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MATH

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DIVISION OF DECIMAL NUMBERS AND EQUIVALENT RATIOS

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ABACUS CO-EDITORS



MARY LOU KESTELL works on K to 12 mathematics professional learning at the provincial level. She has spent 40+ years working in all aspects of Ontario mathematics education and is a past-president of both OAME and OMCA.



KATHY KUBOTA-ZARIVNIJ currently focuses on K to 6 mathematics professional learning at the provincial level. She is a long-time OAME board member and OAME past president. In relation to her work in mathematics education, she uses complexity thinking to interpret mathematics teaching and learning for students and teachers.

How are these problems similar and different?

a) A roll of ribbon is 21.6 m long. How many ribbon pieces, 0.4 m long can be cut from the roll?

b) Alice travels by bus 2.5 km on Mondays, 3.6 km on Tuesdays, 6.75 km on Thursdays and 12.8 km on Saturdays. What is the average distance Alice travels each day? (nearest 0.01 of km)

c) Pine trees grow at a rate of 0.61 m per year. How many years, would it take it to grow to about 18.3 m?

d) Store A sells 24 breakfast bars for $11.04. Store B sells 9 breakfast bars for $4.05. Which is the better buy?

e) The area of a rectangular garden is 6.72 m^2. If one side length is 2.1 m, what is the length of the other side?



All problems have a decimal number dividend. The problems differ in these ways: divisor and quotient is a whole or decimal number; representations of decimal division: (a) quotative division; (b) partitive division; (c) multiplicative comparison – as many times as; (d) unit rate; (e) area and linear dimensions of a rectangle; (f) equivalent ratios.

In this issue, the Research Summary and contextual problems (solved with different methods) focus on division of decimal numbers and equivalent ratios. Problems in this issue are designed for use using a three-part problem-solving lesson.

RESEARCH SUMMARY – DIVISION OF DECIMAL NUMBERS AND EQUIVALENT RATIOS

How did you learn to divide decimal numbers by whole and decimal numbers? Did you use these popular division methods? Did you learn why these methods work? Skemp (1976) would say that if students only know how to follow these decimal division steps, then they are developing a limited understanding of decimal number division, instrumental understanding, without relational understanding.

Method 1 - Divide as Whole Numbers

- Divide the dividend by the divisor without decimal points, 6.6/6 = 66/6
- Put the decimal places back; 6.6 has one decimal place and 6 has no decimal place.
- The quotient (the result of division), 11 is adjusted back to the original dividend of 6.6, having one decimal place, 1.1.

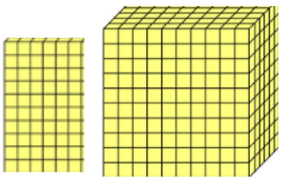
Method 2 - Move the Decimal Point

- Move the decimal point in the divisor to the right (to make it a whole number).
- Move the decimal point in the dividend the same number of spaces as the divisor
- Write the decimal point in the quotient directly above the decimal point in the dividend.
- Divide as whole numbers. Add zeroes in the dividend (after the decimal point) as they are needed.

To develop a relational understanding of decimal division, let's identify the key mathematical concepts, which are implicated in these decimal division explanations.

"Divide the dividend by the divisor without the decimal points, then put the decimal places back."

- This method is about partitioning a whole or dividend (6.6 or 66) into 6 equal groups (*partitive division*) or group sizes of 6 *(quotative division*).
- The quantity 66 dividend is 66 ones or 66 tenths isn't important during the partitioning or grouping, however the final result (*quotient*) must be adjusted according to the number unit (tenths or ones) of the quantity, 66.
- The mathematical explanation of this method using equivalent expressions are:
  (66 x 0.1) ÷ 6 = (66 ÷ 6) x 0.1 → 11 x 0.1 = 1.1 (11 tenths)



If 1 rod = 1 one, and
1 flat = 10 ones, then
66 = 6 flats and 6 rods

If 1 rod = 1 tenth and
1 flat = 10 tenths, then
6.6 = 6 flats and 6 rods

"Move the decimal point in the divisor all the way to the right and move the decimal point in the dividend the same number of spaces as the divisor."

- Isn't this procedure really about the idea of equivalent ratios?
- The quotient (result of division), 1.1 is the *constant of proportionality* for the ratios, 6.6/6 = 6.6 ÷ 6 and 66/60 = 66 ÷ 60
- The dividend, 6.6 is 1.1 times greater than 6 (6 x 1.1 = 6.6) . Similarly, the dividend, 66 is 1.1 times greater than 60 (60 x 1.1 = 66)
- The mathematical explanation of this method using equivalent expressions are:
  6.6/6 = 6.6/6 x(10/10) → (6.6 x 10) ÷ (6 x 10) = 66 ÷ 60 → 66 ÷ 60 = 1.1

To develop a conceptual understanding of decimal number division, students need to build a complex web of connections to other topics in mathematics and to their lived experiences. As division with decimals is introduced, conceptual understanding may be developed by setting up conditions in the problem or in the class discussion where students draw from their understanding of the structure of decimal numbers, conceptual models of division and its relationship to multiplication and subtraction.

Let's examine the use of 2 different methods that show conceptual understanding of division with decimal numbers and equivalent ratios.

Tree Growth Problem - Pine trees grow at a rate of 0.61 metres per year. How many years, would it take it to grow to about 18.3 metres?

Decimal Division as Partial Quotients

$$0.61\text{m}\overline{)\ 18.3\text{m}}\quad \leftarrow \textit{(tree height)}$$

$$\underline{-18.3\text{m}}$$

$$0$$

$$30 \text{ years}$$

$0.61 \times 30 = 61 \times 30 \times 0.01$

$= 1830 \times 0.01$

(growth for every year)

Tree will grow to 18.3 m in 30 years.

Decimal Division as Equivalent Ratios

$$\frac{0.61\text{m}}{1\text{year}} = \frac{18.3\text{m}}{?\text{ year}}$$

$0.61\text{m} \times ___ = 18.3\text{m}$

$61\text{ m} \times 0.01 \times ___ = 1\,830\text{ m} \times 0.01$

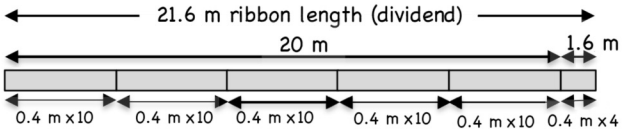
$60\text{ m} \times 30 \times 0.01 = 1\,830\text{ m} \times 0.01$

$$\frac{0.61\text{m} \times 30}{1\text{year} \times 30} = \frac{18.3\text{m}}{30\text{ years}}$$

In 30 years, the tree could grow to 18.3 m.

LINKS TO MANIPULATIVES – BAR MODEL AND AREA MODEL

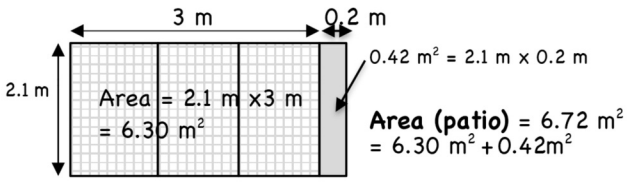
Ribbon Problem - A roll of ribbon is 21.6 m long. How many ribbon pieces, 0.4 m long can be cut from the roll?



Decompose Dividend to Addends → Equivalent Ratios

21.6 m ÷ 0.4 m
= (21.6 m ÷ 0.4 m)
= (21.6x10) m ÷ (0.4x10) m
= 216 ÷ 4
= (200 + 16) m ÷ 4 m
= (200 m ÷4 m) + (16 m ÷4 m)
= 50 + 4 = 54 ribbon pieces

Patio Problem - The area of a rectangular patio is 6.72 m^2. If one side length is 2.1 m, what is the length of the other side?



Decompose Dividend → Partial Quotient

$$2.1\overline{)\ 6.72}\quad 3.2$$

$$\underline{-6.30}\quad 21 \times 3 \times 0.1 = 63 \times 0.1$$

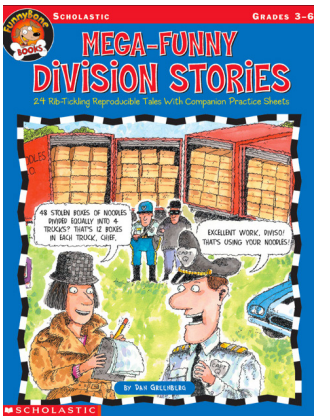
$$.42$$

$$\underline{-.42}\quad 21 \times 0.2 \times 0.1 = 42 \times 0.01$$

$$0$$

$6.72\text{ m}^2 \div 2.1\text{ m} = (3 + 0.2)\text{ m}$

LINKS TO LITERATURE – MEGA FUNNY DIVISION STORIES



Using a children's picture book for learning mathematics concepts provides a real life or situational context for problems. There are numerous teacher resources that have brief story contexts that can be used to develop decimal division problems. An example is *Mega-Funny Division Stories* by Dan Greenberg.

"As a child you loved it. You played in it. And then, somehow you became convinced that dirt was BAD! But it isn't! Just read these words of support. Fran-Swah LeVoo, Gourmet Cook: Where I come from, we make a delicious dish called Dirt Stew. Take 5.24 kg of dirt. Add 4.64 kg of sticks and stones. Now cook for 14 hours or until the mixture turns an ugly mud color. Divide the mixture into 13 bowls. How many kilograms will each bowl contain?"

- Use the numbers from the stories or change them to suit the mathematics learning goals, in terms of the size of the whole number and decimal number units (e.g., tenths, hundredths, thousandths)
- Have the students create decimal number division expressions and create story contexts and problems using different division representations: quotative division; partitive division; multiplicative comparison - as many times as; unit rate; area and linear dimensions of a rectangle; equivalent ratios.

LESSON LEARNING GOALS
Identify the curriculum expectations (MOE, 2005) grade-before and grade-after the grade specific learning goal, in order to:
- see the development of mathematical knowledge, skills, and strategies across grade levels
- determine the mathematical focus of the problems chosen for Before (activation), During (solving problem), and After (Practice) parts of the lesson

BEFORE (Getting Started) – 5 to 10 minutes
- Activating students' mathematical knowledge and experience using a prompt or problem, related to the math in the lesson problem.
- Includes student responses to highlight ideas and strategies.

Travel by Bus Problem 1

If Alice travelled on the bus 125 km over 10 weeks, then how many kilometres did Alice travel in 1 week?

Solution 1

If 100÷10 =10, 20÷10 =2, 5÷10 = 0.5
Then 125÷ 10
= 10+2+0.5 = 12.5 km in 1 week
Alice travels 12.5 km in 1 week.

Solution 2

125 ÷ 10 = 125 x 0.1 (125 tenths)
125 ÷ 10 = 12.5 km in 1 week
Alice travels 12.5 km in 1 week.

Divide any number by 10 (tens) is the same as multiplying the same number by 0.1 (tenths)

125 ÷ 10
= 125 x 0.1
= 12.5

DURING (Working on It) – 15 to 20 minutes
- Understanding the problem – Teacher asks, "What information from the problem are we using to solve it? Explain." Teacher records in a list below the problem, the information that students identify.
- Students solve the problem on chart paper (landscape) with markers (visible for class discussion) in pairs or small groups.
- Teacher circulates to record different student solutions, in addition to the ones the teacher anticipated.

Travel by Bus Problem 2

Alice travels by bus 2.5 km on Monday, 3.6 km on Tuesday, 6.75 km on Thursday and 12.8 km on Saturday. What is the average distance Alice travels each day? (nearest 0.01 of km)

What information are we going to use to solve this problem?
- travelled 2.5 km, 3.6 km, 6.75 km, 12.8 km
- 4 days travelled
- find out average km travelled in 1 day

` AFTER (Consolidation) – 20 to 25 minutes
- Teacher selects 2 or more solutions for class analysis and discussion in a sequence (1st, 2nd, 3rd, etc.) based on mathematical relationships between the solutions and the lesson learning goal.
- Students (authors) explain and discuss their solutions with their classmates; co-construction of success criteria towards the lesson learning goal
- During whole class discussion, solutions are organized (often re-organized) to show mathematical elaboration from one solution to the next and towards lesson learning goal.
- Teacher mathematically annotates (math terms, math symbols, labelled diagrams, concise explanations) on and around solutions to make mathematical ideas, strategies, and models of representation explicit.

factor x factor = product
dividend ÷ divisor = quotient

Decompose the dividend into sum of its parts which are each divided by the divisor

Solution 1

Round Distances to the Nearest Ones:
Monday -about 3 km
Tuesday -about 4 km
Thursday -about 7 km
Saturday -about 13 km

Total km for 4 days travelling on the bus
(3+4+7+13) km
= 27 km

27 km ÷4 days
= (20+4+3) km÷4 days
= (20÷ 4)+(4÷ 4)+
(3÷4) km in 1 day
= 5+1+0.75 km/1 day
= 6.75 km in 1 day

Average distance Alice travelled each day is about 6.75 km.

Solution 2

Round Distances to the Nearest Tenths:
Monday -about 2.5 km
Tuesday -about 3.6 km
Thursday -about 6.8 km
Saturday -about 12.8 km

(25+36+68+128)x0.1 km
= 257 x 0.1 km
= 25.7 km for 4 days

(24+1.6+0.1) km ÷4 days
=(24 ÷4) +(1.6 ÷4)
+(0.1÷4) km/1 day
= (6 +0.4 +0.025)
= 6.43 km/1 day

Alice travelled 6.43 km in 1 day on average.

Distance to Ones Decompose Dividend

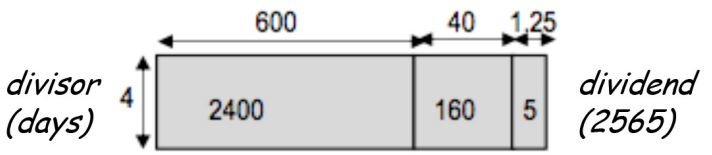
Distance to Tenths Decompose Dividend

Note: Mathematical *annotations* include mathematical vocabulary, symbols, elaborations of mathematical details from solutions, labels describing the method/strategy and questions to further thinking. All *annotations* are records of students' mathematical discussion.

Grade 6	Grade 7	Grade 8
-multiply and divide decimal numbers to tenths by whole numbers, using concrete materials, estimation, algorithms, and calculators	-solve problems involving the division of decimal numbers to thousandths by one-digit whole numbers, using a variety of tools and strategies	-divide decimal numbers by various powers of ten; solve problems involving percents expressed to one decimal place (e.g., 12.5%) and whole-number percents greater than 100

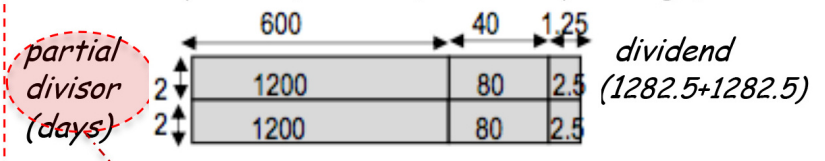
Decompose Dividend into Sum of Its Parts

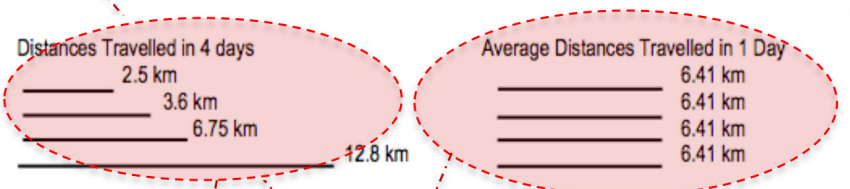
partial quotient (km in 1 day average)



Divide by Partial Divisors

partial quotient (km in 1 day average)





Solution 3

Distance Travelled in 4 Days
(2.5 + 3.6 + 6.75 + 12.8) km
= [(2+3+6+12)
+ (0.5+0.6+0.75+0.8)] km
= (23 +2.65) km
= 25.65 km in 4 days

Average km in 1 Day:
25.65 km ÷ 4 days
= (2565 x0.01) km ÷ 4 days
= (2400+160+5) km x0.1 ÷4 days
= (2400÷4)+(160÷4)+(5÷4)x 0.01 km/1 day
= (600+40+1.25) x 0.01 km/1 day
= 641.25 x 0.01 km/1 day
= 6.41 km / 1 day

On average per 1 day, Alice travelled 6.41 km.

Distance to Hundredths Decompose Dividend

Solution 4

Distance Travelled in 4 Days
2.5 + 3.6 + 6.75 + 12.8) km
= [(2+3+6+12)
+ (0.5+0.6+0.75+0.8)] km
= (23 +2.65) km
= 25.65 km in 4 days

Dividend Divided by Partial Divisor
25.65 ÷ 2 ÷ 2 km/1 day
= (2565 ÷ 2) x0.01 ÷2 km1/day

Multiply Partial Divisor and Quotient to Get Dividend (2565)
[(2x1250)+(2x32)+(2x0.5)] x0.01

Quotient
= (1250+32+0.5) x 0.01
= 1282.5 x 0.01 = 12.82 km in 2 days

1282 km ÷ 2 x 0.01 = 1282÷2 x 0.01 = 641 x 0.01 km/1 day
= 6.41 km/1 day on average

Distance to Hundredths Partial Divisors

AFTER (Highlights/Summary) – 5 minutes

- Teacher and students revisit the student solutions for key ideas, strategies, and models of representation that are related to the lesson learning goal (anticipated success criteria)
- Teacher lists key ideas, strategies, and models of representation so the students can see how the mathematical details from their solutions relate explicitly to the lesson learning goal; summary of co-constructed success criteria

Highlights/Summary

Divide decimal number to hundredths by 1-digit whole number by:

- *decomposing dividend – into parts*
  25.65 = 2565 x0.01 =(2400+160+5)x0.01
  (2565 ÷4) = (2400÷4) +(160÷4) +(5÷4)
- *partial divisors*
  2565÷4 = 2565÷2÷2 = (2400+164+1)÷2÷2
  =[(2400÷2)+(164÷2)+(1÷2)] ÷2
  =(1200+82+0.5)÷2 = 1282.5÷2 = 641

AFTER (Practice) – 5 to 10 minutes

- Teacher chooses 2 or 3 problems, similar to the lesson problem for student to solve in pairs as a scaffold and individually.
- Problems could vary by number (choice, size), problem contexts, or what is unknown or needs to be solved
- Students are asked to solve these problems using a strategy different from the one they used for the lesson problem.

Travel by Bus Problem 3

Jess travels by bus 5.0 km on Monday, 7.2 km on Tuesday, 13.5 km on Thursday and 25.6 km on Saturday. What is the average distance Alice travels each day? (nearest 0.01 of km)

Solution 1

If 6.41 km/1 day as average for 4 days, then double 6.41 km

6.41 km x 2 /1 day
= 12.82 km/1day

Jess travels 12.82 km on average of 1 day

Solution 2

(5.0+7.2+13.5+25.6) km ÷ 4 days
= 51.3 km ÷ 4 days
= (49+3.2+0.1) km ÷ 4 days
= [(48÷4)+(3.2÷4) +(0.1÷ 4)] =
= (12+0.8+0.025) km/1 day = 12.83 km/1 day

Note: A classroom board is longer proportionally than these 2 pages. Due to the space constraints on these pages, the mathematical *annotations* are recorded above the solutions with arrows, rather than on and around the solutions.

LET'S DO MATH – TRAVEL TO THE AIRPORT

Before (Getting Started)

Which is faster 460 km in 4 h or 454.4 km in 4.8 h?

Solution 1 – Estimate Unit Rate (per 1 hour)
Estimate for 460 km in 4 h

460 km in 4 h = 115 km in 1 h → 115 km /h

460 km /4 in 4 h /4 → 460 ÷ 4
= (400 + 40 + 20) ÷4
= (400 ÷ 4) + (40 ÷ 4) + (20 ÷ 4)
= 100+10+5 =115km in 1 h

Estimate for 454.4 km in 4.8 h

454.4 km in 4.8 h is about 450 km in 5 h → 90 km /h
(450 ÷ 5 = 90) The answer will be a little greater than 90 km/h because 450 < 454.4 and 5 > 4.8. So, 460 km in 4 h, is faster.

Solution 2 – Divide Distance by Time
distance ÷ time → rate in km per h

460/4
= 460 ÷ 4 km /h
= (400 + 60) ÷ 4 km/h
= (400 ÷ 4) +(60 ÷ 4)
= 100 + 15 =115 km /h

$$\frac{454.4\text{ km}}{4.8\text{ h}} = \frac{454.4}{4.8} \times \frac{10}{10} = \frac{4544\text{ km}}{48\text{ h}}$$

460 km in 4 h is faster by about 20 km/h.

During (Working On It) – TRAVEL TO THE AIRPORT

Jeremy is flying to Miami and found a cheap airline ticket but he'll have to fly from Buffalo Airport. The trip from Toronto to Buffalo is 185.6 km. One limo service advertises "Get there in 75 min." The bus line says the trip will take 2.5 h. Which service seems like the safer trip?

After (Consolidation)

Anticipating Student Responses:

Solution 1

- 185.6 km in 1.25 h (75 min) seems really fast
- 185.6 km in 2.5 h seems to be a safer speed to travel

Check by estimating…

186km ÷ 1 =186 km > 185.6 km
1 h < 1.25 h (75 min = 1.25 h)

speed of bus < speed of limo
but it is a reasonable speed and safe as it would be greater than 100 km/h

Solution 2

Rate of speed -> kilometres divided by time taken.

Limo Service

$$\frac{\text{kilometres}}{\text{time taken}} = \frac{185.6\text{ km}}{1.25\text{ h}}$$

$$\frac{185.6}{125} \times \frac{100}{100} = \frac{18560}{125}$$

Bus Service

$$\frac{\text{kilometres}}{\text{time taken}} = \frac{185.6\text{ km}}{2.5\text{ h}}$$

$$\frac{85.6}{125} \times \frac{10}{10} = \frac{1856}{25}$$

18560 ÷125 = 148.48 km/h and 1856 ÷25 = 74.25 km/h

Jeremy would be safer travelling by the bus service, that goes about 75 km/h because 148.48 km/h is going over the speed limit.

Coordinating Discussion for Student Learning:

Why might solution 1 be chosen first for student discussion followed by solution 2?

Solution 1 – uses estimates of rates (speed) for limo and bus travel for comparison

Solution 2 – calculates the rates using equivalent ratios

LET'S DO MATH – TRAVEL EXCHANGES

Before (Getting Started)

Jeremy returned from a trip to the United States with $20 USD in his pocket. He researched and found the exchange rate was USD / CND = 1 / 0.80. Should he expect more or less money when he takes $20 back to exchange it for CND money?

Solution 1

I think he will get less money because the US dollar is more valuable than the CND.

$$\frac{\$USD}{\$CND} = \frac{1}{0.8} = \frac{\$20\text{ USD}}{?\ \$\text{ CND}}$$

x 20 (above the middle fraction) and x 20 (below the middle fraction)

? $CND = 0.8 x $20 USD = $16 CND

He should get $16.00 CND for $20.00 USD.

Solution 2

If $USD are worth more than $CND, then he will get less for his $20.

USD	CND
1.00	0.80
10.00	8.00
10.00	16.00

x 10 (between the first and second rows)
x 2 (between the second and third rows)

So, for $20 USD he gets $16 CND.

During (Working On It) – FLIGHT COSTS

Jeremy found online, a return ticket to Miami for $260 USD. He wonders how much more he would have to pay when paying in $CND. At that time, the exchange rate was posted as $CND/$USD =1.25 /1.00. How much would this ticket cost in $CND?

After (Consolidation) *Anticipating Student Responses*

Solution 3

Compare using equivalent ratios.

$$\frac{\$CND}{\$USD} = \frac{1.25}{1.00} = \frac{?\ \$\ CND}{\$260\ USD}$$

1 : 125 = 260 : 260 x 1.25
for every $USD, it costs $1.25 CND

$CND = 1.25 x 260 $USD

The $260 USD ticket will cost $325 CND.

Solution 2

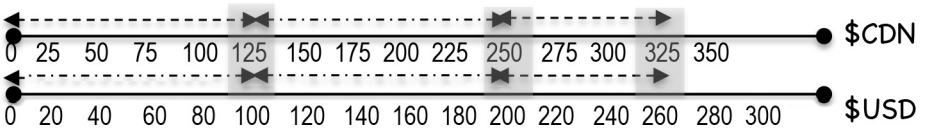
Ratio table relates $CDN for $260 USD

$CND	$USD	
1.25	1.00	x 100
125.00	100.00	÷ 2
62.50	50.00	÷ 5
12.50	10.00	x 100 row
250.00	200.00	by 2
312.50	250.00	add 50 row
325.00	260.00	add 100 row

The $260 USD ticket will cost $325.00 CND.

Solution 1

Line up two scales $125/$100 = $CND/$USD to see multiplicative relationship, 1 : 1.25



$100 USD = $125 CDN

$200 USD = $200 USD x 1.25 CDN/1 USD = $300 CDN (1.25 x 200 = 200 + 100)

$60 USD = $60 x 1.25 CDN/1 USD = $60 x 1.25 = (60+15) = $75CND

So $260 USD = $125 CDN + $125 CDN + $75 CDN = $325 CDN

Coordinating Discussion for Student Learning:

Why might solution 1 be chosen first for student discussion followed by solutions 2 and 3?

Solution 1 – double number line relates $USD to $CDN by a factor of 1.25

Solution 2 – incremental increase of $USD in relation to $CDN = $USD x 1.25 to show pairs of equivalent ratios, starting at 1.25 : 1.00 to $325CN : $260 USD

Solution 3 – equivalent ratios of 1 : 1.25 = 260 : 260 x 1.25

NEXT STEPS FOR YOUR PROFESSIONAL LEARNING

Application to Your Classroom

- Consider how decimal number division problems develop students' understanding of the relationship between a decimal number dividend and decimal number divisor that could result in a whole or decimal number quotient.
- Check that the Divisibility Rules for whole numbers are true with decimal numbers and explain the reasoning of each Divisibility Rule using decimal numbers.
- To suit your students' learning needs, select and design decimal number division problems that are relative to grade specific curriculum expectations in the Number Sense and Numeration (e.g., grade 6 focuses on decimal number (to tenths) dividend and decimal number to tenths, divisor, while grade 7 focuses on decimal number dividends (to thousandths) and decimal number to hundredths, divisor.
- Design decimal number division problems that develop students' experience with different applications in real life situations: (a) quotative division; (b) partitive division; (c) multiplicative comparison - as many times as; (d) unit rate; (e) area and linear dimensions of a rectangle; (f) equivalent ratios.
- Practise noticing the different division methods (decompose dividend, use equivalent ratios) and models of representation (area model) that your students choose to use or not use to solve these decimal number division problems.
- Think ahead of the kinds of mathematical annotations you will record on and around the student solutions to make explicit the details of different models of decimal number division.

Suggested Readings

Some readings used in the Research Summary and in the development of the problems and solutions are listed below.

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CLOSING

Our next issue will focus on the division of fractions, as it relates to the quotative model of division and the comparison of ratios and rates. As well, the next issue will outline the ways that teachers need to know these notions for teaching, as well as, the ways that teachers can use these mathematical ideas and pedagogical strategies within a three-part problem-solving lesson design framework.

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