K-12 Mathematics Linking Cubes



An OAME project
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OAME/AOEM

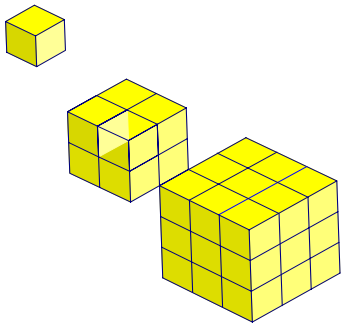
ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

A joint project between OAME and Spectrum for the Leadership Conference 2006.

K-12 LINK CUBES MATH

The Big Cube

Problem: Here is a sequence of cubes. Imagine that each cube is painted on the outside surface. It is then taken apart and sorted into piles according to the number of faces that have paint on them.



How many of the cubes will have paint on

A) no faces b) one face c) two faces d) three faces e) four or more faces?

Size of cube(Lwh)	1	2	3	4	5
# with no painted-faces					
# with one painted face					
# with 2 painted faces					
# with 3 painted					
# with more than 3 painted faces					

Overall Expectations:

Demonstrate facility in the algebraic manipulation of polynomials

Specific Expectations

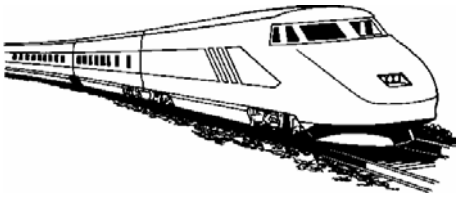
Demonstrate mastery of key algebraic skills

Inside this issue:

Link Cubes Math	1
Kindergarten Activity	2
Grade 1 Activity	3
Grade 2 Activity	4
Grade 3 Activity	5
Grade 7 Activity	5
Grade 4 Activity	6
Grade 5 Activity	7
Grade 6 Activity	8
Grade 8 Activity	9
Grade 9 Activity	10
Grade 10 Activity	11
K-12 the train problem	12
Cube Game Board BLM	13

Model your findings with a graph for each size of cube vs number of painted faces. Explain why this equation is true for all values of n greater than or equal to 2. $n^3 = (n-2)^2 + 6(n-2)^2 + 12(n-2) + 8$. Explain your thinking.

Where do the cubes with three painted faces come from? Where do the cubes with two painted faces come from? Where do the cubes with one painted face come from? Where do the cubes with no painted faces come from? Describe the shape modeled by the expression $n^3—(n-2)^3$ Explain your thinking.



KINDERGARTEN STRAND: PATTERNING AND

Problem:

Which trains are pattern trains?

In pairs, students take turns pulling trains of different lengths out of a bag or box and determine if the cubes are arranged in a pattern or not.

Students sort their train into "Pattern trains", "Not pattern trains" categories.

When all trains have been sorted, each student chooses a pattern train and lengthens the train, continuing the pattern.

Materials:

6-8 interlocking cube trains with cubes in at least 2 different colours, arranged in repeating or growing patterns per pair

4-6 interlocking cube trains with cubes in at least 2 different colours, arranged randomly

sorting mat or sorting rings per pair

a small set of interlocking cubes per pair

Overall Expectations:

Recognize and use patterns

Specific Expectations

Identify and reproduce simple patterns

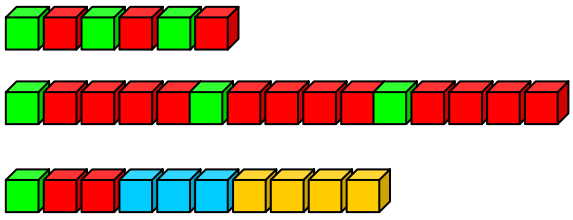
Create and extend simple patterns using a variety of materials or actions

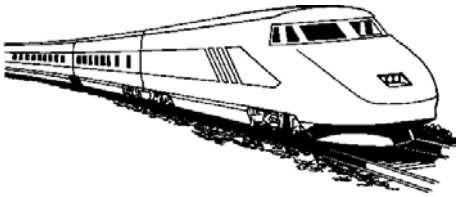
Scaffolded Prompts for Problem:

Express the train pattern using a different material or action.

Create duplicate pattern trains and ask students to match the trains.

Sample trains:





GRADE 1
STRAND: PATTERNING AND ALGEBRA

Problem:

How many different 5-cube trains can you make using up to 2 colours of interlocking cubes?

Place the 5-cube train on the left side of the balance scale.

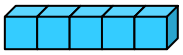
With a partner, place interlocking cubes on the right side of the balance scale to create another 5-cube train. Use one or two colours of cubes.

Build your train and record it on the grid paper.

Materials:

10 interlocking cubes in 2 colours for a total of 20 per pair

1 5-cube train in one colour per pair



interlocking cube grid paper, 1 sheet per pair

1 balance scale per pair

Overall Expectations:

Demonstrate an understanding of the concept of equality, using concrete materials and addition and subtraction to 10

Specific Expectations

Create a set in which the number of objects is greater than, less than, or equal to the number of objects in a given set

Demonstrate examples of equality, through investigation, using a "balance" model

Questions and Prompts:

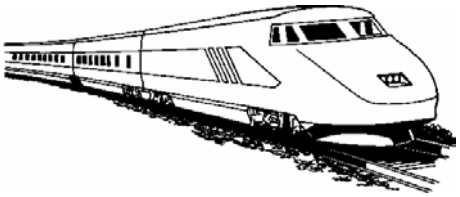
How is the one colour train the same as this 2-colour train? How are they different?

If you placed 3 cubes of one colour in the balance scale, how many more cubes of another colour did you need to make your 5-cube train?

What could a 5-cube train that has more (red) cubes than (green) cubes look like?

Sample 5-Cube Trains





GRADE TWO
STRAND: PATTERNING AND ALGEBRA

Problem:

Rui is building a model train with 6 cars linked together.

Each car will be made of 2 more cubes than the one before.

He plans to use 3 cubes for the first car.

Rui thinks that 20 cubes will be enough to build his train.

Find out if Rui is right.

Use interlocking cubes to show your solution.

Materials:

20 cubes for each student

Make a collection of cubes available to all students

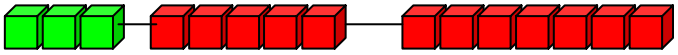
Overall Expectations:

Identify, describe, extend, and create repeating patterns, growing patterns, and shrinking patterns

Specific Expectations

Represent a given growing or shrinking pattern in a variety of ways

Demonstrate, through investigation, an understanding that a pattern results from repeating an operation



Model train with 3 cars

Scaffolded Prompts for Problem:

Build the first car and record how many cubes you used.

Build the second and third car and record how many cubes you used altogether.

How many more cars do you need to build?

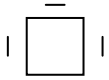
How many cubes will you need to build the 5^{th} car?

Do you think 20 cubes will be enough to build the whole train?

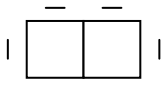
GRADE 3 STRAND PATTERNING AND ALGEBRA

Problem: How many tables are needed for a banquet of 21 people?

Four people can sit around one table .



When two tables are pushed together six people can sit around the tables.



If 21 people want to sit together how many tables have to be pushed together?

Model your solution with interlocking cubes. Remember, one table seats 4 people.



Two tables together seat six people.



Scaffolded prompts for the question:

- Build the third, fourth, and fifth terms with interlocking cubes
- How many people can sit around seven tables that have been pushed together?
- Describe the rule for this pattern.

Overall Expectations:

Decsribe, extend and create a variety of numeric patterns and geometric patterns

Specific Expectations

Extend repeating, growing and shrinking patterns

Represent simple geometric patterns using a number sequence, a number line, or a bar graph.

GRADE 7 LINKING CUBES ACTIVITY

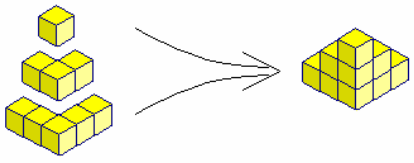
Here are the first three cube constructions in the pattern.



Second construction isometric view



Third construction isometric view



1) How many cubes are in each row?
2) How many cubes in total for each of the first 3 terms?
3) Explain using words how to find the next entry in the pattern.
4) Write the algebraic expression for the number of cubes in each row.
5) Write the algebraic expression for the total number of cubes.

Answers: 1) 1, 3, 5, 7, 9... 2) 1, 4, 9, 16.,,, 4) add =2n –1 5) $t = n^2$

Overall Expectations:

Represent linear growing patterns (where the terms are whole numbers) using concrete materials, graphs and algebraic expressions

Specific Expectations

Develop and represent the general term of a linear growing pattern, using algebraic expressions involving one operation (e.g. the general term for the sequesnce 4,5,6,7… can be written algebraically as n+3, where n represents the term number; the general term for the sequence 5,10,15,20… can be written as 5n, where n represents the term number.

GRADE 4—STRAND PATTERNING AND ALGEBRA

Problem - Who has the fewest cubes left?
Players: 2 to 4

Materials:
BLM Interlocking cube game board (expand or reduce to suit your needs)
10 interlocking cubes in 5 colours for a total of 50 per player
1 number cube

How to Play:
Each player starts the game with 50 interlocking cubes of 5 different colours (e.g., 10 cubes of each colour).
Roll one number cube to see who places the first interlocking cube on the game board. The player who rolls the lowest number chooses the location of the first interlocking cube on the game board.
Players take turns rolling a number cube. The number on the number cube tells you how many interlocking cubes you can place on the game board.
Players may place any combination of colours.
Interlocking cubes may be only placed on the game board if they are continuing a repeating numeric pattern and a geometric pattern that results from a reflection.
Players may use colour, number, shape, and geometric reflection as patterning attributes.
The game ends when interlocking cubes can no longer be placed on the game board to continue a pattern. The player with the least number of cubes wins the game.

Overall Expectations:

Describe, extend, and create a variety of numeric and geometric patterns, make predictions related to the patterns, and investigate repeating patterns involving reflections

Specific Expectations

Extend and describe repeating number patterns.

Make predictions related to repeating numeric patterns.

Extend and create repeating patterns that result from reflections through investigation using a variety of tools

Sample Game Playing

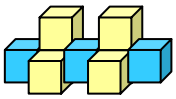
Player 1 rolls 1 and places one blue cube.



Player 2 rolls 3 and places 3 yellow cubes beside the blue one.



Player 1 rolls 5 and places 1 blue cube, 3 yellow cubes and another blue cubes to repeats the pattern that was started.



Scaffolded Prompts for Problem

Show a repeating pattern using interlocking cubes.
Show a reflection using interlocking cubes.
Identify the pattern stem in your repeating pattern.
Where is the best place on the game board to put the first cube?
What pattern attributes are used to make the patterns on the game board?
What strategies are you using to place your interlocking cubes on the game board as part of a pattern?
How early on in the game will you know that you have won the game? Lost the game? Explain.

GRADE 5 - STRAND - PATTERNING AND ALGEBRA

Problem

Lin wants to show that she can model this growing pattern in different ways. The pattern rule is add 2 and start at 1. She predicts that she will need about 30 interlocking cubes if she shows the pattern to the 10^{th} term. Investigate her hypothesis. Model your different solutions using interlocking cubes.

Scaffolded Prompts for Problem

Model a numeric that follows the rule (add 2 and start at 1) and extend it to the 5^{th} term. Use interlocking cubes.

Show a different model of this growing pattern: (add 2 and start at 1). Use interlocking cubes. Consider these attributes (colour, number, shape, and geometric reflection) to create a different model.

Make a table of values for the pattern.

Term	1	2	3	4	5	6	7	8	9	10
Number of cubes	1	3	5							
Total number of cubes used	1	4	?							

How many cubes are needed to include 3 terms? 5 terms? 10 terms?

How many different models of this growing pattern did you make?

How do you know that they are different?

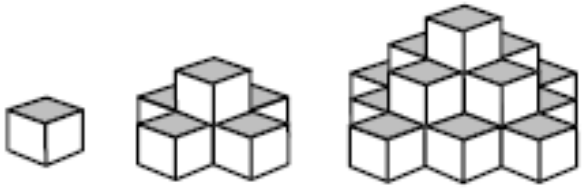
Overall Expectations:

Determine, through investigation using a table of values, relationships in growing and shrinking patterns, and investigate repeating patterns involving translations

Specific Expectations

Extend numeric patterns, using a variety of tools
Make a table of values for a pattern that is generated by adding a number (i.e., a constant) to et the next term or the pattern rule in words
Make predictions related to growing numeric patterns.

GRADE 6 - STRAND- PATTERNING AND ALGEBRA



Problem

Jon plans to build this growing pattern to the 10th term.
He thinks that 200 interlocking cubes is enough.
Find out if Jon's prediction is accurate.
Model your solution using interlocking cubes.

Overall Expectations:

Describe the represent relationships in growing patterns (where the terms are whole numbers).

Specific Expectations

Identify geometric patterns, through investigation using concrete materials or drawings, and represent them numerically

Describe pattern rules (in words) that generate patterns by adding or subtracting a constant or multiplying or dividing by a constant to get the next term.

Scaffolded Prompts for Problem

Build the 4th, 5th, and 6th terms in this growing pattern. Use interlocking cubes.
Record the number of interlocking cubes for each term and total number on a table of values.

Term	1	2	3	4	5	6	7	8	9	10
Number of cubes	1	1+5	1+5+?							
Total number of cubes used	1	6	?							

Predict the number of interlocking cubes for the 7th term.
Build the model to check your prediction.

Predict the number of interlocking cubes for the 10th term. Build the model to check your prediction.

Decide if Jon's 200 cubes are enough to make 10 terms of the pattern.
Describe the rule for this pattern. Use words and numbers.

GRADE 8 LINKING CUBES ACTIVITY

(ADAPTED FROM A MARILYN BURNS ACITIVITY ON THE NCTM WEBSITE)

Task: Make a 10 by 10 linking cube square . Tell the students that the outside cubes on the border must be a different colour from the interior cubes.

Ask the student how many tiles there are in the border. Have the students share all the different ways they could solve the problem. There are at least six different ways to obtain the correct answer (36).

Tell students to record all of the different ways to arrive at the answer. Have the students write the following descriptions using algebraic notation.

Ie. Four times the difference of the side length and one 4(s-1)

- The sum of the length of one side plus two times the length of a side minus one plus one side minus two.
- Four added to four times the side length minus two
- Subtract four from the product of four and the length of each side
- Twice the side length plus twice the side length minus two
- The difference of side length squared and the side length minus two squared

Overall Expectations:

Represent linear growing patterns using algebraic expressions.

Model linear relationships…

Specific Expectations

Model linear relationships using tables of values, graphs and equation.

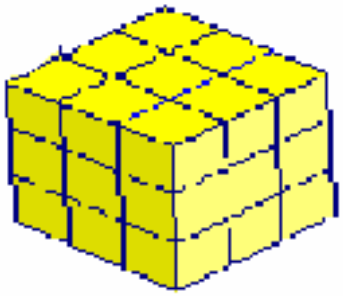
Translate statements describing mathematical relationships into algebraic expressions.

Next have the students complete the following chart

Number of Squares on Edge	Diagram	Number of Squares in Border
3		
4		
5		
6		
7		
8		
9		
10		

Finally have the students answer the following question, *"What do you notice about this pattern?* Explain the increase.

GRADE 9 PAINTED CUBE PROBLEM



A 3 by 3 cube made up of small cubes is dipped into a bucket of red paint and removed. Once the paint dries the cube is taken apart into its 27 smaller cubes.

How many small cubes will have 3 faces painted?

How many small cubes will have 2 faces painted?

Overall Expectations:

- Demonstrate facility in the algebraic manipulation of polynomials

Specific Expectations

Demonstrate mastery of key algebraic skills

Cube Number	3 Faces Painted	2 Faces Painted
3		
4		
5		
6		
7		
8		
n	8	12(n-2)

Extension Questions:

1) If a large cube had 512 cubes with one face painted after being dipped into the paint, how many small cubes is it made of?
2) For an n x n cube, the number of faces with 2 faces painted is 12(n-2). Using cubes to illustrate your reasoning, explain the steps to arrive at this formula.
3) How many surfaces would need glue to paste the 3x3 cube back together?

GRADE 10 PAINTED CUBE PROBLEM

Overall Expectations:

- Demonstrate facility in the algebraic manipulation of polynomials

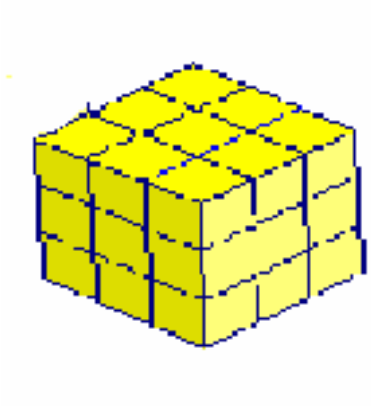
Specific Expectations

Demonstrate mastery of key algebraic skills

A 3 by 3 cube made up of small cubes is dipped into a bucket of red paint and removed. Once the paint dries the cube is taken apart into its 27 smaller cubes.

How many small cubes will have 1face painted?

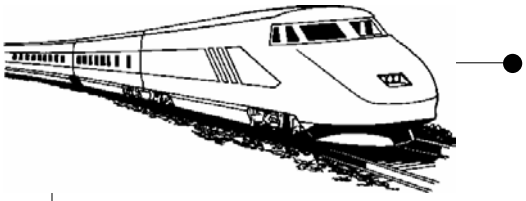
How many small cubes will have 0 faces painted?



Cube Number	1 Faces Painted	0 Faces Painted
3		
4		
5		
6		
7		
8		
n	$6(n-2)^2$	$(n-2)^3$

Extension Questions:

1) Predict what the graph of cube number vs 1 faces painted would look like Verify by plotting the graph.
2) Predict what the graph of cube number vs 0 faces painted would look like. Verify by plotting the graph.
3) How many surfaces would need glue to paste the 3x3 cube back together? Write an equation for the amount of glue



K to 12 Mathematics – Using Interlocking Cubes

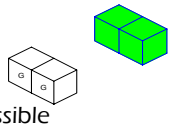
Building Trains with Long Cars and Short Cars

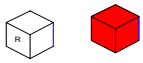
Introduction

In this activity you will consider the types of trains that can be built using two different kinds of cars. These trains can be made from double cars or single cars, or a combination of each. Double cars are made of two single cars joined together. You can use linking cubes to model these trains. Remember that each train has a front and a back. When you build your cube train, place it in front of you and designate either the right or the left as the front of the train

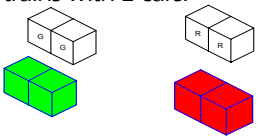
Double cars can be modelled by 2 attached cubes. Choose one colour of cubes to make the long cars. Single cars can be represented by 1 cube. Choose a different colour for short cars.

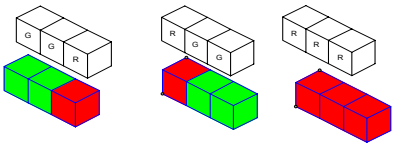
Suppose double cars are represented by green cubes, and single cars are represented by red cubes.

Double Cars 

Single Cars 

Here are the possible trains with 2 cars:





Here are the possible trains with 3 cars

Problem:

Investigate the different trains that you can make for different numbers of cars. Organize your findings. What patterns do you see?

GR 2 Make the 3-car trains. Make as many different 4 car trains as you possible. Record all the different trains on grid paper provided. Predict how many 5 car trains you can make.

GR 5 How many trains can you make with 3 cars? 4 cars? 5 cars? Record all the different trains on grid paper. Explain how you know that you have recorded all possible 5-car trains. Describe any patterns that you see.

GR 8 How many trains can you make with 3 cars? 4 cars? 5 cars? 6 cars? Create a method for organizing your findings. Predict how many trains you can make with 7 cars. Can you test your prediction without actually building all the 7-car trains? Explain your reasoning.

GR 11 How many trains can you make with 3 cars? 4 cars? 5 cars? 6 cars? Can you find a way to determine the number of trains for *n* cars? Explain your reasoning.

Expectations:

GRADE 2 – STRAND – PATTERNING AND ALGEBRA
Overall Expectations

Identify, describe, extend,and create repeating patterns, growing patterns, and shrinking patterns;

Specific Expectations

represent a given growing or shrinking pattern in a variety of ways (e.g., using pictures, actions, colours, sounds, numbers, letters, number lines, bar graphs) (***Sample problem:*** Show the letter pattern A,AA, AAA,AAAA, ... by clapping or hopping.);

demonstrate, through investigation, an undestanding that a pattern results from repeating an operation (e.g., addition, subtraction) or making a repeated change to an attribute (e.g., colour, orientation).

GRADE 5 – STRAND – PATTERNING AND ALGEBRA
Overall Expectations

determine, through investigation using a table of values, relationships in growing and shrinking patterns, and investigate repeating patterns involving translations;

Specific Expectations

build a model to represent a number pattern presented in a table of values that shows the term number and the term;

make predictions related to growing and shrinking geometric and numeric patterns;

GRADE 8 – PROCESS EXPECTATIONS
REPRESENTING

create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representations to solve problems;

GRADE 11 – STRAND—SEQUENCES AND SERIES

Overall Expectations Solve problems involving arithmetic and geometric sequences and series.

Specific Expectations:

determine a formula for the nth term of a given sequence [e.g., the nth term of the sequence 1/2, 2/3, 3/4, . . . is n/(n+1)];

identify sequences as arithmetic or geometric, or neither;

BLM Interlocking Cube Game Board

