8 PAGE INSERT ELEMENTARY MATH

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

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DIVISION OF FRACTIONS

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ABACUS CO-EDITORS



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Which contexts accurately interpret this division expression1, $1\frac{3}{4} \div \frac{1}{3}$?

a) A roll of ribbon is $1\frac{3}{4}$ m long. How many ribbon pieces, which are each $\frac{1}{3}$ m could be cut from the roll?

b) Alice travels by bus $\frac{6}{10}$ km on Mondays, $\frac{8}{10}$ km on Tuesdays, $\frac{35}{100}$ km on Thursday. What is the average distance Alice travels each day? (nearest $\frac{1}{100}$ of km)

c) Pine trees could grow $\frac{1}{3}$ m per year. How many years could it take for a pine tree to reach a height of $1\frac{3}{4}$ m ?

d) Nancy and Patricia brought chocolate cakes to a party. Patricia's cake was $\frac{3}{4}$ the size of Nancy's cake. Nancy and Patricia each had $\frac{1}{3}$ of their cake left over. Which cake was eaten the most?

e) The area of a rectangular garden is $1\frac{3}{4}$ m^2. If one side is $\frac{1}{3}$ m, what is the length of the other side?

All problems have a dividend, which has a fraction number. Problem (b) shows $1\frac{3}{4}$ km ÷ 3, not $1\frac{3}{4} \div \frac{1}{3}$.

Problem (d) is a comparison of $(1 \times \frac{1}{3}) - (\frac{3}{4} \times \frac{1}{3})$. Also, the problems differ by whether the divisor and quotient is a whole or fraction number and different types of division representations, such as: (a) quotative division; (b) partitive division; (c) rate; (d) multiplicative comparison with same and different size wholes; (e) area and linear dimensions of a rectangle.

In this issue, the Research Summary and contextual problems (solved with different methods and models of representation) focus on the division of fractions. The problems in this issue are designed for use within a three-part problem-solving lesson.

RESEARCH SUMMARY – DIVISION OF FRACTIONS

How did you learn to divide fractions by whole numbers and divide fractions by fractions Do you know why those methods worked? Here are two fraction division methods, commonly used.

Method 1 – Division of Fractions
as"Gozinta" for $5 \div \frac{1}{4}$

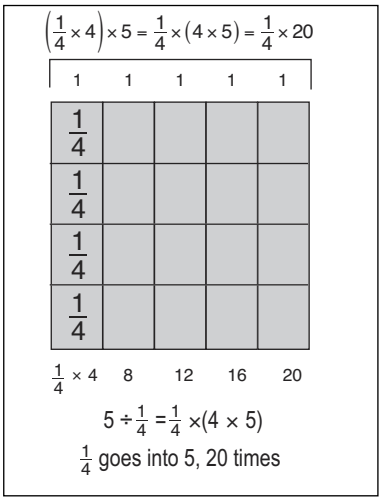
How many times does $\frac{1}{4}$ go into 5?

- $\frac{1}{4}$ goes into 1, 4 times = 4 times x 1
- $\frac{1}{4}$ goes into 2, 8 times = 4 times x 2
- $\frac{1}{4}$ goes into 3, 12 times = 4 times x 3
- $\frac{1}{4}$ goes into 4, 16 times = 4 times x 4
- $\frac{1}{4}$ goes into 5, 20 times = 4 times x 5

Method 2 – Division of Fractions
as Invert and Multiply for $5 \div \frac{1}{4}$

- $5 \div \frac{1}{4} = \frac{5}{1} \times \frac{4}{1} = \frac{20}{1} = 20$
- Multiply the reciprocal of the divisor by the dividend (invert the fraction divisor).
- Multiply the numerators and denominators to get the quotient.

Let's be explicit about the meaning of the concepts for the division of fractions and make connections between methods used for the division of whole numbers and the division of decimal numbers.

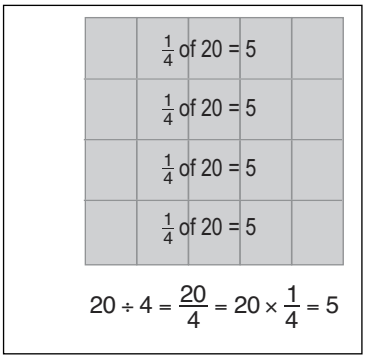


"Gozinta"

- This method is about dividing the dividend, 5 by the divisor, $\frac{1}{4}$ using the quotative model of division; that is, dividing by grouping, rather than fair sharing. How many groups of $\frac{1}{4}$ make 5?
- Multiplication relationships can be used to determine which factors (partial quotients) to multiply by the divisor to get partial products that sum to the dividend.
- Decompose the dividend into a sum of addends
  $5 \div \frac{1}{4} = (1 + 1 + 1 + 1 + 1) \div \frac{1}{4}$
  $= (1 \div \frac{1}{4}) + (1 \div \frac{1}{4}) + (1 \div \frac{1}{4}) + (1 \div \frac{1}{4}) + (1 \div \frac{1}{4}) = 4 + 4 + 4 + 4 + 4 = 20$
- The mathematical summary is:
  $\frac{1}{4}$ x 4 =1, then 1 x 5 = ($\frac{1}{4}$ x 4) x 5 = $\frac{1}{4}$ x (4 x 5) = $\frac{1}{4}$ x 20, then $5 \div \frac{1}{4}$ = 5 x 4
  (If $\frac{1}{4}$ is divided into 1, 4 times; then $\frac{1}{4}$ is divided into 5, (4 x 5) times.)

"Invert and Multiply"

- This method is about the application of the inverse relationships between division and multiplication and focuses on division as partitive (fair sharing).
- Since the equation, 20 ÷ 4 = 5 could mean that 4 groups have 5 in each group, then 20 ÷ 4 = 20 x $\frac{1}{4}$, which means that 20 is divided into
  4 groups or each group is a fourth of the whole, 20. So, each one of the four groups has five or $\frac{1}{4}$ of the whole, 20 is 5.
- The mathematical summary is: If 20 ÷ 4 = 20 x $\frac{1}{4}$, then
  $\frac{20}{1} \div \frac{4}{1} = (20 \div 1) \div (1 \times \frac{1}{4}) = 20 \times \frac{1}{4} = \frac{20}{4} = 5$



Students' understanding of fraction division develops best when connected to the methods of multiplication and division of whole numbers and decimal numbers and everyday contexts. As division with fractions is introduced, students first draw from their understanding of the structure of division (partitive, quotative). Let's examine the use of 2 different methods that show conceptual models of division with fractions (partitive, quotative).

Tree Growth Problem 1

A pine tree grew to $1\frac{3}{4}$m tall over $5\frac{1}{4}$ years. How much did the tree grow each year? *(Partitive Division)*

Solution 1 – Equivalent Multiplication and Division Expressions

$1\frac{3}{4} \div 5\frac{1}{4} = \frac{7}{4} \div \frac{21}{4}$

$\frac{7}{4} \div \frac{21}{4} = \frac{7}{4} \times \frac{4}{21} = \frac{7}{21} \div \frac{4}{4}$

$= \frac{7}{21} = \frac{1}{3}$

Solution 2 – Divide Numerators with Same Denominators

$1\frac{3}{4}\text{m} = \frac{4}{4} + \frac{3}{4} = \frac{7}{4}\text{m}; 5\frac{1}{4} = \frac{21}{4}$ years

$\frac{7}{4} \div \frac{21}{4} = 7 \div 21 = \frac{1}{3}$

$\rightarrow \frac{1}{3}$m in 1 year.

Tree Growth Problem 2

Pine trees could grow $\frac{1}{3}$ m per year. How many years could it take for a pine tree to reach a height of $1\frac{3}{4}$m? *(Quotative Division)*

Solution 1 – Counting by Fraction Unit

$\frac{1}{3} \times \frac{4}{4} = \frac{4}{12}$

$1\frac{3}{4} = \frac{7}{4} \times \frac{3}{3} = \frac{21}{12}$

Counting by $\frac{4}{12}$ and years; $\frac{4}{12}$ is 1 year;

$\frac{8}{12}$ is 2 years; $\frac{12}{12}$ is 3 years; $\frac{16}{12}$ is 4 years;

$\frac{20}{12}$ is 5 years; $\frac{21}{12}$ is $5\frac{1}{4}$ years

because $\frac{1}{12} = \frac{4}{12} \div 4 = \frac{4}{12} \times \frac{1}{4}$

Solution 2 – Divide Numerators with Same Denominators

$1\frac{3}{4}\text{m} = \left(\frac{4}{4} + \frac{3}{4}\right)\text{m} = \frac{7}{4}\text{m}$

$\frac{7}{4}\text{m} \div \frac{1}{3}\text{m per year} = \left(\frac{7}{4} \times \frac{3}{3}\right) \div \left(\frac{1}{3} \times \frac{4}{4}\right)$

$= \frac{21}{12} \div \frac{4}{12} = \frac{21}{4} \times \frac{12}{12} = \frac{21}{4} \times 1$

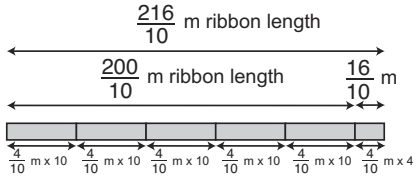
$= \frac{21}{4} = 5\frac{1}{4}$ years

LINKS TO MANIPULATIVES – BAR MODEL AND AREA MODEL

Ribbon Problem

A roll of ribbon is $\frac{216}{10}$m long.

How many ribbon pieces, $\frac{4}{10}$m long can be cut from the roll?



Decompose Dividend to Sum of Addends (Bar Model)

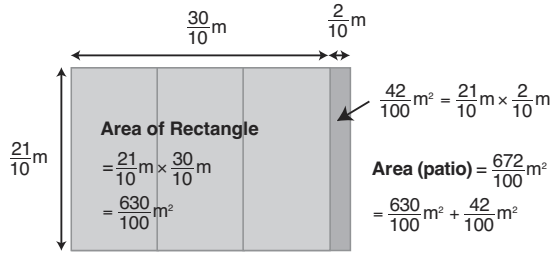
$\frac{216}{10}\text{m} \div \frac{4}{10}\text{m}$

$= (216 \div 4) \times \left(\frac{1}{10}\text{m} \div \frac{1}{10}\text{m}\right)$

= (216 ÷4) ribbon pieces
= [(200 + 16) ÷ 4] ribbon pieces
= (200 ÷ 4) + (16 ÷ 4) ribbon pieces
= 50 + 4 = 54 ribbon pieces

Patio Problem

The area of a rectangular patio is $\frac{672}{100}\text{m}^2$. If one side length is $\frac{21}{10}$m, what is the length of the other side?



Decompose Dividend to Partial Quotients (Area Model)

$\frac{672}{100}\text{m}^2 \div \frac{21}{10}\text{m}$

$= \left[672 \div 21 \times \left(\frac{1}{100} \div \frac{1}{10}\right)\right]\text{m}$

$= \left[(630 \div 42) \div 21 \times \frac{1}{10}\right]\text{m}$

$= \left[\left[(630 \div 3) \div 7\right] + (42 \div 21)\right] \times \frac{1}{10}\text{m}$

$= \left[(210 \div 7) + 2\right] \times \frac{1}{10}\text{m}$

$= (30 + 2) \times \frac{1}{10}\text{m}$

$= \frac{32}{10}\text{m}$ (length of other side)

LESSON LEARNING GOALS
Identify the curriculum expectations (MOE, 2005) grade-before and grade-after the grade specific learning goal, in order to:
- see the development of mathematical knowledge, skills, and strategies across grade levels
- determine the mathematical focus of the problems chosen for Before (activation), During (solving problem) and After (Practice) parts of the lesson

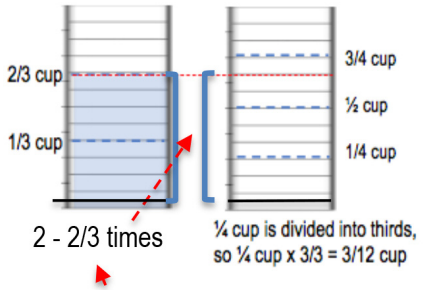
BEFORE (Getting Started) – 5 to 10 minutes
- Activating students' mathematical knowledge and experience using a prompt or problem, related to the math in the lesson problem.
- Includes student responses to highlight ideas and strategies.

Potato Salad Problem 1

You need 2/3 cup of mayo for a recipe. You don't have a 1/3 cup measure; you have a $\frac{1}{4}$ cup measure. How many times should you fill the $\frac{1}{4}$ cup measure?

Solution 1

-same size cup
-divide both cups by unit, 1/12ths
- 2/3 x 4/4 = 8/12
(¼ x 3/3) = 3/12
3/12 x 2 = 6/12
2 - 2/3 times fill ¼ cup



2 - 2/3 times

¼ cup is divided into thirds, so ¼ cup x 3/3 = 3/12 cup

Solution 2

2/3 cup ÷ ¼ cup
= (2/3 cup x 4/4) ÷ ¼ cup x 3/3
= 8/12 cup ÷ 3/12 cup = 2 -2/3 times

Counting number of $\frac{1}{4}$ cup units, where $\frac{1}{4}$ cup is divided into 1/3s

DURING (Working on It) – 15 to 20 minutes
- Understanding the problem – Teacher asks, "What information from the problem are we using to solve it? Explain." Teacher records in a list below the problem, the information that students identify.
- Students solve the problem on chart paper (landscape) with markers (visible for class discussion) in pairs or small groups.
- Teacher circulates to record different student solutions, in addition to the ones the teacher anticipated.

Potato Salad Problem 2

Dicing 1 medium potato is about $\frac{3}{4}$ cup. A 5 kg bag of potatoes is about 32 potatoes. If 18 cups of diced potatoes are used for potato salad, then how many 5 kg bags of potatoes are needed?

What information are we going to use to solve this problem?
- 1 medium potato is $\frac{3}{4}$ cup diced potatoes
- 5 kg bag has 32 potatoes
- 20 cups of diced potatoes used to make salad
- find out how many 5 kg bags needed to make salad

` AFTER (Consolidation) – 20 to 25 minutes
- Teacher selects 2 or more solutions for class analysis and discussion in a sequence (1st, 2nd, 3rd, etc.) based on mathematical relationships between the solutions and the lesson learning goal.
- Students (authors) explain and discuss their solutions with their classmates; co-construction of success criteria towards the lesson learning goal
- During whole class discussion, solutions are organized (often re-organized) to show mathematical elaboration from one solution to the next and towards lesson learning goal.
- Teacher mathematically annotates (math terms, math symbols, labelled diagrams, concise explanations) on and around solutions to make mathematical ideas, strategies, and models of representation explicit.

Quotative Division (fraction divisor)
"18 cups ÷ $\frac{3}{4}$ cup" means how many groups (potatoes) of $\frac{3}{4}$ cup make a total of 18 cups of diced potatoes

Part to Whole Ratio
How many potatoes used compared to how many potatoes in the bag.

Solution 1

How many potatoes?
18 cups ÷ ¾ cup for 1 potato
= 18 x 4/4 cup ÷ ¾ cup
= 72/4 cup ÷ ¾ cup
= (72÷ 3) x (1/4 ÷1/4)
= 24 potatoes

How many 5 kg bags of potatoes?
5kg bag -> 32 potatoes
24 potatoes
32 potatoes in 1 bag
= (24 ÷ 32) ÷ 8/8
= (24÷ 8) ÷ (32 ÷ 8)
= ¾

¾ of 5 kg bag of potatoes is used.

Solution 2

How many potatoes?
18 cups ÷ ¾ cup for 1 potato
= 18 x 4/4 cup ÷ ¾ cup
= 72/4 cup ÷ ¾ cup
= (72÷ 3) x (1/4 ÷1/4)
= [(36+36)÷ 3] x 1
= (12 + 12) x 1
= 24 x 1 = 24 potatoes

How many 5 kg bags of potatoes?
5kg bag -> 32 potatoes
24 potatoes
32 potatoes in 1 bag
= (24 ÷ 32) ÷ 8/8
= (24÷ 8) ÷ (32 ÷ 8)
= ¾ of 5 kg bag of potatoes needed

Common Wholes Divide Numerators

Decompose Dividend into Sum of Addends

Note: Mathematical *annotations* include mathematical vocabulary, symbols, elaborations of mathematical details from solutions, labels describing the method/strategy and questions to further thinking. All *annotations* are records of students' mathematical discussion.

Grade 6	Grade 8	Grade 9
-demonstrate repeated addition of fractions and the multiplication of a fraction by a whole number	-represent multiplication and division of fractions using a variety of tools and strategies	- simplify numerical expressions involving integers and rational numbers, with and without the use of technology

Whole Number and Fraction Operations:

- *Division – fraction represented as partitioning a whole into equal size parts*
  - ➔ *18 ÷ ¾ means how many groups of the size, ¾ in 18 (quotative division)*
- *Multiplication – fraction represented as a multiplier or "how many times as much"*
  - ➔ *5 kg x ¾, where 5 kg is the multiplicand and ¾ is the multiplier (fraction of the whole)*
- *Multiplication and Division as Inverse Operations*
  - ➔ *1 ÷ 2 = 1 x ½ and 1 ÷ 1/2 = 1 x 2/1*
  - ➔ *18 cups ÷ ¾ = 18 x 4/3*
- *Order of Operations*
  - ➔ *5 x ¾ means 5 x 3 ÷ 4*
  - ➔ *5 kg x ¾ = 5 kg x 3 ÷ 4 = 15 kg ÷ 4 = 3-3/4 kg mass of potatoes needed for the potato salad*

Fractions as a Ratio of Part of a Whole

- *potatoes needed : potatoes total*
- *potatoes needed ÷ potatoes total*

Solution 3

How many potatoes?

¾ cup for 1 potato
(¾ cup x12) for (1 x12) potatoes
= (3 x 12 ÷ 4 = 9 cups)
(¾ cup x 12) x 2 = 18 cups for (1 x 12) x 2 = 24 potatoes

How many 5 kg bags of potatoes?

potatoes needed = 24 potatoes
potatoes total 32 potatoes
24 ÷8 = 3 of 5 kg bag
32 8 4

¾ of 5 kg bag of potatoes is used for the potato salad.

Solution 4

How many potatoes?

18 cups ÷ ¾ cup for 1 potato
= 18 cups x4/3 ÷(¾ x 4/3) cups
= (18 x 4 ÷ 3) cups ÷ 1 cup
= (72 ÷ 3)
= 24 potatoes

How many 5 kg bags of potatoes?

potatoes needed = 24 potatoes
potatoes total 32 potatoes
24 ÷8 = 3 of 5 kg bag
32 8 4

¾ of 5 kg bag of potatoes is used for the potato salad.

Proportional Relationship Order of Operations

Multiply by Reciprocal of Divisor to Make Divisor 1

AFTER (Highlights/Summary) – 5 minutes

- Teacher and students revisit the student solutions for key ideas, strategies, and models of representation that are related to the lesson learning goal (anticipated success criteria)
- Teacher lists key ideas, strategies, and models of representation so the students can see how the mathematical details from their solutions relate explicitly to the lesson learning goal; summary of co-constructed success criteria

Highlights/Summary

Divide a fraction by a fraction by:

- *common wholes, divide numerators*
  - 72/4 cup ÷ ¾ cup = (72÷ 3) x (1/4 ÷1/4)
- *decomposing dividend into addends*
  - (72÷ 3) x (1/4 ÷1/4) = [(36+36)÷ 3] x 1
- *proportional relationship and order of operations ->* (¾ cup x12) for (1 x12) potatoes = (3 x 12 ÷ 4 = 9 cups)
- *multiply by reciprocal of divisor*
  - 18 cups x 4/3 ÷ ¾ x 4/3 cups = (18 x 4 ÷ 3) cups ÷ 1 cup = (72 ÷ 3) = 24

AFTER (Practice) – 5 to 10 minutes

- Teacher chooses 2 or 3 problems, similar to the lesson problem for student to solve in pairs as a scaffold and individually.
- Problems could vary by number (choice, size), problem contexts, or what is unknown or needs to be solved
- Students are asked to solve these problems using a strategy different from the one they used for the lesson problem.

Potato Salad Problem 3

Same potato recipe as problem 2, but make 2/3 of the recipe How many 5 kg bags of potatoes are needed?

Solution 1

¾ of bag is 3/3 of the salad recipe
¾ x 2/3 is 2/3 of the salad recipe

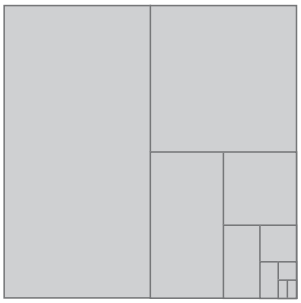
¾ x 2/3 = (3x2)/(4x3)
= 6/12
= (6/12) ÷ (6/6)
= ½ 5kg bag

Solution 2

18 cups ÷ ¾ cup for 1 potato
= 18 cups x 4/3 ÷ (¾ x 4/3) cups x 2/3
= (18 x 4 ÷ 3) cups ÷ 1 cup x 2/3
= (72 ÷ 3) x 2/3
= 24 x 2/3
= 24 x 2 ÷3
= 48 ÷ 3 = 16 potatoes
16/32 = ½ of 5 kg bag

Note: A classroom board is longer proportionally than these 2 pages. Due to the space constraints on these pages, the mathematical *annotations* are recorded above the solutions with arrows, rather than on and around the solutions.





LINKS TO LITERATURE – *THE LION'S SHARE* BY MATTHEW McELLIGOTT

This story is about a lion who baked a cake for his friends. His first friend took half of his cake, then his second friend took half of what was left of the cake and so on until the Lion was left with a crumb. Ant feels guilty and offers to bake another cake. The other friends also offer to bake cakes for the lion, but they each try to outdo each other by promising to bake twice the cakes promised by the previous friend. Very quickly, hundreds of cakes are being promised

and the result is hilarious.

- Write the fraction division equation after each friend eats $\frac{1}{2}$ of the cake left. (e.g., $1 \div 2 = \frac{1}{2}$; $\frac{1}{2} \div 2 = \frac{1}{4}$; $\frac{1}{4} \div 2 = \frac{1}{8}$).
- Model the division of a whole using a rectangle (to model the cake) or along with a 1 m strip of paper, so that the students can visualize the division of fractions in relation to the fraction division equations.
- Change the quotient size (fraction) to suit the mathematics learning goals, such as thirds, fourths, fifths, tenths related to the divisors/multipliers of 3, 4, 5 and 10.

LET'S DO MATH - BUS TRAVELS

Before (Getting Started)

Bus Travel Problem 1

Why is 6 km ÷ 2 days ≠ 6 km ÷ $\frac{1}{2}$ day? Explain using partitive and quotative models.

Solution 1 – Partitive Division

True or False? 6 km ÷ 2 days ≠ 6 km ÷ $\frac{1}{2}$ day

- 6 km divided into 2 days (2 groups) and each day (group) has a size, 3 (3 km in 1 day)
- 6 km divided into 1 day (1 group) has a size of 6 (6 km in 1 day)
- 6 km divided into $\frac{1}{2}$ day ($\frac{1}{2}$ group) has a size of 12 km in 1 day)
- 3 km in 1 day ≠ 12 km in 1 day

Solution 2 – Quotative Division

True or False? 6 km ÷ 2 km ≠ 6 km ÷ $\frac{1}{2}$ km

- 6 km divided by a group size of 2 km, makes 3 days (groups)
- 6 km divided by a group size of 1 km, makes 6 days (groups)
- 6 km divided by a group size of $\frac{1}{2}$ km, makes 12 days (groups)
- 3 days ≠ 12 days

During (Working On It)

Bus Travel Problem 2

Rachel plans to and from the park, which is a total of $7\frac{5}{10}$ km. She needs to take a water break every $1\frac{1}{2}$ km. How many water breaks will she take?

After (Consolidation)

Anticipating Student Responses:

Solution 1 – Counting Fraction Units

$7\frac{5}{10}\text{km} = 7\frac{1}{2} = \frac{15}{2}$; $1\frac{1}{2} = \frac{3}{2}\text{km}$

Breaks	1	2	3	4	5
Km	$\frac{3}{2}$	$\frac{6}{2}$	$\frac{9}{2}$	$\frac{12}{2}$	$\frac{15}{2}$

$\frac{3}{2} \times \#\text{ breaks} = \frac{15}{2} \div \frac{3}{2} \times 5\text{ breaks} = \frac{15}{2}$

Solution 2 – Decompose Divisor (Factors)

$7\frac{5}{10}\text{km} = \frac{75}{10}\text{km}$

$1\frac{1}{2}\text{km} = \frac{3}{2}\text{km} = \frac{3}{2} \times \frac{5}{5} = \frac{15}{10}\text{km}$

$\frac{75}{10} \div \frac{15}{10} = \frac{75}{10}\text{km} \div \frac{5}{10}\text{km} \div \frac{3}{1}$

Rachel needs about 5 water breaks.

Coordinating Discussion for Student Learning:

Why might solution 1 be chosen first for student discussion followed by solution 2?

- *Solution 1* - Counting by fraction unit of $\frac{3}{2}$ km to whole, $\frac{15}{2}$ km, then counting the number of groups of size, $\frac{3}{2}$ km, with common denominator of unit fraction, $\frac{1}{2}$.
- *Solution 2* – Dividing by decomposing divisor with common denominator of $\frac{1}{10}$.

LET'S DO MATH – GARDEN SIZES

Before (Getting Started)

Garden Size Problem 1

A large rectangular community garden is 150 m^2. One side length is $\frac{3}{2}$ the other side length. What are side lengths of the rectangular garden?

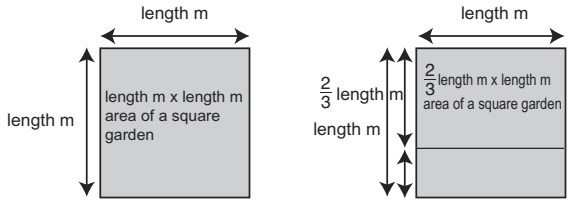
Solution 1

Area of garden is 150 m^2.

- 150 $m^2 \div \frac{2}{3}$ length m = length m
- length m x $\frac{2}{3}$ length m = 150 m^2
- $\frac{2}{3}$ length m x length m = 150 m^2
- length x length = 150 m^2 x 3 ÷ 2
- length x length = 450 $m^2 \div 2$
- length x length = 225 m^2 = 15 m x 15 m
- $\frac{2}{3}$ x 15 m = 10 m

Side lengths are 15 m, 10;
15 m x 10 m = 150 m^2

Solution 2



16 m x 16 m = 256 m^2; (16 m x $\frac{2}{3}$) x 16
= 10.67 m x 16 m = 170.67 m^2

15 m x 15 m = 225 m^2; (15 m x $\frac{2}{3}$) x 15
= 10 m x 15 m = 150 m^2

During (Working On It)

Garden Size Problem 2

Which garden has a greater in area?

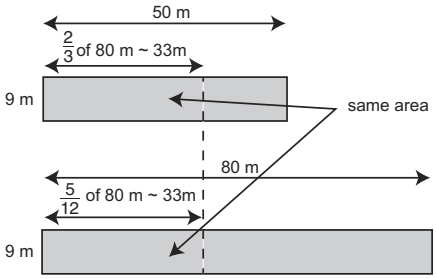
Hydrangea Garden $\frac{2}{3}$ of 450 m^2 or Rose Garden $\frac{5}{12}$ of 720 m^2?

After (Consolidation)

Anticipating Student Responses

Solution 1

Sketch area of rectangle diagrams 1 side length of each rectangle the same, 9m.



Length of other side:

Garden A → 50 m x $\frac{2}{3} = \frac{100}{3}$ ~ 33 m

Garden B → 80 m x $\frac{5}{12} = \frac{400}{12}$ ~ 33 m

Both gardens are about the same size.

Solution 2

Garden A 450 m^2 x2 ÷3 = 900 ÷ 3 = 300 m^2
Garden B 720 m^2 x 5 ÷ 12 = 3600 ÷ 12 = 300 m^2
A is 300 m^2 and 300 m^2 for B

Solution 3

Different size wholes and different size fractions of wholes
450 m^2 ÷ 720 m^2 = 0.625 (0.625 x Rose Hydrangeas)

$\frac{2}{3} \times \frac{4}{4} = \frac{8}{12} > \frac{5}{12}$

$\frac{8}{12}$ of 450 m^2 = (450 x 8) ÷12

$\frac{5}{12}$ of 720 m^2 = (720 x 5) ÷ 12

450 x 8 = (90 x 5 x 8) = (90 x 8 x 5) = 720 x 5

If same dividend, same divisor, then equivalent fraction of a whole expressions

Coordinating Discussion for Student Learning:

Why might solution 1 be chosen first, then solutions 2 and 3 for student discussion?

- *Solution 1* - compare area of rectangles, by showing 1 same side length and the other side length is the same, so area is the same
- *Solution 2* - relates fraction of a whole to division
- *Solution 3* - equivalent fractions to create same divisor (fraction denominator) and compare the dividend

NEXT STEPS FOR YOUR PROFESSIONAL LEARNING

Application to Your Classroom

- Consider how fraction division problems develop students' understanding of the relationship between a whole and fraction number dividend and fraction number divisor that could result in a whole or fraction number quotient.
- To suit your students' learning needs, select and design fraction number division problems that are relative to grade specific curriculum expectations in the Number Sense and Numeration (e.g., grade 8 focuses on fraction division).
- Design fraction number division problems that develop students' experience with different applications in real life situations: (a) quotative division; (b) partitive division; (c) multiplicative comparison - as many times as; (d) rate; (e) area and linear dimensions of a rectangle.
- Practise noticing the different division methods (decompose dividend, use equivalent ratios) and models of representation (area model) that your students choose to use or not use to solve these fraction number division problems.
- Think ahead of the kinds of mathematical annotations you will record on and around the student solutions to make explicit the details of different models of fraction number division.

Suggested Readings

Some readings used in the Research Summary and in the development of the problems and solutions are listed below.

- Braithwaite, D., & Pyke, Aryn. (2017). A computational model of fraction arithmetic. *Psychological Review, 1*-23.
- Bruce, C., Bennett, S., & Flint, T. (2014). *Fractions Operations: Multiplication and Division Literature Review.*
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- Jiang, C., & Chua, B.L. (2009) Strategies for solving three fraction-related word problems on speed: A comparative study between Chinese and Singaporean students. *International Journal of Science and Mathematics Educatio*n.
- Ontario Ministry of Education. (2005). *Ontario Curriculum, Mathematics, Grades 1 to 8.* Toronto: Queen's Printer for Ontario.
- Watanabe, T., Lo, J.J. Son, J.W. (2017). Intended treatment of fractions and fraction operations in mathematics curricular from Japan, Korea and Taiwan. In J.W. Son et al. (eds.), *What matters? Research trends in International Comparative Studies in Mathematics Education,* pp. 33-61.

CLOSING

Our next issue series will focus on absolute and relative comparisons across the five mathematics strands. As well, the next issue will outline the ways that teachers need to know these notions for teaching, as well as, the ways that teachers can use these mathematical ideas and pedagogical strategies within a three-part problem-solving lesson design framework.

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Send your teaching strategy ideas, problems and student solutions to the *Abacus* Co-Editors via email