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Ontario Mathematics Gazette

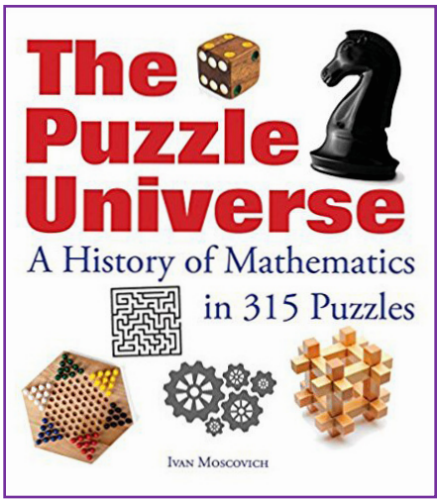
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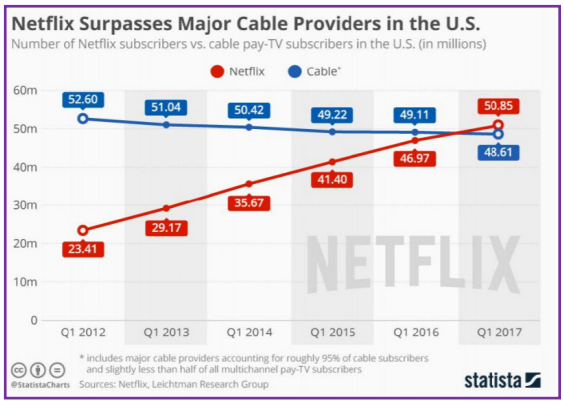
Vol. 56 #4
June 2018

IN THIS ISSUE

- Editor's Report
- President's Message
- Executive Directors' Report
- Letter to the Editor
- What's the Problem? Looking for Liars
- Technology Corner: Graph of the Week
- Provincial Digital Learning Resources – What's New? Set Tool 2.0
- NCTM Report: NCTM benefits to brighten your summer!
- Mb4T (Mathematics by and for Teachers): What's in a Name?
- In the Middle: Fun with Fractions
- Assessment Abby: Addressing Stress
- Fields Institute MathEd Forum Report
- Hey, It's Elementary: Share-Worthy Parent Engagement Materials for Math… No Fake News!
- Balancing Chemical Equations, Using Linear Equations
- A Focus on Student Understanding: Georg Pólya's Problem-Solving Heuristic
- "Mathematics is more important than ever before" – Fields Institute Expands Its Repertoire
- An Honour for Canadian Mathematician Robert Langlands
- BOOK REVIEW: Learning Math History through Puzzles



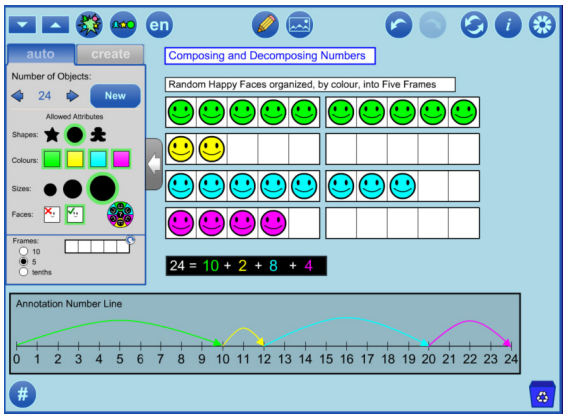
See Book Review



See Technology Corner: Graph of the Week

See Provincial Digital Learning Resources – What's New? Set Tool 2.0

See Editor's Report, The OAME-2018 Organizing Committee

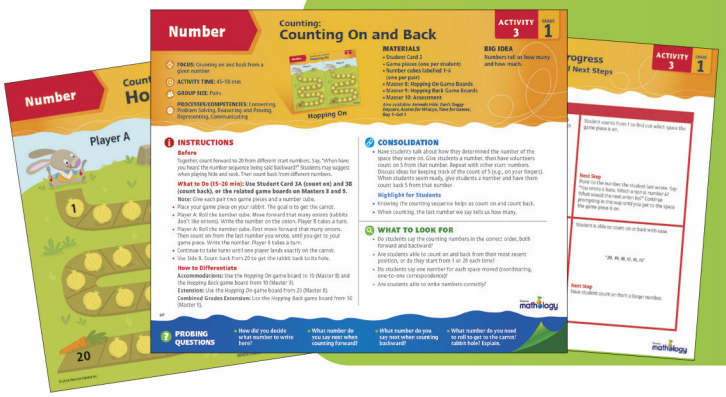




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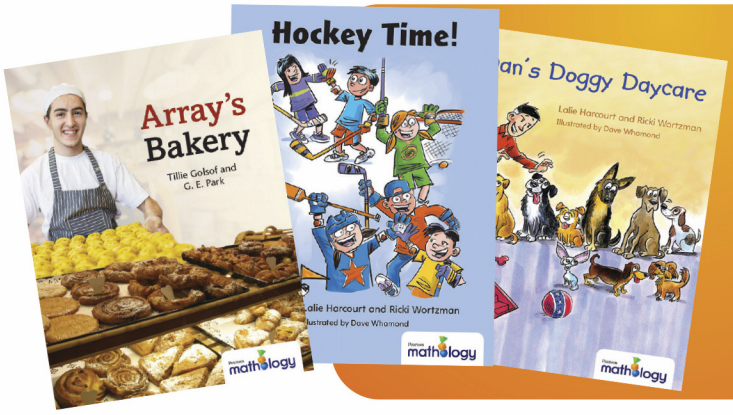
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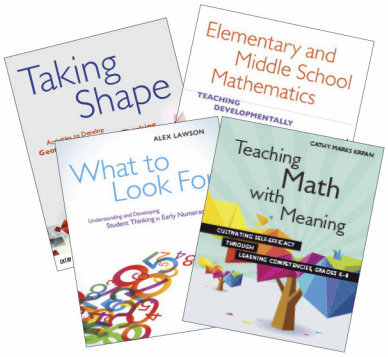
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Submission of Articles

The *Ontario Mathematics Gazette* (OMG) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *Gazette*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles are to be word-processed, MS Word is preferred, and prepared according to the *Publication Manual of the American Psychological Association*, Sixth Edition (2009). However, please use single-line spacing (not double) and only one space after each period. Articles should not exceed five numbered pages of text, and figures, images, and photographs should be placed in the text close to where they belong, with captions. The photographer's permission is required, and for photos of students under the age of 18, the written permission of a parent or guardian is required.

Please submit your article in one blind file (i.e., identity of author is not evident), and include author names, contact information including email and mailing addresses, photos, biographies, and all content removed for blinding in a second file. Please email these two files to Tim Sibbald at **gazette@oame.on.ca**.

Upon review, you will be notified whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected. The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME.

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The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of OAME.

▲ Table of Contents

Up Front

▲ Editor's Report /Timothy Sibbald 2

▲ President's Message /Jill Lazarus 5

▲ Executive Directors' Report /Lynda and Fred Ferneyhough 7

▲ Letter to the Editor 8

Regular Features

▲ The Abacus . . . . . 8-Page Insert

▲ What's the Problem? Looking for Liars /Shawn Godin 11

▲ Technology Corner: Graph of the Week /Mary Bourassa 13

▲ Provincial Digital Learning Resources – What's New? Set Tool 2.0 /Greg Clarke /Agnes Grafton /Markus Wolski /Ross Isenegger 15

▲ NCTM Report: NCTM benefits to brighten your summer! /Jacqueline Hill 21

▲ Mb4T (Mathematics by and for Teachers): What's in a Name? /Jennifer Holm 22

▲ In the Middle: Fun with Fractions /Carly Ziniuk 25

▲ Assessment Abby: Addressing Stress 28

▲ Fields Institute MathEd Forum Report /Angelica Mendaglio 33

▲ Hey, It's Elementary: Share-Worthy Parent Engagement Materials for Math… No Fake News! /Lynda Colgan 37

Articles

▲ Balancing Chemical Equations, Using Linear Equations /Don Vander Klok 30

▲ A Focus on Student Understanding: Georg Pólya's Problem-Solving Heuristic /Jeff Irvine 34

Special Features

▲ "Mathematics is more important than ever before" – Fields Institute expands its repertoire /Judy Mendaglio 9

▲ An Honour for Canadian Mathematician Robert Langlands /Judy Mendaglio 10

▲ BOOK REVIEW: Learning Math History through Puzzles /Reviewed by Tim Sibbald 29

Notices

▲ OAME Graph Sticky Charts 27

▲ OAME Board Members . . . . . .Inside Back Cover

Advertisements

▲ Pearson . . . . . Inside Front Cover

▲ About the *Ontario Mathematics Gazette*

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Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be print ready, and colour advertisements should have no bleeds.

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▲ Editor's Report



TIMOTHY SIBBALD, OCT, PhD
EMAIL: gazette@oame.on.ca

Tim Sibbald is the current Gazette *editor, co-chair of finance for the 2018 annual conference, and a Past President of OAME. He is an associate professor in the Schulich School of Education, Nipissing University, with a focus on mathematics education.*

Summer is clearly coming! Perhaps summer is already here and you are enjoying a restful day that includes catching up on the *Gazette*. As math educators, we definitively share one thing in common—an interest in mathematics. With that in mind, I would like to encourage you to see opportunity in the *Gazette*—to learn from your colleagues in the larger mathematics education community. As summer approaches, consider the possibility of contributing to the *Gazette*. For first-time writers, consider a letter to the editor. A letter is typically less than 200 words and can be an interesting idea or inspiration, a commentary that has been thought through, or perhaps a news item. Often for first-time writers, a book review is a stepping stone. A guide for writing book reviews is available from the editor. Articles about more developed ideas or topics that are of interest to the readership are welcome.

First-Time Writers

Would you like to be notified of an online get-together for first-time authors? Email the editor today!

What I would like to do, specifically for first-time writers, is offer an opportunity to get together and chat with others who share that interest. As editor, I will host the event and can provide insight on the editorial process and what reviewers are tasked to look for.

I want to take a moment to speak about how teachers learn. Many of us began teaching before we had formal credentials. An introductory experience commonly consists of tutoring students or working with youth in different environments such as camps. The experience meaningfully informs the choice to pursue formal qualifications. The route that students take to arrive in faculties of education vary, with some progressing directly through undergraduate studies, others taking a few years to find their way, and others making career shifts from the workforce.

In pre-service teaching, students are exposed to a wide variety of theoretical ideas within faculties and practical

ideas during practicum placements. It is a considerable formative period for teacher candidates, where theory and practice are somewhat meshed, but not entirely. A common discrepancy arises because teachers are quite adept and ingenious on the practical side, while faculty members are informed by research and policies from a wider scope. Often students relay that there is a difference between the "long game" thinking from faculties and practical "in the trenches" reality within schools.

This gap seems to continue as teachers complete their pre-service training and move into school systems, where, to go off on a brief tangent, it needs to be recognized that for faculty members, the systems they see students move into are quite varied and not just in Ontario. At the moment, there is a lot of hiring of educators in British Columbia, and it is not uncommon for teachers to acquire employment in the United Kingdom, China, and other countries. This is often assumed to be due to a lack of jobs in Ontario, which is something of a cliché, as job fairs are seeing some students receive day-of interviews. It remains necessary for teacher candidates to be more flexible than focusing exclusively on their "ideal job," but anecdotally, it seems they do not have to be as flexible as was necessary five years ago.

Novice teachers often engage in additional learning through induction programs. These assist teachers in navigating the complexity of their role. Induction programs are linked to improvements of retention of new teachers, and they also serve to improve the experiences of beginner teachers.

There is a variety of activities for more experienced teachers that are often dubbed "practitioner research." While I was not inspired by that moniker when I was a teacher, I did appreciate that reflective practices, action research, and various activities such as lesson studies, can inform practice. Care should be taken, however, because using well-structured approaches is more likely to lead to a positive end result than approaches that are lax and only mimic the techniques.

Ultimately, there is a need for ideas from beyond the walls of the classroom, school, or school board. The OAME and its chapters are nested in this role and provide a wide range of ideas and resources. There are two choices when you go beyond the classroom or school: the option of being a consumer of learning or learning through providing to others. It is tantamount to the distinction between reading the *Gazette* and writing an article for it. The choice reflects a reality where many teachers undervalue themselves and do not appreciate how much they have to offer.

In terms of consumption, there are professional options with which the OAME membership is typically engaged. However, I would like to speak about graduate schools because I find many teachers, including some who enroll, are not really sure what graduate studies in education are about. The focus of graduate schools is to further the development of one's theoretical understanding of education, for example, exploring how philosophy, psychology, and sociology come together to provide some foundational theoretical notions that provide significant insight into teacher practices. The objective is to address theoretical ideas that require a more substantial practical knowledge of teaching than teacher candidates acquired in pre-service programs.

In my own experience, I found graduate studies added clarity to various disjoint experiences I had as a teacher. There was a sense of coherence gained where classroom practice was clarified by theory, and comprehension of theoretical ideas from graduate classes was informed by classroom-based interpretations. This was informed through sustained discussions with other practitioners from a variety of contexts. The dichotomy of the practical and theoretical worlds blurred and, overall, the classroom experience was improved by gaining a conceptual understanding that could enlighten my practices. The learning also strengthens communication skills and brings some understanding of a much larger landscape of what can and is being done within education as a human endeavour. It is not for everyone, should not be done because of a pay grid, and carries considerable professional value.

The topic of professional learning came to mind because I have always felt the distinction between academia and practitioners has been greater than it should be. I see this as an issue within the field of mathematics at large. It seems unusually difficult for teachers to be informed about modern mathematical discoveries. Consider, for example, that DNA fingerprinting has moved from leading-edge research into a simplified version suited to high schools over the course of the last 30 or 40 years. Is there anything analogous in mathematics education?

To close, I want to offer up a piece of my own professional learning from the last few days. One of my personal informal teachers is our Associate Editor, Anne Yeager. (She is a real teacher, but we are without a formal educational structure.) She pointed out that an early draft of this editorial simply was not an editorial. I appreciate that she took the time to indicate that I might investigate what an editorial actually is. (I try to include a bit of math each time and get carried away with it.) That may sound funny, but all of us are students, and we all need to find those opportunities for growth—I am grateful that Anne suggested the "next step" in my editorship learning. This is something that professionals come to accept, and to the teacher who

believes there is no need to continue learning, I suggest he or she re-evaluate his or her thinking. Anne simply clarified that I am doing something relatively new and that is an opportunity for personal growth. I hope each reader finds time to consider the different ways to continue his or her own life-long learning.

It is the time of year that I have the pleasure of publically expressing thanks for the many educators who have provided peer reviews to ensure the quality of the *Gazette* is maintained.

Did you know… ICM-2018?

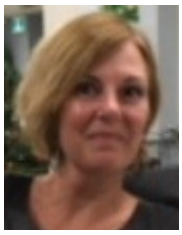
In August, the International Congress of Mathematicians meets. It is the biggest math conference and occurs every four years. This time, it is in Rio de Janeiro, Brazil. Last time was in Seoul, South Korea, where 5193 participants from 122 countries assembled. It was last held in Canada in Vancouver in 1974.

Melissa Baker-Cox
Iain Brodie
Trevor Brown
Leslie Burchat
Cathy Chaput
Ralph Connelly
Andrew Dmytriw
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Matthew Oldridge
David Petro
Jamie Pyper
John Rodger
Peter Saarimaki
Krista Sarmatiuk

Members who would like to be on the notification list when reviewers are needed can email the editor (**Gazette@oame.on.ca**) to be added to the list.

It is through the efforts of the editorial team, reviewers, and ongoing support of the OAME membership that the *Gazette* achieves the status of being a double-blind peer-reviewed publication. This means that authors do not know who reviewed their work, and reviewers do not know the identity of the author of the articles they are reviewing. Every article is reviewed by a minimum of three reviewers.

In addition to the reviewers, once again I would like to express my ongoing thanks to our proofreader, Gitta Berg, who ensures consistency with language, and Penny Clemens, who brings together all the pieces and assembles the final layout for each issue. Lastly, but most certainly not least, are the associate editors. Anne Yeager has been an excellent source of advice over the last year and I sincerely appreciate her ongoing efforts both for advice over the course of my first year in the role and her promotion of excellence—Thank you, Anne! Finally, I am delighted to announce that we have a second associate editor. Jacquie Foster has been quietly becoming more and more enthusiastic about the associate editor role and has been fully involved in this issue.



Jacqueline Foster is a graduate from McMaster University with a Bachelor of Science in Radiation Physics. After two years doing research in the Nuclear Medicine Department at McMaster, she obtained her Bachelor of Education in Mathematics and Science at the University of Toronto. Further qualifications are: Special Education Specialist, Guidance Specialist, and Principal's Qualifications. Jacqueline taught Mathematics and Science for 16 years, and was also involved with Alternative Education, Guidance, Counselling, and Special Education. For the last several years, She has been working as a Special Education Consultant for the Upper Grand District School Board. She has a passion for teaching and an appreciation for how students' think and learn.

In this issue…

In this issue, the Executive Directors, **Fred and Lynda Ferneyhough**, write about volunteers who engage with the annual conference. Figure 1 shows the OAME-2018 organizing team as they transition from OAME duties to Conference work. **Jill Lazarus** writes her final column as President and mentions various aspects of a changing organization. **Angelica Mendaglio** writes about the Fields Institute research day and the Margaret Sinclair Memorial Award, but there is more news from Fields, and **Judy Mendaglio** has written a news piece that is informative about math topics that are currently being worked on. **Judy Mendaglio** has also contributed an acknowledgement of Robert Langlands, who is Canadian and winner of this year's Abel Prize! **Jacqueline Hill** brings the NCTM perspective, and she provides a top-ten list along with noting several events that may be of interest this summer. **Jennifer Holm** presents some "food for pedagogical thought" related to



Figure 1: *The OAME-2018 organizing committee*

geometry of shapes. Just in time for the final days of the school year, **Assessment Abby** addresses assessment stress. **Carly Ziniuk** addresses fractions and makes an interesting math art connection (the connection is part of a whole lot more that I hope she shares in a future column). **Shawn Godin** resolves the problem about liars and truth sayers, and gives us some summer fun of thinking about obtuse integral triangles. The **Provincial Digital Team** reveals details of new tools that facilitate working with virtual ten-frames. **Mary Bourassa** charts a course for having graphs to show and discuss regularly in your classes. **Lynda Colgan** addresses the importance of parents, and details how they can impact students in unanticipated ways. She highlights several resources that could make quite good summer reading and food for thought on how to address parent engagement as a teacher.

This issue also includes **Don VanderKlok**, who demonstrates a method for balancing chemical equations. In talking to a variety of people about the approach, there seems to be some awareness of the method, but it is not very common. It is interesting the way it provides a definitive, math-based approach that is an alternative to educated guess-and-check. **Jeff Irvine** writes about problem solving as an overall process.

Finally, I wish you what I am sure is a well-deserved summer. I hope to hear from some potential new authors and encourage our novice teachers to share some of the challenges they have overcome, as well as encourage our experienced teachers to share their wealth of knowledge. Consider writing a letter to the editor when you discover something you know should be shared—like that challenge I found myself giving a student: Take any Sodoku and circle one of the given numbers. Now ask yourself which, if any, other values could that particular square take, and whether any of those values produces a valid Sodoku.

Best wishes for the summer from the editorial team! ▲

VOLUNTEERS

Thank you!

We couldn't have done it without you!

INFINITE POSSIBILITIES
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MAY 3 - 4
OAME 2018

▲ President's Message



JILL LAZARUS
EMAIL: jill.lazarus@oame.on.ca

Jill Lazarus, the current President of OAME, is a high school teacher in the Renfrew County District School Board. She is also completing a doctorate in Education at the University of Ottawa. Jill has a range of interests in mathematics education, including curriculum design and implementation, student communication in mathematics, educational technologies, classroom assessment, and professional learning.

Writing my final column as the OAME President presents an opportunity to reflect on the journey so far, and on the possibilities that lie ahead. Looking back, it has been an exciting year of growth for the OAME. Various activities and initiatives were offered locally and provincially. Chapters hosted local events. Grades 7 and 8 students participated in the Ontario Mathematics Olympiad. A successful Leadership Conference was held in Ottawa for the first time, for Canada 150, and, for the second year in a row, the Annual Conference sold out quickly.

As I mentioned in the September issue of the *Gazette*, it is the efforts of many individuals who contribute to the success of the OAME. If you have been following the Executive Directors' Reports this year (*OAME Gazette Volume 56*), you know that Lynda and Fred Ferneyhough have been acknowledging the contributions of many people, including board members and chapter representatives, presidents, and conference co-chairs. As the OAME evolves with the times, we continue to reflect on the various roles and responsibilities within this organization.

I look forward to seeing how the roles and committees take shape. More recently, the structure of the role of the President has been under consideration. In a Letter to the OAME Editor in September, Ralph Connelly, who has been the only person to take on this position twice, raised questions about the length of the President's term. He pointed out that quite a few years ago, the two-year term idea got as far as having a group come up with a plan for carrying it out, and that "perhaps it is time to dust it off" (p. 4). From my own experiences as President, I have also wondered whether it is time for a two-year term. Or, since this would require a longer commitment, should we reconceptualize how a team of presidents (Past-President, President, and President-Elect) might share the responsibilities? Or, is it appropriate to do both?





The OAME President is fortunate to be surrounded by the collective wisdom of an Executive Committee that includes two Executive Directors, two Vice-Presidents, a Past-President, and a President-Elect. It was an honour to be in the President position this year and a pleasure to work with Lynda and Fred Ferneyhough, Bill Otto, Sandra-Jean Price, Judy Mendaglio, and David Petro.

David, our incoming President, has an exciting Leadership Conference planned for November 8–9, 2018. This "Learning in the 21st Century" conference, to be held in Windsor, will feature Dan Meyer, Cathy Fosnot, and Graham Fletcher. The OAME 2019 Annual Conference, "All In," will return to Ottawa in May of 2019. Stay tuned for more information and be sure to register early!

As the annual conference has continued to reach capacity for the second year in a row, the complexity of organizing this event has increased. I would like to express my gratitude to the OAME 2018 and OAME 2019 organizing committees for the work that you do to support these events. A special thanks to the conference chairs, Wayne Erdman for 2018, and Marian Small for 2019. With the conference continuing to grow, and reaching more than 2000 registrations for 2018, running this conference takes the time and dedication of many volunteers. Thank you to everyone who has made these events possible.

I would like to take this opportunity to highlight one role that might not be recognized in the OAME Executive Directors' Report. Our Executive Directors, Lynda and Fred, are invaluable to the OAME. It seems that the roles and responsibilities attached to this position have continued to grow. I am fascinated by, and grateful for, Lynda's ability to do everything that she does.

Moving forward, I wish to welcome Paul Alves back to the OAME Executive as President-Elect. Paul is now the second person to take on two terms as President. Sandra Jean Price will be returning this year to the Vice-President role. Finally, Melissa Black returns to a Director position on the Board.

It was an honour to be President of the OAME this year, and a pleasure to be a part of this community. Echoing my message in the September issue of the *Gazette*, I encourage you to consider exploring the many opportunities to get involved in this organization.

Have a wonderful summer. I look forward to seeing what the next year brings! ▲

▲ Executive Directors' Report





LYNDA AND FRED FERNEYHOUGH
EMAIL: eds@oame.on.ca

Lynda and Fred Ferneyhough have been the Executive Directors of OAME since 2010. They taught in the Peel District School Board for over 30 years and had a three-year stint in the United Arab Emirates. Both of them served OAME as Chapter Representatives for CHAMP, Directors, and Vice-Presidents. During their career, they were Department Heads, and authors for McGraw-Hill Ryerson. They are both certified as instructors for Texas Instruments (TI), and Fred continues to coach for TI in the United States.

As I write this, the annual conference is only a few weeks away. Each year, a group of dedicated educators offers to give up their precious time to assist with the organization of the conference. This year, that duty has fallen to members of the OAME Board of Directors, along with a number of others, who were willing to give up a part of their lives to make this an event to remember. For the next three years, organization of the conference is being taken on by different chapters: in Ottawa (COMA 2019), a pairing of the Scarborough and Pine Ridge chapters (SAME/PRMA 2020), and Toronto (TEAMS 2021). If anyone would like to step up for the years beyond that, the Board of Directors would be grateful.

There is immense satisfaction with the implementation of this annual event, but it is demanding work. There are nine committees that begin work approximately three years in advance of the event. The work of different committees kicks in at different times, and it is the job of the Conference co-Chairs to coordinate all of those activities in much the same way that a contractor would oversee the many tradespeople working on a building. It starts with establishing Committee Chairs and choosing a date, facility, a theme, and logo. The Promotions Committee will take this information and order "swag" to advertise the conference, shirts for the workers, and all the large signs. The Facility and Equipment Committee will visit the site(s) and start negotiations for the rooms, parking, and necessary technology. The Finance Committee will set a budget, set registration prices, monitor expenditures and income, and deal with any significant changes in the number of participants. The Hospitality Committee plans for the lunches on site and any special events. The Program and Speakers Committee will decide on the keynote and featured speakers, organize any strands or themes, and put out the call for proposals—more on their work below. The Exhibitors and Sponsors Committee will visit all the exhibitors at an earlier conference to make sure they know of the upcoming conference, to develop a map of the exhibitors' table placements, and monitor reservations. The Registration Committee keeps an eye on the registrations that come in, sets up group codes for school board groups, double-checks that all registrations are also created in MCIS (Mathematics Conference Information System), and answers the multitude of questions that stem from registration attempts. The Volunteers Committee arranges and monitors the large contingent of volunteers needed on site for directing traffic, keeping the speakers happy, and general service requests. The Technology Committee takes care of MCIS, Web support, and the e-conference, while supporting all the Organization Committee needs. Finally, our Executive Directors ensure that the memberships are added or updated for all participants and presenters.

I would like to highlight the work of a few of the committees that I'm familiar with, to give you some idea of the scope of the commitment. Very few large conferences allow for pre-registration into sessions. However, our speakers appreciate the advance knowledge of session sizes, and sometimes circulate resource materials prior to the conference. Participants appreciate the opportunity to reserve a spot in their chosen sessions so that they are not forced to rush. This increases the amount of preparation required by the conference's Program Committee. They spend hundreds of hours negotiating with keynote and featured speakers, and closely examine approximately 400 session proposals from prospective speakers. They are also required to do a lot of fine-tuning, slotting sessions into available rooms, and ensuring that a variety of speakers are available from each panel during each session. This group must somehow ensure that the proposals are in line with the OAME vision, convey a message that can be understood by the prospective conference attendees, line up with the conference theme, and have all the necessary details that are required for speakers. What seems like a relatively easy assignment becomes a massive undertaking that requires at least two dedicated workers (preferably retired) and a cast of others who can contribute as needed. Their work continues as life happens, and some speakers have to cancel, sometimes at the last moment. If you've ever attended a conference and come away amazed at the quality of speakers, please take a moment to thank the Organizing Committee and the Program Committee in particular.

In 2014, I chaired the Facilities Committee with five highly dedicated volunteers. This group handles the parking and manages the issues that arise in various rooms. One unfortunate consequence of their work is that they are on duty almost full-time and do not get a chance to attend any of the sessions. While much of the work involves directing traffic, I was astounded at the number of emergencies that arise during the event. This group of people works from morning to night solidly for two and a half days. They don't need to do as much prior to the event, but when the event begins, they have to be on their toes at all times.

Membership on any of the committees requires dedicated people, who are willing to give up a lot of time to make this event happen. Without their efforts, our annual conference would not be the success that it has been for so long. It just keeps getting bigger and better! ▲

▲ Arithmetic Wordsearch

S	S	E	L	F	O	R	T	Y	F
S	C	Q	F	A	L	H	C	U	I
R	L	U	I	A	I	E	V	I	F
E	Y	A	F	R	T	A	D	D	T
B	T	L	T	W	E	L	V	E	E
M	N	E	Y	W	I	F	E	X	E
U	E	I	N	G	O	F	I	Z	N
N	W	O	N	U	N	S	G	E	S
R	T	H	R	E	E	F	H	R	E
U	N	Y	T	R	I	H	T	O	V
E	R	O	M	N	E	V	E	L	E
S	U	B	T	R	A	C	T	E	N

Words to find: numbers, zero, one, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fifteen, twenty, thirty, forty, fifty, add subtract, equal, less, more

Write the letters that are left over in the spaces below. But write them from left to right and top to bottom.

MYSTERY PHRASE:

"_____ ___ ___"

▲ Letter to the Editor

A letter regarding the editorial column last month.

Thank you for bringing up the topic of arithmetic computational skills in your Editor's report in the March 2018 issue of the *Gazette*. The current curriculum discussions about how students learn computational skills, promoting strategies to understand how adding or subtracting or multiplying or dividing whole numbers, integers, fractions, and decimals, are exciting developments. This, along with the amazing technology available to students, has made me rethink my approach to arithmetic review at Lambton College. But I have not rejected the value of proficiency with no-calculator arithmetic. I continue to believe that this is foundational to mathematical thinking.

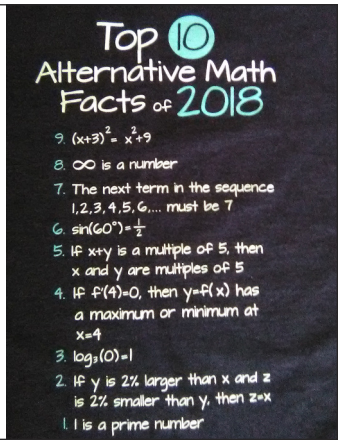
Students from across the province arrive at Lambton College with a variety of backgrounds. Some have latched onto particular computational strategies. Others have never gained one. What they all say is that they have been using a calculator "like forever," so computational strategies are things of a distant past. That is unfortunate, given the no-calculator arithmetic demands of the trades and of nursing, not to mention the no-calculator-allowed first year mathematics courses at university (like the University of Toronto and the University of Western Ontario).

From my college perspective, students need a roadmap in elementary and high school that guides them through no-calculator computational strategies, then consolidates, and then continues to review. To make this work, this roadmap has to have a timeline: time for exploring strategies, and then time to consolidate, and then time to review. There will be some students who do not latch onto and become proficient in any computational strategy and, in my opinion, they will need to be guided to a choice, whether it makes sense to them at the time or not.

Don Vander Klok

▲

From the t-shirts given to markers of Waterloo math contests from Centre for Education in Mathematics and Computing (CEMC)



▲ "Mathematics is more important than ever before" – Fields Institute expands its repertoire



JUDY MENDAGLIO
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Judy Mendaglio is Past President of OAME. She maintains close contact with other organizations involved in mathematics education, including the Fields Institute.

On April 10, Reza Moridi, Ontario's Minister of Research, Innovation, and Science, visited the Fields Institute for Research in Mathematical Sciences to announce funding for a new Centre—the Fields Centre for Quantitative Analysis Modelling (Fields-CQAM). The announcement was accompanied by a $4 million investment to support the two-year pilot.

"In a world of big data, artificial intelligence, and accelerated change, mathematics is more important than ever before," said Dr. Moridi. He continued by describing the landscape of Ontario's economy as rapidly changing, moving away from a manufacturing base and toward a knowledge basis. Managing this change requires the development of new partnerships. At the heart of this multidisciplinary transformation lie the mathematical sciences, providing a framework for describing, understanding, predicting, and navigating the complex relationships with which we are now confronted. Outcomes expected include innovation, training, and economic growth.

Dr. Ian Hambleton, Director of the Fields Institute, said, "The world has lots of problems for us. Challenges." These challenges include gathering the right data, understanding what they tell us, and developing new predictive methods to inform decision making in related industries and disciplines. Those involved in the Centre will learn to tackle problems that are not "pre-digested." They will be trained to listen to and understand perspectives, and will be expected to work together in collaborative teams. These teams will cross university and corporate boundaries, and bring perspectives across various disciplines.

If you have trouble imagining what mathematical research looks like, an example of the work the Fields Institute has done, and will continue to do through this funding, is the analysis they did at the request of St. Michael's Hospital in 2008, when the SARS outbreak hit Toronto. Mathematicians at the Fields Institute modelled the spread of SARS through airline networks. These models are now being used in other applications, for example, as the prediction of infectious diseases such as Ebola.

To continue the excellence of similar research, the CQAM will be structured around labs, linking academic disciplines and industry province wide as follows:

- Financial Data Analytics will be focused on concerns with large data sets.
- Health Analytics and Modelling will focus on prediction of disease progression and wearable devices.
- Infectious Disease Modelling will develop new frameworks and procedures to inform public health policies and vaccine production.
- Human–Machine Symbiosis will target better understanding of human perception, development of new technologies for sensors, and accessibility software.
- Chemical Processes will be researching optimization and process control for energy, waste treatment, and biomedical devices.
- Prediction of Anomalous Events will identify fraudulent financial transactions, diagnose rare cancerous biopsies, and contribute to genome analysis.



At the announcement of the formation of the new Fields Centre for Quantitative Analysis Modelling are Dr. Arvind Gupta (University of Toronto and member of the Board of Directors of the Fields Institute), Dr. Sheila Embleton (York University and Chair of the Board of Directors of the Fields Institute), the Honorable Dr. Reza Moridi, Dr. Ian Hambleton, and Dr. Huaxiong Huang (Deputy Director of the Fields Institute) ▲

▲ An Honour for Canadian Mathematician Robert Langlands



JUDY MENDAGLIO
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Judy Mendaglio is a proud mathie. She describes the world of math as "her happy place," where things slow down and make sense. Reading about math is one of her favourite pastimes. Judy is currently the Past President of OAME and Chair of the Steering Committee of the Fields Institute's MathEd Forum.

Have you spent time trying to find interesting connections among measurement, patterning, and algebra? Are you fascinated by the work that physicists have been doing as they seek the scientific "theory of everything"? Are you aware that the mathematics research community has also been working on a "theory of everything" in mathematics in an attempt to connect seemingly disjoint branches of mathematics? The father of the theory of everything, for mathematics, is Canadian mathematician Robert Langlands. This year, Robert Langlands will be honoured with the Abel Prize in mathematics.

The Abel Prize, which was first awarded in 2003, honours a lifetime of mathematical work and influence. The Norwegian Academy of Science and Letters announced in March that Robert Langlands has been named the 2018 Abel Prize Laureate, in recognition of "his visionary program connecting representation theory to number theory." The King of Norway will present the prize to Langlands at a ceremony in Oslo on May 22, 2018. In addition to the honour, he will receive an amount of 6 million Norwegian Krone, which is roughly equivalent to one million dollars CAD.

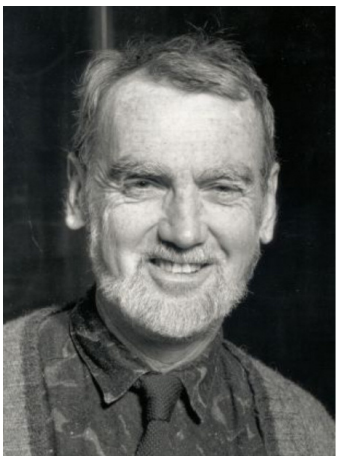


Figure 1: *Robert Langlands* (**www.abelprize.no**)

In the 1960's, Langlands first proposed a theory that could, if developed and proved, connect areas of mathematics that had, until then, been considered disjoint. The seeds of Langlands' insights have grown into what is now known as the Langlands Program, which has provided fertile ground for many areas of mathematical research.

The "theory of everything" in mathematics, the Langlands Program, is a set of conjectures that seek to unify knowledge from different branches of mathematics. The idea is that a problem in one area of mathematics may be very difficult to analyze using the tools available in that area. However, if the structures within the problem can be related to similar structures in a different field, where there are better analytical tools available, then the analysis may be conducted with less difficulty and the results related back to the original problem. In this way, even deeper structures in the original area of mathematics are revealed.

For example, consider a somewhat elementary problem such as finding the properties of solids in four or more dimensions. That is a geometry problem. However, since we have difficulty imagining objects in four or more dimensions, we find analyzing their properties difficult; however, we can relate their geometric structures to algebraic structures that are well understood. We muck about in the world of algebra, have an insight or two, and then relate those insights back to the geometry problem. At the elementary/secondary school level, it could be like needing to know an interior angle of an irregular polygon in geometry and flipping into using algebraic and numerical relationships to figure it out. Interestingly, one of the concepts that seems to connect different branches of mathematics together is symmetry. Thinking about elementary and secondary mathematics, we see symmetry in numbers, figures, and objects, as well as relations and functions. In this way, we might connect number sense, geometry, and modelling.

Of course, the math of the Langlands Program is of a more advanced level than even most mathematicians understand, but, fundamentally, the idea is the same. In the Langlands Program, the branches of mathematics that are being related are those connecting our understanding of numbers and our understanding of change. Think arithmetic meets geometry, and they meet calculus, and then take it up a few dozen notches. The main branches of mathematics that are involved are number theory (the study of properties and relationships of numbers), algebraic geometry (a cousin of analytic geometry), representation theory (which concerns sets and mappings), and mathematical physics. Mathematician and author of *Love and Math*, Edward Frenkel, has called Langlands' theories "the source code of all mathematics."

Langlands' breakthrough theory has generated an enormous playground for mathematical research. Mathematicians who have been honoured because of their work on Langlands' conjectures include Fields Medalists

Laurent Lafforgue (in 2002) and Ngô Bao Châu (in 2010). Langlands himself was honoured with the Wolf Prize for Mathematics in 1995, and subsequently so have Alexander Beilinson and Vladimir Drinfeld (2018). Other well-known recipients of the Abel Prize are Andrew Wiles, who proved Fermat's Last Theorem, and John Nash, known for his work in economic theory and the subject of the Hollywood movie *A Beautiful Mind*.

Robert Langlands was born and educated in British Columbia. He did his undergraduate and master's degrees at the University of British Columbia before moving to Yale University for his doctorate. He has spent most of his career at Princeton University, although he comes to his home in Montreal between semesters. Now 81, Langlands is Professor Emeritus at the Institute for Advanced Study in Princeton.

Sources:

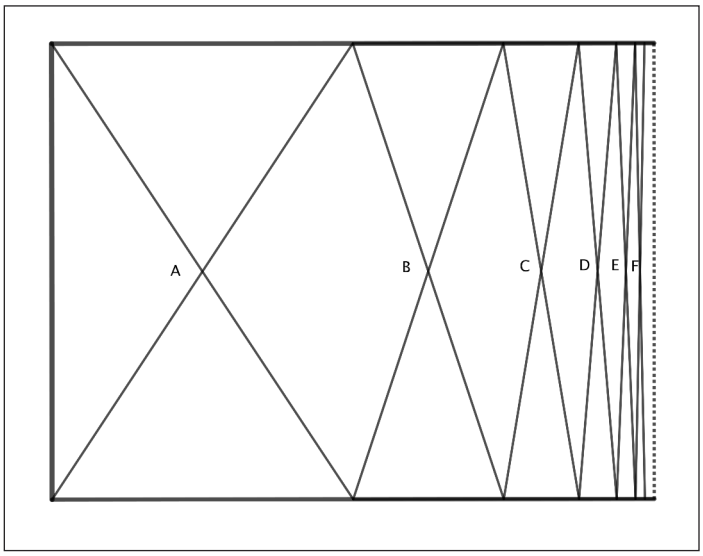
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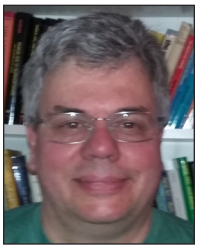
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▲ What's the Problem? Looking for Liars

SHAWN GODIN



Shawn Godin is head of Mathematics, Business, Law, and Computer Science at Cairine Wilson Secondary School in Orleans. He strongly believes in the central role of problem solving in the mathematics classroom. He continues to be involved in mathematical activities: presenting workshops, writing articles, working on local projects, and helping create mathematics contests.

Welcome back, problem solvers. Last time, I left you with the following problem:

> *On a certain island live only knights, who always tell the truth, and knaves, who always lie. One day, you find 10 island natives standing in a circle. Each one states: "Both people next to me are knaves!" Of the 10 in the circle, what is the* minimum *possible number of knights?*

This problem came from the website **brilliant.org**. The website offers free and paid memberships that provide access to mathematics, science, and computer science problems and courses. The free membership allows access to some of the material, which includes scores of problems submitted by Brilliant users. You can check out a TEDx presentation on developing students in STEM subjects by Brilliant's CEO, Sue Khim, at **www.youtube.com/watch?v=EnQCYZ8Oz8Q**.

Even though this problem does not link directly to any particular curriculum topic, logic puzzles encourage students to use their reasoning and communication skills, which are two of the math processes in the curriculum. Later we will extend the problem and make further ties to the curriculum.

This problem lends itself nicely to using manipulatives to help visualize it. I will use letters: a "T" for a truth-telling knight and an "L" for a never-so-truthful knave. Students could use different attribute blocks or tiles of different colours to allow them to quickly try things out. First, we'll examine groups of three, who are part of a circle; the focus is on the person in the centre and who can be beside them. For our knight, who always tells the truth, the only option is to have two knaves beside him: LTL. For a knave, we cannot have two knaves beside him, so we can either have two knights or one knight and one knave: TLT, LLT, or TLL.

From these observations, we see that we cannot ever have two knights beside each other. We can have two knaves beside each other, but not three in a row. Since we want the minimum number of knights, we can start with LLT and repeat it around the circle as many times as possible. After we have done it three times, we have a knight and a knave separated by an empty space that we cannot fill (Figure 1).

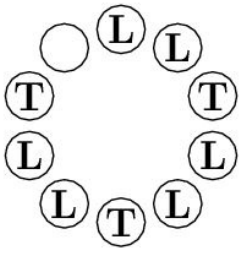


Figure 1: *First attempt at arrangement*

If we try to fill the empty space with a knight, we have two knights beside each other, which cannot happen. Similarly, putting a knave in the open position leaves us with three knaves beside each other, which we also cannot have. Since we must have a knave by a knight, if we fill the empty space with a knave, then we must change our first person to a knight to make things work. If we do so, we get the configuration in Figure 2, which works (changes in red). Thus, our minimum number of knights is 4.

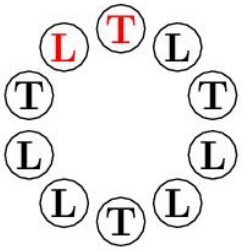


Figure 2: *A valid arrangement*

As so often happens, the problem by itself is cute, but if we sit back and ask, "What if…?," our investigation becomes richer.

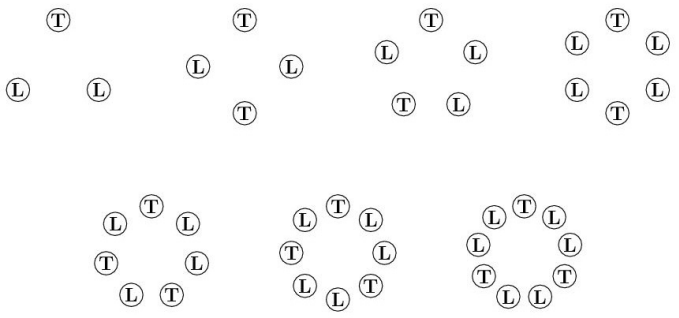


Figure 3: *Valid arrangements for 3 through 9 islanders*

Let's first examine if we can predict, in general, what the minimum number of knights will be for any number of islanders standing in a circle. Generating valid arrangements for 3 through 9 islanders (Figure 3), we can tabulate the results (Figure 4).

islanders	3	4	5	6	7	8	9	10
knights	1	2	2	2	3	3	3	4

Figure 4: *Table of valid arrangements*

Some patterns start to appear in the table. It seems that the maximum number of knights remains unchanged for three different numbers of islanders, starting with 4 islanders and 2 knights. Unfortunately, this doesn't lead directly to a linear relation, yet there are "linear-ish" things going on. Extending the table from Figure 4 and graphing the results, using Desmos, we obtain the graph in Figure 5.

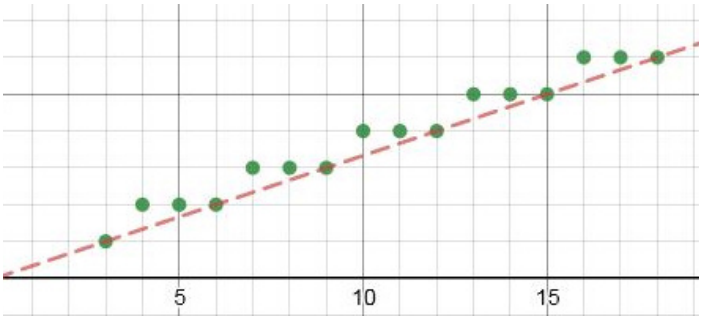


Figure 5: *Graph of minimum number of knights for various numbers of islanders*

The points themselves don't all line up, although it seems that the "last" points in each group do. If we focus on these values, we see that they lie on the line with equation . They correspond to when the number of islanders is a multiple of 3, in which case, we can use of the pattern TLL or LLT a number of times and then "close the loop." For example, for 9 islanders, we could use either TLLTLLTLL or LLTLLTLLT (where we are reading around the circle so the ends get tied together) to maximize the knaves (which minimizes the knights). A little exploration will find that the rest of the points lie on two other lines that are parallel to the first. I will leave these as exercises.

This is a good place to introduce two useful, related functions. The *floor function*, noted as $\lfloor x \rfloor$, is defined as the largest integer n such that $n \leq$. So if x is an integer, the floor function does not change it; for example, $\lfloor 7 \rfloor = 7$, $\lfloor \pi \rfloor = 3$, $\lfloor 8.75 \rfloor = 8$ and $\lfloor -1.2 \rfloor = -2$. The floor function can be thought of a round down to the closest integer. This is also known as the greatest integer function. (It is not the same as the "trunc" function that removes the decimal because trunk (–1.2) = –1 rather than –2.)

Similarly, the *ceiling function*, noted as $\lceil x \rceil$, is defined as the smallest integer n such that . Thus, $\lceil 7 \rceil = 7$, $\lceil \pi \rceil = 4$, $\lceil 8.75 \rceil = 9$ and $\lceil -1.2 \rceil = -1$. So the ceiling function rounds up to the nearest integer. If you look at Figure 5, we can see that the points that don't lie on the line correspond to $\frac{x}{3}$ being a non-integer, and the corresponding value of y is rounded up to the next integer, just like the ceiling function does. If we graph $y = \left\lceil \frac{x}{3} \right\rceil$ (you can get the ceiling function in the list of functions in Desmos or by typing "ceil"), you get the graph in Figure 6. It seems we have found a function to describe our problem. Note that if you scroll along the graph of the step function below, you will find that the rightmost point is part of the step, but the leftmost point is not; that is, (6, 2) is on the graph, but (6, 3) is not. This is denoted using closed and open circles, respectively, which Desmos does not add automatically.

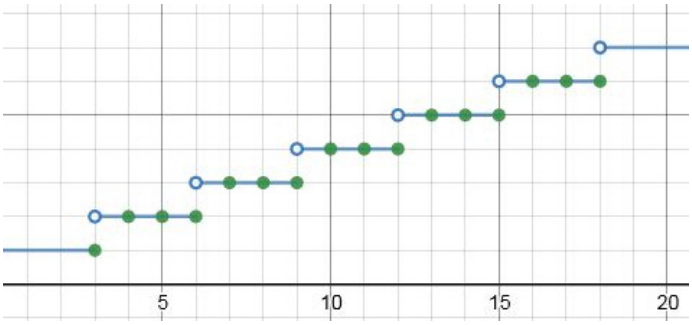


Figure 6: *Graph of* $y = \left\lceil \frac{x}{3} \right\rceil$ *with minimum knight data*

We can further play with the problem by exploring what patterns we see if we are after the maximum number of knights. You could also change what each person says to, "At least one person next to me is a knave!," or, "Considering the 4 closest people next to me, all 4 are knaves!," or anything else of your devising. Have fun with your explorations!

Now for your homework:

> *A (non-degenerate) triangle has side lengths 10, 17, and* x*, where* x *is an integer. For which values of* x *is the triangle obtuse?*

Until next time, happy problem solving! ▲

▲ Technology Corner: Graph of the Week



MARY BOURASSA
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Mary Bourassa teaches mathematics at West Carleton Secondary School in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She has presented workshops internationally, authored mathematics resources, is a Past VP of OAME, and a Past President of COMA. An award-winning teacher, Mary continually strives to learn new and better ways of helping students learn and love mathematics.

We have long been able to find good problems of the week (PoW) from sites like the Centre for Education in Mathematics and Computing at the University of Waterloo (**www.cemc.uwaterloo.ca/resources/potw.php**), the MATHCOUNTS® Foundation (**www.mathcounts.org/resources/problem-of-the-week**), and other online sites. What is new is that there are also some challenges based on graphs and charts that can be used with students in a variety of ways.

Graph of the Week (**www.turnersgraphoftheweek.com/**) is a site created by Kelly Turner that provides students the opportunity to read, understand, and analyze real-world data. An example is shown in Figures 1 and 2. Note that teachers may want to consider that some examples, such as this one, include shows/topics that may not be considered appropriate by some students and/or some households.

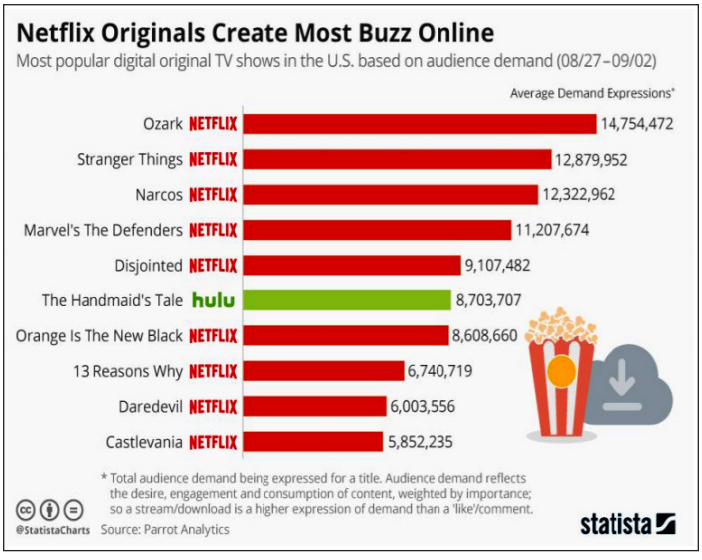


Figure 1

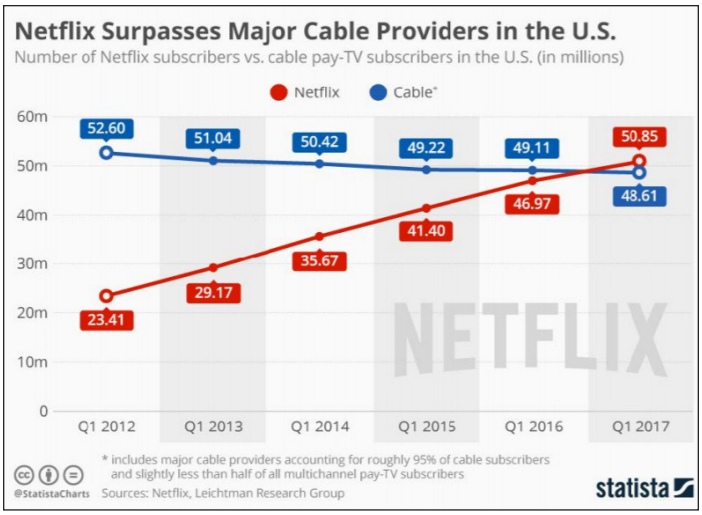


Figure 2

Given these images, students are asked to write a reflection on what information is being provided in the graphs/charts. Students are encouraged to use the following general questions to help guide their reflections.

- What is the topic of the graph?
- What is being compared in the graph?
- What do the *horizontal* axis and *vertical* axis represent?
- What observations can you make about the graph?
- Is there an upward or downward trend? Explain.
- Are there any sudden spikes in the graph? If so, what conclusions can you make about these spikes?
- What prediction(s) can you make about the future?
- What inferences can you make about the graph?

I was immediately reminded of OSSLT (Literacy Test) preparation questions that feature charts and graphs. This resource could be incorporated into many classes as a cross-curricular activity to give students more opportunities to decipher the meaning and trends shown in the chart or graph and to draw conclusions. Kelly also provides a downloadable template for student responses.

After seeing the source of some of the graphs used on *Graph of the Week*, I discovered that Statista, a statistics portal, also has a chart-of-the-day feature (**www.statista.com/chartoftheday/**), along with a treasure trove of statistics. Among the charts is the one shown in Figure 3. I love the idea of using this chart to talk about place value.

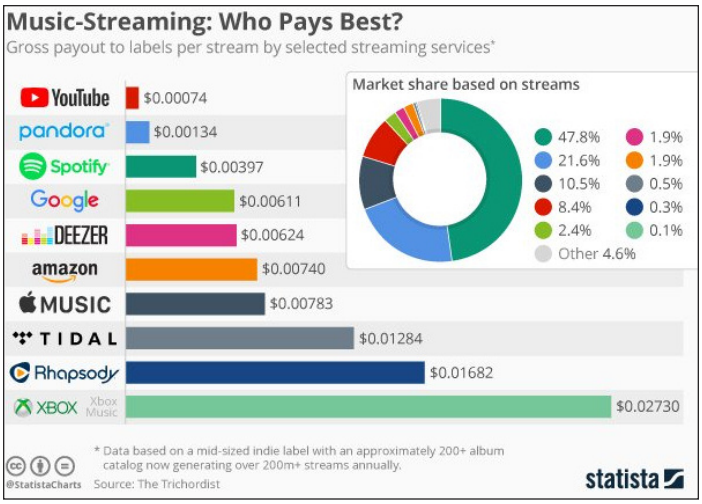


Figure 3

Figure 4 shows another chart from Statista. It would be interesting to cover up the brands and see if students could place the brands in the correct order. Students would also benefit from calculating the percent increase in sales from 2016 to 2017 to create a different ranking of the five brands shown.



Figure 4

Another source of interesting graphs is *The New York Times* with its monthly feature called "What's Going On in This Graph?" (**www.nytimes.com/column/whats-going-on-in-this-graph**). It takes graphs that have appeared elsewhere on **NYTimes.com** and removes some of the text to encourage student thinking to come first. The link to the original article is posted a week after the graph is first released. The questions are simple. What do you notice? What do you wonder? What might be going on in these graphs? You are then encouraged to join the conversation online by posting comments (with a recommendation that students younger than 13 have their teacher post comments for them). Conversation is the goal, through replies to

comments or through the facilitated discussion at the posted date and time.

An example is shown in Figure 5.

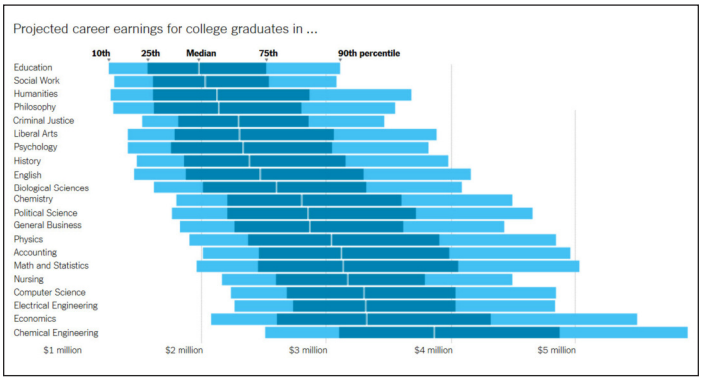


Figure 5

The follow-up to the graph includes definitions of percentiles, quartiles, medians, and boxplots to help students tie these terms to what they are seeing in the graph and deepen their understanding. They also provided links to the data and a time-series graph of the data, asking which is easier to interpret. Readers are given more questions to consider, but this gives an introductory sense.

Figure 6 is another graph for you to consider, taken from Chris Hunter's blog (**reflectionsinthewhy.wordpress.com**). He stripped everything other than the graph itself to create something that will provoke students to notice and wonder. Chris took the graph from the "101 Questions" website (**www.101qs.com**). You can find it there if you search "water consumption"—that's your big hint.

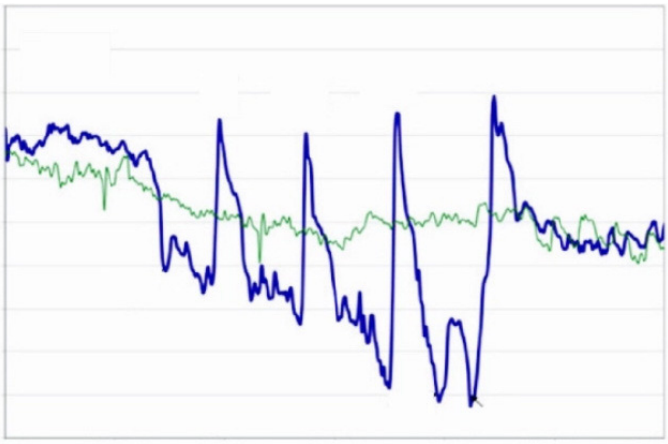


Figure 6

Providing students with opportunities to discuss and analyze graphs and charts is an important aspect of math education. Literacy includes being able to understand and interpret graphs, and the sites shared here will allow your students to develop these skills. Be sure to check them out!

▲

▲ Provincial Digital Learning Resources – What's New? Set Tool 2.0






GREG CLARKE

AGNES GRAFTON

MARKUS WOLSKI

ROSS ISENEGGER

Greg (Simcoe Muskoka Catholic District School Board), Agnes (Brant Haldimand Norfolk Catholic District School Board), Markus, and Ross (Near North District School Board) are all currently on assignment with the Ontario Ministry of Education as Provincial Math Leads, working on the digital resources found at ***mathies.ca****.*

The Set Tool

The Set Tool was updated to version 2.0 in February 2018. This version includes many interface improvements, as well as some requested new features:

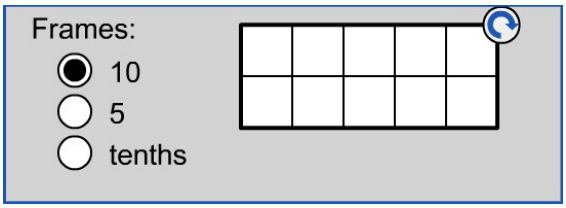
- Five Frames
- Objects snap into number frames
- Number frames can be moved, copied, or rotated (along with their objects)
- *Auto Arrange* button organizes objects into groups of 5 or 10
- *Randomize Attributes* buttons allow configuration of the attributes to change

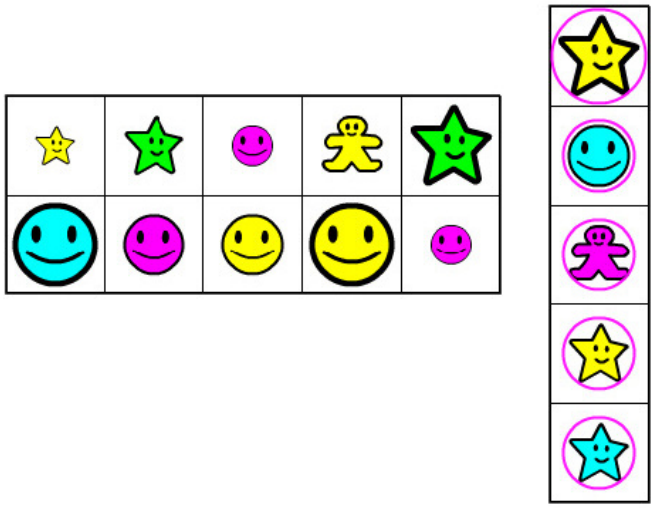
These tool enhancements, together with the updated annotation features (see March 2018 *Gazette* article for details) and the Save/Load capabilities, both of which are being added to all **mathies.ca** apps, truly make this version something worth investigating. These improvements, along with some classroom examples, are explored in this article.

New Features: Five Frames, Snapping to Grid, Auto Arrange

In response to requests from Early Years educators, five frames have been added to the number-frame types. Also new for this version is the ability to rotate the number frames, either before you drag them out of the panel, or afterwards while in the workspace.

The biggest improvement to the new Set Tool is that number frames have built-in snapping logic. Drag and drop objects into a number frame, or release a number frame on top of a set of objects and the objects snap into empty places. To reinforce the counting aspect of the tool, no two objects are allowed to occupy the same frame position. So, if there is a conflict, the extra objects get bounced outside the number frame. Once objects have snapped into a frame, they stay there.





When you select a number frame:

- both the frame and its contents are highlighted
- the contents automatically come along for the ride when dragged

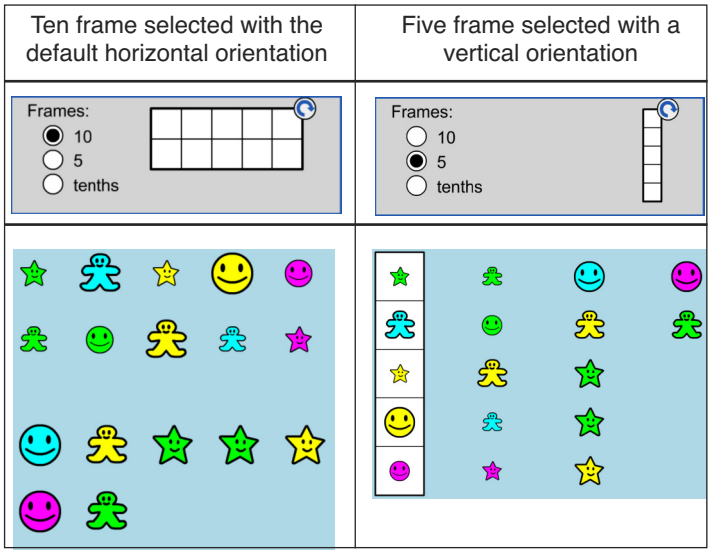
To remove a single object from a number frame:

- start with a non-selected frame
- drag individual objects outside the frame

To remove all the objects from a number frame:

- select the frame (its contents will be selected too)
- click on the frame again to deselect the frame (and just the frame)
- drag all of the highlighted objects away

The functionality of the *Auto Arrange* button has been changed so that it now lines up objects in the workspace in groups of 5 or 10, depending on the choices you have made in the panel. Here is an example of what will happen when a random set of 17 objects is arranged, based on the type and orientation of the number-frame setting. The spacing of the arrangement is designed so that objects fit nicely into a number frame, as shown in the example on the right.



Various arrangements of objects help students to see that numbers can be composed in different ways. In the table, students can see 17 as 1 ten and 7 units, which reinforces place value. They can also see 17 as 3 groups of 5 and 2 more. They might even skip count by 5's to 15 and then add on 2 more.

Mathematical Connections

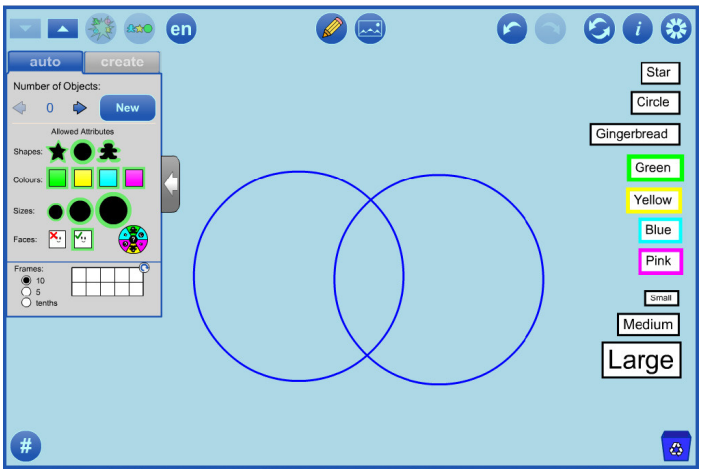
The Set Tool allows for up to four attributes (Shape, Size, Colour, and Face) for each object, making it an ideal tool for a variety of sorting and probability activities. Each object can take one value from each of the following attributes:

Shape:	Star, Circle, and Gingerbread
Size:	Small, Medium, and Large
Colour:	Green, Yellow, Blue, and Pink
Face:	Shown and Hidden

This makes the Set Tool appropriate for exploring counting, conservation of number, patterning, sorting, probability, and modelling mathematical operations. Refer to the support wiki and the *Gazette* article from December 2016 for some additional examples.

Sorting Using the Venn Diagram Template

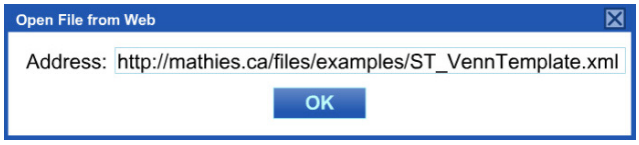
A class is learning about sorting based on two attributes. The teacher decides to create an activity that uses the mathies Venn Template file.



How does the teacher know this file exists? The wiki page for Set (accessed via the *i-button*) now includes a list of sample files for teachers and students to explore. Teachers may also want to look for sample files on the wiki pages for other mathies apps that have the capacity to load files.

How do students access this file?

1) Open the Set Tool.
2) Click on Settings), then the Open from Web button .
3) In the dialog that appears, enter the address for the file.



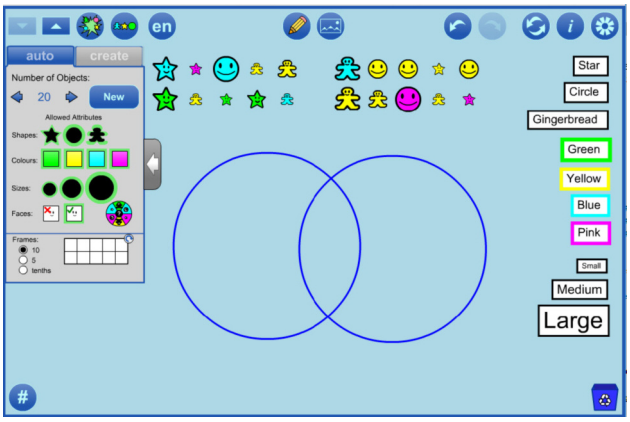
4) Click OK.

Tech Tip: Creating Template Files

How was the ST_VennTemplate file created?

1) The Set Tool was opened in a browser on a desktop computer. The browser window was set to 1024 pixels by 680 pixels so that the saved file would open nicely on most devices, including the iPad and other tablets.
2) The annotation tools were used to create two overlapping circles for the Venn diagram. The text tool was used to create labels that are reflective of the various attributes.
3) The file was then saved by opening *Settings* , clicking the *Save File* button , and supplying a location and file name.

The teacher instructs the students to open the file and generate a set of random objects to sort.

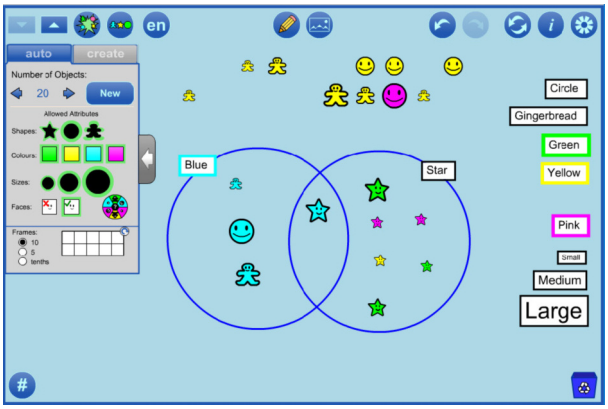


This student created a set of 20 random objects.

How?

1) Click on the *auto* tab.
2) Decide which attributes you would like to include in the randomization. This image shows the default set-up. The green highlights indicate that the set will be generated using all 3 shapes, 4 colours, and 3 sizes with faces showing.
3) Type 20 in the *Number of Objects* input field 20 , then press Enter or click New . Objects will be created according to the selected attributes and scattered around the workspace.
4) Click the *Auto Arrange* button to organize the objects.

Suppose a student decided to sort using *blue* as one attribute and *star* shaped as the second attribute.



How?

1) Click on the pencil button to open annotation.
2) Drag the appropriate label to each circle.
3) To avoid unintentionally dragging the circles or labels, click the pencil button again to close the annotation before sorting.
4) Drag the objects into the appropriate region of the Venn diagram.

This might also be an opportunity to discuss probability. How many blue objects would you expect to see in a random set of 20 objects? How many stars would you expect to see? (Since there are 20 objects, and 4 possible colours, one would expect to see 5 blue objects on average. Likewise, you would expect to see 6 or 7 stars.)

The teacher instructs the students to save their work and then repeat their sorts, using a different set of objects. Alternatively, students could be asked to sort the same set of objects, using different attributes.

To save their work, students press the *Settings* button, then the *Save File* button. This file can then be shared with the teacher or a peer. Students can also open the file at a later date and continue working.

A new set can be generated, using the auto tab, in a variety of ways:

1) Press the *New* button to generate a random set with the same number of objects. The objects generated will be scattered across the workspace.

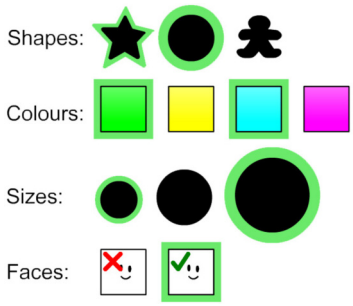


2) Enter a different value or use the stepper arrows to generate a different-sized set. Items will be added to or deleted from the workspace.
3) Press the *Randomize Attributes* in Set button to randomize the attributes of the current objects in the workspace, without changing their location. This is a new feature as of version 2.0.

Teachers might want to engage students in a discussion about whether or not it is advisable to change the number of elements in a set if they then wish to compare the outcomes of the two sorts. How would you go about comparing the results of the two sorts if a different-sized set were used? It would also be important to have a discussion about the advisability of changing the set of allowable attributes for each trial. What might you do if you wanted to skew the outcome of the second sort?

Tech Tip: Allowed Attributes in Auto Tab

The set of allowable attributes is customizable by toggling the icons.



Looking at the green highlights (halo on the exterior, if you are reading this in greyscale) in the given example, all objects would be turned into either stars or circles, either green or light blue, small or large sizes, showing happy faces when the *Randomize Attributes* in *Set* button is clicked. Notice that the appearance of this *Randomize* button changes as different attributes are allowed.

Exploring Experimental Probability

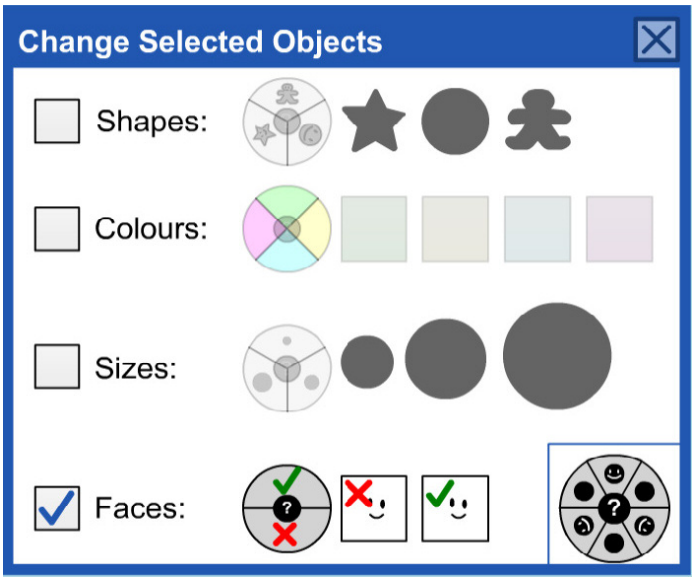
Simulating Coin Tosses

A student wishes to explore the probability associated with the flipping of a coin. She decides to simulate flipping 10 coins at once, and plans to do this multiple times, to compare the results of various trials.

Using the Set Tool, she first changes the multiplier to ×10, then drags 10 circles into the workspace.



While they are selected, she clicks the *Change Selected Objects* button, which opens up the Change Selected Objects dialog.



Since she wants to focus on only whether the flip of a coin reveals heads or tails, she turns on the *Faces* check box and then turns off the *Shapes*, *Colours*, and *Sizes* check boxes.

Clicking on one of the two *Randomize Spinners* in the Faces row of the dialog will randomly "flip" each selected circle, and assign them a happy face or not.

The result of this first random trial is 4 of 10 happy faces (heads).

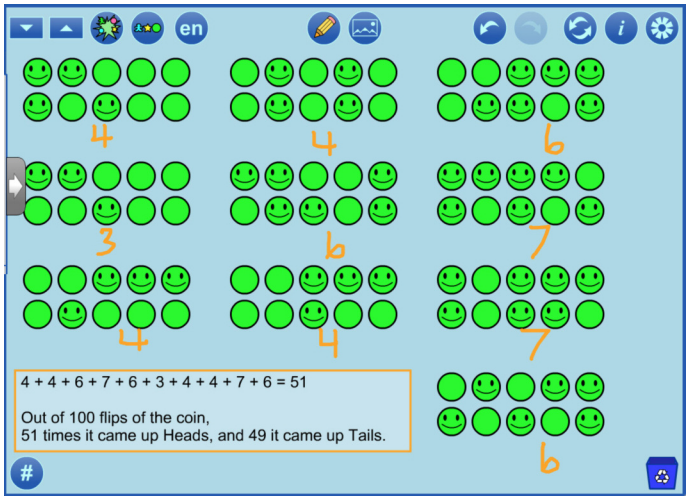


The student then copies these 10 shapes. The Set Tool places them in a neighbouring non-overlapping position. A click of the *Randomize Spinner* flips this new set of 10 circles, which results in a different combination of happy and blank faces.

The student continues this until she has completed 10 trials, and her workspace is filled.

Using the annotation tools, she notes the number of happy faces in each trial. She calculates and records the total.

What is interesting about this student's example is that the overall experimental probability turned out to be $\frac{51}{100}$, which is quite close to the theoretical probability of $\frac{50}{100} = \frac{1}{2}$. However, not a single trial of 10 tosses actually yielded 5 happy faces. Using the *Randomize* button in the Set Tool lets students experience how surprising randomness sometimes feels.



To try the experiment again, she could select all 100 "flipped" circles, and click the *Random Attributes* button one more time, and suddenly have a fresh toss of 100 coins to explore and compare.

Other Simulation Ideas

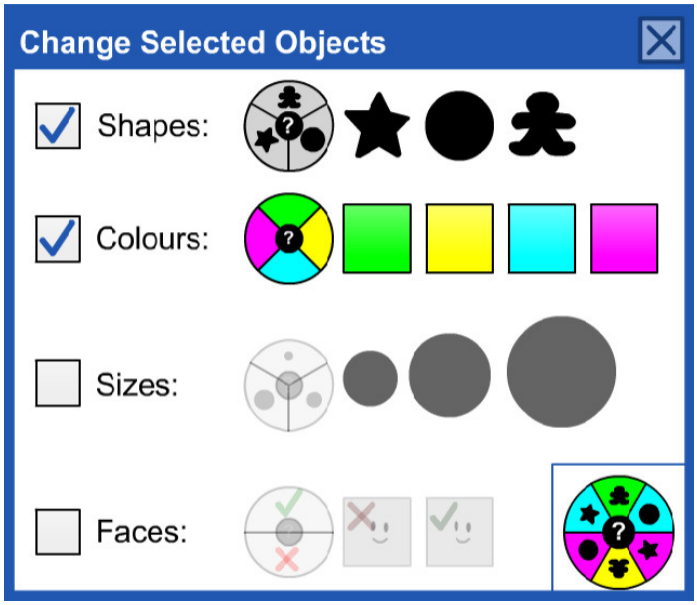
Tech Tip: Simulations Using the Change Selected Objects Dialog

To investigate a different simulation, the *Change Selected Objects* dialog can be customized by adjusting the attribute check boxes.

To create a simulation corresponding to a combinations question about matching 3 tops (shapes) and 4 pants (colours), do the following:

- Check the *Shapes* and *Colours* boxes, and uncheck the *Sizes* and *Faces* boxes.
- Select the objects you wish to randomize.
- Click the *Randomize Attributes* button, and all the selected objects will randomly be assigned one of 3 shapes and one of 4 colours, for a total of 12 different possible shape–colour combinations.

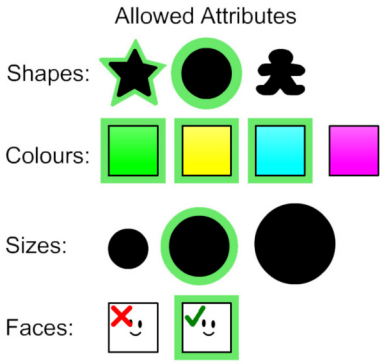
Note: The original size and face attributes will remain unchanged.



Note: Because the size and face attributes are not changed, it might be less confusing for students to start with a single size and face attribute. Before turning off the *Sizes* and *Faces* settings, click on one of the size icons, and one of the face icons. Then, uncheck the *Sizes* and *Faces* boxes.

Tech Tip: Simulations Using the *Auto* Tab

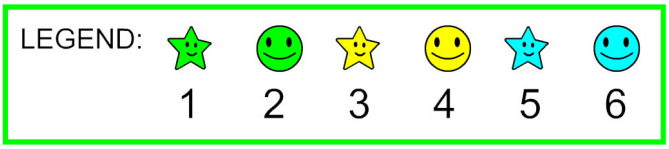
An alternate approach to creating a simulation is to use the *Randomize Attributes* spinner in the *Auto* tab. This allows for further customization of the attributes, since you can specify which shapes, colours, sizes, and faces are allowed in the set.



For example, to create a simulation with six distinct outcomes, we can select 2 shapes and 3 colours:

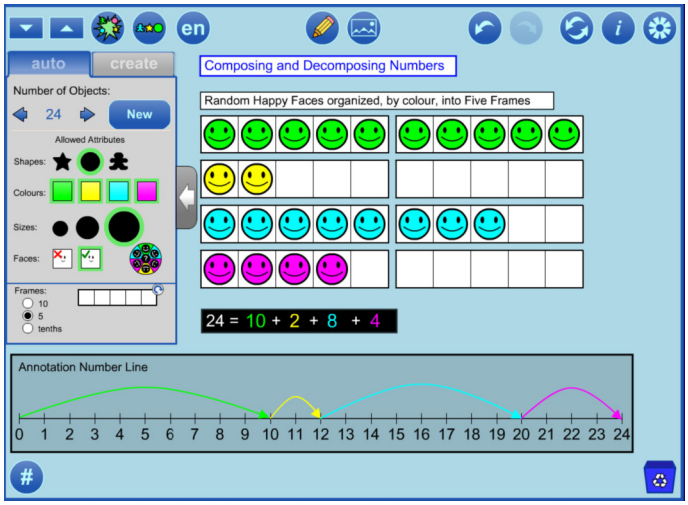
- Click the attribute icons so that there are green highlights around only 2 of the shapes, around 3 of the colours, 1 of the sizes, and 1 of the face choices.
- Click the *Randomize Attributes* button, and all the objects in the workspace will randomly be assigned 1 of the 2 shapes and 1 of the 3 colours.

Note: This legend might be a helpful "mapping," if you wish to simulate tossing a 6-sided number cube.



New Drawing Objects = Multiple Representations

The inclusion of the new drawing objects in the Annotation Tools make it possible to reinforce your understanding by using other representations alongside the Set Tool objects. The annotation number line (shown here) and annotation grid tools are especially versatile.



File Operations: Saving and Loading

A saved file will include all of the steps used to create the representation, as well as a snapshot of the final annotation objects on the screen. If the file is loaded, the undo/redo buttons can be used to step back and forth through the creation of the contents. The annotation objects are part of the saved material (and can be edited/moved/deleted, if desired), but they are not currently part of the undo history. The save/load feature can be used to set up a problem or an activity for students in advance (or by teachers in workshops) as in the Venn diagram example above. Also, students can save their work to come back to at a later time, or to share with their peers or teacher.

On the Wiki Support Page for the Set Tool (accessible from the i button), there are also some sample files that can be loaded. There are links you can click on to directly load the file into a desktop browser. Alternatively, you can open one of the sample files, using the *Open from WWW* button, accessible from the Settings menu, and entering the address. (If you are using a mobile device, this is the only method available to you.)

For more details about load/save, including transferring files between devices, visit the wiki page at **mathclips.wikispaces.com/FileOperations**.

Feedback and Future Requests

Please feel free to send us your feedback about any mathies tool, using the Feedback Form button inside the Information Dialog, accessed from the i button. Visit the support wiki page for more examples and detailed descriptions of the functionality of the tool.



You can also send your comments to **WhatsNew@oame.on.ca**. You can share your experiences on Twitter, using the hashtag #Onmathies, and follow or message us at @ONmathies. There is an increasing set of interesting posts of student and teacher work on Twitter.

To be among the first to find out about the latest digital tool developments, sign up for our email list at **mathclips.ca/WhatsNewEmailList.html**. ▲

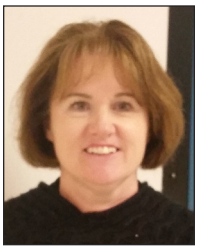
▲ DNA - Daily Numeracy Activity

SUDOKU 1

				5		1	4	
	3		6				9	2
			7					8
	7	2			1			
1								4
			4			7	2	
5					9			
2	9				6		3	
	8	7		2				

▲ NCTM Report: NCTM benefits to brighten your summer!

JACQUELINE HILL



Jacqueline Hill is a Grades 9–12 teacher of mathematics, as well as an online instructor for York University. She is a Past President of OAME and OMCA, as well as the recipient of the award for Exceptional Teaching in Secondary Mathematics. She also wrote the "Director's Dialogue" for the Gazette for a number of years.

Summer is a time of longer days, recharging your batteries and renewing your excitement for teaching and learning.

The NCTM is dedicated and devoted to helping mathematics educators delve deeper into their craft. Whether you are a classroom teacher, retired teacher, university/college professor, or person in a position of responsibility in a school board or the Ontario Ministry of Education, the NCTM is ready to help you make the most of your summer.

In the spirit of summer fun, here is a top ten list for things to do this summer!

Yours in mathematical fun,

Jacqueline

▲

10 Clean your work area. Whether it be a classroom or cubicle, cleaning and organizing will help you start fresh in the Fall.

9 Take the time to organize your computer and print files. Put your binders back in the correct order, and sort the papers lingering on your desk.

8 Take the time to use your scanner to convert bulky print copies into digital documents.

7 Take the time to do a self-inventory and reflect on how your year went. What do you want to STOP, START, and CONTINUE in the coming year?

6 Take time to search out student activities for the fall (or the last days of school). Under the "Classroom Resources" tab of the NCTM website, you will find activities for all ages and levels of mathematics. You may like to try out "Illumination Calculation Nation."

5 Take time to organize your own professional reading list for the summer (as well as a few good beach books). Under the "NCTM Publications" tab, you will find resources such as:

Books

- *Journal for Research in Mathematics Education*
- *Mathematics Teacher*
- *Mathematics Teacher Educator*
- *Mathematics Teaching in the Middle School*
- *Rights and Permissions*
- *Teaching Children Mathematics*
- Write, Review, Referee

4 Take time for your hobbies. If your hobbies include math puzzles, you might like the ARC activities found under the "Classroom Resources" tab; they are activities with rigor and coherence.

3 Take time for family and friends. You might want to take them to a math conference. The NCTM has the following 2018 calendar dates:

- July 16 – *2018 Conference for the Advancement of Mathematics Teaching* (CAMT) in Houston, Texas
- July 24 – *Michigan Council of Teachers of Mathematics Institute and Conferenc*e in Grand Rapids, Michigan
- August 2 – *Colorado Council of Teachers of Mathematics Annual Conference* in Denver, Colorado
- August 6 – *Pennsylvania Council of Teachers of Mathematics Annual Conference* in Harrisburg, Pennsylvania

2 Take time to plan for a successful Fall start-up. Under the "Classroom Resources" tab, there is a "Problem of the Week" link.

1 Take time for you. You are worth it. NCTM has various online communities that enable you to stay in touch with the math community, but with timing and pacing to suit you if you wish to pursue it.

▲ MB4T (Mathematics by and for Teachers): What's in a Name?

JENNIFER HOLM



Jennifer Holm is an Assistant Professor at Wilfrid Laurier University and works with primary/junior and junior/intermediate pre-service teachers, as well as in the field supporting current mathematics teachers. She is interested in developing mathematics knowledge for teaching with both pre-service and in-service teachers. She focuses on the beliefs and opinions that pre-service teachers hold about mathematics and teaching and the connection they have to past experiences. She uses this research to support future teachers in developing beliefs and knowledge that will encourage and support effective teaching practices.

Some of the most fascinating conversations I have with teachers (both in-service and pre-service) focus on the names, definitions, and certain concepts related to geometric shapes. The first conversation tends to arise when asked how many rectangle-shaped gardens are possible in a five by five space. Inevitably the question comes up: "Is a square acceptable to use?" At this point, I pause the discussion and bring it back to the group: "Is a square a rectangle?" Usually there is a split in the conversation about whether it is or is not. Some will confidently say "no" because a rectangle has two long sides and two short sides. After some discussion, it becomes clear that no one in the group can state, unequivocally, what the definition of a rectangle is because there are many common "definitions" that are being expressed. A rectangle is defined as "a four-sided shape where every angle is a right angle (90°) [and] opposite sides are parallel and of equal length" ("Rectangle," 2017). Some dictionaries add in "*especially:* one with adjacent sides of unequal lengths" (Rectangle, 2018, emphasis in original), which makes sense because some individuals have learned that a rectangle has to have unequal lengths. Once it is established that technically, the definition supports that a rectangle does not have to have unequal lengths on the adjacent sides, then the square is by definition a rectangle, but is a special case of rectangle.

When delving further into geometric discussions, the definition of a parallelogram is another that has some misconceptions. Most often, I am told that a parallelogram is "a rectangle that has been tipped over." When returning to the mathematical definition of a parallelogram, and time after time, many are stunned that it is "a quadrilateral with opposite sides parallel and equal" (Parallelogram, 2018). Most are surprised that the angle measurements are not part of the definition thinking that it must include two angles that are less than 90 degrees. After discussion, it now becomes clear that a rectangle can actually be classified as a parallelogram, just a special case of one. I can then make the analogy that a rhombus is a special case of a parallelogram in the same way that a square is a special rectangle, which makes the conversation turn to how a square could also be classified as a special case of a rhombus (in addition to a parallelogram).

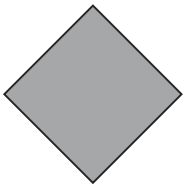


Figure 1: *Rotated shape example*

In an attempt to show how confused many in-service educators and pre-service candidates can be regarding geometry, the image in Figure 1 is presented, and the name of the shape is asked. Most are confident that it is a "diamond," until I rotate the shape 45 degrees. At that point, many have a change of mind, due to the square that they see. Continuing on, I mention that, in fact, the "diamond" is not a mathematical shape name. It can be confusing for students when transformations are introduced if they believe that shapes can change names by rotating them. Many will ask what the correct mathematical name is for a shape that they have typically called "diamond." The term "kite" is used to describe this mathematical shape, and by definition, is "a flat shape with straight sides. It has 2 pairs of equal adjacent sides" ("Kite," 2017). By this definition then, parallelograms with four equal sides (squares and rhombuses) would also be considered kites, but parallelograms where there are two different lengths of parallel sides, would not.

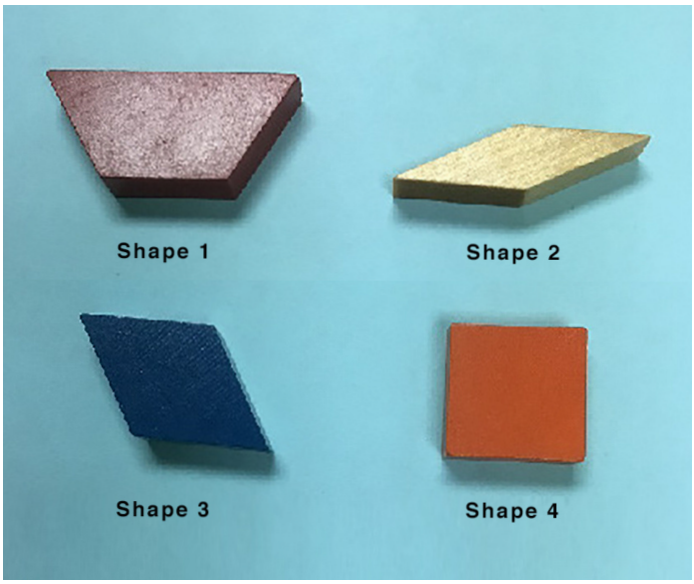


Figure 2: *Four-sided shapes in the pattern block set*

Table 1: Base-shape descriptions			
Shape 1	Shape 2	Shape 3	Shape 4
quadrilateral trapezoid	quadrilateral parallelogram rhombus kite	quadrilateral parallelogram kite rhombus	quadrilateral parallelogram kite rectangle rhombus square

When pulling out the four-sided shapes (technically cylinders, but we will focus on the base shape) in the pattern blocks set (see Figure 2), it is interesting to see how many different names apply to the shapes, especially if they are ordered from something general to most specific. Table 1 shows a list of the different names that could be used to refer to each of the shapes in this subset of the pattern blocks. By looking at all the names that could be used to refer to each of the shapes, it is clear how students could become confused and why some parts of the definitions are left out to make things "easier" for students. This reduces the richness of the discussions that can occur, as well as potentially lead to difficulties later in their mathematical careers. An interesting activity to try with students is to have them create shapes that would *not* be examples of all of the different terms, such as "Draw a quadrilateral that could be classified as a parallelogram, but not a rectangle or rhombus." In this activity, it would become clearer what students know and which shapes are still causing confusion in the classifications of shapes.

YouTube has a lot of different songs to help students remember different mathematical concepts. *Quadrilaterals* by Dylan Peters is one such song that is set to the tune of Radioactive and talks about different quadrilaterals. Although there are some spelling mistakes in the words to the song (which can be used instructionally), some of the mathematical concepts that you can learn (such as the angles in a quadrilateral add up to 360 degrees) are embedded in the catchy tune. What is problematic is that there are also some inaccuracies about the different shapes, which may not be caught unless looked at carefully in advance. For example, one verse in the song maintains that "a rhombus is like a square, if a square is swaying to this beat" (Peters, 2014). In my experience as a teacher, music can be a powerful classroom tool, so my suggestion is not to eliminate catchy songs like this, but instead, play the song and invite students to hunt for the misleading (or even incorrect) information. Another favourite for the classroom, and a great song to use to find the errors, and just to have a laugh, is Wylde Vyrals (2010): a rendition of quadrilaterals to the beat of Pitbull's 1234. It would be interesting to also have classes put together their own videos, using a favourite song to showcase what they have learned about the shapes, which is likely the root of Peters' (2014) video.

When talking about geometric shapes and properties, area is a topic of conversation that is important to the measurement expectations in the curriculum. Length multiplied by width is typically the remembered area formula for a rectangle, $(\text{side})^2$ for a square, and base multiplied by perpendicular height for a parallelogram. At this time, please return to the earlier conversation in this column: rectangles and squares are both special cases of parallelograms. If all three are parallelograms, then why teach students three different formulas? I find at the pre-service level many of these formulas have not had conceptual support and have been learned as being little more than a bunch of memorized letters. A focus on conceptual understanding could potentially avoid this concern.

If students are not convinced that they are all parallelograms, it is easy to help students prove the formula is the same by demonstrating using a parallelogram, such as the one in Figure 3. Ask if anyone can find a way to make different shapes they already know from this shape. Two answers typically arise: 1. both triangles cut off, leaving one rectangle and the two identical triangles; 2. cutting off one triangle and moving it to the other side to make a rectangle of the same area as the initial shape (see Figure 4). If a group gets stuck looking at the two triangles and rectangle, I suggest to the group that it would be easier to have a single formula that looks at the entire shape as one whole shape. In this way, you can guide their thinking toward making a single familiar shape. This makes it clear that a rectangle with the same base and height would have the same area, but what is different is the perimeter of the shape, which in Figure 3 would be longer than the rectangle constructed in Figure 4. Interestingly, giving students a situation with the two shapes and asking questions about the area and perimeter, without giving straightforward measurements (other than noting they are the same length along the floor and then both go to the ceiling), I find many students are convinced that the parallelogram must have a larger area.



Figure 3: *Example of a parallelogram*



Figure 4: Adjusting the parallelogram to show the relation to the rectangle area formula. Cut the triangle along the dotted lines and then move it to the opposite side with the dotted lines.

In addition, there are some interesting connections that can be made with later topics in volume and area that can be simplified by minimizing all the formulas learned. In a December 2012 article of the *Ontario Mathematics Gazette*, Kajander links the parallelogram formula to finding the area of the circle (see Figure 5). The *Pythagorean Math* website also shows how the formula is derived, and she created a YouTube video that can be used in the classroom.

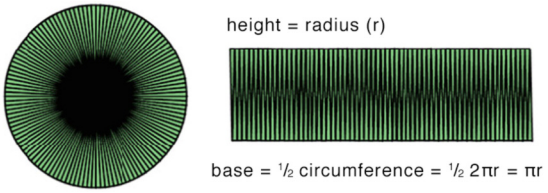


Figure 5: *Area formula for the circle (image from* ***pythagoreanmath.com/complete-explanation-for-area-of-a-circleformula****)*

The simplest connection is probably the approach that uses the area of a triangle. Any of the parallelograms in Figure 6 could be used to illustrate the concept because they can all be cut into two identical triangles. It then becomes clear that the area of triangle would be found by multiplying the base and height and then dividing by two: $A = \frac{bh}{2}$.

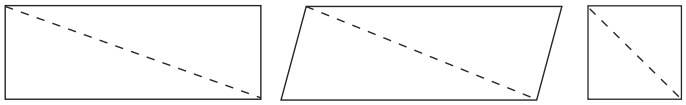


Figure 6: *Parallelogram that could be used to show the relationship of the triangle area formula to a parallelogram. The dotted line shows where the shape can be cut into two equal triangles for demonstration.*



Figure 7: *Trapezoid for building the area formula*

The trapezoid formula can also be built from the basic idea of how to find the area of a parallelogram. Figure 7 shows a trapezoid that is helpful to start with in building the area formula. If there is a suggestion to just cut off the triangle and use the triangle and parallelogram formulas together, remind the group of the previous discussion about making one single familiar shape to work with in creating the formula. If stuck, suggest making a second, identical trapezoid and see what can be done with the two shapes together. By making a duplicate and putting them together (such as Figure 8), it is easy to see where the area formula is derived from finding the base and multiplying by the height of the created parallelogram (in this case, a rectangular parallelogram). The height would be the height of the single shape, with the base being the length of the top and bottom of the shapes added together. Since there are two trapezoids, the formula would have to be divided by two, making the final formula for all trapezoids: $A = \frac{h(a+b)}{2}$. The reason for the choice of using this type of trapezoid is that it can lead to a generalization of the formula. If you use a trapezoid like the one in the pattern block set (Figure 2), some students may attempt to do the same thing as with the parallelogram and realize that you can move the triangle on the right to the left and make a parallelogram. This leads to the mistaken conception that the formula for a trapezoid is the same as a parallelogram ($A = b \times h$), which would not work for all trapezoids.

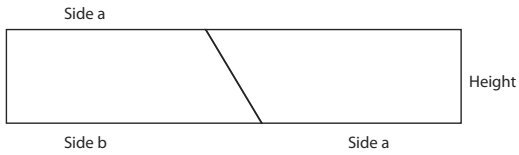


Figure 8: Building the trapezoid area formula

Volume of right prisms and right circular cylinders can all be simplified, once students have focused on the two-dimensional shapes that become the bases of these three-dimensional shapes. The volume of all right prisms and right circular cylinders can be found by determining the area of the base shape and then multiplying it by the height of the structure. A formula for the volume of all of these shapes could then be $V = A_{base} \times h$. This would then decrease the number of seemingly different (and often confusing) formulas to be memorized. In the curriculum, the Ontario Ministry of Education (2005) has noted that in Grade 7, students should be able to create this formula as a generalization for all right prisms, and then in Grade 8, generalize this formula in relation to the cylinder. Notice that the term "right circular cylinder" was used in describing the shape, so you may be asking, "Why not just say cylinder as it says in the curriculum?" Here lies another common misunderstanding or simplification in talking about the cylinder. The description of a cylinder is in fact "two congruent, parallel faces called *bases*. Lines joining corresponding points on the two bases are always parallel" (Van de Walle, Karp, Bay-Williams, McGarvey, & Folk, 2015, p. 407). This would mean that classification as a cylinder includes many more shapes than the "can-like" shape, as you can see in Figure 9. The formula is applicable for all cylinders as long as the height is the perpendicular distance between the two congruent bases. This would also add some confusion to the curriculum expectation: "Determine the relationships among units and measurable attributes, including the area of a circle and the volume of a cylinder" (Ontario Ministry of Education [OME], 2005, p. 113), which presupposes that the only type of cylinder is a "circular" one.

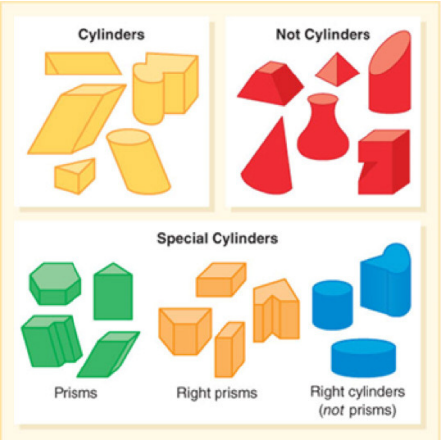


Figure 9: *Examples of cylinders from Van de Walle et al., 2015, p. 407*

In consulting the definition used in the curriculum, the cylinder is defined as "a three-dimensional figure with two congruent, parallel, circular faces and one curved surface" (OME, 2005, p. 122). This definition simplifies the existing mathematical definition of the cylinder, so teachers should be aware of this as they are planning lessons that allow students to explore the mathematical definitions of shapes. Prisms have polygons as their faces: "a polyhedron with two polygonal faces lying in parallel planes and with the other faces parallelograms" (Prism, 2018). By this definition, then, circular cylinders would not be prisms, but prisms would be a special case of cylinders.

The goal here was threefold: to think about teaching students correct terminology and definitions for shapes, to look at ways to make concepts less confusing and more accessible, and to increase the retention of the concepts.

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▲ In the Middle: Fun with Fractions

CARLY ZINIUK



Carly Ziniuk teaches Grade 9 Mathematics, Grade 12 Data Management, and Advanced Placement Statistics at the Bishop Strachan School in Toronto, Ontario, Canada. She is very active in adopting real-life data to engage her students in solving problems.

I have been having lots of conversations with my colleagues lately about fractions. Most of our conversations have been trying to figure out some strategies to help our students be more fluent and more agile with fractions, beyond just performing the routine operations in the Number Sense and Numeration strand. I went hunting for some fraction ideas I had used in the past, that might continue to support and strengthen our students' fraction fortitude. Here are four different activities, ranging from small to large, that you could use to reinforce fractions in other strand focuses.

Fraction War

You can "play" this card game by removing the faces and considering the aces as ones, with black being positive and red being negative (again, you can make all positive, if that is easier, or more appropriate, for your students). Another option is to use an entire deck with the face cards to all be 11, or with values Jack (11), Queen (12), and King (13).

As in the card game, War, each student gets half the deck and both simultaneously flip over two cards. The students then stack their two cards to create a fraction. They can then use these two cards to compare different criteria and determine the winner. For example:

- Greatest fraction
- Least fraction
- Fraction closest to $\frac{1}{2}$ (or $-\frac{1}{2}$)

Students inevitably create some really great "rules" to figure out what to do in the case of a tie (***Extension:*** *What's the chance a tie occurs?*). Each player can flip in one more card and then include them into their existing fraction. They could also flip two new cards each and create new fractions. The winner of this round would get both sets of cards. You might also want to check out our editor's *Gazette* article, "Learning math through card games" (Sibbald, 2010), for some additional suggestions on how to control the length of

the regular game of War.

I advise shuffling after each round. You can also play this with the entire class, using an online card flipper like **www.random.org/playing-cards/**, or on the SMART Board®, using Notebook's Random Generator download and the deck of cards images.

Round #	Name: Noam Number of Cards	Name: Asher Number of cards
1	26	26
2	40	12
3	46	6
4	50	2
5	52	0

As an extension to connect Data Management and Patterning, have the players complete a chart, recording the game results.

Questions follow naturally from this:

- What does it look like to graph each player's pattern (number of cards versus round number on the graph)?
- Do all games have the same graph in the end?
- How do different games' patterns change?
- Does the person who wins the first round always win?
- How do the tables and graphs change if you don't shuffle?

Recipe Reboot

Have each student bring in a favourite recipe (always good to have printed copies of a few of your own favourites). I then discuss or display the following conversion information:

5 mL = 1 teaspoon 15 mL = 1 tablespoon

240 mL = 1 cup 1 L = 4.2 cups

You can have each student work on her or his own recipe, or randomly distribute the recipes. It is fun to try to figure out whose it is.

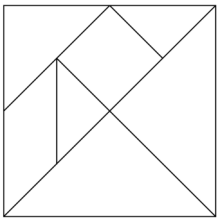
This gives students a chance to use fractions and rates as real chefs often do!

1. Rank the ingredients from most to least.
2. Find the total amount of ingredients. You need to maintain consistency with units (all cups or all teaspoons or all tablespoons or all mL).
3. What is the fraction of the largest ingredient? What is the percentage of that ingredient?
4. What is the fraction of the smallest ingredient? What is the percentage of that ingredient?
5. What fraction of the recipe is one serving size? (If your recipe is for something like cookies, determine what you think a *reasonable* single serving at one sitting looks like!) Determine what the actual amount of each ingredient is in each serving.
6. If you were to make this recipe for the entire class, how much of each ingredient would you need?

Folding Fractions?

I have used tangrams in a lot of different areas of teaching math, beyond just puzzle building, to reinforce spatial sense. Directly using the language of similarity and congruence, we create the tangram pieces, using folding and cutting, rather than measuring or a pattern. Having each student start with a square with different side lengths, determining the original area, and then the fraction of that original area.

Student Instructions



Since each of you started with a different-sized square, the tangram puzzle can be any size! In fact, as long as you know that the tangram pieces form a square with the arrangement you see here, the edges can be bigger or smaller, depending on the size of the original square. Regardless of the side length of the original square, the angles will stay the same and so will the proportions. This is the mathematical concept of *similarity*.

1. Explain how you can make your own tangram puzzle set, using a step-by-step guideline. Remember, you should not need to use measurements (or measuring devices, for that matter). Use mathematical language to help clarify your description.
2. What fraction of the total square is each piece?
3. Which pieces have the same area? Do any have the same perimeter?

Since the proportion of the area of each piece compared to the whole square is independent of the size of the square, it is opportune to note similarity and scale factor. For middle school students, it is an appropriate way to justify the overall area scale factor as a square of the side length.

This is a great opportunity for students to rehearse the rules for multiplying and dividing fractions, particularly when considering the pieces, which fit "inside" each other or the different pieces (shapes) with the same area.

4. What is the approximate probability that if you randomly chose a square millimeter from within that original tangram square, that it would fall inside the

parallelogram piece?

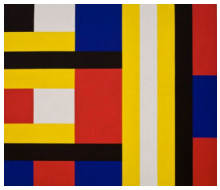
If your students have experienced tangram picture/puzzle building, especially for the animals or letters, approaching these with fractions is another connection, with examples like these:

5. What fraction of the tangram makes up the cat's head?
6. What part of the design will give you the closest to $\frac{1}{5}$ of the entire tangram?
7. What fraction of the puzzle is shaded?

There are a lot of great previous *Gazette* articles that connect tangrams and fractions, if these types of questions intrigue your students (or you!). A couple more ideas to consider are Cornelius de Groot's "Tangrams Are Not Toys" from March 2008, and Bonnie Litwiller and David Duncan's "Probabilities on the Tangram: An Example of Linking Mathematical Ideas" from March 1997. This latter article makes connections beyond my suggestions for the probability introduction here.

Math in Art

Taking time to look at abstract art through fractions can also be a great opportunity for engagement. The National Gallery of Canada has a wealth of online resources teachers can use, both at **www.gallery.ca** and at Cybermuse, **www.cybermuse.gallery.ca**. Through their collection, you can find many paintings by Guido Molinari and Claude Tousignant (often referred to as "les plasticiens"), which are very geometrical and work well for fraction explorations. Not surprisingly a glance at the work of these artists reveals they were inspired by the work of Piet Mondrian, with the geometric shapes at the centre of the artistic design.



These questions were built for the Molinari painting "Vertical Horizontal Blancs" (1960), but could be used with alternates like:

- Molinari's *Poly-relational* (1958)
- Jean Gougen's *Multiple Reds* (1961) or *Dynamic Space* (1959)
- Jack Bush's *Big A* (1968)
- Jean-Paul Jérôme's *Untitled* (1957)

These are great Canadian modern paintings from our own National Gallery for students to consider geometrically, beyond the more famous Mondrian alternative. Quilt designs also have a connection to this approach and you can find some of those at the Textile Museum of Canada (**www.textilemuseum.ca**).

Sample questions for artwork:

1. Name the shapes shown in this painting.
2. Calculate the area for each shape.
3. What is the fraction of the total painting for each piece? What is the fraction for each colour in the painting?
4. What colour do you think has the most impact in the painting? Is it associated with the calculation you have done above?
5. What colour do you think has the least impact in the painting? Is it associated with the calculation you have done above?
6. Do any of the colours have the same area? Do the dimensions of the printed painting affect whether colours have the same area?
7. Another way to do this is to cut the artwork apart and put the same colours together like a puzzle.
   a. Does your answer, done by cutting, agree with the area calculations?
   b. Do you think it makes more sense when you look at it like a puzzle?

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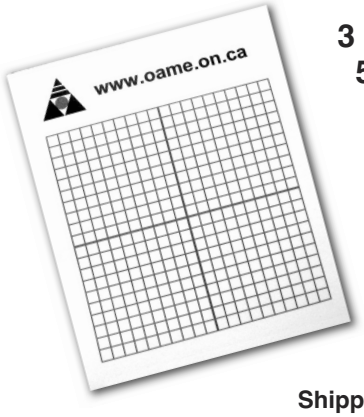
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▲ Assessment Abby Addressing Stress



ASSESSMENT ABBY
EMAIL: assessmentabby@oame.on.ca

Ask Assessment Abby A³ is a regular column in the OAME *Gazette*, where teachers can share concerns and best practices about assessment, evaluation, and reporting of mathematics. Please send your questions to Ask Abby at **assessmentabby@oame.on.ca**.

Dear Assessment Abby,

I find that my students get stressed out during tests, and I worry that I am not getting a true picture of their knowledge and understanding. How can we assess in a way that maintains a healthy mindset and supports student well-being in mathematics?

There is research in psychology and cognitive science that suggests stress does indeed affect student performance in mathematics.

> While important for mathematical processes, working memory is also highly sensitive to interference from stressors. Ashcraft and Kirk (2001) and Ashcraft and Krause (2007) found that cognitive processes can be negatively affected by the interference of negative emotions, such as math anxiety or pressured situations. This suggests how negative feelings might overload the working memory system, thus resulting in a drop in performance (Caviola, Carey, Mammarella, & Szucs, 2017, p. 2).

How can we mitigate the impact of stressors?

Some things we might try have been documented in the recent Ontario Ministry of Education *Capacity Building K–12, Special Edition #48, Yes, I can! Paying Attention to Well-Being in the Mathematics Classroom* (2018).

1. Use a variety of assessment strategies. Provide multiple opportunities and means for students to demonstrate what they know and can do.
2. See the student as a whole person, and pay attention to all developmental domains when you plan instruction, assessment, and learning activities (e.g., provide opportunities to move while learning, plan for supportive social interactions, consider emotional impact of instruction).
3. Student self-assessment is linked to improving student well-being.

Other things to consider:

1. Avoid timed tests.
2. Recognize that repeated use of pencil-and-paper test may not give valid data about what students know and can do. Other forms of assessment may be more appropriate to particular topics or instruction. Collect information in a variety of ways. Try to have conversations with students to assess some expectations and/or learning skills when you observe them working within a group.
3. Set up positive norms in math class. Start the first day! Show that you value problem solving by planning first-week activities that are problem based and fun, rather than pen-and-paper diagnostic testing or a teacher-centred review.
4. Include students in decisions about assessments, such as timing or format.
5. Ensure that students are aware that they will have multiple opportunities to show you their understanding.
6. Take the mystery out of tests. Be sure that your students know what types of questions they will see, and the format that will be asked. If they have a clear sense of what will be assessed, it will minimize anxiety over the unknown.
7. Other than recall of facts and demonstrating understanding with procedural knowledge, which may be assessed through traditional tests, the other categories in the achievement chart (Knowledge & Understanding, Application, Communication, and Thinking) are probably not going to be adequately assessed using a test.

In order to get a true picture of your students' achievements, it is essential to consider reducing their stress during assessments. Hopefully, some of the ideas in this column help you and your students.

Resources

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▲ BOOK REVIEW: Learning Math History through Puzzles

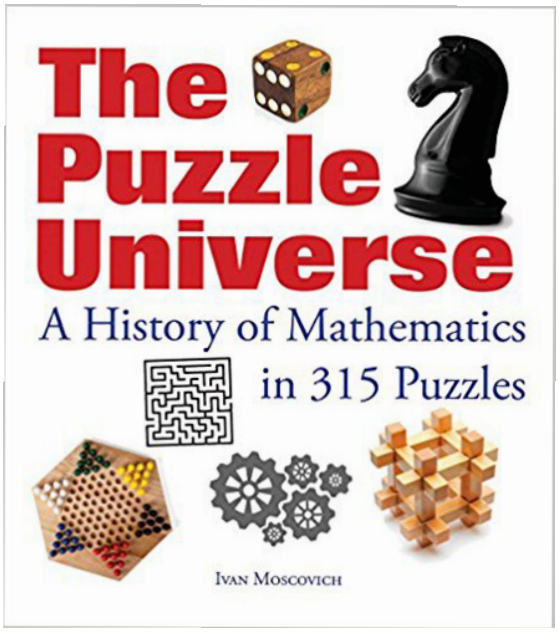


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Tim Sibbald is the current Gazette *editor, co-chair of finance for the 2018 annual conference, and Past President of OAME. He is an associate professor in the Schulich School of Education, Nipissing University, with a focus on mathematics education.*

Moscovich, I. (2015). *The puzzle universe. A history of mathematics in 315 puzzles.* Richmond Hill, ON: Firefly Books Ltd.

If I could only recommend one book for classrooms to provide the opportunity for students to explore some enrichment while their teacher is occupied with other students, then this is the book. There are a number of reasons. Firstly, it is colourful and inviting—a coffee-table book in style. Second, there are the extensive illustrations throughout, with every page in colour. Thirdly, the manner in which problems are provided is accessible to students. The historical component is frequently mentioned along with connections to practical matters, personalities, or other problems.



The range of puzzles includes many commonly known puzzles such as river crossings involving a goat. It includes more challenging and commonly known puzzles such as the Monty Hall problem. It also provides interesting ideas around the Pythagorean theorem that directly extends the curriculum. In addition, it includes direct connections to many branches of mathematics in an accessible manner. The range of available puzzles will facilitate students choosing what interests them as opposed to the teacher having to provide some guidance.

The book does have some explanations that could be clearer. A specific example is Kurschak's theorem, which is noticeable because of a four-colour design. Another is a diagram associated with an explanation of Morley's trisector theorem. It shows a generalization that does not help those who are unfamiliar with the theorem (see *Gazette* 54(2), page 31, for an explanation of the theorem). While the book has occasional issues like these, a significant benefit is that when one looks up Kurschak's theorem, one finds an opportunity for enrichment, as it uses a particular geometric pattern to derive the area of a decagon. Similarly, Morley's trisector theorem is a serious contender for enrichment.

There are circumstances where having answers in the back of the book, allows one to jump too quickly to verify that students should have thought more deeply about a particular problem. Teachers are adept at addressing this sort of issue, and it is not dissimilar to textbooks with answers in the back, except that the book is presenting problems to challenge one's thinking. In terms of enrichment, the answers are needed, but students will need to be encouraged to take a problem away and think about it for a period of time, before looking at the answer.

The final benefit I will mention for this book is the opportunity for the teacher to learn while using the book in the classroom. As I have read through the book, I have learned about a few problems that I was not familiar with and gained new details on some problems with which I was familiar. An example is the problem of how it can appear that a square and rectangle with different areas can be composed of the same combination of congruent shapes. I was familiar with this, but was not aware that it is known as "Curry's square paradox." One I was personally not familiar with is the Kakeya needle paradox. This clearly is an intriguing and somewhat accessible result of modern mathematics. I mean "accessible" in the sense that one can understand the concept, even if the formal mathematical machinery becomes overwhelming in online explanations.

I am confident that this book will provide new ideas and insights to all teachers who read through the pages. Students who are exposed to it will gain from the engaging portrayal of a wide range of ideas. In my opinion, it is best suited to Grades 6 to 12. An enthusiastic teacher can likely use it as a resource in Grades 4 and 5, but more care is needed about selecting suitable problems. ▲

Balancing Chemical Equations, Using Linear Equations

DON VANDER KLOK



Don Vander Klok has taught mathematics for over 30 years. Fifteen of those were in Ontario community colleges. He is a member of the Ontario College of Teachers. He is past president of the Ontario College Mathematics Association and has recently retired from Lambton College in Sarnia.

One of the cautions during Hurricane Irma in 2017 was to avoid using a charcoal grill indoors for heat (**https://onsafety.cpsc.gov/blog/2017/09/08/keep-safe-after-hurricane-irma/**). The danger is that carbon, present in the charcoal, combined with oxygen in the burning process, can produce carbon monoxide, a poisonous gas that is odourless and deadly. Since burning the carbon transforms it into something different, it is called a "chemical change." In chemistry class, this gets represented in an equation: C (carbon) plus O (oxygen) produces CO (carbon monoxide).

The equation shows the ingredients in this chemical reaction at the molecular level. A molecule of carbon C has one atom, so it could be written as C_1 to show this. In general, the subscript 1 is understood and not written. A molecule of oxygen O has two atoms, and the subscript 2 describes this in the representation O_2. They combine in burning to produce carbon monoxide, and that molecule is made up of one carbon atom and one oxygen atom. Again, the subscripts of one (C_1O_1) are assumed, and it is written as CO. Using chemical and mathematical symbols, the equation would be written as $C + O_2 \rightarrow CO$.

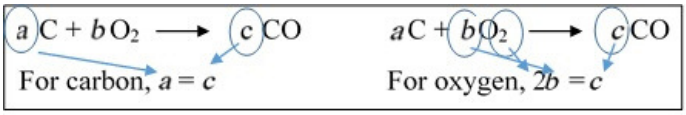
Chemical equations like $C + O_2 \rightarrow CO$ need to be "balanced" so that there are the same number of carbon atoms C and oxygen atoms O on each side of the arrow (the "reactants" and the "product"). As it stands, there is 1 carbon atom on the left side and 1 carbon atom on the right side. That is balanced. For oxygen, there are 2 atoms on the left side and 1 atom on the right side. This is not balanced. To balance the equation, numerical coefficients are used at the beginning of each of the components (elements or compounds) in the equation. If we represent those numbers with lower-case letters—*a*, *b*, and *c*—then we need a balanced equation that will have the form:

$$a\,C + b\,O_2 \rightarrow c\,CO$$

The challenge of balancing the equation is to find the values for *a*, *b*, and *c*.

Trial and error (hopefully combined with "educated guessing") is the most common explanation to figure out what those coefficients are. A better approach that I recommend uses a system of linear equations. The equations are split up by element. The parts of the equations are the coefficients combined with the number of atoms for each element.

Here is what the equations are and how they are derived:



Solving this system of linear equations is different than usual because there are only two equations with three unknowns. To proceed with this set-up, choose a value for one of the variables. I will choose $a = 1$.

Using the equation for carbon, $a = c$, $1 = c$.

Using the equation for oxygen, $2b = c$,

$$2b = 1,$$
$$b = \frac{1}{2}.$$

The balanced equation is $1\,C + \frac{1}{2}\,O_2 \rightarrow 1\,CO$. However, in chemistry, numerical coefficients need to be whole numbers to correspond to whole molecules. To achieve that, multiply the coefficients on both sides by 2 (which is allowed because it scales $a = 1$ to a value that eliminates any fractions).

$$2 \times [1\,C + \frac{1}{2}\,O_2 \rightarrow 1\,CO]$$
$$2\,C + O_2 \rightarrow 2\,CO$$

A coefficient of 1 is generally not written (like "*x*" rather than "1*x*"). The equation now has 2 carbon atoms and 2 oxygen atoms on each side. The equation is balanced!

Balancing $Fe + O_2 \rightarrow Fe_2O_3$

This equation describes the change in iron (Fe) when exposed to oxygen in the air (O) over time. It develops iron oxide, usually called rust (Fe_2O_3). In this example, the elements on the left have the same numerical subscripts as in the first example, but this time, the number of atoms in each of the elements of the product are different. In order to construct the equations for each element, write in numerical coefficient variables:

$$a\,Fe_2 + b\,O_2 \rightarrow c\,Fe_2O_3$$

Set up the linear equations for each element. Since there are two elements, there will be two equations. For iron, $2a = 2c$. For oxygen, $2b = 3c$.

Again, there are two equations with three variables. To solve this system of equations, choose a number for one of the variables, say $a = 1$.

Using the equation for iron, $2(1) = 2c$, $1 = c$.

Using the equation for oxygen and substituting the value $c = 1$, $2b = 3(1)$, $b = \frac{3}{2}$. The balanced equation is

$$1\ Fe_2 + \frac{3}{2}O_2 \rightarrow 1\ Fe_2\ O_3.$$

To ensure that the values for *a*, *b*, and *c* are whole numbers, multiply the coefficients on both sides by 2.

$$2 \times [1\ Fe_2 + \frac{3}{2}O_2 \rightarrow 1\ Fe_2\ O_3]$$

$$2\ Fe_2 + 3\ O_2 \rightarrow 2\ Fe_2\ O_3$$

This results in a balanced equation with 2 × 2 iron atoms on the left and 2 × 2 on the right, as well as 3 × 2 oxygen atoms on the left and 2 × 3 on the right.

Balancing $C_3\ H_8 + O_2 \rightarrow CO_2 + H_2O$

This equation describes propane (C_3H_8) burning with the presence of oxygen (O_2). This produces carbon dioxide (CO_2) and water (H_2O). It is different from the other two examples because it involves three different elements, and there are two reactants along with two products.

To balance the equation, begin by writing in the coefficient variables on each part of the chemical equation:

$$a\ C_3\ H_8 + b\ O_2 \rightarrow c\ CO_2 + d\ H_2O$$

For carbon, $3a = c$.

For hydrogen, $8a = 2d$ (which simplifies to $4a = d$).

For oxygen, $2b = 2c + d$.

Note that you could substitute the equivalent expressions for *c* and *d* at this point:

$$2b = 2(3a) + (4a)$$
$$= 10a$$
$$\text{or } b = 5a$$

This time, there are three equations and four unknowns. To solve this system of equations, choose a number for one of the variables. Again, I will choose $a = 1$.

In the equation for carbon, $3(1) = c$, $3 = c$.

In the equation for hydrogen, $8(1) = 2d$, $4 = d$.

In the equation for $2b = 2(3) + 4$, $2b = 10$, $b = 5$.

So $C_3\ H_8 + 5\ O_2 \rightarrow 3\ CO_2 + 4\ H_2O$.

That is balanced, since there are 3 carbon atoms on both sides, 8 hydrogen atoms on the left side and 4 × 2 on the right, as well as 5 × 2 oxygen atoms on the left and 3 × 2 + 4 = 10 on the right side.

Balancing $Mg\ (OH)_2 + HCl \rightarrow H_2O + MgCl_2$

This equation describes milk of magnesia (magnesium hydroxide $Mg\ (OH)_2$, a common ingredient of stomach acid neutralizers) meeting stomach hydrochloric acid (HCl), producing water (H_2O) to help calm the upset stomach, and magnesium chloride ($MgCl_2$), commonly taken as a mineral supplement. The elements in this equation are magnesium (Mg), oxygen (O), hydrogen (H), and chlorine (Cl). This equation is different from the other examples because of the brackets around the OH. Those brackets are there to show the chemical formula for magnesium hydroxide containing two atoms of "hydroxide."

To balance this equation, think of the brackets and the 2 subscript in the magnesium hydroxide in terms of the distributive property, where $Mg\ (OH)_2 = Mg\ (O_{1 \times 2}\ H_{1 \times 2}) = MgO_2H_2$. Now to insert variables for the numerical coefficients on each of the parts of the equation:

$$a\ MgO_2H_2 + b\ HCl \rightarrow c\ H_2O + d\ MgCl_2$$

For magnesium, $a = d$.

For oxygen $2a = c$.

For hydrogen, $2a + b = 2c$.

For chlorine, $b = 2d$.

This time, there are four elements and four equations. They are linearly dependent and there is a degree of freedom, so a value can be set. Choose $a = 1$.

For magnesium, $1 = d$.

For oxygen, $2(1) = c$, $2 = c$.

For hydrogen, $2(1) + b = 2(2)$, $2 + b = 4$, $b = 2$.

For chlorine, $b = 2(1) = 2$.

So, $MgO_2H_2 + 2\ HCl \rightarrow 2\ H_2O + MgCl_2$, or more properly, $Mg\ (OH)_2 + 2\ HCl \rightarrow 2\ H_2O + MgCl_2$.

This equation is balanced, with 1 magnesium atom, 2 oxygen atoms, 4 hydrogen atoms (1 x 2 + 2 x 1 on the left, 2 x 2 on the right), and 2 chlorine atoms on each side.

Balancing $Cu + HNO_3 \rightarrow Cu\ (NO_3)_2 + NO + H_2O$

This equation describes what happens when copper (Cu) is added to a dilute solution of nitric acid (HNO_3). The resulting products are copper nitrate ($Cu\ (NO_3)_2$), nitric oxide (NO), and water (H_2O). This oxidizing process turns the copper to a different colour and is used as "copper etching" in art. The elements in this equation are copper (Cu), hydrogen (H), nitrogen (N), and oxygen (O). This equation is different from the others, since there are 4 elements (which will result in 4 linear equations) and 5 parts in the chemical equation.

Since there are brackets in the chemical representation of copper nitrate, think of these (as in the previous example) in terms of the distributive property:

$$Cu\ (N_1O_3)_2 = CuN_{1\times 2}O_{3\times 2} = CuN_2O_6.$$

Now to insert variables for the numerical coefficients on each of the parts of the equation:

$$a\ Cu + b\ HNO_3 \rightarrow c\ CuN_2O_6 + d\ NO + e\ H_2O$$

For copper, $a = c$.

For hydrogen, $b = 2e$.

For nitrogen, $b = 2c + d$.

For oxygen, $3b = 6c + d + e$.

As in the other equations, I will choose $a = 1$.

For copper, $1 = c$.

For nitrogen, $b = 2(1) + d$, $b = 2 + d$.

For oxygen, $3b = 6(1) + d + e$, $3b = 6 + d + e$.

This is turning out to be challenging, since the choice of $a = 1$ does not readily produce values for b, d, or e. There are a variety of substitutions that could be used; I shall choose to work with the oxygen equation, turning it into an equation involving only the variable b. To begin, solve for d and e in terms of b:

From the equation for hydrogen, $b = 2e$, so $\frac{1}{2}b = e$.

From the equation for nitrogen, $b = 2 + d$, so $b - 2 = d$.

Substituting these into the oxygen equation:

$$3b = 6 + (b - 2) + (\frac{1}{2}b)$$

$$3b = 6 + b - 2 + \frac{1}{2}b$$

$$\frac{3}{2}b = 4$$

$$b = \frac{8}{3}$$

Substituting this value into the changed hydrogen equation, $e = \frac{1}{2}\left(\frac{8}{3}\right) = \frac{8}{6} = \frac{4}{3}$.

Substituting $b = \frac{8}{3}$ into the changed nitrogen equation, $d = \frac{8}{3} - 2 = \frac{2}{3}$.

Now to use these values to state the balanced chemical equation:

$$1\ Cu + \frac{8}{3}\ HNO_3 \rightarrow 1\ Cu(NO_3)_2 + \frac{2}{3}\ NO + \frac{4}{3}\ H_2O$$

To ensure that the values for a, b, and c are whole numbers, multiply the coefficients on both sides by 3.

$$3 \times [1\ Cu + \frac{8}{3}\ HNO_3 \rightarrow 1\ Cu\ (NO_3)_2 + \frac{2}{3}\ NO + \frac{4}{3}\ H_2O]$$

$$3\ Cu + 8\ HNO_3 \rightarrow 3\ Cu\ (NO_3)_2 + 2\ NO + 4\ H_2O$$

This equation is balanced, with 3 copper atoms, 8 hydrogen atoms, 8 nitrogen atoms ($3 \times 2 + 2$ on the right), and 24 oxygen atoms (8×3 on the left and $3 \times (3 \times 2) + 2 \times 1 + 4 \times 1$ on the right) on either side.

In Summary

I used this method in my high school and college mathematics classes as an application of solving systems of linear equations. Like mixture problems, students needed to construct a system of linear equations for their solution and sometimes use the tools of substitution or addition and subtraction. Students loved this method of balancing a chemical equation and used it in their chemistry classes. Where trial and error left them puzzled, wondering where to start, this method gave them a procedure to follow, so they could start developing a solution right away. ▲

▲ Solution to Arithmetic Wordsearch

S	S	E	L	F	O	R	T	Y	F
S	C	Q	F	A	L	H	C	U	I
R	L	U	I	A	I	E	V	I	F
E	Y	A	F	R	T	A	D	D	T
B	T	L	T	W	E	L	V	E	E
M	N	E	Y	W	I	F	E	X	E
U	E	I	N	G	O	F	I	Z	N
N	W	O	N	U	N	S	G	E	S
R	T	H	R	E	E	F	H	R	E
U	N	Y	T	R	I	H	T	O	V
E	R	O	M	N	E	V	E	L	E
S	U	B	T	R	A	C	T	E	N

NUMBERS,
ZERO, ONE, TWO, THREE, FOUR, FIVE, SIX,
SEVEN, EIGHT, NINE, TEN, ELEVEN, TWELVE,
THIRTEEN, FIFTEEN, TWENTY, THIRTY, FORTY,
FIFTY,
ADD, SUBTRACT, EQUAL, MORE

MYSTERY PHRASE: CALCULATING FOR FUN

▲ Fields Institute MathEd Forum Report



ANGELICA MENDAGLIO

Angelica Mendaglio is an instructional designer at Vretta Inc. in Toronto, Ontario, where she helps to create interactive digital mathematics lessons and activities for middle school students.

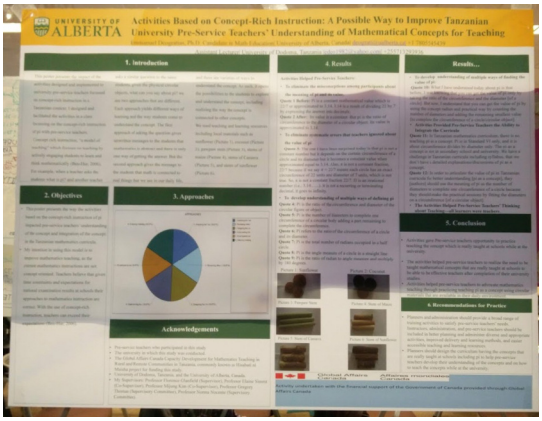
Research Day and Margaret Sinclair Memorial Award Lecture

Every year, the January meeting of the Fields MathEd Forum is focused on current education research conducted by students, teachers, and other education professionals. This includes a program of presentations during the day as usual, but also a collection of research posters at lunchtime for participants to peruse and discuss with the researchers themselves. It is an opportunity to see what questions and challenges in math education are being tackled today, whether it be by university researchers through a provincial-wide study or by a teacher or team of teachers within their own classrooms.

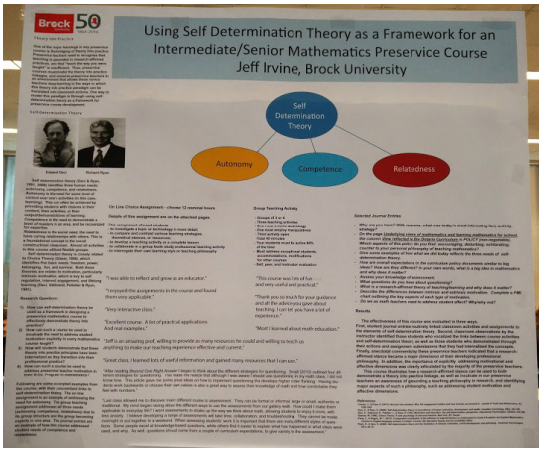
This year, there was a wide variety in the types of research being presented. The first talk, *Building Complexity into Development Projects: A Case Study,* given by Elaine Simmt (University of Alberta), Andrew Binde (University of Dodoma), Florence Glanfield (University of Alberta), and Joyce Mgombelo (Brock University), detailed a development project. The lecture detailed the multi-pronged approach taken by the project, incorporating teacher education, policy networks, and communities to expand capacity for mathematics education in rural and remote communities in three regions of Tanzania. In particular, community-based approaches have been shown to be most effective in similar projects in developing African countries, and helped to strengthen the sustainability of the project.

In the afternoon, Alison Macaulay (Ontario Ministry of Education) gave a talk entitled *Effective Practices in Grade 9 Applied Mathematics,* which looked at the results of a qualitative study of practices in four schools, where Grade 9 Applied math students were consistently performing better on the EQAO tests than the provincial average. The goal of the study was to identify practices which could help students in other schools as well. One characteristic that Alison's study found in these schools is a fight against the stigma that is often associated with Applied classes. In these schools, the perspective on the Applied math course is not that it is a lower level than the Academic course, but rather a different course that caters to a different learning style. This was part of a larger trend that Alison observed, where teachers build student confidence in their Applied classrooms, which often include students who have had negative math experiences in the past. She also found that the teachers in these schools tend to start the school year with geometry and measurement units, as students tend to do well in those areas, and this helps to build student confidence by starting the course with success. Teachers in these classes also celebrate different problem-solving approaches from their students and encourage different ways of thinking, and take advantage of the social nature of students at that age by having them work in groups that facilitate co-operation.

The poster session showcased 13 different research projects from students and teachers on topics ranging from pre-service teachers' beliefs on problem solving to undergraduate perceptions of computational thinking, and from studies of flipped classrooms to pre-service teachers' content understanding in Tanzania.



Poster from Emmanuel Deogratias, doctoral (Ph.D.) candidate from University of Alberta



Poster from Jeff Irvine, doctoral (Ph.D.) candidate from Brock University

The MathEd Forum meets monthly at the Fields Institute in Toronto. You can find more information on upcoming meetings, and register to join either in person or through the live stream at **www.fields.utoronto.ca/activities/workshops**. The call for presenters and posters for the January "Research Day" meeting of the MathEd Forum goes out every November. Teachers are encouraged to share their classroom research with the community!

Following the January meeting was the Margaret Sinclair Memorial Award presentation. This award is presented every year to someone who demonstrates innovation and excellence in math education at the elementary, secondary, college, or university level across Canada. The recipient gives a lecture at the Fields Institute in the following year.

The recipient of the award in 2017 was Dr. Brian Forrest of the University of Waterloo. The focus of Dr. Forrest's talk was the development of the Waterloo Masters of Mathematics for Teachers Program, a three-year online part-time masters degree designed for in-service teachers to improve their subject knowledge. Because of the time constraints that teachers experience, asking teachers to come to a physical classroom (likely taking a year off to do so) was infeasible—a part-time distance education program was ideal to accommodate the busy schedules of teachers. Today, teachers from all over the world are enrolled in the program.



Dr. Brian Forrest being presented the award by Larry Sinclair and Walter Whiteley

The Margaret Sinclair Memorial Award is presented annually, with nominations being accepted until the end of February each year. If you would like to nominate someone for this award, visit **www.fields.utoronto.ca/honours-and-fellowships/margaret-sinclair-memorial-award** for more information. ▲

▲ A Focus on Student Understanding: Georg Pólya's Problem-Solving Heuristic

JEFF IRVINE



Jeff Irvine has been a secondary math teacher in Waterloo and Peel District School Boards, a secondary math department head, and a secondary vice-principal. Jeff has taught at three faculties of education and at Sheridan College. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. He is currently an instructor of mathematics education at Brock University, where he is pursuing a doctorate.

Georg Pólya's problem-solving heuristic is given in the front matter of every mathematics curriculum document in Ontario:

- Understand the problem
- Make a plan
- Do the plan
- Reflect/look back

Pólya's seminal book, *How to Solve It*, expands on each of the four steps. We can see in Pólya's suggestions the foundation of our current problem-solving approach in mathematics. Pólya's heuristic also illustrates the interconnectedness of problem solving with the mathematical processes. These are noted briefly beneath each step.

Understand the problem:

- What are you asked to find or show?
- Can you restate the problem in your own words?
- Can you think of a picture or a diagram that might help you understand the problem?
- Is there enough information to enable you to find a solution?
- Do you understand all the words used in stating the problem?
- Do you need to ask a question to get the answer?

This facilitates connections to the mathematical

processes of representing, communicating, as well as reasoning and proving.

Make a plan:

- Guess and check.
- Make an orderly list.
- Eliminate possibilities.
- Use symmetry.
- Consider special cases.
- Use direct reasoning.
- Look for a pattern.
- Solve an equation.
- Draw a picture.
- Solve a simpler problem.
- Use a model.
- Work backward.
- Use a formula.
- Be creative.
- Use your head ("always use your own brain first"—G. Pólya).

Mathematical processes that can be connected to this component include representing, selecting tools and computational strategies, connecting, communicating, as well as reasoning and proving.

Carry out the plan:

This step is usually easier than devising the plan. In general, all you need is care and patience, given that you have the necessary skills. Persist with the plan that you have chosen. If it continues not to work, discard it and choose another. Don't be misled; this is how mathematics is done, even by professionals.

In terms of math processes, this step encorporates any of the processes for the first two steps.

Reflect/look back/extend:

Much can be gained by taking the time to reflect and look back at what you have done, what worked and what didn't. Doing this will enable you to predict what strategy to use to solve future problems, if these relate to the original problem.

The is related to the mathematical processes of reflecting, connecting, and communicating.

Let's apply Pólya's steps to a problem:

The 12 sticks: In my bag, there are 12 straight sticks. Each stick is a different whole-number length. No matter which 3 sticks I take from my bag, I cannot make a triangle with them. What is the shortest possible length for the longest stick in my bag?



Understand the problem: This is a great time to use a manipulative. Spaghetti is good for this, as it can be broken into different lengths. Most groups start exploring by choosing sticks of length 1, 2, and 3 units. These can't make a triangle, since 1 + 2 = 3, so all you get is a straight line. The first decision is whether a stick of length 4 can be included. Fairly quickly, students decide that the answer is "no," since 2, 3, and 4 do make a triangle. So a stick of length 4 can't be in the bag.

Make a plan/do the plan: Most groups proceed using trial and error. The next length, 5, is included in the bag, since 2 + 3 = 5 is a straight line, and other combinations like 1 + 2 < 5, so no triangle. Continuing in this way, sticks of length 6 and 7 are excluded, since they can make triangles (5 + 2 > 6, 3 + 5 > 7). This relationship is called the triangle inequality: for a triangle of sides a, b, c, $a + b > c$; i.e., the sum of two sides of a triangle is greater than the third side. Continuing in this way, the next included stick is length 8. Now students begin to recognize a recursion pattern, $t_n = t_{n-1} + t_{n-2}$, similar to the Fibonacci sequence. Continuing the pattern allows students to find the value of t_{12}.

Reflect/look back: The late W.W. Sawyer, professor emeritus at the University of Toronto, said, "If you have 2 minutes to solve a problem, the first 90 seconds should be spent in thought." The 12 sticks problem is a great illustration of this statement. Once the problem is really understood, the solution can be found quickly. Pólya's heuristic emphasizes the need to undertand a problem, rather than just rushing into computation. Once the 12 sticks problem has been resolved, students are encouraged to examine other problems that can be solved using patterning, after exploring to gain understanding. This approach has sometimes been called "problem solving by mucking about."

Here are some other problems to which Pólya's heuristic can be applied. The use of manipulatives is a natural method to aid understanding.

Crocodile river: A group of explorers has reached a very long, wide river infested with crocodiles. On their side of the river is a small boat and 2 children. The boat will hold ***either*** 1 adult or the 2 children. How many trips will be needed to take the entire party of 10 adults and the 2 children across the river? Write a formula for the number of trips needed with n adults and 2 children.



Materials: Linking cubes to represent the adults, 2 small cubes for the children, something for a boat.

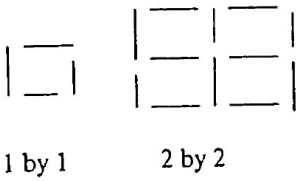
Log pile: Logs are usually piled so that the ends form triangles. For example, 3 logs are piled into a triangle 2 rows high. How



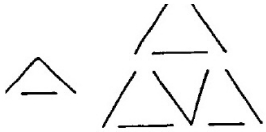
many logs are there in a triangular pile 10 rows high? How many logs are there in a triangular pile *n* rows high? (drinking straws make great logs)

Magic numbers: Find a number that leaves a remainder of 1 when divided by 2; 2 when divided by 3; and 3 when divided by 4. Find as many numbers as you can that fit these requirements. Generate a formula for these numbers. Pose some additional problems, using this idea.

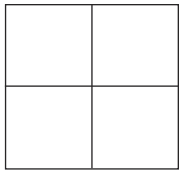
Silly squares: Squares are constructed with toothpicks as shown. How many toothpicks are needed to construct a 6 by 6 square? Write a formula or a recursion relation for the number of toothpicks in an *n* by *n* square.



Toothpick triangles: Construct triangles with toothpicks as shown. Make at least 5 samples. Count the number of toothpicks to make the small (equilateral) triangles in each figure. Find the number of toothpicks needed to make the 20th term in this pattern. Find a formula for the number of toothpicks needed to make the *n*th term in this pattern.



Nested squares: A grid contains squares of various sizes. For example, a 2 by 2 grid contains 5 squares (4 small squares plus 1 large square).



Construct a 3 by 3 grid, a 4 by 4 grid, and so on, and count the total number of squares in each figure. Find the total number of squares in a 10 by 10 grid. Write a formula or recursion relation for the total number of squares in an *n* by *n* grid.

Marble mountain: Oranges and other spherical objects are often piled with triangular bases. The diagram in Figure 1 illustrates some of the smaller piles, separated by layers.



Model this situation, using marbles. You may need rulers or other straight edges to keep the marble pile from rolling away, or you could use Plasticine as a sticky base. Find the total number of oranges in a triangular pile 10 layers tall. Write a formula for the total number of oranges in a pile *n* layers tall.

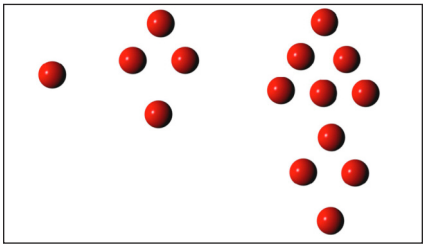
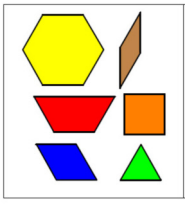


Figure 1: *Showing the layers (vertically) for marble mountains with 1 (left), 2 (middle), and 3 (right) layers*

The equilateral triangle problem: Using only triangular, trapezoidal, and hexagonal pattern blocks, construct an equilateral triangle in which the total areas of all three colours are equal. Then find the next biggest triangle with the areas of all three colours equal. Look for a way to generate more triangles with this property.



The painted cube problem: A cube is built from smaller unit cubes and then dipped into paint. How many of the smaller cubes have:



- exactly one face painted?
- exactly two faces painted?
- exactly three faces painted?
- no faces painted?

Generalize your answers for an *n* by *n* by *n* cube. The activity is well suited to using linking cubes as a manipulative.

There are a multitude of other problems to which Pólya's heuristic can be applied. For example, Shawn Godin's "Parabolic Products" problem from the December 2017 *Gazette* (56(2), pp. 21–22) is a great example of generalizing from a base problem, using technology as a method.

Reference

Pólya, G. (1945). *How to solve it*. Princeton, NJ: Princeton University Press. ▲

▲ Solution to Daily Numeracy Activity

8	2	6	9	5	3	1	4	7
7	3	1	6	8	4	5	9	2
9	4	5	7	1	2	3	6	8
4	7	2	5	9	1	6	8	3
1	6	8	2	3	7	9	5	4
3	5	9	4	6	8	7	2	1
5	1	3	8	4	9	2	7	6
2	9	4	1	7	6	8	3	5
6	8	7	3	2	5	4	1	9

▲ Hey, It's Elementary: Share-Worthy Parent Engagement Materials for Math… No Fake News!

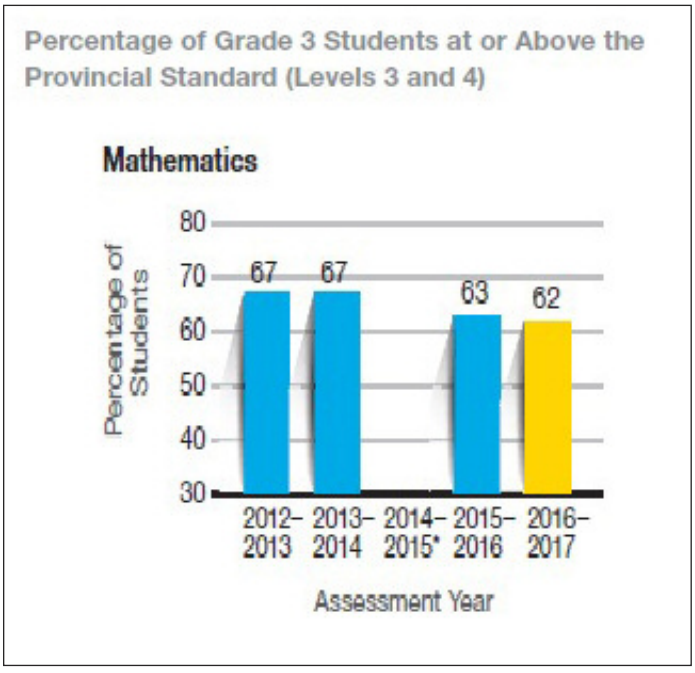
LYNDA COLGAN



Dr. Lynda Colgan is a Professor at the Faculty of Education, Queen's University. In addition to her teaching responsibilities in the BEd and Graduate programs, she is involved in research and knowledge-mobilization projects with the Council of Ontario Directors of Education, TVO/TFO, the Ontario Ministry of Education, and the Mathematics Knowledge Network.

At a recent A Parent Involvement Committee (PIC) conference, I had the privilege and pleasure of speaking to a large group of highly invested parents about the role of parent engagement in mathematics achievement, mindset, disposition, and attitude.

I presented relevant research findings, including recent EQAO data to frame my talk, using the latest infographic summarizing Primary and Junior achievement trends (see Figure 1). I also included a comparison pie chart (see Figure 2) that presents the results of students who were in Grade 6 in 2017, with their results in the Primary-division assessment when they were in Grade 3 in 2014.



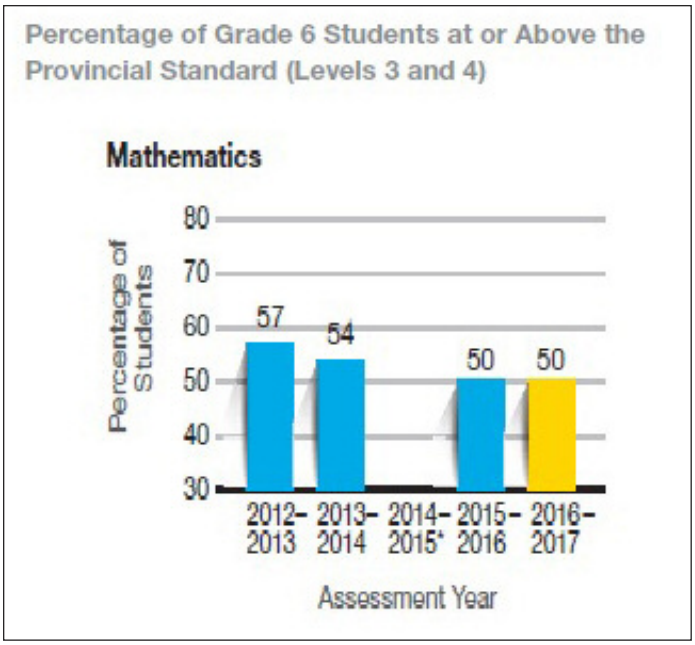


Figure 1: *Summary of Primary and Junior achievement trends*

I explained that for many years, our response to data such as these, as mathematics educators, has centred on improvement within the curriculum, its content, and delivery. We have focused on the development and implementation of outstanding resources for teachers, including the suite of *Effective Guides*. There has been an enormous investment in professional development around powerful instructional strategies like *Number Talks* and *The Three-Part Lesson Structure*.

I reiterated the fact that children spend only 14 percent of their waking hours in school from birth to the end of Grade 12—a calculation that always garners looks of surprise, and a powerful awareness that schools cannot be held solely responsible for the sum total of a child's academic, social, and emotional growth.

I then turned to the research that underscores the powerful influence that families wield in their homes. Through words and actions, they establish the tone of the "educational culture" experienced by their children for a large part of their lives. For each research point, I illustrate with an example or anecdote taken from my own experiences over the years as a classroom teacher, math coordinator, pre-service instructor, and family math mentor/facilitator.

For example:

- The support of parents as "…the single most important contributory factor to student achievement" and noting that "in terms of raising student achievement, parents matter significantly" (Harris, Andrew-Power, & Goodall, 2009, p. 2).

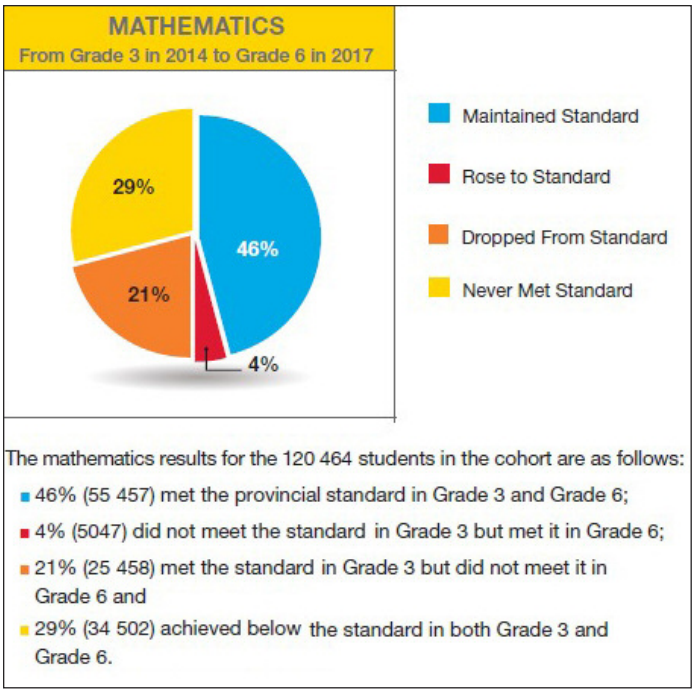


Figure 2: *Results of Grade 6 EQAO compared to Grade 3 for the same students*

- The research over three decades is consistent and persuasive: student success is positively impacted by parent and family involvement, regardless of background, socio-economic status, or parents' level of education (Jeynes, 2005).
- Families and schools are knowledgeable about children and learning, but in different ways—schools having knowledge of pedagogy, subject matter, and policy; parents knowing their children's motivations, skills, and interests (e.g., Cooper, Riehl, & Hasan, 2010, pp. 765–766).
- Parent involvement that is focused on student learning has the greatest impact on student achievement compared to other forms of involvement (Henderson & Mapp, 2002).
- Family support of their children's learning at home is linked to improved student outcomes (Fan & Chen, 2001; Hoover-Dempsey et al., 2005; Jeynes, 2005).
- Parental expectations and aspirations for their children, parent–child communication, and encouragement for learning in mathematics are associated with increased mathematics achievement (e.g., Fan & Chen, 2001; Jeynes, 2005; Vukovic, Roberts, & Wright, 2013).
- Parents' educational beliefs and aspirations for their children stood out as having the greatest influence on academic achievement—over and above the effect of Socio-Economic Status (SES), and are consistent across racial groups and subject areas (Fan, 2001).
- Students whose parents have high expectations for them have "more perseverance, greater intrinsic motivation to learn mathematics, and more confidence in their ability to solve mathematics problems" (OECD, 2012, p. 18).
- Worldwide, disadvantaged students generally scored lower in mathematics, but "resilient" students, i.e., those with greater engagement, drive, motivation, and self-beliefs, break the link between low SES and poor achievement (OECD, 2013, p. 17).
- Parents' ratings of their daughter's mathematics ability were the strongest predictor of the daughter's own mathematics ability ratings; further, parents' ratings of their daughters' English abilities were found to undermine girls' interests in mathematics (Eccles, 2005). Specifically, if the parent believes that a girl is better in English than in math, the child will come to believe she is better in English than math, even though there is no evidence that the girls in the study were any better at English than they were at math (Eccles, 2005, p. 200).
- Parents who attribute their children's success to effort rather than luck, have a greater effect on their children's achievement. Parents who believe that intelligence is malleable are more likely to assume that they, and their children, can control outcomes through their efforts (Hoover-Dempsey, & Sandler, 1997, pp. 24–25).
- In a large-scale field study of Grades 1 and 2 students and their parents, Maloney et al. (2015) found that "when parents are more math-anxious, their children learn significantly less math over the school year and have more math anxiety by the school year's end—but only if math-anxious parents report providing frequent help with math homework" (p. 1).
- Despite parents' good intentions, in the absence of parental "positive math attitudes," their involvement in math homework may contribute to their children's math anxiety and thus inhibit their mathematics growth (Maloney et al., 2015, p. 7).

I closed this portion of the presentation with a quotation:

"You are their idol. They look up to you. They don't do a good job listening to you, but they do a fantastic job imitating you." — Karl Subban on being a role model to your children

This overview of research findings was the catalyst for a rich conversation. Questions and comments were rapid-fire and passionate. *"I never thought that what I said off-handedly about math mattered so much." "How can I help*

when I do not understand the math homework that my child brings to the kitchen table?" Inevitably, the "big" question came: *"So are you saying that parents are responsible for the decline in math achievement over the years?"*

How did I respond?

With a categorical, "No!" I began by sincerely thanking parents for attending the session, reminding them that parents who are themselves motivated to become involved in their children's learning, in turn have an effect on their children's motivation to succeed. I addressed the issue of parental mathematics self-efficacy by explaining that the best support is simple: *"Ask your child to teach you what he or she learned in math today at school."* Alternately, play a math-based board game together for 15 minutes; read a book with a math theme together; or talk about the math that you use in the kitchen or the garden.

At that point, one mother spoke up, "*Where do I begin? Where do I get help? I don't have a clue where to begin…."* I looked out on a sea of nodding heads.

"That is exactly why I am here today!"

I provided walk-throughs of the three most recent parent-engagement toolkits for math by CODE: *Inspiring your child to learn and love math; Math Storytime* (see Figure 3), and *Choose a Path That Includes Math* (see Figure 4).



Figure 3: *The* Math Storytime *resource*

Next, I shared a suite of resources that are being prepared by an amazing team from Queen's University, including Dr. Michelle Searle and MEd candidate, Tyler Ashford, as part of a project called *Building Parent Engagement: A Project to Support the Implementation of Ontario's Renewed Mathematics Strategy.* Our team is working with six schools across Ontario—The Prince Charles School in Napanee, Prince Charles School in Verona, Selby School in Selby, Graham-Bell Victoria School in Brantford, and St. Teresa of Calcutta and Immaculate Conception Schools in Windsor to build a repository of rich materials to support parent engagement in mathematics in innovative and effective ways. The goal of the project is to take the pressure off schools and parents by providing research-based, invitational resources that are ready to use in a variety of ways: newsletters, websites, school council, and PIC-sponsored events. I then distributed four of our colourful infographics (see Figure 5): *The Math of Trees, The Math of the Garden, Math Around the Home;* and *The Mathematics of Games.*

I closed with a quotation from Dr. Kenneth Leithwood, Professor Emeritus, OISE

> *"…half of the achievement we're responsible for as educators happens out there, so if we can do something out there to encourage more productive learning conditions, we can hit some home runs!"* (The Learning Exchange, 2012)

Note to OAME *Gazette* Readers: All the materials are in field-test stage at this time, and after feedback and refinement, based on comments from educators and parents, will find a permanent home on our new website, due to be launched in June 2019. Four are included here as thumbnail visuals (see Figure 5). If you would like to obtain PDF copies for use by your school or school council, we would be happy to share. Email **parentengagement@queensu.ca** and we'll respond as soon as possible.

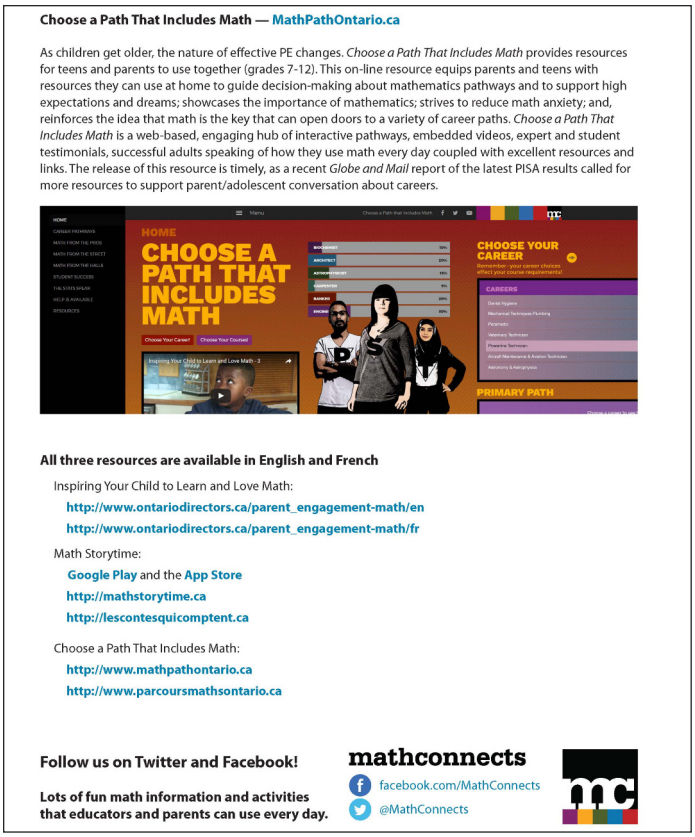


Figure 4: *The* Choose a Path That Includes Math *resource*

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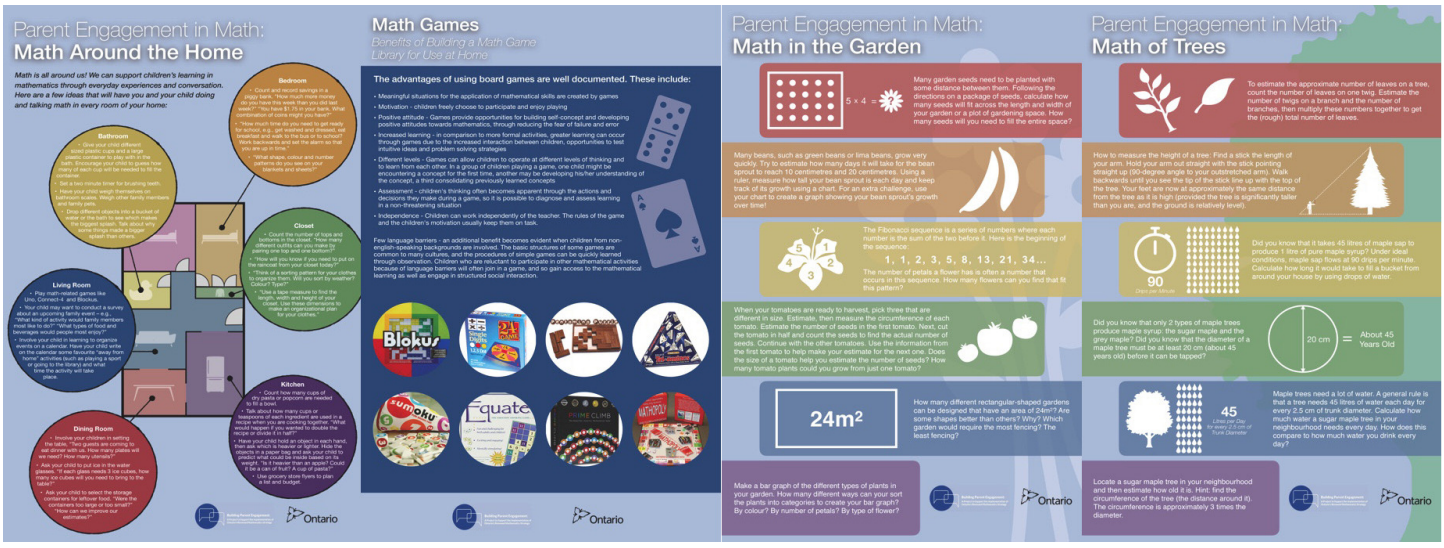


Figure 5: *Mathematics infographics*

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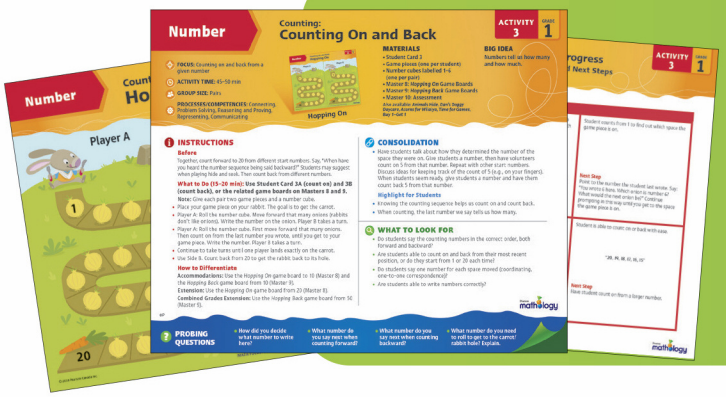
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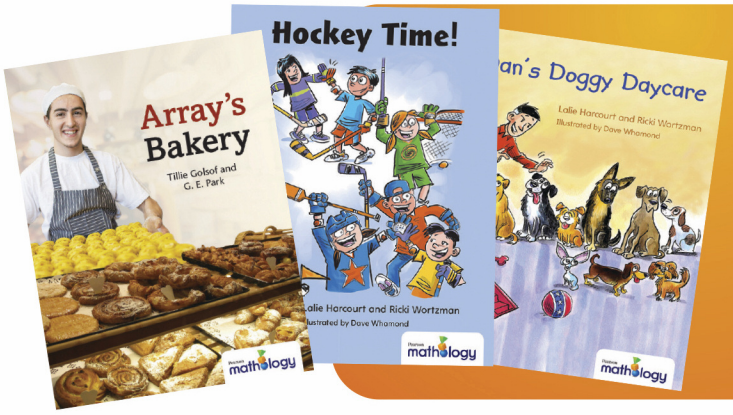
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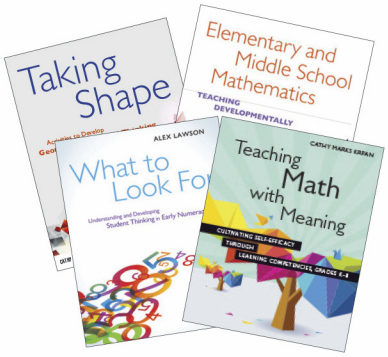
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