▲ COMPUTER ALGEBRA SYSTEMS AND THE ONTARIO CURRICULUM

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Several years ago, *Texas Instruments* developed a new product called the *TI-92*. This was before the orginal *TI-83* had become popular and was targeted to a completely different audience. This handheld looked vastly different than its predecessors. It was much wider and the screen was in a completely different ratio. The device had a full QWERTY keyboard and a cursor wheel that could be directed in eight different directions rather than just four. The *92* also contained two features that previous models were missing – a dynamic geometry package and a computer algebra system. The geometry program was *Cabri Geometry* packaged specifically for this product. The computer algebra system was a subset of DERIVE, long considered one of the easier symbolic manipulators. Over the eight years since its introduction, the *92* has gone through three changes. First, a "Plus" chip was created to allow for upgrades and downloads over the internet. Later, it was declared that the product could not be used on the AP calculus examination due to the QWERTY keyboard, so *Texas Instruments* developed the *TI-89*. This product has a case similar to the *TI83/84* family, but has all of the functionality of the *TI92 Plus*. Finally, a new product, the Voyage 200 was introduced two years ago to replace the *TI92* altogether. For all intents and purposes, the products are functionally equivalent.

Computer algebra systems have been around for a long time. Maple, Mathematica and DERIVE were the main products on the market. Each has its own niche in the research and educational markets. The *92* marked the first time that a hand-held product had been developed with an algebra system as opposed to a computer. A number of teachers were sceptical of its use and fears were expressed (and continue to be

Johannes Buteo (c. 1525)

This French mathematician wrote this in his book Logistica*: If the price of five apples, reduced by the price of one pear is 13 dinar, and the price of 15 pears, reduced by the price of one apple is 6 dinar, what is the price of an apple and a pear?*

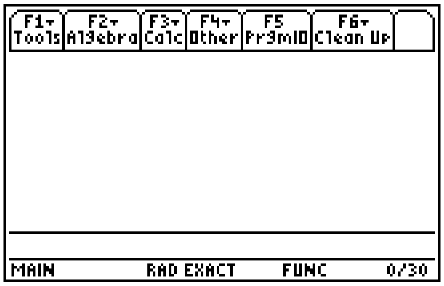
Isaac Newton (1642 - 1727)

This British mathematician wrote in his book Arithmetica Universialis the following: "In my studies I discovered that the actual problems are often of more value than the rules." He posed this problem: Three pastures have an area of 3 HA, 10 HA and 24 HA. The growth condiditons are exactly the sam ein all three pastures. The grass density and yield per area unit are the same. On the first pasture, 12 oxen graze for 4 weeks and on the second pasture 21 oxen graze for 9 weeks, at which time these pastures have been completely grazed. How many oxen could graze on the third pasture for 18 weeks?

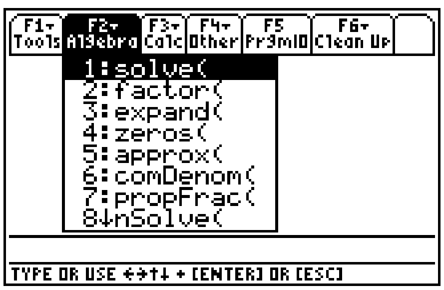
Christian Goldbach (1690 - 1764)

This mathematician was born in Prussia and lived in many countries before settling in Russia. The Goldbach Conjecture claims that every even number (except 2) is the sum of two prime numbers. This conjecture has been verified up to 2 x 10^{10}. Check this yourself for numbers up to 40.
In 1742 Goldbach sent this conjecture to the Swiss mathematician Leonard Euler. This conjecture remains unsolved today.)

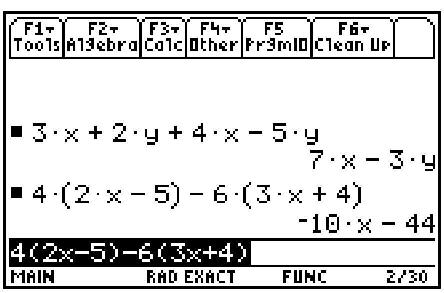
expressed) concerning the erosion of algebraic manipulative skills if these products were used by secondary school students. Let's look at some of the operations that these products will do and address some of the advantages and disadvantages. For this article, screen shots have been taken from the *TI89 Titanium* model.)



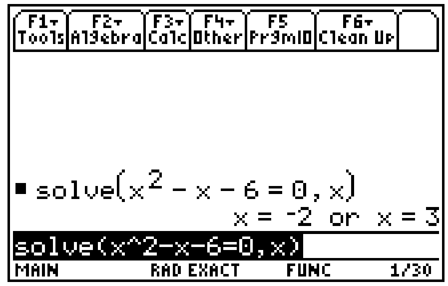
In the home screen, you see that the product presents a completely different appearance than the *TI83/84*. The menu has two items which are capable of striking fear into some math teachers and hope into the hearts of students.



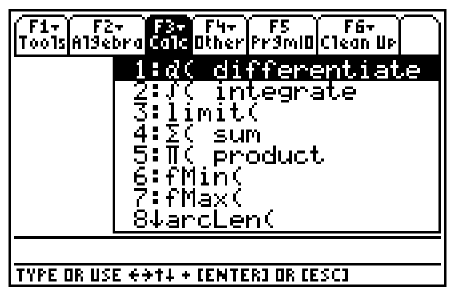
Pressing F2 brings up the Algebra menu. Most of the features here are self-explanatory. The device can solve an equation, factor an expression, expand brackets and perform a large number of manipulations that we historically have required students to perform by hand.



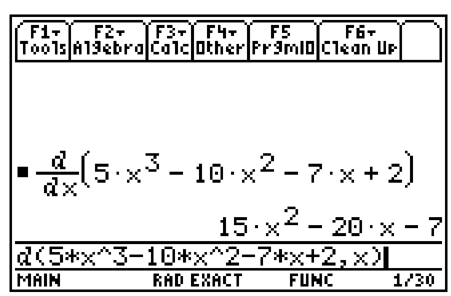
In the screen to the right, the expressions to the right of the square marker were typed on the entry line at the bottom of the screen. When ENTER is pressed, the expression is displayed on one line of the history screen and the results are shown to the right if there is enough room or on the next line if necessary.



In the next screen, the symbolic manipulator has been used to solve a quadratic equation. In this screen and the previous one, it is obvious that the device has more power than some teachers would like students to have. These devices though have been accepted for use on the SAT in the United States and elsewhere in Canada and overseas. Their appearance in the market engenders a similar debate among mathematics educators to that which precipitated the introduction of graphing calculators and even the introduction of calculators themselves. How these devices are integrated into a classroom will form the basis for this discussion and a question will be how this will impact the curriculum in years to come. This could be felt to be similar to how the introduction of the square root key impacted the teaching of Newton's Method for calculating square roots! We hope that this article will help kick start some of the discussion as this technology becomes affordable and accessible for students.



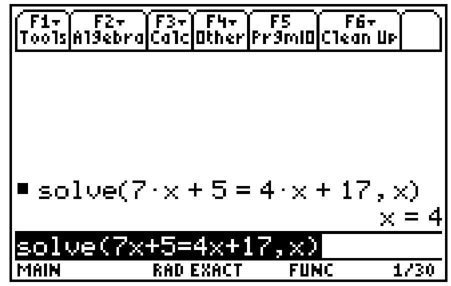
In addition to the algebra menu, the device has a very powerful Calculus menu, capable of finding derivatives, limits, integrals, sums of series, as well as other features that we may not use much in the secondary school curriculum.



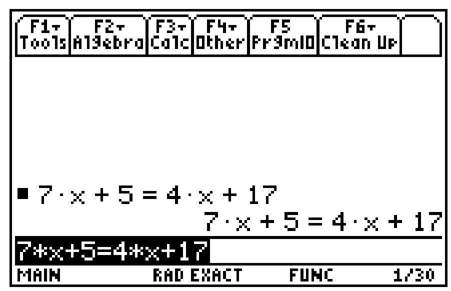
If a student has access to one of these devices, what becomes of expectations that require us to teach these skills? Obviously, a few keystrokes will accomplish a great deal. The derivative shown is a very simple power rule problem, but any expression can be used as the argument for the derivative feature.

Part of the issue is how our students will respond to the use of these devices and another fear is how some teachers will use them in class. Each year, we find students in our classes who pick up a calculator to add subtract, multiply or divide with simple values. Many use their scientific calculator as a crutch and the fear is that they will do the same here is a valid concern. To us, it would be inappropriate to teach any skill using only a piece of technology. One example that shows this is the derivative shown above. We would deem teaching derivatives solely by using a hand held device to be a poor use of technology. In most classrooms, the idea of a derivative is developed from what we call “first principles” where a limit of the slope of a secant is used to find the slope of a tangent line at a particular point. The goal, of course, is to instill an understanding of what a derivative means and not simply the mechanics of how to calculate it. At the end of this approach, the teacher frequently then does several examples involving the derivatives of powers of x in the hope that some will develop a sense of the pattern involved in the power rule. We would call this developing “symbol sense”. Technology does not replace the need to understand the concept but can be a positive tool in developing this symbol sense and investigating patterns, much like elementary students may use a simple calculator to investigate numeric patterns.

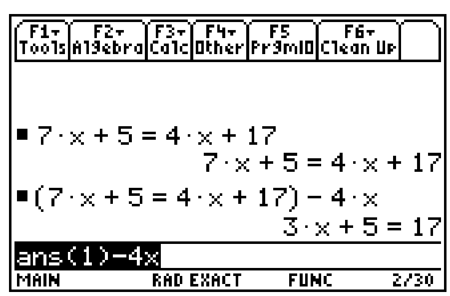
What happens to the students in your class who get so bogged down in the algebraic manipulation that they never stop to try and understand or develop the pattern or symbol sense? How often have we all seen students who, due to a simple arithmetic error, completely miss the pattern due to the error and never correct this problem? Do we continue penalizing those students for the remainder of the course on topics such as related rates or optimization problems if they have very little ability to perform a simple derivative or even worse, they can’t solve a simple equation or factor a simple trinomial? In the past, often students who fail the first test on derivatives were doomed to fail every successive assessment due to their lack of manipulative skill. After several years of working with these devices, we recommend an approach where a skill is taught and evaluated by hand first and then a technology is taught to assist students in getting past the lack of skills. In our experience, many of the students who failed a derivative test did so, not because they didn’t understand what a derivative was, but because they can’t add two terms together any better in grade twelve than they could in grade nine. We suggest that it is an appropriate use of technology to teach the skill, test it and then work with the technology in the future.



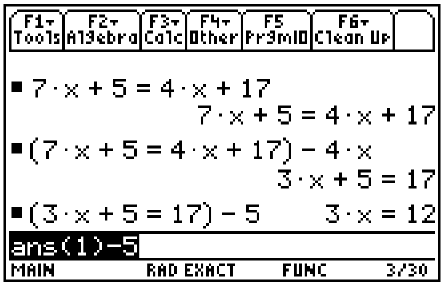
The use of a computer algebra system also lends itself to discovery of a set of steps that kids can follow. Consider the example above. We would never recommend that a student only be taught to solve a linear equation in this manner in grade eight or nine. That would be a disservice to the students. Instead, consider the following approach as an alternative.



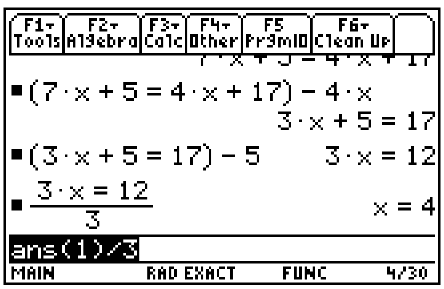
First, type in the equation and press ENTER. At first, it appears that absolutely nothing has been accomplished. In this type of equation, we would normally show our students how to balance this equation using either pencil and paper methods or perhaps algebra tiles to bring like terms together on opposite sides of the equal sign.



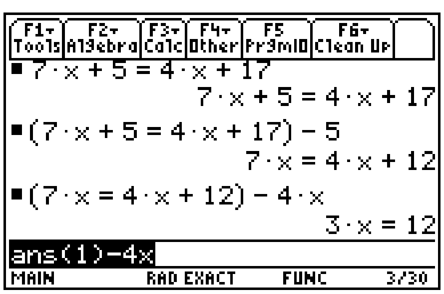
However, when the equation was entered, it is retained in memory as an expression. On the next line the user has entered the algebraic phrase "–4x". This is interpreted as an operation to be performed on the last result stored in memory, which was the original equation. On the entry line, "ans(1)" refers to the original equation. The result shown is exactly what we hope our students would get after subtracting 4x from each side.



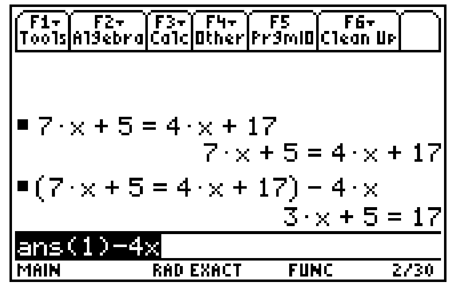
Let's continue this approach a bit further. The next operation that we would have kids perform is to move the constant 5 to the right by subtracting 5 from each side. On the entry line, we accomplish this by entering the instruction "–5" as shown on the screen capture. The result is shown after the device executes the instruction.



Finally, we would instruct our students to divide each side by 3 in order to isolate the variable x. On the entry line, we type in "/3" and this action is performed on the result of the previous operation. The result is shown. What this approach accomplishes is almost identical to what we have kids do with paper and pencil. Basically, the hand held device acts as a number cruncher for the steps that the student must come up with.



Another aspect to this approach comes about when we discuss with students whether it mattered if we moved the 4x or the 5 first. Most of us would state this without showing it because it is a lot of writing for something that we know works – the students should just take our word for it. With the technology, it is a simple matter to clear out the steps performed and go through them again in a slightly different order. In this screen, the two steps have been reversed to get to the same point as before with the variable terms on the left side of the equal sign and the constant terms together on the right side of the equal sign.



Finally, we offer one more advantage to this approach. Many students think that the steps involved in solving an equation are some kind of magic and it really doesn't matter what we do. For example, many kids think that, since I want to get the variable terms together, all we need to do is to combine the 7x and 4x, so let's add them. The result shows that we still have the same number of terms as we started with, so nothing has been accomplished. The technology represents the result of this operation correctly according to the instruction that the student has entered and the student can clearly see that his/her guess was not right. If they do the same operation on paper the result is something like "I don't know what went wrong sir. I did all the right steps, didn't I?" With technology, it is an easy matter to clear the last line, or even to proceed correctly from this point forward.

Properly used, a computer algebra system can help our students to understand algebraic manipulations and more importantly, build symbol sense better than they did in the past and allow them to get over the hump of mistakes in manipulations that prevent them from doing anything constructive with the problems that we ask them to consider as applications and/or TIPS questions. If the entire focus is on manipulations, maybe all that is being accomplished is evaluating their knowledge and allowing the lack thereof to prevent them from taking mathematics a bit further! CAS has the opportunity to be a further equalizer for students as we encourage them to go beyond "mindless manipulation" into applications and problem solving!

We look forward to further discussion on this topic! ▲