Making Data Management a worthwhile *mathematical* topic

Brent Davis
University of Alberta



Recapping: The FRAME

Mathematics-for-Teaching refers to the disciplinary knowledge that teachers invoke when helping learners make sense of the subject matter.

QUALITIES:

- largely tacit/enacted,
- collectively established,
- trans-grade
- never finished
- major contributor to the shape of formal mathematics

Of particular interest to us are the figurative aspects of MfT:

- ✓ metaphors
- ✓ images
- ✓ analogies
- ✓ mnemonics
- ✓ applications
- ✓ gestures

Established and Establishing Knowledge

- "Mathematics" refers to established knowledge
- "Mathematical" refers to processes involved in establishing knowledge
- Schooling has tended to focus more on mathematics than the mathematical.

On the importance of an emphasis on the mathematical:

Humans have an deep desire for consistency that, in part, gets expressed as a quest for patterns …

FAILURE to locate patterns leads to
- categorization,
- memorization,
- procedural knowledge

(all with HUGE cognitive demands)

SUCCESS in locating patterns leads to
- generalization,
- understanding,
- adaptable knowledge

(all with tremendous cognitive economy)

An unsubstantiated assertion:

Data management has tended to be presented in terms of reduction and distillation of information rather than a quest for patterns.

An unsubstantiated assertion:

Data management has tended to be presented in terms of reduction and distillation of information rather than a quest for patterns.

FAILURE to locate patterns leads to
- categorization,
- memorization,
- procedural knowledge

(all with HUGE cognitive demands)

SUCCESS in locating patterns leads to
- generalization,
- understanding,
- adaptable knowledge

(all with tremendous cognitive economy)

Structure of the workshop

TOPICS IN DATA MANAGEMENT	TOPICS IN COMPLEXITY THINKING
✓ Quantity Sense V Number Sense	✓ What/where is knowledge?
✓ Types of Data	✓ What do complex dynamics look like?
✓ Network Theory: A New Mode of Representing Data	✓ How are complex unities structured?

Quantity Sense V Number Sense



Quantity Sense V Number Sense

"Quantity Sense" …

- ✓ is a sense of *muchness*, *size* … that is,
- ✓ it's a *feel* for an amount or magnitude.
- ✓ It appears to be, in part, innate …
- ✓ and shared with many creatures.

"Number Sense" …

- ✓ is an ability to use numbers appropriately,
- ✓ especially when combining them,
- ✓ It appears to be entirely learned (esp. early on in life) …
- ✓ and uniquely human.

Pi in the Sky:

Counting Thinking, and Being

John D. Barrow (1992, p. 168)

… Experiments reveal that some animals possess a rudimentary *notion of quantity* although not the *notion of counting*. … [We] know that … cats and dogs are able to detect a small change in quantity. Experiments with birds reveal that in general they can choose the larger of two piles of seeds if the variation in quantity is 3 and 1, 4 and 2, or 4 and 3, but generally lose track when the choice is between piles of 4 and 5.

Globe and Mail
Friday, August 20, 2004

Life Without Numbers in a Unique Amazon Tribe

Piraha apparently can't learn to count

1+1=2. Mathematics doesn't get any more basic than this, but even 1+1 would stump the brightest minds among the Piraha tribe of the Amazon.

A study appearing today in the journal *Science* reports that the hunter-gatherers seem to be the only group of humans known to have no concept of numbering and counting.

A brief demonstration of *subitizing*

- ✓ from the Latin *subitus*, sudden
- ✓ an ability to distinguish quantities at a glance
- ✓ enumeration without *conscious* counting (Stanislas Dehaene, *The number sense: how the mind creates mathematics*, 1997, p. 68)



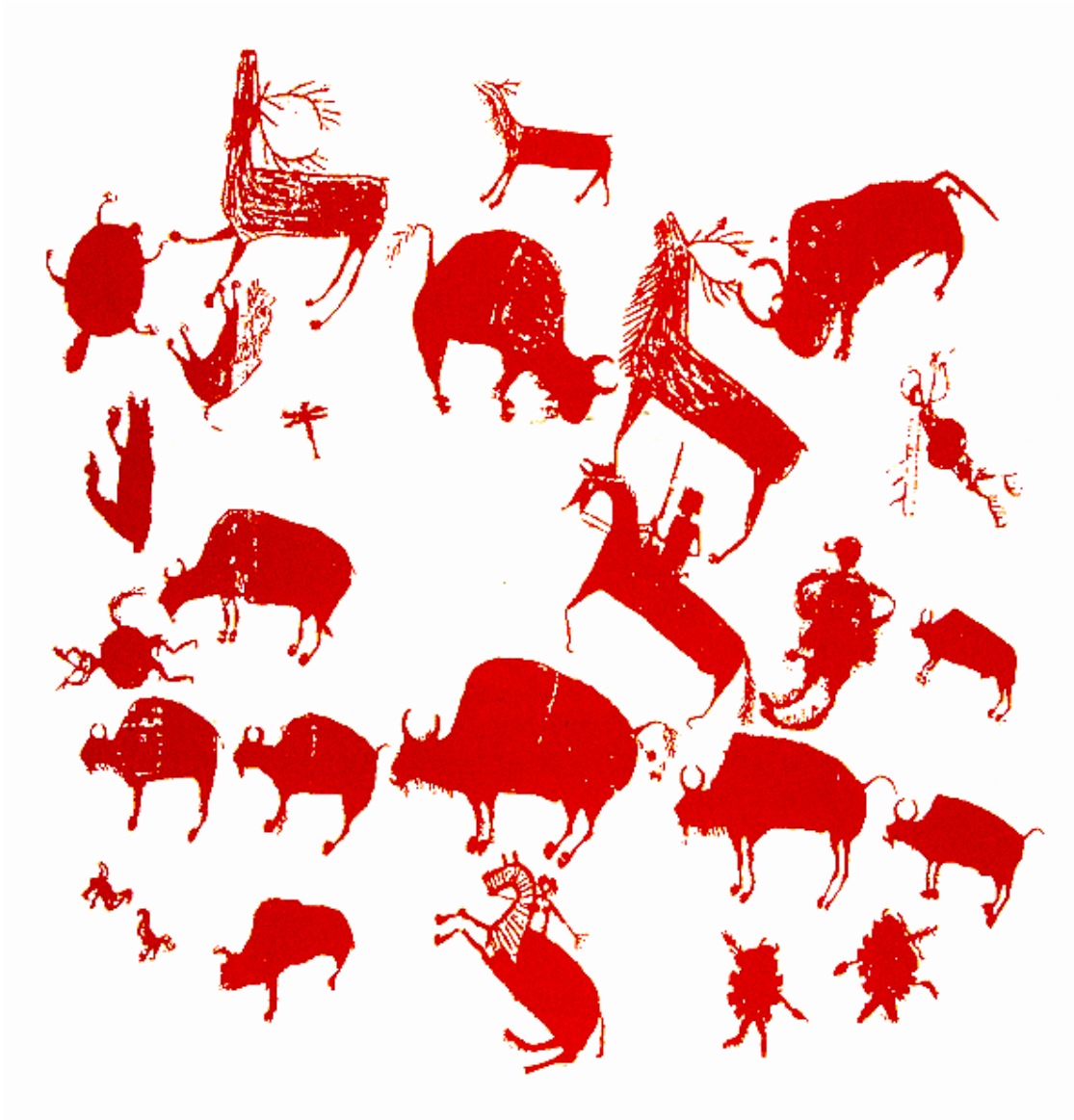










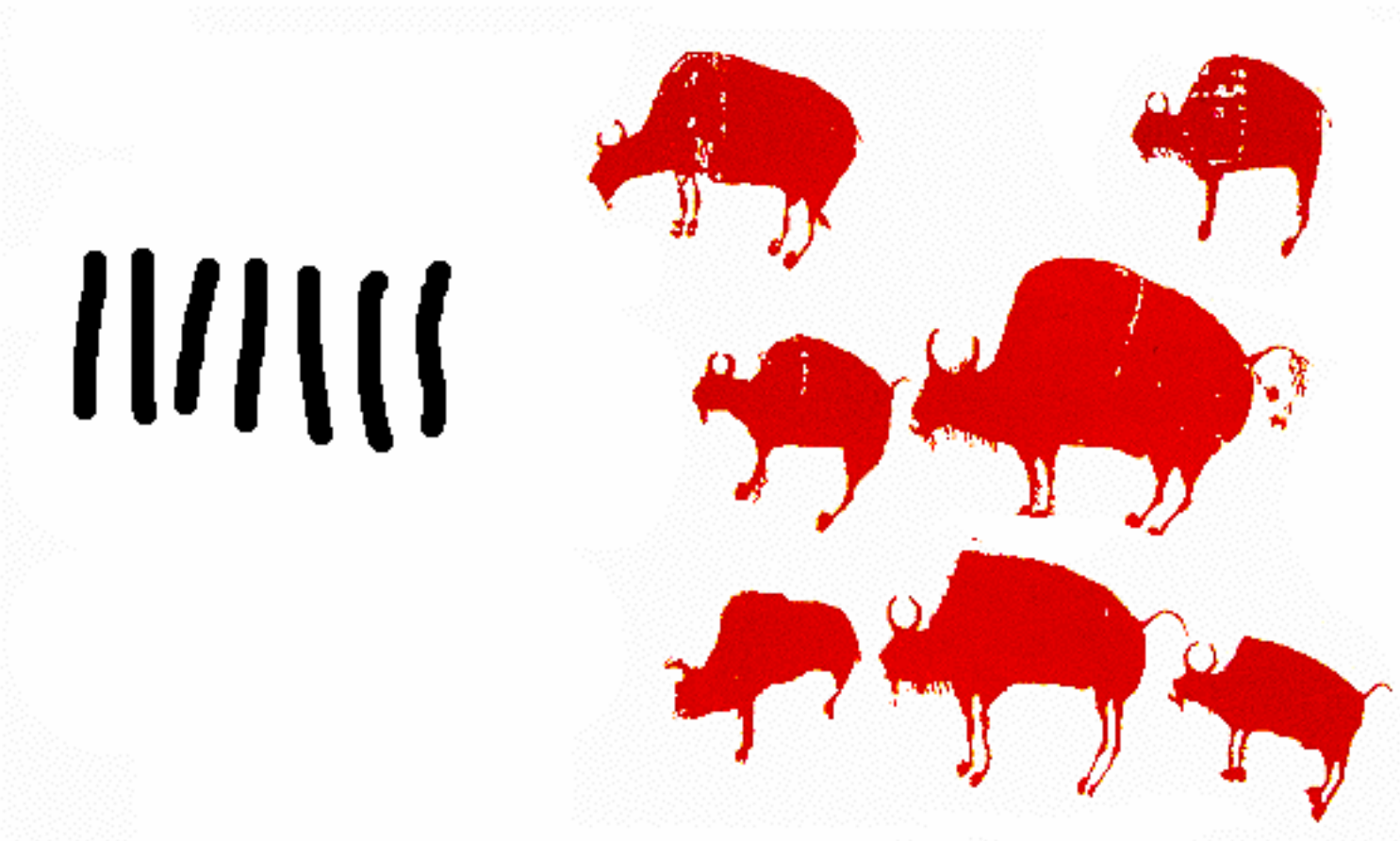


Some comments on *subitizing*

- ✓ Our subitizing limits tend to range from 4 to 7.
- ✓ It does appear that we're counting.
- ✓ Numeration systems have tended to be structured around the "natural" limits of subitization. (Our place value system isn't.)
- ✓ We can learn to be much, much better subitizers through practice …
- ✓ … but our number system actually makes it seem like this skill is unnecessary …
- ✓ … which is what makes data management a REALLY important mathematical topic.

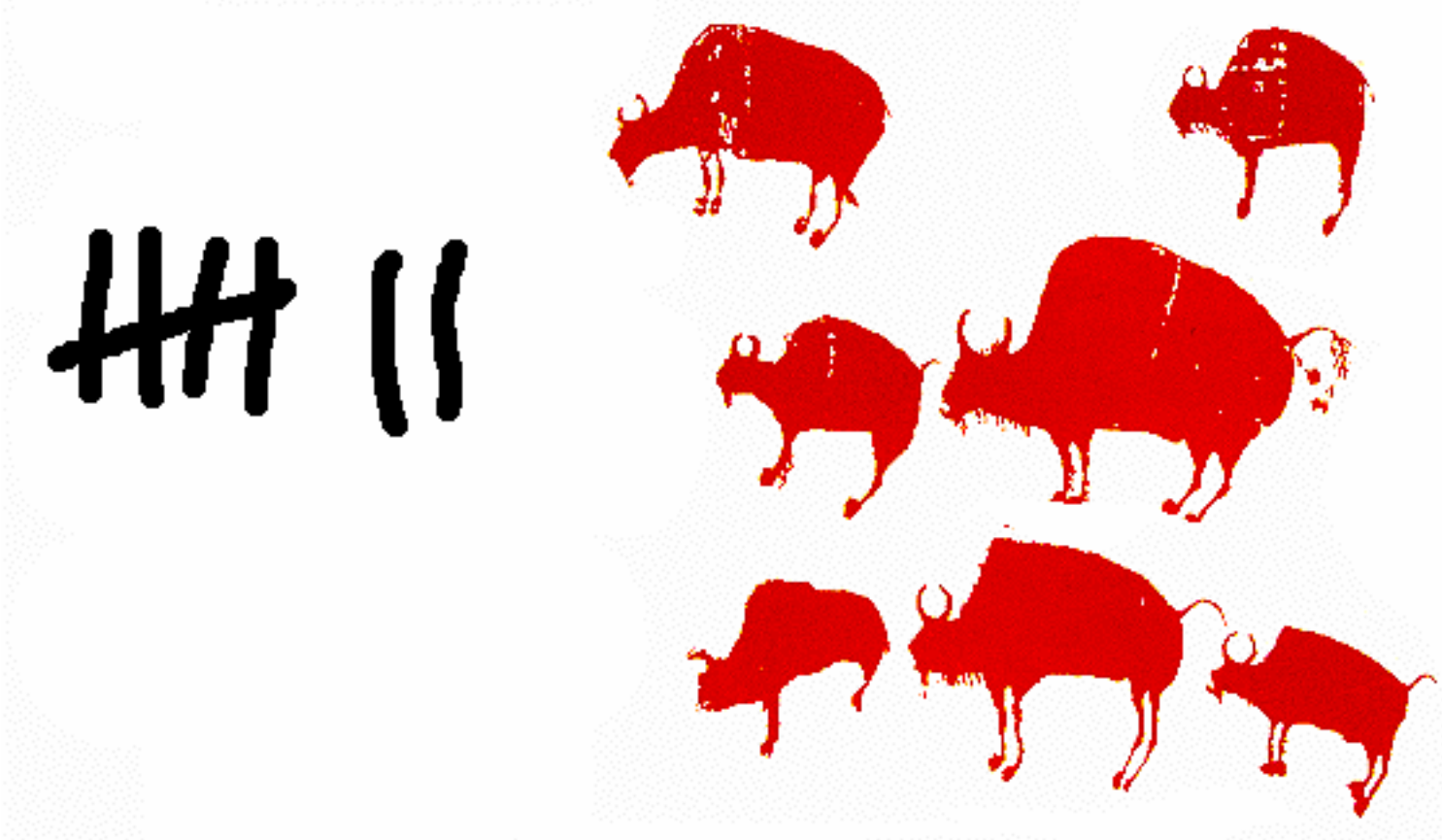
All numeration systems have relied on a reduction of information …

… from simple tallies …



All numeration systems have relied on a reduction of information …

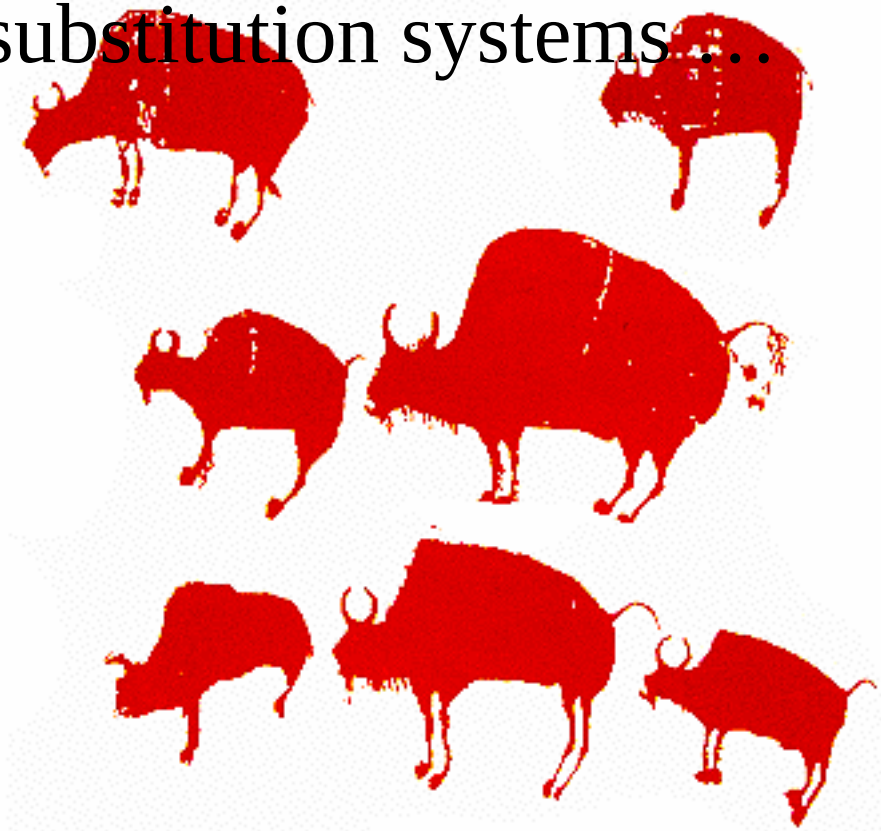
… from simple tallies …
… to more sophisticated, clustered tallies …



All numeration systems have relied on a reduction of information …

… from simple tallies …
… to more sophisticated, clustered tallies …
… to symbol substitution systems …

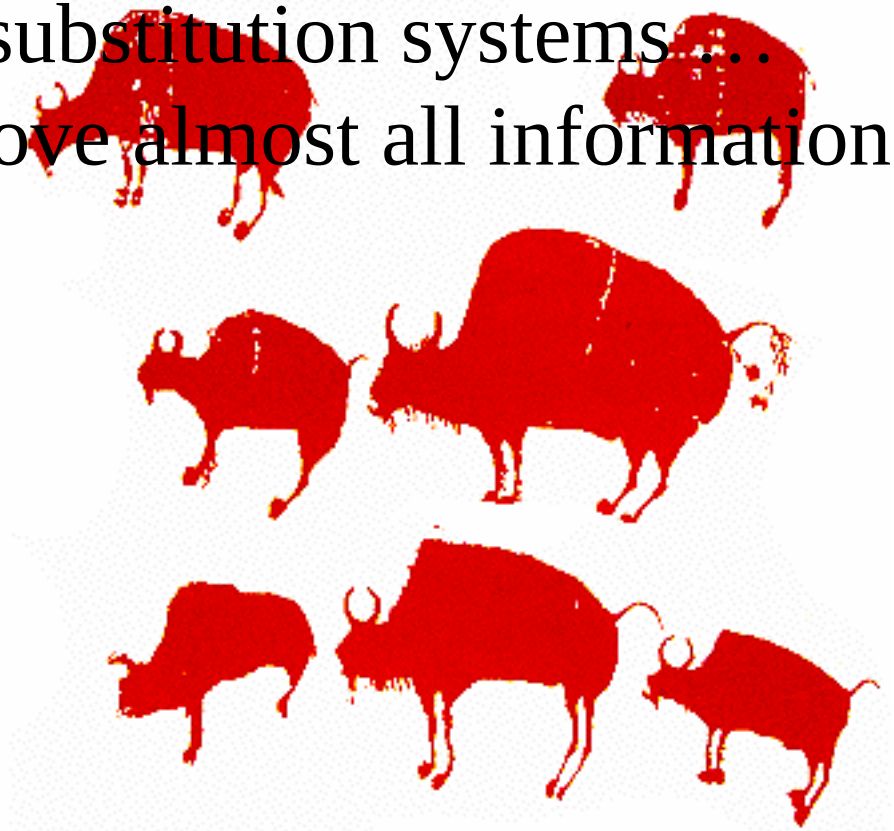
VII



All numeration systems have relied on a reduction of information …

… from simple tallies …
… to more sophisticated, clustered tallies …
… to symbol substitution systems …
… which can remove almost all information.

7



To re-emphasize, our numeration systems are useful because they reduce the amount of information ...

3 200 die in 9/11
60 000 Slaughtered in Rwanda
Up to 1 Billion Malnourished

… which is both powerful and numbing.

To re-emphasize, our numeration systems are useful because they reduce the amount of information ...

60 000 hectares of rainforest
140 species
27 000 children from hunger*

… which is both powerful and numbing.

*These statistics are borrowed from Rena Upitis's keynote address at this conference.

Another example …

A Towering loss

JDS Uniphase reported a loss for the year of $50.56-billion (U.S.).



Another example …

A Towering loss

JDS Uniphase reported a loss for the year of $50.56-billion (U.S.). That's the equivalent of a stack of U.S. $10,000 bills 2.3 times the height of the CN tower.

CN tower = 1,815 ft., 5 inches
JDS year loss = $50.56-billion (U.S.)
Height of 100 $10,000 bills (U.S.) = about 1 inch



… which is powerful … but, personally, I still don't have much sense of the quantity involved.

For me, this separation of quantity sense from number sense cuts to the core of what data management is about …

… which I imagine to be the study of strategies to render numerical data meaningful

The principal issue here is that our number system is exponential

- ✓ In contrast, *tallying systems* are additive,
- ✓ and most *symbol substitution systems* are based on multiplication.
- ✓ The exponential character of our place value system prompts us to understand large quantities in *metaphoric*, not *literal* terms …
- ✓ … e.g., 3 000 000 is 3 millions … where the **3** (not the **millions**) is the meaningful bit of information.

An experience-based strategy: Counting rice

- ✓ How many grains of rice are there in a small bowl?
- ✓ Needed: rice, small bowl, dark background paper, and a large flat surface

Counting rice …
Strategies I've encountered

- ✓ Successive halving and redoubling
- ✓ Counting (e.g., 100), building matching piles, multiplying
- ✓ Counting (e.g., 100), using it as a base for larger amounts (e.g., 1000), building matching piles
- ✓ Measuring (e.g., pen-cap-full—depending on the available technologies), counting, multiplying
- ✓ Spreading out over an area, subdividing, counting, and multiplying
- ✓ Dividing total volume (of, e.g., cone or cylinder) by a subvolume.
- ✓ Random sampling
- ✓ Using mass (rather than volume, area, or number) in one of the above strategies.

Counting rice …
Ties to Data Management

- ✓ How large of container would be needed to hold 1 000 000 grains of rice?
- ✓ 1 000 000 000?
- ✓ Illustrate the probability of winning the Lotto 6/49 jackpot.
- ✓ Illustrate how many people are there in the world.
- ✓ Illustrate the portion of the world's population that lives in Canada.
- ✓ Illustrate Bill Gates' wealth relative to, say, mine. (Note, wealth of richest 447 people = wealth of poorest 2.5 B.)
- ✓ OTHERS?

Counting rice …
Ties to Data Management Techniques

- ✓ Compare the different sorts of errors -- e.g., additive, multiplicative, exponential
- ✓ Relationship (usually inverse) between accuracy and efficiency of methods
- ✓ Discuss meaningfulness (or lack thereof) of different sorts of representations
- ✓ Be critical of the tendency for number-based (*versus* quantity-based) representations
- ✓ Others?

A COMPLEX ISSUE: Where/what is knowledge?

An orienting note: One of the hottest areas of current research is mathematical modeling (or simulation)—which might be described as a sort of data management strategy …

… that differs from strategies emphasized in math curricula* mainly around its dynamism …

… which, one might argue, is a valuable quality when representing complex phenomena.

* Apparent exception: McGraw Hill Gr. 12

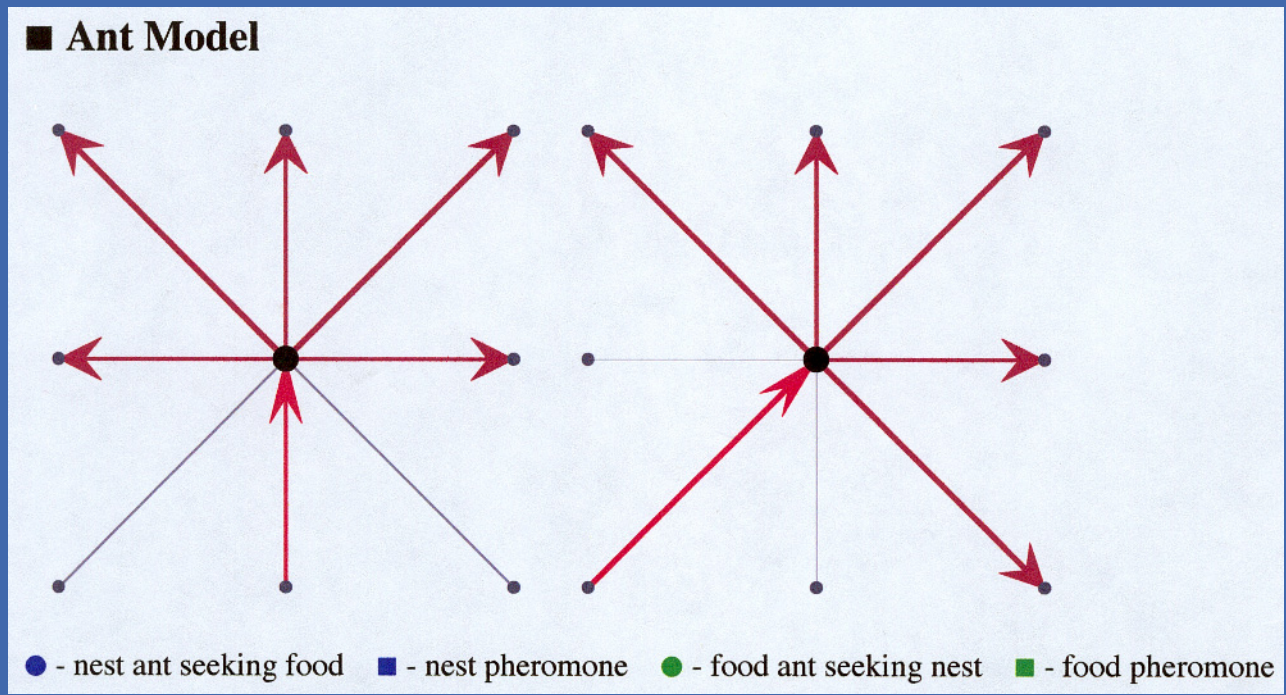
A COMPLEX ISSUE: Where/what is knowledge?

Note:

During the actual session, I made use of Jim Fuite's (of the University of Alberta) "learning ants" dynamic simulation, in which 'ants' that know virtually nothing and that act on the basis of only local information (see the next slide) collectively 'learn' the location of a pile of food.

A COMPLEX ISSUE: Where/what is knowledge?

What does each ant know? …



A COMPLEX ISSUE: Where is knowledge?

SOME POINTS:

- COMPLEX UNITIES = coherences in the stream of matter (or in the flow of activity)
- KNOWLEDGE/MEMORY = established (but mutable) patterns of activity that are *shared* (i.e., *distributed*) among agents
- So … what does that mean for the structuring of classroom activities?

Types of data—wild assertion

- Worrisomely little attention is paid to the sorts of phenomena that underlie the data used in DM units.

Types of phenomena

Complexity researchers identify three broad categories of phenomena:

- SIMPLE (mechanical, involving few parts, describable using Newtonian mechanics)
- COMPLICATED (mechanical, involving many parts, requires probabilistic and statistical tools to describe)
- COMPLEX (adaptive, involving many dynamic parts, requires phenomenon-specific research approach)

Type of phenomenon?

These phenomena have distinctive profiles:

- ✓ SIMPLE (smooth curves)
- ✓ COMPLICATED (smoothable curves)
- ✓ COMPLEX (scale independent curves)

Discussion Task

- For each category, identify two weather-chart-based phenomena that will probably generate an appropriate profile:

a) Simple phenomena
b) Complicated phenomena
c) Complex phenomena

Be prepared to justify your decisions.

Today/Tonight

Cloudy with sunny breaks. Winds northeasterly 15km/h. Tonight, cloudy with clear breaks.
High 14, low 6

Tomorrow

Variably cloudy (pop 40%).
High 17, Low 3

Monday

Variably cloudy.
High 18, Low 1

Tuesday

Variably cloudy (pop 40%).
High 15, Low 0

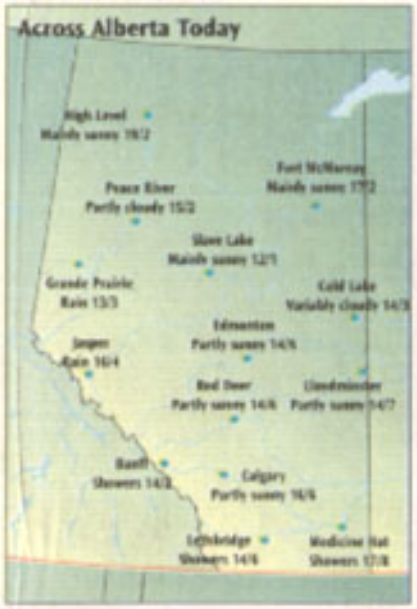
Wednesday

Mainly sunny with cloudy periods.
High 17, Low 0



Nicholas Yee, Grade 5

Across Alberta Today



Edmonton's Almanac

Information from Edmonton Int'l Airport at 7 p.m.

Yesterday's Weather

High Temperature	16.6°
Low Temperature	6.4°
Mean Temperature	12°
Barometer at 7 p.m.	100.86 kPa

Precipitation

Yesterday	0 mm
Total this month	0 mm

(The precipitation total includes both rain and melted snow. 1 mm of water is approximately 1 cm of snow)

Today's City Records

Normal High	16.0°
Normal Low	2.5°
Record High	29.9°/1987
Record Low	-9.4°/1885
Record Precipitation	19.6mm in 1993

Sunrise: 5:46 a.m.
Sunset: 9:15 p.m.

Moonrise: 5:23 a.m.
Moonset: 9:01 p.m.

May 08 New Moon · May 16 First Qrter · May 23 Full Moon · May 30 Last Qrter

Roads .....471-6056/www.amsa.ab.ca Alberta Weather 468-4340

Alberta Forecast

Edmonton: Cloudy with showers ending in the morning, cloudy with sunny breaks in the afternoon with 40% probability of precipitation. High 14. Cloudy with clear breaks tonight. Low 6. Tomorrow, variably cloudy with 40% POP. High 17, Low 3.

Central Alberta: Mainly cloudy in the morning, becoming cloudy with sunny breaks in the afternoon. High 14.

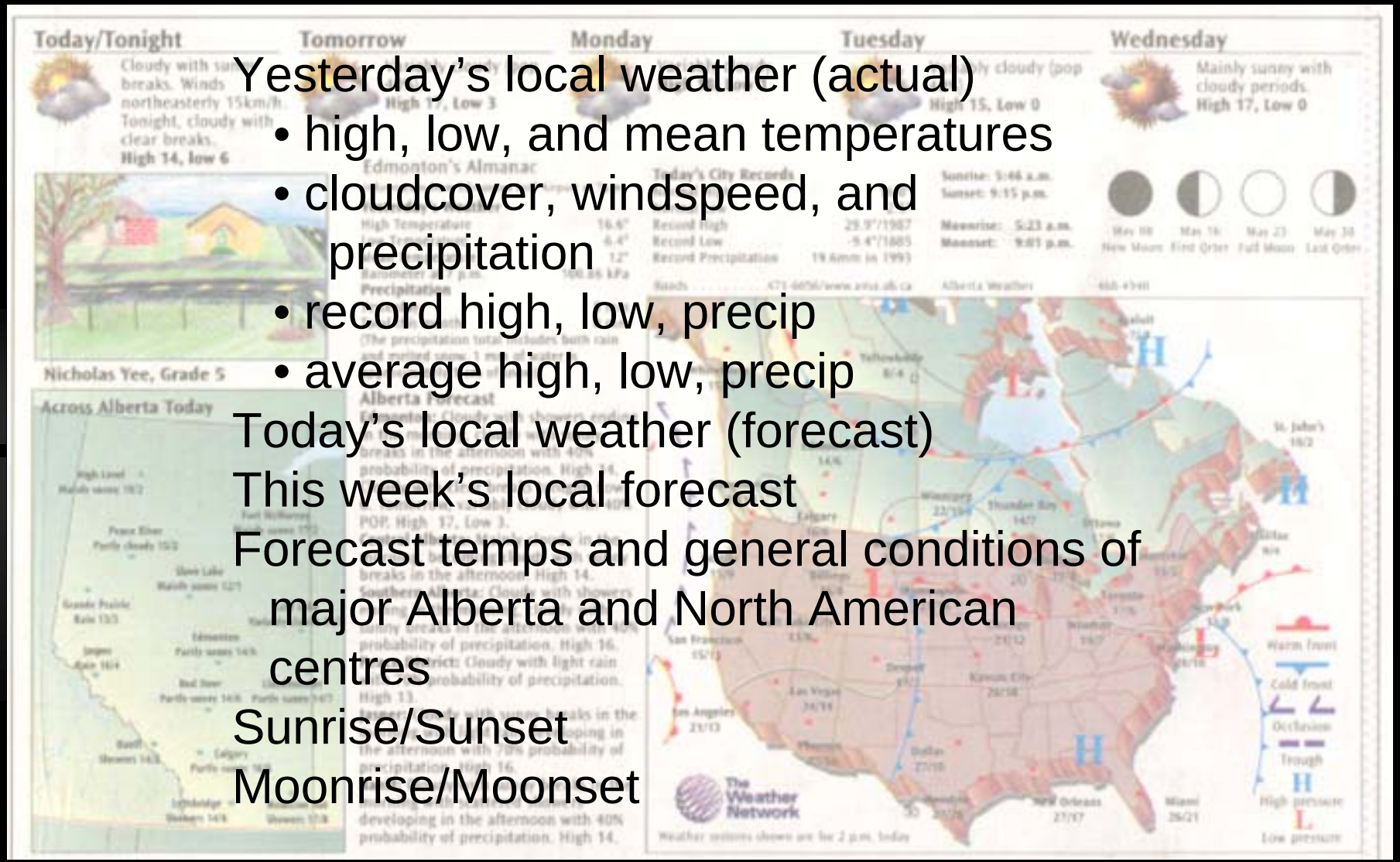
Southern Alberta: Cloudy with showers ending in the morning, cloudy with sunny breaks in the afternoon with 40% probability of precipitation. High 16.

Peace District: Cloudy with light rain with 70% probability of precipitation. High 13.

Jasper: Cloudy with sunny breaks in the morning with light rain developing in the afternoon with 70% probability of precipitation. High 16.

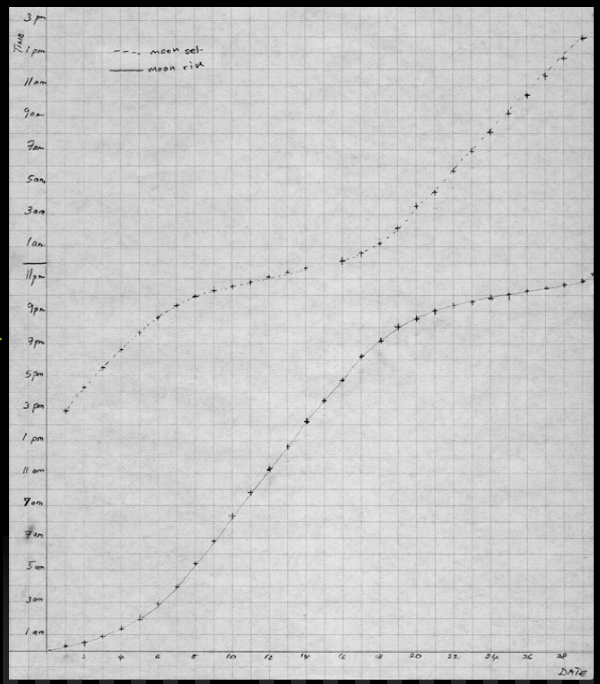
Banff: Cloudy with sunny breaks in the morning with scattered showers developing in the afternoon with 40% probability of precipitation. High 14.

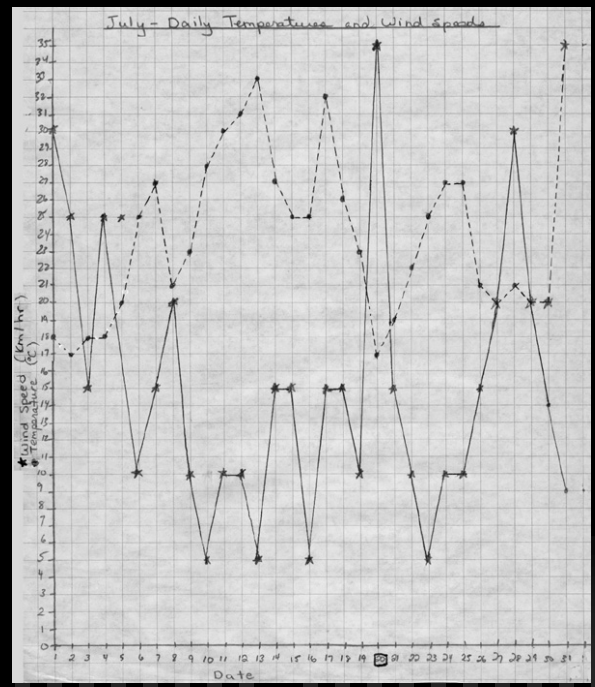




- Yesterday's local weather (actual)
  - high, low, and mean temperatures
  - cloudcover, windspeed, and precipitation
  - record high, low, precip
  - average high, low, precip
- Today's local weather (forecast)
- This week's local forecast
- Forecast temps and general conditions of major Alberta and North American centres
- Sunrise/Sunset
- Moonrise/Moonset

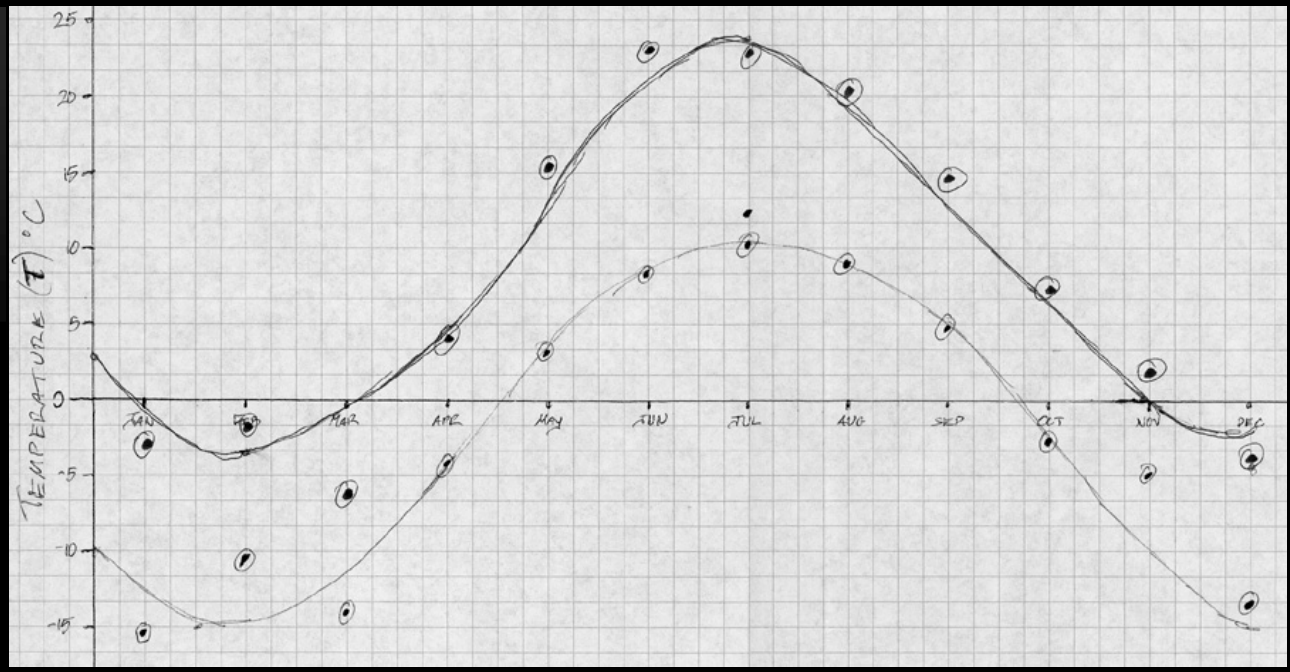
Moon rise/ setting times



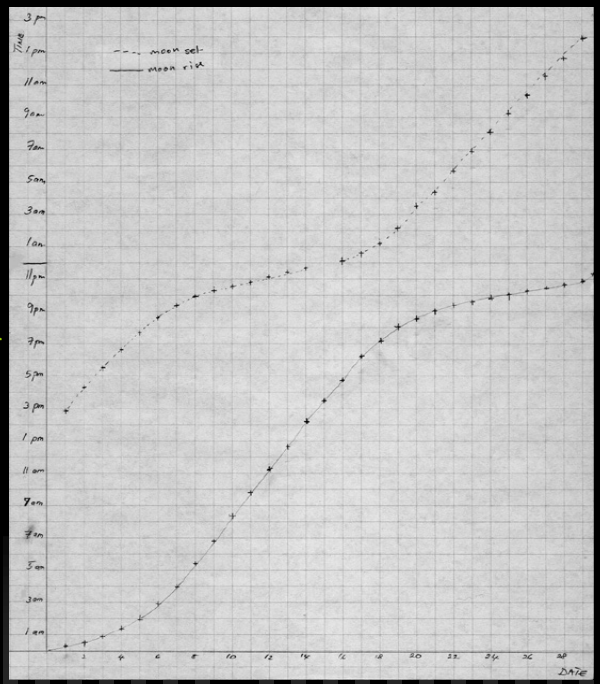


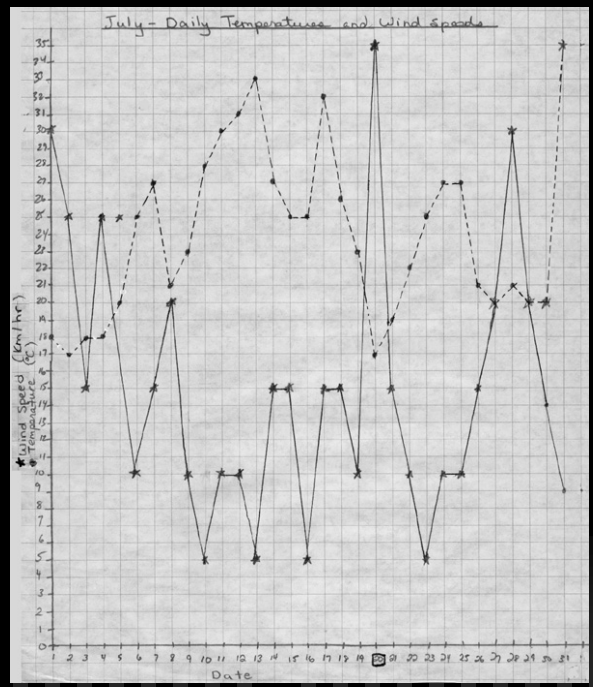
Average daily temps and wind speeds in July

Average daily temps (by month) for the year



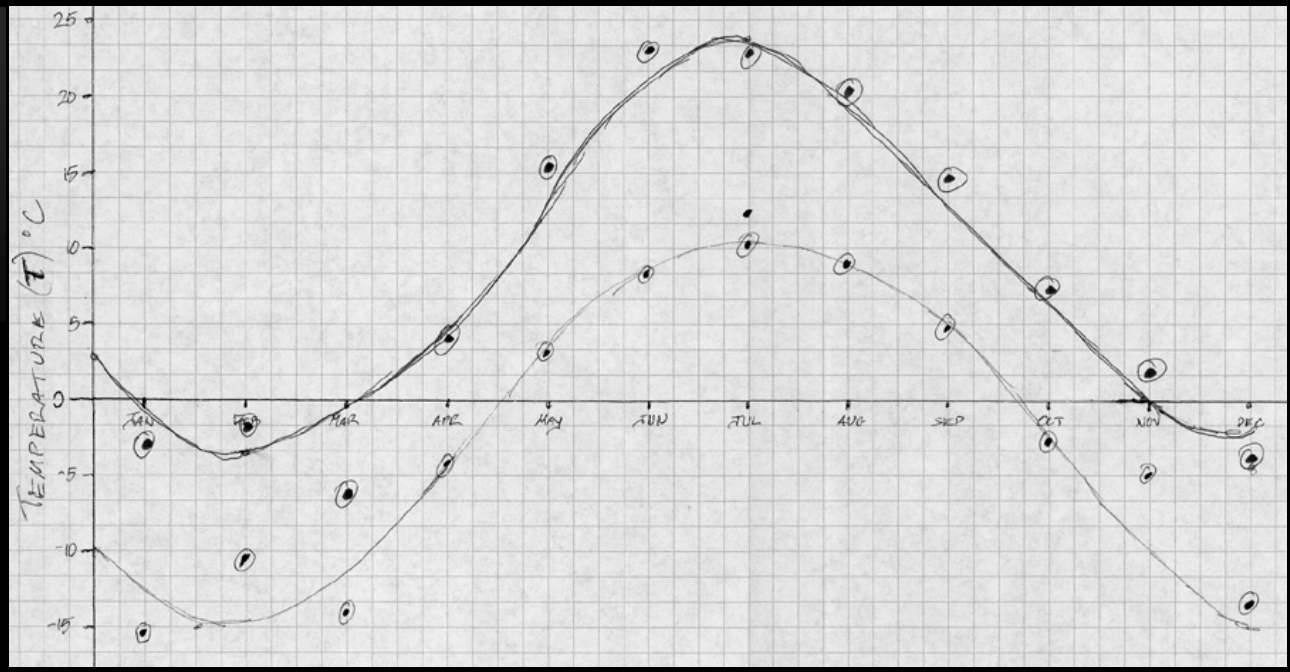
SIMPLE (smooth)





COMPLEX (scale inde-pendent)

COMPLI-CATED (smooth-able)*



*Actually, it this curve doesn't actually smooth out, suggesting that it's actually pointing to a complex phenenon.

A COMPLEX ISSUE:
What do complex dynamics look like?

A simulation …

A COMPLEX ISSUE:
What do complex dynamics look like?

Note

As with the last simulation, during the session I made use of Jim Fuite's 'learning ants' again. In this case, ratios of blue ants to green ants was monitored, as was the spatial entropy—each of which generated a scale independent curve that wouldn't smooth out.

COMPARATIVE DYNAMICS (Darren Stanley)

Which is the image of a healthy heart?

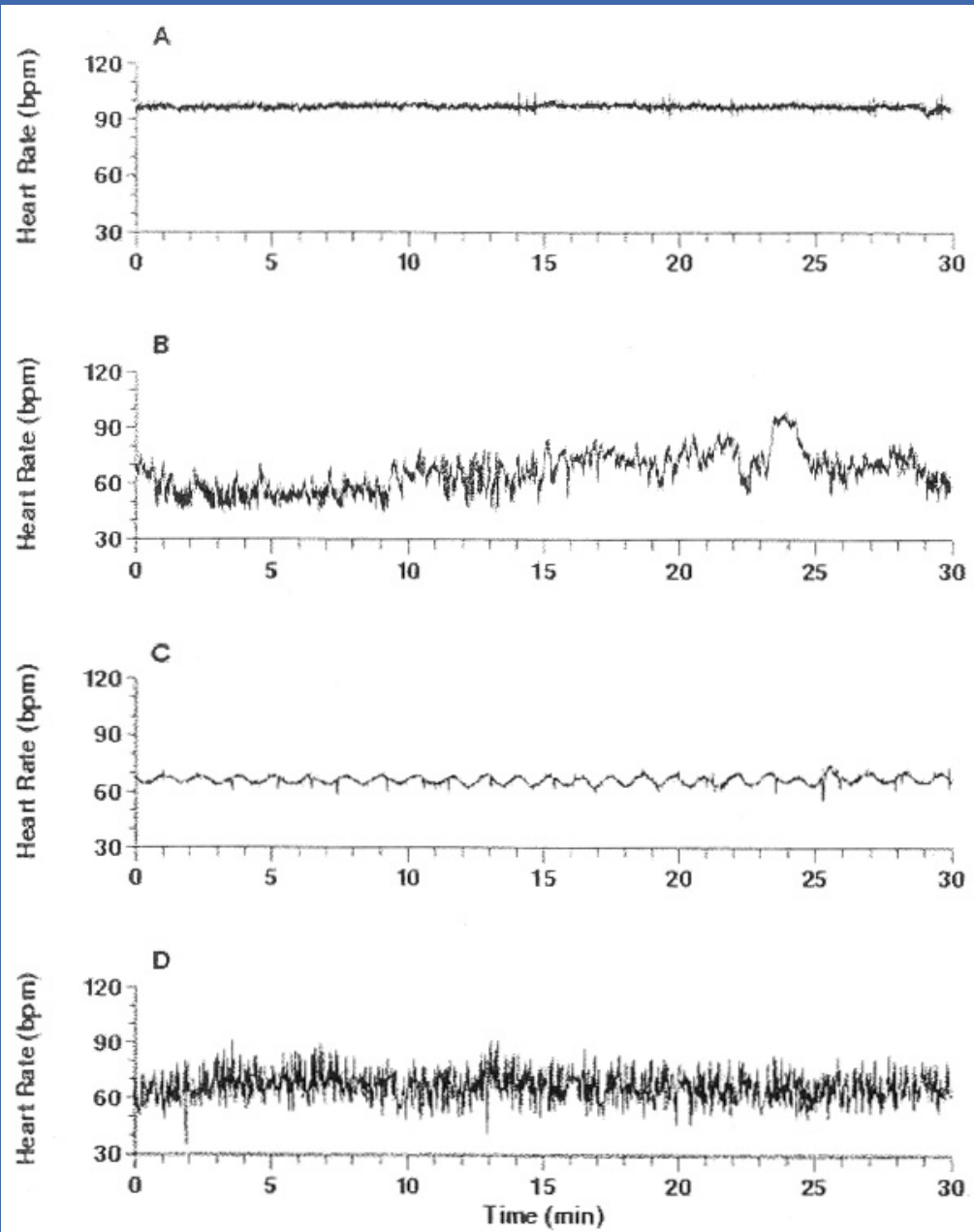
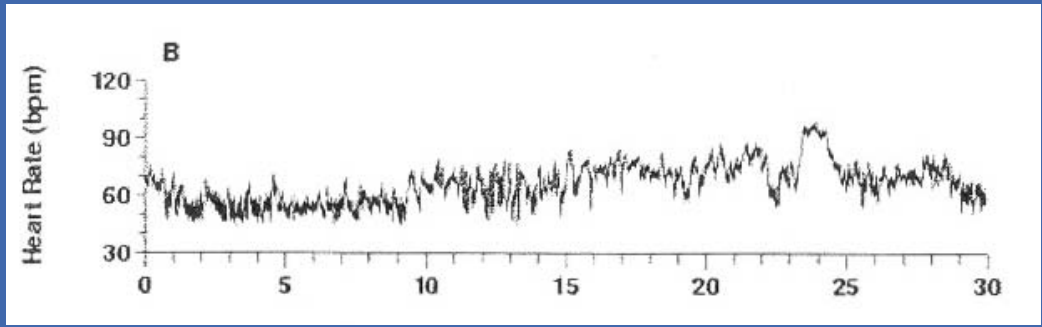


Figure 10: Time series for four representative heart rate recordings

COMPARATIVE DYNAMICS (Darren Stanley)

Which is the image of a healthy heart?



Notice how this image resembles representations of stock markets, reported temperatures, simulated ant activity …

… but does it resemble classroom dynamics? your small group dynamics? the dynamics of this workshop?

A COMPLEX ISSUE:
What do complex dynamics look like?

SOME POINTS:

- It turns out that *all healthy complex phenomena* have a dynamic profile that is fractal
- Darren argues persuasively that this fact can (and should) be used to assess the health of schools and other organizations

A COMPLEX ISSUE:
What do complex dynamics look like?

DISCUSSION TASK:

- What sorts of classroom traits should we be looking for if we want to portray healthy classroom dynamics (i.e., what are the 'vital signs' that are analogous to a healthy heartbeat)?
- Can we (or should we), as educational leaders, rethink the tools to evaluate teachers, based on complex dynamics?

[NOTE: In the following three slides, I report on some of the topics of discussion of the three groups. The first two slides were assembled immediately after the first two discussions and then used to prompt/inform subsequent discussions—so the second and third are elaborations.]

A COMPLEX ISSUE:
What do complex dynamics look like?

DISCUSSION FOCI of the FIRST GROUP:

- **Possibilities**: energy level; perplexity; variety; shifting authority; shifting emotions; physical movement

- **Issues**: temporal dimensions, needs for stability; choice (of groups, activities, difficulty, resources, etc.)

A COMPLEX ISSUE:
What do complex dynamics look like?

DISCUSSION FOCI of the SECOND GROUP:
SOME POSSIBLE VITAL SIGNS OF THE CLASSROOM:

ACTIVITY

- watch accelerated, soundless video; monitor sound level of purposeful noise; monitor room temperature

ENGAGEMENT

- monitor patterns of pauses; monitor tones of voice; watch with infrared for bodily temperature variations; monitor affective responses

NETWORK STRUCTURE

- monitor who speaks … to whom; monitor physical movement; monitor variety, availability, and use of resources

A COMPLEX ISSUE:
What do complex dynamics look like?

DISCUSSION FOCI of the THIRD GROUP:

ACTIVITY

- questioning (i.e., types of questions that teachers ask; types that students ask)
- number of distinct interpretations of a concept represented
- interplay of physical activity and energy in the class

ENGAGEMENT

- nervous behavior (e.g., tapping, fidgeting, withdrawing)
- body language (e.g., crossed arms)
- how quickly the time seems to pass for participants
- voice inflection, tonality, and volume
- what are learners talking about as they leave the room?

COMMENT

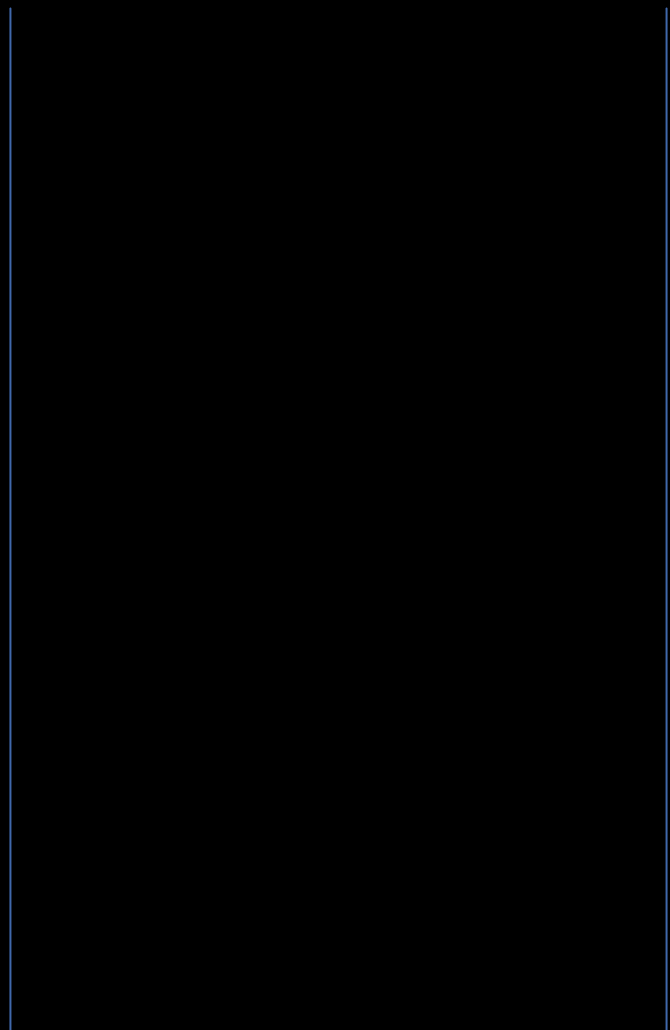
- There are emergent issues at the level of social collectivity (e.g., gender, race, and similar issues) that should compel a certain complexification of a classrooms 'vitals.'

Network Theory

- With the recognition of a new class of (complex) phenomena comes the realization that we need a new means of representing the structures of complex unities …
- Hence: NETWORK THEORY

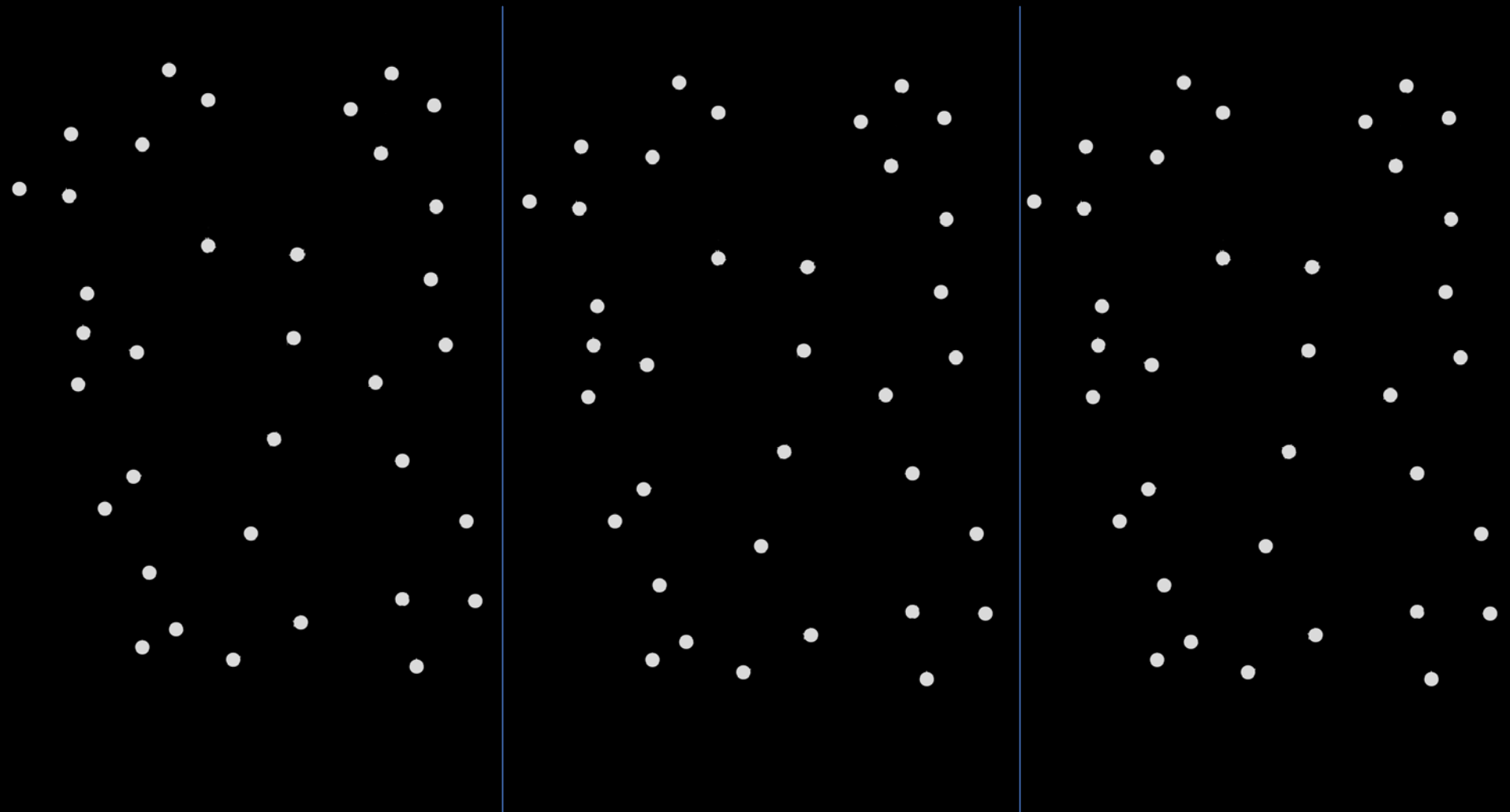
The Structure of Complexity

Theorists have identified three types of networks:



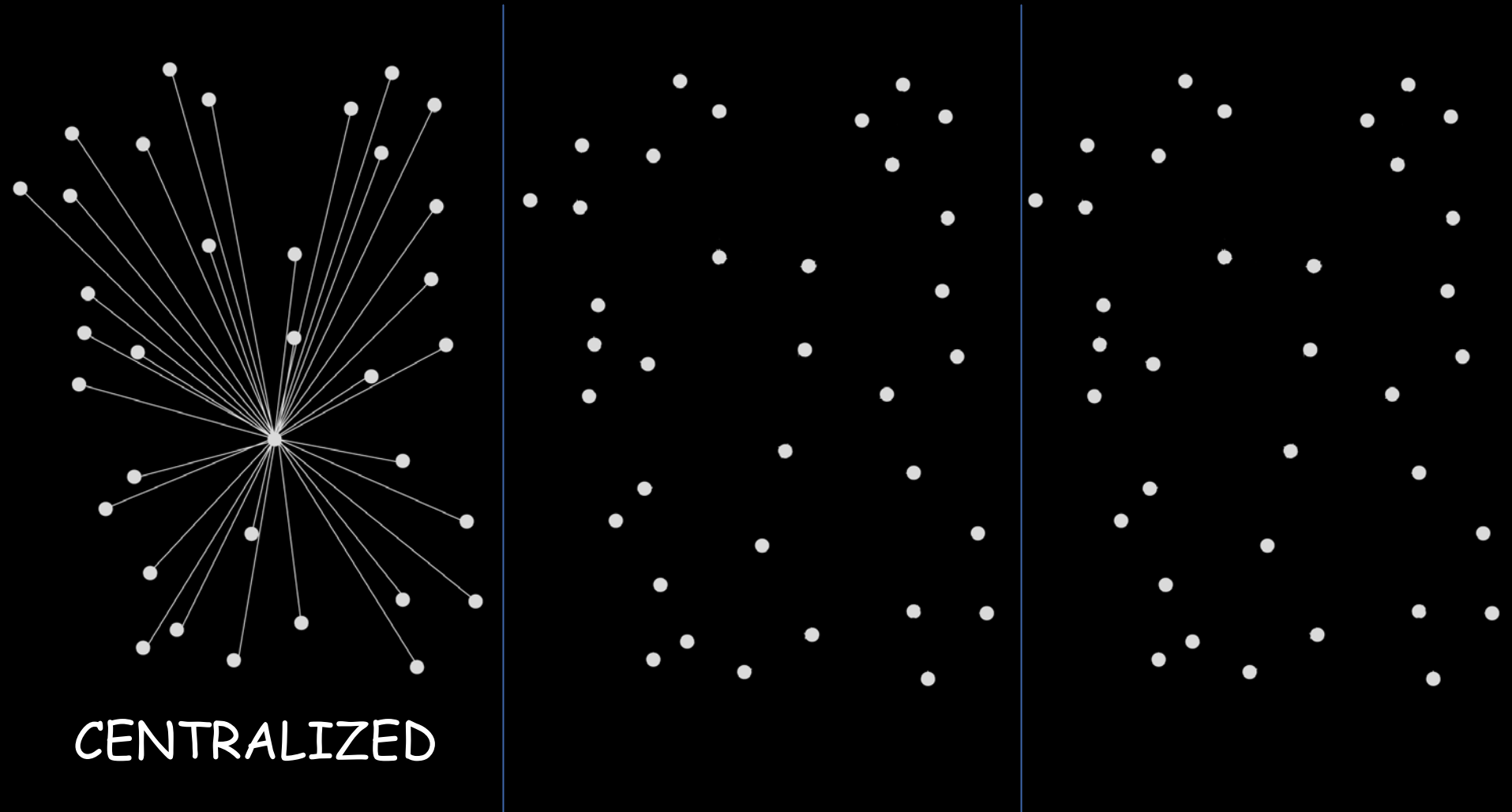
The Structure of Complexity

Theorists have identified three types of networks:



The Structure of Complexity

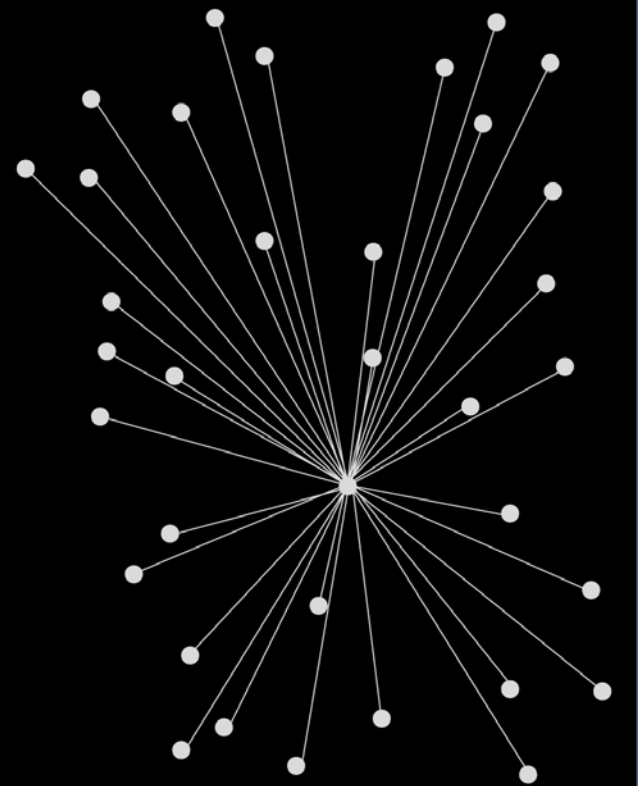
Theorists have identified three types of networks:



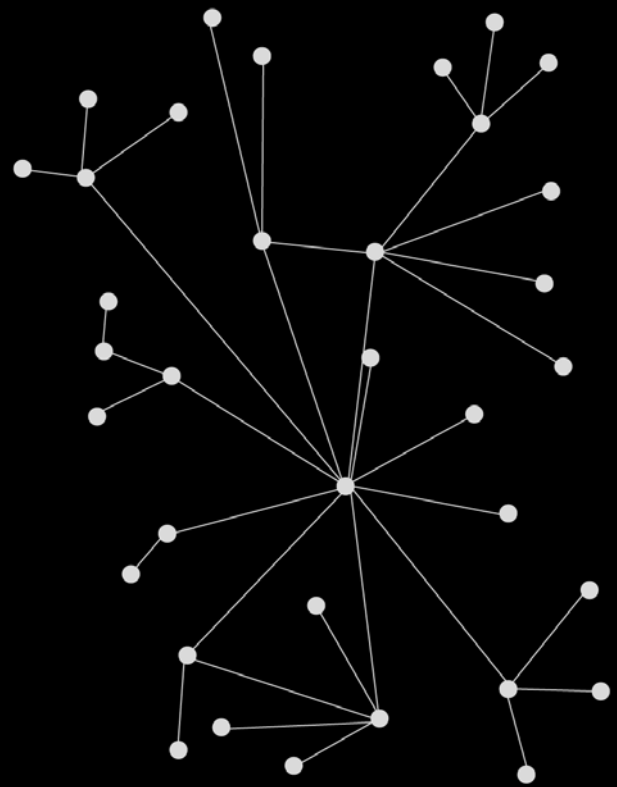
CENTRALIZED

The Structure of Complexity

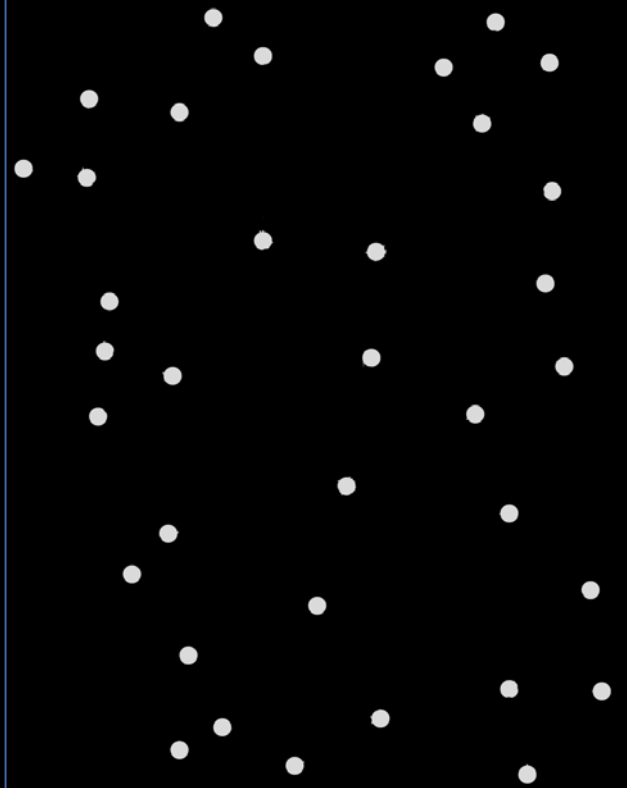
Theorists have identified three types of networks:



CENTRALIZED

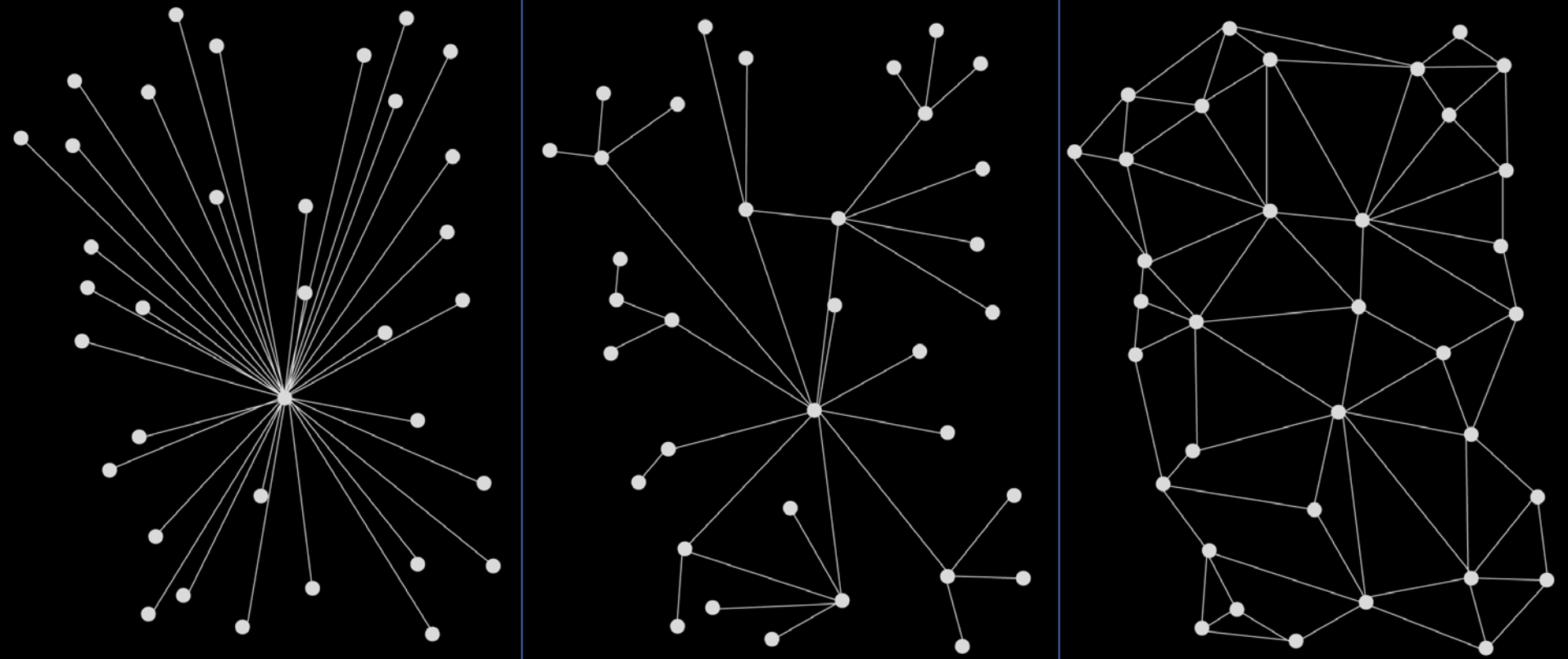


DECENTRALIZED



The Structure of Complexity

Theorists have identified three types of networks:



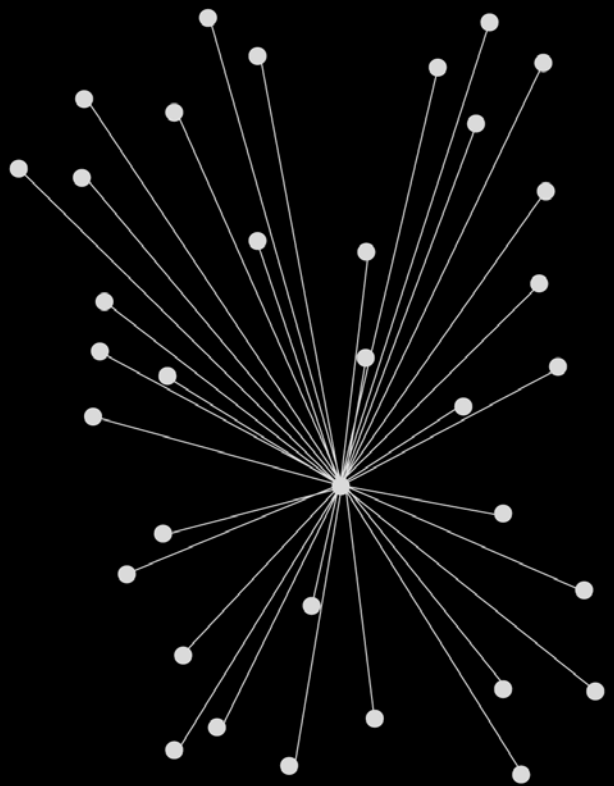
CENTRALIZED

DECENTRALIZED

DISTRIBUTED

The Structure of Complexity

Theorists have identified three types of networks:



CENTRALIZED

The centralized network has the advantage of efficient communication,

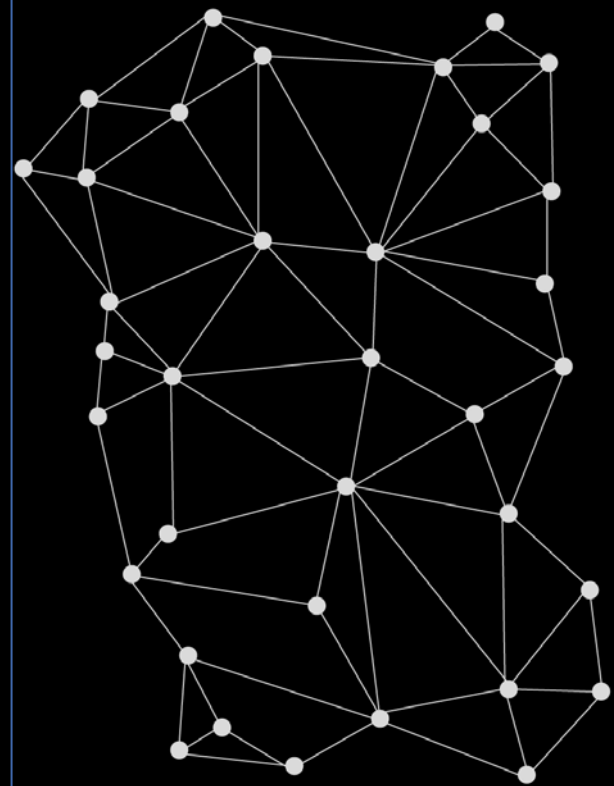
but the disadvantage of not being very robust.

The Structure of Complexity

Theorists have identified three types of networks:

The distributed network has the advantage of being very robust,

but the disadvantage of poor communication.



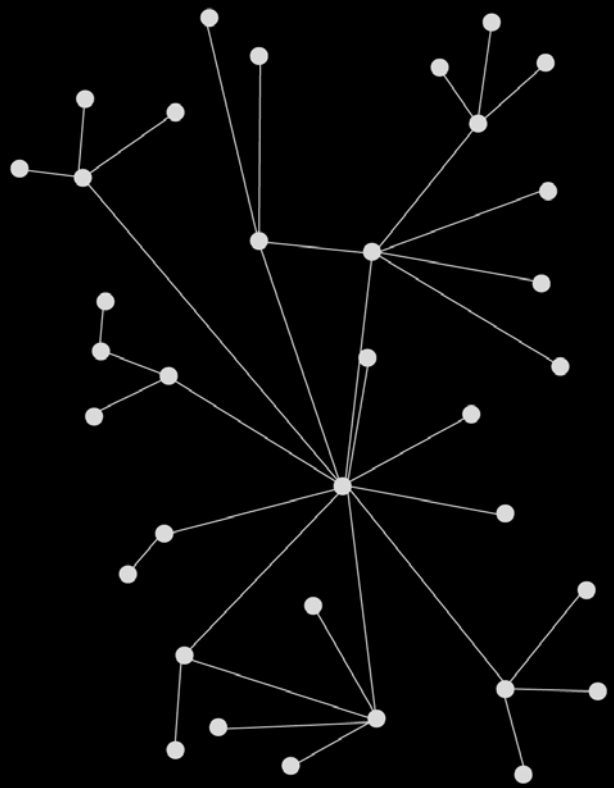
DISTRIBUTED

The Structure of Complexity

Theorists have identified three types of networks:

The decentralized network is the "fingerprint" of a complex (learning) system/collective.

It's "just right" …



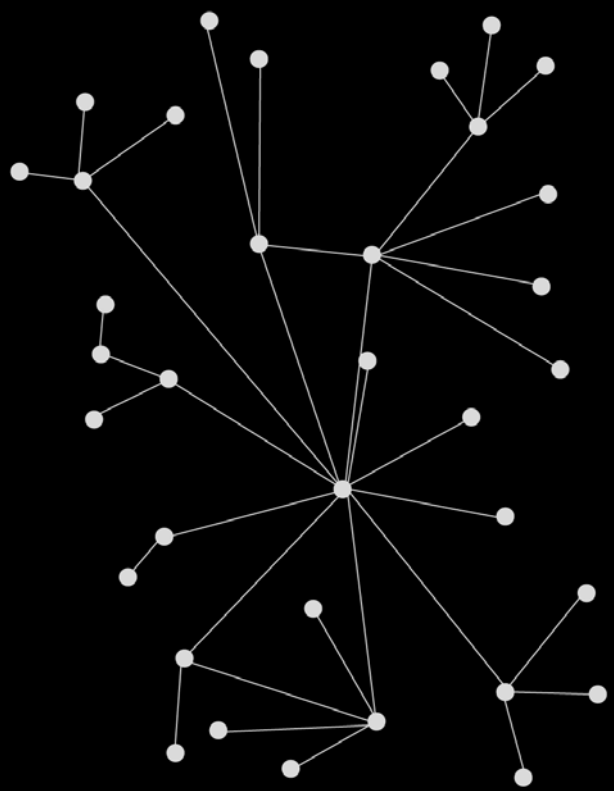
DECENTRALIZED

The Structure of Complexity

Theorists have identified three types of networks:

The decentralized network is the "fingerprint" of a complex (learning) system/collective.

It's "just right" …



DECENTRALIZED

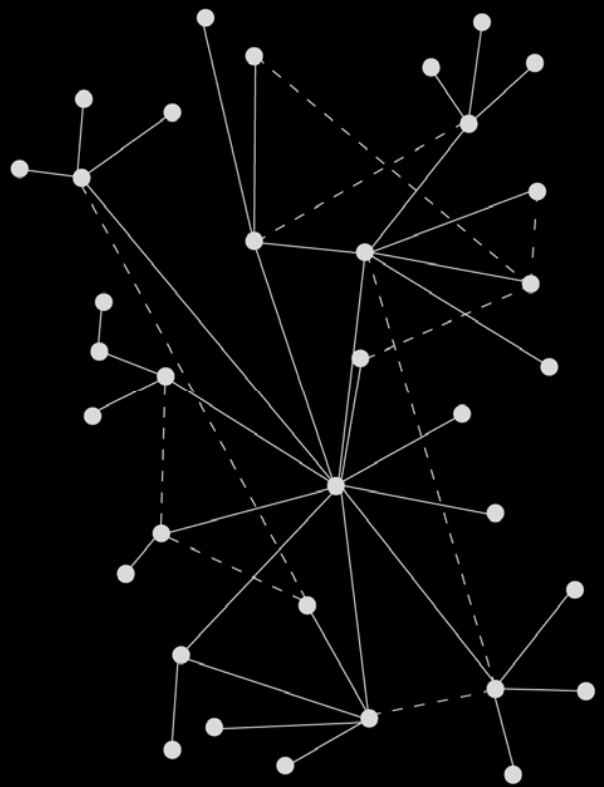
… balancing efficient communication with a robust structure.

The Structure of Complexity

Theorists have identified three types of networks:

The decentralized network is the "fingerprint" of a complex (learning) system/collective.

It's "just right" …

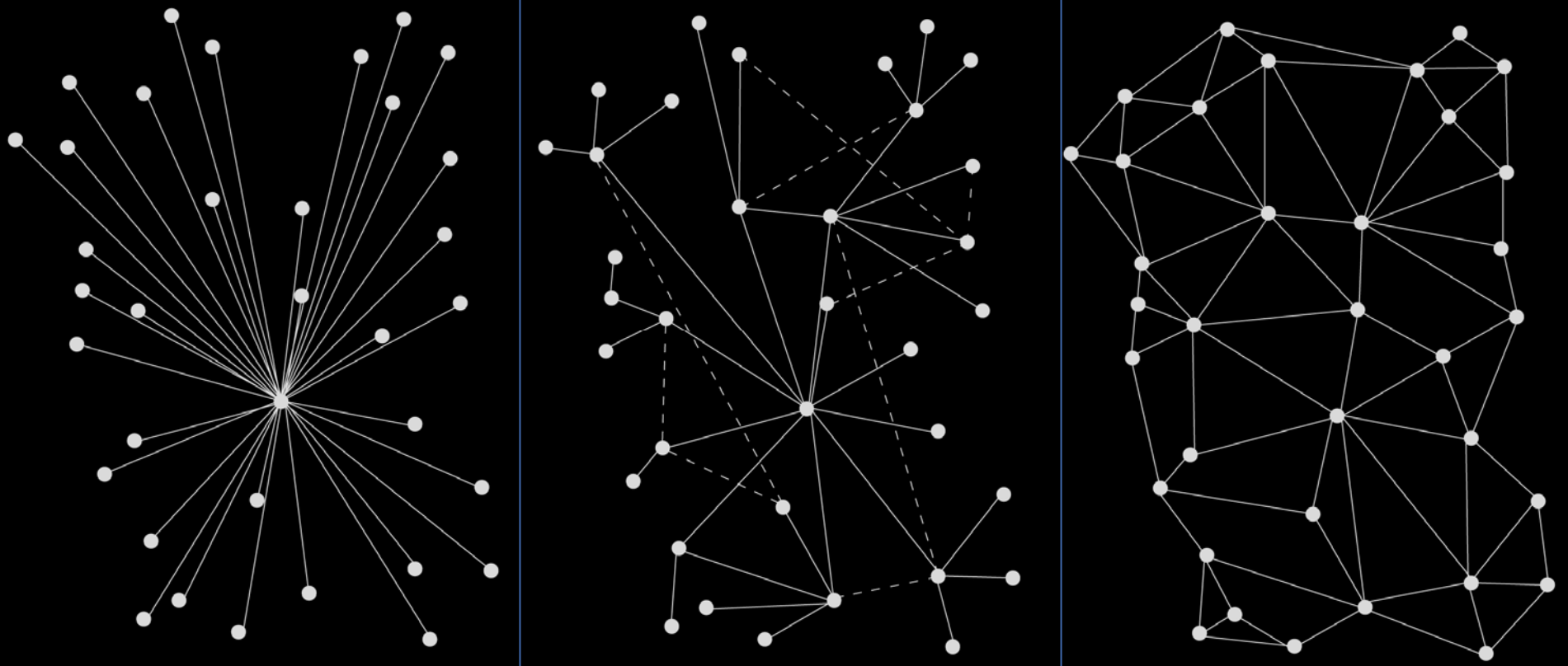


DECENTRALIZED

… balancing efficient communication with a robust structure …

… especially when weak links are figured in.

The Structure of Complexity



CENTRALIZED

DECENTRALIZED

DISTRIBUTED

Increased Stress
Diminished Resources

Decreased Stress
Improved Resources

A network representation exercise

- COLLECTIVELY choose a reasonably prominent mathematics concept,
- Assemble a network of associations.
- NOTE: THIS IS NOT A BRAINSTORMING EXERCISE …
- … it's an academic one, in which each node and link should be justifiable.

Discussion Topics:

What would we look for to decide whether a particular in-service, classroom, and other organizational structure is/isn't decentralized?

Mathematics, language, and other categories of human knowledge map out onto decentralized networks (not hierarchies or chains or nets). What would a decentralized curriculum look like?

Is there a better question I could be asking here?

An obvious implication:

- Neither the leader-centered nor the individual-centered collective is complex;
- Rather, the ‘thing’ at the center is some sort of collective project/possibility/knowledge.
- This is one POWERFUL example of how data management can help us understand our world.

So, where's the teacher is all this?

My answer: The teachers is the consciousness of the collective …

- a metaphor, based on …
- the limited capacities of consciousness,
- the fact that consciousness does not control, but it does orient

So, where's the teacher is all this?

My answer: The teachers is the consciousness of the collective …

… meaning that the teacher selects from the array of interpretive possibilities that present themselves at any moment.

So, where's the teacher is all this?

My answer: The teachers is the consciousness of the collective …

… which is not directing, controlling, guiding, or facilitating, but participating.