

ONTARIO MATHEMATICS OLYMPIAD

SAMPLE QUESTIONS ANSWERS

NOTE: This was an event for two students.

MAGIC SQUARES

Magic squares are mathematical puzzles that date back to 2200 BC, when they were called, Io-shu. In a magic square all the rows, columns and diagonals sum to the same number.

Fill in the following square using only the numbers from 0 to 15 (once each) so that the sum of the rows, columns and diagonals is 30.

15	4	8	3
2	9	5	14
1	10	6	13
12	7	11	0

12 marks

6 for square

6 for combinations

How many interesting number combinations can you find in this square?

1. all rows add to 30
2. all columns add to 30
3. all corners add to 30
4. sum of 4 numbers in quadrant is 30
5. sum of middle numbers in top and bottom rows is 30
6. sum of 4 middle numbers is 30
7. sum of middle numbers in left and right columns is 30
8. sum of corners of 3x3 squares is 30
9. sum of numbers opposite each other within diagonals

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PERFECT METRIC TIME

In a metric perfect world, a full metric day would have 10 metric hours, one metric hour would have 10 metric minutes, one metric minute would have 10 metric seconds and one metric second would have 10 metric miniseconds.

Can you record the following information?

- (a) Average metric time your pair wakes up in the morning.
- (b) Metric time your school starts on a regular day.
- (c) Metric time your school ends.
- (d) If your team has been told to report to the Principal's office at four metric hours, five metric minutes, six metric seconds and seven metric miniseconds, what time would that be in the 24 hour system.
- (e) What could the calendar system look like with this metric perfect time?

12 marks 2 marks for each answer

$$24 \text{ normal hours (h)} = 10 \text{ metric hours (mh)} \quad \therefore h \times \frac{10}{24} = \text{mh}$$

$$(a) 7:00 \text{ am} \quad 7 \times \frac{10}{24} = 2.917 \text{mh} = 2\text{mh } 9\text{mm } 1\text{ms } 7\text{mms}$$

$$(b) 9:00 \text{ am} \quad 9 \times \frac{10}{24} = 3.750 \text{mh}$$

$$(c) 3 \text{ pm} \quad 15 \times \frac{10}{24} = 6.250 \text{mh}$$

$$(d) 4\text{mh } 5\text{mm } 6\text{ms } 7\text{mms} = 4.567 = \frac{10}{24} \times h \quad h = 10.9608 \text{ or about } 10:57$$

$$(e) \text{ your time } \times \frac{10}{24}$$

$$(f) (f) 10 \text{ m day/wk, } 10\text{mwk/month, } 10 \text{ mmonth/year}$$

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The MISSING NUMBER

The Chinese were wonderfully expert at figures. Here is an example.

A Chinese mathematician said to write down any two numbers, provided that in forming them, you use all the digits from 0 to 9 once.

For example, you could write: 342195 and 6087

Then, next he said to add the numbers, and finally, erase the two numbers and any digit in the answer.

$$\begin{array}{r} 342195 \\ + 6087 \\ \hline 348282 \end{array} \quad \longrightarrow \quad \boxed{34 ? 282}$$

The professor glanced at the number 34 ? 282 and immediately said the Missing number was 8.

Can you supply the missing digit in the following, and explain how the Chinese mathematician was able to guess the missing digit in the answer so quickly.

$$1 ? 1341$$

Try to find the missing digits in the following numbers:

$$5 ? 3086 \quad 54 ? 028 \quad ?31346 \quad 10899 ?$$

12 marks 5 marks for answers 7 marks for explanation

The nine digits add up to 45, a multiple of 9. Regardless of those digits and 0 are arranged to make the 2 numbers, the sum of the two numbers will be a multiple of 9. When you add the digits of any multiple of nine, the sum is a multiple of nine. Hence the number is the number to add to the sum of the given digits to get a multiple of nine $3+4+2+8+2 = 19$ one needs to add 8 $19 + 8 = 27 = 9 \times 3$

For 5 ? 3086 number is 5 For 54 ? 028 the number is 9

For ?31346 number is 1 For 10899 ? the number is 0

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LEONARDO PISANO FIBONACCI 12th Century Italy

A certain man put a pair of rabbits in a place surrounded on all sides by a wall. How many rabbits can be produced from that pair in a year, if it is supposed that every month each pair of rabbits begets a new pair which from the second month on becomes productive?

12 marks 1 mark for each month (can add marks if get idea but wrong start)

Start with

1	(1 st month)
1	(2 nd month)
2	(3 rd month)
3	
5	
8	
13	
21	
34	
55	
89	
144	

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ALICE in WONDERLAND

Lewis Carroll, the author of the story of Alice in Wonderland was not only a very talented writer, but he was quite fond of mathematical puzzles. The following puzzle has been attributed to him.

“ A queen and her son and daughter are being held in the top room of a high tower. Outside their window is a pulley with a rope over it, and a basket at each end of the rope. The baskets are of equal weight. The one outside the window is empty, and the other on the ground contains a stone with a mass of 30 kilograms. The stone serves as a counterweight.

There is enough friction in the pulley so that it is safe for anyone to be lowered in one basket provided his/her mass is not greater than the mass of the other basket by more than six kilograms. If the difference is greater than six kilograms, they might come down with such a speed that the bump at the bottom might injure them. Of course, when one basket goes down, the other basket goes up to the window.

The queen’s mass is 78 kilograms, the daughter’s mass is 42 kilograms, and the son’s is 36 kilograms. What is the simplest algorithm (fewest number of steps) by which they all can get to safely to the ground? The basket is large enough to hold any two people, or one person and the stone. No one assists the prisoners in escaping nor can they help themselves by pulling on the rope. In other words, the pulley operates only when the mass in one basket exceeds the mass in the other”

Assuming the last person gets out of the way quickly, make up a diagram of the movements (_____ down; _____ up).

12 marks (may vary) approx. 2 marks per step

1. son down, stays in
2. rock out, daughter down
3. rock is thrown down
4. son down, stays
5. rock thrown down
6. queen goes down, son goes up
7. rock thrown down
8. son down

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BACHET 17th Century France

What is the least number of weights that can be used on a scale pan to weight against any integral number of pounds from 1 to 40 inclusive, if the weights can be placed in either of the scale pans?

12 marks

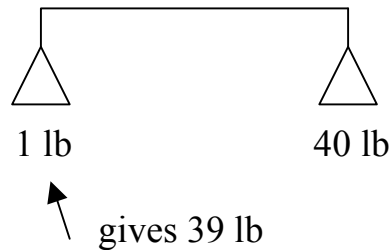
2 marks—weights on both sides (1 mark if 6 – 8 weights)

8 marks for chart, logic, organization, etc

Possible solution: List weights from 1 to 40 and eliminate all others until necessary minimum is found.

A way to solve the problem: trick is to balance pans, weights can go on both sides.

For example to get 39 lb



Minimum of 5 weights: 1 lb 2 lb 7 lb 21lb and 40 lb

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Each question is worth 2 marks

INDIVIDUAL

1. What number am I? First, 8 is added to me, then I am multiplied by 6. Then 40 is subtracted from me. Finally, I am divided by 10. The result is 11. What number am I?

$$[(X + 8) \times 6 - 40] \div 10 = 11 \quad x = 17$$

2. Which numbers have a sum of 123 and a difference of 45?

$$x + y = 123$$

$$x - y = 45 \quad x = 84, y = 39$$

3. On a test consisting of 30 questions. Anita had 50% more right answers than wrong answers. Each answer was either right or wrong.

(a) How many answers did Anita have right? Right 18

(b) What is the ratio of right answers to wrong answers? Ratio 3 : 2

4. A person cashes a cheque at the bank. By mistake, the teller pays the number of cents as dollars and the number of dollars as cents. The person spent \$3.50 before realizing the mistake, then on counting the money finds there is exactly double the amount of the cheque. For what amount was the cheque made out?

$$\text{Cheque is } \$14.32 \quad \text{received } \$32.14 \quad \text{spent } \$3.50 \quad \$32.14 - \$3.50 = \$28.64$$

5. A local restaurant sells chicken nuggets in boxes of 6, 9 and 20.

Determine whether the following amounts of nuggets can be purchased.

Number of desired nuggets: 15, 99, 19, 48, 76

15 1 box of 6 nuggets and 1 box of 9 nuggets

99 11 boxes of 9 nuggets or 15 boxes of 6 nuggets and 1 box of 9 nuggets

19 not possible

48 2 boxes of 6 nuggets and 4 boxes of 9 nuggets

76 2 boxes of 20 nuggets and 4 boxes of 9 nuggets

or 2 boxes of 20 nuggets and 6 boxes of 9 nuggets

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SAMPLE QUESTIONS ANSWERS

INDIVIDUAL (continued)

6. Old Faithful, a geyser in Yellowstone National Park, erupts in regular intervals. The park rangers use this rule to find the number of minutes to the next eruption: the length of the eruption times 4 plus 30 minutes.

Give the time for the next 3 eruptions in the following chart:

<u>TIME</u>	<u>LENGTH OF ERUPTION</u>	<u>NEXT PREDICTED TIME</u>
9:00 am	7 minutes	$4 \times 7 + 30 = 58$ 9:58 am
9:58 am	2 minutes 30 seconds	$4 \times 2.5 + 30 = 40$ min 10:38 am
10:38 am	6 minutes 15 seconds	$6.25 \times 4 + 30 = 55$ min 11:33 am
11:33 am	3 minutes 20 seconds	$3.33 \times 4 + 30 = 43$ min + 19 sec 12:16:19 pm

7. Between noon and midnight, but not counting these times, how often will the minute hand and hour hand of a clock overlap?

10 times between 1 and 2, 2 and 3, 3 and 4, 4 and 5, 5 and 6,
6 and 7, 7 and 8, 8 and 9, 9 and 10, 10 and 11

8. There are 9 red marbles and 10 green marbles in a jar. Chantal plays a strange game. She removes two marbles at a time, with the following rules:
- (a) if the marbles are both green, she puts one white back.
 - (b) if there is one marble of each colour, she puts one green marble back.
 - (c) if the marbles are both red, she puts one green marble back.
- At the end, there will be one marble left in the jar. Which colour must it be?

The remaining marble must be red.

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SAMPLE QUESTIONS ANSWERS

INDIVIDUAL (continued)

9. As you are traveling to the store you unfortunately lose $\frac{1}{3}$ of your money. Then, on the bus you lose $\frac{2}{5}$ of what you had left. When you arrive at the store you have \$1.50. How much money did you originally have?

$$\frac{1}{3}x + \frac{2}{5}\left(\frac{2}{3}x\right) + 1.50 = x \quad x = \$3.75$$

10. What is the second largest number in the set? $\{0.3, 0.9, 0.18, 0.27, .081\}$

$\{0.9, 0.3, 0.27, 0.18, .081\}$ the number is 0.3

11. Find the value of $\frac{0.2}{0.4}$. $\frac{0.2}{0.4} = \frac{2}{4} = \frac{1}{2}$ 0.5

12. If $x = 0.3$, find the value of $\frac{1}{x}$? $\frac{1}{x} = \frac{1}{0.3} = \frac{10}{3}$

13. What is the value of $12 + \frac{6}{3} \times 2 - 1$? $12 + \frac{6}{3} \times 2 - 1 = 12 + 2 \times 2 - 1 = 12 + 4 - 1 = 15$

14. What is 0.75% of 264? $0.75\% \text{ of } 264 = 0.0075 \times 264 = 1.98$

15. If 10% of a number is equal to 25% of 16, what is the number?

$$10\% \text{ of } x = 25\% \text{ of } 16 \quad .1x = .25 \times 16 \quad .1x = 4 \quad x = 40 \quad \text{number is } 40$$

16. Find the average of 6.2 and 0.62

$$\text{Average is } \frac{6.2 + 0.62}{2} = 3.41$$

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INDIVIDUAL (continued)

17. What is the number halfway between $\frac{1}{8}$ and $\frac{7}{12}$? $\frac{\frac{1}{8} + \frac{7}{12}}{2} = \frac{3+14}{48} = \frac{17}{48}$

18. The average of -3 and a second number is 2. What is the second number?

$$\frac{-3+x}{2} = 2 \quad -3+x=4 \quad x=7$$

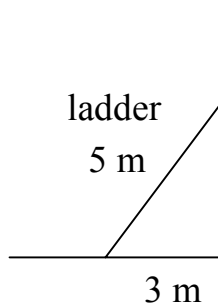
19. A railway engine is 12 metres long. To build a model railway to 1/100 scale, the length of model engine, in cm, should be ...

$$\frac{1}{100} \times 12 \text{ m} = 0.12 \text{ m} = 12 \text{ cm}$$

20. Find the area of the circle with radius, $\frac{1}{\pi}$ $A = \pi r^2$

$$\text{area is } \pi \left(\frac{1}{\pi} \right)^2 = \pi \times \frac{1}{\pi^2} = \frac{1}{\pi}$$

21. A ladder 5 metres in length is leaning against a wall 12 metres high. The foot of the ladder is 3 metres from the base of the wall. What is the distance, in metres, from the top of the ladder to the top of the wall?



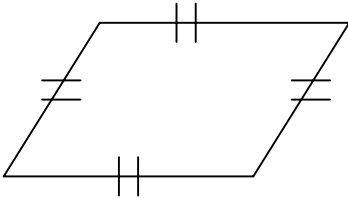
by Pythagoras $3^2 + h^2 = 5^2$
 $h^2 = 25 - 9 = 16$
 $h = \sqrt{16} = 4$
distance from top is $12 - 4 = 8$ metres

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INDIVIDUAL (continued)

22.



A parallelogram has four equal sides and a perimeter of 14 units. If the distance between the parallel sides is 5 units, find the area of the parallelogram.

Perimeter is 14 units each side is $\frac{14}{4} = 3.5$ units
Area, $A = bh$ Area is $3.5 \times 5 = 17.5$ square units

23. Find the sum of the first six multiples of 8, starting with 8.

$$8 + 16 + 24 + 32 + 40 + 48 = 168$$

24. Which of the following number(s) would be counted, if you started at 777 and counted backwards by 7's:

(a) 45 (b) 44 (c) 43 (d) 42 (e) 41

Only multiples of 7 Therefore (d) 42

25. What is the number of pairs of parallel faces on a cube?

3 pairs

26. What is the sum of the numbers on the opposite faces of a die (one of a pair of dice)?

The number is 7