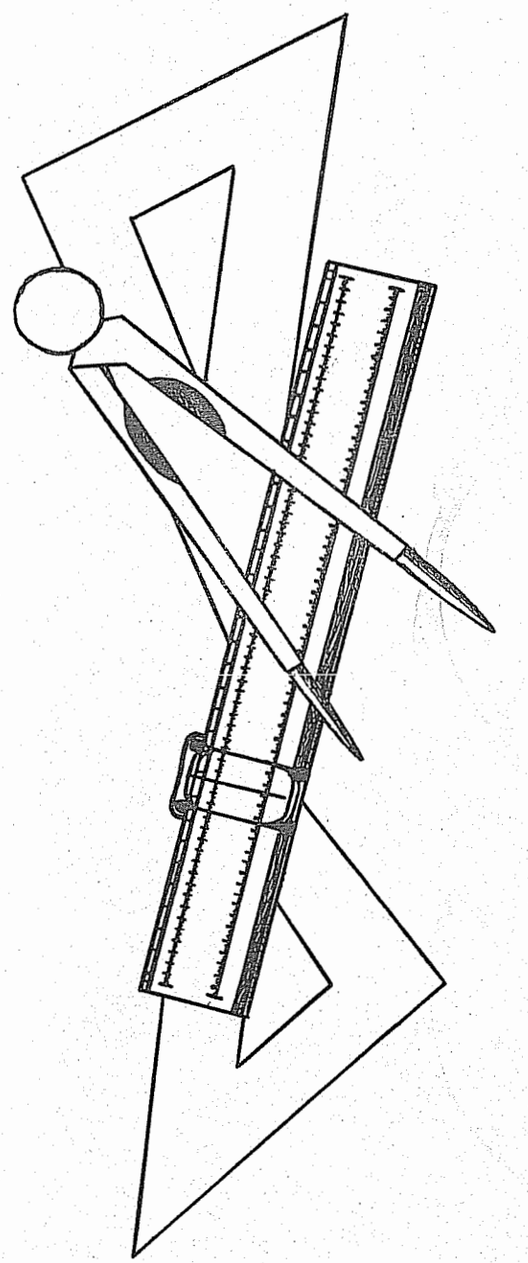


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SCHOOL MATHEMATICS IN THE SEVENTIES*

Professor Ralph Staal+, University of Waterloo

The title of this talk suggests looking down the road toward the future. The first thing I see, where mathematics education is concerned is that we are moving very fast down the road, and the second thing I see is a fork in the road a very short way ahead. There is little doubt in my mind which way is better, but there is very much doubt as to which way we will go.

The Meta-issue

In characteristic mathematical fashion, I would like to begin, not with an issue in the usual sense, but with a meta - issue. This is an issue beyond or above the issues of education proper. It may seem like a rather vague way to begin, and not very down-to-earth, but the more I see of current educational disputes the more I am convinced that this meta - issue simply has to be settled at least to some degree if a large part of the good work and hard thinking that is being done is not to go down the drain. In this sense it is a very down-to-earch matter indeed.

The meta - issue is -- at what level are we to think about the ordinary issues of education? How high or low a resolving power are we to bring to bear on these problems: This is the fork in the road.

During the past few years a great deal has been said and written about educational reform, and I have been impressed and surprised by the extent to which articles on education are divided into two levels of sophistication, with very little appearing in the grey area in between. That there should be differences is not at all surprising, but the sharpness and clarity of the

* A lecture for faculty and students given March 3, 1970 at Althouse College of Education.

+ Past president of the Ontario Mathematics Commission.

differences is, and therein lies a most serious situation.

Educational problems are real-life problems, and hence they are not only difficult, but so difficult, involving a great many parameters, that we are forced to use a well-known procedure of scientific method in our attempts to solve them. We have to replace the real problems by approximating problems: we try to make these sufficiently simple that we can solve them and then we hope that their solutions will approximately solve the real problems. There is of course a great temptation in all this, and some educational commentators have fallen right into it. If one doesn't quite known what one is doing, or has ulterior motives, the result is often a grand oversimplification - the real problems are replaced by problems so simple that a child could solve them, and a solution is triumphantly announced. But, alas! -- the real-life problem is forgotten.

Before going on with this point let me briefly comment on a level so low that until a short time ago I din't realize it existed. It may not be significant, but the mere fact that it seems to exist at all is disturbing. I am tempted to call it the level of the educational psychopath, but this is rather too strong a description: it does however indicate the direction of the tendency which I am describing. If you suspect, as you should, that I am over-reacting, please reserve your judgment until you have heard the following example.

In the teaching of mathematics to very young children there is a well-known classroom experiment, a very rough description of which is as follows. A tall, thin jar and a short, fat jar of equal cubic content are shown to the class. The contents are poured back and forth without comment, but in such a way that the children see the contents of the one full jar exactly filling the other jar. They are then asked "Which jar holds more?", or words to that effect. What is interesting is the extent to which children below a

certain age level tend to say that the tall jar holds more. The conclusion is drawn that below that age level, children tend to confuse linear measurement with volume measurement. Now, like all experiments of this kind, the drawing of the inference is fraught with statistical and other perils: one could disapprove of the manner of a particular performance, or of the words used, one could claim that the reasons for a child's answer might be quite different from what we have in mind, and so on. This sort of criticism one expects - it is part of the game. But now listen to this. The experiment is criticised because it "enables the teacher to be pompous and to demean the pupil". I leave this with you without further comment for the leisurely consideration which it requires.

To return to the two levels of sophistication already mentioned, there is naturally some variation within each, but the lower can be fairly well characterized by describing it as the level of simplistic arguments, of black-and-white analyses, of crudely stereotyped dramatis personae, of issues described in terms which beg the fundamental question which is at issue, and of melodramatic rather than thoughtful presentation. Having said this much, the upper level could be described simply as that which is complementary to the lower: it could perhaps better be described as the level at which the real-life problems are recognized in sufficient fullness of complexity to strongly tax our abilities without a solution being out of the question.

The Hall-Dennis Report

Inevitably we come to consider the position of the Hall-Dennis report, not so much because of what it is, as because of the ways in which it is being used. It is a sufficiently extensive report for an overall evaluation to have little meaning: rather it should be evaluated section-by-section

or aspect-by-aspect. It has been described as "a mixture of sound and wooly thinking", as a "sermon rather than a report" as well as "an inspired document." My own view is that it is a rather confusing mixture of good and bad which is very likely to be used very badly. It contains many sentiments with which I sympathize deeply, and parts of it are thoughtfully written. But it succeeds somehow in putting its worst foot forward, with its best qualities appearing only to the persistent and careful reader, and this is a feature to which more care should have been given. Because of this, and in spite of certain redeeming features, I would place it definitely at the lower of the two levels I have in mind. On the surface, and this is an essential aspect of any report, Living and Learning presents the problem of how to improve our educational system by asking us if we want an authoritarian, regimented, fact-stuffing system which ignores the needs of the individual child. This is a crudely simplistic Aunt Sally approach from which it never entirely succeeds in escaping, and which to my dismay seems to be exactly suited to the tastes of many educational commentators - it makes the problems so easy to solve!

To make the point by contrast, I suggest comparison with publications such as the K-13 Geometry Report put out by OISE, and with the celebrated essays on the aims of education by Whitehead. Perhaps it isn't fair to compare with Whitehead, but on the other hand the resources available to the Hall-Dennis committee were substantial.

So much for the meta-issue. If it isn't settled in favour of the higher level (and the odds at the moment aren't too cheering), what most of us understand by mathematics education may be in for hard times. But if the die falls the other way, the prospects are bright beyond precedent. Three and two with the bases loaded!

Summary of Recent Developments

Before singling out any particular issues, let us briefly review developments in mathematics education in the 1960's: these provide the base from which we will have to work. For most people the name "New Math" comes immediately to mind. This name has by now been used in so many ways that it has little meaning. In its most superficial sense, and this is unfortunately its most widely known sense, examplified by books with titles such as "New Math for Parents" it refers to a copious and, in the opinion of many mathematicians, pretentious, use of the language of sets, and to heavy emphasis on a number of rather special topics such as the use of bases other than ten and the distinction between numbers and numerals. Emphasis has happily been put on "teaching for understanding, rather than by rote," but in many cases this has been marred by the absurd pretence (perhaps the publisher's fault) that teaching for understanding is a newly discovered principle. In a deeper sense, New Math is an effort to bring mathematics teaching up to date by including at least some of the insights gained during the past hundred and fifty years: this is a most laudable aim, as yet only partly achieved, in comparison with what might be done. I submit that by far the most valuable contribution of New Math has been its side-effects. In order to bring it about, the entire school mathematics scene had to be wakened from a very deep slumber - a large part of the credit for this goes to the Ontario Mathematics Commission - and this proved to be a large undertaking. The result ultimately has been an unprecedented receptiveness to new ideas which is surely the major achievement of the 1960's, and one which was substantially realized before the publication of Living and Learning.

Much more has been happening than this, however. At the primary level especially, instruction has been livened up by varieties of the "discovery" approach - this is by no means a new method but the degree of emphasis being put on it is quite new. A real effort is being made to find out how small children learn, and to provide a rich mathematical experience for them in the materials of everyday living, presented in an informal and appealing manner. It has also been discovered that a number of topics formerly reserved for high school are actually, when presented with sufficient understanding and imagination, well within the grasp of many elementary school pupils. I am not referring here to name-dropping exercises where the student shows some familiarity with the vocabulary of "groups" and "ordered pairs", but to such substantial achievements as being able to set up and solve equations.

It is very hard to describe in general terms what has happened in high school mathematics, and I shall only mention a few features. The initial thrust in the early sixties seems to have been largely foundational, or anatomical. The language of sets, and of variables and "open sentences" was brought down from part of university mathematics, and, according to some critics, with too little regard for what is really was used for in the universities and for the change in environment involved in bringing it down. A large part of the initial enthusiasm for this sort of thing has apparently worn off with experience, and the emphasis has shifted to content which is substantial in the sense that it involves theorems and applications. Examples are vectors, complex numbers, the use of matrices, and Calculus. A good deal of pointless virtuosity has been thrown out, such as that connected with the solving of triangles.

There has, however, been a complication which requires a much closer look than we have time for for a satisfactory evaluation. The Department of Education has gradually loosened up requirements and, on the whole, the very noticeable increase in the material available for the brighter students has in some directions been accompanied by a decrease in what is required in material coupled with level of achievement. The experience of the universities and of those administering mathematics competitions has not been at all good. At the University of Waterloo we have had to make available remedial classes for students whose marks, according to regulations established ten years ago, qualify them for honours work. Causes are sometimes buried rather deeply, and we mustn't jump to conclusions, but we certainly have cause for concern. Since the abolition of province-wide Grade 13 examinations, universities have had to face the dim prospect of either ignoring a student's marks at school and relying entirely on a few special entrance tests, or weighting somehow the marks, or other evaluations, coming from each school.

One of the brightest features of the current scene in mathematics is the amount (not necessarily the proportion!) of first-rate literature available. Many eminent mathematicians who twenty years ago would have looked down their noses at writing for the schools have contributed. One of the best examples is the Random House New Mathematics Library. The bright student going to a good school with well-qualified mathematics teachers has never been as well off in this regard.

If this all sounds like a rather confusing mixture of both bright and dim prospects, then it is in that respect an accurate picture. It is precisely this mixed quality which we must recognize if we are to

make the most of the 1970's.

A few specific issues seem to emerge out of all this as natural and useful focal points for discussion and decision. Let us consider just three of them: the issue of child-centred education, the issue of freedom versus discipline, and the issue of the discovery approach to knowledge versus the organized approach.

Child-Centred Education

The low-level Hall-Dennis enthusiast misses entirely the point of what we call "the issue of child-centred education." He conjures up a melodramatic picture of a not too intelligent teacher who is not really interested in children for their own sakes and who sees them as so many sausage-skins ready for stuffing on a mass-production assembly line. We are supposed to react in horror to this Aunt Sally - as I'm sure we do - and cling tightly to the first antidote which is presented to us, namely his. Now, teachers of the kind described unfortunately have existed, and still do, and we should deplore this fact: but what we are proffered is the result of nearsighted and amateurish oversteering resulting from an oversimplified view of the real problem. Certainly the needs of the individual child are our primary concern, and certainly the child's feelings and imagination must be understood, as well as our capabilities permit, in order for teaching to be really effective. But the issue arises from the fact that there is more to it than that: the child is not alone; he lives in a physical world with which he must come to terms; he is part of a society consisting of individuals whose needs and desires are not all the same as his; and it will be enormously to his disadvantage if "child-centred" education encourages him to become self-centred, and if we are so afraid

of imposing upon him that he is denied in large measure the opportunity of profiting from the experience of his elders. The "Well, what do you think?" approach to teaching has great value, and we need more of it, but it isn't enough. The description "child-centred" is unfortunate: what we want is better described as child-benefitting education, the main ingredient of which is concern for the individual child - but this shows itself in many ways besides child-centredness.

Freedom versus Discipline

The issue of freedom versus discipline seems to be especially lively at this time. A healthy reaction to pointless and unproductive rigidity has, through low-level thinking, or thoughtlessness, been allowed to develop into an over-reaction with at times absurd consequences. Authoritarianism has become, not an obstacle, but a bogey, and its equally objectionable opposite has become a fad. The latest example to haunt us is the "swing-ing library." Most libraries built or redecorated in the past twenty years, have been bright, comfortable and serviceable units staffed very largely by attractive young ladies: this has been especially true of the childrens' sections, equipped with such things as oversized stuffed animals and featuring story-telling hours and other entertainments. According to a recent newspaper article, this apparently isn't enough. A very grim picture is painted of a hatchet-faced elderly librarian who delights in admonishing children, working in a dim and dusty vault filled with unread books, and we are told by insinuation that the only alternative to this is a noisy library, with happy children playing merrily (anyone who objects to this is a real creep) and with copying machines that go "beep" (here I am quoting). O tempora!

Whitehead wrote a beautiful and timeless article on "the alternating

rhythm of freedom and discipline". It is deplorable that so many commentators on the problems of education have apparently not heard of Whitehead, one of the great educators, philosophers and logicians of this century. It is dumbfounding that it should apparently be necessary to remind people in positions of responsibility that freedom and discipline are inextricable - the warp and the woof of education. To us here today it is such an obvious fact that it seems rather odd to state it: perhaps therein lies the danger. Even when it is fully recognized, the problem remains of exactly how to weave the two together in a particular subject such as mathematics. There is also the problem of getting the idea across to fairly young children. I have found the example of the skiier very helpful in doing this. If he has a good technique (which takes lots and lots of practice) he can ski almost anywhere - he is <u>free</u>. Otherwise he has to stay on the baby hills and can only go straight down. On the freedom side there is motivation, the background of experience, and the development of curiosity and the imagination. Discipline includes reflection, correction and the skills and organization of knowledge needed for free play at a higher level.

Here we come to some quite specific issues in the teaching of mathematics. When I was in High School in the thirties, techniques were overemphasized: there was quite a lot of pure junk in the curriculum, and I was glad to see it thrown out, but the throwing-out has tended to become hard to stop rather like the Sorcerer's Apprentice in reverse. There really aren't very many basic formulas or techniques in school mathematics. (The situation is unfortunately often made to look worse than it is by a current preoccupation with formal definitions and special terminology when

simple English will do (perhaps English is no longer regarded as simple!): we seem to have brought in a certain amount of junk inadvertently by the back door.) As far as the important techniques are concerned, I feel that sympathy for the complaining type of student is misplaced: he may have legitimate grievances, but this is not one of them. I also disagree with those teachers who feel that "it doesn't matter as long as they can figure it out." This is an important factor where techniques of secondary importance are involved, but if it were applied unthinkingly we would all be counting on our toes. Freedom to enjoy and appreciate mathematics comes only when the really basic techniques are gotten out of the way, and the only way to do this is to learn them so well that they become obedient and inconspicuous servants. In order to do this it is not - repeat, not - necessary to make technique the end of teaching: this would be to fall right back into the trap. Here we must face up to the meta-issue which was mentioned at the start. Are we willing and able to see mathematical technique as being very important without being all-important? This is a more sophisticated attitude to take than what appears to be currently popular. I have been able to peruse a good number of mathematics competition papers marked at our university: these were written by the <u>best</u> students in the province, and I can say with confidence that simple technique is becoming more of a problem than before.

Another aspect of freedom is the freedom to study what one wishes and when, and the freedom of individual school boards, schools and teachers to deal with curricular matters as they see fit. We are just now entering a period when the role of the epartment of ducation will be largely advisory, rather than legislative in this regard. Just as restriction to driving on one side of the road (it doesn't really matter which) results in

smoother traffic flow and hence on the whole increases our freedom to travel where we want, so some restrictions on freedom of choice will be necessary if students are not to end up in a mathematical tangle. From what some people are saying one would think that the issue was purely political, concerned exclusively with human rights and freedom from regimentation: actually, in the case of mathematical learning, it is mathematics. Mathematics is a structured subject, with great degrees of dependence of one topic on another. It is by no means totally ordered, but it is strongly partially ordered, and the explorer who barges ahead enthusiastically with little forethought or guidance by someone who knows the route, will almost certainly have to return to base for re-outfitting at most inconvenient times. Given an abundant supply of tutors, the necessary amount of guidance and control could no doubt be provided in a purely informal way with a minimum of regulations, but another boundary condition on the whole business of this kind of freedom is that the present student-teacher ratio is far too high for this kind of system, and is unlikely to change very much. Even if tutoring is at hand, the tailor-made curriculum needs very careful handling if studying is not to become a very lonely occupation. There are many ingredients here which need the most careful kind of balancing and far more perceptive analyses then many educational reformers are apparently able to provide. May I commend to your attention, for further comments along these lines, the excellent article by Professor Duff of the University of Toronto, appearing in the November 1969 issue of the Ontario Mathematics Gazette (Vol. 8, # 1). The Ontario Mathematics Commission will certainly continue to give serious consideration to the problems of curriculum design in

mathematics - I would hope in the direction he has suggested.

Discovery versus Organization

Finally I come to the issue of discovery versus organization in the acquisition of knowledge. For guidance we turn once again to Whitehead: this time to the pungent remark that "knowledge doesn't keep any better than fish". To be understood and even to be satisfactorily preserved, knowledge must be kept alive. This protest against dead knowledge is a theme to which Whitehead returns time and time again. It has many corollaries, like all major theorems. One of them is the importance of learning by discovery. Another is the deadliness of the merely systematic organization and dissemination of knowledge, without seeing it organically alive in its natural habitat. This is of course a protest against perhaps the most common form of bad teaching, so justifiably attacked by recent reformers. But the remedy for mere organization is not mere disorganization, which is about all that some reformers have to offer us, in the guise of so-called "non-linear" learning.

Whitehead looks much deeper into the matter than this, and summarizes his findings in a threefold pattern of Romance, Precision and Generalization especially where mathematics is concerned. In some ways this can be regarded as a refinement of his views about freedom and discipline. Learning in mathematics begins, not with definitions, but with something like free play - an intuitive and creative exploration with the concepts already developed. This is the stage of Romance: it is the stage where curiosity, the chief motive power for study, is stimulated and the imagination aroused.

Premature precision, that is, pedantry, would kill it. But sooner or later it bogs down and becomes unwieldy, with vaguely con-

ceived entities tangling confusedly with one another. Then is the time for Precision to enter, tidy things up, clarify and give meaning to what has taken place: it is rather closely analogous to weeding and pruning a quickly growing garden.

In mathematics the next stage - Generalization - is especially rewarding. Precision enables one to see clearly the lines which have developed, and one can now extrapolate from them: in other words, generalize. This is where one experiences most strongly the sense of deepened understanding and of the power of mathematical reasoning. The cycle can now repeat itself at a higher level, or seek out new directions at the original level.

None of this is intended to be an infallible or inviolate directive for progress in mathematics, but is abundantly illustrated in the history of the development of mathematical ideas. For example, complex numbers arose, not - repeat, not by consideration of ordered pairs operated on in a certain rather peculiar way, but almost accidentally in attempts to solve real cubic equations. The so-called "imaginary" so-called "number" i was manipulated long before it was given any meaning, and peculiar properties came to the surface, such as the existence of two complex numbers, each of which is the square of the other, and the usefulness of complex points of intersection of non-intersecting circles. The story of the domestication of complex numbers is a fascinating and instructive one, and it is a pity that it is often simply bypassed by beginning the study of complex numbers at the Precision stage with a definition. The Precision stage is an interesting stage in itself, for several definitions arise: the final stage, in most treatments, which describes complex numbers as transformations of the plane leads to a

powerful generalization in the notion of a linear transformation - a notion which runs through virtually all of modern mathematics in one form or another.

Once again, the real-life problem is one of balancing or blending several necessary ingredients. The simple-minded version of it which is being hawked about is a one-ingredient recipe which, of course, anyone can make: to buy it is to take the one path at the fork in the road, to follow Whitehead is to take the other.

Summary

In summary, it seems to me that the seventies will be a period of unprecedented opportunities in mathematics education. The prevailing attitude toward change is receptive - sometimes rather uncritically receptive, in fact. A larger-than-ever portion of the total mathematical community is taking an interest in school mathematics, and no end of resource material is at hand - much of it of very high quality. We will never have a better opportunity for improving our answers to the problem of how best to implement our concern for the individual child, with the balance of ingredients that this involves; how to make the most of the complementary relationship, rather than contradiction, between freedom and discipline; and how to find the most effective combination of free exploration, precision and generalization. First, however, we must pass the test of temptation - the temptation of disposing of these quite difficult problems by oversimplifying them. I hope that as individual teachers, as well as through our professional organizations, we can encourage and promote an adequate and perceptive response to these issues.

ADDRESS PRESENTED AT THE SEMI-ANNUAL MEETING OF THE

ONTARIO MATHEMATICS COMMISSION

Dr. M. K. Straka, Ontario Department of Education

Dr. Crawford, Ladies and Gentlemen:

Thank you for your kind words of introduction. I am afraid I will have to disappoint you. I am in no position to deliver thirty minutes of original thinking; as a matter of fact, most of the problems I am going to mention have been acted upon by the Mathematics Commission, so that all I need to do is to condense and organize some material. Furthermore I am now standing between you and lunch and I think I had better deliver in the time that I was assigned.

To start with, I thought it might be interesting to you to hear what is going on right now at the Colleges. It is well-known that we have twenty Colleges of Applied Arts and Technology, and in addition to that the Ryerson Polytechnical Institute, which is legally not a college but for practical purposes could be looked upon as such. Whenever the colleges discuss some problems or curriculum, Ryerson is usually included and we are on the best terms with them. Now, these twenty colleges are operated on 50 campuses, from Atikoken to places like Lindsay, Peterborough, Pembroke and so on. A staff of about 3,000 teachers works with 30,000 regular day school students and another 30,000 extension students. Right now our enrollment of regular students represent 30% of the Grade 12 output. In a few years we do hope indeed to come to a point where 60%, 70%, or 75% of the total Grade 12 graduates will attend post-secondary colleges.

The range of education we offer is also tremendously large. We have officially 144 technology programs, 64 applied arts programs and 47 business administration programs. If you look at this inofficially, considering the various titles by which the colleges themselves call these programs, you will end up with many more. So we have a total of 255 being offered in this year of study and probably next year the number will be closer to 300.

Our real concern are the problem areas in Mathematics that we have at the Colleges. As I said before, the problems that I am going to mention are neither unknown nor new. I have tried to arrange them reasonably in order of importance.

The largest problem by far that faces the colleges is one that I may call "closing the gap". Now if you will bear with me a few minutes of well-known history of Mathematics developing in Ontario: first of all we had the advent of the "New Mathematics" which shocked everybody, because on the high school level the teachers had to swing around complete and learn the "new approach". At the Institutes of Technology that existe at that time, there was almost near panic because the trend to the purely theoretic mathematics did certainly not serve the ends of the Institutes. After a while this machinery got oiled and running. Then overnight twenty new colleges were created. This again was a little bit of a shock to the high school system because the requirements of the colleges certainl did not match the high school mathematics at this particular time. Over the last two or three years I think you have noticed a slow trend away from the purely theoretical mathematics to a little more of the applied mathematics, that is more useful to the colleges than the general ideas of structure, set theory and so on.

Now among all these changes, I am afraid, the real victim was the student, because there was a tremendous gap at any time between post-secondary and secondary education. The two were always somehow out of phase, mostly 180 degrees. The proper matching represents our major problem today and the consequences are far-reaching: for example, the colleges are forced to introduce anything from 2 to 6 weeks of upgrading sessions in the summer time to account for the mismatch between secondary output and college admission requirements.

Here, I think, like no other place, is a tremendous field of application for the Ontario Mathematics Commission. Here we are, a multi-layer body, covering the whole range of mathematics education, and we have the connections and the means to eliminate the mismatch in this particular area. I know that the concern for this problem has been with us for some time. The Mathematics Commission has immediately responded to this challenge. The fact that there was a committee for the Colleges of Applied Arts and Technology instituted almost as soon as the Colleges were created, bears witness that the Mathematics Commission was, for one, a body which never was 180 degress out of phase, but practically on top of the events. I would like to take this opportunity to congratulate the Presidents and the Executives of this body for providing the leadership instantaneously when it was needed.

But we are still in another sorry state of affairs because the problem is of such immensity. One thing I would like to see is the type of thinking one might call K-15. Excellent reports are available for the span K-13, but there it ends. We are still inclined to assume that everything stops after the secondary level, and whatever comes after that is purely coincidental. This K-15 thinking is one thing I believe the

Commission can introduce into the mind of all our colleagues on the high schools and the colleges.

Secondly, the Ontario Mathematics Commission is in the best positio to advise both partners, as it has been doing for the last few years. W have the Secondary Committee as well as the Colleges of Applied Arts and Technology Committee. These two have been advising both levels on the most recent changes, but with our limited means we could only achieve limited success in the face of an almost overwhelming problem.

Another thing I believe that the Mathematics Commission can do, and as a matter of fact again has already done - I believe that Dr. Crawford will bring this up a few minutes later - is to bring the teachers of thes two levels together. It is one thing for us to sit in here and have five people representing the Colleges and five people representing the seconda system, but there are hundreds of teachers out this way and thousands of teachers out that way. They are the ones that have to bring about the change, they are the people that are dealing with the students, they are on the frontier, and there it is, where we have to concentrate our efforts on the student level. How often does it happen that a student does not like Mathematics. He drops it in Grade 9 and certainly at this time he has no conception how he will be penalized for that act. If, at a later date, he wishes to embark on a technical career - he is out! There is the possibility that every single math teacher, and in the long run every sing high school teacher, can provide guidance for the student. This is most important or we shall face some real critical times. I shall come back to this problem a little later.

Another problem area that we face is the area of business mathematics I really hate to use that term because whenever you say "business mathematics" you think of "commercial mathematics" and commercial means no

* see the article by James Walker in this issue

mathematics (-with due apologies to my colleagues in this particular stream). But whatever has been called mathematics of finance, - interest, compound interest, annuities and so on - was a nice little package that is rather obsolete in today's business world. In the age of the computer who has ever to establish a payment schedule for a finance company from tables? - Furthermore the requirements of the modern businessman are completely different. Open the Financial Post today and you think you are looking at the Ontario Mathematics Gazette: graphs, quotations and figures. Pity the poor fellow who has no conception of mathematics! He is hopelessly lost. Modern economics is another field that is thoroughly penetrated by mathematics, symbols and mathematical operations. If we want to train the businessman of tomorrow, we must change radically our approach to the teaching of mathematics. We must provide the tools that he will need. The businessman needs almost as much mathematics as the technician or technologist today. We cannot propogate this on one level without looking back and inspecting our input: here again is the Problem of the Gap. Whatever one level is trying to do must be supported by the other level or it is going to be futile. The problem is one of communications - whatever is going on in the so-called "commercial subjects" and what is going on at the colleges. If we are to cope with these problems, we have to view them together and see what we can do. The OMC again has here a large freedom of action. We shall certainly have to re-examine the mathematics for the commercial courses. It is no longer sufficient to have the student multiply and let it go at that, but it is necessary to introduce more interest and more emphasis on mathematics as it has been the custom in the past.

Another thing that is most important is to establish a prototype course of study for the Colleges of Applied Arts and Technology. I am

happy to report that the action is again ahead of my talk, that already committees are working on a course of study that might serve as a core content or as a guide line for the colleges engaged in the teaching of the business mathematics.

There is another area which ultimately will influence the teaching of mathematics at all levels; I would like to identify this as the probler of high speed computational equipment. You might notice that I did not come out and say "computer", but chose the expression "high speed computational equipment". Let us just look at two aspects of computing equipment, the speed and the price, and changes over the last few years. I am referring to the range from the electro-mechanical calculators, that we al know were noisy and slow, to the new high speed noiseless electronic displ equipment that gives you instantaneous write-out. The speed has increased from the old clackety-clack 30-seconds-per-operation to a point where the only factor slowing down the machine is the human input device. At the same time the price has gone down from $2,200 three years ago, to about $700. today and the price is still falling. Sooner or later everybody will be in a position to purchase such pieces of equipment for little money and use them universally.

Now I am really going to stick my neck out and project a little farthe into the future. I can see that finally in some time, maybe fifteen or twenty years from now, a child instead of wearing a wrist watch, will have a little computing device strapped around his arm. This is not so far-fetched, when you took at the technological progress that we are making right now and will be making in the future. But surely just this one gadget will revolutionize the teaching of mathematics from kindergarten right on when we do not worry anymore about numerical computation methods

other than pushing a few buttons. We all know how in our own lifetime the development has gone. We have been brought up on logarithm tables, in the case where accuracy was required, and I remember having heard of tables of 14 figure logarithms, which were supposedly to be used in ship design. I still remember with horror the Bruhns tables which were 3 inches thick, which we had to use in university to calculate all sorts of silly things. Now today logarithms are virtually obsolete. Every institution that has some computing to do owns a device featuring 20 significant figure answers almost instantaneously - so who needs logarithms? What is going to be the next step? Already I have pretty well retired my slide rule because it is a lot simpler to punch a few numbers into a calculator and see the result come up with more significant figures than I shall ever use.

How will that development affect the Ontario Mathematics Commission? There is one field in which action has to come and has to come fast. Many colleges have installed what is called a "mathematics lab" or "computation lab" which, depending on the institution, functions almost twenty-four hours a day and takes various pieces of equipment and various staffing. We are fortunate here to have one of the experts, Mrs. Glyn-Jones, from Niagara College who has established a lab a long time ago and is, I believe, one of the most experienced persons in the colleges on this type of mathematics teaching. I would feel that it is very necessary to establish guidelines for computing labs in terms of equipment, philosophy, staffing and various details. All the other colleges could then profit quickly from the experiences of the few who are already very deeply in it. Most of the colleges are very seriously thinking of purchasing equipment to get these labs under way. In almost every college there is already a quiet corner where two or three electronic machines are used continuously by the students in their quest for speedier computational aids.

Again, notice, I have not mentioned the computer, which is another can of worms. I am happy to report that the Mathematics Commission has again taken the lead in this field as well: about a year ago, a set of problems, designed for the introduction to computers, was published by the OMC. But since computers represent such a large and complicated area, and since everybody is talking computers these days anyway, I felt it is really unnecessary to elaborate on this particular topic.

Another problem that we face and I think we have to devote a lot of energy to, if we are not going to be obliterated, is the area of "mathematics for other than technical courses". It is one of my pet peeves that, whenever I go to a party, some humanistically or literary inclined man is going to show off his intricate knowledge of literature. He bombards me with questions like "Have you read the latest book by Max Smith? Isn't it a beaut?", Upon my reply: "Max Smith who?" he turns away with disgust. When I turn around and ask him: "Have you read Gauss lately?" he says "Never heard of the fellow" and feels good about it. It is perfectly alright to be ignorant on the scientific section but it is a deadly sin to be ignorant on the fine arts scene. However, it is my conviction that a few mathematicians have contributed more to our high standard of civilization than the whole great bunch of fiction writers combined, may they be dead or still living. This problem, I feel, is one of the most significant ones in the long run. We are all aware of male students who are afraid of mathematics, and of a large section, about 50% of the student population, which is convinced that being born a girl makes them automatically unfit for any mathematical thought at all. Ignorance usually leads to fright of the unknown and we can already see the effects of this fright of mathematics and scientific things genrally: The enrolment in engineering and technological courses across the North American continent is dropping rapidly while enrolment in Applied Arts and Humanistic subjects

is increasing at an unprecedented rate. I am convinced that a lot of it is due to the simple ignorance of people on matters mathematical and scientific. I do not think it will help to recite integration tables backwards in ones sleep, but I do think everybody should have a literacy in mathematics and science. The fact that today technology is blamed for all the evil in the world, - which I in turn would like to blame on the human shortcomings, - is bad enough. But it gives us a good clue that somewhere along the line we have failed to recognize our educational responsibilities and have not placed enough importance on the teaching of appropriate mathematics to everybody.

What can the Mathematics Commission do? Immediately I can not see an action that would rectify this tragic state of affairs overnight, but I think we would have to keep plugging away at the problem. We can prepare a prototype course of study that could be used at the colleges for the Applied Arts field, that makes mathematics interesting, fascinating and at the same time puts the vital points across. At the same time we cannot do anything on a third level that is not prepared on a first and second level. So there again all the practitioners of education are involved, the elementary as well as the secondary, to bring about a change in attitudes towards mathematics. The "new math" in essence was an attempt to change this attitude and I am sure it has done a lot to improve things, but we are far from being even adequate in this respect and I suggest that we have no limit for our activities in this field.

As a last problem area I would like to identify one where I have to stick my neck out - and probably get it chopped off on Monday morning by superiors - the area of teacher training. At the Colleges of Applied Arts and Technology, we do not require certification as a teacher. This regulation was made only out of bare necessity: we had to create 20 colleges

and staff them, mostly from commerce and industry. At the time no one could have asked the new staff members for one complete year of training at O.C.E. or other Colleges of Education. So we are in the unfortunate position of having people teach without adequate exposure to the basics of teaching. The job of teacher training is left to the individual colleges and some of them have done extraordinary things and some of them have just about shrugged their shoulders; some colleges have established a training program that is quite excellent and some of them have done literally nothing. Now we only talked about the teachers in general, but let us specify teachers of mathematics. If a person was fortunate enough to come to a college where an excellent program has been established, it was still in terms of general teaching, not in terms of specifics, how to teach mathematics.

Mathematics in the colleges has been in another unfortunate situation: there is an engineer being hired to teach Applied Mechanics and he has five hours left over in his timetable, so the administration feels, since he is an engineer he can either teach mathematics or physics. It is not in the best interest of those subjects that an engineer is being given literally the freedom of running a program without a chance to get acquainted with the specifics of teaching mathematics or physics. On the other hand I must say most of my colleagues who are engineers have done an admirable job under the circumstances.

What can the Ontario Mathematics Commission do? It is a very touchy problem for the Commission to say, "Well, come on you colleges all, we offer facilities for teacher training in mathematics". The colleges are very concerned about their autonomy and any hint of intruding upon their internal affairs is high treason. However the necessity for teacher training in mathematics still exists and I would hope that ways could be found out of this dilemma. Bodies like the Mathematics Commission could

be given the chance to organize teacher training on a long term base. Again this is one field where my talk comes after the facts. The Mathematics Commission has already undertaken steps in this direction and is discussing problems of this sort.

Outside of training new teachers there is still a large area of updating and even up-grading teachers, that might be done by involvement or by initiative of the Ontario Mathematics Commission. We might be able to have certain universities and certain colleges of education giving special courses to Mathematics Teachers at the tertiary level. We may run all sorts of work-shops or seminars to discuss pertinent questions.

I have indicated some of the problem areas at the post-secondary level, but most of them have been linked intimately to lower levels of education so that it is hard to separate a problem at the college from a problem in education. The Mathematics Commission for the last decade has been active in providing leadership and has specific achievements to its credit. I for one am most confident that for the next decade the Mathematics Commission will go on to even greater glory and greater achievements because the problems seem to multiply rather than to reduce. There is ample necessity for us to exist, to act and to help ourselves and our colleagues on the way to the goal of ever improving the teaching of mathematics on all levels and providing the best service to our students.

BUSINESS MATHEMATICS IN COLLEGES OF APPLIED ARTS AND TECHNOLOGY

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The advent of the colleges of Applied Arts and Technology in 1966-67 brought to Ontario several new problems in the teaching of Mathematics and Statistics. Part of the underlying philosophy of these Colleges was the provision of job-related training for students not qualified to enter University, and the Business Divisions of the Colleges were asked to provide such training in the fields of, primarily, Accounting, Data Processing, and Marketing.

The Business Mathematics teachers were aware that they had no reliable precedents to guide them, and even now, after three years of operations, are still forced to find their way through curricula of uncertain value. It is a time for experiment, for trial and - we hope - not too much error.

The problems being encountered at the College level are many and varied, but they can be classified roughly into three categories: aptitude, attitude, and exposure. Let me deal with them in turn.

Since the Colleges draw their Business students from those high school graduates who, for one reason or another, are not university candidates, the levels of mathematical sophistication are generally much lower than in a university; so much so, in fact, that there are students currently applying for admission to Business courses to whom the most

rudimentary arithmetic presents insurmountable problems. Many of these students find the multiplication of two negative numbers beyond their capacity; simple operations with fractions become an impenetrable jungle; even those who have had some training in algebra are unable to work in any variables other than x and y.

The diversity of aptitudes presents difficulty when attempts are made to raise students to a common level of mathematics so that they can enter their specialised courses in the second year. First year classes are likely to contain simultaneously students who have to be re-taught the fundamentals and others who are ready to tackle calculus, and the teacher faced with such a mixture finds it impossible to hold the attention and interest of the whole class. The usual compromise consists of teaching the middle third of the class and hoping that the bright don't get bored, and the dull don't desert.

Nevertheless, during the first two semesters, some common denominator must be found, so that the students can be passed on to the second-year instructors with some identifiable level of achievement to which the specialised subjects can be keyed. This struggle unfortunately leaves the high-aptitude students with nothing to show for six months' work, and the low-aptitude students with a sense of complete frustration or worse.

The differences in aptitudes show up, not in ability to perform the operations of mathematics, but in the ability to recognise problems as being susceptible to the techniques of mathematical analysis. The modern educational concepts of allowing the student to 'discover' his knowledge, rather then feeding knowledge to him, contribute nothing to the solution of this dilemma. There simply is not time to let the student grope his way towards

an appreciation of the strength of mathematical tools; he has to be shown. He has to be taken by the hand through problem after problem, and repeatedly told that 'this is the way to solve it'. It took the best brains of mankind centuries to develop the Simplex method in Linear Programming, and the second-year teachers will not wait centuries while the student figures it out for himself. They will want him to come equipped in a few months' time.

There just isn't much anyone can do about this problem, except resort to 'force-feeding' techniques for the poorer students. Attempts to stream students according to aptitude have largely failed, since such attempts lead only to a widening of the gap by the end of the first year, and create major headaches in the specialist courses. The new 'promotion by subject' approach in the high schools' mathematics programmes is just another way of 'streaming by aptitude', and does nothing to help the Colleges; quite the reverse, in fact, since all it does is shift the problem forward in time, and aggravate it by the time the student enters College. So it looks as if we are to be stuck with this one for some time.

The problem of attitude is perhaps the worst problem the Colleges face. Thetypical student entering College is quite decidedly 'turned off' on mathematics. Many students who might possess considerable ability in mathematics if properly encouraged deliverately duck certain programmes because they see mathematics listed as compulsory subjects. They are either scared stiff, or completely disinterested, and it is difficult to say where, when, or how this attitude is acquired.

The first tendency for the College teacher is to blame the high schools.

but this is not altogether fair. Questions directed at high school teachers show that they have the same problem. Under the 'free curriculum' system, high schools are having a difficult time persuading students in non-science areas to go ahead with mathematics courses, so we must look further back than the high schools.

A tentative explanation was recently advanced by Dr. W. W. Sawyer, University of Toronto, who pointed to the unconscionable amount of time spent at the pre-high school level on addition, subtraction, multiplication and division. He is not surprised that the student rapidly becomes bored. Constant repetition of the basic drills gradually wears down the students' resistance and leads them to expect that all future courses in mathematics will have the same grinding effect. There is nothing fundamentally exciting about addition.

Most of the new programmes which are currently being touted are applicable at the high school level; only very seldom does one hear about innovations at earlier stages of education in mathematics. Certainly we have had the infamous 'rods' for some time now, but it is now opportune to enquire whether they have done anything for the student's attitude, as distinct from effects on his aptitude.

So perhaps part of the solution lies in a review and renovation of the elementary school system. The Colleges are to be faced soon with an intake from high schools in which the student has been able to opt out of mathematics from Grade IX onwards, and has so opted out. These mathematical semi-literates will expect and demand that the doors of the Colleges' Business Divisions be opened to them, and standing the present terms of reference of

the Colleges, the doors must be kept open. If we think we have problems now, in a few years we may well be looking back at 1970 and fondly wishing that it were still around.

While the students' attitude to mathematics persists, of course, it is impossible to cater to the wish that they should be 'led' to mathematics; while that may be a desirable aim with totally motivated students, it can only lead to absolute chaos with students who are just 'not interested'. Such students must initially be 'driven' to mathematics. Note the word 'initially'. It is not suggested that the student's entire sojourn in a College should be in an atmosphere of compulsory mathematics, but at least in the first year he should be made to apply himself to the discipline. It is then up to the College teacher - unless and until things improve at the pre-College level - to catch the interest of the student to the extent that he will pick up some momentum of his own in the second and subsequent years.

The third major problem - that of exposure - seems to vary in its degree of severity largely according to the particular high school from which the student has come. Some high schools are turning out students who are considerably more aware than others of the tremendous power of mathematical tools in solving practical problems. Many high schools, on the other hand, are regrettably turning out students whose mathematical exposure has been limited to reiteration of proofs of propositions in algebra, geometry and trigonometry, few of which are of any significant value to prospective Business students. It wouldn't be too bad if the stress had been placed on the beautiful logic of such proofs; then at least the student's reasoning

powers would have been alerted and possibly exercised. But all too often the emphasis has been on memorisation, most of it, unfortunately, inexpertly done so that retention is minimal.

Insufficient attention has apparently been given to teaching the student to translate word problems into mathematics. Solution of word problems and performing mathematical operations have been kept separate, so that the student has little or no appreciation of his own abilities apart from the ability to pass exams.

A recent example of the sort of backlash which can result from this type of limited exposure occurred in Durham College in a class which had just been led to a state of considerable efficiency in the use of logarithms. It did not occur to the teacher that it was necessary to explain that logarithms are a powerful tool in the avoidance of 'bulk arithmetic'. Some two weeks later, in a review test on annuities, all the students in the class went through agonies multiplying, subtracting, and dividing several multi-digit numbers by the long method. Not a single student completed the test in the time allowed, and when tackled with having wasted time by not using logarithms they replied, "But sir, it wasn't a question on logarithms!"

Despite all the beautifully-arranged curricula displayed by high schools when asked to show what they are proposing to teach, experience in Colleges with the high school graduate tends to show that in many cases the provisions of those curricula are more often observed in the breach. Unless <u>all</u> the students are lying in their teeth, somewhere between 30% and 40% of the topics in the curriculum are never reached, or are so scantily treated as to leave no lasting impression on the mind of the student. It is almost

inconceivable that a student reaching College after 12 or 13 years in the pre-College system should be unable to calculate what 10% of $100 is, but many are in that state. One tends to wonder what future they would have had in business had they gone there straight from high school.

Remember that these remarks do not apply to all high schools, but the proportion is significant enough to require that at least half of a first-year College class must be seriously considered as mathematically illiterate. Students from the four-year B. & C. stream who seem to have been bred to a 'second-class citizens' state comprise a large number of these illiterates, and display serious lacks of exposure which lead in turn to deplorable attitudes towards mathematics.

The College teacher must make up his mind that he is going to have to live with these problems for a long time to come. What can he do about it?

First, he must fix in his mind the boundaries he must touch in the first-year course. These will be determined by two things; the mathematics which the student will require in the business world, which are essentially 'commercial mathematics', and a short-term goal; and the mathematical groundwork which must be laid for further courses in the second year. This entails a great deal of pre-planning, and a high degree of cooperation between faculty. It requires diligent attempts to integrate mathematics with all other relevant subject areas. (The mathematics teacher who has not taken part in such attemp previously will be astonished to learn just how much assistance he can give teachers of other subjects by introducing appropriate topics on a planned timetable designed to coincide with or precede parts of other courses.)

His own experience of the business world - and teachers in Colleges of Applied Arts and Technology usually are required to have had business experience - coupled with his researches into the uses of mathematics in other areas, will lead him rapidly to identifying the necessary bounderies. How he reaches those boundaries is his own affair, but they must be reached if teachers of second and subsequent years are to be able to rely on getting students properly equipped with the required skills.

Second, he must polish his techniques to the point where mathematics becomes enjoyable for the students. He must be able to build the students' enthusiasms and catch their imagination. His presentation must be flawless, so that nothing he does distracts the students' attention from the mathematics. While he is on his feet in class he must generate so much emthusiasm himself that it spills over into the class. This is not easy to do. No one at the end of a trying day can easily become lyrical about inverting a matrix, but the effort must be made.

Third, he must make 'Related' a key word in his presentations. He must be prepared at each and every point to justify the effort he is asking from the students by relating theory to practice. And further, the practice must not only be related to the theory; it must also be related to the students' experience. For example, the theory of probability can be illustrated by experiments of drawing coloured balls from boxes; but it can be better illustrated by calculating the chances of being dealt a perfect hand at euchre, or determining the expected value of the prizes offered by cigarette companies. This approach is vitally important in Statistics, where the text-books stick to the sizes of skulls found in the Western Desert,

whereas the student is much more likely to be inflamed by an analysis of his own gas mileage or the prices of books in the College bookstore.

And finally, College mathematics teachers must be "available". The nine-to-five attitude is great for graduate school where students are basically motivated, but the vast differences in the students in College mathematics classes demand that when time is needed by the slower students it must be available. This is the only way in which provision can be made in Colleges for the 'slow learner'. He cannot be allowed to work at his own rate. There is just too much to be covered in the all-too-short two or three years of his course. If at first he must be driven to catch up or keep up, then he must be driven. Once he has generated some enthusiasm it can only be sustained by continuous encouragement and the ready availability of help, advice and guidance when he needs it. The teacher who is on top of his job will be aware of the effort that has gone into awakening that enthusiasm, and he will be prepared to do his utmost to ensure that nothing happens to discourage it once it appears. And nothing will discourage it faster than confining his efforts to official classroom appearances.

There is no pedagogic Aspirin which can be taken to make these problems go away. They have to be faced and challenged. There are - indeed there must be - many other solutions besides those suggested above, and they must be diligently sought after. If they are not, mathematics may well become, at least in the Colleges, math-emetics.

A CLASS PROJECT

Harold S. Martin, Queensmount Senior Public School, Kitchener

The project took the form of a survey of homes by a Grade 8 class, in the Forest Hill Subdivision in Kitchener, on November 26, 1969.

Planning Stage

When planning a survey of this nature it is of prime importance to motivate the students so that they are willing to give their full effort and co-operation. The method which I used to achieve this, and found to be very effective, was to informally discuss the geographical area which lay adjacent to our school. In doing this, questions were raised and differences in opinion evolved. It was at this point that I suggested doing a survey of the area and actually finding the answers to some of the questions.

The class suggested approximately 25 possible questions, of which we actually used 12. The teacher should play the leading role in the final choice of questions to insure a proper balance of information which will be collected. If you hope to get full co-operation from the people you survey, you should be very careful not to ask questions which are considered personal.

To demonstrate to our superintendent that this project was worthwhile, I was asked to define the values which my students would gain from this experience. I presented this problem to the class in such a way that they were to come up with arguments as to why they should be allowed to do the survey and how it would be beneficial to them. Following are the ideas which the students presented:

1. It will be good experience for us to take the responsibility of calling on homes where we will, in many cases, have to use our own initiative.
2. It will help to promote a good image of teenagers by having us conduc ourselves in an adult manner.
3. It will help us to develop good judgement and tact.
4. We can apply the information from the survey in the classroom in the following ways:
   (a) plot the information in graphs
   (b) summarize the information in chart form.
   (c) use the information in working with per cents and ratios.
   (d) compare the results of this survey with known facts throughout the city and the country.
   (e) use the information to work with averages, means and medians.
   (f) use the information to project the development of shopping areas in the city.
   (g) compare our own homes with the information we get from the survey.
   (h) write compositions on such topics as "how the homeowners reacted to the survey," "our impressions of the survey", "an interesting experience at a particular home", etc.
   (i) use the information for general class discussions.

By using a city map we made a copy of the Forest Hill area and put it on the overhead projector. The students each chose a partner so that they would be going out in pairs. A particular street, or section of a street, was then assigned to each pair of students, making sure that no student surveyed the immediate area where he or she lived.

A letter was also made up which introduced the students to the home-

owners and stated their purpose in being there. This letter bore the school's name, as well as the signatures of the principal and myself.

The Surveying

The students left the school at approximately 9:10 in the morning and visited homes until about 11:45. Each pair of students visited approximately 20 homes and as a result a total of 331 homes were called on. The names of the people interviewed were not recorded and they were informed of this fact before they answered the questions. The questions asked were as follows: (I have also included some of the results as a matter of interest but did not include the answers of those which name specific products.)

1. What make of car do you have? How many? 1 car - 201, 2 or more, 127.
2. Which of the following magazines do you get on a regular basis?
3. Do you own a snowmobile? yes - 16, no - 315.
4. Do you own a cottage? yes - 55, no - 276.
5. Where will you do most of you Christmas shopping? Downtown - 116, Plaza - 148, Both - 67.
6. What make of television do you have? Do you have a colour set? Colour - 87, Black & White - 339.
7. Where do you do most of your grocery shopping?
8. Do you agree with the idea that animals from the Humane Society should be used for scientific purposes? yes - 180, nO - 113, No comment - 38.
9. Do you think the voting age should be lowered to 18? yes - 123, nō - 199, no comment - 9.
10. Should foods which contain cyclamate be taken off the market? yes - 158, no - 134, no comment - 39.
11. Do you think the drinking water in Kitchener should be fluoridated? yes - 205, no - 108, no comment - 18.

12. Do you think that children to-day are getting a better education than what children did 25 years ago? yes - 306, no - 18, no comment - 7.

It should be noted that the students got excellent co-operation from almost every home that was visited. In no case were they treated rudely or had a "door slammed in their face."

Follow-Up

The first task on returning to the classroom was to summarize the 331 survey sheets. The class was divided into 12 groups (because there were 12 questions), and each group was responsible for summarizing a particular question.

Following this they divided themselves into groups and chose the question or questions which their group wanted to analyse. It should be noted that the first time they simply summarized the information, whereas this time their task was to analyse and display the information. This took considerable planning and effort on their part. Some went back through the 331 sheets to get further information from various questions with the idea of seeing how they related to each other.

The final outcome was that most of the information was displayed eithe in the form of three-dimensional models, charts, or graphs. We also translated the answers to all the questions in the forms of per cents. One char depicted the average home in Forest Hill.

After analysing all the information, letters were sent to various people in the city. The letters were sent to people who we felt would find a particular part of the survey relevant to their interests. The letters were sent to the following: the local member of the federal parliament, the local member of the provincial legislature, the Kitchener Water Commiss the Humane Society, a local maufacturer of television sets, the Downtown

Business Men's Association, and various local grocery stores. In every case we got a reply to our letters, and in a few cases some very worthwhile information which related to our survey.

A letter was also made up by the class which summarized the information from the survey and thanked the people for their co-operation. The students then went out in pairs again and left a copy of this letter at each home which they had visited earlier. Originally this was not in our plans, but we decided to do this because of numerous phone calls to the school requesting information about the results of the survey.

The final step in this project was to have the students evaluate the survey as they saw it. Basically, I asked them to comment on what they thought was particularly good about the project, and also what changes they would suggest if they were doing it again. The most usual comments from this were:

1. Use questions which present more possibilities for making comparisons between questions.
2. Survey two socioeconomic areas and compare the two.
3. They appreciated working in pairs rather than going out alone.
4. They enjoyed sending letters to various companies and organizations, and particularly enjoyed getting return letters from them.
5. Many wished they could have visited more homes so that the survey would have been more meaningful.
6. They appreciated having a letter from the school which introduced them to the householders.
7. Some suggested doing another survey in which they would visit various industries and retail locations.
8. Many said they would have liked to spend more time in analysing the questions and displaying the information.

9. A number said it gave them an opportunity to show responsibility and to be polite.

Further Suggestions

I feel that a survey of this type could be done on a much wider basis if a number of schools in the city took part. For example, each senior school in the city could do a survey in their own geographical area with the same questions being asked throughout the city. The first step would then be for each school to summarize and analyse the information for their particular area. Following this, representatives from each school would meet at one place and analyse the information for the whole city. This would create considerable interest because the students would be working with people from the other schools. The survey would of course be more meaningful because many more homes over a wider area would be surveyed.

Concluding Comments

By this time it has become very obvious that a survey of this type relates to many subjects which are on the regular course of study. I did the survey from a mathematics and English base because these are the two subjects which I teach. However, I feel it is particularly relevent to geography, and also to science in the field of enviromental science. Even though I did this project with an enrichment class, I feel confident that it could be done equally well with most grade 8 classes. I am certain of the fact that it has been a very worthwhile educational experience, for both my students and myself.

THE IMPLEMENTATION OF A MATHEMATICS CLUB

Douglas S. Ailles, Kipling Collegiate Institute, Etobicoke

There have in the past been many so called extra curricular activities in junior and senior high schools. Usually they take the form of sports events such as basketball games, track and field, and football which is still the major role in many schools. There is a slow revolution gradually gaining momentum in the academic subject fields as well. These are small clubs which have come into being in comparison with the athletic association. One of these which I feel can play a major role in the school is the Mathematics club. Essentially Mathematics is challenging to students or should be presented in such a way to enhance the inquisitive mind. It is not always possible to provide this challenge to the honour students in the classroom because basic subject matter must be kept within the reach of all the students.

Students of high calibre could be brought together for intensive investigation of an optional, but vital concept by these individuals. An advanced programme could be devised to meet the needs of more able students. The organization of a mathematics club within the bounds of a large or small high school may just be the key necessary to provide a relaxed atmosphere or foster the many challenges sought and seldom solved by students.

Students form the backbone of any school, let alone any club. The cohesiveness of this group if prided by its members will be quite high. In fact a high degree of interest displayed by a few dedicated ones may soon bubble over and multiply while membership may have to be limited if the nature of the club activities provides opportunities for students not

usually offerred. Of course membership should be completely voluntary. Mainly those students who do very well in the classroom are the first to join. But this need not be the case. Students tend to do better in a subject if they have a high interest in it. I try to encourage the averag and slow learners to participate as well. Who is to say that the student who is fighting to achieve passing gradings in grades eight, nine, and ten mathematics may not be a future mathematician? If his latent interest is aroused, he then seeks to discover a fact that inspires him.

A good recruiting programme might be necessary to initiate such a clul if one has not been in existance. I have found that one way to tackle thi problem is to plan a social meeting in which prospective members have an opportunity to discover some probable activities. Thirty-five mm. slides provide an excellent way to foresee what may be in store for new members. If you discover that too many students want to join, divided the club into two smaller groups - i.e. junior and senior. If you wish to limit members ask all the interested to submit a short article on mathematics to be presented later to other members at some future meeting.

You may find it difficult, almost impossible to organize a mathematics club and operate it effectively without a yearly budget. Funds should be available to set up a math library, for concrete material, filming, film rental, math models, charts, subscription to magazines math projects, etc

Organization of the club is advisable - a list of officers duly electe a basic constitution drawn up and planned meetings. It need not be formal by any means.

Try not to have a preconceived idea of what will interest the student The type of programme must appeal to them. In fact it would mean more to them if they had some say or choice in their programme. What special activities might be introduced? Will the entire student population be

invited to hear guest speakers brought into the school sponsored by the mathematics club? Who will make arrangements for social meetings? What special project will be submitted to a Science or a Math Fair?

Possible suggestions for activities are: - class presentations by individuals, video tape for class demonstrations, preparing materials, and mathematics models.

It is not recommended that students be used to grade papers or prepare bulletin boards.

Talks could be made by college students explaining needs for university participation or by college mathematicians in industry. Visits could be arranged to mathematics centres, to other clubs, to mathematics and science fairs, to computer centres, to local industry, making super eight films - topological concepts, construction of 3 dimensional shapes, design and architecture, presentations by other staff members; and a student presentation of a topic of his choice.

Some questions to consider when setting up a mathematics library are: Where should it be located? Who will look after it? How should it be arranged? Should it be a part of the regular school library or what other relation to it? How will the math library be used? How could you compose sources of games (math), recreations and puzzles?

A tutorial service can be organized as one of the activities of the club. Students who achieve well in Mathematics in previous year's work in senior grades are quite willing to assist those students who are having difficulties in mathematics on an individual basis. A minimal tutorial fee may be charged for their service to compensate the reliable eager student, willing to give up some of his leisure time.

This service encourages many slow learners to join the mathematics club.

NOTE: This is not to be scheduled at a regular meeting, but at a time

mutually decided by parties concerned. The slow learners by being in contact with better students in this subject may then wish to become involved in other mathematical activities. In this manner, they could be incorporated into a Mathematics club.

Some students prefer to work with other students rather than go to a teacher for assistance.

If there exists an individualized type programme, honour students could be available for assistance during study periods.

Thus I feel that many students who partake in a Mathematics club either directly or indirectly will have a new vision into the world of Mathematics.

CURRICULUM AND METHODOLOGY

The following books have been prepared by Study Committees of the Ontario Mathematics Commission:

K - 13 MATHEMATICS: SOME NON-GEOMETRIC ASPECTS,

Part I Statistics and Probability

(Curriculum Series, No. 6) Price to be announced

GEOMETRY, Kindergarten to Grade Thirteen

(Curriculum Series, No. 2) Price $2.50

INTERMEDIATE MATHEMATICS METHODOLOGY

(Curriculum Series, No. 4) Price $2.00

These books may be obtained from:

Publication Sales
The Ontario Institute for Studies in Education
102 Bloor Street West, Toronto 5, Ontario
Toronto 5, Ontario
Telephone: (416) 635-2787

Copies of the Experimental Course for Grade 12 Mathematics in the 4-year program are also available from O.I.S.E.

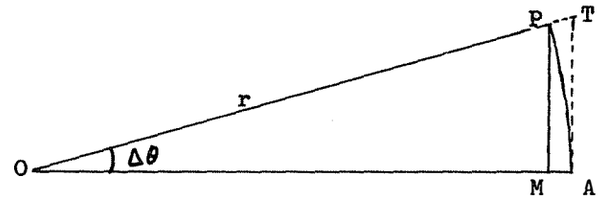
MATHEMATICAL NOTES

ON TEACHING $\lim_{\Delta\theta \to 0} \frac{\sin \Delta\theta}{\Delta\theta}$

Edward Evans, Guelph

Many Grade 13 students who elect Math A come to the course with limited mathematical maturity and a minimum confidence in the manipulation of trigonometric formulae. For this reason, it is highly desirable to present this topic with a simple, direct picture, in order to carry conviction and to aid in recalling the result.

The following method therefore appears to provide such a picture and demonstration, more readily grasped than the one in the present* textbook.



Using the solid line part of the above diagram, note that PM = r sin $\Delta\theta$, arc PA = $r\Delta\theta$, AM = r(1 - cos $\Delta\theta$).

Now, if we hold PM fixed, and let the point O move to the left as we increase the radius r, it is easy to visualize what happens as $\Delta\theta \to 0$. The point A approaches the point M, and the arc PA flattens and approaches the line PM.

(Note: the fixed length of PM need cause no real difficulty as we allow $\Delta\theta$ to decrease, as it is the relative lengths of PM and r that we are considering, and $\Delta\theta$ can be made as near zero as we wish by choosing increasingly larger values of r.)

Hence,

$$\lim_{\Delta\theta \to 0} \frac{\sin \Delta\theta}{\Delta\theta} = \lim_{\Delta\theta \to 0} \frac{PM}{\text{arc } PA} = 1, \quad \text{and} \quad \lim_{\Delta\theta \to 0} \frac{1 - \cos \Delta\theta}{\Delta\theta} = \lim_{\Delta\theta \to 0} \frac{AM}{PA} = 0$$

Furthermore, a proof similar to that in the text can readily be devised by adding the dotted lines in the diagram and squeezing the arc PA between the lines PM and TA, while the area of the sector OPA and of the triangle OTA approach the value of the area of the triangle OPM more and more closely as $\Delta\theta$ approaches zero.

One is then able to develop the derivative of $y = \sin x$ directly by

$$\Delta y = \sin (x + \Delta x) - \sin x$$

$$= \sin x \cos \Delta x + \cos x \sin \Delta x - \sin x$$

and $$\frac{\Delta y}{\Delta x} = \sin x \frac{\cos \Delta x - 1}{\Delta x} + \cos x \frac{\sin \Delta x}{\Delta x}$$

The derivative of $y = \cos x$ can then be similarly developed as an exercise by the student.

* Calculus - by Elliott, Fryer et al. (p. 168).

ON DIFFERENTIATING $[f(x)]^{g(x)}$

Miss Leslie Stewart,* Toronto

While experimenting with the derivative of the function $y = x^x$, I stumbled on the following result. If the function is differentiated first by considering the base constant and then by considering the exponent constant the sum of the two results equals the derivative obtained by using logarithms and partial differentiation.

Using logarithms:

$y = x^x$

$\ln y = x \ln x$

$\ln y - x \ln x = 0$

$$\frac{dy}{dx} = -\frac{F/\ x}{F/\ y}$$

$$= -\frac{-\ln x - x(\frac{1}{x})}{1/y}$$

$$= (\ln x + 1)\, x^x$$

$$= x^x \ln x + x^x$$

Using the exponent method:

$y = x^x$

$$\frac{dy}{dx} = x(x)^{x-1} + x^x \ln x$$

$$= x^x + x^x \ln x$$

I tried this method on several other functions with the same result. Finally I tried this method on the general case $y = [f(x)]^{g(x)}$ with the following results.

Using logarithms:

$\ln y = g(x) \ln f(x)$

$\ln y = g(x) \ln f(x) = 0$

$$\frac{dy}{dx} = \frac{-[-g'(x) \ln f(x) - g(x)\, f'(x)\, (\frac{1}{f(x)})]}{1/y}$$

* Written while a Grade 13 student at Jarvis Collegiate, Toronto.

PROBLEMS AND SOLUTIONS

Edited by W. Fraser

Problems for inclusion in the problem section are welcomed by the editors. Please submit both your problems (preferably accompanied by a solution), and your solutions to published problems to Prof. W. Fraser, Department of Mathematics and Statistics, University of Guelph, Guelph, Ontario.

Problem 70.04. Show that for all positive integers n,

$$\frac{1}{\sqrt{4n+1}} < \frac{1.3.5....(2n-1)}{2.4.6.....(2n)} \leq \frac{1}{\sqrt{3n+1}}$$

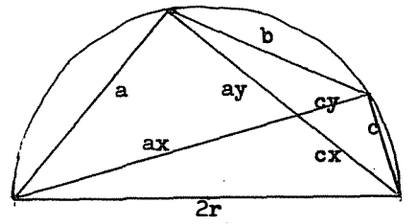
Problem 70.05. Of all triangles cut off from a fixed angle C by straight lines passing through a fixed point H in the interior of the angle, which has the least area?

Problem 70.06. Where a and b are real constants such that $0 < a < 3$, find the maxima and minima of $x^3 - ax - b$ for $0 \leq x \leq 1$. Choose values $a = a^*$ and $b = b^*$ such that these extremes are equal in absolute value. Show that the choice a^*, b^* leads to the smallest possible value for $\max\ x^3 - ax - b$ in the interval $0 \leq x \leq 1$.

Solutions

Problem 10, November, 1969.

The following solution for problem 10 was submitted by Mr. E. Evans, Guelph, who commented that in the general case (part(c)), while an equation was readily obtained, it did not appear to be readily solved.



$$\frac{b}{2r} = \frac{y}{x} \qquad (1)$$

$$a^2 + (ay + cx)^2 = 4r^2 \qquad (2)$$

$$c^2 + (ax + cy)^2 = 4r^2 \qquad (3)$$

Elimination of x and y leads to

$$4r^3 - r(a^2 + b^2 + c^2) - abc = 0.$$

Problem 70.02, March, 1970.

Solutions to this problem have been submitted by P. Weygang of Levack High School, T. J. Griffiths of North Lambton Secondary School, and M. A. Laframboise of Detroit. The following approach, used in one of the solutions, seemed to involve the least effort.

If the second of the relations

$$\cos x + \cos y + \cos z = 0,$$

$$\sin x + \sin y + \sin z = 0,$$

is multiplied by i and in turn added and subtracted from the first, we get the relations

$$e^{ix} + e^{iy} + e^{iz} = 0$$

and

$$e^{-ix} + e^{-iy} + e^{-iz} = 0.$$

For brevity of notation write $e^{ix} = u$, $e^{iy} = v$, and $e^{iz} = w$. Then $u + v + w = 0$, and $1/u + 1/v + 1/w = 0$.

Hence $0 = (u + v + w)^2 = u^2 + v^2 + w^2 + 2(uv + vw + wu)$

$$= u^2 + v^2 + w^2 + 2uvw(1/u + 1/v + 1/w)$$

$$= u^2 + v^2 + w^2. \qquad (1)$$

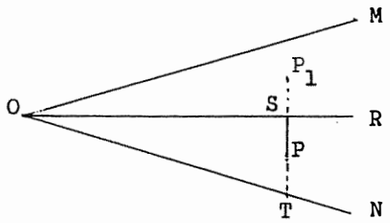
Similarly $0 = (1/u + 1/v + 1/w)^2 = 1/u^2 + 1/v^2 + 1/w^2.$ (2)

The relations (1) and (2) imply the desired result.

Problem 70.03, March 1970.

Solutions to this problem have been submitted by Jacques Kahale of

Ecole Secondaire Theriault, R. Kostash of North Lambton Secondary School, and M. A. Laframboise of Detroit. The solution follows.



1. Construct the bisector OR.
2. From P draw a perpendicular to the bisector OR meeting it at S and extend to P_1 so that $P_1S = SP$
3. P and P_1 are two points on the required circle. For a third point extend P_1P to cut ON at T. Find the mean proportional to TP and TP_1. With centre T mark a point P_2 on ON such that the length TP_2 is equal to the mean proportional. There will be two such points. The circle through P, P_1, P_2 fulfils the required conditions and, as noted above, there will be two such circles.

Elementary Mathematics 4. by R. Hendren, W. C. McClure and J. Routledge. Toronto: W. J. Gage Ltd., 1967. 280 pages.

Elementary Mathematics 4, Teacher's Commentary. Hendren, Kirkpatrick, McClure, Routledge. Toronto: W. J. Gage Ltd., 1967. iv + 122 pages.

Elementary Mathematics 4 is the fourth book of a series of an elementary mathematics program. It is a textbook appropriate to the average grade four pupil and can be used as a one pupil text or for small group instruction, but is to be considered as just one of the aids to the total program.

Its major strength lies in its emphasis on problem solving, one of the major goals of this programme as stated in the introduction to the Teacher's Commentary. "Development of insight into problem solving is achieved by the systematic use of distinctive methods of teaching. First, the pupil is helped to recognize what is happening in the problem. Second, he is taught to translate the problem from words into the symbols of a number sentence. Third, he is shown how to select the proper mathematical process to solve the number sentence. Fourth, he is taught to translate the completed number sentence back into the physical world of the problem."

Problem exercises are used as well to provide more practice in the use and reinforcement of basic facts. The authors recognize that a reasonable facility in arithmetical computation is needed to progress in the study of mathematics and for this a sound knowledge of the basic facts is essential. Frequent use of basic facts is made in the introduction of new concepts. Various ways are included for practice and reinforcement, not only of the basic facts, but for the new concepts

taught, particularly under the section "Making Sure." A series of tests are included as checks on a pupil's achievement.

The discovery approach is more evident in the area of geometry and measurement and in the special enrichment section "Explorations" where pupils are encouraged to make their own investigations.

The section titles are more appealing to the children and within their comprehension. e.g. Moving ahead, Putting Numbers to Work, Pencils Down, Do You Remember.

For greater emphasis and attention number symbols used for examples are written in enlarged print and new terms and important facts are enclosed in coloured boxes.

Some new topics are introduced such as symmetry, co-ordinates on number plane, angles, measurement of regions, construction of polyhedrons and the metric system. There appears to be much work, especially paper-pencil work, on the abstraction of geometry (lines, points, angles, etc.) but the guide points out that this work is to be done only after a thorough study of the solids and always in relation to them. Too much abstraction too early draws away from the fundamental enjoyment and appreciation of geometry.

The use of ordered pairs in the work of ratios prepares a good beginning to the work which is to come.

Although this is purported to be a modern elementary mathematics programme unfortunately the textbook is still a textbook within the traditional format. Expense is a great problem which cannot be overlooked. The first two pages contained actual photographs but after that only sketched illustrations are used. This provides for simplicity in detail but what about using photographs of children at work and true

examples (spelling mistakes and all) of their work? Is it possible to have more colour or even picture problems?

The Teacher's Commentary is an invaluable part of the programme. It is well-written, brief and easily understood. No teacher, experienced or inexperienced, should attempt to follow the text without first reading the philosophy and general over-all plan. The guide is a good reference to provide mathematical background for the teacher. Each lesson is organized under the headings Objectives, Comments and Lesson suggestions. Specific objectives are defined in terms of the pupil's expected achievement. The Comments provide essential mathematical information as well as the mathematical and psychological reasons for some of the topics. Lesson suggestions outline introductory activities, teaching procedures and follow-up activities. The Teacher's Commentary makes a most useful and helpful guide for the classroom teacher.

J. Stemerowicz, Kenora, Ontario

Mathematics, by Samuel Rapport and Helen Wright. The New York University Library of Science Series.

It is one of the disadvantages of teaching mathematics that background reading is rarely a passive pursuit. One envies other disciplines where simply reading a novel may provide a deeper understanding of essential aspects of study - social values, character motivation, or simply geography.

This volume attempts a fairly painless excursion into the world of mathematics for the layman. As usual this means the good high school

mathematics student, at least, despite the statement (or because of it ?) that a minimal knowledge of mathematics is required. The additional claim that "scholary apparatus, such as footnotes has been omitted" is simply not true.

The territory to explore is so large that only the odd glimpse here and there is possible, and on the whole the selection is an interesting one. Perhaps, however, too much is attempted. The first section consists of biographies of six great mathematicians. (Try to predict - five out of six for honours.) Fascinating as the lives of mathematicians are, the time for their study is probably after a close acquaintance with their work, rather analogously to discovering the "feet of clay" of the great composers, it is dubious how much illumination of their work is provided or how much motivation to study is obtained.

This first section occupies one third of the book. The second section consists of three essays on mathematical method, the third, five articles on assorted topics of pure mathematics and finally another seven on its applications. The latter two sections contain much of the familiar material on large numbers, topology, calculus, probability, non-Euclidean geometries etc., but perhaps the most important pages are the acknowledgements, easily missed at the beginning of the book, where reference is made to the source of the material. The editors of this volume would certainly have achieved their object if the reader's curiosity were aroused sufficiently to explore these, and, although this book is uneven in texture and can only be read in snippets, it is nevertheless readable and could well achieve that end.

Martin Cooper, Southmount Secondary School,
Hamilton

NOTES ON MEETINGS

During the "Premier Congres International de l'Enseignement Mathematique",* Lyon, August 24-30, 1969, the representative of the U.S.S.R. gave a report about the new mathematics program in the schools of the Soviet Union which will take effect in 1969-1970.

Every student must attend school from age seven to seventeen and mathematics is obligatory in all classes to all students. In the first eight school years, six hours per week of mathematical instruction will be given and in the remaining two years five hours per week. Gifted students may take additional mathematical instruction from their fourth school year on.

In the instruction much emphasis is to be placed on numerical calculations and applicational aspects to the natural sciences, such as Physics, Chemistry and Biology. Modern abstract concepts are to be only cautiously introduced as it is felt that mathematics should not appear as an arbitrary abstract system of conventions. The abstract symbolism of mathematics should mainly be left for the specialist.

H. P. Heinig

(Abstracted from a report in "Elemente der Mathematik, Vol. 25, (1) 1970).

* See the report in Vol. 8 #2 by J. Egsgard.

MEETINGS

A National Mathematics Convention (N.C.T.M.) hosted by the Manitoba Teachers' Mathematics Association, will be held in Winnipeg on October 15 - 17, 1970. The purpose of this convention is to familiarize the teachers with new ideas in the development of mathematical content and in the new approaches to the teaching of mathematics.

One feature of this convention will be a display of various student projects in mathematics. Students from kindergarten to grade twelve are being encouraged to submit mathematical projects.

For more information write: William P. Solypa, Chairman,
Student Exhibit Committee,

A joint conference (CAST - STAO) of science teachers combining the First Canada Wide Conference of the Canadian Association of Science Teachers, and the Second Annual Autumn conference of the Science Teachers of Ontario, will be held Saturday, Sunday and Monday, October 31 to November 2, 1970 at the Ontario Science Centre, Toronto and the Holiday Inn, Don Mills. For more information write to: Mr. G. R. Spink,
Birchmount Park C. I.,

No. 1

Executive Officers of the Ontario Association of Teachers of Mathematics

1970-1971

Dr. K. D. Fryer, Honorary President
Associate Dean,
Faculty of Mathematics,
University of Waterloo,
Waterloo, Ontario.

D. A. Fox, President
Ontario Department of Education,
Ottawa 5, Ontario.

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North Bay, Ontario.

M. B. MacGregor,
Publicity Representative,
Devine Street School,
Sarnia, Ontario.

No. 2

An Invitation to Join the O.A.T.M.

Each fall the principal of every school in Ontario receives application forms from the Ontario Educational Association to be filled out and returned by the members of his staff. The O.A.T.M. asks that you give serious thought to joining the association and if mathematics is your "thing" check the square after Mathematics on the application form. The cost of joining is $6.00 per year. At each of the Regional Conferences there will be a member of the association present to discuss the benefits of joining our organization. They will also provide you with the

application forms which may be filled out and returned while you are attending the conference.

No. 3

Regional Conferences of the O.A.T.M. 1970-1971

The Ontario Association of Teachers of Mathematics is continuing this year its programme to sponsor regional conferences with items of interest for both elementary and secondary teachers.

Laurentian University Sudbury - October 3 (Secondary)
Strathroy - October 3 (Elementary)
Althouse College London - October 24 (Secondary)
Wingham - October 17 (Elementary)
Windsor Teachers' College - October 17 (Elementary) with OEA
Brantford - October 31 (Elementary)
Chatham - October 31 (Elementary)
Kitchener - Fall Meeting date to be announced (Secondary)
Ottawa Teachers' College - October 31 (Elementary) with O.E.A.
St. Lawrence College, Kingston - September 26
St. Catherines Teachers' College - November 7

1971 ONTARIO ASSOCIATION OF TEACHERS OF MATHEMATICS CONFERENCE -
Niagara Falls - March 21-23, 1971

Details to follow

ANNOUNCING

A HANDBOOK FOR TEACHERS OF MATHEMATICS IN THE INTERMEDIATE GRADES.

A new book prepared and published by the Ontario Mathematics Commission.
152 pages of helpful and informative material for use in the classroom for Grades 7 to 10.
The book covers a wide variety of topics, some new, some old, but looked at in a fresh light.
Some of the topics covered are: Mathematics in Man's Environment, in Economics, in Biology, in Art, in Statistics.
Geometry - a new look at old problems
Numbers, Number planes, graphs and vectors
Number Patterns and Algorithms
Problem solving.

Individual copies are available FREE from the Ontario Mathematics Commission (in the case of bulk orders for Mathematics Departments, some arrangements will have to be made for mailing).

Please complete the application form below.

- -

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To: The Secretary-Treasurer,
Ontario Mathematics Commission,
1260 Bay Street,
Toronto 185, Ontario.

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Please check:

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