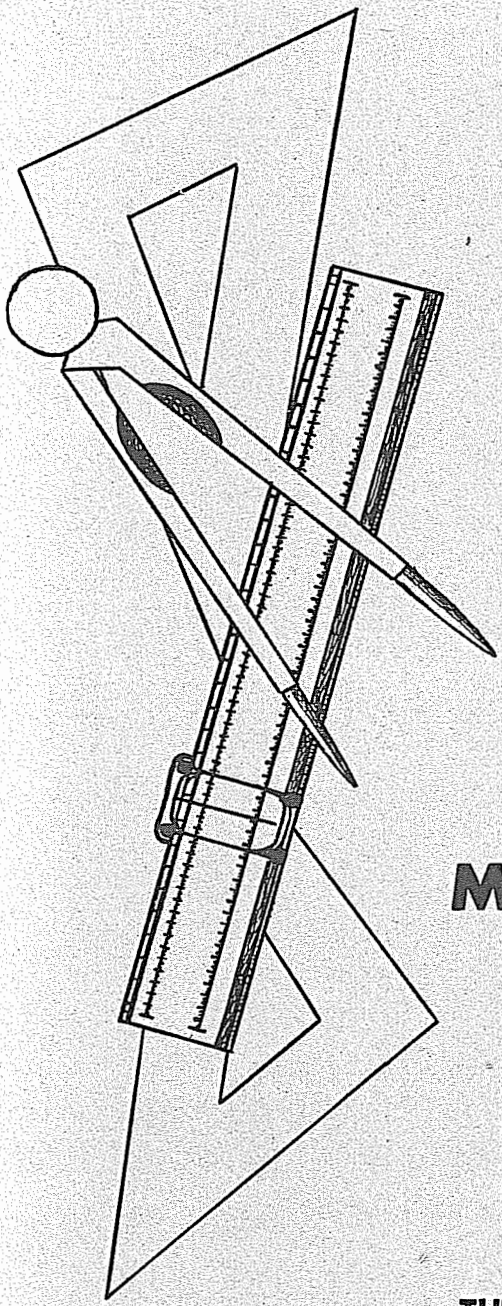


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ONTARIO MATHEMATICS GAZETTE

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E D I T O R I A L

The focus in this issue is on teacher education in mathematics. At long last, educators in Ontario are beginning to realize that the teacher is the most important person in education (apart from the pupil!) The Hall Dennis Report lends its weight to this point of view in no uncertain fashion.

> "A child's best guarantee of a good education is an inspiring teacher ....."

and

> "Changes in education, no matter how sweeping, profound, or ideal, are barren unless they bring about changes in the classroom; their effectiveness is determined almost entirely by the teacher."
>
> (Hall Dennis Report P.121)

The same report makes a number of significant recommendations about teacher education - both pre-service and in-service - and recognizes though not as urgently as one might have wished, the need to expand and develop in an integrated manner the continuing education of teachers.

The main thrust of the articles that follow is aimed at pre-service education at the secondary level. It is hoped to deal more specifically with elementary and inservice education in a future issue.

The first article discusses the emerging philosophy of helping a teacher to develop flexible and varied approaches to teaching. By way of contrast, the second describes an actual practical experiment in teacher education carried out in the high school itself as an application of one of the approaches being advocated in one College of Education.

The third article deals with the exciting "concurrent" approach to teacher education being tried out with considerable success at the University of Waterloo.

As a kind of epilogue, or in a mood of reflection, an extremely interesting survey of teachers' attitudes to their training and their views on what forms teachers in education should take, forms the theme of the final major article.

There is interesting material in each. Taken together, it is our hope that you as readers will be stimulated and encouraged to look to the future as one in which innovation and experiment will continue to take place in teacher education in mathematics. Do write in with your comments and criticisms. We all have a part in ensuring that the teacher image gained by the pupil is one of inspiration and high quality rather than dullness and mediocrity.

D.H.C.

ACHIEVING A BALANCED TEACHING PROGRAM

by

W. B. MacLean, D. L. Mumford and D.W.Alexander

The College of Education, University of Toronto.

Introduction

Many teachers of mathematics in Ontario schools employ a variety of teaching techniques to provide their students with a meaningful, stimulating, and constructive experience in learning. The following material is presented in the hope that it may encourage other teachers to experiment with different techniques and hence build up a balanced teaching program of experiment, investigation, discussion, individual study...

A balanced teaching program makes appropriate use of the following:

1. small group investigation, study or discussion
2. full class participation lessons
* 3. individual study
4. socratic discussion.

* In this article, individual study has not been explicitly discussed.

How to conduct a small group investigation

A small group may consist of from two to six persons depending on the nature of the investigation to be made.

The organization of a class into small groups requires careful consideration. Grouping according to achievement level is probably the poorest way. Groups that do not have the stimulating affect of the imagination and ideas of any of the better achievers may become frustrated rather than encouraged. Heterogeneous groupings on the whole seem best. Here the more imaginative can learn to assist and encourage the lesser and each gains from the experience. Some people work better together than others and this may be a good criterion for choice on some occasions. It may happen that some students will show a special or sustained interest in a topic and thus naturally form a special group for further exploration of the topic. On the other hand, there may be times when grouping according to the need for remedial work is best. "Flexibility" is a keyword in grouping.

In conducting a small group investigation the following eight points all require consideration.

(1) The investigation is a combined effort. Each person is expected to contribute in a constructive and organized way.

(2) The appointing of a chairman. This should be done immediately the group is constituted. The chairman should promote the necessary organization and

control, and should superintend all the work of the group.

(3) Reading the instructions for the investigation. The chairman's first task is to see that each member of the group reads the instructions completely and carefully. This may be done individually or by the group as a whole.

(4) Developing a plan of attack. This includes an appreciation of the investigation as a whole; agreement on how it is to be carried out and recorded; and the distribution of tasks among the members of the group. Further discussion may be required as the work progresses.

(5) Doing the investigation. The necessary materials, aids and equipment should be readily available. The chairman should keep in touch with the overall work of his group.

(6) Preparing a report on the investigation. In general, investigations should be reported on by the group. With this in mind the investigators should be recording their work in such a way that its presentation in a report to the class as a whole will be facilitated. This may involve the preparation of large graphs, models, posters, or transparencies for use on the overhead projector.

(7) Giving the report. The giving of the report is a most important part of the experience providing a means for the class to share the experiences of the various groups and stimulating further discussion. Through this experience students gain confidence and ability to communicate ideas. The report should be a joint effort of the whole group and is best presented when each member has a part in it.

(8) Discussion. Class discussion should round out or integrate the experience into a useful whole and place it into its proper perspective in the general situation. When necessary the teacher should stimulate the class discussions and draw out meaningful conclusions from the reports.

Some investigations may take two or three class periods to complete. When this is the case it is necessary to provide space in which each group can neatly store its working material for continued work the next day.

The responsibility for good behavior lies heavily with the students. If they and the teacher are not accustomed to this learning technique, a period of developmental training is necessary. While some teachers will be able to move directly into group activity with good success most are advised to introduce it gradually.

One way to do this is to introduce a topic through a short (10 minute) period of discussion in groups of 2 or 3. Such

periods of work can gradually be extended until whole periods or several successive periods are occupied with active investigation.

Another way to introduce group methods is through the use of full class participation lessons. In this type of lesson, although the teacher has withdrawn from the front of the class and the freedom of action of the students is considerably extended, the lesson is structured by the teacher and remains under his direct control.

Full class participation lesson.

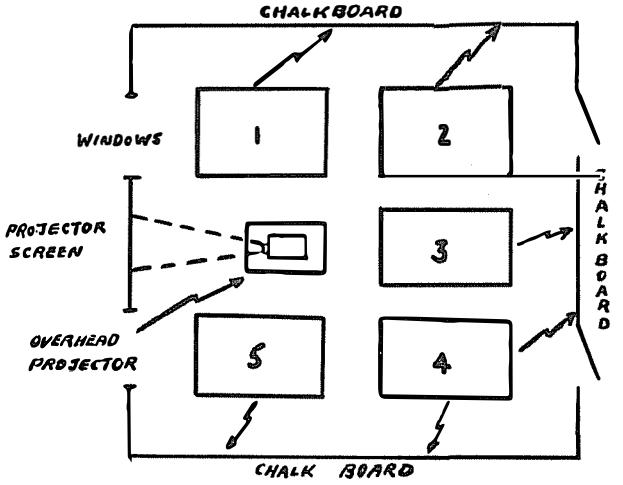
The basic idea is that the whole class operates as a unit under the guidance of the teacher rather than in groups of three or four students. The method permits the teacher to control in a fairly direct manner the activity of the class so that some particular purpose may be achieved. It ensures that every member of the class participates in all that is done. It promotes communication among the students because each is responsible for giving assistance to others when required. It assists in developing an ability to enquire and investigate depending to some extent on the ingenuity and imagination of the teacher in planning the lesson. It should develop an appreciation for pattern in mathematics. It provides an opportunity for drill while at the same time, development of new ideas or extensions of old ones is taking place.

The method involves all the aspects of good learning experience. It begins with an evaluation of the students read-

iness to proceed to an extension of his knowledge and understanding. This evaluation is made by both the student himself and by the teacher. It lays the ground work for the extension of knowledge and gives the student an opportunity to develop, to discover, or to determine the extension for himself with as little suggestion as necessary. It provides the necessary consolidation in a useful way: at each stage the teacher is aware of the amount and direction of discussion that is needed to develop, clarify, or consolidate before the next stage in development is presented. Whatever is obtained by each student is a product of his own experience.

A classroom set up as indicated in the diagram probably provides the most useful arrangement allowing for

(1) the use of the overhead projector to present instructions to the students. The position indicated allows instructions to be seen by all the students in the room and does not eliminate chalkboard space.



(2) one or two representatives from each group to work at the chalkboard

(3) the remaining students of each group to work at their seats, to discuss problems with each other, to observe work done at the board, and to assist the worker at the board if required. It is possible that a whole group may find itself working together at the board.

The following example, which might be used to teach how to expand the indicated product of two binomial expressions in which the variables are integers, illustrates the method.

For this lesson it is assumed that

(1) the integer number system (I, +, x) has been developed;

(2) there is agreement on terms of an expression involving integers.

$$3a + 2b - 3c \iff (3a) + (2b) + (-3c)$$

Signs of quality; a part of each term

Addition signs of operation, understood;

(3) addition and subtraction of expressions is understood;

(4) the distributive property (D) has been taught for $(N_o, +, x)$ and introduced as a property (I, +, x);

(5) D is the basic requirement for the expansion process.

The lesson guide may be a sequence of transparencies (or dittoed reproductions) containing exercises or patterns to guide the student to the ideas of the lesson.

Transparency 1.

COMPLETE:

	Expression	Simplification	
		Operation in brackets first	Using the Distributive property
1.	98(100 + 10)		
2.	19(98 + 2)		
3.	48(10 + ½)		
4.	72(1/3 + ¼)		
5.			5 x 7 + 5 x 9 = 80
6.			12 x 3 + 12 x 8=?
7.		36 x 20 = ?	36 x 4 + ? = ?
8.			
9.			
10.			

After whatever brief comment the teacher proposes to make, a representative from each group is sent to the board and transparency 1 is presented.

Students at the board and at their seats proceed to do the work as instructed. Students are encouraged to work quickly and to discuss with each other any difficulties or questions which arise. Those at their seats should keep an eye on their representatives at the board and when the need arises go to the board to discuss the problem and make suggestions.

The instructions in transparency 1 have been set out in a table, a very convenient and clear method. Instructions, of course, may be given orally, presented on mimeographed sheets or in any manner or form suitable to the topic or circumstances.

This set of exercises reviews the distributive property in a known context. It contains enough information to guide students. It sets out in each example three expressions which are equivalent in that they represent the same number. It gets the student thinking about and using the distributive property, laying the ground work for the problem at hand. It allows both student and teacher to assess the readiness of the group to go on. The teacher may find complete readiness or he may find that it is necessary to do more preparation work. Not much will be achieved if new work is introduced before the class is ready for it.

The teacher may stop the procedure at any appropriate time for discussion. On the blackboard are many examples of the present understanding of the students. These will clearly indicate the discussion necessary when the students have completed their work and returned to their seats. Students can be asked to explain their thinking for a particular example and discussion ensues. The questions are numbered to facilitate reference.

Questions 8, 9, 10 (transparency 1) are for the student to make up. If the student can make up examples of his own correctly it is obvious that understanding is achieved. Slower students may complete only the examples listed.

If the work is progressing in a completely favourable manner it may not be necessary to complete all the work in the table.

Transparency 2

COMPLETE:

	Expression	Simplification (Distributive Property)
1.	2(a + b)	
2.	3(x + 2y)	= 3(?) + 3(?) =
3.	(x + (-2y))5	= (?)5 + (?)5 = ? + ? = ? - ?
4.	7(2a + (-5b))	= = =
5.	(3x - 2y)(-4)	
6.		
7.		
8.		

New representatives should be sent to the board.

The examples illustrated by transparency 2 review D in the context of I and its relationship to terms in expressions in which the variables are integers.

A good deal of assistance is given in this example. A

teacher knowing his own class will organize the examples to meet their need. For a class of brighter students one might just say: "Investigate the product of a monomial and a binomial expression in which the variables are integers."

This leaves the initiative and ingenuity in choosing examples to the students. All possible types of situation should be investigated. Note how this is done in the table. The arrangement ensures that the students make up examples of their own.

Transparency 3

COMPLETE:

	Expression	Simplification (Distributive Property)
1.	a(b + 3)	
2.	(a+3)(b+4)	= (a+3)(?) + (a+3)(?) =
3.	(x+4)(y+6)	= =
4.	(5-d)(6+m)	
5.		= (x+7)p + (x+7)(-q) = =
6.		
7.		
8.		

Transparency 3 introduces an expansion for the indicated product of two binomial expressions involving integers.

The expansions result in an expression of four terms. Only one step in simplification is required.

The objective is not so much the obtaining of a correct answer but an understanding of the principles involved in the procedure.

Students should have the feeling that there are ideas and relationships to be understood in the work they are doing and should attack the work accordingly. The problems they make up themselves reflect this attitude.

The students working at the board should be changed fairly often so that as many students as is reasonably possible get an opportunity to work at the board. In this way the teacher and others have an opportunity to observe and test each student. Test is used here in the sense that the teacher,by constant observation, acquaints himself with the achievement level of each student in terms of skill, understanding, imagination, creativity and confidence. He is able to take immediate and constant action to provide assistance for those in difficulty and to provide more challenging work for the more competent. This kind of testing is of far greater value and significance than periodic tests at the end of each month or period of work.

Transparency 4

COMPLETE:

	Expression	Simplification
1.	$(x + 3)(x + 2)$	$= (x+3)? + (x+3)?$ $=$? $=$? $=$?
2.	$(x+5)(x-5)$	
3.	$(x+3)\ (x+3)$	
4.	$(x+y)\ (n-y)$	
5.	$(x + y)^2$	
6.		
7.		

The expansions in this set of examples (transparency 4) may be taken to several stages of simplification using the commutative and associative properties to justify the result. The distributive property is used in the sense

$$ap + bp = (a + b)p$$
$$= p(a + b),$$

that is for factoring.

Students should be able to justify the simplifications they make when discussing their work.

It might be advisable to add a column to the table for this purpose.

Expansion	Simplification	Authority

Note the introduction in this table or some special products. The results are interesting. Some expansions simplify to 4 terms, some to 3 terms, some to 2 terms. The factors have special properties which some students will immediately observe. A first look at these is given here. The objective of the lesson, however, is only to practice obtaining expressions equivalent to the indicated product of two binomials.

It is interesting to note that 34 examples have been worked by each student in this lesson. This "drill" is done in the development of a new idea. But it is still practice in the drill sense. Students are not conscious,however, of the deadening feeling of doing 34 repetitions of the same manipulation.

When students discuss their problems and ideas with each other they are forming a firm basis for learning. When teachers listen to what students are thinking or know about the things they are to learn about they are forming a firm basis for teaching. Where the preceeding type of procedure has been used, learning has become much more a community effort with interest, understanding, and enthusiasm reaching high levels.

Socratic discussion

Useful discussion most often takes place after the participants have had some experience with the topic under consideration. Students who have made a group investigation and reported on it are ready for discussion with the teacher and other students to whom the report has been made. It is at this time that useful constructive contribution can be made to a discussion designed to round out the experience and firm up conclusions (possibly under the guidance of the teacher).

When students have completed one phase of a full class participation lesson, they are ready to discuss the concepts involved if necessary and many examples are available on the blackboard to provide the basis for discussion.

After students have made an independent study of some topic, they are ready to discuss it back and forth with others and the teacher. Indeed discussion at this stage is necessary to expand and integrate concepts related to the topic involved.

So often in the classroom we begin discussion with students before they have made the necessary preparation. Provide them with some opportunity to become involved with the subject matter, to raise questions in their minds before investigating discussion.

Teachers working as a team.

The production of worthwhile investigations and full class participation lessons requires a good deal of time, ingenuity, imagination, and research. Production, evaluation, and control

are all aided when a group of teachers work together to plan, organize, and prepare the mathematical experiences for the particular grades which they teach. Team discussion first provides an evaluation of student readiness, then an outline plan of how this further learning is to be promoted. Individual teachers can then prepare parts of the presentation which, after further discussion, become a presentable program. The real evaluation comes when the reaction of the students to various aspects of the program is observed. Further discussion now leads to revision to eliminate weaknesses. Teachers should, whenever possible, visit each others classrooms both to observe and to assist in the teaching. This kind of team teaching effort makes optimum use of staff potential and provides the very best type of experience for the students.

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A Co-operative Venture in Teacher Education

by D. H. Crawford, D. B. Wing, J. L. Boyd and
Type A Students at McArthur College of
Education, Queen's University, Kingston.

INTRODUCTION

This article describes a short unit of work on graphing. At the suggestion of Dr. D. H. Crawford, assisted by Professor D. B. Wing and the Type A Mathematics students of McArthur College of Education, a series of laboratory lessons was conducted in the classroom of Mr. J. L. Boyd at Loyalist Collegiate, Kingston. The pupils involved were a 4 year grade 10 S.T.T. class.

The purpose of the project was two-fold. First, it offered the teachers in training an opportunity to design their own experiments and to test, in a real classroom situation, the suitability of the material and the practicality of the approach. Second, it gave the pupils a chance to develop the ideas of graphing a relation in a non algebraic framework by producing graphs from physically generated data.

THE EXPERIMENTS

The experiments were designed by the Type A students subsequent to some similar work in their McArthur Seminar. Descriptions of these experiments appear later in the article. They were designed with the following ideas in mind:

(i) No elaborate equipment would be necessary

(ii) The experiments would be performed in an ordinary classroom as opposed to a properly equipped laboratory.

(iii) The experiments would be short enough in duration that a group of 5 boys could carry out the experiment, tabulate the results and begin to draw up a graph in a period of 40 minutes.

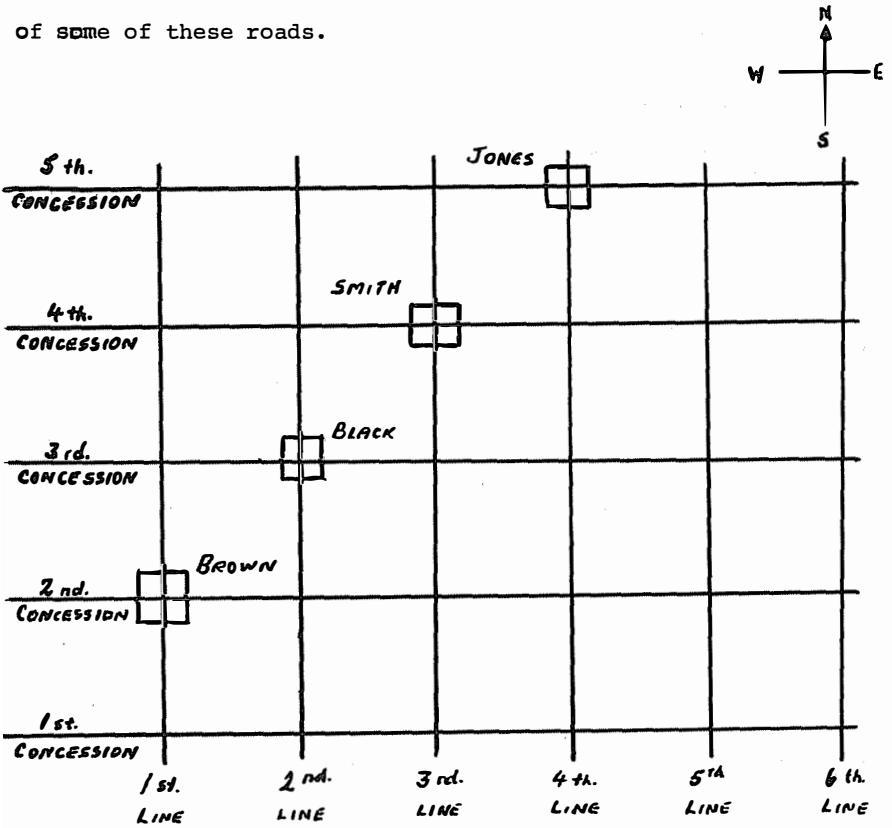
(iv) The experiments would produce graphs that were approximately linear, parabolic and hyperbolic in shape.

THE INTRODUCTORY LESSON

Since the pupils were being asked to do experiments which would lead to tables of values and ultimately graphing these results, it seemed necessary to the student teachers to given an introductory lesson on the plotting of points. This particular class (4 year S.T.T.) for one reason or another, were known to have experienced difficulties with mathematics in the past and for most of them, the idea of co-ordinates would be new. Accordingly, a lesson was devised based on a grid system of line and concession raods with stragetically placed farms. One student teacher spent ten minutes introducing this idea. The class was then given a grid and asked to complete a number of questions that lead to the idea of an ordered pair (pages 171 and 172). A second student-teacher spent another ten minutes establishing the ideas suggested by the questions. A second set of questions was presented designed to put across the idea of plotting points after having selected axes. (page 173). Finally, a third student-teacher summarized what had been done. A class discussion eventually brought forth the notion that such a grid system might be useful in displaying data obtained from experiments.

INTRODUCTION TO GRAPHS AND PLOTTING POINTS

Here is a map of a farming community. The lines represent roads. Concession Roads run east and west. Line roads run north and south. Farmers live at the intersections of some of these roads.



Mr. Black lives at the corner of 2nd Line and 3rd Concession.

Question 1: Describe where the other farmers live.

An easy way to describe where these farmers live is to write pairs of numbers in a table.

Question 2: Complete the following table:

Farmer	Line Road Number	Concession Road Number
Black	2	3
Brown		
Smith		
Jones		

Question 3: How many numbers are required to describe where one farmer lives?

Question 4: In the table above does it matter which column the numbers are put in? Explain.

Three new farmers just moved into this community. I can describe where they live in the following way.

Line Road Numbers → ← Concession Road Numbers

Mr. Brock (3, 2)

Mr. Kent (4, 3)

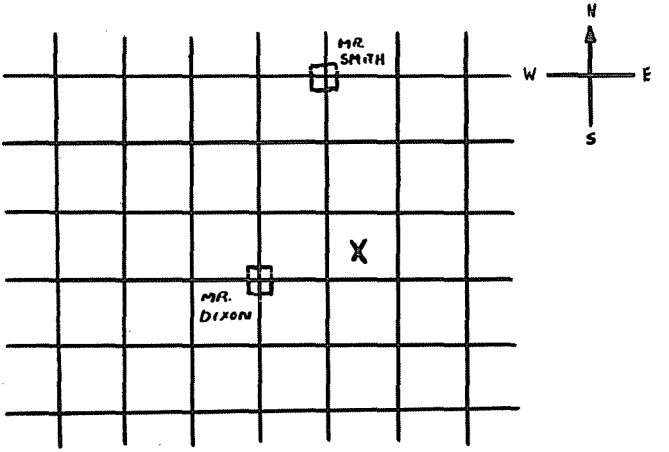
Mr. White (5, 1)

Question 5: Put the names of these farmers on the map at the place where they live.

Ordered Pair: The pair of numbers 3,2 when put into the order 3 coming first followed by 2, makes up the ordered pair (3,2) which describes where Mr. Brock lives.

PLOTTING POINTS

(1) On pages 171 and 172 it was agreed that the line road numbers came first in the ordered pairs. In the plan below, Mr. Dixon lives at (3,2), and Mr. Smith at (4,5). Number and name the roads so that this is true.



(2) Mr. Keller moves into the empty farm at (5,1). Draw in Mr. Kellers farm.

(3) Where could Mr. Cole move in order to live in a straight line with Mr. Dixon and Mr. Keller? Draw that location on the map above. Describe that location as an ordered pair.

(4) Mr. Dixon's son has an accident with his snowmobile. It is in a field marked by position "x", but cannot be seen from the ground. How could an aircraft pilot describe to the search part on the ground where Mr. Dixon's son and snowmobile are located?

THE LABORATORY LESSONS

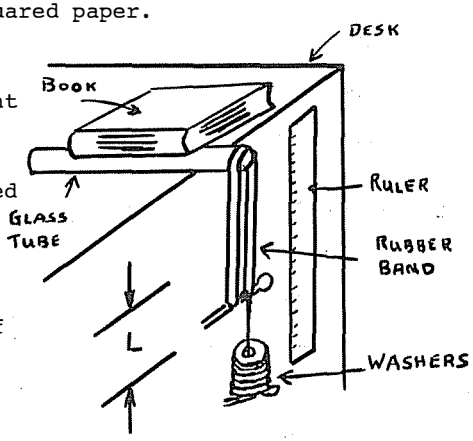
The pupils were issued with five "kits" each day. A "kit" consisted of a sheet of instructions plus the appropriate apparatus. (pages 175-81). A chairman and a recording secretary were duly elected for each of the five groups of five boys. Each group did one experiment illustrating a linear relationship and two illustrating non linear relationships. Experiment G was abandoned as unsatisfactory. The outline of each experiment was prepared by a student teacher or a pair of student teachers. As such, instructions as to procedure varied in depth of detail. However, on any day of experimentation at least four adults were present to give any assistance necessary. It was quite noticeable that after the first day the pupils became very self reliant. The emphasis at this stage was on attaining sufficient data so that it could be plotted on a sheet of graph paper. No attempt was made to formally "write-up" the experiments. One student, though was expected to put the table of results and the graph on an acetate for subsequent presentation to the class.

EXPERIMENT A HOOKE'S LAW

Materials: Equipment for experiment A, ruler (with cm. scale), squared paper.

Procedure:

(1) Set up the equipment as shown.



L is the distance the rubber band is stretched by the weight of the washers.

W is the weight on the rubber band in terms of the number of washers.

(2) Measure L for W = 0, 2, 4, ..., 20 and record the measurements in the following table.

W	0	2	4	6	8	10	12	14	16	18	20
L											

(3) Plot these number pairs on a piece of graph paper with W along the horizontal axis and L along the vertical axis.

Problems:

(1) Should the graph be a continuous curve? If so, connect the points by a smooth curve.

(2) What is the shape of the curve you obtained?

(3) From the graph read L for W = 5, 17.

(4) How would the graph be different if a rubber band had been used which stretched more easily?

EXPERIMENT B

LABORATORY EXPERIMENTS ON GRAPHING AND RELATIONSHIPS

Equipment: Ball

Tape measure

Squared paper.

Procedure:

1. Drop the ball from various heights and measure the height of the bounce.
   (Suggestion - Obtain the average of three readings for each height.)
2. Using the height of drop and height of bounce as the two components of an ordered pair, plot the results on squared paper.
3. Join the points with a smooth line.

Questions:

1. Describe the curve thus formed.
2. What relationship, if any, does the curve show?

EXPERIMENT C. HEIGHT OF PENDULUM V.S. ANGLE

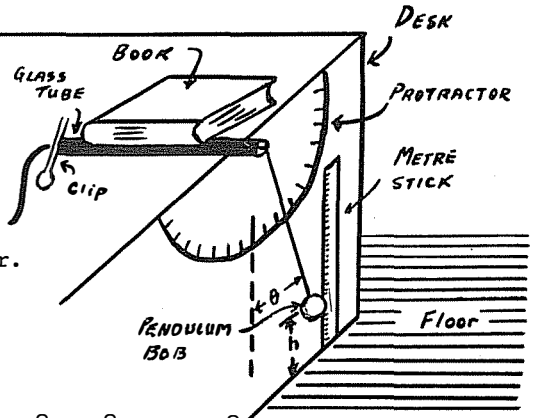
Materials: Equipment for experiment C, protractor, metre stick, squared paper.

Procedure:

(1) Set up the equipment as shown.

θ is the angle which the string of the pendulum makes with the vertical.

h is the height of the pendulum bob from the floor.



(2) Measure h for $\theta = 0^{o}$, 10^{o}, 20^{o}, --- 90^{o} and record the measurement in the following table.

θ	0^{o}	10^{o}	20^{o}	30^{o}	40^{o}	50^{o}	60^{o}	70^{o}	80^{o}	90^{o}
h										

(3) Plot these number pairs on a piece of graph paper with along the horizontal axis and h along the vertical axis.

Problems:

(1) Should the graph be a continuous curve? If so, connect the points by a smooth curve.

(2) How would the graph be different if a longer (or shorter) pendulum had been used?

(3) Read from the graph the height for $\theta = 35^{0}$, 75^{o}.

EXPERIMENT D

Equipment: compass

squared paper

graph with axes.

Instructions:

On the squared paper construct a circle of radius ½ inch using your compass. The area of this circle may be estimated by counting the number of squares of the paper inside the circle. Find and record below the area of this circle. (Just inside the circumference of the circle will be pieces of squares. Add bits of one to bits of others in your area estimation.) Also record the diameter of the circle. Find areas and diameters for circles of radius 1 inch, 1½ inches, 2 inches, 3 inches as above. Record. Plot your results on the graph paper provided. Make a smooth curve.

RADIUS	DIAMETER (IN INCHES)	AREA (IN SQUARES)
½"		
1"		
1½"		
2"		
3"		

Questions:

(1) Can the points be joined in a straight line?

(2) Describe the curve.

(3) From your graph can you tell what area a circle of radius 2½" has?

(4) What is the area of a circle radius 4"? If a circle has area 10, what is its radius.

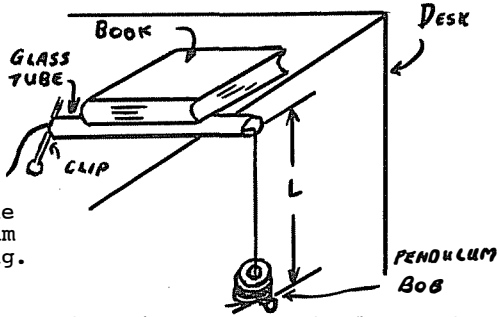
(5) Is there a formula for finding area of a circle?

EXPERIMENT E

Materials: Equipment for experiment E, watch (or wall clock) with sweep second hand, squared paper, metre stick.

Procedure:

(1) Set up the equipment as shown.



L is the length of the pendulum.

P is the period and is the time it takes the pendulum to make one complete swing.

Example: If it takes the pendulum 24 secs. to make 10 complete swings then P = 24 ÷ 10 = 2.4 secs.

(2) Measure P for L = 8, 16, 24, ···, 72 cm. and record the measurements in the following table.

L(cm)	0	8	16	24	32	40	48	56	64	72
P(sec)										

(3) Plot these number pairs on a piece of graph paper with L along the horizontal axis and P along the vertical axis.

Problems:

(1) Should the graph be a continuous curve?
If so, connect the points by a smooth curve.

(2) From the graph read P for L = 28, 60 cm.

(3) Does P depend on - the mass of the pendulum bob?
- the amplitude of the swing?

EXPERIMENT F ROLLING A BALL DOWN A TRACK.

Materials: stop watch, ball, track, meter stick.

Procedure:

A. 1. With the track on the floor, raise one end 3 inches.

2. Start the watch when the ball is released at the top of the track.

3. Stop the watch when the ball rolls off the bottom of the track.

4. Record the height of the track at the raised end and the time on the stop watch.

B. Repeat point A raising the track 4 inches, then 6", 9", 12", 15", 18", 24", 36".

C. From parts A & B you have found 9 ordered pairs of numbers. The first number is the time, the second number is the height of the track.

Fill in the following table: and plot these ordered pairs on graph paper.

TIME	HEIGHT OF TRACK
	3 inches
	4
	6
	9
	12
	15
	18
	24
	36

Questions:

1. Could we have found more ordered pairs? How many more?

2. Read from the graph the time required when the track is raised 10 inches.

EXPERIMENT G

Given: 3 tin cans of various sizes labelled one to three

string

1 soft drink bottle

graph paper

ruler

Instructions: Starting with tin #1, by wrapping the string once around and using your ruler find and record below the circumference of tin #1. Now measure the diameter of tin #1 and record it in the appropriate place below. Next measure the diameter and circumference of tin #2 - record. Do similarly for all the tins. Using the provided graph paper and the given set of axes plot diameter of tins versus circumference of tins. Join the points.

TIN	DIAMETER (INCHES)	CIRCUMFERENCE (INCHES)
1		
2		
3		
BOTTLE	DIAMETER	CIRCUMFERENCE
SMALL END		
BIG END		

Measure and record the circumference and diameter of the circles at both ends of the pop bottle. Plot diameter versus circumference on your graph.

Questions: 1. Can the points be joined in a straight line?

(2) From your graph what is the circumference of a circle radius 3½"? 5"?

(3) What radius does a circle circumference 36" have?

(4) Someone says a circle of radius 2¼" has a circumference of 27". Is he correct? If not what is the circumference?

(5) Is there a formula for the circumference of a circle with known radius?

FOLLOW UP LESSONS

Each experiment was performed by the instructors so that they would have a basis for comparison with the student's results. During the days following the experiments Dr. Crawford led a class discussion and analysis of them. Briefly, a student would describe his experiment for the benefit of those who did not perform that particluar one. The shape of the graph, continuity errors and the like were among the many ideas discussed. Then Dr. Crawford showed his results on the overhead projector and the two graphs were compared. If any very noticeable discrepancies occurred, he made an attempt to reconcile the two. One notable occasion came during the follow-up to experiment A.

The pupils produced the following table together with its graph. (Page 183)

W	0	2	4	6	8	10	12	14	16	18	20	Washers
L	9.5	9.7	10	10.2	10.4	10.7	10.7	11.5	11.7	12	12.5	cms

Whereas the instructors produced the following table and graph (Page 184)

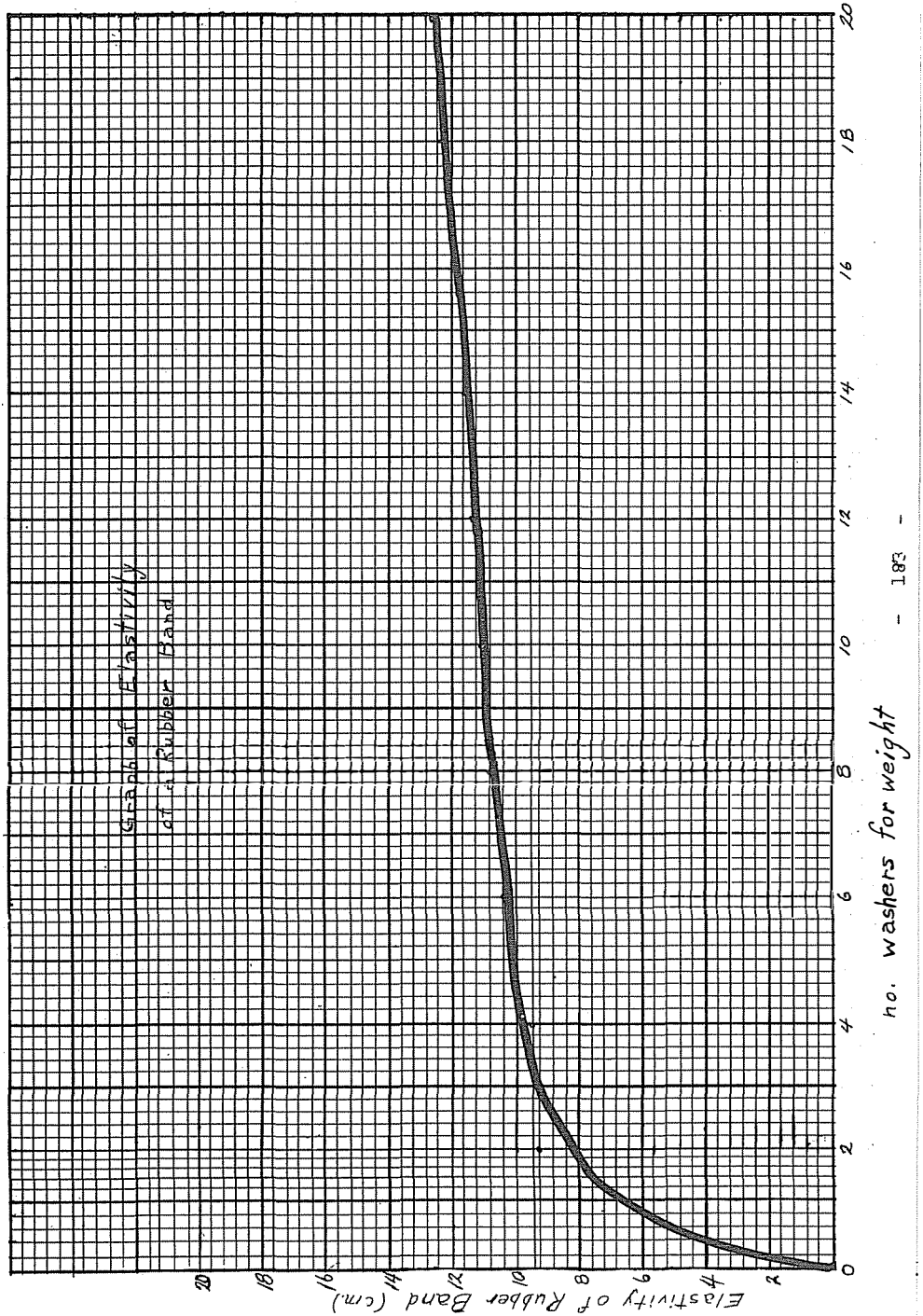
W	0	2	4	6	8	10	12	14	16	18	20	Washers
L	0	1	3	4	7	9	12	14	16	19	21	1/16 of inch

The pupils failed to see any similarity. They admitted that they might have made an error especially as L = 10.7 for both 10 and 12 wahsers was unlikely. Eventually, after a valuable discussion on standardisation the following table was drawn up

W	0	2	4	6	8	10	12	14	16	18	20	Washers
L(pupils)	0	.08	.2	.28	.36	.48	.48	.8	.88	1	1.2	ins.
L(inst)	0	.06	.19	.25	.44	.56	.75	.88	1	1.18	1.31	ins.

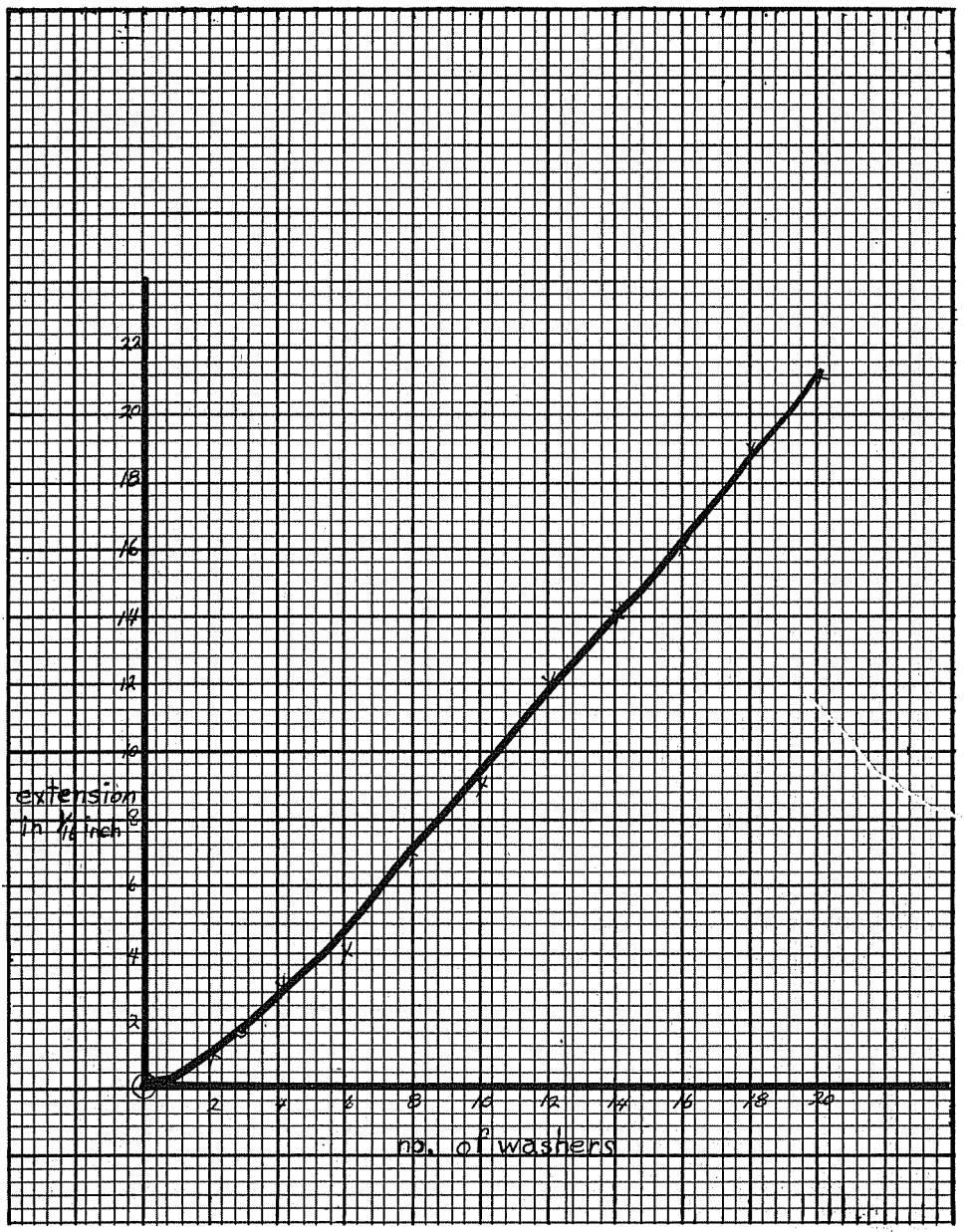
GRAPHICAL SOLUTIONS

(For instructions see reverse side)



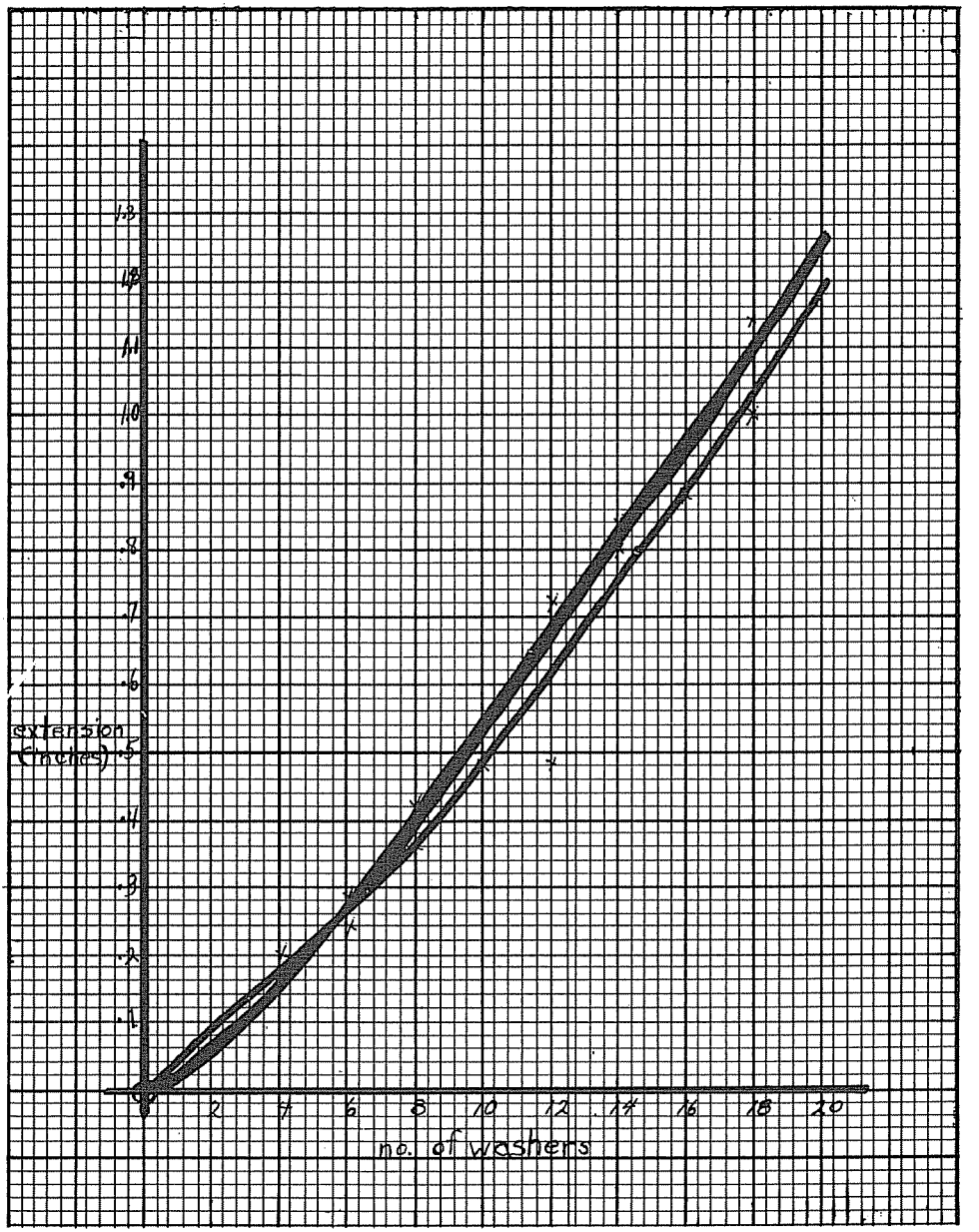
GRAPHICAL SOLUTIONS

(For instructions see reverse side)



GRAPHICAL SOLUTIONS

(For instructions see reverse side)



The pupils have difficulty following even the most elementary instructions and hence require supervision and assistance. Equipment must be pre-assembled and careful planning is required.

Although the material might not appear directly on any course of study, we feel that its inclusion in a four year course is amply justified since it can serve as an introduction to many important topics. Simple formulas and their graphs is an obvious one. For example, "A = lw", plotting A against W with l constant, or W against l with A constant, or $T = 2\pi\sqrt{\frac{l}{g}}$, plotting T against l after suitable substitutions for π and g. These will illustrate graphs similar to the graphs previously obtained with the fundamental difference that these are produced from mathematically generated data. Relations and their graphs, analytical geometry and the graphical solution of two linear equations in two unknowns are other topics which could follow these experiments.

Summing up, the authors feel that this unit on experimental graphing offers a section of work that is student generated, features student activity and leads naturally to several key portions of a four year course in Mathematics. Conceivably, it could be helpful in science and some of the shop subjects. The experimental activity did generate better student interest and participation. The work of subsequent topics did follow more easily than those topics studied prior to this unit. We suggest you try something akin to this with your 4 year students.

THE UNIVERSITY OF WATERLOO CO-OPERATIVE TEACHER TRAINING PROGRAMME

by R. G. Dunkley
Faculty of Mathematics
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A new venture in training of secondary school mathematics teachers is being conducted by the University of Waterloo. In this programme students spend alternating four month terms in the University and in secondary schools. Currently there are forty students employed by twenty-one school boards across the province.

The programme originated because of the shortage of mathematics teachers and because many excellent candidates, by the time they graduate, seek occupation in industry rather than attend the year programme at Colleges of Education. It is the belief of the originators of the programme that teaching, like other professions such as medicine, requires substantial preparation, not only academic, but also in mental preparation and acquiring the experience necessary to deal with the youth of today. We feel that the practical training of current training institutions is inadequate for many students and that four month terms in schools offers far more opportunity for developing the necessary skills. This is not an indictment of current practices. Indeed, many students seeking admission to the programme are advised to pursue conventional training if it is felt that they will not benefit from this programme.

A strong point of the programme is the great amount of student contact inherent in it. This allows great opportunity

for observing (and practicing) the introduction of a subject to a class, motivation of students, helping students with their problems - in other words, with total education.

The programme commences in Year 2 of the student's programme. This provides maturing time for the student, ensures that he has one and one-half years of university mathematical knowledge, and also allows the University, through a series of interviews with classroom teachers, to be sure that the student is really desirous of becoming a teacher before he is placed in a school. Even then the student cannot stay in the programme if no school is willing to hire him.

University preparation of the student includes a course in Psychology, one in Sociology, and one in Philosophy of Education. These courses, while open to all students in the University, are primarily designed for persons aspiring to teaching as a career. As well, discussions involving the students and teachers from surrounding areas are held. Throughout the aim is discussion and self-appraisal of the art of teaching. For example, the three courses mentioned earlier all involve extensive discussion of the experiences the students have accumulated in their four-month work terms.

The mathematics content of the programme is identical with that of the regular programme, thus ensuring that a graduating student has a legitimate degree in mathematics. Students are encouraged to take courses which will be beneficial in teaching, such a projective geometry, History of Mathematics, etc. All students take at least one course in computer science.

During work terms the student is assigned to assist a mathematics teacher, normally the head of the department. Initially, he spends his time in observing different teachers, discussing the operation of a school with school officials, getting to know as many students as possible, and acquiring familiarity with the various mathematics courses. He provides assistance through tutoring of individual students, assisting in problem sessions, helping with administrative detail, marking tests, etc. As he becomes more familiar with the students in particular classes he assumes some of the teaching of those groups. This is a gradual development, depending on the student and his supervisor. In some cases the student teaches a full section of the course under supervision. In others, he teaches scattered lessons. It is clearly established that the student takes greater interest if allowed to pursue a topic for a substantial period of time.

There are, as one would expect, many variations on these suggestions. In a few instances students have completely taken over some areas, notably introduction of computer science in the school, and coaching of the Junior Mathematics Contest team. In some schools they have even given computing courses for the teachers. Experience to date indicates that there are numerous areas where these young people can be of extensive use in the classroom.

By the end of the first work term, the student is expected to be conversant with the operation of the school in general, and with the operation of the mathematics department in particular.

He is expected to have acquired a definite feel for the skills of teaching and to have exhibited a definite desire to work with, and help, young people in their mathematics. If he has not achieved these things, he is not asked to return to the school for subsequent work terms. Thus there is a built-in success requirement; if a student is not satisfying the demands of teaching in the classroom, he does not continue in the programme. There is , of course, the problem of personality clashes and unfortunate situations from the student view-point. In such cases, the student is transferred to another school.

In subsequent terms in the schools, the student is allowed to develop his skills further. In some instances a student works with three or four different teachers, easing the load of the teacher in each case. In all cases, there is a substantial reduction of the supervision required, with a resultant increase in time for the teacher concerned to work on course development or other special products.

The response of participating schools leaves no doubt that these people are extremely useful in the schools, nor any doubt that the extensive training period is extremely beneficial. The student-teachers have in almost all cases been labelled as superior prospects. This is undoubtedly partly due to the original screening process, but certainly the fact that they spend lengthy periods in the classroom and work with the students in a normal teaching manner must be considered a major contribution to this success. The originators of the programme are convinced that for some students this is a superior form of preparation for the teaching profession.

THE EDUCATION OF MATHEMATICS TEACHERS: A SURVEY OF TEACHER ATTITUDES

by

The Althouse College of Education

(Editor's Note: In the winter of 1967-68 a committee was set up under the auspices of Althouse College of Education with the co-operation of the McArthur College of Education and the College of Education, University of Toronto, to study the training of mathematics teachers in Ontario. The idea of conducting such a study originated with the Department of Mathematics, A.C.E. Unfortunately, due to budget considerations, the committee was unable to carry out the original plan for a two week study period, in June 1968, and had to content itself with three single day meetings at intervals of two months. The committee identified certain aspects of teacher education in mathematics which in its view merited study. They included

1. The Problem of Recruitment.
2. Alternative Schemes for Teacher Training.
3. Ways and Means of Improving Present Programs.

A major portion of the approach to item 3 consisted of designing, distributing and analysing the responses of a questionnaire dealing with the views on teacher education of practicing teachers in Ontario. The questionnaire was distributed to 150 schools across Ontario, selected more or less at random from "Who's Where". Replies were received from 110 schools. This very good response about 73%, seemed to indicate to the Committee both a high degree of

professional responsiblity on the part of those teachers who answered, and also a high degree of interest and concern in the matter of professional training of Mathematics teachers. This article is a slightly edited version of the report.

THE QUESTIONNAIRE

Committee on the Training of Mathematics Teachers

Dear Colleague:

The above committee is meeting this year to consider ways in which the training of Mathematics teachers in Ontario can be improved. I hope you will agree that this is a most worthwhile and necessary task, for which we should very much appreciate your help. The committee is sponsored by Althouse College of Education with the cooperation of the McArthur College of Education and the College of Education, University of Toronto.

One of the sub-committees is concerned with contacting as many Ontario Mathematics teachers as possible to discover their views on this matter. We realize that your time is precious and have therefore made out the following form to facilitate your giving the more factual ideas as quickly and as easily as possible. However, we should be grateful if you would also give your impressions, ideas and suggestions in the space provided, and these will, I assure you, be considered at least as carefully as the more statistical data.

Number of years teaching experience..... (a) Elementary .....
(b) Secondary .....
(c) Other (specify) .....

Certification: (Please check the appropriate item)

Type A .....
Type B Endorsed.....
Type B .....
T.S.S. .....

A. Academic Education

1. Which topic or topics taken by you at University do you consider are most helpful to you in teaching? .....

2. Which topic or topics taken by you at University do you consider are least helpful to you in teaching? .....

3. Which further topic or topics not taken by you at University do you consider would be of help to you in teaching? .....

B. Professional Education

1. For your professional preparation did you attend

   (a) a College of Education .....

   (b) a Summer School .....

   (c) Other (specify) .....

2. Which topic or topics taken by you in the education course do you consider are most helpful to you in teaching? .....

3. Which topic or topics taken by you in the education course do you consider are least helpful to you in teaching? .....

4. Which further topic or topics do you think should be offered in the education course? .....

5. Estimate the number of weeks you spent in practice teaching. .....

6. How long do you consider practice teaching should last in a teacher training programme? .....

7. Should a student in a College of Education be allowed to take only one teaching option, but with the possibility of taking a second one if he chooses? Yes..... No.....
   Should he be required to practice teach in this second option? Yes..... No.....

8. What suggestions would you make concerning the improvement of a teacher training programme?

   .....

   .....

   .....

   .....

   .....

   (You may sign this form if you wish).....

   (continue on reverse side if necessary)

ANALYSIS OF QUESTIONS A1 THROUGH B7

Replies were received from 286 teachers, 99 of whom were Type A, 115 Type B, 38 T.S.S. and 34 of whom had elementary school experience in addition to their secondary school qualifications and experience.

All figures stated in the following analysis of answers to the preceding questionnaire are percentages of the total number of topics mentioned in that particular question, and only those topics mentioned a significant number of times are included.

Question: A 1
- 36% Calculus, Algebra or Co-ordinate Geometry
- 27% Modern Algebra, Sets, Number Theory, Matrices
- 8% Psychology
- 5% Vectors
- 5% Probability and Statistics
- 5% Physics

A 2
- 37% Language or Arts courses
- 12% Probability and Statistics
- 7% Psychology

A 3
- 19% Computer Science or Numerical Analysis
- 17% Psychology
- 16% Advanced Modern Algebra, Sets, Number Theory or Matrices
- 14% History of Mathematics
- 10% Advanced Calculus, Algebra or Co-ordinate Geometry
- 5% Probability and Statistics

B 2
- 47% Methodology
- 12% Psychology
- 10% Practice Teaching
- 7% Philosophy
- 5% Administration, P.P.A.

B 3
- 23% Psychology
- 21% Philosophy
- 21% Administration, P.P.A.
- 18% History of Education

B 4
- 46% Advanced Methodology, Evaluation Techniques
- 25% Psychology, Discipline, Adolescent Psychology
- 12% More Practice Teaching
- 8% Visual Aids

B 1 40% College of Education
54% Summer School
6% Other

B 6 The average length of time mentioned was 11.5 weeks

B 7 Only one teaching option - 80%
If a second option is taken, 73% thought the student teacher should practice teach in it.

ANSWERS TO QUESTION B8

In answer to the last question, B 8, many suggestions were offered concerning the improvement of a teacher training programme. Unfortunately, it is possible to include only a few which are representative. Since the summer course in its present form is to be abolished, no comments dealing with those courses have been included. It should be understood also that this is simply a report, not an evaluation, and consequently the opinions expressed are those only of the teachers who stated them and do not imply either the approval or disapproval of the Committee.

Many comments dealt with the present situation at Colleges of Education.

- There is too great a step down from university to College of Education -- both in terms of material and treatment of students (from adult to children).

- More stress on "methods" than "content". The content should already be mastered.

- A further stress on the knowledge of the matter and content of the course to be taught. With but one option,

one could stress the content much more fully, I have found many new teachers who did not know their material.

- I feel the major object of a College of Education in training prospective Mathematics teachers should be to show students as many different methods of presenting mathematics lessons as possible. Students should then discuss the relative merits of these different methods. Course content should be known by all students accepted for the mathematics courses and methodology should take precedence over course content.

- FIRST - Emphasize training! One doesn't become a professional golfer by reading books. Assume that a teacher knows his field. Let him observe the experts teach; let him criticize them, have him teach and reteach until he can teach an acceptable lesson.

Correct him while teaching. In other words show him rather than tell him; let him do it and then point out the mistakes as he goes. Get him away from so much formal examination of the "teacher student". To me, what is desired of a teacher is that he be able <u>to teach</u> and that he have an acceptable philosophy of education. It is ridiculous to take a math major and test him on his knowledge of History or Grade 9 Botany.

Any teaching programme should make the student aware of different teaching techniques. Do not ask him to list them. Let him perform them in the classroom.

As far as philosophy is concerned, ten sentences of philosophy from a live and active teacher has more impact than the googols of words written by the philosophers of old. (Not that they can't be appreciated.)

- 1. More teaching assignments and observational teaching with discussion at the end.

2. More seminars and if possible workshops for student teachers to investigate new methods of teaching.

- Smaller classes with a seminar approach.

- Smaller lecture groups. More opportunity to discuss problems which arise during practice teaching.

- More practice teaching and considerably more discussion of the 2-year and 4-year stream students.

- More concentration on 4-year students. In depth treatment of present rather than presentations of new-fangled predictions of the future. Techniques for enriching course for students.

- Suggest ways of using new-arrangements; i.e. team teaching, experiments, tables instead of desks, math labs, instead of row by row face the front desks. Just saying "do team teaching" is of little use. Concrete actions and techniques should be examined and discussed. The role of machines in the classroom, field projects, (specific details), with logical follow-throughs covering all phases of project-marking, etc. included. All this might be included in 'General Methodology'??

Marking schemes, objective and subjective testing--not just 'set up a test and marking scheme' needs more emphasis. Beginning teachers need lots of help in this area. It never seems to be well covered at a training school.

- Course on how to test and evaluate tests for a class.

- Should emphasize creation of student interest and creation of a working atmosphere, also methods of evaluating a student's work and of rewarding a student for work well done.

Also some training in discipline.

- Improve methodology courses by not allowing methodology lecturers to dwell on their individual method. Perhaps a rotary system of lecturers so that different approaches may be observed. "Field trips" to different kinds of

schools; two year, tech. and comm.

- I would include lectures by those who are active in a given subject.

- More consideration given to what you want to achieve in the mathematical experience of a pupil by the time he graduates. This would necessitate a comprehensive over-view of mathematics. This would allow the beginning teacher to see the "woods" as well as the "trees" as regards courses of study.

- The greatest problems facing the "new" teacher are in the fields of discipline and "transfer of ideas". A meaningful course in sociology and psychology might enable one to recognize and understand the problems faced by some students more readily; perhaps methods dealing with "typical" problems could be presented.

- Discuss more philosophy of teaching and get a good general feeling of where you're going in the next 10-15 years. This carry-over lets teachers get a grasp on the future and not carry on with an uninteresting past tradition.

- We recognise now that we only learn well what we do and little of what we hear. We recognize also that students

of any age put forth maximum effort to solve problems which they meet. The effort is somewhat less if the problem is hypothetical.

Of the many and various topics mentioned in the questionnaire, some arose with considerable frequency. The emphasis on methodology as compared with content was considered to be of great importance. Many people suggested that a discussion or seminar approach was far superior to lectures. Other topics felt to be of importance were the study of audio-visual aids, modern techniques, the study of adolescents with emphasis on difficult and lower ability children and the 2-year and 4-year courses.

A training in how to establish discipline was mentioned by several teachers. This, however, was considered by the Committee to be a field which could not be taught, but was an attribute gained by experience in the classroom, personality and pure hard work.

The following comments are representative of many dealing with practice teaching.

- More time should be spent on actual classroom presentations with the student teacher remaining at the same school with the same class for a period of one month (without interuption).

- In-service training where a student teacher goes to one school and spends 3 months observing, teaching, preparing tests, working with teaching material and doing other remedial work.

- Prospective teachers should be assigned to different members of a particular staff - in their subject and in related subjects, for a longer period of time so that they are incorporated into the school. I'm sure that new teachers learn 90% of their skill from the competent personable, experienced teachers on the staff of their first school.

This is where good math teachers are developing. This is where potentially good teachers are lost.

- The student teacher should be more involved in the actual teaching of a class. He should not teach isolated topics but be responsible for leading a class through an entire topic and then testing same.

- The trainee should spend most of his practice teaching time in his primary subject with only one or two weeks in his second subject.

- Allow student to conduct a class with no other teacher present to handle discipline.

1.- The student teacher should be required to observe the very best teaching available in a normal classroom situation.
2.- The trainee should be required to teach some four-year material, and be made aware of the difficulties involved.

3.- The practice teaching time should be greatly extended. Dental trainees and medsmen spend all of their final year in practical work. The teacher should face a similar program. Perhaps the best area for this exposure is the four-year A & S program, where the maximum amount of baby-sitting and minimum of learning is taking place in Ontario's Secondary Schools, and where the course of study is open to experimentation.

- We emphasize doing and discovering in Math, yet it is all lecture at O.C.E., or was. More practice teaching, particularly to non-academic or four-year students. Very few of the O.C.E. studentd are prepared to work in our school which is 1/2 four year and 2/3 male population.

- Prospective teachers should have more exposure to teaching situations. The regular teacher, during practice teaching, should be more critical and helpful - most simply say that a good job was done. Students might be asked to give a confidential, criticism of the student teacher.

- Associates should try to evaluate student's teaching without imposing his own approach. Too many student teachers remark, "Use his method to get a good mark. In the classroom I am going to adopt my own."

The most frequent suggestions here were for more practice teaching, more freedom and responsibility for the student teacher during his practice teaching and more teaching to four-year classes.

Another large group of suggestions concerned the revision of the present College of Education set-up.

- I think teacher apprenticeship would be an excellent idea. College of Education courses are more meaningful after you have tried teaching. You understand the problems then and hence the solutions are more meaningful.

- More of an apprenticeship idea -- teach a light load (2-3 classes) during the year (in more than one school, if possible), then have lectures in the summer or in a co-operative type program. New teachers should not teach a full load. No other occupation expects full output at first, must train some on job.

- An internship period of 1 month (May) or more, followed by teaching for a year 1/2 to 3/4 timetable in an area where a centre can be set up to assist the trainee.

- Some sort of system whereby a prospective teacher is 'assigned' to an experienced teacher and given a very light load of teaching responsibilities but have lots of opportunity to observe and assist in the preparation and teaching of the lessons.

- I think that Waterloo with its new option in Co-operative Mathematics is wonderful.. Some students can leave university and step right into teaching, others need the practice teaching so I think that the course should be made more on an individual pattern. Let all prospective teachers take lectures for the first three, four or five months and write exams, then put them in the schools. The schools can gradually give them more and more responsibility, let the Head of the Department decide this. Also they should be paid as their responsibility increases.

- I believe in a program which takes students directly from university and places them in the Secondary School system. After some experience (3 months) they should then return to the College of Ed. to have their problems answered (3 months). Back to their school and apply these new methods (3 months). Return to the College of Ed. for final session (3 months). It seems to me that this method is very worthwhile and not too hard to implement.

- I am, in principle, in favour of the teacher training idea put forward by the University of Waterloo. I don't think that 8 months training is sufficient for the teaching profession.

- academic education and professional education should take place simultaneously.

- I have always felt that to be truly professional a teacher should have courses during his university training in Education, and that he should do some practice teaching along with it, and that this course should be rewarded with a degree at the end of 4 years, and with a teacher's certificate at the end of 5 or 6 years if need be. Other professions such as doctors and lawyers spend this amount of time and more. Why not a teacher? If so, respect, remuneration and all other ills would be alleviated. This summer school business or one winter course does not suffice to make a professional.

- I felt the practice teaching was the most valuable part of my year in the College of Education. For this reason I feel that perhaps the University of Waterloo's new co-operative program may be a very effective method of training teachers. Perhaps there should be a special summer course for graduates of this program, with emphasis on Psychology and History of Education, School Management and Law, etc. rather than methods. The present system of a full year's training in education however would remain a necessity for others.

- I suggest an eight month period of teacher training, six months in the schools and two months in the colleges of education on a rotating group basis.

Student teachers assigned to schools or better hired at a partial salary by schoolboards, would be under the supervision of the department head and good teachers with a timetable duplicating that of the supervisor. However, duties throughout the school would be assigned to student teachers, for example the resource centres, guidance departments and "free textbook" rooms, vice-principal attendance routines, etc. A student teacher would teach one class continuously from September lst. The teaching load increased gradually as the year continues and according to his efficiency to a maximum of probably two preparations and four periods per day.

The time spent at the college of education would set up the courses as outlined above, would gather the new methods used in the various schools, discuss and distribute them the work would be to a great extent on a personal assistance basis, rounding out each student teacher's knowledge of society. With the small number of students at one time this might be possible.

- They should have the opportunity to teach and sit in on regular classes in different parts of the province.

- The teacher-training program should be part of a course aimed at a teaching degree. It should be part of a University's curriculum with practice teaching in the final year.

- Extension courses leading to a master's degree whereby a teacher could gain further depth in his particular subject. More specifically -- a Master's degree in the teaching of Math.

- Most important - Colleges of Education must follow up their products after training completed, and act as "coaches" until certification is made permanent.

- Teacher training in Ontario needs to undergo radical changes and rapidly if it is going to claim any effectiveness at all.

Ideally Colleges of Education should be abolished. A student would then graduate from university and begin a one year apprenticeship at any high school. During this year he would receive a salary of $3500 to $4000 and would participate in every aspect of school teaching and administration, from helping a custodian change a fuse to assisting a principal in administration. He would start out and observe various teachers, the good and the not so good - then as he gained more experience, be given more and more teaching responsibilities, set marks, prepare report cards, consult with students - in short do everything that a teacher does in as close as possible a true situation. At the end of the year, the prospective teacher would be in an excellent position to decide if he wanted to

teach as a career. If not he could enter another field and feel that his apprenticeship year was not wasted. (This system would also have a tremendous effect on the improvement of present teachers who tend to fall into a rut.)

However I am not so naive to think my proposal will be even considered somewhat seriously. In its place I will offer a second suggestion.

The whole philosophy of teacher training needs to be revamped and updated. Colleges of Education must look to the future, not dwell in the past, as is currently the practice. Who cares about Plato, Socrates and Kant, etc. except for a very cursory discussion. The philosphy of education must turn to the ideas of today and the men of today and tomorrow. Analyze and predict the future trends - do not tear apart the past. Psychology courses should and must deal with to-day's student, his social and economic enviroment, and his ambitions, and this child should live in Ontario, not United States or Germany. The necessary information in Professional Practice and Administration could be read from a book and the lectures disbanded. A few interesting seminars of discussion periods could be enjoyed, but let's not overwork them. I am also convinced that the content of History of Education would make enjoyable and interesting light reading, but it is terrible when it must be studied!

As for a second teaching option, this, in most cases, is ludicrous. We live in a time of specialization. Gone is the teacher who is called upon to teach 5-7 subjects, and we must recognize this.

As I see it, students should spend the majority of their time in the schools and do the things teachers do. They must not be expected to waste their time in the artificial situation of our present practice teaching situation. These students could then return to the College of Education for a maximum of 2 months and consider the future of education in Ontario.

The most common comments suggested some form of internship plan, or indicated approval of some integrated academic and professional training at University, such as the University of Waterloo Cooperative plan.

The overall impression obtained from the replies was a plea for practicality rather than an academic and theoretical approach in teacher training programmes, that the art of teaching is learned in the classroom and in discussion, not in the lecture hall.

THE 1969 O.A.T.M. CONVENTION

by

W. D. Sokoluk
Pub. Rep. O.A.T.M.

The activities of the Ontario Association of Teachers of Mathematics were again climaxed this year at the 78th Annual Convention held March 16, 17 and 18 at the Skyline Hotel in Toronto. It was the unanimous opinion of those present that the program was one of the best ever and that the facilities of the Skyline were excellent.

Dr. E. Snapper, the keynote speaker from Dartmouth College, presented two outstanding lectures: The Euclidean Group and The Affine Group. Dr. Snapper instructed us in a precise and logical manner. We were challenged and enriched intellectually, and delighted by his sense of humour. The effect of his lectures will be felt in the classrooms of Ontario.

The convention was fortunate to procure the services of such outstanding men as Dr. J. Hogarth, Mr. W. W. Bates and Mr. A. Scroggie to report on topics of current interest in the province. Dr. Hogarth of Queen's University discussed the implications of the Hall-Dennis Report in school mathematics. Mr. Bates, Director of Mathematics in Toronto, led a stimulating session in which "A Balanced Program in Mathematics" was discussed. Mr. Scroggie's presentation "The Evolution of Curriculum", gave us an insight into the complexity of the development of curriculum and the interplay of all concerned in the learning process. He also reported on the present state of grade 11, five year curriculum changes.

Once again the success of the convention was largely due to the excellent work done in the seminar sessions in which all participate directly. Seminars were held in the following topics:

Making the 4-year Mathematics Interesting (Mr. W.Auglin)

Use of Television (Mr. F. Ebos)

Student Involvement Program (Mr. V. Mathewson)

After The Mathematics Workshop-What? (Mr. E. Pipher)

Desk Calculators (Mr. H. L. Ridge)

We were fortunate to share in the experiences of these outstanding teachers in the experimental programs.

The final presentation of the convention was made Dr. K. E. Iverson of I.B.M. Dr. Iverson gave a lively demonstration of a computer at work, having the I.B.M. computer connected to his console directly by telephone. He illustrated computer language adapted to school mathematics and the cost-share program proposed by his company. His use of closed circuit television in his presentation was an interesting teaching technique.

The Exhibitors this year were greater in number with larger and more colourful displays. The many books, calculators and assorted mathematical devices available to teachers were amply displayed and courteously attended by helpful representatives.

An area beside the Exhibitors was filled with entries (to the Math Fair) from all parts of the province. Comments that the Fair was better than most seen at N.C.T.M. conventions attest to the fact that it was one of the highlights of the convention.

On the lighter side of the convention the guests enjoyed the sociability of the Early Bird Reception the first night. An excellent dinner banquet was held the next night at which Dr. T. S. Webster of Queens University delighted his audience with a humourous talk about the modern student. Many guests enjoyed the dance later in the Cloud Room. On Tuesday the delegates appreciated the I.B.M. luncheon. Throughout the formal proceedings a special program for wives included activities such as swimming, shows and shopping excusions.

A reminder for next year:

The 1970 O.A.T.M. convention will be held in Ottawa at the Chateau Laurier Hotel during the winter break. This will be the first time the convention will be held outside Toronto so that we are anxious to make it an outstanding success. Make your plans now. See you in Ottawa!

PROBLEM NO. 9 (Professor Norman Miller)

The essential content of this theorem was known to and proved by Archimedes (287 - 212 B.C.) see Goussat-Hedrick, Mathematical Analysis, Vol. I p.134. As, however, it appears in few, if any, High School texts, it may interest teachers to supply a proof by methods that are less than 22 centuries old.

Tangents to a parabola at B and C meet at A. Prove that (a) the area bounded by the parabola and the chord BC is two thirds that of the triangle ABC, (b) the line joining the mid points of AB and AC is tangent to the parabola at the mid point of the line.

SOLUTIONS

Problem No. 7 (Vol. 7, No. 1, 1968, P.82). Prove the identity

$$\binom{n}{n} + \binom{n+1}{n} + \binom{n+2}{n} + \ldots + \binom{n-1}{n} = \binom{n}{n+1}$$

Solution by Mr. E. Evans

(a) Since the well known formula $\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$

can be readily proved, $\binom{n}{r} = \binom{n+1}{r+1} - \binom{n}{r+1}$

Hence, $\binom{n-1}{r} = \binom{n}{r+1} - \binom{n-1}{r+1}$

$$\binom{n-2}{r} = \binom{n-1}{r+1} - \binom{n-2}{r+1}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\binom{r+1}{r} = \binom{n-(n-r-1)}{r} = \binom{n(n-r)}{r+1} - \binom{n-(n-r-1)}{r+1} \text{ or } \binom{r+1}{r+1}$$

$$\binom{r}{r} = \binom{n-(n-r)}{r} = \binom{n-(n-r-1)}{r+1} \text{ Since } \binom{r}{r} = \binom{r+1}{r+1} = 1$$

Hence the sum of the terms on the left $= \binom{n}{r+1}$

Problems No. 7, by a second method

The number of choices can be more easily considered without loss of generality by taking a particular case of choosing a committee of 4 from 9 men, and later extending it to the general case of choosing r+1 out of n men.

Let us suppose that each man of the 9 has the privilege of accepting or declining appointments to the committee.

(a) If less than 4 men accept, there is no committee.

(b) If only 4 accept, we have our committee in 1 way, or $\binom{4}{4}$ ways.

(c) If, now, one of the men who had previously declined to serve is persuaded to accept, the number of possible committees is increased by the number of ways he can be on the committee, and the 3 others chosen from the 4 who have already accepted, i.e. in 1 x $\binom{4}{3}$ ways, thus giving a total of $\binom{4}{3} + \binom{4}{3}$ ways.

(d) Again, if another man who had previously declined, now accepts, the number of possible committees is increased by taking him and choosing the other 3 members from the 5 who have already accepted, or 1 x $\binom{5}{3}$ ways,
giving a total of $\binom{5}{3} + \binom{4}{3} + \binom{4}{4}$ ways.

The process can be continued until only one man declines to serve. If he now accepts, the number of committees is increased by 1 x $\binom{8}{3}$. But this exhausts all the possible committees of 4 out of the 9 men, or $\binom{9}{4}$ ways.

Therefore $\binom{9}{4} = \binom{8}{3} + \binom{7}{3} + \ldots\ldots + \binom{5}{3} + \binom{4}{3} + \binom{4}{4}$

The above argument applies equally to choosing r+1 men out of n men, thus

$$\binom{n}{r+1} = \binom{n-1}{r} + \binom{n-2}{r} + \ldots + \binom{r+2}{r} + \binom{r+1}{r} + \binom{r+1}{r+1}$$

But since $\binom{r+1}{r+1} = 1 = \binom{r}{r}$, we can replace the last term above by $\binom{r}{r}$ and the formula is proved.

BOOK REVIEW

Title: Contemporary Teaching of Secondary School Mathematics by Stephen S. Willoughby. Published by John Wiley & Sons, Inc. New York, 1967.

The author has stated his purpose in writing this book clearly and explicitly in the preface, a manner which sets the tone for the rest of the book. As he points out, Mr. Willoughby intended the book particularly for three groups of readers. The first group consists of those experienced teachers of mathematics who have been unable to keep up to date on recent developments in the mathematics curricula in secondary schools. One gets the feeling that he considers this to be the largest of his three audiences.

The second category of readers are the inexperienced, or relatively new teachers who have a strong, recent background in mathematics. Mr. Willoughby's intention is to relate the teaching of mathematics at the secondary level to the concepts of the pure mathematics courses these teachers have just finished.

The last (and in the author's hopes, least) audience are those new or prospective teachers who do not have an adequate background in mathematics. His objectives for this group are low - merely to show them "...where their major weaknesses are and what they must do to remedy their shortcomings."[1]

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[1] Page v: op. cit: Contemporary Teaching of Secondary School Mathematics by Stephen S. Willoughby John Wiley & Sons, Inc. New York, 1967.

Mr. Willoughby presents his information to these diverse groups by dividing the content into two main types of chapters. Firstly, there are a number of chapters on general topics related to the subject. (Chapters 1,2,3,15). Such considerations as a short review of the history of mathematics education in the United States, a definitive account of the reasons for the modern curriculum, and an excellent chapter on useful tips for the classroom are contained in this group of chapters.

The remaining chapters are devoted to specific areas of mathematics. Some examples are number and numeration systems, geometry, intuitive geometry, logic and sets, probability and statistics, and calculus.

The organization of the chapters is worthy of comment. All are written in the best traditions of a good textbook. Each of those in the former group closes with a list of suggestions for further study and questions for group discussion, while each in the latter group closes with a section on teaching hints specific to the topic and a list of exercises to give the interested reader some actual work with the mathematics being considered. Finally, all chapters contain a list of references pertinent to the chapter topic including Mr. Willoughby's personal comments on each. This allows each of the various readers to select references which are useful to his particular level of knowledge.

Undoubtedly the most useful chapter in the group on general topics is Chapter Three, entitled Teaching and Learning: Hints for the Classroom. This is not a complete "how-to-be-a-master-teacher-in-one-easy-lesson" approach but rather an almost annecdotal chapter loaded with nuggets of teaching hints which

would be useful, I suspect, to many experienced teachers as well as to the novice. The author's comments and advice I found to be remarkably practical, particularly in the section dealing with grading. It is refreshing to read a pedagogical text which recognizes the realties of student behavior as well as the time and numbers limitations on the teacher, and makes recommendations based on those realties. This section suggests, for example, that homework not be marked on a regular basis, since this task is time-consuming and often merely encourages copying. The author also presents a strong argument for using grading as a psychological tool particularly in the early part of the year. That is, assigning grades with a large measure of weighting in favour of such considerations as motivations of individual students, realizing the need to encourage weaker students who are making the effort to continue to do so. Along with these and other hints, the author provides a basis for assigning marks, not merely by grading tests and exams. He then concludes the section on grading in his usual practical style by pointing out the pitfalls of assigning grades and then allowing students to convince the teacher that their mark is too low. He suggests that having arrived at a grade, it is better to stick to it, unless a definite clerical error is involved. Otherwise the teacher merely ends up in long arguments with students and usually rewards complainers. Similarly, the section on discipline in this chapter is a realistic list of techniques applicable to the teaching of any subject. Each piece of advice is supported with a description of how the procedure is carried out in actual situations. One noteworthy hint is the usefulness of advance planning of

disciplined problems. That is, having thought out in advance how to handle any discipline problem that generally might arise, the teacher then can deal with a problem in the best way and not just "off the top of his head".

Two sections of this chapter stand out above the others. These are the section on the discovery approach and the last section headed simply People. The former presents a clear idea of exactly what the discovery approach is, what it isn't, when it should be used and when it should not be used. In a few pages, the author dispells many misconceptions about the technique. He also presents an actual lesson excellently taught in this manner by a gifted teacher. The knowledge "discovered" and material covered in this one lesson is remarkable.

The section on People is similarly enlightening. Having dealt with the people who are the teacher's main concern, namely the students, Mr. Willoughby now makes a few comments on the various other types of people with whom a teacher must deal in the course of his career. In doing so, he minces no words, pointing out exactly the sort of difficulties and assistance a teacher can expect to get from such individuals as parents, the P T A, principals, department heads, and his colleagues. In short, this chapter is one of the most useful treatments of this topic that I have encountered.

Defining which of the chapters in the second group is the most important or even the least important is impossible. This would depend entirely on the reader's viewpoint. Judged independently of this factor, each of the chapters is about the same. All contain not only explanations of some of the basics of the

topic area, but also some new (to the reviewer) problems and techniques which would provide a source of ideas for the classroom teaching of the subject. In each chapter the stress is on understanding the mathematical structure underlying the topic, and on possible approaches to take in the teaching of these topics. Each of these chapters also contains a series of exercises which are not only useful as learning work for the less knowledgable teacher, but also form in themselves a valuable storehouse of ideas and problems to use in the classroom.

As a reader who fits somewhere between Mr. Willoughby's second and third categories of audiences, the reviewer found this book very valuable for exactly the author's stated intentions. The material is well presented, in a very readable and at the same time mathematically precise language. Mr. Willoughby has succeeded in writing about a potentially dry subject in an enjoyable,and at times, even humourous manner without sacrificing accuracy. In fact the only errors I found in the book were not errors in Mr. Willoughby's mathematics but rather in his proofreading. While the typesetting is a very readable size and style, it does contain several typographic errors. Of these, I only noticed two which could possibly affect the reader's understanding, but only those readers who fall into group three.

Mr. Willoughby suggests that his book could be used as a text for a course in a teacher's college. I would not recommend that it replace the text currently used in McArthur. However, I strongly recommend that it be used as a supplementary text for the course. The usefulness of Contemporary Teaching of Secondary School Mathematics to students at both the Type A and Type B levels can

only be truly appreciated by the students themselves.

D. M. Borrowman
McArthur College of Education
Queen's University.

Title: Modern Mathematics, Vol. #1 by Georges Papy

Published 1963 by Marcel Didier, Brussels and Paris.

Published 1968 by Collier-MacMillan Canada Ltd.

Translator - Frank Gorner.

THE CONTEXT OF THE BOOK

Simply put, this book talks about sets, relations, numbers and groups. The book is aimed at the 12 - 13 year old youngsters and would be suitable for Ontario grade IX mathematics classes.

The first five chapters deal with introductory set theory. The notion of set itself along with "belonging to," "equality of," and "emptyness" are carefully brought out in the first chapter. Chapter two is entitled "Subsets" and includes properties of inclusion; chapter three deals with "intersection", "union" and "difference." Elementary work on the algebra of sets including ideas of associativity and distributivity are forthcoming in chapter four, while chapter five discusses partitions.

Chapter six introduces the first elements of geometry - points, lines, planes, parallelism and perpendicularity-in terms of the previously developed set theory. Euclid's geometry as much as takes the form of a set theoretical model instead of its classical approach as an independent theory.

Chapter seven marks the beginning of the work on relations. They are introduced simply using games and situations familiar to the students. These easy to follow relations lead to the more complex study of special types, $<, \subset$, etc. and cartesian products.

Chapter eight investigates relational properties such as reflexivity, antireflexivity, symmetry, antisymmetry and transitivity.

Chapter nine leads the students from familiar children, parents, grandparent relationships to the study of general compositions of relations. Equivalence relations and order relations are the topics of chapters ten and eleven respectively. Chapter eleven concludes with an introduction to convexity.

Chapter twelve discusses functions. Types of functions such as injective, surjective, bijective and composition are brought out in the study of this important type of relation. The logical extension of the function, the permutation, is introduced in chapter thirteen.

The next two chapters, fourteen and fifteen, investigate the nature of transformations and projections. Constant and identity transformations, parallel and coordinate projections make up chapter fourteen and parallel ordered pairs, work on half planes and Pasch's theorem constitute chapter fifteen.

Number theory begins in chapter sixteen with work on equipotence, natural numbers, infinite sets and order of the cardinals. Chapter seventeen discusses addition. It is studied under the topics of commutativity, associativity, identity element and order; chapter eighteen extends these topics to the definition of multiplication with distributivity and divisivility introduced as well.

Binary numbers (chapter nineteen) are studied in the light of the previous two chapters and the first arithmetic manipulations are begun by solving for binary numbers representing natural numbers and vice versa.

An interesting treatment of negative numbers is presented in chapter twenty. The binary numbers of chapter nineteen are utilized to form two armies in a battle of extermination. Properties, equations and subtraction in $\mathbb{Z}$ are studied next ending the chapter with a discussion of $\mathbb{Z}$, + as a ring.

Chapters twenty-one, twenty-two, and twenty-three introduces notions of equipollence, translations, vectors, and central symmetries. The work is consistent with its preceding chapters in the sence that it is for the most part theoretical in nature and not computation oriented.

The book's closing chapter is intitled simply "Groups". Identities, inverses, and cycles form the major part of the treatment which is the climax of the book.

DISCUSSION OF THE BOOK

It is easy to find nice things to say about Papy's book. I shall discuss them under three groupings, format, content and Papy diagrams as the last topic deserves a section of its own.

The first thing one notices when thumbing through this book is that some its pages are white and some are brown. It is easy therefore to distinguish the exercises from the theory as they are written on the brown paper, but even more important it is easy to distinguish the theory from the exercises.
Immediately one has the feeling that the author is on the students' side helping him to organize his work.

Paragraphs are separated; a simple device, yet very effective in separating thoughts. Definitions are in dark black rectangles and stand out smartly. Dark lettering is used to emphasize the main points

of sentences and helps the non-English major understand how to read the sentence. (e.g. page two-instead of saying that an object is an element of a set, we say that the object belongs to the set.) The words "an element of a set" and"belongs" jump out at you and you are sure to gether the correct inference. Italics are used to distinguish propositions but are also teamed up with dark lettering, capitals and rectangular "blocking" for special effects. Note the impact of this statement found on page 156:

DEFINITION - Every relation which is reflexive, transitive,and symmetric at the same time is called an EQUIVALENCE RELATION.

Linked with the impressive list of useful format conventions above is what I consider to be a super student -helper. The following appears at the top of page 156:

156 EQUIVALENCE RELATIONS [CH.10

On page 157 one reads:

§2] PARTITION DEFINED BY A RELATION 157

How many times have I and other mathematics students searched through textbooks and felt that they were alien and even hostile because (a) the chapters weren't named or (b) the chapter numbers weren't printed or (c) the chapter divisions were not printed or (d) the pages were numbered at the bottom. Every Mathematics text should be as helpful as this text is in helping students to use it.

How lovely reading mathematics can be if diagrams and subsequent discussion are facing the student simultaneously. How annoying and hostile of a text to expect students to keep turning back the page every step in an argument to verify that step in the diagram.

Papy has taken great pains to make mathematics readable. Where long arguments must go over the page he has often resupplied the appropriate diagram. Pages always end with complete thought paragraphs and exercise questions never extend even to the opposite facing page.

Even if Papy's content were inconsistent, improperly structured and riddled with atrocious mathematical error (which it certainly is not) his book is a significant contribution to mathematics in that he has shown how a mathematics textbook should be organized - a format with maximum psychological appeal.

CONTENT AND PEDAGOGY

Now that the student has a textbook that he can control it would be helpful if the content were well structured. As you might expect by now, here too Papy doesn't let us down.

One of his teaching devices is an interesting recurrence of themes. For example, "reflexivity" is defined for formal usage on page 126. By this time however it is not an unfamiliar word as the student met this idea at least informally on page four and again on page twenty-one. Properties of $\mathbb{N}$ and $\mathbb{Z}$ are pretty extensive when viewed for the first time but by the time these properties are studied the student has long since mastered concepts such as commutativity, distributivity, and associativity.

As an extra, Papy has added a one page bibliography of Cantor, Euclid, Cayley, Galois (mynemesis) and Liebniz, each with the particular section of work for which he has been credited. Mathematics is not exclusive of history and is in fact an integral part of it. It is for the betterment of the student's personality if he can integrate

mathematics with all of his other subjects and vice-versa.

The exercises are plentiful, colourful, relevant and well structured. They are well laid out (format) and vary from fill-in answers, use the above example as an analogy for solving the following, to straight-forward example explanations. On the whole these exercises take on the aspect of programmed learning and as such have the built-in good qualities and vices that naturally come from this type of learning process.

Fortunately the worst vice of nostalgically slow learning steps does not appear.

Papy has a gift for easing students from their reality into the abstract. Concepts such as infinity, loci and relations are introduced with diagrams, games - fun! Examples are inevitably picked from the students' enviroment.

Deductive reasoning is a type of abstraction which demands reasonably high levels of mathematical maturity. How nice it would be if students could be eased into it. Papy has found a middle step in the form of the mathematical film. Oh no! Not a movie, but an "imagninary" film that the student can make for himself to aid his thinking and from which he can obtain his deductions. Note the "proof" of proposition 10 (page 369).

PAPY DIAGRAMS

So far I have presented this as merely an excellent mathematics book. I am ready now to present the device which makes this book brilliant. It is the "Papy diagram" - an incomparable aid in the understanding of sets. It is a concrete model which the student

can use to study the nature of sets. It is a vehicle with the use of which the student can study abstract algebraic systems.

As a model it is colourful (i.e. note the seven different colours used in the Papy diagram on page twenty-eight.)

It is a simple model that in this book is represented in large diagrams (i.e. note how the Papy diagram on page 35 all but fills the page.)

On page 131 Papy has used a diagram to make a denotative definition of antisymmetry.

Humour is his purpose on page 136 when a diagram looks suspiciously like a person's face.

His diagrams are in a very subtle sense artistically sound. For example, on page 148 the composition of vector "yellow" with vector "blue" yields vector "green". Any artist mixing his paints will tell you that and so the diagrams become even a little bit more interesting to someone to whom before math was a drag. Subtleties such as these make Papy diagrams tremendous eye-catchers and terrific interest arousers.

Disregarding the psychological merit of his diagrams, the non-associativity of set difference, not readily apparent in abstract form, is child's play in one 4" x 5" Papy diagram on page 35.

SOME SHORTCOMINGS

Papy's book does have a few shortcomings but most are either unavoidable or not too serious.

For instance, the book, mostly for reasons of excellent format, may seem a little large to some students - after all it is 460 pages, and its weight and awkwardness may be reasons for some

less mathematically oriented students to leave this text in their school locker instead of taking it home.

A more serious criticism is that the book contains no exercise answers thereby placing a greater load on the teacher and introducing uncertainty to the student. With the type of questions asked however it would surely take another 100 to 150 pages to supply all of the answers.

Some arguments against the book for lack of computational skill development can probably be justified but this was not the author's intent.

Problems in translation inevitably occur when the original work is written in a foreign tongue and girls' names like Michelle, cars like Peugots, Citroens, and expressions such as "has a father" may seem a little strange and uninviting when first encountered. I suppose these things could lead to a small link between Math and French should the mathematics teacher be cultured enough to want to draw attention to this link.

It can be criticized as well that the study of natural numbers and integers should logically lead to study of reals.

The most distressing thing that I found about the book was its rather strict copyright law. I quote "All rights reserved. No part of this book may be reproduced or utilized in any form or by any means, electronic or mechanical, including photocopying, recording or by any information storage and retrieval system, without permission in writing from the Publisher." Not to be able to plagiarize such a beautiful book is indeed a crime!

CONCULSION

In conclusion I am very impressed with Papy's book "Modern Mathematics". It is modern in every sense of the word. The Papy diagram has made it a revolutionary approach to mathematics teaching. It is a book that every mathematics teacher should read and every school board should seriously consider.

Leonard Minty
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ANNOUNCEMENTS

AN APOLOGY

Did you have difficulty reading the article on Convexity in the last issue of the Gazette? Inadvertently the paging of pages 90 - 92 was involved in a 1 - 1 transformation whereby the correct Page 90 was displaced two pages, and appeared as Page 92.

Please accept our apologies.

MICHIGAN COUNCIL OF TEACHERS OF MATHEMATICS

is holding a Name-of-Site Meeting in Kalamazoo, Michigan, August 11-13, 1969. Well known speakers such as Phillip S. Jones, John Wagner and Frank Allen will be featured and special tours are available to conference participants. You are invited to plan your summer vacation round this event in "The Water Wonderland". For further information, write to

Mr. Charles Zoet,
380 W. Liberty,
Plymouth, Michigan 48170.