Learning Systems: The Case of Mathematics-for-Teaching



OAME 2006

Leadership Conference

Using Complexity to Structure In-Service Experiences for Mathematics Teachers



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We Acknowledge

Our mentor Tom Kieren; our colleagues, Dennis Sumara, David Reid, Joyce Mgombelo, Florence Glanfield, Deborah Osberg; our former students, Immaculate Namukasa, Darren Stanley; and our current students, Helena Miranda, Mary Beiseigel, Jim Fuite, Elizabeth Mowat, Jerome Proulx

Among the topics announced in the titles:

- **mathematics-for-teaching**
  - what effective math teachers (need to) know
- **complexity thinking**
  - a means to make sense of MfT
- **in-service experiences**
  - prompting complex learning systems to promote MfT
- **leadership**
  - putting it all (and keeping it) together

TOPIC 1: Mathematics-for-Teaching

Definition: the *not-necessarily-explicit* knowledge that teachers draw on to render the subject matter meaningful for learners

What is addition?

Four grounding metaphors of arithmetic (Lakoff & Núñez, 2000):

- collection of objects
- construction of objects
- use of measuring sticks
- motion

A **grounding metaphor** is a means to project from everyday experiences onto abstract concepts:

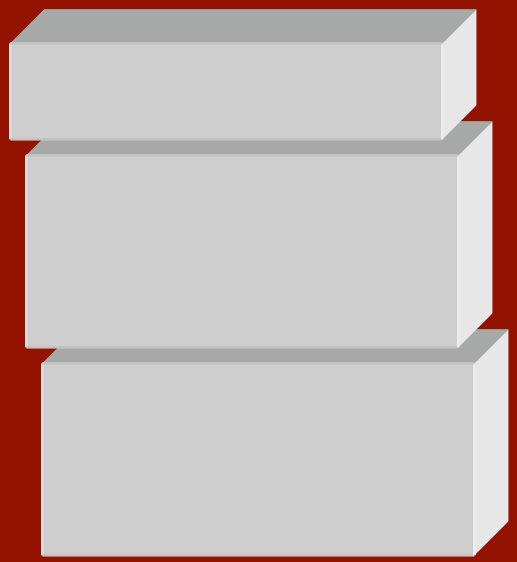
- e.g., addition as collecting objects:



1 + 2 = 3

A **grounding metaphor** is a means to project from everyday experiences onto abstract concepts:

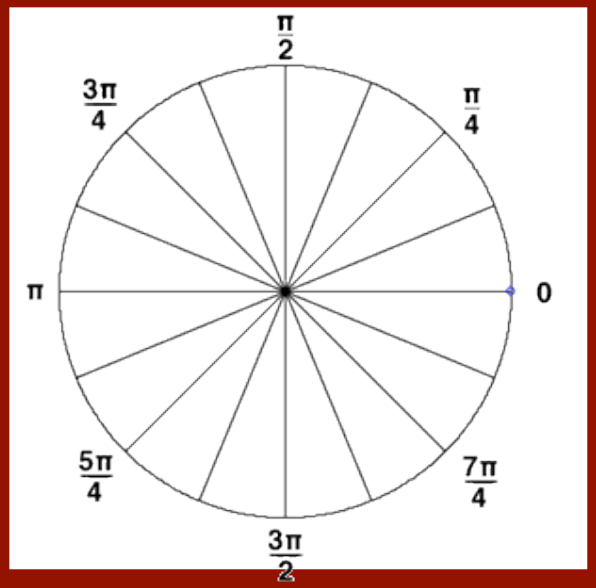
- e.g., addition as object construction:



2 + 0.5 = 2.5

A **grounding metaphor** is a means to project <u>from</u> everyday experiences <u>onto</u> abstract concepts:

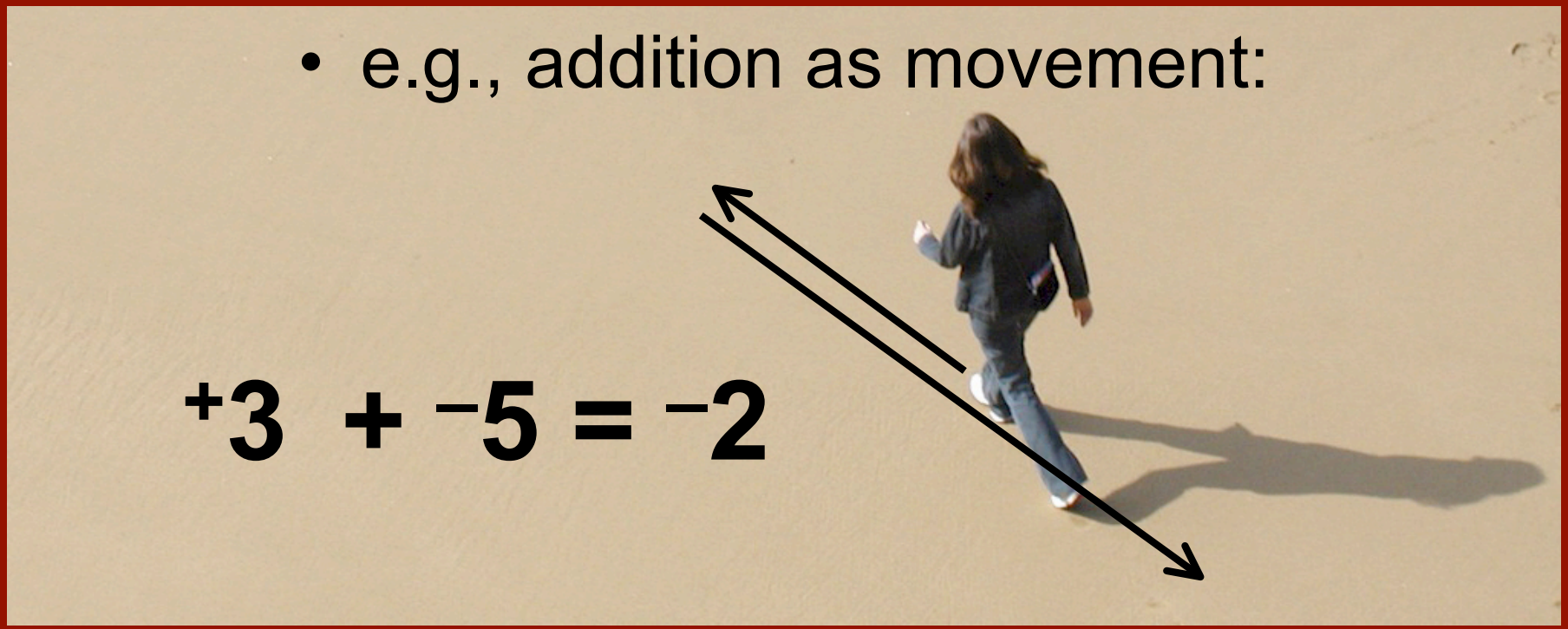
- e.g., addition as measures:



$\pi + \pi = 2\pi$

A **grounding metaphor** is a means to project <u>from</u> everyday experiences <u>onto</u> abstract concepts:

- e.g., addition as movement:



What's 1.2 ÷ 0.5 ?

Show how you know.

Recursive Elaborations

Multiplication (increasing by equal groups)

Division (sharing into equal groups)

Addition (increasing)

Subtraction (taking away)



Object Collection

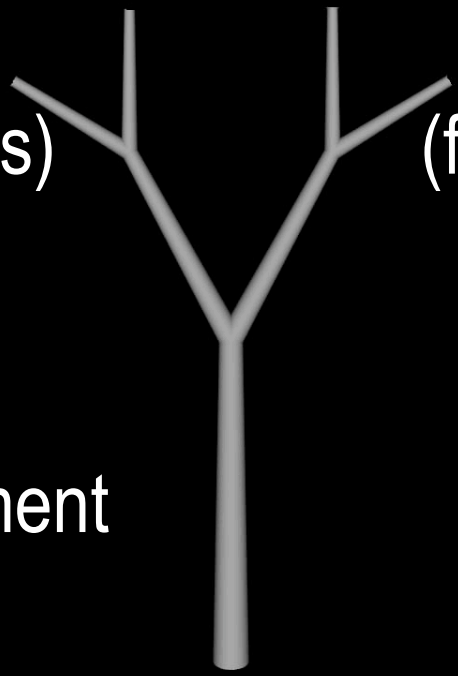
Recursive Elaborations

Multiplication (jumping dimensions)

Division (creating ratios)

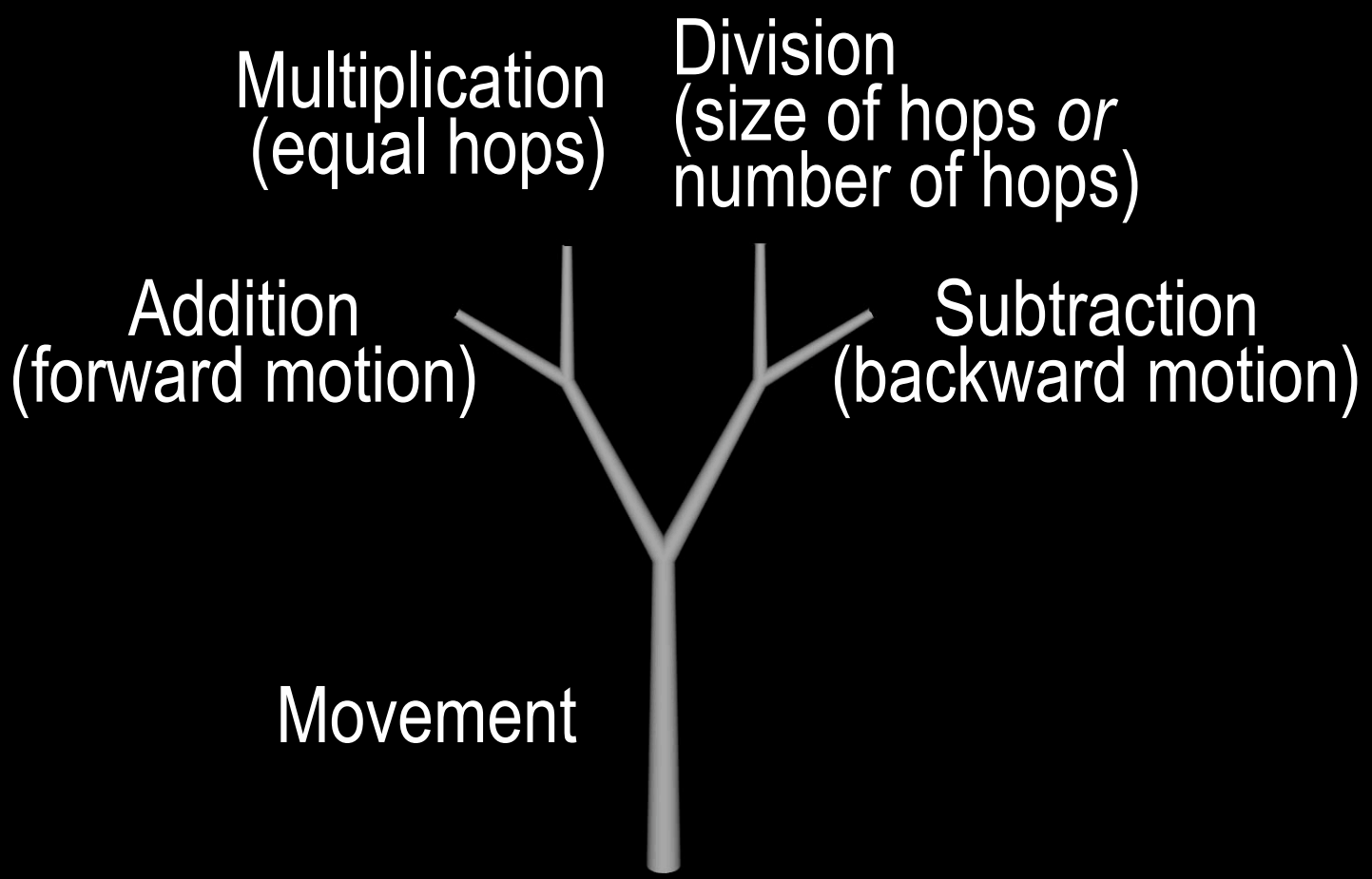
Addition (combining distances)

Subtraction (finding differences)



Measurement

Recursive Elaborations



Recapping TOPIC 1: Mathematics-for-Teaching

Definition: the *not-necessarily-explicit* knowledge that teachers draw on to render the subject matter meaningful to learners

TOPIC 2:
Complexity Thinking

Definition: the study of ***learning systems***—that is, self-organizing, adaptive phenomena

Complex (Learning) Systems include

bodily sub-systems,
individuals,
social collectives,
bodies of knowledge,
societies,
species,
the biosphere …

Complex (learning) systems are identified by certain qualities, including …

- recursively elaborative,
- nested,
- bottom-up emergent,
- self-transforming,
  disequilibrated,
- scale independent,
- permeably bounded.

Complex (learning) systems are identified by certain qualities, including …

- recursively elaborative,
- nested

are the two we look at in this session.

Recursive Elaboration



Complex forms evolve through processes that are constant, but that constantly transform the forms.

Recursive Elaboration

Examples:

- ***subjective*** understandings (e.g., as new experiences/ metaphors expand the notion of addition)



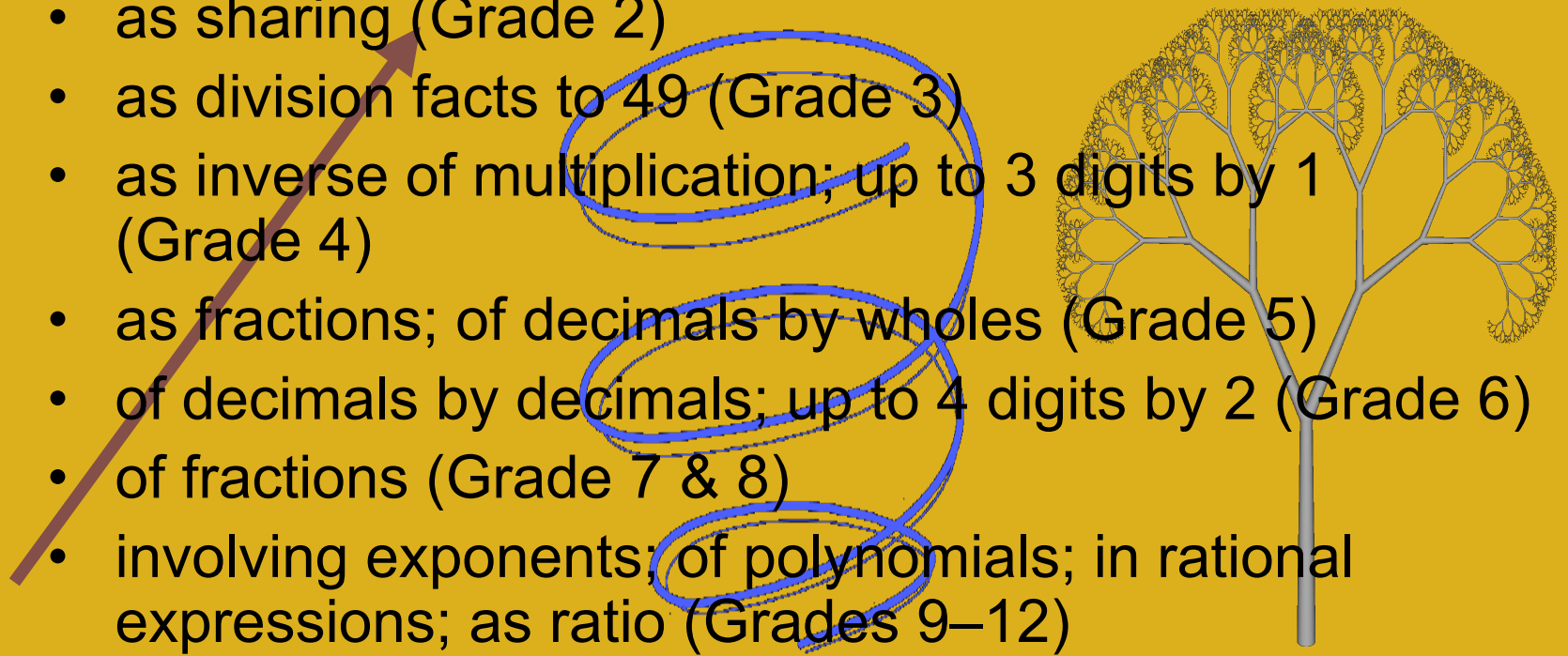
- ***objective*** concepts (e.g., as new number systems are created by extending established ones)



Recursive Elaboration

Division …

- as sharing (Grade 2)
- as division facts to 49 (Grade 3)
- as inverse of multiplication; up to 3 digits by 1 (Grade 4)
- as fractions; of decimals by wholes (Grade 5)
- of decimals by decimals; up to 4 digits by 2 (Grade 6)
- of fractions (Grade 7 & 8)
- involving exponents; of polynomials; in rational expressions; as ratio (Grades 9–12)



What are some of the popular synonyms for ‘division’?

Synonyms for Division

SEPARATION

- analysis, apportionment, autopsy, bisection, breaking, breaking down, breaking up, carving, concession, contrasting, cutting up, demarcation, departmentalizing, detaching, detachment, diagnosis, disconnection, disjuncture, dismemberment, disparting, disseverance, dissolution, distinguishing, distribution, disunion, disuniting, dividing, divorce, parceling, parting, partition, reduction, rending, rupture, segmentation, selection, separating, severance, splitting up, subdivision, vivisection

Synonyms for Division

PART

- affiliate, associate, border, boundary, branch, category, chunk, class, compartment, concession, cut, degree, demarcation, department, divide, dividend, divider, dividing line, divvy, end, fraction, fragment, grouping, head, kind, lobe, lump, member, moiety, offshoot, parcel, partition, piece, portion, rake-off, ramification, section, sector, segment, share, slice, sort, split, subdivision, wedge

Synonyms for Division

ESTRANGEMENT

- breach, conflict, difficulty, disaccord, disagreement, discord, disharmony, dispute, dissension, dissent, dissidence, dissonance, disunion, feud, rupture, split, trouble, variance, words

Synonyms for Division

REGION

- belt, block, city, county, division, domain, dominion, enclosure, field, kingdom, locality, neighborhood, parcel, patch, plot, precinct, principality, quarter, section, sector, sphere, square, state, stretch, territory, township, tract, turf, vicinity, ward, zone

Synonyms for Division

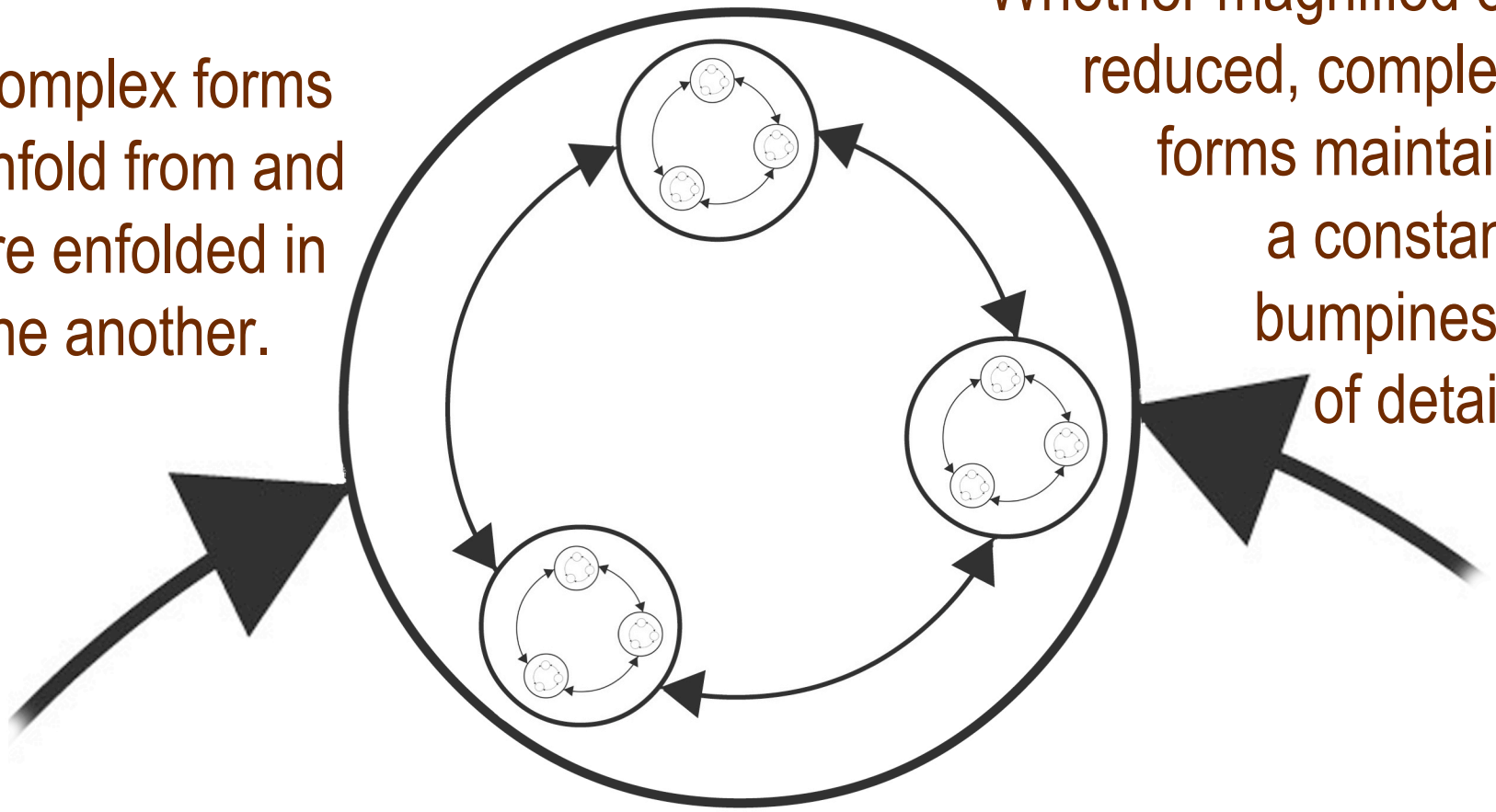
SOLDIERS

- armed force, artillery, battalion, battery, brigade, cavalry, column, command, company, corps, detail, division, flight, formation, infantry, legion, men, outfit, patrol unit, platoon, regiment, soldiers, soldiery, squad, troops, wing

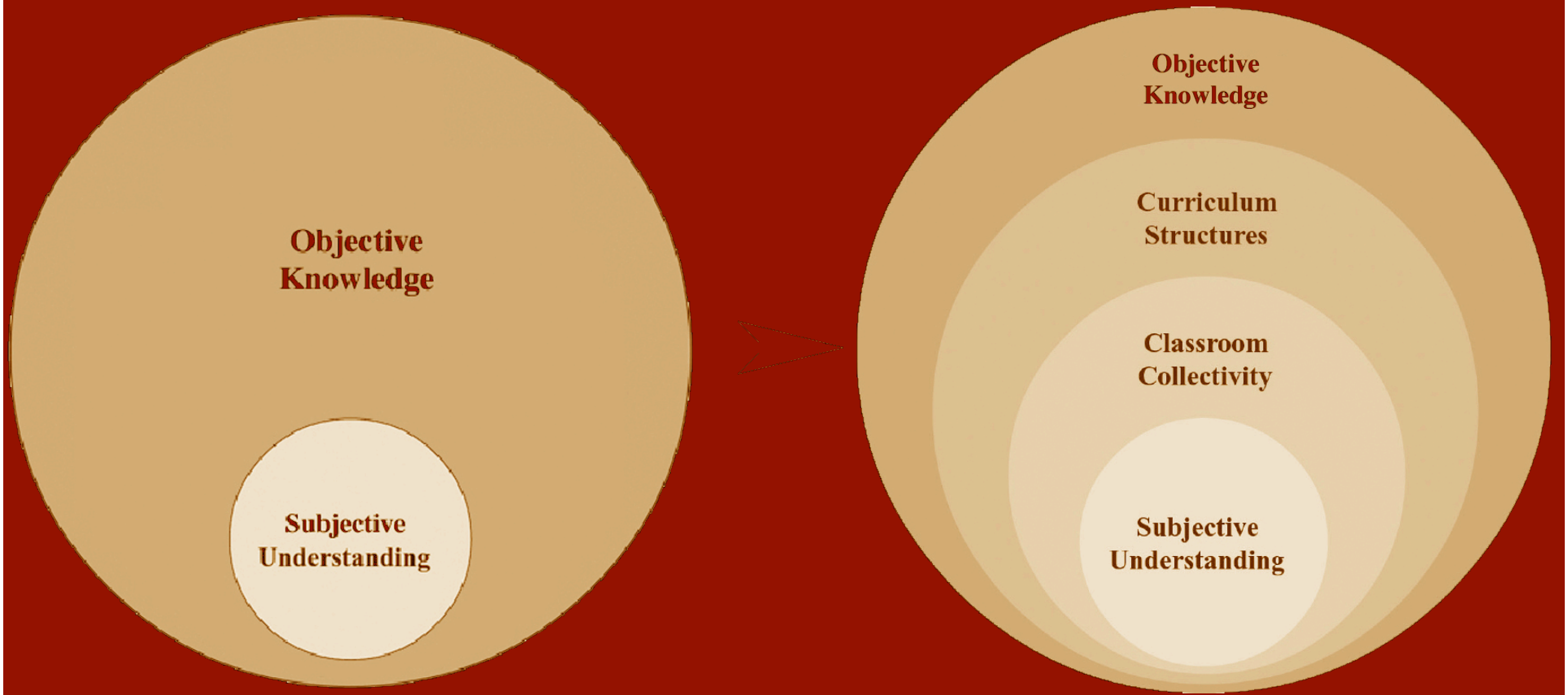
Nested & Scale Independent

Complex forms unfold from and are enfolded in one another.

Whether magnified or reduced, complex forms maintain a constant bumpiness of detail.

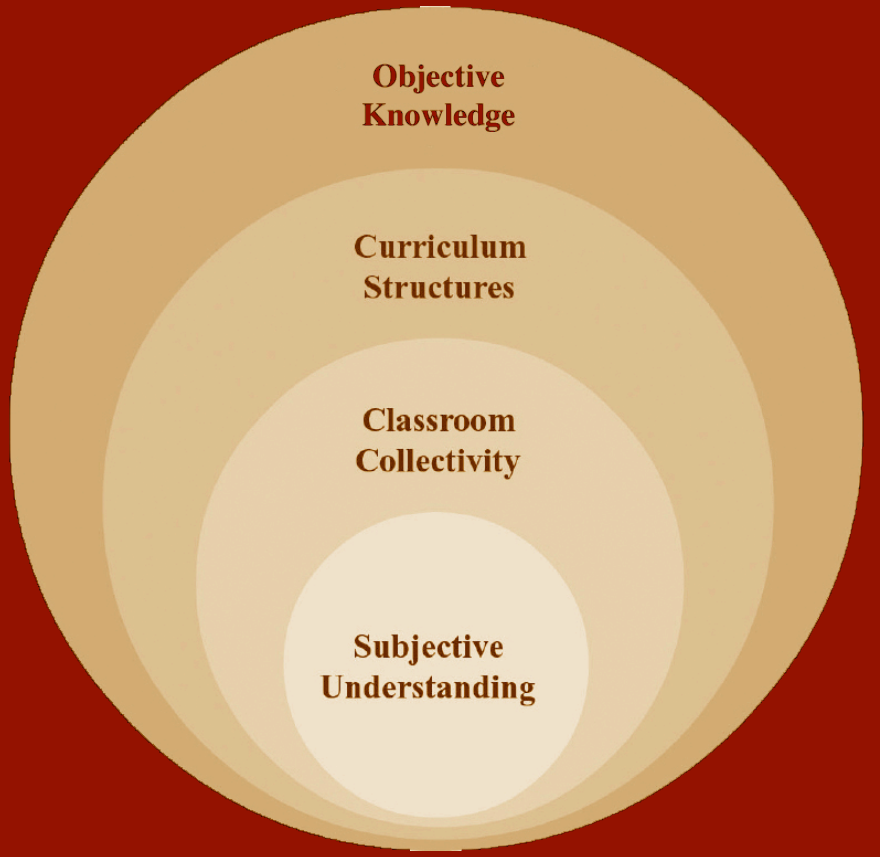


Nested & Scale Independent



Nested & Scale Independent

Mathematics-for-Teaching



The Temporal Dimension of Learning Systems



Personal
(seconds)

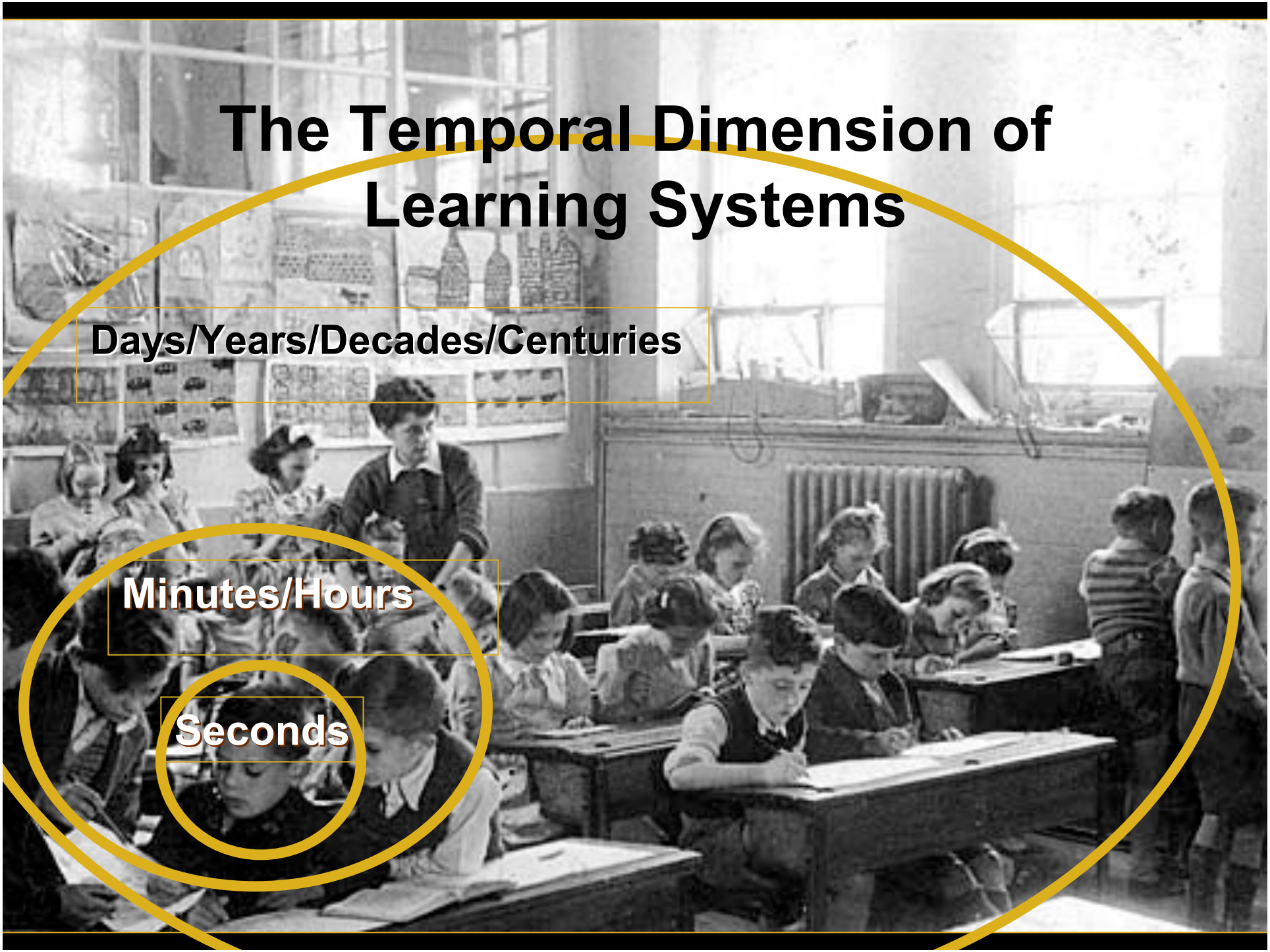
The Temporal Dimension of Learning Systems



Social
(minutes & hours)

The Temporal Dimension of Learning Systems




The Temporal Dimension of Learning Systems
Days/Years/Decades/Centuries
Minutes/Hours
Seconds

Recapping TOPIC 2: Complexity Thinking

Definition: the study of ***learning systems*—that** is, self-organizing, adaptive phenomena

TOPIC 3:
In-Service Experiences

Comment: [The] impulse for participation has upended the economy and is steadily turning the sphere of social networking—smart mobs, hive minds, and collaborative action—into the main event. (Kevin Kelly)

Our orienting belief:

- The ‘learner’ that we should focus on is not the individual, but the collective …
- … whether talking classrooms, teachers, schools, school districts, or whatever …
- simply because, as participants, we can more directly influence over the collective.

Intelligent Collectives

- The collective *can* be more intelligent than the most intelligent in the collective.
- You do not need consensus—in fact, consensus can encourage tepid, LCD solutions.
- Rigid hierarchical, multilayered organizations discourage the free flow of information.
- Decisions about local problems should be made, as much as possible, by people close to the problem.
- Paradoxically, the best way for a group to be smart is for each person to act as independently as possible.

Examples of individual-focused education

- University teacher education programs
- In-service workshops
- Professional development conferences and conventions
- One-off university mathematics classes
- Graduate programs

What's wrong with these pictures?

1) concerned with specific grade level only
2) individualistic conception of learning
3) focused on explicit, formal knowledge
4) radical separation of knowing and knowledge
5) radical separation of research mathematics and mathematics-for-teaching
6) scripted (prescriptive) curriculum

Alternatives—concerned with collectivity

- Cohort programs
- Communities of practice (informal and formal)
- Inquiry-based conferences and workshops
- This meeting?

A new picture

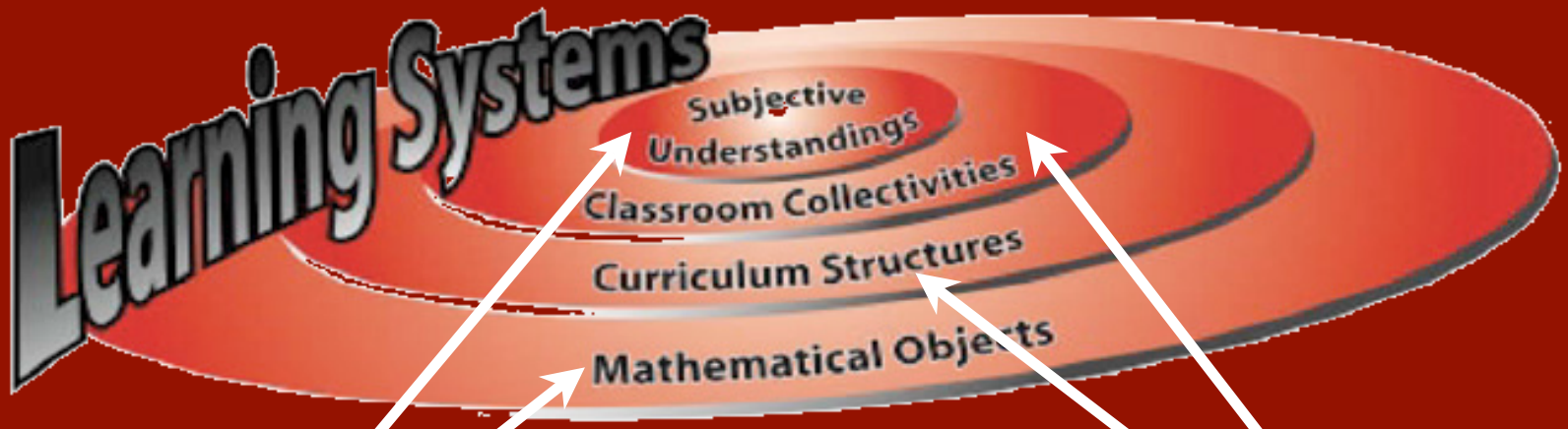
1) concerned with ~~specific grade level only~~ **K-12**
2) ~~individualistic~~ **collective** conception of learning
3) focused on ~~explicit, formal~~ **tacit, enacted** knowledge
4) ~~radical separation~~ **multi-plied conception** of knowing and ~~knowledge~~
5) ~~radical separation of~~ **refusal to separate** research mathematics and mathematics-for-teaching
6) ~~scripted (prescriptive)~~ **emergent (proscriptive)** curriculum

Recapping TOPIC 3: In-Service Experiences

Comment: [The] impulse for participation has upended the economy and is steadily turning the sphere of social networking—smart mobs, hive minds, and collaborative action—into the main event. (Kevin Kelly)

TOPIC 4:
Leadership

Comment: In complexity terms, the effectiveness of leaders is linked to their capacities to trust the collective and to curb the desire to control outcomes. (Roger Lewin and Birute Regine)

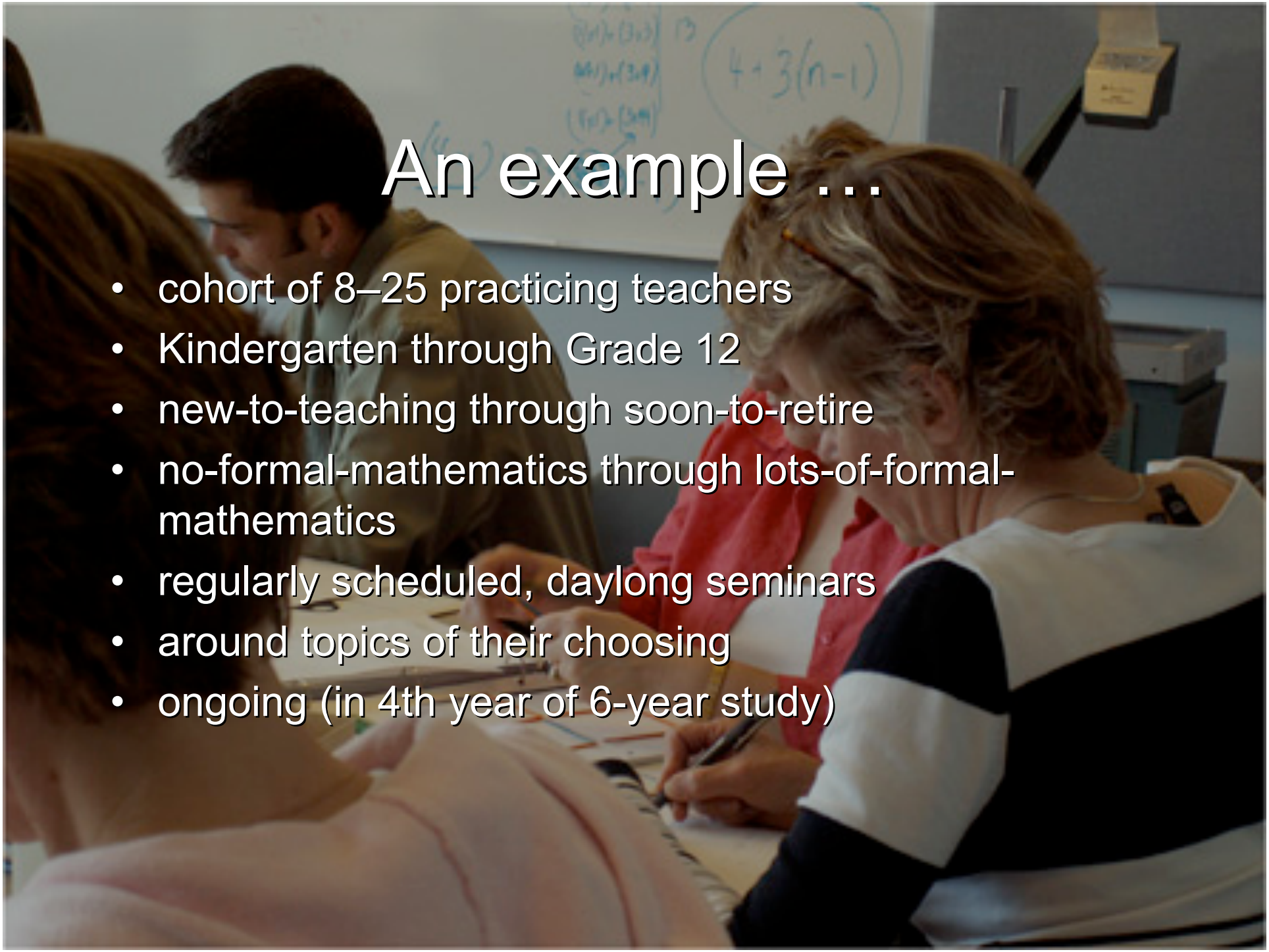


Where discussions of math teaching are usually focused

Where we think discussions should be focused

Some of what we've learned about effective teaching/leading

- ensure redundancy among participants
- find ways for diverse ideas to be represented
- make ideas bump against one another
- decentralize control
- nurture ALL LEVELS of learning/learners simultaneously



An example …

- cohort of 8–25 practicing teachers
- Kindergarten through Grade 12
- new-to-teaching through soon-to-retire
- no-formal-mathematics through lots-of-formal-mathematics
- regularly scheduled, daylong seminars
- around topics of their choosing
- ongoing (in 4th year of 6-year study)

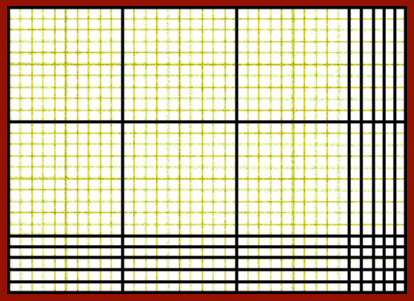
Multiplication involves …

- repeated addition;
- grouping;
- sequential folding;
- many-layering (literal meaning of ‘multiply’);
- the basis of proportional reasoning;
- grid-generating;
- dimension-changing;
- intermediary of adding and exponentiating;
- opposite/inverse of division;
- stretching or compressing of number line;
- rotating a number line.

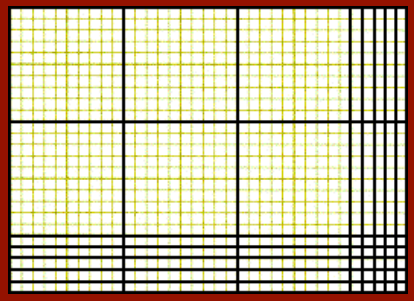
K–Grade 3:
$2 \times 3 = ?$



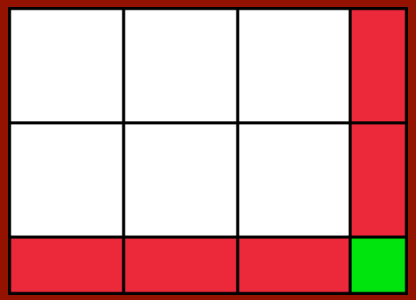
Grades 4–5
$25 \times 35 = ?$



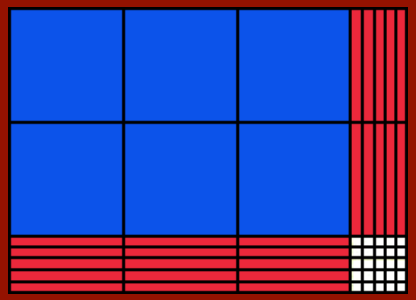
Grades 5–6
$2.5 \times 3.5 = ?$



Grades 7–8
$2\,{}^{1}\!/_{2} \times 3\,{}^{1}\!/_{2} = ?$



Grades 9–10
$(2x + 5y)(3x + 5y) = ?$



The topics:

- **mathematics-for-teaching**
  - what effective math teachers (need to) know
- **complexity thinking**
  - a means to make sense of MfT
- **in-service experiences**
  - prompting complex learning systems to promote MfT
- **leadership**
  - the purpose of this conference