1500^o)$

(iii) $\sin^2 \frac{\pi}{3} + 3 \cos\frac{2\pi}{3}$

Sketch the graph for the following functions and determine amplitude, period and phase shift.

(i) $y = 3\sin\theta$, $-2\pi \leq \theta \leq 2\pi$

(ii) $y = 4\sin\frac{\theta}{2}$, $-4\pi \leq \theta \leq 4\pi$

(iii) $y = 2\cos(\theta - \frac{\pi}{6})$, $2\pi \leq \theta \leq 2\pi$

19) Law of Sines and Cosines.

(i) Solve triangle ABC given $A = 66^o$, $B = 36^o$, $AB = 8.4$ in.

(ii) In triangle ABC , $a = 70$, $b = 80$, $c = 100$, find the smallest angle to the nearest degree.

(iii) Two roads diverge from a village at a 15^o angle. Two boys leave at the same time from the village, one at 3 mph and the other at 5 mph. Find to the nearest tenth mile, their distance apart after 2 hours.

20) Sequences, arithmetic and geometric series.

(i) Write the first four terms of the sequence 'a' such that $a_n = 2n + 3$.

(ii) Write the first four terms of the sequence 'f' such that

$$f_1 = 1$$

$$f_{n+1} = 2n(n+2)+1$$

(iii) Write the recursive definition of the sequence 'f' having terms

$$1, \frac{1}{1.2}, \frac{1}{1.2.3}, \frac{1}{1.2.3.4}, \ldots, \frac{1}{1.2.3,\ldots,n}, \ldots$$

(i) Find the 13th term of the arithmetic sequence 2,9,16,23,...

(ii) Find the sum to 20 terms of the arithmetic series whose nth term is (3n-1) .

(iii) Find the number of terms of the arithmetic sequence -1,3,7,11,... which must be added to yield the sum 405.

(i) Find the 7th term of the geometric sequence 2,6,18,54,...

(ii) Find the sum of all natural numbers less than 100 which are multiples of 9 .

(iii) The sum to 26 terms of an arithmetic series is 325 and the sum to 17 terms is 340. Find the 29th term.

21) Exponential and logarithmic functions.

Laws for exponents.

(i) Simplify $\dfrac{a^x . a^{-2x} . a^{3x}}{a^{2x-3}}$

(ii) Evaluate $(\frac{4}{9})^{3/2} \times 2^0 \times 1000^{-2/3}$

(iii) Simplify $\frac{5^y \times 25^{y-2}}{125^{y-1}}$

(i) Make a table of values for the powers of 2 and using it calculate 64×16.

(ii) Solve $x = \log_2 8^3$

(iii) Make a table of values and sketch the graphs for $y = 5^x$ and $y = \log_5 x$.

II. MICHAEL REPORT

To obtain an indication of the present mathematical competence of grade 12 graduates of the 4-year S.T.T. Course, a questionnaire consisting of 36 problems in various topics, as tabulated below, was compiled. This questionnaire was then mailed to 70 of the largest composite and technical secondary schools in the Province.

Replies from 51 schools are summarized below. Nine schools replied that they could express no opinion as they were not yet offering grade 12 of this Course.

This summary could be interesting and useful to Mathematics Instructors in both the Secondary Schools and the CAAT's.

A Selection of Comments

The ability and training of students in the electronics course that I teach cannot be differentiated as to a 4 or 5 year program. The primary difference in their response to the course is due to the presence or absence of an electronics background.

Their problem solving and reading ability is universally terrible.

The manipulation of formulae, cross multiplying and factoring should be known to the point that it becomes a mechanical operation, freeing the operators mind to concentrate on the significance of the problem.

K. Nairn, M.App.Sc., Confederation College.

I have no comment on the 5 year or 4 year program.

My students have a great deal of difficulty because they are weak in the basic arithmetic. (That is fractions, signed

numbers, etc.). Their problem solving ability is not very good. I have students who have taken the "business program" and their problem solving ability is quite adequate. The students who have taken the "science program" have no problem solving ability whatsoever.

The basic rules of arithmetic have to be learned so that they can be applied without error. I believe it is this area that should be stressed along with the ability to solve written problems.

Glenn R. Smith, P. Eng.,
Confederation College.

The five year students have some grounding in theory. The four year students haven't had any theory. The five year students prefer a few difficult problems while the four year like large numbers of easy problems.

The four year students realize that they are behind in mathematics and are putting forth extra effort to catch up.

J.R. MacLead, Dean, Porcupine Campus.

This questionnaire would be more appropriate if handed out at the end of the academic year.

The Four Year Curriculum is more suitable as a background for Technology students.

However I find that the four year students have a more difficult time in math.

The Five Year student seems to be a little better off having just covered the new Gr. 12 academic course.

Topics such as exponents, radicals, quadratics, logarithms, trig, analytic geometry are relatively fresh in his mind,

whereas the four year student may have covered these at one time or another they are not as fresh.

Good Text books at the C.A.A.T. level in mathematics are badly needed. Can you do something about it?

K. Darroch, St.Clair College of Applied Arts, Tech.

The students are literally afraid of numbers and their relation to problems. They are almost incapable of developing and solving their own algebraic problems. Even if they establish a relationship between two variables they dare not pursue the matter further.

They have a great reliance on memorizing long and complicated formulae hoping that this can be used "as is". They will memorize three versions of one formula if they feel there is a need for them. They can then use one followed by an incorrect variation on the theme.

G.B. Allan, Lambton College of App. Arts, Tech.

Seems to lack in development of formulae using definitions and fundamentals. They accept formulae and basic relations as rules instead of facts.

A.K. Kownpur

The graduates of the five year program are very well prepared. Those students who have attended the mathematics courses offered in Grade 13, even though they did not successfully complete the course, are much better prepared than those who enter directly from Grade 12.

The graduates of Grade 12 of the 5 year program seem better prepared than the grade 12 graduates of the 4 year program.

K.E. Miles, Algonquin College.

The assessments overleaf are based on what the standard "should be" upon graduation from Secondary School judging by the pronouncements of the various Secondary School curriculum documents.

Generally the statement that "I have had Graphing" does not indicate at all that students can in fact graph (for example).

Students do not realize where their knowledge of mathematical techniques can help them in their other studies. They seem to have been subjected to mathematics courses where skills etc. have been taught for their own sake rather than as useful techniques for solving problems.

Two areas especially are in need of a great deal of work.

(1) Graphical analysis
(2) The basic concept of a Function.

T.G. Semadeni, Cambrian College, Sudbury

I do not know which students are from the four-year stream and which from the five-year stream--I have only second and third year students this year. From previous experience the vocational school graduates were customarily weaker in mathematics than those from the academic stream. This year many of the vocational school students have gone into technician courses rather than technology courses. My general impression is that the students I get in the second year this term are definitely poorer in mathematics than those of four or five years ago.

A.A. Knight, St. Clair College.

A four year student has always taken "Applied Mathematics" and finds it easier to approach a practical problem than a student more orientated to the 5 year "Theoretical Mathematics".

I receive many more ridiculous answers from 5 year students than 4 year students. They are not trained to estimate answers and many 5 year students had no previous instruction on a slide rule.

My opinion is perhaps biased a bit as we normally get top 4 year students in Technology while the 5 year student is more apt to be an average student. The four year student is at a slight disadvantage in mathematics, but he can easily overcome this.

W.G. McConnell, P.Eng., Beal Sec. School

A good four year student coming out of our own School, (H.B. Beal), is as well prepared to enter the technology course as a student from the five year course grade 12. Both students have holes in their background. At present our four year students have had greater amounts of drill and can do certain parts of the Math well. Example, logarithm computations, quadratic equations. The five year Grade 12 has a course of greater breadth but has not had the drill. Often times the four year student approaches the course with a better attitude. Some secondary schools do not give as strong a course as we do to their four year students. These students have a great deal of difficulty with the technology Math.

R. Holmes, H.B. Beal, Sec. School.

2. After only 2 months of experience several things are becoming clear:

(a) Many grade 13 graduates lack fundamental knowledge and fundamental skills.

(b) Most 4 yr. B. and C graduates are deficient in basic algebraic knowledge.

(c) Several high schools have allowed their 4 yr. B. and C students to drop mathematics in favour of commercial arithmetic.

(d) There is a general lack of sense of logic.

(e) Few students are aware of the value of proper work habits, (Graphs drawn on regular paper without ruler, etc.)

(f) Many students have a profound distrust of mathematics and believe that they are not able to do this subject.

When one takes the time to analyze where this feeling got started, it is often found that they missed just a minor fact, sometimes as long ago as their elementary school days. Consequently they always obtained incorrect answers and became discouraged while their teachers did not bother to find out why.

(g) The secondary school should prepare students for CAAT's by:

1. Supplying more algebra (basics) in 4 yr. programs.
2. Placing more value on correct answers than method.
3. Teaching Math. as a tool and not as an end in itself.
4. Try to find why so many dislike Math, so that in turn the high schools can go back to public schools to solve some "understanding" problems at that level.

R. May, Humber College

3. Questions answered "no comment" since these parts of work have not been discussed yet.

There is a significant difference in the students' training, not necessarily ability, from the four-year program. In some cases, the mathematical preparation is very weak or non-existent.

J. W. Speight

The outstanding feature of the training and abilities of these students is the tremendous range of backgrounds and preparations. On the one hand you have grade 13 graduates who did well on the department Math A and Math B exams, on the other hand there are some who cannot do the most fundamental operations in Algebra. However, between these extremes there is a good number with adequate preparation for the course offered.

M. Baumgartner

Note:

Course offered is Math of Finance to Business students - involves AP and GP, annuities and bond values (second term), also simple interest and discount problems and logs (first term).

A. Knight

C.A.A.T. QUESTIONNAIRE RE MATHEMATICAL ABILITY
OF GRADE 12 GRADUATES OF 4-YEAR S. T. & T. Course

Summary of Replies Received from 51 Technical and Large Composite Schools in the Province of Ontario.

PROBLEM	Could be Solved Correctly By (Check One) 80%	50%	20% of Class	Topic Not Taken
1. Factor Fully: $4a^3 - 36ab^2$	39	11		1
2. Simplify $\frac{x^2 - 16}{x^2 - 2x - 8} \times \frac{x - 2}{2x + 8}$	23	21	6	1
3. Simplify $\frac{4a + 3}{7} - \frac{2a - 5}{3} + \frac{6a - 7}{5}$	20	25	5	1
4. Solve $\frac{3x - 2}{10} - \frac{x - 1}{6} = \frac{1}{2}$	25	22	4	
5. Given $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ Solve for v in terms of f and u.	4	28	16	3
6. Solve $\frac{a}{3} - \frac{b}{5} = 2$ $\frac{2a}{3} + \frac{3b}{5} = 9$	22	23	4	2
7. The cost of printing a number of posters follows the formula $c = pn + s$ where p is the price per poster, n is the no. of posters, s is the cost of setting the type and c is the total cost. If a printer quotes \$17.00 for 200 posters or \$32.00 for 500 posters, find p and s.	11	18	17	5

	Could be Solved Correctly By (Check One)			
	80%	50%	20% of Class	Topic Not Taken
8. A boy can row 10 miles downstream in 1 hr. 40 min. The return trip takes 5 hrs. What is his speed in mph in still water, and what is the speed of the current.	2	11	30	8
9. If $f(x) = \frac{x^2 - 5x + 6}{x - 5}$ then $f(-3) =$	7	6	4	34
10. Solve correct to one decimal place $3x^2 - 11x - 7 = 0$	30	16	4	1
11. The sum of a number and 15 times its reciprocal is equal to 8. Find the number	9	26	13	3
12. Sketch the graph for $\{(x,y) \mid 2x - y > 8,\ x,y \in R\}$	4	8	1	38
13. Find the sum of the series $85 + 83 + 81 + \ldots$ to 85 terms	8	7	4	32
14. Solve for x : $\log 4x^2 - \log x = 3$	1	5	14	31
15. Solve for y : $32^y = 64$	3	9	18	21
16. Simplify: $\frac{5^0 \times 5^{\frac{3}{2}} \times 10^{-2}}{5^{-\frac{1}{2}} \times 15^0}$	9	17	18	7
17. Solve: $x + \sqrt{x + 4} = 8$	12	27	8	4

	Could be Solved Correctly By (Check One)			
	80%	50%	20% of Class	Topic Not Taken
18. The distance travelled by a moving object, starting from rest, varies as the square of the time. If the object travels 64 ft. in the first 2 seconds, how far will it travel in the first 5 seconds?	1	14	21	15
19. Name the curves for each of the following equations, and sketch them using the intercepts only.				
(a) $4x - 9y - 36 = 0$	20	4		27
(b) $4x^2 - 9y - 36 = 0$	7	8	7	29
(c) $4x^2 + 4y^2 - 36 = 0$	7	6	6	32
(d) $4x^2 + 9y^2 - 36 = 0$	4	5	6	36
(e) $4x^2 - 9y^2 - 36 = 0$		6	7	38
(f) $xy - 36 = 0$		9	6	36
20. Find the equation of the line passing through (-4,0) and (3,2)	4	4	2	41
21. Calculate the distance between the points - A(-2,5) and B(5,9) .	8	4	5	34
22. A ladder 32 ft. long is placed against a vertical wall. What angle does the ladder make with the ground if its foot is 7 feet from the wall. (Tables provided).	46	3	2	
23. Without using tables, evaluate- (a) $\sin 60^\circ$ (b) $\cos 210^\circ$	27	17	2	5

	Could be Solved Correctly By (Check One)			
	80%	50%	20% of Class	Topic Not Taken
24. Sketch $y = \cos \theta$ for $0^\circ < \theta < 360^\circ$	9	14	8	20
25. Two ships leave port at the same time. One sails N. 32°W. at 15 knots, and the other sails N. 78° E. at 18 knots. How far apart are the ships two hours after leaving the port? (A knot is one nautical mile per hour).	11	29	8	3
26. A wheel makes 1750 revolutions per minute. Through how many radians does it revolve in one second?	4	14	8	25
27. ABC is an isosceles triangle in which $\angle A = \angle C$ and $\angle B = 74^\circ 36'$; Calculate the number of degrees in $\angle A$.	39	11	1	
28. What is the area of a square whose diagonal is 30 inches?	19	27	5	
29. Find the angle between the bisectors of the two angles of a trapezoid which are formed by the bases and one nonparallel side. (Assume definition of trapezoid given).		13	20	18
30. Find the area of the three-cornered section enclosed by three mutually tangent 15 in. circles.	1	6	18	26
31. Two sides of a parallelogram are 10 in. and 12 in. long, and the contained angle is 135°. Find the length of the longer diagonal.	22	20	6	3
32. Find the length of the shadow of a man 5 ft. 10 in. high, standing 9 ft. from the foot of a street lamp which is 17 ft. above the ground.	24	17	8	2

	Could be Solved Correctly by (Check One)			
	80%	50%	20% of Class	Topic Not Taken
33. Evaluate using the slide-rule. $\frac{414 \times 294 \times 714}{252 \times 760}$	33	13	3	2
34. Evaluate using logs $\sqrt[5]{0.0617}$	30	19	2	
35. How many square inches of sheet metal will be required to make an 8" stovepipe 5' long. Neglect any overlap.	34	15	2	
36. What size solid sphere could be made by melting down a solid copper cylinder 10" in diameter and 20" in height?	7	16	21	7

37. Other topics taken but not listed above.

Statics & Mechanics	21 schools
Machines (pulleys, gears, etc.)	6
More time on Logs	9
More time on Trigonometry	5
Compound Interest & Bus. Math.	4
Deductive Geometry	2
Construction Geometry	3

38. Any comments you may wish to make.

(a) A Technical School in a large city has 8 streams of gr. 12-4 yr. S.T.&T. students

(b) The average graduate is good with arithmetic calculations but has little ability in algebraic manipulation. He can substitute in formulas but

doesn't understand their derivations.

(c) The present Dept. of Education outline is inadequate for C.A.A.T.'s.

(d) Ind. Chemistry and Electronics students are usually superior in mathematical aptitude and knowledge to those in Drafting and Auto Mechanics.

Summarizing Comments:

(a) The schools replied that 50% or more of the class could do 21 of the 36 questions listed. The other 15 questions could be handled by much less than 50% of the class.

(b) This summary could be useful to both the C.A.A.T.'s and the Secondary School instructors

Summary dated: Jan. 30/68

III. QUESTIONNAIRE TO C.A.A.T. INSTRUCTORS

The C.A.A.T. Committee of the Ontario Mathematics Commission has undertaken as one of its projects a study to improve correlation between Secondary School training in Mathematics and requirements of the 3-year Technology program of the C.A.A.T.'s. It would be a great help to us if you would take time to complete the following questionnaire, using the results of your entrance tests and/or classroom work this year. Please indicate your opinion of general student ability in the topic indicated by placing an X in the appropriate column. If you have no opinion, please place an X in the first column.

	No opinion on this item	Excellent	Very Good	Good	Satisfactory	Barely adequate	Poor	No previous instruction
1. Arithmetic Computations								
(a) Whole Numbers	3	3	3	6	12	1		
(b) Fractions		1		5	10	11	1	
(c) Order of Operations		1	1	5	11	5	5	
(d) Estimating Answers	1			3	3	7	12	2
2. Algebraic Manipulations								
(a) Formula Manipulation		1	3	6	11	7		
(b) Factoring	2			3	9	8	6	
(c) Positive and Negative Numbers	3	1	3	7	9	2	2	1
(d) Rational Expressions	6			3	9	7	2	1
(e) Exponents	2		1	1	6	15	3	
(f) Surds	7		1	1	4	12	3	
3. Solving Equations								
(a) 1st Degree in one Variable	5	1	5	7	6	3	1	
(b) 1st Degree in 2 or 3 Variables	9		2	4	5	6	2	
(c) Quadratics in one Variable	9		2	3	4	5	5	
4. Logarithms								
(a) Arithmetic Calculations	8	1	2	3	4	5	4	1
(b) Theory of Logs	8		1	1		5	11	2

		No opinion on this item	Excellent	Very Good	Good	Satisfactory	Barely adequate	Poor	No previous instruction
5.	Problem Solving								
	(a) Interpretation	2			2	4	7	12	1
	(b) Use of Data	3			2	6	3	13	1
6.	Trigonometry								
	(a) Basic Definitions	9			6	7	2	3	1
	(b) Solving Triangles	10			5	4	3	5	1
	(c) Identities	14				3	5	3	3
	(d) Graphs of Trig.Functions	11			1	5	4	5	2
7.	Slide Rule								
	(a) Multiplication and Division	6	2	1	4	1	3	5	6
	(b) Roots and Powers	7		2	2	1	3	7	6
8.	Relations and Functions								
	(a) Graphing	9				3	12	4	
	(b) Linear Functions	9			1	5	10	3	
	(c) Quadratic Functions	12				1	6	8	1
9.	Geometry								
	(a) Basic Ideas - Angle, Congruency, Pythagorean	6		1	4	12	3	2	
	(b) Similar Triangles	8		1	3	12	2	1	1
	(c) Circle Facts	7		1	2	9	6	2	1
10.	Analytic Geometry								
	(a) Straight Line	16			2	3	2	3	2
	(b) Circle	18				2	1	3	4
	(c) General Comprehension	17			1	1	4	3	2
11.	On the back of this sheet, comment on the ability and training of the graduates of the 5 year and 4 year programs of the secondary schools, as you have found them this year.								

PLEASE RETURN TO: Ontario Mathematics Commission,
1260 Bay Street, Toronto 5.

MATHEMATICS I
A PROVISIONAL FIRST YEAR COURSE OF STUDY
for
3 YEAR TECHNOLOGY COURSES
in
C.A.A.T.'s

Prepared by
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ONTARIO MATHEMATICS COMMISSION

This Mathematics outline for the common first year of the 3 year-technology programme is intended as a typical example. Each individual college will ultimately develop its own course, suited to its local needs by changes in emphasis and timing, omission or addition of minor topics etc.

This outling is by no means a prescribed, inflexible course of study. On the other hand, there is no reason why it could not be used in the classroom right away as is, with changes deferred to the next year of study.

Major changes in the programme are not recommended for one individual college alone, in order to retain some degree of uniformity for the common first year throughout the system. Such changes should be made with the consent of a curriculum committee or conference only.

INTRODUCTION

The present course of study for first year Engineering Technology has been changed from the last (1966) edition due to two principal experiences:

(a) only a teacher very familiar with the systems and the course could actually teach the full content

(b) the students in first year have a markedly decreased ability to handle numerical work compared to the ones who graduated under the "old" maths.

Consequently the two major changes are: a reduction in material, (which is mostly transferred into the second year) and an extra emphasis on review and teaching of numerical methods.

One unavoidable difficulty of a first year course is the difference in background of the individual students. The attempt to produce an equal level of knowledge before "teaching the course" has been found unsatisfactory. Not only does it overtax the weaker students in the first week, and discourages them, it also bores the better students to tears and "unmotivates" them. Furthermore does it require too much time out of the available hours.

A better system seems to be the "review-when-needed". It is agreed that the poorest students require an upgrading programme (mostly in algebra) from the first day. (A diagnostic achievement test may be used to identify the candidates). This upgrading could be done in small tutorial groups after hours, and by individual textbook and programmed instruction assignments. Some of the topics are, in effect, straight review (e.g. functional values, algebraic operations, etc.)

All the other necessary reviewing is done as an introduction to the topic for which it is needed. The definition and laws of powers precede the chapter on logarithms, basic geometry of the angle and triangle is

taught before trig etc. Weak students should be given the forementioned library-text-and programmed assignments to bring them up to the standards.

The Mathematics programme is to be complemented by instructions in computer language and programming preferably outside of those hours assigned to the teaching of Mathematics. It ought to be offered as soon as reasonably possible in first year, so that assignments in the applied subjects can be given.

The General Objectives of the Course:

1. To provide the student with a thorough understanding of the language of Mathematics to enable him to express engineering problems in mathematical language and to interpret results obtained by mathematical analysis.
2. The question "How to solve a problem?" will be answered by the detailed discussion as to logical development and methods of analysis of many problems selected from different fields of engineering. This will emphasize at the same time the generality of mathematical tools in approaching problems of great diversity.
3. Besides being familiar with the available methods, students should be skilled and efficient in using mathematics. The challenge of selected class and home assignments will help the student to acquire the required skills and efficiency.
4. The course should provide the foundation for more advanced courses in Mathematics of the second and third year studies.

Time Schedule:

The time allotted is based on a 30 week course 5 periods per week. The first term is usually 14 weeks long,

the second 16. Furthermore it does require a modification of team teaching, a division of 3 hours lecture interspersed with 2 tutorial classes per week has been found quite effective.

		Suggested time in periods	
1.	Computations with Measured Quantities	11	
2.	The Language of Mathematics . . .	9	
3.	Exponents and Logarithms	15	
4.	Trigonometric Functions	11	
5.	Analytic Trigonometry	10	
6.	Complex Numbers	9	
			65
7.	Equation of First Degree	12	
8.	Quadratic Equations	10	
9.	Differential Calculus	26	
10.	Integral Calculus	22	
			70
			135

Texts:

1. "Basic Technical Mathematics with Calculus" by A. Washington (Addison-Wesley).
2. "Five-Figure Logarithmic and other Tables" by Castle (MacMillan).
3. "Handbook of Mathematical Tables and Formulas" by Burington (McGraw-Hill).

Other References:

1. "Basic Mathematics for Science and Engineering" by Andres, Miser and Reingold (Wiley).
2. "Basic Mathematics for Engineering and Science" by Van Voorkis and Haskins (prentice-Hall).
3. "Industrial Algebra and Trigonometry" by Wolfe, Mueller and Mullikin (McGraw-Hill).
4. "A First Course in Mathematics" by Mainardi, Konove and Baker (Van Nostrand).
5. "Mathematics for Technical Students" Vol. I, II, & III, by Gasson.
6. "Technical Mathematics with Calculus" by Rice and Knight (McGraw-Hill). 2nd edition
7. "Applied Calculus" by Adams (Wiley).
8. "A Practical Calculus" by Proudfoot (MacMillan).
9. "Practical Mathematics for Technical Students" by Gwilliam and Straka (Prentice Hall)

Computations With Measured Quantities

Exact and approximate numbers; significant figures

The difference between general and approximate solution and the importance of the latter in technology. Rules of determing the amount of significant figures with special consideration of zero digits.

Scientific Notation

Short review of the law of indices, powers of ten and their application to scientific notation.

Error and Relative Error

A short review of percentage calculations, percentage error, margin of accuracy in the $\pm$ notation.

Use of Slide Rule and Mathematical Tables

The reading of the slide rule scales, multiplication, division, squares and cubes, square root and cube root including a discussion of using the appropriate decade. (or explanation of the R_1,R_2 scales). The general technique of interpolation when using tables. Familiarization with the prescribed book of mathematical tables.

Estimation

The importance of an "educated guess" in technology. Estimations of the magnitude (power of ten) of the result. Exercise in assuming information which is not specifically given. Estimation of the position of the decimal point in slide rule operations. (use scientific notation).

Operations involving Physical Dimensions

Defintion of the term "dimension" (mass, length, time) and "unit" (elg. cm/sec). Dimensional analysis on simple examples of mechanics. Discussion of the metric system and its prefixes, (10^6 = mega).

The Language of Mathematics

Functional Values

The difference between functions and relations in physical applications. Computing the numerical values of the dependent variable and the general expression for algebraic abscissas. Example: $f(x) = 3x + 1$, find $f(3)$ and $f(a+1)$, Graphs of functions and relations.

Factorial notation, sigma notation

The application of the two symbols as found in finite and infinite series and power series.

Algebraic operation

A short review of the laws applying to equations.

The equation as the shorthand statement of a physical problem. A drill in translation from worded problems to equations.

Handling of equations

Application of the transposition rules to engineering formulae. A drill in solving for any arbitrary variable in the formula, excluding powers and surds. Stress the fact that the dependent and independent variable may be represented by any letter in the alphabet not only by x and y.

Exponents and Logarithms

Applications of the laws of exponents

Continuation of the transposition exercises including roots and exponents. The fractional exponent notation. Drill in converting root symbols to exponents and vice versa.

The definition of a logarithm

The laws of indices permit reduction of multiplication to addition etc. In order to utilize this simplification every number must be expressed as a power of the same base. The advantage of the base 10.

$$n = b^{\log_b n} \qquad n = 10^{\log n}$$

Memorization of the definition and constant drill.

The laws of logarithm

Multiplication, division and powers. Very brief numerical exercises to reinforce the defintion of a logarithm and the logarithmic laws. Do not teach logarithmic computation in great details since this method is virtually obsolete in today's industry.

The logarithmic function

y = log x. Graph of the function and discussion of log 0. The linear scale (actually log scale) on the slide rule.

The Exponential function

The equivalent writing of y = log x and $x = 10^y$. Logarithmic and exponential function are inverse to each other. The number e as the number of natural growth,

$$e = \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$$

and its importance in problems involving hyperbolae. A logarithmic system based on the number e. Definition of the natural log function, and its graph. Tables of natural logs and their unfitness for numerical work. Exercise in looking up positive and negative exponentials and natural logs.

Logarithmic systems of different bases.

The mathematical and graphical significance of the constant factor relating two systems, and its numerical value. The special case of the bases e and 10.

Applications

Simple exponential and logarithmic equations from technology and physics, such as capacitor discharge, transmission line impedances etc. Stress numerical answers.

Trigonometric Functions

The Radian Measure

Definition of radian measure as the ratio of arc to radius. Radian is not a dimension, but a pure number. Conversion from degrees to radians and back. Use radian measure from now on whenever practical.

Application of the radian measure

The length of an arc, the area of a circular sector, angular velocity, (note: the dimension of a.v. is 1/time). Conversion of a.v. into rpm.

Review

Review of the defintion of six trig. ratios and the graphs of sin x, cos x, and tan x. Basic trig identities such as the pythagorean indentities, $\sin\left(\frac{\pi}{2} - x\right)$.

Use of trig. tables and slide rule. Angles of any value between 0 and 360°.

The Right Angle Triangle

Engineering problems involving right angle triangles (simple vector compositions and addition) provided such exercises have not yet been done in the Applied Mechanics course. Stay away from standard "course of plane" and "surveying" problems.

Inverse Trig Function

Developing the concept of inverse functions arrive at the notation y arc sin x. Mention, but avoid $\sin^{-1}x$ notation. The graphs of arc sin x, arc cos x and arc tan x. Introduce the concept of the so-called "principal value". Applications to power factor, angle of polar co-ordinates and vectors etc.

Sin and Cos Law

Review of the four laws of congruency. Derivation of the sin and cos law and its application in the four cases of congruency. Applied problems of increasing difficulty. Note that for mechanical students a high degree of proficiency is required. The use of the co-tangent law and other time saving devices is optional but not necessary.

Analytical Trigonometry

The Addition Theorem

The formula for sim $(\alpha \pm \beta)$ and cos $(\alpha \pm \beta)$. Application of these formulae to find angles in all four quadrants. Developing of the sum formula for tan $(\alpha \pm \beta)$ from sin and cos.

Double and Half Angle Formula

Application of the addition theorem to 2 equal angles: sin 2x, cos 2x, tan 2x. Rewriting of the double angle formula in the half angle form. Avoid triple and quadruple angles.

Sum Into Product, Product Into Sum

Derivation of these formulae from the addition theorem. Memorization of all analytical trig. identities should be avoided. The student may be given a mimeographed sheet containing all desirable identities or may be permitted to use his formulae collection in class and during tests.

Applications

The use of the fore-mentioned identities in engineering. Mechanical applications: stress analysis, projected throw, vibrations. Electrical: derive the identity

$A \sin x + B \cos x = \sqrt{A^2 + B^2} \sin (x + \alpha)$, $A = \text{arc tan}\frac{B}{A}$

and apply to ac theory.

Complex Numbers

The j operator

The problem of extracting a square root from a negative number. Introduction of the symbol $\sqrt{-1} = j$ (or i). A quick review of the number system including real numbers. The equation $x^2 + 1 = 0$ has no real root.

Imaginary numbers

Numbers of the type bj and a + bj. Expansion of the number system to include complex numbers.

Operation with comples numbers

Equality condition, addition, multiplication. Conjugate complex numbers, their sum and product. Division. Note that the set of complex numbers is closed under all algebraic operations.

Vectors and complex numbers

The Gaussian plane. Representation of Vectors by complex numbers. Make sure that the difference between the unit vectors i and j and the imaginary unit j is well comprehended. Vector addition is equal to the addition of complex numbers.

Conversions

Vectors expressed in polar coordinates, the application of de Moivre's theorem to the polar form to yield the exponential form. Conversion from polar into complex form gives the trigonometric form of a vector. Drill of conversions into various forms.

Powers and Roots

Definition of powers of a complex number. Conversion of the complex form into the trig and polar form and the rule: Power of magnitude and multiple of the angle. The root of a complex number by reversion of this rule. Stress the fact that

multiplication and powers of complex numbers are not applicable to vectors, but to phasors used in electrical work. This is very important for later work on dot and cross products of vectors.

Applications

Applications of the techniques of complex numbers to vector addition problems and problems of ac theory. Interpretation of the j operator as a 90° rotation. The existence of functions having a complex variable may be mentioned and illustrated on an example.

Equations of First Degree

Linear Functions

Definiton of the linear relation between two variables and the graph of this relation. Start from the proportionality $y = kx$ and discuss direct variation. Work up to the general form $ax + by + c = 0$. At every step of the way, provide ample illustrations from physics.

The straight Line graph

Linear functions have always straight line graphs. The significance of the constants of the equation for the graph. Stress the concept of slope, first as the tangent of the angle of inclination, then as a dimensioned quantity in physics. (velocity is the slope of a distance--time diagram). The 2-point form, the point-slope and the 2-intercept form. Drill with applied problems.

Systems of Linear Equations

Interprete one equation in one variable as $y = f(x)$ when $y = 0$. Review shortly conventional solution methods and conditions of compatibility. Introduce the augmented matrix of the system as its representative. Define determinants of a matrix, restricting the scope to 3×3 determinants. State Cramer's

rule. (Demonstration of the general 2 x 2 case is optional). Applications for Engineering such as equilibrium conditions, network loop equations, etc.

Quadratic Equations

Quadratic Functions

The graphs of the functions $y = x^2$, $y = ax^2$, $y = ax^2 + bx$ and $y = ax^2 + bx + c$. Statement that a second order relation of this kind between two variables will always produce a parabola. The relations $x = y^2$ etc. The quadratic equation produced from the quadratic function by letting y be equal to 0. Point out the two real and different solutions and the double root from the graph.

The Quadratic Formula

Derivation of the quadratic formula using the standard type $x^2 + px + q = 0$ and extending the solution to the standard form and $ax^2 + bx + c = 0$. No other method of solution is practical for technological work. Discriminant of an equation and its relation to the type of roots. The relation of the sum and product of the roots to the coefficients of the first standard form. Factoring of a quadratic term by using the roots of a quadratic equation. Applications from engineering problems.

Other Quadratic Equations

Higher order equations reducible to quadratics including some trigonometric equations. Systems of quadratic equations including two quadratics and one quadratic and one linear term. Graphical interpretation of such systems and conditions of real solutions.

Radical Equations

Solution by proper arrangement of the square root terms and squaring. Do not exceed two surds in one equation. Discussion of the appropriate sign given to a square root to justify the computed solutions. (This way the term "Extraneous Root" becomes superfluous). Applications.

Differential Calculus

Review

Functional notation especially of the form f(a + b), finite summation.

Limits

The meaning of the symbols infinity and 0 as encountered in limiting processes. The fact that infinity is not a number. The symbols $\lim_{x \to 0} x$ and $\lim_{x \to \infty} x$ express tendencies of a change in the variable.

$$\lim_{x \to 0} \frac{\sin x}{x}, \quad \lim_{x \to 0} \frac{1 - \cos x}{x}$$

The tangent

Defintion of the tangent as a limiting case of the secant. Description of the slope of the tangent as a limit of the slope of the secant.

Use both symbols $\lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$ and $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$

The rate of change

The slope of the tangent indicates the rate of change of the function. Discussion of the physical

meaning of certain time rates such as a velocity, acceleration, induced voltage, current through capacitor etc.

The Derivative

Definition of the derivative as the slope of the tangent at the point P(x,y). Parallel Treatment of the simple cases y = constant and y = kx by formula and graph. $y = x^2$ by formula, verification on the graph.

Formal Differentiation

Derivation of differentiation rules by much intuition and little rigor. Powers of x (binominal theorem and formula). Sin x and Cos x by intuition and graph. Proof by Formula optional, but not necessary. Exponential by numerical analysis (find slope for several points by numerical methods and show that the slope is equal to the ordinate at this point). Formal proof optional. Include the negative exponential differentiation rule. The natural log as the inverse of the exponential must follow analog rules. Note that outside of x^n no formal proof is necessary!

Differentiation of operations

Sums and differences, constant multiplier y = kf(x) or y/k = f(x) (dividing y by a constant k corresponds to stretching the y axis k times. This yields to k times the slope). Several special cases of function of a function such as sin ax, cos ax, exp ($\pm$ ax) may be stated for practical work without further proof.

Applications

Motion analysis, simple harmonic motion, ac problems of voltages and currents in inductances and capacitances, Faraday's induction law, Newton's first law of motion, (dp/dt = f) etc.

Implicit Differentiation

The implicit form of a function and its derivative, using the chain rule for members containing y. Applications: Tangents to conics (path velocity of circular and elliptic motion).

Small Increments

Approximation of small increments by differentials. Extimates applied to problems involving heat expansion, resistance changes due to temperature, voltage fluctuations in simple circuits, etc.

Integral Claculus

Area under a Curve

Approximation of area by the trapezoidal rule and Simpson's rule. Discussion of the errors involved and their numerical estimation and methods of reduction.

The exact area under a Curve

The exact area as a limiting process applied to an approximatimg step function. Intuitive conclusion that the error will tend toward 0 with the width of the strip. Definition of the integral sign as the symbol of infinite summation. Stress the existence of the term dx in the symbolic writing.

Anti-differentiation

Differentiation as an operation must have an inverse operation. Use any appropriate symbol such as D^{-1} or AD as an operator symbol. Invert all known differentiation rules. (the power rule is most difficult and ought to be done as the last). Stress the exception of AD x^{-1} . Exercises in anti-differentiation with engineering problems similar to those used in differentiation. Intuitive proof that anti-differentiation can perform infinite summations. Use of the integration

symbol from now on. Stress conversions from one form into another (if $P = dE/dt$, then $E = \int P\ dt$).

Definite Integration

Introduce defintie integrals as representatives of an area and drill computation with definite integrals. Stress the difference between a definite and a general (indefinte) integral. Use negative areas in the problems also.

Applications

Discuss slowly and carefully the meaning of an integral symbol in physics. e.g. In dc theory, $Q = It$. If I becomes a function of t, this formula is no longer valid. It will remain valid, however, if the time interval during which the current flows, tends toward 0. If I flows for and interval dt, only a differential of charge dQ will be transferred. The total charge Q can be found by summing up over all differentials dQ or I dt. $Q = \int I\ dt$ is the appropriate ac formula. It is of greatest importance that the student understands this type of reasoning and is able to interprete any integral symbol he may encounter in physics in this way. Further applications: volumina of revolution, simple moments of inertia, centroids of elementary figures, average and rms value tec.

PROBLEMS AND SOLUTIONS

Send problems and solutions to the Problems Editor, Ontario Mathematics Gazette, Department of Mathematics, Queen's University, Kingston, Ontario.

Problem No. 7 (Prof. Norman Miller, Queen's University (Ret.))

Prove the identity

$$\binom{r}{r} + \binom{r+1}{r} + \binom{r+2}{r} + \ldots + \binom{n-1}{r} = \binom{n}{r+1} ,$$

(a) by formal manipulation

(b) by an argument based on the interpretation of the combinatorial symbols.

Problem No. 8 (Mr. E.Evans, Guelph, Ontario)

Let S and T be linear transformations in the plane, of the form

$$\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} ax+by \\ cx+dy \end{bmatrix}$$

The transformations TS and ST will not in general be equal, but they always transform the unit circle $x^2 + y^2 = 1$ into ellipses with the same area. Under what conditions will these ellipses have the same eccentricity?

THE ONTARIO INSTITUTE FOR STUDIES IN EDUCATION

INTERMEDIATE MATHEMATICS METHODOLOGY
Curriculum Series No. 4 $1.50

Report of the Intermediate Mathematics Methodology Committee of the Office of Development. The report deals with the implications of the many new instructional methods for teaching of mathematics in elementary and secondary schools, as well as testing and evaluation, instructional environment, teacher education, and school organization.
Pp. v + 93.

(Editor's Note:

The above curriculum report has now been published and is recommended as required reading for the school teachers of mathematics.)

PLEASE NOTE - RE SUSCRIPTIONS

The rate for individual subscriptions as from September 1968, including renewals is $2.00.

THE SAMUEL BEATTY FUND

Following Dr. Samuel Beatty's retirement in 1952 as Head of the Mathematics Department and Dean of the Faculty of Arts, University of Toronto, a number of graduates of Mathematics and Physics, of the University of Toronto, organized a scholarship fund to recognize the great contribution that Dr. Beatty made to the education of students of Mathematics and Physics. The primary purpose of the fund was and is "to promote and encourage the study of mathematics in the said University and throughout the Province of Ontario."

The Editorial Board and all concerned with the publication of the Ontario Mathematics Gazette appreciate the annual grants received from the Samuel Beatty Fund in support of the publication of the Gazette.

ONTARIO MATHEMATICS COMMISSION

TRAVELLING LIBRARIES

A second Travelling Library available for display at educational meetings has been sponsored by the Advisory Committee for Elementary Education of the Ontario Mathematics Commission.

Each library consists of several dozen books supplied by the Canadian Text Book Publishers' Council, National Council of Teachers of Mathematics, Ontario Institute for Studies in Education, individual publishers and others interested in Mathematics.

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