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Ontario Mathematics Gazette

OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

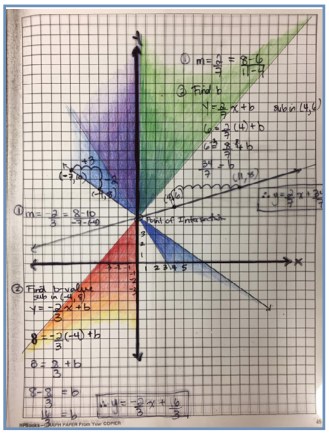
AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

Vol. 57 #2
Dec 2018

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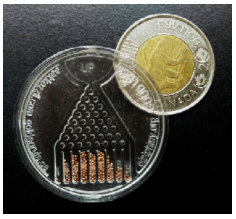


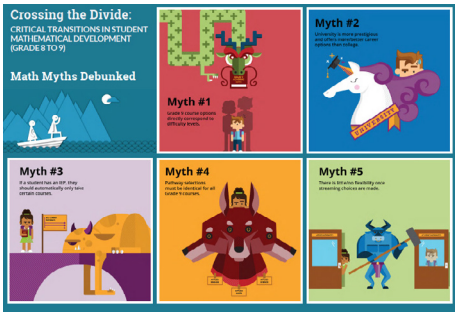




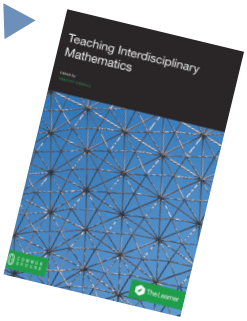
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See Mb4T (Mathematics by and for Teachers): Supporting Understanding of Fraction Operations

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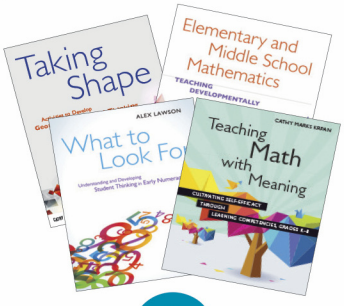


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Submission of Articles

The *Ontario Mathematics Gazette* (OMG) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *Gazette*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles are to be word-processed, MS Word is preferred, and prepared according to the *Publication Manual of the American Psychological Association*, Sixth Edition (2009). However, please use single-line spacing (not double) and only one space after each period. Articles should not exceed five numbered pages of text, and figures, images, and photographs should be placed in the text close to where they belong, with captions. The photographer's permission is required, and for photos of students under the age of 18, the written permission of a parent or guardian is required.

Please submit your article in one blind file (i.e., identity of author is not evident), and include author names, contact information including email and mailing addresses, photos, biographies, and all content removed for blinding in a second file. Please email these two files to Tim Sibbald at **gazette@oame.on.ca**.

Upon review, you will be notified whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected. The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME.

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The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of OAME.

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Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** by email or at the above address. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be print ready, and colour advertisements should have no bleeds.

Advertising Rates
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▲ Editor's Report



TIMOTHY SIBBALD, OCT, PhD
EMAIL: gazette@oame.on.ca

Tim Sibbald is the current Gazette *editor and a Past President of OAME. He is an associate professor in the Schulich School of Education, Nipissing University, with a focus on mathematics education in both pre-service and graduate programs.*

It never ceases to amaze me how many things are actively percolating in math education. There is no end of activity, and often there is a feeling of bouncing from one issue to the next, only to be reminded of another issue after that. In my opinion, this is the reason why taking stock and focusing on personal goals is important—it is easy to get lost in the rhetoric of the day. In my position, where I am currently on a sabbatical, taking stock is a curious mix of stepping away, but also of applying systematic research thinking. The same approach applies as teachers, though within the confines of that role—stepping away is often in the summer, but professional reading/research/PD is systematically worked into a busy role. However, this editorial is an attempt to capture the eclectic mix of educational matters we contend with, in order to make the point of how varied and vast the scope of our task is.

The latest EQAO results have come out. I question whether the public publishing of results is genuine accountability or not. There are numerous difficulties with publicly exposing the results, for example, stories of EQAO results being used as real-estate considerations (CBC, 2017). However, this issue is not new, and there seems to be a firmly entrenched interpretation of "accountability" that I argue is being mistaken for transparency. What concerns me when I think about it in a systematic manner is not the public foray into information that is made available to them—kudos to the public for paying attention. The concern is the focus on descriptive statistics that are based on single individuals or single schools. This belies the challenges of the complexities of the parameters with which teachers deal. Simply put, it is well established that student achievement and socio-economics are correlated, and the concern is that the EQAO could be exacerbating that relationship.

In a recent conversation, I was asked, out of the blue, if I knew what the "GAINS" in EduGAINS stood for. While I made the connection to the precursor MathGAINS, I confess to being uncertain of the meaning. Checking the "About"

feature of the website and searching the website did not reveal this detail easily. However, in a fuzzy logo on a document, one can make out "Growing Accessible Interactive Networked Supports." While that reflects, in a broad manner, what the project logo encapsulated as the overall direction of years ago, it is worth contemplating if the resources have grown beyond that original direction, or if it still reflects what it is. But, alas, we move on….

A fascinating article about taking apart photos, using online tools, was published recently. It is not appropriate as it stands, to the classroom, but is informative for teachers about the power of geometry combined with technology. The techniques and details can be worked into classroom geometry, especially if you use local satellite views or, in some municipalities (such as London), aerial photographs of the city are available. It would be ideal to have an article in the *Gazette* that summarizes the principles on which the article is based. The article can be found at **www.cbc.ca/news/the-investigators-bbc-cameroon-1.4843440**.

Recently I learned of the Farey sequence. Students can benefit from learning about this curiosity. The first element of the series is $\left\{\frac{0}{1},\frac{1}{1}\right\}$. Yes, the element is a set of fractions. The second element is all fractions from zero to one that have whole-number numerators and denominators of 2 or less, presented in order from smallest to largest: $\left\{\frac{0}{2},\frac{1}{2},\frac{2}{2}\right\}$. Note that I have omitted equivalent fractions. Unusual details happen as the sequence continues, such as in the seventh set, with the collection $\frac{1}{4},\frac{2}{7},\frac{1}{3}$. There are many patterns patterns underlying the sequence, and your students can be challenged to find and articulate one. Consider, for example, that Farey sequences have the property that $\frac{a}{b}<\frac{a+c}{b+d}<\frac{c}{d}$.



Figure 1: *A Pi-cence plate*

About this point, time and intrigue in math take over with a raft of small items. Whether it is the Pi-cence plate (see Figure 1) that was made using **www.acme.com/licensemaker/licensemaker.cgi?state=Ontario&text=III+I+IV&plate=1994&r=843377272**, or wondering about the decay pattern of a Mayan book, shown in Figure 2, that has been declared authentic (see **www.cbc.ca/news/technology/mexico-maya-modex-1.4806032**). There is also a trail of broken spaghetti after testing out the claim that adding a significant twist in a stick of spaghetti prevents it breaking into three and will only produce two pieces (see **http://news.mit.edu/2018/mitmathe maticians-solve-age-old-spaghetti-mystery-0813**)—that might be something for your students explore on the last day of school before the holidays, not to mention the question of whether a promise by Revenue Canada to plant a tree would persuade you to e-file instead of using paper. The campaign led to 22.5 percent e-filing compared to a control groups at 21.2 percent, but what you need to know to compare these is that the trial group was 25 000 people, and the control group was 6250 people (see **www.cbc.ca/news/politics/nudge-online-cra-experiment-tax-filing-environment-trees-letters-1.4751769**).

By this point, you have likely begun to wonder where this editorial is going. That is the point—teachers live in an ongoing stream of change. It usually burbles, but sometimes rages, and there is no end of redirections and diversions that draw our attention, which brings me to the message I want to convey—that a task we rarely speak about is the importance of looking after ourselves. There is a need to catch one's breath, to jump out of the stream, and consider the priorities of the flow of ideas. It is not simply a passive "holiday" style task; rather, it is an established, researched need called *reflection*.

It has been 35 years since Donald Schon wrote *The Reflective Practitioner*, where he spoke about teachers as reflecting on action, but also reflecting in action. The notions he brought forward resonated with teachers, and research on the topic is alive and well because of its importance. Among the notions that have been added to the topic is "critical-reflection," which begins with stepping back from the action so that one can gain some coherence to one's thinking about events. So, while I wish everyone Happy Holidays, I also hope that you find time to revitalize yourself by reflecting on your practice. It is an effective form of professional development, but there are no formal awards for taking on what appears to be a private task. The reward, for those who engage in reflective practice, is a quiet, subtle improvement in the coherence of what one is doing.


Figure 2: *What is the damage pattern for this Mayan book?*

In This Issue

The Executive Directors reflect on their many years of math teaching experience, as well as share some of their own personal perspectives on contemporary issues. It is interesting to juxtapose this opinion piece with that found in the *President's Message*. Along with this is a brief report from an ad hoc committee that was tasked with taking a detailed look at the annual conference. Angelica Mendaglio brings news from the Fields MathEd Forum and causes one to wonder about the relationship of Plinko boards and the Galton board. Jacqueline Hill considers how the online accessibility of NCTM resources makes them available to teachers in smaller northern communities (and you too).

Shawn Godin takes a turn into the Ulam spiral—a fascinating piece of evidence that the prime numbers have structure. Carly Ziniuk raises the topic of speed climbing, and one hopes that isn't why Ron Lancaster takes us to the Blue Mountains. By the way, an apology for any eyestrain caused by Figure 1 in Ron's column last issue—please find that figure reproduced in Figure 3 with a readable font! Jennifer Holm chops up fractions for a rewarding pedagogical approach. Mary Bourassa shows how Delta Math can make a difference, while the Provincial Digital team charts new tools for us. Ann Arden has a small chat with Dr. Marian Small.

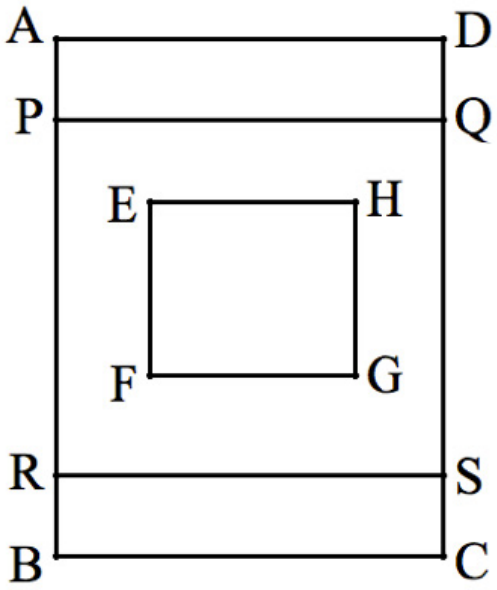


Figure 3: *Ron Lancaster's Figure 1 from last issue*

New this month is David Costello's column, which will focus on how math and literacy are intertwined—personally I look forward to learning more about this combination. Missing this month is *Hey, It's Elementary!*, and I hope the coming year is easier on our longest-running columnist, Lynda Colgan.

The Last Words

I owe considerable thanks to many of my colleagues. The columnists were asked to move their usual deadline ahead five days to suit my travel schedule. What is more, the editorial team managed to jump several hurdles to meet schedule and medical details along the way. I want to thank Jimmy Pai for stepping into an associate editor role to help in a time of need. I also want to thank Jacquie, Anne, Gitta, and Penny for their efforts. Lastly, as I disappeared before the final proofreading of the issue was complete, I need to thank Dr. Dan Jarvis for stepping in as I left on my travels. Thank you to each of you for supporting the worthy cause that the *Gazette* is.

Reference

Boisvert, N. (2017, May 24). *No pressure! Realtors say EQAO test scores can have a 6-digit impact on housing prices*. Retrieved from **www.cbc.ca/news/canada/toronto/no-pressure-realtors-say-eqao-test-scores-can-have-a-6-digit-impact-on-housing-prices-1.4129638** ▲

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OAME 2019 is calling for volunteers!

This is a great opportunity for students, including those in high school, university, and pre-service areas. Know any students who are interested in being a part of the most exciting math conference in the province? Want to get experience and connect with colleagues, network with other teachers and boards, and hear from some of the top leading mathematics educators out there? The Volunteer Committee for the OAME 2019 annual conference is looking for volunteers! Volunteers can sign up for half-day, one-day, or two-day time slots.

Along with volunteer time, there will be opportunities to register for a session or two on us! If you are interested, or if you know any students who would be interested in being a part of this engaging and meaningful experience and would like more information, please contact us at **volunteers@ oame2019.ca** by January 31, 2019.

President's Message



DAVID PETRO

David Petro, the current President of OAME, is the math, science, and IT consultant at the Windsor Essex Catholic District School Board. He is a large proponent of exploiting technology for the educational benefit of students.

What are the "basics" in math? We hear it a lot in the media, from our politicians, from our parents, and even from some of our own colleagues, but what does that mean? As you read this, it's December, and as I write it, it's October, and we have just started the year with a new motto: "Focus on the Fundamentals."

In the summer, before school started, we had tweets giving us a possible glimpse into what that might mean:

> *Ontario's math curriculum is failing our students. This week, we announced education consultations, which will include moving away from Discovery Math to traditional formulas and memorization techniques. Let's get our kids back to basics with proven methods of teaching that work.* (@FordNation — **https://twitter.com/fordnation/status/1033803567751024640**)

For those who didn't look any further, that might be either good or bad, but the fact is, the Ontario Ministry of Education had released their teacher (**www.edu.gov.on.ca/eng/teachers/teacher_guide_math_en.pdf**) and parent (**www.edu.gov.on.ca/eng/parents/parent_guide_math_en.pdf**) guides for focusing on the fundamentals, and these weren't seismic shifts in what was already going on. It wasn't new curriculum. It wasn't a direction to move away from "discovery" math. It wasn't throwing out strands in the elementary grades. It wasn't instructions for children to start memorizing more. It was basically a call to focus a bit more on number sense in the elementary grades, with some clarification (bolded in blue in the teacher guide) as to things that should be practised constantly from grade to grade. And as for memorization, well, right in the teacher guide, there is a quotation from Alex Lawson:

> *Children should learn their number facts. However, they would benefit from learning these facts by using an increasingly sophisticated series of strategies rather than by jumping directly to memorization* (p. 3).

They further clarify that "Repeated practice, or 'drill,' by itself may improve speed, but it does not contribute to understanding and it is not sufficient to guarantee immediate recall" (p. 3). So clearly, it matters where you get your information from.

That being said, I go back to my original question: What are the "basics" of math? I don't think that we all agree on what they are. In my job, I talk to a lot of teachers and often ask that question with varied results. We can generally agree that knowing multiplication facts is a basic skill. We can usually agree that the values up to 12 x 12 are typically accepted (see Sibbald, 2018 for commentary), but sometimes I don't agree on the methods to help students know those facts. As we continue the conversation, things get muddied even more. A lot of times, I hear "Fractions? Kids need to know their fractions." I often press further as to what that means exactly. Does it only mean that students need to understand what a fraction is and what it represents, or does it mean they need to be able to mechanically use operations with fractions fluently?

Maybe we even need to be more specific about asking what the basics are. That is, are we talking about basics that will help students be successful and productive in life, or successful and productive in school? I mean, there is a lot in our curriculum and most of it is just "stuff." Most of the curriculum is not really too important for daily life (I love the quadratic formula, but really, who needs to know it?). And even though the individual things may not be important, the act of learning is important for daily life. Solving problems in different situations, learning new strategies, struggling through difficult things—those are all elements that help build neural connections that we use later to solve the real problems we encounter.

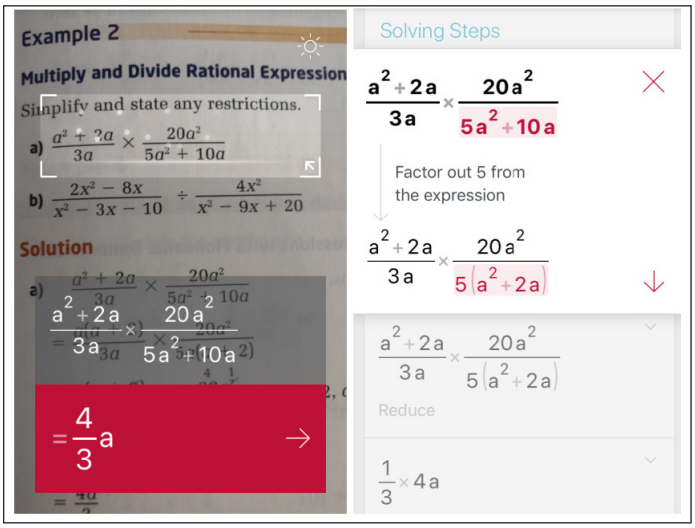


Figure 1: *Photomath solves a question*

To complicate things further, technology is quickly making things that we do obsolete. That is, if students can google the answers, is it worth teaching in the first place? Now before any negative responses are written, please let me clarify. Based on what I said above about creating new neural pathways, yes, teaching just about anything is a good thing. However, take an app like Photomath shown in Figure 1. You literally snap a picture of a question, and instantly it not only gives you the answer, but also shows you the steps. You can complain about this app's existence, but the reality is that it already exists. If students have access to this, how does it change your teaching practice? This is not very different from one claiming that with the ubiquity of calculators, this may be why students don't know their math facts. In reality, this might be on us. Nowhere in any curriculum document does it say "replace mental math with calculator math."

Where does the OAME come in? Since its inception, the OAME has always been about finding ways to connect mathematics to students. We are about providing information to our members about provincial directions in education, and the direction that math education is moving locally and globally. If you go to our website, you will see our guiding principles, including our mission statement. That mission includes our goal to promote excellence in mathematics education throughout the province of Ontario. We envision a learning environment where all students see, hear, do, and touch mathematics in a profound and meaningful way. We have been promoting these ideas for years in the conferences that we organize, the projects that we get involved with, and local OAME chapters that we support. We are moving forward to bring more value to you as OAME members in the coming months and years. Whether it be leveraging new technologies or remixing old methods, OAME will always be there to help support teachers.

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"I found that as I engaged in inquiry, my discoveries changed both my teaching and me" (Marin, 2014, p. 24).

Marin, K. (2014). Becoming a teacher educator: A self-study of the use of inquiry in a mathematics methods course. *Studying Teacher Education, 10,* 1, 20–35.

NOMINATIONS FOR OAME/AOEM BOARD OF DIRECTORS 2019-2020

Elections are currently being held for a President-Elect (four-year term), a Vice President (three-year term), an Emeritus Executive member (two-year term), and four Directors (up to a three-year term), of the OAME/AOEM.

- Due to recent changes in the OAME Constitution, By-Laws & Terms of Reference, the term of office for the OAME President has been changed to two years BEGINNING on September 1, 2020. In order to implement this change, with the election of the upcoming President-Elect for the 2019-2020 OAME Board year, that individual will still serve for one year. However, that term will be followed by a two-year term as President for the 2020–2021 & 2021–2022 OAME Board years, followed by a one-year term as Past President for the 2022–2023 OAME Board year.
- OAME/AOEM members running for the Emeritus Executive member must have experience necessary to offer a historical perspective for all matters before the Executive.
- OAME/AOEM members running for Director will declare their intent to work on either Grades JK–6, Grades 7–12, College, or University issues based on their personal experience, expertise, and interests.

Nominations for these positions are to be submitted electronically by completing an online Nomination Form by 5:00 p.m. on January 18, 2019.

A nominee must be a member of the OAME/AOEM. By March 2019, the Nomination Committee will publish a slate of candidates, including candidate bios, for consideration by the membership. Voting is open to all OAME/AOEM members. The election results will be released at the Annual Conference of OAME/AOEM in May 2019.

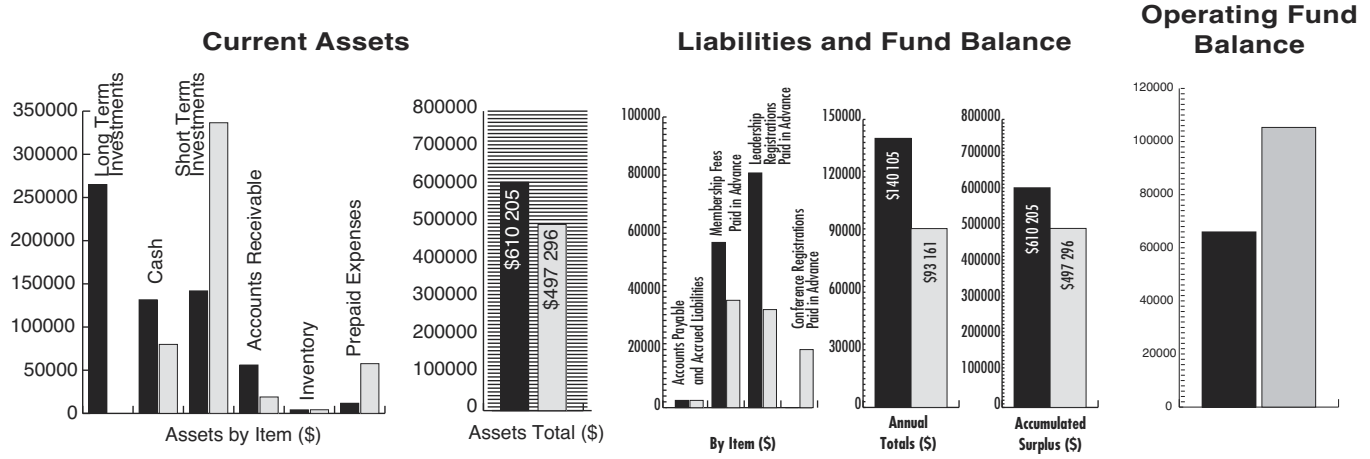
Please contact Jill Lazarus (**jill.lazarus@oame.on.ca**) for a link to the Nomination Form or for more information.
Thank you for submitting your nomination!

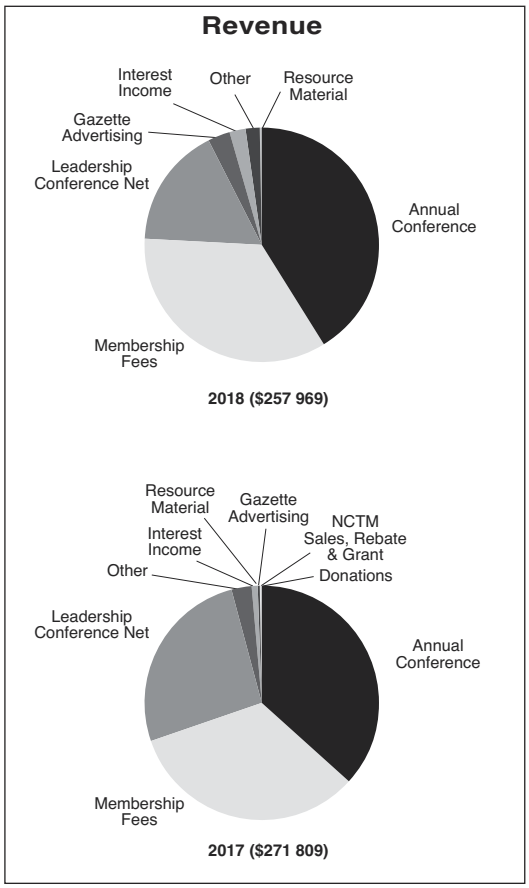
▲ OAME Financial Statement 2017-18

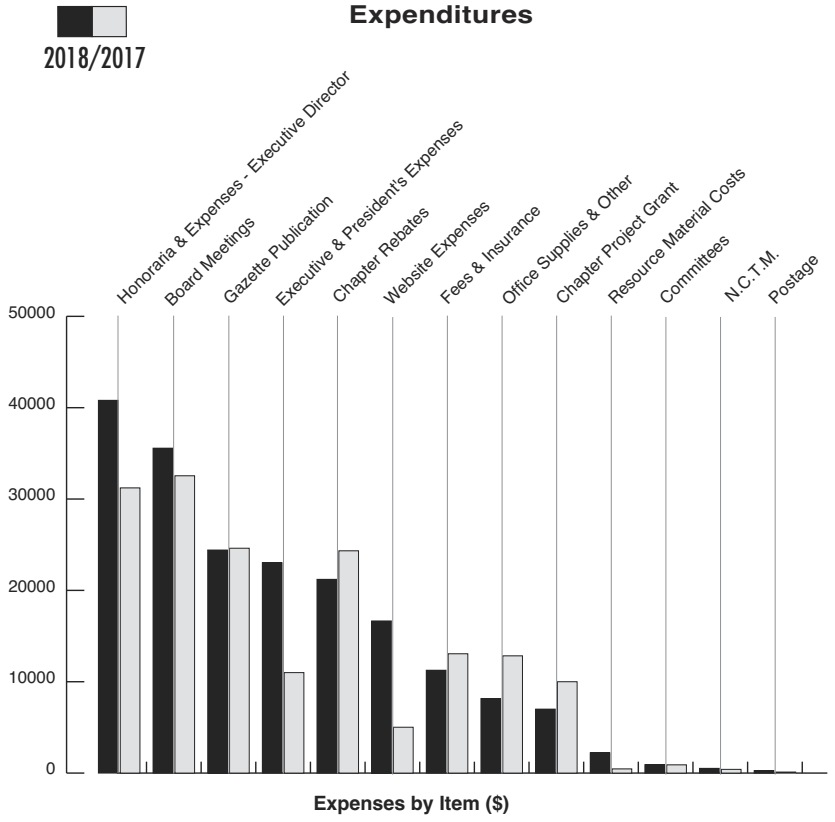
Notice to Readers

On the basis of information provided by management, we have compiled the statement of financial position of Ontario Association for Mathematics Education (Association Ontarienne pour L'Enseignement des Mathématiques) as of August 31, 2018, and the statement of operations, changes in net assets, and cash flow for the year then ended. We have not performed an audit or a review engagement in respect of these financial statements and, accordingly, we express no assurance thereon. Readers are cautioned that these financial statements may not be appropriate for their purposes.

London, Ontario, September 12, 2018, SKRYPNYK GROUP, Professional Corporation, Authorized to practise public accounting by CPA of O.







▲ Executive Directors' Reflections on Practice





LYNDA AND FRED FERNEYHOUGH
EMAIL: eds@oame.on.ca

Lynda and Fred Ferneyhough taught in the Peel District School Board for over 30 years, as well as experiencing a 3-year teaching stint in the United Arab Emirates. During their careers, they have served as Department Heads, as authors for McGraw-Hill Ryerson, and as certified instructors for Texas Instruments (TI). Fred continues to coach for TI in the United States. They have both served OAME as Chapter Representatives (CHAMP), as Directors, as Vice-Presidents, and as Executive Directors (2010-present).

Teaching mathematics has always been a challenge—yeah, that's an understatement. When Lynda and I started teaching in 1976 (no, it was not the dark ages, but the dinosaurs had only recently been wiped out), we were in schools within the then Peel Board of Education which, like almost all schools within our board, had nine periods per day, each of which were 45 minutes in length. We taught six classes all year long. Semestering was an approach that was being tried in a few schools, but, to my knowledge, it did not make an appearance in Peel until 1979, when a new school opened in Brampton. Over time, we saw the format of the school day change several times until every school in Peel finally adopted the semester system. One of the most unusual formats was a scheme at Mayfield Secondary School that featured five periods per day that were an hour in length plus a one-hour lunch period. We saw each class five times out of an eight-day cycle.

We have seen five different curricula. I believe that it was called the "credit system" when we started and the courses that we taught in the mid-seventies were similar to the ones we took in secondary school in the late sixties. A change was made to adopt a transformational approach to Geometry concepts in the late seventies. Then, along came *Ontario Schools: Intermediate and Senior* (OS:IS) in the mid-eighties, which changed Grade 13 to the Ontario Academic Credits (OACs). The changes in 1999 introduced a "problems first" approach, which completely changed the way we taught and assessed. It also phased out the OAC credits. This was fine-tuned in 2005 to the curriculum that we currently have in place.

Assessment was vastly different in Peel District at the time. We had three exam periods each year, one in November, one just before the March break, and a set of final exams in early June. This left lots of time for professional development at the end of the school year. In our board there were interesting formulae for determining whether or not a student was required to write the final exam or was exempt, and students were highly motivated to earn that exemption.

In Peel, we were expected to teach to the textbooks. For the most part, marks were based upon a skills-based test at the end of each chapter. There were very few assignments included in grade calculations. There were no performance tasks, no grade negotiations, no calculators, and no rubrics.

It was the dawning of the age of the hand-held calculator, and the cost of a four-function device was much higher than what students pay for a decent scientific calculator today. Math textbooks contained tables in the back of the book for trigonometric functions and logarithms. It used to take two to three weeks to teach a unit on logarithms, and the only aid that students had access to were those tables or slide rules. (I wonder how many readers know what a characteristic and mantissa are.) A well-equipped classroom had a seven-foot

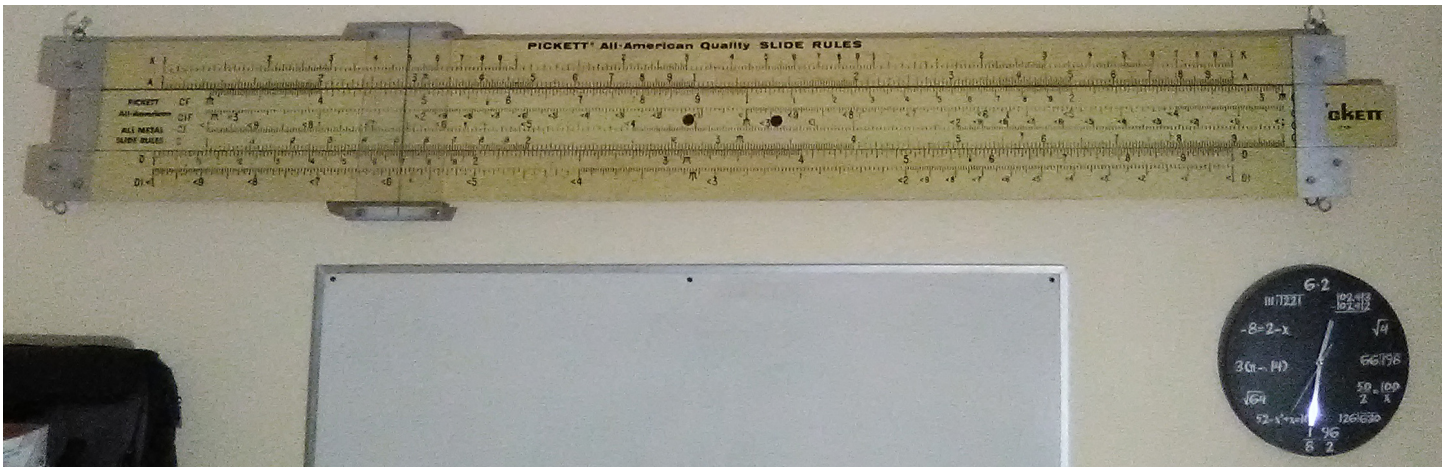


Figure 1: *Demonstrator slide rule with one-foot diameter clock on right*

demonstrator slide rule (see Figure 1) at the front, which teachers used to show students some of the features of those devices. It took years before many schools permitted the use of scientific calculators in mathematics classrooms.

Summer school also changed significantly in Peel over the years. It used to be a six-week program. Students who were taking a new credit attended all day (up to five hours), and students who were taking makeup classes attended all six weeks, but could take two classes for half of the day. If you finished a course in summer school, you were not guaranteed a credit. Somehow, that changed to a much shorter time frame, with the approach understood by almost everyone in the board that completion of the course resulted in an automatic granting of a credit. As a department head, I used to shudder looking at the changes in marks that students earned at summer school, knowing full well that most of them increased their course mark in a way that did not reflect their ability, and, in my opinion, set the student up for failure in the next course.

Characteristic and Mantissa of a Log

Since $\text{Log}_{10}(124) = 2.093$, the characteristic is the whole number part, i.e., 2, and the mantissa is the part to the right of the decimal point i.e., 0.093.

The classroom is a vastly different world now than it was in 1976, and it seems to me that the demands on teachers have increased substantially. I can recall having a high failure rate in a Grade 9 class in 1977 and being asked to justify those results. That made sense, and I had a very productive conversation with my administration about those results. I wonder how many teachers today feel as if they are being told by administrators to "just pass them" along? Are teachers under pressure for being too demanding with their students? Does this, along with other factors, lead to increased stress in the workplace?

So, what's coming up on the horizon? There are two events that have taken place recently that lead me to believe there will be more difficult days ahead.

First, the province has elected a government that has indicated it wants to do away with "discovery math." I have yet to see any definition of that term, and I doubt that we ever will, but there is an implied intent to take us "back to basics." There is no doubt, in my opinion, that a firm understanding of number concepts makes life in the math classroom much easier for students, but I hope we are not proposing to have students memorize addition and multiplication tables in the belief that this is going to solve all of our issues. The danger inherent in this approach is that a lot of good teaching methods are going to be attacked and possibly discarded in the process.

Second, in mid-September, a television network forced the engineering faculty at the University of Waterloo to reveal the data on schools it uses for admission purposes. As a result, a list of schools was published, which stated that some schools' grades were inflated to the point where the engineering faculty discounted the grades for students from those schools. I think many teachers in the province suspected the universities made some form of adjustment to grades from schools based upon historical performance, but this was the first time schools were named. Unfortunately, the 20 schools that UW discounted were listed for everyone to see. You have to wonder what the reaction of principals and superintendents is going to be for teachers at those schools, not to mention parents and students. In addition, I have to wonder what will happen in other schools in the vicinity of the 20 that were named. Will there be a sense of relief that they were not included? Or will there be a fear that "We could be next," and, "What do we have to do to prevent this?" It is not going to be a fun time.

Grade inflation has become an issue that we need to take seriously. In case you've never seen it, the following is found in the admissions policy for the faculty of mathematics at the University of Waterloo.

> If you repeat a required course or take a course outside of your regular day school, you may jeopardize your admissibility to the Faculty of Mathematics. Take all of your courses in regular day school and do well in your first attempt. (See **https://uwaterloo.ca/math/future-undergraduates/applying/admissions**.)

Both of these issues have the potential for a backlash from politicians, school board officials, parents, and school administrators. In my opinion, it's going to be an interesting, and possibly stressful time to be a math teacher in Ontario, but in the words of Red Green, "Remember, I'm pulling for you; we're all in this together!" ▲

Skovsmose said that "mathematics as a school subject refers to a rather well-defined body of knowledge parcelled out in bits and pieces to be taught and learned according to preformed criteria. Mathematics could, however, also refer to domains of knowledge and understanding that are not institutionalized through research priorities or curricular structures" (2016, p. 1).

Skovsmose, O. (2016). Mathematics: A critical rationality? In P. Ernest, B. Sriraman, & N. Ernest (Eds.), *Critical mathematics education: theory, praxis, and reality* (pp. 1–22). Charlotte, NC: Information Age Publishing, Inc.

Ad Hoc Committee Report: Hosting the OAME Annual Conference



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Jill Lazarus, the Past President of OAME, is a high school teacher in the Renfrew County District School Board. She is also completing a PhD in Education at the University of Ottawa. Jill has a range of interests in mathematics education, including curriculum design and implementation, student communication in mathematics, educational technologies, classroom assessment, and professional learning.



DWIGHT STEAD

Dwight Stead is a retired Grades 7–12 mathematics consultant and teacher from the Dufferin-Peel Catholic District School Board. He is currently a tutor for TVO Mathify and a Provincial Mathematics Professional Learning Facilitator. He was also co-chair of the 2014 OAME annual conference, past OAME Director, and former president of CHAMP. He currently lives in Vineland, Ontario.

The OAME Annual Conference is a highlight for many math educators in the province and beyond. The conference has grown in size and scope over the years. This article is intended to provide a historical perspective on the conference, to outline some of the challenges of holding the conference each year, and to share the response of OAME to those challenges.

Traditionally, the conference has been hosted by one of OAME's 15 chapters. The 15 chapters that make up OAME are a critical connection to the membership. The chapters provide key learning opportunities for math educators in their area. The experience of having the conference run by various chapters has been and continues to be an opportunity to network with teachers in different regions of the province, and provides vital resources to the hosting chapter. The conference has truly been a provincial conference. It has travelled from its first hosting site in Kingston (1974) to Windsor (most recently in 2011), to Ottawa (coming back to the nation's capital again in 2019), to North Bay (1998), to the Niagara region (2010), and many points in between.

To host a conference, a chapter submits a proposal to the OAME Executive at least two years in advance. The hosting chapter needs to ensure that it has a venue in place and an organizing committee prepared to take on the challenge of an annual conference. If the chapter presents a strong application, it will be selected to host the annual conference by the OAME Executive.

After being granted the conference hosting privileges, the work begins in earnest, with the chapter requiring as much time as needed to put all of the pieces in place and to ensure that all of the necessary features of a conference are considered. The hosting chapter is completely comprised of volunteers, including teachers who are in the classroom, retired teachers, students from nearby faculties of education, and students from local schools. This conference has been possible because of the commitment of volunteers who take on the significant task of completing the many features of the conference.

A conference committee is comprised of different subcommittees summarized below:

- A promotions committee: oversees marketing and promotions to spread the word about the conference
- A facilities committee: ensures that the other committees' needs will be accommodated at the venue
- A hospitality committee: plans for ancillary events during the conference and considers dietary needs of attendees
- A program committee: encourages past and first-time speakers to submit session proposals, and books keynote and/or featured speakers
- Website and registration committees: design website and look after the registration process, and communicate with delegates and speakers
- A volunteer committee: makes sure that there are plenty of helpful volunteers to assist during the conference
- A finance committee: keeps all committees updated on their budgets
- An eConference committee: responsible for broadcasting portions of the program digitally to remote participants
- A conference chair or co-chairs: coordinates all the aspects of the conference

Hosting the annual conference is a two-year commitment and will require many volunteers. Given the size of the conference (the conference in 2018 had 2000 registered delegates and presenters), it is a challenge for some chapters to host without a strong volunteer base in place. When a conference year does not have a volunteer host within two years of the date, the OAME Board of Directors takes responsibility for planning and hosting the event. This is what happened in 2015 and 2018.

This set of circumstances spurred the OAME executive to consider the challenges of hosting the conference, especially given the growth in registration. (Note: Registration closed early for a recent conference because it had reached its maximum capacity.) The OAME Executive created the Ad Hoc Conference Committee in 2016 and provided it with terms of reference that were wide in scope. The terms involved looking at all aspects of the conference and bringing back recommendations to the Board of Directors within two years. The Ad Hoc Conference Committee was comprised of then OAME President Jill Lazarus, past OAME President Paul Alves, and past OAME Board of Directors member and OAME 2014 Conference Co-Chair Dwight Stead.

The primary focus of the OAME Executive and the Ad Hoc Conference Committee is to ensure that the annual conference is well positioned to maintain the momentum that has made it the premier math professional development event of the school year. The committee consulted the OAME Board of Directors, past conference organizing committees, and the OAME membership to gather insights and opinions on the critical features of the conference, along with considering how the important conference features can be maintained within the existing framework. The Ad Hoc Conference Committee also considered other possible frameworks that could make it easier for hosting chapters and the OAME organization in the future.

The Ad Hoc Conference Committee presented its final report to the OAME Board in June 2018. Within the report, the committee proposed recommendations that honoured its mandate of preserving the core of what makes the OAME conference important to the membership, but also balanced those needs with what can be provided by a group of dedicated volunteers. In the coming months and years, the organization will share those recommendations with the chapters and members. The overall impact of the recommendations will not change what was regarded as important by the the Board of Directors and membership, but could change how the conference is administered and who is administering the conference. It is the hope of the OAME Board of Directors and the Ad Hoc Conference Committee that these changes will ensure the annual conference remains the marquee professional development event in the future, as it is today. ▲

▲ Linking Literacy and Math: Using Writing Strategies to Support Learning

DAVID COSTELLO



Dr. David Costello is a vice-principal who has previously held many roles. He taught in the primary division before assuming roles of numeracy interventionist, numeracy coach, numeracy leader, and curriculum consultant for Prince Edward Island. David has also instructed university courses focused on curriculum, differentiation, mathematics, and language arts. He facilitates professional learning sessions in the areas of mathematics and school development. His research situates mathematics within a literacy perspective and examines leveraging literacy-based instruction in mathematics.

Prior to this year, I worked as a consultant and leader at both the board and department levels for seven years. During this time, my professional learning of mathematics deepened. This deepening was not only because of the reading and research I did, but also because of the opportunity to work alongside teachers—as co-planner, co-teacher, and reflective practitioner.

A significant part of my work, while with the board and department, was viewing math as a form of literacy. Over the years, the definition of literacy has evolved to mean more than reading and writing. I am careful to not use the phrase "math and literacy," as this excludes mathematics from overlapping with literacy. Instead, language arts and mathematics are part of a broader definition of literacy. Being literate is about meaning-making and communication. Meaning-making, which I refer to as comprehension, and communication are the foundational aspects of mathematics. Comprehension and communication are our goals for reading and writing. However, aren't reading and writing what we do in mathematics?

When we view mathematics in the context of literacy, we approach instruction and learning holistically, something that makes sense for students. It is not about the parts of a process that students should memorize (i.e., multiplication facts); it is about students approaching, working through, and making meaning with a concept. Taking this one step further, it is about understanding what they are engaged in, why they have taken a certain approach, and knowing when

they are on track or not. It is about making meaning as they work through a task, whether it be to completion or not. It is about students being engaged with the mathematical concept and developing the mindset of a mathematician.

So, what are the implications of viewing mathematics in the context of literacy? Within such a view, the goal of mathematics should be for students to think mathematically; to approach a situation from a meaning-making perspective. It is imperative that students engage with the concept, strategy, and/or process. During such engagement, students apply their previous learning to new situations. This application results in the synthesis of new learning. It is not enough that students simply *do* or *perform math*, something that is called "word calling" or "decoding" in reading. What is the point of crunching numbers if the student does not understand what he or she is undertaking? Understanding must support the work; without this, math is little more than numbers doodled on a page.

When taking this approach to mathematics, I have seen students move to a conceptual understanding instead of rote procedure. In my initial experience as a teacher, many students could arrive at an answer, but were unable to describe their thinking process or why their approach provided a solution. I knew there was more to do as a teacher, but I didn't want to add things that would be a make-work project for students. I wanted opportunities for comprehension to be embedded in their thinking. Over time, as my instructional approach reflected the perspective of mathematics within literacy, students started to conceptualize their approach, work toward a solution, self-monitor comprehension, and communicate their learning journey. It was a move from decoding to comprehension; from doing to thinking; from following directions to meaning-making. It was during this move that my understanding of student thinking deepened. I was more aware of why students were making the decisions they were making. I was digging deeper instead of wider.

Instead of giving numerous problems for students to solve, I would provide one problem that would require students to dig deep to work through. I went from counting the number of correct responses to examining the work of students for their understanding and misconceptions. This was a change for me. Initially, I struggled with not having numerous assigned problems to look through. However, what these numerous problems provided me with was the opportunity to see if students were right or wrong. Instead, by assigning one strategically crafted problem, I could delve deeper into student thinking, to observe the thinking patterns of students, identify stumbling blocks, determine what students did when they were stuck, and examine how students worked through a problem from start to finish.

As a consultant and leader, I facilitated numerous sessions on mathematical content and instructional practices. I received feedback from teachers and principals regarding my messages and how they were enacted it in the school. However, I was now the teacher with my own students. Instead of dropping into a classroom or school for a short period of time, I was the one who was tasked with the daily responsibility of supporting student learning. I now had a much more in-depth awareness of the students I would be working with, and the exceptionalities that they brought with them to school. It was my task to be responsible for their entire year of mathematics.

It was August when I started to look through my student data to get an understanding of their learning needs, I remember thinking—I provided structures that teachers were encouraged to use at the beginning of a school year. I was now faced with being in the position of using these structures. As I looked through the data, I noticed how well my students were using place value. This was important, as my school had a strong history of mathematical achievement. So, I mapped out my first month. We would focus on building this strong foundation of place value. What happened next was something I was not fully expecting.

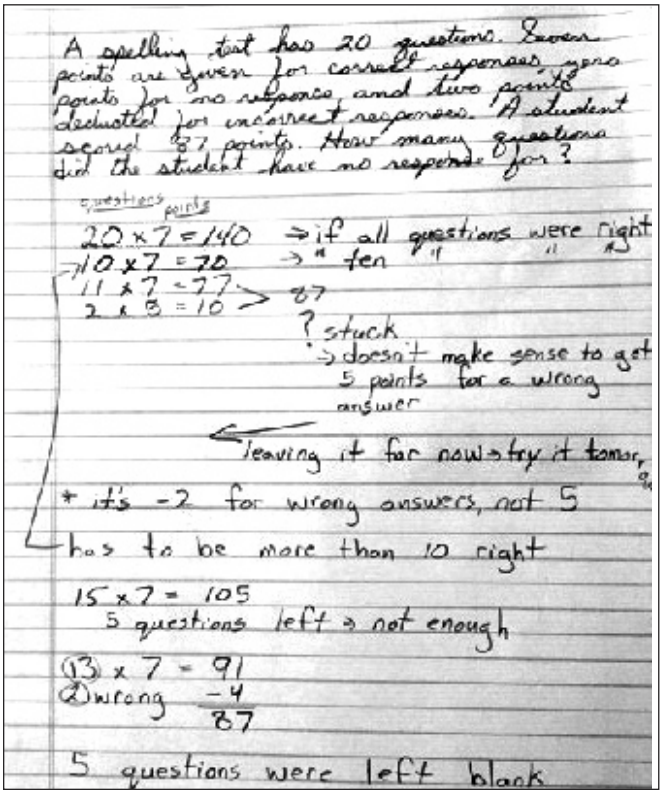
During the first week of school, I provided students with tasks that would enable me to assess their understanding of place value, while providing them opportunities to deepen this understanding. At the end of the week, I felt something was missing. Students were "getting the answer," but were unable to explain to me how they arrived at the solution. It was like a large part of my work at the board and departmental level; the students were decoding the process, but not displaying the comprehension necessary in understanding the concept. I needed to dig deeper, while not creating a make-work project. So, I relied on a writing strategy that would support students in monitoring their comprehension, while also providing me with an opportunity to assess their thinking points within the problem—margin notes. This was a strategy that I used in a writer's workshop when I taught in the classroom before moving to the board and department.

In the writer's workshop, students would often use sticky notes to document their thinking as they revised and edited their writing. Students relied on a variety of symbols to represent thinking, such as "?" for questions about a particular aspect of the text, "!" for important parts of the text, "*" for important ideas for revision, and "←" when they planned to return to the text at a later date. What I witnessed was how students used margin notes as a form of self-monitoring their writing. This was consistent with writing in mathematics being a form of reflection that can support students in consolidating their thinking (National Council of

Teachers of Mathematics, 2000). Realizing this, I decided to apply margin notes to mathematics. This strategy also coincides with the work of Mason, Burton, and Stacey (2010), when they reference students recording keywords as they engaged with a problem.

When working through a problem, students used a "?" at the specific part of the problem that caused them difficulty. Keywords would accompany the "?" as a method of describing what students found difficult. This could be "estimation" to indicate students were unsure of an appropriate estimate, or "start" to indicate students were unsure how to start the problem. Similar to language arts, a "*" represented a connection, and "←" represented instances when the student identified the need to return to the problem at a later date. A "√" reflected that students reviewed their work to ensure that there were no errors and that they addressed the question being asked.

The writing-in-the-margins strategy provided me with insight into which students understood the concept and which students were working toward this understanding. What was interesting was the fact that some of the students who *solved* the task were unable to record their thinking. It was as if they were simply following a rote process, without understanding the concept supporting it. By encouraging students to use this writing strategy, I could determine their level of understanding and how this developed during task completion. I was pleased with this realization. However, there was more learning to come.



This was the turning point for me. It was not until I encouraged students to use this strategy again that I recognized they may have been working through this process by rote. It was just that I had provided a new structure, without expectations to dig deeper. Students became aware of where to add a few strategic details. So, I was left with having to decide how to move this writing strategy forward.

What I did was twofold. First, I incorporated opportunities for dialogue among students. My goal was for students to share their work with classmates and to respond to the writing in the margins. I wanted students to examine other students' work for similarities and differences. This writing strategy served as a springboard for classroom discourse and student learning.

Second, I wanted to use this writing strategy as a tool for discussion between myself and the student. I used the work sample to meet with students to discuss their thinking points, not necessarily the solution. What evolved from this was a discussion that focused more on student thinking than on whether or not the response was correct. The discussion was more open-ended and provided students the opportunity to decide on a learning goal for the next few days. It was the springboard for comprehension, not decoding.

Viewing math from a literacy stance provided me with the system to recognize that, as teachers, we need to go beyond decoding. We need to promote and support student understanding, and for this to happen, students need to be active participants. Within my example, students were using their writing strategy not as an end, but as a means to gain conceptual understanding.

What I learned was that the tool, instructional practice or question, is not the be all and end all. Writing in the margins was a helpful strategy, but did not solve all my concerns. Instead, I needed to go back to my perspective of math as overlapping with literacy to understand what it meant to make-meaning—it wasn't about decoding or getting the correct response. It was about having students communicate their understanding in a way that allowed me to assess their understanding. I needed to dig deeper to go beyond the surface level, and sharing that is the goal of this new column.

References

Mason, J., Burton, L., & Stacey, K. (2010). *Thinking mathematically* (2nd ed.). Essex, UK: Pearson Education.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author. ▲

▲ Mathematical Snapshots: A Visit to the Blue Mountains

RON LANCASTER



*Ron Lancaster is an Associate Professor at the University of Toronto, where he teaches mathematics courses for pre-service middle and high school teachers. He has over 20 years of experience teaching Grades 7–12 mathematics. Ron's professional activities include consultations and conference presentations in North America, Asia, England, the Middle East, Africa, India, and Europe. Ron is an author for the NCTM (*The Mathematical Lens*) and member of the Advisory Board for the Museum of Mathematics in New York. He is the recipient of the 2015 Margaret Sinclair Memorial Award Recognizing Innovation and Excellence in Mathematics Education awarded by the Fields Institute.*

In the September 2018 column, Ron Lancaster took us to Niagara-on-the-Lake for some noticing, wondering, and questioning. In this issue, he takes us to the town of The Blue Mountains and The Blue Mountain Village, nestled within the town. Next issue, the focus will be on another place in Ontario. If you want your hometown to be the focus, send Ron math photos from your area or invite him for a visit.



Photograph 1: *Hatley Scoops Ice Cream store in Blue Mountain Village*

Notice there are 21 flavours listed in two columns. Wonder what other ways the flavours could be listed. Wonder which flavours are popular. Wonder why the flavours are listed in the order shown. Wonder which ones would taste good together on a cone with 2, 3, or 4 scoops. Wonder if any has ever asked for a 5-scoop cone to share with friends and how much it would cost. Notice that over 50 percent of the flavours have names made up of two words. Wonder what the percentage is for one-word and three-word names. Wonder if flavours of ice cream usually have two-word names. Wonder if the Kiddie cone has less than a scoop of ice cream. Wonder if a scoop of ice cream at Hatley Scoops is the same as a scoop at another ice cream place. Notice the prices for the different sizes. Wonder if there is a pattern.

Photograph 1: Mathematical questions

Question 1

(a) Calculate the difference in cost between a:
3-scoop and a 2-scoop cone
4-scoop and a 3-scoop cone

(b) Based on your calculations from part (a), suggest a price for a 5-scoop cone and a 1-scoop cone.

(c) Find a mathematical model for the price of a cone as a function of the number of scoops. Use your equation to suggest a price for a 5-scoop cone and a 1-scoop cone. How do these prices compare with your answers to part (b)? Are they the same? If not, why not?



Photograph 2: *Gate on the property of a home in the town of Blue Mountain*

Notice the letters "O" and "S." Wonder what they mean. Wonder who lives here. Notice the symmetry in the design of each door and in the overall design. Notice that both doors would be mirror images of each other if the letters "O" and "S" were each replaced by a "T." Wonder what other letters could be used so that the doors are mirror images of each other. Notice the different shapes that appear on the doors. Wonder how many more shapes there would be if the diagonals of each door were completed. Wonder how the area of one of the circles compares to the area of one of the doors.

Photograph 2: Mathematical questions

Question 2

Pretend that Figure 1 shows a scale diagram of the two doors. Suppose that the diagonals for both doors were not cut off by the circles as they are in real life, and that the circles remain in the design. In creating this figure, using the

Geometer's Sketchpad®, the author found that the ratio of AB to BE is 9:11, and the ratio of AB to OP is 9:1. The dimensions of both doors are the same and circles have the same radii.

(a) How many triangles are there within the overall design? How do you know you have found them all?

(b) Decide by estimating, which of the following is closest to the area of the circle expressed as a percentage of the area of the door.
   (i) 6%
   (ii) 8%
   (iii) 10%
   (iv) 12%
   (v) 14%

(c) Calculate and compare the areas of the circle and door in the figure. Is your result consistent with your answer to part (b)? If not, measure the ratios and see if you can determine which are not accurate.

(d) Suppose the designer of the door wanted a design for which the area of the circle is half the area of the door. Estimate and then calculate the radius of the circle as a fraction or percent of the door width for this design.

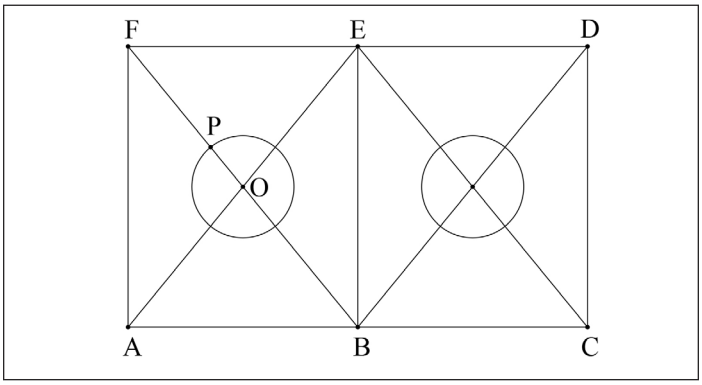


Figure 1: *Diagram of the doors*



Photograph 3: *Identical objects in front of a restaurant in Blue Mountain Village*

Notice the unusual objects in front of the restaurant. Wonder what they are and what this has to do with the restaurant. Wonder what type of food is served in this restaurant. Notice the patio set up with chairs and tables. Wonder what the time of the day was when this photograph was taken and what the month was. Notice there are more chairs than tables. Wonder what the ratio of chairs to tables is and if it ever changes. Wonder if the restaurant hires more people when the patio is open. Notice the angles formed by these two objects and the fence. Wonder what angles are the same. Wonder what the angle is between each of these objects and the ground. Wonder if these angles are the same.

Photograph 3: Mathematical questions

Question 3

Figure 2 shows a geometrical diagram of the two objects and the top, bottom, and side railings of the fence. The objects are imagined to be two line segments and assumed to make an angle of 45˚ with the top and bottom railings. The author recalls measuring the angles with his Magnetic Protractor Angle Locator, manufactured by Johnson Hand Tools and that it was close to 45˚.

(a) Identify pairs of angles in Figure 2 that you are certain are equal to each other and those you hypothesize are equal. How do you know with certainty that some are the same? What was your hypothesis?

(b) If you were there in person, suggest at least three different strategies for measuring angle FCD if you did not have access to the device used by the author.

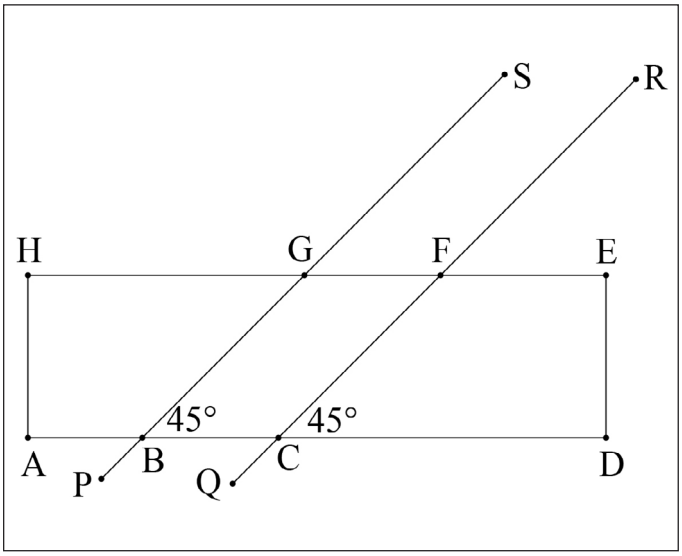


Figure 2: *Geometry of the objects*



Photograph 4: *Logo for Columbia Sportswear Company*

Notice the shapes. Notice how the corners are rounded. Wonder if small circles are used at each corner. Wonder how we could find the area of these shapes. Wonder why they used eight of these shapes for their logo. Notice the pattern of the shapes and the symmetry. Wonder how the pattern could be extended if the designer wanted to use more shapes.

Photograph 4: Mathematical questions

Figure 3 shows a possible method for how a designer could start with a rectangle and add arcs from a circle to create the shape in the logo for the store.

(a) Write a step-by-step description of how to make the shape shown at the right by starting with a rectangle. Provide enough details so that someone could make the shape without having to ask for any clarifications.

(b) In what ways does this shape not provide an accurate representation of the real shape in the logo? Suggest another method for making a new shape that is closer in appearance to the shape in the logo.

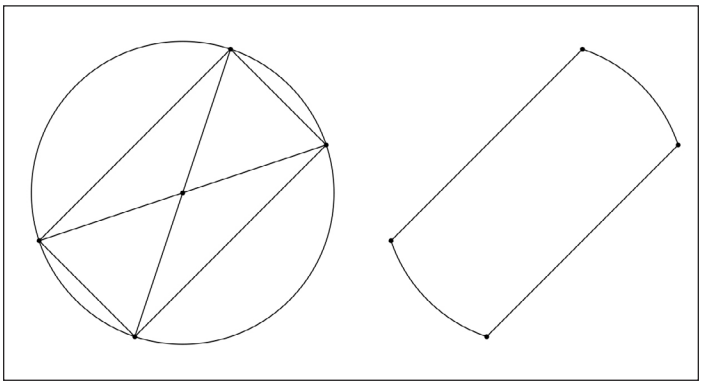


Figure 3: *Geometry of a design*

As a final note, answers to specific questions are not going to be provided, as this column is intended to encourage the reader to think about how to ask good questions. However, if you would like further insight on any particular question, please talk to your colleagues, raise it with your chapter, or tweet about it at "OAMEwrites," and whether it is resolved or not, you are always welcome to send a letter to the editor. ▲

▲ Technology Corner: DeltaMath.com



MARY BOURASSA
EMAIL: mary.bourassa@oame.on.ca

Mary Bourassa teaches mathematics at West Carleton Secondary School in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She has presented workshops internationally, authored mathematics resources, is a Past VP of OAME, and a Past President of COMA. An award-winning teacher, Mary continually strives to learn new and better ways of helping students learn and love mathematics.

DeltaMath.com is a website that was created by Zach Korzyk nine years ago. Zach, who is a Math for America Master Teacher in New York City, wanted an online math tool to help his students practise the material they were learning. He wanted to ensure that his students were not copying each other's work, so he built his site to generate a multitude of variations of each question type. His website has grown so that it now includes math questions from middle school through Grade 12, and there is an additional category for computer science. Teachers can create assignments that are automatically graded and tailored to each of their classes. It is all free.

Once you have logged into your teacher account, you can create your first assignment. Click "Create/Edit," then choose "Create Standard Assignment," give your assignment a name, and select the period(s) to which it pertains. Next, click on the "Skills" tab. The modules are sorted into middle school, algebra 1, algebra 2, geometry, precalculus, calculus, and computer science. You can explore modules such as the one shown in Figure 1, or use the search box to find a specific topic.

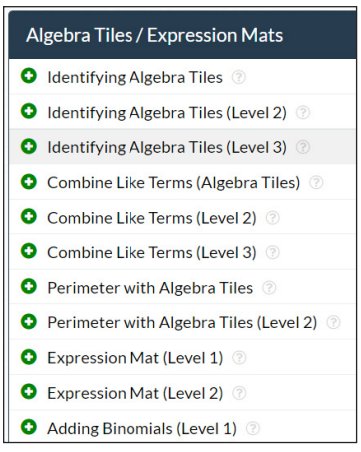


Figure 1: *Some modules from which to choose*

As you can see, certain modules have different difficulty levels. To add a module to your assignment, click on the plus sign. An assignment can consist of one or more modules, or can feature a mixture of problems from multiple

modules. Some modules are guided, some are timed, and some are multiple choice; each has a special notation when shown in the list. In Figure 2, you can see that "Simplifying Radicals" can be guided (more on this below), and "Basic Radical Operations" is timed, with the time limit set by the teacher. Modules that have multiple-choice questions are denoted with "(MC)."

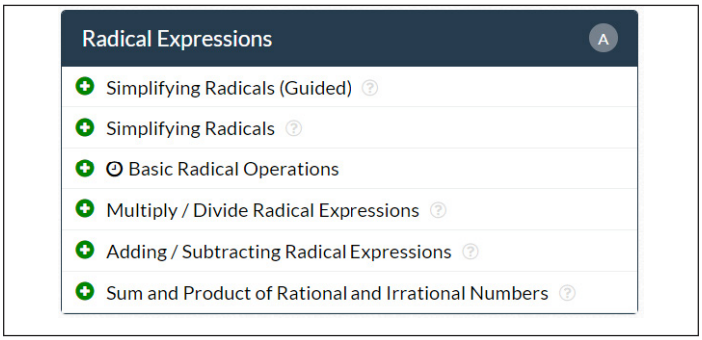


Figure 2: *Modules in an assignment*

The guided questions are beneficial for students who are in the midst of learning new material. The questions are scaffolded, allowing students to receive feedback at each step of their solution. Figure 3 shows a question from "Simplifying Radicals" that is not guided.

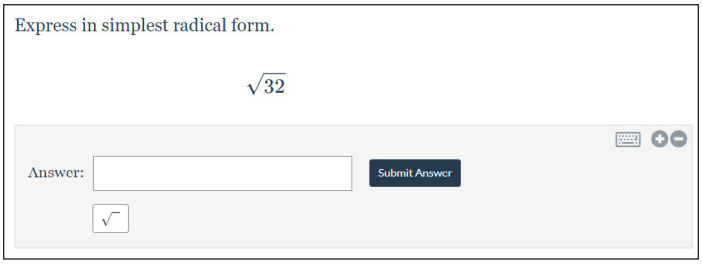


Figure 3: *Sample question from "Simplifying Radicals"*

In contrast, Figures 4–7 show the progression of a similar question from the guided section.

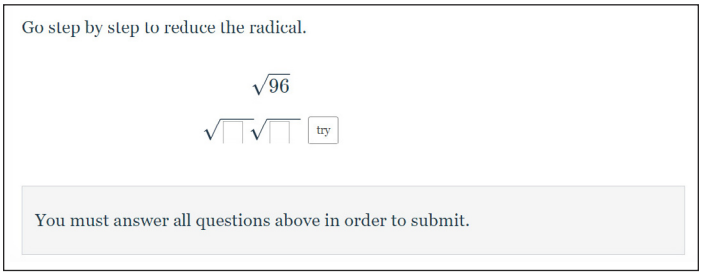


Figure 4: *First step in guided question progression*

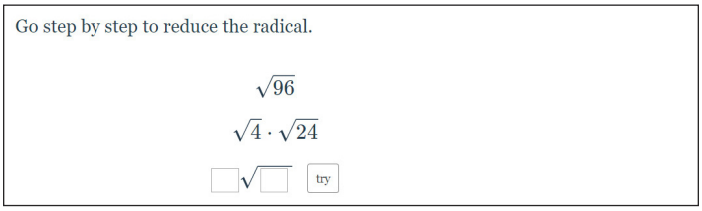


Figure 5: *Second step in guided question progression*

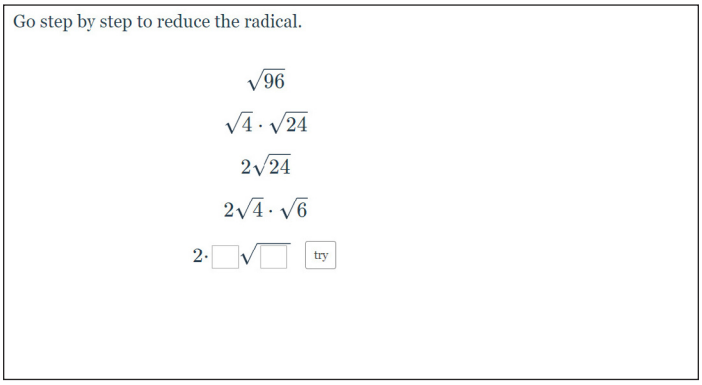


Figure 6: *Third step in guided question progression*

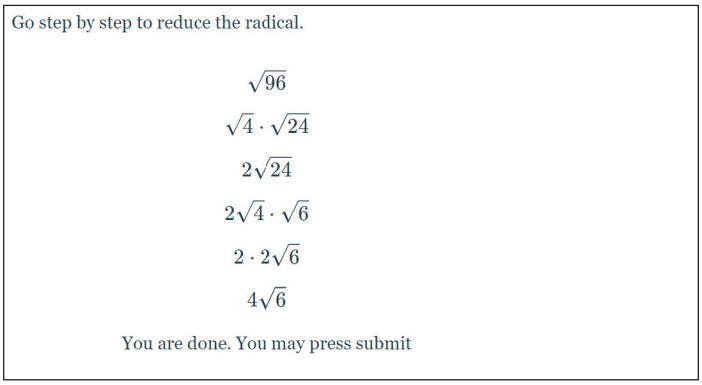


Figure 7: *End result of guided question progression*

Once the student has submitted the answer, he or she receives the feedback shown in Figure 8.

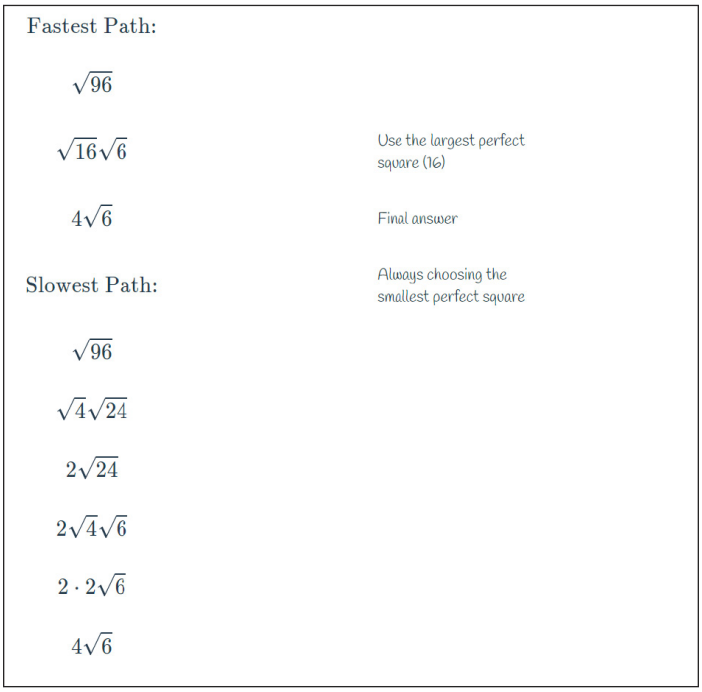


Figure 8: *Sample feedback for a guided question*

If a wrong step is entered at any point during a guided question, a pop-up window will state that there is an error,

with a quick explanation of why it is wrong. Students can try again as many times as they need until they are successful.

If a wrong answer is submitted for a non-guided question, the student will receive feedback, as shown in Figure 9. However, the teacher can choose whether a student can have multiple attempts at a question.

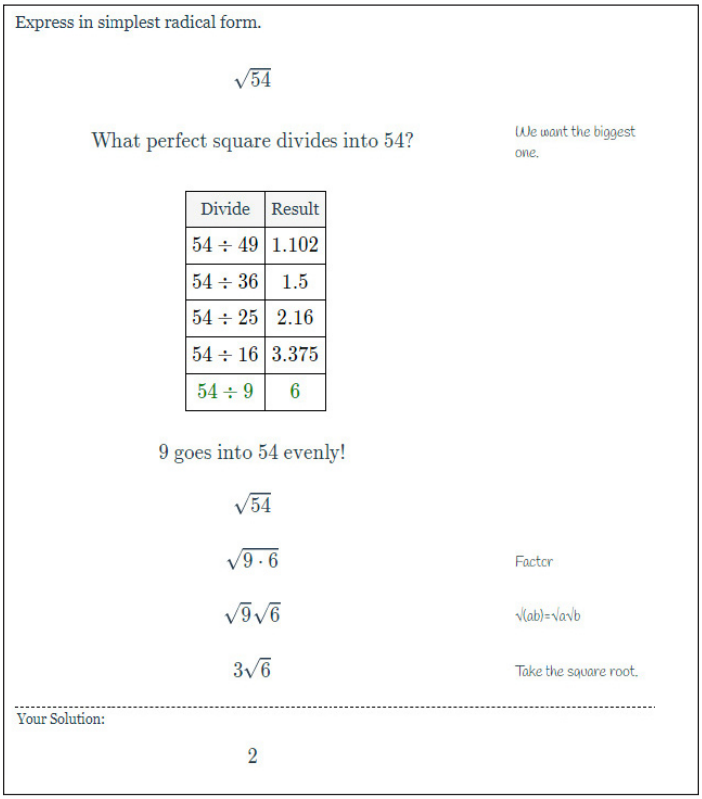


Figure 9: *Sample feedback for a non-guided question*

Once you have chosen the modules for an assignment, you can choose the number of questions that each student must answer correctly, and the penalty for an incorrect answer. If the penalty is set to "1 off," that means that the student will have to correctly answer an additional question in order to meet the required number for that module. You can set the penalty to 0. If you click on "more," as shown in Figure 10, you will see additional options.

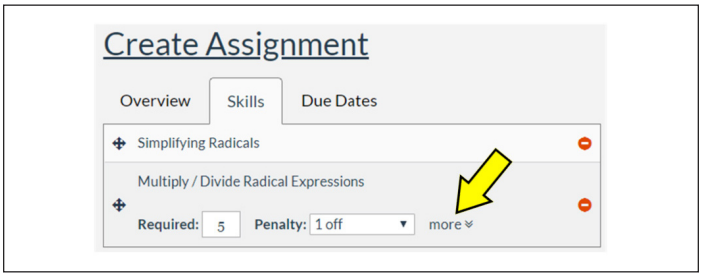


Figure 10: *Locating the "more" option*

You should now see the option of showing an example, which is valuable for students who may be struggling as they work through a new skill, along with the weight of that module and whether multiple attempts will be permitted.

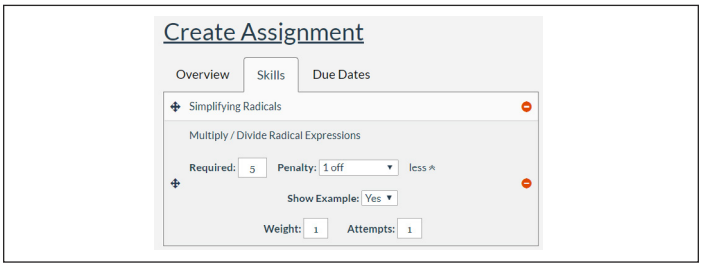


Figure 11: *Additional options*

Another option is to create a mixed-practice assignment, as shown in Figure 12. This allows you to create one assignment with questions from multiple modules.

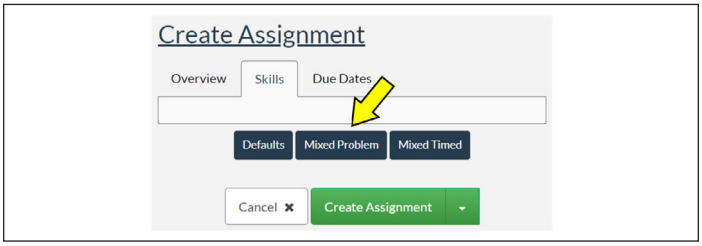


Figure 12: *Creating a mixed-module assignment*

The assignment shown in Figure 13 will feature ten questions from five modules on solving equations. As there is no penalty for incorrect answers, students must answer ten questions correctly to complete the assignment. Questions from two of the modules will appear twice as frequently as the others, as shown on the right of the module name. No examples will be shown, so students will have to show that they are able to distinguish between the different types of problems. Students will be allowed two attempts to get each question right. (If any of the modules had multiple-choice questions, those would only be given one attempt.)

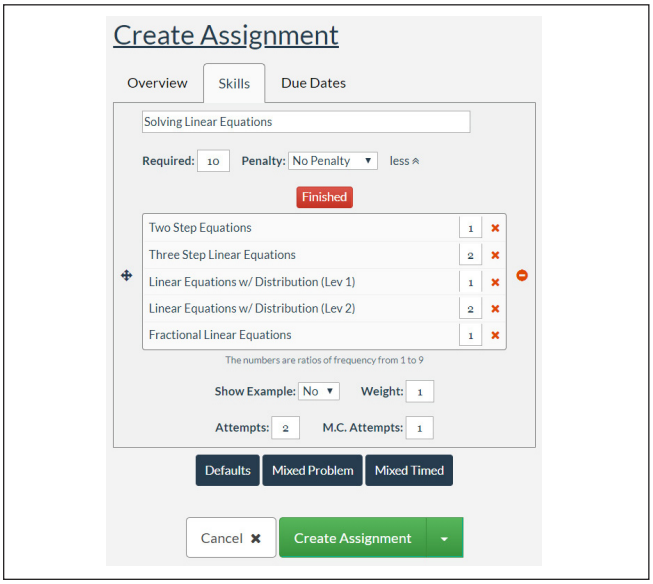


Figure 13: *Defining a multi-module assignment*

When a student works through this assignment, he or she will get a notice if an answer is incorrect (because two attempts are allowed), and will get feedback after each question in the form of a complete solution and either a check mark if the answer was correct or an "x" if the answer was incorrect. There may also be a reason given, along with an incorrect answer such as, "Your answer was not simplified."

The teacher can see the results by clicking on "Student Data," then "View Assignments." Figure 14 shows a sample. If you scroll over each of the check marks/"x"'s, you will see how long the student took to answer that question. Clicking on each will bring up that particular question.

First	Last	Period	Record	Score	Errors	History
Mary	Bourassa	Period 2	10/10	10	0	✔ ✔ ✔ ✔ ✖ ✔ ✔ ✔ ✖ ✔ ✔ ✔ ✖

Figure 14: *Sample feedback*

Also under "Student Data" is the option to see "Problem Logs." These allow the teacher to see all the specific questions the student answered, along with the answers. There are many more features when looking at student data, including downloading the data to Excel.

Another interesting feature of **DeltaMath.com** is a test correction. It is explained on the website as follows.

> *A test correction is when the teacher links each question on a test to a specific skill on Delta Math. On their account, the student inputs which questions they got wrong and Delta Math creates a custom assignment based on the problems they missed. This way the students have to demonstrate proficiency on similar problems to complete the correction. The teacher can optionally set the test correction to automatically give students a certain percentage of the points they lost back, and the student can see their test score gradually rising as they complete the assignment.*

Along with all the excellent and ever-improving features of the website is the high quality of the questions. I will share one more example from calculus in Figure 15.

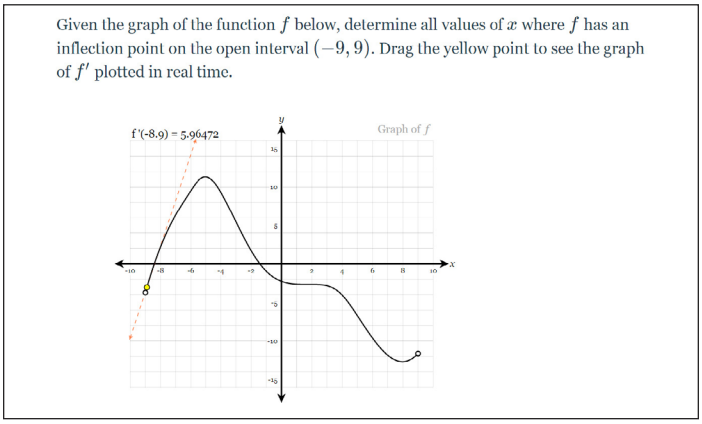


Figure 15: *A sample calculus question*

Figure 16 shows the results of dragging the point. The derivative function is graphed in orange.

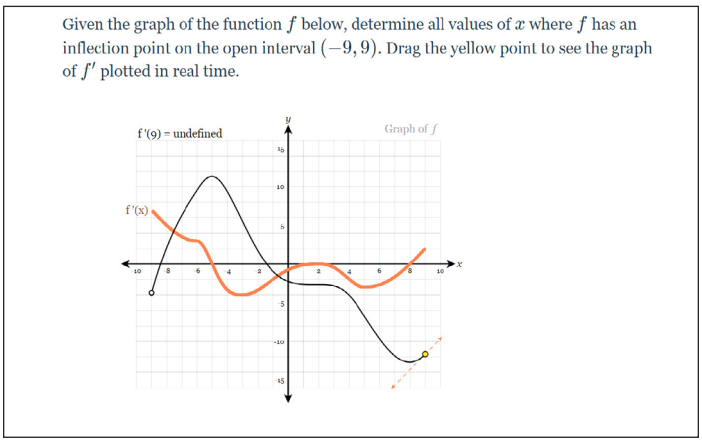


Figure 16: *The graph of a function, along with its derivative*

Figure 17 shows my answer, and Figure 18 is the explanation that appears after the answer is submitted.



Figure 17: *My answer to the question*

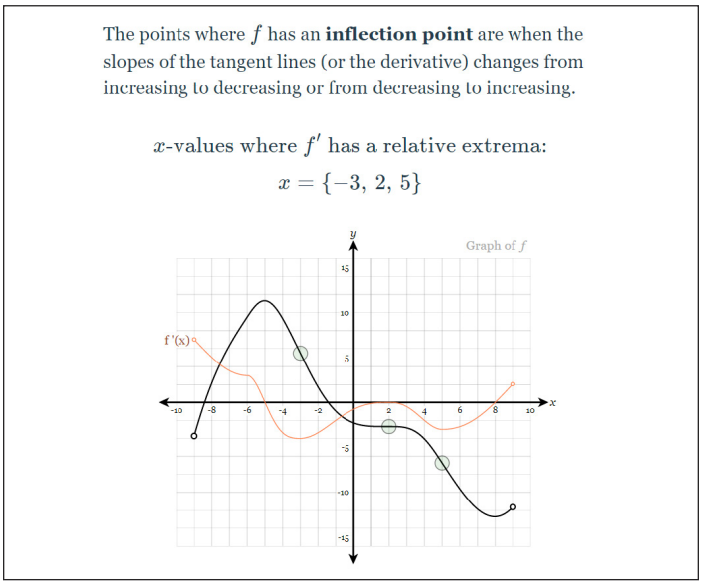
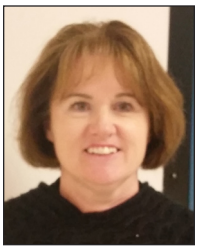


Figure 18: *The question feedback*

To help you learn more, the site has a help page with videos explaining everything from getting started to closing a term. Zach also answers questions in a very timely manner on Twitter @MrDeltaMath. **DeltaMath.com** can be used as a tool to help students fill gaps, as a supplement to help them learn content that they have missed or did not understand, or as an evaluation tool. The ability to tailor the options to your requirements, the vast repertoire of questions that are searchable, and the quality of the questions themselves make this a tool well worth checking out. ▲

▲ OAME/NCTM Report: Around the Province... in NCTM Style

JACQUELINE HILL



Jacqueline Hill is a Grades 9–12 teacher of mathematics, as well as an online instructor for York University. She is a Past President of OAME and OMCA, as well as the recipient of the award for Exceptional Teaching in Secondary Mathematics. She also wrote the "Director's Dialogue" for the Gazette *for a number of years.*

A shout-out around the province! It doesn't matter if you are in the far north or western area of the province, with an NCTM membership you have access to everything on the NCTM website. The NCTM brings mathematically sound materials and ideas right to your devices. Not a member yet? Here are some things to explore on the website (**www.nctm.org**)—even if you live in Ignace, Moonbeam, Fort Severn, or anywhere else in Ontario.

Our first shout-out is to IGNACE and MOONBEAM in the far north of our great province! Under the "Classroom Resources" tab, "Problems of the Week" has grade-relevant problems that will excite the student in all of us.

Our second shout-out is to FORT SEVERN! It is on "Featured" books from NCTM! The first book, *A Pleasure to Measur*e, consists of tasks for teaching measurement in the elementary grades. A second featured book is *Unpacking Fractions* for the middle school classroom. They are tested strategies to build student understanding. The Grades 9–12 book, *Taking Action: Implementing Effective Mathematics Teaching Practices*, has eight effective practices. It contains narrative cases, classroom videos, and real student work.

Our third shout-out is to GOGAMA! Under the "Publications" tab, the NCTM *Illuminations* have brainteasers and success stories. The brainteasers can be used at all levels in the curriculum. "Candies" is one such example:

> A bowl contains 75 candies, identical except for colour. 20 are red, 25 are green, and 30 are brown. Without looking, what is the least number of candies you must pick in order to be absolutely certain that 3 of them are brown? Solution: You could pick all of the red candies, of which there are 20, and all of the green ones, of which there are 25, and still not have a single brown candy in your hand. That's already 45 candies. You'll then need to pick 3 brown ones, giving you a total of 48. This is the number you must pick to be absolutely certain that there are 3 brown candies. However, it's certainly possible—even likely—that you would have 3 brown candies if you picked fewer. You just can't be absolutely certain that you'll have 3 brown ones.

The success stories give personal perspectives on how students can be actively engaged in their mathematics.

Our final shout-out is to WINDSOR! If you check the "Research Advocacy" tab on the website, you will be able to explore research briefs and clips. There are articles on topics such as "Video Games in the Math Classroom," to "Using Data," to "Benefits of Discussion," to "Algebraic Thinking in Arithmetic."

NCTM important dates to note:

- April 1–3, 2019 – San Diego, California – NCTM Research Conference
- April 3–6, 2019 – San Diego, California – NCTM Annual Conference

Yours in mathematical fun,

Jacqueline ▲

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The Art of Math

LAURA BRISCOE
JENI VAN KESTEREN



Innovation, project-based learning, and global citizenship are at the heart of Laura Briscoe's teaching philosophy. She stimulates imagination and empowers people to make relevant connections through the arts, with the world around them. Laura is a visual arts department head, global competencies facilitator, and a member of the Thames Valley District School Board Innovative Team and the School Improvement Team at Oakridge Secondary School.



Experiential learning and improving the understanding of mathematics are at the core of Jeni Van Kesteren's approach to teaching. She teaches mathematical concepts, using innovative approaches to enhance learning and build confidence. She has been recognized as the Team Leader in a provincial grant through the Teacher Learning and Leadership Program (TLLP), used to explore the impact of collaborative learning on student success in mathematics. Jeni is a math coach, student achievement leader, student success teacher, and a member of the Thames Valley District School Board Innovative Team at Oakridge Secondary School.

How can we build student confidence in math, and provide creative flexibility to their learning approaches, while also developing a deeper purpose in their motivation? We believe that one possible answer lies in creating opportunities for students to explore interdisciplinary connections.

The Thames Valley District School Board (TVDSB) has an innovative team with a variety of stakeholders from senior administration to classroom teachers. This team asked teachers, "If you could have a group of secondary students for more than one period a day, what would your day look like? What credits would your students earn?"

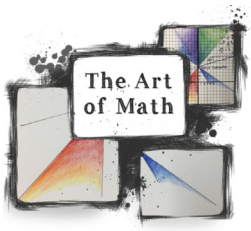


Figure 1

We came together to respond to these questions. With the support of our principal at the school level, and the team at the board level, we launched the Art of Math program in September 2017 at Oakridge Secondary School in London, Ontario.

The Art of Math is a bundled course that falls under the umbrella of what the TVDSB calls "Sch(school)ool." Students in the program earn their Grade 9 Visual Arts credit simultaneously with either their Grade 9 Academic or their Grade 9 Applied Math credit. Providing both pathways provides flexibility for parents/guardians who may not know which math option is best for their child going into high school. The structure of the program makes it possible to switch pathways early on, if needed.

The program provides students the opportunity to learn by doing, creating, and inventing. Through exploration and experimentation with traditional and non-traditional materials, students gain a deeper understanding of mathematical concepts.

Students are together for the entire morning; they do not move from math to art. The curricula are blended and intertwined to create deeper and more deliberate connections. Projects allow students to demonstrate learning expectations from both Art and Math curricula and are developed by both subject teachers to reinforce the importance of subjects aligning. Rubrics for assessment are created with student input to ensure that students are clear about expectations.

For example, think about what you see in the sample work in Figure 2. How do you interpret it?



Figure 2: *Sample work*

A math teacher can take an artwork to introduce shapes with connections to perimeter and area or focus on lines, curves, and equations. Students might be asked to measure the shapes directly, or import an image of the artwork into Desmos to describe the different functions by determining their equations. Using the same piece, an art teacher could discuss the elements and principles of design explored to express emotions and individuality.

Sometimes the art project and math components are connected directly, as in the example above. Other times, the art is used as a way to analyze or visualize a math concept after it is completed.

For example, students worked with soapstone to create their own sculptures. Students then determined the mass

and volume of their sculpture (Figure 3) to add to a class data set. Points were plotted, and a line of best fit was used to estimate the average density of the stone used. This connected to our unit in math on linear relations.

We team-taught lessons and intertwined the disciplines within instruction. For the pilot year, we had 40 students with us for the morning. Projects drive the learning process while incorporating art and math. Sometimes students are split into smaller groups for math or art instruction, while the rest of the class works on a interdisciplinary project that combines math and art together. Our goal is to take a creative, hands-on approach to deepening student understanding of both visual art and mathematical concepts. Projects are flexible enough to challenge the learning of each student. For some, this means pushing their understanding of mathematical concepts and asking questions beyond curriculum expectations.



Figure 3: *The mass and volume of student soapstone sculptures were collected. Data were plotted on a Cartesian plane, and the line of best fit was used to describe the average density of the stone we used.*

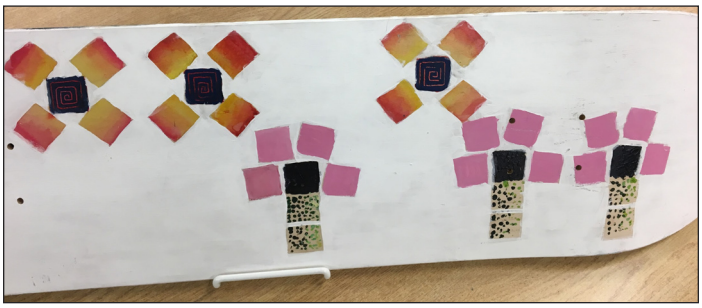


Figure 4: *A student pattern showing three "position numbers." Tiles are filled in using different painting techniques like pointilism, sgraffito, wash, dry-brush, etc.*



Figure 5: *A decreasing linear pattern with corresponding graph and table of values. Students used their own creative freedom to fill in the background. On display at the Art of Math show in January 2018.*

For example, when we were first introducing the concepts of linear growing patterns in math, students were learning different painting techniques in art. We followed some of the introductory lessons in the book *From Patterns to Algebra*, and students created different patterns, using the tiles, and copied them into their sketchbooks "blank." They then used different painting techniques to fill them in. This led into our Skatepark unit, where students were given a skateboard. On one side, they had a linear pattern (Figure 4) filled in, using different painting techniques (Figure 5). They also included the corresponding graph and table of values. On the other side, they completed a landscape painting (Figure 6).



Figure 6: *On the other side, students completed a landscape painting of their choice.*

When we started linear patterns, the focus was on linear growing patterns. Some students used their skateboard project as a way to investigate decreasing linear patterns before we covered this in class.

During this unit, we also revisited the Pythagorean theorem and combined this knowledge with our understanding of slope to design skatepark ramps for the art show. Students were put into groups to design, build, and paint their own ramp. The ramps needed to have an appropriate slope that was researched by the group. Cut lists were made so that materials would be the correct length, and then the ramps were painted with a different art history era. Other classes came to complete an art history lesson, using our ramps (Figure 7).

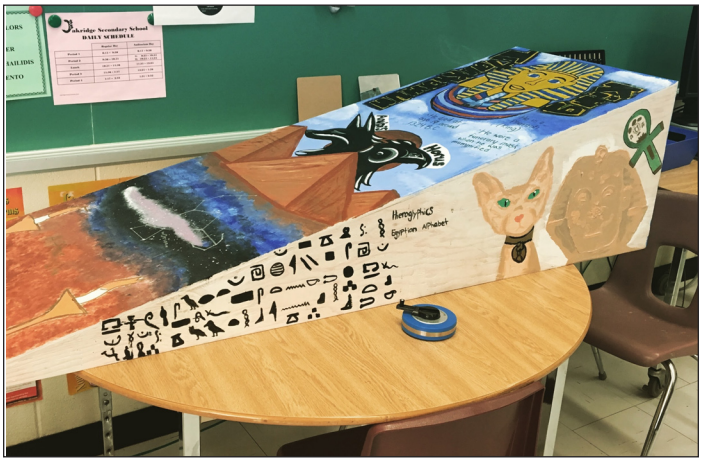


Figure 7: *An example of the ancient Egypt ramp*

Our units were defined by the art project on which students were working. While students worked on their unit projects, they were also given instruction on specific art and math concepts. Figure 8 is an example of a lesson where students were reviewing colour theory and working with colour mixing. Students were also asked to determine the equations of their lines by selecting two points.

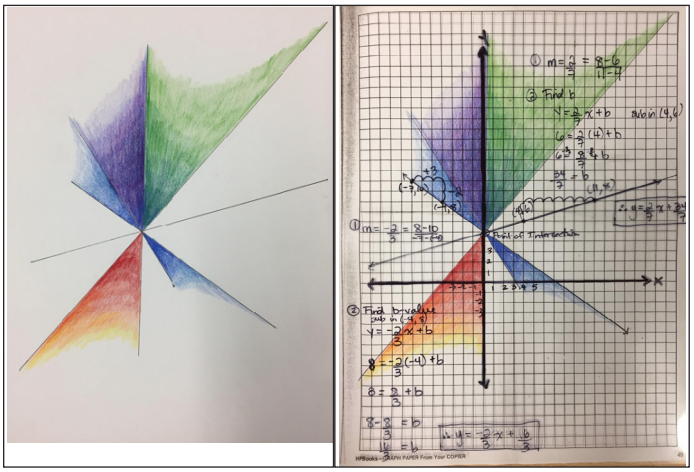


Figure 8: *Left is a colourful lines drawing, where students reviewed colour mixing and shading techniques. Right: Students used grid paper photocopied onto transparancies to determine the equations of their lines.*

When we reached concepts in math that were more algebraic in nature, we would break out into smaller groups as needed. We had an art project connection to each of the overall expectations in both the Grade 9 Applied and Academic courses. In addition, students had math homework and written assessments to ensure that students would be prepared for EQAO and their next math courses.

At first, we weren't sure how we would link solving linear equations and working with exponents to an art project. These topics ended up being connected to our unit on printmaking (Figures 9 and 10). Students practised with brayer rollers and ink to create a linear equation with one variable. Students then worked to create their own linoleum prints and used these as the base in an expression involving powers. Because students were creating their own questions, some ended up working with negative exponents and began to ask how to simplify their expressions when it didn't seem to follow the "rules." We also found that students were creating questions that were more challenging than what they were willing to tackle in their homework.

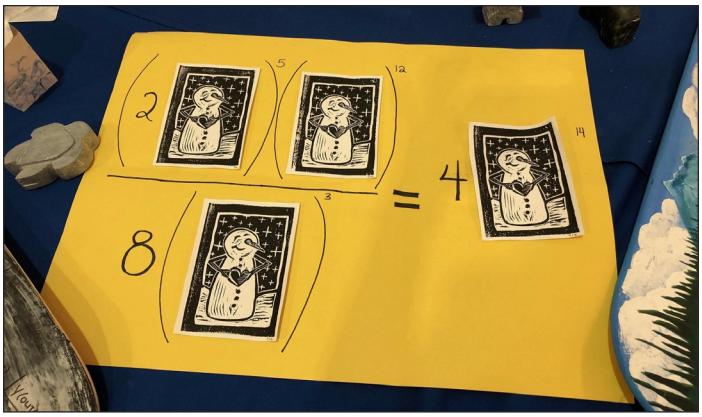


Figure 9: *Students continue working with printmaking, and use their art to demonstrate their understanding of simplifying expressions with powers of the same base.*

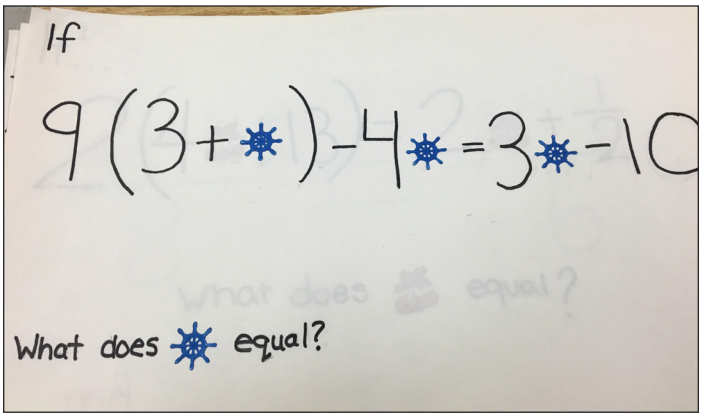


Figure 10: *Students are introduced to the concept of printmaking, while studying linear equations with one variable.*

We are often asked how we got started with the Art of Math program. Prior to considering this program, we joined forces for a cross-curricular project joining Jeni's Grade 10 Academic math students and Laura's Grade 11 Art students. The experience made us realize just how much students enjoyed blending the two disciplines. We discovered the possibility of team-teaching concepts together in a more intentional, long-range manner.

For Jeni, the motivation to make a change came from a desire to try to help students see math in a different way, and to provide students the opportunity to connect math with the world around them. In her years teaching visual arts, Laura found that there were many hard-working, dedicated honour-roll students, who would express how they dreaded going to math class or how they were anxious about an upcoming test. These students were really lacking confidence. She also wanted art students to gain an appreciation of how mathematical concepts can strengthen and support their innovative approaches to develop their ideas and designs.

Early on in the semester, the class went on a field trip to an art gallery. A guide prompted students to describe what they saw in a piece of art. The first student to speak began by describing the ratio of land to sky; another student commented on the use of lines and the different shapes that were combined together. While these were brief comments, for Jeni, they were a highlight of the trip. Students were making connections to math in a context where she would have fully expected them to be thinking only about art.

Artist critiques became opportunities for students to share and learn from one another about both art and math. Students were coming to class prepared to talk about math through something that they are comfortable with, something they created, their art of math projects. Completing their projects gave them confidence in their understanding of concepts and put them in a better mental state.

This ability to make connections continued through the semester. Students began using math as a way of communicating and analyzing what they were creating in art. Or they began to use their art to express mathematical concepts. As a result, Jeni found a decrease in the level of stress that she would usually find in a Grade 9 math course. In fact, some students expressed that they couldn't possibly be learning math because they didn't feel "stressed." One student described the course as "one of those classes that gets you up in the morning, and you're excited to go because you never really know what's going to happen… and it's really, really fun. You learn different ways of doing what you thought would be hard."

A memorable moment for Laura occurred when she was speaking with a few students after the first day of EQAO testing. One student expressed that he must have "actually learned some math this semester. I don't know where the answers were coming from. I just knew what to write!" It became clear that the learning was happening so naturally, it didn't feel like work.

By providing students with the opportunity to make connections and see things differently, it empowers their learning. Connecting subjects together allows students to create with purpose.

When students are motivated and empowered, they accomplish amazing things!

To learn more about our story, follow us on Twitter @ArtOfMathEd, YouTube search: Art of Math TedX, or subscribe to our YouTube channel at **https://bit.ly/2MMVh1k**. ▲

Reminder: Don't forget the photo contest!

See Editor column in the September *Gazette* for details.

▲ Fields Institute MathEd Forum Report

ANGELICA MENDAGLIO



Angelica Mendaglio is an instructional designer at Vretta Inc. in Toronto, Ontario, where she helps to create interactive digital mathematics lessons and activities for middle school students.

The Fields MathEd Forum kick-started another year of presentations and discussions on current research and practices in mathematics education at the end of September. The first meeting of the new school year was terrifically engaging, and centred around the theme of *Meaningful, Emotional, Humanistic, Mathematical Experiences.*

This phrase, shortened to MEHMEs (pronounced "memes"), was coined by Burke Brown. The September meeting of the Forum was, in a sense, a tribute to Burke, who was a longtime attendee of the Forum and a staunch advocate of the importance of probability and statistics for everyone. Burke believed that MEHMEs were of critical importance to developing a love of mathematics, and that we should be working to incorporate them into our classrooms.

The first talk of the Forum, given by Charles Anifowose from Vretta, focused on MEHMEs and why Burke thought they were of critical importance in math education. Charles also spoke about Burke's other ideas of ways to bring discussions on mathematics into the popular discourse, including discussing issues of social inequality through mathematical modelling.

The second talk at the September meeting was given by Parker Glynn-Adey, a teacher at the University of Toronto. Parker expanded on another of Burke's ideas called mathematical "eipiphinies." The odd spelling of this word is because it is combining the elements of the famous Euler's Equation:

$$e^{i\pi} + 1 = 0$$

This equation is held by many mathematicians to be among the most beautiful equations in math because of how it combines the imaginary and transcendental numbers e, i, and π, and brings them into the realm of the more ordinary numbers 1 and 0. This is the mathematical beauty that Burke wanted more people to see, and he used the word "eipiphiny" to describe these experiences. Parker had also designed a card game that used the same symbols present

in Euler's Equation, which the Forum attendees had a chance to play and then to discuss ways that the game could be used in a classroom to spark mathematical discussions.

After lunch, Forum attendees heard form Ray Hall, most well known for his Instagram account @physicsfun (**www.instagram.com/physicsfun/**), where he posts simple demonstration videos of objects that make math and physics visible. Though very short and without any words, Ray's videos are extremely interesting, and can make well-known properties visible, or exhibit seemingly impossible behaviour that can leave students and adults alike puzzled. Ray shared some examples of these fascinating "physics toys," and gave some insight into how their mysterious behaviours are still governed by the laws of physics. He also gave a behind-the-scenes look at how he creates his videos, and how his simple demonstrations have sparked the imagination of over 1 million people.

The final talk of the day was given by Philip Poissant, who created what is known as Galton Boards in different form factors so they could be easily shared. A Galton Board, similar to the Plinko game on *The Price is Right*, can be used to demonstrate concepts in probability and statistics, such as the normal curve. Philip helped to design a Galton Board that would fit on a desktop, as well as a smaller version that is the size of a coin.

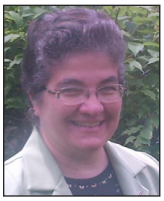


Philip Poissant spoke about creating a Galton Board the size of a coin.

This year of Math Ed Forum meetings is off to a great start, with many more engaging meetings to come. The Forum convenes at the Fields Institute in downtown Toronto on the last Saturday each month from September to November, and from January to April. If you are not able to travel to Toronto to join in person, you can also watch the presentations remotely through the Fields website.

Dates for the upcoming year can be found at **www.fields.utoronto.ca/activities/18-19/mathed-forum**.

▲ Provincial Digital Learning Resources – What's New? Number Chart – Focus on Number Sense Fundamentals







AGNES GRAFTON

ROSS ISENEGGER

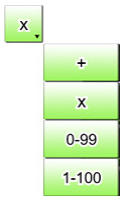
MARKUS WOLSKI

*Agnes (Brant Haldimand Norfolk Catholic District School Board), Ross, and Markus (Near North District School Board) are Project Leads, working on the development of the digital resources found at **mathies.ca**.*

The mathies interactive Number Chart tool includes two number charts (0–99 and 1–100), an addition chart (to 12 + 12), and a multiplication chart (to 12 x 12). These charts are ideal tools to support students as they develop a variety of fundamental math skills (see the Ontario Ministry of Education's (2018a) recent *Focusing on the Fundamentals of Math* document). Specifically, number charts provide opportunities for students to work with numbers in a variety of ways, including counting, comparing, and composing and decomposing numbers. They can be used to support the development of a variety of operation strategies and mental math skills, as well as the recognition of number properties (e.g., identity, commutative). The operation charts can be used to support students in mastering math facts by providing immediate feedback during practice.

The tool opens, showing the multiplication chart. We invite you to go to **mathies.ca**, open the Number Chart tool, and play along as you read this article.





Select the type of chart you would like to use from the Chart Type Options.

Number Charts

0-99 or 1-100?

Two different number charts are available. The charts are similar in many ways. Smaller values appear at the top of the chart and increase as you move right and down the chart. Moving to the right by one card is equivalent to adding 1, while moving left by one card is equivalent to subtracting 1. Moving up or down by one row is equivalent to subtracting or adding 10. So, why have two charts? The 1–100 chart might be preferred for counting activities, since 0 is not a counting number, or when dealing with questions that involve 100, such as 86 + 14. On the other hand, the 0–99 chart is needed when dealing with questions such as 16 – 16. This chart supports the idea that zero is a number in its own right, not just a digit in numbers such as 50. It is also interesting to note that on the 0–99 chart, all the numbers in each decade appear in the same row (e.g., 20, 21, 22,... 29). Values in any given row all contain the same number of groups of ten and then some number of ones. Seeing all the values from the same decade in the same row helps to reinforce place-value concepts.

Counting and Numeral Recognition

The number chart can be used to support young students as they make connections between oral counting and written numerals.



Hide all the values in the chart by selecting *Hide All* from the Flip Options. Notice there are also options to *Show All* or *Flip All*. These options affect the entire chart.

Flip a few numbers in the first row of the 1–100 chart by clicking on the individual cards. Ask students to determine the missing numbers in the row.



Flip each card as the missing number is said so students can see the numeral that they have just named. Use additional rows in the chart to support students in learning the corresponding numerals as they learn to count to 20, to 50, and so on.

The number chart can also be used to reinforce skip-counting concepts. Have students mark each number as they skip-count by various values. This next image shows skip counting by 2 and uses the circle stamp to highlight each number in the sequence. Encourage students to look for patterns related to different skip-counting values and eventually name the numbers in the sequence as multiples.



Eventually, encourage students to start skip-counting at numbers other than the typical 1, 2, 5, or 10 so they can see the patterns geometrically shift on the chart.

Tech Tip:

Using the Annotation Tools

- Click the pencil, eraser, or circle button to enable annotation mode.
- A grey mask with a yellow border appears to indicate the tool is in annotation mode.



- To exit annotation mode and resume interacting with the number chart, click the "x" in the top right corner of the mask. Alternatively, click again on the highlighted button to exit annotation mode.

Draw freehand by clicking and dragging.

Erase freehand drawings and circles by clicking and dragging the eraser around the number chart.

Either show or hide a circle on cards by:clicking individual cardsclicking and dragging over multiple cards.

Patterning

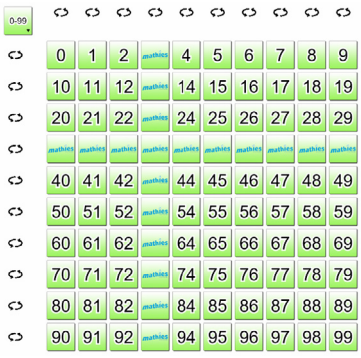
Number charts can be used to create a range of patterns from simple AB patterns to growing or shrinking patterns. Students can use the charts to help identify, extend, and

create patterns. Working with patterns provides students an opportunity to develop and practise their mental math skills, leading to proficiency with operations.

Here are some patterning questions to try with your students:

- Hide all the cards. Flip two cards, skip one, flip two, skip one. Ask students to extend this repeating pattern. Can they predict what values will be on the hidden cards before flipping them? Once the pattern has been extended for the entire chart, ask students to think about the values on the cards that remain hidden. Students can then use the *Flip All* option to flip all the cards in the chart. What pattern do they notice in the cards that are now showing? Have them try a different repeating pattern of their own.
- Give students pattern challenges to mark on their boards. Have them determine the pattern rule and the next number(s) in the pattern, e.g., 19, 16, 13, 10… or 21, 28, 35, 42….
- Consider patterns that involve moving down/up a row and one or two squares to the right/left (i.e., adding/subtracting 11 or 12). Help the students link this to their mental math skills.
- Using the 0–99 chart, start with all the values face up. Flip all the numbers with a 3 as the ones or tens digit and look for a pattern.

  Repeat with 7 and compare. Try other numbers. What happens? Why?



Tech Tip:

Click on the flip button to the left of a row or at the top of a column to flip the entire row or column.

- Cover all the numbers in the 0–99 chart that satisfy a certain rule, and look for patterns. For example, cover all numbers with:
  - digits that add to 9, or
  - a tens digit that is larger than the ones digit, or
  - a tens digits that is smaller than the ones digit.

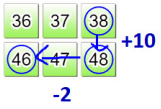
Operations

Number charts can be used to help students develop an understanding of a variety of mental math strategies such as decomposition or compensation.

- Compute sums such as 23 + 36 by decomposing addends into friendly numbers.
  - Example: Start with 23. Decompose 36 into 30 + 6. Add 23 + 30 by moving down three rows of ten, then add on 6 more by moving to the right six cards.



- Compute sums such as 38 + 8 by compensating (adding and then subtracting the same amounts).
  - Example: Think of 38 + 8
    as 38 + (8 + 2) - 2
    or 38 + 10 - 2.



Number charts can also be used to demonstrate various number properties. For example, demonstrate the associative property of addition by determining (7 + 6) + 4 and showing the result is the same as 7 + (6 + 4).

Students can think about the relationship between the multiples of a number on the 1–100 chart and where they appear on the multiplication chart (e.g., the fifth multiple of 3 occurs in the fifth row and third column of the multiplication chart).

Problem Solving

A quick Google search provides a plethora of suggested uses for a Hundred Chart. Denise Gaskins's (2018) Let's Play Math blog offers *30+ Things to Do with a Hundred Chart*, which suggests a variety of games and activities. The following activity provides students an opportunity to practise their addition skills as they look for patterns and make predictions.

- A *Cross pattern* is a square plus the four squares directly up, down, left, and right from it.
- An "X" *pattern* is a square plus the four touching it diagonally.





- Choose any square that is not on an edge of the hundred board (use either hundreds chart). Find its Cross and "X" patterns and add up their sums. Can you explain why they add up to the same number? Can you find any other patterns that work that way? Can you figure out how to predict the Cross or "X" pattern sum for any number?

The *Guess the Number* resource (Kawas, 1994) has sets of clues that include the use of money and measurement, as well as basic number facts and properties. Teachers and students can create their own sets of clues. For example:

- The number is greater than 5 x 7.
  The number is less than 120 – 30.
  The number is not a multiple of 10.
  The sum of the numbers' digits is 11.
  I say this number when I skip-count by 5.
  What is the number?

Have students start with a number chart showing all the values. As they read each clue, have them flip over the cards that have been excluded (similar to *Guess Who*, the popular children's game). Students continue until only one number remains.

Operation Charts

The addition and multiplication charts can be used to support students as they develop proficiency with operations. Using the charts helps them to visualize and understand a variety of strategies for adding and multiplying.

Addition

Some suggested activities include the following:

- The EduGAINs site has a collection of Whole Number supports including *Developing Proficiency with Addition and Subtraction* (Ontario Ministry of Education, 2018b), which identifies strategies, provides examples, and shows visual summaries based on a number chart (at **EduGAINS.ca**, search the site for "Proficiency Addition").

  Explicitly discuss various addition strategies. Indicate the addition facts where these strategies apply on the addition chart. Some addition facts can be determined using more than one strategy. Look for visual patterns for each strategy.
  - One-more-than and two-more-than facts (e.g., 1 + 5 and 7 + 2)
  - Doubles (e.g., 2 + 2 and 8 + 8)
  - Near doubles (e.g., 6 + 7 and 5 + 4)

    This strategy involves sums where one addend is one more than the other.

    The strategy is to double the smaller number and add one more or to double the larger number and take away one.
  - Anchor-of-Ten Facts (e.g., 1 + 9 and 8 + 2)
  - Make-Ten Facts (e.g., 3 + 8 and 9 + 5)

    This strategy involves adding onto either 8 or 9 to make 10, then adding on the rest.

    Example: For 9 + 5, start with 9 and add on 1 to make 10, then add on 4 more to get 14.
- How many different ways can you find two numbers that add to 12? Mark each sum on the addition chart. Repeat using a different sum. What patterns do you notice?
- Demonstrate the commutative property of addition. Example: 5 + 8 gives the same result as 8 + 5.
- Show that the sum of two odd numbers is always even. What about the sum of two even numbers? The sum of an even and an odd number? Convince a friend that these generalizations are true.
- Practise addition facts.
  - Start by hiding all the cards in the chart. Choose an addition fact. Determine the sum.
  - Flip the corresponding card to check.

Multiplication

Some suggested activities include the following:

- Determine missing factors.
  - Start by hiding all the numbers in the chart.
  - Focus on one factor by considering one row or one column at a time.
  - Expose some of the numbers (products) and determine each of the missing products.
  - Flip the card to check.
  - Think about the strategies used. Discuss.
    Example: Consider products where one factor is 6. The products 6, 12, 30, and 60 are showing. Determine the remaining multiples of 6.



- Hidden Multiplication Facts (WINS, 2018)
  - Start by hiding all the numbers in the chart.
  - Randomly select a digit from 0 to 9.
  - Flip over all product cards that contain a digit in the product that matches the randomly selected digit.
  - Discuss: How do you know you found all the products containing the indicated digit? What strategies did you use?
- Demonstrate the commutative property of multiplication.
  - Mark pairs of questions such as: 3 x 4 and 4 x 3, 1 x 10 and 10 x 1, 9 x 6 and 6 x 9.
  - Mark several more pairs of questions on the chart. What pattern do you notice?
- Demonstrate the associative property of multiplication. Example: Determine (3 x 2) x 5 and show the result is the same as 3 x (2 x 5).
- Practise multiplication facts.
  - Start by hiding all the cards in the chart.
  - Choose a multiplication fact.
  - Determine the product.
  - Flip the corresponding card to check.

The EduGAINS site also has a Whole Number support for multiplication (Ontario Ministry of Education, 2018c).

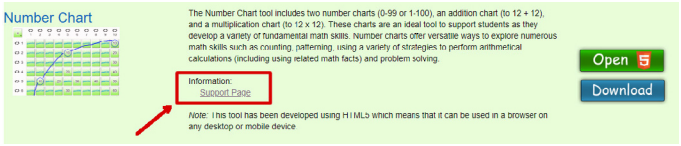
"Understanding how numbers work is foundational to all aspects of mathematics" (Ontario Ministry of Education, 2018a). The interactive mathies Number Chart tool provides a platform for a surprisingly rich set of activities. The activities suggested here provide opportunities for students to recognize and understand various number properties and relationships as well as to develop and practise strategies. As the document states, "Fluency with basic math facts is fostered through instruction that highlights strategies for remembering facts, focuses on making sense, and integrates math-fact learning into other aspects of math learning" (Ontario Ministry of Education, 2018a). Students can use the Number Chart to visually make sense of facts and strategies.

We would like to thank Kathleen Corrigan and Kyla Kadlec for their contributions to the Number Chart Support Page (2018), on which this article was based.

Support Pages – New Home

You might have recently tried to access the mathies support information from inside the Information dialog of one of the other tools. Unfortunately, you would have been greeted with a Farewell message from **wikispaces.com**, since they have discontinued their free service.

Have no fear, the wealth of material previously housed on Wikispaces has been migrated to **support.mathies.ca**. Links from the tools to the support pages will be updated as each tool is revised. In the meantime, you might want to use the Support Page link found on the "mathies Learning Tools" (2018) page.

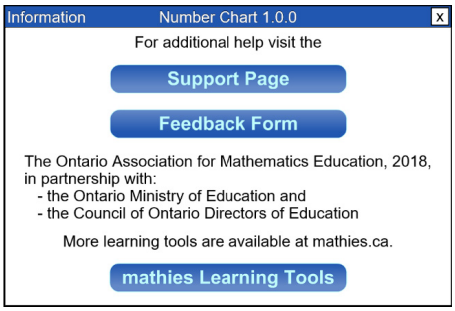


The link to the Support Page in the Number Chart's information dialog does point to the correct location (see below).

Also, notice the shiny new HTM5 button on mathies. Number Chart is the first mathies tool to be developed in HTML5, which means it can be used in a browser on any desktop or mobile device. We expect that by the time this goes to print, there will be an "Ouvrir" button to access a French version of the tool.

Feedback and Future Requests

Please feel free to send us your feedback about any mathies tool, using the Feedback Form button inside the Information Dialog, accessed from the button.



You can also send your comments to **WhatsNew@oame.on.ca**. You can share your experiences on Twitter, using the hashtag #ONmathies, and follow or message us at @ONmathies. There is an increasing set of interesting posts of student and teacher work on Twitter.

To be among the first to find out about the latest digital tool developments, sign up for our email list at **http://mathclips.ca/WhatsNewEmailList.html**.

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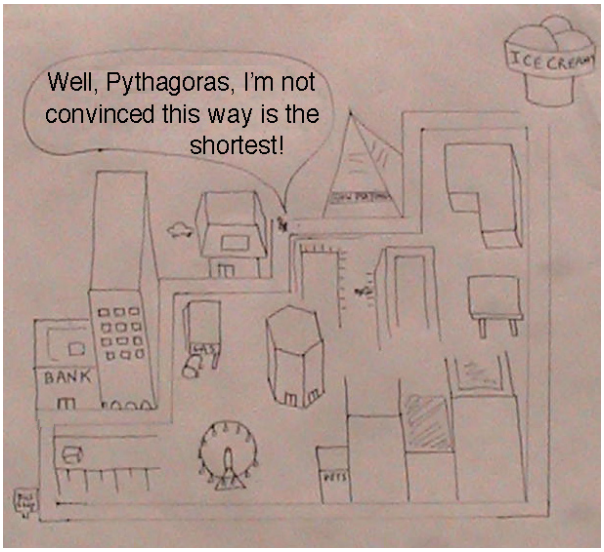
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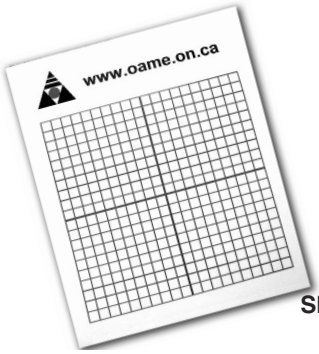
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▲ In the Middle: Speed Climbing and Thinking Routines



CARLY ZINIUK

Carly Ziniuk teaches Grade 9 Mathematics, Grade 12 Data Management, and Advanced Placement Statistics at the Bishop Strachan School in Toronto, Ontario, Canada. She is very active in adopting real-life data to engage her students in solving problems.

Last spring, a student showed me a movie entitled *Two Nineteen Forty Four*, which she thought I would find interesting. Although I had very little knowledge of competitive rock climbing, within the first minute, I was captivated. The 7-minute video showed the record-setting climb of Brad Gobright and Jim Reynolds up El Capitan, a well-known ascent in Yosemite Valley. The 2-hour, 19-minute, 44-second ascent (hence the title) in October 2017 was captured using time-lapse filming.

Their route up El Capitan, known as the Nose, was once considered impossible to climb. It has more recently become the most popular route for competitions to log the fastest time in both free climbing (with minimal aids) and speed climbing (either solo or pair). *National Geographic* (June 6, 2018) indicated that from October 2017 to June 2018, the record for climbing the Nose was broken four more times, including breaking the two-hour mark. The documentary *Free Solo*, which played at TIFF 2018, portrayed Alex Honnold's attempts to free-climb El Capitan over 3000 near-vertical feet (900 metres).

This video inspired me to consider some of the ways to promote richer thinking in my classroom, especially with broad mathematical content like the El Capitan speed-climbing records. I spent a few hours this summer learning more about speed and free climbing, and finding related data. I pondered a lot of areas where I could connect this to my classroom, including linearity, slope, rates, and ratios. With this problem starting the school year, I have been using Project Zero's Visible Thinking Routines. I have been using these routines with my students to make their thinking more readily communicated. This also has helped me assess students, using observations and conversations. Project Zero presents more than 30 different routines to motivate students to show their understanding, make meaningful

connections between math and everyday life, and to reflect more explicitly on their thinking.

If you haven't used the Visible Thinking Routines (**www.visiblethinkingpz.org/**), these are a series of strategies that can be used throughout the year in middle school classrooms to extend and deepen student thinking. These are the routines I use most often and find most effective for open-ended problems or rich situations like the speed-climbing experiences of El Capitan. It is handy to have each routine copied onto coloured cue cards to help remind groups of the routine instructions.

Connect-Extend-Challenge

CONNECT:	How are the ideas and information CONNECTED to what you already knew?
EXTEND:	What new ideas did you get that EXTENDED or pushed your thinking in new directions?
CHALLENGE:	What is still CHALLENGING or confusing for you to get your mind around? What questions, wonderings, or puzzles do you now have?

I gave the students this table of record times for El Capitan's The Nose Speed Climbing from Wikipedia, and gave the instructions from Project Zero for Connect-Extend-Challenge.

I like to use this thinking routine with groups of four students and give them relatively short 1- to 2-minute time constraints per stage (Connect-Extend-Challenge). When I gave this to my Grade 9 students, they were quick to "connect" this to a graph to see how fast the times had changed in the past 40 years. They then "extended" to look at how to represent the last few points carefully, despite the scale change. Other students were looking at the change per year, representing it as a rate, but were not sure how to deal with the most recent times. Many students were "challenged" by the 1984 time as a record, but then also whether they should just break the records into parts and look at them as separate clumps where the records seemed to jump. I gave them a bit of a peek into piece-wise functions when I put these data into technology. Many of the students were also "challenged" by a decreasing graph in general, and found that confusing, despite understanding the problem.

Date	Climbers	Time (hh:mm:ss)
2018-6-6	Tommy Caldwell, Alex Honnold	1:58:07
2018-6-4	Tommy Caldwell, Alex Honnold	2:01:50
2018-5-30	Tommy Caldwell, Alex Honnold	2:10:15
2017-10-21	Jim Reynolds, Brad Gobright	2:19:44
2012-6-17	Hans Florine, Alex Honnold	2:23:46
2010-11-6	Dean Potter, Sean Leary	2:36:45
2008-10-12	Hans Florine, Yuji Hirayama	2:37:05
2008-7-2	Hans Florine, Yuji Hirayama	2:43:33
2007-10-8	Alexander and Thomas Huber	2:45:45
2007-10-4	Alexander and Thomas Huber	2:48:30
2002-9-29	Hans Florine, Yuji Hirayama	2:48:55
2001-11	Dean Potter, Timmy O'Neill	3:24:20
2001-10	Hans Florine, Jim Herson	3:57:27
2001-10	Dean Potter, Timmy O'Neill	3:59:35
1992	Hans Florine, Peter Croft	4:22
1991	Peter Croft, Dave Schultz	4:48
1991	Hans Florine, Andres Puhvel	6:01
1990	Peter Croft, Dave Schultz	6:40
1990	Hans Florine, Steve Schneider	8:06
1986	John Bachar, Peter Croft	10:05
1984	Duncan Critchey, Romain Vogler	9:30
1975	Jim Bridwell, John Long, Billy Westbay	17:45

This thinking routine works as a starter between topics to connect one idea to a new concept. I have used it with quadratic data when the students are used to dealing with first differences and linear models. I have also used it to have students create a rule for negative exponents by keeping the powers with a given base and decreasing the integer exponents. Students have "challenged" the fractional/decimal exponents as the result of this thinking routine as well.

Think-Puzzle-Explore

Similar to the routine above, I had students watch the *Two Ninety Forty Four* video and note some of the key rates it contains. They noted some of following:

- Reynolds and Gobright climb approximately 7–8 m per minute.
- 42 minutes from the start, they arrive at El Cap Tower, which is 366 m from the ground.
- They start at 6:57 a.m. and reach the 2000-foot (610-metre) point, referred to as the Great Roof, at 8:16 a.m.

1. What do you **think** you know about this topic?
2. What questions or **puzzles** do you have?
3. How can you **explore** this topic?

I use Think-Puzzle-Explore with students, individually or in groups, using sticky notes for brainstorming. I then have them come together between phases to compare their "thinking," "puzzling," and "exploring." The students were very keen to determine whether the climbing rate for the record-setters was consistent. I had them watch the video several times, pausing and rewinding to collect and check their data. The "thinking," in most groups, was that it would not be consistent because surely some parts of the climb would be more difficult and others easier. Many of the students posed a "Puzzle" and began creating a table with 7–8 m per minute to show reaching 366 m from the ground, while others just looked at the rate between two points. When the class came together, I was able then to have them look at the trip from beginning to end and see whether the individual segments they had checked matched the overall rate. This was a natural development from their calculations and "explorations."

I have used this thinking routine with a variety of Pythagorean theorem videos, constructions, and applets to have students Think, Puzzle, and Explore with different visual proofs. I have also used it for transformations of two-dimensional shapes in Grades 6 and 7, using labelled coordinates, and asked them to develop rules they could use.

What Makes You Say That?

While watching the video, you can see the actual climbing time elapsed. I asked students to consider what the "frame rate" of the video means. With this question, I give students the simple thinking routine, "What Makes You Say That?"

1. What's going on with the timing of the video and the actual climbing time?
2. What do you see that makes you say that?

Climbers call the section of a climb's route a *pitch*, and they are often between two ledges. The Nose route of El Capitan has 31 pitches. In free climbing, the grade is a numerical code referring to athletic difficulty, often referring to how much time a climber is expected to take on that section of the climb. The Yosemite Decimal System (YDS) is used for rating the difficulty level of pitches, with Classes 1 to 5 being respectively least to most difficult free climbing.

The **Climber.org** website indicates that there are subdivisions of Class 5. The list, shown below, provides an opportunity for students to try to relate different mapped routes and their grades. Using Thinking Routines including "What Makes You Say That?" to consider why the different pitches have different grades provides a concrete connection for middle school students.

5.0 to 5.4 – There are two handholds and two footholds for every move; the holds become progressively smaller as the number increases.

5.5 to 5.6 – The two handholds and two footholds are there, obvious to the experienced, but not necessarily so to the beginner.

5.7 – The move is missing one hand- or foothold.

5.8 – The move is missing two holds of the four, or missing only one, but is very strenuous.

5.9 – The move has only one reasonable hold, which may be for either a foot or a hand.

5.10 – No hand- or footholds. The choices are to pretend a hold is there, pray a lot, or go home.

5.11 – After thorough inspection, you conclude this move is obviously impossible; however, occasionally someone actually accomplishes it. Since there is nothing for a handhold, grab it with both hands.

5.12 – The surface is as smooth as glass and vertical. No one has really ever made this move, although a few claim they have.

5.13 – This is identical to 5.12, except it is located under overhanging rock.

I have also had students consider the mathematical definition of slope as compared to the climbing definition of pitch. We used the "What Makes You Say That?" routine for this comparison as well.

Conclusion

Thinking Routines fit integrated learning in middle school classrooms quite well. They are effective for responding to student interests and can help connect their interests to math. The routines I've described can help keep conversations focused and constrained. I often use them as starters or as journal prompts to wrap up the day's thinking. This video was a wonderful prompt to remind me how many interesting directions my students' thinking can go in with great source material.

Resources

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from **https://news.nationalgeographic.com/2018/06/alex-honnold-tommy-caldwell-speed-record-el-capitan-nose-yosemite-culture/?ex_cid=SigDig**

Vasarhelyi, E.C., & Chin, J. (2018). *Free Solo* [Video file]. Retrieved from **www.nationalgeographic.com/films/free-solo/about/**

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Visible Thinking [Video file]. (n.d.). Retrieved from **www.visiblethinkingpz.org/VisibleThinking_html_files/VisibleThinking1.html**

Visible Thinking [Video file]. (n.d.). Retrieved from **www.visiblethinkingpz.org/VisibleThinking_html_files/03_ThinkingRoutines/03d_UnderstandingRoutines.html**

The Yosemite Decimal System [Video file]. (n.d.). Retrieved from **http://climber.org/data/decimal.html** ▲

Team Canada at the International Mathematical Olympiad

Congratulations to Team Canada, who placed 16th out of 107 countries during the competition in Romania. The team brought home five silver medals and a bronze—a result that shows the result of hard work, perseverance, and talent.

The team consisted of Toronto's Howard Halim and Victor Rong; Richmond Hill's William Zhao; London's Zixiang Zhou; and two members, Thomas Guo and Nicholas Sun, who study in the United States.



▲ A Graph Tells a Story

JEFF IRVINE

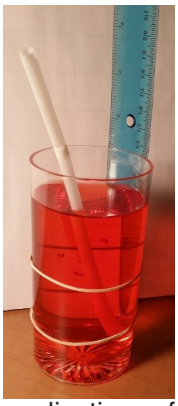


Jeff Irvine has been a secondary math teacher in Waterloo and Peel District School Boards, a secondary math department head, and a secondary vice-principal. Jeff has taught at three faculties of education and at Sheridan College. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. He is currently an instructor of mathematics education at Brock University, where he is pursuing a PhD.

As mathematics teachers, we are familiar with the relationships among graphs, equations, pictures/diagrams, and words. This multiple-representation mindset is often lacking in our students, who sometimes fail to see the connections that we take for granted. The advantage of different representations of the same situation is that some representations provide information that may not have been readily seen in another representation. For example, a graph provides information about linearity versus non-linearity, as well as information about changes in slopes and intercepts. A table of values provides information about patterns in data. An equation provides information as a representation well suited to forecasting (extrapolation), as well as the ability to represent patterns in a concise manner. Pictures and words give us physical or written descriptions of reality that can be represented mathematically in other forms.

A major takeaway for students is that a graph tells a story and can provide information that is not readily available from other representations. This is a big idea in mathematics. We routinely use straight lines to provide information about cellphone plans or identify break-even points for a business; parabolas show us maximum area or minimum cost. But the best way for students to learn "the graph tells a story" concept is with student-generated data, followed by the "What if?" thinking, which results from changing the parameters of the original situation. This is a great opportunity for students to think critically and justify their ideas. It also allows other students to challenge the results and have students explain their thoughts.

I first saw the basic problem in *The Mathematics Teacher* almost 20 years ago. A student takes equal-sized sips of



liquid (I like cranberry juice for this, since it makes reading the heights easier). In my experience, the student will need some practice sips to attempt to have all sips the same size. After each sip, the height of liquid remaining is recorded. Students conjecture the shape of the graph, then graph the experimental data, and find the equation of the line of best fit. This activity has a number of very useful attributes: it is clearly real life; it's fun, it shows an application of mathematics that students can readily understand; it uses student-generated data, so students are likely to be more engaged with the problem; it involves *fuzzy* data, since it is unlikely that all sips will be exactly the same size; and most importantly, it relates a physical situation to a graph. The graph tells a story about the real-life situation. I have used this activity in Grade 9 to address expectations around linear relations, scatter plots, lines of best fit, and finding equations of lines.

The base problem, which is useful in its own right, is a straightforward modelling problem, where students generate a linear graph based on data. The mathematics of multiple representations occurs when students consider changes to the base conditions of the problem. This is an excellent way to address Grade 9 expectations around linear versus non-linear relations. By relating changes in physical situations to changes in the resulting graphs, his activity reinforces the difference between constant versus non-constant changes (slope versus curvilinear). I have also used this activity in Grade 10, to reinforce the differences between linear and non-linear relationships and to introduce the unit on quadratic functions (even though none of the resultant graphs are quadratic).

All the changes come under the heading, "What would the graph look like if…?" I usually break these condition changes into several levels. For example, if you use cylindrical glasses with stems and have other glasses with the same diameter, the intercept will change, while the slope remains the same. Changing the diameters of the cylinders changes the slope, but possibly not the intercept (see Figure 2). Changing the shape of the glass will result in a non-linear graph, assuming that the sip size remains the same (Figure 3). Several different glasses are placed on each group's table, and students can deal with them in any order they choose.

Level 1

These changes involve different-diameter cylinders, or cylinders that have a stem base. Different diameters change the slope of the line, since all sips are assumed to be the same size as the base problem. Glasses with stems change the intercept on the height axis.

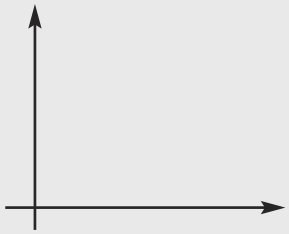


For each glass at a student station, the group completes a think sheet (Figure 1).

Liquid Assets Think Sheet

Sketch your glass:

Sketch a graph of what happens to the liquid's height:



Written justification for your graph:

Figure 1: *The think sheet for the activity*

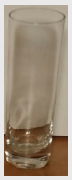
Level 2

This is the first set of glasses for which graphs will be non-linear, since these are no longer true cylinders. For most of the glasses, the height will decrease more slowly at first, and then decrease more rapidly as the diameter of the glass decreases.



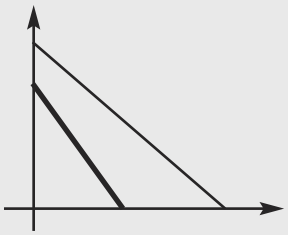
Liquid Assets Think Sheet

Sketch your glass:



Sketch a graph of what happens to the liquid's height:

The new graph is shown in **bold**.



Written justification for your graph:

The sip size is the same, but because the cylinder's diameter is smaller, it will take fewer sips to empty the glass. So the line will still have a negative slope, but will be steeper. Also, since the volume of the cylinder is less, the *y*-intercept will be lower.

Figure 2: *Think sheet for a level 1 glass*

Level 3

These glasses usually combine two conditions, such as a decreasing diameter and a stem.



Level 4

These are the most complex situations, typically with two or more conditions changed. For example, the glass in the middle of the picture above has a stem plus a very wide diameter at the top, getting smaller gradually until it reaches the much smaller portion at the bottom of the glass, which then also gets smaller.



Each student station will have a variety of glasses from all four levels. For each glass, the group completes a think sheet. These can be posted for a gallery walk and critiqued by other students.

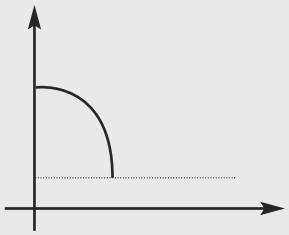
When I evaluate this activity, students are allowed to choose any glass in the classroom. This differentiated assessment allows every student the opportunity to be successful. Students know that if they choose a relatively simple glass to model, their justification for their graph will need to be superior. Students who choose relatively complex glasses will still need a solid justification for their graphs. I have also had considerable success using student-generated rubrics for the evaluation.

Liquid Assets Think Sheet

Sketch your glass:



Sketch a graph of what happens to the liquid's height:



Written justification for your graph:

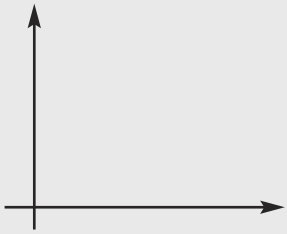
The glass has a stem, so it is shifted up. The graph will be a curve, since the glass is not a cylinder and changes widths. At the top, the glass is wide, so it will empty slowly. But when the liquid level hits the small part, the glass will empty really quickly, so the graph becomes steeper. The graph won't hit the x-axis because of the stem. The bottom of the graph will only go to the level where the stem is. I have shown the stem height on the graph with a dotted line.

Figure 3: *Think sheet for a level 4 glass*

Liquid Assets Evaluation

Sketch your glass:

Sketch a graph of what happens to the liquid:



Written justification for your graph:

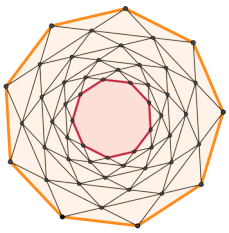
Figure 4: *Evaluation sheet for liquid assets activity*

Another fun and useful activity is to give students a graph and ask them to sketch the shape of the glass, with written justification. We do this in a think/pair/share format, where each student generates a graph and then trades with his or her partner, whose job it is to sketch the shape of the glass, and explain his or her thinking.

This activity illustrates the power of "What if?" It also involves students in higher-order reasoning as they think mathematically, which I believe is one of our goals when teaching mathematics. The changes in the shapes of the glasses require students to analyze new situations, drawing logical conclusions based on their prior experiences with less complex glasses. This provides the opportunity to explain their reasoning and justify the resulting graphs. While certainly not the only activity that requires higher-order thinking, this is a fun activity with a low floor and high ceiling.

Reference

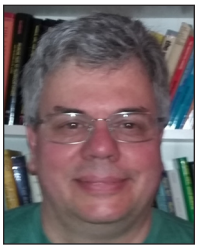
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Nonogon Flower

▲ What's the Problem? Spiral Sums

SHAWN GODIN



Shawn Godin is head of Mathematics, Business, Law, and Computer Science at Cairine Wilson Secondary School in Orleans. He strongly believes in the central role of problem solving in the mathematics classroom. He continues to be involved in mathematical activities: presenting workshops, writing articles, working on local projects, and helping create mathematics contests.

Welcome back, problem solvers. Last time, I left you with the following problem:

> *The rectangular spiral is constructed as follows. Starting at (0, 0), line segments of lengths 1, 1, 2, 2, 3, 3, 4, 4,… are drawn in a clockwise manner, as shown in the diagram (Figure 1). The integers from 1 to 1000 are placed, in increasing order, wherever the spiral passes through a point with integer coordinates (that is, 1 at (0, 0), 2 at (1, 0), 3 at (1, -1), and so on).*

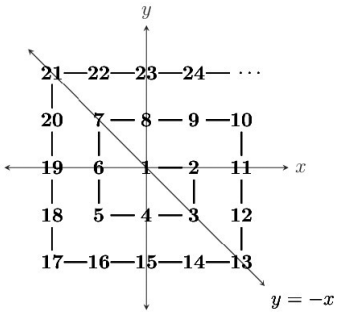


Figure 1: *Spiralling the positive integers*

> *What is the sum of all of the positive integers from 1 to 1000 that are written at points on the line* y = –x*?*

This problem was question A5 from the 2017 Canadian Senior Mathematics Contest (CSMC). The CSMC is run by the Centre for Education for Mathematics and Computing (CEMC) at the University of Waterloo. You can check out past mathematics contests and other CEMC activities and resources on their website: www.cemc.uwaterloo.ca.

This problem is related to the *Ulam spiral*. The Ulam spiral was created by mathematician Stanislaw Ulam in the 1960's. The story goes that Ulam was at a presentation that he found boring, so he started creating a spiral of numbers like the one in Figure 1 (except it wasn't on the coordinate plane and it was a mirror image of ours, with turns going to

the left instead of the right). Then he started circling the prime numbers, and he noticed that they seemed to lie on certain horizontal, vertical, and "diagonal" lines (with slope ±1). In Figure 2, our spiral has been extended out to 1000 (as per our problem), and the primes have been coloured black. Following Ulam, notice the line with equation $y = x$ for negative values of x, we get the following primes on our line: 5, 17, 37, 101, 197, 257, 401, 577, and 677.

Typically, the Ulam spiral is depicted without the numbers written, just the primes coloured. In Figure 3, our spiral has been extended just beyond 10 000, with the primes indicated in black. Notice the unusual texture of black line segments, but all of them seem to be broken up. The Ulam spiral lets us see that the set of prime numbers has some structure to it and that patterns exist in them. However, as all of our "lines of primes" are broken at seemingly random places, we can tell that the primes are not ready to give up all their secrets just yet. I could go on forever about the primes, but I suppose we should get back to the problem at hand.

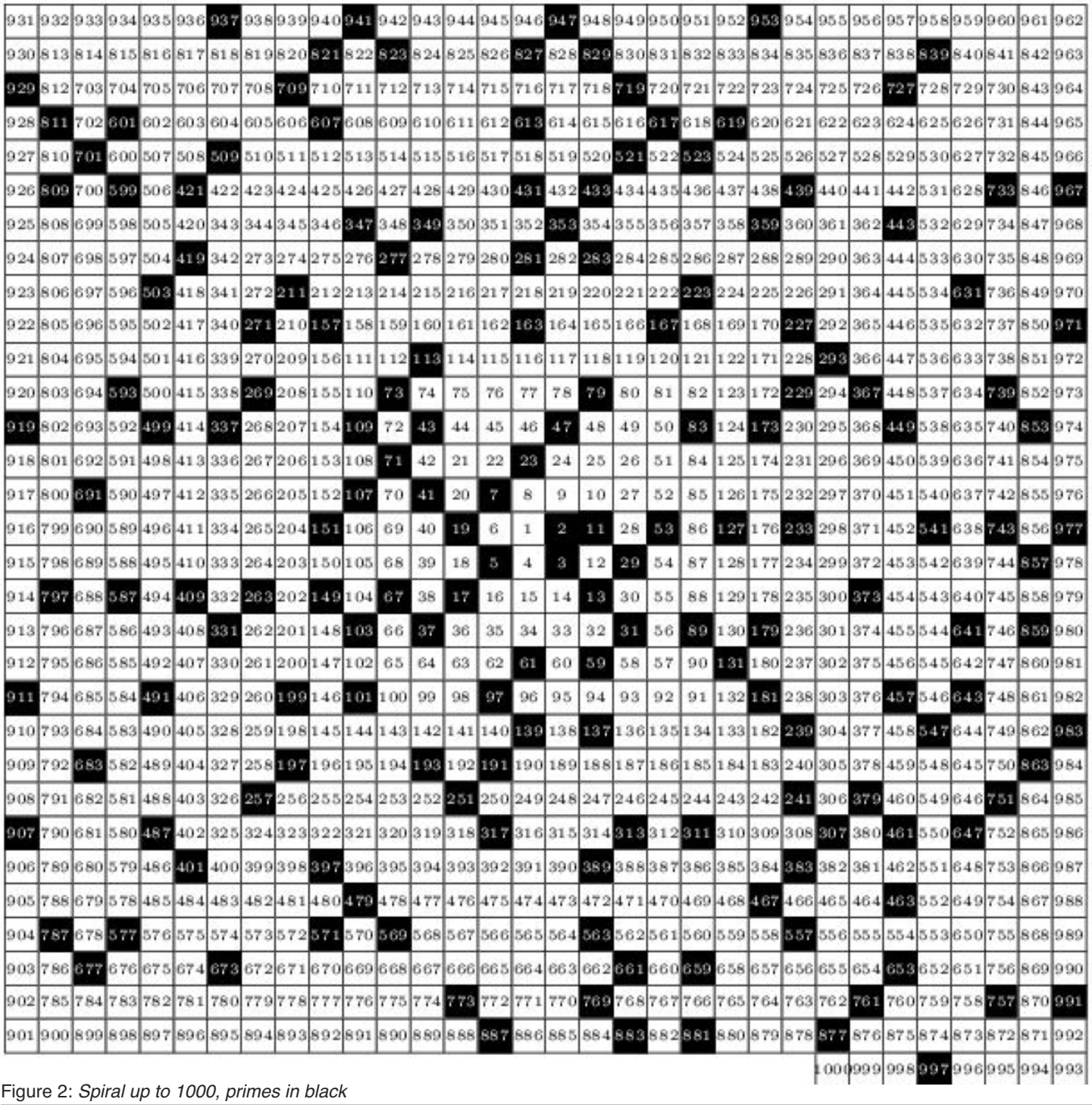


Figure 2: *Spiral up to 1000, primes in black*

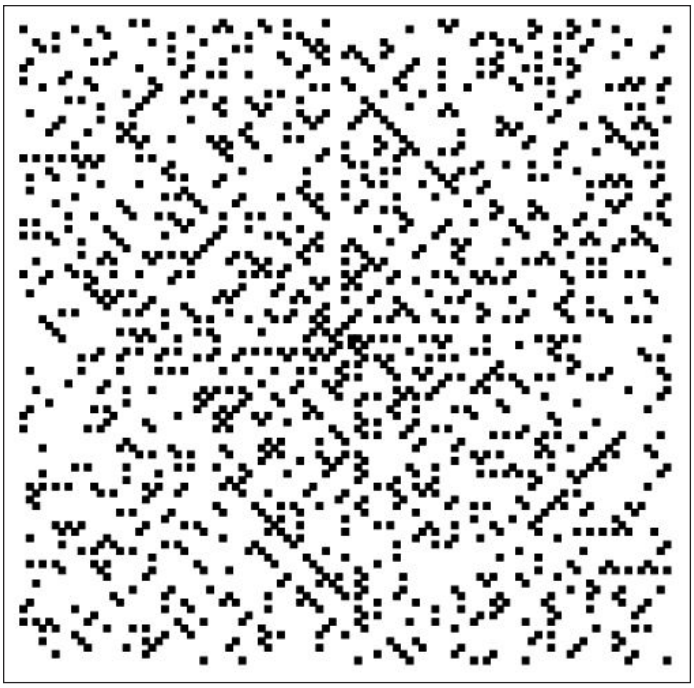


Figure 3: *Spiral up to 10 000, primes coloured black*

If we were ambitious (or bored) enough to complete the spiral in Figure 2, we could just list the numbers that we wanted (the primes are in bold): 1, **3**, **7**, **13**, 21, **31**, **43**, 57, **73**, 91, 111, 133, **157**, 183, **211**, **241**, 273, **307**, 343, 381, **421**, **463**, 507, 553, **601**, 651, 703, **757**, 813, 871, 931, 993. The problem could then be solved by adding them up to get 10 944, but what fun would that be?

We would like a more efficient way of finding and adding all the numbers on the given line. We can start with the solutions from Figure 1 and putting them into a table of values we see in Figure 4. A quick analysis of the differences shows that we seem to have a quadratic relation. We can convince ourselves of this by returning to Figure 1 and noting that, to get from 1 to 3, we travel down two sides of a square of side length 1. To go from 3 to 7, we go down two sides of a square of side length 2. Continuing the pattern, we see that the first differences will be 2, 4, 6, 8, 10, 12, 14,… just by how the spiral was constructed. Hence, our pattern in the first and second differences holds and it is a quadratic relation.

x	y
1	1
2	3
3	7
4	13
5	21

Figure 4: *Table of the first five numbers on the line*

Next, we need to determine the quadratic function that yields our table. Since the second differences are always 2, our leading coefficient will be 2 ÷ 2 = 1. We could use the second differences to determine that $f(0) = 1$; hence our constant term is also 1. Thus, the function is of the form $f(x) = x^2 + bx + 1$. We can solve for b by plugging in a value for x, and using our table in Figure 4 to get an equation that can then be solved for b to determine that $f(x) = x^2 - x + 1$.

Next we need to find how many terms we need to add together. Since we want to go up to 1000, set $f(x) = 1000$ and solve. The quadratic equation yields $= \frac{1 \pm \sqrt{3997}}{2}$, or $x \doteq 32.1$ (since we are only interested in positive values of x). This means that the 32nd number would be under 1000, and the 33rd will be greater than 1000, which can be easily checked. We can now just plug in 1, 2, 3,… 32 for x, to produce the list we found earlier, and then add them up.

Again, it would be nice to be able to find a quicker way to solve this problem. If, instead of working out the values of $f(x)$ we kept them in "expanded" form, then $f(1) + f(2) + \cdots + f(32)$ could be written as

$$(1^2 - 1 + 1) + (2^2 - 2 + 1) + (3^2 - 3 + 1) + \cdots + (32^2 - 32 + 1)$$

or as

$$(1^2 + 2^2 + 3^2 + \cdots + 32^2) - (1 + 2 + 3 + \cdots + 32) + (1 + 1 + 1 + \cdots + 1).$$

The last sum we can easily see is 32. If we have studied arithmetic series, we will know that the middle sum is

$$\frac{32 \times 33}{2} = 528.$$

There are several ways we can attack the first sum; we will look at it algebraically. Let's consider the function

$$S(n) = 1^2 + 2^2 + 3^2 + \cdots + n^2.$$

If we create a table of values for this function, you will find that the first differences are just the perfect squares. This makes sense because as we increase n by one, we add the next square to our sum. This means that our second differences will be linear and our third differences will be constant. Thus, our function is cubic and is of the form

$$S(n) = an^3 + bn^2 + cn + d.$$

Substituting $n = 1, 2, 3, 4$ into our definition and into our equation, we get the following system of equations:

$$a + b + c + d = 1,$$
$$8a + 4b + 2c + d = 5,$$
$$27a + 9b + 3c + d = 14,$$
$$64a + 16b + 4c + d = 30.$$

If we solve the system (I will leave the details to you) we get

$$a = \frac{1}{3}, b = \frac{1}{2}, c = \frac{1}{6}, d = 0,$$

which give us

$$S(n) = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n = \frac{n(n+1)(2n+1)}{6},$$

the formula for sum of the first n squares. Thus, substituting

$n = 32$ yields 11 440 for the first sum identified earlier. Adding the other two terms our final answer is

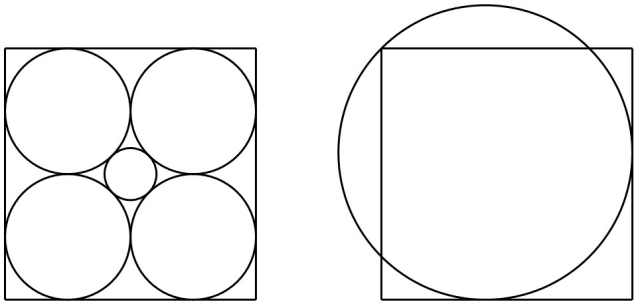
$$11440 - 528 + 32 = 10944,$$

as we found earlier.

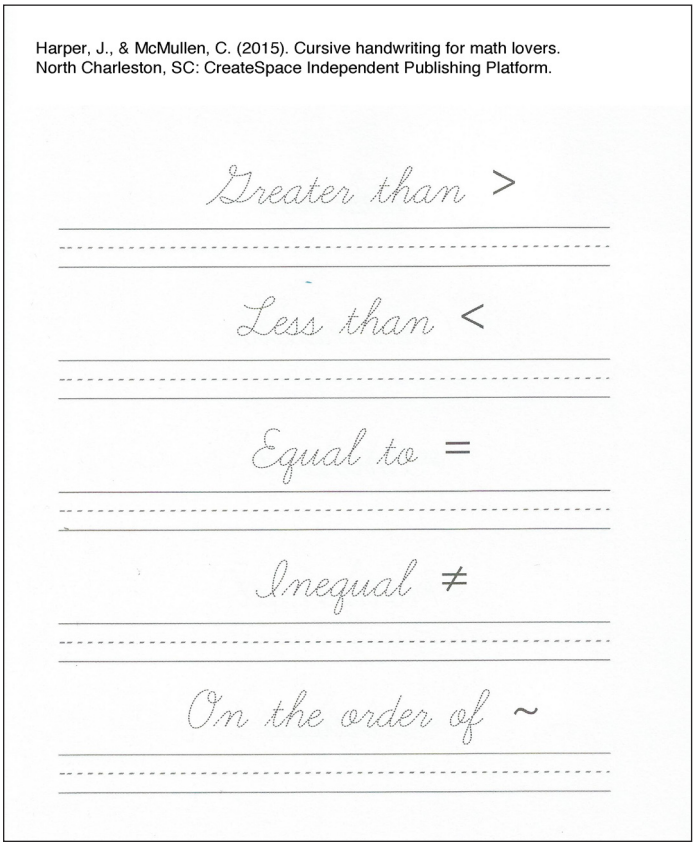
If students haven't encountered arithmetic series yet, they can use the same method (i.e., solving a system of equations) to come up with the formula for the sum of the squares as well. It is interesting to note that at the start of the contest, students were given the formulas for the arithmetic sum and the sum of squares. For those who don't know the formula for the sum of the first n cubes, you can practise the technique on that problem (and you may have a pleasant and curious surprise when you do!).

Now it's time for your homework:

Determine all you can about the squares and circles drawn in the figures below:



Until next time, happy problem solving! ▲



▲ MB4T (Mathematics by and for Teachers): Supporting Understanding of Fraction Operations

JENNIFER HOLM



Jennifer Holm is an Assistant Professor at Wilfrid Laurier University and works with primary/junior and junior/intermediate pre-service teachers, as well as in the field supporting current mathematics teachers. She is interested in developing mathematics knowledge for teaching with both pre-service and in-service teachers. She focuses on the beliefs and opinions that pre-service teachers hold about mathematics and teaching and the connection they have to past experiences. She uses this research to support future teachers in developing beliefs and knowledge that will encourage and support effective teaching practices.

> *Learning about fractions in the upper elementary grades is hard. Really hard. Fractions are hard not only for children to learn but for teachers to teach* (Burns, 2001, page ix).

As Burns notes, fractions are one of the more difficult concepts for students and adults to learn, and it is challenging to gain a deep understanding of their operations. Fraction operations present challenges because the procedures themselves do not follow the same processes and strategies that apply to whole numbers and decimals. Without a firm understanding of the meaning of fractions, their operations can become a memorized set of rules that have no conceptual meaning and are therefore easily forgotten. Building a base of understanding for operations can help mitigate misunderstandings and improve retention of the underlying concepts. Experiences with models or different representations can lead to a deeper understanding, and support an understanding of operations that goes beyond memorized procedures. For more on the benefits of modelling, please see Holm (2018).

There are many different manipulatives that can be used to build understandings of fractions, including fraction pieces, pattern blocks, counters, Cuisenaire rods, number lines, and technology-based ideas (see Grafton, Isenegger, Wolski, & Clarke, 2017a). In order to support students in developing an understanding of fractions and fraction operations, having students make fraction kits provides a start that involves direct experiences with fractions. As

noted, there are many different options for beginning understandings of fractions; however, the fraction kit allows for student ownership of the manipulatives and can be created in a way that allows for different opportunities to understand fractions. In my experience, pre-made kits or manipulatives sometimes lead to students looking at only the fractions written on the bars, whereas an initial experience of creating kits facilitates deeper conversations with students about the underlying meaning, which I expand upon throughout this article.

Although kits and manipulatives are used in many classrooms, this column proposes a particular approach to fraction kits and fraction activities, with a goal of supporting development of understanding the operations prior to teaching the operations of fractions. The activities suggested help to set up foundational knowledge in the junior grades and can also be used to activate knowledge in the intermediate grades to continue support while developing understandings of fraction operations.

To start, it is suggested that multiple different colours of papers are not used for the fraction kits. This eliminates the possibility of students relying on the colours when describing their actions, when discussing the activities, or responding to questions about the tasks. In addition, keeping students from writing the fraction values on the pieces allows for flexibility in the activities and ensures students rely on the relationship of the piece with the whole instead of simply looking at what is written on the paper. This continued reliance on examining the size of the paper and determining the value in relation to the whole piece reinforces a deeper understanding of fractions.

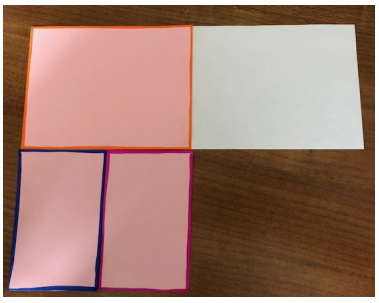


Figure 1: *One-half on top of one whole (which is rectangular), showing the relationship with two one-quarter pieces. Note: The one whole is a different colour, and the fractions are outlined only to allow for contrast and ease of viewing for the images. In the kit, they would all be the same colour without an outline.*

A particular order is proposed herein, following the suggestion of Marilyn Burns (2001). In order to prepare for the lesson, each student needs five equal-sized, identical coloured strips of paper. Have students choose one strip that will be left in one piece to be designated one whole. Now students are to take another whole strip and divide it into "half." At this point, it is important to discuss how students know that this is "one-half," focusing on the language that there are two equal-sized parts to make the whole. Oftentimes, the next move is to make thirds following the pattern in the fraction bars kits. Instead, have students make quarters out of the next fraction strip because the link to the relationship between the fractions follows more naturally. Now when discussing with students that the portion is "one-quarter," there is also a relationship with one-half that can be seen (see Figure 1). One-quarter is one of four pieces in a whole, and it is also one of two pieces in one-half (or half of a half). Many students will create the quarters by folding halves again and then folding them in half to make quarters.

The experience of creating the kit naturally moves students to experience the relationships between the fractions, which could be missed or may have to be directly taught when using pre-made fraction bars. Also, by building this language, you are setting the stage for conversations in multiplication ($\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$, which is the same as saying one-half of one-half). From here, build the concepts of eighths and then sixteenths. Since each subsequent one is half of the proceeding fraction, you can continue building the relationships among the fractions. The discussions can also focus on other relationships, such as one-sixteenth is one-quarter of one-half ($\frac{1}{4}\times\frac{1}{2}=\frac{1}{16}$). This focus on the language is helpful for both building relationships among the fractions, but also sets the groundwork for discussions about multiplication.

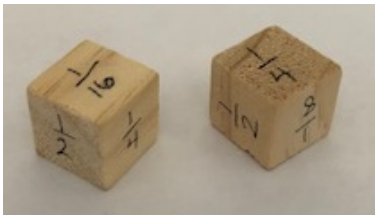


Figure 2: *Fraction dice*

After building the kits and discussing the relationships among the fractions, the next helpful step is to build an understanding of equivalent fractions through different games and activities. For the following games and activities, it is helpful to have created dice so that each student in the class can have one. The wooden craft blocks at the Dollar Store make helpful game pieces. On each die, put the fractions $\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{8},\frac{1}{16},\frac{1}{16}$ on the six faces (see Figure 2). Notice that the kits and the activities are based on a special set of fractions, where denominators are all related by factors of two. This design allows for the activities to continue building the understandings that were established by making the kit.

The first game is *Cover up* from Burns (2001), and uses one fraction die for the pair. Each student should have his or her own fraction kit. In pairs, the students take turns rolling the fraction die. For each turn, the fraction that comes up on the die tells what piece of the kit to place on the whole strip. After the student places the piece, have him or her check with a partner to be sure they both agree with the play. Once

agreement has been reached, the other person has a turn to roll the die. The first player to cover his or her whole strip *exactly* wins. This is an important piece of the rules so that students are focusing on what makes one whole. As students are playing, the teacher can facilitate by asking questions such as the following:

- "How much of your whole is currently filled? How do you know that?"
- "Can you complete your game board in one move?"
- "What roll(s) would be needed to complete the game board?"
- "How much do you have uncovered? How do you know?"

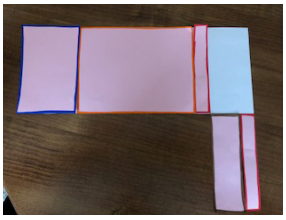


Figure 3: *The top image shows the game board with $\frac{1}{4}, \frac{1}{2}, \frac{1}{16}$ rolled. The second image shows how a student may figure out that two rolls $\left(\frac{1}{8}, \frac{1}{16}\right)$ will fill the board.*

Here is where it becomes important that the pieces are all the same colour. Students are not able to say, "Well, I need the green piece to win." Instead the focus is on knowing that the $\frac{1}{8}$ fraction piece is a particular strip because there are 8 of them that make up one whole. In the top image of Figure 3, there is a game board that is shown after a player has rolled $\frac{1}{4}, \frac{1}{2}, \text{and } \frac{1}{16}$. The second image in Figure 3 shows how a student might answer the question, "How much more is needed to fill the whole?" By placing the $\frac{1}{8}, \frac{1}{16}$ under the end portion, it is clear that at least two more moves are needed. These questions and discussions during the activity can help students pay attention to the values of the fractions and facilitate building an understanding of the different ways to make one whole.

Following the game, create a whole-class consolidation to discuss the activity and allow generalizations to be made that support learning. Again, a key idea is building the fraction language. Endeavour to build class understandings of how they can determine how much of the whole is covered. Do this by looking at what piece(s) fill the end or by finding out the equivalent pieces. For example, returning to Figure 3, in thinking about how much of the whole is covered, knowing that the amount that is not yet covered is equal to $\frac{3}{16}\left(\frac{2}{16} = \frac{1}{8}\right)$ makes it clear that the covered portion is $\frac{3}{16}$. By having students leave their last "winning" game board intact, a focus can also be on different ways the game board was filled. It provides an opportunity for students to see all the ways that one whole can be made. Once all the different ways are identified (perhaps summarize them on an anchor chart), have students look for patterns or similarities. For example, if the student in Figure 3 happens to finish the board, the equation might be $\frac{1}{4}+\frac{1}{2}+\frac{1}{16}+\frac{1}{8}+\frac{1}{16}=1$. Another student may have $\frac{1}{4}+\frac{1}{16}+\frac{1}{8}+\frac{1}{16}+\frac{1}{8}+\frac{1}{16}+\frac{1}{16}+\frac{1}{4}=1$. Discussions could include looking at which fractions are the same or how the two equations can be rearranged to show the same amount because of equivalent parts. Another interesting observation can be that whenever $\frac{1}{16}$ is rolled, you have to have an even number of them to complete the board. The goal of discussion is to reinforce the definition of equivalent fractions, which is going to be helpful when adding and subtracting fractions later.

As a note, by not writing the symbolic fraction name, a shorter version of the game could be played by using the one-half strip as the "whole," and then playing the game from there. The die would then have two "free spots," since there would not be a $\frac{1}{16}$ piece in this new, shorter game.

To continue building understandings of equivalent fractions, the next activity is starting with a version of *Uncover It* (Burns, 2001). The game begins with each person placing two one-half pieces on the whole. As the players take turns, there are three options to choose from on each turn: they can remove a piece if they have the size indicated by the fraction facing up on the die they rolled; they can exchange any of the pieces currently on the whole strip for equivalent pieces; or they can do nothing. For the first round, it would mean that the only roll that would allow a piece to be removed would be to roll a $\frac{1}{2}$. The first move would likely be to exchange the one-half for equivalent pieces in order to increase the odds of having a play. The goal is to be the first player who removes all of his or her pieces from the whole strip. There are two caveats for the game that increase the challenge and support fraction understandings. First, a player can only remove a piece or exchange it on a single turn, not both, and second, a player has to go out exactly (see Figure 4). While students are playing the game, the teacher can circulate and ask questions, encouraging students to explain their strategies, using fraction language. Following the activity, it is helpful to have a consolidating discussion to allow students to share their strategies and what they have learned about equivalent fractions with the class (i.e., "What are different ways you can make one-half?").



Figure 4: *This shows the* Uncover It *game with only a $\frac{1}{8}$ and $\frac{1}{16}$ left. A roll of $\frac{1}{4}$ cannot be used to win, even though it is larger than both the fractions left. In order to go out with this board, the player must roll a $\frac{1}{8}$ and $\frac{1}{16}$ (or exchange the $\frac{1}{8}$ for two $\frac{1}{16}$ and then roll three $\frac{1}{16}$).*

Once the students have had some practice with the *Uncover It* activity, they can try a second version of *Uncover It*. The directions for this activity are all the same, except they have the option of removing one or more pieces from the board, as long as they add up to the fraction that is facing up on the rolled die. In the discussion, students may end up deciding that the best strategy is to convert all the pieces to sixteenths so that for the majority of the activity, they can always remove something on each roll. Again, the focus is on equivalent fractions and which pieces compose each of the fractions on the die.

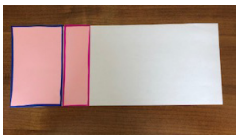


Figure 5: *Train made from $\frac{1}{4}$ and $\frac{1}{8}$. Have students record the starting amounts of the fractions in order to keep track of the information for the discussion.*

A final activity has stronger links to actually using fraction operations without having to provide direct instruction about addition yet. To set this up, consider using the following precursor activity. Have students see how many different ways they can make a single piece (e.g., $\frac{1}{2}=\frac{2}{4}=\frac{4}{8}=\frac{8}{16}$). This will help students identify equivalent fractions and set up the method for finding a common denominator when adding and subtracting fractions. Burns (2001) calls the activity that builds on this *Pick Two*. The point of the activity is to pick two pieces that are different sizes and to make a "train" on the whole (see Figure 5). The next step is to use the train to make one that has all the same-sized pieces and is equal to the length of the train. Students can be challenged to see how many different ways they could make the trains and record the final fractions (see Figure 6 for an example). By having the students record the fractions, the follow-up discussion could be to determine how the fractions are being added to create the new train. Using the image in Figure 6, when discussing the trains, the equation $\frac{1}{4}+\frac{1}{8}=\frac{3}{8}=\frac{6}{16}$ can be built.



Figure 6: *Shows that the train of $\frac{1}{4}$ and $\frac{1}{8}$ is equal to $\frac{3}{8}$ and $\frac{6}{16}$*

The discussion can then extend to examining the two equivalent fractions and how they might relate to the numbers in the equation. A further discussion could have students "predict" what trains of equivalent pieces can be made by only looking at the two fractions that are being combined (i.e., whether they can predict a common denominator). By having a discussion with the class, students have an opportunity to discover the "method" for finding a common denominator by looking at the patterns in the related fractions. This activity can be extended by having students use three or four pieces, where at least two pieces are different sizes. The follow-up discussion focuses on having students look at the relationship between the fractions that are combined in the initial train and the pieces that are chosen to make the same-sized train. This increases the understanding of finding common denominators in addition.

After experiences with the fraction kit described in this column, a second kit can be created with thirds, sixths, twelfths, and twenty-fourths. The same activities can be played in order to learn the relationships among the different fractions in this kit. Further activities can be used in order to develop the relationships between the two kits. Another option instead of physically making the next kit could be to use digital fraction strips resources, such as the one explored in Grafton, Isenegger, Wolski, and Clarke (2017a, 2017b), in order to continue building relationships among the fraction values. Although this column focuses on the activities from the Marilyn Burns fraction series, there are many other places to find activities that can be done with fraction kits. You can find some of them in the additional activities section of the column. Some of the sites listed modify similar activities from this column or have created classroom-friendly instructions that may be helpful for working with fraction kits. In the end, the most important element described here is ensuring that students are building language that will be useful for them when looking at fraction operations, and will help them develop a deep understanding of how the fractions relate to each other.

Additional Activities

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Holm, J. (2018). MB4T (Mathematics by and for teachers): Examining the use of manipulatives and models for understanding. *Ontario Mathematics Gazette, 57*(1), 31–35. ▲

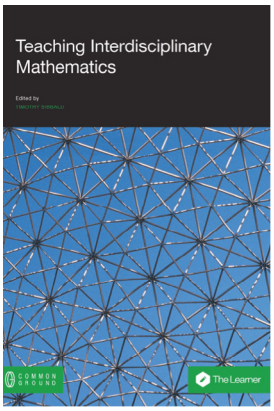
▲ BOOK REVIEW: CREATING DYNAMIC LEARNING EXPERIENCES

JUDI AGAR



Judi Agar is a Science and Math teacher with the Simcoe County District School Board (SCDSB). She is currently on the LTO list, and has also taught in the Night School and Summer School programs with the SCDSB. Judi is also a Professor at Canadore College, teaching a General Education course in Astronomy.

Sibbald, T. (Ed.). (2018). *Teaching interdisciplinary mathematics.* Champaign, IL: Common Ground Research Networks.



Recently there has been a push throughout the education field to integrate science, technology, and mathematics to provide varied learning opportunities for our students. With the prevalence of acronyms like "STEM" and "STEAM," teachers have been searching for new and interesting ways to integrate mathematics with other disciplines, and create dynamic learning experiences. Personally, I am always looking for new and exciting ways to teach mathematical concepts, and ways of checking for student understanding. *Teaching Interdisciplinary Math* provides chapters that explore different ideas of how mathematics can be integrated with other subjects to provide an interdisciplinary approach. Authors of the various chapters come from different countries and different educational backgrounds, which leads to an interesting perspective.

One of the most interesting chapters, in my opinion, discusses ways of integrating math with English, by asking students to incorporate math into their writing. This serves as an interesting tool in gauging student understanding of the mathematical concepts. It allows teachers to check if students have described concepts correctly in their writing, and the depth to which they incorporate the mathematical concepts. As a science and math teacher, I have never thought to incorporate English in this way. It seems like an interesting way to give students another avenue to demonstrate their understanding, perhaps lending to one of their strengths.

All of the chapters in this book provide interesting information about how to incorporate various disciplines into mathematics teaching. From the lens of a high school math teacher, some of the chapters are more easily applicable than others. While the information on post-secondary program structure in other countries provides an interesting read, it is difficult to imagine how ideas from these programs would be implemented in a secondary school classroom.

In my own teaching practice, I always like to find ways to integrate my subjects, as well as ways to link material that students may be learning in class to "real world" situations. The chapters in this book have provided me with new ways to approach mathematics in physics, and expand my teaching repertoire. I have encountered new ideas for assessments and thought more deeply about how to incorporate subjects such as English and history into mathematics lessons. In conclusion, I have found this book to be a useful resource that is full of interesting ideas. I suggest this book is a good fit for Intermediate and Senior teachers looking for new ways to integrate Mathematics with other subjects. ▲

THANKS TO OATM

TOM GRIFFITHS
EMAIL: tgriffit@uwo.ca

An invited opinion from a long-standing member of OAME (and OATM).

On first coming to Canada in 1966, I was head of the Math department at North Lambton Secondary School in Forest, Ontario. This was a country school of about 700 students. Through the *Mathematics Bulletin*, a journal published by the Mathematics Department at the University of Waterloo, I became aware of and involved in the Ontario mathematics community. In 1969, I was elected as a councillor to the OATM (Ontario Association of Teachers of Mathematics), which later became OAME.

With my department, I attended the annual conferences and met many other math teachers who also loved what they did. I found a peer group of like-minded people, who shared their ideas and friendship. At the conference each Spring, I also experienced a very valuable recharging of enthusiasm. My acceptance into the Ontario mathematics community has been greatly enhanced by my membership in this organization. I was fortunate to be a council member for 21 years, and the fourth president of OAME.

I urge any new teachers to join the organisation and attend the conferences to experience the support of their peers. ▲



Interview with Marian Small

ANN ARDEN



Ann Arden is a math teacher in the Ottawa-Carleton District School Board and is currently an instructional coach. She has also taught in the Faculty of Education at the University of Ottawa, and her son's teacher this year was a former student! Ann is very interested in assessment to improve learning, especially through the use of conversational and observational evidence. Ann is on Twitter as @annarden.

Ann Arden interviewed Marian Small, Professor Emeritus of the University of New Brunswick and independent consultant, on August 22, 2018 in Ottawa.

What were your own experiences with school mathematics?

I was lucky to have a string of fantastic high school math teachers. It wasn't my family. My dad was a history teacher and can't do any mathematics whatsoever. The one I remember the most was an old navy guy or something, and this was a second career, and he didn't know much content, but he was amazing with kids. When I was 15 years old, which was quite a lot of years ago, he named a theorem after me, and I still remember that day. So that's a powerful teacher. You don't remember a "day" without a powerful teacher. I feel it's better if a teacher actually does know their stuff and can bring a lot to the table. But in the end, you also need that person who makes you feel that this is a great thing to be engaged in. And it's the passion they bring, or whatever that is.

Most of the work I do is aimed at teachers, not kids. Because I'm convinced they are the key. So a good teacher just makes it all work for a kid. They've had one teacher that turns it all around for them in a positive way. Teachers are very powerful people, and I can't explain it, but it seems in math that the emotions run higher. I've never heard a person hate social studies forever because of one bad teacher, but I actually have heard them hate math forever because of one. I can't explain it. I totally don't understand, but emotions run high. It seems to me, our role as being a positive agent for that kid is huge.

A big theme in your work has been helping teachers see the big ideas in the curriculum, and you've done this in many ways. Through publications, through textbook writing, through resource support, through workshops, and other ways. In your 2011 book, "Big Ideas with Dr. Small," you wrote: "Upon reading the curriculum outcomes or expectations they must cover, teachers might also feel overwhelmed with the many small pieces of material to cover, rather than seeing a coherent whole with fleshed-out details, a perspective that would be much more manageable" (p. ix). Can you talk a little more about that?



When I talk, particularly to elementary school teachers, when you look at the Language curriculum, they actually think of it as more global. There are little skills that they have to do, but they really think of it globally. The way the math curriculum is written, pretty much everywhere, is very teeny-weeny pieces which seem really hierarchical: "You're not allowed to do this until you've done that." I feel it boxes them. They feel they don't see a bigger picture. It feels to me that we've written math curriculum to keep teachers "in the weeds." I want out of the weeds. I want us to do all these "baby-sized things," but as examples of something bigger.

One of the big ideas I often talk about is that when you represent numbers different ways, you see different things about the number, but the curriculum doesn't say that. It just says "represent numbers a lot of ways." Teachers don't know why they're doing it. They're just doing it because it says it. The kids are doing it because the teacher said to. Everyone's doing it, but they're not seeing the point of things. They're not seeing that when I do it with numbers, it's kind of the same thing as when I do it with graphs; it's kind of the same thing as when I do it with algebraic expressions. There's this big picture thing. When you represent something differently, you just have a different insight. That's why you do it. Teachers don't talk like that. They just talk about, "We do this and I turn it into a graph, I turn it into a table," and they chunk it all into miniscule things.

I'm not saying there's anything evil about it, but I'm saying it doesn't help kids. There are kids who can't even buy in unless they know why they're doing things. They're completely out of it then. And the other part of it is, if you see the connections, it helps you do it because, "Oh, it's just like that! Oh, yeah. I can do it." [I recently worked with a group of teachers] and we went through all the curriculum expectations and we said, "And what do we really want them

to know? What idea do we really want to come out of this one and this one? And where are those ideas?" It was a wonderful two days and the teachers now feel they have better direction. So this is about direction, but it's also about making decisions—"This is where you should spend your time." Yes, they [expectations] are equal in the list, but they're not equal. When you read it, what do you really want to get out of it because it's a little bit of a mystery sometimes—what is the intention of it? For me, a lot of this big idea stuff sounds nice, but it's not just because theoretically I want them to have big ideas. I think they help direct you. So what I said to teachers this morning is that when I write a learning goal for a lesson, it has to evolve from a bigger idea, or it's not the learning goal for my lesson.

I was just about to ask about some of these buzzwords. I know that there's been a lot of conversation in school districts and through RMS and through different Ministry initiatives about "learning goals" and "success criteria," and I feel sometimes myself a little bit stuck in the weeds.

For me there are big ideas, umbrella ideas, that encompass everything. Those big-big ideas don't really help you write learning goals for a lesson. So then I have a "tier-2." My tier-2 is something called "essential understandings," which are derived from big ideas, but just one layer down. For example, you might have a big idea about representations, but one step down is something like this one: "When we use place values to describe decimal numbers, we're standardizing the way we decompose numbers. You can't use any fractions you like anymore; they have to be tenths, hundredths, thousandths, etc. So that's not a huge idea, but it's pretty big. So that's in my list of essential understandings. When I'm teaching a lesson and I'm going to create a learning goal, learning goals for me have to derive from an essential understanding, and when I write the consolidation of my lesson, if I'm not asking a question that comes back to the essential understanding, then that question doesn't get asked. In a lesson, for example, about all that place value stuff, it could be, "If you weren't using a place value system, how might you break up a half? But if you were using a place value system, then what could you do?" I'm focusing kids on ideas that are bigger all the time. So that's how learning goals happen.

Success criteria, for me, [are] a different animal. And, to me, that's "Did it work?" If I have a task, it "works"—if the kids did all the stuff they were supposed to, and it "works," if they came away with what I'm going to call these "generalizations." I've been suggesting to teachers that math is more about getting to those generalizations, and that every individual problem we do, in and of itself, is useless unless it's part of this bigger picture. Because nobody will ever see that problem again in their entire life. So, teachers… I was telling them this morning, and I feel super strongly about this, teachers spent a lot of time in the last ten years, in good ways, focusing on how you got answers, and I don't want to do that. The problem is semi-irrelevant. It was a vehicle to get to an idea. I've [teacher] got to be talking about that idea, and that's what's not happening. So my goal is to teach teachers to teach ideas.

That's really helpful. Every time I see you getting to ideas of questions and the complex endeavour of teaching, you've got a really wonderful way of getting to the "meat" with reasonably few words. How did you develop that?

I had a grad student maybe 20 years ago. It was a teacher whom I was very fond of and we were close. One of the things she said to me this one day was, "Marian, you always ask the best questions." And she ended up doing her Master's thesis about questioning and it made me realize this is what I do. And it took me to a direction. So it was almost happenstance that somebody said it out loud. For me, it really is about, "How do you ask the right questions?" And the "right questions" have two parts. One is *it's the right question for that child*, and the other is that *it's the right question for the math I really care about*. So I think that many teachers who teach math, even high school teachers, don't really know what matters. They all know a lot of stuff, but they make all this stuff equal and it's not. There's some stuff—who cares anyway. And there's some stuff that's big. And nobody's forcing them, whether they're in training or whether they're part of the system, to really reflect on what really does matter. What should we spend our time on? I just think that as a profession, educators don't want to rock the boat, so they take whatever anybody tells them. I don't want people to be mean. I do want people to be respectful, but I think we could use more boat-rockers.

In your experience working with districts and teachers, what are some things that our readers can latch onto? If I'm a classroom teacher and I'm keen to become a better math teacher, what advice do you have? What can I do if I'm in my first five years as a new math teacher and I'm keen to learn more?

The truth for many people is you need the right colleague. Some people can, but not many of us are great solo reinventing the universe. So you really need a mentor who is not just fashionable, but who is a true mentor. So the mentor has to be a person who is cultivating you, but it's a two-way street. It's not like, "I'm just telling you stuff"; there's a relationship. I think you have to learn from someone else. It could be a human being, but for individuals who are in places that are hard or whatever, it could be books, but it's not being fashionable and fitting the mould and saying what everybody's saying. It's truly reflecting on, "What do I do that

makes kids' eyes light up?" What do I do where a kid is like, "Oh, I get it now." What are the moves that I make that do and don't make a difference?"

I know it takes time, but if you don't reflect, you're going to get nowhere. I actually believe reflection is the most important thing a teacher can do. But you have to have some kind of mentor, book, or person to help you know what to reflect on. I have an interesting experience when I work in what we usually call "lesson studies" with groups of teachers. We're all in a room together and we see stuff happen, come out, we talk about it. I'm often disappointed in what they notice and what they don't. Yes, you do care that this girl was too bossy, or this kid was this. Yeah, I know all that and you do, but that isn't really why we are there. We're there to figure out: "Are they really learning stuff?" and "What are they learning?" Teachers honestly haven't been forced by "the system" to focus on that. We haven't learned to focus on the educational enterprise as well as we should. I actually believe that they need the right mentor, especially if you're in a school with a culture where that is communal, that could make all the difference. So when a school turns around, it's everybody. When a teacher turns around by themselves, it's hard if they're by themselves.

What kinds of structures do you see working for teachers' learning around mathematics education?

The principals I know, for example, plan [so] their teachers have collaborative time together to plan. Otherwise, it's just hit and miss. You know, her French teacher comes in there, so that's when she's free, but hers comes there…. Nobody can talk to anybody and they're just busy marking papers. Part of it is just collaborative time. Part of it is, I think, and I think this is about structure, is making teachers, in a culture, believe that they're called "teachers," not "assessors," and that their main job and their main focus should be about learning situations. How do I create them? How do I make them stronger? And yeah, you have to assess, but that is not your main gig. So part of it is structural changes in what you attend to, and I think those are structures. So if you think of yourself as mostly a "teacher," and a little bit of an "assessor," you behave differently than if you behave the other way.

I'd like to ask about your work on questioning. You wrote in one of your books, "More Good Questions – Great Ways to Differentiate Secondary Mathematics," that "Open questions need just the right amount of ambiguity. They may seem vague and that might initially bother students, but the vagueness is critical to ensuring the question is broad enough to meet the needs of all students…." (p. 9)

There are lots of takes on that. One camp of people, who don't love me too much, talk about "how vague she is." I am. I know it and it's on purpose. I want enough room for a kid to come to it the way they want to come to it, not the way I intend for them to come to it, so to create that, you need a bit of ambiguity. Part of it is for that reason, so you can decide you meant something different than me, and it's all good, and it's usually a richer conversation. One of the examples that I often use when I am talking about proportional reasoning is, "Imagine one group grew from two to five and another group grew from six to ten." I don't say, "Which grew a greater percent?" I say, "Which grew more?" 'Cause I totally want both things to happen. I want one kid to take it one way; I want one kid to take it another way, so one will be a percent increase… a multiplicative increase, and one will be an additive increase and that's deliberate. It wasn't that I didn't know what I was doing. It's that I want that conversation to happen. So kids see you can always do that—there's always two ways you can look at it. You just make a choice about what way makes sense in a particular situation. So part of it is that vagueness allows for richer conversations. Part of it is that vagueness.

This is another one I talk to teachers about a lot: Suppose I wanted little tiny kids to know that you could add numbers in any order. If everybody in the same room does the same four examples and it worked, that's not a lot of data to generalize from. Say, "If you picked any two numbers, could you do it in any order?" We'll at least have a 100 pieces… so in a way, it makes for stronger induction or stronger generalizations. So part of it is actually [that] you have more data from which to draw conclusions. Tons of teachers let students draw conclusions from hardly any data at all. And later, they criticize them for doing exactly the same thing… there's only two examples, and they draw a conclusion they shouldn't have and they get mad at them.

It has to do with access to kids. It has to do with taking strong kids and pushing them further. I often will ask, in an elementary school, "Give me a fraction just a tiny bit more that a half." Well, what is tiny? It's in the eye of the beholder. For some kids, three-quarters does it. But there will be a kid who will say "501 over 1000." Then there's another kid who says, "No, I can do better," and you have... [a situation that is] rich. One of the things that's interesting about "a tiny bit more" is that it's really about infinity as well. If you have a number line that goes from zero to one, and you say "three-quarters" and you say "half," a lot of kids don't buy that that's a tiny bit more. But then if you say to them that the number line went from zero to ten million, so think about where zero, a half is, and where three- quarters is... they're darned close! So these bigger things, these are not just baby-sized things. So if you teach in what I'm going to call a "more convergent way," those opportunities just never come up. Because you're going to say, "Which is more?" There you go—that's

the end of the story. There's nothing else to talk about.

You are the Chair of OAME 2019! And thank you for taking on that giant task. It's not up to one person to dictate the aims and goals, but we're in a period right now of interesting times in mathematics education. What do you see as the role of OAME for supporting teachers?

There really is a core that's big—not one person—of people who are very integral, who really are committed, who really have leadership roles. It's a wonderful organization. I love being here. It's a big province with a lot of fantastic math educators. I think OAME has played a significant role in that. It's a wonderful organization for teachers. The *Gazette* is a great thing. The conference is great. I tell people from other places that OAME is better than most NCTM regional conferences or even the [NCTM] Annual [conference]. And I mean it. You really learn from your colleagues. I actually think we have a lot of talent in the province and OAME is part of the thing that cultivates it. It really is a facilitator to make things happen. The reason I took on this thing [chairing OAME 2019] is just a payback because I really appreciate all of the work that they've done to be a real network here, who meet each other, know each other, get together, [and] help each other.

Wonderful. Thank you very much for your time! ▲

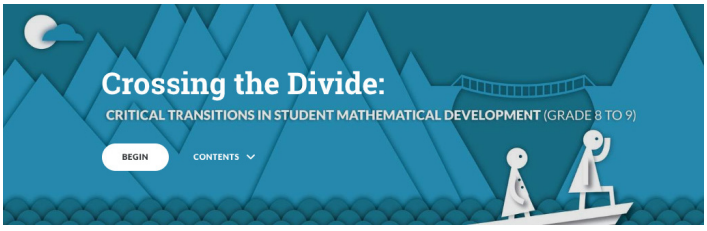
▲ Crossing the Divide: Critical Transitions in Student Mathematics Development (Grade 8 to 9)

DR. DANIEL JARVIS

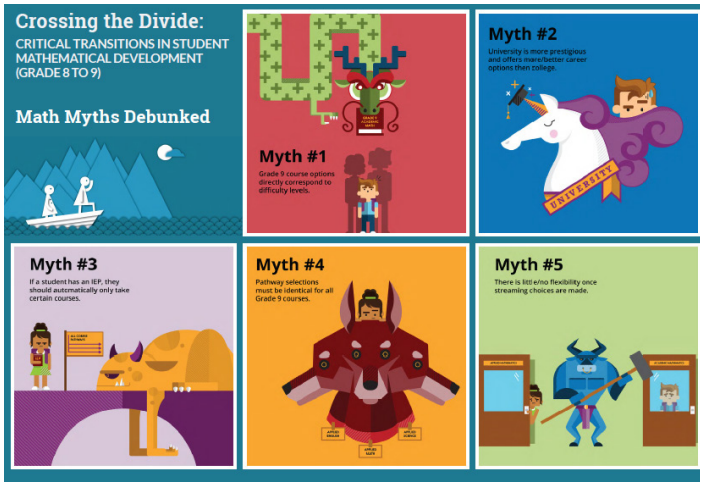


Dr. Daniel Jarvis is a Professor of Graduate and Mathematics Education in the Schulich School of Education at Nipissing University. With a unique background in the areas of mathematics and visual arts, he has taught both topics at the secondary and post-secondary levels. His research interests include instructional technology, mathematics of the workplace, integrated curricula, innovation education, and teacher professional learning.

Dr. Jarvis has recently launched a new online (**www.mkncrossingthedivide.ca**) resource, *Crossing the Divide: Critical Transitions in Student Mathematical Development (Grade 8 to 9)*, which was designed as a support tool for teachers, students, and parents/guardians. This brief is to draw attention to what the website has to offer, but for more details, readers will need to explore the resource.



The online resource features five sections: (i) introductory welcome videos in English and French; (ii) a brief literature review; (iii) transition support strategies organized chronologically (i.e., Grade 8 strategies, interim summer strategies, Grade 9 strategies); (iv) five common math-related myths with corresponding rebuttals; and (v) a list of links to 12 other online resource and support sites. Within three of the above-mentioned sections, a reproducible infographic has also been made available for potential discussion use in professional development contexts.



This project is part of the Mathematics Knowledge Network (MKN), which represents a four-year grant from the Ontario Ministry of Education via the Knowledge Network for Applied Education Research. The MKN is comprised of four distinct Communities of Practice: Mathematics Leadership, Indigenous Knowledge, Computational Thinking, and Critical Transitions. Dr. Jarvis is serving with the Critical Transitions team under the project leadership of Dr. Lynda Colgan of Queen's University, and he is specifically focusing on the experiences of teachers, students, and parents/guardians as students transition from elementary to secondary school mathematics learning environments. ▲

OAME/AOEM AWARDS FOR EXCELLENCE IN MATHEMATICS EDUCATION



The purpose of the Ontario Association for Mathematics Education's Awards is to formally recognize individuals and educators who demonstrate an outstanding contribution to mathematics education in Ontario. The successful candidate of each award enhances the learning environment for all students, providing opportunity for students to do, see, hear, and touch mathematics in a profound and meaningful way. If you know someone who qualifies for this honour, please nominate her/him/them for one of the following awards.

OAME/AOEM Life Membership Award: This award recognizes an educator who has contributed in a significant way to OAME/AOEM, demonstrates outstanding leadership in mathematics education, and has accumulated 10 or more years of membership in OAME/AOEM.

Award for Exceptional and Creative Teaching in Elementary Mathematics: This award recognizes an exceptional and creative elementary educator who demonstrates excellence in mathematics education and contributes to the overall development of students.

Award for Exceptional and Creative Teaching in Secondary Mathematics: This award recognizes an exceptional and creative secondary school mathematics educator who demonstrates excellence in mathematics education and contributes to the overall development of students.

Elementary School Staff Award for Exceptional and Collaborative Mathematics Teaching: This award recognizes an elementary school (a school accommodating any or all of grades K–8) whose staff fosters collaborative and excellent mathematics teaching, contributes to the overall development of student learning of mathematics, and demonstrates leadership in the mathematics community.

Secondary School Mathematics Staff Award for Exceptional and Collaborative Teaching: This award recognizes a secondary school mathematics staff which fosters collegiality, team work, and excellent classroom teaching, contributes to the overall development of students, and demonstrates leadership in the mathematics education community.

Award for Leadership in Mathematics Education: This award recognizes an educator who has demonstrated leadership by contributing in a significant way to the development of mathematics educators and the enhancement of mathematics education in Ontario through innovation and/or research related activities.

Secondary School Department Award for Exceptional and Collaborative Mathematics Teaching: recognizes a secondary school mathematics staff, who foster collegiality, teamwork, and excellent classroom teaching; contributes to the overall development of students; and demonstrates leadership in the mathematics education community.

Award for Outstanding Contribution to OAME/AOEM and Mathematics in Ontario: This award recognizes an individual who has made a significant contribution to OAME/AOEM and contributed directly or indirectly to the improvement of mathematics education in Ontario but who is not necessarily an educator or a member of OAME/AOEM.

OAME/AOEM Awards Nomination Process

1. Click on the Google form link, found at **www.oame.on.ca**, to access the nomination form.
2. Please email up to three letters of support for this award to **EDs@oame.on.ca** by January 15, 2019
   Ensure all letters of support:
   - address the criteria for the award
   - include the name and position of the supporter
   - are in ONE (1) PDF document with the nominee's name and award in the file name (e.g., Karl Gauss - Lifetime Membership Award).
3. Include the Curriculum Vitae, of up to five pages, for Lifetime, Leadership and Outstanding Contribution award nominees.

DEADLINE: Nominations are due by January 15, 2019 to the Executive Directors (contact info above).
The awards will be presented at the Annual OAME Conference in May 2019.

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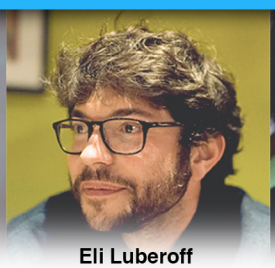
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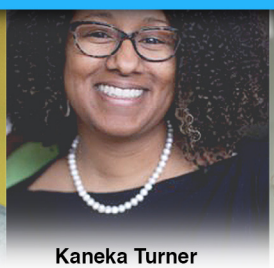
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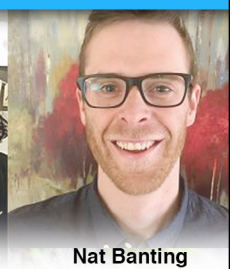

Eli Luberoff

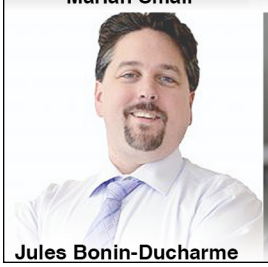

Kaneka Turner


Jennifer Wilson

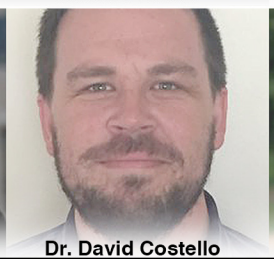


Sameer Shah & Matt Baker


Nat Banting


Jules Bonin-Ducharme


Tracey Zagerv


Dr. David Costello


Graham Fletcher


Lisa Lunney Borden


Amy Lin