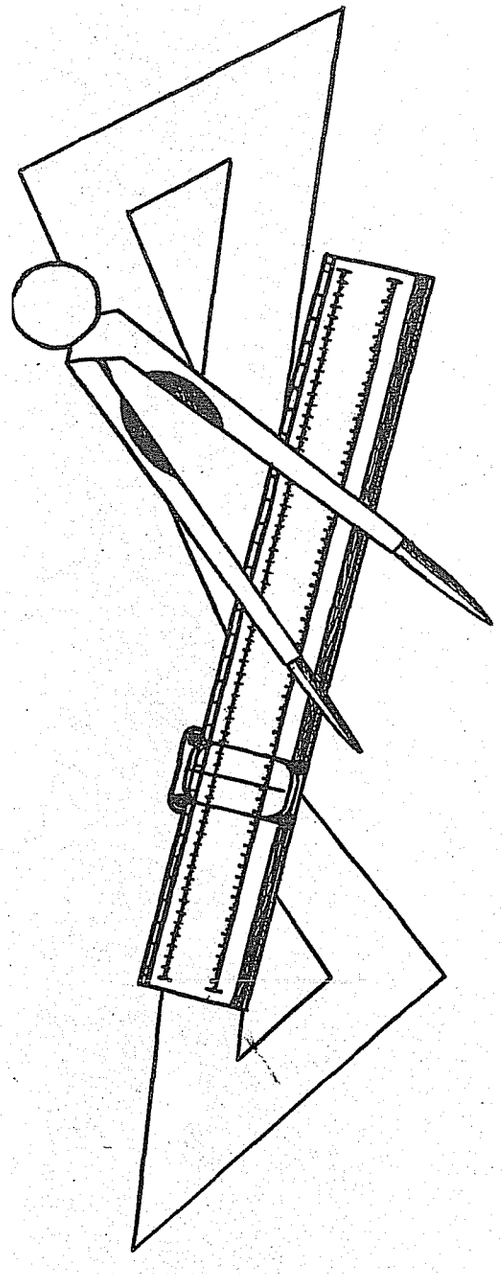
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GEOMETRY IN ELEMENTARY SCHOOLS, K-6

J. J. Del Grande*
Mathematics Coordinator, Borough of North York

Geometry, beyond a doubt, has been taught and learned by more people over the centuries than any other subject. Geometry has withstood the test of time and we have inherited it, without change, from great geometers such as Pythagoras and Euclid. There is evidence to show that geometry in a primitive form was known in 20,000 B.C.! Geometry has survived because of its usefulness and because it could be utilized as a logical deductive system with which the human mind could grapple. Traditionally geometry has been reserved for study by 15 and 16 year old children. Why can't geometry be taught to 5 year old children?

The geometry of Euclid is too profound and too subtle for young children. Recent developments in mathematics and the teaching of geometry have shown that geometry can be taught to children if the approach is by means of motion or transformation geometry. In the report Geometry: Kindergarten to Grade Thirteen[1] appears the following: "Motion geometry is an attractive subject to teach. It deals with ideas that come very naturally to children - for example, symmetry.

* Mr. Del Grande was the guest speaker at the Annual General Meeting of the O.M.C. on 19th September, 1970. The above article is, with his permission, taken from a publication of the Mathematics Department for North York.

(1) K-13 Geometry report (O I S E, 1967)

A child is probably aware of the symmetry of a butterfly before the concept of distance has become fully clarified. Much of the work with motion geometry appeals to the artistic side of children - for example, the classification of ornamental patterns for their symmetry interests - and is accessible to children in Grades 4 to 6 , while continuing to hold interest for much older students."

Not only can geometry be taught meaningfully to 5 year olds but geometry is perhaps one of the most critical considerations in early education. If we conceive of geometry as exploration of space, and as relationships in space, then it is apparent that geometry is the vehicle through which a child can orient himself in space. Hence the study of geometry establishes the security of the child because he is exploring and appreciating the space in which he lives, i.e. his environment.

Children enjoy working with geometric notions and physical geometry at a very young age but unfortunately, geometric activity often ceases after a brief excursion into the topic. Perhaps this occurs because courses of study encourage students to put away "childish things" and get down to the business of arithmetic and memorizing facts. This sequence is contary to the findings of Piaget and Inhelder (1956) who discovered that children can understand geometric notions long before they understand many arithmetic ideas. By handling geometric shapes constructed of paper or wood the student acquires an insight into notions of corners, points, square corners, straight

edges, flat faces, roundness of discs and spheres. With these beginnings children gain a degree of understanding to go on and to grow in geometric readiness.

The geometry of form will help children perceive, compare, measure and generalize. In this activity they will sharpen their intuition without plunging too far into abstractions. Children learn by doing.

Experience shows that young children can comprehend a fairly high level of geometry if the media of presentation can be found. The media involves verbalization. Thus, it is important to use the language of children rather than precise mathematical language. A child readily accepts the name 'corner' but would be perplexed and hesitant to use the name 'vertex'. As the child matures he will seek new mathematical words (the names the grown-ups use) to add to his vocabulary. Children learn best by utilizing all their senses. Auditory, visual and tactile senses come to play in an activity approach. This approach encourages the child to learn by doing and enables him to experience the joy of discovery. Group activity, independent work and progressing at one's own rate are natural outcomes in the activity approach.

Geometry in the elementary grades should be informal and should be a study of shapes, sizes, patterns and designs - these are the things from which children discover concepts. By studying forms children discover geometric relationships and by making constructions they learn geometric facts.

Geometric readiness comes at an early age. Children enjoy fitting geometric shapes into their proper places on a puzzle board. Children quickly discriminate between a square and a rectangle, between an equilateral triangle and a right triangle and between a trapezium and a parallelogram. The geometry developed for K - 6 is designed to meet these requirements. Great emphasis is placed on the use of concrete materials and conclusions drawn from experimentation. Geometry properties are obtained inductively; but great care must be taken to test conclusions drawn from experimentation.

What criteria should one use to include geometric topics in an elementary school program? Three criteria are listed.

(1) The topic fits in a logical sequence of development in a mathematic "strand".

(2) The topic involves the kinds of behaviours that a child can be expected to exhibit.

(3) The topic is useful in developing mathematical intuition, strategies, knowledge and maturity either at his present level or at a higher level.

There are four aspects of geometry recommended for elementary schools.

(1) Shape, often called non-metric geometry, deals with the geometric figures, patterns and spatial relationships.

(2) Transformations primarily deal with translations, reflections, rotations and glide reflections which are

distance preserving. These are called slides, flips and turns. Transformations involving similarity, dilations and projections are introduced in a primitive way.

(3) Measurement, often called metric geometry, deals with the assignment of number to geometric objects. For example, the volume of a solid, the area of a region and the length of a line segment. Measurement in the objectives is treated as a separate strand.

(4) Coordinates and graphs is sometimes called the geometry of place. Coordinates and graphs are treated in a separate strand in the objectives.

SHAPE (Non-metric Geometry)

A child's world is three-dimensional. His early experience with geometry involves spheres, cubes, cylinders, etc. which he may call "ball", "box" and "can". Any study of geometry must start from these real objects. Children should learn to identify the basic qualities by observing the attributes for which two objects are alike and the attributes for which they are different. Two-dimensional figures are outcomes of the child's three-dimensional experiences. A study of the interrelationship of shapes is also included through the idea of congruent figures. The topics of Shape and Measurement follow.

(a) Figures in 3 dimensions: Classifies and identifies ready-made 3-D figures such as cones, cylinders, spheres, cubes, rectangular solids, pyraminds, prisms.

(b) Figures in 2 dimensions: Classifies and identifies 2-D figures such as circle, square, triangle, rectangle, parallelogram, pentagon and hexagon.

(c) Points, lines and angles: Identifies open and closed simple curves, straight line, line segment, ray, endpoint, midpoint, parallel lines.

(d) Construction of 2-D figures: Construction of 2-D figures by means of geoboards, straws and pipe cleaners, meccano and ruler constructions by joinig dots.

(e) Construction of 3-D figures: Construction of 3-D figures by means of straws and pipe cleaners, soft wire, sticks with glue, cardboard, ready-made polygons, paper and plasticine.

(f) Tiling: space filling patterns using a 2-D shape.

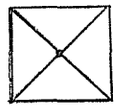
(g) Curves: A curve as a set of points such as line and circle. Curves formed by curve stitching, straight-edge construction, string and pencil constructions, straight-edge and compasses construction and sections of solids such as sphere, cylinder and cone.

(h) Topology: Extensions of experience with shape by means of topological topics such as inside and outside of curves and surfaces. Mobius strip, 2-colour problem, 4-colour problem, torus compared to sphere, and tracing of simple networks (unicursal problems).

Transformations

(a) Transformations are studied as slides, flips and turns of physical objects including body movements, geoboards and paper cut-outs. These transformations are sometimes called isometries because, under these transformations, distance is preserved. That is, the length of a line segment does not change under an isometry.

(b) Symmetry is studied as a transformation that maps a figure onto itself. A symmetry may be a reflection, in which case the figure has an axis of symmetry and the figure can be reflected onto itself in a mirror line. This property can be illustrated by means of a semi-transparent plastic mirror. A symmetry may be a rotation. For example, if a square is rotated about its centre through 90^{o}, the square is brought back to its original position.



(c) Tiling - Space filling patterns using transformations of a 2-D shape.

(d) Projections and shadows - Introduction to projections by means of a light source or the sun.

(e) Scale drawings in 2-D of maps, and scale models.

MEASUREMENT

(a) Arbitrary units: Measurement of length, area, volume and

angle using non-standard units.

(b) Standard units: Measurements of length, area, volume using English and metric units of measure. Angle using degree measure.

(c) Rotation Angles: Measure of a rotation angle using non-standard unit, (fractions of a full turn, clock measures). Degree measure with angles up to 360^{o}. Rotation using the points of a compass.

(d) Lengths: Perimeters of polygons using non-standard units. English measure and metric measure. Estimating lengths and comparing lengths of curves and boundaries.

(e) Area: Area using non-standard units, English measure and metric measure. Area using dissection and transformations. For example, cut a figure into pieces and move pieces around.

(f) Volume: Volumes using non-standard units, English measure and metric measure. Comparison of volumes of similar and dissimilar 3-D containers.

(g) Measurement in Transformations: Measurement with translations as vectors, rotations related to angles and reflections.

(h) Volume and Area formulas: Development of formulas for volume and area of simple figures or solids through experimentation.

(i) Circle properties: Discovery of the relationship between radius, diameter, circumference, and area of a circle.

GRAPHS

(a) Graphs in measurement can be used for recording data. Graphs should be utilized for the information that they convey.

(b) Coordinate graphs may be introduced at a very low level using concrete materials such as coloured discs. Emphasis should be placed on naming a square in the plane. Transformations, such as slides and flips may be illustrated by using these same materials.

(c) Naming points in the plane first using coloured discs and then integers.

THE SECOND ROUND OF REVISIONS TO THE SECONDARY SCHOOL 5 YR. A & S MATHEMATICS COURSES - A FEW OBSERVATIONS*

R. L. Behan, Lisgar Collegiate Institute. (Prepared for a meeting of Heads of Mathematics, Ottawa Board of Education.)

Since the advent of the New Mathematics, Annual Revision of the Mathematics Courses is something that Mathematics Teachers have come to accept as inevitable. Course changes cause disruptions to both preceding and subsequent courses and compensating revisions are then required. This process has been going on uninterrupted since 1964. All of the Mathematics courses have been revised and the revision of the revisions has passed through Grades 9, 10, 11 and soon Grade 12. The Grade 10 course has not been in existence 2 years. The Grade 11 course is in its initial year and the latest Grade 12 course will be introduced this September (1970).

What has been the result of these revisions? Have they been necessary? Are we satisfied that the resulting courses are the best for the needs of our students?

A feature of the Mathematics' Courses recently revised is the introduction at the Grade 9 and 10 level of topics such as Groups, Fields, Matrices, Vectors and Transformations. These topics were previously not Secondary School subject material. At the early stage they are isolated, have limited application, and are not understood or retained by the

* Reprinted from "MATH MEMO", May 1970; a publication of the Mathematics Subject Council District 26.

average Grade 9 or 10 class. The introduction of Groups for example to Grade 10 last year proved to be a waste of time. The only application of Groups appeared in the Chapters on Vectors and Transformations. The topic of Groups is only supplementary to the work on Vectors and Transformations not vital to their introduction or understanding. Thus the time devoted to Groups might better have been spent in consolidating the work in Geometry as preparation for the Grade 11 Course.

When a course is revised there is usually a great haste or panic to revise the subsequent courses because of the new topics being introduced. This haste is unwarranted. The revisions are not so revolutionary that we cannot take the time to properly assess the good points and weaknesses of one course and its implications for subsequent courses before proceeding to a new one. I have reached the point where I would welcome a respite from course revision, a change from change, a time to carefully consider where we are going and how best to get there. To me a revised course no longer means discovering what is new. I now ask myself "What has been left out?". I seem to spend a good deal of time on repair work, work sheets, review sheets and preparing extra problems to cover weak and neglected areas and I do more of this each year.

The new courses increase the "Operational Skills" of the students by presenting ideas such as Absolute Value,

Complex Numbers, Inequality and restrictions in division, earlier than in former years. At the same time the new courses fail to acknowledge the need to consolidate these ideas by providing the time for it. This is a criticism that is made about all the New Mathematics Courses.

Concerning Grade 9 we benefit from the teaching of New Mathematics in the elementary schools. The introductory work on Sets is especially well retained by the students entering Grade 9. They are also well aware of the Whole Numbers, Natural Numbers and Integers but they lack real understanding of the properties of these number systems. This is not a criticism of the elementary schools. Much time is taken in Grade 9 in the study of these properties but it is not until operations with all of the Number Systems have been studied that their properties are really understood. For this reason this study may well be postponed until Grade 10.

As much time as possible should be spent with the various operations on algebraic expressions, on factoring, equation solving and problem solving. While Absolute Value, Inequalities, Radicals and Graphing on the Number Lines should be introduced; Vectors, 3-Space, Density and Order should be postponed.

Usually Geometry is the last topic covered and some of the slower classes may not receive an introduction to Geometry. Since it is necessary as preliminary work for the Grade 10 course, the following topics should be covered in Grade 9:

- Definitions of angles, triangles, and parts of the circle.
- The 6 Basic constructions and construction problems resulting from them.
- Angle relationships for a transversal between 2 parallel lines.
- An introduction to the idea of congruence.

Although it is not part of the Grade 9 course, a review of the Multiplication tables, Long Division, Decimal Fractions and percent, is time not lost.

In Grade 10 the properties of all the Number Systems should be studied after the students understand the difference between Rational and Irrational Numbers and have studied operations on them.

Geometry is an important part of the course. Some review of construction problems will be necessary. Before entering Grade 11 the student should have lots of practice in the proof of deductions.

Vectors are introduced leading to Transformations and vector proof of deductions in Geometry. If Groups are to be taken, this topic should be studied as supplementary to Vectors where there are applications and not as an isolated topic remote from application.

Fields, 3-Space topics and Matrices may well be omitted.

Attention should be directed to the History of Mathematics and this is an excellent time to introduce Mathematics Projects. At Lisgar we require a short Historical Essay

from the Grade 10 students, a model or demonstration project from Grade 11 and the Grade 13 "MATH A" students are required to submit an eassay on a topic beyond the Grade 13 Course.

Grade 11

With the emphasis on Geometry it is important that students have a good background in Grade 9 and 10 Geometry and have sufficient practice in analysis and proof of deductions before entering Grade 11.

There should be agreement about the deductions which are important enough to be called Theorems. At present each Textbook author has his own ideas not only on what deductions to call Theorems but also the names for the theorems and these ideas may differ considerably from those of a rival author. This makes it very difficult for the student transferring to a new school if it involves a new textbook. The situation is even more confusing when the same publisher calls a theorem in one text what is merely a deduction in the following year's text. Since some uniformity must be reached at least within the school the Department Head and his teachers must decide on a policy for theorems that teachers and students may follow.

Emphasis should be placed on a correct presentation of proofs for deductions. Students should be responsible for the proofs of a certain specified number of Theorems. This

does not impose an unreasonable burden on the students and it will have the benefit of improved solutions of problems not only in Geometry but in Algebra as well.

The work on Vectors, and Transformations is continued in Grade 11 and here again the idea of Groups if introduced in Grade 10 should be reinforced through applications.

Grade 12

The Grade 12 Course is the best Mathematics Course in Secondary School and is excellent preparation for Grade 13. Until a revised Grade 13 course compels it, there is no need for revision of the present Grade 12 course. Certainly nothing in the revised Grade 11 course makes a revision of Grade 12 Mathematics necessary.

The Grade 12 textbooks display a remarkable lack of uniformity in the Mathematical tables used for solution of problems requiring Logarithms and Trigonometric Ratios. Some texts have no Mean Difference Tables and the tables of Trigonometric Ratios are given to Degrees only. The message is clear, "practical mathematics has no place in the 5 yr. A & S programme, the student may ignore completely the precision of a number and significant digits. This was formerly the responsibility of the Mathematics Department but now we need be concerned no longer." It is of concern, however, to the Science Department and here we have the responsibility of complementing the Science courses where possible. It is up to

us to teach for example Ratio and Proportion - not the Science Teacher. This implies that we are acquainted with the material being taught in the Science courses. With the rate of change of the Mathematics courses and Science courses it is becoming increasingly difficult to bring about this desirable state.

Grade 13

The de-emphasis of application for Mathematics is particularly prominent in the Trigonometry portion of the Math A course in Grade 13. While the Winding Function is an excellent method of introducing the topic, all of the Grade 13 Trigonometry remains theoretical. Students would have real difficulty in relating the trigonometry of Grade 13 to the Trigonometry of Grade 11 where the Trigonometric Ratios are introduced by the ratios of the sides of a Right Angle Triangle. The practical applications of Trigonometry in Grade 11 are confined to the use of The Law of Sines and the Law of Cosines in a section too brief to be of value.

A Comment on Texts

With the course revisions has come the emphasis on discovery in the textbooks. Texts with this emphasis are almost useless for review or if a student is catching up after an absence from school. The need remains for texts with clear examples showing the solution to types of problems.

AN EXPERIMENT IN PROGRAMMED LESSONS IN MATHEMATICS AT GLOUCESTER HIGH SCHOOL*

Melba F. Christie, Gloucester High School

How many times has every teacher been faced with the hopeless situation of having six or more students come for help on different topics all at one time? Each one expects in one half hour or less some magical illumination that would clearly reveal all the ideas of lessons missed through illness, field trips, sports events, lack of attention, or lack of ability to comprehend. Besides trying to cope with this difficulty, the conscientious teacher is groping with the trend of letting each student progress at his own speed.

Is there a solution? Yes, programmed lessons.

We are all more or less familiar with this type of lesson. Industry is using them to advantage. Many programs are being written in the United States. In Ontario they are growing in popularity on the elementary level, but for Highschool students there are few available in Mathmatics adapted to the Ontario course of study.

There are books available, but they are costly, bulky, and inadequate. If every Highschool course in Mathematics for both four-year and five-year programs were written in lesson units and labelled, then when a student missed a day,

* Reprinted from "MATH MEMO", May 1970; a publication of the Mathematics Subject Council District 26.

a week, a month, or needed extra help, he could be provided with these lessons and allowed to proceed on his own. The lessons could also be used as a complete course.

With these ideas in mind, the Mathematics Department of Gloucester Highschool is trying out in seven classes of Level II in the Four-Year Program a set of eight lessons on Trigonometry written by the author of this article.

These lessons are written in a conversational style as a teacher would speak to a student, describing, asking questions, giving encouragement. They are given out one at a time. A student feels a sense of accomplishment every time he hands in one and takes another. The teacher scans, but does not mark the lessons. The student likes the idea that the teacher is going to see his work. After the eight lessons are completed, a diagnostic test will be given and marked.

It should be pointed out that the teacher does no formal teaching for these lessons. He or she is entirely a resource person allowing each student to proceed at his own rate and to ask for help only when it is needed. If a library were set up, as afore mentioned, a student could proceed unit by unit, repeating a unit as necessary, or completing two years work or a fraction thereof in one. Mathematics, which is so highly organized and structured is especially adaptable to this type of treatment.

With the wide range of ability so many teachers find in each class, no longer would the student who needs extra help be left helpless and hopeless, nor would the keen student be held back by repeated explanations. Each student could find in programmed lessons a Mathematics suited to his needs, a subject at which everyone could succeed on his own level. Teachers would be relieved of the problems created by the bored and lost, and could truly shine in giving help just where it is needed and in preparing enrichment material.

Programmed lessons have a great future!

MATHEMATICAL NOTES

REMOVING BRACKETS FOR SIMPLIFYING EXPRESSIONS*

D. Wilson, Hillcrest Highschool

I will assume that the students have developed, by use of vector diagrams and intuition, the ability to add and subtract integers with raised signs, and that they have discovered the sign conventions for multiplication of integers.

Furthermore, the student should be well acquainted with the following properties of the mathematic system (I,+)

(a) Associativity under addition

(b) Commutativity under addition

(c) Distributive property of multiplication or addition

(d) Property of opposites or additive inverse.

(Let me stress that -x should be interpreted as 'the opposite of x' and NOT 'negative x'.)

Let me now consider a number of examples which I suggest progress in difficulty and would be covered over a series of lessons. The intent here is to give the student a consistent approach to operating with '_' signs and brackets. Since subtraction is the inverse operation of addition, I choose to interpret all expressions in terms of addition and rely on the properties stated above.

* Reprinted from "MATH MEMO", May 1970; a publication of the Mathematics Subject Council District 26.

EXAMPLE 1 (Elimination of raised signs)

Simplify

$^{+}2 + {}^{+}3 + {}^{-}1$

Teaching solution

We will agree to write $^{+}2$ as +2

$^{-}1$ as -1

etc.

$^{+}2 + {}^{+}3 + {}^{-1}$

= (+2) +(+3) + (-1)

Further, let us leave out the 'addition' signs. This convention allows us to remove the brackets and thus simplify the appearance of the expression.

= +2 +3 -1

(The student may read this as 'positive 2' and 'positive 3' and 'negative 1').

We may also write +2 as 2 since number is represented by the integer +2 and the whole number 2.

= 2 + 3 - 1

= 5 - 1

= 4

Note: Here we are using the associative property of addition so that it is reasonable to interpret

2 + 3 - 1

as (+2) + (+3) + (-1)

and not to consider subtraction.

Efficient solution

$^{+}2 + {}^{+}3 + {}^{-}1$

= 2 + 3 - 1

= 5 - 1

= 4

EXAMPLE 2 (Orderly collection of terms)

Simplify (+1) + (-2) - (+3) - (-4)

Teaching solution

(+1) + (-2) - (+3) - (-4)

Let us use the rule for subtraction and change to addition by 'adding the opposite'

= (+1) + (-2) + (-3) + (+4)

= 1 - 2 - 3 + 4

= 1 + 4 - 2 - 3

I suggest that the students be orderly and collect the positive terms together and then the negative terms in the order they occur from left to right.

Note that we are applying the commutative properties of addition here so that it is wise that the student interpret

1 - 2 - 3 + 4

as '1' and '-2' and '-3' and '4'

i.e. (1) + (-2) + (-3) + (4)

continuing the solution we have

= 5 - 5

= 0

Efficient Solution

(+1) + (-2) - (+3) - (-4)

= 1 - 2 - 3 + 4

= 1 + 4 - 2 - 3

= 5 - 5

= 0

EXAMPLE 3 (Expressions involving variables)

Simplify 5b - 3 - 7b + 4 + 2b

Teaching Solution

Once again the students should interpret this as

(+5b) + (-3) + (-7b) + (+4) + (+2b)

or (5b) + (-3) + (-7b) + (4) + (2b)

if we agree to write 2 for +2 etc.

Then 5b - 3 - 7b + 4 + 2b

= 5b - 7b + 2b - 3 + 4

We collect the terms involving variables and then the number terms in an orderly way. It is expected that it is not necessary to write

5b - 7b + 2b

as 5b + 2b - 7b (i.e. collecting positive and negative terms)

at this stage of development.

The student should arrive at the following result immediately.

$= 0b + 1$

$= 1$

Efficient solution

$5b - 3 - 7b + 4 + 2b$

$= 5b - 7b + 2b - 3 + 4$

$= 0b + 1$

$= 1$

EXAMPLE 4 (Distributing other than $\pm$ 1).

(a) $3x + 2(x + 5)$ (b) $3x + 2(x - 5)$

Teaching solution

$3x + 2(x + 5)$	$3x + 2(x - 5)$
We interpret as	or
$(3x) + (2)\left[(x) + (5)\right]$	$(3x) + (2)\left[(x) + (-5)\right]$ *
$= (3x) + (2)(x) + (2)(5)$	$= (3x) + (2)\ (x) + (2)\ (-5)$
$= 3x + 2x + 10$	$= 3x + 2x - 10$
$= 5x + 10$	$= 5x - 10$

*Note - We distribute multiplication over addition and therefore choose to interpret

$x - 5$

as $(x) + (- 5)$

This is consistent with the previous results.

Efficient solution

$$3x + 2(x + 5)$$
$$= 3x + 2x + 10$$
$$= 5x + 10$$

$$3x + 2(x - 5)$$
$$= 3x + 2x - 10$$
$$= 5x - 10$$

(c) $3x - 2(x + 5)$

(d) $3x - 2(x - 5)$

Teaching solution

$$3x - 2(x + 5)$$
$$= (3x) + (-2)\left[(x) + (5)\right]$$
$$= (3x + (-2)(x) + (-2)(5)$$
$$= 3x - 2x - 10$$
$$= x - 10$$

$$3x - 2(x - 5)$$
$$= (3x) + (-2)\left[(x) + (-5)\right]$$
$$= (3x) + (-2)\ (x) + (-2)\ (-5)$$
$$= 3x - 2x + 10$$
$$= x + 10$$

Efficient solution

$$3x - 2(x + 5)$$
$$= 3x - 2x - 10$$
$$= x - 10$$

$$3x - 2(x - 5)$$
$$= 3x - 2x + 10$$
$$= x + 10$$

EXAMPLE 5(Distributing $\overset{+}{-}$)

(a) $3x + (x + 5)$

(b) $3x + (x - 5)$

Teaching solution

$$3x + (x + 5)$$
$$= (3x) + (+1)\left[(x) + (5)\right]$$
$$= (3x) + (+1)\ (x) + (+1)\ (5)$$
$$= 3x + x + 5$$
$$= 4x + 5$$

$$3x + (x - 5)$$
$$= (3x) + (+1)\left[(x) + (-5)\right]$$
$$= (3x) + (+1)\ (x) + (+1)\ (-5)$$
$$= 3x + x - 5$$
$$= 4x - 5$$

Efficient solution

$3x + (x + 5)$	$3x + (x - 5)$
$= 3x + x + 5$	$= 3x + x - 5$
$= 4x + 5$	$= 4x - 5$

Let us interpret this as adding the terms inside the bracket as they appear. We can consider

$3x + (x - 5)$

as '3x' and 'x and negative 5'

(c) $3x - (x + 5)$ (d) $3x - (x - 5)$

Teaching solution

$3x - (x + 5)$	$3x - (x - 5)$
$(3x) + (-1)[(x) + (5)]$	$(3x) + (-1)[(x) + (-5)]$
$= (3x) + (-1)(x) + (-1)(5)$	$= (3x) + (-1)(x) + (-1)(-5)$
$= 3x - x - 5$	$= 3x - x + 5$
$= 2x - 5$	$= 2x + 5$

Instead of using the distributive property it seems more convenient to 'add the opposite' and we obtain immediately the

Efficient solution

$3x - (x + 5)$	$3x - (x - 5)$
$= 3x + (-x) + (-5)$	$= 3x + (-x) + (+5)$
$= 3x - x - 5$	$= 3x - x + 5$
$= 2x - 5$	$= 2x + 5$

Step two could be conveniently omitted.

EXAMPLE 6 Application

Simplify $x(x - 2) - 2x(-x + 1) - (x - 3)$

Teaching solution

Interpretation of the expression:

$(x)\left[(x) + (-2)\right] + (-2x)\left[(-x) + (1)\right] - (x - 3)$

the opposite

$= (x)(x) + (x)(-2) + (-2x)(-x) + (-2x)(1) \underline{- x + 3}$

$= x^2 - 2x + 2x^2 - 2x - x + 3$

$= x^2 + 2x^2 - 3x - 2x - x + 3$

$= 3x^2 - 5x + 3$

Efficient solution

$x(x - 2) - 2x(-x + 1) - (x - 3)$

$= x^2 - 2x + 2x^2 - 2x - x + 3$

$= x^2 + 2x^2 - 2x - 2x - x + 3$

$= 3x^2 - 5x + 3$

ABSOLUTE INEQUALITIES

David B. Wing, McArthur College, Kingston

Consider the two sets A and B defined as follows:

(i) $A = \{x \in R \mid |2x-1| \leq S-x\}$

If $2x - 1 \geq 0$, then by definition $|2x - 1|$ can be replaced by $2x - 1$.

So $(2x-1) \leq 5-x$

$\iff 3x \leq 6$

$\iff x \leq 2$

Since $2x-1 \geq 0 \iff x \geq \frac{1}{2}$, then $x \leq 2$ PROVIDED $x \geq \frac{1}{2}$. Possible solutions are given by

$\frac{1}{2} \leq x \leq 2$.

OR

If $2x - 1 < 0$, then by definition $|2x - 1|$ can be replaced by $-(2x - 1)$. Thus

$-(2x - 1) \leq 5 - x$

$\iff -2x + 1 \leq 5 - x$

$\iff -x \leq 4$

$\iff x \geq -4$

Since $2x-1 < 0 \iff x < \frac{1}{2}$, then $x \geq -4$ PROVIDED also that $x < \frac{1}{2}$. Possible solutions

(ii) $B = |x \in R| |2x - 1| \geq S-x$

If $2x - 1 \geq$, then by definition $|2x-1|$ can be replaced by $2x - 1$

So $(2x - 1) \geq 5 - x$

$\iff 3x \geq 6$

$\iff x \geq 2$.

Since $2x - 1 \geq 0 \iff x \geq \frac{1}{2}$, then if $x \geq 2$, certainly $x \geq \frac{1}{2}$. Possible solutions are given by

$x \geq 2$.

OR

If $2x-1 < 0$, then by definition, $|2x - 1|$ can be replaced by $-(2x-1)$ Thus

$-(2x - 1) \geq 5 - x$

$\iff -2x + 1 \geq 5 - x$

$\iff -x \geq 4$

$\iff x \leq -4$.

Since $2x-1 < 0 \iff x < \frac{1}{2}$, then if $x \leq -4$, then certainly $x < \frac{1}{2}$. Possible solutions are given by

are given by

$-4 \leq x < \frac{1}{2}$.

$x \leq -4$.

Therefore if $x \in A$, x must satisfy $\frac{1}{2} \leq x \leq 2$ or $-4 \leq x < \frac{1}{2}$.

$\iff -4 \leq x \leq 2$

$\iff -4 \leq x$ AND $x \leq 2$

i.e. $A = \{x \in R \mid -4 \leq x$ AND $x \leq 2\}$.

Therefore if $x \in B$, x must satisfy $x \geq 2$ at $x \leq -4$

i.e. $B = \{x \in R \mid x \leq -4$ OR $x \geq 2\}$.

It would be tedious to go through either of these solutions each time one of these problems is encountered. So we agree, having seen a formal explanation such as the one above, to replace anything of the form $|x| \leq a$, $a \in R^+$ by

$-a \leq x$ and $x \leq +a$ or, more briefly

$-a \leq x \leq a$,

and

$|x| \geq a$, $a \in R^+$ by

$x \leq -a$ OR $x \geq a$.

The justification for these past two statements follows the pattern outlined above.

RESOLVING THE "AND - OR" DIFFICULTY IN ABSOLUTE INEQUALITIES

Frank Sweet, Bruce Peninsula D.H.S., Lion's Head, Ontario

When faced with the problem of solving absolute inequalities, students are often confused as to why in "less than" problems " and" (set intersection) is used, while "or" (set union) is used in connection with inequalities of the "greater than" type.

It is the purpose of this note to point out that "or" is, in fact, correct <u>in both cases</u>, although for the "less than" inequality the set union may be reduced to an intersection.

To illustrate the argument we shall solve

$$|x| < 2 \quad (x \in R) \qquad (1)$$

The formal definition

$$|x| = x \text{ if } x \geq 0$$
$$= -x \text{ if } x < 0$$

leads us to consider two cases:

(a) If $x \geq 0$, then $|x| < 2$

$$\iff x < 2$$

$$\therefore x \geq 0 \text{ } \underline{\text{and}} \text{ } x < 2$$

$$\text{so that } 0 \leq x < 2 \qquad (2)$$

(b) If $x < 0$, then $|x| < 2$

$\Longleftrightarrow \quad -x < 2$

$\Longleftrightarrow \quad x > -2$

$\therefore\ x > -2$ and $x < 0$

so that $-2 < x < 0$ (3)

The difficulty seems to arise through the neglect of the two-sided nature of inequalities (2) and (3). If $x < 2$ and $-2 < x$ are used, then one must (apparently arbitrarily) introduce set intersection ("and") to achieve the correct solution. This is not necessary, however.

To continue: Since $x \geq 0$ or $x < 0$ $(x \in R)$ the solution set of inequality (1) is the union of the two (disjoint) sets

$$\{x \in R \mid -2 < x < 0\}\ ,\quad \{x \in R \mid 0 \leq x < 2\}$$

i.e. $\{x \in R \quad -2 < x < 0 \ \underline{\text{or}}\ 0 \leq x < 2\}$

That this union is equivalent to the (intersection) set

$$\{x \in R \mid -2 < x < 2\}$$

is clear and it is at this point that a "practical rule" (the "and" rule) may be introduced, if desired.

We now solve

$$|x| > 2 \quad (x \in R) \qquad (4)$$

Again there are two cases:

(a) If $x \geq 0$, then $|x| > 2$

$$\iff x > 2$$

$$\therefore x \geq 0 \text{ and } x > 2$$

so that

$$x > 2. \qquad (5)$$

(b) If $x < 0$, then $|x| > 2$

$$\iff -x > 2$$

$$\iff x < -2$$

$$\therefore x < -2 \text{ and } x < 0$$

so that

$$x < -2 \qquad (6)$$

Since $x \geq 0$ or $x < 0$ $(x \in R)$

the solution set of inequality (4) is the union of the two (disjoint) sets

$$\{x \in R \mid x < -2\}, \quad \{x \in R \mid x > 2\}$$

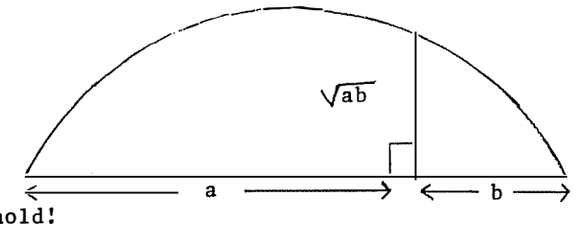
i.e. $\{x \in R \mid x < -2 \text{ or } x > 2\}$.

In this case there is no convenient reduction of the union to an intersection.

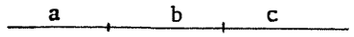
INEQUALITIES

(Based on a talk by Dr. L. Woodby of Michigan State, Grand Rapids, at the Milwaukee NCTM; submitted by J. LeSage, Program Consultant, Region #7)

1. Prove geometric mean $\leq$ arithmetic mean

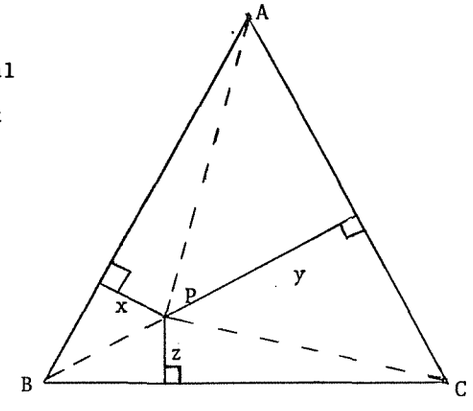


Behold!

2. 

Break a stick arbitrarily into three parts. What is the probability that they will form a triangle? First consider another problem.

$\triangle ABC$ is equilateral and P is any point in or on $\triangle ABC$



$2\Delta ABC = BC \cdot h$ where h is the altitude.

$2\Delta PBC = BC \cdot z$

$2\Delta PAC = AC \cdot y$

$2\Delta PAB = AB \cdot x$

$\therefore x + y + z = h$

Now consider the original problem.

In order to form a triangle,

$a < \frac{1}{2}\ell$ and $b < \frac{1}{2}\ell$

and $c < \frac{1}{2}\ell$ where ℓ is the length of the stick. If we look at ΔABC, the corresponding restriction is

$X, Y, Z < \frac{1}{2} h.$

If $x < \frac{1}{2} h$, then P must lie in the region ABDE where E,H,D are the midpoints of CA, CF, and CB respectively.

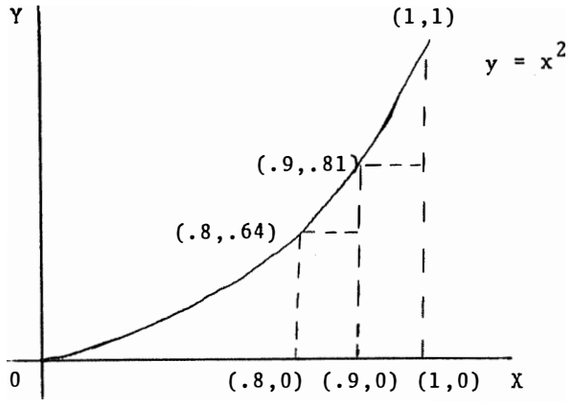
Similarly P must also lie in the regions AFDC and BCEF.

$\therefore$ P must lie in the intersection of the three regions, i.e., ΔDEF.

$\therefore$ Probability that P be such that $X, y, z < \frac{1}{2} h$ is $\frac{\Delta DEF}{\Delta ABC} = \frac{1}{4}$.

$\therefore$ In the original problem, the restriction that $a, b, c < \frac{1}{2}\ell$ imposes a probability of $\frac{1}{4}$.

3.



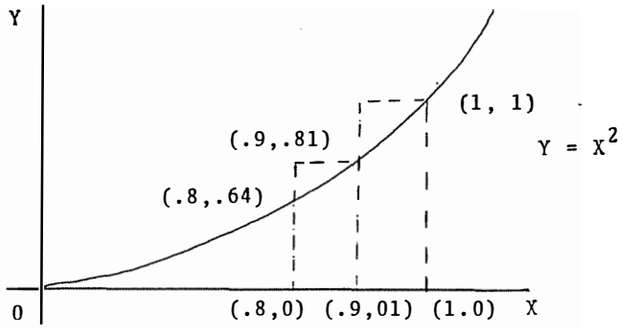
Find the area under the curve A. Notice that $A < \frac{1}{2}$.

Draw ten rectangles with base 0.1.

Their area is $0.1\ (0.9^2 + 0.8^2 + \ldots + 0.1^2)$

$0.001\ (9^2 + 8^2 + \ldots + 1^2) < A < \frac{1}{2}$.

Now consider the rectangles that are above the curve.



$\therefore .1\ (1^2+0.9^2+0.8^2+\ldots+0.1^2) > A$

$\therefore 0.001\ (1^2+2^2+\ldots 9^2) < A < 0.001\ (1^2+2^2+\ldots+9^2+10^2$.

Notice that the difference between the two expressions is $0.001 \times 10^2 = 0.1$.

So, we already have a reasonably accurate expression for A.

Recalling that $\sum_{1}^{n} x^2 = \frac{n(n+1)}{2}\cdot\frac{(2n+1)}{3}$

$0.285 < A < 0.385$.

Now, let the increment be 0.01 instead of 0.1

$\therefore 0.01(0.01^2+0.02^2+\ldots+0.99^2) < A < 0.01(0.01^2+\ldots+0.99^2+1^2)$.

$\therefore 10^{-6}(1^2+2^2\ldots+99^2) < A < 10^{-6}(1^2+\ldots+99^2+100^2)$

$0.32835 < A < 0.33835$

$A \doteq 0.3$.

Continue the process and obtain $0.3328335 < A < 0.3338335$ and we notice an interesting pattern developing on both sides.

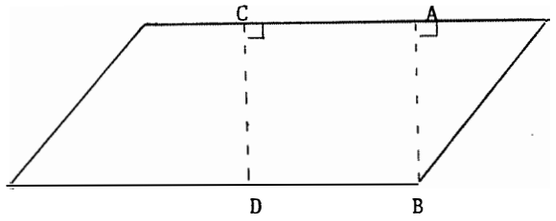
Among other things, we see an extra 3 appearing in the tenth place on both sides each time.

$\therefore \lim A = \frac{1}{3}$.

TIPS

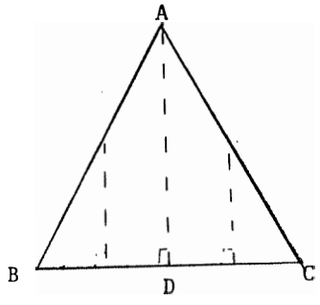
(Based on a talk by Professor Dale Jungst, Northern Illinois University, Dekalb, Illinois, at Milwaukee NCTM; submitted by J. LeSage, Program Consultant, Region #7)

1. Area of Parallelogram



Instead of cutting the parallelogram along AB and rearranging the pieces to make a square, try cutting along any line CD (perpendicular to one of the sides).

2. Area of Triangle



Fold on altitude AD.

Fold B to D, C to D and A to D to form a rectangle with base ½BC and height ½ AD whose area is ½▽ ABC.

∴ ½Δ ABC = ½ BC. ½AD

∴ ½ Δ ABC = ½ BC.½ AD.

3. GEO BOARD

Glue a piece of indoor/outdoor or kitchen carpet to the back of the geoboard. It makes it quieter and you can use it for a nail board. Somebody in the audience suggested that wig pins were good pins to use. (I wouldn't know - I use toupee pins.)

PROBLEMS AND SOLUTIONS

Edited by W. Fraser

Please send solutions and problems you would like to suggest for inclusion in this section to Professor W. Fraser, Department of Mathematics and Statistics, University of Guelph, Guelph, Ontario.

Problems

Problem 70.07. Using cents, nickels, dimes, quarters and half-dollars, in how many different ways is it possible to provide change for one dollar?

Problem 70.08. Given the equations of three lines in the plane which do not intersect in a point, show how to find the coordinates of the centre of the inscribed circle without the need of plotting the lines, or of finding coordinates of the vertices of the triangle. A desirable solution would be considered as one that could be programmed for a computer. In comparing solutions the one which calls for the less effort would be considered better.

Solutions

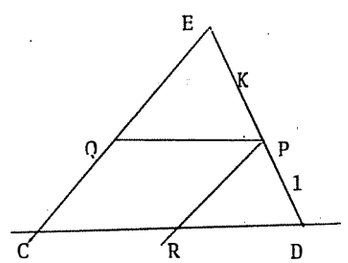
Solutions to problem 70.04 have been submitted by T. J. Griffiths, North Lambton Secondary School, and

Romesh Singh, Arthur District Highschool. The easiest procedure to apply appeared to be the use of induction. For this purpose let

$$u_n = \frac{1.3.5....(2n-1)}{2.4.6....(2n)}.$$

The assumption of the truth of both parts of the given inequality for an integer n thus leads by easy algebra to the implications of this truth for the integer (n + 1). The relation is easily verified to be true for n = 1 and the result follows.

Solutions to problem 70.05 have been submitted by P. Weygang, Levack Highschool, and R. Koslash and T. J. Griffiths of North Lambton Secondary School. The latter solution is presented below.



Given the lines CQE and CRD, and the point P, complete the parallelogram CRPQ, and let ED be a line through P with the ratio

$$\frac{|PE|}{|PD|} = \frac{K}{1}.$$

Let the area of $\triangle RDP = A$ and the area of the parallelogram CRPQ equal B, a constant. Note that the triangles RDP, QPE and CDE are similar. It follows that the area of triangle $QPE = K^2A$ and the area of $\triangle CDE = (1 + K)^2A = \Delta$, the area to be minimized. We have

$B = (1 + K)^2A - K^2A - A = 2KA$, hence $A = \frac{B}{2K}$.

Therefore $\Delta = A(1 + K)^2 = \frac{B}{2K}(1 + K)^2 = \frac{B}{2}(\frac{1}{K} + 2 + K)$.

This letter expression is minimized for K = 1, which tells us that the required line DE should be drawn so that P is its mid-point.

Introduction to Applied Mathematics, Vol. 1. By J. Genin and J. Maybee. Holt, Rinehart and Winston (1970).

This is primarily a textbook on ordinary and partial differential equations catering to an undergraduate level course. As the name implies the text is practically oriented, not only in the worked out examples, but also in the selection of problems found at the end of the various sections. The problems given are quite numerous and challenging.

Electrical and mechanical examples are primarily used in the exposition of the first two chapters on ordinary differential equations. The questions of uniqueness and stability are also referred to. Simultaneous differential equations are presented along with a brief but adequate introduction to linear algebra. Matrices are subsequently applied to the solution of ordinary differential equations.

In the remaining three chapters partial differential equations are applied to boundary value problems in heat conduction, diffusion, vibration of bars, strings, etc. The method of separation of variables is used in the final chapters as the unifying link between ordinary and partial differential equations. In addition to this, alternative methods of solving partial differential equations are compared such as the energy integral, separation of variables, finite-differences and influence functions.

The authors periodically refer to the practical question regarding the adequacy of the mathematical model in representing the actual physical system.

In conclusion, the text is written very lucidly and it should prove to be a stimulating source of applied problems.

L. Marzec,
Mohawk College.

Introduction to Computational Science and Data Processing, 2nd edition. By Richard N. Schmidt and William E. Meyers. Holt, Rinehart and Winston (1970). xxix + 389.

Introduction to Computer Science and Data Processing presents a large body of material in a well prepared textbook. Reading of the textbook is facilitated by the layout of the pages, the clearly constructed diagrams, and the legibility of the various print forms employed. The accompanying soft covered Instructor's Manual introduces the purpose of each chapter together with suggestive aids, teaching outlines, and answers to questions at the end of each chapter.

In revising their original textbook and publishing this second edition, the authors' objectives, as presented in the preface, have been achieved. These objectives are:

(1) to acquaint the students with the large body of material which computer science and data processing encompasses
(2) to prepare those students who decide to enter the computer field with a firm foundation so that they can advance in

The rules of integral calculus are stated with little or no proofs and are then applied to the calculations of simple area, work and pressure problems.

The last topic treated in the textbook is the development of functions using the Maclaurin's and Taylor's theorems.

As the title suggests this book is technically oriented with the emphasis being placed on the solution of practical problems. Involved and lengthy proofs are minimized and numerous illustrative examples with step-by-step solutions are given. There are many drill type questions as well as word problems provided.

The nature and the scope of this text would make it suitable for use in the calculus portion of Mathematics for most of the Two-year Technician Programs of the Community College level. It would also be useful reference text for Grade XIII.

V. Frankovich,
Mohawk College.

Program Flowcharting. By George A. Gleim. Holt, Rinehart and Winston (1970).

The author notes that most students have difficulty in developing good flow charts for problem solving using computers, and that the writing of good flow charts is essential for complete understanding of the problem, efficient use of the computer, and providing unambiguous documentation.

This small reference guide (70 pages) is an attempt to develop this facility by using simple, but "real-life" examples of problems with which the student could be confronted as a programmer or analyst in industry. It would serve as a useful reference guide for the computer programmer/analyst student. The flow charting symbols conform closely to both IBM standards and USASI (United States of America Standards Institute).

The material by chapter heading covers: Flowcharting Symbols, Looping, Flowcharting Guidelines, Simplification of Flowcharts, Heading Routines, Inventory Control, The System Flowchart, Personnel File Search, File Maintenance Simulation, and Scientific Examples.

Each chapter contains one or two well chosen examples for illustration, and a few practice exercises. No solutions are presented for those practice exercises.

It is emphasized in the concluding chapter that "a flowchart is independent of the language in which it is to be coded". An illustrative problem is shown coded in COBOL and FORTRAN.

P. Fletcher,
Mohawk College.

Geometric Vectors. Copp Clark (1968). 42 pp. $1.60.

Geometric Vectors is one of the series of seven booklets published by Copp Clark (1968) to handle the seven optional

topics established for the Grade 10 four year programme. These booklets are intended to supplement the text Mathematics 410.

Geometric Vectors lacks an index or table of contents, however the following topics are covered:

(1) scalar and vector quantities
(2) geometric representation of scalar and vector quantities
(3) addition of two collinear vectors
(4) addition of two non-collinear vectors
(5) parallelogram and triangle laws for vector addition
(6) addition of three or more non-collinear vectors
(7) multiplication of a vector by a scalar.

The format is clear and use is made of two colours to emphasize important relations and to distinguish resultant vectors from their components.

Problems are provided at the end of each section and are as practical as possible. There is also a review exercise at the end of the booklet and answers are provided for most of the questions.

Unfortunately the booklet contains several annoying inconsistencies such as referring to weight in one section as a scalar and in another section as a force. A vaguely stated problem results in two possible solutions when only one was intended. One question asks to determine the magnitude of a vector to the nearest pound but the answer given is to the nearest $\frac{1}{10}$ pound. Some answers are incorrect and so on.

Despite these annoyances I feel the booklet would be useful to help the teacher cover the topic of vectors at the Grade 10 four year level.

K. Jones,
Anderson High School,
Whitby.

Managing Personal Property. Copp Clark.

Unfortunately this option booklet "Managing Personal Property" was reviewed in isolation and not with the core text or the other option booklets. The observations made keep this in mind.

The booklet is simply bound and contains a good quality paper which should survive a reasonable length of time if handled respectfully. The type is easily read with adequate spacing between sub-topics, examples and problems. The underlining and "boxing" of statements and formulae is excellent and not overdone. Drawings, charts and tables are clear and easily scrutinized.

The written material in each section of the booklet is concise and the presentations are immediately reinforced through examples and a variety of problems. A reasonable range of skills is contained within the problems and although I do not necessarily agree with the approach taken, the skills must be mastered if the student is to be successful.

The major weakness of this particular option booklet is the topics chosen by the authors, "Personal property takes many forms-" this cannot be disputed but the "forms" presented are unrealistic, impersonal and lack imagination for this age group. I cannot see a student at this level relating to:

- deposits, withdrawals and bank balances in the thousands of dollars.
- investing $10,000 in a mortgage,
- purchasing a $5,000.00, 20 payment life insurance
- insuring a home for $19,000.00 at 76¢ per $100.00 for three years.

Mathematical skills, concepts and attitudes are equally important and are grasped, learned, mastered more efficiently if they are related to the student's experience and interest.

The two sections which are meaningful are the last two in the booklet, namely "Automobile Insurance", and "Cost of Automobile Ownership". These are of interest to today's teenager; they are personal; they are realistic; they could do a lasting service for students in helping them manage their personal property.

The underlying principles inherent within this volume could be taught as easily and with greater student interest if the authors related these principles to the property of their parents.

Are we to prepare the student who takes this option for a vocation or are we to assist him to live and learn?

I. M. Fraser,
Fisher Heights Public School.

Integration of Ordinary Differential Equations. By E. L. Ince. Oliver and Boyd Limited, Edinburgh, Seventh Edition (1963). 146 pp.

Professor Ince in his preface states that "the object of this book is to provide in a compact form an account of the methods of integrating explicitly the commoner types of ordinary differential equations." There is no doubt that he attains his objective. Perhaps there are times when clarity is sacrificed to compactness, but too much should not be made of this - the text is, after all, designed for university students who may be assumed to have attained some degree of mathematical maturity.

In general, the text is extremely well written and the language is clear, unambiguous, and refreshingly free from grammatical errors. The steps taken in the solution of the various types of ordinary differential equations are clearly indicated, by cross references where necessary. Sufficient examples are given of each type and these should help the student to see the application of the general method to specific equations.

Because the purpose of the book is to provide the subject matter in a compact form, the type used for the examples is small. This, surprisingly enough, is almost always quite clear even, for example, when an exponent has the form $\int p\,dx$. There were, considering the small type, very few ambiguities and in

almost all the cases I noted there was no difficulty in identifying the correct symbol from the context.

The text is rich in examples - 300 equations - and the answers to all the questions are given. The examples are roughly in the order in which the equations are discussed. This should enable the student to become expert in the solution of equations and at the same time should test his ability to recognise the various types which is, of course, a necessary condition for success in this area. Though I did not attempt to answer all 300 questions, I found no errors in the solution of those I worked out.

I would recommend this book as either a text or a reference book for those students who need the minimum amount of theory but who do wish to obtain a practical knowledge of the methods of solution of the ordinary differential equation.

E. Glyn-Jones,

Niagara College of
Applied Arts and
Technology.

Mathematiques Contemporaines 4. By Brouillerd, Basque and Labelle. Holt, Rinehart and Winston. $5.50.

Vaudrait-il la peine d'adopter le nouveau manuel Mathématiques contemporaines pour une classe de 10e?

Avant de repondre a cette question, analysons ce manuel sous differents aspects:

1. Connaissance du sujet: Le manuel montre que les auteurs sont d'excellents mathematiciens. La principale erreur qu'ils ont faite sur ce point est de commencer la géométrie avec rapports et proportions. Ils auraient dû au moins donner une revue substantielle de la géométrie depuis la définition d'un point jusqu'au sujet de rapports et proportions.

2. Precision des informations: Très bien, quoique couchée en des termes trop difficiles pour l'étudiant moyen. Il y a cependant trop d'erreurs et de répétitions qui n'ont pas encore été corrigées.

3. Style, qualités littéraires: Très bien, exagérés meme, ce qui rend le texte difficile pour n'importe quel élève non brillant.

4. Méthode de présentation: La présentation est claire, les résumés de chapitre sont excessivement bien faits. Cependant on n'a fait aucun usage de couleurs ni de gravures, ce qui rend le texte un peu rébarbatif.

5. Format: Bien choisi, plutôt attrayant.

6. Présentation artistique: Mathématiquement froide. Ni couleurs, ni illustrations, ni portraits, ni humour.

7. Typographie et impression: Bien choisies, très lisibles.

8. Reliure et couverture: Belles, solides, attrayantes.

9. Adaptation a la 10e année: Là est le plus grand désappointement de ce manuel pour nos dixièmes années francaises

en Ontario. On semble avoir préparé ce volume comme un compromis entre les programmes de l'Ontario et de ceux du Québec, dans l'espoir sans doute de profiter des deux marchés. Malheureusement cette dualité d'allégeance a considérablement nui à l'exploration du sujet tel qu'il devrait être couvert dans les programmes ontariens, au point qu'il manque de grandes sections appartenant à la 10e année et que d'autres sujets sont couverts qui devraient l'être seulement en douzième année.

Après ces différentes considérations, il est facile de répondre à la question du début. Il n'est pas à mon avis recommendable d'adopter les Mathématiques contemporaines pour une classe ordinaire de 10e année. Tout au plus peut-on se risquer à l'employer dans une classe avancée, et même là, je ne le recommenderais qu'a titre de référence.

Il semble que la meilleure solution pour trouver un texte francais convenant aus 10e années serait d'en écrire un soi-meme. Avis aux intéressés.

Robert L. Couture,
Academie de la Salle.

MEETINGS

Spring in Niagara Falls

The Annual Conference of the O.A.T.M. will be held at the Sheraton Brock Hotel in Niagara Falls, Ontario on March 21, 22, 23, 1971.

The practice of having the Conference held in a different centre each year, started in Ottawa in 1970, is thus continued. This will enable the delegates eventually to become acquainted with all areas in Ontario, and this, it is hoped, will result in a broadening of the membership.

The Keynote Speakers this year will be Dr. Z. Usiskin of the University of Chicago and Mr. Arthur Damon of the Department of Education (North Bay). Dr. Usiskin will be speaking on "The New Mathematics: Have we Gone Far Enough?" and Mr. Damon's topic will be "The Development of Elementary School Mathematics".

Make a New Year's Resolution now. Come to Niagara Falls in March.

For more information write to:

Mr. Alex Prokich

St. Joseph's School

Crimsby, Ontario.

O.A.T.M.

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