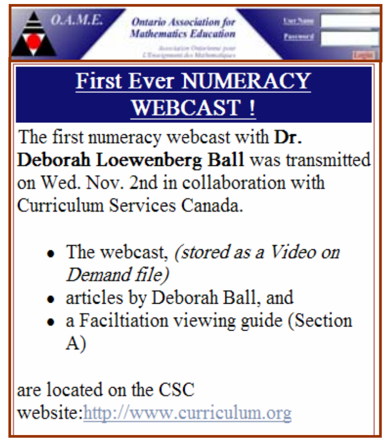
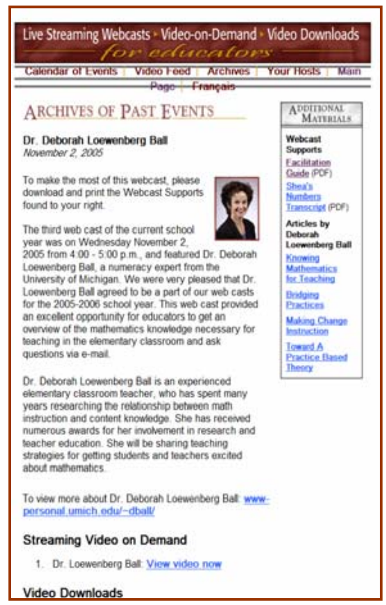
OAME Gazette – President's Message #4 (for the 2006 issue #2)
Knowing and Learning Mathematics for Teaching

Have you had a chance to view The Literacy and Numeracy Secretariat's webcast that featured Dr. Deborah Loewenberg Ball? Dr. Loewenberg Ball is a mathematics educator from the University of Michigan whose research includes the study of improving mathematics teaching through policy, reform initiatives, and teacher education and mathematics knowledge for teaching. Tens of thousands of Ontarians have viewed this webcast since its initial presentation in November 2005. You can still watch it through video streaming at the Curriculum Services Canada site, http://www.curriculum.org.





During my final president's message, I'd like to spend a few moments with you thinking through Dr. Ball's idea of knowing mathematics for teaching and discussing a few ways that you and your colleagues could explore the practices of learning mathematics for teaching.

Knowing Mathematics for Teaching K to 12

What do you think a teacher needs to know and be able to do, specifically, to teach mathematics?

Ball (2000) explained that there is a distinction between knowing how to do math and knowing math in ways that are usable in the practice of teaching. She identified two central aspects for the work of teaching.

"First is the capacity to deconstruct one's own knowledge into a less polished and final form where critical components are accessible and visible. This feature of teaching means that paradoxically, expert personal knowledge of subject matter is often, ironically inadequate for teaching. Because teachers must be able to work with content for students in its growing, unfinished state, they must be able to do something perverse: work backward from mature and compressed understanding of the content to unpack its constituent elements" (p. 245).

The second aspect is to be able to use that knowledge for teaching. This specialized mathematics knowledge; that is, mathematics knowledge for teaching, often referred to as "pedagogical content knowledge" (Schulman, 1986, 1987; Wilson, Shulman, & Richert, 1987) is a combination of knowledge that links mathematics content and pedagogy. "Included here is knowledge of what is typically difficult for students, of representations that are most useful for teaching a specific idea or procedure, and ways to develop a particular idea … This kind of knowledge is not something a mathematician would necessarily have, but neither would it be familiar to a high school social studies teacher. It is quite clearly mathematical, yet formulated around the need to make ideas accessible to others. Pedagogical content knowledge highlights the interplay of mathematics and pedagogy in teaching. Rooted in content knowledge, it comprises more than understanding the content oneself" (pp. 245-256).

Therefore, let's think about new ways for teachers to develop knowledge of mathematics for teaching. Let's focus our thinking on what teachers need to know, how they have to know it, and ways that teachers can learn to use these ideas and strategies through classroom-based inquiries.

Doing Mathematics as a Teacher

I've noticed that it is not common practice for teachers to solve the lesson's mathematics problem prior to teaching the lesson. Some teachers do not read the teaching and learning strategies and suggestions from a teacher's guide prior to teaching a lesson. I continue to hear that the teacher's guide is more often used as an answer book rather than as a pedagogical resource guide. So, I'm wondering how teachers come to know the breadth of mathematics possible from solving a mathematics problem and the probable and diverse ways that students will demonstrate their mathematical thinking and knowing when solving a division of fractions problem.

There are many approaches to teaching the division of fractions through solving problems. Typically, teachers have students practise calculating the division of fractions in worksheets and in word problems, using a division algorithm. Ma (1999) reported that teachers often relate mathematics to every day life events so that students' mathematical thinking and doing was situated within a meaningful context. In Ma's study of Chinese and American teachers, she explained that the Chinese teacher's mathematical knowledge was rooted and intertwined with real world contexts. Yet, she noted that the American teachers identified contexts for problems that were often superficially connected to the mathematics. Fosnot and Dolk (2002) explained that context problems need to be connected closely to the students' lives. Such problems are designed to anticipate and develop students' mathematical modeling of the real world. As well, these problems have built-in constraints to support and stretch the students' mathematizing.

So, as you solve this problem, think about the aspects of division of fractions that are typically difficult for students. Think about the different representations that are most useful for teaching division of fractions and ways to develop students' understanding. Think about solving this problem, in terms of doing mathematics as a teacher.

What story contexts and models are appropriate for solving $1\frac{3}{4} \div \frac{1}{3}$ in different ways?

Understanding the Problem

What does the problem ask you to do? What mathematical ideas and details from the problem will you include in your plan to solve this problem?

Anticipating the Range of Possible Solutions from Students

As you make sense of the problem, what mathematical solutions are you seeing as being possible? In terms of the stories, are you thinking of real life contexts (e.g., sharing cookies, using ratios to predict the amount of total driving time, measuring lengths of rope using imperial units)? Or, were you thinking of a jingle like "Your job is not to reason why. Just invert and multiply." The solutions to this problem focus on these concepts: measurement model (quotative or grouping – how many ⅓s are in 1¾), partitive model (ratio or sharing – finding a number such that ⅓ of it is 1¾). Though there is one numerical answer, it turns out that there are several different mathematical solutions representing each model. I think that learning to anticipate a range of possible solutions to a problem stems from solving the problem yourself with colleagues and noticing the breadth and depth of mathematics related to the problem. Now that you know the range of possibility, get your mathematical thinking tools together (e.g., manipulatives, grid paper, pencil) and generate many different solutions and story models for this problem. So, what did you do?

- Did you provide a numerical answer, one solution, or different solutions within the predicted range of possibilities?
- What different ways did you represent your solutions to the division of fractions problem?
- How do your stories relate to your mathematical models?
- How do your stories relate to your personal experiences? To your students' personal experiences?
- How is your story model useful for developing an understanding of division of fractions?

Keep talking, sharing mathematical thinking, and building on each other's ideas to develop several different solutions. Also, think about the possible solutions that students will provide and the mathematical reasons for their thinking.

Recognizing and Understanding the Mathematical Issues

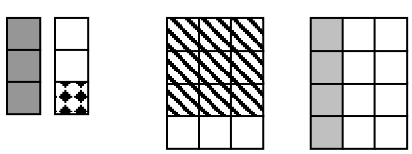
If you had your students (elementary or secondary) or colleagues at your school solve this problem, notice the mathematics in their story models and representations of division of fractions. Here are some sample solutions. A few of the solutions include typical errors.

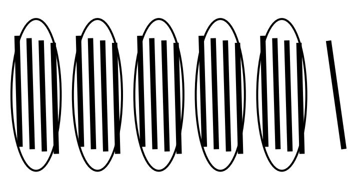
You want to split 1¾ m of rope evenly among 3 children. How much should each child get?

You have $1.75 and may soon triple your money. How much money would you end up with?

You are making a recipe for cookies that needs 1¾ cups of butter. How many sticks of butter (each stick is = ⅓ cup) will you need?

$1\frac{3}{4} \div \frac{1}{3}$ is the same as $1 \div \frac{1}{3}$ and $\frac{3}{4} \div \frac{1}{3}$
As ratios 1/(1/3) and (3/4)/(1/3)
are equivalent to 3:1 and 9:4,
$3/1 + 9/4 = 3 + 2\frac{1}{4} = 5\frac{1}{4}$





thirds	1	2	3	4	5	$\frac{1}{4}$
quarters	4	8	12	16	20	21

$1\frac{3}{4} \div \frac{1}{3} = 7/4 \div 1/3$
$= 21/12 \div 4/12 = 21 \div 4 = 5\frac{1}{4}$

⅓	⅓	⅓(3)	1
$1\frac{3}{4}$	7/4	(7/4)(3)	21/4

$1\frac{3}{4} \div \frac{1}{3} = 1\frac{3}{4} \times 3/1 =$
$7/4 \times 3/1 = 21/4 = 5\frac{1}{4}$

Think about:

- How is the mathematics similar and different among the solutions?
- Which representations were more useful for learning division of fractions?
- How do the story models provide possibility for the use of particular mathematical models and strategies for dividing fractions?
- Which mathematical ideas and strategies were represented more frequently and less frequently among the collection of solutions?
- What aspect of division of fractions is typically difficult for students and teachers?
- What possible errors and/or omissions typically occur in the solutions? Why?

Learning Mathematics for Teaching – A Framework for Professional Learning

If we were only interested in calculating the numerical answer, then calculating $1\frac{1}{4} \div \frac{1}{3}$ might appear to be an insignificant contribution to the goal of learning mathematics for teaching. However, the experience of solving this division problem is intriguing because the problem requires the development of multiple solutions (i.e., mathematical and story models), thus expanding the diverse possibilities for mathematical thinking and doing. Yet, the problem is sufficiently constrained with problem details (i.e., $1\frac{1}{4} \div \frac{1}{3}$) to ensure that the mathematical ideas are not too diverse and can be interrelated. When the teacher is aware of the mathematical possibilities in the problems posed to students, he/she can determine whether significant mathematics learning can emerge.

So, what would a professional learning framework look like that focused on learning mathematics for teaching, as described by Ball (2000, 2003, 2005)? What would a professional learning framework look like that was designed using the five key conditions of complex learning systems that Davis & Simmt (2003, 2005) outline: internal diversity, internal redundancy, organized randomness, decentralized control, neighbour interactions? This is the question that we should consider as we participate in and organize professional learning experiences for teachers in our OAME chapters and in our school boards.

Continuing Our Collective OAME Work

Through the individual work of OAME members and the collective practices of the OAME community, our shared goal to improve student achievement through improved instruction is being realized. At our upcoming annual conference, the OAME community will come together as a provincial unity to share our collective knowledge and practices of mathematics.

Come join our mathematics learning community at our 2006 OAME provincial mathematics conference, *Every 1 Counts.* It will be held at Fanshawe College, London, Ontario from May 11 to 13, 2006. This conference will include administrator and teacher workshops that provide practical ideas and strategies for improving mathematics teaching and learning in classrooms. As well, OAME's latest professional development resource, *Growing Up Mathematically*, will be presented. I hope to meet you there!   KKZ

References

Ball, D. l. & Bass, H. (2003). Toward a practice-based theory of mathematical knowledge for teaching. In B. Davis & E. Simmt (Eds.), Proceedings of the 2002 Annual Meeting of the Canadian mathematics Educatiion Study Group, (pp. 30-14). Edmonton, AB: CMESG/GCEDM.

Ball, D. & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), Multiple perspectives on the teaching and learning of mathematics (pp. 83-104). Westport, CT: Ablex.

Ball, D. Hill, H., and Bass, H. (2005). Knowing Mathematics for Teaching: Who knows mathematics well enough to teach third grade, and how can we decide? American Educator, pp. 14, 16, 17, 20-22, 43-46.

Davis, B. and Simmt, E. (2005). Mathematics-For-Teaching: An ongoing investigation of the mathematics that teachers (need to) know. Educational Studies in mathematics, pp. 1-27.

Davis, B. & Simmt, E.. (2003). Understanding learning systems: mathematics education and complexity science. In Journal for Research in Mathematics Education, vol. 34, no. 2: 137 -167.

Fosnot, C.T. & Dolk, M. (2002).Young mathematicians at work: Constructing fractions, decimals, and percents. Portsmouth, NH: Heinemann.

Ma, L. (1999). Knowing and teaching elementary mathematics. NJ: Lawrence Erlbaum.

Shulman, L.S. (1987). Knowledge and teaching: Foundations of the new form. Harvard Educational Review, 57, 1-22.

Wilson, S.M., Shulman, L.S., & Richert, A. (1987). 150 different ways fo knowing: Representations of knowledge in teaching. In J. Calderhead (Ed.), Exploring teacher thinking (pp. 104-124). Sussex, UK: Holt, Rinehart & Winston.